Natural leptogenesis and neutrino masses with two Higgs doublets

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The minimal Type I seesaw model cannot explain the observed neutrino masses and the baryon asymmetry of the Universe via hierarchical thermal leptogenesis without ceding naturalness. We show that this conclusion can be avoided by adding a second Higgs doublet with \( \tan \beta \geq 4 \). The models considered naturally accommodate a standard model-like Higgs boson and predict TeV-scale scalar states and low- to intermediate-scale hierarchical leptogenesis with 10\(^3\) GeV \( \lesssim M_{N_1} \lesssim 10^8\) GeV.

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\[ M_{N_1} \gtrsim 5 \times 10^8 \text{ GeV} \left( \frac{v}{246 \text{ GeV}} \right)^2, \tag{4} \]

where \( v \) is the vev that enters the seesaw of Eq. (3).

The ability of the minimal Type I seesaw model to simultaneously explain neutrino masses and the BAU is certainly intriguing. However, Vissani observed [16] that the model is incapable of doing so without generating a naturalness problem.\(^1\) Equation (4) is simply incompatible with the conservative naturalness requirement that corrections to the electroweak \( \mu^2 \) parameter of Eq. (1) not exceed 1 TeV\(^2\). With three flavors of hierarchical right-handed neutrinos, this requires [17]

\[ M_{N_1} \lesssim 3 \times 10^7 \text{ GeV} \left( \frac{v}{246 \text{ GeV}} \right)^\frac{3}{2}. \tag{5} \]

The incompatibility is exemplified in Fig. 1; nowhere at \( v = 246 \text{ GeV} \) is it possible to simultaneously fulfill the Davidson–Ibarra and Vissani bounds.

A sensible question is then: in what minimal ways can this incompatibility be overcome? Figure 1 suggests three conspicuous (but not mutually exclusive) options: (1) Modify the correction to \( \mu^2 \), e.g., by restoring supersymmetry or by partly cancelling the correction from the heavy fermion loop [18,19]; (2) lower the Davidson–Ibarra bound, e.g., by considering resonant leptogenesis [20], an alternative mechanism [21], or by introducing new fields which allow an increased CP asymmetry in the right-handed neutrino decay; (3) seek an extension of the canonical seesaw for neutrino mass, i.e., reduce the (possibly effective) \( v \) entering the seesaw Eq. (3).

In this paper we will consider the third option. Specifically we will examine alternative seesaw possibilities when the minimal Type I seesaw model is extended

\(^{1}\text{Naturalness is admittedly an aesthetic requirement of a model, and the possibility remains that nature is just fine-tuned.}\)
by a second Higgs doublet $\Phi_2$. We are motivated by the following observation: if the seesaw neutrino mass of Eq. (3) is evaluated at $v \lesssim 30$ GeV, then Eqs. (4) and (5) become compatible, as is clear from Fig. 1. Thus, we expect that two-Higgs-doublet models with right-handed neutrinos ($\nu$HDMs) and $\tan \beta = v_1/v_2 \gtrsim 8$, where $\Phi_2$ is responsible for a tree-level seesaw, can naturally accommodate leptogenesis and neutrino masses. In fact, we find that $\tan \beta \gtrsim 4$ is possible, since the extra scalar states can be naturally TeV scale and the Vissani bound can be relaxed.

Upon examining the $\nu$2HDM scenarios which will succeed, we rediscover the radiative Ma model [22] as the only possibility when $v_2 = 0$. Otherwise, in models without a significant radiative neutrino mass component, we require $0.3 \lesssim v_2/\text{GeV} \lesssim 60$. The potentially small vev is made natural by softly breaking a $U(1)$ or $Z_2$ symmetry, which also automatically results in one SM-like $CP$-even state. One advantage of this scenario is that it can work in all $\nu$2HDM types, greatly increasing the opportunity for model building.

The paper is organized as follows. In Sec. II we build the $\nu$2HDM models of interest, describe the scalar states, and briefly review the relevant experimental constraints. In Sec. III we pay particular attention to naturalness limits on the extra scalars; we verify that a natural $\nu$2HDM of any type is still allowed by experiment. We discuss neutrino masses in Sec. IV and leptogenesis in Sec. V. The region of parameter space which naturally achieves hierarchical leptogenesis is identified. We conclude in Sec. VI.

II. $\nu$2HDM MODEL

A. Lagrangian

The scalar content of the model contains two doublets $\Phi_{1,2}$ each with hypercharge +1. For simplicity we consider the softly broken, $CP$-conserving, $Z_2$-symmetric potential (see, e.g., Ref. [23])

$$V_{\text{2HDM}} = m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - m_{12}^2 (\Phi_1^\dagger \Phi_2 + \Phi_2^\dagger \Phi_1)$$

$$+ \frac{\lambda_1}{2} (\Phi_1^\dagger \Phi_1)^2 + \frac{\lambda_2}{2} (\Phi_2^\dagger \Phi_2)^2$$

$$+ \lambda_3 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_4 (\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1)$$

$$+ \frac{\lambda_5}{2} (\Phi_1^\dagger \Phi_2)^2 + (\Phi_2^\dagger \Phi_1)^2, \quad (6)$$

where all the parameters are real. To explain observations, at least one of these doublets must obtain a nonzero vev. We consider $m_{11}^2 < 0$ and a $CP$-conserving vacuum,

$$\langle \Phi_1 \rangle_0 = \frac{1}{\sqrt{2}} \left( \begin{array}{c} 0 \\ v_1 \end{array} \right), \quad \langle \Phi_2 \rangle_0 = \frac{1}{\sqrt{2}} \left( \begin{array}{c} 0 \\ v_2 \end{array} \right), \quad (7)$$

where $v_1 > 0$, $v_2 \geq 0$, and $v_1^2 + v_2^2 = v^2 \approx (246 \text{ GeV})^2$.

