Improving the Safety of Forklifts Through Automation of Task Execution

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A thesis submitted in fulfilment of the requirements for the degree of

Master of Philosophy

in the

Department or Mechanical Engineering

THE UNIVERSITY OF MELBOURNE

August 2016

Produced on archival quality paper.
Abstract

Forklift vehicles are used throughout the world, in many industries to move stock from location to location. Unfortunately these vehicles are prone to toppling, possibly causing stock damage, injuries and loss of income. There are fewer safety interventions applied to forklifts than would be expected of other wheeled vehicle, with the industry relying on operator training and experience to prevent accidents.

This thesis proposes a method of measuring the toppling propensity, and a trajectory generation algorithm to allow the autonomous use of forklift vehicles. The zero moment point and a barycentric coordinate system will be used to measure the dynamic balance. The measure will allow analysis of forklift behaviour and may be used to provide information of the state of balance to the operator with the intent of aiding judgement and to improve accountability.

The obtained dynamic balance measure will then be used as an inequality constraint during the construction of a non-holonomic trajectory. This will be used to ensure the trajectory complies with safe manoeuvring of the forklift vehicle. The autonomy of the vehicle will allow the operator to distance themselves from the vehicle, interacting only through high level decisions as opposed to low level handling of the vehicle, with the intent of removing the human element of error from the toppling problem.

An analysis of forklift behaviour given differing loading scenarios will be discussed, with similar scenarios computationally simulated to produce sample trajectories of the new algorithm. The results are compared to intuitive notions of movement and to safety literature.
Declaration of Authorship

I, Leng Benjamin Vongchank, declare that this thesis titled, 'Improving the Safety of Forklifts Through Automation of Task Execution' and the work presented in it are my own. I confirm that:

■ This work was done wholly or mainly while in candidature for a research degree at this University toward the degree.

■ Acknowledgement has been made to all sources and material used or adapted.

■ I have acknowledged all main sources of help.

■ Where the thesis is based on work done by myself jointly with others, I have made clear exactly what was done by others and what I have contributed myself.

■ This thesis is less than 50,000 words in length.

Signed:  
__________________________________________________________________________

Date:  
__________________________________________________________________________
Acknowledgements

I would like to first thank my supervisors Associate Professor Denny Oetomo and Associate Professor Allison Kealy for their help and patience during my candidature. Their help was integral in getting started and finding the most applicable literature. Without your support and guidance I would not have been able to complete this work.

I would also like to thank our industry collaborators Speedshield Technologies. Speedshield provided valuable insights into the forklift industry and help with the construction of the scale model forklift. Without your participation, defining a research goal relevant to the industry may have been difficult.

I extend my gratitude towards the University of Melbourne robotics lab for providing an encouraging space to conduct research, providing discussions on topics, both relevant and irrelevant, and for allowing me to vent my frustrations. I would also thank both the Robotics research group and control research group for listening to my presentations and providing feedback on both the presentation and my work.

Finally, I would like to thank my friends, family and in particular my partner for providing endless non-sensical humour involving potatoes and elephants, helping to colour the slower days of my candidature... and cake. This work would not have been possible without cake and all of you.
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<td>Forklift Vehicle</td>
</tr>
<tr>
<td>SKU</td>
<td>Sock Keeping Unit</td>
</tr>
<tr>
<td>AS/RS</td>
<td>Automatic Storage / Retrieval System</td>
</tr>
<tr>
<td>VLM</td>
<td>Vertical Lift Machine</td>
</tr>
<tr>
<td>DB</td>
<td>Dynamic Balance</td>
</tr>
<tr>
<td>DBM</td>
<td>Dynamic Balance Margin</td>
</tr>
<tr>
<td>BDBM</td>
<td>Barycentric Dynamic Balance Margin</td>
</tr>
<tr>
<td>BC</td>
<td>Barycentric Coordinates</td>
</tr>
<tr>
<td>WC</td>
<td>Wachspress Coordinates</td>
</tr>
<tr>
<td>MVC</td>
<td>Mean Value Coordinates</td>
</tr>
<tr>
<td>ZMP</td>
<td>Zero Moment Point</td>
</tr>
<tr>
<td>SP</td>
<td>Support Polygon</td>
</tr>
<tr>
<td>CC</td>
<td>Continuous Curvature</td>
</tr>
<tr>
<td>SCC</td>
<td>Simple Continuous Curvature</td>
</tr>
<tr>
<td>mSCC</td>
<td>modified Simple Continuous Curvature</td>
</tr>
<tr>
<td>1CC</td>
<td>Single (1) Continuous Curvature</td>
</tr>
<tr>
<td>2CC</td>
<td>Double (2) Continuous Curvature</td>
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## Nomenclature

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<th>Unit</th>
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<tr>
<td>$N$</td>
<td>Number of links</td>
<td></td>
</tr>
<tr>
<td>$M$</td>
<td>Number of vertices</td>
<td></td>
</tr>
<tr>
<td>$i$</td>
<td>Link number</td>
<td></td>
</tr>
<tr>
<td>$j$</td>
<td>Vertex number</td>
<td></td>
</tr>
<tr>
<td>$m$</td>
<td>Total Mass</td>
<td>kg</td>
</tr>
<tr>
<td>$m_f$</td>
<td>Mass of FLV</td>
<td>kg</td>
</tr>
<tr>
<td>$m_l$</td>
<td>Mass of payload</td>
<td>kg</td>
</tr>
<tr>
<td>$m_i$</td>
<td>Mass of link $i$</td>
<td>kg</td>
</tr>
<tr>
<td>$G_i$</td>
<td>Centre of gravity of link $i$</td>
<td>m</td>
</tr>
<tr>
<td>$r_{G_i}$</td>
<td>Position of centre of gravity of link $i$</td>
<td>m</td>
</tr>
<tr>
<td>$\dot{r}_{G_i}$</td>
<td>Velocity of centre of gravity of link $i$</td>
<td>m s$^{-1}$</td>
</tr>
<tr>
<td>$\ddot{r}_{G_i}$</td>
<td>Acceleration of centre of gravity of link $i$</td>
<td>m s$^{-2}$</td>
</tr>
<tr>
<td>$r_O$</td>
<td>Position of body frame 1</td>
<td>m</td>
</tr>
<tr>
<td>$\dot{r}_O$</td>
<td>Velocity of body frame 1</td>
<td>m s$^{-1}$</td>
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<tr>
<td>$\ddot{r}_O$</td>
<td>Acceleration of body frame 1</td>
<td>m s$^{-2}$</td>
</tr>
<tr>
<td>$v_w$</td>
<td>Linear velocity of the driven wheel</td>
<td>m s$^{-1}$</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Steering angle</td>
<td>rad</td>
</tr>
<tr>
<td>$\dot{\alpha}$</td>
<td>Steering velocity</td>
<td>rad s$^{-1}$</td>
</tr>
<tr>
<td>$\ddot{\alpha}$</td>
<td>Steering acceleration</td>
<td>rad s$^{-2}$</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Heading</td>
<td>rad</td>
</tr>
<tr>
<td>$\dot{\theta}$</td>
<td>Angular velocity of FLV</td>
<td>rad s$^{-1}$</td>
</tr>
<tr>
<td>$\ddot{\theta}$</td>
<td>Angular acceleration of FLV</td>
<td>rad s$^{-2}$</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
<td>Unit</td>
</tr>
<tr>
<td>--------</td>
<td>------------------------------------------</td>
<td>------------</td>
</tr>
<tr>
<td>$\theta_m$</td>
<td>Heading of mast</td>
<td>rad</td>
</tr>
<tr>
<td>$\psi_m$</td>
<td>Tilt angle of mast</td>
<td>rad</td>
</tr>
<tr>
<td>$\psi_c$</td>
<td>Tilt angle of carriage</td>
<td>rad</td>
</tr>
<tr>
<td>$w$</td>
<td>Wheelbase</td>
<td>m</td>
</tr>
<tr>
<td>$l$</td>
<td>Track</td>
<td>m</td>
</tr>
<tr>
<td>$h$</td>
<td>Lift</td>
<td>m</td>
</tr>
<tr>
<td>$d$</td>
<td>Reach</td>
<td>m</td>
</tr>
<tr>
<td>$e$</td>
<td>Fork/carriage eccentricity</td>
<td>m</td>
</tr>
<tr>
<td>$R$</td>
<td>Radius of curvature</td>
<td>m</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>Curvature</td>
<td>m$^{-1}$</td>
</tr>
<tr>
<td>$R_{\text{min}}$</td>
<td>Minimum radius of curvature</td>
<td>m</td>
</tr>
<tr>
<td>$\kappa_{\text{max}}$</td>
<td>Maximum curvature</td>
<td>m$^{-1}$</td>
</tr>
<tr>
<td>$s$</td>
<td>Distance travelled</td>
<td>m</td>
</tr>
<tr>
<td>$t$</td>
<td>Time</td>
<td>s</td>
</tr>
<tr>
<td>$\dot{H}$</td>
<td>Torque</td>
<td>N·m</td>
</tr>
<tr>
<td>$r_{\text{ZMP}}$</td>
<td>Position of ZMP</td>
<td>m</td>
</tr>
<tr>
<td>$C$</td>
<td>Set of vertices (corners) of length M</td>
<td></td>
</tr>
<tr>
<td>$c_j$</td>
<td>Vertex $j$</td>
<td></td>
</tr>
<tr>
<td>$p$</td>
<td>Point in Cartesian space</td>
<td></td>
</tr>
<tr>
<td>$I$</td>
<td>Moment of inertia</td>
<td>kg·m$^2$</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Barycentric coordinate, tuple of length M</td>
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<td>Barycentric coordinate of the ZMP</td>
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<tr>
<td>$\kappa_2$</td>
<td>Curvature of last turn</td>
<td>m$^{-1}$</td>
</tr>
<tr>
<td>$K$</td>
<td>Set of $\kappa_1, \kappa_2$</td>
<td></td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Turning rate</td>
<td>rad·s$^{-1}$</td>
</tr>
<tr>
<td>$\sigma_{\text{max}}$</td>
<td>Maximum turning rate</td>
<td>rad·s$^{-1}$</td>
</tr>
<tr>
<td>$\sigma_{\text{eff}}$</td>
<td>Effective geometric turning rate</td>
<td>rad·s$^{-1}$</td>
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Chapter 1

Introduction

This chapter will introduce the reasoning motivating the study of forklift vehicles (FLV) and the intended outcomes. The characteristics of FLVs will be introduced as well as the associated safety concerns related to FLV operation. Section 1.2 will outline the aims, highlighting the outcomes of each aim and concluding with the intended contributions. The chapter will conclude with an outline of the thesis.

1.1 Background and Motivation

Forklift Vehicles (FLV) are a common materials handling unit used in many different types of working conditions. However, throughout operation of FLVs, there is considerable risk of injury – to both operators and pedestrians – and damages to stock, infrastructure, and the FLV itself. The intent of this thesis is to reduce the safety risks associated with rideable FLV operation, where these aims will be further developed in Section 1.2. This study is conducted in collaboration with Speedshield Technologies under the Australian Research Council’s Linkage Projects funding scheme (project number LP130100113).

FLVs can be generally characterised by a pair of forks, also known as tynes, mounted on the non-steered side of the FLV. These forks are often used to lift pallets of stock, where the
pallets are designed to allow entry of the forks below the stock and a surface for the forks to lift from. FLVs used for freight handling have special attachments for freight containers in place of forks, where the container is rigidly attached to the forks, rather than the payload sitting atop a lifting surface. Following the differences in FLV necessity and construction, there exists broad classifications to describe the nature of a type of FLV. There are a total of seven classes of powered industrial vehicle noting that all classes, excluding VI, are FLVs [1]. Class VII FLVs cover the set of atypical designs, including freight handling trucks and forklift attachments for other vehicles and will hence be omitted from this study. Section 2.1 will elaborate on the definition of the classes.

FLVs are common in many workplaces, being used in both outdoor and indoor applications, ranging from freight handling to moving stock in shopping centres. In 2011, there was a reported total of 1,088,366 FLVs ordered worldwide [2], these numbers include Classes I-V FLVs. Of the 1,088,366 FLVs over 70% are classed as ridable FLVs. Although FLVs are widely used, FLV safety practices are not comprehensive or well regulated [3-5].

To compound the safety issue, the general design of an FLV is inherently easy to unbalance [5], this stems from designing the FLV to maintain high versatility and compactness. One example of this compactness are reach trucks, a class II FLV designed for narrow aisles, weighing approximately 3 tonnes, with a footprint of approximately 1.5 m$^2$ and a maximum payload capacity of approximately 45% of its weight [6]. The compactness becomes problematic when the system is carrying heavy loads or is in motion. Another possible reason for the lack of safety features, believed by our industry collaborators, may be due to legal concerns in the USA, where by adding any ‘safety feature’ there is potential for litigation if the feature is not 100% successful. Although this may be confined to the USA, the USA is a large market, where some leading manufacturers, including Hyster, are based in the USA. This has been considered to result in a market inertia that opposes the adoption of new safety features.

To overcome potential dangers in operating a FLV, licenses are required for all classes of FLVs that are ridable [3]. The training for such a license covers safety, assessed with a written test, and a practical component. Although, from anecdotal sources, there is a
potential that these tests are treated as formalities, favouring experience in driving over understanding of safety material. These courses may be as little as two days long [7].

For the purpose of this study we consider two major types of FLV operational failure: collision due to obstructed view and toppling due to FLV dynamics. The former form of failure results from the payload being carried in the general direction of motion. The payload may be large, thus obstructing the view in the direction of travel. Collisions may occur between the FLV and pedestrians, storage racking or stock. The latter type of failure results from the aforementioned small footprint and heavy weight resulting in easily toppling the vehicle. Toppling may result from unexpected load distributions of the payload or through operator negligence. This thesis will consider operator negligence to include poor operator behaviour, flaws in situational assessment or poor operator performance, e.g. operator is fatigued. Section 2.1 will discuss failures in more detail.

The study proposes the use of semi-autonomous forklifts to remove the human element from the repetitive, and dangerous, task of driving while also removing this notion of ‘safety feature’ to reduce the risk of toppling. We must note here that this thesis covers only a portion of the overall intended goal stated. To achieve this, the thesis aims to complete a dynamic model of a FLV and construct trajectories that satisfy these safety constraints. This is with the intention of creating a foundation for future work, where higher levels of automation and intelligence may be implemented, and allow experimental testing. To allow use in an industrial setting, there will be consideration into minimising the computational requirement where possible and minimising modifications to allow retrofitting of existing vehicles.

1.2 Aims and Outcomes

This thesis will address the issue of toppling raised in 1.1 by focusing on two aims, described in detail below. Here we will also describe the intended outcome as a result of each aim, specifically,

Aim 1. Model and analyse the dynamics of an FLV
Chapter 1. \textit{Introduction}

\textbf{Aim 1.1.} Define a method to analyse toppling of an FLV

\textbf{Aim 1.2.} Develop a scalar inequality to quantify the toppling propensity

Aim 1 focuses on studying the dynamic behaviour of the FLV. By defining clearly the FLV behaviour with respect to its balance, we will be able determine the toppling propensity given knowledge of the current state. The model to be developed is intended to cover the majority of FLVs currently in use, allowing adaptation of our further algorithms for use this class. Analytical models for balance are described in Section 2.2, with specific focus on the concept of the Zero Moment Point (ZMP)[8], with Chapter 2 adapting existing techniques for the case of an FLV. Chapter 3 will develop a measure to identify the toppling propensity.

\textbf{Aim 2.} Construct a trajectory constrained by the dynamic behaviour of FLVs, ensuring balanced motion along the entirety of the path.

\textbf{Aim 2.1.} Modify a distance optimal geometric path planner to allow FLV like manoeuvres and time dependent considerations.

\textbf{Aim 2.2.} Incorporate the results of Aim 1 to ensure dynamic balance along the trajectory.

Aim 2 applies the results of Aim 1 to generate trajectories between two terminal poses. We will use the Simple Continuous Curvature (SCC) path [9] as a geometric planner forming the basis of our balanced trajectory. The choice of a geometric planner and details regarding the SCC path will be explained in Section 2.3. The formulation of the SCC path will be modified to allow time dependent variables to act as the shaping variables. An additional constrained optimisation process is required to complete the construction of the geometry and ensure dynamic constraints are satisfied. This approach will be detailed in Chapter 5.

The resulting contributions are as follows

- A scalar measure of toppling propensity for robots with planar motion based on the ZMP.
- A trajectory planner for a non-holonomic vehicle satisfying balance constraints with consideration for implementation in an industrial setting.
1.3 Chapter Outline

The thesis is organised as follows: Chapter 2 will cover forklift safety, dynamic balance concepts and motion planning. The state of forklift safety and autonomy will be reviewed, including the technologies currently available on the market. The concept of dynamic balance will be introduced through the Zero Moment Point (ZMP) and theories associated with measuring this balance. Nonholonomic motion planners will then be introduced with a class of geometric planners based on the Dubins’ Path in focus.

Chapter 3 seeks to establish the necessary dynamic models used for the class of three wheeled forklift in our study. The kinematic model will be derived, stating appropriate assumptions. A simplified model of the ZMP will be introduced for use with FLVs.

Chapter 4 and 5 will cover a new method to characterise the dynamic balance and how it may be used. A dynamic balance margin will be developed and applied to a trajectory. The formulation will be then used as a means to calculate a trajectory that is balanced.

