CONTACT FORCE GENERATED BY IMPACT OF BOULDER ON CONCRETE SURFACE

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Abstract

Severe damage to dwellings and other types of structures can be caused by the impact of a moving, or falling, object in the extreme events of rockfalls, landslides, severe windstorms, and hailstorms. Such damage which is usually localised in nature is controlled by the magnitude of the impact force occurring at the point of contact and is referred herein as the contact force. The magnitude of the contact force can be many times higher than the design impact force (which is essentially the equivalent quasi-static force derived from momentum-energy principles) as per stipulations by codes of practices. Currently, the compressive stiffness properties of the colliding materials controlling the conditions of contact has not been factored into code models.

Non-linear visco-elastic behaviour of the contact spring forming part of the two-degree-of-freedom (2DOF) analytical lumped-mass model has been proposed in previous investigations for predicting the magnitude of the contact force that is generated by the impact of hail or windborne debris based on idealising objects into spheres. To ensure realistic modelling outcomes, gas-gun experimentations involving machining, or moulding, impactor specimens into spheres would be required in order that contact stiffness parameters can be obtained by calibrating against results from high-velocity impact. This article presents the development, and experimental validation, of an extension of this deterministic modelling methodology for predicting the magnitude of the contact force generated by much larger impactors such as boulders. Impact testing involving real large boulders would be very costly. An important innovation presented in this article is compression testing (by a common test rig) of cylindrical specimens cored from representative boulder, and concrete, samples to determine their compressive stiffness properties. This eliminates the need of any impact experimentations and yet achieves the desired modelling outcomes of accurately

46 predicting contact force generated by the impact of an idealised boulder on a concrete
47 surface. Details of the extensive experimental validation of the proposed deterministic
48 model are reported. Size effects of the impactor on contact force behaviour have also been
49 investigated. A design chart is introduced for predicting the peak amplitude of the contact
50 force for given boulder size and velocity of impact.

51 **Keywords:** contact force; compression test; impact; rock; concrete

1. Introduction

Natural disasters, such as rockfalls, landslides, hurricanes, and hailstorms can cause significant damage to buildings, or other types of infrastructure, and can have significant consequences to life safety and the economy. For example, buildings in the proximity of hill slopes and roadways (and train tracks) navigating through mountainous terrains can be vulnerable to rockfall hazards [1]. It is of a challenge to engineers in the design of protective structures such as rockfall barriers amid uncertainties over the magnitude of the impact forces that can be generated in projected boulder impact scenarios.

A design impact force which is derived, typically, by employing momentum and energy principles as per stipulations by many codes of practices is essentially the equivalent static force (Eurocode 1, Australian standard AS 5100.2, Japanese code by Japan Road Association) [1-4]. Impact actions causing global movement (bending, sliding and overturning) can be represented by the analysis of an equivalent static force [5-7] whereas the local actions of the impact causing denting, local crushing, punching, and perforation would need to be analysed using a different approach – an approach which involves consideration of the amount of force which is generated at the point of contact (referred herein as “contact force”) [8, 9]. Contact force lasts only for a very short duration, but the magnitude of the force can be many times higher than that of the equivalent static force [10, 11]. Unlike an equivalent static force, a contact force cannot be derived purely from momentum-energy principles because of the sensitivity of the magnitude of the contact force to the compressive stiffness properties of the impactor and that of the surface of the target which is subjected to the strike. The main difference between the two types of analyses is the manner in which inertia forces generated by the motion of the target (which can influence the mode of failure) is taken into account in the analysis. For example, the stem

77 wall of an impact-resistant barrier risks being punched through should the magnitude of the
78 contact force so generated by the impact exceeds the punching resistance of the concrete.
79 However, punching failure can be deemed unlikely according to the results of analysis of an
80 equivalent static force which simulates the impulsive action of impact. This indicates that
81 the stem wall is dominated by flexure in the quasi-static representation.

82
83 Finite element simulations have also been employed for analysing the damaging potentials
84 of an impact. Software packages such as LS-DYNA and ABAQUS are commonly used by
85 researchers for undertaking finite element simulations [12, 13]. However, materials library
86 in the software is mostly suited to modelling mainstream construction materials (such as
87 concrete, steel, bricks, mortar and the like) and may not always provide sufficient coverage
88 for naturally occurring materials (such as rocks and hailstones). The properties of naturally
89 occurring materials are difficult to generalise with certainty unless representative data from
90 the testing of samples of materials involved in the collision is available. When undertaking
91 numerical simulations, the sensitivity of the modelling outcomes to the mesh size of the
92 finite elements would also need to be accounted for. For these reasons, it is cautioned herein
93 that results from finite element simulations on their own should not be relied upon in design
94 practices for determining the nature and extent of potential damage that is generated in a
95 projected impact scenario.

