Optimal Monetary Policy with Capital and a Financial Accelerator

James Hansen

PII: S0165-1889(18)30124-6
DOI: [10.1016/j.jedc.2018.04.006](https://doi.org/10.1016/j.jedc.2018.04.006)
Reference: DYNCON 3590

To appear in: *Journal of Economic Dynamics & Control*

Received date: 30 March 2017
Revised date: 8 January 2018
Accepted date: 18 April 2018

Please cite this article as: James Hansen, Optimal Monetary Policy with Capital and a Financial Accelerator, *Journal of Economic Dynamics & Control* (2018), doi: [10.1016/j.jedc.2018.04.006](https://doi.org/10.1016/j.jedc.2018.04.006)

This is a PDF file of an unedited manuscript that has been accepted for publication. As a service to our customers we are providing this early version of the manuscript. The manuscript will undergo copyediting, typesetting, and review of the resulting proof before it is published in its final form. Please note that during the production process errors may be discovered which could affect the content, and all legal disclaimers that apply to the journal pertain.

Optimal Monetary Policy with Capital and a Financial Accelerator

James Hansen

The University of Melbourne, Level 4, 111 Barry Street, FBE building, Victoria 3010, Australia

Abstract

Whether there is a trade-off between price and financial stability is an open question. This paper characterises optimal monetary policy analytically in a New Keynesian economy with capital and a financial accelerator. In addition to stabilising inflation, there is an incentive to smooth volatility in the net worth of borrowers and this presents a trade-off for monetary policy. This trade-off can be eliminated if policymakers can additionally choose an optimal transfer to borrowers. Enriching the model with fire sales and countercyclical bankruptcy costs, there is also an incentive to smooth volatility in default and the capital stock. An optimal transfer to borrowers is not sufficient to eliminate the trade-off in that case. With fewer instruments than targets, optimal transfers are state contingent and countercyclical, while monetary policy stabilises inflation.

Keywords: Optimal monetary policy, Linear-quadratic analysis, Capital, Financial accelerator, Fire sales.

JEL Codes: E44, E52

Email address: james.hansen@unimelb.edu.au (James Hansen)

¹Phone: +61 4 1343 8591

1. Introduction

How should monetary policy be shaped in the presence of capital and financial frictions? Prior to the 2007–08 Crisis, the consensus was that monetary policy should provide a nominal anchor and stabilise domestic prices. By mitigating volatility in domestic inflation, price dispersion is reduced and inefficient production is minimised (Clarida et al., 1999; Woodford, 2003; Galí and Monacelli, 2005). The gains in departing from an inflation target, for example responding to volatility in asset prices, were considered small or negative (Bernanke and Gertler, 2001; Gilchrist and Leahy, 2002; Faia and Monacelli, 2007; Nutahara, 2014, 2015).

Notwithstanding, the effects of the Crisis have been severe. In the eighteen months from December 2007, the labour market deteriorated sharply: total non-farm employment fell by almost 8 million and the unemployment rate almost doubled from 5 to 9.5 per cent. The subsequent recovery in jobs growth has been slow. There is debate as to whether an alternative monetary policy would have improved outcomes. Had central banks chosen tighter monetary policy earlier, would this have reduced leverage prior to the Crisis and its severity?

More generally, whether an inflation target is sufficient for managing financial shocks in the presence of a financial accelerator remains contested (Cecchetti et al., 2000; Dupor, 2005; Gambacorta and Signoretti, 2014; Nisticó, 2016). It has been argued that inflation targeting itself could contribute to a higher likelihood of financial instability (Borio and Lowe, 2002; Borio and White, 2004) and that it may be sub-optimal for a central bank to ignore the distribution of financial tail risks (Woodford, 2012). In short, it may be optimal for a central bank to accept greater inflation volatility if it means reduced financial volatility.² By contrast, the ‘modified’ Jackson-hole consensus argues that monetary should mitigate volatility in inflation only (Svensson, 2010; Smets, 2014).³

²For example, by “leaning against the wind” to prevent the build-up of leverage or risk within the economy.

³More specifically, monetary policy should only respond to financial shocks once they arrive, even if unconventional policy tools are required.

An important factor in this debate is to establish precisely what the objectives of monetary policy are when both capital and financial frictions matter. Under what assumptions does it hold that a central bank can focus solely on stabilising inflation, and under what alternatives will it be optimal to take financial volatility into consideration as well?

The central bank's objective has been derived in a New Keynesian (NK) economy with capital, but without a financial accelerator (Edge, 2003; Takamura, Watanabe, and Kudo, 2006); and with a financial accelerator applied to the payment of labour, but where capital is assumed fixed (Carlstrom, Fuerst, and Paustian, 2010; De Fiore and Tristani, 2012). The implications of a financial accelerator together with investment volatility, as is common in quantitative business cycle models, have not been considered.⁴ This is the contribution of this paper.

There are reasons to think that capital with a financial accelerator could be important. The collapse in investment during the Crisis is a key channel through which financial volatility propagated to the real economy. Financial frictions and investment volatility are also relevant for the business cycle more generally (Gertler and Kiyotaki, 2010; Gilchrist and Zakrajsek, 2012; Jermann and Quadrini, 2012). By characterising optimal monetary policy in this environment, one can consider the optimal response to financial shocks, whether there is a trade-off between price stability and financial stability and whether additional instruments can help resolve any trade-off faced.

I study the objectives of monetary policy in three NK economies. The first is a simple NK economy with capital and exogenous investment efficiency shocks.⁵ The second additionally includes a financial accelerator with costly state verification. Developed by Carlstrom and Fuerst (1997), and similar to the model proposed by Bernanke, Gertler, and Gilchrist (1999), I focus on this model because it is suf-

⁴Gertler and Karadi (2011) is one exception, although the focus there is on unconventional policy rather than the optimal choice of the nominal interest rate. The other is Collard, Dellas, Diba, and Loisel (2017), who study optimal Ramsey policy without deriving an analytic objective and with a different financial wedge (socially inefficient lending).

⁵Examples of analysis of this economy without investment efficiency shocks include Edge (2003) and Takamura et al. (2006). Like these papers, I assume that capital is perfectly substitutable across firms. The only friction is that a random subset of firms are able to choose their prices optimally each period (Calvo pricing). This economy provides a benchmark for establishing the objectives of policy in the presence of capital and investment efficiency shocks, but not financial frictions.

ficiently rich to have both capital and a non-trivial financial accelerator, while still remaining analytically tractable when studying optimal policy. The implied debt contract in this economy is also optimal (Gale and Hellwig, 1985).⁶ The third model I consider is a second-generation financial accelerator model that also includes asset fire sales and countercyclical bankruptcy costs (Choi and Cook, 2012).⁷

In the standard NK model with fixed capital, no financing frictions and absent household heterogeneity, it is well known that disturbances to demand (for example, shocks to utility from consumption) or productivity (labour augmenting technical change) do not create a trade-off for monetary policy. There is no distinction between stabilising volatility in real quantities and inflation, and so the “divine coincidence” holds with zero inflation optimally chosen in all periods (Clarida, Gali, and Gertler, 1999).⁸

I examine whether the same result holds with capital and a financial accelerator. In addition to demand and productivity shocks, I also consider shocks to the marginal efficiency of investment (in the model with capital), to net worth and bankruptcy costs (in the model with CSV) and to volatility in investment returns (in the model with fire sales). These shocks have been emphasised in previous quantitative analysis.⁹

To understand the central bank’s objectives, I use the Linear–Quadratic (LQ) approach and study the full commitment solution, otherwise known as optimal policy from a ‘timeless perspective’ (Benigno and Woodford, 2012). Deriving an explicit welfare-theoretic loss provides maximum transparency on policy objectives and solving the LQ problem provides analytical insight into any trade-offs faced. In particular, the frictions in the economy that a central bank should be concerned with, and how these might affect any policy trade-off.

⁶Optimal policy is less tractable in the BGG model and the implied debt contract is not necessarily optimal (Dmitriev and Hoddenbagh, 2017). While the BGG model provides a better quantitative match to the data, one would expect the optimal policy insights to be similar given the similarities between the two models.

⁷The advantage of these two models is that they capture some key dimensions of the interaction between real and financial volatility. However, they do abstract from frictions in the raising of funds by financial intermediaries, such as agency problems between banks and their depositors (Gertler and Karadi, 2011) or bank runs (Angeloni and Faia, 2013), which can also be important.

⁸Only in response to inefficient shocks (such as cost-push or markup shocks), is it optimal to deviate from a zero-inflation equilibrium.

⁹See, for example, Carlstrom and Fuerst (2001), Faia and Monacelli (2007), Justiniano et al. (2010, 2011) and Choi and Cook (2012).

Focusing on a central bank with access to a single instrument, the nominal interest rate, I find:

- (i) Introducing capital in an otherwise standard NK economy does not affect the divine coincidence. In response to all shocks – demand, productivity and the marginal efficiency of investment – the flexible-price equilibrium remains optimal and there is no trade-off between stabilising real variables and inflation.
- (ii) Introducing a second friction, costly state verification (CSV), breaks the divine coincidence. Volatility in the net worth of borrowers becomes an additional objective for monetary policy and there is a non-trivial trade-off between stabilising volatility in it and inflation.
- (iii) In the model with CSV, net worth is a sufficient statistic for capturing the extent to which monetary policy deviates from a flexible-price equilibrium. Volatility in credit spreads and asset prices only matter to the extent they affect net worth and are not sufficient statistics in their own right.
- (iv) Allowing for fire sales and countercyclical bankruptcy increases the set of variables relevant to the central bank's objective and again the divine coincidence does not hold. Volatility in default, the capital stock and net worth are all relevant to optimal monetary policy and there is a trade-off between price and financial stability.

Changes in investment efficiency are viewed as consistent with financial frictions that are important at business cycle frequencies (Justiniano et al., 2010, 2011). Result (i) implies that when these changes are exogenous, the conventional wisdom holds – monetary policy need only consider financial volatility to the extent that it influences forecasts of inflation and real variables. Nevertheless, the second result makes clear that when changes in investment efficiency are micro-founded and sensitive to changes in interest rates, inflation, investment activity, asset prices and default, then financial variables become relevant to the optimal policy calculus.¹⁰ In particular, with CSV

¹⁰This is not simply to forecast the output and inflation. Volatility in financial variables becomes part of the objective.

the primitive of interest is net worth. With fire sales, volatility in default and the capital stock also become relevant.