A general 2HDM will have flavor-changing neutral currents at tree level. These can be avoided if right-handed fermions of a given type ($u_R^i, d_R^i, e_R^i$) couple to only one of the doublets [24,25]. Although not strictly necessary, we will assume that this is realized and adopt the convention that only $\Phi_1$ couples to the $u_R^i$. In a $\nu$2HDM, if we assume this also applies for the $\nu_R^i$, then there are eight possibilities. As mentioned in the Introduction, the seesaw constraint Eq. (3) can be made consistent with naturalness and leptogenesis if the vev contributing to the seesaw is sufficiently small. Since we would like our model to remain perturbative, and already $y_\nu \approx 1$ for $v \approx 246$ GeV, we anticipate that $\Phi_2$ obtains the small vev, and thus we couple it to the $\nu_R^i$. Remaining are four possible $\nu$2HDMs which we refer to by their conventional types as listed in Table I.\footnote{Type I $\nu$2HDMs with $v_2 \sim e\text{V}$ were considered in Refs. [26–29]. We will end up considering $v_2$ of $O(0.1 – 10)$ GeV.}

TABLE I. The four models with no tree-level flavor-changing neutral currents and allowing for a GeV-scale vev to provide the seesaw while preserving perturbativity of $y_\nu$.

<table>
<thead>
<tr>
<th>Model</th>
<th>$u_R^i$</th>
<th>$d_R^i$</th>
<th>$e_R^i$</th>
<th>$\nu_R^i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type I</td>
<td>$\Phi_1$</td>
<td>$\Phi_1$</td>
<td>$\Phi_1$</td>
<td>$\Phi_2$</td>
</tr>
<tr>
<td>Type II</td>
<td>$\Phi_1$</td>
<td>$\Phi_2$</td>
<td>$\Phi_2$</td>
<td>$\Phi_2$</td>
</tr>
<tr>
<td>LS</td>
<td>$\Phi_1$</td>
<td>$\Phi_1$</td>
<td>$\Phi_2$</td>
<td>$\Phi_2$</td>
</tr>
<tr>
<td>Flipped</td>
<td>$\Phi_1$</td>
<td>$\Phi_2$</td>
<td>$\Phi_1$</td>
<td>$\Phi_2$</td>
</tr>
</tbody>
</table>

The Yukawa Lagrangian is then given by

$$-\mathcal{L}_Y = y_i \bar{Q}_L \Phi_1 u_R + y_i \bar{Q}_L \Phi_1 d_R$$

$$+ y_i \bar{U}_L \Phi_2 e_R + y_i \bar{T}_L \Phi_2 \nu_R$$

$$+ \frac{1}{2} M_N (\nu_R^i)^\dagger \nu_R + \text{H.c.}, \quad (8)$$
where $I, J$ depend on the model type, and family indices are implied.

### B. Scalar masses and mixings

Consistency with experiments requires the extra scalar states to have masses at least $\gtrsim 80 \text{ GeV}$. To construct models with potentially TeV-scale scalars with a naturally small $v_2$, we will consider $m_{22}^2 > 0$ and $m_{11}/m_{22}^2 \ll 1$ [26]. This is technically natural, since in the limit $m_{11}^2/m_{22}^2 \to 0$ a $U(1)$ or $Z_2$ symmetry is restored if $\lambda_5 = 0$ or $\lambda_5 \neq 0$, respectively.

For $m_{11} < 0$, the vevs are given by

$$v_1 \approx \sqrt{-2m_{11}^2/\lambda_1},$$

$$v_2 \approx \frac{1}{1 + \frac{v_1^2}{2m_{22}^2}} \lambda_{345} m_{22}^2 v_1,$$

where $\lambda_{345} = \lambda_3 + \lambda_4 + \lambda_5$. These relations become exact when $m_{11}^2 = 0$ and when $\lambda_2 = 0$, respectively. For $m_{22}^2 \gg \lambda_{345} v_1^2$, $\tan \beta = v_1/v_2 \approx m_{22}^2/m_{11}^2$.

