A Direct Regression Approach to Decomposing Socioeconomic Inequality of Health

Abstract

This paper presents a new regression-based decomposition of socioeconomic inequality of health that is more direct than other approaches. The method can be applied to both rank-dependent and level-dependent indicators of inequality. The response variable of our regression model is a simple reformulation of the measure of overall performance of an individual in the health and socioeconomic domains. Regression results are described in terms of marginal effects of the explanatory variables, but also in terms of their logworths or importance values. We illustrate our method, and compare it with alternatives, using Australian health and income data.

Keywords: Bivariate inequality, Socioeconomic health inequality, Decomposition

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1 Introduction

Research on the measurement of socioeconomic inequality of health is increasingly focused on the identification of the factors that contribute to this type of inequality and account for its evolution over time. Various decomposition techniques have been proposed to better understand and explain the observed levels of inequality at a given time for a given country and in comparisons over time and across countries. Some of these techniques have been reproduced from well-known results in the fields of inequality measurement (e.g., Rao, 1969; Bourguignon, 1979; Shorrocks, 1980) and labour economics (e.g., Blinder, 1973; Oaxaca, 1973). Others have been developed specifically for the analysis of bivariate inequality (e.g., Wagstaff, Van Doorslaer and Watanabe, 2003; Clarke, Gerdtham and Connelly, 2003; Wagstaff, 2005b; Abul Naga and Geoffard, 2006; Van Ourti, Van Doorslaer and Koolman, 2009; Makdissi, Sylla and Yazbeck, 2013; Allanson and Petrie, 2013). Van Doorslaer and Van Ourti (2011), O'Donnell, Van Doorslaer and Wagstaff (2012), and Van Ourti, Erreygers and Clarke (2014) provide useful overviews of the state of the art on the analysis of socioeconomic inequality of health.

Almost all of this work is centered around rank-dependent indicators of inequality, which are the dominant measures of socioeconomic inequality of health. They are defined in terms of the ranks of the socioeconomic variable and the levels of the health variable. Recently, however, Erreygers and Kessels (2017) argued in favour of using level-dependent indicators which are based on the levels of both the socioeconomic and the health variable and therefore capture more information.

In empirical analyses of socioeconomic inequality of health, the regression-based decomposition methods proposed by Wagstaff, Van Doorslaer and Watanabe (2003), henceforth referred to as WDW, have become widely used. The WDW framework was developed with the aim of studying three types of questions: (i) What are the causes of socioeconomic inequality of health in a given society at a given moment of time? (ii) What explains the differences between societies and the changes over time in socioeconomic inequality? and (iii) What is the effect of policy changes? Here we focus exclusively on the first of these issues. The study of differences, changes and policy impacts, often based on Oaxaca-Blinder decomposition techniques, is beyond the scope of our paper. With regard to the determination of the causes of socioeconomic inequality of health, the WDW approach consists of two steps: the first is to run a regression in order to identify the factors deemed to be related to health; the second is to assess to what extent the associations of these factors with socioeconomic status contribute to the observed level of socioeconomic inequality. It is important to note that in the first step only the

covariates of health are explored; those of socioeconomic status are ignored. For this reason we have characterized the WDW decomposition method as ‘one-dimensional’. To recognize the bivariate nature of rank-dependent indicators, Erreygers and Kessels (2013) presented various two-dimensional decomposition methods, where the covariates of *both* health and socioeconomic rank were considered. Using structural equation modeling, Kessels and Erreygers (2016) even introduced a feedback mechanism between the two indicator variables. As shown in these papers, the decomposition results may differ substantially according to the specific regression-based method one adopts.

In recent years, decomposition techniques based on recentered influence function (RIF) regression have been developed as alternatives. These techniques have a wide range of applications (see Firpo, 2007), and have been used to analyse inequality and poverty (see, e.g., Essama-Nssah and Lambert, 2012, 2016). Heckley, Gerdtham and Kjellsson (2016) were the first to formulate a general RIF regression framework for the decomposition of rank-dependent indicators of socioeconomic inequality of health. Rather than being based on a regression of health and/or socioeconomic status, this method focuses on the influence that each observation of the two variables has on the indicator. First, a vector of individual RIF values is computed, expressing every individual’s influence on the indicator. Next, these RIF values are regressed on a set of explanatory variables using OLS, yielding the marginal effects of the variables on the influence they exert on the indicator.

In this paper we propose a direct regression approach that concentrates on the individual *components* of the indicator rather than on the *influences*. Our method exploits the fact that the indicator can be reformulated as the mean of a composite variable which measures the joint outcome in the two dimensions of interest. This variable compares each individual’s performance with respect to health and socioeconomic position to the average attainments in these domains. The decomposition is based on the regression of the composite variable on the explanatory variables. Similar to the RIF decomposition, we describe the regression results in terms of the marginal effects of the explanatory variables on the response.

The introduction of this new and simple regression-based method for the decomposition of socioeconomic inequality of health is the major contribution of our paper. We highlight the differences between our method and the two other regression-based approaches, WDW and RIF. In addition, we introduce the logworth statistic as a means to assess the importance of the explanatory variables in each of the regressions.

We start by presenting and briefly comparing the indices of socioeconomic inequality of health we are considering here. Next, we discuss the WDW and

RIF regression approaches for decomposition analysis as well as our new and more direct regression approach. We illustrate our method and compare it to the current approaches using an empirical study of income-related health inequality in Australia.

2 Indices of Socioeconomic Inequality of Health

2.1 Basic Notation and Assumptions

For a population of n individuals ($i = 1, 2, \dots, n$) we observe the levels each person attains in the socioeconomic (y_i) and health (h_i) domains. For convenience, we designate socioeconomic achievement as “income”, but it goes without saying that socioeconomic status can be measured in a variety of ways (by consumption levels, by years of education, etc.). We assume that both y_i and h_i are ratio-scale variables with well-defined lower bounds greater than or equal to 0. Income is typically an unbounded variable. Health, by contrast, is often a bounded variable and possibly of the binary type.

Our formulas for the rank- and level-dependent indices will be expressed in terms of relative incomes and relative fractional ranks. The relative income of an individual $\tilde{y}_i$ is the ratio of this individual’s income to average income μ_y . We denote the rank of individual i in the income distribution as r_i . If there are no ties in the income distribution, the rank of the poorest person in society is equal to 1, that of the second poorest person equal to 2, etc. If a group of $k + 1$ individuals are tied in position g , the rank of each of these individuals is equal to $g + (k/2)$. For simplicity, however, we assume there are no ties in the income distribution, and that individuals are ranked according to their incomes.¹ The fractional rank f_i is defined as $f_i \equiv \frac{1}{n} (r_i - \frac{1}{2})$, and varies between $\frac{1}{2n}$ and $1 - \frac{1}{2n}$. The average fractional rank is $\mu_f = \frac{1}{2}$. The relative fractional rank $\tilde{f}_i$ is the fractional rank divided by its average, i.e. $\tilde{f}_i = 2f_i$, and varies between $\frac{1}{n}$ and $2 - \frac{1}{n}$.

2.2 Bivariate Linear Indices

The indices of bivariate inequality considered in this paper belong to a broad class of linear inequality measures. These indices are closely related to the family of linear measures of univariate inequality defined by Mehran (1976). We focus on bivariate indices where income serves as the weighting variable

¹Appendix A specifies how the formulas must be adapted when there are ties, and when not all individuals have the same population weight. In footnote 5 of Section 3.2 we explain how the RIFs must be defined in the presence of income ties.

and health as the weighted variable. In formal terms, the absolute version of these indices takes the following form²:

$$I(y, h) = \frac{1}{n} \sum_{i=1}^n w_i(y) h_i \quad (1)$$

The relative version of our indices is obtained by dividing the index by the mean of the weighted variable:

$$I^r(y, h) = \frac{I(y, h)}{\mu_h} \quad (2)$$

Other versions of the indices have been proposed in the literature, e.g. in order to deal with binary variables (Wagstaff, 2005a; Erreygers and Van Ourti, 2011). For bounded variables, whether they are binary or of another type, we suggest the use of the index proposed by Erreygers (2009):

$$I^b(y, h) = \frac{A}{(h_{\max} - h_{\min})} I(y, h) \quad (3)$$

where $h_{\min}$ and $h_{\max}$ stand for the lower and upper bounds of the health variable, and A is a constant.