96
97 Localised damages caused by impact on aluminium panels and glass have been reported in
98 recent publications [8, 9]. As for concrete, there is a lack of theory behind analytical models
99 that are available for predicting localised damage. Commonly used analytical models are
100 empirical equations (e.g. modified Petry formula, ACE formula, BRL formula and Amman
101 and Whitney formula) which were derived from results obtained from experiments featuring

perforation of concrete slabs by a high-speed missile impact or by an explosion [14, 15]. Ref. [14] recommends that the modified NDRC formula can be extended for analysing a low-velocity impact of a large moving (or falling) object. Dynamic tests revealing severe localised damage to reinforced concrete resulted from the impact of a fallen object have also been reported in the literature [16, 17].

In the dynamic experimentation of a rockfall barrier, many boulder samples of (nearly) identical material and geometrical properties would be required for repetitive testing given that the boulder needs to be replaced once it is damaged in an impact. An impact experiment employing solid steel (with a hemispherical contact surface) as the impactor has the advantage of repeatability and reproducibility. The same impactor can be used repeatedly across a large number of tests without compromising its capacity to deliver the desired level of impact. Reproducing the tests by an independent research group for verifying the reported findings is also possible. In impact tests conducted by the authors using torpedo shaped impactor objects made of solid steel, the extent of localised damage inflicted by the impact on a reinforced concrete surface was studied. It was found that slight damage to the concrete surface as shown in Fig.1 can result from an impact scenario in which the impactor object weighing 280 kg was accelerated to an impact velocity of 5.1 m/s. Further details of the experimental setup are presented in Appendix A.

Figure 1. Solid steel impactor weighing 280 kg striking a reinforced concrete surface

123

124 In situations where the barrier is designed to resist impact by fallen objects other than solid
125 steel (e.g. fallen rocks/boulders) it is required to:

126 (1) develop, and verify, a methodology for predicting the intensity of the contact force for
127 the projected impact scenarios (where the impactor is boulder rather than solid steel).

128 (2) correlate the intensity of impact with the damage state of the concrete.

129

130 This paper is concerned with developing a deterministic methodology featuring simple and
131 inexpensive experimentation coupled with analytical modelling to fulfil the first requirement
132 which is to quantify the intensity of the contact force that can be generated in projected
133 scenarios of boulder impact. Input parameters into the analytical model are: the mass of the
134 impactor, its shape and dimensions, the velocity of impact and relevant mechanical
135 properties (compressive stiffness) of the impactor and target object. Given that the
136 mechanical properties of concrete are well documented, the remaining challenge is to
137 identify the relevant (compressive stiffness) properties of the boulder material based on
138 testing of cored rock samples. In deriving the deterministic relationships, the boulder is
139 idealised as a spherical object. Thus, the effects of random shape irregularities are not within
140 the scope of the study reported herein.

141

142 Meeting the second requirement involves developing correlations between the residual
143 deformation of the target (the concrete) and the contact force along with the area of contact.

144 The experiment depicted in Fig. 1 (showing slight damage from a 280 kg projectile with
145 diameter 300 mm hitting a concrete target. This impact resulted in a contact force of 800 kN
146 and a contact area of 1800 mm²) is a contribution to the development.

147

In summary, the main thrust of the study described in this article was in predicting the forcing function generated by the impact of an idealised boulder. The mechanics of concrete in the damage state deserves separate treatment to fulfil the second requirement and is outside the scope of the article.

2. Contact force predictive methodologies

2.1 Experimental approach

The contact force between two colliding objects can be obtained in a direct manner by conducting impact experiments. However, impact test apparatus such as gas-guns and drop hammers are usually not available in a mechanical testing facility. Where an impact test equipment is available, the test results would only represent specific scenarios that have been subject to the impact testing [11, 16-18]. Static testing by a load rig for evaluating the stiffness parameters of a material is expected to be more readily adopted in common practice than machining, or moulding, specimens into a sphere for impact testing by use of either a gas-gun or other types of impact test devices. This is because, applying a quasi-static load onto cylindrical specimens of concrete and rocks is commonly done in a material testing laboratory to find mechanical properties such as tensile strength, compressive strength as per procedures stipulated in regulatory documents (eg. ASTM C873, ASTM D7012) [19, 20].

2.2 Contact force model of Hunt and Crossley

The non-linear elastic contact force model (i.e. the well-known “Hertz” model [21]) would not take into consideration energy dissipation in the course of the impact, thereby resulting in a significant overestimation of the magnitude of the contact force. A two-degree-of-freedom (2DOF) lumped-mass system which incorporates a non-linear visco-elastic model as proposed by Hunt and Crossley and defined by Eq. (1a) can be employed in numerical

simulations of contact forces [22]. The damping coefficient D_n in Eq. (1a) can be expressed as a function of the coefficient of restitution as shown by Eq. (1b) [23]:

$$F_c = k_n \delta^p + D_n \delta^p \dot{\delta} \quad (1a)$$

$$D_n = (0.2p + 1.3) \left(\frac{1-COR}{COR} \right) \frac{k_n}{\dot{\delta}_0} \quad (1b)$$

where F_c is the contact force (in units of N), δ is the indentation (in units of m), p is the exponent, k_n is the contact stiffness (in units of N/m^{*p*}), $\dot{\delta}$ is the indentation velocity (in units of m/s), COR is the coefficient of restitution and $\dot{\delta}_0$ is the initial indentation velocity (in units of m/s).

