Using a New Entropy Loss Analysis to Assess the Accuracy of RANS Predictions of an HPT Vane

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ABSTRACT

Entropy loss is widely used to quantify the efficiency of components in turbomachines, and empirical relations have been developed to estimate the contribution of different mechanisms. However, further analysis is still needed to not only get a deeper insight of the physics, but also to more accurately quantify the loss generation caused by different terms. In the present study, the entropy transport equations based on averaged flow quantities are first derived, and the entropy generation process is fully decomposed into several terms representing different physical mechanisms, such as mean viscous dissipation, turbulence production, mean and turbulent heat flux, etc. This decomposition framework is then applied to high-resolution LES and RANS results of a VKI LS-89 HPT vane, and a detailed quantification of different entropy generation terms is obtained. The results show that the entropy generation caused by mean flow features like mean viscous dissipation and mean heat flux are in close agreement between LES and RANS, indicating that RANS provides an overall good prediction for the mean flow. Furthermore, we find that turbulence production plays an important role in entropy generation as it represents the energy extracted from the mean flow to

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turbulent fluctuations. However, the difference between RANS and LES results for the turbulence production term is not negligible, particularly in the wake region. This implies that the failure of RANS to predict the correct total loss might be largely caused by errors in capturing the correct turbulence production in the near wake region.

INTRODUCTION

An efficient use of energy is one of the most important goals for designing modern turbomachines. Based on the second law of thermodynamics, entropy generation is widely used to represent loss of efficiency [1, 2]. Generally, viscous effects and irreversible heat flux are taken as major sources for entropy generation in turbomachines, and various empirical relations have been derived to estimate the entropy generated from different origins such as the viscous effects in the boundary layer, viscous effects in wake mixing, and heat transfer across temperature differences [3]. However, the accuracy of these correlation based methods is limited in many cases. Therefore, further analysis is still needed not only to get a deeper insight of the physics, but also to more accurately quantify the loss generation caused by different terms.

Thanks to the fast development of numerical algorithms, turbulence modelling techniques, and computational power in the past decades, computational fluid dynamics (CFD), despite some limitations, is playing a more and more important role in turbomachine designs [4]. Calculating entropy generation based on CFD flow fields can provide not only the total entropy variation, but also the spatial distribution of local entropy generation. Therefore, CFD analysis promises to be a valuable tool for the prediction of efficiency in turbomachinery components, which is very useful for the design community [5, 6].

In previous studies, different methods have been proposed to quantify entropy generation mechanisms based on CFD calculations. Moore and Moore [7] first proposed a method to calculate mean entropy generation with viscous dissipation and increase of turbulent kinetic energy (TKE), and the method was then applied, showing satisfactory results for entropy generation in a two-dimensional turbulent boundary layer. Camberos [8] presented a strategy to derive entropy balance equations for general thermodynamic processes based on the second law, and entropy generation in instantaneous flow fields was presented. Reynolds averaging techniques were applied to the instantaneous entropy generation equations, and the modelling of turbulence closures in entropy transport equations for viscous, incompressible flows was proposed [9]. More recently, the entropy generation in a high-pressure turbine (HPT) vane was obtained by post-processing direct numerical simulation (DNS) data, and the individual contribution from the different sources were quantified [10]. Lengani et al. [11] derived an entropy balance equation expressed as a

function of total quantities, and a proper orthogonal decomposition procedure was applied to a large-eddy simulation (LES) of a low-pressure turbine (LPT) to determine the dominating modes for the entropy generation. It is noted that the entropy breakdown closures derived from previous studies have been presented in various forms. The lack of consistency, however, limits the applicability of the entropy analysis.

Here, we present a complete derivation of the entropy balance equation for compressible flows in turbomachines. The closure is derived from basic flow governing equations and thermodynamic laws, which guarantees the framework can be applied to both high-fidelity data and RANS modelling results. Applying this entropy analysis to a LS-89 HPT vane, the accuracy of RANS predictions will be assessed based on high-resolution LES data. The error of RANS modelling for turbulence closures are quantified based on entropy generation, and the major error source within the vane is located.

ENTROPY BALANCE EQUATIONS

Based on the second law of thermodynamics, the total entropy S of a system satisfies

$$\dot{S} + I_S = \int_V G_s dV \geq 0, \quad (1)$$

where $\dot{S}$ is the time-rate-of-change of the system entropy, I_S is the entropy current with external environment, and the right-hand side term represents the entropy generation rate G_s integrated over the volume V [8]. For most thermodynamic processes, the generation of total entropy is greater than zero, while the equivalent holds only when the process is reversible.

For single phase fluid flows, the entropy balance equation can be written as

$$\frac{\partial(\rho s)}{\partial t} + \frac{\partial F_i}{\partial x_i} \equiv G_s \geq 0. \quad (2)$$

Here, ρ is density of the local fluid, s is the specific entropy, and F_i is the entropy flux caused by mass flux and heat transfer [9]. It is noted that the specific form of F_i and G_s depends on the governing equations for the flow. For example, the entropy generation mechanisms for viscous or inviscid flows will be significantly different.

