Characteristics of Reynolds shear stress in Adverse Pressure Gradient Turbulent Boundary Layers

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Abstract The focus of the present work is to characterize the features of the turbulent inertia term (the wall-normal gradient of Reynolds shear stress) through the mean momentum balance and the Reynolds shear stress correlation coefficient (ρ_{uv}). Effects of the Reynolds number and Clauser pressure-gradient parameter, β , are discussed. Large eddy simulations of low Reynolds number adverse pressure gradient turbulent boundary layers from Bobke et al. (2017), low Reynolds number experimental data from Vila et al. (2017) and Volino (2020), and newly acquired experimental data at higher Reynolds number from the Flow Physics Facility at The University of New Hampshire are utilized for this analysis. Observations are compared to zero pressure gradient turbulent boundary layer direct numerical simulations of Schlatter and Örlü (2010) and Sillero et al. (2013), and experimental data from Zimmerman et al. (2019) and Zimmerman (2019). These cases show that the correlation coefficient (ρ_{uv}) decreases in magnitude with increasing Reynolds number and β . However, from these initial observations we find that ρ_{uv} is more sensitive to changes in the Reynolds number in comparison to the examined range of β . We also find that the location of zero-crossing of the turbulent inertia term seems to scale with $\sqrt{\delta^+}$ while the minimum of ρ_{uv} scales with δ .

1 Introduction

The effects of pressure gradients on turbulent boundary layers are often parameterized in terms of the Clauser pressure-gradient parameter $\beta = (\delta^*/\tau_w)(dP/dx)$, where δ^* is the displacement thickness, τ_w is the wall shear stress, and dP/dx is the

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streamwise pressure gradient. Positive and negative values of β respectively reflect adverse pressure gradient (APG) and favorable pressure gradient conditions, while $\beta = 0$ reflects zero pressure gradient (ZPG) conditions. The Clauser parameter can also be characterized as a ratio of turbulence time scale $t_{turb} = \Delta/u_\tau$ to pressure gradient time scale $t_{PG} = 1/|dU_\infty/dx|$, where Δ is the Rotta-Clauser thickness and u_τ the friction velocity. For the APG flow to be in equilibrium, the ratio of two time-scales, i.e., $|\beta| = t_{turb}/t_{PG}$, has to be a constant. Therefore, under a non-constant β , the flow would take time to reach equilibrium, a history-effect would be present. This history-effect is anticipated to be different when β increases or decreases and also depends on how rapidly β is changing. In order to avoid these potentially confounding history-effects, this paper focuses on cases where β is nearly constant in numerical simulations taken from [1] as well as experimental cases where β is only increasing mildly with Reynolds number obtained from [2] and [3] and newly acquired data at the Flow Physics Facility of the University of New Hampshire.

1.1 Mean Momentum Balance

The mean momentum balance (MMB) is the time-averaged statement of Newton's second law. The MMB for the ZPG turbulent boundary layer (TBL) consists of three terms: viscous force (VF), turbulent inertia (TI), and mean inertia (MI):

$$\underbrace{\frac{\partial^2 U^+}{\partial y^{2+}}}_{\text{VF}} + \underbrace{\frac{\partial \overline{-uv}^+}{\partial y^+}}_{\text{TI}} + \underbrace{\left[-U^+ \frac{\partial U^+}{\partial x^+} - V^+ \frac{\partial U^+}{\partial y^+} \right]}_{\text{MI}} = 0. \quad (1)$$

In (1) x and y respectively represent the streamwise and wall-normal coordinate directions and the superscript '+' denotes normalization by the mean wall shear stress and the fluid kinematic viscosity, or 'viscous' scales. The VF term is the gradient of the viscous stress and the TI term is the gradient of the Reynolds shear stress. The zero-crossing of the TI term corresponds to the peak location of the Reynolds shear stress. Prior to the zero-crossing, the TI term acts as a source of momentum (i.e., $\text{TI} > 0$), whereas after the zero-crossing the TI term becomes negative and acts as a momentum sink. These source and sink regions balance to zero, (i.e. it must integrate to zero across the boundary layer), which implies that the TI term is merely redistributive. The MI term reflects boundary layer development, and so when cast as a force represents a source of momentum. The MMB for the APG TBL includes a fourth term, the pressure gradient (PG) term, which is constant and negative:

$$\underbrace{\frac{\partial^2 U^+}{\partial y^{2+}}}_{\text{VF}} + \underbrace{\frac{\partial \overline{-uv}^+}{\partial y^+}}_{\text{TI}} + \underbrace{\left[-U^+ \frac{\partial U^+}{\partial x^+} - V^+ \frac{\partial U^+}{\partial y^+} \right]}_{\text{MI}} + \underbrace{U_\infty^+ \frac{\partial U_\infty^+}{\partial x^+}}_{\text{PG}} = 0. \quad (2)$$

