Estimation of grain elasticity properties from ultrasonic measurements on dry granular pack

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ABSTRACT

The elastic properties of crystalline rocks can be estimated from ultrasonic measurements on the powders of crushed rocks produced by the drilling process. To determine the elastic properties of grains from properties of powder packs, we study the dependence of their ultrasonic wave velocities on pressure. From this dependency, using the Hertz-Mindlin (HM) theory, we can calculate the effective ratio of the grain shear modulus to one minus the Poisson ratio. The HM theory requires the knowledge of grain coordination number as a function of pressure, which can be obtained using an empirical relation based on published numerical simulations. Previous work has shown that this approach gives an

accurate prediction of the effective bulk modulus of glass beads but produces a significant discrepancy for sand. This discrepancy may be attributed to the angularity of sand grains. To overcome this problem, we introduce a shape factor into the empirical relation for the coordination number. This new shape factor allows us to reconcile the rock physics model with laboratory measurements. We show that the shape factor varies from 1 to 2.5 for different grain shape angularity and sorting (grain size distributions). The modified theory allows us to estimate a combination of elasticity parameters of the grains from the measured dependence of P-wave velocity in the pack on the pressure.

Keywords— Rock physics, Mining, Ultrasonic velocities

INTRODUCTION

For over the last two decades, the elastic response of materials consisting of packed grains has been of interest to scientist and engineers due to the vast spread of such materials. In exploration geophysics, there are two primary objectives of studies of the elastic or acoustic response of granular media. The first objective is the understanding of the acoustic properties of unconsolidated (granular packs) from lab measurements (Prasad and Meissner, 1992; Zimmer et al., 2007; Dutta et al., 2010). And the second objective is to interpret the sonic and the seismic data from the field (Bachrach et al., 2000; Bachrach and Avseth, 2008). These studies show some agreements between theoretical predictions of bulk modulus and experimental data, as well as discrepancies between theoretical predictions of shear modulus and experimental data. In this paper, we want to use the theoretical models and experimental measurements on granular packs to estimate the elastic properties of the grain. This is important for example for estimation of the elasticity properties of rock in mineral exploration using the drilling powders when only the drilling powders are available for inspection and are of interest for fluid substitution in poorly consolidated reservoirs. The elastic response of packs is also scientifically interesting because of its non-linearity, even though the grain contacts (at the microscopic level) are behaving as linearly elastic.

Although there are studies of the elastic properties of the pack as a function of the grain properties and hydrostatic pressure (Zimmer et al., 2007), there is not any research that looks at the inverse, where we want to determine the elastic properties of the grains from the response of the pack.

Herein, we will use the existing theories predicting the P- and S-velocities as functions of hydrostatic pressure and grain properties. Hertz-Mindlin (HM) model (Mavko et al.,

2009; Makse et al., 2004; Zimmer et al., 2007) describes the velocity-hydrostatic pressure relationship of the granular pack. The HM model shows that the bulk modulus of the pack is proportional to the third root of pressure (i.e. $K \propto P^{1/3}$) due to the effect of Hertzian contact (Landau and Lifshitz, 1970; Johnson, 1987). Makse et al. (2004) confirm the HM model prediction of bulk modulus of glass beads with experimental data by using an empirical dependence of coordination number, i.e. the average number of grains in contact, and porosity on pressure from numerical simulations (Zhang and Makse, 2005). Garcia and Medina (2006) confirm the dependence of coordination number and porosity to the hydrostatic pressure and thus validate the HM model. Zimmer et al. (2007) have predicted the bulk modulus of glass beads, based on the empirical equation of coordination number, with good agreement with experimental data, although their empirical equation of coordination number is different from Makse et al. (2004) and Garcia and Medina (2006). By assuming their empirical expression for the coordination number, they found the stress-dependency of moduli varies from the fourth root to the sixth root of pressure. While the predicted bulk modulus is in good agreement with experimental data in a limited range of hydrostatic pressures, there is a discrepancy between the estimated shear modulus of the frictional grain's contact model and the experimental data. Zimmer et al. (2007) show the no-friction contact model, i.e. Walton's model (Mavko et al., 2009), can predict the experimental shear modulus. This agreement, however, can not provide any extra information of grain's elastic moduli. Dutta et al. (2010) proposed an expression for the empirical coordination numbers by fitting extended Walton model with an ad hoc coefficient (α equal to 0.6) to overcome to the discrepancy of shear modulus (and the discrepancy of shear-wave velocity) predictions. In this paper, we focus on the estimation of bulk modulus of unconsolidated samples with angular grains, and we propose an extra fitting parameter to take the angularity of grains