Results (ii) to (iv) highlight that a financial accelerator constrains investment allocations relative to a frictionless equilibrium. On the supply side, costly default raises the cost of supplying new capital, which affects marginal costs. On the demand side, costly default reduces the expected return from investment to entrepreneurs and thus the resources available for them to consume. The latter affects the demand for goods and changes the distribution of consumption between entrepreneurs and households.¹¹ Both effects imply a trade-off for monetary policy that can either stabilise inflation to reduce price dispersion, or allow for some non-zero inflation to help mitigate the effects of socially costly default. The divine coincidence does not hold either in the CSV model or with fire sales.

These results apply when the central bank uses a single instrument, the nominal interest rate. Two additional instruments are also considered: a transfer to entrepreneurs financed by a lump-sum tax on households (in the CSV and fire-sales models); and a subsidy on liquidation services (in the fire-sales model only). These instruments are considered because they align with the underlying financial distortions associated with each model. One can think of a central bank choosing them, or alternatively a separate fiscal authority or prudential regulator choosing them and who shares the same welfare-theoretic loss.¹²

The first instrument is equivalent to a time-varying subsidy (or tax) on borrowing and is therefore similar to a policy of choosing optimal liquidity requirements. The second, also referred to as ‘heterodox policy’ (Choi and Cook, 2012), captures how a time-varying subsidy (or tax) can be used to help stabilise impaired asset prices. With access to these additional instruments:

- (v) In the CSV model, a transfer to entrepreneurs restores the divine coincidence and the flexible-price equilibrium is optimal in this case.

¹¹This feature of the model aligns with findings by Cúrdia and Woodford (2015) and Nisticó (2016) who also find dispersion in consumption between borrowers and lenders is relevant to social welfare.

¹²While these instruments are often thought of as fiscal instruments, being financed through taxation, the asset purchase and lending programmes implemented by central banks during the Crisis had elements of these policies. They effectively subsidised credit (transferred resources) to borrowers and creditors by providing loans at terms that private markets were not willing to offer.

- (vi) In the model with fire sales, access to transfers and liquidation subsidies are not sufficient to restore the divine coincidence.
- (vii) With fewer instruments than targets in the model with fire sales, it is optimal to increase transfers and liquidation subsidies in response to increased volatility in investment returns or declines in net worth. It is optimal to reduce transfers and liquidation subsidies in response to higher demand or productivity. The optimal choice of nominal interest rate offsets the effects of these policies and the underlying shocks on inflation. However, the divine coincidence does not hold.

Result (v) highlights that access to transfers can fully restore the optimality of a central bank's focus on inflation in the CSV model (the modified Jackson-hole consensus holds). Result (vi) makes clear that further instruments are required with fire sales. Indeed, allowing for transfers to entrepreneurs and liquidation subsidies is not sufficient for restoring the divine coincidence as there are four objectives (volatility in default, the capital stock, net worth and inflation), but only three instruments. Result (vii) makes clear that with fewer instruments than targets, optimal transfers and liquidation subsidies are countercyclical. However, it is also optimal for the nominal interest rate to offset the effects of those policies on inflation. That is, optimal 'prudential' and monetary policy offset one another.

While earlier literature has emphasised the role of credit spreads with financial frictions when capital is fixed (Carlstrom et al., 2010; Cúrdia and Woodford, 2010), that emphasis is not as strong in the models with time-varying capital considered here. Credit spreads, default rates and investment all matter, either through their effects on net worth in the CSV model, or as separate objectives in their own right when fire sales are included. Credit spreads alone are not sufficient for capturing the optimal deviation from a flexible-price equilibrium. Asset prices, a focus of previous literature on interest rate rules (Bernanke and Gertler, 2001; Gilchrist and Leahy, 2002; Faia and Monacelli, 2007; Nutahara, 2014, 2015), also do not capture all relevant information when optimally deviating from the flexible-price equilibrium.

These results connect to a broader literature on how monetary policy should be conducted optimally in the presence of financial frictions. Cúrdia and Woodford

(2015) and Nisticó (2016) study optimal monetary policy with incomplete financial markets and consumer heterogeneity (again with fixed capital), and find that consumption dispersion becomes an additional objective for monetary policy.¹³ Collard et al. (2017) study optimal Ramsey policy, jointly monetary and prudential policy, in a model with capital and socially inefficient lending. By setting capital requirements sufficiently high, all inefficient lending is removed and welfare is maximised. Faia and Monacelli (2007) study optimal interest rate rules numerically in a first generation financial accelerator model, similar to that proposed by Carlstrom and Fuerst (1997), and find that constrained efficient interest rate rules that respond to asset price volatility are sub-optimal.

With capital and financial frictions and using an LQ approximation of optimal policy, I find that neither credit spreads, asset prices, or consumption dispersion are sufficient for capturing the underlying trade-off for monetary policy. In a model with capital and CSV, it is the net worth of borrowers that is most relevant for capturing this trade-off. With fire sales additionally included, volatility in default and capital also become relevant. These results do not require lending to be socially inefficient, or that prudential policy need also be optimal.¹⁴

The next section describes the three economies in which optimal monetary policy is studied analytically. Section 3 discusses the main findings and compares them with existing literature. The final section concludes.

2. The Economies

2.1. The NK Economy with Capital

The environment with capital, but no credit friction reflects a standard NK economy with a flexible rental market for capital. As this economy is well known, I discuss it only briefly. There is a continuum of ex ante identical households on the unit interval. Each infinitesimal household supplies labour to one infinitesimal firm, consumes and can invest in capital and government bonds. Household utility is additively separable in consumption, $C_{i,t}$, and labour effort, $H_{i,t}$, and there is perfect consumption

¹³In the former case, information about consumption dispersion can be captured through credit spreads.

¹⁴In particular, optimal monetary policy is characterised for *any* given prudential policy.

insurance across households ($C_{i,t} = C_t$ for all i). The usual first-order conditions are:

$$K_{t+1} = (1 - \delta) K_t + \Upsilon_t I_t \quad (1)$$

$$U_c(C_t, \Psi_t) = \beta E_t \left(U_c(C_{t+1}, \Psi_{t+1}) (1 + i_t^b) \frac{P_t}{P_{t+1}} \right) \quad (2)$$

$$\frac{Q_t}{P_t} = \beta E_t \left(\frac{U_c(C_{t+1}, \Psi_{t+1})}{U_c(C_t, \Psi_t)} \left(R_{t+1} + (1 - \delta) \frac{Q_{t+1}}{P_{t+1}} \right) \right) \quad (3)$$

$$W_{i,t} = \frac{V_H(H_{i,t})}{U_c(C_t, \Psi_t)} \quad (4)$$

where K_t is the capital stock held in period t , I_t is investment, U_c the marginal utility from consumption, i_t^b the nominal interest rate, P_t the price of consumption, Q_t the shadow value of an extra unit of capital (asset prices), R_t the real rental return to capital, V_H is the marginal disutility from working, and $W_{i,t}$ is the real wage paid to household i given labour effort $H_{i,t}$ – that is, labour is firm-specific. The rate of time preference is given by β and δ denotes the rate of depreciation in capital. Ψ_t is a demand shock to the marginal utility of consumption and Υ_t a shock to the marginal efficiency of investment. Here, the assumption of perfect substitutability of capital across firms ($R_{i,t} = R_t$ for all i) has been imposed. This completes the description of the demand side of the economy.

The supply side of this economy is also standard. There is a representative (perfectly competitive) final goods producer that transforms a bundle of intermediate goods into final output that can be consumed or invested. The usual Dixit-Stiglitz aggregator is assumed and the implied demand for intermediates is:

$$Y_t = \left(\int_0^1 X_{i,t}^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}} \quad (5)$$

$$X_{i,t} = \left(\frac{p_{i,t}}{P_t} \right)^{-\theta} Y_t \quad (6)$$

where $X_{i,t}$ denotes intermediate good i , Y_t is final output, θ is the elasticity of substitution between intermediate goods and $p_{i,t}$ is the price of intermediate i .

For intermediate production, there is a unit measure of monopolistically competitive firms who use labour that is specific to them, but generic capital that can be costlessly reallocated across firms. Consistent with the standard NK model, I assume Calvo pricing so that only a fraction of firms $(1 - \gamma)$ can optimise their price in any

given period. The technology used by intermediate producers, their optimal capital to labour ratio, and real marginal cost ($S_{i,t}$) are given by:

$$X_{i,t} = K_{i,t}^\alpha (Z_t H_{i,t})^{1-\alpha} \quad (7)$$

$$\frac{K_{i,t}}{H_{i,t}} = \frac{W_{i,t}}{R_t} \frac{\alpha}{1-\alpha} \quad (8)$$

$$S_{i,t} = \frac{1}{Z_t^{1-\alpha}} \left(\frac{W_{i,t}}{1-\alpha} \right)^{1-\alpha} \left(\frac{R_t}{\alpha} \right)^\alpha \quad (9)$$

where $K_{i,t}$ denotes capital demanded by firm i at time t , α denotes the steady state share of factor payments that accrues to capital and Z_t is a common labour augmenting technology (productivity) shock. For firms able to choose their price, their optimal price ($p_{i,t}^*$) is:

$$\frac{p_{i,t}^*}{P_t} = (1-\chi) \frac{\theta}{\theta-1} \frac{E_t \sum_{\tau=t}^{\infty} (\beta\gamma)^{\tau-t} \frac{U_c(C_\tau, \psi_\tau)}{U_c(C_t, \psi_t)} P_\tau^\theta Y_\tau S_{i,\tau}}{E_t \sum_{\tau=t}^{\infty} (\beta\gamma)^{\tau-t} \frac{U_c(C_\tau, \psi_\tau)}{U_c(C_t, \psi_t)} P_\tau^\theta Y_\tau} \quad (10)$$

where χ captures any production subsidies (or taxes). The aggregate price index is:

$$P_t \equiv \left(\int_0^1 p_{i,t}^{1-\theta} di \right)^{\frac{1}{1-\theta}} = ((1-\gamma) p_t^{1-\theta} + \gamma P_{t-1}^{1-\theta})^{\frac{1}{1-\theta}} \quad (11)$$

where all firms able to do so choose the same optimal reset price $p_{i,t}^* = p_t$ in equilibrium, and firms unable to do so simply retain their price from the previous period. With all markets clearing (goods, labour, capital and bonds), the aggregate resource constraint and aggregate capital stock are given by:

$$Y_t = C_t + I_t \quad (12)$$

$$K_t = \int_0^1 K_{i,t} di \quad (13)$$

This completes the description of the simple NK economy with capital.