In the following, we will present the derivation of entropy generation equations for turbine flows.

Entropy Generation with Navier-Stokes Equations

This part presents the process to derive the local entropy balance equation based on the three dimensional compressible Navier-Stokes (NS) equations. First of all, the conservation of mass, momentum and energy can be written as

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u_j)}{\partial x_j} = 0, \quad (3)$$

$$\frac{\partial(\rho u_i)}{\partial t} + \frac{\partial(\rho u_i u_j + p \delta_{ij})}{\partial x_j} = \frac{\partial \sigma_{ij}}{\partial x_j}, \quad (4)$$

$$\frac{\partial(\rho e_0)}{\partial t} + \frac{\partial[u_j(\rho e_0 + p)]}{\partial x_j} = \frac{\partial(\sigma_{ij} u_i)}{\partial x_j} - \frac{\partial q_j}{\partial x_j}, \quad (5)$$

with u_i , p , e_0 representing velocity components, pressure, and specific energy, respectively. Here, the fluid is assumed to be Newtonian with second viscosity set to $-2/3\mu$, so that the viscous stress is written as

$$\sigma_{ij} = \mu \left(-\frac{2}{3} \frac{\partial u_k}{\partial x_k} \delta_{ij} + \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right). \quad (6)$$

Moreover, using the Fourier law, the heat flux vector components are given by

$$q_j = -\frac{c_p \mu}{Pr} \frac{\partial T}{\partial x_j}, \quad (7)$$

where c_p is specific heat at constant pressure, and Pr is the Prandtl number.

Subtracting the kinetic energy from the total energy equation, *i. e.* (5) $- u_i \times$ (4) $+ \frac{1}{2} u_i u_i \times$ (3), the

equation for the internal energy $e = e_0 - 1/2 u_i u_i$ can be derived as

$$\frac{\partial(\rho e)}{\partial t} + \frac{\partial(\rho u_j e)}{\partial x_j} + p \frac{\partial u_j}{\partial x_j} = \sigma_{ij} \frac{\partial u_i}{\partial x_j} - \frac{\partial q_j}{\partial x_j}, \quad (8)$$

Combining the first law of thermodynamics

$$de = T ds - p d \frac{1}{\rho} \quad (9)$$

with Eqn. (8), the entropy balance equation can be derived

$$\begin{aligned} \frac{\partial(\rho s)}{\partial t} + \frac{\partial(\rho u_i s)}{\partial x_i} + \frac{\partial}{\partial x_i} \left(\frac{q_i}{T} \right) \\ = \frac{1}{T} \sigma_{ij} \frac{\partial u_j}{\partial x_i} + \frac{c_p \mu}{Pr} \frac{1}{T^2} \left(\frac{\partial T}{\partial x_i} \right)^2. \end{aligned} \quad (10)$$

Compared to the generalized entropy balance equation Eqn. (2), the entropy flux for compressible viscous flow is caused by mass flux and heat flux as

$$\frac{\partial F_i}{\partial x_i} = \underbrace{\frac{\partial(\rho u_i s)}{\partial x_i}}_{\text{mass flux}} + \underbrace{\frac{\partial}{\partial x_i} \left(\frac{q_i}{T} \right)}_{\text{heat flux}}, \quad (11)$$

and the entropy generation mechanisms are related to viscous dissipation and irreversible heat transfer

$$G_s^{NS} = \underbrace{\frac{1}{T} \sigma_{ij} \frac{\partial u_j}{\partial x_i}}_{\text{vis. dissip.}} + \underbrace{\frac{c_p \mu}{Pr} \frac{1}{T^2} \left(\frac{\partial T}{\partial x_i} \right)^2}_{\text{heat trans.}}. \quad (12)$$

We note that the above term G_s^{NS} applies to the local entropy generation in the instantaneous flow fields governed by Navier-Stokes equations. Taking the average of the instantaneous entropy generation Eqn. (12), we can decompose the viscous effect into mean flow viscous dissipation and turbulent dissipa-

tion [10]. However, other terms related to turbulent fluctuations, such as the turbulent heat flux, cannot be directly derived. Furthermore, due to the extremely complex environment in turbomachines, flows experience highly unsteady, non-equilibrium thermodynamic processes through turbine vanes, particularly with strong turbulence and non-uniform distribution of the fluid. Therefore, a volume integral of the local entropy generation term in Eqn. (12) might be biased for the total entropy generation of the whole system.

