Robyn Pierce & Kaye Stacey

Teaching with new technology: Four 'early majority' teachers

Affiliation and address for both authors
Melbourne Graduate School of Education
Level 3 234 Queensberry St
University of Melbourne
VIC 3010
Australia

The e-mail address, telephone and fax numbers of the corresponding author

r.pierce@unimelb.edu.au

Phone: +61 3 8344 8519

Fax: +61 3 3742

Abstract (150-250 words) 248 words

This paper explores how four good teachers, who do not have a special interest in technology, meet the challenge of introducing the rapidly developing mathematics analysis software (e.g. spreadsheets, function graphers, symbolic algebra manipulation and dynamic geometry) into their classrooms. These teachers' practice is viewed through the lens of Roger's framework for the diffusion of innovation and Pierce and Stacey's pedagogical opportunities map. Data on teachers' views and practices was collected over two years. 'Pedagogical Maps' give a picture of the teachers' perception and uptake of pedagogical opportunities. New practices have been added slowly to each teacher's repertoire and their increasing fluency in practical ability to teach with the technology resulted in some changes to the classroom didactic contract. Overall, new technology seemed to have been absorbed into current practice, more than changing practice. At this stage of their development, these teachers do not identify the distinctive new mathematical capabilities as contributing to the major relative advantage of the innovation. Instead they see the relative advantage mostly in the incremental improvements to capabilities of earlier calculators, and meeting the need for students to be up to date. One of the current challenges is that significant changes in both software and hardware design have been happening so rapidly that these early majority teachers felt almost constantly hampered by the need to learn and teach new technical skills and so continue to make limited progress in taking advantage of opportunities to approach mathematics concepts in new ways.

Keywords

computer algebra systems; dynamic geometry; spread sheets; calculators; mathematics education innovation; mathematics with technology

Teaching with new technology: Four 'early majority' teachers

Introduction

Mathematics analysis software that may be used with handheld devices and computers is widely available to schools and individuals. It supports any or all of the following: arithmetic and scientific calculations, function graphing, dynamic geometry, exact arithmetic, symbolic algebra and statistics (for examples see Figures 1, 7 and 8). Mathematics analysis software (MAS) was originally designed for professionals to ease the burden of calculation but over the past four decades has found a place in many classrooms around the world. This rapid development of technology, from four function calculators to multi-representational connected devices, for a discipline that has evolved over thousands of years with a classroom tradition of teacher-centred exposition raises many questions and challenges for mathematics teachers. This paper explores how four good teachers who do not have a special interest in technology meet this challenge.

In his 1962 book, the sociologist Rogers described the diffusion of innovations in society (Rogers, 1995). Innovations are first taken up by brave 'innovators' and then enthusiastic 'early adopters', eager to explore the capabilities of a new artefact. Following them is the 'early majority'. They accept the necessity for change but wish to move forward carefully and thoughtfully, learning from the experience of the early adopters rather than paving the way. Rogers says that this group usually has contact with early adopters, but that they seldom hold positions of opinion leadership in the area. The late majority and the laggards follow in adopting the innovation. The teachers in this study are part of the early majority in the adoption of MAS technology for teaching.

The research focus of the study is how some early majority teachers take up the pedagogical opportunities offered by the new MAS, specifically TI-Nspire(CAS). By following the teachers over two years, we are able to observe their growth and changing practices. Studying the early majority is important because it is much larger than the combined groups of innovators and early adopters, and because it may give insight into some of the integration issues that will soon be met with the similarly large 'late majority'. It is of special interest now because many of the basic features of MAS have been available for a decade, steadily improved and extended.

The place of MAS in teaching has been the subject of much debate and its uptake has been very varied (see for example Pierce and Ball, 2010; Thomas, 2006). Many studies of learning to teach with new technology have been case studies of innovators or early adopters, although some of the early adopters have achieved this status by selection for a special project, rather than individual initiative. See for example, Berry, Graham, Honey, and Headlam (2007), Drijvers, Doorman, Boon, Reed, and Gravemeijer (2010), Kendal, Stacey, and Pierce (2005). In each of the research studies mentioned above, the pioneer teachers were generally positive about the use of technology in their teaching. However, as noted by many researchers (see, for example, Drijvers et al., 2010), the diffusion of new teaching practices with MAS has been slow to meet the expectations foreshadowed by researchers such as Artigue (2002) or Stacey, Asp, and McCrae (2000). Trouche and Drijvers (2010) observe that diffusion has been greater with hand held technology than with computers.

In the sections which follow we describe some background to this study from two points of view; first situating it in Roger's theory and secondly giving a theoretical framework for the pedagogical opportunities which teachers can adopt. The article then describes the context and method of the study and outlines the results of four case studies, individually and together. The final section examines the implications for research and for teacher professional development for the potentially large group of early majority teachers.

Diffusion of Innovations

Here we situate the present study in terms of Roger's theory of diffusion of innovations. The study was conducted after an invitation to the researchers from a school mathematics leadership team to assist the school to adopt new calculators (TI-Nspire(CAS)). The mathematics leaders made this decision because the new calculators were perceived to have more features than the graphic calculators used at the school. They were seen as the technology for the future; something which they would inevitably adopt at some stage. Adoption now was also seen to be especially valuable for students who would go on to complete mathematics through to Year 12 when they could use it for the end-of-school state examinations. In terms of Roger's theory, adopting the new technology was an organisational decision, and the school was moving into the third and fourth of Roger's 5 stages of adoption of an innovation by an organisation: redefining the work, and clarifying what using the new technology meant in practice. The school mathematics leaders were early adopters of the technology, and by implementing their decision for school change, the other mathematics teachers are in the early majority.

If new technology is to have an impact on teaching and learning in mathematics classroom then along with bringing the artefacts into the school (the school decision), there needs to be diffusion of new ideas and practices into classrooms. Roger's (1995) work proposed that four key factors influence the spread of an innovation: the innovation itself, communication channels, time, and a social system. In this study the innovation involves accessing the pedagogical opportunities afforded by new technology for teaching mathematics. Communication is most especially related to sharing within the school. The school leaders had attended professional learning programs, which brought the innovation to the school, but for the purpose of the study of the early majority teachers, intra-school communication is the most important. The social system involves the school, its leaders, teachers, parents and students, and others in the education system, especially those responsible for external assessments. Broadly speaking, the social system supported this innovation. The school provided teachers' equipment and recommended the purchase of calculators to parents.

Rogers defines several intrinsic characteristics of innovations that influence decisions to adopt or reject: relative advantage; compatibility; complexity or simplicity; trial-ability and observe-ability. The relative advantages from the (organisational) point of view of the school have been described above. In the case studies, the relative advantage is to be examined from the point of view of classroom teaching and learning and in comparison to their previous scientific and graphics calculators. For compatibility, we consider how this technology fits with or requires change to usual classroom practices. Complexity relates to both mathematical and teaching issues. TI-Nspire(CAS) is indeed a complex tool that can make some aspects of mathematics simple, but can occasionally make some simple mathematics seem complicated. Additionally, how complex is it to use in teaching? In this study, the main obstacles to trial-ability have been dealt with in advance, since the

equipment is available to all students and teachers.. The observe-ability characteristic relates to whether an individual's adopting the innovation can be seen by others: in this case the teachers worked together to develop a school program and this provided a certain degree of observe-ability. Thus, the first three intrinsic characteristics are of most interest here.

