Title: The Dissolution of a Stationary Spherical Bubble Beneath a Flat Plate

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Abstract

The dissolution of a single stationary bubble held in place by a horizontal plate is commonly observed experimentally. For several decades the standard approach to the analysis of such dissolution data has been to apply a correction factor of $\ln(2)=0.69$ to the Epstein-Plesset equation for an isolated bubble. In this paper, the transport equations for a stationary bubble touching a plate are solved numerically for the common case where the flow field caused by the change in system volume as the bubble dissolves can be neglected. It is found that the total bubble lifetime is not well characterised by the use of the $\ln(2)$ factor. However, in most experimental situations, the initial stages of bubble dissolution are not captured. For low gas solubilities the use of a correction factor of 0.69 to the Epstein-Plesset equation is appropriate once the initial transients have dissipated. The correction factor varies from 0.69 to 0.77 across the full range of situations described in this paper. The mathematical model is validated by comparison to experimental data.

Keywords: single bubble, mass transfer, dissolution, diffusion coefficient

Introduction

The dissolution of stationary bubbles has been widely investigated in the past for commercial, biological and fundamental interests such as removal of bubbles in glass (Doremus, 1960; Greene & Gaffney, 1959) and polymer melts (Favelukis, Tadmor, & Talmon, 1995; Kontopoulou & Vlachopoulos, 1999) and in the determination of the diffusion coefficient of gases in liquids (Houghton, Ritchie & Thomson, 1962; Krieger et al., 1967; Wise & Houghton, 1966). Most recently, lipid, protein or surfactant coated microbubbles are being utilised as ultrasound contrast agents (Christiansen & Lindner, 2005), drug and gene delivery vehicles (Bekeredjian et al., 2005) and blood substitutes (Van Liew & Burkard, 1995), and an understanding of the behaviour of these coated microbubbles is becoming important.

The dissolution of a bubble involves solution of gas into the liquid at the interface, and diffusion of dissolved gas away from the interface into the bulk liquid. It is assumed that the former is fast compared to the latter, hence the dissolution process is assumed to be diffusion controlled. Several workers (Frade, 1983; Duda and Vrentas, 1971) have numerically solved the fundamental problem of an isolated spherical bubble of initial radius R_0 with the following assumptions:

- a) The bubble contains a uniform and constant gas density. This assumption implies that there is a negligible effect of the changing Laplace pressure on the gas density as the bubble shrinks. This effect is found to be important only when the bubble radius becomes very small and surface tension forces are large.
- b) Transfer of gas from the bubble is controlled by Fickian diffusion (dissolution of gas into the liquid is fast) and the concentration of dissolved gas in the liquid at the interface is constant at the equilibrium concentration C_E .

- c) The diffusivity, temperature and pressure of the system are constant.
- d) The partial specific volumes of solute (v) and solvent are constant but not necessarily equal.
- e) The velocity field in the liquid is purely radial, and the bubble is a perfect isolated sphere so that the concentration field is spherically symmetrical.

Other workers have developed more sophisticated numerical solutions for isolated bubbles that incorporate the effects of multicomponent solute species (Yung et al. 1989, Cable and Frade 1987) and surface tension (Cable & Frade, 1988).

The governing equations are those of advection-diffusion of solute in the liquid together with a kinematic condition at the moving bubble surface that describes the change in bubble radius in terms of the diffusion flux there. Duda and Vrentas (1969, 1971) expressed the problem in a dimensionless form containing the two non-dimensional parameters

$$N_a = \frac{C_E - C_0}{\rho_g(1 - vC_E)} \quad (1)$$

and

$$N_b = \frac{v(C_E - C_0)}{1 - vC_E} \quad (2)$$

The dimensionless parameter N_a indicates the solubility of the solute in the fluid whereas:

$$\varepsilon = N_a - N_b = N_a(1 - v\rho_g) \quad (3)$$

indicates the importance (relative to diffusion) of advection of solute due to the liquid velocity field which results from the volume change of the system as the bubble dissolves.