Entropy Loss with Favre-Averaged NS Equations

The efficiency of turbomachines is generally assessed in a time-averaged sense. Therefore, the entropy generation should be calculated based on mean flow fields, *i. e.* RANS results or averaged fields from high-fidelity data. Accordingly, the Favre-averaged conservation equations [12] need to be considered.

$$\frac{\partial \bar{\rho}}{\partial t} + \frac{\partial(\bar{\rho} \tilde{u}_j)}{\partial x_j} = 0, \quad (13)$$

$$\frac{\partial(\bar{\rho} \tilde{u}_i)}{\partial t} + \frac{\partial(\bar{\rho} \tilde{u}_i \tilde{u}_j + \bar{p} \delta_{ij})}{\partial x_j} = \frac{\partial \sigma_{ij}(\tilde{u})}{\partial x_j} + \frac{\partial \tau_{ij}^R}{\partial x_j}, \quad (14)$$

$$\begin{aligned} & \frac{\partial}{\partial t} (\bar{\rho} \tilde{e} + \frac{1}{2} \bar{\rho} \tilde{u}_i \tilde{u}_i + \bar{\rho} k) + \frac{\partial}{\partial x_j} [\tilde{u}_j (\bar{\rho} \tilde{e} + \frac{1}{2} \bar{\rho} \tilde{u}_i \tilde{u}_i + \bar{\rho} k + \bar{p})] \\ &= \frac{\partial}{\partial x_j} (\tilde{u}_i \sigma_{ij}(\tilde{u}) + \tilde{u}_i \tau_{ij}^R) - \frac{\partial}{\partial x_j} (\tilde{q}_j + q_j^R) \\ &+ \frac{\partial \mathcal{F}_j}{\partial x_j} + \frac{\partial}{\partial x_j} [\mathcal{V}]. \end{aligned} \quad (15)$$

Here, $\bar{\mathcal{C}}$ is the time-average of $\mathcal{C}$, and $\tilde{\mathcal{C}}$ denotes a Favre-averaged quantity as

$$\tilde{\mathcal{C}} = \frac{\bar{\rho \mathcal{C}}}{\bar{\rho}}. \quad (16)$$

The Reynolds stress is

$$\tau_{ij}^R = -\bar{\rho}(\widetilde{u_i u_j} - \widetilde{u_i} \widetilde{u_j}), \quad (17)$$

and the turbulent heat flux is

$$q_j^R = \bar{\rho} c_p (\widetilde{T u_j} - \widetilde{T} \widetilde{u_j}). \quad (18)$$

These two terms represent the contribution of fluctuations to the mean flow, which are typically modelled in RANS calculations. Furthermore, the term $\mathcal{V}$ is the viscous effect related to fluctuation velocities as

$$\mathcal{V} = \overline{u_i'' \sigma_{ij}}(\widetilde{u}) + \widetilde{u_i} \sigma_{ij}(\overline{u''}), \quad (19)$$

which is negligible for most cases. Moreover, the term $\mathcal{F}_j$ is the total flux for the TKE $k = 0.5 \overline{\rho u_i'' u_i''} / \bar{\rho}$, which can be decomposed as

$$\mathcal{F}_j = -\overline{\rho u_j'' \frac{1}{2} u_i'' u_i''} - \overline{p' u_i'' \delta_{ij}} + \overline{u_i'' \sigma_{ij}(u'')}. \quad (20)$$

Here, $u_i'' = u_i - \widetilde{u_i}$, and $p' = p - \bar{p}$. Note that the volume integral of $\int_V \frac{\partial \mathcal{F}_j}{\partial x_j} dV$ is usually negligible in turbine flows, so that the TKE flux terms can be neglected for the entropy calculations in the present study.

For time-averaged flow fields, the time derivative terms in Eqns. (13), (14) and (15) equal to zero, and we can assume that the mean flow governed by the equations satisfies the first thermodynamic law as

$$d\tilde{e} = \tilde{T} d\tilde{s} - \bar{p} d\frac{1}{\bar{\rho}}. \quad (21)$$

Furthermore, discarding the negligible terms with $\mathcal{V}$ and $\mathcal{F}_j$ in Eqn. (15), we can derive the entropy balance

equation for the mean flow via a similar process for the instantaneous equation

$$\begin{aligned} \frac{\partial(\bar{\rho}\tilde{u}_i\tilde{s})}{\partial x_i} + \frac{\partial}{\partial x_i}\left(\frac{\tilde{q}_i}{\tilde{T}}\right) &= \frac{1}{\tilde{T}}\sigma_{ij}(\tilde{u})\frac{\partial\tilde{u}_j}{\partial x_i} + \frac{c_p\mu}{Pr}\frac{1}{\tilde{T}^2}\left(\frac{\partial\tilde{T}}{\partial x_i}\right)^2 \\ &+ \frac{1}{\tilde{T}}\tau_{ij}^R\frac{\partial\tilde{u}_j}{\partial x_i} - \frac{1}{\tilde{T}}\frac{\partial}{\partial x_i}(q_i^R) - \frac{1}{\tilde{T}}\frac{\partial(\bar{\rho}\tilde{u}_ik)}{\partial x_i}. \end{aligned} \quad (22)$$

