Spatial averaging effects on the streamwise and wall-normal velocity measurements in a wall-bounded turbulence using a cross-wire probe

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Abstract The spatial averaging effects due to a cross-wire probe on the measured turbulence statistics in a wall-bounded flow are investigated using a combined approach of Direct Numerical Simulation data, theoretical methods and experiments. In particular, the wire length (l), spacing (Δs_y) and angle (θ) of a cross-wire probe configured to measure the streamwise and wall-normal velocities are systematically varied to isolate effects of each parameter. The measured streamwise velocity from a cross-wire probe is found to be an average of the filtered velocities sensed by the two wires. Thus, in general, an increase in the sensor dimensions when normalised by viscous units leads to an attenuated variance for the streamwise velocity ($\overline{u^2}$), resulting from a larger contribution to the spatial averaging process from poorly correlated velocities. In contrast, the variance for the wall-normal velocity ($\overline{w^2}$) can be amplified, and this is shown to be the result of an additional contributing term (compared to $\overline{u^2}$) due to differences in the filtered wire-normal velocity between the two wires. This additional term leads to a spurious wall-normal velocity signal, resulting in an amplified variance recorded by the cross-wire probe. Compared to the streamwise and wall-normal velocity variances, the Reynolds shear stress ($-\overline{uw}$) perhaps surprisingly shows less variation when l , Δs_y and θ are varied. The robustness of Reynolds shear stress to the finite sensor size is due to two effects. (I): Reynolds shear stress is devoid of energetic contributions from the near-isotropic fine scales unlike the $\overline{u^2}$ and $\overline{w^2}$ statistics, hence cross-wire probe dimensions are typically sufficiently small in terms of viscous unit to adequately capture the $\overline{uw}$ statistics for a range of l and θ investigated. (II) The Δs_y dependency arises due to cross terms between the filtered velocities from two wires, however, it turns out that these terms

cancel one another in the case of Reynolds shear stress, but not for the $\overline{u^2}$ and $\overline{w^2}$ statistics. We note that this does not, however, suggest that $\overline{uw}$ is easier to measure accurately normal stresses; on the contrary, in a companion paper (Baidya *et al.* 2019 *Meas. Sci. Technol.* **30** 085301) we show that $\overline{uw}$ measurements are more prone to errors due to uncertainty in probe geometry and calibration procedure.

Keywords cross-wire, spatial resolution, wall-turbulence

1 Introduction

In an experimental investigation of turbulent flows, finite sensor dimension effects remain an important consideration when interpreting the data. This is because all physical sensors occupy a finite dimension, and therefore the velocity *measured* by the sensor does not correspond to a velocity at a point, but rather a spatially filtered velocity over the region occupied by the sensor. Therefore, the sensor acts as a filter with velocity fluctuations that occur within a region smaller than the sensor dimension being insufficiently resolved, leading to a dependence of the measured turbulence statistics on the dimension of the sensor used.

The spatial attenuation of the streamwise velocity measured using a single normal hot-wire is perhaps the most well-studied case of the finite sensor dimension effects, with considerable theoretical (Dryden *et al.*, 1937; Uberoi & Kovasznay, 1953; Wyngaard, 1968), numerical (Chin *et al.*, 2011; Suzuki & Kasagi, 1992) and experimental (Hutchins *et al.*, 2009; Johansson & Alfredsson, 1983; Ligrani & Bradshaw, 1987) investigations performed to date. The addition of a second wire in cross-wire probes (hereafter referred to as $\times$ -probes) means that the measured velocities are not only functions of the sensor dimension, but also the relative positions of the two wires, which leads to a significant increase

in the complexity of spatial resolution investigations. Quantifying the full effects of the entire parametric space constituting the sensor dimension and the relative positions of the two wires is difficult. Therefore, most investigations to date have concentrated on determining the effect of one parameter in isolation. For example, the wire separation effects have been studied in isolation both experimentally (Browne *et al.*, 1988; Strohl & Comte-Bellot, 1973) and numerically (Suzuki & Kasagi, 1992). While Tagawa *et al.* (1992) investigated the effects of the wire inclination angle numerically. Here, we present extensive numerical studies using a direct numerical simulation (DNS) database where multiple parameters related to the dimension and relative position of the sensors are systematically considered, and their effect on the measured turbulent stresses and energy spectra are investigated. Furthermore, the results from the numerical studies are compared against an analytical expression derived theoretically and against experimental results for some selective sensor configurations. In this paper, we use the coordinate system x , y and z to refer to the streamwise, spanwise and wall-normal directions, respectively; while u , v and w denote the corresponding fluctuating velocities. Capitalisation and overbars indicate time-averaged quantities, while superscript '+' is used to denote normalisation by viscous units (e.g. $U^+ = U/U_\tau$, $z^+ = zU_\tau/\nu$, where U_τ and ν denote the mean friction velocity and kinematic viscosity, respectively.)

Here, the focus of our investigation will be on an $\times$ -probe configured to measure the streamwise and wall-normal velocity (hereafter referred to as a u - w $\times$ -probe), to complement the study by Philip *et al.* (2013b) who investigated $\times$ -probes configured for measuring the streamwise and spanwise velocity (u - v $\times$ -probe). For turbulent shear flows (and especially in wall-bounded turbulence), the Reynolds shear stress ($-\overline{uw}$) is one of the most important turbulent quantities, appearing in the mean streamwise momentum equation. It should be noted that, in a wall-bounded flow, due to mean wall-normal shear, the flow is inhomogeneous along the individual hot-wires in the u - w $\times$ -probe, unlike the u - v $\times$ -probe where it is homogeneous. Here, we extend the analysis of Philip *et al.* (2013a,b), to the case where spatial averaging is along the inhomogeneous direction. The paper consists of four major sections, with the first part detailing a model for the finite sensor size. This model is then used to simulate the measured velocity using an $\times$ -probe given a known input velocity (the measured velocity is not the same as the input velocity due to spatial averaging that occurs across the finite sensor). In the second section, in order to investigate the effect of spatial averaging along the inhomogeneous direction in isolation, a simpler case of single inclined wire in a wall-bounded flow is studied. This investigation is then extended to $\times$ -probe in the third section; where an additional parameter, the spacing between the two incli-

ned wires, is considered. In the final section, comparison of these models against experimental $\times$ -probe data are provided.

2 Modelling the finite sensing elements in the $\times$ -probe

The spatial filtering of velocities across the sensing element of length l is modelled as a box filter. To obtain the filtered velocity, the velocities along the sensor are convolved with a box function with a value of $1/l$ ranging from $-l/2$ to $l/2$. This implies that the hot-wire is equally sensitive to the velocities over its entire length, which is a good approximation for a hot-wire with a large length to diameter ratio (Chin *et al.*, 2011). This approach has been previously employed by Chin *et al.* (2011) and Philip *et al.* (2013a), who have demonstrated that the predicted attenuation of the $\overline{u^2}$ profile from the box-filtered velocity closely resembles the experimental results up to a wire length of $l^+ \approx 40$. Furthermore, the effects of non-linear filtering and wire misalignment have been theoretically studied by Segalini *et al.* (2011), who find that for a realistic misalignment angle ($< 5^\circ$) the non-linear effects alone do not change the trends in the measured mean and variance, with respect to the sensor size.

For a linear filter such as the one utilised in the current study, the total velocity can be decomposed into the mean (U) and fluctuating components (u) as (Philip *et al.*, 2013a)

$$\tilde{U} = \frac{1}{l}(U * b), \quad \text{and} \quad \tilde{u} = \frac{1}{l}(u * b), \quad (1)$$

where, $*$ denotes the convolution integral and b the box filter function. Here, tilde is used to denote filtering along the wire length. Hence, the filtered mean velocity only depends on the unfiltered mean velocities along the sensor, while the filtered fluctuating velocity depends only on the unfiltered fluctuations along the sensor.

To resolve two components of velocity instantaneously, a probe containing a pair of hot-wires is required. To measure the streamwise and wall-normal velocities, the $\times$ -probe is configured such that the two inclined wires are parallel to the x - z plane, separated by a finite distance Δs_y in the spanwise direction, as illustrated in figure 1(a).

It is convenient to express the heat convected away from an inclined hot-wire using an *effective velocity*, U_{eff} . This is because, in a constant temperature anemometer, the heat convected is equal to Joule heating from the wire, which in turn has a one-to-one correspondence with the output voltage from the anemometer. Hence, use of effective velocity allows velocity-voltage mapping functions across multiple inclination angles to be described using a single E - U_{eff} relationship (Bruun, 1995). For typical measurements, the contribution from the tangential velocities is empirically observed to be minimal (Champagne *et al.*, 1967). This agrees

Fig. 1: A schematic of $\times$ -probe configure to measure the u and w velocities. (a) Isometric and (b) x - z plane projections. The red and blue wires shown in (a) are located on the respectively coloured planes, while the effective velocity for the two wires (given here by the normal component to the sensor, $u_{A\perp,w1}$ and $u_{A\perp,w2}$) corresponding to the measured velocity, u_A , is shown in (b).

with the investigations by Philip *et al.* (2013b), who compared a simulated $\times$ -probe with and without inclusion of the tangential velocity contributions finding only slight quantitative differences in the results. Therefore to simplify the present analysis, we will assume that the inclined hot-wire only senses velocity perpendicular to the sensor. For, figure 1(b), only the normal component of the velocity vector u_A , $u_{A\perp,w1}$ and $u_{A\perp,w2}$ (the subscripts w1 and w2 are used to distinguish between the wires) contributes to the heat convection for wire 1 and 2, respectively.