Theoretical Framework of Pedagogical Opportunities

The framework used to guide the analysis of the data collected in this project is a taxonomy of the pedagogical opportunities that are supported by the use of MAS. These are summarised in the Pedagogical Opportunities Map (see Figure 2). This map is discussed in detail in Pierce and Stacey (2010), and a brief outline is given here. As indicated on the map the pedagogical opportunities are underpinned by functional capability (fast calculation, correct algebraic manipulations, exact graphing with flexible zooming and accurate geometric constructions as illustrated in Figures 1a and 1b below).

Figure 1 Two examples of MAS output: Fig 1a: Perfect squares showing symbols, graphs and algebraic spreadsheet using TI-Nspire. Fig 1b: Display of problem with Geogebra dynamic diagram and graph of area.

Functional use of MAS allows students to find answers more easily, including to problems normally considered to be beyond their pen-and-paper mathematical skills. Such functional use is unlikely to impact on students' understanding unless it is incorporated into new pedagogy. The pedagogical map specifies 10 broad pedagogical opportunities: ways in which technology use might be adopted to benefit learning. Opportunities might or might not be taken up by a teacher: the physical presence of technology is a necessary but not sufficient condition. The map organises opportunities at three levels that reflect the perspective of teachers thinking about:

- the tasks they will set for their students (the bottom row)
- the style of classroom interaction which they desire (the middle row)
- what perspectives on the subject (i.e. mathematics as a whole or a topic) they should promote (the top row).

Figure 2 Pedagogical map for mathematics analysis software (Pierce and Stacey 2010, p. 6)

At the task level, the map draws attention to five different types of opportunities. There are opportunities to use MAS to assist in the learning of pen-and-paper skills, for example by encouraging immediate on-demand checking; taking over the mechanical work on one part of a task to release students' attention to focus on another part; or careful analysis of many correct examples. MAS can allow students to work with realistic problems set in real situations, where details may be distracting or calculations may be beyond a student's pen-and-paper skills. A variety of capabilities of MAS, such as easy use of sliders, allows students to explore variability and consider generalisations. MAS allows students to analyse simulated data or collect data from working diagrams of real situations, and finally different representations of the same mathematical concept may be dynamically linked and viewed simultaneously thus allowing students to build strong connected schema.

At the classroom level, there are cognitive and social opportunities. The presence in the classroom of a new, alternative and emotionally neutral mathematical authority (a machine doesn't look angry or disappointed or comment publicly) can present opportunities to change the didactic contract (the cognitive aspect - see Pierce, Stacey & Wander, 2010 or Brousseau 1997) or can support social dynamics that discourage students from being passive observers. In an MAS environment the didactic contract includes the unwritten, reciprocal responsibilities and expectations of the teacher and students with respect to knowledge of mathematics and of using the technology to do mathematics (which button to push, which menu to select etc.)

At the subject level, the teacher thinks about how using MAS can impact on students' views of mathematics as a discipline and its role in society. Teachers may choose to use points of difference between the target ideal mathematics and the machine mathematics which students see in front of them, perhaps to emphasise certain epistemic aspects of mathematics; they may choose to rework the balance of skills, concepts and applications perhaps to allow more emphasis on conceptual understanding or application; then finally MAS may be used to approach topics in new ways that

allow students to gain an overview of the purpose of the topic before emphasising detail. Although it is impossible to claim that the pedagogical opportunities map catalogues all of the opportunities, past use has shown it to be a robust framework which can describe the practice of many teachers in a way which is accessible to both researchers and teachers. In this study the researchers have used the map to analyse and represent teachers' reflections on their own practice.

This Study: Context and Method

The research focus of the study is to examine how early majority teachers take up the pedagogical opportunities offered by the new MAS, specifically TI-Nspire(CAS). As noted above, the school mathematics leaders invited the researchers to assist with introduction of the new calculators. The school is a girls' secondary school in a middle class area, with strong leadership, a stable staff and a good team ethos in mathematics. Year 10 (students aged 15 – 16) was selected as the area which could benefit most from the development of new practices (Roger's third and fourth stages). Data were collected over a two year period while the research team worked with seven teachers. The subset of data reported in this paper comes from the four teachers (pseudonyms Fran, Gina, Hari and Iris) who taught Year 10 in both years. We conceptualise this study as a set of four linked descriptive case studies in which we look for evidence of the adoption of new ideas and practices.

The lesson study professional development program

Over the two years, the Year 10 teachers participated in a professional learning program developed around 'lesson study' and run by the researchers in conjunction with the mathematics leaders (see Pierce & Stacey, 2011, for details of this extended intervention). In the first year of the study TI-Nspire(CAS) was new for teachers and students. The program began with an introductory session on basic calculator operation, and included many lesson observations by the team and by teachers, and data collection including surveys and formal interviews. There were also numerous informal interviews associated with lesson observation (sometimes no more than a conversation with notes made immediately after).

The main feature of the lesson study was that three 'research lessons' (this is the lesson study terminology) were developed by the research team to exemplify new opportunities for teaching with TI-Nspire. Teachers observed a colleague teach the research lesson first, followed by a focus group where all the teachers analysed lesson features. In the following few days each teacher then taught her own class the same research lesson with some teachers and researchers observing. This was again followed by reflective discussion of their experiences. In addition to the research lessons largely prepared by the research team, some teachers shared their own lessons with colleagues and observed each other teaching them. It is important to note that the school mathematics leadership team had major influence on the research lesson design, to create lessons which they considered would be acceptable to the teachers. Using Roger's term, this increased the compatibility of the innovation with current practice, smoothing the path for adoption.

Data collection

Data were collected from surveys, observations of research and other lessons with associated follow-up conversations, and formal interviews as listed in Table 1. The MTUS usage survey gave background information about each teachers' prior experience of teaching with technology. The

MTPS perceptions survey tracked over time the intentions, attitudes and perceptions that would likely influence their use of the technology. The P-Map survey monitored teaching practices over time. Each lesson observed by the university team was audio or video recorded with digital photos of teachers' and students' work. There were at least two observers from the research team for research lessons.

