**Dynamic Longevity Hedging in the Presence of Population Basis Risk: A Feasibility
Analysis from Technical and Economic Perspectives**

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Abstract

In this paper, we study the feasibility of dynamic longevity hedging with standardized securities that are linked to broad-based mortality indexes. On the technical front, we generalize the dynamic ‘delta’ hedging strategy developed by Cairns (2011) to incorporate the situation when population basis risk exists. On the economic front, we discuss the potential financial benefits of an index-based hedge over a bespoke risk transfer. By considering data from a large group of national populations, we found evidence supporting the diversifiability of population basis risk. We further propose a customized surplus swap – executed between a hedger and reinsurer – to utilize the diversifiability. As standardized instruments demand less illiquidity premium, a combination of a dynamic index-based hedge and the proposed customized surplus swap may possibly be a more economical (and equally effective) alternative to a bespoke risk transfer.

Introduction

The market for longevity risk transfers started about 10 years ago. Since then, the market has seen some significant developments, most notably in terms of the number and size of deals (Blake et al., 2014). However, relative to the size of the global longevity risk exposure, the present longevity risk transfer market is still very small. A small market not only impedes longevity risk management, but also poses systemic concerns, because when longevity risk is shifted from the corporate sector to a limited number of (re)insurers, with global interconnections, there may

be systemic consequences in the case of a failure of a key player (Basel Committee of Banking Supervision, 2013).

The underdevelopment of the longevity risk transfer market may be attributed to the marked imbalance between demand and supply. To date, most of the longevity risk transfers executed are insurance-based, typically in the form of pension buy-ins, pension buy-outs or bespoke longevity swaps. While the insurance industry has the scope and financial stability to assume longevity risk, it does not generate sufficient supply for acceptance of the risk because of its capacity constraints. Using the assets for pension plans, in excess of 31 trillion USD, as a proxy for demand and the assets of 2.6 trillion USD held by the global insurance industry to cover non-life risks as a proxy for supply, Graziani (2014) concluded that the demand for acceptance of longevity risk exceeds supply by a multiple of 10. Michaelson and Mulholland (2014) also reached a similar conclusion by comparing the potential increase in pension liabilities due to unforeseen longevity improvement with the aggregate capital of the global insurance industry.

The growth of the longevity risk transfer market therefore depends highly on the creation of supply, most likely by inviting participation from capital markets, which are capable of assuming a larger portion of the longevity risk exposures from pension plans around the world.¹ The longevity asset class offers capital market investors a risk premium, plus potential diversification benefits due to its very low correlation with other asset classes. However, drawing interest from such investors requires the longevity risk transfer market to package the risk as standardized products that are structured like typical capital market derivatives and linked to broad-based mortality indexes. The act of standardization is important in part because it fosters the development of liquidity, and in part because it removes the information asymmetry arising from the fact that hedgers (pension plans) have better knowledge about the mortality experience of their own portfolios.

Towards the goal of standardization, the market for longevity risk transfers has to overcome two technical challenges which discourage hedgers from using standardized hedging instruments. The first challenge is to find out how standardized instruments can be used to form a hedge that can eliminate a meaningful portion of the hedger's longevity risk exposure. Hedging strategies have to be developed so that hedgers know the type and notional amounts of hedging instruments they need to acquire. The second challenge is to understand and more importantly mitigate the residual risks that are left behind by a standardized, index-based longevity hedge. Of the residual

¹According to Roxburgh (2011), the total value of the world's financial stock, comprising equity market capitalization and outstanding bonds and loans, is 212 trillion USD at the end of 2010.

risks the most significant constituent is population basis risk, which arises from the difference in future mortality improvements between the population associated with the hedger's own portfolio and the population(s) to which the standardized instruments are linked. However, as explained below, the research questions on longevity hedging strategies and population basis risk are still open.

A significant portion of the existing literature on longevity hedging strategies focuses on static hedging (Cairns, 2013; Cairns et al., 2006, 2014; Coughlan et al., 2011; Dowd et al., 2011; Li and Hardy, 2011; Li and Luo, 2012). Static hedging strategies are generally subject to the shortcoming of the need for long-dated hedging instruments. For example, in an illustrative static hedge for a 30-year pension liability, Li and Luo (2012) used five securities, of which the longest time-to-maturity is 25 years. Such long-dated securities do not seem appealing to capital market investors. A few researchers including Cairns (2011), Dahl (2004), Dahl and Møller (2006), Dahl et al. (2008) and Luciano et al. (2012) proposed dynamic longevity hedging strategies. Except the work of Cairns (2011), the existing dynamic longevity hedging strategies were developed from continuous-time models, which provide mathematical tractability but are not straightforward to implement in practice. Further, although some existing static hedging strategies include an adjustment for population basis risk (Dowd et al., 2011; Li and Hardy, 2011; Li and Luo, 2012), none of the aforementioned dynamic longevity hedging strategies takes population basis risk into account.

For the problem of population basis risk, researchers have recently contributed significantly to the development of multi-population stochastic mortality models (Ahmadi and Li, 2014; Cairns et al., 2011; Dowd et al., 2011; Hatzopoulos and Haberman, 2013; Jarner and Kryger, 2011; Li and Hardy, 2011; Li and Lee, 2005; Yang and Wang, 2013; Zhou et al., 2013, 2014). Such models can be regarded as a pre-requisite for understanding population basis risk, because they allow users to gauge the range of possible mortality differentials between two related populations, with biological reasonableness taken into consideration. Researchers have also introduced metrics for quantifying population basis risk, for example, reduction in expected shortfall (Ngai and Sherris, 2011), reduction in portfolio variance (Coughlan et al., 2011; Li and Hardy, 2011) and minimal required buffer (Stevens et al., 2011). However, to our knowledge, little attention has been paid to how population basis risk can be mitigated.