By neglecting the advection term, Epstein and Plesset (1950) derived an approximate analytical solution for a stationary bubble which yielded the following equation for the dimensionless bubble radius a in terms of dimensionless time t^*

$$\frac{da}{dt^*} = -N_a \left(\frac{1}{a} + \frac{1}{(\pi t^*)^{1/2}} \right) \quad (4)$$

The $1/(\pi t^*)$ term in Equation 4 can be ignored for large dissolution times, thus allowing the equation to be solved with an approximate analytical solution:

$$a^2 = 1 - 2N_a t^* \quad (5)$$

Other workers (Bankoff, 1964; Krikaldy, 1956) discuss a more sophisticated quasi-steady state solution, valid for small values of N_a and long times given by

$$a^2 = 1 - \frac{2N_a t^* \ln(1 + N_b)}{N_b} \quad (6)$$

This equation approaches Equation 5 for small N_b .

Other studies (Liebermann, 1957; Houghton et al., 1962; Wise & Houghton, 1966; Wigman, Evans & Galvin, 2001, Duncan and Needham, 2004) consider the dissolution of a single bubble that is held in place beneath a stationary flat plate, and attempt to account for this by modifying the Epstein-Plesset solution (Equation 5) for an isolated bubble. Liebermann (1957) introduced a correction factor of $\ln(2) = 0.693$ into the Epstein-Plesset solution. This factor was taken from the theory of electrostatics where the capacitance of a conducting sphere drops by a factor of $\ln(2)$

when it is placed next to an ungrounded infinite plane. Thus in the presence of a flat plate, Equation 5 becomes

$$a^2 = 1 - 2\ln(2)N_a t^* \quad (7)$$

Wise and Houghton (1968) confirmed this correction factor numerically for a point contact using potential theory assuming quasi-steady diffusion. However, both Manley (1960) and Wise and Houghton (1966, 1968) comment that the bubble is likely to have a finite area of contact with the plate. This will both reduce the volume of the bubble below that of a perfect sphere and further disrupt the concentration distribution around the bubble. For an air bubble resting below a Plexiglas plate, Wise and Houghton (1966) suggest that the bubble volume be reduced by a factor of 0.947 and the concentration field by a geometry factor of 0.650, giving an overall correction factor of 0.686.

Cable (1967) points out that the $\ln(2)$ correction factor will be valid only for low solubilities when the concentration boundary layer is essentially infinite in size. This will only occur for very small values of N_a . As the solubility increases, the shell of liquid around the bubble in which diffusion occurs (i.e. the boundary layer) becomes thinner, and the plate “cuts off” a smaller fraction of the spherical shell (see Figure 1). In the limit of infinite solubility the diffusion boundary layer is so thin that it does not intersect the plate and the diffusion behavior becomes that of an isolated bubble. Thus the correction factor should move from 0.693 towards 1.0 as the solubility increases, provided the effect of liquid velocity remains small.

Takemura et al. (1996) appear to be the only authors to have considered these wall effects through numerical modeling. These authors find that the correction factor does indeed increase as the

solubility increases. In the absence of natural convection, they find that the correction factor to the Epstein-Plesset equation moves from 0.72 to around 1.3 as N_a (their l/b) moves from 0.001 to 1.0. However, the kinematic condition of the bubble interface used in this model is somewhat simplistic and this may have affected the results at the upper end of this range.

It is apparent from the above account that a complete mathematical understanding of the influence of a flat retaining plate on bubble dissolution is lacking in the literature. The aim of this paper is to present a finite difference numerical model of bubble dissolution that explicitly accounts for the presence of the boundary condition on the flat plate, and to compare the model predictions with experimental data and with predictions of existing models.

Numerical Model

This model assumes that the bubble makes a point contact with the wall (horizontal plate). Like many studies of isolated bubbles (Epstein & Plesset 1950, Houghton et al. 1962, Krieger et al. (1967), Subramanian and Weinberg, 1981) this work assumes that the effect of liquid velocity can be ignored ($\varepsilon = 0$), and that the bubble is small enough to remain spherical. However, the concentration distribution is no longer spherically symmetric because of the presence of the wall. Rather than solve for the concentration in the physical domain with a moving boundary (the bubble surface), it is convenient instead to solve in a transformed domain that is square with fixed boundaries. A suitable coordinate transformation for a point P in Figure 2a is given by

$$\eta = 1 - \exp(-\beta(r - R(t))) \quad (8)$$

$$\mu = \cos \theta = x / (x^2 + \sigma^2)^{1/2} \quad (9)$$

Equation 8 was first proposed by Duda and Vrentas(1969) for conditions of spherical symmetry (an isolated bubble) and Equation 9 results from simple geometry considerations.