2.2. The Model with CSV

I now augment the economy with a friction in the financing of investment using the approach proposed by Carlstrom and Fuerst (1997, 2001) and that is similar to the first generation financial accelerator models studied by Bernanke et al. (1999) and

Faia and Monacelli (2007). The key change is to assume that households no longer undertake investment directly. Instead, a second group of agents, entrepreneurs, are required to produce new capital. Entrepreneurs are financially constrained and must borrow to finance their investment. There is continuum of them of measure ζ . They are risk neutral and have access to a risky investment technology: entrepreneur j , is able to purchase $I_{j,t}$ units of final output and invest it in a project that yields $\omega_{j,t}I_{j,t}$ units of final capital, which is then sold to households at price Q_t . Uncertainty over the investment payoff is governed by the random variable, $\omega_{j,t}$, which is independently identically distributed with CDF $\Phi_\omega(x)$. This distribution is common to all entrepreneurs with mean 1 and variance $\bar{\sigma}^2$.

The financial friction is that entrepreneurs have limited collateral (hereafter net worth) and require financial intermediaries (banks) to leverage their project. The amount they borrow is defined by the total value of their investment, $P_t I_{j,t}$, less their net worth, $NW_{j,t}$. Net worth is determined by the value of capital held and a small government transfer (F_t^e) financed through a lump-sum tax on households:

$$NW_{j,t} = P_t F_t^e + (P_t R_t + Q_t (1 - \delta)) K_{j,t}^e \quad (14)$$

where $K_{j,t}^e$ is the stock of capital held by entrepreneur j . The government transfer provides start-up capital for entrepreneurs and can be viewed as a substitute for entrepreneurs supplying labour inelastically and receiving a wage.¹⁵ This assumption can be relaxed without affecting the results discussed below.¹⁶ Initially, I will assume the government transfer is fixed and time invariant. Later on, the central bank can choose the transfer optimally and it will be an additional policy instrument.¹⁷

Entrepreneurs borrow from competitive banks. The threshold at which an entrepreneur is not able to repay their loan and must default, $\bar{\omega}_{j,t}$, is defined by:

$$\bar{\omega}_{j,t} \equiv \frac{(1 + R_t^L) (P_t I_{j,t} - NW_{j,t})}{Q_t I_{j,t}} \quad (15)$$

¹⁵See De Fiore and Tristani (2012) – it ensures that they have some starting capital with which to invest in the case of default or when a new entrepreneur replaces a retiring entrepreneur.

¹⁶See online Appendix B.2.

¹⁷One can analogously think of an optimal subsidy (or tax) on borrowing that is similar to the optimal choice of liquidity requirements (Choi and Cook, 2012).

where R_t^L is the interest rate charged by banks on loans to entrepreneurs. Entrepreneurs can only repay their loans when their investment return exceeds the default threshold $\omega_{j,t} \geq \bar{\omega}_{j,t}$. In the event that $\omega_{j,t} < \bar{\omega}_{j,t}$, entrepreneurs default and receive no payoff. While it is costless for entrepreneurs to observe their investment return, it is costly for banks to do so (Carlstrom and Fuerst, 1997). Banks must pay a verification or bankruptcy cost, $\nu_t \in [0, 1]$, proportional to the nominal resale value of an investment project. In the CSV model, ν_t is exogenous and so bankruptcy costs are acyclic.

Following Carlstrom and Fuerst, the optimal contract consists of the default threshold and investment level, $\{\bar{\omega}_{j,t}, I_{j,t}\}$, chosen by the entrepreneur to maximise their expected return subject to the bank's participation constraint.¹⁸ Aggregating over the mass of entrepreneurs, the optimal default threshold and investment are given by:

$$q_t f(\bar{\omega}_t) = \frac{f_\omega(\bar{\omega}_t)}{g_\omega(\bar{\omega}_t, \nu_t)} (q_t g(\bar{\omega}_t, \nu_t) - 1) \quad (16)$$

$$\zeta I_t = \frac{\zeta n w_t}{1 - q_t g(\bar{\omega}_t, \nu_t)} \quad (17)$$

where $q_t \equiv \frac{Q_t}{P_t}$ is the real price of capital, aggregate investment is $\zeta I_t \equiv \int_1^{1+\zeta} I_{j,t} dj$, nominal net worth is $\zeta N W_t \equiv \int_1^{1+\zeta} N W_{j,t} dj$ and real net worth defined by $n w_t \equiv \frac{N W_t}{P_t}$. $f(\bar{\omega}_t) \equiv \int_{\bar{\omega}_t}^{\infty} (\omega_t - \bar{\omega}_t) d\Phi(\omega_t)$ is the expected share of investment income accruing to entrepreneurs and $f_\omega(\bar{\omega}_t) \equiv \frac{\partial f(\bar{\omega}_t)}{\partial \bar{\omega}_t}$ its partial derivative with respect to the default threshold, and $g(\bar{\omega}_t, \nu_t) \equiv 1 - \nu_t \Phi(\bar{\omega}_t) - f(\bar{\omega}_t)$ is the expected share of investment income accruing to banks and $g_\omega(\bar{\omega}_t, \nu_t) \equiv \frac{\partial g(\bar{\omega}_t, \nu_t)}{\partial \bar{\omega}_t}$ its partial derivative again with respect to the default threshold.

Entrepreneurs are risk neutral with rate of time preference β^e , face stochastic retirement with probability $(1 - \kappa)$, and those who do not default are able to consume after their investment proceeds have been realised.¹⁹ Consistent with Carlstrom and Fuerst (2001) and Choi and Cook (2012), I focus on an equilibrium where entrepreneurs are sufficiently patient that they save all of their investment returns

¹⁸Under the optimal contract, banks only pay the verification cost when an entrepreneur defaults (Gale and Hellwig, 1985)

¹⁹Stochastic retirement prevents self-financing on the part of entrepreneurs.

each period in order to re-invest prior to retirement ($K_{j,t+1}^e = (\omega_t - \bar{\omega}_t) I_{j,t}$, $C_{j,t} = 0$ for $t < \underline{R}$), and only consume their total worth upon retirement ($K_{j,t+1}^e = 0$ and $C_{j,t} = q_t (\omega_t - \bar{\omega}_t) I_{j,t}$ for $t = \underline{R}$) where $\underline{R}$ denotes the generic period in which an entrepreneur is stochastically forced to retire.²⁰ In aggregate, entrepreneur saving and consumption are therefore:

$$q_t \zeta K_{t+1}^e = f(\bar{\omega}_t) q_t \zeta I_t - \zeta C_t^e \quad (18)$$

$$\zeta C_t^e = (1 - \kappa) f(\bar{\omega}_t) q_t \zeta I_t \quad (19)$$

where $\zeta C_t^e \equiv \int_1^{1+\zeta} C_{j,t}^e dj$ denotes aggregate entrepreneur consumption and $\zeta K_t^e \equiv \int_1^{1+\zeta} K_{j,t}^e dj$ aggregate entrepreneur saving. Again, all markets clear so that the resource constraint, accumulation of capital, and distribution of capital are:²¹

$$Y_t = C_t + \zeta C_t^e + \zeta I_t \quad (20)$$

$$K_{t+1} = (1 - \delta) K_t + \zeta I_t - \nu_t \Phi(\bar{\omega}_t) \zeta I_t \quad (21)$$

$$K_t = K_t^h + \zeta K_t^e \quad (22)$$

where it should be noted capital is now held by both households and entrepreneurs and $\nu_t \Phi(\bar{\omega}_t) \zeta I_t$ is the forgone investment associated with costly monitoring by banks. The household demand and goods production side of the economy in the CSV model are otherwise identical to the simple NK economy with capital discussed in Section 2.1.

2.3. The Model with Fire Sales

One of the limitations of the model with CSV is that bankruptcy costs are acyclic, which is at odds with the stylised fact that these costs rise in recessions and fall in booms. The third economy I study incorporates fire sales so that bankruptcy costs become endogenous and countercyclical. Almost all of the previous equilibrium conditions in the CSV model still apply, but the substantive changes are: a) bankruptcy

²⁰ A full description of the entrepreneurs' optimal consumption and savings decision is provided in online Appendix B.

²¹ Note that $K_t^h \equiv \int_0^1 K_{i,t}^h di$.

costs, ν_t , are now a state-contingent function of asset prices; and b) costly default is now expensed from output rather than investment.²²

To include fire sales, I follow the approach developed by Choi and Cook (2012), and assume the price of investment projects in default, impaired assets, declines as default rates increase. Like Choi and Cook, I assume a fixed measure of liquidators who liquidate investment projects in default. Liquidators purchase foreclosed projects from banks at fire-sale prices, FS_t . These assets are repaired and re-sold to households at their full capital value, Q_t , but require liquidation services, νM_t , where M_t is the marginal cost of liquidation and ν the proportion of the project requiring liquidation.

With Bertrand competition, fire-sale asset prices are: $FS_t = Q_t - \nu M_t$. Liquidators use a diminishing returns technology to transform final output purchased, N_t , into liquidation services, $L_t = L(N_t)$. The inverse of this function, the amount of final output required to produced a given quantity of liquidation services, is quadratic and has derivatives $\frac{\partial N_t}{\partial L_t} = 0$ and $\frac{\partial^2 N_t}{\partial L_t^2} = \varphi_l$ when evaluated in steady state. The real marginal cost of liquidation services is:

$$\frac{M_t}{P_t} = \varphi_l \left(\frac{L_t - L}{L} \right) \quad (23)$$

where $L \equiv \nu \zeta I \Phi(\bar{\omega})$ is the steady state level of liquidation services. Total demand for liquidation is proportional to the total expected value of projects in default:

$$L_t = \nu \zeta I_t \int_0^{\bar{\omega}_t} \omega_t d\Phi(\omega_t; \sigma_t) \quad (24)$$

where $\sigma_t \equiv var(\omega_t)$ denotes the variance of investment returns. Returns volatility is now assumed to be the time varying exogenous source of financial volatility in the fire-sales model,²³ but the CDF Φ is otherwise identical to that specified in the model with CSV.