Eq. (1a) of the Hunt and Crossley model has two terms: the first term $k_n \delta^p$ is in the form of the ‘‘Hertz’’ equation and the second term is the damping term $D_n \delta^p \dot{\delta}$ which takes into account energy dissipation. Ignoring the second term can result in a gross overestimation of the amount of deformation caused by the impact. The application of the Hunt and Crossley model for predicting the magnitude of contact forces generated by the impact of hail and windborne debris, and the experimental validation of the methodology have been well documented in the literature [10, 11, 18]. This article is aimed at presenting an extension of the methodology for predicting the magnitude of the contact force generated by the impact of a much larger impactor such as a boulder, on a concrete surface, and the experimental validation of the methodology. It is proposed herein to require only quasi-static testing of cylindrical (cored) specimen of the boulder (along with moulded specimens of concrete) on a standard test machine for determining dynamic stiffness. This method is much preferred to machining a boulder specimen into a sphere for impact testing as it is very costly.

2.3 Compression testing of cylindrical specimens

In the proposed test setup, two cylindrical (boulder and concrete) specimens of similar in diameter are to be placed orthogonally on the test rig for compression testing as shown in Fig. 2a based on a technique introduced in Ref. [24]. This testing configuration which is designed to simulate the condition of point contact between a sphere and the planar surface of a half-space (Fig. 2b) waives away the need to prepare spherical specimens for testing [24].

Figure 2. Schematic diagram of cylinders crossed at right angle to simulate the conditions of contact between a sphere and a planar surface [24]

Fig. 3 shows a photograph of the typical setup for the compression test as described. The boulder specimen that was placed in the middle was 83 mm in diameter whereas the pair of half-cylindrical concrete specimens (at top and bottom) were of 32 MPa in strength and were both 100 mm in diameter. The analytical model presented in Ref. [24] contains provisions to deal with situations where the cylinders in contact are of different diameters. A modification factor of 0.97 which was calculated from Ref. [24] was applied to make a minor adjustment to the predicted value of α to allow for the difference in diameter between the specimens. Compression tests were conducted using a high-speed testing machine (Instron) at different rates of loading (varying in between 2.5 m/s and 8 m/s) for determining the value of the contact stiffness parameters between the two colliding objects, that is, boulder and concrete. Fig. 4 shows the force-displacement relationships corresponding to

217 five different rates of load increase in displacement-controlled conditions of load
218 application.

Figure 3. Compression test setup

Figure 4. Test results for transverse compression between 83 mm dia. granite rock core and
100 mm dia. concrete cylinders

3. Boundary effects and boundary conversion factor

3.1 The need of a boundary conversion factor

It is noted that the boundary conditions imposed by the loading platens in the test rig on the boulder-concrete specimens are different to that of the real scenario of impact between the two materials (i.e. there is no top load in actual impact condition). Thus, a boundary conversion factor would need to be applied to allow for this difference. The accuracy of this recommended factor has been confirmed by finite element analyses of the test setup using the commercial program LS-DYNA. Details of the finite element model are described next.

3.2 Finite element simulations of compression tests

Key input parameters specified in the finite element model for rock specimen, concrete specimens, and steel platens are listed in Table 1.

Table 1. Key input parameters of the finite element model

Parameters	Input		
Part name	Rock	Concrete	Steel platens
Element	Constant stress 8-node hexahedron solid elements		
Material model	MAT_ELASTIC	MAT_WINFRITH_ CONCRETE	MAT_RIGID
Density (kg/m ³)	2700	2400	7850
Young's modulus (GPa)	80	30	200
Poisson's ratio	0.24	0.2	0.3
Uniaxial compressive strength (MPa)	N/A	32	N/A
Uniaxial tensile strength (MPa)	N/A	2	N/A

Fracture energy ^[25] (Nm/m ²)	N/A	75	N/A
Aggregate size (radius in mm)	N/A	5	N/A
Hourglass control type ^[26]		4	
Hourglass coefficient ^[26]		0.03	

238 N/A: Not Applicable

239 The AUTOMATIC_SURFACE_TO_SURFACE contact algorithm was specified to model
240 the contact between the cylindrical specimens of rock and concrete, and between the
241 concrete specimens and the steel platens. Cylindrical rock specimens were modelled with
242 4743 elements (5454 nodes) whereas each half cylindrical concrete specimens and steel
243 platens were modelled with 2035 (2544 nodes) and 8000 (8946 nodes) elements
244 respectively.