In this paper, we attempt to address the limitations of the current literature by investigating how a dynamic, index-based longevity hedge can be performed when population basis risk is present and how the residual risks left behind by the hedge can be mitigated. One part of the

framework is a dynamic hedging strategy with which a pension plan can transfer the ‘trend risk’ (i.e., the risk surrounding the trend in longevity improvement) to capital markets, even if the securities available are linked to a broad-based mortality index. Another part of the framework is a specially designed reinsurance treaty, called a ‘customized surplus swap’, which transfers the residual risks to a reinsurer who collectively manages the residual risks from the index-based longevity hedges of various pension plans.²

The dynamic hedging strategy we propose is obtained by generalizing the dynamic ‘delta’ hedging strategy of Cairns (2011) to incorporate the situation when the populations associated with the hedger’s portfolio and the hedging instruments are not the same. The generalization is derived on the basis of the augmented common factor (ACF) model (Li and Lee, 2005), under which the mortality dynamics of different populations are non-trivially correlated. Because the ACF model contains only one time-varying stochastic factor that is shared by all of the populations being modeled, the proposed hedging strategy requires the hedger to hold only one hedging instrument at a time. Also, because of dynamic adjustments, the hedging instrument used can be shorter-dated. The former property helps the market to concentrate liquidity, while the latter property better meets the appetite of capital market investors. Adding further to the contribution of Cairns (2011) is a study of the robustness of the dynamic hedging strategy relative to different factors including model risk, small sample risk and the properties of the hedging instruments used.

The customized surplus swap we design eliminates all residual risks that are left behind by the dynamic longevity hedge. Therefore, the combination of a dynamic longevity hedge and customized surplus swap should produce the same hedge effectiveness as a typical bespoke longevity swap. Using real mortality data from 25 different populations, we demonstrate that the residual risks can potentially be diversified away when a reinsurer writes customized surplus swaps with a range of hedgers. A reinsurer should thus have a much larger capacity to write customized surplus swaps than contracts such as pension buy-outs which involve significant systematic risk. Overall, our proposed risk management framework is likely to be more economical than traditional longevity risk transfers that are entirely insurance-based, because in theory it is less costly to transfer the systematic trend risk through liquidly traded standardized securities than tailor-

²A similar concept was mentioned by Cairns et al. (2008). In their set-up, hedgers transfer all their longevity risk exposures by writing bespoke longevity swaps with a special purposed vehicle (SPV), and the SPV in turn issues a standardized longevity bond which transfers the trend risk to the bondholders. The residual risks are borne by the SPV manager.

made (re)insurance contracts.

The rest of this paper is organized as follows. The “Dynamic Longevity Hedging Strategy” section presents the technical details of the proposed dynamic hedging strategy. The “Analyzing the Dynamic Longevity Hedge” section illustrates the proposed dynamic hedging strategy and evaluates its robustness relative to various factors. The “Managing the Residual Risks” defines the proposed customized surplus swap and demonstrates the diversifiability of the residual risks. The last section concludes the paper with some suggestions for future research.

Dynamic Longevity Hedging Strategy

The Assumed Model

The dynamic hedging strategy requires an assumed stochastic mortality model, from which quantities such as hedge ratios can be derived. In the single-population set-up of Cairns (2011), the original Cairns-Blake-Dowd model (a.k.a. Model M5) was assumed. In our generalization, we assume the augmented common factor (ACF) model proposed by Li and Lee (2005). The ACF model concurrently models the mortality dynamics of multiple, say P , populations as follows:

$$\ln(m_{x,t}^{(i)}) = a_x^{(i)} + B_x K_t + b_x^{(i)} k_t^{(i)} + \epsilon_{x,t}^{(i)}, \quad i = 1, \dots, P,$$

where $m_{x,t}^{(i)}$ represents population i 's central rate of death at age x and in year t , $a_x^{(i)}$ is a parameter indicating population i 's average level of mortality at age x , K_t is a time-varying index that is shared by all P populations, $k_t^{(i)}$ is a time-varying index that is specific to population i , parameters B_x and $b_x^{(i)}$ respectively reflect the sensitivity of $\ln(m_{x,t}^{(i)})$ to K_t and $k_t^{(i)}$, and $\epsilon_{x,t}^{(i)}$ is the error term that captures all remaining variations. The procedure we use to estimate the model is detailed in Online Appendix A.

The trend in K_t determines the evolution of mortality over time for all populations being modeled. As in the original Lee-Carter model (Lee and Carter, 1992), K_t is assumed to follow a random walk with drift: $K_t = C + K_{t-1} + \xi_t$, where C is the drift term and $\{\xi_t\}$ is a sequence of i.i.d. normal random variables with zero mean and constant variance σ_K^2 .

Departures from the common time trend are captured by the population-specific index $k_t^{(i)}$, which is assumed to follow a first order autoregressive process: $k_t^{(i)} = \phi_0^{(i)} + \phi_1^{(i)} k_{t-1}^{(i)} + \zeta_t^{(i)}$, where $\phi_0^{(i)}$ and $\phi_1^{(i)}$ are constants, and $\{\zeta_t^{(i)}\}$ is a sequence of i.i.d. normal random variables with zero mean and constant variance $\sigma_{k,i}^2$. We require $|\phi_1^{(i)}| < 1$ so that the process for $k_t^{(i)}$

is mean-reverting. This property ensures that the resulting forecasts are coherent, which means the projected mortality rates for different populations do not diverge indefinitely over time. To incorporate any correlation that is not captured by the common trend K_t , we further assume that $\zeta_t^{(i)}$ and $\zeta_t^{(j)}$ for $i \neq j$ are constantly correlated, despite such correlations not being taken into account in the original ACF model.

The Set-up

We let

$$S_{x,t}^{(i)}(T) = \prod_{s=1}^T (1 - q_{x+s-1,t+s}^{(i)}) \quad (1)$$

be the *ex post* probability that an individual who is from population i and aged x at time t (the end of year t) would have survived to time $t + T$, where $q_{x,t}^{(i)}$ denotes the probability that an individual from population i dies between time $t - 1$ and t (during year t), provided that he/she has survived to age x at time $t - 1$. When computing $q_{x,t}^{(i)}$ from $m_{x,t}^{(i)}$ (on which the ACF model is based), we use the approximation $q_{x,t}^{(i)} \approx 1 - \exp(-m_{x,t}^{(i)})$. It is clear from the definitions that $S_{x,t}^{(i)}(T)$ is not known prior to time $t + T$, while $q_{x,t}^{(i)}$ is not known prior to time t .