Finally, Chapter 6 will summarise the major outcomes and propose future improvements, providing comments on the necessary steps toward the goal of semi-autonomous FLV fleets.
Chapter 2

Literature Review

The focus of this Chapter is to review the necessary literature and industry information to achieve the goals set in Chapter 1. This chapter will cover the basic construction of a forklift vehicle (FLV) and identify the necessary components for this study. Some of the existing technologies relating to safety and automation of materials handling and FLVs will be reviewed, revealing where this study fits in the industry. The vehicle dynamics and terminology will then be presented in this chapter and a brief comment on motion planners will be made. Section 2.3 will cover a class of non-holonomic geometric path planners that are suited to this study.

2.1 Forklift Vehicles: a non-holonomic mobile manipulator

Forklift vehicles (FLV) are non-holonomic vehicles bearing many similarities to a robotic mobile manipulator, a field of research which is applicable to this work. FLVs are a class of powered industrial vehicle and materials handling unit. The term ‘Powered industrial vehicle’ is used only for classification with use in standards, while the term ‘materials handling unit’ refers to a class of machinery used to move stock.

Powered industrial vehicles are divided into 7 classes [1], all but one class, class VI, describes an FLV. Class VI vehicles refer to a type of tow vehicle and will not be considered in this
thesis. The classes are described from I to VII, summarised in Table 2.1. Examples of
the different classes can be found in Figure 2.1 [10]. Lift codes are used to further classify
vehicles of each class, further information can be found in [1].

<table>
<thead>
<tr>
<th>Class</th>
<th>Description</th>
<th>Figure</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>Electric motor rider trucks</td>
<td>2.1a</td>
</tr>
<tr>
<td>II</td>
<td>Electric motor narrow aisle trucks</td>
<td></td>
</tr>
<tr>
<td>III</td>
<td>Electric motor hand trucks or hand/rider trucks</td>
<td>2.1c, 2.1d</td>
</tr>
<tr>
<td>IV</td>
<td>Internal combustion engine trucks (solid/cushion tyres)</td>
<td>2.1b</td>
</tr>
<tr>
<td>V</td>
<td>Internal combustion engine trucks (pneumatic tyres)</td>
<td></td>
</tr>
<tr>
<td>VI</td>
<td>Electric and internal combustion engine tractors</td>
<td>2.1e</td>
</tr>
<tr>
<td>VII</td>
<td>Rough terrain forklift trucks</td>
<td>2.1f</td>
</tr>
</tbody>
</table>

Table 2.1: Summary of powered industrial vehicle classifications [1]

Rider trucks are those with a car-like interface: a steering wheel and pedals, whereas hand
(including hand/rider) trucks use a lever like mechanism as shown in Figure 2.1c and 2.1d.
Rider trucks are to be considered in more detail in this thesis as safety requirements are
more stringent, requiring training before use [1, 7, 11], and the maximum load capacity is
generally greater. Classes I, IV - VII are considered rider trucks.

Classes I-III vehicles are generally for indoor use, while class IV-VII vehicles are for outdoor
use, where the main factor separating use are the vehicles emissions. Class I, IV and V
vehicles are for similar applications, general purpose stock relocation, including to and
from trucks, and to picking/packing stations. Class IV and V vehicles are generally used
outdoors, where combustion emissions are less concerning. Class I vehicles are used in
and around warehouses or loading areas. Class II vehicles are specialised vehicles, used
almost exclusively in warehouse-like environments, where the surface is flat with no or
little inclines, and shelving, with limited space, is used to store stock. Class III vehicles
are typically used in pedestrian heavy environments, for moving ground stock between
locations, for example a class II vehicles may bring a pallet to the picking/packing station
and a class III vehicles used to move it to allow pickers/packers easy access. Class VII
vehicles are generally special use vehicles, including forklift attachments on tractors and
the freight lifting reach stacker.
(a) Class I Electric counterbalanced truck, carrying pallets with a rear tilted mast.

(b) Class IV Counterbalanced internal combustion engine truck. The LPG tank is visible on the rear of the vehicle.

(c) Class III electric hand/rider truck provides standing room for operator.

(d) Class III electric hand (walkie) pallet truck requires the operator to walk with the truck.

(e) Class VI electric tow tractor moving trolleys loaded with stock.

(f) Class VII reach stacker used for freight handling.

Figure 2.1: Examples of different classes of powered industrial vehicles.

The focus of this thesis will be on Classes I, II, IV and V vehicles. The four mentioned classes are common place in the materials handling industry, with over 750,000 ordered in 2011 worldwide [2]. These four classes exhibit similar physical properties with a few exceptions, one being the articulated forklift where the mast section may rotate independently from the main body. These similarities will be described in Section 3.1 and will be used to define the necessary parameters for a model of forklift dynamics.
2.1.1 Existing Safety Interventions

Currently there are few standard safety interventions applied to FLV operation [12]. The few that are implemented are behavioural interventions, seeking to address the behaviour of the operator or the environment which the FLV must operate in [4, 12]. The former is addressed through driving training, requiring both written and practical tests. The training period may be as little as 2 days [7]. Upon informal questioning of FLV operators, it has been mentioned that the written test is treated with little respect, where the invigilator may leave the room allowing sharing of answers. The latter is more strictly enforced in the form of infrastructure and procedures. These changes to the environment may be likened to road rules or traffic management, requiring pedestrian crossings, specified laneways for vehicles. However, these interventions fall short when the FLV is required to work in close proximity with pedestrians, for example in picking/packing locations, or with other structures, for example storage shelves and trucks [13]. There exist a number of safety interventions that are commercially available intended to augment the standard procedures.

Speedshield Technologies have a number of products that help to mitigate and manage safe practice for FLVs [14]. One of the active devices is a speed limiter capable of detecting RFID zones in which different limits may be imposed. This device may work without input or knowledge from the operator. The second is a seatbelt interlock device that attempts to decide, through logic, whether a seatbelt has been used correctly, i.e. the operator is seated and seatbelt is used on the operator as opposed to the seatbelt left permanently buckled in an attempt to circumvent the safety protocol. This enforcement requires the compliance of the operator. The aforementioned logic was designed to prevent misuse; however favours a no false positive reporting policy thus there are known loopholes in the logic. Speedshield Technologies also offer management tools, operator identification, pre-drive check-lists and impact sensing and automatic shut down in case of incident. These systems are designed to report issues to management or a safety officer. These solutions target the operator behaviour by enforcing accountability, using limited intelligence to prevent circumvention.

Toyota takes a different approach to safety by modifying the driving system. The Toyota System of Active Stabilisation (SAS) [15] focuses on affecting the mechanics of the FLV.
This is achieved through several processes: mast speed reduction, tilt control and position locking, and body stiffening. The first two affect lifting at heights by preventing forward tilting (position locking) and reducing speeds to improve control. The last process is designed to prevent roll-over. A hydraulic actuator, named *swing lock cylinder*, is placed on the rear (drive) axle, seen in Figure 2.2 [16], and is stiffened when the body begins to roll. This works on the notion that the rear axle is designed as a pin-joint resulting in a triangular base, by stiffening the actuator, the body can no longer roll with respect to the axle, effectively providing a larger base. These systems react to the operators input and act in some predefined manner.

![Figure 2.2: Location of hydraulic (circled) used in Toyota SAS.](image)

Safety in the industry has been primarily improved through management and improving the FLV system. Safety through management has a focus on the accountability of operators and pedestrians, with procedures defining acceptable behaviour. The improvements to the FLV are of a practical nature and based upon reacting to the current state. Due to the nature of the problem, i.e. an unpredictable input, a reactive approach is, perhaps, the only method to improve safety; however there is an assumption on the operator using the FLV in a sensible manner. For example if an FLV was restricted by velocity and acceleration with respect to curvature, the path desired by the operator will be maintained, but the speed may be limited depending on the desired turn. However, it is possible to circumvent this by driving at high velocities in a straight line (zero curvature) then take an unexpected, sharp turn. Due to the quick change in state, it may not be possible to prevent toppling. This can only occur when there is zero curvature and thus all velocities are valid. A fast
deceleration may cause tip-over, but not slowing enough may cause roll-over. Changing the safety strategy to limiting curvature with respect to velocity and acceleration may result in a dangerous path, e.g. unable to avoid shelves or pedestrians due to larger curvatures at higher speeds. These two examples are extreme cases; however it does illustrate the possibility of bypassing safety interventions. It should be noted that there are compromises between usability and ‘fool proofing’ when designing these reactive systems.

2.1.2 Materials handling automation

The overview of current safety interventions in Section 2.1.1 describes systems that, in the most part, are reactive. Some requirements in the situation must be fulfilled before the device or system takes action. This is common for FLVs fitted with safety devices. On the other end of the spectrum are fully automated devices, commonly referred to as Automated Storage / Retrieval Systems (AS/RS). These systems forgo the human element in the transport of goods in a facility, hence eliminating a large source of error. Humans are still required to load new stock or remove stock from the system; however a clear interface is defined, separating the machine and human components.

KIVA Systems LLC provides a materials handling system that uses fully automated mobile robots, named pods seen in Figure 2.3 [17], and a central server to move and organise stock. The Kiva Mobile-robotic Warehouse Automation System has been used in many retail order fulfilment centres, with Amazon.com being the most notable customer. For the purpose of this thesis we will use ‘KIVA Systems’ to refer to the company while ‘Kiva system’ to refer to the product. One white paper [18] believes the Kiva system to be most suited to this retail fulfilment role without a large number of stock keeping units (SKU). This white paper outlines the system and makes recommendations towards the use in small sized orders that require different stock, where the number of orders is consistent across time. Larger orders are not handled very well by the small scale of the shelving, these are seen in distribution centres for retail outlets. For orders that require very few stock items, then the logistic capabilities of the Kiva system is under utilised. Well defined peak periods cause a problem in costs, both capital and running, to account for the peak periods, where
many of the pods will be underutilised in the off-peak periods. KIVA systems does provide a rental option to allow for these situations.

There are a number of solutions utilising a carriage mounted on rails known as vertical lift modules, VLM, moving the stock to either a handling area, shelving or conveyors. Figure 2.4 [19] shows a conceptual rendition of a AS/RS system in a warehouse. Companies such as Bastion Solutions, Vahle Inc. and Westfalia Technologies Inc. provide AR/AS solitions of utilising VLMs. The main benefits of this type of system is the increase in stock density with respect to floor space, where the height of SKUs are not limited as is the case with non-specialised FLVs. The major downside is the associated costs. Capital costs to construct a specialised warehouse for such as solution is high compared to that of normal warehouses. Furthermore, without the resources to run another facility while bringing the AR/RS online, the cost of downtime may be especially harmful to companies, especially those that are small to mid sized [18]. These solutions have very little flexibility, when one section is not operational. For example, a line of shelving uses a single VLM, if this unit fails then the entire shelving line is no longer usable. This differs from the Kiva system where another pod can take over.

These automated systems focus much attention on efficiency with little work displayed on safety measures or balance [20–22]. The systems are designed in such a manner that they are enclosed with a single area for exchange between automated to manual systems. The
other common FLV failures, including load slippage or toppling, are not considered due to the design of such systems inherently not allowing such failures to occur. For example a gantry cannot have load slippage as the payload moves perpendicular to the direction of the forks unless loading or unloading the payload. Furthermore, gantries do not topple, a failure of this manner will be more catastrophic, where the structural integrity of the gantry is lost. With regard to the KIVA system, each unit moves slowly (walking pace) with relatively light and small loads.

These AS/RS have a number of common points, including: dedicated hardware and environment, efficiency of space usage, and high capital costs. The dedicated environments are problematic as there can be no gradual change of the environment, requiring a new space or downtime to rebuild a current space. Speedshield Technologies notes that the downtime and capital acts as a barrier to adopting this new technology.
2.2 Dynamic Balance and Mobile Manipulators

The term dynamic balance refers to a dynamic system which is not undergoing an uncontrolled fall. One definition for an uncontrolled fall is described as no moments about the convex hull encompassing the systems ground contacts, this definition will be described in more detail in Section 2.2.1. The term stability is often used to describe dynamic balance [23]. This thesis will refrain from using stability in this manner and adopt the terminology of dynamic balance as in [23]. The word stability is used in control theory, and where dynamic systems and control meet, as in robotics, there should be a distinction in these terms. For example, one can imagine a legged robot about to fall, in this case the system is not dynamically balanced, however the control over the joints must remain stable to allow correction of the fall, namely accurately and precisely moving a leg to compensate. In this sense neither term is interchangeable and this definition provides clarity.

Traditionally legged robots make use of dynamic balance; however the notion can be extended to wheeled vehicles and mobile manipulators [24–26]. Wheeled vehicles can become unbalanced generally in two directions, the sagittal direction (forward and back) and lateral (side to side). For the purpose of this thesis tip-over and roll-over will be used respectively and toppling used to refer to both cases.

Tip-over is often not a concern for non-industrial vehicles due to their geometry; however roll-over is a concern when considering high speeds and sharp turns. Toppling is a concern for mobile manipulators where high accelerations, velocities and sharp turns are concerned but added to this is the ability to move a manipulator(s) and handle a payload. Handling a large payload may unbalance the mobile manipulator without considering any other dynamic effects. On the other hand, if the mobile manipulator can handle such a load and not become unbalanced, then tip/roll-over conditions may be exacerbated by this increase in mass and heightened centre of gravity (CoG). FLVs may be considered a non-holonomic mobile manipulator, where the mast and forks are a simple manipulator.
2.2.1 Zero Moment Point

The Zero Moment Point (ZMP) is a method to determine if a system is dynamically balanced. The ZMP was first introduced to allow robotic gait synthesis [8] . The notion of dynamic balance is inherent to the definition of the ZMP, where the ZMP may only exist when the system is dynamically balanced [23]. The ZMP can be described as a point on the ground plane that has a sum of zero moments with respect to the system. If such a point exists then the system is balanced, if the point does not exist then system is said to be unbalanced. Traditionally this has been used for planar surfaces with no incline. More recently, there have been works to generalise the ZMP to arbitrarily incline planes [24, 27, 28].

The notion of the existence of the ZMP is tied to the Support Polygon (SP), the convex hull of the robots ground contacts. The ZMP may only exist when there is zero moment (as the name implies). As a moment appears about the SP, the ZMP tends to the edge of the SP and degenerates to non-existence. If a theoretical ZMP is calculated that is on the edge or outside of the SP, then we name this the fictitious ZMP (FZMP). The FZMP and ZMP are mutually exclusive, with the existence of the FZMP indicating a loss of dynamic balance. The FZMP is useful for planning future movements, particularly in robotic gait synthesis where the SP may change.

The ZMP can be calculated empirically by using force sensors on the ground contacts. The point of action of these forces are calculated to reveal the ZMP. However, this is not always practical. Wheeled vehicles cannot easily measure the force on the tyres due to the design of the wheel joint and also due to the complex and highly non-linear behaviour of the tyre itself. We will use the d’Alembert formulation of the ZMP described in 2.2.1.1. The d’Alembert formulation allows the existence of the FZMP, where the theoretical calculation may, indeed, find a point outside the SP. Additional calculation is required to check for the existence of the ZMP or FZMP, a novel method will be described in 3.
2.2.1.1 Derivation of the Zero Moment Point

Assume a point $P = [x_P, y_P, 0]^T$ exists such that the moment about $P$, $M_P = [0, 0, M_z]^T$, where $M_z$, or the yaw motion, can be non-zero. Let $A_i$ be the location of the CoG of the rigid body $i$, where $i = 1, 2, \ldots, n$.

The force, $F_{A_i}$, and moment, $M_{A_i}$, represent the body forces acting on rigid body $i$ passing through the point $A_i$, where:

$$F_{A_i} = m_i a_i$$

$$M_{A_i} = \dot{H}_i = I_i \ddot{\omega}_i + \omega_i \times I_i \dot{\omega}_i$$

Where $a_i$ is the acceleration, $\omega_i$ is the angular velocity and $H_i$ is the angular momentum of the body $i$.

The force, $F_P$, and moment, $M_P$, is the ground reaction force passing through the point $P$ where:

$$F_{A_i} = -F_P$$

Assuming there is no motion, the sum of moments about $P = 0$:

$$0 = M_P - \sum_{i=1}^{n} (r_{PA_i} \times F_{A_i} - \dot{H}_i)$$

$$M_P = \sum_{i=1}^{n} (r_{PA_i} \times F_{A_i} - \dot{H}_i) \quad (2.1)$$

where:

$$r_{PA_i} = r_{A_i} - r_P$$

$$r_{A_i} = \begin{bmatrix} x_{A_i} \\ y_{A_i} \\ z_{A_i} \end{bmatrix} ; r_P = \begin{bmatrix} x_{ZMP} \\ y_{ZMP} \\ 0 \end{bmatrix}$$
Expand (2.1) into the $x$ and $y$ components. Note the $z$ component can be ignored as $z_{ZMP} = 0$.

\[ 0 = \sum_{i=1}^{n} \left( m_i a_i y_{A_i} - m_i a_i y_{ZMP} \right) - m_i a_i z_{A_i} - z_{ZMP} + (\dot{H}_i)_x \]  \hspace{1cm} (2.2)

\[ 0 = \sum_{i=1}^{n} \left( m_i a_i x_{A_i} - m_i a_i x_{ZMP} \right) - m_i a_i x_{A_i} - x_{ZMP} + (\dot{H}_i)_y \]  \hspace{1cm} (2.3)

where $(\dot{H}_i)_x$ and $(\dot{H}_i)_y$ are the components along the $x$ and $y$ axis respectively.