Bankruptcy costs are now a function of fire-sale asset prices and are therefore

²²I follow this approach to keep the model as close as possible to that developed by Choi and Cook (2012). One could relax b) and alternatively assume liquidation is expensed from investment. The results discussed below would be similar.

²³Whereas the CSV model assumes exogenous shocks to bankruptcy and held the volatility of returns was constant, the fire-sales model allows for time varying volatility to capture exogenous changes in investment uncertainty (Bloom, 2009).

counter-cyclical:

$$\nu_t = 1 - \frac{FS_t}{Q_t} \quad (25)$$

The economy-wide resource constraint with costly liquidation is:

$$Y_t = C_t + \zeta C_t^e + \zeta I_t + \frac{\varphi_l}{2L} (L_t - L)^2 \quad (26)$$

where $\frac{\varphi_l}{2L} (L_t - L)^2$ denotes final output absorbed through liquidation. As these services absorb final output, rather than investment, capital accumulation is now simply:

$$K_{t+1} = (1 - \delta) K_t + \zeta I_t \quad (27)$$

The rest of economy – households, entrepreneurs' consumption and borrowing decisions, and the supply of intermediate and final goods – is otherwise identical to the CSV model discussed in Section 2.2. The online Appendix provides a full characterisation of the decentralised equilibrium for the model with fire sales, for the model with CSV, and for the simple NK economy with capital.

3. Optimal Monetary Policy

I now consider optimal monetary policy in these three economies. I focus on the LQ problem and the optimal commitment solution from a timeless perspective (Woodford, 2003; Benigno and Woodford, 2012).²⁴ An important assumption that facilitates tractability in the LQ approximation of models with capital is the assumption of a non-distorted (efficient) steady state.²⁵ In the model with capital only this achieved by assuming a subsidy to the provision of intermediate goods ($(1 - \chi) \frac{\theta}{\theta - 1} = 1$); in the CSV model by assuming that steady state bankruptcy costs are small ($\bar{\nu} \searrow 0$);²⁶

²⁴Quantitatively, it is equivalent to a log-linear approximation of the solution to the optimal Ramsey problem for the central bank with restrictions on initial conditions that ensure that only the time invariant component of alternative policies are ranked.

²⁵Specifically, it facilitates iteration of terms that are multiplicative in capital out of the respective loss functions.

²⁶ $\bar{\nu} \rightarrow 0$ from above is sufficient for the steady state capital to output ratio, real return to capital and real shadow value of capital to be the same as in an RBC economy with endogenous labour supply and that is efficient. Under the additional assumption that $\kappa \rightarrow 1$ from below, steady state social welfare in the CSV economy will be the same as that for an RBC economy as well (see online Appendix D).

and in the fire-sales model by assuming that only the deviation of liquidity services from their steady state value ($L_t - L$) absorbs final output (see Equation 26).

Although these assumptions are important for the quantitative properties of each model, they are less likely to be important when studying the insights from optimal policy analytically.²⁷ One can view the optimal policy exercises discussed here as those in which the underlying economy is efficient, but the central bank has to stabilise perturbations away from the efficient allocation in the presence of both pricing and financial frictions.

For social welfare, the maintained assumption is that the central bank maximises an aggregate measure of welfare, incorporating the welfare of both households and entrepreneurs:

$$SW_{t_0} = E_{t_0} \sum_{t=t_0}^{\infty} \left[\beta^{t-t_0} \left(U(C_t, \Psi_t) - \int_0^1 V(H_{i,t}) di \right) + \Lambda_t \int_1^{1+\zeta} (\kappa \beta^e)^{t-t_0} C_{j,t}^e dj \right] \quad (28)$$

where SW_{t_0} is expected welfare conditional on all available information and shocks at time $t = t_0$ and Λ_t is the weight placed on the aggregate welfare of entrepreneurs in social welfare. The reason for weighting the welfare of households and entrepreneurs is that the welfare of both borrowers and lenders is implicit in many central bank mandates – i.e. neither group is favoured *a priori* over the other. Focusing solely on the welfare of households would imply a first-order incentive to redistribute consumption away from entrepreneurs and towards them. To avoid such redistributive policies, I normalise the weight on entrepreneurial welfare to $\Lambda_t = \bar{U}_c (\beta/\kappa \beta^e)^{t-t_0}$.²⁸

In what follows, I derive second-order approximations of Equation (28) that result in loss functions of the form:

$$SW_{t_0} = -E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} (\mathcal{L}_t) + t.i.p + O(\|\vartheta\|^3) \quad (29)$$

²⁷For example, since a large distortion associated with monopolistic competition introduces a non-zero target for the output gap, one conjecture is that a large financial friction will introduce additional non-zero terms in the targets for financial variables, in addition to those already discussed below.

²⁸This choice ensures there is no first-order redistribution of consumption due to different attitudes to risk or preferences over time between households and entrepreneurs, and that asymptotically both groups are weighted equally.

where third-order terms or higher are grouped in $O(\|\vartheta\|^3)$ and *t.i.p* denotes any additional terms that are independent of monetary policy (i.e. not a function of the central bank's choices). These approximations provide second-order accurate rankings of welfare and since first-order terms are eliminated from them, can be minimised subject to first-order approximations of the constraints describing the relevant economy's decentralised equilibrium (Woodford, 2003; Benigno and Woodford, 2012).

It should be clear that the LQ approximations of optimal policy solve for the constrained-efficient equilibrium in each economy. That is, an optimal allocation subject to the constraints imposed by economic behaviour, expectations and the different frictions affecting each economy. Alternative instruments that would otherwise allow the policymaker to eliminate or offset the assumed frictions (beyond those specified in each respective policy problem) are assumed not to be available.²⁹

Following Woodford (2003), I also reformulate the optimal policy problem for each economy using the benchmark of the flexible-price equilibrium. Defined *relative* to each economy studied, the flexible-price equilibrium is the hypothetical allocation obtained where no firm has any incentive to change its price and so the allocation obtained would be the same as that obtained as if all prices were flexible. This is a natural benchmark for comparison as price stability holds in this equilibrium and it measures the extent to which a policymaker has an incentive to deviate from fully stabilising inflation.

It should be noted that the three economies studied here all converge to the same equilibrium in the absence of financial frictions or perturbations to investment efficiency. If Calvo pricing is also absent, they all converge to an RBC model.³⁰

3.1. The NK Economy With Capital Only

I now state the first result relating to optimal policy in the simple NK economy with capital, but no financial accelerator (all proofs are in the online Appendix).

²⁹For example, if the policymaker had access to a costless monitoring tool and can directly monitor investment projects in the economy with costly-state-verification (or could simply provide a subsidy sufficiently large to entrepreneurs that they never need to borrow) these would be examples of instruments that would facilitate the 'first-best' allocation (i.e. the socially efficient allocation in the absence of any distortions). In the economies studied below, we will assume such instruments do not exist, and so the 'first-best' allocation may not be feasible. Put differently, financial frictions will bind and have the potential constrain optimal policy in some way.

³⁰See Remark 1 in online Appendix E.

Proposition 1. *In an NK economy with capital but no financial accelerator, productivity, demand and investment efficiency shocks can be optimally stabilised by choosing the flexible-price equilibrium. That is, the divine coincidence holds.*

(i) *The loss function can be approximated by:*

$$\mathcal{L}_t^{Capital} = \omega_x \frac{x_t^2}{2} + \omega_g \frac{g_t^2}{2} + \omega_j \frac{j_t^2}{2} + \omega_\pi \frac{\pi_t^2}{2} - \omega_{gx} g_t x_t - \omega_{jx} j_t x_t \quad (30)$$

where $x_t = y_t - y_t^n$, $g_t = i_t - i_t^n$, $j_t = k_t - k_t^n$ denote the output, investment and capital gaps respectively or deviations from their corresponding flexible-price equilibrium values, and with loss coefficients $\omega_x = \left(\frac{\alpha+\eta}{1-\alpha} + \sigma\right)$, $\omega_g = \sigma s^2 \delta^2$, $\omega_j = \frac{\alpha(1+\alpha\eta)}{1-\alpha}$, $\omega_\pi = \frac{(\theta^{-1}-1+\frac{\eta+1}{\alpha\eta+1})\theta^2\gamma}{(1-\gamma)(1-\beta\gamma)}$, $\omega_{gx} = s\sigma\delta$, $\omega_{jx} = \frac{\alpha(1+\eta)}{1-\alpha}$, where $s = \frac{K}{Y}$, $\eta = \frac{V_{HH}H}{V_H}$, and $\sigma = -\frac{U_{cc}Y}{U_c}$.

(ii) *The policy $\{x_t = 0, g_t = 0, j_{t+1} = 0, \pi_t = 0\}_{t \geq t_0}$ and $\{r_t^b = r_t^{bn}\}_{t \geq t_0}$, where r_t^{bn} is the natural rate of interest, is an optimal policy.*

Proposition 1 establishes that although output, investment and capital are all present in the loss function in the economy with capital, the inclusion of capital per se does not alter the optimal policy prescription of implementing the flexible-price equilibrium. That is, the divine coincidence holds. One implication is that if exogenous changes in investment efficiency capture financial volatility (Justiniano et al., 2010, 2011), and these are orthogonal to changes in the real economy, then monetary policy need only account for financial volatility to the extent that it changes forecasts for real quantities and inflation – i.e. the “modified Jackson-hole consensus” holds.

This result re-enforces a point made by Goodfriend and King (1997) and Woodford (2003). If nominal price rigidity is the only distortion that prevents a rational-expectations equilibrium from achieving its optimal allocation, then choosing a policy that minimises the effect of that distortion is optimal. In the NK model with capital, the flexible-price equilibrium is optimal because under that policy no firm has an incentive to re-optimize its price, even if it had the ability to do so. I now consider the economies that also incorporate financial accelerators so that investment efficiency is endogenous, with costly default and heterogeneity in borrowing and lending.

3.2. The CSV Model

Proposition 2 establishes that the same result does not apply once costly state verification is introduced.