into account.

Zimmer et al. (2007) show that the estimated bulk modulus of dry-sand samples (Pomponio Beach sand) has shown larger exponent of pressure dependency, and they conclude that the angularity of grains may be the reason for this discrepancy. That means the micro-mechanics of contacts of angular grains is not obeying the Hertzian contact mechanics assumptions. They also predict if HM theory of bulk modulus has been used to fit dry-sand data by using the coordination number as a fitting parameter, the coordination number is almost 1.5 times larger than the empirical equation of the coordination number of spherical grains (refer to Figure 9 in Zimmer et al. (2007)). Dutta et al. (2010) propose an empirical generalized coordination number to overcome the discrepancy between measured and predicted of P- and S-wave velocities, or bulk and shear modulus. Their generalized coordination number is a function of pressure and α as $C(P, \alpha) = (a + bP^c)/(d + \alpha^c)$, where a, b, c and d are fitting parameters. α is defined as an ad hoc coefficient of friction, it is equal to zero for non-friction, and one for perfect adhesion. Thus the angularity of grains can change the micro-mechanics of contacts, the friction of inter-granular contacts, and the coordination number of angular grains (irregular grains) can be different from the spherical grains. Our approach needs only one extra fitting parameter related to the angularity of the grains in addition to the parameters required to describe the dependency of the spherical coordination number on the pressure. In our studies, we assume that the coordination number of angular grains differs from the coordination number of spherical grains by multiplication.

In the following sections, we use these theoretical formulations to invert for grain properties from measurements of P- and S-wave velocities in the pack as functions of hydrostatic pressure.

THEORY

The Hertz-Mindlin model is a mean field approximation that takes both the normal and the tangential grain contact force into account. The predicted bulk modulus is in agreement with the Walton model (and extended Walton model) and the hydrostatic approximation of Johnson et. al. (Mavko et al., 2009). These theories show the effective bulk modulus as a function of hydrostatic pressure P :

$$K_{eff} = \frac{1}{3} \left[\frac{3C^2(1-\phi)^2}{2\pi^2} \left(\frac{\mu_g}{1-\nu_g} \right)^2 P \right]^{1/3}, \quad (1)$$

where μ_g and ν_g are grain shear modulus and Poissons ratio, respectively. C is the coordination number, and ϕ is the porosity of the pack. In these models, the coordination number and porosity are the unknown parameters. Makse et al. (2004) have shown that these geometrical parameters are also functions of hydrostatic pressure and for glass beads the relationship is:

$$C(P) = C_0 + \left(\frac{P}{10 \text{ MPa}} \right)^{0.30(\pm 0.05)} \quad (2)$$

and

$$\phi(P) = \phi_0 - \left(\frac{P}{14 \text{ GPa}} \right)^{0.62(\pm 0.06)}. \quad (3)$$

In figure 1, we plot the variation of porosity of glass bead (■) and Ottawa sand packs (▲) from Domenico (1977). The solid line represents equation (3), with $\phi_0 = 0.385$. The Ottawa sand pack deviates from the empirically predicted relationship of the porosity-pressure curve (equation 3) by less than 1%, which we consider using the porosity-pressure empirical equation as shown in equation (3) for all powder packs with different grain sorting and shapes.