The final step is to obtain an expression for the measured velocity by the $\times$ -probe, in terms of effective velocity from the individual wires. In an experiment, due to uncertainties over wire inclination angles and contributions from the tangential velocities to the total heat convention, this is achieved through a calibration procedure where the $\times$ -probe is placed in a uniform flow with a known velocity at various angles α to construct a map between the flow angle and effective velocities (Bruun, 1995). However, for a simulated $\times$ -probe, where we neglect tangential velocities (see (Moin

& Spalart, 1989) for the case when the tangential velocity contributions are retained) these uncertainties are not present and an exact expression can be obtained between the measured velocities (u and w) and the wire-normal velocities ($u_{\perp,w1}$ and $u_{\perp,w2}$) from the geometry shown in figure 1(b). That is,

$$u = \frac{\tilde{u}_{\perp,w2}}{\sin(\theta_{w1} + \theta_{w2})} \cos \theta_{w1} + \frac{\tilde{u}_{\perp,w1}}{\sin(\theta_{w1} + \theta_{w2})} \cos \theta_{w2}, \quad (2a)$$

$$w = -\frac{\tilde{u}_{\perp,w2}}{\sin(\theta_{w1} + \theta_{w2})} \sin \theta_{w1} + \frac{\tilde{u}_{\perp,w1}}{\sin(\theta_{w1} + \theta_{w2})} \sin \theta_{w2}. \quad (2b)$$

Here, tilde again denotes filtered velocity from the individual wires. Since the measured fluctuating velocities can be described fully in terms of fluctuating velocities along the sensors as shown in equation (2), the variance measured using the $\times$ -probe will not be affected by the local mean velocities experienced by the two wires, at least in the context of linear filtering. Furthermore, $\times$ -probes are typically configured to be symmetrical about the x -axis (as is the case for the experimental investigation carried out in § 7), and therefore both wires have length l (cf. figure 1b), while $\theta_{w1} = \theta_{w2} = \theta$. Substituting the wire-normal velocities with the filtered u and w components along the wire (e.g. $\tilde{u}_{\perp,w1} = \tilde{u}_{w1} \sin \theta + \tilde{w}_{w1} \cos \theta$) in (2) results in

$$u = \frac{\tilde{u}_{w1} + \tilde{u}_{w2}}{2} + \frac{\tilde{w}_{w1} - \tilde{w}_{w2}}{2 \tan \theta}, \quad (3a)$$

$$w = \frac{\tilde{w}_{w1} + \tilde{w}_{w2}}{2} + \frac{(\tilde{u}_{w1} - \tilde{u}_{w2}) \tan \theta}{2}. \quad (3b)$$

In order to simulate an $\times$ -probe in a wall-bounded flow, a Direct Numerical Simulation (DNS) channel flow database from del Álamo *et al.* (2004) at $Re_\tau = (U_\tau h)/\nu = 934$ is used, where U_τ , h and ν are the mean friction velocity, half-channel height and kinematic viscosity of the fluid. The DNS database has a domain size of $8\pi h$, $3\pi h$ and $2h$, in the x , y and z directions respectively. A Fourier, Fourier and Chebyshev spatial discretization scheme is used in x , y and z directions, with a grid size corresponding to $3072 \times 2304 \times 385$. The available database has been de-aliased and interpolated at a grid resolution of $\Delta x \times \Delta y \approx 7.6 \times 3.8$ viscous units, while Δz varies from 0.03 at the wall to 7.6 viscous units at the channel centreline.

3 Filtering effects from individual inclined hot-wires

In the previous section, we have shown that the measured velocity from an $\times$ -probe can be expressed in terms of the (filtered) velocities from the two inclined hot-wires. For an $\times$ -probe measuring the u and w velocities, the velocity vectors along the wire that are being filtered are inhomogeneous, as opposed to an $\times$ -probe configured to measure the u

and v velocities. Thus, the effect of spatial averaging along an inclined wire in the x - z plane is investigated here in isolation, since spatial averaging effects along the inhomogeneous direction for a single wire itself is not well understood (prior studies have largely concentrated on spatial averaging along the homogeneous direction, see e.g. [Chin et al. \(2011\)](#); [Philip et al. \(2013a\)](#); [Wyngaard \(1968\)](#)). The knowledge gained here will provide a foundation for the investigation of the $\times$ -probe conducted in the proceeding section as the finite dimension effects on the measured velocity from an $\times$ -probe turns out to be an average of filtered velocities from two inclined wires plus additional (non-trivial) wire separation effects (see § 4 for further details). Hence, filtered results for the individual velocity components are investigated here, although in practice only the combined effective velocity from all components can be determined from a single inclined wire.

In order to formulate the effect of finite sensor size, consider two arbitrary points A and B along the sensor, each a distance ξ and ζ respectively from the wire centre, as shown in figure 2. Hence, the positional vector of point A and B from the centre of the wire, $\mathbf{P}_A$ and $\mathbf{P}_B$, is given by

$$\mathbf{P}_A = \xi \cos \theta \hat{\mathbf{i}} - \xi \sin \theta \hat{\mathbf{k}}, \quad \text{and} \quad \mathbf{P}_B = \zeta \cos \theta \hat{\mathbf{i}} - \zeta \sin \theta \hat{\mathbf{k}}, \quad (4)$$

for the inclined wires in the x - z plane, where $\hat{\mathbf{i}}$ and $\hat{\mathbf{k}}$ denote the unit vector in the streamwise and wall-normal directions, respectively. Then, the turbulent stresses obtained from the box-filtered velocities can be expressed as

$$\begin{aligned} \langle \tilde{u}_i \tilde{u}_j \rangle &= \frac{1}{l^2} \left\langle \int_{-l/2}^{l/2} u_{i,A} d\xi \int_{-l/2}^{l/2} u_{j,B} d\zeta \right\rangle \\ &= \frac{1}{l^2} \int_{-l/2}^{l/2} \int_{-l/2}^{l/2} \langle u_{i,A} u_{j,B} \rangle d\xi d\zeta, \end{aligned} \quad (5)$$

where $u_{i,A}$ and $u_{j,B}$ correspond to the i^{th} component of velocity at point A and the j^{th} component of velocity at point B, respectively, while the angle brackets (e.g. $\langle u \rangle$) indicate ensemble averaging. It should be also noted that, the terms $u_{i,A}$ and $u_{j,B}$ in equation (5) are interchangeable with the terms $u_{j,A}$ and $u_{i,B}$, respectively.

Using transformation of coordinates, $r = \zeta - \xi$, equation (5) can be rewritten in terms of the two-point correlation function $R_{ij}(\xi, r)$ as ([Comte-Bellot & Foss, 2007](#), see also figure 2):

$$\begin{aligned} \langle \tilde{u}_i \tilde{u}_j \rangle &= \frac{1}{l^2} \int_{-l}^l \int_{-l/2-r}^{l/2-r} \langle u_{i,A} u_{j,B} \rangle d\xi dr, \\ &= \frac{1}{l^2} \int_{-l}^l \int_{-l/2-r}^{l/2-r} R_{ij}(\xi, r) d\xi dr, \end{aligned} \quad (6)$$

Fig. 2: Notation used to uniquely describe the locations of two points located on a sensor. (a) The relationship between ξ , ζ and r . Here, symbols $\bullet$ and $\bullet$ denote the two points (point A and B), while symbol $\bullet$ corresponds to the centre of the sensor. (b) The transformation between the coordinate systems (ζ, ξ) and (r, ξ) . The valid domain where the two points lie on the sensor is shown shaded grey.

where r equals the relative distance between the points A and B. Hence, the ratio between the filtered and original statistics can be expressed as

$$\frac{\langle \tilde{u}_i \tilde{u}_j \rangle}{\overline{u_i u_{j\text{DNS}}}} = \frac{1}{l^2} \int_{-l}^l \int_{-l/2-r}^{l/2-r} \frac{\sigma(u_{i,A}) \sigma(u_{j,B})}{\overline{u_i u_{j\text{DNS}}}} \mathcal{R}_{N_{ij}}(\xi, r) d\xi dr. \quad (7)$$

Here, $\sigma(u_{i,A})$ indicates the standard deviation of the signal $u_{i,A}$, while $\mathcal{R}_{N_{ij}} = R_{ij} / (\sigma(u_{i,A}) \sigma(u_{j,B}))$ corresponds to the normalised two-point correlation function. Since the normalised two-point correlation is a measure of similarity between the velocities located at two points, it is understandable that $\langle \tilde{u}_i \tilde{u}_j \rangle$ can be fully described in terms of $\mathcal{R}_{N_{ij}}$ and l . Consequently, in wall-bounded flows, the spatial-averaging effects from an inclined wire can be well parameterised by considered l^+ , θ and z_c^+ (viscous-scaled wall-normal location of the wire centre). This is because the filtering effect is most severe for the small-scales, which scale with the inner-viscous units (e.g. spanwise spacing of the near-wall streak is $O(100U_\tau/\nu)$ ([Kline et al., 1967](#))). In other words, when evaluating (7), the dependency of $\mathcal{R}_{N_{ij}}$ on r^+ remains approximately the same across multiple Re , at a fixed z_c^+ .

Figures 3(a)-(c) show the effect of spatial filtering across the sensor on the turbulent stresses for a single inclined wire in the x - z plane for various sensor lengths (l) and inclination angles (θ). The centre of the inclined wire is fixed at $z^+ \approx 15$ for all cases. The figures show relative differences ($\mathcal{RD}$) between the filtered ($\langle \tilde{u}_i \tilde{u}_j \rangle$) and the unfiltered ($\overline{u_i u_{j\text{DNS}}}$)

Fig. 3: Relative difference ($\mathcal{RD}$) between the turbulent stresses from the filtered and original velocities for an inclined wire in the x - z plane. $\mathcal{RD}$ of (a) $\overline{u^2}$, (b) $\overline{w^2}$ and (c) $\overline{uw}$ statistics. The symbols indicate different inclination angles: $\bullet$ $\theta = -30^\circ$, $\blacktriangle$ $\theta = -45^\circ$, $\blacksquare$ $\theta = 45^\circ$, $\blacklozenge$ $\theta = -60^\circ$ and $\star$ $\theta = -90^\circ$. The symbols $\times \rightarrow \times$ correspond to the unfiltered DNS statistics where the grid resolution varies from 1.3 to 7.6 viscous units.

DNS statistics, which is evaluated as

$$\mathcal{RD}_{\tilde{u}_i \tilde{u}_j} = \frac{\langle \tilde{u}_i \tilde{u}_j \rangle - \overline{u_i u_j}_{\text{DNS}}}{\overline{u_i u_j}_{\text{DNS}}}. \quad (8)$$

Thus, $\mathcal{RD} = 0$ corresponds to when the filtered and original turbulent stresses are equal, while $\mathcal{RD} = 1$ corresponds to 100% overestimation, and similarly $\mathcal{RD} = -1$ corresponds

to 100% underestimation. It should be noted that we limit our investigation to the effect of spatial filtering on the $\overline{u^2}$, $\overline{w^2}$ and $\overline{uw}$ statistics, since only these turbulent stresses can be evaluated using a u - w $\times$ -probe.