Proposition 2. *In an NK economy with capital and costly state verification, the divine coincidence does not hold and the flexible-price equilibrium is not optimal.*

(i) *The new primitive of interest to the central bank is net worth and the loss function can be approximated by:*

$$\begin{aligned} \mathcal{L}_t^{CSV} = & \omega_x \frac{x_t^2}{2} + \omega_\pi \frac{\pi_t^2}{2} + \omega_g \frac{g_t^2}{2} + \omega_j \frac{j_t^2}{2} - \omega_{xg} x_t g_t - \omega_{xj} x_t j_t \\ & + \underbrace{\omega_n \frac{(n_t - n_t^*)^2}{2} - \omega_{xn} x_t n_t + \omega_{gn} g_t n_t}_{\text{New terms in net worth}} \end{aligned} \quad (31)$$

where the new worth gap is defined as $n_t = nw_t - nw_t^n$, and with additional loss coefficients $\omega_n = \sigma (s^e)^2$, $\omega_{xn} = \sigma s^e$ and $\omega_{gn} = \sigma \delta s^e$, where $s^e = \frac{\zeta C^e}{Y}$ and all other terms are the same as those defined in Proposition 1. The net worth gap target is:

$$n_t^* = \sigma^{-1} (s^e)^{-2} \left(s \delta \phi(\bar{\omega}) \frac{f(\bar{\omega})}{f_{\bar{\omega}}(\bar{\omega})} \xi_t + s^e (\sigma y_t^n - s \delta i_t^n - v \psi_t - \sigma s^e c_t^{en}) \right)$$

with $v = \frac{U_{c\psi}}{U_c}$, $\xi_t = \ln(1 - \nu_t)$ and $\psi_t = \ln \Psi_t$ are renormalised shocks to bankruptcy costs and demand, $\bar{\omega}$ is the steady state default threshold, $\phi(x) = \frac{d\Phi(x)}{dx}$ is the pdf of investment returns, and c_t^{en} denotes the flexible-price equilibrium value of aggregate entrepreneur consumption.

(ii) *In response to shocks to productivity, demand, net worth or bankruptcy costs, the policy $\{x_t = 0, g_t = 0, j_{t+1} = 0, n_t = 0, \pi_t = 0\}_{t \geq t_0}$ and $\{i_t^b = r_t^{bn}\}_{t \geq t_0}$ is not an optimal policy.*

Corollary 2.1. *In the absence of endogenous variation in net worth, but still in the presence of costly state verification, the objective collapses to that of an NK economy with capital. The divine coincidence holds.*

Proposition 2 and Corollary 2.1 establish that introducing a financial friction in the supply of investment goods does alter the objectives of monetary policy. In particular, it is the net worth of borrowers, in real terms, that becomes the primitive

for determining the optimal deviation from the flexible-price equilibrium. The reason that net worth is important is that it governs the borrowing needs of entrepreneurs who invest and thus the size of the deadweight loss associated with default (costly monitoring). By smoothing volatility in net worth around an appropriately defined target (n_t^*) – a function of the shocks hitting the economy – these deadweight losses can be partially mitigated and investment allocations that are closer to those obtained in an equilibrium without costly state verification can be obtained.

Intuition can be gained by comparing this result to that for the model with capital only. With capital only and a single distortion the flexible-price equilibrium is optimal as nominal price rigidity is the only source of inefficiency in the economy. Thus, the socially efficient or ‘first-best’ allocation can be obtained. With CSV, there is a second distortion implied by costly monitoring for banks. The presence of asymmetric information on their part raises the cost of borrowing for entrepreneurs and constrains the amount of funds lent to them. This restricts investment allocations relative to what would be achieved in an equilibrium without CSV. The presence of costly-state-verification implies that the socially optimal allocation in the model with capital only and without financing frictions is not within the feasible set of allocations from which the central bank can choose. In other words, the central bank has to trade off price stability against financial stability.

The primitive variable that affects the borrowing required by entrepreneurs, and thus the social costs of CSV and the trade-off faced, is net worth. If entrepreneurs have more (or less) pledgeable collateral they require more (less) loans and so changes in net worth have a direct effect on the extent to which the CSV friction distorts the economy from its socially efficient allocation. This time varying source of inefficiency implies that a central bank has an incentive to use interest rates, that affect asset prices, rental returns and capital accumulation, and thus the value of net worth, to help mitigate the effects of the CSV distortion.

Up to a first-order approximation, the net worth gap can be written as: $n_t = \frac{f_{\bar{\omega}_t}(\bar{\omega})\bar{\omega}}{f(\bar{\omega})}\hat{\bar{\omega}}_t^g + g_t$ where $\hat{\bar{\omega}}_t^g$ and g_t are the default threshold and investment gaps respectively. This expansion makes it clear why simple interest-rate rules augmented with

asset prices fail to replicate optimal monetary policy.³¹ Although asset prices affect the loss function indirectly, through their effect on the target value for net worth, they do not capture all relevant information for capturing the social cost of default.

Interestingly, credit spreads share the same property. Measuring spreads as the real net external finance premium (or real spread between borrowing and lending rates), $\frac{1+R_t^L}{P_t} - 1$, Corollary 2.2 makes clear that smoothing credit spreads around an appropriately defined target does not capture all relevant information for the optimal deviation from the flexible-price equilibrium. Unlike other models of credit frictions in economies without capital (Carlstrom et al., 2010; De Fiore and Tristani, 2012; Cúrdia and Woodford, 2015), the central bank must also consider volatility in investment, which separately contributes to the loss and is a function of net worth and default.

Corollary 2.2. *Stabilising credit spreads is not isomorphic to stabilising volatility in net worth. Reformulating the objective in terms of spreads, an additional target for investment enters the loss:*

$$\begin{aligned} \mathcal{L}_t^{CSV} = & \omega_x \frac{x_t^2}{2} + \omega_\pi \frac{\pi_t^2}{2} + \underbrace{\tilde{\omega}_g \frac{(g_t - g_t^*)^2}{2}}_{\text{Additional target}} + \omega_j \frac{j_t^2}{2} - \tilde{\omega}_{xg} x_t g_t - \omega_{xj} x_t j_t \\ & + \underbrace{\tilde{\omega}_n \frac{(\hat{s}p_t^g - \hat{s}p_t^*)^2}{2} + \omega_{gs} g_t \hat{s}p_t^g - \tilde{\omega}_{ns} x_t \hat{s}p_t^g}_{\text{Terms with credit spreads}} \end{aligned}$$

where $\hat{s}p_t^g = \hat{s}p_t - \hat{s}p_t^n$ is the credit spread gap, $\hat{s}p_t^* = \frac{\Xi f_{\omega}(\bar{\omega})}{f(\bar{\omega})\bar{\omega}} n_t^*$ is the target value for spreads, $g_t^* = \frac{\omega_n}{\omega_g} n_t^*$ is the target value for investment and with modified loss weights $\tilde{\omega}_g = \omega_g + 2\omega_{gn} + \omega_n$, $\tilde{\omega}_{ns} = \omega_{nx} \frac{f_{\omega}(\bar{\omega})(\bar{\omega})}{f(\bar{\omega})\Xi}$, $\tilde{\omega}_n = \omega_n \left(\frac{f_{\omega}(\bar{\omega})(\bar{\omega})}{f(\bar{\omega})\Xi} \right)^2$, $\omega_{gs} = \omega_n \frac{f_{\omega}(\bar{\omega})(\bar{\omega})}{f(\bar{\omega})\Xi} + \omega_{gn} \frac{f_{\omega}(\bar{\omega})}{f(\bar{\omega})\Xi}$ where $\Xi = \left(1 + \frac{f_{\omega}(\bar{\omega})}{1-f(\bar{\omega})} \right)$ and $\omega_g, \omega_{gn}, \omega_n$ and ω_{nx} are the same coefficients defined in Proposition 2.

In which direction should monetary policy optimally respond to shocks in the CSV model? Figure 1 displays key responses to exogenous one per cent increases in demand, productivity, bankruptcy costs and the same decline in net worth.³² It highlights that a small amount of inflation is optimal in response to *all* shocks.³³

³¹See for example Faia and Monacelli (2007).

³²The calibration used is in line with previous literature (see online Appendix G).

³³Quantitatively, the deviations from a zero-inflation target are small reflecting the assumption of

Figure 1: Optimal Policy in the CSV Model

Notes: The impulse response functions are in response to separate 1 per cent increases in demand, productivity and bankruptcy costs, and a one per cent decline in net worth. * Sub-optimal inflation denotes the inflation outcome for a central bank ignoring the presence of the optimal target value for the net worth gap when minimising the loss. The horizontal axes are measured in quarters.

By allowing positive inflation, net worth, investment and output are all higher relative to the flexible-price equilibrium and this mitigates the financial distortion

a non-distorted steady state. Focusing on numerical results with larger distortions results in larger inflation responses under optimal policy.

associated with costly monitoring. However, reducing this distortion comes at the expense of higher price dispersion, and so there is a meaningful trade-off for monetary policy. Figure 1 (bottom right panel) also highlights the optimal inflation response if the target value for the net worth gap is ignored by the central bank. In this case a zero-inflation policy is perceived as optimal, but is not consistent with optimal policy.

3.3. The Model with Fire Sales

I now consider what optimal monetary policy looks like in the economy with fire sales and bankruptcy costs that are countercyclical. The next result establishes that incorporating fire sales increases the complexity of the loss:

Proposition 3. *In the economy with fire sales and countercyclical bankruptcy costs, the divine coincidence does not hold.*

(i) *The loss function can be approximated by:*

$$\begin{aligned}
 \mathcal{L}_t^{FS} = & \omega_\pi \frac{\pi_t^2}{2} + \omega_x \frac{x_t^2}{2} + \omega_n \frac{(n_t - n_t^{**})^2}{2} - \omega_{xj} x_t j_t - \omega_{xn} x_t n_t - \tilde{\omega}_{xg} x_t g_t + \tilde{\omega}_{gn} g_t n_t \\
 & \underbrace{+ \tilde{\omega}_g \frac{(g_t - g_t^{**})^2}{2} + \omega_\omega \frac{(\hat{\omega}_t^g - \hat{\omega}_t^{**})^2}{2} + \omega_j \frac{(j_t - j_t^{**})^2}{2}}_{\text{New targets with fire sales}} \\
 & + \underbrace{\omega_{\omega g} g_t \hat{\omega}_t^g + \omega_{n\omega} n_t \hat{\omega}_t^g - \omega_{x\omega} x_t \hat{\omega}_t^g}_{\text{New volatility terms with fires-sales}}
 \end{aligned} \tag{32}$$

where the targets for net worth (n_t^{**}), the capital stock (j_t^{**}), the default threshold ($\hat{\omega}_t^{**}$) and investment (g_t^{**}) are functions of the flexible-price equilibrium values and the underlying shocks. These values and the loss coefficients are defined in Appendix A.