Rearranging (2.2) and (2.3):

\[ y_{ZMP} = \frac{\sum_{i=1}^{n} (y_{A_i} m_i a_i - z_{A_i} m_i a_i + (\dot{H}_i)_x)}{\sum_{i=1}^{n} (m_i a_i)} \]  \hspace{1cm} (2.4)

\[ x_{ZMP} = \frac{\sum_{i=1}^{n} (x_{A_i} m_i a_i - z_{A_i} m_i a_i - (\dot{H}_i)_y)}{\sum_{i=1}^{n} (m_i a_i)} \]  \hspace{1cm} (2.5)

\[ z_{ZMP} = 0 \]  \hspace{1cm} (2.6)

Equations (2.4)-(2.6) define the position of the ZMP in Cartesian space; however, contains
no information as to whether the calculated ZMP lies within the polygon. Chapter 4 will
describe a method to confirm the existence of the ZMP or FZMP for arbitrary convex poly-
gons. Furthermore the ZMP calculated provides no direct notion of toppling propensity.
We require a quantifiable measure, which will be discussed in Section 2.2.2.

2.2.2 Dynamic Balance Margin

The dynamic balance margin (DBM) is a scalar measure of tip-over propensity. In the
literature the term stability margin or criteria is used to define dynamic balance margin
[29–31]. For consistency, this thesis will replace 'stability' with 'dynamic balance' as in
Section 2.2. It should also be noted that 'stability margin', not just 'stability', is a term
also used in control theory and will be clearly differentiated in this work.

The DBM can be roughly categorised in two groups: energy based methods, and geometric
based methods [30, 32]. In general, the energy based methods attempt to calculate the
energy required for tip-over, while geometric methods use the dynamics in Cartesian space
to find the distance, of some physical dimension, to tip-over. One such geometric mea-
sure utilises the ZMP and measures the distance to the closest edge [29]. For particular
symmetric geometries there exist regions of the highest balance measure. In the case of
a rectangle, this is a portion of the axis of symmetry along the longest edge Figure 2.6.
Here we note that the centroid of the rectangle should be the most balanced point, as any
deviation along the long axis of symmetry will result in less forces required to unbalance
the agent over one of the shorter edges. This work will provide a DBM that will consider
all edges of the polygon, as opposed to the closest edge.

![Figure 2.6: Example of a rectangular SP. The inner lines represent the furthest distance from an edge.](image)
Chapter 2. Literature Review

The Foot Rotation Index [30] and many energy based methods require the separate calculation of the DBM, outside of the ZMP. One goal of this work is to reuse the already processed ZMP to further calculate the DBM, as such, these methods are not considered but are recognised as capable counterparts.

Section 4.1 describes a novel method to calculate a DBM using the ZMP. Unlike other methods, this method requires use of the established ZMP theory. Using the ZMP and this novel barycentric based DBM retains the computational benefits and information provided by the ZMP while adding a method to quantify the dynamic balance described by the ZMP.

2.3 Non-holonomic geometric path planning

Motion planners can be categorised into holonomic and non-holonomic motion planners. The non-holonomic case poses significant problems due to the system being non-integrable [33] and is of more importance in regards to this study. There exist a class of geometric motion planners that require fixed terminal configurations utilising a parameterised form, this is appropriate for the FLV problem, where terminal configurations are fixed. Additionally geometric path planners of this type reduce the search space of a path, compared to sample or search based methods, due to the parameterisation. Such parameterisations often trade optimality for a reduction of computational resources. Geometric path planners can ensure continuity known as geometric $G^n$ continuity. $G^1$ continuity corresponds to tangential continuity along the entire path. $G^2$ continuity corresponds to continuity in the curvature or normals of the path.

Geometric path planners, are inherently unable to achieve the $C^2$ continuity, continuity in acceleration, which may be used in lieu of the non-holonomic constraint. The achieved geometric continuity is a weaker condition on continuity compared to $C^n$ continuity. $C^n$ continuity requires $G^n$ continuity as a necessary condition.

$$C^n \Rightarrow G^n$$
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To achieve a trajectory a class of geometric paths based on the Dubin’s path will be described in this section, with the notion of a fixed end pose and non-holomicity in mind. Additionally a continuous velocity and acceleration profile will be imposed on the FLV to achieve $C^2$ continuity.

2.3.1 Dubins’ path

Dubins’ path characterises a path with terminal and maximum curvature constraints. The path has typically been used as a motion planner for simplified car-like vehicles. The original formulation by [34] was a proof for geodesic minimal curve lengths given constraints on terminal positions and tangents, and curvature. The described proof showed that paths consisted of words made from straight segments $S$, and curved left $L$ and right $R$ segments could be used to find the minimal length curve. The result was all paths could have a resulting minimal path from six defined words: $RLR$, $LRL$, $RSL$, $LSR$, $RSR$ and $LSL$. These can be rewritten using the curved segments $C$ and straight segments $S$ for brevity, resulting in the words: $CCC$ and $CSC$.

The simplified car-like vehicle can be defined by the C-space $C = \mathbb{R}^2 \times S^1$. It follows that the prescribed use of $S$ and $C$ segments will accomplish this C-space. This, however, does not address the non-holonomic constraint. The Dubins’ path is only $C^1$ continuous, where a non-holonomic path requires at least $C^2$ continuity in the path geometry. There are a number of works that do not explicitly handle the curvature discontinuity [35–37], either assuming sufficiently large $\dot{\omega}$ that results in minimal deviation of the path or that the robot must stop at each turn.

To construct the Dubins’ path, the initial pose $q_i$, final pose $q_f$ and the minimum radius of curvature $R_{\text{min}}$ (or maximum curvature $\kappa_{\text{max}} = \frac{1}{R_{\text{min}}}$) is required. The circle of radius $R_{\text{min}}$ is centred at $O_1$ and $O_2$ for the initial and final turns. $O_3$ is reserved for the intermediate turn in the CCC type path. Refer to Figure 2.7 for the geometry adapted from [38].
Algorithm 1 summarises the construction of the Dubins’ path seen in [34]. If the initial and final heading are equal \( \theta_i = \theta_f \) then only a straight line is necessary. Reference circles of radius \( R_{\text{min}} \) are constructed tangent to both \( q_i \) and \( q_f \), where two tangential circles are possible for both poses. CCC type can only be constructed if a circle of radius \( R_{\text{min}} \) can be fitted tangent to any pair of initial and final circles. If the distance \( O_1O_2 = 2R_{\text{min}} \) then there is only one intermediate circle, if \( O_1O_2 < 2R_{\text{min}} \) then there are two circles, otherwise only CSC type paths exist. Where \( O_1O_2 \geq 2R_{\text{min}} \) CSC type is possible but there may be self intersection. The S segment is calculated by finding matching the tangent angles between the C segments. When all possible paths are found, determine the respective lengths and choose the shortest.

**Algorithm 1 Summary of Dubins’ Path construction**

1: if \( \theta_i = \theta_f \) and \( \angle q_iq_f = 0 \) then
2: Shortest path is a straight line
3: else
4: Construct a total of 4 circles of radius \( R_{\text{min}} \), tangent to \( q_i \) and \( q_f \).
5: for all Combinations of initial and final circles do
6: if \( O_1O_2 < 2R_{\text{min}} \) then
7: CCC type possible
8: Construct circles tangent to initial and final circles
9: Choose shortest path passing through 3 circles
10: end if
11: CSC type possible
12: Connect a straight line, tangent to initial and final circles and continuous \( \theta \)
13: end for
14: Choose word that results in the shortest path
15: end if
16: Output final shortest path

The construction of the Dubins’ path is simple, with the path construction consisting of matching angles from circles. Note that this algorithm does have an optimisation; however the optimisation runs with a worst case constant complexity of 8, where the 6 words are constructed and 2 extra for each CCC.

The Dubins’ path ensures optimal distance with little computation, however, cannot consider arbitrary angular accelerations \( \dot{\omega} \), where \( \dot{\omega} \) is considered to be either 0 or \( \infty \). This results in \( G^1 \). There is an additional assumption regarding linear velocity \( v \in \{0,1\} \) where
Figure 2.7: Example of Dubins’ path. a), b) CSC type, and c) CCC type paths.

$v = 0$ to achieve the discontinuous turn. This assumption is an artefact of adapting a mathematical proof for shortest distance into a path for a non-holonomic robot. This indicates stopping is required for every turn. If this behaviour is unwanted then the Dubins’ path is not truly holonomic, but is ‘friendly’ to non-holonomic robots if $\dot{\omega}_{\text{max}}$ is sufficiently large, such that deviations from the designed path is small.

2.3.2 Simple Continuous Curvature path

The Dubins’ path, although optimal and computationally cheap, does not strictly abide by the non-holonomic constraint due to the discontinuous curvature at the beginning and end of the circular arcs or turns. This issue was addressed by [39], creating the notion of continuous curvature (CC) turns to replace the circular arcs. However, by doing so the assumptions and geometry is changed and thus the optimality is lost. The computational benefits remain, with an increase in complexity depending on the type of CC turn used. This section will first describe the geometry of a CC turn, then the geometry of the path before providing an algorithm to construct the path.
A CC turn is constructed from 2 symmetric spirals and a circular arc. The start and end of the turn have 0 curvature, allowing straight segments or other CC turns to be appended with little issue. The spiral segments can use any spiral that has one end with 0 curvature. The two spirals used in recent literature are the clothoid or Euler spiral [39] and the Fermat’s spiral [40], a type of Archimedean spiral. This thesis will use clothoids with the form

\[
x = \sqrt{\frac{\pi}{\sigma_{\text{max}}}} F_{\text{RC}} \left( \sqrt{\frac{\theta_{\text{lim}}}{\pi}} \right)
\]

(2.7)

\[
y = \sqrt{\frac{\pi}{\sigma_{\text{max}}}} F_{\text{RS}} \left( \sqrt{\frac{\theta_{\text{lim}}}{\pi}} \right)
\]

(2.8)

where the Fresnel integrals are defined as

\[
F_{\text{RC}}(x) = \int_{0}^{x} \cos \frac{\pi}{2} u^2 du
\]

(2.9)

\[
F_{\text{RS}}(x) = \int_{0}^{x} \sin \frac{\pi}{2} u^2 du
\]

(2.10)

The clothoid is shaped by the two parameters: turning rate \( \sigma_{\text{max}} \) and the clothoid turning limit \( \theta_{\text{lim}} \). \( \sigma_{\text{max}} \) is the maximum turning rate of the vehicle, while \( \theta_{\text{lim}} = \frac{\kappa_{\text{max}}}{\sigma_{\text{max}}} \) represents the maximum angle a pair of symmetric clothoids with the maximum curvature \( \kappa_{\text{max}} \) can achieve. If \( \theta_d > \theta_i + \theta_{\text{lim}} \) where \( \theta_d \) is the desired heading then the CC turn will self intersect. In this case, assuming \( \theta_{\text{lim}} < \theta_{\text{lim}}^{\text{max}} \approx 1.46262 \pi \) [39], then an elementary path will be used.

The symmetry is ensured by the two construction circles centred at \( \Omega \), where one has the radius \( r_{\text{min}} \) corresponding to the circular segment of the CC turn and the second, larger circle with the radius \( R_t \). The ends of the CC turn start and finish on the circle \( \Omega R_t \) where the path at this point is not tangent to the circle. The angle \( \gamma \) between the heading at the end of the turn and the tangent to the circle \( \Omega R_t \) is used to determine \( q_3 \).

\[
R_t = \sqrt{(x_\Omega - x_3)^2 + (y_\Omega - y_3)^2}
\]

(2.11)

\[
\gamma = - \arctan \left( \frac{x_{\Omega R_t}}{y_{\Omega R_t}} \right)
\]

(2.12)
The junctions between spiral and circular arc, defined in [9], is illustrated in Figure 2.8. The configurations $q_s$, $q_1$, $q_2$, $q_3$ denote the initial configuration, the transition between spiral to circular arc, circular arc to spiral and spiral to next segment respectively.

Configuration $q_1$, Equation 2.13, is calculated using the definition of the clothoid, given the desired heading $\theta$ and turning rate $\sigma$. The clothoid is preserved throughout the path. This allows one calculation of the clothoid, requiring only translation and rotation to fit the section $q_2-q_3$

$$q_1(\beta) = \begin{cases} 
    x_1 = \sqrt{\frac{\pi}{\sigma}} \text{FrC} \left( \sqrt{\frac{\theta}{\pi}} \right) \\
    y_1 = \sqrt{\frac{\pi}{\sigma}} \text{FrC} \left( \sqrt{\frac{\theta}{\pi}} \right) \\
    \theta_1 = \frac{\theta_{\text{in}}}{2} \\
    \kappa_1 = \kappa_{\text{max}}
\end{cases} \quad (2.13)$$

The end of the CC turn $q_3$ may be calculated using the symmetry of the CC turn and the interconnecting heading $\beta$. The calculation of $\beta$ differs depending on the type of path used, TTT or TST.

$$q_3(\beta) = \begin{cases} 
    x_3(\beta) = R_t[\sin(\beta - \gamma) - \sin \gamma] \\
    y_3(\beta) = R_t[\cos \gamma - \cos(\beta - \gamma)] \\
    \theta_3(\beta) = \beta \\
    \kappa_3 = 0
\end{cases} \quad (2.14)$$

CC turns are constructed at $q_s$ and $q_f$. By equating the headings between segments TTT and TST paths can be constructed. The TST turn requires the distance between the circle centres to be sufficiently far $\Omega_1\Omega_2 > 2R_t$. It is sufficient to match the headings between the two CC turns to create the straight segment. The angles $\alpha_1$, Equation (2.16), and $\alpha_2$, Equation (2.15), describes the straight segment in relation to $\Omega_1$ and $\Omega_2$. The angle
\[ \alpha_\Omega = 2 \cos \gamma \frac{R_t}{\Omega_1 \Omega_2} \] (2.15)

\[ \alpha_1 = \alpha_2 - \gamma - \frac{\pi}{2} \] (2.16)

\( \alpha_{\Omega} \) is the angle between the horizontal axis and the line \( \Omega_1 \Omega_2 \), this, in conjunction with \( \alpha_1 \) will provide the heading of the S segment in the inertial frame. The TTT type turn will not be discussed in detail as it will not be used in the proposed trajectory planner in Section 5.2. The geometry of the path can be seen in Figure 2.9 [39]. Note that the path can be specified by finding the intermediate poses, and \( \Omega_1 \) and \( \Omega_2 \), allowing partial path construction during the optimisation (choice between words) process.

Figure 2.8: A CC turn. \( q_1 \) is the end of the first clothoid and start of the circular arc. \( q_2 \) is the end of the circular arc and start of the second symmetric spiral. Finally \( q_3 \) is the end of the second spiral, ending with 0 curvature.
The path has now been described geometrically and the algorithm Algorithm 2 is a summary of the construction provided in [39].

Due to the requirement of continuous curvature, the resulting SCC path is longer than the Dubins’ path, as seen in Figure 2.10. Here the length of the Dubins’ path $s_D$ is shorter than the length of the SCC path $s_{SCC}$, for a turning rate $\sigma_{max} = 0.1$. The difference in length varies with $\sigma_{max}$, the shape of the desired path may also change due to the choice of spiral.

The SCC path still assumes $v \in [0, 1]$, but addresses the other issues of the Dubins’ path noted in 2.3.1. The SCC path relaxes the Dubins’ path curvature constraint $\kappa \in \{0, \kappa_{max}\}$, allowing $C^2$ continuity. Since the SCC path is now $C^2$ continuous it fulfills our requirements to use as a basis for a non-holonomic trajectory planner. The SCC path will be extended in Chapter 5 to remedy the remaining velocity assumption.

2.3.2.1 Spirals

This thesis will use the following definition of a spiral: any curve that has a positive continuously changing curvature. The result is a non-integrable curve that will, in the
Algorithm 2 Summary of SCC Path construction

1: if \( \theta_i = \theta_f \) and \( \angle q_s q_f = 0 \) then
2: Shortest path is a straight line
3: else
4: Construct a clothoid using the equations (2.7) and (2.8)
5: for all Combinations of initial and final left and right clothoids do
6: Transform the clothoids appropriately
7: Find \( q_3 \) and \( q_4 \)
8: if \( O_1 O_2 < 2R_t \) then
9: TTT type possible
10: Construct circles tangent to initial and final circles of radius \( R_t \)
11: Construct a CC turn with terminal headings matching \( \theta_3 \) and \( \theta_4 \)
12: Choose shortest path
13: end if
14: TST type possible
15: Connect a straight line, tangent to initial and final circles and continuous \( \theta \), using the angle \( \alpha_1 + \alpha_O \)
16: end for
17: Choose word that results in the shortest path
18: end if
19: Output final shortest path

sense of a path, result in a left turn. Simply negating the the spiral function will result in a right turn. Note that this is not a typical definition of a spiral but a useful form in the motion planning sense. The following spirals will be discussed and are interchangeable in the SCC path:

- Clothoid or Euler spiral [39]
- Archimedean spiral [40]

And lastly a spiral of note that is not interchangeable:

- Logarithmic spiral

Note that this is not an exhaustive list and the provided definition does not represent all spirals, but provides a useful form.
Comparison between Dubins’ and SCC paths

$$\begin{array}{c|c|c}
\text{Dubins’ path} & \text{SCC path} & \text{terminal state} \\
\hline
s_D & s_{SCC} & q_s \rightarrow q_f \\
\end{array}$$

Figure 2.10: Comparison between Dubins’ and SCC paths. The distances for the Dubins’ and SCC path are $$s_D = 16.14$$, $$s_{SCC} = 22.08$$, respectively.

Example of an Euler Spiral

Example of an Archimedean Spiral

Example of a Logarithmic Spiral

Figure 2.11: Examples of different spirals. Note that Euler and Fermat’s spirals begin with zero curvature at the centre and continue in a clockwise direction. The logarithmic spiral is inward spiralling and does not have this zero initial curvature property. The Fermat’s spiral is a type of Archimedean spiral.