Define by $\mathcal{F}_t$ the information about the evolution of mortality up to and including time t . Due to the Markov property of the assumed stochastic processes, the value of $E(S_{x,u}^{(i)}(T)|\mathcal{F}_t)$ for $u \geq t$ depends only on the values of K_t and $k_t^{(i)}$ but not the values of K_v and $k_v^{(i)}$ for $v < t$. Hence, we have

$$p_{x,u}^{(i)}(T, K_t, k_t^{(i)}) := E(S_{x,u}^{(i)}(T)|K_t, k_t^{(i)}) = E(S_{x,u}^{(i)}(T)|\mathcal{F}_t).$$

We call $p_{x,u}^{(i)}(T, K_t, k_t^{(i)})$ a spot survival probability when $u = t$ and a forward survival probability when $u > t$.

Let us suppose that the hedger intends to hedge the longevity risk associated with a pension plan for a single cohort of individuals, who are all from population H and aged x_0 at time 0. The plan pays each pensioner \$1 at the end of each year until death. It follows that the time- t value of the pension plan's future liabilities (per surviving pensioner at time t) can be expressed in terms of spot survival probabilities as

$$FL_t = \sum_{s=1}^{\infty} (1+r)^{-s} p_{x_0+t,t}^{(H)}(s, K_t, k_t^{(H)}),$$

where r is the interest rate for discounting purposes.

The hedging instruments are q-forwards that are associated with population R . A q-forward is a zero-coupon swap with its floating leg proportional to the realized death probability at a certain reference age during the year immediately prior to maturity and its fixed leg proportional to the corresponding pre-determined forward mortality rate. In this application, the hedger should participate in the q-forwards as the fixed-rate receiver, so that he/she will receive a net payment from the counterparty when mortality turns out to be lower than expected.

Consider a q-forward that is linked to reference population R and age x_f . Suppose that the q-forward is issued at time t_0 and matures at time $t_0 + T^*$. The payoff from the q-forward depends on the realized value of $q_{x_f, t_0 + T^*}^{(R)}$. The corresponding forward mortality rate q^f is chosen so that no payment exchanges hands at inception (time t_0). It is assumed that $q^f = E(q_{x_f, t_0 + T^*}^{(R)} | \mathcal{F}_{t_0})$, which is equivalent to saying that no risk premium is given to the counterparty accepting the risk.³ At $t = t_0, \dots, t_0 + T^* - 1$, the value of the hedger's position of the q-forward (per \$1 notional) can be expressed as

$$\begin{aligned} Q_t(t_0) &= (1+r)^{-(t_0+T^*-t)}(q^f - E(q_{x_f, t_0+T^*}^{(R)} | \mathcal{F}_t)) \\ &= (1+r)^{-(t_0+T^*-t)}(q^f - (1 - E(S_{x_f, t_0+T^*-1}^{(R)}(1) | \mathcal{F}_t))) \\ &= (1+r)^{-(t_0+T^*-t)}(q^f - (1 - p_{x_f, t_0+T^*-1}^{(R)}(1, K_t, k_t^{(R)}))). \end{aligned}$$

Under our pricing assumption, we have $Q_{t_0}(t_0) = 0$. Note that both FL_t and $Q_t(t_0)$ are related linearly to values of $p_{x,u}^{(i)}(T, K_t, k_t^{(i)})$, where $i = H, R$ and $u \geq t$.

The main idea behind the dynamic hedging strategy is that at each discrete time point t , the q-forward portfolio is adjusted so that FL_t and the adjusted q-forward portfolio have similar sensitivities to changes in the underlying common mortality index K_t . Hence, at each discrete time point t , we need to compute FL_t and $Q_t(t_0)$ and their partial derivatives with respect to K_t . However, because of the way in which $S_{x,t}^{(i)}(T)$ depends on K_u and $k_u^{(i)}$ for $u = t+1, \dots, T$, the values of $p_{x,u}^{(i)}(T, K_t, k_t^{(i)})$ for $u \geq t$ (and thus FL_t and $Q_t(t_0)$) cannot be computed analytically. It follows that nested simulations are required, making the dynamic hedging framework strategy computationally challenging.

In more detail, let us assume that the hedging horizon is Y years and that the q-forward portfolio is adjusted annually. Suppose that N sample paths of future mortality (i.e., values of

³Because the counterparty accepting longevity risk from the hedger deserves a risk premium, in practice q^f should be smaller than $E(q_{x_f, t_0+T^*}^{(R)} | \mathcal{F}_{t_0})$, so that payoff to the counterparty is positive in expectation terms. However, because our focus for now is the technical aspects rather than the associated costs, we assume $q^f = E(q_{x_f, t_0+T^*}^{(R)} | \mathcal{F}_{t_0})$ for simplicity.

K_t , $k_t^{(H)}$ and $k_t^{(R)}$ for $t = 1, \dots, Y$) are used to evaluate the hedge's performance. For each of these N sample paths, we need to evaluate, at each time point t for $t = 1, \dots, Y$, FL_t and $Q_t(t_0)$ on the basis of the realized values of K_t , $k_t^{(H)}$ and $k_t^{(R)}$ in that particular sample path. If we calculate each FL_t and $Q_t(t_0)$ with M sample paths of mortality beyond time t , then in total we need to generate $N \times M \times Y$ sample paths. Because N and M are typically very large, say 10,000, the computational burden is huge.

To reduce computation burden, in the next subsection we derive formulas to approximate $p_{x,u}^{(i)}(T, K_t, k_t^{(i)})$ for $u \geq t$ so that the need for some of the simulations can be avoided. The accuracy of the approximation formulas is evaluated in Online Appendix B.

The Approximation Methods

The approximation formula for $p_{x,u}^{(i)}(T, K_t, k_t^{(i)})$ depends on whether $u = t$ or $u > t$.