For all wire inclination angles examined in figure 3, an increase in l results in increasingly attenuated turbulent stresses. This is because a longer viscous scaled wire length leads to a spatial averaging process with a larger representation from poorly correlated velocities as evident from the decrease in $|\mathcal{R}_{N_{ij}}|$ that occurs away from the origin, $r = 0$, in figures 4(a)-(c). Further, for a fixed l the attenuation increases as the wire inclination is changed from horizontal ($\theta = 0^\circ$) to vertical ($\theta = -90^\circ$), and this can be attributed to a faster decrease in $|\mathcal{R}_{N_{ij}}|$ in z compared to the x direction. It should be noted that, since $\mathcal{R}_{N_{ij}}$ in the x - y and x - z planes share the same characteristics, trends of the filtered statistics with respect to l and θ observed here are also seen for an inclined single wire in the x - y plane (Philip *et al.*, 2013a). However, unlike in the x - y plane, $\mathcal{R}_{N_{ij}}$ in the x - z plane are not symmetric about the origin, and therefore, the filtered statistics from the inclined wire at $\theta = -45^\circ$ and $\theta = 45^\circ$ are distinct as shown in figure 3 using symbols $\blacktriangle$ and $\blacksquare$, respectively. For a more detailed discussion of the similarity and difference between the $\mathcal{R}_{N_{ij}}$ behaviour in the homogeneous and inhomogeneous plane, the reader is referred to appendix B. The lesser attenuated $\overline{u^2}$ and $\overline{w^2}$ statistics for an inclined wire at $\theta = -45^\circ$ compared to the 45° case is consistent with $\mathcal{R}_{N_{ij}}$ shown in figure 4, where two points at a negative (following the right-hand rule) inclination angle are better correlated than the positive counterpart, for a given r . Furthermore, while $\mathcal{R}_{N_{uu}}$ and $\mathcal{R}_{N_{ww}}$ is maximum at $r = 0$ and becomes increasingly uncorrelated with increasing r , this is not necessarily the case for $\mathcal{R}_{N_{uw}}$. For example, in figure 4(c) the w velocity is best correlated with $u(x^+, z^+ = 15)$ at an offset $(r_x^+, r_z^+) \approx (-5, 10)$, as indicated by the grey square ($\square$). Moreover, the r_x locations where $-\mathcal{R}_{N_{uw}}$ is maximum for a given r_z (denoted by a thin dashed line), display a non-trivial behaviour which include switches of sign that leads to the phase shift between the reference, $u(z^+ = 15)$, and the $w(z^+)$ signals alternating between a lead ($r_x < 0$) and a lag ($r_x > 0$). Consequently, spatial averaging of the velocities along the inclined wire (shown as grey line) leads to an inclusion of u and w velocities which are better correlated than at the centre of the wire and hence have a positive contribution to $-\langle \tilde{u} \tilde{w} \rangle$. Thus, the attenuation of the $\langle \tilde{u} \tilde{w} \rangle$ statistics is smaller compared to $\langle \tilde{u}^2 \rangle$ and $\langle \tilde{w}^2 \rangle$ for a fixed l^+ , as evident from figures 3(a)-(c).

4 $\times$ -probe

A parametric study on how the turbulent stresses measured using an $\times$ -probe are affected by differing wire length (l),

Fig. 4: Normalised two-point correlation of an unfiltered DNS velocity on a streamwise-wall-normal plane about $z^+ \approx 15$. (a) Autocorrelation of u velocity, $\mathcal{R}_{N_{uu}}$, (b) autocorrelation of w velocity, $\mathcal{R}_{N_{ww}}$, and (c) cross-correlation of the u and w velocities, $-\mathcal{R}_{N_{uw}}$. For reference, an inclined wire with configurations $l^+ = 20, \theta = -45^\circ$ is also shown in grey, while $\bullet$ denotes the origin ($r_x = r_z = 0$) of the two-point correlation functions. The thick (black) solid, dotted (shown only for c), dash-dotted and dashed lines correspond to the contour levels 0.25, 0.45, 0.5 and 0.75, respectively. The thin dashed line in (c) denotes the locations where $-\mathcal{R}_{N_{uw}}$ is maximum for a fixed r_z as a function of z , while the grey square indicates the location of the global maxima.

Fig. 5: DNS simulation of a u - w $\times$ -probe at $z^+ \approx 15$ for three different sensor parameters: sensor length (l^+), spanwise spacing between the sensors (Δs_y^+) and sensor angle with respect to the x direction (θ). Left column (a,d,g): $\Delta s_y^+ = 0$, centre column (b,e,h): $\Delta s_y^+ = 5.2$ and right column (c,f,i): $\Delta s_y^+ = 19.6$. Top row (a-c): streamwise variance $(\overline{u^2})^+$, middle row (d-f): wall-normal variance $(\overline{w^2})^+$ and bottom row (g-i): Reynolds shear stress $(-\overline{uw})^+$ distributions. --- $\theta = 30^\circ$, — $\theta = 45^\circ$ and -.- $\theta = 60^\circ$. The symbols $\times$ — $\times$ correspond to the unfiltered DNS statistics where the grid resolution varies from 1.3 to 7.6 viscous units. It should be noted that, while the ordinates for (a)-(c) and (g)-(i) are the same, they differ for (d)-(f).

wire angle (θ) and wire separation (Δs_y) is reported in this section. Furthermore, as with § 3, the centre of the sensing elements is maintained at $z^+ \approx 15$.

Figures 5(a)-(c), (d)-(f) and (g)-(i) show the measured streamwise variances $\overline{u^2}$, wall-normal variance $\overline{w^2}$ and Reynolds shear stress $-\overline{uw}$, respectively as functions of l , Δs_y and θ . An increased attenuation of the $\overline{u^2}$ and $\overline{w^2}$ statistics is observed with increasing l due to the velocity along the wire becoming increasingly uncorrelated. Although both $\overline{u^2}$ and $\overline{w^2}$ follow a similar trend (i.e. they both decrease) with increasing l , opposite behaviour occurs when Δs_y is increased. While an attenuation of $\overline{u^2}$ is observed with increasing Δs_y , an increasingly amplified $\overline{w^2}$ is obtained with increased wire separation. The amplification of $\overline{w^2}$ at $\Delta s_y \neq 0$ is the result of additional contribution from the $\tilde{u}_{w1} - \tilde{u}_{w2}$ term (cf. equation 3b) existing in the near-wall region ($z^+ < 100$). The amplification of $\overline{w^2}$ is also dependent on the wire angle, with a five times increase observed when the wire angle is changed from 30° to 60° when $\Delta s_y^+ \approx 20$. However, the effect of wire angle on $\overline{u^2}$ and $\overline{w^2}$ when $\Delta s_y = 0$ is significantly less, as shown in figures 5 (a) and (d). Unlike the $\overline{u^2}$ and $\overline{w^2}$ statistics, the Reynolds shear stress shows no clear trend against any of the three parameters investigated here. The trends (or lack of one) in the $\overline{u^2}$, $\overline{w^2}$ and $\overline{uw}$ statistics observed with respect to l , θ and Δs_y is explained in the following section through a closer look at the contributing terms to the measured turbulent stresses.

4.1 Special case: $\theta = 45^\circ$

Setting $\theta = 45^\circ$, and expressing the measured u and w velocities in terms of spatially filtered velocities along the wire, with a provision that $l^+ \sim O(10)$ leads to (see appendix C for derivations, adapted from Philip *et al.* (2013a))

$$\frac{\overline{u^2}}{\overline{u^2}_{\text{DNS}}} \approx \underbrace{F_{uu}}_{l \text{ effect}} \underbrace{\left[1 - \frac{1}{2} \left(1 - \tilde{\mathcal{R}}_{N_{uu}}(\Delta s_y) \right) \right]}_{\Delta s \text{ effect}}, \quad (9)$$

filtering of the $\tilde{u}$ velocities
across the two wires

$$\frac{\overline{w^2}}{\overline{w^2}_{\text{DNS}}} \approx \underbrace{F_{ww}}_{l \text{ effect}} \underbrace{\left[1 - \frac{1}{2} \left(1 - \tilde{\mathcal{R}}_{N_{ww}}(\Delta s_y) \right) \right]}_{\Delta s \text{ effect}} + \underbrace{\left[\frac{1}{2} \left(1 - \tilde{\mathcal{R}}_{N_{uu}}(\Delta s_y) \right) \frac{\langle \tilde{u}^2 \rangle}{\langle \tilde{w}^2 \rangle} \right]}_{\Delta s \text{ effect}}, \quad (10)$$

filtering of the $\tilde{w}$ velocities
across the two wires

contribution from the differences in
the $\tilde{u}$ velocities across the two wires
amplification

and

$$\frac{\overline{uw}}{\overline{uw}_{\text{DNS}}} \approx \underbrace{F_{uw}}_{l \text{ effect}} \times \underbrace{1}_{\text{no } \Delta s \text{ effect}}, \quad (11)$$

where the function F_{uiuj} corresponds to the ratio of turbulent stresses from the spatially filtered velocity and the original DNS statistics, or

$$F_{uu} = \frac{\langle \tilde{u}^2 \rangle}{\overline{u^2}_{\text{DNS}}}, \quad F_{ww} = \frac{\langle \tilde{w}^2 \rangle}{\overline{w^2}_{\text{DNS}}}, \quad \text{and} \quad F_{uw} = \frac{\langle \tilde{u} \tilde{w} \rangle}{\overline{uw}_{\text{DNS}}}. \quad (12)$$

Therefore $F_{uiuj} = \mathcal{RD}_{\tilde{u}_i \tilde{u}_j} + 1$, where $\mathcal{RD}_{\tilde{u}_i \tilde{u}_j}$ corresponds to the relative difference between the filtered and the original statistics as defined in § 3. Furthermore, $\tilde{\mathcal{R}}_N$ denotes the normalised two-point correlation function between the filtered velocities, or

$$\tilde{\mathcal{R}}_{N_{uu}}(\Delta s_y) = \frac{\langle \tilde{u}(\mathbf{x}) \tilde{u}(\mathbf{x} + \Delta s_y \mathbf{j}) \rangle}{\langle \tilde{u}^2 \rangle},$$

$$\tilde{\mathcal{R}}_{N_{ww}}(\Delta s_y) = \frac{\langle \tilde{w}(\mathbf{x}) \tilde{w}(\mathbf{x} + \Delta s_y \mathbf{j}) \rangle}{\langle \tilde{w}^2 \rangle}. \quad (13)$$

The terms F_{uiuj} are solely dependent on the spatially filtered velocities along the two inclined wires, which have already been investigated in isolation in § 3. It should be noted that, similar expressions for equations 9 and 10 have been previously obtained by Suzuki & Kasagi (1992), for a more general case where the axial sensitivity coefficient of the hot-wire sensor is not equal to zero.