Table 1

Data collection summary

Data source	Time of data collection	Details of instrument
Mathematics with Technology Use Survey (MTUS)	Beginning of year 1	Pierce and Ball (2010)
Mathematics with Technology Perceptions Survey (MTPS)	Beginning of year 1, end of year 2	Pierce and Ball (2009)
P-Map survey	Beginning of year 1, end of year 2	New survey (Appendix)
Formal interviews	At start of project and end of year 2	Semi-structured. Verbal discussion of open response questions.
Lesson Observations (recorded, with associated informal interviews)	At least 2 lessons per teacher per year, ending some months before final interview.	Pierce and Stacey (2009)

Creating pedagogical maps

A pedagogical map was constructed from the P-Map Survey (see Appendix) to give a picture of each teacher's perceptions of their teaching with TI-Nspire(CAS), at the beginning and again at the end of the project. This is not a precise measure but rather gives a global impression of the opportunities each teacher perceived and accessed. The maps are intended to show both differences over time and also between teachers. They are not a judgement of *quality* of practice. The survey is structured around the boxes in Figure 2. Responses to Likert (Appendix Part A) and open response items (Appendix Part B) were weighted through coding according to the degree to which an opportunity had been taken up. The teachers' responses to the P-Map Survey (see appendix) were coded for Part A using two red dots for "No opportunity" (implying that they had not perceived any opportunity); one red dot for "didn't choose to do" (opportunity was perceived but not accessed); one blue dot for "Tried didn't work well"; two blue dots for "Tried went OK"; and three blue dots for "Tried went well". Responses to open questions in Part B were similarly coded. For each box, the ratio (r) of total blue to total red dots was calculated and shading assigned as follows: 0 no shading; $0 < r \leq 1$ light shading; $1 < r \leq 5$ medium shading and $r > 5$ dark shading. The precise ratios should not be compared between boxes because the number of items linked to each box is not always the same. In the

resulting maps (e.g. Figure 3 discussed in the next section) darker shading indicates a greater uptake of the particular aspect of pedagogical opportunity. Because the cut-offs for the shading are broad, we can be confident that a difference in shading between the before and after maps represents a change over time in the teacher's reports of their practice.

Results: Four case studies

The results of the project are presented in two parts. In this first section are summary descriptions for each case study teachers and reported change over time. The descriptions are based on the surveys and pedagogical maps, with selected illustrations drawn from the other interactions (lesson observations, interviews). In section 4, we will report from the viewpoint of the pedagogical opportunities, combining data on uptake from all teachers.

All interactions with the school showed that these teachers were part of well led, cohesive team. All data showed that they all aimed to do their jobs conscientiously and this included implementing both school policy and the collective intentions of their mathematics group as well as possible. Consequently they all willingly took on the task of learning to teach with the new MAS. They were concerned to prepare their students for the use of these calculators in later years including in senior secondary statewide examinations (VCAA, 2010) and hoped to use TI-Nspire(CAS) to teach some topics better than before.

Fran

Previous use of MAS and goals. Fran was a very experienced and respected teacher. At the beginning Fran stated that she had found technology for mathematics most helpful for teaching graphing, statistics and simultaneous equations. The main technologies she had used in previous years were the CD accompanying their text book, a scientific calculator for all classes, and statistical and graphing software for Year 11 students (16-17 year olds). She had also made limited use of spreadsheets, symbolic calculation, data logging and simulations. In summary, she is an experienced graphics calculator user, who is newly adapting to a technology which includes symbolic manipulation, dynamic geometry and a mathematically-able integrated document system (Pierce and Stacey, 2009). At the beginning Fran wrote that her goal was that the Year 10 students would learn to use all the facilities of this technology that are relevant for their work and so move into senior school mathematics as confident CAS users. It seemed that her main aim was for her students to gain *technical facility*.

Uptake of pedagogical opportunities. Fran's pedagogical maps are shown in Figure 3. Although in the first interview, she made no specific mention of using the technology to enhance teaching or support students' mathematical learning, her first P-map survey indicated several such uses, and the second substantially more. It seems that as her technical skills with the new MAS improved so did her uptake of the various pedagogical opportunities especially at the task and class levels. This type of change at the task level was not evident in the lessons we observed, but (for Fran and the other teachers) this may be because we observed only a few lessons whereas Fran is answering the survey cognisant of her whole teaching practice. In the later interviews, Fran gave many examples. Two examples were of using the contrast between ideal graphs and machine generated graphs in a finite viewing window to deepen understanding of function behaviour; one occasion she reported began

when students were surprised to see an apparently straight line graph for an exponential function. She reported having students explore regularity and variation with circle theorems illustrated with dynamic geometry which she judged to be “really effective”, “a lot faster” and “very dynamic and visual”.

Figure 3 Maps of Fran's reports of her teaching with MAS

Gina

Previous use of MAS and goals. Gina was relatively new to teaching but had already won the approval of her colleagues. She reported that in her limited experience she had made a little use of scientific calculators with Years 8/9 and a lot with Years 10/ 11, and done a little data logging with year 10. She used the CD with the text book herself but not with students. From experience before teaching, she valued graphics calculators for the extra features they offered beyond a regular scientific calculator. She had recently started using an interactive whiteboard and felt it offered potential for showing students calculator steps and exploring applets, especially so students could see visual images. She commented in the interview that it was complex to learn to use the TI-Nspire, a sentiment we believe was shared by the other teachers and was behind the request of the school for the assistance of the researchers. Gina's goal for Year 10 was to use TI-Nspire(CAS) to help students explore mathematical problems and to work with graphs. She thought the calculator would help students perform graph calculations quickly (e.g. gradient). Gina also hoped this new technology would assist in building students' confidence and motivation for mathematics.

Uptake of pedagogical opportunities. Mapping Gina's use of MAS from her P-Map surveys shows that she accessed a wider range opportunities in the second year. Generally we see increases across the map in Figure 4. Gina's final interview included many concrete examples. For example, in year 1 she commented that she could use surprising results on the calculator as a “learning experience”. At the end of Year 2, she recalled classroom discussions on unexpected rearranging of formulas by the CAS and commented:

Gina: *I suppose it really opens up those kinds of conversations, which might last 30 seconds or a minute, and other conversations where you can get a really good debate and arguments going.*

Gina also commented on a move towards more interpretation of mathematical results:

Gina: *I've moved towards more of a focus on the real-life interpreting questions, getting the information, drawing parabolas [...] [Asking] what does the x-intercept mean? Is that a feasible answer or not?*

In her final interview, Gina described how she had come to appreciate that technology could be very useful in all lessons. In year 2, some of her classrooms did not have projection facilities, and she commented:

Gina: *Last year I would flick it up just for a couple of minutes and show a solution, whereas now [.....] I still do it, but it takes a lot more effort. So I can see why certain teachers, if they are not big fans of technology, could just push it to the side.*

Gina's lessons were always teacher-led, but the year 2 lessons had greater student participation. The final year 2 research lesson started off in much the same way as the year 1 research lesson, with a strongly teacher led episode but the second half of the lesson included far more student contributions. She seems to have achieved some change in the social dynamics and didactic contract.

Figure 4 Maps of Gina's reports of teaching with MAS

Hari

Previous use of MAS and goals. Hari reported that prior to this project she had frequently used technology for statistics with Year 12 students and for all year levels, a lot of use of scientific calculators and the CD accompanying the text book. She had made a little classroom use of graphs,

algebra, spreadsheets, and dynamic geometry. Hari especially identified as valuable the following uses of technology: doing statistical analysis with her senior students; drawing accurate graphs quickly and hence examining graphical features; using dynamic geometry to teach Pythagoras' theorem. Hari's goals for Year 10 were to support her students' learning of mathematics by assisting them exploring graphs and doing statistical and algebraic calculations. She drew attention to the major new mathematical capability of TI-Nspire(CAS) to work with algebraic symbols. Perhaps surprisingly, Hari was the only teacher to mention this. In terms of Roger's criterion of relative advantage, it does not appear that the teachers saw symbolic algebra as an important advantage of the new technology, at least at this early stage and for Year 10.