The clothoid is a spiral that has a curvature proportional to the length of the curve eqn. (2.17), where $$a$$ is an arbitrary parameter that defines the growth of curvature. This property is useful for finding arc lengths; however, the clothoid itself requires the calculation of Fresnel integrals. Note the eqns. (2.9) and (2.10) are specifically designed for the SCC
Chapter 2. Literature Review

Figure 2.12: Comparison of spiral curvatures. Euler and Fermat’s spirals begin with zero curvature, suitable for use in the SCC path. The Fermat’s spiral reaches high curvatures more quickly compared to the Euler spiral. The Euler spiral is able to reach any arbitrarily large curvature, unlike the Fermat’s spiral.

Path.

\[ \kappa = \frac{d\theta}{ds} = as \]  
(2.17)

Arithmetic spirals are less complex to calculate than the clothoid with the form in eqn. (2.18). Arithmetic spirals have the convenient property of equal distances between successive turns. This results in an initially high curvature, reducing as the spiral grows. A Fermat’s spiral, a type of Arithmetic spiral with \(a = 0, b = 1, c = 2\), is used in [40] to achieve sharper turns than through the use of clothoids.

\[ r = a + b\theta^\frac{1}{c} \]  
(2.18)

Logarithmic spirals differ from Arithmetic spirals in that the distance between successive turns increases. Logarithmic spirals are self-similar and found commonly in nature. The
logarithmic spiral is often used for biologically inspired, or representative, mechanisms or constraints. The notion most similar to our study is directional sensors, where the sensor is fixed and constrained to scan in an arc, requiring a logarithmic spiral motion to maintain perception of the target in the sensor arc. The equiangular property allows the sensor to maintain the same direction while approaching the target, where the angle between the tangent of the spiral and the line to the centre is the same at any point. The drawback of the logarithmic spiral is that the spiral does not begin with zero curvature. If the spiral is reversed (inward rotating) to gain zero curvature at the initial state then the spiral itself is slower to reach the same curvature as the previously mentioned spirals. The logarithmic spiral is not suitable for the purpose of a CC turn, however remains worthy of note.

2.3.3 Double Continuous Curvature path

The SCC path addresses issues with curvature discontinuity of Dubins’ path while resulting in a sub-optimal path. The Double Continuous Curvature (2CC) path [41] improves on the distance optimality by removing the symmetry constraint of clothoid pairs, optimising for the shortest length of the path. This, however, removes the unique form the SCC provides in favour of a family of solutions that may be more optimal. It must be noted that the 2CC path is used for path tracking, allowing correction from path deviations.

The Double Continuous Curvature path describes a concatenation of two Single Continuous Curvature turns. Originally these were termed DCC and SCC, though also citing Simple Continuous Curvature (SCC). To avoid this naming conflict, this thesis will use 1CC and 2CC for Single and Double Continuous Curvature respectively. Figure 2.13 provides an example of the 2CC path [41]. This family of paths use a similar set of segments as SCC paths; however the clothoids are not constrained to be symmetric or equal for both turns. Due to this arbitrary choice of clothoid the problem now becomes an optimisation problem and more closely resembles the optimal path of unbounded concatenations of clothoids [13]. 2CC paths outlined in optimises for distance (2.19) where \( n = 9 \) is the number of segments.
in the path.

\[ Q^* = \arg \min_Q \sum_{i=1}^{n} |l_i| \quad (2.19) \]

Figure 2.13: Example of a 2CC path.

The 2CC path is used for line tracking in AGVs. The system uses a look ahead approach, where a camera captures information on the desired path. When new features of the desired path are revealed the generated path is recalculated. This recalculation may occur from any arbitrary pose. This differs from the SCC path, where it requires zero heading at the initial and final poses. The 2CC is designed to generate a valid path given a reference without prior knowledge, whereas the SCC path generates a path given a desired destination. The 2CC path will not be used in this thesis, but a variant may be used in future for dynamic environments.

2.4 Summary

This literature review has revealed issues in the current state of safety interventions used for FLVs and deficiencies in the use and implementation of fully automated warehouses. The current safety interventions focus on the operator behaviour, enforcing accountability
of the operator through procedures. Reactive safety interventions can help within some operational bound, but may fail in extreme situations or if the operator does work within these bounds. The reactive system may, in fact, be a much more difficult problem than full automation, requiring prediction of inputs. Arguably, the fact that there is an operator results in accidents. AS/RS remove the operators entirely using fully automated systems. The current, commercially available AS/RS are expensive and require a purpose built environment. This proves to be a limitation due to the capital costs and solutions may not be suitable for particular industries.

This thesis will address the middle ground, semi-automation using retrofitted FLVs. Retrofitting vehicles is potentially less costly, and aimed to reduce changes in the environment. This aims to increase the availability to small and mid-sized companies who may not be able to afford AS/RS. The dynamic balance and path planners are used to automate the driving component, with higher level decisions left to an operator, who may be remotely operating a fleet of FLVs. To safely achieve motion, that is no toppling, we consider the dynamic balance of the system. We specifically outline the ZMP due to it’s ability to calculate the dynamic balance with only d’Alembert forces and similarities to some aspects of FLVs, which we will be further described in Section 3.4.
Chapter 3

Model of Forklift Vehicle Dynamics

This chapter addresses the physical model and the methods used to evaluate the dynamics of the FLV. The kinematic model is derived in Sections 3.2 and 3.3, with the latter specifically addressing the kinematics of the payload. This is followed by a description of dynamic balance and the subsequent derivation of the Zero Moment Point in Section 3.4. The Zero Moment Point is derived for arbitrary n linked systems; however, the focus in this work is an FLV with \( n = 1 \).

3.1 Overview of Forklift Vehicle Dynamics

The outcome of this study is intended for practical use in the industry, and so it follows that the mathematical model derived should sufficiently describe a large number of FLV types for a common work environment. The assumptions used throughout this thesis are listed below:

**Assumption 1.** Working environment is in a warehouse. Horizontal (planar) motion only, with a structured and known environment.

**Assumption 2.** Sufficient friction between tyres and ground, such that there is no slip.

**Assumption 3.** Load is rigid and there is sufficient friction to prevent slip.
Assumption 1 is to simplify the problem down to a common environment where FLVs are used. Warehouses typically require FLVs to move stock between transportation and storage, and for human workers to (un)load stock. These environments are highly structured with dynamic elements, such as pedestrians, stock placement, and other vehicles. For this study we will use only static environments with horizontal (no inclines) surfaces for FLV operation.

Assumption 2 ignores slipping between the contact patch of the tyre and the surface that is driven on. The no slip condition results in a more conservative estimate of the tipping propensity, where the slip may be beneficial by reducing the magnitude of accelerations.

Assumption 3 assumes once a payload has been lifted it cannot slip or change shape. Generally stock is bound such that the contents cannot move with respect to the pallet, hence the rigid load assumption is acceptable. Load slippage will not be covered in detail and not be considered in the trajectory planning algorithm. Section 4.6.2 will examine a simple model of load slip to compare modes of failure.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Parameter Name</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w$</td>
<td>track</td>
<td>Distance between the wheels in the lateral direction</td>
</tr>
<tr>
<td>$l$</td>
<td>wheelbase</td>
<td>Distance between the wheels in the sagittal direction</td>
</tr>
<tr>
<td>$m_b$</td>
<td>FLV mass</td>
<td>Mass of the FLV body only</td>
</tr>
<tr>
<td>$m_l$</td>
<td>Mass of payload</td>
<td>Mass of payload, static after lifting</td>
</tr>
<tr>
<td>$m$</td>
<td>Combined mass</td>
<td>Combined mass of FLV body and payload</td>
</tr>
<tr>
<td>$h$</td>
<td>Lift</td>
<td>Height of forks along the mast</td>
</tr>
<tr>
<td>$d_m$</td>
<td>Reach (mast)</td>
<td>Forward extension of the mast</td>
</tr>
<tr>
<td>$d_f$</td>
<td>Reach (fork)</td>
<td>Forward extension of the forks/carriage</td>
</tr>
<tr>
<td>$e$</td>
<td>Eccentricity</td>
<td>Lateral eccentricity (alignment) of forks</td>
</tr>
<tr>
<td>$\psi_m$</td>
<td>Mast angle</td>
<td>Tilt angle of mast</td>
</tr>
<tr>
<td>$\psi_f$</td>
<td>Fork angle</td>
<td>Tilt angle of forks and carriage</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Steering angle</td>
<td>Angle of the rear steered wheel</td>
</tr>
<tr>
<td>$\theta_m$</td>
<td>Mast heading</td>
<td>Angle of mast with respect to the FLV heading</td>
</tr>
</tbody>
</table>

Table 3.1: Summary of FLV parameters

The parameters listed in Table 3.1 are used to describe the current state of the FLV. Not all of the parameters are applicable for all classes of FLV, for example, $\psi_m$ and $\psi_f$ are mutually exclusive and used for the same purpose. The dynamic behaviour, however, differs between
the two, hence it is necessary to distinguish the two. Any unnecessary parameter can be set to an appropriate static value and will have no further effects on the system.

The mast and fork parameters will help define the CoG of the load and, depending on the type of model, the dynamics of the mast/load. The CoG of the load requires either
prior knowledge of the load or additional sensors. Assumptions can be made depending on working environment, for example warehouses typically have evenly distributed cartons stacked in a known pattern and to specified height, if the weight and CoG of a carton is known, then there is enough information to estimate the pallets weight and CoG. It is required that load not shift in this case, this assumption is reflected in many situations where the pallet is wrapped to prevent the cartons from movement.

The FLV to be considered in this thesis is the R1.4 reach truck, a class II truck. The reach truck is a type of electric forklift with a forward extending mast. The battery is used as a counterbalance. Reach trucks are designed for use in narrow-aisle warehouses, utilising the reach capability and small footprint to allow reduced aisle widths. Reach is used to describe a forward translating carriage or mast. Translating carriages are also know as pantograph reach trucks, with the carriage mounted on a pantograph, useful for deep shelves. The reach capability allows for smaller collision radius when the drawn in, while allowing the FLV to easily load and unload from shelves by extending. Extensive use of a reach truck in a warehouse allows design of aisles with possibly more storage space. The only feature the reach truck does not possess is the variable mast angle.

3.2 Kinematics of a Forklift Vehicle

The three wheeled and four wheeled forklifts are known to have a ‘stability triangle’ mentioned in forklift safety literature [5, 11]. This idea of a triangle for a three wheeled vehicle is simply due to the wheel arrangement. The four wheel case, the triangle results from arrangement of the rear axle, where a pin joint is used to connect the rear axle. The steering geometry can be reduced into a single effective steering angle, hence we choose a tricycle model to describe the kinematics, see Figure 3.2. Unlike a traditional tricycle model, this study will use the passive wheels as the front wheels, while the drive and steering wheel will be the rear.

It should be noted that the definition of turning radius here and that of FLV specifications differs. This thesis will use the turning radius to mean the arc at which the frame of the
Figure 3.2: Tricycle model of an FLV. Note this model uses the passive wheels as the front of the vehicle.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Parameter Name</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>Steering angle</td>
<td>Angle of the steered wheel</td>
</tr>
<tr>
<td>$R$</td>
<td>Turning radius</td>
<td>Radius with which the vehicle is turning</td>
</tr>
<tr>
<td>$\dot{\theta}$</td>
<td>Angular velocity</td>
<td>Angular velocity of the FLV</td>
</tr>
<tr>
<td>$\dot{\alpha}$</td>
<td>Steering angular velocity</td>
<td>Angular velocity of steered wheel</td>
</tr>
</tbody>
</table>

Table 3.2: Summary of kinematic parameters

FLV travels. FLV specifications [10] use the turning radius $W_\alpha$ to represent the minimum arc that the body turns when turning on the spot, for collision avoidance. The difference is illustrated in Figure 3.3 Collision avoidance will not be discussed in this thesis.

$R$ is defined as the distance from $O$ to the intersection between rays cast in the perpendicular direction of the wheels.

$$R = d \tan \delta$$

where $\delta = \frac{\pi}{2} - \alpha$. 
The linear wheel velocity $v_w$ is the tangential velocity calculated using the wheel radius $r_w$ and the wheels angular velocity $\omega_w$. We assume $\omega_w$ is known.

\[ v_w = \omega_w r_w \]

The angular velocity at the midpoint between the two undriven wheels is:

\[ \dot{\theta} = \frac{v_w}{R_w} \quad (3.1) \]

The velocities in the FLV frame $x_1y_1z_1$:

\[ v_{x_1} = v_w \cos \alpha \quad (3.2) \]
Assuming a zero slip condition:

\[ v_{y_1} = 0 \]  

(3.3)

Converting (3.2) and (3.3) into the inertial frame \( XYZ \) results in:

\[
\begin{align*}
\dot{v}_X &= v_{x_1} \cos \theta + v_{y_1} \sin \theta \\
&= v_w \cos \theta \cos \alpha \\
\dot{v}_Y &= v_{x_1} \sin \theta + v_{y_1} \cos \theta \\
&= v_w \sin \theta \cos \alpha
\end{align*}
\]

(3.4) (3.5)

We will now represent the kinematic model in configuration space, using equations (3.1), (3.4), (3.5):

\[
\begin{bmatrix}
\dot{X} \\
\dot{Y} \\
\dot{\theta} \\
\dot{\alpha}
\end{bmatrix} =
\begin{bmatrix}
\cos \theta \cos \alpha & 0 \\
\sin \theta \cos \alpha & 0 \\
1/R & 0 \\
0 & 1
\end{bmatrix}
\begin{bmatrix}
v_w \\
\dot{\alpha}
\end{bmatrix}
\]

(3.6)

We have now established the kinematic model in eq. (3.6); however, the risk toppling has yet to be considered. Section 3.4 will derive the zero moment point (ZMP) as a method to consider the toppling behaviour. The kinematics of the payload can be considered independently from the body and will be discussed in Section 3.3.

### 3.3 Kinematics of Forklift Vehicle Payload

The payload of an FLV is mounted on a moveable carriage, the components are described in Table 3.3. The mast is used to translate the carriage to allow reach of stacked or shelved goods. Eccentric movement of the carriage is used to align the forks with the pallet. The payload can be tilted in the \( y_1 \) axis to prevent the load slipping while decelerating or
operating on inclines; however, to the best of my knowledge, only the mast or carriage can be tilted, not both.

<table>
<thead>
<tr>
<th>Sections</th>
<th>Description</th>
<th>Moving parts</th>
</tr>
</thead>
<tbody>
<tr>
<td>Forks or Tynes</td>
<td>Payload is mounted on forks</td>
<td>Forks can be placed in different positions on the carriage, must be manually moved.</td>
</tr>
<tr>
<td>Carriage</td>
<td>Holds the forks</td>
<td>Carriages can be moved in the $z_2$ and, on some FLVs, in the $x_1$ direction. Some carriages can rotate in the $y_3$ axis.</td>
</tr>
<tr>
<td>Mast</td>
<td>Holds the carriage</td>
<td>Some masts can rotate in the $y_2$ axis or the $z_2$ direction.</td>
</tr>
</tbody>
</table>

**Table 3.3:** Summary of terminology and function FLV payload handling system.

The configuration of the tilt mechanism is important to calculate the position of the load CoG. The tilt angle for the mast $\psi_m$ and for the carriage $\psi_c$, height of tilt mechanism $h_\psi$ and finally the position of the payload CoG with respect to the carriage $3^2_rG_l$. Although it has been mentioned that the carriage and mast can not both tilt, the kinematics are defined assuming both are able. Both are defined individually for clarity.

$$
\begin{align*}
3^2_rG_l = \begin{pmatrix}
T \left(\begin{bmatrix}0, \psi_m, 0\end{bmatrix}^T, [0, 0, h]^T\right) & T \left(\begin{bmatrix}0, \psi_c, 0\end{bmatrix}^T, [0, 0, h_\psi]^T\right)
\end{pmatrix}
\end{align*}
$$

(3.7)

where the transform matrix $T$ is defined with the three dimensional rotation matrix $R(\beta)$ and translation $\mathbf{p}$:

$$
T(\beta, \mathbf{p}) = \begin{bmatrix}
R(\beta) & \mathbf{p} \\
0 & 1
\end{bmatrix}
$$

The kinematics defined in this section assumes the CoG of the load relative to the carriage can be estimated. This is achieved in practice by payloads stacked in a known and repeatable manner. However, the accuracy cannot be guaranteed. An additional source of error may occur when the payload is loaded onto the forks, where the load can very in both $x_1$
Figure 3.4: Schematic of the mast. Frame 1 is attached to the FLV as in Figure 3.2. The mast can extend with reach $d$ and lift $h$. Two tilt angles are defined $\psi_m, \psi_f$, where a forward tilt is defined as negative.

and $y_1$ directions. One method to solve this problem is to instrument the forks. Work on a three dimensional approximation of the payload CoG using strain gauges installed on the forks has been carried out by [42]. The payload kinematics will be used for the analysis of load slippage in Section 4.6.

### 3.4 Zero Moment Point and Forklift Vehicles

The ZMP derived in Section 2.2.1.1 considers the general case, a system with $N$ links and a SP with $M$ vertices. FLVs require only a limited number of links and vertices to sufficiently
describe the system.