245

246 To simulate impact actions in the test rig, the upper loading platen was prescribed with a
247 velocity of motion (V_0) to impose a high rate of loading on the cylindrical specimens (refer
248 to Model 1 in Fig. 5). The simulated test results based on Model 1 were then validated by
249 comparing with data from physical experimentation. An example of the comparison based
250 on a velocity of 2.5 m/s is shown in Fig. 6. The analysis was repeated to simulate an impact
251 scenario. In this second simulation, the upper part of the test setup in the test rig was first
252 removed. The rock specimen was then prescribed with a velocity to simulate the dynamic
253 actions of a vertical impact immediately following initial contact between the boulder and
254 the concrete (refer to Model 2 in Fig. 5). With Model 2, no load is imposed from above. The
255 cylindrical rock specimen is instead modelled to move with an initial impact velocity V_0 to
256 simulate the impact action.

Figure 5. Finite element simulation models

Figure 6. Experimentally recorded and finite element simulated results at 2.5 m/s

It is revealed in the comparison of the simulated results between Models 1 and 2 (based on a velocity of 2.5 m/s) that the change in the boundary condition is translated into a 1.8 times increase in slope of the force-displacement relationship as illustrated in Fig. 6. The boundary conversion factor has been found to be consistently close to 1.8 showing insensitivity to changes in the rate of loading. The force-displacement relationships that have been obtained from physical experimentation employing the test rig (Fig. 4) were adjusted accordingly by

a factor of 1.8 to simulate the force-displacement relationships in impact conditions. The adjusted curves were employed to obtain stiffness parameters through a curve-fitting process as presented in section 4.

4. Loading rate effects and dynamic factor

4.1 Modelling loading rate effects

The Hunt and Crossley model as defined by Eq. (1) is used herein to characterise the hysteretic behaviour of the non-linear visco-elastic contact spring forming (the upper) part of the two-degree-of-freedom (2DOF) lumped-mass model (Fig. 7).

Figure 7. Two-degree of freedom (2DOF) lumped-mass system

The input parameters: m_1 , m_2 , k_2 , $x_1(t)$ and $x_2(t)$ as shown in Fig. 7 denote the mass of the impactor, the generalised mass of the target, the stiffness of the target (or the rear spring), the displacement of the impactor, and the displacement of the target at time t respectively. Methods to determine the values of m_2 and k_2 are provided in detail in Refs. [27, 28]. The 2DOF model as described can be executed on an Excel spreadsheet or on MATLAB to simulate the contact forcing function. Details of the program algorithm and demonstrations of its use can be found in Ref. [29].

Values of parameters characterising the compressive stiffness properties of the frontal spring (k_n and p) can be determined by curve-fitting the simulated force-displacement

relationships. A few examples of the curve-fits are shown in Fig. 8. The other parameter (D_n) in the Hunt and Crossley model characterising the hysteretic behaviour of the frontal spring can be related to the coefficient of restitution (COR) [23]. The value of COR can in turn be linked to the area (A) enclosed by the force-displacement curves on loading and unloading as shown in Fig. 9.

Figure 8. Calibration of 2DOF model against measurements from physical testings with boundary conversion (or from FE simulations) of force-displacement curves at different loading rates

Figure. 9. Shaded area equals energy dissipated during impact at 2.5 m/s

Energy dissipated by viscous damping = Area of the force-displacement curve

$$\oint D_n \delta^p \dot{\delta} d\delta = A \quad (2a)$$

$$\frac{(1 - \text{COR}^2) k_n \delta_m^{p+1}}{(p + 1) \text{COR}} = A \quad (2b)$$

$$\text{COR} = \frac{-A(p + 1) + \sqrt{[A(p + 1)]^2 + 4(k_n \delta_m^{p+1})^2}}{2k_n \delta_m^{p+1}} \quad (2c)$$

Eq. (2a) for defining A (the value of which can be found in MATLAB using the “trapz” command in units of Nm) was first identified as per the Hunt and Crossley model. Eq. (2b) was then derived in Ref. [23] as a solution to the integral shown in Eq. (2a). Algebraic manipulation of Eq. (2b) resulted in Eq. (2c) which can be used for calculating the value of COR (hence D_n) for a given value of A and maximum shortening of the frontal spring (δ_m). All the parameters in Eqs. (2a-2c) follow the same SI units mentioned in section 2.2. For example, when $A = 0.4$ Nm and $\delta_m = 0.0003$ m at an impact velocity of 2.5 m/s (Fig. 9), results of the calibration are $k_n = 370$ MN/m^{1.355}, $p = 1.355$ and $\text{COR} = 0.78$. The calibration can be executed in EXCEL, for example, using the *Solver* command.