Figure 2 illustrates this transformation from the axi-symmetric physical domain in cylindrical coordinates (σ, x) . The right quadrant of the (σ, x) generating plane extending from the moving bubble surface to infinity is mapped onto a unit square in the (η, μ) plane. The primes in Figure 2 denote transformed points. For example, the points B and C at infinity are transformed to points B' and C' at finite locations in the (η, μ) plane.

When the transport by the liquid velocity can be ignored, the axi-symmetric evolution of concentration in the liquid is given in cylindrical coordinates by

$$\frac{\partial C}{\partial t} = D \left(\frac{1}{\sigma} \frac{\partial}{\partial \sigma} \left(\sigma \frac{\partial C}{\partial \sigma} \right) + \frac{\partial^2 C}{\partial x^2} \right) \quad (10)$$

where the bubble motion is determined by the diffusion flux integrated over the bubble surface S as follows:

$$\rho_g (1 - \nu C_E) 4\pi R^2 \frac{dR}{dt} = D \iint_{r=R(t)} \frac{\partial C}{\partial r} dS \quad (11)$$

The form of the left side of Equation 11 can be found in Duda and Vrentas (1969) for example.

We set $C = C_0$ initially, $C = C_E$ at the bubble surface, and $C = C_0$ at infinity.

The equations corresponding to (10) and (11) in transformed coordinates can be shown to be

$$\begin{aligned} \frac{\partial C}{\partial t} - \beta(1-\eta) \left(R' \left(\frac{X}{r} + 1 \right) + D \left(\frac{2}{r} - \beta \right) \right) \frac{\partial C}{\partial \eta} + \frac{2D\mu}{Y^2} \frac{\partial C}{\partial \mu} \\ = D\beta^2(1-\eta)^2 \frac{\partial^2 C}{\partial \eta^2} + \frac{D(1-\mu^2)}{Y^2} \frac{\partial^2 C}{\partial \mu^2} - \frac{2D\beta R(1-\mu^2)(1-\eta)}{rY} \frac{\partial^2 C}{\partial \mu \partial \eta} \end{aligned} \quad (12)$$

$$\frac{dR}{dt} = \frac{2D\beta}{\rho_g(1-\nu C_E)} \int_0^1 \mu \left. \frac{\partial C}{\partial \eta} \right|_{\eta=0} d\mu \quad (13)$$

where $R' = \frac{dR}{dt}$, $X = x - R$, and $Y = (x^2 + \sigma^2)^{1/2}$. In terms of η and μ , the variables r , X , and Y are

$$r = R - \frac{1}{\beta} \ln(1-\eta) \quad (14)$$

$$X = \mu(r^2 - R^2(1-\mu^2))^{1/2} - R(1-\mu^2) \quad (15)$$

$$Y = (r^2 - R^2(1-\mu^2))^{1/2} + R\mu. \quad (16)$$

The boundary conditions for $C(\eta, \mu, t)$ become

$$C(0, \mu, t) = C_E \quad (\text{bubble surface}) \quad (17)$$

$$C(1, \mu, t) = C_0 \quad (\text{infinite distance from the bubble}) \quad (18)$$

$$\frac{\partial C}{\partial \mu} = \beta(r^2 - R^2)^{1/2} (1-\eta) \frac{R}{r} \frac{\partial C}{\partial \eta} \quad \text{at } \mu = 0 \text{ (zero flux across the wall)} \quad (19)$$

$$\begin{aligned} \frac{\partial C}{\partial t} = & \left(D \left(\frac{2}{r} - \beta \right) + 2R' \right) \beta(1-\eta) \frac{\partial C}{\partial \eta} - \frac{2D}{(R+r)^2} \frac{\partial C}{\partial \mu} + D\beta^2(1-\eta)^2 \frac{\partial^2 C}{\partial \eta^2} \\ & \text{at } \mu = 1 \text{ (symmetry at } \sigma = 0) \end{aligned} \quad (20)$$

The initial condition is

$$C(\eta, \mu, 0) = C_0 \quad (21)$$

We solve a non-dimensional form of the equations in terms of dimensionless quantities

$F = (C - C_0)/(C_E - C_0)$, $t^* = Dt/R_0^2$, $a = R/R_0$, $X^* = X/R_0$, $Y^* = Y/R_0$ and $r^* = r/R_0$. Equation

(11) for the bubble surface motion becomes

$$\frac{da}{dt^*} = 2\beta^* N_a \int_0^1 \mu \left. \frac{\partial F}{\partial \eta} \right|_{\eta=0} d\mu \quad (22)$$

Time integration was performed numerically using an explicit Euler scheme with second order spatial differencing. As required for numerical stability of such a system, the time step was chosen to be small enough to satisfy the diffusion time step limitation and the Courant condition (see e.g. Fletcher, 1991).