There is a useful constraint on $m_{22}^2$, which is derived as follows. The minimization conditions give

$$m_{22}^2 + \frac{1}{2} \lambda_2 v_2^2 = \tan^2 \beta \left( m_{11}^2 + \frac{1}{2} \lambda_1 v_1^2 \right),$$

where $\lambda_1 v_1^2 \approx m_{11}^2 \approx (125 \text{ GeV})^2$ (see below). In the limit $m_{22}^2 \gg \lambda_2 v_2^2/2$, it can be seen that $m_{22}^2$ is bounded above by

$$m_{22}^2 \lesssim \frac{1}{2} m_{11}^2 \tan^2 \beta \quad (12)$$

if $m_{11}^2$ is to remain negative. Figure 2 illustrates how $m_{11}^2$ deviates from its standard value of $-\langle 88 \text{ GeV} \rangle^2$ as $m_{11}^2$ approaches this bound. For $m_{22}^2$ above this bound, $m_{11}^2$ very quickly grows to values $> v_2^2$, and $v \approx 246 \text{ GeV}$ is only explained by a miraculous balance of $m_{11}^2$ against $m_{22}^2/\tan^2 \beta$, which constitutes a fine-tuning. Thus, we adopt Eq. (12) as a consistency condition.

The charged scalar and pseudoscalar (neutral scalar) mass-squared matrices are diagonalized by a mixing angle $\beta (\alpha)$. The neutral mass eigenstates are

$$h = \rho_1 \cos \alpha + \rho_2 \sin \alpha,$$

$$H = \rho_2 \cos \alpha - \rho_1 \sin \alpha,$$

$$A = \eta_2 \sin \beta - \eta_1 \cos \beta,$$

where $\rho_i = \sqrt{2 \text{Re}(\Phi_i^0)} - v_i$ and $\eta_i = \sqrt{2 \text{Im}(\Phi_i^0)}$. The masses are given by

$$m_h^2 = \lambda_1 v_1^2 + O \left( \frac{m_{12}^4}{m_{22}^2} v_1^4 \right),$$

$$m_H^2 = m_{22}^2 + \frac{1}{2} \lambda_{345} v_1^2 + O \left( \frac{m_{12}^4}{m_{22}^2} m_{22}^2 \right),$$

$$m_A^2 = m_{22}^2 + \frac{1}{2} \left( \lambda_{345} - 2 \lambda_3 \right) v_1^2 + O \left( \frac{m_{12}^4}{m_{22}^2} m_{22}^2 \right),$$

$$m_{H^+}^2 = m_{22}^2 + \frac{1}{2} \lambda_3 v_1^2 + O \left( \frac{m_{12}^4}{m_{22}^2} m_{22}^2 \right),$$

i.e., the same as in the inert doublet model [23] up to corrections proportional to $m_{12}^4/m_{22}^4$, which we provide in Appendix A. Clearly, if $m_{22}^2 \gg v_2^2$, the mass scale of extra scalar states is $\approx m_{22}^2$.

In the alignment limit $\cos(\alpha - \beta) \to 0$, the couplings of $h$ to SM particles become SM-like. We calculate

$$\cos^2(\alpha - \beta) \approx \frac{m_{12}^4 v_1^4}{m_{22}^4 m_{11}^2 \left( 1 - \frac{v_1^2}{2m_{22}^2} \left( \lambda_4 - \lambda_{345} \right) \right)^2 \left( 1 + \frac{v_1^2}{2m_{22}^2} \lambda_{345} \right)},$$

suppressed by the approximate $U(1)$ or $Z_2$ symmetry ($m_{11}^2/m_{22}^2 \ll 1$) as well as the usual decoupling limit suppression ($v_1^2/m_{22}^2 \ll 1$) [30]. Thus, the model naturally accommodates a SM-like neutral scalar state.

### C. Constraints

With $M_N > m_{22}$ the constraints (and search strategies) for a Type-II HDM of a given type are largely identical to those for a 2HDM of the same type, for which there is extensive literature (see references henceforth). The Type-II HDM potential Eq. (6) is subject to a few standard theoretical constraints [23]. The necessary and sufficient conditions for positivity of the potential in all directions are [31–33]

$$\lambda_{1,2} \geq 0,$$

$$\lambda_3 \geq -\sqrt{\lambda_1 \lambda_2},$$

$$\lambda_3 + \lambda_4 - |\lambda_5| \geq -\sqrt{\lambda_1 \lambda_2}. \quad (16)$$
Vacuum stability of the potential minimum is more difficult to evaluate. An inequality which ensures a global minimum, missing possible metastable vacua, is presented in Ref. [34].

Tree-level perturbative unitarity of scalar-scalar scattering is ensured by bounding the eigenvalues of the scattering matrix [23,35–37]. Perturbativity of the $\lambda_i$ can also be demanded [38]. At the very least, these bounds should be implemented at the mass scale of the scalar states. In addition they may be demanded up to some high scale under the renormalization group evolution, which results in nontrivial constraints on $\tan\beta$. For Type I and flipped 2HDMs, even additional scalars with masses down to 80 GeV may still have evaded detection.

In summary, for moderate to large $\tan\beta$ and $\cos(\alpha - \beta) \approx 0$, experiments are most constraining for the Type II and flipped 2HDMs, with $m_{22} \geq 480$ GeV necessary (implying $v_2 \leq 45$ GeV). For Type I and LS 2HDMs, even additional scalars with masses down to 80 GeV may still have evaded detection.