Physical experimentations conducted in the study were restricted to loading rates of up to 8 m/s due to limitations of the test rig. Thus, the database of test results was augmented by computer simulations obtained using LS-DYNA to provide coverage for higher velocity values of 14 - 20 m/s (Figs. 8 and 10). It is noted that the finite element model employed in the simulations had been validated by comparison against test results based on lower velocity conditions (Fig. 6). The calibrated values of the Hunt and Crossley model parameters (characterising the frontal spring in the 2DOF model) are used for plotting Figs. 10a - 10c to show correlations with impact velocity (V_0). Expression for determining the value of k_n , p and COR as derived from curve fitting are listed in Eqs. (3a) – (3c), where V_0 is in units of m/s. The size effects of the colliding bodies have not been incorporated in the relationships derived for dynamic parameters in this section. Size effects on the dynamic parameters will be illustrated in section 5.

● Physical test ■ Finite element simulation — Best-fit curve

Figure 10. Correlation of calibrated Hunt and Crossley model parameter values with compression test velocity values for rock-concrete impact: (a) k_n ; (b) p ; (c) COR

$$k_n = 84.273V_0 + 160.86 \quad (\text{MN/m}^p) \quad (3a)$$

$$p = 0.0292V_0 + 1.3191 \quad (3b)$$

$$\text{COR} = 1.3355e^{-0.23V_0} \quad (3c)$$

4.2 The dynamic factor

The above conducted compression testings of an array of cylinders are at a loading rate exceeding 2 m/s. However, commonly available test rigs which are in use in material laboratories do not possess the capability of attaining such a high rate of loading. This section extends the proposed procedure to accommodate the use of a test rig which can only apply a low loading rate of 50 mm/min. Although certain types of static test rigs can go higher than 50 mm/min in their loading rate, this study was carried out at 50 mm/min in view of the very limited maximum crosshead speed of typical compression testing machines which are commonly used in practice.

Following an adjustment to the curve obtained from static test (50 mm/min) by the boundary conversion factor of 1.8 (as derived in section 3) and comparison with other adjusted experimental curves (or finite element simulated curves at 14 m/s and 20 m/s), a further increase in stiffness from static to dynamic loading rates was observed. This factor of increase is denoted as the *dynamic factor* (DF) in the rest of the paper. An example is given below to show the derivation of the dynamic factor for an impact velocity of 8 m/s (Fig. 11).

Figure 11. Illustration of the effects of boundary condition and loading rate

As illustrated in Fig. 11, a dynamic factor of 1.75 needs to be applied to modify the curve (which was recorded from the test at a low loading rate of 50 mm/min) to make allowance for a much higher loading rate corresponding to an impact velocity of 8 m/s. The dynamic factors for other considered impact velocities have also been derived using the same approach and given by Fig. 12.

Figure 12. Correlation of dynamic factor with compression test velocity

4.3 Experimental validation of model

Impact experiments have been conducted to validate the accuracies of predictions generated by the proposed modelling procedure. The impact experiments were carried out by releasing a 100-mm diameter impactor which was made of granite from drop heights ranging between 0.25 m and 2 m (test setup is shown in Fig. 13). A polyvinyl chloride (PVC) tube, which was drilled with holes to reduce the air drag, was used to guide the impactor onto a miniature (Grade 30) concrete slab specimen of dimensions: 300 mm × 300 mm × 40 mm. Given that the interest is on localised actions, the quality of the results derived from these tests would not be compromised by the limited size of the slab specimen.

An accelerometer (measurement range: ± 2500 g and sensitivity: 1.04 mV/g) was attached rigidly to the impactor for recording the acceleration time-histories which when multiplied by the mass of the impactor give the contact forcing functions. Multiple tests were conducted to ensure the retest reliability of the experimental data.

Figure 13. Experimental setup

In Fig. 14, the list of COR values derived from the Hunt and Crossley model (using Eqs. (2a)-(2c)) is shown alongside COR values that were obtained directly from the drop test based on observing the change in velocity of the impactor on re-bounce (the velocity of motion was inferred from the recorded acceleration time-histories). The comparison demonstrating consistencies between the two sets of results serves the purpose of verifying the Hunt and Crossley model.

In parallel with the physical experimentation, numerical simulations of the drop tests have also been undertaken using the finite element program LS-DYNA employing the input parameters stated in section 3.2 (noting that both the impactor and the target were of the same type of materials as described in section 3). Target concrete slab specimens and spherical granite impactor specimens were modelled with a total number of 28800 (33489 nodes) and 23625 (24376 nodes) solid elements respectively. Comparisons between the experimentally recorded, and simulated, results reveal good agreement across the whole range of impact velocities that was covered by the investigation (Fig. 15).

Figure 14. Comparison of **COR** values from drop tests and from Hunt and Crossley model for 100 mm dia. granite sphere impacting concrete

The non-linear visco-elastic contact spring which is characterised by parameters defined by Eqs. (3a) – (3c) and Fig. 10 (forming part of the 2DOF spring connected lumped-mass model) was used for determining the magnitude of the contact force for comparison with experimental measurements (in conjunction with the augmented database of results from FE simulations to cover for the higher velocity impact scenarios that could not be reproduced physically in the authors' laboratory because of headroom limitations). Close agreement between results from physical testings and predictions from the use of the 2DOF model adopting parameter values defined by Eqs. (3a) – (3c) (and with finite element simulations) is well demonstrated in Fig. 15. Input values used in the 2DOF model for m_1 was 1.4 kg, m_2 was 2.2 kg (equals to 0.25 times of the total mass of the slab) and the stiffness of the target as measured using 50 kN MTS machine was 2760 kN/m [28].