Uptake of pedagogical opportunities. The maps (see Figure 5) of Hari's use of MAS show increased uptake of opportunities at the task level and key changes in the didactic contract and exploitation of the contrast between ideal and machine mathematics. In the year 1 interview, Hari said that when unexpected answers arose she would advise students to open a new document and start again. In contrast, in year 2 Hari said that she would ask "What's happening here?" and prompt discussion as appropriate. She gave an example of using a calculator set in exact mode which produces a complicated fractional multiple of π or an exact surd (square root) when students are expecting a decimal answer. In year 1, these answers were regarded as an obstacle to students, but in year 2, Hari saw value in contrasting the exact and approximate answers and understanding the meaning of both. She had recognised and exploited the contrast between ideal and machine mathematics to benefit learning. Hari gave other examples in the final interview which support the additional use of pedagogical opportunities evident in Figure 5, such as exploring the regularity in calculations with negative and fractional indices, which she said would be "too clunky on another sort of calculator". After giving another example of this pedagogical opportunity, Hari commented:

Hari: *You get the feeling that they aren't just being polite when they say they get it, as opposed to when they're faced with a lot of algebra. That idea of recognition of pattern is a better learning tool.*

Early Year 1

Late Year 2

Figure 5 Maps of Hari's reports of teaching with MAS

Iris

Previous use of MAS and goals. Iris came into the project with less experience in using MAS than the other three teachers. She reported having previously made a lot of use of scientific calculators and a little use of graph, symbol, spreadsheet, simulation and data logging software. Based on previous experience, Iris valued scientific calculator features [for speed and accuracy] and graphing software because it saved time and allowed quick and easy exploration of graphs (specifically changes in a , b , c and m on the graphs of $y = mx + c$ and $y = ax^2 + bx + c$). This is a very popular activity in Australian schools. Iris said that when teaching science she found applets very helpful, but she made no reference to using applets in mathematics classes. Iris's goal for Year 10 was to make graphing easier for students, and to incorporate the new calculator thoroughly into her teaching (e.g. by having students make written notes on the calculator procedures for each topic).

Uptake of pedagogical opportunities. Based on the mapping (see Figure 6) it seems that Iris's take up of pedagogical opportunities was fairly similar in year 1 and year 2. This matches our observations but we were surprised to see the number and strength of opportunities Iris reported accessing as this matched neither our classroom observations (admittedly limited) nor our interviews. Iris mentioned some activities which were commonly done with graphics calculators (such as observing the effect of parameters on graphs), more attention to real-life applications, and also noted how students who were weak in algebra could pass procedures such as equation solving over to CAS when trying to learn new topics that required those skills.

At her final interview Iris told us that her use of MAS had been greatly limited by her allocation to classrooms without easy access to a data projector, interactive white board or even overhead projector, which prohibited her publically showing calculator use. Special efforts had to be made to ensure that she had appropriate equipment for the observed lessons, but Iris noted that when she had to bring equipment to a room it often took about 15 minutes to set up; far too long in the context of a normal teaching day.

Early Year 1	Late Year 2
--------------	-------------

Figure 6 Maps of Iris's reports of teaching with MAS

Results: Lesson observations, focus groups and interviews

This section reports how the teachers took advantage, or not, of the pedagogical opportunities provided by MAS. Lessons, focus groups and interviews were analysed for evidence of choices of how the teachers used technology. As noted above, several lessons by each teacher were observed. For the four case study teachers, these included all three research lessons and others of their own choosing. Analysis of the videos for each teacher showed strong similarities between the enacted lessons, and so the lesson observations of the four teachers are discussed together. The similarities between these teachers appeared stronger than between other groups of teachers that we have studied. This may be because these teachers worked together at the same school sharing the same students and school culture, or possibly it is a characteristic of early majority teachers – we do not have evidence on this point.

We will show both use and change by reporting observations of the first and last research lessons: a quadratics application lesson taught after the teachers had had about four months working with TI-Nspire(CAS) and a lesson on transformations just over a year later. Each research lesson was designed to take advantage of selected task-level pedagogical opportunities of MAS: the quadratics lesson to make use of *simulation* and to *link representations*; the transformations lesson to *explore regularity and variation*, *link representations* and to support *learning pen-and-paper skills*. Teachers were provided with lesson notes, student worksheets (at the request of the school) and prepared multi-page TI-Nspire files for teacher and student use, including dynamic diagrams and prepared spreadsheets etc. As noted above, other lessons by all teachers were observed and analysed. The observations presented below are consistent with the observations of the full set of lessons.

The quadratics lesson is fully described by Wander and Pierce (2009). It completed their unit of work. Briefly, students found an algebraic expression for the area of a fish shape (a dynamic diagram) as a function of the 'body length' (see Figure 7), found the same function by data capture, and interchanged algebraic forms using CAS and/or pencil and paper and found the minimum value. The topic of the transformations lesson was finding equations of the image of translated or reflected parabolas. Introducing the language and notation for transformations was an important goal of the lesson for the school. The MAS was mainly used in this lesson to demonstrate translation and reflection, controlled by sliders (see Figure 8). Students could see the transformation taking place, and observe its effects on the graph and the equation.

Data from all sources are combined in the results sections below. We recognise that any set of lesson observations can provide only a snapshot of a teacher's repertoire, so we do not expect a tight relationship between the data from observation and teachers' self-reports. Examining them together gives a fuller picture.

Increased skill in using technology

As Gina commented at her final interview, the calculator and software were complex to learn.

Gina: It's hard at the start. Really hard at the start! But the confidence I had this time last year and the confidence I have now - it's totally different. The only thing is that there's so many features [.....] All the different functions and the different features and settings are really valuable.

The observations reported in this section will demonstrate that over the life of the project, all teachers made substantial progress and became comfortable using the MAS in class.

In the quadratics lesson of year 1, it was obvious that the teachers needed to concentrate on getting the technology to work. For some, this included managing the relatively new task of projecting a high resolution computer/calculator display and incorporating it well with handwritten work on the traditional whiteboard. For example, Hari turned on the interactive white board but taught by writing on it rather than using it as a learning tool for visualisation. The students used handheld TI-Nspire(CAS) while Hari gave verbal instructions. For all four teachers, there were often moments when the lesson flow was disrupted by technical glitches that seem trivial in retrospect, such as having to locate a different page in the TI-Nspire(CAS) multi-page document, or clicking in the wrong window to collect data from a dynamic diagram.

A year later at the transformations lessons, all teachers' technology skills had improved considerably. All appeared very comfortable in using the MAS for demonstrations to the class and they incorporated the projected calculator output image (on traditional or interactive whiteboard) seamlessly into their lesson presentation, alternating between it and the electronic student worksheet. Even Iris, who explained that she rarely had access to projection facilities in her classrooms and so might have had less practice, used the technology for mathematics and presentation fluently.

However, there was still more for the teachers to learn. For example, other teachers provided assistance at the beginning of all the research lessons to ensure that all students had correct access to the prepared TI-Nspire(CAS) document. To our knowledge, the teachers did not use prepared files of their own making but generally used the MAS for immediate calculation like previous graphics and CAS calculators without a document facility. Learning to use such complex technology takes time. In summary the project reinforced for the researchers, the magnitude of the task of learning to use new technology for teaching mathematics, and the need for support, time and accumulated teaching experience to accomplish this change.