### III. NATURALNESS

In the SM, the renormalization group equation (RGE) for the electroweak $\mu$ parameter [as in Eq. (1)] is dominated by the top quark Yukawa,

$$\frac{d\mu^2}{d\ln \mu_R} \approx \frac{1}{(4\pi)^2} \frac{\alpha_e^2}{\beta_0^2} \mu^2,$$

where $\mu_R$ is the renormalization scale. At low energy we measure $\mu^2 \approx -(88 \text{ GeV})^2$, and under SM running it is apparent that $|\mu|$ remains $\sim 100$ GeV even up to the Planck scale $M_{Pl} \sim 10^{18}$ GeV. Thus, there is no measurable naturalness problem in the SM alone; there is no fine-tuning of any measurable parameter at a high scale, only the cancellation of an unmeasurable bare parameter against an unphysical cutoff scale, which should be assigned no physical significance. With this understood, it is clear that a measurable naturalness problem can only arise when $d\mu^2/d\ln \mu_R \gtrsim \mu^2$. Indeed, this is exactly how the Vissani bound in the Type I seesaw model can be interpreted [16,17]. Let us now examine when the $\nu$2HDM encounters such a problem.

In practice, the naturalness considerations can be divided into two distinct calculations: the influence of $m_{22}$ on $m_{11}$ and the influence of $M_N$ on $m_{22}$. These influences will be considered in turn.

#### A. Corrections to $m_{11}^2$

If $m_{11}^2 \ll m_{22}^2 \tan^2 \beta/2$, then $m_{11}^2$ sets the mass of the observed SM-like Higgs via Eqs. (9) and (14). The one-loop RGE for the $m_{11}^2$ parameter is [23] (see Ref. [41] for a recent two-loop calculation).

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3Note that some of the bounds derived in these papers do not apply to the softly broken $Z_2$-symmetric case and also do not apply to the $Z_2$-symmetric case when one of the vevs vanishes.

4We refer the reader to Refs. [41,43–48] for allowed parameter space as a function of $\cos(\alpha - \beta)$ and $\tan\beta$ in all 2HDM types.

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\[
\frac{dm_{11}^2}{d \ln \mu_R} = \frac{1}{(4\pi)^2} \left[ (4\lambda_3 + 2\lambda_4) m_{22}^2 + \mathcal{O}(m_{11}^4) \right].
\]

which compares percentage variations of two (in principle) measurable parameters. Let us now estimate how such a bound might constrain \(m_{22}\).

For simplicity, and anticipating that the \(m_{22}\) scale is not far above the electroweak scale, we will evolve the dimensionless parameters using the \((\nu)\text{2HDM}\) RGEs from the \(m_{2}\) scale. First, the one-loop gauge coupling RGEs [23,59] can be solved analytically. Upon substitution into the \(\lambda_{3,4}\) RGEs [Eqs. (19)], and considering only the pure gauge contribution, the \(\lambda_{3,4}\) running can be solved for given initial conditions. For simplicity we take \(\lambda_3(m_{22}) = \lambda_4(m_{22}) \equiv \lambda_{3,4}(m_{22})\) and consider it a free parameter.

Next we solve Eq. (18) for \(m_{11}^2(\mu_R)\) with the initial condition \(m_{11}^2(m_{22}) = -(88 \text{ GeV})^2\) (neglecting any RGE evolution of \(m_{22}^2\)). With these simplifications \(\partial m_{11}^2(\mu_R)/\partial m_{11}^2(\Lambda_h) = 1\), and the fine-tuning measure is given simply by \(\Delta(\Lambda_h) = |m_{11}^2(\Lambda_h)/(88 \text{ GeV})^2|\).

Note that in setting the initial condition \(m_{11}^2(m_{22}) = -(88 \text{ GeV})^2\) we have implicitly assumed that \(m_{22}^2 \ll m_{21}\tan^2 \beta/2\) [see Eq. (12) and Fig. 2]. This is conservative for negative \(m_{21}^2\), since \(|m_{21}^2(m_{22})|\) shrinks as \(m_{21}^2\tan^2 \beta \rightarrow m_{11}^2/2\) and the naturalness constraint would become more stringent. In some circumstances we will obtain naturalness bounds on \(m_{22}^2\) which exceed \(m_{21}^2\tan^2 \beta/2\), which just indicates that the naturalness constraint is weaker than the consistency condition Eq. (12).

In Fig. 3 we show \(\Delta = 10\) and \(\Delta = 100\) contours as a function of \(\Lambda_h\) and \(\lambda_{3,4}(m_{22})\). These represent naturalness upper bounds on \(m_{22}\). The cupshaped structures of apparently low fine-tuning in \(m_{11}^2\) occur when \(m_{11}^2\) runs negative before turning and passing through \(m_{11}^2 = 0\), which just corresponds to a fine-tuning in \(\lambda_{3,4}(\Lambda_h)\). A stringent naturalness constraint is obtained by demanding \(\Delta < 10\) at \(\Lambda_h = M_{\text{Pl}}\); from Fig. 3 it is clear that this implies \(m_{22} \lesssim \text{few} \times 10^3 \text{ GeV}\).