Experimental Techniques

A rectangular cell of dimensions 0.04 m x 0.04 m x 0.1 m was designed and constructed to study the dissolution rate of a single stationary bubble in an aqueous solution, pictured in Figure 3. The cell is made of stainless steel material, with rubber seals to prevent any leakage of gas and liquid into and out of the cell. A removable lid allows liquid to be transferred into the cell, this also allows easy cleaning of the cell interior. A stainless steel frame that supports a glass plate is positioned at the centre of the cell. A 26 gauge needle with a bevel edge is inserted through a septum on the side of the cell wall with the tip positioned directly beneath the glass plate.

The glass plate was chemically cleaned before each experiment to remove any contaminants on the surface. This also makes the surface hydrophilic and allows the bubble resting underneath the glass to remain spherical. To clean the glass plate, it was immersed in a mixture of approximately 50 vol % hydrogen peroxide and 50 vol % purified water (biofiltered and de-ionised to conductivity $< 10^{-6} \text{ S cm}^{-1}$) with a few drops of ammonia solution and heated until the solution began to boil. The solution was then removed from the heat until the bubbling ceased. Once cooled, the glass plate was rinsed thoroughly with similarly purified water.

Purified water was degassed using a Javac Vacuum pump and stored in an airtight jar overnight. The liquid was then carefully transferred to the cell and the concentration of dissolved air in the liquid was measured by measuring the dissolved oxygen content with an YSI 5739 Field Probe, with a stated accuracy of 0.2 %, and converting the concentration to percentage of air saturation. The dissolved air concentrations of 26 %, 43 %, 35 % and 54 % saturation was reached by leaving the system open to the atmosphere and monitoring the dissolved oxygen concentration until the desired concentration was reached. The dissolved oxygen concentration was measured before and after the experiment to ensure that the dissolved gas concentration in the bulk liquid remained constant. Because the dissolved gas concentration was measured before the lid was placed, it was possible for gas to dissolve while the lid was being fitted and this sometimes led to an increase in the measured gas concentration at the end of the experiment. Consequently, a number of runs were conducted and those with a concentration change above 2 % were deleted. At least three runs were retained at each concentration level. The temperature was not controlled, but monitored and found to be 21.5 ± 1.5 °C.

At the beginning of each experiment an air bubble was generated at the tip of the needle and released. The bubble rose to the plate and was allowed to settle and become stationary over the ensuing one to two minutes. Data recording commenced 1 to 5 minutes after bubble generation with a bubble radius of 0.030 ± 0.005 cm. The image of the bubble was captured using a SciTech CCD 100 camera and an image grabber program from a PC was used to capture the bubble at specified time intervals until the bubble was no longer visible. A sequence of images captured in this manner is shown as Figure 4. After the dissolution of a bubble the solution was changed to a fresh sample. The bubble size was measured from the captured image using a commercial word processing package by drawing a rectangle around it. Two diameters were determined from the

rectangle dimensions and these were averaged. The average error between the two dimensions was 0.00024cm.

Results/discussion

The model equations were solved numerically on a 120×120 grid in the (η, μ) plane. For the calculations shown the transformation parameter $\beta^* = 0.5$. This gave a suitable non-uniform distribution of computational mesh cells in the physical domain with a high mesh density near the bubble surface where the concentration gradients are highest, and much lower mesh density at large distances from the bubble where the gradients are smaller. Doubling the number of cells in each direction in the (η, μ) plane to form a 240×240 grid produced a negligible difference in the predictions. A contour plot showing the concentration field around the dissolving bubble when it is at around 50% of its original radius is shown in Figure 5.