Taking up task level pedagogical opportunities

This section discusses how the identified task level pedagogical opportunities were enacted in the observed lessons. The quadratics lesson especially highlighted the use of multiple representations, intending to link the dynamic fish geometry, the symbolic expression for the area, and its graph (see Figure 7). The teachers did not make explicit links between the geometric shape, values in the table and the graph. They seemed to assume that students would link the representations themselves. Only Iris asked the students to be systematic in their data collection.

Iris: *Make the body small; all the way over to making the tail really small; and collect at least ten points along the way.*

Once the data had been captured and plotted, all four teachers asked the students about the shape of the resulting curve and linked the turning point with the minimum total area.

Gina: *What is that shape?*

Students: *Parabola*

Gina: *If looking for the minimum area, what characteristic of the graph [is required]?*

Students: *Turning point.*

Figure 7 The triple screen from quadratics lesson

The prepared files for the transformations lesson offered pedagogical opportunities to explore regularity and variation. This was strongly inhibited by the focus on definite answers (in contrast to exploration) that is a key part of the didactic contract for mathematics, discussed below. For example, none of the teachers publically displayed the general effect of a horizontal translation by h units with free movement of the h slider provided. Two possible positions of the translated source are shown in Figure 8. Instead, they only showed positioning of the translated source directly above the desired image parabola ($h = -5$), ready for moving down. Perhaps they considered that free sliding was too obvious to be shown to students. Another possible instance where the slider offered a pedagogical opportunity of regularity and variation was to show how the turning point of the image parabola can be represented by $(3+h, 2)$ regardless of whether h is positive or negative. Some students struggled with the conflict between adding or subtracting a negative h . This result was stated, and checked against the example, rather than explored in generality.

Figure 8 Screen dump from transformations file, with added horizontal translations (dotted)

Looking beyond the two lessons, the case study maps in Figures 3 to 6 suggest that there was considerable expansion of recognising task level opportunities over the two years. We attribute these changes to the experience of the research lessons which modelled some pedagogical opportunities, teachers' growing facility with using the technology, their experience of the lessons shared amongst the group and their own accumulating experience of using the calculators in class. The main pedagogical opportunities mentioned are linking representations, exploring and discovering regularities, and some use of technology to support the development of pen-and-paper skills (e.g. by checking), using functional capabilities of graphing, statistics and very occasional symbolic algebra. Teachers made little mention of being able to use real data in problems (which is a capability shared with scientific calculators) and almost none of the greatly expanded capacity for simulation, with only Hari reporting any use. Our explanation is that using a simulation requires some 'programming' of the TI-Nspire(CAS) (e.g. using the dynamic geometry) to make a prepared file for students, and even by the end of the project, teachers were not equipped to do this for themselves. When discussing the pedagogical opportunities in the final interviews, the main example the teachers gave is the now classic graphics calculator lesson showing how the graph of a quadratic function changes as the coefficients are changed, which benefitted from improved presentation features such as better screen resolution, sliders etc. Just as Hari introduced some new technology syntax to her students in the context of familiar mathematics, it seems that these teachers are adapting familiar tasks to a new technology. This seems to be a sensible approach that should be capitalised on. Overall, however in these case studies we saw little evidence of spontaneously using the new mathematical and document features of the MAS, even by the end of the second year.

Taking up classroom level pedagogical opportunities

In this section, we report on two interlinked aspects of the classroom - the social dynamics and the didactic contract. Because the story of the didactic contract for mathematics is different to that -for technology, these are treated separately. In all classrooms, there was a friendly and co-operative atmosphere, with good relations between teachers and students. For all four teachers, with both lessons, the general pattern of the lessons was for the teacher to read out loud the worksheet text

and questions, providing additional explanation where required. The students were often asked to volunteer publically an answer to the question, which the teacher commented on and recorded the correct answer for all to see. While students worked individually for short time periods to complete a few items, the teacher was available for assistance. Students appeared to expect individual assistance: for example, there were almost no questions asked publically.

Changing social dynamics

The medium and heavy shading at the classroom level of the pedagogical maps for the four teachers show that all perceived some opportunities to change the social dynamics of the classroom. There were a few comments related to this in the interviews. Iris commented that when there was considerable use of technology, as in the research lessons, the lessons would be inherently more complex so *“there’s more likely to be interactions among students... it certainly generates that kind of co-operative interaction”*. Fran also commented on greater student engagement in the research lessons, which she attributed to the teachers being *“more interactive”* with the students, and students feeling *“they want to keep up”* with the lesson, in contrast to individual work from the textbook. In the MTPS perceptions survey, all teachers reported a neutral or positive effect on students’ enjoyment of mathematics over the project life.

However, in our lesson observations, we observed no significant change in the social dynamics from the beginning to the end of the project. We interpret this to mean that such change was not desired by these teachers. Other classroom organisation to that described above was possible with the worksheets supplied, and teachers from other schools used the MAS to encourage more independent work as well as to encourage pair-wise discussion through working with shared screens.

Changed didactic contract with regard to technology

In an MAS environment the didactic contract is about the reciprocal responsibilities and expectations of the teacher and students with respect to both mathematical and technical (“how to use the calculator”) knowledge (Pierce & Stacey, 2010). This section examines the contract for technical knowledge. The comparison of the quadratics and transformations lessons showed a considerable change here, with greater independence expected of the students. We attribute this to the increased technology skills of the teachers (see above), as well as the better skills of the second student cohort, who had used the calculators more consistently than the first cohort.

For all four classes, in the quadratics lesson, the teachers and the students expected that the teacher would be the authority with regard to the technology. Each teacher described each new calculator action in detail and the lesson flow was frequently paused to make sure that all students had pressed the correct buttons or conquered actions such as clicking and dragging. Some teachers moved around the classroom to ensure that each student had exactly the same screen. Nearly all the students waited for this guidance from the teacher. Figure 7 provides an example. Data capture was used to collect values of the area of a fish shape from a dynamic diagram (simulation) as the body length changed. The ordered pairs were automatically stored in a spreadsheet and plotted. This resulted in the triple screen (see Figure 7) of the dynamic fish diagram, a spreadsheet of captured points (body length, area) and a scatterplot of this data. The teacher’s emphasis on the technical issue of how to capture points is evident in the spreadsheet of Figure 7, where we can see that the same point ($b = 8.232$) has been captured many times as she demonstrated the process.

By the transformations lesson, all four teachers gave considerably fewer instructions on technology use. Fran provides an example of the change over time. In the quadratics lesson, Fran took responsibility for technical knowledge, leading the students keystroke by keystroke. We observed very little student-to-student technology help; they mainly waited for the teacher. In the transformations lesson, Fran expected that this new group of students would take initiative in learning to use the technology themselves. Fran still gave technical demonstrations but quickly. Students also proposed alternative calculator methods, which were accepted by Fran. Instead of leading key stroke by key stroke as before, Fran said she would show 'one-way' and allowed the students to choose their own calculator method.