Figures 6(a)-(c) show the relative difference between the measured $\overline{u^2}$, $\overline{w^2}$ and $\overline{uw}$ statistics using an $\times$ -probe and the original DNS statistics, respectively. It is useful to rewrite $\overline{u^2}$ in terms of F_{uu} and $\tilde{\mathcal{R}}_{N_{uu}}$ (cf. equation 9) such that the effects from l and Δs_y can be examined in isolation. Thus, while the $\overline{u^2}$ statistics are increasingly attenuated with increasing l , due to spatial averaging processes along the wire as discussed in § 3, an increase in Δs_y leads to even further attenuation. Since $\overline{u^2} \gg \overline{w^2}$ in wall-bounded flows, equation (3a) reduces to $u \approx 0.5 \tilde{u}_{w1} + 0.5 \tilde{u}_{w2}$, meaning an additional averaging process between the two wires is invoked when determining u using an $\times$ -probe. Since the two filtered velocities become increasingly uncorrelated with an increase in the wire separation distance, this leads to a $\overline{u^2}$ dependency on Δs_y , as shown in figures 5(a)-(c) and 6(a).

For the $\overline{w^2}$ statistics, in contrast to $\overline{u^2}$ none of the terms in equation (3b) can be neglected. This leads to an additional contributing term to $\overline{w^2}$ which is associated with the Δs_y effect, as shown in equation (10). Since $1 - \tilde{\mathcal{R}}_{N_{uu}} = 0$ when $\Delta s_y = 0$ and approaches unity as $\Delta s_y \rightarrow \infty$, the additional term leads to an increase in the $\overline{w^2}$ statistics with increasing Δs_y , an opposite effect to the $\tilde{\mathcal{R}}_{N_{ww}}$ term in (10). Further, it should be noted that, to obtain equation (3b), we assumed

Fig. 6: Distribution of relative difference ($\mathcal{RD}$) of a u - w $\times$ -probe. (a) Streamwise variance ($\mathcal{RD}_{uu}$), (b) wall-normal variance ($\mathcal{RD}_{ww}$) and (c) Reynold shear stress ($\mathcal{RD}_{uw}$). Four wire separation cases shown correspond to $\Delta s_y^+ = 0$, $\Delta s_y^+ = 5.2$, $\Delta s_y^+ = 12.1$ and $\Delta s_y^+ = 19.6$. As with figure 5, $\times$ — $\times$ denotes the variation in the grid resolution for the DNS volume used when simulating the $\times$ -probe. Symbols $\bullet$ and $\blacksquare$ correspond to two configuration where $\mathcal{RD}_{uw} \approx 0$.

that the two wires experience an identical velocity vector, which is not necessarily satisfied inside the turbulent flow. For example, any differences in the $\tilde{u}_\perp$ velocities between the two wires leads to a non-zero spurious w , resulting in increased $\overline{w^2}$ statistics. A larger Δs_y leads to a poorer correlation between $\tilde{u}_\perp$ from the two wires, leading to increasingly spurious w . In addition, the fact that $\overline{u^2}/\overline{w^2} \gg 1$ in wall-

bounded flows, means that the additional $\overline{w^2}$ contribution from the last term in equation (3b) can be sufficiently large to suppress the attenuating effect from the other two terms, resulting in an overall amplification compared to the original $\overline{w^2}_{\text{DNS}}$ value, as observed in figures 5(d)-(f) and 6(b).

It is evident from equation (11) that unlike $\overline{u^2}$ and $\overline{w^2}$, the $\overline{uw}$ statistics have no contribution from the $\tilde{R}_{N_{ij}}(\Delta s_y)$ terms. This is the result of a fortuitous cancellation of these terms in the expansion process due to equal and opposite contributions from the two wires (cf. appendix for details). This suggests that the measured Reynolds shear stress is not a function of wire separation, at least in the context of linear filtering, unlike the measured streamwise and wall-normal turbulence intensities. However, it should be noted that the $\tilde{R}_{N_{ij}}(\Delta s_y)$ terms are only equal and opposite when considering the mean over sufficient ensembles and thus the instantaneous uw signal recorded by the $\times$ -probe may still exhibit a Δs_y dependency, as illustrated in the following section.

4.2 Other considerations

We have observed that with increased wire separation, $\overline{u^2}$ decreases, while $\overline{w^2}$ increases. However, with increases in the wire length, increased attenuation of $\overline{u^2}$ and $\overline{w^2}$ occur. Hence, it should be noted that, although $\mathcal{RD}$ results shown in figures 3 and 6 correspond to the most frequently used measure of error for study of the spatial averaging effects from sensors (Philip *et al.*, 2013b), $\mathcal{RD} = 0$ does not in any way guarantee that the two signals are identical. For example, the opposing l and Δs effects for $\overline{w^2}$ can exactly cancel each other in a time-averaged sense leading to an identical variance even when instantaneously the w velocity signal is different. An alternative indicator that considers the error in instantaneous velocity is $\langle \mathcal{E}_{u_i} \mathcal{E}_{u_j} \rangle / \overline{u_i u_j}_{\text{DNS}}$, where $\mathcal{E}_{u_i}(t) = u_i(t) - u_{i,\text{DNS}}(t)$ corresponds to the error in the i^{th} component of the velocity. This is because $\langle \mathcal{E}_{u_i} \mathcal{E}_{u_j} \rangle / \overline{u_i u_j}_{\text{DNS}}$ contains an additional term corresponding to the correlation between the measured and the original velocity, $\langle u_i u_{j,\text{DNS}} \rangle$, which is absent in $\mathcal{RD}$ (Philip *et al.*, 2013b). Hence,

$$\frac{\langle \mathcal{E}_{u_i} \mathcal{E}_{u_j} \rangle}{\overline{u_i u_j}_{\text{DNS}}} = \mathcal{RD}_{u_i u_j} + 2 \left(1 - \frac{\langle u_i u_{j,\text{DNS}} \rangle + \langle u_{i,\text{DNS}} u_j \rangle}{2 \overline{u_i u_j}_{\text{DNS}}} \right) \quad (14)$$

The errors in the instantaneous velocity measured using an $\times$ -probe are quantified as function of l^+ and Δs^+ in figure 7. For the majority of cases considered for $\langle \mathcal{E}_u^2 \rangle / \overline{u^2}$ and $\langle \mathcal{E}_w^2 \rangle / \overline{w^2}$ in figure 6, substantial differences exist between the variance from the simulated $\times$ -probe and the original DNS statistics, and therefore the trends in figure 7(a) and (b) closely follow that observed for $\mathcal{RD}_{uu}$ and $\mathcal{RD}_{ww}$ in figures 6(a) and (b), respectively. However, for the $\langle \mathcal{E}_u \mathcal{E}_w \rangle$ results shown in figure 7(c), the correlation terms $\langle u_i u_{j,\text{DNS}} \rangle$ and

Fig. 7: Error in the measured velocity obtained using an $\times$ -probes with $\theta = 45^\circ$. Errors in (a) streamwise velocity ($\mathcal{E}_u$), (b) wall-normal velocity ($\mathcal{E}_w$) and (c) $\langle \mathcal{E}_u \mathcal{E}_w \rangle$ for a u - w type $\times$ -probe. The symbols are as in figure 6.

$\langle u_{j,\text{DNS}} u_i \rangle$ in (14) are significant and therefore $\langle \mathcal{E}_u \mathcal{E}_w \rangle / \overline{uw}$ differ substantially from $\mathcal{RD}_{uw}$. As an illustration, figure 8 shows comparison of instantaneous u^+ , w^+ and u^+w^+ signals from a u - w $\times$ -probes corresponding to (i) $l^+ = 14$, $\Delta s_y^+ = 5.2$ and (ii) $l^+ = 25$, $\Delta s_y^+ = 5.2$; as indicated by the dashed and dotted lines, respectively. While the u^+w^+ signals from both cases have an identical mean value to that from the original DNS velocity, shown as a solid line, the instantaneous Reynolds shear stress from the $\times$ -probe with longer wire is more erroneous. A clear degradation in the performance of an $\times$ -probe to capture the correct instantaneous uw signal with an increase in l^+ occurs for the $\Delta s_y^+ = 0$ and 5.2 cases in figure

Fig. 8: Comparison of measured fluctuating velocity from an $\times$ -probe. — original DNS signal; - - $\times$ -probe: $l^+ = 14$, $\Delta s_y^+ = 5.2$; . . $\times$ -probe: $l^+ = 25$, $\Delta s_y^+ = 5.2$.

7(c), even though this is not necessarily evident when only $\mathcal{RD}_{uw}$ is considered.

5 l , Δs_y and θ effects on the spectra

In terms of techniques to capture flow velocities, the hot wire anemometer (HWA) has high dynamic range, good spatial and temporal resolutions; and ability to sample very large turn-over times (Comte-Bellot & Foss, 2007). HWA is often the technique of choice when examining contributions to velocity fluctuations from a range of scales at high Re . In this case, the ratio between the smallest and the largest energy containing motions can be four orders of magnitude or more. Hence, the effects of finite sensor size on the measured spectra is an important consideration when designing a hot-wire experiment.

In this section, we will investigate the effects of wire length, separation and angle on the scale-by-scale contributions to the turbulent stresses measured using $\times$ -probes. Similar to § 3 and 4, the effects are examined at a single wall-normal location, with the centre of the $\times$ -probe now lo-

Fig. 9: Effect of wire length (l), spacing (Δs_y) and angle (θ) on measured pre-multiplied spectra using an $\times$ -probe. Left column (a,d,g): u spectra, centre column (b,e,h): w spectra and right column (c,f,i): uw cospectra. The insets in (a,d,g) show the absolute difference between the measured and original u -spectra, $|\Delta k_x^+ \phi_{uu}^+|$, as a function of λ_x^+ . Top row (a,b,c): $\Delta s_y^+ = 7$ and $\theta = 45^\circ$, while $---$ $l^+ = 10$, $-.-$ $l^+ = 20$ and $...$ $l^+ = 30$; middle row (d,e,f): $l^+ = 20$ and $\theta = 45^\circ$, while $---$ $\Delta s_y^+ = 0$, $-.-$ $\Delta s_y^+ = 7$ and $...$ $\Delta s_y^+ = 30$; bottom row (g,h,i): $l^+ = 20$ and $\Delta s_y^+ = 7$, while $---$ $\theta = 30^\circ$, $-.-$ $\theta = 45^\circ$ and $...$ $\theta = 60^\circ$. The original DNS spectra at $z^+ \approx 100$ are denoted using $—$, while the vertical dashed lines correspond to $\lambda_x^+ = 100$.

cated at $z^+ = 100$ (since the finite dimension effects for a typical $\times$ -probe are too severe at $z^+ = 15$ to obtain a reliable spectra). Figures 9(a)-(i) show the effect of l , Δs_y and θ on the measured u , w spectra and uw cospectra (denoted as ϕ_{uu} , ϕ_{ww} and ϕ_{uw} , respectively) for the $\times$ -probe. It should be noted that, in order to isolate individual effects, one parameter is systematically varied in each row of figure 9. While, the u and w spectra is observed to depend on l , Δs_y and θ (the dependency is discussed further in § 5.1-5.3), the uw cospectra remain comparatively unchanged, despite increases in l , Δs_y and θ . The robustness of the uw cospectra to the finite sensor effects is discussed further in § 5.4.