Garcia and Medina (2006) have shown that this power law fits coordination and porosity results in their relationship for glass beads:

$$C(P) = C_0 + 9.7(\phi_0 - \phi(P))^{0.48}. \quad (4)$$

These geometrical parameters are empirically estimated from the simulation of random spherical packs. Zimmer et al. (2007) are using a different coordination and porosity relationship for regular identical spherical grains, namely:

$$C(P) = 24.00 \exp(-2.547\phi(P)) - 0.3731. \quad (5)$$

The coordination number relations proposed by Garcia and Medina (2006) and Zimmer et al. (2007) are not showing any explicit dependency on hydrodynamic pressure. They have only related the porosity (see equations (4) and (5)) which itself is a function of pressure. Zimmer et al. (2007) show the porosity is changing during the cycle of compaction tests (see the Figure 3(b) in their paper). Herein, we use the explicit pressure dependent empirical equations (2) and (3) to analyse our experimental data.

The effective bulk modulus of a pack equation (1), depends on the pressure independent combination of the grain elasticity parameters $\mu_g/(1 - \nu_g)$ that can be expressed as:

$$\frac{\mu_g}{1 - \nu_g} = 3\pi\sqrt{2(1 - \phi(P))} \frac{[\rho_s(V_P^2 - 4/3V_S^2)]^{3/2}}{C(P)P^{1/2}}. \quad (6)$$

Thus, one way of validating the presented theory is to check whether $\mu_g/(1 - \nu_g)$ is independent of pressure. To do so, we plot the velocities measured by Domenico (1977) at different hydrostatic pressures for glass beads, together with the derived moduli in Figures 2a-2e. In particular, Figure 2c shows $V_P^2 - 4/3V_S^2$, which is proportional to the bulk modulus, together with the cubic root function (solid line). The figure justifies this power law for the

medium range of hydrostatic pressures. Figure 2e shows the pressure dependency of the ratio $\mu_g/(1 - \nu_g)$ as given by equation (6) with two alternative assumptions. The red triangles show the results for fixed coordination and porosity ($C = C_0 = 4$ and $\phi = \phi_0 = 0.38$), while black squares correspond to the results with coordination number and porosity given by equations (2) and (3), respectively. Also, figure 2e indicates that the change of micro-geometry during compaction is important and shows the coordination number from equation (2) can be used to estimate the ratio of $\mu_g/(1 - \nu_g)$ of glass beads by using the equation (6). The lower values of coordination number and porosity ($\blacktriangle$ in the plot) overestimate the value of the ratio of $\mu_g/(1 - \nu_g)$.

For most granular materials the assumption of mono-disperse sphere pack and non-angularity is not valid. Shaebani et al. (2012) show that the poly-dispersity would modify the equation (1) by a scalar, which is a function of different moments of radii of spherical grains. The scalar is a function of moments of poly-dispersity radius distributions. On the other hand, there is not any theoretical prediction of angular (non-spherical) grains in literatures for 3D packs. However, it has been shown the wave velocity propagating through the angular granular packs is slower at low pressure (Prasad and Meissner, 1992; Pilbeam and Vainys, 1973). Szarf et al. (2011) simulate 2D irregular grain pack and show that the coordination number increases as a function of the shape irregularity.

Dutta et al. (2010) use the extended Walton model, which introduces a parameter α to represent the friction of grain contacts. This ad hoc parameter is zero for non-frictional contacts and is one for adhesive contacts. They found the P-wave coordination number is, empirically, a function of α parameter as:

$$C(P, \alpha) = \frac{a + bP^c}{d + \alpha^c}, \quad (7)$$

Where a, b, c, d are fitting parameters. In the P-wave velocity case study, c parameter is equal to 0.28 . This exponent is very close to the exponent in the equation (2). Therefore the first approximation to take the grain shape angularities (irregularities) into account is to modify the coordination number in equation (2) by multiplying it by a scalar parameter as:

$$C(P, \gamma) = \gamma \left(C_0 + \left(\frac{P}{10 \text{ MPa}} \right)^{0.30} \right), \quad (8)$$

where γ can be a function of heterogeneity based on non-spherical grain shape (angularity) and poly-dispersity. Comparison of equation (8) to equation (7) shows that the γ parameter can be assumed as a function of α ($\gamma = a/C_0(d + \alpha^c)$). This also can be seen from Zimmer et al. (2007), where they have shown that the bulk modulus of unconsolidated pack of Pomponio Beach sand can be estimated from HM theory only if we assume the coordination number is 1.5 larger than the coordination number of spherical grains (In our notations this translate to $\gamma = 1.5$) from equation (5). Figures 3a and 3b show the μ -CT images of spherical (ellipsoidal) grains packs and Fontainebleau powders packs under ambient pressure. The comparison of the figures shows the mean coordination number of Fontainebleau powders is greater than spherical packs, as expected. In the next section, we estimate γ by calculating $\mu_g/(1 - \nu_g)$ versus hydraulic pressure for five types of rock powders.

EXPERIMENTAL DATA

To verify the use of single parameter and to study feasibility of estimation of the grain properties from the P- and S-wave velocities in the pack, we use results previously reported by Domenico (1977) and Bilenko et al. (2015). We also performed experiments on different powders, namely: 1) crushed Fontainebleau sandstone (Bilenko et al., 2015); 2) crushed quartz-albite-muscovite schist (ameta-morphosed sedimentary rock) from (Bilenko et al., 2015), 3) intact sand from Esperance Beach, Western Australia (this paper); 4) crushed sand of Esperance beach (sample crushed after applying 150 MPa vertical stress) (this paper); and 5) Ottawa sand sample (Domenico, 1977). Bilenko et al. (2015) analyzed the powders by using a scanning electron microscope (SEM), portable X-ray Fluorescence (pXRF) and X-ray diffraction (XRD) to study the mineral and chemical composition of the powders and document the particle size and shape distribution. Their studies show that the Fontainebleau powders and sandstone powders are almost pure quartz (SiO_2), and quartz-albite-muscovite powder consists of 26.1% quartz, 22.1% albite, 12.2% biotite and 39.7% muscovite. They show that powders are homogeneous fine-grained media with an average particle size less than $100\mu\text{m}$ (Figures 4a, and 4b). Sands of the Esperance beach is 99.5% quartz.

Figure 5 shows the schematic of the experimental set-up used in this study. Set up is similar to that reported previously by Bilenko et al. (2015). Dry powder sample is placed inside a poly-methyl methacrylate (PMMA) cylindrical chamber. Two polyether ether ketone (PEEK) plastic pistons are used to apply vertical stress to the powder sample. The sample is compacted in the axial direction by applying stress using a hydraulic actuator operated by a manual hydraulic pump. Piezoelectric transducers V-153A (Panametrics)

are placed at the both ends of PEEK pistons, and transducers are used to measure P- and S- waves velocities propagating through the sample in the axial direction at the central frequency of 1 MHz. Implementation of PEEK pistons allows to reduce waves reflections inside the sample by matching acoustic impedances between sample and piston; moreover, such pistons allow to separate P- and S- waves and decrease uncertainty in the determination of the S- wave arrival. Details of ultrasonic measurements are described in Lebedev et al. (2013). All the experimental measurements are performed at room temperature. The applied axial stress varies from 2 to 50 MPa with the increment of 4 MPa for loading and unloading cycles.