2.3 Rank-Dependent Indices

Rank-dependent indices rely exclusively on income ranks to define the weights $w_i(y)$; no other information on the income distribution enters into the calculation of the weights. There exists a great variety of these indices, expressing different degrees of distributional sensitivity (Erreygers, Clarke and Van Ourti, 2012). In this paper we limit ourselves to indices which are characterized by a weighting function which is linear in the relative fractional income ranks $\tilde{f}_i$. This weighting function and the associated bivariate index R are defined as follows:

$$w_i^R = \tilde{f}_i - 1 \quad (4)$$

$$R = \frac{1}{n} \sum_{i=1}^n w_i^R h_i \quad (5)$$

The weights w_i^R steadily increase as $\tilde{f}_i$ goes from (almost) 0 to (almost) 2. If the group of the ‘poor’ is defined as those who have negative weights and the

²If there is no confusion possible, we will drop the arguments and simply write I instead of $I(y, h)$, etc.

group of the ‘rich’ as those who have positive weights, the weighting function (4) puts the boundary between the two groups exactly in the middle of the population. Those with relative fractional ranks smaller than 1 have negative weights, and those with relative fractional ranks larger than 1 positive weights. The negative weights sum to $-n/4$ and the positive weights to $n/4$.³ This motivates the choice of $A = 4$ for the bounded version of the rank-dependent index, as suggested by Erreygers (2009). The relative version of the index is equal to the standard Concentration Index.

2.4 Level-Dependent Indices

In contrast to rank-dependent indices, the weights of level-dependent indices are based upon income levels rather than income ranks. Also for level-dependent indices different versions can be defined (Erreygers and Kessels, 2017), but here we consider only the basic version L characterized by a weighting function which is a simple linear function of relative income⁴:

$$w_i^L = \tilde{y}_i - 1 \quad (6)$$

$$L = \frac{1}{n} \sum_{i=1}^n w_i^L h_i \quad (7)$$

The weights w_i^L are proportional to the deviations of the incomes from the mean. If there is a high degree of income inequality as shown by a positively skewed income distribution, typically there will be a lot of individuals with income levels below the mean, who will therefore have negative, but in absolute terms rather small weights. On the other side of the spectrum, those who have very high incomes will have positive, and in absolute terms quite large weights. Hence, a small change in the health level of a relatively well-off individual will probably have a more pronounced influence on the index than a comparable change in the health level of an individual who is less well-off.

All the negative weights sum to $-(n/2)M$, and all the positive weights to $(n/2)M$, where M stands for the Mean Absolute Deviation of $\tilde{y}$:

$$M = \frac{1}{n} \sum_{i=1}^n |\tilde{y}_i - 1| \quad (8)$$

³Strictly speaking, this result holds only when n is even. When n is odd, the sum of the negative weights is $-[(n-1)(n+1)]/(4n)$, which for large n is approximately equal to $-n/4$. And similarly for the positive weights.

⁴The L index is related to the κ index proposed by Abul Naga and Geoffard (2006: 365). For the specific choice of their parameters $\alpha = \beta = 1$, it turns out that we have $L = (\kappa - 1)\mu_h$. It needs to be pointed out that in their social welfare framework, inspired by Tsui (1999), the parameters are constrained by the condition $\alpha + \beta \leq 1$.

Since M is at most equal to $2(n-1)/n$, it follows that these two amounts are bounded by $1-n$ and $n-1$. For the bounded version of the level-dependent index, the natural choice for the value of A is therefore $A = 1$.

Although the weighting function of index L looks very similar to that of index R , the two indices are not identical (see Erreygers et al., 2018). A striking difference is that the pivotal position of the rank-dependent index (meaning the position for which the weighting function equals zero and changes sign) corresponds to the median income level, while that of the level-dependent index is constituted by the mean income level.

3 Regression Approaches to Decomposition

3.1 The Conventional Approach

Regression-based decomposition methods are now widely used in the analysis of socioeconomic inequality of health. The WDW approach has paved the way for a stream of studies in which the Concentration Index is decomposed in two steps based on regressions of health. The idea is to first start from a simple equation which explains the health variable in terms of the independent variables $x_1, x_2, \dots, x_q$:

$$h_i = \lambda_0 + \lambda_1 x_{1,i} + \lambda_2 x_{2,i} + \dots + \lambda_q x_{q,i} + \varepsilon_i \quad (9)$$

where ε_i represents the error term for which the zero conditional mean assumption applies, i.e. $E(\varepsilon_i | x_{1,i}, \dots, x_{q,i}) = 0$. The right-hand side is then plugged into the formula for the Concentration Index instead of h_i . Applying the procedure to (5), we obtain:

$$R(\tilde{f}, h) = \sum_{j=1}^q \hat{\lambda}_j R(\tilde{f}, x_j) + R(\tilde{f}, \hat{\varepsilon}) \quad (10)$$

Often the terms $\hat{\lambda}_j R(\tilde{f}, x_j)$ are identified as the contributions of the variables x_j to the measured amount of inequality $R(\tilde{f}, h)$, and the term $R(\tilde{f}, \hat{\varepsilon})$ as the unexplained part of the inequality.

Obviously, the same procedure can be applied to (7), which then yields:

$$L(\tilde{y}, h) = \sum_{j=1}^q \hat{\lambda}_j L(\tilde{y}, x_j) + L(\tilde{y}, \hat{\varepsilon}) \quad (11)$$

In previous work we have taken a critical look at a few aspects of this approach, and advocated caution in the interpretation of the results (Erreygers

and Kessels, 2013; Kessels and Erreygers, 2016). Our main criticism is that an explanation of the association between two variables which is based on the determinants of only one of these variables, appears to be partial and incomplete, and may therefore lead to biased conclusions. Remaining in the framework of rank-dependent indicators, we also suggested a few alternatives. In this paper we go a step further and generalize one of the alternatives we suggested into a regression method which can be applied to both rank- and level-dependent indicators, but which has a slightly more intuitive interpretation in the latter case. Before we present this new regression method, we briefly look at the RIF regression method which has been introduced recently.

3.2 The RIF Regression Approach

Heckley, Gerdtham and Kjellsson (2016) applied the concept of RIF regression to the decomposition of rank-dependent indices. They built on the work by Firpo et al. (2007, 2009) who derived and decomposed RIFs for univariate measures of income inequality such as the variance, the unconditional quantile and the Gini index. The RIF is based on the influence function (IF), which is rooted in robust statistics (Hampel et al., 1986) and provides an approximation of the derivative of a statistic. For each individual a RIF value can be computed that denotes the individual's influence on the statistic. This value indicates how the statistic would change if that individual were to be removed from the sample. To decompose a particular rank-dependent index, one can compute the corresponding vector of RIF values using the formulas of Heckley, Gerdtham and Kjellsson (2016), which, in its turn, serves as the response variable in a linear regression model. As a result, one can estimate the marginal degrees of influence the explanatory variables have on the index. The approach has been applied by Cai, Coyte and Zhao (2017), who use the RIF regression decomposition method to examine income-related health inequality in China.