(ii) *In response to shocks to demand, productivity, net worth or the volatility of investment returns, the policy $\{x_t = 0, g_t = 0, j_{t+1} = 0, n_t = 0, \hat{\omega}_t^g = 0, \pi_t = 0\}_{t \geq t_0}$ and $\{i_t^b = r_t^{bn}\}_{t \geq t_0}$ is not an optimal policy.*

Proposition 3 establishes that there are more targets once fire sales and countercyclical bankruptcy costs are included in the model. In this case, there are targets for investment, net worth, the default threshold and the capital stock, although only three of them are linearly independent.³⁴

³⁴The targets for investment and default are linearly dependent.

Comparing this result with the results for the CSV model, it is clear that there are now three distortions – nominal price rigidity, asymmetric information for banks monitoring entrepreneurs, and costly liquidation services. Thus, the central bank must optimally weight the social cost associated with each friction when choosing the optimal interest rate, rather than targeting any one of them. Moreover, the set of variables in the optimal policy calculus is now richer reflecting that investment, net worth, capital and default rates jointly determine the amount of loans required by entrepreneurs, bankruptcy costs in the event of default, and liquidation services required for impaired assets. As established for the CSV model, the divine coincidence does not hold and the flexible-price equilibrium is no longer optimal.

Figure 2 reports the key responses in the model with fire sales. In response to a one per cent increase in investment volatility, or the same decline in net worth, it is optimal to choose positive inflation and a negative real interest rate gap – that is, a more expansionary path for interest rates than in the case of the flexible-price equilibrium. This means that the central bank offsets social costly financial volatility by allowing inflation to help insulate the effects of these shocks on the financial sector. By choosing a lower interest rate path, relative to the flexible-price equilibrium, asset prices and thus entrepreneurial net worth are higher, and again the investment and output allocations are closer to those that would be obtained in a frictionless economy. The magnitude of the deviation from the flexible-price equilibrium is captured in the target values in the loss, with positive target values for the net worth, capital and default threshold gaps (Figure 3).

In response to positive demand or productivity shocks, it is optimal to choose a more contractionary path for monetary policy relative to the flexible-price equilibrium (refer to Figure 2).³⁵ These relatively more contractionary policies help to stabilise financial volatility associated with higher asset prices, investment, default and liquidation. In response to a positive demand shock, it is optimal to choose negative values for all three targets (Figure 3). For productivity shocks, the targets are all close to zero.³⁶

³⁵In the case of demand shocks interest rates rise by more than in the zero-inflation case, and for productivity shocks they fall by less.

³⁶This is because an increase in productivity is accompanied by a decline in investment volatility

Figure 2: Responses Under Optimal Policy with Fire Sales

Note: The impulse response functions are in response to separate 1 per cent increases in demand, productivity and investment volatility, and a one per cent decline in net worth. The horizontal axes are measured in quarters.

and default and so the need to respond to financial volatility is weaker (see online Appendix G).

Figure 3: Optimal Targets with Fire Sales

Note: The quarterly percentage change in the optimal targets are in response to separate 1 per cent increases in demand, productivity and investment volatility, and a one per cent decline in net worth.

3.4. Comparing the loss functions

It is helpful to review the relationships between each of the loss functions derived, and how they relate to a standard New Keynesian economy (Woodford, 2003). Proposition 4 makes clear that as each friction is removed, the loss functions converge to each other ($\mathcal{L}_t^{FS} \rightarrow \mathcal{L}_t^{CSV} \rightarrow \mathcal{L}_t^{Capital}$) and then to the loss for the standard New Keynesian economy as the capital share approaches zero. This highlights the mapping between the frictions and the additional terms derived in each loss (Table 1).

Proposition 4. (i) In the absence of bankruptcy cost shocks ($\xi_t = 0$) and shocks to the volatility of investment returns ($\hat{\sigma}_t = 0$), the loss function with fire sales converges to the loss function with CSV as marginal liquidation costs approach zero:

$$\mathcal{L}_t^{FS} \rightarrow \mathcal{L}_t^{CSV} \text{ as } \varphi_l \searrow 0$$

(ii) In the model with CSV, as entrepreneur capital holdings approach zero the loss function with CSV converges to the loss function in the model with capital:

$$\mathcal{L}_t^{CSV} \rightarrow \mathcal{L}_t^{Capital} \text{ as } K_t^e \searrow 0$$

(iii) In the model with capital, as the capital share approaches zero, the loss function converges to that of the standard New Keynesian model:

$$\mathcal{L}_t^{Capital} \rightarrow \mathcal{L}_t \text{ as } \alpha \searrow 0$$

Table 1: Comparison of the Loss Functions

Standard:	$\mathcal{L}_t = \tilde{\omega}_x \frac{x_t^2}{2} + \tilde{\omega}_\pi \frac{\pi_t^2}{2}$
With capital:	$\mathcal{L}_t^{capital} = \omega_x \frac{x_t^2}{2} x_t + \omega_\pi \pi_t + \underbrace{\omega_g \frac{g_t^2}{2} + \omega_j \frac{j_t^2}{2} - \omega_{xg} x_t g_t - \omega_{xj} x_t j_t}_{\text{New terms due to capital}}$
With CSV:	$\mathcal{L}_t^{CSV} = \omega_x \frac{x_t^2}{2} + \omega_\pi \frac{\pi_t^2}{2} + \omega_g \frac{g_t^2}{2} + \omega_j \frac{j_t^2}{2} - \omega_{xg} x_t g_t - \omega_{xj} x_t j_t$ $+ \underbrace{\omega_n \frac{(n_t - n_t^*)^2}{2} - \omega_{xn} x_t n_t + \omega_{gn} g_t n_t}_{\text{New terms due to costly state verification}}$
With fire-sales:	$\mathcal{L}_t^{FS} = \omega_x \frac{x_t^2}{2} + \omega_\pi \frac{\pi_t^2}{2} + \underbrace{\tilde{\omega}_g \frac{(g_t - g_t^{**})^2}{2} + \tilde{\omega}_j \frac{(j_t - j_t^{**})^2}{2}}_{\text{New non-zero targets due to fire-sales}}$ $- \tilde{\omega}_{xg} x_t g_t - \tilde{\omega}_{xj} x_t j_t + \omega_n \frac{(n_t - n_t^{**})^2}{2} - \omega_{xn} x_t n_t + \tilde{\omega}_{gn} g_t n_t$ $+ \underbrace{\omega_\omega \frac{(\hat{\omega}_t^g - \hat{\omega}_t^{**})^2}{2} + \omega_{\omega g} g_t \hat{\omega}_t^g + \omega_{n\omega} n_t \hat{\omega}_t^g - \omega_{x\omega} x_t \hat{\omega}_t^g}_{\text{New terms due to fire-sales}}$

Notes: $\tilde{\omega}_x = \lim_{\alpha \searrow 0} \omega_x$ and $\tilde{\omega}_\pi = \lim_{\alpha \searrow 0} \omega_\pi$. Other loss coefficients are as previously defined.

Table 1 highlights that introducing capital does not introduce any additional targets (and so the flexible price equilibrium remains optimal), but it does bring investment and capital gap volatility, as well their interaction with volatility in the output gap, into the objective. This should not be surprising. Since the primitive for house-

hold welfare is consumption volatility, it is natural that both output and investment volatility are relevant in a model with capital.

Turning to the loss with CSV we see that the additional terms associated with volatility in net worth and a non-zero target for it arise from the presence of asymmetric information in financial markets (costly monitoring). The primitive that captures the social cost of this friction is entrepreneurs' net worth. Finally, with fire-sales additionally included, we see that the loss also includes terms associated the volatility of default and non-zero targets for capital, investment and default.

3.5. Access to Multiple Instruments

Thus far, the results condition on access to a single instrument – the nominal interest rate. A natural question is whether access to additional instruments could eliminate the trade-off between price stability and financial stability? I first focus on the CSV model with acyclic bankruptcy costs and allow the transfer to entrepreneurs (F_t^e) to be optimally chosen by the central bank, in addition to the nominal interest rate. As mentioned earlier, transfers could equivalently be chosen by a fiscal authority or a prudential authority who shares the same welfare theoretic loss.

Proposition 5. *With an additional policy instrument, transfers to entrepreneurs, $\widehat{F}_t^e$, the flexible-price equilibrium again becomes optimal in the CSV model and the divine coincidence holds. This equilibrium results in higher welfare than in an equilibrium where only the nominal interest rate can be chosen. An optimal sequence of transfers and interest rates can be set as follows:*

1. *The target value for net worth is computed in a flexible-price equilibrium under a policy of a constant (steady state) transfer;*
2. *A sequence of transfers is chosen such that the target value for net worth (computed in the flexible-price equilibrium in 1.) is reset to zero in all periods $\{n_t^* = 0\}_{t \geq t_0}$. Denote the sequence that achieves this, $\{\widehat{F}_t^{en*}\}_{t \geq t_0}$, and the corresponding sequence of perturbed natural interest rates, $\{r_{t|n_t^*=0}^{bn}\}_{t \geq t_0}$; and*
3. *Implement the perturbed flexible-price equilibrium by choosing a sequence of nominal interest rates that is equal to the perturbed sequence of natural interest rates $\{i_t^b = r_{t|n_t^*=0}^{bn}\}_{t \geq t_0}$.*

Proposition 5 makes clear how nominal interest rates and transfers can be chosen optimally. The optimal sequence of transfers is designed to eliminate the incentive for the central bank to target a non-zero gap for net worth, and eliminates the incentive for the central bank to take net worth into consideration when setting interest rates. Nevertheless, the optimal sequence of transfers changes the underlying natural rate of interest in the economy by affecting the demand for borrowing. The central bank optimally takes account of this and implements a flexible-price equilibrium by choosing a nominal interest rate sequence that is equivalent to the sequence of perturbed natural interest rates. This solution has a natural delineation between the optimal transfer, that stabilises the effects of costly default, and optimal interest rates that stabilise inflation.