5.1 The effects of the wire length, l

An increased attenuation of small-scale energy from a single normal hot-wire with increase in the sensor length has been previously demonstrated by Ligriani & Bradshaw (1987) and Hutchins *et al.* (2009). Furthermore, Hutchins *et al.* (2009) find that the small-scale contribution to $\overline{u^2}$ remains approximately fixed when scaled in viscous units at all Re , and is

only altered when l^+ is changed. Figures 9(a)-(c) show the result of simulations performed with a fixed wire angle of $\theta = 45^\circ$ and a fixed wire spacing of $\Delta s_y^+ = 7$ at three different wire lengths $l^+ = 10, 20$ and 30 . Similar to single normal hot-wires, attenuation of small-scale u fluctuations observed from $\times$ -probes, with longer l^+ sensors resulting in increased attenuation of the small-scale streamwise energy contribution. At $z^+ = 100$, this is particularly evident as a reduced premultiplied energy, in the range $\lambda_x^+ \lesssim 1000$ in figure 9(a). Further, the small-scale w energy is similarly attenuated when a longer l^+ sensor is used (see figure 9(b)).

5.2 The effects of the wire spacing, Δs_y

In a wall-bounded flow, the u velocities obtained from an $\times$ -probe are well approximated by the average of filtered u velocities along the two wires, as shown in § 4. Therefore, increased attenuation of energy, similar to that observed with increase in l occurs when the wire separation is increased, as shown in figure 9(d). However, in contrast to the effect of l^+ which is isolated in figure 9(a), 9(d) indicates that attenu-

ation due to increasing Δs_y^+ is not solely restricted to the small-scales, but is evident at all scales for $\Delta s_y^+ = 30$.

For the w component, in addition to the filtering effect due to wire length, there is an additional potential amplification of w energy due to the differences in the measured normal component of velocity between the two wires. As Δs_y increases, this amplifying effect is exacerbated. The additional contribution, due to the fact that the wires are at an increased separation, leads to an overestimation of the w spectra over a wide range of wavelengths, with amplification in $\lambda_x^+ \gtrsim 500$ evident for the $\Delta s_y^+ = 20$ case shown in figure 9(e). The energy attenuation for $\lambda_x^+ \lesssim 500$ is due to the finite wire length ($l^+ = 20$) used for all Δs_y cases.

5.3 The effects of the wire angle, θ

Figures 9(g)-(i) show the effect on the measured ϕ_{uu} , ϕ_{ww} and ϕ_{uw} from the $\times$ -probe with $l^+ = 20$ and $\Delta s_y^+ = 7$, as the wire angle θ , is varied to 30° , 45° and 60° . Since the u velocity in a wall-bounded flow is better correlated in the x direction than in the z direction, the filtering effect along the wire is well characterised by the quantity $l \sin \theta$, for wire angle where $\tan \theta \sim O(1)$. Since the quantity $l \sin \theta$ reduces as the wire angle decreases from 60° to 30° , less attenuation of the small-scale u spectra occurs.

The additional contribution to the w spectra from differences in the filtered u velocities between the two wires includes a term with $\tan \theta$. At a low value of θ , $\tan \theta \rightarrow 0$ in equation (3), leading to a suppression of the contribution from the $\tilde{u}_{w1} - \tilde{u}_{w2}$ term to the measured w velocities using the $\times$ -probe. Therefore at a low value of θ , the amplification effects on the measured w velocity due to a non-zero wire separation diminishes, while at a large θ an overestimate of energetic contributions across a wide range of scales may occur.

Although, at $z^+ = 100$, the difference between the u and w energy for wire angle $\theta = 30^\circ$, 45° and 60° is marginal as shown in figures 9(g)-(i), from § 4 we know that closer to the wall at $z^+ \approx 15$, the differences are exacerbated. At low values of θ , the errors in $\overline{u^2}$ reduces, primarily due to the u velocity being significantly better correlated in the x direction compared to z , and hence leading to a smaller attenuation as shown for the inclined wires in § 3 (cf. figure 3). Furthermore, the w velocity evaluated from the $\times$ -probe is less susceptible to a mismatch between the filtered wire-normal velocities of the two wires. However, there are other disadvantages to smaller angles θ . This wire angle effectively sets the cone angle between which the $\times$ -probe is able to uniquely resolve the two components of velocity. Thus a trade-off must be made, as a low value of θ results in small cone angle, while a high value of θ leads to a magnified error in the measured $\overline{w^2}$ statistics. With care, it is likely that different $\times$ -probes could be designed for different wall-normal

locations (since high cone angle flow only typically occurs close to the wall).

5.4 The robustness of Reynolds shear stress cospectra to the finite sensor effects

From figure 9, it is evident that a negligible energetic content exists in the $\lambda_x^+ < 100$ regime for the Reynolds shear stress, in contrast to the $\overline{u^2}$ and $\overline{w^2}$ statistics. These fine-scale motions are near isotropic and therefore do not contribute to $\overline{uw}$. Hence, even at $l^+ = 30$, the longest wire length considered in figure 9, the $\times$ -probe is able to resolve most of the uw containing motions, but not the u and w contributions from the near isotropic fine-scale motion. This leads to the uw cospectra being unchanged when l is made smaller or θ varied, as observed in figures 9(c) and (h), respectively. It should be noted that, at sufficiently large l ($l^+ \approx 100$) an attenuation of small-scales do occur for the uw cospectra, but such a large sensor is rarely used in practice due to severe attenuation of the u and w spectra. Furthermore, since the Reynolds shear stress recorded using an $\times$ -probe is devoid of wire separation effects, at least in the context of linear filtering as discussed in § 4, the uw cospectra also remain unchanged when Δs_y is varied.

6 The effects of the wall-normal position

As we have pointed out in the earlier sections, the two-point correlation function of the individual velocity components determines the magnitude of attenuation or amplification that occur for each of the turbulent stresses. Since the two-point correlation function changes with the z location, this leads to a varying attenuation/amplification with z . Here, simulations for selective $\times$ -probe configurations for $\theta = 45^\circ$ are performed at multiple z locations to obtain complete turbulence profiles.

Figure 10 shows the measured turbulence statistics from the simulated u - w $\times$ -probe with configurations corresponding to $l^+ = \Delta s^+ = 11$; $l^+ = \Delta s^+ = 20$; $l^+ = 20$, $\Delta s^+ = 7$ (equal to the $\times$ -probe dimensions in viscous units constructed for experiments, cf. § 7); and $l^+ = \Delta s^+ = 7$; which are shown by the symbols $\bullet$, $\blacksquare$, $\blacklozenge$ and $\blackstar$ respectively. From figure 10(a), it is evident that the attenuation in $\overline{u^2}$ decreases with increasing z in the region $z^+ \gtrsim 15$. This is the result of two points at a fixed r becoming better correlated with increasing z and therefore the filtered velocities are a more accurate representation of the original velocity.

In contrast, the $\overline{w^2}$ statistics transition from amplification to attenuation as z is increased, as shown in figure 10(b). This is because unlike $\overline{u^2}$, in addition to contributions from the filters velocities along the wires $\tilde{w}_{w1}$ and $\tilde{w}_{w2}$, contributions from differences between $\tilde{u}_{w1}$ and $\tilde{u}_{w2}$ exist for the

Fig. 10: The effects of wall positions on the measured turbulent stresses using a u - w $\times$ -probe with $\theta = 45^\circ$. (a) streamwise variance, (b) wall-normal variance and (c) Reynolds shear stress statistics. — original DNS statistics, $\bullet$ $l^+ = \Delta s_y^+ = 19$, $\blacksquare$ $l^+ = \Delta s_y^+ = 11$, $\blacklozenge$ $l^+ = 20$, $\Delta s_y^+ = 7$ and $\star$ $l^+ = \Delta s_y^+ = 7$. The shaded region shown in (b) corresponds to the $\overline{w'^2}$ contribution from the filtered w velocities along the wires, for the $\blacklozenge$ case.

$\overline{w'^2}$ statistics (cf. equation 10). As shown in § 4.1, the former leads to an attenuation, while the latter amplifies $\overline{w'^2}$. Further, the contributions from these two mechanisms behave differently as functions of z . Specifically, as the z position is increased from $z^+ \approx 15$, the contribution from the filtered u velocity responsible for the amplified $\overline{w'^2}$ statistic diminishes, while the contribution from the average of filtered

w velocities remains approximately similar. Therefore, at $z^+ \approx 15$, the term $\widetilde{u}_{w1} - \widetilde{u}_{w2}$ dominates in the equation (3b), leading to a large overestimation in $\overline{w'^2}$. However, as the z location is increased, the contribution from the term $\widetilde{u}_{w1} - \widetilde{u}_{w2}$ decreases while the term $\widetilde{w}_{w1} + \widetilde{w}_{w2}$ remains approximately the same. This is illustrated in figure 10(b), where the contribution to $\overline{w'^2}$ from the filtered w velocities along the wires $(\widetilde{w}_{w1} + \widetilde{w}_{w2})^2/4$ is shown as a shaded region. With increasing z the measured $\overline{w'^2}$ approaches the shaded region resulting in attenuated $\overline{w'^2}$ statistic for $z^+ > 150$. It should be noted that, there exists a z location where the extra contribution from the terms $\widetilde{u}_{w1}$ and $\widetilde{u}_{w2}$ is identical to attenuation of the $\overline{w'^2}$ statistic from the spatial filtering effects, resulting in a match between the measured $\overline{w'^2}$ and the original DNS statistics at that z location. Also, depending on the $\times$ -probe geometry, the z location where the measured and original statistics match changes. However, the fact that an $\times$ -probe locally returns the exact turbulent stress values does not guarantee that the recorded instantaneous velocity signal is devoid of errors.

Out of all the turbulent stresses examined here, the Reynolds shear stress shows the least sensitivity to the l and Δs effects when $\theta = 45^\circ$, across all measured z positions as shown in figure 10(c). It should be noted that here, we have only considered errors introduced from the spatial averaging due to finite sensor dimensions. In the next section, the simulated $\times$ -probe results are compared against measured statistics from experiments.

7 Comparisons against experimental results

The qualitative trends observed in the simulation study are compared against the experimental results obtained using custom built sub-miniature $\times$ -probes in the High Reynolds Number Boundary Layer Wind Tunnel (HRNBLWT), at the University of Melbourne. For further details of the experiment, the readers are referred to Baidya et al. (2017).