Results for a range of N_a values are provided in Table 1. The values in the range $0.0092 < N_a < 0.015$ correspond to an air bubble dissolving in water at 22°C at saturation levels of 54 to 26%. It is increasingly unrealistic to ignore convective effects as N_a increases outside this range. Hence no calculation results are shown for $N_a > 0.1$. For comparison, the Epstein-Plesset approximate solution (Equation 5) is also given. Further, the equations for an isolated bubble, as presented by Duda and Vrentas (1971), were also solved without advection ($\varepsilon = 0$). For these latter equations, the grid size was set to 200 in the radial direction.

As shown in Table 1, the total bubble lifetime in the presence of a wall can be estimated from the simple Epstein-Plesset Equation if the dimensionless time is divided by a correction factor of between 0.76 and 0.77 for the range $0.0092 < N_a < 0.015$. The results are consistent with the value of 0.762 provided by Takemura et al. (1996) (including advection but not natural convection) for $N_a = 0.01$. As expected, the correction factor increases slightly as N_a increases. Similarly, the effect of a boundary wall can be estimated from the numerical solution of the Duda and Vrentas (1971) equations if the dimensionless time is divided by a correction factor of 0.72. Complete bubble dissolution curves for $N_a = 0.0092$ are given in Figure 6. As expected the present model with the wall boundary condition predicts significantly longer dissolution times than the other models representative of an isolated bubble. Furthermore, when the predictions from the present model are multiplied by a factor of 0.722 they are seen to closely approach the values predicted by the numerical model of Duda and Vrentas (1971) for an isolated bubble (they are essentially coincident), as is expected from the discussion above.

The above comparison of the Duda and Vrentas (1971) solution for an isolated bubble and the present solution for a bubble touching a wall shows that the appropriate correction factor to the dimensionless time which converts one prediction to the other is 0.72 since both solutions are based on the full equations (without advection). If this factor, instead of 0.76-0.77 (Table 1), was applied to the Epstein-Plesset model, it would overpredict the dissolution time. Note that for very short times, Figure 6 shows that the dissolution rate of a bubble near a boundary wall is actually lower than that predicted by the Epstein- Plesset model for an isolated bubble. This is a consequence of the approximate nature of the Epstein- Plesset model.

The predicted correction factor (0.76-0.77) required to account for the wall using the simple Epstein- Plesset formula is higher than the historical value of $\ln(2) = 0.693$. This is because the full effect of the initial transients is included in our estimate. However, in many experimental situations, these transients are not observed since there is a finite time between bubble creation and the commencement of data recording. For bubble radii of less than around 80% of the initial starting value, a plot of a^2 versus time is essentially linear, consistent with the Epstein-Plesset formulation. In Table 2 and Figure 7, the predictions are reformulated and presented again, but with time = 0 set when the bubble radius is at 80% of its original value. It is clear that there is now a much greater consistency between the Epstein-Plesset prediction and solution of the full equations over this time period for N_a values typical of the air-water system. Table 2 shows that the Epstein-Plesset correction factors are also much closer to the historical value of $\ln(2)= 0.693$.

It is common to present experimental data in the form of a^2 versus t^* as this provides a linear relationship for the Epstein-Plesset Equation (Equation 5). The experimental data as well as the theoretical curve using the present model is illustrated in Figure 8(a) to (d). The experimental data presented on each curve represents the results of at least three separate experiments. A value of $1.75 \times 10^{-9} \text{ m}^2/\text{s}$ for the diffusion coefficient provides the best fit to the data, within the range of values quoted in the literature for diffusion of air in water at ambient temperature (from 1.75 to $2.0 \times 10^{-9} \text{ m}^2/\text{s}$ (Himmelblau 1964)). With this model, it is necessary to specifically allow for the time delay between bubble formation and data capture. While this delay was not accurately recorded it corresponds to a relatively short dimensionless time of around 2 and so this value was used. Also shown is the Epstein-Plesset Equation, corrected by factors as provided in Table 1.

The experimental data displays some curvature, indicative of non-linear transient behaviour. It is evident that the full model presented in this work provides this curvature and gives an excellent fit to this data. While the Epstein-Plesset equation is linear, this equation also provides a very good fit to the data, if an appropriate correction factor is used.