We present the discrete version of the formulas of Heckley, Gerdtham and Kjellsson (2016) for the IF and RIF for the rank-dependent index R . The IF is defined as

$$IF_i^R = \mu_h - h_i - 2R + \tilde{f}_i h_i - 2glp_i \quad (12)$$

where glp_i stands for the generalized Lorenz point of individual i , i.e. the absolute concentration curve coordinate of the individual. Given that individual i does not belong to a tied income group (recall that we assume there are no income ties), the glp_i value is defined as follows:

$$glp_i = \frac{h_i + 2 \sum_{j=0}^{i-1} h_j}{2n} \quad (13)$$

where we take $h_0 = 0$.⁵

The RIF for the rank-dependent index R is obtained as the sum of the IF and the value for R , i.e.

$$RIF_i^R = IF_i^R + R \quad (14)$$

The regression equation for RIF^R is given by

$$RIF_i^R = \phi_0 + \phi_1 x_{1,i} + \phi_2 x_{2,i} + \dots + \phi_q x_{q,i} + \psi_i \quad (15)$$

where ϕ_0 is the intercept, $\phi_1, \phi_2, \dots, \phi_q$ represent the marginal degrees of influence the explanatory variables have on R and ψ_i is the error term where $E(\psi_i | x_{1,i}, \dots, x_{q,i}) = 0$. As shown by Heckley, Gerdtham and Kjellsson (2016: 97), under the assumptions of a model form linear in the parameters with an additive error term and zero conditional mean of the error term, the regression coefficients denote marginal effects and the regression model (15) is optimally estimated using OLS.

We apply a similar procedure as in Heckley, Gerdtham and Kjellsson (2016) to compute the formulas for the IF and RIF for the level-dependent index L . We start from the covariance definition of L (Erreygers and Kessels, 2017: Equation (26)):

$$L = \frac{Cov(y, h)}{\mu_y} \quad (16)$$

We then apply the chain rule for influence functions to obtain the IF:

$$IF_i^L = (\tilde{y}_i - 1)(h_i - \mu_h) - \tilde{y}_i L \quad (17)$$

The corresponding RIF is computed by adding the index L to the IF so that

$$RIF_i^L = (\tilde{y}_i - 1)(h_i - \mu_h - L) \quad (18)$$

Finally, the regression equation for RIF^L is

$$RIF_i^L = \chi_0 + \chi_1 x_{1,i} + \chi_2 x_{2,i} + \dots + \chi_q x_{q,i} + \tau_i \quad (19)$$

where χ_0 is the intercept, $\chi_1, \chi_2, \dots, \chi_q$ represent the marginal degrees of influence the explanatory variables have on L and τ_i is the error term where $E(\tau_i | x_{1,i}, \dots, x_{q,i}) = 0$. Also here, the model is estimated using OLS.

⁵If individual i belongs to a tied income group with $k+1$ members, $\{g, g+1, \dots, g+k\}$, then the glp_i value of every individual of this group is equal to:

$$glp_i = \frac{\sum_{j=g}^{g+k} h_j + 2 \sum_{j=0}^{g-1} h_j}{2n}$$

3.3 A Direct Approach

The WDW and RIF regression approaches to the decomposition of socioeconomic inequality of health are specific applications of techniques which can be used for the analysis of a wide variety of inequality and poverty indicators. By contrast, the method which we now present is developed specifically for the analysis of bivariate inequality indicators of the covariance type, and it cannot in any simple way be extended to other indicators. The method consists of a one-step approach that can be implemented directly from rewriting the index.

Let us begin with the (absolute version of the) level-dependent index. Observe that we can write index L as:

$$L = \frac{1}{n} \sum_{i=1}^n (\tilde{y}_i h_i - \mu_h) \quad (20)$$

Defining $d_i = \tilde{y}_i h_i - \mu_h$, we arrive at the following extremely simple expression of the index:

$$L = \frac{1}{n} \sum_{i=1}^n d_i = \mu_d \quad (21)$$

The value of the d variable can be interpreted as a numerical indicator of an individual's deviation from what we call the 'reference outcome' in the income-health domain. The reference outcome is defined as the situation in which an individual attains in each dimension the mean or expected level, i.e. when $(y_i, h_i) = (\mu_y, \mu_h)$. If individual i reaches the reference outcome, we have $d_i = \frac{\mu_y}{\mu_y} \mu_h - \mu_h = 0$. If an individual performs better than average on both income and health, then $d_i > 0$; in the opposite case, we have $d_i < 0$. If one of the two is higher than average but the other lower, the sign of d_i is undetermined. If $\frac{y_i h_i}{\mu_y \mu_h} > 1$, we have $d_i > 0$, and this can be interpreted as indicating a situation which is on the whole better than the reference outcome. If, however, $\frac{y_i h_i}{\mu_y \mu_h} < 1$, we have $d_i < 0$, which can be seen as a situation which is on the whole worse than the reference outcome.

We treat an individual's d value as a measure of how this individual performs overall, i.e. jointly for income and health, relative to society as a whole. If an individual performs worse than average in one dimension, this can be compensated by a strong performance in the other dimension. Since the inequality index L is additive in the individual d values, we consider these values to be the individual components of the index. It should be kept in mind that these components are of a relative nature; a change in an individual's health or income level affects not only the component of this

individual, but also that of all other individuals, through its effect on average health or income.

The new decomposition method we propose here is based on a regression of the individual d values. We see this as a more natural and direct procedure to explain the correlation between income and health than the one based on a regression of health only or the one based on the RIF regression. As a matter of fact, the d values take into account both income and health, they constitute the individual components of the index, and by means of their average they capture the degree of association between income and health. Assume that we explain d by means of the independent variables $x_1, x_2, \dots, x_q$. More specifically, let us consider the linear regression equation:

$$d_i = \gamma_0 + \gamma_1 x_{1,i} + \gamma_2 x_{2,i} + \dots + \gamma_q x_{q,i} + \varphi_i \quad (22)$$

where φ_i is the error term for which we assume zero conditional mean, $E(\varphi_i | x_{1,i}, \dots, x_{q,i}) = 0$. Similar as for RIF regression, the regression coefficients represent marginal effects and the regression specification (22) is optimally estimated using OLS. In general, the sign and significance of the coefficients γ_j express the effect which the variables have on the individual components of the index. One certainly expects variables which are positively correlated with both income and health to have positive coefficients, since higher values of income and health are equivalent to higher values of d . By contrast, variables which are negatively correlated with both income and health are expected to have negative coefficients.

Plugging the estimates of (22) into (21), and taking into account that $\mu_{\hat{\varphi}} = 0$, we obtain the following result:

$$L = \hat{\gamma}_0 + \hat{\gamma}_1 \mu_{x_1} + \hat{\gamma}_2 \mu_{x_2} + \dots + \hat{\gamma}_q \mu_{x_q} \quad (23)$$

Expression (23) can be used to assess the expected marginal effect of each independent variable on L . The estimated effect of a small ceteris paribus change $\Delta \mu_{x_j}$ of μ_{x_j} on L is equal to $\hat{\gamma}_j \Delta \mu_{x_j}$. For comparable changes $\Delta \mu_{x_j}$ and $\Delta \mu_{x_l}$ of the variables x_j and x_l , it is therefore possible to determine which of the two has the largest marginal effect on the index. What we definitely should not do, is to claim that the product $\hat{\gamma}_j \mu_{x_j}$ constitutes the contribution of variable x_j to the observed level of inequality. If that were true, the contribution of any mean centered variable would automatically be zero. However, we can gain some insight into the importance of the independent variables by looking at the logworth values for regression equation (22). The logworth statistic, defined as $-\log_{10}(P\text{-value of the } F\text{-test})$, measures the importance of one variable or a group of variables as a whole for which an F -

test has been performed.⁶ Logworth values are logarithmic transformations of the P -values from F -tests in order to compose a ranking of the variables' importances. They prove especially useful when handling F -tests that are extremely significant, which happens with large datasets. Logworth values are always positive. A logworth value of zero corresponds to an insignificant P -value of 1, a logworth value of 1.3 to a P -value of 0.05 and any logworth value above two to a P -value below 0.01. Logworth values of 100 and more correspond to extremely small P -values. Generally, larger effects lead to more significant P -values and larger logworth values, but this relationship does not always hold because significance also depends on the error variance. In effect, logworth values are based on F -tests that compare the explanatory power of the variable(s) to the error variance. The error variance is thus embedded in their computation.