Typically, experiments are performed at Reynolds numbers that are an order of magnitude greater than the DNS. Therefore, the finite sensor dimension effects can only be simulated at a low Re , and the effects need to be inferred at a higher Re . For a single normal hot-wire, since the attenuation occurs at small scales whose contribution to $\overline{u'^2}$ remains fixed when scaled in the viscous units (Hutchins et al., 2009), the amount of near-wall attenuation is found to be well parameterised by l^+ and z^+ (Chin et al., 2014; Smits et al., 2011). Furthermore, since the small-scale behaviour is universal in the pipe, channel and boundary layer flows (Monty et al., 2009), similar attenuation of $\overline{u'^2}$ occurs and the same equation describing the attenuation in terms of l^+ and z^+ works well across all three types of flow (Smits et al., 2011). For example, Chin et al. (2014) have found that when

Fig. 11: The differences, $\mathcal{D}_{u\bar{u}}$, between the measured turbulence statistics from a u - w $\times$ -probe and the corresponding statistics from DNS. The experimental results, shown as circles, are from boundary layer flow at $Re_\tau \approx 2,500$, while the results from the simulated probes with matched resolution, shown in lines, are from channel flow at $Re_\tau = 934$. The reference statistics are obtained from DNS of Sillero *et al.* (2013) and del Álamo *et al.* (2004), respectively. Differences in the (a) streamwise velocity variance, (b) wall-normal velocity variance and (c) Reynolds shear stress.

the l^+ correction obtained for channel at $Re_\tau = 934$, is applied to experimental boundary layer data at $Re_\tau \approx 7000$, a good collapse of the profiles occurs for different l^+ probes. Hence, even though the individual profiles may not match due to differences in flow geometry and Re , the deviations in the viscous scaled measured statistics (indicated here as $\mathcal{D}$, e.g. $\mathcal{D}_{u^2} = \bar{u}_{\text{measured}}^2 - \bar{u}_{\text{original}}^2$) at fixed z^+ due to finite sensor dimension effects should remain approximately the same in the near-wall region across all wall-bounded flow, provided that the sensor dimensions remain fixed in viscous units. Furthermore, the small-scale invariance when scaled by viscous units observed for the u velocity, also occurs for the v and w velocities (Baidya, 2016; Baidya *et al.*, 2017). Consequently, Lee *et al.* (2016) have shown that the deviations for the w^2 statistics observed in boundary layer particle image velocimetry experiments agree well with that predicted from the filtered channel DNS statistics when the averaging was performed across all three dimensions to match the spatial resolution expected from the experiments.

Figures 11(a) and (b) show the deviations in the variances observed for the simulated and experimental u - w $\times$ -probes. Note that, the experiments are conducted at $Re_\tau \approx 2500$, and therefore the statistics from boundary layer DNS at a matched Re ($\delta_{99}^+ = 2000$) from Sillero *et al.* (2013) are taken as a reference from which the disparity is measured. For the $\bar{u}^2$ and $\bar{w}^2$ statistics, the simulated $\times$ -probe is able to provide a good estimate of the deviations in the measured statistics from experiments, as observed in figures 11(a) and (b), respectively. This suggests that finite sensor dimension effects are the dominant source of error for the $\bar{u}^2$ and $\bar{w}^2$ statistics measured using an $\times$ -probe. However, unlike the $\bar{u}^2$ and $\bar{w}^2$ statistics, a significant discrepancy exists between the simulated and measured Reynolds shear stresses, particularly in the region $z^+ \lesssim 100$ as shown in figure 11(c). Thus, while the $\bar{uw}$ statistics are robust to the finite sensor effects, it is found to be highly sensitive to other sources of

errors such as those introduced by wire misalignment and calibration procedures. Since the focus of this paper is on investigating the effects of spatial averaging due to finite sensor dimensions, the other error sources are investigated in a companion paper (Baidya *et al.*, 2019). In appendix A, for completeness we show the comparison of the statistics from the experimental u - v $\times$ -probe with that from a simulation. The good agreement between the two, as seen in figure B1, further reinforces the validity of the theoretical approach taken here to understand spatial resolution issues in wall-turbulence.

8 Summary and conclusions

A systematic study of the wire length (l), spacing (Δs_y) and angle (θ) effects for a cross-wire probe (referred to here as an $\times$ -probe) configured to measure the streamwise and wall-normal velocities (u - w $\times$ -probe) showed that a longer wire in viscous units lead to more attenuated $\bar{u}^2$ and $\bar{w}^2$ statistics. This is because, at increased l^+ , a larger portion of the velocities being averaged is poorly correlated, leading to attenuated filtered statistics. However, when Δs_y^+ is increased, although $\bar{u}^2$ become attenuated, the $\bar{w}^2$ statistics can be amplified as shown in figures 5(d)-(f). This is because, for the w velocity it turns out that a difference between filtered wire-normal velocities from the two wires, due to local anisotropy from turbulence, leads to a non-zero spurious w being recorded by the $\times$ -probe. This results in an amplified $\bar{w}^2$ statistic, and the magnitude of amplification increases as the wire spacing is increased since the filtered wire-normal velocities from the two wires deviate more as Δs_y is increased. For a fixed l , a larger wire angle θ leads to an increase in the x dimension occupied by the $\times$ -probe while the z dimension decreases. In a wall-bounded flow, since the velocities at two points are better correlated in x than z for a fixed distance between the points, an increase in θ leads to a lar-

ger portion of poorly correlated velocities being included in the spatial averaging process, again leading to an attenuated statistic. The sensitivity of measured turbulent stresses to the finite sensor effects show that time-averaged Reynolds shear stress is significantly more robust to variation in l , Δs_y and θ compared to the streamwise and wall-normal velocity variances. The robustness of the $\overline{uw}$ statistics when $l^+ \lesssim O(10)$ is a result of small-scales with characteristic wavelengths below 100 viscous units not contributing to the $\overline{uw}$ stress, while those same scales make a significant contribution to $\overline{u^2}$ and $\overline{w^2}$ statistics. Further, as shown in § 4, due to a fortuitous cancellation of cross terms involving the filtered velocities from the two wires, the measured time-averaged Reynolds shear stress is not a function of wire separation distance Δs_y , at least in the context of linear filtering.

A summary of the main conclusions and recommendations based on the simulated dual hot-wire probe studies is as follows:

1. The wire separation, Δs^+ , should be kept as small as possible. That is, the usage of $\times$ -probes is recommended over $\vee$ -probes (Philip *et al.*, 2013b), at least in wall-bounded flows. This is because, at a comparable wire length and wire separation dimensions (i.e., a majority of dual hot-wire probes used in experiments), the errors due to non-zero wire separation dominate.
2. Misalignment of the sensor plane (a plane parallel to both hot-wires) and the streamwise–spanwise (x - y) or streamwise–wall-normal (x - z) planes should be avoided. Thus, the usage of inclined generic probe to allow near-wall access is strongly discouraged, as this can lead to a substantial error, up to 10 % for the turbulent stresses, to be introduced in the log region and beyond, until half the boundary layer thickness (Baidya *et al.*, 2019). Instead, a custom $\times$ -probe, where the sensor plane can be designed to accommodate the inclination angle, such that it remains parallel to the x - y or x - z planes should be considered.
3. A well-designed calibration procedure is vital. The Reynolds shear stress is found to be extremely sensitive (more than twice the other stresses) to any uncertainties introduced during the calibration procedure (Baidya *et al.*, 2019). In this regard, an *in situ* two-dimensional calibration conducted with an articulated probe holder in the freestream region of the wind tunnel (e.g., Österlund, 1999) may be considered the best practise. For a probe holder restricted to traversing in the wall-normal direction, an *in situ* two-dimensional calibration using an articulated jet flow combined with an additional zero angle calibration in the freestream region of the wind tunnel to account for any angular misalignment between the jet and the wind tunnel (e.g., Baidya *et al.*, 2017) is recommended.

4. Spurious measurements of the spanwise and wall-normal velocities due to finite sensor dimensions can lead to amplified variances (see figure 10b, where an overestimation in excess of 100 % is recorded in the near-wall region). This is in contrast to the streamwise velocity, where the finite dimension effects result in attenuated variances.
5. A good prediction of the finite sensor effects can be obtained from a simulated probe utilising direct numerical simulation database (at a lower Re) with matched parameters in viscous units. In the current study, it allowed the dominant sources of errors to be separated between intrinsic limitations due to a particular sensor dimensions for the normal stresses, and the result of uncertainties in the probe geometry and calibration procedure for the Reynolds shear stress.
6. Although in a time-averaged sense, the Reynolds shear stress is more robust to the finite dimension effects compared to the normal stresses, the instantaneous Reynolds shear stress signal increasingly deviates from the original signal for the larger dimension probes. This needs to be taken into account if one is interested in triple or higher order correlations requiring the streamwise and wall-normal velocities.

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A Comparisons against u - v $\times$ -probe experiments

Figure B1 compares deviations in $\overline{v^2}$ statistic observed from experiments, against predictions from the spatial filtering procedure outline here applied to a u - v $\times$ -probe (see Philip *et al.* (2013b) for details).

B Spatial filtering for an inclined hot-wire

In this section, the two-point correlation behaviour in the homogeneous and inhomogeneous plane is investigated. The similarities and differences between the two are examined, to develop a better understanding of spatial filtering effects in the inhomogeneous plane.

B.1 Spatial filtering along the homogeneous direction

For a case when the inclined wire lies in a *homogeneous* plane (x - y plane in a wall-bounded flow) $\mathcal{R}_{N_{ij}}$ in (7) is only

Fig. B1: As figure 11, but for $\overline{v^2}$ statistic measured using a u - v $\times$ -probe.

dependent on r . This is illustrated in figure B2, where a map of $\mathcal{R}_{N_{uu}}$ for two points on the inclined wire which lie on the x - y plane (at $z^+ \approx 15$) is shown. Hence in the x - y plane,

$$\int_{-l}^l \int_{-l/2-r}^{l/2-r} \mathcal{R}_{N_{ij}}(\xi, r) d\xi dr = \int_{-l}^l \mathcal{R}_{N_{ij}}(r) \left(\int_{-l/2-r}^{l/2-r} 1 d\xi \right) dr. \quad (\text{B.1})$$

Further, both $\sigma(u_{i,A})$ and $\sigma(u_{j,B})$ are independent of ξ and r in the x - y plane, and therefore equation (7) simplifies to

$$\begin{aligned} \frac{\langle \tilde{u}_i \tilde{u}_j \rangle}{\overline{u_i u_j}_{\text{DNS}}} &= \frac{1}{l^2} \frac{\sigma(u_i) \sigma(u_j)}{\overline{u_i u_j}_{\text{DNS}}} \int_{-l}^l \mathcal{R}_{N_{ij}}(r) \left(\int_{-l/2-r}^{l/2-r} 1 d\xi \right) dr, \\ &= \frac{1}{l^2 \rho_{ij\text{DNS}}} \int_{-l}^l \mathcal{R}_{N_{ij}}(r_x, r_y) (l - |r|) dr, \end{aligned} \quad (\text{B.2})$$

where r_x and r_y correspond to the displacement in the x and y direction between two points separated by r along the wire, while ρ_{ij} denote the turbulent stress correlation coefficients, i.e. $\rho_{ij} = \overline{u_i u_j} / (\sigma(u_i) \sigma(u_j))$. For a single normal hot-wire measuring the streamwise turbulence intensity, equation (B.2) reduces to the form obtained by Dryden *et al.* (1937).