○ Physical test - - - Finite element simulation — 2DOF model

Figure 15. Comparison of contact forcing functions for 100 mm dia. granite spheres impacting concrete from physical testings, FE analyses and use of 2DOF model at impact velocities of:

(a) 2.2 m/s; (b) 3.1 m/s; (c) 3.8 m/s; (d) 4.4 m/s; (e) 5.4 m/s; (f) 6.3 m/s

5. Size effects of impactor object

This section illustrates the size effects of colliding materials on the 2DOF model parameters (k_n , p and COR). Size of the specimens and testing velocity which can be employed in the compression testing machines in the authors' laboratory have limitations. As a consequence, the size effects on dynamic parameters have been studied from impact experiments along with FE simulations in this section. However, the methodology provided in this paper can be used for different size of samples to study the size effects on the dynamic parameters from compression testing [30].

The main focus was on contact conditions at the rock-concrete interface. Rock samples tested in this study were derived from (unweathered) granite which is common in Hong Kong where the core was obtained. Thus, unweathered granite and concrete have been used as the colliding materials in the investigation. Test results are expected to vary with rock types and the degree of weathering [1].

Impact experiments using custom made gas-gun equipment

Impact experiments were carried out using a custom-made gas-gun apparatus which was fitted with a contact force measurement device (Fig. 16a). Granite spheres of 50 mm in diameter were used as the impactor and 90 mm × 90 mm × 60 mm concrete specimens attached to steel lumped-mass were used as target (both the impactor and the target were of

the same type of materials as described in section 3). The steel lumped-mass was 90 mm in diameter and 40 mm long weighing 2.16 kg. The stiffness of rear spring which connects the steel lumped-mass to a rigid support at the back was 51.7 kN/m.

Displacement of the target lumped-mass was assumed to be equal to the shortening of the rear spring (x_2). An accelerometer (measurement range: $\pm 20,000$ g and sensitivity: 0.24 mV/g) was attached to the steel lumped-mass and accelerations along the direction of impact were measured. A high-speed camera operating at a rate of 20,000 frames per second was used to measure the velocity of movement of both the spherical flying impactor object (the granite specimen) and that of the spring connected lumped-mass target (on which a concrete specimen was attached). The input values for m_2 and k_2 were 3.362 kg (lumped mass of the steel and the concrete specimen) and 51.7 kN/m respectively. The contact force measurement device is shown by the photo of Fig 16a along with the schematic diagram of Fig. 16b which explains the operational principles of the device. Impact experiments that have been conducted had a velocity of impact ranging from 9.5 m/s to 26.3 m/s.

Fig. 16. (a) Camera capture of an impactor striking the contact force measurement device
(b) schematic diagram illustrating the operational principle of the device

Values of the Hunt and Crossley model parameters (k_n and p) were calibrated (along with values of COR measured from the high-speed camera captures) to match with the recorded contact force time-histories as shown by Fig. 17. Calibrated values of the parameters have been listed in table 2.

Figure 17. Comparison of measured and 2DOF simulated contact force for 50 mm dia. granite sphere impacting concrete specimen at impact velocities: (a) 9.52 m/s; (b) 11.72 m/s; (c) 14.2 m/s; (d) 17.2 m/s; (e) 20.8 m/s; (f) 23.8 m/s; (g) 26.3m/s

Table 2. Calibrated Hunt and Crossley model parameters (k_n and p along with measured COR values) for 50 mm diameter granite spheres

Impact velocity V_0 (m/s)	Contact stiffness k_n (MN/m ^{p})	Exponent p	Coefficient of restitution COR
9.52	700	1.5	0.567
11.72	800	1.54	0.48
14.22	1100	1.6	0.386
17.2	1150	1.62	0.315
20.81	1200	1.67	0.29
23.8	1500	1.7	0.25
26.3	1600	1.728	0.22