Can the same approach also be applied to rank-dependent indicators? It can, with one important change. Let us start by observing that the rank-dependent index R may be expressed as:

$$R = \frac{1}{n} \sum_{i=1}^n (\tilde{f}_i h_i - \mu_h) \quad (24)$$

The mean relative fractional income rank of an individual is equal to 1, which corresponds to the individual with median income. By analogy with our approach of the level-dependent index, we can define the reference outcome as the situation in which an individual attains the average level of both relative fractional income rank and health. For an individual who reaches the reference outcome, the value of the expression between brackets in (24) is 0. Defining $u_i \equiv \tilde{f}_i h_i - \mu_h$, we can interpret $u_i > 0$ as an indication that individual i attains a situation in the income rank-health space which is on the whole better than the reference outcome, and $u_i < 0$ as indicating a situation which is on the whole worse than the reference outcome.

Since we now have:

$$R = \frac{1}{n} \sum_{i=1}^n u_i = \mu_u \quad (25)$$

what we should do next is to explain u by means of a set of independent variables. Suppose that we estimate the following regression using OLS:

$$u_i = \beta_0 + \beta_1 x_{1,i} + \beta_2 x_{2,i} + \dots + \beta_q x_{q,i} + \eta_i \quad (26)$$

⁶The name *logworth* was coined in 1995 by Padraic Neville in the specific context of decision tree criteria. The logworth statistic has been implemented in the JMP software product from SAS Institute, Cary, NC, USA.

where we assume $E(\eta_i|x_{1,i}, \dots, x_{q,i}) = 0$. We then end up with the following counterpart of (23):

$$R = \hat{\beta}_0 + \hat{\beta}_1\mu_{x_1} + \hat{\beta}_2\mu_{x_2} + \dots + \hat{\beta}_q\mu_{x_q} \quad (27)$$

Also here, logworth values associated with regression equation (26) provide information on the importance of the (groups of) variables. They can be used for ranking the variables based on their impact on the model.

An important difference between the decomposition of L and that of R , is that the reference outcome of individual i in the first case corresponds to the mean income μ_y and the mean health μ_h , but in the second to the mean income *rank* (and hence to the *median* rather than the mean income) and the mean health.

3.4 Comparing the Different Approaches

Before we apply the three different regression-based decomposition methods in practice, we wish to dig a little deeper to discover what distinguishes them. A first observation is that the response variables of their regressions are of a different nature: ‘health’ in the case of the WDW decomposition, ‘influence on the indicator’ in the case of the RIF decomposition, and ‘overall outcome with respect to income and health’ in the direct decomposition. This criterion clearly sets apart the WDW decomposition, since it looks at one of the variables only – health. In previous work (Erreygers and Kessels, 2013; Kessels and Erreygers, 2016) we have argued why we believe that a one-sided regression approach could yield biased decomposition results. The RIF and direct decompositions, by contrast, consider both variables jointly, and are therefore more promising.

This brings us to a second observation: there is an important difference between the response variables of the RIF and direct regressions. We show this by comparing the RIF values and the composite variable values for the level-dependent index L , respectively RIF_i^L and d_i , and by providing a graphical illustration. From the definition of RIF_i^L (see (18)) it follows that $RIF_i^L > 0$ in two cases: high income and good health (i.e., when $\tilde{y}_i > 1$ and $h_i > \mu_h + L$), and low income and bad health (i.e., when $\tilde{y}_i < 1$ and $h_i < \mu_h + L$). Likewise, we have $RIF_i^L < 0$ for individuals with high income and bad health ($\tilde{y}_i > 1$ and $h_i < \mu_h + L$), or with low income and good health ($\tilde{y}_i < 1$ and $h_i > \mu_h + L$). Figure 1 represents the situation of an individual with low income and bad health. Her RIF_i^L is equal to the area of the shaded rectangle defined by the point representing her situation $(\tilde{y}_i, h_i)$ and the point representing the reference situation $(1, \mu_h + L)$. For the composite

variable we get something substantially different. As already explained, we have $d_i > 0$ when $\tilde{y}_i h_i > \mu_h$, and $d_i < 0$ when $\tilde{y}_i h_i < \mu_h$. For individuals with high income and good health (which now means $\tilde{y}_i > 1$ and $h_i > \mu_h$) we have $d_i > 0$, but for individuals with low income and bad health ($\tilde{y}_i < 1$ and $h_i < \mu_h$) we have $d_i < 0$. Figure 2 represents the same individual as the one of Figure 1. Her d_i value is a measure of the distance between two rectangular hyperbola. The first is defined by the equation $\tilde{y}h = \mu_h$ and consists of all combinations of relative income $\tilde{y}$ and health h for which $d = 0$, such as $\tilde{y} = 1$ and $h = \mu_h$. The second hyperbola is defined by the equation $\tilde{y}h = \tilde{y}_i h_i$ and consists of all combinations of relative income $\tilde{y}$ and health h for which $d = d_i$, such as $\tilde{y} = \tilde{y}_i$ and $h = h_i$. The d_i value corresponds to the vertical distance between the second and the first hyperbola measured at the location $\tilde{y} = 1$.

What this analysis reveals is that there is an asymmetry between the RIF response and the composite variable response with regard to individuals with low income and bad health. The RIF values for these individuals are positive, just as those for individuals with high income and good health. In other words, the RIF response does not distinguish people who are doing badly in both dimensions and people who are doing well in both dimensions. The values of the composite variable, by contrast, are negative for individuals with low income and bad health and positive for individuals with high income and good health. This means that the composite variable response does not treat these individuals similarly; in fact, it treats them oppositely. By focusing on the *influence* of individual observations on the observed inequality measure, the RIF decomposition ignores whether this influence is exerted by individuals who are well-off in all respects or by those who are struggling. The direct decomposition does take this difference into account, and concentrates on the *overall income-health outcome* of individuals.

This has important implications for the coefficients of the regression equations. Suppose one of the explanatory variables is positively associated with both income and health. High values of this variable therefore tend to go together with positive values of the composite variable, and low values with negative values of the composite variable. As a result, the covariate and the response variable of the direct decomposition regression are definitely positively correlated, and we can expect the estimated coefficient of the variable to be positive. By contrast, since both high and low values of the covariate are associated with positive and similar values of the RIF response variable, the covariate is only weakly correlated with the RIF response variable, if at all. Hence, it is unclear what to expect from the sign of the regression coefficient. A similar conclusion holds in the case of an explanatory variable which is negatively associated with both income and health. On the whole, it

Figure 1: Illustration of the RIF_i^L value for an individual with relative income $\tilde{y}_i$ and health h_i .

Figure 2: Illustration of the d_i value for an individual with relative income $\tilde{y}_i$ and health h_i .

seems that the direct regression is better capable of revealing which variables have an undoubtedly positive or negative marginal effect on socioeconomic inequality of health than the RIF regression.

The choice is therefore between an influence-oriented method which decomposes income-related inequality based on an explanation of the statistical impact of individual observations, and a direct decomposition method based on an explanation of the overall positions of individuals in the income-health domain. We will now illustrate our findings by means of an empirical analysis.