Unchanged didactic contract with respect to mathematics

This section examines the didactic contracts with respect to the mathematics rather than the technology. In the quadratics lesson there were several key points where the students might have taken more responsibility for their own mathematics learning. Early in the lesson, students saw a dynamic diagram (on a prepared file on each student's calculator) in the shape of a fish. A slider allowed the user to observe the effect on the size and shape of the fish of varying the body length. This opportunity to engage students by making conjectures about the nature of the functional relationship and create interest in solving the problem was only taken up by Gina. Instead there was a focus only on finding the minimum area. Iris did not discuss the dynamic diagram. Fran and Hari showed that different body lengths produced fish that looked different and were content that the students seemed to enjoy using the slider to create different images. (In classes at other schools, teachers encouraged conjecturing about the relationship between body length and area.)

Fran: Does the total length of the [fish] change?

Student : No

Fran: As you move it [the slider] along, the body gets smaller and the tail gets larger.

Only Gina encouraged any conjecture about the way the area of the fish varied:

Gina: How many different possibilities [for the areas and shapes] do you think there are?

Several students responded at the same time: one commented that there is only one possibility (misunderstanding the intention of the question) and others responded "lots", "many". Gina then 'funnelled' the responses to get her desired answer of "infinity" and the lesson continued with students individually manipulating the slider to find the exact minimum area.

All four teachers directed the students to move the slider to find the minimum area but there was no class discussion of what they observed, relating the total area to the body length or the nature of the functional relationship. Instead discussion focussed on manipulating the diagram to get the most accurate minimum.

A rare instance of students publically conjecturing occurred in Fran's transformations lesson. The worksheet asked students to find and describe the links between the horizontal translation of -5 units and vertical translation of -6 units, the source parabola $y = (x-3)^2 + 2$ and the image parabola $y = (x+2)^2 - 4$ and coordinates of the turning points (3, 2) and (-2, -4). The teacher wanted students to notice that when the turning point (3, 2) is translated to (-2, 4), the translation along the x-axis is to 2

- 5 = -3 and along the y-axis is to 2 - 6 = -4. The 1 minute 15 second transcript below shows how a correct response was accepted (the small add/subtract error of S1 is ignored at this stage), and the false conjecture of S2 was immediately dismissed.

Fran: [Reads question to whole class and begins to give answer]. *So what we do is ...* [notices students with hands up]. *Yes?*

S1: *That -2 and +3, both the x-values of the turning point, if you add them together you get -5.*

Fran: *Yes, that's right, that's right. And what else?*

S2: *If you times them together, you get -6.*

Fran: *Well...you are confusing the [x-axis] translation with the y-axis translation. We might not combine them, but just look at what we did to +2 and -4. Or, how we getthe -6 from the +2 and the -4.*

S3: *Added together.*

Fran: *Yes, added together, yes $2 + (-6) = -4$.* [Fran goes on to write correct answers on the whiteboard, with explanation about adding and subtracting negative numbers].

The example above shows the teacher's control of the class mathematics, and the emphasis on arriving at correct answers quickly. The other observed lessons were like this, for all teachers at the school, and for the duration of the project. To some extent, this may have been exaggerated by the lesson study format: *"When I have many other people observing I find that I focus on them and lose a little bit of my adventurous spirit when tackling problems."*

In the initial interviews, the aspect of changing the didactic contract which the teachers were most interested in was to encourage more active participation (e.g., more questioning) on the part of students. From the teachers' comments in the final interviews and our observations, there was some increase in questioning over the time of the project along with a growing independence of students, but this was very much linked to practical questions about the use of technology.

Gina observed that her students seemed embarrassed to ask about mathematics that they don't understand whereas they feel freer to ask about technology. This may be related to their level of understanding: it is harder to formulate questions about a mathematical issue than about practical button pressing with technology. Additionally, it may be less confronting to self-image to ask a question about a new technology.

Gina: *They seem to ask a lot more questions when they have the technology. ...I think sometimes my students feel a bit embarrassed that they can't do some of the work. [...]* [When they are using technology, the calculator gives them part of the answer], *so they're a bit more confident in seeking assistance because they don't feel as embarrassed.*

The evidence above shows that the didactic contract with respect to mathematics was constant. Teachers were in charge of the mathematics; they requested contained answers to well-formulated questions and responded quickly; only rarely did any student offer an idea about the mathematics in public. In the observed lessons, technology was used to find the correct answer (functional use),

often passing over the opportunity for it to assist students to explore or make and test conjectures themselves. In all the lessons, the dominant aspect of the teacher's mathematical role was to lead the lesson in a lock-step way and to provide correct answers and explanations.

Summary - classroom level opportunities

The lesson analysis and the maps suggest some changes at the classroom level for all four teachers. However, overall it seems that the teachers at this well functioning school are generally content with both the social and cognitive (didactic contract) aspects at the classroom level and are not seeking major changes. At the final interview, each teacher commented that the students expected their teacher to teach them how to use the technology and to constantly troubleshoot. They made comments about their moves (and we observed their success) to encourage independence in an area where students appear very dependent, despite being of the "digital native" generation. In Roger's terms, the new technology is compatible with many classroom didactic contracts and social organisation: there are opportunities, but change in is not inherent in adopting the technology.

Subject level opportunities

At the subject level the maps above indicate steady or increasingly positive responses in each category. For example, there was a clear change for Gina and Hari in exploiting the contrast between machine mathematics and ideal mathematics. To explore this, in the interviews we asked teachers to elaborate on how they dealt with unexpected answers from MAS. Teachers started to deliberately provoke mathematical discussion by, for example, Hari contrasting exact and approximate answers and Fran and Gina discussing unexpected algebraic forms. We have often observed that teachers beginning to use technology are very concerned to protect students from the puzzling or unfamiliar mathematical responses of the MAS. It was interesting to see that this reality was no longer regarded as an extreme negative by the teachers, but was seen to have some potential benefits. In Roger's terms, teachers began to see some relative advantage in the algebraic capability of the new technology.

Conclusion

We introduced our four teachers as part of the 'early majority' for adopting the new technology of TI-Nspire(CAS). They made the change because of a decision of their school, which they supported and followed keenly, but they were not the initiators. Of course, these good teachers may have been initiators in other aspects of school life. This early majority status distinguished them in several ways from many of the teachers who had been studied in other research projects. They were dependent on the school for the provision of equipment and for professional development in mathematics, such as the lesson study with the university. The goals they identified for the innovation were practical and closely related to their daily teaching tasks, and did not spring from any dissatisfaction with current practice or a deep urge for change. These teachers worked confidently and competently within the system, aiming to meet the requirements of the school and the education system, rather than serving mathematical goals of their own making. By the end of the second year, all were very competent at using the technology as required, but none of them expressed to us a strong interest to explore more of what the technology could do. Whilst some of them enjoyed showing the dynamic fish image and got a 'wow' reaction from the students, they did not actively seek out programs such as this. Their concern lay very directly in the classroom and specified

curriculum. The 'maps' give a picture of the teachers' perception and uptake of pedagogical opportunities that indicates change over a two year period. New practices were slowly added to each teacher's repertoire. One of the current challenges may be that significant changes in both software and hardware design have been happening so rapidly that these early majority teachers will feel almost constantly hampered by the need to learn and teach new technical skills and so continue to make limited progress in taking advantage of opportunities to approach mathematics concepts in new ways.