Similar to Eqn. (11), the left-hand side terms represent entropy variations caused by mass and heat fluxes with external environments. On the right-hand side, the entropy generation mechanisms for RANS flow fields can be written as

$$\begin{aligned} \tilde{G}_s^{RANS} &= \underbrace{\frac{1}{\tilde{T}}\sigma_{ij}(\tilde{u})\frac{\partial\tilde{u}_j}{\partial x_i}}_{V_M} + \underbrace{\frac{c_p\mu}{Pr}\frac{1}{\tilde{T}^2}\left(\frac{\partial\tilde{T}}{\partial x_i}\right)^2}_{H_M} \\ &+ \underbrace{\frac{1}{\tilde{T}}\tau_{ij}^R\frac{\partial\tilde{u}_j}{\partial x_i}}_{P_R} - \underbrace{\frac{1}{\tilde{T}}\frac{\partial}{\partial x_i}(q_i^R)}_{H_R} - \underbrace{\frac{1}{\tilde{T}}\frac{\partial(\bar{\rho}\tilde{u}_ik)}{\partial x_i}}_{A_k} \end{aligned} \quad (23)$$

which include entropy generation caused by the mean viscous dissipation V_M , mean flow irreversible heat flux H_M , the turbulence production related to the Reynolds stress P_R , turbulent heat flux H_R , and advection of TKE A_k . We remark that the terms P_R , H_R and A_k stand for the contribution of turbulence fluctuations to the irreversible process of entropy generation.

Sub-Grid Scale Terms with LES Equations

High-fidelity simulations, including DNS and LES, can be used for calibration of RANS solutions [13, 14]. As DNS is usually too expensive for most turbomachinery flows, LES data seems to be more widely applied. In order to derive entropy generation terms for LES fields, the LES equations need to be investigated.

Defining a spatial filtering operation $\langle \cdot \rangle$, a Favre-filtered field is written as

$$\hat{C} = \frac{\langle \rho C \rangle}{\langle \rho \rangle}. \quad (24)$$

Applying the Favre-filtering operation to Eqns. (4) and (5), the LES equations can be written as:

$$\frac{\partial(\langle\rho\rangle\widehat{u}_i)}{\partial t} + \frac{\partial(\langle\rho\rangle\widehat{u}_i\widehat{u}_j + \langle p\rangle\delta_{ij})}{\partial x_j} = \frac{\partial\widehat{\sigma}_{ij}}{\partial x_j} + \frac{\partial\tau_{ij}^{SGS}}{\partial x_j}, \quad (25)$$

and

$$\begin{aligned} \frac{\partial(\langle\rho\rangle\widehat{e}_0)}{\partial t} + \frac{\partial[\widehat{u}_j(\langle\rho\rangle\widehat{e}_0 + \langle p\rangle)]}{\partial x_j} &= \frac{\partial(\widehat{\sigma}_{ij}\widehat{u}_i)}{\partial x_j} + \frac{\partial(\tau_{ij}^{SGS}\widehat{u}_i)}{\partial x_j} \\ &\quad - \frac{\partial\widehat{q}_j}{\partial x_j} - \frac{\partial q_j^{SGS}}{\partial x_j}. \end{aligned} \quad (26)$$

Here, two extra terms are introduced to account for the contribution of the sub-grid scale (SGS) fluctuations to the resolved flow, *i. e.* the SGS stress

$$\tau_{ij}^{SGS} = -\langle\rho\rangle(\widehat{u_i u_j} - \widehat{u}_i\widehat{u}_j), \quad (27)$$

and the SGS heat flux

$$q_j^{SGS} = \langle\rho\rangle c_p(\widehat{T u_j} - \widehat{T}\widehat{u}_j). \quad (28)$$

When we calculate entropy generation based on LES mean flow fields, these SGS terms also have a direct impact.

It is common to assume that taking the Favre-average of a filtered field is equivalent to taking the Favre-average of the original field

$$\widetilde{\widetilde{\mathcal{C}}} = \widetilde{\mathcal{C}}. \quad (29)$$

Therefore, the entropy balance equation for Favre-averaged LES flow fields is similar to Eqn. (22), only with

two extra SGS terms:

$$\begin{aligned}
 \frac{\partial(\bar{\rho}\tilde{u}_i\tilde{s})}{\partial x_i} + \frac{\partial}{\partial x_i}\left(\frac{\tilde{q}_i}{\tilde{T}}\right) &= \frac{1}{\tilde{T}}\sigma_{ij}(\tilde{u})\frac{\partial\tilde{u}_j}{\partial x_i} + \frac{c_p\mu}{Pr}\frac{1}{\tilde{T}^2}\left(\frac{\partial\tilde{T}}{\partial x_i}\right)^2 \\
 &+ \frac{1}{\tilde{T}}\tau_{ij}^R\frac{\partial\tilde{u}_j}{\partial x_i} - \frac{1}{\tilde{T}}\frac{\partial}{\partial x_i}(q_i^R) - \frac{1}{\tilde{T}}\frac{\partial(\bar{\rho}\tilde{u}_ik)}{\partial x_i} \\
 &+ \frac{1}{\tilde{T}}\tau_{ij}^{SGS}\frac{\partial\tilde{u}_j}{\partial x_i} - \frac{1}{\tilde{T}}\frac{\partial}{\partial x_i}(\tilde{q}_i^{SGS}).
 \end{aligned} \tag{30}$$