4 Empirical Application

4.1 Description of the Data

The data come from Wave 13 of the Household, Income and Labour Dynamics in Australia (HILDA) survey, and were collected in 2013-2014.⁷ Our observations are individuals aged 15 years or older. Our income variable is equivalized income, calculated using the modified OECD equivalence scale (Australian Bureau of Statistics, 2013, Appendix 3). Each member of a household is assigned an equivalized income equal to the household's disposable income divided by its equivalence factor. This factor is obtained by giving the first adult of the household a score of 1, every other adult a score of 0.5, every child a score of 0.3, and then adding up all the scores. Although children younger than 15 years are not included in the sample, they are incorporated in the calculation of the equivalized incomes. Our health variable is the SF-6D health score. Since this is a bounded variable, we use the bounded version of the inequality indices, R^b and L^b . Given that the bounds of the variable are $h_{\min} = 0$ and $h_{\max} = 1$, it is easy to check that $R^b = 4R$ and $L^b = L$.

For our decomposition analysis we consider the following explanatory variables: sex, age, ancestry, occupational status, marital status, number of children, housing type, self-rated prosperity⁸, life satisfaction, satisfac-

⁷More information on the HILDA database can be found here: <https://www.melbourneinstitute.com/hilda/>.

⁸This means people's satisfaction with their financial situation, which is fairly unrelated to income, as few Australians feel prosperous despite having a high income (see Hamilton and Barbato, 2005). The survey asked respondents the following: 'Given your current needs and financial responsibilities, would you say that you and your family are: prosperous, very comfortable, reasonably comfortable, just getting along, poor, very poor?' We grouped the first three categories into the category "living comfortably" and the last two categories into the category "living poorly".

tion with weight, and lifestyle variables including smoking, time stress, sleep quality and physical activity. Our sample contains 14,728 individuals.⁹ The empirical work in Erreygers et al. (2018) has shown that this is a fairly large and representative sample, which does not require sample weights in the calculations. The composition of our sample is described in Table B1 in Appendix B.

4.2 Regression Approaches to Decomposition: A Comparison

To begin with, we emphasize that the three regression approaches considered in this paper (the WDW, RIF and direct methods) yield results that are not all of the same nature. The conventional WDW approach is often used to estimate the relative contributions of different factors to the measured degree of socioeconomic inequality of health. These contributions, which can be either positive or negative, usually do not add up to 100%. The difference, which again can be positive or negative, is accounted for by the residual term, which may be quite substantial. On the other hand, the RIF and the direct regression approaches differ from the WDW approach in the sense that they allow to estimate the marginal effects of the factors (and in particular their categories in the case of categorical factors) on the RIF of the observed inequality measure and on the inequality measure itself, respectively. In addition to their marginal effects, we are able to assess their importances by looking at the logworth values.

For the comparison of the three regression-based decomposition approaches, we need to estimate five regression equations. Each regression equation involves the same set of independent variables. While the dependent variable of the WDW regression is health h , those of the two RIF regressions are the RIFs for the bounded versions of the rank- and level-dependent indices, respectively $RIF-R^b$ and $RIF-L^b$, and those of the direct regressions the combined relative fractional income rank-health variable and the combined relative income-health variable, respectively u and d . The independent variables are those mentioned in Table B1. Except for age, the number of children aged 0-4 and aged 5-14, and life satisfaction, which are numerical variables, all variables are categorical and enter into the regression equation as dummy variables. The reference situation is that of a female non-indigenous person,

⁹Note that we excluded individuals with a negative income. Furthermore, we classified 3,192 missing observations for ancestry as “non-indigenous”, 4 for occupational status as “not in labour force”, 104 for self-rated prosperity as “just getting along”, 80 for satisfaction with weight as “neither satisfied/dissatisfied”, 81 for smoking as “non-smoker”, and 23 for time stress as “sometimes stressed”.

who provides a professional service, is married, lives in a house, lives comfortably, is neither satisfied/dissatisfied with her weight, does not smoke, is sometimes stressed for time, has a fairly good sleep quality, and does frequent physical activity. Table B2 in Appendix B provides summary statistics for the variables. Similar as in Erreygers et al. (2018), the values for the indices R^b and L^b equal 0.0657 and 0.0144, respectively.

4.2.1 WDW Regression Results

The results of the regression for the health variable h can be found in Table 1, with the logworth values for the independent variables included for completeness. For the categorical variables, we provide these values for the associated dummy variables as a whole. The logworth values show that life satisfaction, sleep quality and occupational status are the most important determinants of health. They are followed by physical activity, time stress and age. Also, self-rated prosperity, satisfaction with weight and having young children (aged 0-4) have a considerable effect on health. Of minor importance are having older children (aged 5-14), smoking, ancestry, marital status, sex and housing type. The regression coefficients $\hat{\lambda}_j$ suggest that being male, a manager or professional, having more children, a very good sleep quality, almost no stress, a greater life satisfaction, and being (very) satisfied with one's weight have positive effects on health; all other variables either have negative effects, or are insignificant at the 5% level.

The associated decompositions of the R^b and L^b indices are shown in Table 2. The contribution of variable x_j to R^b and L^b depends on the estimated coefficient $\hat{\lambda}_j$ in Table 1 and on the indices $4R(\tilde{f}, x_j)$ and $L(\tilde{y}, x_j)$. In the last column of each decomposition, the contributions are expressed as percentages; for the variables with more than two categories, such as housing type, occupational status and self-rated prosperity, these percentages are combined over the categories. Note that the choice of the reference category for the categorical variables does not have any impact on the percentages. The results for index R^b are quite similar to those for index L^b . The largest contribution amounts to 41% for the R^b index and to 37% for the L^b index and is due to occupational status. This contribution is followed by self-rated prosperity (12% for R^b and 13% for L^b) and by life satisfaction (8% for R^b and 11% for L^b). Not surprisingly, this ranking is somewhat different from the one indicated by the logworth values for the health regression in Table 1. Also, the dummy variables of occupational status, except for the managers and professionals dummy, and those of self-rated prosperity tend to be negatively associated with health (as can be seen from the negative sign of the regression coefficients $\hat{\lambda}_j$) and characterized by a strong pro-poor distribution

(as revealed by the indices $4R(\tilde{f}, x_j)$ and $L(\tilde{y}, x_j)$). The managers and professionals dummy as well as life satisfaction show an opposite pattern. As far as the other variables are concerned, time stress and having a higher number of children contribute negatively to socioeconomic inequality. It must also be observed that the so-called unexplained part of the decomposition is quite substantial, viz. 23% for the R^b index and 25% for the L^b index.

4.2.2 RIF Regression Results

Tables 3 and 4 contain the decomposition results of the RIF regressions. Similar to what we did for the health regression, we calculated the logworth values and grouped the category dummies of the categorical variables together. A first observation is that the RIF regressions do not explain much of the variation in the RIFs, as indicated by the very small R^2 values (5% for $RIF-R^b$ and 4% for $RIF-L^b$) and the small logworth values, some of which are smaller than 1.3 indicating insignificance at the 5% level. Correspondingly, the estimation errors of the regressions are extremely large.¹⁰ Looking at the logworth values for $RIF-R^b$ and $RIF-L^b$, we observe similar importance rankings of the variables. Self-rated prosperity and employment status are most important for explaining the RIFs, followed by physical activity.

A study of the regressions' marginal effects shows that having more young children and a greater life satisfaction, being almost always, rarely and never stressed and very dissatisfied with one's weight, tend to have a negative effect on both $RIF-R^b$ and $RIF-L^b$, while all other variables have positive effects or are insignificant at the 5% level. Following the interpretation by Heckley, Gerdtham and Kjellsson (2016), this means, for instance, that an equal marginal increase in the life satisfaction for everyone in the population would have a negative impact on socioeconomic inequality of health. For most other variables (or their categories in the case of categorical variables) the impact would be positive or negligible.