If any new physics comes in below \(M_{\text{Pl}}\), then the running of \(m_{11}^2\) could change, and these bounds do not apply. If that is the case, then it is more appropriate to consider \(\Lambda_h\) at the scale of the new physics, which weakens the bound, as is clear from Fig. 3. In the \((\nu)\text{2HDM}\) this new physics scale is the right-handed neutrino scale \(M_N\), after which the right-handed neutrinos can contribute to the running of \(m_{11}^2\) through \(m_{22}^2\) at one loop.

We have so far ignored the RGE contributions from possibly large Yukawas. There are two situations in which the Yukawas play a significant role. The first is in Type II, LS, and flipped \(\nu\text{2HDMs}\) with \(\tan \beta\) large enough such that an early Landau pole is induced (see Sec. II C), and the second is in Type II and flipped \(\nu\text{2HDMs}\) with
moderate to large $\tan \beta$ when the pure Yukawa term $\pm \frac{1}{(4\pi)^2} 12y_D^2 y_{\tau}^2$ induced by a quark box diagram contributes significantly to the $\lambda_{3,4}$ RGEs [Eqs. (19)]. In Fig. 4 we show how the $\Delta = 10$ contours change as a function of $v_2$ in an example Type II $\nu_2$HDM. For this figure we have numerically solved the full set of one-loop RGEs [41] including the top/bottom/tau Yukawas, taking the following values at the scale $M_Z$: $g_2 = 1.48$, $\lambda_1 = \lambda_2 = 0.26$, and $y_t = 0.96/\sin \beta$, $y_b = 0.017/\cos \beta$, $y_\tau = 0.010/\cos \beta$. Comparing to Fig. 3 it can be seen that the pure Yukawa term has a noticeable effect when $v_2 \lesssim 20$ GeV. It is also apparent from Fig. 4 that nearing $v_2 \approx 3.6$ GeV (below which a Landau pole is induced before $M_P$) can act to degrade or improve the naturalness bound. The $v_2 = 3$ GeV bound in Fig. 4 shows the effect of hitting the Landau pole at $\sim 10^9$ GeV. We note that this only signals the breakdown of perturbation theory, and of our one-loop RGEs; we cannot calculate $m_{22}(\mu_R)$ above this scale, though it is perfectly possible that the theory remains natural.

In a repeated full one-loop RGE analysis, we found that the flipped $\nu_2$HDM gave essentially the same results as the Type II $\nu_2$HDM in Fig. 4, and there was no noticeable Yukawa effect in the LS $\nu_2$HDM until the Landau pole was reached. Thus, we found that the stringent naturalness bounds of Eq. (25) and Fig. 3 are applicable at all times in the Type I $\nu_2$HDM, for $v_2 \gtrsim 2$ GeV in the LS $\nu_2$HDM, and for $v_2 \gtrsim 20$ GeV in the Type II and flipped $\nu_2$HDMs. Otherwise, Yukawa effects must be taken into account. Either way, the important point is now clear: a TeV-scale $m_{22}$ can be both completely natural and, as was discussed in the previous subsection, is experimentally allowed in all $\nu_2$HDM types.

### B. Corrections to $m_{22}^2$

Let us now consider the influence of the right-handed neutrinos. The one-loop RGE for $m_{22}^2$ is [17,60]

$$\frac{dm_{22}^2}{d\ln \mu_R} = \frac{1}{(4\pi)^2} \left[ -4 \text{Tr}[y_D^2 D_M^2 y_{\tau}^2] + O(m_{22}^2) \right].$$

A conservative naturalness bound is obtained by bounding the running as we did in Eq. (23),

$$\frac{1}{4\pi^2} \text{Tr}[y_D^2 D_M^2 y_{\tau}^2] \leq \Lambda_{\text{bound}}^2,$$

where taking $\Lambda_{\text{bound}} = 1$ TeV gives the Vissani bound on $M_{N_1}$ of Eq. (5) [17]. However, now we are bounding corrections to $m_{22}^2$ rather than $m_{11}^2$, which may be TeV...
scale. Thus, depending on the mass of the extra scalars, it is possible that we can sensibly take \( \Lambda_{\text{bound}} > 1 \) TeV, in which case the the naturalness bound is somewhat relaxed; in Fig. 5 we show a relaxed Vissani bound for \( \Lambda_{\text{bound}} = \min(10 \, \text{TeV}, 10 \sqrt{m_y^2 \tan^2 \beta / 2}) \), where we have kept in mind the consistency condition Eq. (12). The Vissani bound still represents the unnatural area of parameter space if \( m_{22} \) is closer to 100 GeV.