Since the FLV has two sections that may be moved with significant inertia, the chassis and mast, then $N = 2$. The payload may be combined with the inertia with the mast under the assumption that the payload will not move with respect to the forks, i.e. no load slippage. This thesis will simplify the problem to $N = 1$. This will reduce the number of measurements and sensors required. This simplification requires the assumption that the payloads dynamics relative to the FLV is small.

\begin{align*}
y_{ZMP} &= \frac{y_{A}ma_{z} - z_{A}ma_{y} + \dot{H}_{x}}{ma_{z}} \\
x_{ZMP} &= \frac{x_{A}ma_{z} - z_{A}ma_{x} - \dot{H}_{y}}{ma_{z}} \\
z_{ZMP} &= 0
\end{align*}

To complete the ZMP description we must describe the support polygon SP. The SP may be sufficiently described with $M = 3$ for the class of FLV described in Section 3.1. The three point contact model was found suitable to describe FLVs in Section 3.2 due to the placement of wheels in 3-wheeled FLVs and due to the physical arrangement of 4-wheel FLVs, where the rear axle is attached by a pin joint oriented in the forward direction, resulting in a similar triangular configuration when considering toppling. Here we observe parallels between the ZMP and the currently taught ‘stability triangle’ mentioned in FLV safety literature [5, 43]. The safety courses describe the prevention of toppling by calculating the effective CoG with respect to the safety triangle, taking into account loads and their height. This is a simplified, static version of the ZMP.

3.5 Summary

This section has established a kinematic model and the ZMP for use with an FLV. The kinematic model is based on the tricycle model with a modification of the forward direction.
Chapter 3. Model of FLV Dynamics

The choice of the tricycle model is suitable for Class I-IV FLVs, where the steering geometry may be modelled with three wheels and corresponds with the stability triangle. The ZMP is modelled with only one link and uses an SP with three ground contacts. The concept of the ZMP is similar to existing safety guidelines, where a safety triangle is observed to check toppling. The current guidelines is useful only in static situations, where it is possible to calculate by the operator. The ZMP extends this notion to include vehicle dynamics.

The usage of d’Alembert’s forces to calculate the ZMP results the need to distinguish between the ZMP and FZMP. The existence of the ZMP can be determined by relating the calculated point $r_{ZMP}$ to the SP. Chapter 4 describes a method of relating the SP to $r_{ZMP}$ using a barycentric coordinate system, and further extending this to construct a novel dynamic balance margin, providing a scalar measure of toppling propensity. The dynamic balance margin will be used to evaluate trajectories and states, and will be later used to calculate safe trajectories in Chapter 4.
Chapter 4

Dynamic Balance Margin

The model formulated in Chapter 3 will be utilised in this chapter. The Zero Moment Point (ZMP) derivation, Eqn. (3.8) - (3.10), can not distinguish between the fictitious ZMP (FZMP) or the real ZMP. This issue will be solved by relating the ground contacts of a system to the formulated ZMP through the use of a barycentric coordinate (BC) system. This coordinate system will then be used to formulate a scalar measure of toppling propensity: a dynamic balance margin. Section 4.5 will use this measure to evaluate known trajectories. This chapter aims to develop a scalar toppling propensity measure, usable with the ZMP derived in Chapter 3.

4.1 Dynamic Balance Margin and the Zero Moment Point

The dynamic balance margin (DBM) is termed stability margin in [30, 32, 44]; however for consistency we will replace 'stability' for 'dynamic balance'. Previous work regarding DBM focused on just finding the DBM. The intention in this thesis is to design a DBM using information that can be used for other calculations. By doing so, computational resources can be saved by reuse of information.

The DBM $\Xi$, described previously in Section 2.2.2, provides a scalar measure of toppling propensity. Due to the existence of the FZMP, a relationship between $r_{ZMP}$ and the ground
contacts of the FLV $C$, where $M$ is the number of vertices in $C$, is required, such that $\Xi$ is bound by:

$$\Xi = f(r_{ZMP}, C), \; \Xi \in [0, 1]$$

(4.1)

where:

$$f : (\mathbb{R}^3, \mathbb{R}^M) \to \mathbb{R}$$

For $\Xi = 0$ the system is considered dynamically unbalanced and $\Xi = 1$ is the most balanced state. $\Xi \leq 0$ corresponds to the existence of the FZMP, that is when $r_{ZMP}$ is on the exterior, or edge of the SP defined by $C$.

The problem is now framed as a normalised system that requires a point to be found within some convex polygon. One solution to this problem is to use a barycentric coordinate system (BCS). BCSs define a normalised coordinate system in the interior of a convex polygon. Outside of the polygon the BCS is not necessarily well defined, except for the case of a triangle $M = 3$. The ground contacts $C$ may be used as the vertices of the BCS polygon, with $r_{ZMP}$ mapped to the BC $\lambda$; hence we obtain a relationship between $r_{ZMP}$ and $C$. Section 4.2 will describe the BCS in more detail, while Sections 4.3 and 4.4 will be used to define $\Xi$.

### 4.2 Barycentric Coordinates

Qualitatively, a BCS uniquely describes the interior of a convex polygon with the number of coordinates equal to the number of vertices of the polygon $M$. The BCS is normalised with respect to the boundary of the polygon. The Cartesian point $p$ is mapped to the barycentric coordinate $\lambda$.

The barycentric coordinate $\lambda = \{\lambda_1, \lambda_2, \ldots, \lambda_M\}$ describes a point in relation to $C = \{c_1, c_2, \ldots, c_M\}$ of the convex polygon with $M$ vertices. Each coordinate $\lambda_j$, where $j = \ldots$
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1, 2, \ldots, M, corresponds to the normalised distance from the vertex \( c_j \). Note that ‘distance’ is used loosely here and does not mean the Euclidean distance. When \( \lambda_j = 1 \) and \( \lambda_k = 0, \forall k \neq j \), then \( p \) is on the vertex \( j \). When two adjacent coordinates \( \lambda_j = \lambda_{j+1} \neq 0 \) and \( \lambda_k = 0, \forall k \neq \{j, j+1\} \) then \( p \) lies on the edge \( c_j c_{j+1} \).

The point \( p \) is related to the barycentric coordinate system by the following linear equation:

\[ p = \sum_{j=1}^{M} \lambda_j p_j \tag{4.2} \]

Since barycentric coordinates are considered a normalised coordinate system, \( \lambda \) must satisfy:

\[ \sum_{j=1}^{M} \lambda_j = 1 \tag{4.3} \]

Barycentric coordinates are well defined strictly within the convex polygon and are, thus, considered valid only if:

\[ \forall j \lambda_j \in (0, 1] \tag{4.4} \]

Points lying outside or on the edge of the convex polygon do not satisfy the Equation (4.4). Section 4.4 will make use of this property to determine if the ZMP or FZMP exists. This thesis will explore two generalised BCSs and finally a simpler formulation for the case of a triangle. \( \Xi \) will be defined in two forms to exploit the properties of the BCS, each with its own particular uses.

### 4.3 Generalised Barycentric Coordinates

A generalised BC system can be used to define any arbitrary polygon of \( M \) vertices. This thesis will not discuss the implications of non-convex polygons in relation to the BCS as
the SP is strictly a convex polygon.

There is no unique BC system for \( M > 3 \) and are termed generalised BC systems. This thesis will explore two of these generalised systems: Wachspress coordinates (WC) [45] and mean value coordinates (MVC) [46]. The construction of these coordinate systems will be described before comparing their use in calculating \( \Xi \). The construction of each coordinate system will only be discussed briefly. Generalised barycentric coordinates is defined in [47] as:

\[
\lambda_j = \frac{w_j}{\sum_{k=1}^{M} w_k}
\]  

Coordinate systems differ with the definition of \( w_j \), which may be loosely be interpreted as the weights of each vertex, including the previously mentioned WC and MVC systems. These systems will be defined in Sections 4.3.1 and 4.3.2 respectively.

Before describing the two methods, some notation must first be defined. Figure 4.1 graphically represents the necessary parameters. The vertices are ordered in a counter-clockwise direction, with each vertex \( c_j \) and \( p \) defining the signed areas \( A_j, B_j, C_j \), and three angles \( \alpha_j, \beta_j, \gamma_j \) are defined with respect to the point \( p \) and vertices \( c_j \).

**Figure 4.1:** Notation used for convex polygons, adapted from [47]. Three signed areas \( A_j, B_j, C_j \) and three angles \( \alpha_j, \beta_j, \gamma_j \) are defined with respect to the point \( p \) and vertices \( c_j \).
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(4.6), where $\Delta$ is the area of a triangle and $u, v$ are vectors comprised of pairs three adjacent vertices. The angles $\alpha_j, \beta_j, \gamma_j$ can be calculated using Equation (4.7), where $\delta$ is the angle $\angle uv$.

$$\Delta = |u \times v|$$  \hspace{1cm} (4.6)

$$\delta = \cos^{-1}\left(\frac{u \cdot v}{||u|| ||v||}\right)$$  \hspace{1cm} (4.7)

Sections 4.3.1 and 4.3.2 will present the BCS using three quadrilaterals, from a regular square to a highly skewed quadrilateral, and additionally an irregular hexagon. These polygons will be used to investigate the behaviour of the two generalised BCS. The contour plots found in Sections 4.3.1 and 4.3.2 are limited to a contour magnitude of two. Magnitudes greater than two are ignored for readability, where the peaks of infinite magnitude provide little information for our purposes.

4.3.1 Wachspress Coordinates

The Wachspress coordinates makes use of signed areas to define $w_j$, Equation (4.8) [47]. The areas $A_j$ and $B_j$ are defined in Figure 4.1 using Equation (4.6).

$$w_j = \frac{A_j + A_{j-1} - B_j}{A_j A_{j-1}}$$  \hspace{1cm} (4.8)

The WCS is affine invariant [47] providing a predictable mapping, particularly useful with the scaling transformation. For use with FLVs, the scaling transformation can correspond to a change of units, most commonly between metric and US customary systems.

Figure 4.2 compares three different quadrilaterals and two of their coordinates, $\lambda_1$ and $\lambda_3$. The value of each individual coordinate begins at 1 at the associated vertex $c_i$, reducing to 0 for all edges not adjacent to $c_i$. The distribution of $\lambda_1$ in Figures 4.2a-4.2e are approximately evenly distributed over the polygon, whereas $\lambda_3$ is more concentrated towards
Figure 4.2: Comparison of WC contours for quadrilaterals of varying skew for vertices $c_1$ (a-c) and $c_3$ (d-f). The black circle represents the vertex corresponding to the coordinate used for the contours.

c_3$, a feature particularly emphasised in Figure 4.2f. This can also be seen in an irregular hexagon, Figure 4.3, at vertices $c_2$, $c_5$, $c_6$ and to a lesser extent at $c_3$.

The irregular hexagon case, Figure 4.3, highlights the behaviour of the WC system outside the polygon, and the complexity of the manifolds. The proximity of the inflection points to the polygon edge may cause issues with a practical implementation of an optimisation, where local minima may be too close to each other for the optimisation to perform correctly.

The WC have undesirable zero manifolds outside of the polygon, seen in both Figures 4.2 and 4.3. These result in non-unique coordinates outside of the polygon, which are not desirable for use in gradient based optimisation. This undesirable quality will be discussed in more detail in Section 4.3.3, after the coordinates are processed into a dynamic balance margin. The next section will explore the MVC, which does not have the issue of non-unique coordinates.
Figure 4.3: WC of an irregular hexagon shown by the dotted line. The respective vertex is marked by circle. The contours are limited to the range $[-2, 2]$. The coordinate is strictly decreasing from the respective vertex to the edge of the polygon. However, outside the polygon, the coordinates are not well defined. Saddle points can be seen in each of the coordinates, these indicate non-unique values.
4.3.2 Mean Value Coordinates

Mean Value Coordinates (MVC) are an approximation of harmonic coordinates using the mean value theorem [47]. MVC has a number of interesting properties, those of importance to this work is listed below:

1. Similarity invariance

2. $C^\infty$ everywhere excluding the vertices $C$

The MVCs are calculated using the following equation:

$$w_j = \frac{\tan(\alpha_j/2) + \tan(\alpha_{j+1}/2)}{||c_j - p||} \quad (4.9)$$

Similarity invariance, property 1, is defined by [46] as a transformation using ‘a translation, rotation, reflection, uniform scaling or combination of these’. Translations and rotations are particularly useful for vehicles, where invariance of the contours may be assumed regardless of pose or scaling factors. Property 2 indicates there are no discontinuities outside the polygon, unlike the WC.

One issue with using the MVC is the lack of a well defined centroid. Here ‘well defined centroid’ is defined as $\lambda_j = \lambda_k$. This property is used to calculate a normalising factor for $\Xi$. However, we note that this becomes a problem only when the shape is highly skewed or irregular.

Previously with WCs, an uneven distribution of $\lambda$ is observed for skewed or irregular polygons and non-unique coordinates on the exterior of any polygon. The uneven distribution is observed to a greater extent in the MVC, Figures 4.4 and 4.5. However the MVC are unique over the entire space. The MVC requires the calculation of tangents, which may result in slower speeds compared to the calculation of the cross product in WCs. The slight increase in computation time may impact negatively on the system if the coordinates must be calculated in large numbers, as is possible in the iterations of an optimisation or if the system uses a short sample time.
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The WC has a more even distribution over the interior of skewed polygon, while the MVC has a well-defined exterior. These behaviours will have implications when applied to the two proposed BDBM in the following section. The behaviour of the four systems will be examined, with a brief analysis.

### 4.3.3 Comparison of Dynamic Balance Margins

The formulation of the Dynamic Balance can be varied by changing the BC system used and by how the coordinates $\lambda$ are combined to form the scalar measure. We defined two
Chapter 4. *Dynamic Balance Margin*

Figure 4.5: Mean Value Coordinates of an irregular hexagon shown by the black dotted line. The respective vertex is marked by a black circle. The contours are limited to the range $[-2, 2]$. The coordinate is strictly decreasing from the respective vertex to the edge of the polygon. Does not contain saddle points as in WC. For $\lambda_2$, $\lambda_5$, $\lambda_6$ the coordinates are observed to be heavily weighted towards the coordinate.
novel barycentric DBM (BDBM) below:

\[ \Xi_p = A \prod_{i=1}^{m} \lambda_i \]
\[ = An^m \prod_{i=1}^{n} \lambda_i \] (4.10)
\[ \Xi_m = A \frac{\min(\lambda)}{n-1} \]
\[ = An \min(\lambda) \] (4.11)

where the negativity test \( A \):

\[ A = \begin{cases} 
-1 & \exists \lambda_j \leq 0 \\
1 & \text{otherwise} 
\end{cases} \] (4.12)

The two BDBM defined in Equation (4.10) and (4.11) are normalised using the presumed centroid value \( \lambda_i = \frac{1}{n}, \forall i \in [1, m] \). This centroid value is true for triangles, and regular arbitrary \( M \) regular polygons using WC and MVC systems. Here the normalisation factor is chosen for a triangular system, which is representative of FLVs. The analysis will focus on the differences between \( \Xi \) and the two BC systems for quadrilaterals.

\( \Xi_m \). Zero only along the edge of the polygon and positive in the interior.

\( \Xi_p \). Smooth contours. Zero along the lines that make up the polygon.

**Figure 4.6:** BDBM using WC system for a regular quadrilateral.
(a) $\Xi_m$. Zero only along the edge of the polygon. Contours have smaller gradient compared to WC case.

(b) $\Xi_p$. Smooth contours. Zero along the edge and away from the vertex. Zero lines away from polygon do not intersect, important for polygons with more vertices.

**Figure 4.7: BDBM using MVC system for a regular quadrilateral.**

Figures 4.6 and 4.7 show a contour map of a regular quadrilateral while Figures 4.8 and 4.9 are contour maps of irregular quadrilaterals. Each figure is divided into two, with plots of both $\Xi_p$ and $\Xi_m$. $\Xi_p$ provides a smooth contour. As $\Xi_p \to 0$ or $\Xi_p \to 1$ the gradient tends to zero.

The product operation used in $\Xi_p$, Figures 4.6b, 4.7b, 4.8b and 4.9b, results in lines of zero extending from the polygon edge. The lines of zero for WCs are collinear with the edges of the polygon. The MVC case has zero lines extending from the vertex away from the polygon. These zero lines may intersect when $M > 4$ for WCs. Optimisations may have reduced performance due to the existence of zero outside the polygon.

The maximum value presented in Figures 4.8 and 4.9 are not 1 suggesting the expected centroid value $\lambda_j = \lambda_k$ is not true for arbitrary $M$ polygons. Note that these calculated maximum values are subject to discretisation errors. Larger discrepancies result in difficulties comparing the DBM between systems, since the maximal value of the system is not known.

The degree of stability, DoS, detailed in [29] will produce contours of concentric SPs and is simply calculated by $\Xi_{DoS} = \min(D_{ZMP})$, where $D_{ZMP}$ is the set of distances to the edges of the SP from ZMP. The DoS is similarly easy to calculate and readily normalised.
(a) $\Xi_m$. Contours on the exterior are notably different, with uneven gradients. Maximum value is 0.98, relatively small discrepancy in centroid value.

(b) $\Xi_p$. Zero lines intersect, this becomes more pronounced for increased vertex counts. Maximum value is 0.99.

**Figure 4.8:** BDBM using WC system for an irregular quadrilateral.

(a) $\Xi_m$. Maximum value is 0.95, suggests that there is no clear centroid. Discrepancy becomes larger with more vertices.

(b) $\Xi_p$. Maximum value is 0.99. Discrepancy with expected centroid value is small.

**Figure 4.9:** BDBM using MVC system for an irregular quadrilateral.

as in the proposed BDBM; however lacks consideration of all edges simultaneously. Figure 4.10 compares the case of $\Xi = 1$, where the BDBM provides more information regarding balance in the direction of the rectangles long axis.