Thus the entropy generation for LES flow fields can be written as

$$\begin{aligned}
 \tilde{G}_s^{LES} &= \underbrace{\frac{1}{\tilde{T}}\sigma_{ij}(\tilde{u})\frac{\partial\tilde{u}_j}{\partial x_i}}_{V_M} + \underbrace{\frac{c_p\mu}{Pr}\frac{1}{\tilde{T}^2}\left(\frac{\partial\tilde{T}}{\partial x_i}\right)^2}_{H_M} \\
 &+ \underbrace{\frac{1}{\tilde{T}}\tau_{ij}^R\frac{\partial\tilde{u}_j}{\partial x_i}}_{P_R} - \underbrace{\frac{1}{\tilde{T}}\frac{\partial}{\partial x_i}(q_i^R)}_{H_R} - \underbrace{\frac{1}{\tilde{T}}\frac{\partial(\bar{\rho}\tilde{u}_ik)}{\partial x_i}}_{A_k} \\
 &+ \underbrace{\frac{1}{\tilde{T}}\tau_{ij}^{SGS}\frac{\partial\tilde{u}_j}{\partial x_i}}_{V_S} - \underbrace{\frac{1}{\tilde{T}}\frac{\partial}{\partial x_i}(\tilde{q}_i^{SGS})}_{H_S},
 \end{aligned} \tag{31}$$

where V_S and H_S represent the entropy generation related to the viscous dissipation caused by SGS stress and the SGS heat flux.

NUMERICAL SIMULATIONS

The entropy generation analysis is applied to a LS-89 HPT vane [15] operating at an isentropic exit Reynolds number of $Re = 0.57 \times 10^6$ and an exit Mach number of $Ma = 0.9$, which is representative of a modern transonic HPT nozzle [10]. As the flow field shows complex flow physics including boundary layer transition, shock waves and vortex shedding in the blade wake, the selected case is challenging for RANS calculations. In the present study, the available high-resolution LES data [16] is first post-processed, and then RANS calculations are performed and analysed. The error source for the RANS predictions are quantitatively investigated by comparing to the LES results.

Fig. 1. Sketch of the LS89 HPT blade geometry and domain boundaries.

LES Data

The LES was performed using the in-house compressible Navier-Stokes solver HiPSTAR, with the standard WALE model [17] applied as the SGS model. The numerical solver has been extensively verified for turbomachinery flows [18]. The details of the data was presented in a previous study [16], and only a brief summary of the key features of the case is presented here.

The geometry of an axial-pitch plane cut of the HPT computational domain is shown in Fig. 1. The axial chord C_{ax} is used as the reference length scale, and the blade surface is set as an iso-thermal wall. Periodic conditions are applied at the boundaries in the pitchwise direction (y), and a zonal characteristic boundary-condition [19] is applied to attenuate acoustic reflections at the axial (x) outlet boundary. At the inlet boundary, Riemann invariant boundary conditions have been used, along with additional turbulence fluctuations generated by the digital filter method [20]. The integral length scale of the turbulence fluctuations is $20\%C_{ax}$ and the intensity TI is $6\%u_{\infty}$. The turbulence intensity is defined as $TI = \frac{\sqrt{2/3k}}{u_{\infty}}$, where the reference velocity u_{∞} is set as the inlet mean velocity. The HPT is statistically two dimensional as periodic conditions are applied at the boundaries in the spanwise direction. The spanwise domain size is set as $L_z = 80\%C_{ax}$, which allows for the spanwise correlation of the large turbulence length scale. The wall normal spacing of the LES mesh is below $y^+ = 2$, and the spacing in the spanwise and axial directions are around 20 and 50 wall units, respectively. This extraordinary high-resolution guarantees the accuracy of the LES data, which has been compared against [16] the DNS results of Wheeler *et al.* [10].

RANS Setup

RANS calculations of the HPT case have been performed using the finite volume solver TRAF [21]. The two dimensional Reynolds-averaged Navier-Stokes equations are solved on multi-block structured grids.

Table 1. RANS grid convergence parameters.

Grids	O-Grid	H-Grid	Total Points
Coarse	565×57	81×101	40,386
Mid	705×65	101×161	62,086
Fine	1081×113	161×320	173,673

Fig. 2. Pressure coefficient distribution across the blade.

The convective fluxes are discretized by a 2^{nd} order TVD-MUSCL strategy build on Roe's upwind scheme, and the viscous fluxes are solved by a central difference scheme. For turbulence closures, the $k - \omega$ SST with $\gamma - Re_\theta$ transitional correlation [22] is applied. The computational domain was discretized using an O-H grid with 705×65 , and 101×161 mesh points, respectively. The mesh was selected on the basis of a grid-refinement analysis. The dimensions for the three different grids to establish grid independence are listed in Tab. 1.

The RANS results are compared to the LES data. The pressure coefficients C_p , obtained from these simulations, are plotted versus the fraction of axial chord in Fig. 2. Starting from the leading edge, the suction-side pressure drops quickly, while the decrease of C_p on the pressure-side initiates near the trailing edge. All the RANS cases capture the pressure decrease on both sides, and a good agreement is shown between the RANS and LES results. Moreover, the RANS results show a slight deviation from the LES data near the suction-side trailing edge, which is presumably due to the boundary layer transition not being captured correctly by the RANS.