Roger's theory of diffusion is very useful for analyzing the professional development of teachers that is required for an innovation such as the use of MAS to be adopted in schools and we recommend that it is used more often in mathematics education to guide the thinking of educational systems, researchers, and school leaders. First it points out that the early adopters and innovators will have different characteristics to teachers in the other categories. This study revealed some of these characteristics for the early majority, as summarised in the paragraph above. From the point of view of school leadership or a centralised school system, understanding the 5 stages of adoption of an organizational innovation from Roger's theory can assist in planning strategies for improvement. From this study we learned something of the timescale (longer than might be expected) and mechanism of the stages of redefining the work and clarifying what using the new technology meant in practice. Importantly, Roger's diffusion features of communication and the social setting were well met in this school, enabling us to examine intrinsic characteristics of the innovation itself.

In particular, the intrinsic characteristic of complexity of the new technology for these teachers (calculator and classroom display, technical and mathematical knowledge) was a considerable obstacle, which was largely but not fully overcome in the 20 months of the project. This factor is easy to under-estimate, as early experiences can be derailed by obstacles that in retrospect are trivial. Gina's observation that uncommitted teachers would not persist is likely to be true - this points to the need for in-house support on all aspects of the innovation. In the present study, it was provided by colleagues, backed by the university team. However, it is clear to us that the third stage of innovation was far from complete at the end of the project. Although the teachers became confident technology users, they still used only a limited range of its capabilities.

Apart from complexity, Rogers identified four intrinsic characteristics of innovations that influence an individual's decision to adopt or reject an innovation: relative advantage; compatibility, trial-ability and observe-ability. Trial-ability was provided here by the school making the MAS equipment available to teachers and students (a significant financial decision) and observe-ability was put in place through the lesson study program and by a culture of sharing amongst teachers. As reported elsewhere (Pierce & Stacey, 2011), the lesson study was successful in that it gave teachers a supported experience with both functional and pedagogical uses of the new technology, but also provided the observability challenge of performing in front of their peers.

In Roger's terms, the new technology is compatible with many types of classroom didactic contracts and social organisation: there are opportunities, but change at the classroom level is not inherent in adopting the technology. The new technology seemed to have been absorbed into current practice, more than changing practice. Perhaps this is because at this school, the teachers are generally satisfied with their teaching. The features used and teaching methods adopted were mainly those compatible with previous (graphics calculator based) practice and with the mainly unchanged

didactic contract for mathematics. A simpler, but no less important, aspect to compatibility is the extent to which the physical requirements for using the new technology are met by conditions in the school. Limited access to data projectors made trialling difficult in some instances, showing that the innovation was not entirely compatible with these teachers' workplace. The experiences of the teachers with carrying equipment and losing lesson time re-emphasise for us the need for schools to provide the necessary equipment in all classrooms, if there is to be a realistic expectation that teachers will use technology regularly. Even this apparently well-equipped school did not provide what all the Year 10 teachers needed to work effectively with MAS.

The relative advantage of the new technology for teaching Year 10 classes came from a general desire to teach some topics better. In the interviews, Fran and Hari (who had more experience with previous calculators) saw many improvements of the new technology over the old (e.g. better handling and formatting of indices, some statistical processes, presentation features), thus supporting the school decision for change. Experience at the initial professional development sessions indicated to the researchers that there were essentially five new capabilities of the TI-Nspire(CAS) that were useful in Year 10 mathematics: dynamic geometry and the possibility of data capture from the diagrams; symbolic algebra capability (new to most); using pre-prepared files and the integrated document system, having exact values and algebra capability in the spreadsheet. However, when the teachers summarized what worked well in their own lessons, only exact values and dynamic geometry were spontaneously mentioned. At this stage of their development, these teachers do not identify the distinctive new capabilities as contributing to the major relative advantage of the innovation. Instead they see the relative advantage in the classroom mostly in the incremental improvements to capabilities of earlier calculators, and meeting the need for students to be up to date.

The lack of obvious enthusiasm of teachers for using symbolic algebra, the main new mathematical capability of the new technology, is interesting. We hypothesise that this is because they see their main task as teaching by-hand algebra, and the didactic contract for mathematics, which focuses on getting answers, reduces its exploratory uses. Additionally there is not yet a bank of well-known lessons using this new capability. The second research lesson used technology for building by-hand skills with irrational square roots (mostly as a checking device) and this was well accepted. The relative advantage of the new technology over the old was mainly seen as better graphics calculator features (e.g. clarity of display, easier ways to access features, added capability), and so high compatibility with existing practice was built into the implementation.

In summary, professional development based around lesson study can be recommended both for teacher development and as a setting for studying teacher change. The theory of diffusion of innovations provides an excellent framework for the analysis and it also highlights the importance of research broadening beyond early adopters and innovators. The study has revealed many ways in which the early majority are different, as well as the long timeframe for building up their expertise. The pedagogical maps have provided a useful framework for analyzing practice and changes in practice, and can also be used to present opportunities for teachers to consider. The study indicates that a strategy for implementation must take a multi-year approach, beginning with teachers using improved capabilities to accomplish familiar tasks, but following a steady plan to redesign the learning environment, lesson by lesson, to incorporate new features and new pedagogical opportunities.

Acknowledgments

We wish to thank the teachers and students who participated in this study, Roger Wander, Lynda Ball, and guest observers of the research lessons, and we acknowledge the financial support of Texas Instruments Australia.

References

- Artigue, M. (2002). Learning mathematics in a CAS environment: The genesis of a reflection about instrumentation and the dialectics between technical and conceptual work. *International Journal of Computers for Mathematical Learning*. 7, 245–274.
- Berry, J., Graham, T., Honey, S., & Headlam, C. (2007). A case study of the issues arising when teachers adopt the use of a new form of technology in their teaching for the first time. *The International Journal for Technology in Mathematics Education*. 14(3), 150-160.
- Brousseau, G. (1997). *Theory of didactical situations in mathematics : didactique des mathématiques, 1970-1990*. Edited and translated by N. Balacheff, M. Cooper, R. Sutherland, V. Warfield. Dordrecht, The Netherlands: Kluwer.
- Drijvers, P., Doorman, M., Boon, P., Reed, H., Gravemeijer, K. (2010). The teacher and the tool: instrumental orchestrations in the technology-rich mathematics classroom. *Educational Studies in Mathematics*. 75(2), 213-234.
- Kendal, M., Stacey, K., & Pierce, R. (2005). The influence of a computer algebra environment on teachers' practice. In D. Guin, K. Ruthven, & L. Trouche (Eds.), *The didactical challenge of symbolic calculators: Turning a computational device into a mathematical instrument*. (pp. 83-112). Dordrecht, The Netherlands: Kluwer.
- Pierce, R. & Ball, L. (2009) Perceptions which may affect teachers' intention to use technology in secondary mathematics classes. *Educational Studies in Mathematics*. 7(1), 229-317.
- Pierce, R. & Ball, L. (2010). Secondary Teachers' Use of Technology for Teaching Mathematics In *Making a difference. Proceedings of the 2010 Conference of the Australian Association for Research in Education*. Retrieved 28th February 2011 from <http://aare.edu.au/10pap/2214PierceBall.pdf>
- Pierce, R., & Stacey, K. (2009). Researching principles of lesson design to realise the pedagogical opportunities of mathematics analysis software. *Teaching Mathematics and Its Applications*. 28, 228-233.
- Pierce, R., & Stacey, K. (2010). Mapping pedagogical opportunities provided by mathematics analysis software. *International Journal of Computers for Mathematical Learning*. 15(1), 1- 20.
- Pierce, R & Stacey, K. (2011). Lesson Study for Professional Development and Research. *Journal of Science and Mathematics Education in Southeast Asia*. 34(1), 26 – 46.
- Pierce, R., Stacey, K, & Wander, R. (2010). Teaching with technology and the didactic contract. *ZDM*. 42(7), 683-695.
- Pierce, R., Stacey, K., Wander, R., and Ball, L. (2011). The design of lessons using mathematics analysis software to support multiple representations in secondary school mathematics. *Technology, Pedagogy and Education*. 20(1), 95-112.
- Rogers, E. (1995) *Diffusion of Innovations*. New York: The Free Press.
- Stacey, K., Asp, G., & McCrae, B. (2000). Goals for a CAS-active senior mathematics curriculum. In M. O. J. Thomas (Ed.), *Proceedings of TIME 2000 An International Conference on Technology in Mathematics Education*. December 11-14, 2000 (pp. 244-252). Auckland, New Zealand: University of Auckland and Auckland University of Technology.