In the model with fire sales and countercyclical bankruptcy costs, the solution is less straightforward. The next proposition makes clear that at least two further instruments are necessary to restore the divine coincidence.

Proposition 6. *In the model with fire sales, at least two additional instruments, in addition to the nominal interest rate and transfers to entrepreneurs, are necessary (although not sufficient) for restoring the optimality of a flexible-price equilibrium.*

Proposition 6 is an application of the Tinbergen principle. With three linearly independent targets in the loss function in addition to inflation – one each for capital, net worth and default – at least two further instruments are required for the flexible-price equilibrium to be optimal. This result highlights that even small increments in financial complexity can substantially complicate the loss function and the number of instruments required for monetary policy to focus exclusively on stabilising inflation. Put differently, the view that nominal interest rates should be used because they can “get in all the cracks” and affect all financial prices and costs of financial intermediation (Smets, 2014), has merit relative to other more targeted instruments, which only partially capture the social cost of fire sales.

What does optimal monetary policy look like in the model with fire sales and access to multiple instruments? I first allow the central bank to choose nominal interest rates and transfers to entrepreneurs. Figure 4 makes clear that higher transfers are optimal in response to increases in investment volatility or to a decline in net worth.

They imply higher net worth for investment, which helps to implement investment allocations closer to those that would be obtained in a frictionless equilibrium.

Figure 4: Optimal Policy with Two Instruments

Note: The quarterly impulse response functions are in response to separate 1 per cent increases in demand, productivity and investment volatility, and a one per cent decline in net worth.

Interestingly, a *higher* real interest rate gap – a policy that is more contractionary than in the case of a flexible-price equilibrium – is also called for. This more than offsets the effects of higher transfers on inflation and suggests it can be optimal for monetary and ‘prudential policy’ to work in opposite directions. In the case of positive demand or productivity shocks, the signs are reserved, it is optimal to reduce transfers and offset this with a negative real interest rate gap. In effect, transfers are countercyclical and monetary policy again works to offset this. Nevertheless, a flexible-price equilibrium is not optimal.

I also consider a third instrument, liquidation subsidies (Choi and Cook, 2012), which reduce the marginal cost of liquidation in the event of default as opposed to changing the value of collateral held by entrepreneurs. Figure 5 highlights that

even with three instruments, it remains optimal to choose non-zero inflation. For exogenous declines in net worth and increases in investment volatility, it is optimal to raise transfers and increase liquidation subsidies. Higher real interest rate gaps are then chosen to offset the effects of those policies on inflation. For increases in demand and productivity, reduced transfers and a tax on liquidation services are optimal in conjunction with a negative real interest rate gap.

Figure 5: Optimal Policy with Three Instruments

Notes: The impulse response functions are in response to separate 1 per cent increases in demand, productivity and investment volatility, and a one per cent decline in net worth. The instruments available include the nominal interest rate, a transfer to entrepreneurs and a subsidy on the marginal cost of liquidation.

These findings build on previous literature where prudential (or macro-prudential) rules are found to insulate the real economy from financial shocks (Choi and Cook, 2012; Angeloni and Faia, 2013). The results here make clear that countercyclical transfers and liquidation subsidies are also consistent with minimisation of a welfare-theoretic loss and should focusing on stabilising volatility in default, the capital stock

and net worth.³⁷ The results also align with findings in Collard, Dellas, Diba, and Loisel (2017) where optimal prudential policy is countercyclical, while optimal monetary policy stabilises inflation. There are some important differences, however. The models considered here do not assume that lending need be socially inefficient or that prudential policy must be jointly optimal. The welfare-theoretic objectives derived apply regardless of the delegation of instruments between a central bank, prudential authority or fiscal authority. Moreover, the objectives can be used to study monetary policy for *any* given prudential policy.

4. Conclusion

This paper provides an analytic characterisation of optimal monetary policy in New Keynesian economies where both capital and a financial accelerator matter. Previous literature has considered capital, but abstracted from a financial accelerator, or considered financial accelerator effects on the demand for labour but held capital fixed. The objectives of monetary policy in economies where there is a credit friction on the supply of new capital, which captures salient features of the 2007-08 Crisis and business cycle volatility more generally, have remained unclear.

Studying a class of models with capital and a financial accelerator that are commonly embedded in quantitative business cycle models, the introduction of capital per se does not introduce a trade-off for monetary policy. However, the introduction of a financial accelerator does matter. In a model with CSV, the net worth of borrowers becomes the primitive of interest to the central bank and there is a trade-off between stabilising it and inflation. Nonetheless, access to a single additional instrument, a transfer to borrowers, is sufficient for restoring the divine coincidence.

Enriching the model with fire sales, there are additional targets in the loss (targets for default, and capital also matter), but the underlying intuition for the trade-offs faced in addressing price versus financial stability remains similar. One difference in the model with greater financial complexity is that more instruments are required to restore the divine coincidence – a transfer to borrowers and a subsidy on liquidation

³⁷Transfers to entrepreneurs are similar to other macroprudential policies, such as time-varying liquidity (Choi and Cook, 2012) or capital (Angeloni and Faia, 2013) requirements, which also affect the costs of investment and default.

are not sufficient. With fewer instruments than targets, it is optimal for transfers and liquidation subsidies to stabilise volatility in default, net worth and the capital stock. Changes in the nominal interest rate are used to mitigate the effects of these policies on inflation, although the divine coincidence does not hold.

References

- Angeloni, I., Faia, E., 2013. Capital Regulation and Monetary Policy with Fragile Banks. *Journal of Monetary Economics* 60 (3), 311–324.
- Benigno, P., Woodford, M., 2012. Linear-Quadratic Approximation of Optimal Policy Problems. *Journal of Economic Theory* 147 (1), 1–42.
- Bernanke, B., Gertler, M., 2001. Should Central Banks Respond to Movements in Asset Prices. *The American Economic Review* 91 (2), 253–257.
- Bernanke, B., Gertler, M., Gilchrist, S., 1999. The Financial Accelerator in a Quantitative Business Cycle Framework. Vol. 1, Part 3 of *Handbook of Macroeconomics*. Elsevier, pp. 1341–1393.
- Bloom, N., 2009. The Impact of Uncertainty Shocks. *Econometrica* 77 (3), 623–685.
- Borio, C., Lowe, P., July 2002. Asset Prices, Financial and Monetary Stability: Exploring the Nexus. Working Paper 114, Bank for International Settlements.
- Borio, C., White, W. R., February 2004. Whither Monetary and Financial Stability? The Implications of Evolving Policy Regimes. Working Paper 147, Bank for International Settlements.
- Carlstrom, C. T., Fuerst, T. S., 1997. Agency Costs, Net Worth, and Business Fluctuations: A Computable General Equilibrium Analysis. *The American Economic Review* 87 (5), 893–910.
- Carlstrom, C. T., Fuerst, T. S., June 2001. Monetary Shocks, Agency Costs, and Business Cycles. *Carnegie-Rochester Conference Series on Public Policy* 54 (1), 1–27.

- Carlstrom, C. T., Fuerst, T. S., Paustian, M., 2010. Optimal Monetary Policy in a Model with Agency Costs. *Journal of Money, Credit and Banking* 42, 37–70.
- Cecchetti, S. G., Genberg, H., Lipsky, J., Wadhvani, S. B., 2000. Asset Prices and Central Bank Policy. *Geneva Reports on the World Economy* 2.
- Choi, W. G., Cook, D., 2012. Fire Sales and the Financial Accelerator. *Journal of Monetary Economics* 59 (4), 336–351.
- Clarida, R., Gali, J., Gertler, M., 1999. The Science of Monetary Policy: A New Keynesian Perspective. *Journal of Economic Literature* 37 (4), 1661–1707.
- Collard, F., Dellas, H., Diba, B., Loisel, O., January 2017. Optimal monetary and prudential policies. *American Economic Journal: Macroeconomics* 9 (1), 40–87.
- Cúrdia, V., Woodford, M., 2010. Credit Spreads and Monetary Policy. *Journal of Money, Credit and Banking* 42, 3–35.
- Cúrdia, V., Woodford, M., December 2015. Credit Frictions and Optimal Monetary Policy. Working Paper 21820, National Bureau of Economic Research.
- De Fiore, F., Tristani, O., 2012. Optimal Monetary Policy in a Model of the Credit Channel. *The Economic Journal* 123 (571), 906–931.
- Dmitriev, M., Hoddenbagh, J., 2017. The Financial Accelerator and the Optimal State-dependent Contract. *Review of Economic Dynamics* 24 (Supplement C), 43–65.
- Dupor, B., 2005. Stabilizing Non-fundamental Asset Price Movements Under Discretion and Limited Information. *Journal of Monetary Economics* 52 (4), 727–747.
- Edge, R., 2003. A Utility-based Welfare Criterion in a Model with Endogenous Capital Accumulation. Finance and Economics Discussion Series 2003–66, Board of Governors of the Federal Reserve System (U.S.).
- Faia, E., Monacelli, T., 2007. Optimal Interest Rate Rules, Asset Prices, and Credit Frictions. *Journal of Economic Dynamics and Control* 31 (10), 3228–3254.