As before, we could instead bound a quantity which measures the fine-tuning in \( m_{22} \), at some high scale \( \Lambda_h \). In this case, the fine-tuning measure of Eq. (24) is

\[
\Delta(\Lambda_h) = 1 + \frac{1}{4\pi^2} \sum_{i,j} (y_{\nu,i})_j M_{22}^2 y_{\nu,j}^i \ln(M_j/\Lambda_h) \frac{m_{22}^2(M_j)}{m_{22}^2}.
\]

Taking \( m_{22}(M_j) \sim 1 \) TeV and demanding \( \Delta(M_{\nu}) < 10 \) gives a similar bound to Vissani [Eq. (27) with \( \Lambda_{\text{bound}} = 1 \) TeV]. Note that there is no naturalness bound on \( M_N \) in the \( y_{\nu} \to 0 \) limit. This is the technically natural limit corresponding to an enhanced Poincaré symmetry in which \( v_R \) decouples from the theory [61].

In summary, there are up to three scales in the \( \nu \)HDM: \( v, m_{22}, \) and \( M_N \). We have described the conditions under which \( v^2 \) (or \( m_{12}^2 \)) is protected from \( m_{22}^2 \), and \( m_{22}^2 \) from \( M_N \).

Under such conditions it follows that \( m_{12}^2 \) is also protected from \( M_N^2 \) and the model is entirely natural.

### IV. Neutrino Masses

If \( m_{12}^2 = 0 \) and \( \lambda_5 = 0 \), then a \( U(1) \) lepton number symmetry can be defined, and neutrinos remain massless. Let us now consider turning each nonzero in turn.

#### A. \( m_{12}^2 > 0, \lambda_5 = 0 \)

If \( m_{12}^2 > 0 \) then the \( U(1) \) lepton number symmetry is softly broken; i.e., the breaking does not force us to insert a nonzero \( \lambda_5 \) term in order to introduce a divergent counterterm, and it is consistent to consider \( m_{12}^2 > 0, \lambda_5 = 0 \).

In this situation the neutrino mass matrix is given by the seesaw formula,

\[
m_{\nu} = \frac{v^2}{2} y_\nu D_M^{-1} y_\nu^T \approx \frac{v^2}{\tan^2 \beta} \frac{y_\nu D_M^{-1} y_\nu^T}{2},
\]

where \( \tan \beta \approx m_{22}^2 / m_{12}^2 \) for \( m_{22}^2 \gg \lambda_4 \delta_{12}^2 \) [see Eq. (10)].

The analogous Davidson–Ibarra and Vissani bounds are given by the standard Eqs. (4) and (5) with the replacement \( v \to v_2 \). These bounds are depicted in Fig. 5. If \( v_2 \lesssim 30 \) GeV (\( \tan \beta \gtrsim 8 \)), then both bounds are satisfied. As well, as discussed in Sec. III, if \( m_{22} \) is TeV scale, the Vissani bound can be relaxed, and the required \( CP \) asymmetry needed to reproduce the BAU via leptogenesis may be naturally achieved for \( v_2 \lesssim 60 \) GeV (\( \tan \beta \gtrsim 4 \)). In the Type I \( \nu \)HDM, \( v_2 \) can be naturally \( \ll \) GeV. Otherwise, requiring a perturbative theory up to \( M_N \) restricts \( v_2 \gtrsim 1 \) GeV (\( v_2 \gtrsim 2 \) GeV) for the LS (Type II/fliped) \( \nu \)HDM in the parameter space region of interest, as depicted in Fig. 5.

#### B. \( m_{12}^2 = 0, \lambda_5 \neq 0 \)

In this situation \( v_2 = 0 \), and a \( Z_2 \) symmetry remains unbroken; this is the scenario of Ma [22]. The model yields a radiative neutrino mass and a dark matter candidate. This is only possible in the Type I \( \nu \)HDM, since in any other type the unbroken \( Z_2 \) forbids a Dirac mass term for any charged fermion coupling to \( \Phi_2 \). Note that the limit \( \lambda_5 \to 0 \) is technically natural, since in that limit the \( U(1) \) lepton number symmetry is reinstated.

If \( M_N^2 \gg m_{22}^2 \), \( v^2 \) the radiatively induced neutrino mass matrix is

\[
(m_{\nu})_{ij} \approx \frac{v^2}{2} (y_\nu)_{ik} (y_\nu^T)_{kj} \lambda_5 \frac{M_N}{8\pi^2} \left( \frac{2M_N^2}{(m_{H_1}^2 + m_{H_2}^2)} \right) - 1 \right).
\]

The analogous Davidson–Ibarra and Vissani bounds are given by the standard Eqs. (4) and (5) with the intuitive replacement \( v^2 \to v^2 \frac{\lambda_5}{8\pi^2} (\ln [2M_N^2/(m_{H_1}^2 + m_{H_2}^2)] - 1) \). This assumes that there is no fine-tuning in the complex \( y_\nu \) parameters to reproduce the observed neutrino masses (see Appendix B for details). These bounds are depicted in Fig. 6, where the Davidson–Ibarra bound has been
evaluated at $m_{22} = 500$ GeV as an illustrative example (the bound is only mildly sensitive to $m_{22}$). We find that the Ma model with $\lambda_5 \lesssim 0.5$ can naturally achieve the required $CP$ asymmetry to reproduce the BAU via hierarchical leptogenesis.\(^6\)

C. $m_{12}^2 > 0$, $\lambda_5 \neq 0$

In this case both the tree-level seesaw and the radiative mechanism will contribute to the neutrino mass. Both contributions are calculable, and either might dominate. Note that it is still technically natural to take $\lambda_5 \to 0$ in this case, since it restores a softly broken $U(1)$ symmetry. In other words, the $\lambda_5$ RGEs to all orders will be multiplicative in $\lambda_5$, indicative of the fact that the soft-breaking term can only generate finite $U(1)$-breaking corrections.