Approximating $p_{x,u}^{(i)}(T, K_t, k_t^{(i)})$ when $u = t$

Following Cairns (2011), we approximate $p_{x,t}^{(i)}(T, K_t, k_t^{(i)})$ by applying a Taylor expansion to its probit transform, $f_{x,t}^{(i)}(T, K_t, k_t^{(i)}) := \Phi^{-1}(p_{x,t}^{(i)}(T, K_t, k_t^{(i)}))$, where Φ denotes the standard normal distribution function. The Taylor expansion is made around $\hat{K}_t = E(K_t|K_0)$ and $\hat{k}_t^{(i)} = E(k_t^{(i)}|k_0^{(i)})$. We consider a second-order approximation, $f_{x,t}^{(i)}(T, K_t, k_t^{(i)}) \approx \tilde{f}_{x,t}^{(i)}(T, K_t, k_t^{(i)})$, where

$$\begin{aligned} \tilde{f}_{x,t}^{(i)}(T, K_t, k_t^{(i)}) &:= D_{x,t,0}^{(i)}(T) + D_{x,t,1}^{(i)}(T)(K_t - \hat{K}_t) + D_{x,t,2}^{(i)}(T)(k_t^{(i)} - \hat{k}_t^{(i)}) \\ &\quad + \frac{1}{2}D_{x,t,3}^{(i)}(T)(K_t - \hat{K}_t)^2 + \frac{1}{2}D_{x,t,4}^{(i)}(T)(k_t^{(i)} - \hat{k}_t^{(i)})^2 \\ &\quad + D_{x,t,5}^{(i)}(T)(K_t - \hat{K}_t)(k_t^{(i)} - \hat{k}_t^{(i)}), \end{aligned}$$

and

$$\begin{aligned} D_{x,t,0}^{(i)}(T) &= f_{x,t}^{(i)}(T, \hat{K}_t, \hat{k}_t^{(i)}), & D_{x,t,1}^{(i)}(T) &= \left. \frac{\partial f_{x,t}^{(i)}(T, K_t, \hat{k}_t^{(i)})}{\partial K_t} \right|_{K_t = \hat{K}_t}, \\ D_{x,t,2}^{(i)}(T) &= \left. \frac{\partial f_{x,t}^{(i)}(T, \hat{K}_t, k_t^{(i)})}{\partial k_t^{(i)}} \right|_{k_t^{(i)} = \hat{k}_t^{(i)}}, & D_{x,t,3}^{(i)}(T) &= \left. \frac{\partial^2 f_{x,t}^{(i)}(T, K_t, \hat{k}_t^{(i)})}{\partial K_t^2} \right|_{K_t = \hat{K}_t}, \\ D_{x,t,4}^{(i)}(T) &= \left. \frac{\partial^2 f_{x,t}^{(i)}(T, \hat{K}_t, k_t^{(i)})}{\partial k_t^{(i)2}} \right|_{k_t^{(i)} = \hat{k}_t^{(i)}}, & D_{x,t,5}^{(i)}(T) &= \left. \frac{\partial^2 f_{x,t}^{(i)}(T, K_t, k_t^{(i)})}{\partial K_t \partial k_t^{(i)}} \right|_{K_t = \hat{K}_t, k_t^{(i)} = \hat{k}_t^{(i)}}. \end{aligned}$$

The values of $D_{x,t,j}^{(i)}(T)$ for $j = 1, \dots, 5$ are computed numerically by finite difference approximations (see, e.g., Cairns, 2011, Appendix B). For a fixed t , the finite difference approximations require nine sets of M sample mortality paths, which are respectively based on nine

different sets of starting values, including $(K_t = \hat{K}_t, k_t^{(i)} = \hat{k}_t^{(i)})$, $(K_t = \hat{K}_t + h, k_t^{(i)} = \hat{k}_t^{(i)})$, $(K_t = \hat{K}_t, k_t^{(i)} = \hat{k}_t^{(i)} + h)$ and so on. For a hedging horizon of Y time steps, the number of sample paths required to generate the partial derivatives is $9 \times M \times Y$.

Suppose again that N mortality scenarios are used to evaluate the hedge's performance. Because the partial derivatives are independent of these N mortality scenarios, the total number of sample paths we need to generate is $N + 9 \times M \times Y$, which is significantly smaller than $N \times M \times Y$ when N and M are large.

Approximating $p_{x,u}^{(i)}(T, K_t, k_t^{(i)})$ when $u > t$

Using a first-order approximation, it can be shown that

$$p_{x,u}^{(i)}(T, K_t, k_t^{(i)}) \approx \Phi \left(\frac{-E(V_u^{(i)}|\mathcal{F}_t)}{\sqrt{\text{Var}(V_u^{(i)}|\mathcal{F}_t)}} \right).$$

A proof of the above approximation formula and the expressions for $E(V_u^{(i)})$ and $\text{Var}(V_u^{(i)}|\mathcal{F}_t)$ are provided in Online Appendix C.

Deriving Hedge Ratios

Our goal is to ensure that at each discrete time point t , the q-forward portfolio and the pension plan's future liabilities have similar sensitivities to changes in the underlying common mortality index K_t . To achieve this goal, the hedge ratio h_t (i.e., the notional amount of the q-forward) at time t is chosen in such a way that

$$\frac{\partial FL_t}{\partial K_t} = h_t \frac{\partial Q_t(t_0)}{\partial K_t}.$$

Because we match the first derivatives only, only one q-forward contract is needed at each t . For the same reason, our hedge may be considered as a 'delta' hedge. In principle, one may create, for example, a 'gamma' hedge by matching also the second order derivatives. It should also be noted that the requirement of only one q-forward contract in the delta hedge is a consequence of the fact that the ACF model contains only one common time-varying factor. If a mortality model with multiple common time-varying factors is assumed (e.g., the M-CBD model described in Online Appendix D), then the delta hedge would require more than one q-forward.

The partial derivative of FL_t with respect to K_t is computed as follows:

$$\begin{aligned}
\frac{\partial FL_t}{\partial K_t} &= \sum_{s=1}^{\infty} (1+r)^{-s} \frac{\partial p_{x_0+t,t}^{(H)}(s, K_t, k_t^{(H)})}{\partial K_t} \\
&= \sum_{s=1}^{\infty} (1+r)^{-s} \frac{\partial \Phi(f_{x_0+t,t}^{(H)}(s, K_t, k_t^{(H)}))}{\partial K_t} \\
&\approx \sum_{s=1}^{\infty} (1+r)^{-s} \phi \left(\tilde{f}_{x_0+t,t}^{(H)}(s, K_t, k_t^{(H)}) \right) \\
&\quad \times \left(D_{x_0+t,t,1}^{(H)}(s) + D_{x_0+t,t,3}^{(H)}(s)(K_t - \hat{K}_t) + D_{x_0+t,t,5}^{(H)}(s)(k_t^{(H)} - \hat{k}_t^{(H)}) \right),
\end{aligned}$$

where ϕ represents the probability density function for a standard normal random variable.