This section has explored different implementations of a BDBM and has shown the freedom of choice, depending on the necessary contours required or desired. Other BC systems or formulation of $\Xi$ may be introduced independently with few changes to the algorithm, if
any, required. The BDBM provides more information than the DoS; however is unlikely to provide any computational improvements due to the simplicity and similarity of calculations. The estimate of the centroid remains useful for \( \Xi_p \), but may cause issues when polygons are more greatly skewed. The estimation of the centroid remains useful with its simplicity. BCs are unique for \( M = 3 \) and have a well defined centroid. This triangular system will be discussed in the following section.

4.4 FLVs and Triangular Barycentric Coordinates

General purpose FLV are most commonly constructed with no suspension and an articulated rear axle with a single point of connection, resulting in this notion of the ‘stability triangle’ in forklift safety literature [5, 7, 11]. In this case, the SP will be constructed from the wheel contacts with the ground; however the contact patch of the wheel is not a simple polygon. Using the centres of the wheel, a conservative approximation of the SP can be made while creating a triangle. A triangle is useful in the formulation of the barycentric coordinate as there is a unique solution. Another reason for defining a triangular coordinate system is to make use of a simplified formulation, which will be outlined in this section.

The triangular barycentric coordinates for the point \( p \) is defined as:

\[
p = \lambda_1 c_1 + \lambda_2 c_2 + \lambda_3 c_3
\] (4.13)
Since $\lambda_1, \lambda_2, \lambda_3$ must satisfy the unity constraint:

$$\lambda_1 + \lambda_2 + \lambda_3 = 1$$  \hspace{1cm} (4.14)

Thus $\lambda_3$ can be found by rearranging (4.14):

$$\lambda_3 = 1 - \lambda_1 - \lambda_2$$  \hspace{1cm} (4.15)

Expanding (4.13) into its $x$ and $y$ components:

$$x = \lambda_1 x_1 + \lambda_2 x_2 + \lambda_3 x_3$$

$$y = \lambda_1 y_1 + \lambda_2 y_2 + \lambda_3 y_3$$

Where $x_i$, $y_i$, and $z_i$ correspond to the Cartesian coordinates of the vertex $c_i$. 

**Figure 4.11**: Example of a triangular barycentric coordinate system. The direction a point outside the triangle can be determined by the sign of the coordinates.
Substituting (4.15) results in a mapping from $\lambda$ to $p$:

\[
x = \lambda_1 x_1 + \lambda_2 x_2 + (1 - \lambda_1 - \lambda_2)x_3
\]

(4.16)

\[
y = \lambda_1 y_1 + \lambda_2 y_2 + (1 - \lambda_1 - \lambda_2)y_3
\]

(4.17)

Letting $p = p_{ZMP}$ and solving (4.16) and (4.17), we are able to find $\lambda_1$ and $\lambda_2$ in terms of the ZMP coordinates $(x_{ZMP}, y_{ZMP})$, subsequently finding $\lambda_3$:

\[
\lambda_1 = \frac{(y_2 - y_3)(x_{ZMP} - x_3) + (x_3 - x_2)(y_{ZMP} - y_3)}{(y_2 - y_3)(x_1 - x_3) + (x_3 - x_2)(y_1 - y_3)}
\]

(4.18)

\[
\lambda_2 = \frac{(y_3 - y_1)(x_{ZMP} - x_3) + (x_1 - x_3)(y_{ZMP} - y_3)}{(y_2 - y_3)(x_1 - x_3) + (x_3 - x_2)(y_1 - y_3)}
\]

(4.19)

\[
\lambda_3 = 1 - \lambda_1 - \lambda_2
\]

(4.20)

The product BDBM, Equation (4.10), will be used for the triangular system:

\[
\Xi_p = A \frac{\lambda_1 \lambda_2 \lambda_3}{3}
\]

(4.21)

Where $A$ is defined in Equation (4.12).

This formulation creates an upper bound $\Xi \leq 1$ with any $\Xi \leq 0$ equating to a tip over condition. Reiterating Section 4.1, $\Xi = 0$ denotes the degeneration of the ZMP and $\Xi < 0$ denotes the FZMP. Directionality may be obtained through $\lambda$. Direction of failure is not used directly in this work; however is observed in Section 5.3. Sections 4.5 and 4.6 will explore different practical uses of the BDBM.

### 4.5 Trajectory evaluation using the Dynamic Balance Margin

One potential use of the BDBM is to evaluate known trajectories. Evaluating trajectories may be used in a post processing stage for ensuring planned trajectories do in fact fulfil
the dynamic constraints or to evaluate a logged trajectory. The latter is usable in industry, where a logged trajectory may be used to evaluate an operator’s performance. For example, if the operator was involved in an accident or near miss incident then the logged data may be used to determine the accountability of the operator by identifying past driving behaviour.

This section will examine trajectories generated using a polynomial of order seven, Figures 4.12 and 4.13, and nine, Figures 4.14 and 4.15. The trajectory is constructed using known terminal states. The derivatives of the polynomial are used as velocities and accelerations. This trajectory is not intended to account for the dynamic balance constraints but does fulfil the $C^2$ constraint.

The polynomials are constructed using a general polynomial equation of order $n = 7, 9$:

$$x = a_0 + a_1 t + a_2 t^2 + \ldots + a_n t^n$$

(4.22)

Coefficients $a_0 - a_5$ is used to define the terminal configurations: position, velocity and acceleration. Coefficients $a_6 - a_7$ are used to shape the path. Figure 4.12 and 4.13 contains two turns and almost three almost straight sections linking the turns. The second turn is sharper than the first, with both turns causing the DBM condition to fail.

In the two cases, using the same coefficients but assigning a longer time to complete the path, Figure 4.13 and 4.15, allows the FLV is able to achieve the turns without violating the DBM. This suggests that a simplistic approach would to plan a trajectory would be to use a polynomial and increase the time the time component as necessary until the system does not violate the dynamic constraints. Consequently, long completion times may be planned, with little ability to control completion time.

Similar to the previous case, Coefficients $a_6 - a_5$ is used to define the terminal configurations, while coefficients $a_6 - a_9$ are used to shape the path. Figure 4.14 and 4.15 contains one continuous left turn. Two sections of the trajectory have increasing curvatures, with the later turn, at approximately $10.1 s$, resulting in failure. Increasing the completion time to $15 s$ results in a safe path.
Figure 4.12: Trajectory formed by a polynomial of order seven. *Left* The path followed by FLV (triangle), with highlighted sections indicating an unbalanced state. *Right* dynamic balance over time. The vertical highlights represent an unbalanced state. The trajectory is completed over 12 seconds.

Figure 4.13: Trajectory formed by a polynomial of order seven. *Left* The path followed by FLV (triangle). *Right* dynamic balance over time. The trajectory is completed over 12.7 seconds. The longer completion time resulted in no unbalanced state.
Figure 4.14: Trajectory formed by a polynomial of order nine. Left path followed by FLV (black triangle), bolded red lines indicate an unbalanced state. Right dynamic balance over time. The trajectory is completed over 12 seconds.

Figure 4.15: Trajectory formed by a polynomial of order nine. Left path followed by FLV (black triangle). Right dynamic balance over time. The trajectory is completed over 15 seconds. No unbalanced state detected.
This section has described one possible method of use for the DBM, as an evaluator of a known trajectory. This is a typical use case for motion planners, where the construction of a trajectory may be iterated over using the DBM as a constraint. However, the DBM may be used in an industrial setting, evaluating telemetry of real vehicles in the same manner that one would evaluate a computational trajectory, allowing safety interventions to be developed. Evaluating a known trajectory is, however, a reactive approach, where the state evaluated is the current or past state. Reacting to an event restricts what safety interventions may be implemented.

One implementation is a warning indicator for the operator, where the current DBM can be actively displayed. In this implementation, it may be useful to use $\Xi_p$, due to the smaller gradients as $\Xi_p \to 0$. This may provide an earlier indicator of approaching danger. Another possible usage is to determine the behaviour of an operator as an evaluation of performance or during an investigation after an incident. If the state of the vehicle is logged, the data may be used to indicate whether the operator consistently drives safely or not.

The next section will describe the use of the DBM to computationally study the changes in behaviour of an FLV. The ability to study the behaviour computationally may be useful for designing FLVs and motion planning algorithms.

### 4.6 Dynamic Balance Margin as an analysis tool

The DBM is a useful tool in an offline setting. The DBM can be used to study the effects of load changes with a simplistic representation. One possibility is to study new motion planning strategies or evaluate vehicle designs with respect to a set of required configurations.

This section will analyse two aspects of FLV operation: dynamic balance sensitivity to load position and comparison of deceleration failure conditions. The sensitivity to load positioning will explore changes in load height, reach and eccentricity and the effects on the envelope of dynamically balanced operation. This analysis can be used to determine
operating procedures when moving loads. Another aspect that is explored is the degree of change that occurs if the loads CoG is not where it is predicted. The load eccentricity is of particular importance due to the resulting asymmetry in the feasible inputs.

The second analysis focuses on two types of deceleration failure: forward tip-over and load slippage. Although the two modes of failure are coupled, we will consider the instance of one condition occurring to result in failure. Failure as a result of either mode will result in a less desirable state, with the position of the combined CoG moving forward in both cases. Although it is possible to recover from such a situation, the temporary loss of control, of the load or due to wheel lift, is deemed dangerous and unwanted. This analysis provides compares deceleration and mast tilt angle, given a load configuration.

The analysis will be conducted using the model FLV that is currently being built with the help of Speedshield Technologies and James Hancock, shown in Figure 4.16. The model has a mass of 8kg and is expected to carry a maximum of 4kg, a similar ratio to that of real FLVs. The physical limitations of the model FLV, e.g. maximum lift and reach, will be disregarded during the analysis to emphasise certain behaviours of the trajectory generation algorithm.

### 4.6.1 Effects of loading scenarios on Dynamic Balance

The load on an FLV can be translated in three directions $x_1$, $y_2$, $z_2$ and rotated in one direction $\psi$. Changes in $y_2$ and $\psi$ is often small in comparison to the translations in $x_1$ and $z_2$ directions. For convenience, Table 4.1 summarises the names used for each translation and rotation.

<table>
<thead>
<tr>
<th>Direction</th>
<th>Name</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_1$</td>
<td>Reach $d$</td>
</tr>
<tr>
<td>$y_2$</td>
<td>Eccentricity $e$</td>
</tr>
<tr>
<td>$z_2$</td>
<td>Lift $h$</td>
</tr>
<tr>
<td>$\psi$</td>
<td>Tilt Angle</td>
</tr>
</tbody>
</table>

Table 4.1: Summary of translations and rotations and their respective names.
Figure 4.16: Schematic of model FLV.
The analysis in this section will examine the lift $h$ and eccentricity $e$ with respect to acceleration $\ddot{r}$ and velocity $\dot{r}$. The steering angle $\alpha$, turning rate $\sigma$ will not vary in an individual figure. Four loading scenarios will be explored, summarised in Table 4.2. The effects of tilt angle will be explored separately in Section 4.6.2.

Scenario 1, Figures 4.17a - 4.17d, represents the model FLV without a load. When $\alpha = 0^\circ$, the FLV is more prone to tip-over with a positive acceleration than a negative acceleration. This is a result of the rear placement of the CoG used to counterbalance the vehicle when handling loads. While turning, there is symmetry across $\alpha = 0^\circ$. The plot for $\alpha = 90^\circ$ is not shown as this case is equivalent to $\alpha = 90^\circ$ with negative velocity. When $\alpha = \pm 45^\circ$ acceleration provides the same effect as seen in the $\alpha = \pm 90^\circ$ case; however, the range of acceptable velocity is significantly reduced. As $\alpha \rightarrow \pm 90^\circ$, the range of acceptable velocities increases.

When $\alpha = 0^\circ$ there is no rotational dynamics, resulting in the velocity limits of $\pm \infty$. The triangular shape of the workspace at $\alpha = \pm 45^\circ$, is a result of the triangular SP. The triangular base impedes turning performance when accelerating forward, but not when reversing. This is seen in Figures 4.17b and 4.17d as a narrowing velocity limit as acceleration increases. When $\alpha = \pm 90^\circ$ the FLV will turn with $R = 0$ m, with the acceleration and velocity components only affecting the angular dynamics. Since $R = 0$, the case when $\alpha = 90^\circ$ and $\dot{r} > 0$ is equivalent to $\alpha = -90^\circ$ and $\dot{r} < 0$, resulting in the symmetry seen in Figure 4.17a.

Scenario 2 adds a load, of mass $m = 4$ kg, placed at $h = 0.1$ m and placed $d = 0.1$ m, seen in Figure 4.17e - 4.17h. From a glance, the acceptable range of inputs has increased with the addition of the load. The range of acceleration at $\alpha = 0^\circ$ is markedly improved, particularly

<table>
<thead>
<tr>
<th>Scenario</th>
<th>$h$</th>
<th>$e$</th>
<th>$m_l$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0</td>
<td>0.0</td>
<td>0.00</td>
</tr>
<tr>
<td>2</td>
<td>0.1</td>
<td>0.0</td>
<td>4.00</td>
</tr>
<tr>
<td>3</td>
<td>0.9</td>
<td>0.0</td>
<td>4.00</td>
</tr>
<tr>
<td>4</td>
<td>0.9</td>
<td>0.2</td>
<td>4.00</td>
</tr>
</tbody>
</table>

Table 4.2: The loading scenarios used in Figures 4.17 and 4.18
Figure 4.17: Comparison of dynamic balance margin. Scenario 1 and 2 is represented on the left and right columns respectively. Red corresponds to $\Xi = 0$ while blue corresponds to $\Xi = 1$. 
allowing more forward acceleration. The velocity limits seen at $\alpha = \pm 45^\circ$ is marginally larger than those in Scenario 1. The improvement in forward acceleration capabilities is seen across all $\alpha$. The velocity limits are also increased, with minor improvements at $\alpha = \pm 45^\circ$, to a more significant 3 m/s at $\alpha = \pm 90^\circ$, across all acceleration ranges.

There is an overall improvement in the FLV's capabilities when the load is added. The additional load also shifts the most balanced location toward the positive acceleration direction, corresponding to moving the CoG to the front of the vehicle resulting in increasingly poorer reversing capabilities. The improvement in the velocity range corresponds to the ability to turn at higher velocities.

Scenario 3, Figures 4.18a - 4.18d, increases the height of the load to $h = 0.9$ m, with all other factors remaining the same. This scenario shows a clear degradation in performance, with all limits reduced compared to Scenario 2. It should be noted the acceleration and deceleration capabilities become very limited, especially while turning. The trends between steering angle remain the same.

Eccentricity in the load placement is introduced in Scenario 4, Figures 4.18e - 4.18h. A break in the symmetry over $\alpha = 0^\circ$ is observed. Since the eccentricity is to the left of the vehicle, a corresponding left turn is difficult, resulting in small operating region when $\alpha = 45^\circ$, Figure 4.18h. Conversely a right turn has a larger acceptable operating region, seen in Figure 4.18f. Something not seen in other Scenarios, is the divide between the most balanced (blue) regions. This may be interpreted as while not moving, including at rest and beginning/stopping motion, the vehicle is not well balanced. Furthermore, when $\alpha = |90^\circ|$, the model FLV has almost no ability to turn in a counter-clockwise direction (left turn).

The behaviour discussed in this section can be explained in more general terms. Since the model FLV is counterbalanced to allow heavier loads to be added, the vehicle itself is inherently unbalanced, with the CoG toward the rear of the vehicle. The addition of a load at a low position improves the dynamic balance, by moving the CoG forward while
Figure 4.18: Comparison of dynamic balance margin. Scenario 3 and 4 is represented on the left and right columns respectively. Red corresponds to $\Xi = 0$ while blue corresponds to $\Xi = 1$. 

(a) Scenario 3, $\alpha = -90^\circ$

(b) Scenario 4, $\alpha = -90^\circ$

(c) Scenario 3, $\alpha = -45^\circ$

(d) Scenario 4, $\alpha = -45^\circ$

(e) Scenario 3, $\alpha = 0^\circ$

(f) Scenario 4, $\alpha = 0^\circ$

(g) Scenario 3, $\alpha = 45^\circ$

(h) Scenario 4, $\alpha = 45^\circ$
remaining low in the $z$-direction. Increasing the height of load raises the CoG and results in an increased sensitivity to movement.

The other notable behaviour regards the eccentricity of the load and the subsequent favour of one turning direction. This effect is practically applied by motorcyclists and snowboarders, using a technique known as counter-steering. Counter-steering uses a shift in body weight to allow sharper turns. In an FLV it is possible to use this behaviour; however it may not be practical and is often determine by the load distribution rather than the operator. Knowledge of this behaviour remains useful for motion planning algorithms.

This section examined the dynamic balance behaviour of a model FLV using different loading scenarios. The tilt angle was not examined in this section and will be discussed in Section 4.6.2, using only straight line motion. The analysis in this section may be used to better design the FLV, mechanically, or the algorithms used to control an FLV.

4.6.2 Effects of deceleration and tilt angle

One common FLV accident that has yet to be examined is load slippage. This mode of failure competes directly with tip-over in the forward direction of the vehicle; fast deceleration may cause either the load to slip or tip-over. The event of one mode of failure may exacerbate the conditions for the other mode of failure: a load slipping forward will move the CoG forward, possibly causing tip-over; while a tip-over will cause the effective tilt angle to increase, possibly cause load slippage.