4.2.3 Direct Regression Results

The regression results for the composite variables u and d are reported in Tables 5 and 6. Compared to the RIF regressions, the direct regressions have substantially larger R^2 values (35% for u and 21% for d), larger logworth values (mostly much larger than 1.3 and thus greatly below the significance

¹⁰As an extra confirmation, the Root Mean Square Error (RMSE) of the regression for $RIF-R^b$ is 0.2773 which almost equals the RIF's standard deviation of 0.2849, and the RMSE of the regression for $RIF-L^b$ is 0.0780 which also practically matches the RIF's standard deviation of 0.0796.

level of 5%) and larger marginal effects, all indicative of greater explanatory power.

The logworth values from both direct regressions point toward the same important duo of explanatory variables as in the other decompositions, namely occupational status and self-rated prosperity. However, for the direct decompositions, marital status comes third. As far as the marginal effects are concerned, being male, a manager or professional, having a very good sleep quality and a greater life satisfaction, and being dissatisfied with one's weight, tend to have a positive effect on an individual's achievement in the income rank-health and income-health domains; all the other variables either have negative coefficients or are insignificant at the 5% level. We observe that these signs largely correspond to the ones of the indices $4R(\tilde{f}, x_j)$ and $L(\tilde{y}, x_j)$ in Table 2. This implies that the directions of the associations between income and each of the explanatory variables seem to determine the signs of the direct regression coefficients. With regard to the magnitudes of the marginal effects, for both indices, we can say that the effects are the largest for individuals living in non-private dwellings, living poorly or who are just getting along, who are not in the labour force or unemployed, and a manager or professional. It deserves to be pointed out that the effects of these variables are all negative, except for the effect of the manager and professional dummy. This means that living in non-private dwellings, to give just one example, is associated with a pro-poor change of the indices. By contrast, being a manager or professional is associated with a pro-rich change of the indices.

4.2.4 Comparing the RIF and Direct Response Variables

In Section 3.4 we pointed out that there are profound differences in the way certain combinations of income and health are translated into values of the response variables of the RIF and direct regressions. This entails a lack of correlation between the two response variables. Table 7 shows the six correlation coefficients between the RIF responses and the composite variable responses used in the direct decompositions. While the correlations among the two RIF responses as well as among the two composite variable responses are large (82% and 80%), the correlations between these two groups of responses are small or negligible.

There is an additional observation which deserves to be made when comparing the RIF and direct decomposition methods. When presenting the results of our empirical application, we pointed out that the RIF regressions have very low explanatory power. The R^2 value of the $RIF-R^b$ regression is a meagre 0.0548 and that of the $RIF-L^b$ regression even lower, 0.0405. To put these numbers into perspective: the R^2 values of the regression of u

Table 1: The health regression

Variable	$\hat{\lambda}_j$	Prob > $ t $	Prob > F	Logworth
Male	0.0045	0.0090		2.048
Indigenous	-0.0209	< 0.0001		4.111
Age	-0.0010	< 0.0001		71.557
Not married	-0.0051	0.0071		2.151
Children 0-4	0.0174	< 0.0001		22.800
Children 5-14	0.0074	< 0.0001		9.188
Semi-detached house	-0.0057	0.0827		
Flat	-0.0037	0.1586		
Non-private dwelling	-0.0338	0.0404		
Other dwelling	-0.0103	0.3733	0.0575	1.240
Managers & professionals	0.0055	0.0164		
Manual workers	-0.0092	0.0035		
Unemployed	-0.0331	< 0.0001		
Not in labour force	-0.0547	< 0.0001	< 0.0001	164.348
Living poorly	-0.0403	< 0.0001		
Just getting along	-0.0219	< 0.0001	< 0.0001	35.975
Smoking	-0.0121	< 0.0001		7.205
Very good sleep quality	0.0278	< 0.0001		
Fairly bad sleep quality	-0.0369	< 0.0001		
Very bad sleep quality	-0.0794	< 0.0001		
Not reported	-0.0428	< 0.0001	< 0.0001	178.856
Almost always stressed	-0.0341	< 0.0001		
Often stressed	-0.0199	< 0.0001		
Rarely stressed	0.0218	< 0.0001		
Never stressed	0.0401	< 0.0001	< 0.0001	86.646
Life satisfaction	0.0199	< 0.0001		204.244
Very satisfied with weight	0.0146	< 0.0001		
Satisfied with weight	0.0038	0.0998		
Dissatisfied with weight	-0.0054	0.0144		
Very dissatisfied with weight	-0.0310	< 0.0001	< 0.0001	30.829
No physical activity	-0.0593	< 0.0001		
Some physical activity	-0.0202	< 0.0001	< 0.0001	102.235
Constant	0.6946	< 0.0001		
R^2	0.3665			

Table 2: WDW decompositions of R^b and L^b

Variable	$4R(\tilde{f}, x_j)$	$4\hat{\lambda}_j R(\tilde{f}, x_j)$	%	$L(\tilde{y}, x_j)$	$\hat{\lambda}_j L(\tilde{y}, x_j)$	%
Male	0.0540	0.0002	0.37	0.0116	0.0001	0.36
Indigenous	-0.0294	0.0006	0.93	-0.0067	0.0001	0.97
Age	-5.8245	0.0056	8.57	-0.6138	0.0006	4.13
Not married	-0.2147	0.0011	1.67	-0.0493	0.0003	1.76
Children 0-4	-0.0642	-0.0011	-1.70	-0.0177	-0.0003	-2.14
Children 5-14	-0.0626	-0.0005	-0.71	-0.0164	-0.0001	-0.84
Semi-detached house	-0.0054	0.0000		0.0011	0.0000	
Flat	-0.0204	0.0001		-0.0004	0.0000	
Non-private dwelling	-0.0067	0.0002		-0.0014	0.0000	
Other dwelling	-0.0066	0.0001	0.61	-0.0014	0.0000	0.40
Managers & professionals	0.3587	0.0020		0.0885	0.0005	
Manual workers	-0.0143	0.0001		-0.0083	0.0001	
Unemployed	-0.0464	0.0015		-0.0105	0.0003	
Not in labour force	-0.4245	0.0232	40.84	-0.0817	0.0045	37.44
Living poorly	-0.0644	0.0026		-0.0135	0.0005	
Just getting along	-0.2567	0.0056	12.49	-0.0631	0.0014	13.38
Smoking	-0.1105	0.0013	2.03	-0.0274	0.0003	2.31
Very good sleep quality	0.0182	0.0005		0.0067	0.0002	
Fairly bad sleep quality	-0.0280	0.0010		-0.0086	0.0003	
Very bad sleep quality	-0.0267	0.0021		-0.0058	0.0005	
Not reported	-0.0091	0.0004	6.15	-0.0020	0.0001	7.31
Almost always stressed	0.0282	-0.0010		0.0048	-0.0002	
Often stressed	0.1031	-0.0020		0.0205	-0.0004	
Rarely stressed	-0.1087	-0.0024		-0.0222	-0.0005	
Never stressed	-0.0303	-0.0012	-10.03	-0.0056	-0.0002	-8.89
Life satisfaction	0.2666	0.0053	8.07	0.0780	0.0016	10.79
Very satisfied with weight	-0.0029	0.0000		0.0012	0.0000	
Satisfied with weight	-0.0299	-0.0001		-0.0065	0.0000	
Dissatisfied with weight	0.0736	-0.0004		0.0183	-0.0001	
Very dissatisfied with weight	-0.0066	0.0002	-0.53	-0.0038	0.0001	0.07
No physical activity	-0.0888	0.0053		-0.0180	0.0011	
Some physical activity	0.0010	0.0000	7.98	-0.0028	0.0001	7.82
Residual term	0.0153	0.0153	23.24	0.0036	0.0036	25.13
Total		0.0657	100.00		0.0144	100.00