B.2 Spatial filtering along the inhomogeneous direction

For an inclined wire which lies in an *inhomogeneous* plane, due to flow heterogeneity along the sensing element, the correlation map between the two points on the wire becomes dependent on z as well as the r location where the correlation is evaluated. To illustrate this point, figure B3 shows the $\mathcal{R}_{N_{uu}}$ map for two points on the inclined wire which lie on the x - z plane, calculated using the DNS database (for $z_c^+ \approx 15$, where z_c is the wall-normal location of the wire mid-point). Due to flow heterogeneity along the sensing element, the isocontours shown in figures B3(a) and (b) no longer follow the $r = \text{constant}$ lines, unlike for the homogeneous case. However, the departure from a $\mathcal{R}_{N_{uu}}$ which is independent of ξ (i.e. function only of r) due to inhomogeneity turns out to be relatively small ($\sim 10\%$ in the case of figure B3). Thus,

the two-point correlation function at the wall location $z + \delta z$ may be modelled to a good approximation by only considering the linear term in the Taylor expansion. That is,

$$\mathcal{R}_{N_{ij}}(r) \Big|_{z_A + \delta z} \approx \mathcal{R}_{N_{ij}}(r) \Big|_z + \frac{\partial \mathcal{R}_{N_{ij}}(r)}{\partial z} \Big|_z \delta z \quad (\text{B.3})$$

for a small wall-normal offset, δz .

Let us now reconsider the term $\langle u_{i,A} u_{j,B} \rangle$ in (6) (point A and B are located at a distance ξ and ζ from the mid-point of the inclined wire). Since, $\langle u_{i,A} u_{j,B} \rangle = R_{ij}(\zeta - \xi) \Big|_{z=z_A} = R_{ji}(\xi - \zeta) \Big|_{z=z_B}$, based on (B.3)

$$\begin{aligned} \frac{\langle u_{i,A} u_{j,B} \rangle}{\overline{u_i u_j}_{\text{DNS}}} &\approx \frac{1}{2 \rho_{ij\text{DNS}}} \left[\left(1 + \mathcal{A}_j(r_z) \right) \mathcal{R}_{N_{ij}}(r) \Big|_{z=z_c} + \right. \\ &\quad \left(1 + \mathcal{A}_i(-r_z) \right) \mathcal{R}_{N_{ji}}(-r) \Big|_{z=z_c} + \left(1 + \mathcal{A}_j(r_z) \right) \frac{\partial \mathcal{R}_{N_{ij}}(r)}{\partial z} \Big|_{z=z_c} \xi_z \\ &\quad \left. + \left(1 + \mathcal{A}_i(-r_z) \right) \frac{\partial \mathcal{R}_{N_{ji}}(-r)}{\partial z} \Big|_{z=z_c} \zeta_z \right], \end{aligned} \quad (\text{B.4})$$

where, $r = \zeta - \xi$, while $\xi_z = -\xi \cos \theta$ and $\zeta_z = -\zeta \cos \theta$ correspond to the wall-normal component of ξ and ζ , respectively. Furthermore,

$$\mathcal{A}_i(r_z) = \frac{\sigma(u_i(z_c + r_z)) - \sigma(u_i(z_c))}{\sigma(u_i(z_c))}, \quad (\text{B.5})$$

where, $\sigma(u_i(z))$ denotes the standard deviation of $u_i(z)$.

Figure B4 show a linearly approximated $\mathcal{R}_{N_{uu}}$ for the same inclined wire for which an exact $\mathcal{R}_{N_{uu}}$ is shown in figure B3(b). The estimates obtained from the linear approximation are observed to be good, and small deviations that exist are confined to the extreme r values. Furthermore, although not shown here for brevity, similar result holds for $\mathcal{R}_{N_{ww}}$. Compared to $\mathcal{R}_{N_{uu}}$ and $\mathcal{R}_{N_{ww}}$, the normalised cross-correlation between the u and w velocities (cf. figure B5a) display a more pronounced departure from the two-point correlation behaviour seen in a homogeneous plane, and consequently the errors associated with the linearly approximated $\mathcal{R}_{N_{uw}}$, shown in figure B5(b) is larger compared to the autocorrelation counterparts.

Fig. B2: Normalised autocorrelation function map of u from two points residing on an inclined wire ($\theta = 45^\circ$) in the streamwise–spanwise plane at $z_c^+ \approx 15$. The u autocorrelations are shown as function of (a) the distances ξ and ζ which defines the two points from the centre of the wire and (b) ξ and the relative distance between the two points r ($r = \zeta - \xi$). The dash, dot-dashed and solid lines correspond to the contour levels 0.75, 0.85 and 0.95, respectively.

Fig. B3: Normalised autocorrelation function map of u , $\mathcal{R}_{Nuu}$, from two points residing on an inclined wire ($\theta = -45^\circ$) in the streamwise–wall-normal plane at $z_c^+ \approx 15$ (location of the centre of wire). The u autocorrelations are shown as function of (a) ξ and ζ and (b) ξ and r ($r = \zeta - \xi$). The dash, dot-dashed and solid lines correspond to the contour levels 0.75, 0.85 and 0.95, respectively.

Substitute (B.4) into (7) to obtain

$$\begin{aligned}
 \frac{\langle \widetilde{u}_i \widetilde{u}_j \rangle}{\overline{u_i u_j}_{\text{DNS}}} &= \frac{1}{2l^2 \rho_{ij\text{DNS}}} \left[\overbrace{\int_{-l}^l (\mathcal{R}_{Nij}(r) + \mathcal{R}_{Nji}(-r))(l - |r|) dr}^{\text{I}} \right. \\
 &+ \overbrace{\int_{-l}^l (\mathcal{A}_j(r_z) \mathcal{R}_{Nij}(r) + \mathcal{A}_i(-r_z) \mathcal{R}_{Nji}(-r))(l - |r|) dr}^{\text{II}} \\
 &+ \overbrace{\int_{-l}^l \left(1 + \mathcal{A}_j(r_z)\right) \frac{\partial \mathcal{R}_{Nij}(r)}{\partial z} \bigg|_{z=z_c} \left(\int_{-l/2-r}^{l/2-r} \xi_z d\xi \right) dr}^{\text{IIIa}} \\
 &- \overbrace{\int_{-l}^l \left(1 + \mathcal{A}_i(-r_z)\right) \frac{\partial \mathcal{R}_{Nji}(-r)}{\partial z} \bigg|_{z=z_c} \left(\int_{-l/2-r}^{l/2-r} \zeta_z d\zeta \right) dr}^{\text{IIIb}} \\
 &\left. + \overbrace{\mathcal{O}(l^2)}^{\text{IV}} \right] \quad (\text{B.6})
 \end{aligned}$$

Figures B6(a)–(c) show the relative magnitudes of terms I–IV in (B.6). Here, symbol $\blacktriangle$ show contribution from $\mathcal{O}(l^2)$ terms and therefore corresponds to errors as a result of the linear approximation. The $\mathcal{O}(l^2)$ errors for the Reynolds shear

stress are significantly larger than that obtained for the u and w variances, but this is to be expected based on the differences in errors resulting from the linear approximation observed in figures B2, B4 and B5. Further, symbols $\bullet$, $\blacksquare$ and $\blacklozenge$ in figure B6 denote contribution of terms I–III, respectively.

For a case, when $\mathcal{A}_i(r_z) = 0$ and $\partial \mathcal{R}_{Nij} / \partial z = 0$, terms II–IV in (B.6) banishes, and therefore term I is equivalent to the expression obtained for filtered turbulent stress statistics in a homogeneous flow. From comparison between figures B2 and B3, it is evident that for the streamwise component the two points residing on an inclined single wire are correlated with each other to a similar degree regardless of the wire being oriented in the x - y or x - z plane. Hence, the contributions from term I ($\bullet$) and the filtered u statistic from an equivalent wire in the x - y plane (denoted by dotted lines) are similar, as shown in figure B6(a). Thus, as is the case for the inclined wire in the homogeneous plane, a higher degree of correlation between the velocities that are being spatially filtered along the sensing element leads to a smaller deviation between the filtered and original statistics. Hence, the attenuation of $\langle \widetilde{u} \widetilde{w} \rangle$ is smaller compared to that observed for

Fig. B4: Linear approximation of $\mathcal{R}_{N_{uw}}$ shown in figure B3b (an identical colour map is used).

Fig. B5: Normalised cross-correlation function map of u and w , $\mathcal{R}_{N_{uw}}$, from two points residing on an inclined wire ($\theta = -45^\circ$) in the streamwise–wall-normal plane at $z_c^+ \approx 15$ (location of the centre of wire). (a) and (b) show comparison between the exact $\mathcal{R}_{N_{uw}}$ and its linear approximation. The dash, dot-dashed and solid lines correspond to the contour levels 0.35, 0.425 and 0.5, respectively.

the $\langle \tilde{u}^2 \rangle$ and $\langle \tilde{w}^2 \rangle$ statistics, for a given l^+ (see § 3 for further discussions).