The load slippage is modelled using the standard friction model for the static friction $\mu_s = 0.5$ \cite{48}, corresponding to wood contacting steel. To simplify the problem, the complex interaction between slip and tip conditions are ignored, taking the state of overcoming the static friction to be the point of failure.

$$ g (\mu_s \cos \psi - \sin \psi) + \ddot{r}_x \cos \psi > 0 $$

(4.23)
Note that Equation (4.23) does not depend on mass, hence the slip profile is the same for this analysis, seen in Figures 4.19 and 4.20.

The following analysis focuses only on the deceleration while travelling in a straight line \( \theta = 0 \). In this case, the velocity and angular components do not need to be considered for the dynamic balance. Two payload masses are considered, 2 kg and 4 kg. The deceleration and tilt angles are varied.

Figures 4.19 and 4.20 represent the regions which failure occur, classifying the two modes of failure as load slippage and forward tip-over. Typical tilt angles are shown as vertical lines to provide a sense of applicable regions. The figures extend from \([-\pi/4, \pi/4]\) rads to show behaviour outside the typical regions.

From the figures, we can confirm, intuitively, the expected behaviour of the system. When the reach is extended (load is moved forward), the system begins to tip more easily when not decelerating, seen in Figures 4.19 and 4.20 by the orange region covering a larger area of the negative acceleration. Two things to be noted: max deceleration is lowered, and the allowable tilt angle is moved toward the negative angles. Note that the effects of loading scenario and deceleration has been seen previously in Section 4.6.1; however now the tilt angle is now included.

Considering only the effects of slip, it can be seen that positive tilt angles result in slip at lower decelerations. The slip condition does not change with the loading scenario, or the type of tilt used (mast or carriage), which is not seen in these figures. The result is a static bound to deceleration. For implementation purposes, it is necessary to choose between the slip or dynamic balance conditions to bound acceleration.

It is possible to infer one failure condition will exacerbate the other condition. For example, in Figure 4.19c the region near 0 rad tilt angle contain both failure conditions in a small area. If the load causes tipping, the slight increase in tilt angle may result in the load slipping. If we consider the load slipping, then the effective reach will increase, possibly resulting in the tipping condition to occur.
(a) Load placed at \([0.2, 0, 0.8]^T \text{ m.} \)

(b) Load placed at \([0.8, 0, 0.8]^T \text{ m.} \)

(c) Load placed at \([0.2, 0, 1.2]^T \text{ m.} \)

(d) Load placed at \([0.8, 0, 1.2]^T \text{ m.} \)

Figure 4.19: Varying deceleration and tilt angle of an FLV carrying load. Load mass is fixed at 2 kg. The load placed at \([x, y, z]^T \text{ m.} \) from the centre of the front axle, with \(x\) the distance in front of the axle, \(y\) the eccentricity to the left and \(z\) the height from the ground. The regions indicated refer to the safe region or the failure condition of forward tip-over, load slippage, or both tip and slip. Load slippage is only considered for the direction away from the mast. Typical tilt angles seen in FLVs include \([-4, 2]^\circ\), \([-5, 5]^\circ\) and \([-10, 10]^\circ\). These ranges are seen in reach trucks, counterbalanced electric and combustion engine FLVs respectively. The area circled in (c) highlights a case where both modes must be considered as either type may cause failure.
Chapter 4. Dynamic Balance Margin

Figure 4.20: Varying deceleration and tilt angle of an FLV carrying load (continued). Load mass is fixed at 4 kg. The load placed at $[x, y, z]^T$ m from the centre of the front axle, with $x$ the distance in front of the axle, $y$ the eccentricity to the left and $z$ the height from the ground. The regions indicated refer to the safe region or the failure condition of forward tip-over, load slippage, or both tip and slip. Load slippage is only considered for the direction away from the mast. Typical tilt angles seen in FLVs include $[-4, 2]^{\circ}$, $[-5, 5]^{\circ}$ and $[-10, 10]^{\circ}$. These ranges are seen in reach trucks, counterbalanced electric and combustion engine FLVs respectively.
This section has shown the BDBM agree with the relevant current operating procedures and provides an insight into how deceleration limits can be devised. The next section will summarise the contributions of this chapter.

4.7 Summary of contributions

This chapter has explored a new DBM, utilising the ZMP and BC. The BDBM is then used to analyse safety in typical operation of FLVs. Two different BC were used implemented, providing choice of implementation for polygons with $m > 3$. The triangular BC was also derived for use on FLVs. A summary of each coordinate system is listed in Table 4.3. The distribution of contours within the polygon and the behaviour outside the polygon were consider in detail. This work does not make a recommendation of which BC for general use as the choice must be informed using the properties of each BC.

<table>
<thead>
<tr>
<th>Wachspresse</th>
<th>Mean Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>- More evenly distributed $\lambda$ for irregular polygons</td>
<td>- More evenly distributed $\lambda$ for regular polygons</td>
</tr>
<tr>
<td>- More predictable barycentre for highly skewed polygons</td>
<td>- Well defined outside of polygon</td>
</tr>
</tbody>
</table>

Table 4.3: Benefits of each BC for BDBM.

The use of the BDBM as an analysis tool has been demonstrated in Section 4.6, using a scale model FLV for the analysis. The result is the confirmation of known operating procedures, and quantifying the effects of changing loading conditions. 4.4.

The two BDBMs $\Xi_p$ and $\Xi_m$ have differing properties, useful to different situations and hardware. For example, when $m = 3$ the computational saving is small, while the unique coordinates outside the polygon is more suitable for gradient based optimisations. A summary of the benefits of each type are described in Tables 4.4.

The BDBMs were used in Section 4.6 to analyse FLV behaviour under different loading scenarios and during breaking manoeuvres. The results follow intuitive sense and commonly taught safety protocols; however also provide a quantification of the vehicle’s limitations.
Chapter 4. *Dynamic Balance Margin*

<table>
<thead>
<tr>
<th>$\Xi_p$</th>
<th>$\Xi_m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>- Computational fast for arbitrary $M$</td>
<td>- Contours match polygons of $M = 3$</td>
</tr>
<tr>
<td>- Small gradient as $\Xi_p \to 0$ or $\Xi_p \to 0$</td>
<td>- No lines of $\Xi_m = 0$ outside of polygon</td>
</tr>
</tbody>
</table>

*Table 4.4: Benefits of each $\Xi$ for BDBM.*

The BDBM in this section has only been used for offline analysis. Chapter 5 will apply the BDBM during the calculation of trajectories to construct balanced trajectories online for the purpose of autonomous motion.
Chapter 5

Balanced trajectory generation for non-holonomic vehicles

The DBM introduced in Chapter 4 will be used to constrain a trajectory based on the SCC path. The new algorithm will consider dynamic constraints and time during the construction of the path, allowing a safe trajectory to be generated. This provides the benefit of knowing the placement of the FLV without requiring deviations to adjust during run time. This is useful for obstacle avoidance. The chapter is concluded with a computational illustration of possible trajectories given defined and configurations. The illustrations highlight the resulting features of the algorithm.

5.1 Introduction

Trajectory generation for non-holonomic vehicles has traditionally focused on ‘car-like’ vehicles and fixed-wing aircraft [37, 49]. These vehicles can traditional only produce velocity in one axis, and require turning to achieve a new pose. Car-like vehicles are considered to be vehicles which have two control inputs: steering angular velocity and forward velocity. Negative velocities, indicating a reversing procedure, is generally considered an extension.
The result is a vehicle that is not locally controllable [49]. Physically this means the vehicle cannot achieve all poses close to the initial pose without having to first leave the proximity.

The Dubins’ path, described in Section 2.3.1, makes use of piecewise circular arcs and straight segments in some combination results in discontinuous curvature which may only be achieved if the vehicle stops at each discontinuity to correct the current heading. However, the Dubins’ path may be used to approximate a non-holonomic path for a forward moving agent, removing the need to stop at each turn. To approximate a path, the steering rate must be sufficiently large, such that the error resulting from entering and existing a turn, is minimised. The result is the implementation will have some positive finite error in distance. The SCC path adds to the Dubins’ path by adding non-holonomic transitions in the form of spirals [39, 40]. This thesis will use the SCC path as a basis for the trajectory generator.

The trajectory generation algorithm uses the following assumptions:

1. Terminal configurations are known and are well chosen
2. A controller exists with some known acceleration characteristics
3. The vehicle operates on planar surfaces with sufficient friction and no inclines

The trajectory generation algorithm is designed to work with a motion planner, which will provide terminal configurations, and a controller to realise the trajectory. Assumption 1 considers that a motion planner calculates well chosen way points that provide the terminal configurations necessary for the trajectory generator.

Assumption 2 relates to providing the controller with realisable inputs. ‘Well chosen’ here means: a solution exists between configurations and does not violate any constraints. By creating acceleration constraints based on the controller used, flexibility is given to the design of the controller and the choice of vehicle.

The final assumption restricts the operating environment to warehouse like environments. It should be noted that warehouse environments may include inclines, in the form of ramps.
The ZMP has been extended for inclines in [24], as such this extension will not be discussed in this thesis.

The following considerations will be made for the design of the algorithm:

1. Minimise computational requirements
2. Trajectory may be calculated in a sufficiently small time, in the order of seconds
3. Dynamic behaviour handled during construction of the trajectory

The first two considerations relate to the application of the algorithm in the field. Minimising computational requirements reduces the total time required to plan a motion. In turn, this reduces the required computational resources, possibly allowing a reduction in total cost. The total calculation time need not be instant but must not require excessively long wait times for an order to be fulfilled. Consideration 3 prevents the need to analyse the trajectory after construction to ensure constraints are met. This can be done by enforcing constraints on the rules governing the construction of the trajectory. This thesis will exploit the construction of the basis SCC path such that the entire path need not be explicitly calculated until the trajectory is used.

Section 5.2 will detail the changes to the SCC path algorithm to achieve a dynamically balanced trajectory.

5.2 Modified SCC trajectory

This thesis proposes the following additions to the SCC path:

- Relaxing curvature constraints, Section 5.2.1
- Introducing time dependencies, Section 5.2.2
- Use an optimisation process to resolve the larger search space, Section 5.2.4
The goal of these additions is to incorporate time dependent components into the construction. Since the time dependent components are resolved during construction, the dynamic behaviour may also be considered. Section 5.2.3 will summarise the necessary changes to the geometry, detailed in [39], as a result of curvature relaxation and introduction of time dependencies.

### 5.2.1 Relaxing curvature constraints

The curvature constraints used in the SCC path are highly restrictive, where the maximum curvature must be met and only one curvature is possible. This assumption is okay for vehicles with little change in dynamic behaviour throughout operation. However, this is not applicable for FLVs, where the load can dramatically change the dynamic behaviour.

Two changes to the current curvature constraints are considered:

1. Curvature is not restricted to only using the maximum curvature: \( \kappa \leq \kappa_{\text{max}} \)

2. Two maximum curvatures \( K = \{\kappa_1, \kappa_2\} \) corresponding to each turn

The first change follows from the creation of an SCC path, where the transition T segment is designed to reach the maximum curvature. In some cases this is not necessary. Take the trivial example of a straight line motion: the two T segments should be degenerate cases; however the path may be equivalently constructed with T segments that loop back to the terminal configurations. This trivial case can be solved by a simple colinearity test, but there are other cases where \( \kappa_{\text{max}} \) is much larger than \( \Delta\theta \) which may result in similar behaviour.

The second change addresses the differing sensitivity with respect to the states of the system, specifically velocity and accelerations. A single trajectory from one configuration \( q_A \) to \( q_B \) is likely to have an overall increase, or decrease, to the velocity. As such, the two turns will have differing operating regions.
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The change in geometry as a result of this will be discussed in the next section as the effective turning rate effects the geometry similarly.

### 5.2.2 Effective turning rates

One result of the unit velocity assumption is a fixed turning rate with respect to the geometry. The turning rate \( \sigma = \frac{d\theta}{dt} \) is mapped from a time dependence to distance dependence such that the geometric turning rate \( \frac{d\theta}{ds} \) using the unit velocity assumption. A fixed turning rate with respect to geometry is undesirable, moreover, it is an unrealistic representation of most vehicles. By not using the unit velocity assumption, but rather the current velocity, the same mapping can be used to achieve the effective geometric turning rate \( \sigma_{\text{eff}} \). For brevity, effective geometric turning rate will be shortened to effective turning rate.

\[
\sigma_{\text{eff}} = \frac{\sigma}{r} \quad (5.1)
\]

This results in covering a larger distances with higher velocities, compared to lower velocities, to achieve the same heading. This is a more natural expectation of turning behaviour. Another benefit of \( \sigma_{\text{eff}} \) is a positive effect on dynamic balance and performance. Reducing the effective turning rate at high velocities improves balance, while increasing the turning rate at low velocities improves performance.

The combination of the relaxed curvature constraints, Section 5.2.1, and the effective turning rates results in the inability to achieve a TTT turn due to a change in the expected geometry.

### 5.2.3 Change in geometry

Equations (2.11) - (2.15) must be modified to accommodate \( K \) and \( \sigma_{\text{eff}}_k ; k = 1, 2 \). Below is a guideline to the necessary changes to allow the construction of the new trajectory.

\[
\theta_{\text{lim}}_k = \frac{K_k}{\sigma_{\text{eff}}_k} ; k = 1, 2 \quad (5.2)
\]
Chapter 5. *Balanced Trajectory Generation*

To begin, $\theta_{\text{lim}}$ acts as a basis for the clothoids’ placement and shape, equations (5.3), (5.4) and (5.5). Two $\theta_{\text{lim}}$ is defined for the two turns. The turn centre $\Omega_k$ must be updated to accommodate the new $\theta_{\text{lim}_k}$. The final position is then updated in equation (5.5).

$$\Omega_1 = \left(x_1 - \frac{\sin \theta_1}{\kappa_1}, y_1 - \frac{\cos \theta_1}{\kappa_1}\right)$$  \hspace{1cm} (5.3)

$$\Omega_2 = \left(x_5 - \frac{\sin \theta_5}{\kappa_2}, y_5 - \frac{\cos \theta_5}{\kappa_2}\right)$$  \hspace{1cm} (5.4)

$$q_1(\beta) = \begin{bmatrix} x_1 = \sqrt{\frac{\sigma}{\sigma_{\text{eff}}_1}} \text{FrC} \left(\sqrt{\frac{\theta_{\text{lim}_1}}{\pi}}\right) \\ y_1 = \sqrt{\frac{\sigma}{\sigma_{\text{eff}}_2}} \text{FrC} \left(\sqrt{\frac{\theta_{\text{lim}_2}}{\pi}}\right) \\ \theta_1 = \frac{\theta_{\text{lim}_1}}{2} \\ \kappa_1 = \kappa_{\text{max}_1} \end{bmatrix}$$  \hspace{1cm} (5.5)

$$q_3(\beta) = \begin{bmatrix} x_3(\beta) = R_{T_1} [\sin(\beta - \gamma) - \sin \gamma] \\ y_3(\beta) = R_{T_1} [\cos \gamma - \cos(\beta - \gamma)] \\ \theta_3(\beta) = \beta \\ \kappa_3 = 0 \end{bmatrix}$$  \hspace{1cm} (5.6)

$q_5$ is modified similarly to equation (5.5).

$$\gamma_k = -\arctan \frac{x_{\Omega_k}}{y_{\Omega_k}} \hspace{1cm} k = 1, 2$$  \hspace{1cm} (5.7)

$$\alpha_2 = \arcsin(R_{T_1} \cos \gamma_1 + R_{T_2} \cos \gamma_2)$$  \hspace{1cm} (5.8)

Finally the angle $\alpha_2$ is found, equation (5.8). These modifications are all that is necessary to construct TST type trajectories. Since $\gamma_1 \neq \gamma_2$, it is not possible to construct a TTT path in the manner defined in Section 2.3.2. Although this reduces the set of feasible terminal configuration pairs, in practice little is lost. Often the resulting turns require a large amount of space, and the requirement of terminal positions being ‘close’ often allows slowing down of the vehicle to achieve a more direct trajectory.

To demonstrate the changes made by Sections 5.2.1 and 5.2.2, an example path is provided, Figures 5.1 - 5.4. The SCC path is first presented, Figure 5.1, assuming the worst case
The curvature of the first turn is increased from $\kappa_1 = 0.2$ to $\kappa_1 = 0.4$. The circular arc is small, nearing the degenerate limit.

The curvature constraints are relaxed in Figure 5.2. Here, the first turn is allowed a larger curvature of $\kappa_1 = 0.35$, assuming that the first turn is made at lower velocities and accelerations. The circular arc becomes small and nears the point of degeneracy. This is neither a positive or negative trait; however the limit is pronounced in this example. The elementary path [9] – a pair of symmetric clothoids where $\kappa \leq \kappa_{\text{max}}$ – was designed for such a degenerate scenario; however in many cases this solution is not necessary, given the relaxation of requiring maximum curvature to be reached.

Similarly, the effective turning rate will change depending on the current velocity. Since the first turn is assumed to have a lower velocity, a larger $\sigma_{\text{eff}}$ is allowed. The resulting path is shorter and wider range is allowed before the degenerate case occurs, resulting in greater flexibility of motions at lower velocities.

**Table 5.1: Summary of geometric variables for SCC case.** Both turns are identical in their parameterisation.

<table>
<thead>
<tr>
<th>Geometry</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\kappa_1$</td>
<td>0.20 m(^{-1})</td>
</tr>
<tr>
<td>$\kappa_2$</td>
<td>0.20 m(^{-1})</td>
</tr>
<tr>
<td>$\sigma_1$</td>
<td>0.10 rad s(^{-1})</td>
</tr>
<tr>
<td>$\sigma_2$</td>
<td>0.10 rad s(^{-1})</td>
</tr>
</tbody>
</table>
Chapter 5. Balanced Trajectory Generation

Figure 5.2: The curvature of the first turn is increased from $\kappa_1 = 0.2$ to $\kappa_1 = 0.4$. The circular arc is small, nearing the degenerate limit.