Table 3: The regression for $RIF-R^b$

Variable	$\hat{\phi}_j$	Prob > $ t $	Prob > F	Logworth
Male	0.0056	0.2428		0.615
Indigenous	0.0190	0.2001		0.699
Age	0.0007	< 0.0001		5.492
Not married	0.0154	0.0041		2.388
Children 0-4	-0.0159	0.0011		2.954
Children 5-14	0.0060	0.0759		1.120
Semi-detached house	0.0352	0.0001		
Flat	0.0240	0.0011		
Non-private dwelling	0.0856	0.0655		
Other dwelling	0.0436	0.1811	< 0.0001	4.721
Managers & professionals	0.0261	< 0.0001		
Manual workers	-0.0136	0.1242		
Unemployed	0.0045	0.7135		
Not in labour force	0.0665	< 0.0001	< 0.0001	27.327
Living poorly	0.1523	< 0.0001		
Just getting along	0.0330	< 0.0001	< 0.0001	31.545
Smoking	0.0091	0.1486		0.828
Very good sleep quality	0.0029	0.6408		
Fairly bad sleep quality	0.0077	0.2090		
Very bad sleep quality	0.0697	< 0.0001		
Not reported	0.0215	0.3120	< 0.0001	5.821
Almost always stressed	-0.0372	< 0.0001		
Often stressed	-0.0041	0.4847		
Rarely stressed	-0.0238	0.0002		
Never stressed	-0.0549	0.0001	< 0.0001	7.170
Life satisfaction	-0.0073	< 0.0001		4.326
Very satisfied with weight	-0.0062	0.4915		
Satisfied with weight	-0.0014	0.8260		
Dissatisfied with weight	-0.0039	0.5279		
Very dissatisfied with weight	-0.0269	0.0039	0.0569	1.245
No physical activity	0.0709	< 0.0001		
Some physical activity	0.0017	0.7346	< 0.0001	18.678
Constant	0.0385	0.0372		
R^2	0.0548			

Table 4: The regression for $RIF-L^b$

Variable	$\hat{\chi}_j$	Prob > $ t $	Prob > F	Logworth
Male	0.0021	0.1236		0.908
Indigenous	0.0081	0.0525		1.280
Age	0.0002	< 0.0001		5.969
Not married	0.0047	0.0020		2.706
Children 0-4	-0.0036	0.0081		2.094
Children 5-14	0.0005	0.5733		0.242
Semi-detached house	0.0082	0.0016		
Flat	0.0076	0.0002		
Non-private dwelling	0.0269	0.0399		
Other dwelling	0.0096	0.2950	< 0.0001	4.452
Managers & professionals	0.0090	< 0.0001		
Manual workers	-0.0036	0.1523		
Unemployed	0.0055	0.1152		
Not in labour force	0.0173	< 0.0001	< 0.0001	23.530
Living poorly	0.0352	< 0.0001		
Just getting along	0.0086	< 0.0001	< 0.0001	22.226
Smoking	0.0031	0.0759		1.120
Very good sleep quality	-0.0003	0.8709		
Fairly bad sleep quality	0.0007	0.6645		
Very bad sleep quality	0.0146	< 0.0001		
Not reported	0.0025	0.6805	0.0012	2.926
Almost always stressed	-0.0076	0.0022		
Often stressed	-0.0006	0.6935		
Rarely stressed	-0.0056	0.0015		
Never stressed	-0.0123	0.0023	< 0.0001	4.178
Life satisfaction	-0.0015	0.0033		2.481
Very satisfied with weight	0.0015	0.5446		
Satisfied with weight	-0.0011	0.5491		
Dissatisfied with weight	-0.0018	0.2935		
Very dissatisfied with weight	-0.0053	0.0449	0.2227	0.652
No physical activity	0.0140	< 0.0001		
Some physical activity	0.0004	0.7786	< 0.0001	9.161
Constant	0.0024	0.6457		
R^2	0.0405			

Table 5: The regression for u

Variable	$\hat{\chi}_j$	Prob > $ t $	Prob > F	Logworth
Male	0.1059	< 0.0001		4.082
Indigenous	-0.4627	< 0.0001		7.624
Age	-0.0118	< 0.0001		43.927
Not married	-0.5479	< 0.0001		73.560
Children 0-4	-0.3080	< 0.0001		28.794
Children 5-14	-0.2149	< 0.0001		29.399
Semi-detached house	-0.1011	0.0488		
Flat	-0.1601	< 0.0001		
Non-private dwelling	-1.2411	< 0.0001		
Other dwelling	-0.6246	0.0006	< 0.0001	9.461
Managers & professionals	0.7282	< 0.0001		
Manual workers	-0.3766	< 0.0001		
Unemployed	-0.9536	< 0.0001		
Not in labour force	-1.1957	< 0.0001	< 0.0001	537.449
Living poorly	-1.0707	< 0.0001		
Just getting along	-0.8882	< 0.0001	< 0.0001	192.907
Smoking	-0.2211	< 0.0001		9.509
Very good sleep quality	0.0816	0.0176		
Fairly bad sleep quality	-0.0896	0.0084		
Very bad sleep quality	-0.2335	0.0007		
Not reported	-0.1828	0.1233	< 0.0001	4.881
Almost always stressed	-0.0612	0.2152		
Often stressed	-0.0074	0.8207		
Rarely stressed	-0.1101	0.0018		
Never stressed	-0.1663	0.0369	0.0084	2.075
Life satisfaction	0.0957	< 0.0001		20.677
Very satisfied with weight	0.0232	0.6445		
Satisfied with weight	-0.0219	0.5425		
Dissatisfied with weight	0.1165	0.0008		
Very dissatisfied with weight	0.0411	0.4288	0.0007	3.154
No physical activity	-0.3524	< 0.0001		
Some physical activity	-0.1689	< 0.0001	< 0.0001	17.231
Constant	0.8566	< 0.0001		
R^2	0.3547			

Table 6: The regression for d

Variable	$\hat{\chi}_j$	Prob > $ t $	Prob > F	Logworth
Male	0.0260	0.0033		2.488
Indigenous	-0.0967	0.0004		3.418
Age	-0.0015	< 0.0001		7.027
Not married	-0.1145	< 0.0001		30.566
Children 0-4	-0.0653	< 0.0001		12.526
Children 5-14	-0.0450	< 0.0001		12.480
Semi-detached house	0.0065	0.6999		
Flat	-0.0007	0.9611		
Non-private dwelling	-0.2691	0.0016		
Other dwelling	-0.1340	0.0249	0.0044	2.361
Managers & professionals	0.2266	< 0.0001		
Manual workers	-0.0893	< 0.0001		
Unemployed	-0.1702	< 0.0001		
Not in labour force	-0.2250	< 0.0001	< 0.0001	279.915
Living poorly	-0.2272	< 0.0001		
Just getting along	-0.2163	< 0.0001	< 0.0001	103.965
Smoking	-0.0481	< 0.0001		4.521
Very good sleep quality	0.0232	0.0398		
Fairly bad sleep quality	-0.0283	0.0113		
Very bad sleep quality	-0.0547	0.0155		
Not reported	-0.0505	0.1951	0.0006	3.213
Almost always stressed	-0.0197	0.2252		
Often stressed	-0.0049	0.6489		
Rarely stressed	-0.0302	0.0090		
Never stressed	-0.0276	0.2919	0.0895	1.048
Life satisfaction	0.0301	< 0.0001		18.983
Very satisfied with weight	0.0252	0.1268		
Satisfied with weight	-0.0072	0.5408		
Dissatisfied with weight	0.0342	0.0028		
Very dissatisfied with weight	-0.0022	0.8994	0.0012	2.910
No physical activity	-0.0886	< 0.0001		
Some physical activity	-0.0470	< 0.0001	< 0.0001	10.845
Constant	0.0368	0.2773		
R^2	0.2111			

Table 7: Coefficients of correlation between regression responses

	$RIF-R^b$	$RIF-L^b$	u	d
$RIF-R^b$	1.00	0.82	0.01	0.04
$RIF-L^b$		1.00	0.00	0.16
u			1.00	0.80
d				1.00

is 0.3547 and that of the regression of d 0.2111. It seems that the low R^2 values of the RIF regressions are due to a lack of variation in the response variables. The standard deviation of the RIF_i^R variable is 0.2849, while that of the u variable is 1.9224; yet both response variables have the same mean, viz. $R^b = 0.0657$. Likewise, the standard deviation of the RIF_i^L variable is 0.0796, much lower than that of the d variable, 0.5711, even though both have the same mean, viz. $L^b = 0.0144$. This appears to be a general rule and not a peculiarity of the empirical case we have studied. It can even be shown that in some cases the RIF response variable is a constant, which means that the RIF regression is powerless.¹¹

5 Conclusion

Summing up, in order to explain socioeconomic inequality of health, the direct decomposition method seems at least as promising as the WDW and the RIF decomposition methods. It improves upon the WDW method by relying on a regression of the joint outcome of income and health instead of on a regression of health only. And it is superior to the RIF method because it focuses on the deviation of the joint outcome from a reference position instead of on the influence of individual observations on the inequality index. A limitation of our paper is that we have restricted ourselves to the basic versions of the rank- and level-dependent indices. We are confident, however, that it will be possible to extend the approach to more general versions of the indices.