INTERPRETATION

Figures 6a shows the measured P- and S-wave velocities for two different cycles (loading and unloading process) at different pressures for Fontainebleau crushed rocks samples. The Fontainebleau powders grains are angular as shown in the SEM images in Figure 4a, and 2D image of μ -CT shown in Figure 3b. It is interesting to see that the plot of $V_P^2 - 4/3V_S^2$ in Figure 6c for these cycles has the same pressure dependency in 5MPa to 20 MPa range. The solid line in the figure represents the best fit of the third root function. This indicates that the bulk modulus of the granular pack in that range is independent of loading and/or unloading path and behaves as the HM predicted for the bulk modulus of powders (1). To estimate the ratio of grain shear modulus to $1 - \nu_g$ from equation (6), the empirical equation of porosity (3) has been used, and the modified coordination number , i.e. equation (8), has been assumed with various γ . We plot the data in the Figure 6e with γ values of 1.0, 1.5, 2., and 2.5. This plot presents the idea that the wrong assumption of spherical grains ($\gamma = 1.0$) can overestimate the grains modulus (the grains modulus is presented as the solid line).

Figure 6b (similar to Figure 6a) shows the measured P- and S-wave velocities for two different cycles at different pressures for quartz-albite-muscovite schist crushed rocks samples. The quartz-albite-muscovite schist crushed rocks samples grains are more poly-disperse compared to the Fontainebleau crushed rocks sample. These can be seen in the SEM images in Figure 4a and Figure 4b. The plot of $(V_P^2 - 4/3V_S^2)$ has been shown in the Figure 6d for the loading and unloading cycles. The solid line in the plot represents the best fit of the third root function. Similar to Fontainebleau crushed sample, the bulk modulus of quartz-albite-muscovite schist granular pack behaves as predicted by equation (1). In the Figure 6f, we plot equation (6) for various γ . The solid line is calculated from the Hashin-Shtrikman bounds of minerals of quartz-albite-muscovite schist rock sample. This plot is similar to the Figure 6e, and it shows that the γ parameter can be assumed as a function of the angularity and/or poly-dispersity of the granular pack.

Figures 7a and 7b show the measured P- and S-wave velocities for loading cycle, after 5th loading and unloading cycle process, at different pressures for Esperance sand samples, respectively, for before and after the crushing process. The Esperance crushed grains are finer and angular. It is interesting to see that the plot of $V_P^2 - 4/3V_S^2$ in Figure 7c and 7d has the same pressure dependency in 5MPa to 20 MPa range. The solid line in the figure represents the best fit of the third root function. The ratio of grain shear modulus to $1 - \nu_g$ is estimated as before from equation (6), the empirical equation of porosity (3) has been used with the modified coordination number, i.e. equation (8). We plot the data in Figures 7e and 7f with γ values of 1.0, 1.5, 2., and 2.5. These plots show that the fitted γ value has increased after crushing the grains.

This different behaviour would suggest different values of γ for these samples. To confirm this, we plot in Figures 6e, 6f, 7e and 7f equation (6) by using equation (8) for γ equal to

Samples	GB	OT	FT	Q-A-MU	intact Esp.	crushed Esp.	Pom.
Grains shape	Sph.	Ang.	Ang.	Ang. and poly-dispers	Ang.	Ang. and poly-dispers	Ang.
γ	1	≈ 0.9	2.0	≈ 2.5	$\approx 1.$	1.5	1.5

Table 1: This table is a summary of the estimated γ for glass beads(GB), Ottawa sand packs(OT) (Domenico, 1977), Fontainebleau crushed rocks sample (FT), quartz-albite-muscovite schist granular pack (Q-A-MU) (Bilenko et al., 2015), Esperance intact (int. Esp.) , crushed grains (crushed Esp.) and Pomponio Beach sand (Pom.) (Zimmer et al., 2007). The grain shapes are listed as spherical (Sph.), angular (Ang.).

1.0, 1.5, 2.0 and 2.5. It varies for the samples, which suggests different micro-mechanical and/or micro-geometrical properties of the different samples. These values are also close to the value of 1.5 as calculated for Pomponio Beach sand by Zimmer et al. (2007). The summary of estimated γ , the parameter has been listed in Table 1. It is shown that by increasing the sorting of grain size, and angularities of grains the γ value is increasing.