Finite element simulations

The experiments discussed above were conducted using small granite and concrete specimens. Coefficient of restitution values (COR) reported in [31], employing 1 m-diameter (1300 kg) granite spheres are for impact between two granite spheres at speeds up to 1.5 m/s which distinct from the scope of this paper (which is focused on the impact between rock and concrete). Although there are many publications on rockfall impact, rockfall protection measures and behaviour of concrete structures under impact loading [32, 33], the data available on rockfall impact were mostly using steel and concrete blocks as impactors as opposed to actual boulder. Thus, experimental results obtained from the large-scale impact tests on a concrete wall measuring 3 m long, 1.5 m high and 0.23 m thick (a photo of which is shown in Fig. A1 of Appendix A) have also been made use of to develop a finite element

model of the stem wall. Hence, the size effect on the contact force and on the Hunt and Crossley model parameters can be investigated. In the finite element model, spheres of diameter 50 mm (875 elements and 976 nodes), 100 mm (7000 elements and 7351 nodes), 500 mm (23625 elements and 24376 nodes) and 1000 mm (56000 elements and 57301 nodes) with rock material properties as specified in Section 3.2 was used to simulate the impactors and grade 30 concrete was assumed for the wall (135000 elements and 149188 nodes). The reinforcement bars were modelled as Hughes-Liu beam elements (4650 elements and 9346 nodes) with the yield strength of 540 MPa, tangent modulus of 1.13 GPa, and **are** constrained with concrete using LAGRANGE_IN_SOLID. AUTOMATIC_SURFACE_TO_SURFACE contact algorithm was employed for modelling the contact between the impactor and the concrete surface.

The finite element simulated results were used accordingly for calibrating values of the Hunt and Crossley parameters (Figs. 18 – 21) to augment the database of values obtained earlier from calibration against experimental measurements. Values of the COR have also been inferred from velocity values simulated from the finite element model.

Figure 18. Comparison of FE and 2DOF model simulated contact force-time history (using 50 mm-diameter impactor with 0.18 kg mass) at impact velocities of: (a) 14 m/s; (b) 24 m/s

Figure 19. Comparison of FE and 2DOF model simulated contact force-time history (using 100 mm-diameter impactor with 1.4 kg mass) at impact velocities of: (a) 5.4 m/s; (b) 15 m/s

Figure 20. Comparison of FE and 2DOF model simulated contact force-time history (using 500 mm-diameter impactor with 173.5 kg mass) at impact velocities of: (a) 1 m/s; (b) 5 m/s

--- Finite element simulation — 2DOF model

Figure 21. Comparison of FE and 2DOF model simulated contact force-time history (using 1000 mm-diameter impactor with 1387.5 kg mass) at impact velocities of: (a) 1 m/s; (b) 2.5 m/s

The combined database of calibrated values of k_n and p (and measured/simulated values of COR) was then used for plotting graphs to be presented in the later part of this section. The

510 scaling relationship of the parameter k_n for size effects of a spherical impactor object was
 511 studied by the introduction of the normalised stiffness N_{k_n} which is defined by Eq. (4):

$$N_{k_n} = \frac{k_{nD}/k_{n100}}{\sqrt{D/100}} \quad (4)$$

512 where k_{nD} (in units of N/m^p) is the stiffness of an object of diameter D (in units of mm),
 513 and k_{n100} (in units of N/m^p) is the stiffness of an object of 100 mm in diameter. Results
 514 from calibration involving the impact testing of impactor objects and FE simulations with a
 515 range of sizes are presented in Fig. 22. The value of N_{k_n} is shown to be insensitive to
 516 changes in the velocity of impact and size of the impactor and hence may be taken to be
 517 equal to unity. Thus, k_n can be expressed as a function of the impactor size and material
 518 type. Eq. (5) is accordingly the expression for estimating the value of k_n for any spherical
 519 impactor object when the value of k_n of a 100 mm-diameter object made of the same
 520 material is known. Eq. (5) can be re-written into the generalised format of Eq. (6). The
 521 values of k_n for different size of impactors can be obtained using the Eq. (6) once the value
 522 of k_n is found (from the proposed compression testing methodology) for a given size of
 523 impactor.

Figure 22. Variation of normalised k_n values with impact velocity

$$k_{nD} = k_{n100} \sqrt{D/100} \quad (5)$$

$$k_{nD_2} = k_{nD_1} \sqrt{D_2/D_1} \quad (6)$$

526 The value of the exponent p was found to be sensitive to both the size of the impactor (D)
 527 and the impact velocity (V_0). Eqs. (7a) and (7b) were accordingly derived from a multiple
 528 regression analysis to estimate the value of p for given values of D (in units of mm) and V_0
 529 (in units of m/s). The goodness of fit was evaluated using the Leave-one-out Cross-
 530 Validation (LOOCV) method [34].

$$p = 1.3192 + x \quad (7a)$$

where, $x = 0.0175V_0 + 0.0005D$ (7b)

531 The developed model has a low relative prediction error (RPE) and standard error of cross
 532 validation (SECV) with a value of 0.04 and 0.06 respectively and a high value of
 533 cross-validated coefficient of determination (Q^2) with 81% showing a very good fit
 534 (Fig. 23).

Figure 23. Variation of exponent with size and impact velocity

Similarly, the value of COR was also correlated with the size of the impactor (D) and impact velocity (V_0). Eq. (8) was accordingly derived by combining the dual parameters into the single parameter: the kinetic energy of impact (KE_0 , in units of kJ).

$$COR = 0.068 \times KE_0^{-0.433} \quad (8)$$

The goodness of fit is characterised by a RPE, SECV and Q^2 value of 0.01, 0.04 and 95% respectively, and is also well demonstrated in Fig. 24.