¹¹Here is an example. Assume that n is even and that there are no income ties. Let the $n/2$ individuals with the lowest incomes all have health level h^- , and the $n/2$ individuals with the highest incomes health level h^+ , with $h^- \neq h^+$. Then we have $IF_i^R = 0$ and therefore $RIF_i^R = R$ for *all* individuals in society. The u_i values are not constant, however.

Appendix A. Sample Weights and Ties

This appendix describes how the weights w_i must be defined when working with datasets in which not all individuals have the same sample weight, and when there are ties between individuals in the income distribution.

Let us assume that the individuals of our dataset $(1, 2, \dots, n)$ are ranked according to their individual income, i.e. $y_1 \leq y_2 \leq \dots \leq y_n$. We denote the sample weight of individual i by σ_i , and we assume that $\sum_{i=1}^n \sigma_i = 1$.

If there are ties in the income distribution, we define k groups of individuals $G_1, G_2, \dots, G_k$ such that everyone in group G_j has income y_{G_j} , and moreover $y_{G_1} < y_{G_2} < \dots < y_{G_k}$. The sample weight of group G_j is $\sigma_{G_j} = \sum_{i \in G_j} \sigma_i$. The relative fractional rank of group G_j is defined by the formula $\tilde{f}_{G_j} \equiv \sigma_{G_j} + 2 \sum_{l=0}^{j-1} \sigma_{G_l}$, where we take $\sigma_{G_0} = 0$ and $j = 1, \dots, k$.

Let individual i belong to group G_j . Then the weight to be used for this individual in the calculation of the rank-dependent index is equal to:

$$w_i^R = n\sigma_i(\tilde{f}_{G_j} - 1) \quad (\text{A.1})$$

As far as the level-dependent index is concerned, there is no need to consider the sample group weights. The weight of individual i is equal to:

$$w_i^L = n\sigma_i(\tilde{y}_i - 1) \quad (\text{A.2})$$

Appendix B. Sample Statistics

This appendix provides the tables with the sample statistics of the dataset used in the empirical application of Section 4.

Table B1: Descriptive statistics of the sample (N = 14,728)

Numerical variables	Mean	S.D.	Min	Max
Age	44.87	18.57	15	96
Number of children				
Aged 0-4	0.18	0.51	0	4
Aged 5-14	0.30	0.71	0	7
Life satisfaction (0, 1, ..., 10)	7.93	1.41	0	10
Categorical variables (in %, with reference indicated in bold)				
Sex		Marital status		
Female	53.18	Married		64.18
Male	46.82	Not married		35.82
Ancestry		Housing type		
Indigenous	2.50	House		81.28
Non-indigenous	97.50	Semi-detached house		6.74
Smoking		Flat/apartment		11.24
Yes	17.57	Non-private dwelling		0.24
No	82.43	Other dwelling type		0.50
Occupational status		Time stress		
Managers & professionals	24.57	Almost always		8.73
Service & sales workers	29.87	Often		26.76
Manual workers	9.02	Sometimes		40.89
Unemployed	4.04	Rarely		20.77
Not in labour force	32.50	Never		2.85
Sleep quality		Satisfaction with weight		
Very good	19.39	Very satisfied		8.98
Fairly good	54.97	Satisfied		26.69
Fairly bad	20.48	Neither satisfied/dissatisfied		24.19
Very bad	3.97	Dissatisfied		31.47
Not reported	1.19	Very dissatisfied		8.67
Physical activity		Self-rated prosperity		
No	11.11	Living poorly		3.48
Some	37.27	Just getting along		26.61
Frequent	51.62	Living comfortably		69.91

Table B2: Summary statistics for the variables

Variable	Health		Income (\$)	
	Mean	S.D.	Mean	S.D.
Sex				
Female	0.7508	0.1244	50,014	34,726
Male	0.7718	0.1216	52,394	35,102
Ancestry				
Indigenous	0.7309	0.1375	37,429	21,193
Non-indigenous	0.7614	0.1231	51,480	35,133
Marital status				
Married	0.7658	0.1195	55,058	35,959
Not married	0.7512	0.1300	44,087	31,791
Number of children*				
Aged 0-4	0.7821	0.1145	47,050	26,270
Aged 5-14	0.7679	0.1210	48,811	29,359
Housing type				
House	0.7625	0.1225	51,259	33,281
Semi-detached house	0.7523	0.1260	51,958	41,341
Flat/apartment	0.7553	0.1281	50,966	41,969
Non-private dwelling	0.6772	0.1408	21,399	14,107
Other dwelling type	0.7280	0.1253	36,750	23,051
Occupational status				
Managers & professionals	0.7896	0.1056	69,557	40,895
Service & sales workers	0.7852	0.1094	53,184	29,142
Manual workers	0.7771	0.1146	46,415	20,954
Unemployed	0.7406	0.1313	37,811	23,340
Not in labour force	0.7140	0.1351	38,273	32,631
Self-rated prosperity				
Living poorly	0.6497	0.1437	31,339	17,358
Just getting along	0.7222	0.1294	39,006	20,940
Living comfortably	0.7808	0.1134	56,728	38,156

*Results for the number of individuals with children aged 0-4 (1,976) and with children aged 5-14 (2,789).

Table B2 (*Continued*)

	Health		Income (\$)	
Variable	Mean	S.D.	Mean	S.D.
Smoking				
Yes	0.7328	0.1314	43,153	28,402
No	0.7665	0.1210	52,828	35,932
Sleep quality				
Very good	0.8159	0.1060	52,898	40,367
Fairly good	0.7719	0.1117	52,031	34,993
Fairly bad	0.7076	0.1255	48,982	29,942
Very bad	0.6225	0.1363	43,665	29,095
Not reported	0.7076	0.1440	42,443	26,528
Time stress				
Almost always	0.7149	0.1293	53,941	31,630
Often	0.7473	0.1217	55,036	34,747
Sometimes	0.7667	0.1186	51,445	34,167
Rarely	0.7807	0.1227	45,672	35,299
Never	0.7912	0.1459	41,024	45,665
Satisfaction with weight				
Very satisfied	0.8027	0.1156	51,810	38,589
Satisfied	0.7748	0.1214	49,876	34,736
Neither satisfied/dissatisfied	0.7643	0.1225	49,196	32,567
Dissatisfied	0.7525	0.1187	54,093	36,922
Very dissatisfied	0.6924	0.1286	48,910	29,205
Physical activity				
No	0.6752	0.1416	42,845	33,565
Some	0.7505	0.1185	50,740	34,244
Frequent	0.7862	0.1132	53,192	35,420
All	0.7606	0.1236	51,129	34,922

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A Direct Regression Approach to Decomposing Socioeconomic Inequality of Health

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