The transition on the blade can be presented by the wall friction coefficients shown in Fig. 3. For the pressure-side and the early suction-side boundary layer, the RANS calculations provide an accurate

Fig. 3. Wall friction distribution across the blade.

prediction of the wall friction. Moreover, the onset of the suction-side transition is captured by the $\gamma - Re_\theta$ model. However, the region in close proximity to the trailing edge is not predicted well. Despite the deviation near the trailing edge, the grid convergence is demonstrated with the RANS meshes.

Furthermore, Fig. 4 shows the kinetic wake loss profiles at $20\%C_{ax}$ downstream of the blade trailing edge as indicated by the green dashed line in Fig. 1. The kinetic wake loss is defined as

$$\omega(y^*) = \frac{p_{t,i} - p_{t,o}(y^*)}{p_{t,i} - p_o}, \quad (32)$$

with the normalized pitchwise coordinate $y^* = (y - y_{min})/(y_{max} - y_{min})$. Here, $p_{t,i}$, p_o , and $p_{t,o}$ denote the inlet total pressure, outlet static pressure, and outlet total pressure, respectively. We can see that the RANS calculations significantly over-predict the wake peak and under-predict the wake width. The failure of capturing the wake mixing in turbine flows is known as a common weakness for traditional RANS models [14], and its impact will now be investigated based on the entropy analysis.

ENTROPY GENERATION ANALYSIS IN HPT VANE

Quantification of Entropy Generation Terms

The entropy analysis shown in the present study is applied to both LES and RANS flow fields. The LES data have been averaged in time and in the spanwise direction, and the entropy generation terms shown in Eqn. (31) are calculated based on the averaged LES statistics. On the other hand, the turbulence-related terms from RANS are calculated based on the turbulence models. Therefore, the method provides a way

Fig. 4. Wake loss profiles at 20% downstream of the trailing edge ($x = 1.2C_{ax}$).

Table 2. Quantification of entropy generation terms in LES and RANS.

Terms ($JK^{-1}s^{-1}$)	LES int.	RANS int.	Error
V_M	0.0367	0.0344	0.0023
H_M	0.0116	0.0096	0.0020
P_R	0.0598	0.0398	0.0200
H_R	-0.0003	0.0015	-0.0018
A_k	-0.0199	-0.0070	-0.0129
V_S	0.0026	N.A.	N.A.
H_S	0.0008	N.A.	N.A.
G_s	0.0913	0.0783	0.0130
$-\dot{m}R\ln(P_{t,o}/P_{t,i})$	0.0926	0.0797	0.0129

to quantify the modelling error from RANS by comparing to LES.

In order to quantify the loss increase caused by different terms, volume integrals of the entropy generation equations Eqn.(23) and Eqn.(31) are performed. The control volume can be selected as the vane between the inlet and outlet planes as shown in Fig. 1. This volume integral is applied to both the LES data and the RANS results, and the total entropy generation by different terms are listed in Tab. 2.

These terms can be divided into three groups. The first group represents the mean flow effects including the mean viscous dissipation V_M and the mean heat flux H_M . The second group includes the entropy generation induced by turbulence fluctuations, containing the turbulence production P_R , the turbulent heat flux H_R , and the advection of TKE A_k . And the last group consists of the SGS stress terms V_S and the SGS

heat flux term H_S , which only apply for the LES data.

Among these groups, the contributions from the mean flow and the turbulence fluctuations are on roughly the same level, and the viscous effects in each group have much higher amplitudes than the heat flux terms, as $V_M > H_M$ and $P_R > H_R$. On the other hand, the contribution of the SGS terms is relatively small, as $(V_S + H_S)/G_s \approx 3.7\%$. We note that the amplitude of the entropy generation caused by the SGS terms mainly depends on the grid resolution and overall turbulence intensity in the flow field. The small SGS contribution implies the present LES flow is highly-resolved. Nevertheless, to fully avoid any uncertainty, a DNS might be desirable.

As the averaged flow field is time-invariant, the total entropy generation G_s integrated over the whole HPT vane should equal to the entropy flux with the environment. The volume integrals of the left-hand side entropy flux terms from Eqn. (22) or Eqn. (30) can be simplified as the mass-flux term

$$\begin{aligned} \int_V \frac{\partial(\bar{\rho}\tilde{u}_i\tilde{s})}{\partial x_i} dV &= \int_A \bar{\rho}\tilde{u}_i\tilde{s} dA \\ &= \dot{m}(\tilde{s}_{out} - \tilde{s}_{in}), \end{aligned} \quad (33)$$

and the heat transfer term

$$\int_V \frac{\partial}{\partial x_i} \left(\frac{\tilde{q}_i}{\tilde{T}} \right) dV = \int_{A_b} \frac{\tilde{q}_i}{\tilde{T}} dA. \quad (34)$$

As we can see, the mass-flux induced entropy transport is represented by the specific entropy difference between the outlet and inlet plane multiplied by the mass flow rate $\dot{m}$, whereas the entropy transport caused by heat transfer is dominated by the heat flux on the blade surface A_b . These two terms depict the overall entropy transfer between the HPT vane with the environment, *i. e.* the upstream and downstream stages and the turbine blade.