Figure 24. Variation of COR with initial kinetic energy

6. Prediction of contact force

Correlation relationships have been developed in the previous sections for deriving expressions for determining the values of parameters: k_n , p and COR (of the Hunt and Crossley model) as a function of the size of the impactor object and the velocity of impact. Solving for the contact force for any projected unweathered granite boulder impact scenario would involve making use of the correlation relationships of Eqs. (6) – (8) to find the respective parameter values for substitution into Eqs. (1a) and (1b). The time-history of the contact force (i.e. the forcing function) can be found by employing the numerical procedure presented in Ref. [29]. Fig. 25 presents an example simulation of a 500 mm-diameter

555 (200 kg) boulder striking a concrete barrier at 5.1 m/s. The peak contact force of around
 556 1200 kN is found accordingly for the considered scenario. In a parametric study covering a
 557 range of impact scenarios wherein the boulder size ranged between 100 mm and 1000 mm in
 558 diameter, and velocity of impact varied up to 20 m/s, the procedure was repeated for
 559 determining the forcing function, and hence peak contact force for each of the considered
 560 scenarios. Results of the peak contact force are presented in the form of a design chart
 561 (Fig. 26) in which the vertical axis is the peak contact force in logarithmic scale (in units of
 562 kN) and the horizontal axis is the velocity of impact in normal scale (in units of m/s).

563
 564 Figure 25. Contact force time-history for the example scenario of a 500 mm-diameter
 565 (200 kg) boulder striking a concrete barrier at 5.1 m/s
 566

567
 568 Figure 26. Peak contact forces estimated for impact scenarios of different sizes of
 569 unweathered granite spheres on concrete surface

570

571 The use of the expressions developed in this study and the design chart of Fig. 26 can result
572 in over-stating the contact force as the results presented were derived from the testing of
573 unweathered rock.

574

575 **7. Closing remarks**

576 A new modelling methodology for predicting the magnitude of the contact force generated
577 by the impact of a boulder on a concrete surface is presented in this article. The use of
578 expressions derived from momentum-energy principles would only provide estimates of the
579 magnitude of the equivalent static force which can be much lower than the impact force
580 induced at the point of contact between the colliding bodies. Importantly, the magnitude of
581 the contact force is very sensitive to the compressive stiffness properties of the colliding
582 materials. Central to the proposed methodology is analytical modelling of the contact spring
583 forming part of the 2DOF spring-connected lumped-mass model and the adoption of the
584 non-linear visco-elastic model to define its hysteretic behaviour. The novelty of the
585 proposed modelling procedure is the use of simple compression testing of cylindrical (cored)
586 specimens of boulder and concrete samples for determining the values of parameters to
587 characterise the properties of the contact (frontal) spring forming part of the 2DOF system.
588 Accurate information of the compressive stiffness properties of the colliding materials can
589 be obtained by this experimentation methodology which only requires compression testing
590 to be conducted on a standard test rig (by virtue of applying corrections to allow for changes
591 in boundary conditions and rate of loading). The proposed procedure for simulating time-
592 history of the contact force can be operated conveniently by use of Excel spreadsheet, or by
593 MATLAB, given the simplicity of the 2DOF lumped-mass system. The proposed
594 methodology has been well validated by comparison of results from impact testing of

dropping a spherical specimen of granite onto a concrete slab from different drop heights. The loading rate effects and size effects have been incorporated using high-speed gas-gun experiments and finite element simulations. A validated numerical model operated by the finite element program LS-DYNA has also been used to augment the results from physical testings in order that the results database can be extended to a high impact velocity of 20 m/s. A design chart which can be convenient for estimating peak contact force in impact scenarios of a boulder on a concrete surface without the need to undertake numerical simulations nor dynamic experimentations has been derived.

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Appendix A – Large-scale impact experiments on RC barrier

This appendix is to provide a brief description on the large-scale impact tests stated in the main sections of this paper. Pendulum style impact tests were carried out on a reinforced concrete barrier (3 m × 1.5 m × 0.23 m) employing torpedo shaped steel impactors (with a hemispherical contact surface). Experimental setup is shown in Fig. A1.

Figure A1. Experimental setup

A 280 kg and a 1020 kg hemispherical headed steel impactors with a diameter of 300 mm and 400 mm respectively were raised to a desired height and released to strike with the wall. Fig. A2 shows the comparison of contact force time - history from experimental and finite element simulated results when 280 kg impactor hit the target wall at 250 mm from the top

713 edge along the vertical centre line of the wall at 5.1 m/s velocity as measured from
714 high-speed camera recording. Although the maximum contact force generated was around
715 800 kN, measured indentation at the vicinity of contact was low with a value of 3.5 mm
716 (Fig. 1).

717
718 Figure A2. Comparison of contact force-time history at 5.1 m/s