The partial derivative of $Q_t(t_0)$ with respect to K_t depends on the value of t relative to the q-forward's maturity date $t_0 + T^*$. If $t = t_0 + T^* - 1$,⁴

$$\begin{aligned}
\frac{\partial Q_t(t_0)}{\partial K_t} &= (1+r)^{-1} \frac{\partial p_{x_f,t}^{(R)}(1, K_t, k_t^{(R)})}{\partial K_t} \\
&= (1+r)^{-1} \frac{\partial \Phi(f_{x_f,t}^{(R)}(1, K_t, k_t^{(R)}))}{\partial K_t} \\
&\approx (1+r)^{-1} \phi(\tilde{f}_{x_f,t}^{(R)}(1, K_t, k_t^{(R)})) \\
&\quad \times \left(D_{x_f,t,1}^{(R)}(1) + D_{x_f,t,3}^{(R)}(1)(K_t - \hat{K}_t) + D_{x_f,t,5}^{(R)}(1)(k_t^{(R)} - \hat{k}_t^{(R)}) \right).
\end{aligned}$$

If $t = t_0, \dots, t_0 + T^* - 2$,

$$\begin{aligned}
\frac{\partial Q_t(t_0)}{\partial K_t} &= (1+r)^{-(t_0+T^*-t)} \frac{\partial p_{x_f,t_0+T^*-1}^{(R)}(1, K_t, k_t^{(R)})}{\partial K_t} \\
&\approx (1+r)^{-(t_0+T^*-t)} \frac{\partial}{\partial K_t} \Phi \left(\frac{-E(V_{t_0+T^*-1}^{(R)} | \mathcal{F}_t)}{\sqrt{\text{Var}(V_{t_0+T^*-1}^{(R)} | \mathcal{F}_t)}} \right) \\
&= (1+r)^{-(t_0+T^*-t)} \phi \left(\frac{-E(V_{t_0+T^*-1}^{(R)} | \mathcal{F}_t)}{\sqrt{\text{Var}(V_{t_0+T^*-1}^{(R)} | \mathcal{F}_t)}} \right) \frac{-D_{x_f,t_0+T^*-1,1}^{(R)}(1)}{\sqrt{\text{Var}(V_{t_0+T^*-1}^{(R)} | \mathcal{F}_t)}}.
\end{aligned}$$

Evaluating the Hedge

As previously mentioned, N mortality scenarios are simulated to evaluate the effectiveness of the dynamic hedge.

⁴In the rest of this paper, it is assumed that each q-forward used is closed one year after the launch date t_0 . Therefore, the actual calculations involve only the approximation formula for $t = t_0, t_0 + 1$. The approximation formula for $t = t_0 + T^* - 1$ is provided just for completeness.

Define by PL_t the time-0 value of all pension liabilities, given the information up to and including time t ; that is,

$$PL_t = E \left(\sum_{s=1}^{\infty} (1+r)^{-s} S_{x_0,0}^{(H)}(s) \middle| \mathcal{F}_t \right) = \begin{cases} FL_0, & t = 0 \\ \sum_{s=1}^t (1+r)^{-s} S_{x_0,0}^{(H)}(s) + (1+r)^{-t} S_{x_0,0}^{(H)}(t) FL_t, & t > 0. \end{cases} \quad (2)$$

The value of PL_0 is non-random, as it is simply a function of K_0 and $k_0^{(H)}$ whose values are fixed. For $t > 0$, the values of PL_t are different under different simulated mortality scenarios. In particular, the values of $S_{x_0,0}^{(H)}(s)$ for $s = 1, \dots, t$ depend on the realized values of K_s and $k_s^{(H)}$ for $s = 1, \dots, t$, whereas the value of FL_t depends on the realized values of K_t and $k_t^{(H)}$.

It is assumed that at each time point t , the hedger writes a new q-forward contract (i.e., a q-forward with inception date $t_0 = t$) with a notional amount of h_t . The value of this position is $h_t Q_t(t) = 0$ at time t and becomes

$$\begin{aligned} h_t Q_{t+1}(t) &= h_t (1+r)^{-(T^*-1)} (q^f - E(q_{x_f, t+T^*}^{(R)} | \mathcal{F}_{t+1})) \\ &= h_t (1+r)^{-(T^*-1)} (E(q_{x_f, t+T^*}^{(R)} | \mathcal{F}_t) - E(q_{x_f, t+T^*}^{(R)} | \mathcal{F}_{t+1})) \\ &= h_t (1+r)^{-(T^*-1)} (p_{x_f, t+T^*-1}^{(R)}(1, K_{t+1}, k_{t+1}^{(R)}) - p_{x_f, t+T^*-1}^{(R)}(1, K_t, k_t^{(R)})) \end{aligned}$$

at time $t+1$.⁵ At time $t+1$, the position written at time t is closed out, and another new q-forward contract is written. The process repeats until the end of the hedging horizon Y is reached. For simplicity, we assume that all q-forwards used over the hedging horizon have the same maturity T^* and reference age x_f .

Let PA_t be the time-0 value of the assets backing the pension plan at time t . We assume that $PA_0 = PL_0$. For $t = 1, \dots, Y$, we have

$$PA_t = PA_{t-1} + (1+r)^{-t} h_{t-1} Q_t(t-1).$$

If PA_t is very close to PL_t for $t = 1, \dots, Y$, then the dynamic hedge can be said as successful. The potential deviation between PA_t and PL_t is the residual risk that is not mitigated by the hedge. Using this reasoning, we measure hedge effectiveness by the following metric:

$$HE_t = 1 - \frac{\text{Var}(PA_t - PL_t | \mathcal{F}_0)}{\text{Var}(PL_t | \mathcal{F}_0)}.$$

A value of HE_t that is close to one indicates the hedge is effective.

⁵The second step is due to our pricing assumption.

Analyzing the Dynamic Longevity Hedge

Assumptions

The following assumptions are used in the baseline calculations.