As an example, figure 1 shows the distribution of the different terms in (1) and (2). Changes in the MMB indicate structural changes in the APG TBL flow from the ZPG TBL flow. In (1) the effect of turbulence is represented by the Reynolds shear stress $\overline{uv}$ via the TI term. The focus of the present work is to understand changes

in the TI term through the Reynolds shear stress $\overline{uv}$ and the associated Reynolds shear stress correlation coefficient $\rho_{uv} = \overline{uv} / (u_{rms} v_{rms})$, where u_{rms} and v_{rms} are streamwise and wall-normal fluctuating r.m.s. velocities. The Reynolds shear stress correlation coefficient represents an “efficiency” of turbulent momentum transport while the Reynolds shear stress itself may be affected by the available fluctuation energy (as represented by u_{rms} and v_{rms}). We want to know if the larger changes in the Reynolds stresses due to Re_τ (the friction Reynolds number) and β are accompanied by a change to this efficiency.

Fig. 1 (a): MMB of ZPG TBL at $Re_\tau = 490$, from [5]. (b): MMB of APG TBL at $Re_\tau = 490$ at $\beta = 1$, from [1].

1.2 Experimental and Computational Data Summary

The APG cases are compared to canonical ZPG TBL data to provide context for the APG TBL analysis. A summary of the computational and experimental data is given in the table below.

Type of flow	Type of Data	Source of Data	Data Symbol	Re_τ	β
ZPG TBL	DNS	[5]	—	250 - 1300	0
ZPG TBL	DNS	[6]	—	1300 - 2000	0
ZPG TBL	Experiment	[3]	▷	1800	0
ZPG TBL	Experiment	[7] and [8]	○	3300 - 10000	0
APG TBL	LES	[1]	—	360-830	0.9 - 4.5
APG TBL	Experiment	[3]	▷	1800	1.1
APG TBL	Experiment	[2]	*	1900	1.3-2.4
APG TBL	Experiment	recent experiments	○	7100	1.1

Table 1 Summary of data sets and corresponding symbols.

2 Results

The effects of Reynolds number and β on the peak location of the Reynolds shear stress, i.e. the zero-crossing of the TI term of the MMB, and the minimum of ρ_{uv} are discussed in the following sections.

2.1 ZPG TBL for increasing Reynolds number

Before moving to the APG cases, the ZPG case is observed over a range of Reynolds numbers in figure 2. As the Reynolds number increases ρ_{uv} decreases in magnitude; see figure 2 (left). This decrease in magnitude reflects the increasing intensity of the u and v fluctuations in the outer region while $-\overline{uv}^+$ gets closer to its theoretical maximum value of 1 [4]. The peak of $-\overline{uv}^+$ and minimum of $-\rho_{uv}$ are marked by $\diamond$ and $\diamond$ symbols, respectively. As can be seen in figure 2 (right) the peak in ρ_{uv} scales nominally with δ , which is why it appears to move outward when plotted relative to y^+ or $y^+/\sqrt{\delta^+}$. The maximum of $-\overline{uv}$ however, seems to scale with $\sqrt{\delta^+}$. Note that $\delta^+ := \delta u_\tau/\nu =: Re_\tau$, and we interchangeably use δ^+ and Re_τ . There is more scatter for the high Reynolds number markers due to the difficulty of identifying the peak locations within the experimental data. This scaling is well-established for ZPG TBLs despite the scatter in the present experimental data.

Fig. 2 (left): $-\overline{uv}^+$ and $-\rho_{uv}$ at $\beta = 0$. Reynolds number increasing from dark to light colors. $250 < Re_\tau < 2000$, from Refs. [5] and [6] plotted with — lines. $3300 < Re_\tau < 10000$ from Refs. [7] and [8] plotted with $\circ$ symbols. Maximum of $-\overline{uv}^+$ and minimum of $-\rho_{uv}$ marked by $\diamond$ and $\diamond$ symbols, respectively. (right): The corresponding wall-normal locations plotted versus Reynolds number.