From basic thermodynamics, the specific entropy of the averaged single phase fluid can be written as

$$\tilde{s} - \tilde{s}_{ref} = c_p \ln(T_t/T_{t,ref}) - R \ln(P_t/P_{t,ref}), \quad (35)$$

where R is gas constant. Conventionally, the kinetic loss in the turbine vane can be represented by ignoring the blade heat flux and the temperature term [3], as

$$-\dot{m}R\ln(P_{t,o}/P_{t,i}) \approx \int_V \tilde{G}_s dV. \quad (36)$$

On the left-hand side, the $-\dot{m}R\ln(P_{t,o}/P_{t,i})$ term is the entropy variation related to the total pressure difference between the outlet and inlet plane, which is calculated based on the LES and RANS flow field and presented in Tab. 2. This entropy difference is closely related to the total entropy generated in the vane as indicated by the right-hand side term $\int_V \tilde{G}_s dV$. We can see from Tab. 2 that the balance in Eqn. (36) is approximately satisfied, which indicates that the entropy generation terms presented in this study are quantitatively linked to the kinetic loss in the vane.

Furthermore, we can compare the terms from LES and RANS calculations in Tab. 2. It is shown that the major difference is caused by the turbulence production term P_R . The turbulence production represents the energy extracted from the mean flow and passed on to turbulent fluctuations. Negative production is possible in LES when the TKE inversely cascades to mean kinetic energy, but it is not allowed by the RANS approach. Most of the TKE will not be available for mechanical work but will gradually dissipate away. Therefore, the TKE may be considered as one kind of ‘pseudo internal energy’ in the present entropy calculations, and the advection term A_k represents the redistribution of TKE, which also induces entropy variation. It is noted that the contribution of A_k for the whole vane is negative as presented in Tab. 2.

Spatial Distribution of Entropy Generation

The spatial distribution of the entropy generation in the turbine vane is investigated to further understand the physical mechanisms. The LES spatial distribution of the entropy generation caused by the mean flow viscous dissipation V_M is first presented in Fig. 5. We can see that the mean flow viscous dissipation is mainly in the blade boundary layer, and the amplitude in the rest of the vane is negligible.

In addition, a contour of the turbulence production term P_R from LES is shown in Fig. 6. The entropy generated by turbulence production concentrates in two regions, including the late suction-side boundary layer, related to the laminar-turbulent transition, and the wake region, which corresponds to the vortex shedding downstream the trailing edge. For conciseness, the contours for the other terms in Eqn. (31) are not shown here. However, we can infer from Fig. 5 and Fig. 6 that the mean flow effects mainly manifest

Fig. 5. Distribution of the entropy generation caused by mean viscous dissipation V_M .

Fig. 6. Distribution of the production-induced entropy generation P_R .

themselves in the blade boundary layer, while the turbulence effects are caused by the suction-side turbulent boundary layer and the strong vortex shedding in the wake.

The rise in entropy generation by the mean flow effects is quantified in Fig. 7 by integrating over the volume between the inlet plane to a downstream axial plane at x . As the mean flow entropy generation occurs mainly in the blade boundary layer, the increase of V_M and H_M are distributed between the blade leading edge ($x = 0$) and trailing edge ($x = 1$) as indicated by the yellow dashed lines in Fig. 7. The rate of increase of H_M at the leading edge is larger, indicating a stronger temperature gradient in the boundary layer. On the other hand, the rate of increase of the viscous dissipation in the boundary layer is increasing along the blade surface, because the shear is getting stronger as the flow accelerates and the Reynolds number increases. It is noted that the entropy generation terms from these mean flow effects remain approximately invariant downstream the trailing edge, and the mean viscous dissipation has a much larger

Fig. 7. Integrals of the entropy generation caused by mean flow.

amplitude compared to the heat flux effect.

In addition, LES and RANS results in Fig. 7 can be directly compared. It is shown that the mean viscous dissipation V_M from RANS agrees well with the LES result until the blade trailing edge, implying that the velocity boundary layer is accurately predicted by the RANS calculation. A relatively small deviation exists after the blade trailing edge, which is presumably because the RANS-predicted shear layer in the wake close to the trailing edge is not as strong as the shear layer from LES [10]. Moreover, the RANS-predicted H_M deviates from the LES result, while the curves have a similar trend. The difference implies that the RANS thermodynamic boundary layer around the blade surface is not as accurate as the velocity field, suggesting a model form error of the heat flux model [23].

Furthermore, the entropy generation terms related to turbulence effects are shown in Fig. 8. The turbulence-production related entropy generation is dominating for both the LES and RANS results. From the blade leading edge, the production caused entropy generation barely increases until the suction-side boundary layer transition. The RANS calculation provides a satisfactory prediction around the blade surface, including the evident increase of entropy generation in the turbulent boundary layer. However, the major error between the LES and RANS seems to be due to the turbulence production downstream of the blade trailing edge. Compared to RANS, the LES shows a more significant turbulence production just downstream of the trailing edge. Nevertheless, the production rate further downstream is negligible for both cases.