Term II in (B.6) is associated with flow heterogeneity that arises due to varying $\overline{u_i u_j}$ but an unaltered $\mathcal{R}_{N_{ij}}$ along the sensing element. In this scenario, the degree to which the two points residing on the sensing element are correlated with each other remains the same, for a given spacing, despite the presence of flow heterogeneity. To illustrate the

Fig. B6: Relative contributions from terms I - IV in (B.6) to (a) $\langle \tilde{u}^2 \rangle$, (b) $\langle \tilde{w}^2 \rangle$ and (c) $-\langle \tilde{u} \tilde{w} \rangle$ for an inclined hot-wire in the x - z plane, with $\theta = -45^\circ$ and at $z_c^+ \approx 15$. The symbols $\bullet$, $\blacksquare$, $\blacklozenge$ and $\blacktriangle$ denote contributions from terms I - IV, respectively, while the shaded regions correspond to the total contribution from the four terms. The dotted line in (a) shows the total contributions for an equivalent inclined wire in the x - y plane.

effect of the wall-normal variation in $\overline{u_i u_j}$, consider a case when the centre of an inclined wire probe coincides with a local minimum in $\overline{u^2}$ (as function of z) as illustrated in figure B7. Since, the filtered velocity along the wire includes more energetic velocity from the neighbouring z locations in the spatial averaging process, this leads to a filtered statistic that exceeds the original DNS statistic at the z location corresponding to the wire centre. For all three turbulent stresses

Fig. B7: The effect of flow heterogeneity along the sensor on the measured $\overline{u^2}$. The solid line corresponds to the original u^2 , which varies along the sensor of length l , while the dashed line corresponds to the filtered statistics measured by the sensor. Since the measured statistic is attributed to be at the mid-point of the sensor (indicated by the symbol ●), the measured $\overline{u^2}$ (◻) exceeds the original value (<) when u^2 has a local minimum.

considered in figure B6, the variation in $\overline{u_i u_j}$ with z leads to a further decrease in the filtered statistics. For $\langle \tilde{u}^2 \rangle$ and $\langle \tilde{w}^2 \rangle$, inclusion of term II leads to a better estimate of the filtered statistics, however the attenuation of $\langle \tilde{u} \tilde{w} \rangle$ is over predicted since substantial changes in $\mathcal{R}_{N_{uw}}$ that occur along the sensing element remain unaccounted for in term II.

Term III (sum of terms IIIa and IIb in equation B.6) arises due to variations in $\mathcal{R}_{N_{ij}}$ from the wall-normal heterogeneity, which are accounted for by $\partial \mathcal{R}_{N_{ij}}(r_x, r_z) / \partial z$ terms. The near zero value for ♦ in figure B6(a) suggests that insignificant contribution to the u variances occurs from term III. This is consistent with the observation that for a sensor of $O(10)$ viscous unit in length, $\mathcal{R}_{N_{uu}}$ as a function of r at all points on the sensor (even in the inhomogeneous direction) is well approximated by the normalised correlation function at the centre of the wire, as illustrated in figure B3(b) (i.e. $\partial \mathcal{R}_{N_{uu}} / \partial z \approx 0$). Similarly, for the $\langle \tilde{w}^2 \rangle$ statistic, the contribution from the $\partial \mathcal{R}_{N_{ww}} / \partial z$ term is observed to be near zero. However, a non negligible contribution from the $\partial \mathcal{R}_{N_{uw}} / \partial z$ terms exist for the $\langle \tilde{u} \tilde{w} \rangle$ statistic.

C Expressions for measured turbulent stresses using an x -probe

Here, details of an adapted procedure from Philip *et al.* (2013a) applied to a u - w x -probe is presented.

We start from an equivalent expression for equation (3) applicable to the u - w x -probe and set $\theta = 45^\circ$. Thus, the

measured turbulent stress terms can be expressed as

$$\overline{u^2} = \frac{1}{4} \left\langle \left(\tilde{u}_{w1} + \tilde{u}_{w2} \right)^2 + 2 \left(\tilde{u}_{w1} + \tilde{u}_{w2} \right) \left(\tilde{w}_{w1} - \tilde{w}_{w2} \right) + \left(\tilde{w}_{w1} - \tilde{w}_{w2} \right)^2 \right\rangle, \quad (\text{C.1})$$

$$\overline{w^2} = \frac{1}{4} \left\langle \left(\tilde{u}_{w1} - \tilde{u}_{w2} \right)^2 + 2 \left(\tilde{u}_{w1} - \tilde{u}_{w2} \right) \left(\tilde{w}_{w1} + \tilde{w}_{w2} \right) + \left(\tilde{w}_{w1} + \tilde{w}_{w2} \right)^2 \right\rangle, \quad (\text{C.2})$$

and

$$\overline{uw} = \frac{1}{4} \left\langle \left(\tilde{u}_{w1} + \tilde{u}_{w2} + \tilde{w}_{w1} - \tilde{w}_{w2} \right) \left(\tilde{u}_{w1} - \tilde{u}_{w2} + \tilde{w}_{w1} + \tilde{w}_{w2} \right) \right\rangle. \quad (\text{C.3})$$

Due to unsymmetrical nature of the correlation functions in the streamwise-wall-normal plane, the filtered turbulent stresses along the wire inclined at $\theta = -45^\circ$ and $\theta = 45^\circ$ are not identical, however for $l^+ \sim O(10)$

$$\langle \tilde{u}_{w1} \tilde{u}_{w1} \rangle \approx \langle \tilde{u}_{w2} \tilde{u}_{w2} \rangle \approx \langle \tilde{u}^2 \rangle \quad (\text{C.4})$$

holds, and in the similar manner for the wall-normal variance and the Reynolds shear stress as shown in figures 3(a)-(c). Furthermore, the cross terms between the two wires can be denoted in terms of the two-point correlation function of filtered velocities as

$$\langle \tilde{u}_{w1} \tilde{u}_{w2} \rangle = \tilde{R}_{uu}(\Delta s_y). \quad (\text{C.5})$$

Now, expand equation (C.1)-(C.3) to obtain

$$\overline{u^2} \approx \frac{1}{2} \left[\langle \tilde{u}^2 \rangle + \langle \tilde{w}^2 \rangle + \tilde{R}_{uu}(\Delta s_y) - \tilde{R}_{ww}(\Delta s_y) \right], \quad (\text{C.6})$$

$$\overline{w^2} \approx \frac{1}{2} \left[\langle \tilde{u}^2 \rangle + \langle \tilde{w}^2 \rangle - \tilde{R}_{uu}(\Delta s_y) + \tilde{R}_{ww}(\Delta s_y) \right], \quad (\text{C.7})$$

and

$$\overline{uw} \approx \langle \tilde{u} \tilde{w} \rangle. \quad (\text{C.8})$$

The R.H.S of equation (C.8), unlike (C.6) and (C.7) contains no correlation terms due to fortuitous cancellation in the expansion process, while the terms $\langle \tilde{u}_{w1}^2 - \tilde{u}_{w2}^2 \rangle$ and $\langle \tilde{w}_{w1}^2 - \tilde{w}_{w2}^2 \rangle$ are near zero since equal and opposite contribution from the two wires occurs for $l^+ \sim O(10)$.

Equation (C.6) when normalised by the original streamwise variance from DNS at the centre of the u - w x -probe

$(\overline{u^2}_{\text{DNS}})$, can be expressed as

$$\frac{\overline{u^2}}{\overline{u^2}_{\text{DNS}}} \approx \frac{1}{2} \left[\frac{\langle \widetilde{u^2} \rangle}{\overline{u^2}_{\text{DNS}}} + \frac{\langle \widetilde{w^2} \rangle}{\overline{w^2}_{\text{DNS}}} \frac{\overline{w^2}_{\text{DNS}}}{\overline{u^2}_{\text{DNS}}} + \frac{\widetilde{R}_{uu}(\Delta s_y) \langle \widetilde{u^2} \rangle}{\langle \widetilde{u^2} \rangle \overline{u^2}_{\text{DNS}}} - \frac{\widetilde{R}_{ww}(\Delta s_y) \langle \widetilde{w^2} \rangle}{\langle \widetilde{w^2} \rangle \overline{w^2}_{\text{DNS}}} \frac{\overline{w^2}_{\text{DNS}}}{\overline{u^2}_{\text{DNS}}} \right], \quad (\text{C.9})$$

$$\approx \frac{1}{2} \left[F_{uu} + F_{ww} \frac{\overline{w^2}_{\text{DNS}}}{\overline{u^2}_{\text{DNS}}} + \widetilde{R}_{N_{uu}}(\Delta s_y) F_{uu} - \widetilde{R}_{N_{ww}}(\Delta s_y) \frac{\overline{w^2}_{\text{DNS}}}{\overline{u^2}_{\text{DNS}}} F_{ww} \right], \quad (\text{C.10})$$

$$\approx F_{uu} \left[\frac{1}{2} + \frac{1}{2} \widetilde{R}_{N_{uu}}(\Delta s_y) + \left(\frac{1}{2} - \frac{1}{2} \widetilde{R}_{N_{ww}}(\Delta s_y) \right) \frac{\overline{w^2}_{\text{DNS}}}{\overline{u^2}_{\text{DNS}}} \frac{F_{ww}}{F_{uu}} \right] \quad (\text{C.11})$$

$$\approx F_{uu} \left[1 - \frac{1}{2} (1 - \widetilde{R}_{N_{uu}}(\Delta s_y)) + \frac{1}{2} (1 - \widetilde{R}_{N_{ww}}(\Delta s_y)) \frac{\langle \widetilde{w^2} \rangle}{\langle \widetilde{u^2} \rangle} \right], \quad (\text{C.12})$$

where

$$F_{uu} = \frac{\langle \widetilde{u^2} \rangle}{\overline{u^2}_{\text{DNS}}}, \quad \text{and} \quad F_{ww} = \frac{\langle \widetilde{w^2} \rangle}{\overline{w^2}_{\text{DNS}}}, \quad (\text{C.13})$$

and

$$\widetilde{R}_{N_{uu}} = \frac{\widetilde{R}_{uu}}{\langle \widetilde{u^2} \rangle}, \quad \text{and} \quad \widetilde{R}_{N_{ww}} = \frac{\widetilde{R}_{ww}}{\langle \widetilde{w^2} \rangle}. \quad (\text{C.14})$$

F_{uu} and F_{ww} are the ratios of spatially filtered variance to the original variance accounting for attenuation due to wire length, while $\widetilde{R}_{N_{uu}}$ and $\widetilde{R}_{N_{ww}}$ are normalised correlations between the spatially filtered velocities over the wire length l . It should be noted that in a wall-bounded flow, $\overline{w^2}/\overline{u^2}$ is of order 0.1, and therefore the contribution from the term $[1 - \widetilde{R}_{N_{ww}}(\Delta s_y)] [\langle \widetilde{w^2} \rangle / \langle \widetilde{u^2} \rangle]$ in equation (C.12) is insignificant and can be neglected (Philip *et al.*, 2013b). Similar expressions can be obtained for the normalised $\overline{w^2}$ and $\overline{uw}$ statistics, given by

$$\frac{\overline{w^2}}{\overline{w^2}_{\text{DNS}}} \approx F_{ww} \left[1 - \frac{1}{2} (1 - \widetilde{R}_{N_{ww}}(\Delta s_y)) + \frac{1}{2} (1 - \widetilde{R}_{N_{uu}}(\Delta s_y)) \frac{\langle \widetilde{u^2} \rangle}{\langle \widetilde{w^2} \rangle} \right], \quad (\text{C.15})$$

and

$$\frac{\overline{uw}}{\overline{uw}_{\text{DNS}}} \approx F_{uw}. \quad (\text{C.16})$$

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