2.2 ZPG TBL and APG TBL at matched Reynolds number

A ZPG case is compared to several APG cases at matched Reynolds number, $Re_\tau = 490$, for $0.99 < \beta < 4.5$ in figure 3 (left). Note that when comparing data sets at the same Re_τ one cannot determine whether certain features occur at fixed y^+ , $y^+/\sqrt{\delta^+}$, or y/δ . One clear feature that distinguishes the Reynolds shear stress of the APG cases from the ZPG case is the emergence of a peak that exceeds unity, i.e. $-\overline{uv}^+ > 1$. The peak of $-\overline{uv}^+$ is marked by $\diamond$ symbols. The increased peak magnitude is enabled by the presence of the pressure gradient; the maximum Reynolds shear stress is not capped by the maximum viscous stress (i.e. u_τ^2) as in the ZPG case. Surprisingly, there is relatively little change in ρ_{uv} at the outer minimum, marked by $\diamond$ symbols, despite the emergence of outer peaks in u_{rms}^+ and v_{rms}^+ with increasing β . The insensitivity of this ‘efficiency’ to β in the outer region suggests that the observed outer peaks in u_{rms}^+ and v_{rms}^+ are more reflective of changes of the scaling factor

u_τ than of fundamental changes to the local turbulence. The wall-normal location of the outer minimum of ρ_{uv} remains fairly constant with increasing β , as seen in figure 3 (right) for $Re_\tau = 490$ in diamond symbols. Over the range of positive β examined, the peak of $-\overline{uv}^+$ remains nominally constant. However, the peak of the APG cases has shifted outwards from the ZPG case. This indicates that the TI term has remained positive further away from the wall and is acting as an additional source of momentum in the APG cases.

Fig. 3 (left): $-\overline{uv}^+$ and ρ_{uv} . ZPG TBL and APG TBL at constant Reynolds number, $Re_\tau = 490$, from Refs. [5] and [1], respectively. Maximum of $-\overline{uv}^+$ and ρ_{uv} marked by $\diamond$ and $\diamond$ symbols, respectively. (right): The corresponding wall-normal locations plotted versus β for $360 \leq Re_\tau \leq 830$. For $Re_\tau = 1900$: ZPG TBL from Ref. [6] and APG TBL from Ref. [2].

2.3 ZPG TBL and APG TBL: Effects of β and Re_τ

ZPG cases are compared to APG cases for $\beta \approx 1$. The corresponding $\overline{uv}^+$ in figures 4 (b, d) indicate that the peak location of $\overline{uv}^+$ maintains $\sqrt{\delta^+}$ scaling for both the ZPG and APG cases. Note that the outer minimum of ρ_{uv} remains at nominally the same wall-normal location when normalized by δ in figure 4 (c). In figures 4 (a, c) there is minimal change between the ZPG and APG $-\rho_{uv}$ profiles at matched Reynolds number as β increases from 0 to ≈ 1 . In comparison, both the ZPG and APG ρ_{uv} curves decrease in magnitude as the Reynolds number increases. It appears that ρ_{uv} is more sensitive to changes in Reynolds number than the change from $\beta = 0$ to $\beta \approx 1$.

3 Conclusion

We find that ρ_{uv} is more sensitive to changes in Reynolds number in comparison to the examined range of β . Overall there is little change in the ‘efficiency’ of turbulent momentum transport as characterized by ρ_{uv} , for increases in β . It is reasoned that the effects of β diminish as Reynolds number increases, i.e., the same effect requires a larger β . Furthermore, the location of the zero-crossing of the TI term scales with $\sqrt{\delta^+}$ while the minimum of the Reynolds shear stress correlation coefficient scales with δ .

Fig. 4 (a, c) $-\rho_{uv}$ of $\beta = 0$ and $\beta \approx 1$ for increasing Reynolds number. (b, d): $-u^+$ of $\beta = 0$ and $\beta \approx 1$ for increasing Reynolds number. $Re_\tau = 490$: $\beta = 0$ and $\beta = 1.0$ plotted with — and — lines from [5] and [1], respectively. $Re_\tau = 1800$: $\beta = 0$ and $\beta = 1.1$ plotted with $\blacktriangleright$ and $\blacktriangleleft$ symbols from [3]. $Re_\tau = 7800$: $\beta = 0$ and $Re_\tau = 7100$: $\beta = 1.1$ plotted with $\circ$ and $\bullet$ symbols from [8] and recent experiments, respectively.

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