1. The hedger wishes to hedge the pension liabilities that are payable to a single cohort of individuals, who are all aged $x_0 = 60$ at time 0. The mortality experience of these individuals is identical to that of the UK male insured lives.
2. The pension plan pays each individual \$1 at the end of each year until death or age 90, whichever the earliest.⁶
3. The hedging horizon is $Y = 30$ years (i.e., the hedge stops when the liabilities have completely run off).
4. The q-forwards used are linked to the England and Wales (EW) male population. They all have a time-to-maturity (from inception) of $T^* = 10$ years and a reference age of $x_f = 75$.
5. The market for the q-forwards is liquid and no transaction cost is required.
6. The interest rate for all durations is $r = 4\%$ per annum. The interest rate remains constant over time.
7. The hedger can invest or borrow at an interest rate of $r = 4\%$ per annum.
8. The values of $D_{x,t,j}^{(i)}(T)$ for $i = H, R$ and $j = 0, \dots, 5$ are computed from an ACF model that is estimated to the data from the populations of EW males and UK male insured lives over the period of 1966 to 2005 and the age range of 60 to 89.⁷
9. To match the end point of the data sample period, time 0 is set to the end of year 2005.
10. The evaluation of hedge effectiveness is based on $N = 10,000$ mortality scenarios that are generated from the model described in Assumption (8).
11. There is no small sample risk.

⁶We assume that no pension is payable beyond age 90, because the upper limit of the age range to which the ACF model is fitted is 89. This assumption may be relaxed if one assumes a parametric curve to extrapolate death probabilities beyond age 89.

⁷The data for EW males are provided by the Human Mortality Database (2014), while the data for UK male insured lives are obtained from the Institute and Faculty of Actuaries by a written request.

Figure 1: Fan charts showing the distributions of $(PL_t - PL_0)|\mathcal{F}_0$ (in gray) and $(PL_t - PA_t)|\mathcal{F}_0$ (in green) when population basis risk is present (the left panel) and hypothetically absent (the right panel).

Baseline Results

The gray (larger) fan chart in the left panel of Figure 1 depicts the distributions of $(PL_t - PL_0)|\mathcal{F}_0$ (or equivalently $PL_t|\mathcal{F}_0$) for $t = 0, \dots, 30$ under the baseline assumptions. When there is no longevity hedge, the time-0 value of the assets backing the pension plan is always FL_0 , because we assume $PA_0 = PL_0 = FL_0$. Hence, $PL_t - PL_0$ can be regarded as the shortfall in assets in the absence of a longevity hedge.

The green (smaller) fan chart in the left panel Figure 1 shows the distributions of $(PL_t - PA_t)|\mathcal{F}_0$ for $t = 0, \dots, 30$ under the baseline assumptions. We can regard $PL_t - PA_t$ as the shortfall in assets when a dynamic longevity hedge is in place. Over the entire hedging horizon, $(PL_t - PA_t)|\mathcal{F}_0$ is significantly less dispersed than $(PL_t - PL_0)|\mathcal{F}_0$, indicating that the longevity hedge is effective. Over the hedging horizon, the value of HE_t is consistently larger than 90%.

To assess the extent of population basis risk, we repeat the calculations by assuming, hypothetically, that q-forwards linked to the population of UK male insured lives are available and used. The hedging results are shown in the right panel of Figure 1. The degree of population basis risk can be observed from the difference in the widths of the green fan charts in left and right panels of Figure 1.

Robustness

We have tested the robustness of the dynamic hedging strategy relative to various factors. The tests performed are detailed in Online Appendix D. The conclusions drawn from the tests are summarized below.

- When the simulation model contains two stochastic factors, the hedging result is not as good as the baseline result; however, the value of HE_{30} is still above 90%.
- In line with the results produced by Li and Hardy (2011), the impact of small sample risk is modest ($HE_{30} = 86.19\%$) when the initial number of pensioners is 10,000.
- Regardless of which simulation model is used, the hedge effectiveness remains high if the q-forward's reference age (x_f) is between 70 to 80.
- The relationship between hedge effectiveness and the q-forward's time-to-maturity (T^*) depends on the simulation model. For all simulation models considered, the hedging results are still satisfactory even if the time-to-maturity is as short as 5 years.

Managing the Residual Risks

In this section, we explain how the residual risks from a dynamic, index-based longevity hedge can be managed through a reinsurance mechanism.

Assumptions

As we expand our analysis to include more than two populations, some of the previously made assumptions have to be modified accordingly. Below we list the assumptions that are used in this section.

- (i) There are 25 pension plans wishing to hedge their longevity risk exposures. The 25 pension plans have respectively identical mortality experience to 25 different male populations. The chosen populations are the same as the 25 populations that are classified as the 'males West-cluster' by Hatzopoulos and Haberman (2013).

- (ii) Each pension plan contains initially $l_{60} = 3,000$ pensioners who are all aged $x_0 = 60$. For $x = 61, 62, \dots, l_x$ follows the binomial process described in the “Robustness Relative to Small Sample Risk” section in Online Appendix D.
- (iii) At any time point during the hedging horizon, the only hedging instrument available is a q-forward that is linked to EW male population with a time-to-maturity (from inception) of $T^* = 10$ years and a reference age of $x_f = 75$.
- (iv) The values of $D_{x,t,j}^{(i)}(T)$ for $i = 1, \dots, 25$ and $j = 0, \dots, 5$ are computed from an ACF model that is estimated to the data from the 25 male populations under consideration.⁸
- (v) To match the end point of the data sample period, time 0 is set to the end of year 2009.
- (vi) The evaluation of hedge effectiveness is based on $N = 10,000$ mortality scenarios that are generated from the model described in Assumption (iv).

Assumptions (2), (3), (5), (6) and (7) stated in the “Analyzing the Dynamic Longevity Hedge” section remain unchanged.

Customized Surplus Swaps

A dynamic longevity hedge is implemented for each of the 25 pension plans. The hedging results are reported and discussed in Online Appendix E.

The values of HE_{30} range from 37% to 79%. The results indicate that the dynamic longevity hedge may leave substantial residual risks, which include small sample risk and population basis risk. We hereby propose a customized surplus swap that permits pooling of the residual risks from different dynamic longevity hedges. When implementing such a swap in tandem with a dynamic index-based hedge, the pension plan would in theory be immunized from longevity risk.