The deviation between the turbulence production downstream of the blade trailing edge will result in different amplitudes of TKE in the wake. The TKE distributions are plotted along the wake center line indicated by the red dashed arrow in Fig. 1. It is presented in Fig. 9 that the RANS calculation has a much lower TKE peak compared to the LES result, which is mainly due to the smaller amplitude of turbulence

Fig. 8. Integrals of the entropy generation caused by turbulence effects.

Fig. 9. The TKE distribution along the wake center line.

production. Furthermore, the distribution of TKE has an effect on the advection term A_k . As shown in Fig. 8, the TKE advection A_k from RANS calculation significantly deviates from the LES result, which is because of the difference of the TKE distributions near the blade trailing edge. However, the error of A_k gradually decreases further downstream, so that the contribution for the total entropy variation is not as significant as P_R .

The turbulence heat flux term indicates the heat transfer induced by turbulent fluctuations. Although the contribution compared to the turbulence production term is negligible, the difference between the RANS and LES results implies the limitation of the turbulent heat flux modelling [23], particularly in the near trailing edge region with strong vortex shedding.

CONCLUSION

In the present study, we present the derivation of the entropy balance equations for Favre-averaged flow fields. The equations are developed from basic flow-governing equations and thermodynamic laws, so that the framework can be applied to a wide range of data sets including RANS and high-fidelity simulations. The entropy generation, which is closely related to the loss of efficiency of the system, is decomposed into different terms such as mean viscous dissipation, turbulence production, mean and turbulent heat flux, etc.

This decomposition framework is then applied to available high-resolution LES and RANS results of the transonic HPT case. A quantitative analysis of these terms shows that the contributions from the mean flow terms and the turbulence terms are on a similar level, while the SGS terms from the LES data only have a weak effect in this HPT case. Adding all these terms up, the total entropy generated in the whole HPT vane provides a good approximation for the kinetic loss.

Furthermore, the spatial distributions of the entropy generation terms have been investigated, and differences between the LES data and the RANS calculations have been quantitatively analysed. It is shown that the entropy generated by the mean flow effects concentrate in the blade boundary layer, while the turbulence fluctuations induce entropy increase in the suction-side turbulent boundary layer and the wake region with vortex shedding. In particular, the turbulence production term, which represents the energy extracted from the mean flow to turbulent fluctuations, plays an important role in entropy generation. However, the difference between RANS and LES results for the turbulence production term is not negligible, especially in the wake region near the blade trailing edge. This implies that the error of RANS prediction is mainly caused by the inaccurate turbulence production modelled in the near wake region.

ACKNOWLEDGEMENTS

This research used resources of the Oak Ridge Leadership Computing Facility, which is a DOE Office of Science User Facility supported under Contract DE-AC05-00OR22725. This work was also supported by the resources provided by the Pawsey Supercomputing Centre with funding from the Australian Government and the Government of Western Australia.

NOMENCLATURE

General Symbols

$C_{a,x}$ Blade axial chord

C_p Pressure coefficient

C_f	Wall friction coefficient
c_p	Specific heat at constant pressure
e	Internal energy
e_0	Total energy
k	Turbulent kinetic energy
Ma	Mach number
$\dot{m}$	Mass flow rate
Pr	Prandtl number
p	Pressure
q_i	Heat flux vector
R	Gas constant
Re	Reynolds number
T	Temperature
TI	Turbulence intensity
x	Axial coordinate, normalized by axial chord
y	Pitchwise coordinate, normalized by axial chord

Symbols Related to Entropy

A_k	Entropy generation by TKE advection
F_i	Entropy flux vector
G_s	Entropy generation rate
H_M	Entropy generation by mean flow heat flux
H_R	Entropy generation by turbulent heat flux
H_S	Entropy generation by SGS heat flux
P_R	Entropy generation by turbulence production
S	System total entropy
s	Specific entropy
V_M	Entropy generation by mean viscous dissipation
V_S	Entropy generation by SGS stress dissipation

Greek Letters μ Dynamic viscosity μ_t Eddy viscosity ρ Density ω Kinetic wake loss σ_{ij} Viscous stress tensor τ_{ij}^R Reynolds stress tensor τ_{ij}^{SGS} Sub-grid scale stress tensor**Accents** $\bar{\cdot}$ Time-average $\tilde{\cdot}$ Favre-average $\hat{\cdot}$ Favre-filter**Acronyms**

CFD Computational fluid dynamics

DNS Direct numerical simulation

HPT How-pressure turbine

LES Large-eddy simulation

LPT Low-pressure turbine

RANS Reynolds averaged NavierStokes

SGS Sub-grid scale

TKE Turbulent kinetic energy

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Title:

Using a New Entropy Loss Analysis to Assess the Accuracy of RANS Predictions of an High-Pressure Turbine Vane

Date:

2020-08-01

Citation:

Zhao, Y. & Sandberg, R. D. (2020). Using a New Entropy Loss Analysis to Assess the Accuracy of RANS Predictions of an High-Pressure Turbine Vane. Journal of Turbomachinery, 142 (8), <https://doi.org/10.1115/1.4046531>.

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