Thomas, M.O.J., (2006). Teachers using computers in mathematics: a longitudinal study. In J. Novotná, H. Moraová, M. Krátká & N. Stehliková (Eds.) (Vol 5. pp 265 - 272). *Proceedings of the 30th Conference of the International Group for the Psychology of Mathematics Education*. Prague, Czech Republic:IGPME

Trouche, L. & Drijvers, P. (2010). Handheld technology for mathematics education: flashback into the future. *ZDM*, 42, 667–681

VCAA (2010). *VCE mathematics study design. Revised edition 2010*. Retrieved from Victorian Curriculum and Assessment Authority Web site: <http://www.vcaa.vic.edu.au/vce/studies/mathematics/mathstd.pdf>

Wander, R. & Pierce, R. (2009). Marina's fish shop: A mathematically- and technologically-rich lesson. *Australian Mathematics Teacher*. 65(2), 6-12.

Appendix: P-Map Teacher Survey

P-Map Teacher Survey Part A: Likert Items

Teachers were asked to respond to the following items using the 5 point scale below

No opportunity	Didn't choose to do this	Tried didn't work well	Tried went OK	Tried went well
----------------	--------------------------	------------------------	---------------	-----------------

*P-Map Link Codes: F: functional use T: task; C: classroom; S: subject; PP: scaffolding pen- and paper; RD: real data; E: explore; S: simulation; LR: link representations; SD: social dynamics; DC: didactic contract; IM: ideal & machine; RB: rebalance; MO: metacognition & overview

P-Map link*	Item No.	Item
		Let students use technology to:
F T PP	A1	check each step of their work.
F T	A2	do 'hard' or 'long' arithmetic calculations (e.g. divide by 0.A323)
F T	A3	do 'hard' or 'long' algebra.
F T	A4	sketch 'hard' graphs
F T	5	draw/construct accurate geometric diagrams
		Set tasks which required students to:
T RD	6	conduct experiments and collect real data.
F T RD	7	work with 'messy' real world data provided by the teacher.
T RD	8	think about a real world context but using data that has been modified to make calculations 'simpler'.
T PP E	9	do many similar algebraic examples to spot a pattern.

T P P E	10	draw many graphs to identify the features of various families of curves.
T E	11	change the coefficients in an algebraic example to see what happens. (e.g. try various values of c to see effect when solving $y = ax^2 + bx + c$ for x)
T E	12	draw many graphs to identify the effect of varying a coefficient or constant.
		Set tasks which required students to:
T S	13	use simulated data I had provided (eg. by using random number generator)
T S	14	collect data from a statistical simulation
T S	15	collect data from a geometric simulation of a real world problem.
T LR	16	use a graph to solve an equation
T LR	17	use tables / spreadsheets to solve an equation
T LR	18	used tables / spreadsheets and statistical graphs (charts or plots) to solve a problem
		As a teacher I:
S MO	19	used technology to give students an overview then went back to look at details
S IM	20	used non-standard output produced by technology to deepen students' understanding
S IM	21	deliberately set tasks which I knew would produce non standard or unexpected output.
C DC	22	encouraged students to use technology to check answers instead of asking me.
C DC	23	set problems that my students could not solve without using technology.
C SD	24	made use of technology when I was introducing a new topic
C SD	25	used technology via a viewscreen / data projector to illustrate a mathematical concept
C DC	26	used technology via a viewscreen / data projector to demonstrate solution strategies
C SD	27	encouraged students to use technology via a viewscreen / data projector to show their solution strategies to the rest of the class

*P-Map Links Codes: F: functional use T: task; C: classroom; S: subject; PP: scaffolding pen- and paper; RD: real data; E: explore; S: simulation; LR: link representations; SD: social dynamics; DC: didactic contract; IM: ideal & machine; RB: rebalance; MO: metacognition & overview

P-Map Teacher Survey Part B: Open Response Items

P-Map link*	Item No.	Item
F T S	1	What use is currently made of TI-Nspire(CAS) in your class? Please give details/examples.
DC	2	For your year 10 class, has the availability of TI-Nspire(CAS) prompted any change in what <i>you expect of students</i> during a lesson? or out of class? Please give details/ examples.
DC	3	For your year 10 class, has the availability of TI-Nspire(CAS) prompted any change in what <i>students expect of you</i> during a lesson? or out of class? Please give details/ examples.
SD	4	In classes where considerable use of TI-Nspire(CAS) takes place as compared to a class where no technology is used: Have you observed any change in the interactions between students or between the students and teacher? Please give details/ examples.
SD	5	TI-Nspire(CAS) , like other CAS calculators, can produce unexpected results such as unexpected format, unexpected error messages, unexpected setting out etc. How do you deal with this in class? Please give details/ examples.
RB	6	Has the availability of TI-Nspire(CAS) had any impact on the emphasis or sequencing of topics you teach in year 10 ? Or on the details within the topic? Please give details/examples.
RB	7	Are there any other ways in which the availability of TI-Nspire(CAS) has made you think differently about how to teach a particular topic? Please give details/ examples
	8	Anything else you think we should know about teaching with TI-Nspire(CAS) ...

*P-Map Links Codes: F: functional use T: task; C: classroom; S: subject; PP: scaffolding pen- and paper; RD: real data; E: explore; S: simulation; LR: link representations; SD: social dynamics; DC: didactic contract; IM: ideal & machine; RB: rebalance; MO: metacognition & overview

Minerva Access is the Institutional Repository of The University of Melbourne

Author/s:

Pierce, R;Stacey, K

Title:

Teaching with new technology: four 'early majority' teachers

Date:

2013-10-01

Citation:

Pierce, R. & Stacey, K. (2013). Teaching with new technology: four 'early majority' teachers. JOURNAL OF MATHEMATICS TEACHER EDUCATION, 16 (5), pp.323-347. <https://doi.org/10.1007/s10857-012-9227-y>.

Persistent Link:

<http://hdl.handle.net/11343/282662>