A pension plan is immunized from longevity risk over the hedging horizon if $PA_t - PL_t = 0$ for $t = 1, \dots, Y$. We can regard $|PA_t - PL_t|$ as the pension plan’s surplus if $PA_t > PL_t$ and short fall in assets if $PA_t < PL_t$. The swap we design has a maturity of one year and is written at each time point when the dynamic index-based hedge is established or adjusted. We call it a ‘surplus’ swap, because its net cash flow at maturity is derived from the surplus process $PA_t - PL_t$, $t = 1, \dots, Y$, of the pension plan.

⁸The mortality data for all 25 male populations are obtained from the Human Mortality Database (2014). The data used cover a sample period of 1959 to 2009 and a sample age range of 60 to 89.

Our goal is to ensure that $PA_t - PL_t = 0$ for $t = 1, \dots, Y$. We let NCF_t be the net cash flow (payable at time t from the reinsurer to the pension plan) for the customized surplus swap that is written at time $t - 1$. With the swap in place, the recursion formula for PA_t can be rewritten as

$$PA_t = PA_{t-1} + (1+r)^{-t}(h_{t-1}Q_t(t-1) + NCF_t), \quad t = 1, \dots, Y, \quad (3)$$

where $PA_0 = PL_0$. Using equations (3) and (2), we obtain

$$\begin{aligned} PL_t - PA_t &= PL_{t-1} - PA_{t-1} + (1+r)^{-t}(S_{x_0,0}^{(H)}(t)(1+FL_t) - (1+r)S_{x_0,0}^{(H)}(t-1)FL_{t-1}) \\ &\quad - (1+r)^{-t}(h_{t-1}Q_t(t-1) + NCF_t), \quad t = 1, \dots, Y. \end{aligned}$$

To stipulate $PA_t - PL_t = 0$ for $t = 1, \dots, Y$, we require

$$NCF_t = S_{x_0,0}^{(H)}(t)(1+FL_t) - h_{t-1}Q_t(t-1) - (1+r)S_{x_0,0}^{(H)}(t-1)FL_{t-1}, \quad t = 1, \dots, Y.$$

The expression for NCF_t is intuitive. It says that there is no net cash flow from the swap if what the pension plan has at time $t - 1$ accumulated with interest (i.e., $(1+r)S_{x_0,0}^{(H)}(t-1)FL_{t-1}$) plus the proceed from the index-based hedge at time t (i.e., $h_{t-1}Q_t(t-1)$) is just sufficient to cover the plan's financial obligations at time t (i.e., $S_{x_0,0}^{(H)}(t)$) and beyond (i.e., $S_{x_0,0}^{(H)}(t)FL_t$).

Given $\mathcal{F}_{t-1}$, the value of $(1+r)S_{x_0,0}^{(H)}(t-1)FL_{t-1}$ is known, but the values of $S_{x_0,0}^{(H)}(t)(1+FL_t)$ and $h_{t-1}Q_t(t-1)$ are random as they both depend on the values of K_t , $k_t^{(H)}$ and $k_t^{(R)}$ which are not known until time t . It follows that for a customized surplus swap written at time $t - 1$, the fixed and floating legs should be set to $(1+r)S_{x_0,0}^{(H)}(t-1)FL_{t-1}$ and $S_{x_0,0}^{(H)}(t)(1+FL_t) - h_{t-1}Q_t(t-1)$, respectively.

Given how the cash flows are defined, the following should be incorporated into the terms of a customized surplus swap written at time $t - 1$:

- the method and assumptions used to calculate FL_{t-1} and FL_t ;
- the hedge ratio h_{t-1} ;
- the rate r at which the cash flows are discounted;
- the forward mortality rate q^f associated with the q-forward written at time $t - 1$.

For simplicity, we assume that the swap is costless in the following illustration. In practice, of course, the reinsurer demands a reward for taking on the risk and therefore a fixed payment (the risk premium) has to be paid by the pension plan to the reinsurer at either inception or maturity.

An Illustration

We now revisit the index-based longevity hedges for the 25 pension plans. Let us suppose that on top of the index-based hedges, all 25 pension plans write customized surplus swaps with the same reinsurer to eliminate their exposures to the residual risks. Assume further that the assumptions used in formulating the index-based hedges also apply to the terms of the customized surplus swaps.

The fan charts in Figure 2 show the distributions of $NCF_t|\mathcal{F}_0$ for the 25 pension plans. Several comments on the fan charts are made in Online Appendix F.

To study the diversifiability of the residual risks from the index-based hedges, let us consider the cash flows from the viewpoint of the reinsurer who writes customized surplus swaps with the 25 pension plans. The fan chart in the left panel of Figure 3 depicts the distributions (conditioned on $\mathcal{F}_0$) of the average net cash flows payable to each pension plan over the hedging horizon. The variability of the reinsurer's average net cash flows is small compared to the variability of NCF_t for individual pension plans. The diversifiability can be observed more clearly from the right panel of Figure 3, which compares the variances of the reinsurer's average net cash flows with the variances of the net cash flows arising from individual customized surplus swaps.

Discussion and Conclusion

In this paper, we consider a risk management framework in which longevity risk is split into trend risk and residual risks. With the proposed dynamic hedging strategy, a pension plan can transfer its trend risk exposure to capital markets through standardized instruments. Using the proposed customized surplus swap, the pension plan may also transfer the residual risks left behind by the dynamic hedge to a reinsurer, who collectively manages the residual risks from various pension plans. As a whole, our risk management framework allows pension plans to completely eliminate their longevity risk exposures, just as what they can achieve from traditional, entirely insurance-based pension de-risking solutions.

What we propose allows the longevity risk transfer market to package the trend risk as standardized products that are structured like typical capital market derivatives. Compared to products such as pension buy-ins, standardized mortality derivatives are more appealing to capital market investors who generally desire liquidity and transparency. When put in practice, our risk

Figure 2: Fan charts showing the distributions of $NCF_t|\mathcal{F}_0$ for the 25 pension plans under consideration.

management framework may attract participation from capital markets, thereby ameliorating the demand and supply imbalance in the present market for longevity risk transfers. The enhancement of liquidity through standardization may also result in lower risk management costs to pension plans, as the illiquidity premium payable to the counterparty can be reduced. Although there is no sufficient data to test the inverse relationship between liquidity and compensation to investors (typically measured by the Sharpe ratio) in the longevity risk market, there is profound evidence for such an inverse relationship in several financial markets.