There appears to be a slight trend in the data with respect to air saturation, with the data at 54% saturation showing longer dissolution times while that at 26 % saturation gives shorter dissolution times relative to the model predictions. We considered a number of possibilities for this discrepancy. Firstly, the surface tension will tend to increase the equilibrium saturation at the bubble radius, C_E , as the bubble shrinks due to the Laplace pressure. Cable and Frade (1988) present a quasi-steady state solution to the bubble dissolution problem that accounts for these surface tension effects that is valid for $N_a < 0.01$. Use of this solution under the present experimental conditions provides only a very slight correction to the Epstein-Plesset model at small bubble radii indicating that surface tension effects are minimal under these conditions.

Secondly, the mathematical analysis given here assumes a single component gas whereas air is clearly a multicomponent system. This would be a significant issue if the initial aqueous solution was prepared by adding different proportions of each component than would be expected in a saturated solution (see Cable and Frade 1987, Yung, de Witt & Brockwell, 1989). However, our aqueous solutions were prepared by the addition of air to an evacuated solution. As the diffusivities of the two major components (O_2 and N_2) are comparable, the amount that will enter the liquid phase during this solution preparation is predominantly a direct function of their saturation solubility. Thus a solution that is '54% saturated' will contain around 54% of the saturation solubility of oxygen and 54% of the saturation solubility of nitrogen. Consequently, N_a is also

unaffected. The effect of water vapor within the bubble was also considered (Yung, de Witt & Brockwell, 1989) but this would again cause only very minor deviations.

Takemura et al. (1996) argue that under experimental conditions such as those described here, natural convection effects will lead to more rapid dissolution of a bubble (5 to 12% faster) under a plate than predicted by the present model. These authors implicitly assume a Boussinesq approximation of buoyancy with a coefficient:

$$\alpha = -\frac{1}{\rho_l} \frac{d\rho_l}{dC} = 1 \quad (23)$$

However, this overstates the effect of natural convection as it fails to include specific physical property information (such as the specific volume of the solute) in the calculation of α . For the present work, $\alpha = 0.3$ based on a partial specific volume of air, $v = 0.0013 \text{ m}^3/\text{kg}$ indicating that natural convection makes only minor changes (1 – 3%) to the model predictions.

Finally, while the oxygen field probe has an accuracy of 0.2% there was no practical means of confirming the precision of this meter. Slight calibration errors could readily account for the current model deviations. For example, adjustment to the saturation level from 54% to 57% would allow for the best possible fit between the experimental data and the model at this saturation level.

Conclusion

A mathematical model for the dissolution of a stationary spherical bubble below a flat plate has been developed. Analysis of the dissolution curves provided by this model suggests that the $\ln(2)$ correction factor commonly used to correct for the wall boundary condition is truly valid only for a limited range of conditions, namely at long times and low gas solubilities. At short times and for

higher gas solubilities the correction factor diverges from this value. When a full numerical solution to the isolated bubble problem is available, then for an air bubble in water, the total bubble lifetime can be predicted by this solution if the dimensionless time is increased by division by 0.72 (0.716 - 0.723 depending upon gas saturation levels and the time lag between bubble generation and data recording). Conversely, if the data is to be fitted to the simpler Epstein-Plesset equation, the correction factor can vary between 0.695 and 0.773 depending upon the same factors of gas saturation and lag time.

The mathematical model has been fitted to experimental data of a bubble dissolving beneath a plate. The model provides an extremely good fit to these experimental results. There is a slight trend with air saturation level that cannot be explained by the model. We conclude that this results primarily from errors in calibration of the dissolved oxygen probe, with natural convection also a minor contributing factor.

The present model was developed to describe experimental data obtained with air bubbles of 300 micron size making a point contact with a wall. Applications involving situations of higher gas solubility requires the development of more sophisticated models that consider advection effects. Bubbles much smaller in size would require consideration of surface tension effects. Similarly, bubbles any larger than those used here, or conditions of lower air saturation in the bulk fluid would require additional consideration of natural convection effects.