- Gale, D., Hellwig, M., 1985. Incentive-Compatible Debt Contracts: The One-Period Problem. *The Review of Economic Studies* 52 (4), 647–663.
- Galí, J., Monacelli, T., 2005. Monetary Policy and Exchange Rate Volatility in a Small Open Economy. *The Review of Economic Studies* 72 (3), 707–734.
- Gambacorta, L., Signoretti, F., 2014. Should Monetary Policy Lean Against The Wind?: An Analysis Based on a DSGE model with Banking. *Journal of Economic Dynamics and Control* 43, 146–174.
- Gertler, M., Karadi, P., 2011. A Model of Unconventional Monetary Policy. *Journal of Monetary Economics* 58 (1), 17–34.
- Gertler, M., Kiyotaki, N., 2010. Financial Intermediation and Credit Policy in Business Cycle Analysis. *Handbook of Monetary Economics* 3, 547–599.
- Gilchrist, S., Leahy, J. V., 2002. Monetary Policy and Asset Prices. *Journal of Monetary Economics* 49 (1), 75–97.
- Gilchrist, S., Zakrajšek, E., June 2012. Credit Spreads and Business Cycle Fluctuations. *The American Economic Review* 102 (4), 1692–1720.
- Goodfriend, M., King, R., March 1997. The New Neoclassical Synthesis and the Role of Monetary Policy. In: *NBER Macroeconomics Annual 1997, Volume 12*. NBER Chapters. National Bureau of Economic Research, pp. 231–296.
- Jermann, U., Quadrini, V., 2012. Macroeconomic Effects of Financial Shocks. *The American Economic Review* 102 (1), 238–271.
- Justiniano, A., Primiceri, G. E., Tambalotti, A., 2010. Investment Shocks and Business Cycles. *Journal of Monetary Economics* 57 (2), 132–145.
- Justiniano, A., Primiceri, G. E., Tambalotti, A., 2011. Investment Shocks and the Relative Price of Investment. *Review of Economic Dynamics* 14 (1), 102–121.
- Nisticó, S., 2016. Optimal Monetary Policy and Financial Stability in a Non-ricardian Economy. *Journal of the European Economic Association* 14 (5), 1225–1252.

- Nutahara, K., 2014. What Asset Prices Should be Targeted by a Central Bank? *Journal of Money, Credit and Banking* 46 (4), 817–836.
- Nutahara, K., 2015. Do Credit Market Imperfections Justify a Central Bank's Response to Asset Price Fluctuations? *Journal of Economic Dynamics and Control* 61, 81–94.
- Smets, F., 2014. Financial Stability and Monetary Policy: How Closely Interlinked? *International Journal of Central Banking* 10 (2), 263–300.
- Svensson, L. E., 2010. Monetary Policy After the Financial Crisis. In: Address at the Second International Journal of Central Banking Fall Conference, Tokyo (17 September).
- Takamura, T., Watanabe, T., Kudo, T., 2006. Optimal Monetary Policy at the Zero Interest Rate Bound: The Case of Endogenous Capital Formation, Understanding Inflation Dynamics of the Japanese Economy Working Paper No. 3.
- Woodford, M., 2003. *Interest and Prices*. Princeton University Press, New Jersey.
- Woodford, M., April 2012. Inflation Targeting and Financial Stability. Working Paper 17967, National Bureau of Economic Research.

Acknowledgements

This paper is a substantial revision of the first chapter of my PhD thesis for which I received financial support from the Reserve Bank of Australia (RBA). I am also very grateful to Klaus Adam, Kosuke Aoki, Gianluca Benigno, Tommaso Monacelli, Bruce Preston, Kevin Sheedy and to two anonymous referees for all of their helpful suggestions, and to seminar participants at the London School of Economics, Workshop on Macroeconomic Dynamics, RBA and Quantitative Economics workshops. The views expressed in this paper and any errors are my own.

Appendix

Appendix A. Full Representation of Proposition 3

Proposition 3. *In the economy with fire sales and countercyclical bankruptcy costs, the divine coincidence does not hold.*

(i) *The loss function can be approximated by:*

$$\begin{aligned} \mathcal{L}_t^{FS} = & \omega_\pi \frac{\pi_t^2}{2} + \omega_x \frac{x_t^2}{2} + \omega_n \frac{(n_t - n_t^{**})^2}{2} - \omega_{xj} x_t j_t - \omega_{xn} x_t n_t - \tilde{\omega}_{xg} x_t g_t + \tilde{\omega}_{gn} g_t n_t \\ & \underbrace{+ \tilde{\omega}_g \frac{(g_t - g_t^{**})^2}{2} + \omega_\omega \frac{(\hat{\omega}_t^g - \hat{\omega}_t^{**})^2}{2} + \omega_j \frac{(j_t - j_t^{**})^2}{2}}_{\text{New targets with fire sales}} \\ & + \underbrace{\omega_{\omega g} g_t \hat{\omega}_t^g + \omega_{n\omega} n_t \hat{\omega}_t^g - \omega_{x\omega} x_t \hat{\omega}_t^g}_{\text{New volatility terms with fires-sales}} \end{aligned} \quad (\text{A.1})$$

where the new targets for net worth (n_t^{**}), the capital stock (j_t^{**}), the default threshold ($\hat{\omega}_t^{**}$) and investment (g_t^{**}) are functions of natural equilibrium values (and the shocks). The optimal target values are given by:

$$\begin{aligned} n_t^{**} &= \frac{\sigma y_t^n - s\sigma \delta i_t^n - v\psi_t}{\sigma s^e} - \frac{\phi(\bar{\omega}, \bar{\sigma}) \nu \varphi_l}{g_{\bar{\omega}_t}(\bar{\omega}, \bar{\nu}, \bar{\sigma})} \left(i_t^n + l_{\bar{\omega}_t}(\bar{\omega}, \bar{\sigma}) \hat{\omega}_t^n + l_{\sigma_t}(\bar{\omega}, \bar{\sigma}) \hat{\sigma}_t \right) - \hat{n} w_t^n \\ j_t^{**} &= \frac{(1-\alpha)(1-\delta)s}{(1+\alpha\eta)\alpha} \left(\hat{n} w_t^n - \left(\frac{f_{\bar{\omega}_t}(\bar{\omega}, \bar{\sigma})}{f(\bar{\omega}, \bar{\sigma})} - \frac{\phi(\bar{\omega}, \bar{\sigma}) \nu \varphi_l}{f_{\bar{\omega}_t}(\bar{\omega}, \bar{\sigma})} \phi(\bar{\omega}, \bar{\sigma}) \bar{\omega} \right) \bar{\omega} \hat{\omega}_t^n \right. \\ & \quad \left. - \left(1 - \frac{\phi(\bar{\omega}, \bar{\sigma}) \nu \varphi_l}{f_{\bar{\omega}_t}(\bar{\omega}, \bar{\sigma})} \right) i_t^n - \left(\frac{f_{\sigma_t}(\bar{\omega}, \bar{\sigma})}{f(\bar{\omega}, \bar{\sigma})} - \frac{\phi(\bar{\omega}, \bar{\sigma}) \nu \varphi_l l_{\sigma_t}(\bar{\omega}, \bar{\sigma})}{f_{\bar{\omega}_t}(\bar{\omega}, \bar{\sigma})} \right) \bar{\sigma} \hat{\sigma}_t \right) \\ \hat{\omega}_t^{**} &= \frac{(s^e)^2 \sigma \tilde{\varphi}_l n_t^{**} - \varphi_l s_l \left(i_t^n + l_{\bar{\omega}_t}(\bar{\omega}, \bar{\sigma}) \hat{\omega}_t^n + l_{\sigma_t}(\bar{\omega}, \bar{\sigma}) \hat{\sigma}_t \right)}{l_{\bar{\omega}_t}(\bar{\omega}, \bar{\sigma}) (\varphi_l s_l + (s^e)^2 \sigma \tilde{\varphi}_l^2)} \\ g_t^{**} &= \frac{(s^e)^2 \sigma \tilde{\varphi}_l n_t^{**} - \varphi_l s_l \left(i_t^n + l_{\bar{\omega}_t}(\bar{\omega}, \bar{\sigma}) \hat{\omega}_t^n + l_{\sigma_t}(\bar{\omega}, \bar{\sigma}) \hat{\sigma}_t \right)}{s^2 \sigma \delta^2 + \sigma s^e s \delta \tilde{\varphi}_l + \varphi_l s_l + (s^e)^2 \sigma \tilde{\varphi}_l^2} \end{aligned}$$

with loss coefficients:

$$\begin{aligned} \tilde{\omega}_g &= s^2 \sigma \delta^2 + \sigma s^e s \delta \tilde{\varphi}_l + \varphi_l s_l + (s^e)^2 \sigma \tilde{\varphi}_l^2 \\ \omega_\omega &= (\varphi_l s_l + (s^e)^2 \sigma \tilde{\varphi}_l^2) l_{\bar{\omega}_t}(\bar{\omega}, \bar{\sigma})^2 \\ \omega_{\omega g} &= (\varphi_l s_l + (s^e)^2 \sigma \tilde{\varphi}_l^2) l_{\bar{\omega}_t}(\bar{\omega}, \bar{\sigma}) \\ \tilde{\omega}_{xg} &= \sigma s \delta + \sigma s^e \tilde{\varphi}_l \end{aligned}$$

and:

$$\tilde{\omega}_{gn} = (s^e)^2 \sigma \tilde{\varphi}_l + \sigma s^e s \delta$$

$$\omega_{n\omega} = (s^e)^2 \sigma \tilde{\varphi}_l l_{\bar{\omega}_t}(\bar{\omega}, \bar{\sigma})$$

$$\omega_{x\omega} = \sigma s^e \tilde{\varphi}_l l_{\bar{\omega}_t}(\bar{\omega}, \bar{\sigma})$$

where:

$$\begin{aligned} \tilde{\varphi}_l &= g_{\bar{\omega}_t}(\bar{\omega}, \bar{\nu}, \bar{\sigma})^{-1} \phi(\bar{\omega}, \bar{\sigma}) \nu \varphi_l \\ l_{\bar{\omega}_t}(\bar{\omega}, \bar{\sigma}) &= \left(\int_0^{\bar{\omega}} \omega_t \phi(\omega_t; \bar{\sigma}) d\omega_t \right)^{-1} \phi(\bar{\omega}, \bar{\sigma}) \bar{\omega}^2 \\ l_{\sigma_t}(\bar{\omega}, \bar{\sigma}) &= \left(\int_0^{\bar{\omega}} \omega_t \phi(\omega_t; \bar{\sigma}) d\omega_t \right)^{-1} \int_0^{\bar{\omega}} \omega_t \phi_{\sigma_t}(\omega_t; \bar{\sigma}) d\omega_t \bar{\sigma} \end{aligned}$$

and $\bar{\sigma}$ is the steady state variance of investment returns.

(ii) In response to shocks to demand, productivity, net worth or the distribution of investment returns, the policy $\{x_t = 0, g_t = 0, j_{t+1} = 0, n_t = 0, \hat{\omega}_t^g = 0, \pi_t = 0\}_{t \geq t_0}$ and $\{i_t^b = r_t^{bn}\}_{t \geq t_0}$ is not an optimal policy.