V. LEPTOGENESIS

The observed BAU is achieved analogously to standard hierarchical thermal leptogenesis [13]; the out-of-equilibrium $CP$-violating decays of the lightest right-handed neutrino $N_1 \to \ell \Phi_2$ create a lepton asymmetry which is transferred to the baryons by the electroweak sphalerons above $T \sim 100$ GeV.

The details of the leptogenesis are largely defined by the decay parameter

$$K = \frac{\Gamma_D}{H_{|T=M_1}|} = \frac{\tilde{m}_1}{m_*},$$

comparing the rate for decays and inverse decays to the expansion rate at the time of departure from thermal equilibrium. Here, the rates

$$\Gamma_D = \frac{1}{8\pi} (y_{\nu \nu}^+)_{11} M_1,$$

$$H \approx \frac{17T^2}{M_{pl}},$$

are typically rescaled and expressed in terms of an effective neutrino $\tilde{m}_1$ and an equilibrium neutrino mass $m_*$,

$$\tilde{m}_1 = \frac{(y_{\nu \nu}^+)_{11} v^2}{2M_1},$$

$$m_* \approx 1.1 \times 10^{-3} \text{eV} \left(\frac{v}{246 \text{GeV}}\right)^2,$$

where $v$ is the vev that enters the seesaw Eq. (2). In the $\nu$2HDM with $\lambda_5 = 0$ (with $m_{12}^2 = 0$, $\lambda_5 \neq 0$), the analogous definitions make the replacement $v^2 \to v_2^2$ ($v^2 \to v^2 \frac{\lambda_5}{8\pi} (\ln [2M_{N_1}^3/(m_H^2 + m_A^2)] - 1)$). Note that for the scenarios we are interested in (e.g., $v_2 \ll v$) $m_*$ is smaller than its usual value in standard leptogenesis.

When only decays and inverse decays are considered, leptogenesis for given $K$ proceeds exactly as in standard hierarchical thermal leptogenesis (see, e.g., Ref. [63] for a review). In the weak washout regime $K \ll 1$, the baryon asymmetry strongly depends on the initial asymmetry and the initial $N_1$ abundance, with $N_1$ decays occurring at $T \ll M_1$. The strong washout regime $K \gg 1$ is independent of the initial conditions, and the asymmetry is generated as the $N_1$ fall out of thermal equilibrium.

The $2 \leftrightarrow 2$ scatterings with $\Delta L = 1$ (see, e.g., Ref. [64]) provide a correction to the simple decays plus inverse decays picture; they act to increase $N_1$ production at $T > M_1$ and contribute to washout at $T < M_1$. In standard hierarchical thermal leptogenesis, the scattering contributions involving the top quark and the gauge bosons are roughly equal. In the present model, the gauge boson contribution is the same as in the standard scenario. However, by construction, the $\Phi_2$ involved here in leptogenesis does not couple directly to the top quark, and thus the usual $s$-channel $(N_1 \leftrightarrow t q)$ and $t$-channel $(N_1 \leftrightarrow l q, N_l \leftrightarrow t l)$ scattering contributions do not occur. Instead, at large $\alpha \beta$ they can be replaced by the analogous contribution from other charged fermions, i.e., the bottom quark in Type II and flipped $\nu$2HDMs and/or the tau lepton in Type II and LS $\nu$2HDMs. A large tau lepton Yukawa will also introduce new $s$-channel $(N\Phi_2 \leftrightarrow \tau\Phi_2)$ and $t$-channel $(N\Phi_2 \leftrightarrow \tau\Phi_2, \tau N \leftrightarrow \Phi_2\Phi_2)$ scattering contributions. All of these processes are proportional to $(y_{\nu \nu}^+)_{11}$ and hence $M_1 \tilde{m}_1/v^2$, with the appropriate $\nu$2HDM replacement for $v^2$. Therefore, they scale with the decays and inverse

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\(^6\)A similar observation was made in a recent paper [62].

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FIG. 6 (color online). As in Fig. 1 but for the Ma model. The Vissani and relaxed Vissani bounds are evaluated at $m_{22} = 100, 1000$ GeV respectively. The Davidson–Ibarra bound and strong $\Delta L = 2$ washout region are shown for $m_{22} = 500$ GeV, though they are only mildly sensitive to $m_{22}$. The observed BAU is achieved analogously to standard hierarchical thermal leptogenesis [13]; the out-of-equilibrium $CP$ asymmetry and the electro-weak decay parameter $\lambda$ is transferred to the baryons by the electroweak sphalerons above which is calculated, and either might dominate. When only decays and inverse decays are considered, leptogenesis for given $K$ proceeds exactly as in standard hierarchical thermal leptogenesis (see, e.g., Ref. [63] for a review). In the weak washout regime $K \ll 1$, the baryon asymmetry strongly depends on the initial asymmetry and the initial $N_1$ abundance, with $N_1$ decays occurring at $T \ll M_1$. The strong washout regime $K \gg 1$ is independent of the initial conditions, and the asymmetry is generated as the $N_1$ fall out of thermal equilibrium.