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Notation

a	dimensionless bubble radius = R/R_0	
C	The concentration of the dissolved gas in the liquid	kg/m^3
F	Dimensionless concentration $((C-C_0)/(C_E-C_0))$	
C_0	The gas concentration in the bulk liquid	kg/m^3
C_E	Gas concentration at saturation	kg/m^3
D	Diffusion coefficient of the dissolved gas	m^2/s
N_a	Dimensionless variable defined by Equation 1	
N_b	Dimensionless variable defined by Equation 2	
r	radial co-ordinate	
R	Radius of the bubble at time t	m
R_0	Initial radius at time zero	m
r^*	dimensionless radial co-ordinate = r/R_0	
t	Time	s
t^*	Dimensionless time = Dt/R_0^2	
x	Axial co-ordinate in the physical domain	m
X	$x - R$	m
X^*	X/R_0	
Y	$(x^2 + \sigma^2)^{1/2}$	m
Y^*	Y/R_0	
Greek Letters		
α	$\alpha = -\frac{1}{\rho_l} \frac{d\rho_l}{dC}$	
β	transformation parameter	m^{-1}
β^*	βR_0	
ε	dimensionless parameter defined by Equation 3	
η	co-ordinate in the transformed domain	
μ	co-ordinate in the transformed domain	
ν	partial specific volume of solute	m^3/kg
ρ_g	Density of the gas bubble	kg/m^3
ρ_l	Density of the bulk liquid	kg/m^3
σ	cylindrical radial co-ordinate in the physical domain	m

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Figure 1 –Schematic showing how the size of the concentration boundary layer influences the dissolution of a bubble at a plate. (a)At low N_a , approximately one half of the boundary layer is cut off by the plate. (b)At high N_a a much smaller fraction of the boundary layer is disturbed.

Figure 2: A schematic showing the transformation of the symmetric quadrant of the physical domain (a) into a square computational domain (b)

Figure 3: Experimental Cell

Figure 4 – Images of a bubble dissolving within the experimental cell. The last image shows the injection needle used to calibrate distance within the cell environment.

Figure 5 - Image of the concentration field around a dissolving bubble with a wall boundary condition. $N_a = 0.0092$, grid size = 120. Dimensionless time $t^* = 50$ and dimensionless radius $a = 0.51$. The outermost contour shown is for $F = 0.1$ with each contour representing a decrease in concentration of 0.05 through to the bubble surface at $F=1.0$.

Figure 6 – Dissolution of an air bubble in water of air saturation level 54% ($N_a=0.0092$).

Figure 7 – Dissolution of an air bubble in water of air saturation level 54% ($N_a=0.0092$) from 80% of its original size.

Figure 8 – Experimental data of dimensionless bubble size squared (a^2) versus t^* and model curves

based on both the present model and a corrected Epstein-Plesset equation (assuming $D =$

$1.75 \times 10^{-9} \text{ m}^2/\text{s}$ and that data recording commences after 2 dimensionless time units).

- (a) at 54% air saturation level ($N_a = 0.0092$) (b) at 43% air saturation ($N_a = 0.0114$) (c) at 35% air saturation ($N_a = 0.0130$) and (d) at 26% air saturation ($N_a = 0.0148$)

Table 1 - Dimensionless time t^* for total dissolution of a bubble as calculated for a range of mathematical models. Also shown are the correction factors determined from the present model. Dividing the total time calculated by either the Epstein-Plesset(1950) or the Duda and Vrentas(1971) model by this factor accounts for the boundary wall condition.

N_a	Total Dissolution Time			Correction Factor:	
	Epstein – Plesset (Eqn 5)	Duda & Vrentas(1971) $\varepsilon = 0$	Present Model	To Epstein- Plesset (Eqn 5)	To Duda & Vrentas(1971) $\varepsilon = 0$
0.0092	54.2	51.5	71.3	0.761	0.722
0.01	50	47.4	65.6	0.762	0.723
0.015	33.8	31.6	43.7	0.773	0.723
0.1	5.0	4.29	5.87	0.852	0.731

Table 2 - Dimensionless time t^* for dissolution of a bubble from 80% of its original radius as calculated for a range of mathematical models. Also shown are the correction factors determined from the present model. Dividing the total time calculated by either the Epstein-Plesset(1950) or the Duda and Vrentas(1971) model by this factor accounts for the boundary wall condition.

N_a	Dissolution Time			Correction Factor:	
	Epstein – Plesset (Eqn 5)	Duda & Vrentas(1971) $\varepsilon = 0$	Present Model	To Epstein- Plesset (Eqn 5)	To Duda & Vrentas(1971) $\varepsilon = 0$
0.0092	34.7	35.6	49.7	0.698	0.716
0.01	32.0	32.9	45.9	0.697	0.716
0.015	21.6	22.3	31.1	0.695	0.716
0.1	3.20	3.32	4.68	0.688	0.716