DISCUSSION

A potential application of this work is to estimate the shear modulus of the grain material if the Poisson's ratio of grains is known from other physical measurements, and use the grain shape factor as a fitting parameter. Here, Figure 8a and 8b show, respectively, the estimated shear modulus calculated from equation (6) without and with taking the γ correction factor into consideration. The solid line is the most likely prediction of the shear modulus, and the dotted lines are representing $\pm 10\%$ error. The symbols $\bigcirc$, $\square$, $\diamond$, $\times$, $\triangleright$ and $\triangle$ show

different samples: glass beads (Domenico, 1977), Ottawa sand packs (Domenico, 1977), Fontainebleau crushed rocks sample (Bilenko et al., 2015), quartz-albite-muscovite schist granular pack (Bilenko et al., 2015), Esperance intact and Esperance crushed, respectively. This comparison shows that the γ correction factor can be assumed as a free parameter to estimate, within 10% error, the grain shear modulus from compaction of powders.

Any further application of the presented work needs to consider the assumptions that were used during its developments. Here we summarise the key assumptions:

- We assume HM theory is valid for the mid-range of stress, where the stress is well below the level required to crush the grains, but sufficient to allow some grain movement or rotation.
- We apply the theory of hydrostatic pressure dependence of elastic modulus, whereas in our experiments the stress is applied in one direction while strain in other directions is kept at zero. For fluids these conditions would be equivalent to hydrostatic stress but for grain packs there may be some differences between stresses.
- We ignore the effect of air moisture. Previous μ -CT studies showed that the moisture would be aggregated in grain contacts and can affect the contact stiffness.
- We calculate the coordination number using an empirical equation obtained from numerical simulations on spherical grain assemblies. Another possibility is to estimate this parameter from μ -CT images of powder packs.

The results shown in Figures 2, 6 and 7 can be divided into two groups. The first group, which includes glass beads, Fontainebleau powder, Esperance intact sand and Esperance crushed sand, shows agreement with the Hertz-Mindlin model within a significant stress

range. The second group, comprising Ottawa sand and quartz-albite-muscovite, appears to deviate from this relationship. We speculate that for the materials of the second group the pressure may have affected their micro-structure.

CONCLUSION

In this paper, first, we studied the elastic modulus of a glass bead pack versus the loading stress, and we identified a stress range within which the Hertz-Mindlin theory can be used to estimate bulk modulus of the grain pack. Then we calculated the ratio $\frac{\mu_g}{1-\nu_g}$ and showed its good agreement with the expected value for quartz. The parametric studies of the coordination number show that the empirical equation, equation (2), can predict correct stress dependency of the elastic moduli of spherical packs. For the rock samples, we estimate $\frac{\mu_g}{1-\nu_g}$ from equation (6), by taking a grain shape factor (γ) into account, which depends on the rock type. The grain shape factor for different powder samples has been shown to be from 1 to 2.5 (refer to Table 1).

The main conclusion is that we can use this method as an inversion algorithm for estimation of grain shear modulus if we have independent information about grain's bulk modulus or grain Poisson ratio. This information can potentially be obtained by analysing the effective shear modulus of the granular pack within extended Walton framework. It may also be useful to consider elastic wave attenuation in powder packs. Knowledge of the moduli of the grains of the drilling powders can help reconstruct the moduli of the intact rock.

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1 This graph shows porosity as a function of pressure of glass bead (■) and Ottawa sand (▲) (Domenico, 1977). The solid line is equation (3) by assuming the initial porosity is $\phi_0 = 0.385$.