For stock markets, the inverse relationship between liquidity and risk-adjusted rate of return was identified by Lo et al. (2003), who introduced liquidity to the standard mean-variance portfolio optimization framework by constructing three-dimensional mean-variance-liquidity frontiers on the basis of several measures of liquidity including trading volume and percentage bid/offer spreads. They studied the tangency portfolios of the liquidity-constrained mean-variance-liquidity efficient frontiers for some randomly selected stocks, and found that the Sharpe ratio of the tangency portfolio reduces as the liquidity threshold becomes less stringent.

Figure 3: An illustration of the diversifiability of the residual risks from the index-based hedges. The left panel shows the distributions (conditioned on $\mathcal{F}_0$) of the average net cash flows payable from the reinsurer to each pension plan over the hedging horizon. The right panel shows the values of $\text{Var}(NCF_t|\mathcal{F}_0)$, $t = 1, \dots, 30$, for the individual customized surplus swaps (the dotted lines) and the corresponding variances of the reinsurer's average net cash flows payable to each pension plan (the solid line).

For mutual fund markets, the relationship between liquidity and risk-adjusted rate of return was revealed by Idzorek et al. (2012), who investigated whether mutual funds that hold less liquid stocks tend to outperform those that hold more liquid stocks. They first grouped the population of mutual funds under consideration by size and valuation, and further categorized the funds in each group into five liquidity levels on the basis of the stock-level ‘turnover’ measure. It was found that, on average, mutual funds that held less liquid stocks possessed higher Sharpe ratios than those that held more liquid stocks.

For hedge fund markets, Getmansky et al. (2004) studied the potential relationship between liquidity and returns on hedge funds by developing an econometric model from which smoothing-adjusted Sharpe ratios were calculated. It was found that among 908 hedge funds from the TASS Hedge Fund Combined databases, the most illiquid hedge funds (e.g., fixed income directional) had the highest smoothing-adjusted Sharpe ratios, supporting the inverse relationship between liquidity and risk-adjusted required rate of return.

It has been argued that the market for longevity risk transfers has many similarities compared to a typical financial market (Loeys et al., 2007). Hence, it is reasonable to conjecture that the inverse relationship between liquidity and Sharpe ratio found in stock, mutual fund and hedge

fund markets also applies to the market for longevity risk transfers. If this conjecture holds, then our risk management framework would be more economical than the comparable entirely insurance-based methods, because it could transfer the trend risk at a lower cost. A reduced cost may encourage more pension plans to transfer their longevity risk exposures, thereby not only facilitating market growth but also strengthening the stability of the pension industry.

To focus on the design and execution of the proposed risk management methods, we have made no attempt to estimate the associated costs. It thus warrants a separate study to investigate how much the proposed risk management methods may cost. To determine the cost associated with the dynamic longevity hedge, one may replace q^f with a forward mortality rate that is derived from the pricing methods proposed by Chuang and Brckett (2014), Deng et al. (2012) and Li et al. (2011). As a reinsurance treaty, the customized surplus swap may be priced under the Solvency II framework. In particular, its profit margin may be calculated by multiplying the present value of the solvency capital requirements with the spread over risk-free rate which the reinsurer is required to earn on its equity (see Zhou et al., 2014). Also, to understand the value for money of our risk management framework, it would be interesting to compare the total cost required by the proposed risk management methods with that required by a full pension buy-out.⁹

As the proposed dynamic hedging strategy matches only the first partial derivatives (with respect to the only common stochastic factor K_t in the ACF model), it requires the hedger to hold only one hedging instrument at a time. This property may be seen as advantageous, because it helps the market to concentrate liquidity. In future research, one may extend the proposed dynamic hedging strategy to match also the higher order derivatives, and to investigate whether such an extension would lead to an improvement in hedge effectiveness that worths the dilution of liquidity arising from the use of additional instruments.

We acknowledge that the proposed hedging strategy takes no account of the correlation between $\zeta_t^{(H)}$ and $\zeta_t^{(R)}$. Thus, it may not have fully unleashed the potential of a dynamic index-based longevity hedge. Because both FL_t and $Q_t(t_0)$ are not functions of $\text{corr}(\zeta_t^{(H)}, \zeta_t^{(R)})$, it is not trivial to incorporate $\text{corr}(\zeta_t^{(H)}, \zeta_t^{(R)})$ into traditional Greek hedging methods that are based on the partial derivatives of FL_t and $Q_t(t_0)$. However, we may possibly incorporate the correlation between $\zeta_t^{(H)}$ and $\zeta_t^{(R)}$ by considering the generalized state-space hedging method (Liu and Li, 2014, 2015), in which the optimal hedging strategy depends on the correlation matrix of the random

⁹Mercer provides pension buy-out indexes, which track the estimated cost of a full pension buy-out in the US, the UK, Ireland and Canada over time.

innovations in the processes for the hidden states (i.e., the time-varying stochastic factors). This extension is left for future research.

The results of various robustness tests indicate that the effectiveness of the dynamic longevity hedge is reasonably robust relative to model risk and the parameters of the q-forwards used. They also offer some useful insights to market participants. For example, because the dynamic hedge still yields satisfactory hedging results even if the time-to-maturity of the q-forwards is only five years, the market may choose to launch shorter-dated q-forwards, which are more likely to attract capital market investors. As robustness is important in gaining trust from various stakeholders, we believe that future research warrants a more extensive analysis of robustness which considers additional aspects of the longevity hedge (e.g., hedge ratios) and additional factors that may affect hedge effectiveness (e.g., parameter risk).

In illustrating the customized surplus swap, mortality data from a group of distinct national populations are used. In reality, however, a reinsurer may possibly write customized surplus swaps with pension plans that are located in the same country, so it is also important to understand the diversifiability of residual risks across sub-populations with the similar geographical locations but different social-economic statuses. Such an understanding may be developed by considering the Club Vita data set of UK occupational pension schemes that was used by Haberman et al. (2014). We suggest revisiting the study of the customized surplus swap when the Club Vita data set becomes available from the public domain.

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With basis risk; $HE_{30} = 0.9032$

Without basis risk; $HE_{30} = 0.9997$

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