Table 5.2: Summary of geometric variables allowing variable curvatures.

<table>
<thead>
<tr>
<th>Geometry</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\kappa_1$</td>
<td>0.35 m$^{-1}$</td>
</tr>
<tr>
<td>$\kappa_2$</td>
<td>0.20 m$^{-1}$</td>
</tr>
<tr>
<td>$\sigma_1$</td>
<td>0.10 rad s$^{-1}$</td>
</tr>
<tr>
<td>$\sigma_2$</td>
<td>0.10 rad s$^{-1}$</td>
</tr>
</tbody>
</table>

Figure 5.3: Path using relaxed curvature constraints and effective turning rates.

Table 5.3: Summary of geometric variables allowing variable curvatures and modifying turning rates depending on expected velocity.

<table>
<thead>
<tr>
<th>Geometry</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\kappa_1$</td>
<td>0.35 m$^{-1}$</td>
</tr>
<tr>
<td>$\kappa_2$</td>
<td>0.20 m$^{-1}$</td>
</tr>
<tr>
<td>$\sigma_1$</td>
<td>0.35 rad s$^{-1}$</td>
</tr>
<tr>
<td>$\sigma_2$</td>
<td>0.10 rad s$^{-1}$</td>
</tr>
</tbody>
</table>
5.2.4 Resolving time dependencies through optimisation

The addition of time considerations, including variation of $\kappa$ results in a larger search space and redundancy in achieving the task. An optimisation process is, therefore, necessary to resolve these redundancies. However, our aim is not to find the globally optimal path, preferring minimised computational load over optimality; hence only a sub-optimal solution is sought.
For an FLV in an industry setting, minimising time taken and distance travelled is a good measure of performance. Since the optimisation is for a single vehicle’s path, the minimisation may be kept simple.

The optimisation is performed over the segment transitions, not the entire path, to reduce complexity. The parameters to be optimised are the six internal velocities $v$ and the two curvatures $K$.

One cost function is proposed:

$$Q^* = \arg \min_{V,K} w_s s(V,K,\sigma_{\text{max}},q_s,q_f) + w_t t(V,S,q_s,q_f)$$

(5.9)

Where $t(\cdot)$ is the average time taken given the distance $s(\cdot)$ and the internal velocities $v$. Since the path is constructed using known segments, $s(\cdot)$ may be computed easily; however, $t(\cdot)$ cannot be computed directly without full knowledge of the acceleration profile, hence an estimation, requiring a known acceleration characteristic, is used. The weights $w_s$ and $w_t$ control the contribution of $s(\cdot)$ and $t(\cdot)$ respectively.

The set of velocities $V$ and curvatures $K$ are determined through the use of the BDBM at each iteration of the optimisation. Subsequently, the set of distances $S(V,K)$ may be calculated. For brevity, the independent variables of $S$ are not included in the Equation (5.9).

The nominal distance $s_{\text{nom}}$ is calculated using the Euclidean distance between the terminal positions. This can be considered the minimal path length possible between the terminal poses. This value is unique given a pair of terminal poses, as such is a good candidate for a normalisation factor.

Similarly, the nominal time to completion $t_{\text{nom}}$ should be chosen such that it is unique normalisation factor. Using the unit velocity assumption applied to $s_{\text{nom}}$ results in the candidate $t_{\text{nom}} = s_{\text{nom}}$. By using $t_{\text{nom}} = s_{\text{nom}}$ no extra calculations are necessary.

The combination of using an estimation and few iterations often results in the choice of a sub-optimal paths. The next section will provide example paths given a single pair.
of terminal configurations and differing loading scenarios. The resulting trajectories are compared to the results in Section 4.6.1.

### 5.3 Computational illustration

This section will provide examples of the mSCC algorithm given varying loading scenarios. The scenarios presented will be similar to those seen in Section 4.6.1.

The purpose of the computational illustration is to highlight the effects of the dynamic balance on the resultant generated trajectories. The illustration is intended to exaggerate the exhibited behaviour by using extreme scenarios, to clearly present the behaviour of the mSCC algorithm.

The trajectories will all be generated from the initial configuration \( q_s \) to the final configuration \( q_f \). The terminal configurations are standard across all scenarios. The velocity at \( q_s \) begins at 3 ms\(^{-1}\) up to 5 ms\(^{-1}\) at \( q_f \). The velocity is assumed to not increase beyond the range of velocities defined by the terminal velocities. Table ?? summarises the 4 scenarios studied in this chapter.

The optimisation is run using MATLAB's \texttt{fmincon}, with constraints set on maximum and minimum velocities and accelerations using hardware limits. Since this is purely computational, the limits were increased such that exaggerated trajectories may be calculated. The optimisation is run 10 times with new random seeds. This is to help reduce the selection of local minima that result in poor performance.

The cost function, Equation (5.9), minimises the sum of the normalised distance and completion time. The normalising factor uses the distances between the terminal configurations \( s_{nom} \), with the time component assuming unit velocity \( t_{nom} \), such that \( s_{nom} = t_{nom} \). The weights are evenly set with \( w_s = w_t = 1 \).
\[ Q^* = \arg \min_{V,K} \frac{s(V,K,\sigma_{\text{max}},q_s,q_f)}{s_{\text{nom}}} + t(V,S,q_s,q_f) \]  

(5.10)

The following section will present the results of the illustration and discuss the observed behaviour. The observed behaviour will be compared to previous results in Section 4.6.1.

5.3.1 Results and discussion

The results of the five scenarios, described in Table ??, are presented in this section. Scenarios 1-4 presented here are the same as those used in Section 4.6.1, with scenario 5 being the only addition, with the opposite eccentricity to scenario 4. This section will first present the results of the 5 scenarios and then a discussion of the results. ‘Performance’ will be used to reference both distance and time measures, with a good performance indicating shorter distances and completion times.

The results of the 5 scenarios are summarised in Table 5.4. The two scenarios resulting in the best performance are 2 and 4. Scenario 2 has a low load, while scenario 4 has a high load placed to the left of the centre. The eccentrically placed load allows improved left turn performance, which has a positive effect in this case due to the trajectory containing two left turns. Scenario 5 results in the worst performance, with the unloaded scenario a close second. Scenario 3 contains a load that is placed high with no eccentricity. The result is a slightly longer trajectory than scenario 2.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Time t (s)</th>
<th>Distance s (m)</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>15.37</td>
<td>63.50</td>
<td>7.05</td>
</tr>
<tr>
<td>2</td>
<td>12.46</td>
<td>47.13</td>
<td>5.33</td>
</tr>
<tr>
<td>3</td>
<td>12.77</td>
<td>49.96</td>
<td>5.61</td>
</tr>
<tr>
<td>4</td>
<td>15.51</td>
<td>64.17</td>
<td>7.13</td>
</tr>
<tr>
<td>5</td>
<td>12.46</td>
<td>47.13</td>
<td>5.33</td>
</tr>
</tbody>
</table>

Table 5.4: Summary of distance, time travelled and optimisation cost for each scenario.

Note the results of scenario 2 and 4 are the same trajectory.
Chapter 5. Balanced Trajectory Generation

Figure 5.5: Unloaded scenario (scenario 1). The FLV begins at the initial configuration $q_s$, represented by an open circle, beginning at $v_i = 3 \text{ m/s}^2$ and accelerating to $v_f = 5 \text{ m/s}^2$ at the terminal configuration $q_f$, represented by a cross. The FLV travelled $d_1 = 63.50 \text{ m}$ in $t_1 = 15.37 \text{ s}$. Note the required terminal velocities result in exaggerated trajectory completion times and distances.

Scenario 1, Figure 5.5, shows a long path with $t_1 = 15.37 \text{ s}$ and $d_1 = 63.50 \text{ m}$. Although the length of the path may seem excessive, this is due to the high speeds the vehicle is forced to travel at, with $\dot{r}_s = 3 \text{ ms}^{-1}$ and $\dot{r}_f = 5 \text{ ms}^{-1}$. Reiterating, these speeds are exaggerated for the purpose of illustrating the behaviour of the mSCC trajectory. Typically, the unloaded scenario is considered less balanced than a loaded scenario, assuming a relatively low load.
The counterbalance is the cause of the lower balance measure, where a large weight is often placed toward the rear of the vehicle. This results in the CoG being close to the edges of the SP, at the rear of the vehicle, and where the angular dynamics have a greater role in loss of balance. Additionally the maximum positive acceleration would be limited as a result of the rear located CoG. Similar conclusions can be made by studying Figure 4.17.

**Figure 5.6:** Comparison of scenario 1, 2 and 3. Both loaded scenarios perform better than the unloaded scenario. Scenario 2 has a load placed lower than in scenario 3, resulting in better performance. Scenario 2 results in $d_2 = 47.13$ m and $t_2 = 12.46$ s, while scenario 3 results in $d_2 = 49.96$ m and $t_2 = 12.77$ s.
The centrally loaded scenarios, 2 and 3, are compared to the unloaded scenario in Figure 5.6. The lower load in scenario 2, $h_2 = 0.1\, \text{m}$, results in better performance than in scenario 3, $h_3 = 0.9\, \text{m}$. This is an intuitively expected result. As previously mentioned, the counterbalance causes the unloaded scenario to perform worse than the scenarios 2 and 3. This is an intentional design choice, where the counterbalance placed toward the rear of the vehicle allows heavier loads to be carried while minimising the total weight; however at the cost of dynamic balance when unloaded.

Figure 5.7 presents the two eccentrically loaded scenarios 4 and 5. Both scenarios 4 and 5 are loaded to the same height as in scenario 3, with the load off centre by $e_4 = 0.2\, \text{m}$ and $e_5 = -0.2\, \text{m}$. Scenario 4 aids in the left turning capability, while scenario 5 detrims left turns. Similar behaviour is observed when counter-steering on motorcycles and while surfing or snowboarding. Figure 4.18 presented this behaviour with the reduction of the allowable region on one side and a larger region on the other. When the eccentricity becomes large, it is possible to have the FLV dynamically balanced only when in motion. The beginning of this effect can be seen in Figure 4.18F, where the most balanced region is not at the origin. The two equivalent trajectories of scenarios 4 and 5 may be a result of the dynamic balance not limiting the motion for the prescribed terminal configurations.

The scenarios were run for 100 trials using the Dubins’ path, SCC path and mSCC and timed using MATLAB’s timer running on MATLAB 2012b and an Intel i7-4700MQ 2.4 GHz CPU. The resulting times are summarised in Table 5.5. Both the Dubins’ and SCC paths run at negligible times, whereas the mSCC trajectory slows considerably with the use of the \texttt{fmincon} function in MATLAB with 57.36 s dedicated to \texttt{fmincon}. The increase in time is certainly to be expected; however the computational time remains small.

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dubins’</td>
<td>0.477</td>
</tr>
<tr>
<td>SCC</td>
<td>1.067</td>
</tr>
<tr>
<td>mSCC</td>
<td>59.421</td>
</tr>
</tbody>
</table>

Table 5.5: Summary of timing results running 100 trials on MATLAB 2012b and an Intel i7-4700MQ 2.4 GHz CPU
Chapter 5. *Balanced Trajectory Generation*

Figure 5.7: Comparison of scenario 4 and 5. Scenario 4 is equivalent to scenario 2, where the load placed on the left of the vehicle improves the ability to turn left with $d_4 = 47.13\, \text{m}$ and $t_4 = 12.46\, \text{s}$. Scenario 5 has the load placed on the right of the vehicle, resulting in poor performance, with the performance marginally worse than scenario 1 with $d_5 = 64.17\, \text{m}$ and $t_5 = 15.51\, \text{s}$.

The conclusions on FLV behaviour drawn in Section 4.6 can be seen here, in the results of the computational illustration. The optimisation allows the comparison of trajectories; however, does not indicate if the solution is optimal. Furthermore, multiple iterations of the optimisation were used to reduce the effects of local minima.
5.3.2 Summary of Contributions

This chapter has introduced an algorithm combining the SCC path with the systems dynamic constraints, including the DBM, to produce the mSCC trajectory. The additional search space is considered through the use of a coarse optimisation process, focusing only on the states at the transitions between path segments. The optimisation is not intended to find the optimal solution, but rather to remain computationally inexpensive.

Several example paths are shown, studying the effects of load placement on the resulting trajectory. These graphs show a consensus with known FLV behaviour and observations in Section 4.6, noting that the intention of the scenario provided is to accentuate the features of the algorithm given extreme situations.
Chapter 6

Conclusion and future work

6.1 Conclusion

This thesis has constructed an algorithm to create dynamically balanced trajectories for an FLV. The trajectory generation algorithm provides a step toward automation of FLVs. Chapter 2 revealed the dangers of the use FLVs but also the necessity of using such a vehicle. The current state of safety focuses on operator competence, which this thesis regards as the main contributor of failures.

Analysis of toppling (Aim 1.1) is addressed in Chapter 3 using the ZMP and assuming only planar motion on a horizontal surface, suitable for warehouse environments. The ZMP is then combined with BC to form an inequality used to measure toppling propensity (Aim 1.2). Different BCs are compared in Chapter 4 to show the benefits of each form. Furthermore, two DBMs are compared for similar reasons.

Analysis of dynamic behaviour was conducted in Chapter 4 using the DBM. Two studies were conducted: comparison of loading scenarios with respect to instantaneous velocity and acceleration, and comparison of load slipping with tipping scenarios. The former provides insight into FLV behaviour, suitable for aiding design of motion planners and the construction of FLVs. The latter may be used to determine whether load slip or
tipping should be considered, if not both; in many cases tipping or slipping dominates the behaviour.

The mSCC path was developed in Chapter 5 using the results of Aim 1. The mSCC algorithm produces a trajectory considering the dynamic balance, using the DBM produced in Aim 1.2.

The mSCC path combined with a constrained optimisation contributes to Aim 2. The DBM designed for Aim 1.2 is used with a modified geometric path planner, allowing for time considerations and constraint enforcement over the construction of the trajectory.

An illustration of agent behaviour is presented in Chapter 5. Generally the expected result occurs, with high loads causing longer and slower trips compared to loads held lower. Less intuitively the agents may choose a less direct path to achieve the desired configurations. This result is most appreciable when higher velocities are desired, or when a heavy load is held high, where the extra distance allows fulfilment of curvature constraints or acceleration limitations.

6.2 Future work

This section will be divided into two: improvements to the proposed algorithm, and future work toward the goal of the overarching project to automate FLVs. Deficits of the proposed algorithm will be discussed, with suggestions to improve the algorithm. Secondly, the progress toward the final goal of automating a fleet of existing FLVs will be examined.

6.2.1 Improvements of work presented

The reversing procedure may be improved through the use of the Reed-Shepp’s car [50] over the Dubins’ car. Currently, reversing requires the linking of two mSCC trajectories where the terminal pose common to both trajectories have zero velocity and acceleration, and the second trajectory must initially return along the first. The use of Reed-Shepp’s
car with the SCC path is detailed in [51]. Using the Reed-Shepp’s car does require more computational time, where there are 48 words to determine an optimal path [50], compared to 6 in the case of the Dubins’ car. Whether this extra computation outweighs the benefits of adding the reversing procedure to the construction of the trajectory should be analysed.

The dynamics of the payload is not considered separately in this work, assuming the load moves slowly and the effects are negligible. It may be necessary to consider the load dynamics separately in the case of larger freight trucks or specialised narrow aisle trucks. Additional, Speedshield has noted that efficiency is critical with these larger vehicles due to their fuel consumption and operating environment. The ZMP model discussed in Section 3.4 has been initially derived for multiple links, where the payload may be considered an individual link. Combining the additional link with fuel efficiency in the optimisation is one solution. This work has begun and is documented in [42].

Closer consideration to the optimisation process should be made. The design in this thesis aimed to reduce the search space of the optimisation; however no rigorous study was performed on the optimisation process, hence it is currently unknown what practical improvements can be made with a more intensive optimisation process.

Verification of the model using a scale and real FLV is necessary. The design of the scale model can be found in [52]. The scale model is necessary for two purposes: provide confidence in the theoretical model to allow usage on real FLV, where risk and expense is much greater, and allow testing of dangerous behaviour in a small controlled environment. Verification using the model FLV will require the development of an appropriate controller to track the trajectory produced by the mSCC algorithm. One major complication arises from the mSCC trajectory’s requirement of $R=0$ at terminal poses, where recalculation of the trajectory cannot occur arbitrarily. This will require either a way to rejoin the path after a disturbance, or a method to transition to an appropriate configuration.
6.2.2 Steps toward automation of a fleet of existing FLVs

The intention of this thesis is to establish a trajectory generation algorithm as a step toward automating an existing fleet of FLVs. To achieve this final goal, the following is required:

- Design of a suitable controller
- Verification on a model FLV
- Retrofitting a real FLV and subsequent testing
- Design an autonomous task fulfilment system, with an operator proving tasks to the FLV
- Design an FLV fleet management system

The first three goals were discussed in the previous section, while the final two are far reaching goals. No detailed plans for these two goals have been devised.
Bibliography


Chapter 6. Conclusion and future work


Chapter 6. Conclusion and future work


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Title:
Improving the safety of forklifts through automation of task execution

Date:
2016

Persistent Link:
http://hdl.handle.net/11343/115206

File Description:
Improving the Safety of Forklifts Through Automation of Task Execution