2 Figures (a) and (b) shows the P-wave and S-wave velocities of glass bead and the Ottawa sand (Domenico, 1977). Figures (c) and (d) show the $V_P^2 - \frac{4}{3}V_S^2$ from the experimental data and the solid curve line is the best fit power law curve as $P^{1/3}$ to the experimental data. Figure (e) predicts the ratio of $\mu_g/(1 - \nu_g)$ that is estimated from the equation (6), and the solid line is calculated from the modulus of grains. Also, Figure (f) shows the $\mu_g/(1 - \nu_g)$ by assuming γ is equal to 0.9, 1 and 2.5, respectively, presented as ►, ▲ and • symbols. The ■ symbol is calculated based on measured porosity from experiments and coordination number assumed from 8, with $\alpha = 1$.

3 Plots (a), and (b) show the 2D slice of the μ -CT image of ellipsoidal grains, and Fontainebleau powders sample (an example of angular grains), respectively.

4 These are SEM images of the crushed Fontainebleau powders, and the crushed Quartz-albite-muscovite powders, respectively presented in Figures (a) and (b). (Bilenko et al., 2015)

5 Experimental set-up.

6 Figures are the same as Figure(2) for the Fontainebleau granular packs and for the quartz-albite-muscovite granular packs. Also, Figure (e) and (f) show the $\mu_g/(1 - \nu_g)$ by assuming γ is equal to 1., 1.5, 2. and 2.5, respectively, presented as ▲, ►, ■ and • symbols. The solid line in Figure (e) is calculated of from quartz elastic modulus. Also, in Figure (f), the solid line is calculated averaged modulus of the Hashin-Shtrikman bounds.

7 Figures are the same as Figure(2) for the Esperance intact sand packs and the

Esperance crushed sand packs. Also, Figure (e) and (f) show the $\mu_g/(1 - \nu_g)$ by assuming γ is equal to 0.9, 1, 1.5 and 2., respectively, presented as ►, ▲, ■ and • symbols . The solid line in Figure (e) and (f) is calculated of from quartz elastic modulus.

8 These figures show predicted shear modulus of grains versus true value. Figure (a) and (b), respectively, are give estimated shear modulus of samples without and with correction term γ . The solid line is a reference to the true value of shear modulus, and the dotted lines show the 10% error from the reference.

Figure 1: This graph shows porosity as a function of pressure of glass bead (■) and Ottawa sand (▲) (Domenico, 1977). The solid line is equation (3) by assuming the initial porosity is $\phi_0 = 0.385$.

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Figure 2: Figures (a) and (b) shows the P-wave and S-wave velocities of glass bead and the Ottawa sand (Domenico, 1977). Figures (c) and (d) show the $V_p^2 - \frac{4}{3}V_s^2$ from the experimental data and the solid curve line is the best fit power law curve as $P^{1/3}$ to the

Figure 3: Plots (a), and (b) show the 2D slice of the μ -CT image of ellipsoidal grains, and Fontainebleau powders sample (an example of angular grains), respectively.

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(a)

(b)

Figure 4: These are SEM images of the crushed Fontainebleau powders, and the crushed Quartz-albite-muscovite powders, respectively presented in Figures (a) and (b). (Bilenko et al., 2015)

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Figure 5: Experimental set-up.

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Figure 6: Figures are the same as Figure(2) for the Fontainebleau granular packs and for

the quartz-albite-muscovite granular packs. Also, Figure (e) and (f) show the $\mu_g/(1-\nu_g)$ by
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Figure 7: Figures are the same as Figure(2) for the Esperance intact sand packs and the Esperance crushed sand packs. Also, Figure (e) and (f) show the $\mu_g/(1 - \nu_g)$ by assuming

γ is equal to 0.9, 1, 1.5 and 2., respectively, presented as $\blacktriangleright$, $\blacktriangle$, $\blacksquare$ and $\bullet$ symbols. The solid line in Figures (e) and (f) is calculated of from quartz elastic modulus.

Figure 8: These figures show predicted shear modulus of grains versus true value. Figure (a) and (b), respectively, are give estimated shear modulus of samples without and with correction term γ . The solid line is a reference to the true value of shear modulus, and the dotted lines show the 10% error from the reference.

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