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decays so that they represent only a minor (but obviously important) departure from the standard leptogenesis scenario.

The 2 ↔ 2 scatterings with ΔL = 2 mediated by the right-handed neutrinos (Φ2↓ ↔ Φ2↑, Φ2Φ2 ↔ ll) occur as they do in the standard scenario. These processes are proportional to $Tr(\{(\nu_i^c)^T\nu_i^c\})^3$ and hence $M_i^2\bar{m}^2/v^4$ where $\bar{m}^2 = \sum m_i^2$ is the neutrino mass scale $\gtrsim 0.05$ eV. Comparing this rate to the decay/scattering rates proportional to $Tr(\{(\nu_i^c)^T\nu_i^c\})^3$ and hence $M_i^2\bar{m}^2/v^4$ they do in the standard scenario. These processes are lower bound 10^4 GeV ≲ $M_{N_i}$ ≲ 10^6 GeV and 10^{-5} ≲ $\lambda_5$ ≲ 0.5. Formally more important than in the standard case. For $\lambda_5 = 0$, these scatterings will become comparably more important than in the standard case. For $T \lesssim M_1/3$ the thermally averaged ΔL = 2 scattering rate is well approximated by [63]

$$\Gamma_{\Delta L=2}/H \approx \frac{T}{2.2 \times 10^{13} \text{GeV}} \left(\frac{246 \text{ GeV}}{v}\right)^4 \times \left(\frac{\bar{m}}{0.05 \text{ eV}}\right)^2,$$

where the previously described ν2HDM replacements for ν² hold (see Appendix B). In Fig. 5 (Fig. 6), we show the region in the $\lambda_5 = 0$ ($m_{12}^2 = 0$, $\lambda_5 \neq 0$) ν2HDM where these scatterings are still in equilibrium at $T \lesssim M_1/3$. This is the region where strong ΔL = 2 scatterings can potentially wash out the generated asymmetry, depending on the details of the leptogenesis (e.g., in a weak washout scenario with $N_1$ decays at $T \ll M_1$, this washout may be avoided). Demanding that the scatterings fall out of equilibrium before sphaleron freeze-out at $T \sim 100$ GeV provides a lower bound $v_2 \gtrsim 0.3$ or $\lambda_5 \gtrsim 10^{-5}$; this is represented by the strong ΔL = 2 scattering washout regions in Figs. 5 and 6, respectively. We note that this calculation has been performed in the context of a perturbative theory. This is reliable for the Type I ν2HDM but not for Type II, LS, or flipped ν2HDMs with sufficiently small $v_2$, when perturbativity breaks down.

Putting this all together, we can now read off from Figs. 5 and 6 the regions of parameter space which can achieve natural hierarchical thermal leptogenesis. For ν2HDMs with $m_{12}^2 > 0$ and $\lambda_5 = 0$, we find 10^3 GeV ≲ $M_{N_1}$ ≲ few × 10^7 GeV is viable for Type I ν2HDMs, and 10^3 GeV ≲ $M_{N_1}$ ≲ few × 10^7 GeV for all other types if they are to remain perturbative. For the Ma model with

$\bar{m}_{12}^2 = 0$ and $\lambda_5 \neq 0$, we find viable parameter space for 10^3 GeV ≲ $M_{N_1}$ ≲ 10^8 GeV and 10^{-5} ≲ $\lambda_5$ ≲ 0.5.

Lastly we note that the lightest scalar state in the Ma model is stable. It is therefore possible that this state, if it is neutral, constitutes some or all of the observed dark matter. During the leptogenesis epoch, Φ2 is produced in abundance in $N_1$ decays. Overproduction of dark matter is of no concern as long as Φ2 efficiently thermalizes at or below the temperatures when $N_1$ decays occur, which suggests $m_{22} \ll M_{N_1}$. In this case the lightest state is a thermal relic dark matter candidate.

**VI. CONCLUSION**

The minimal Type I seesaw model is unable to explain neutrino masses and the BAU via hierarchical thermal leptogenesis without ceding naturalness. The main conclusion of this paper is the observation that a second Higgs doublet can avoid this problem. These ν2HDM models provide a natural solution by reducing the (possibly effective) vev entering the seesaw formula. This can be done radiatively, or by having the second Higgs doublet provide a tree-level seesaw with a small vev $v_2$, kept natural by softly breaking a $U(1)$ or $Z_2$ symmetry.

The models naturally accommodate a SM-like Higgs and predict the existence of approximately TeV-scale extra scalar states in order to remain natural. We rediscovered the radiative Ma model as the only possibility when $v_2 = 0$; in that case we found 10^3 GeV ≲ $M_{N_1}$ ≲ 10^8 GeV and 10^{-5} ≲ $\lambda_5$ ≲ 0.5 could simultaneously explain neutrino masses and the BAU via leptogenesis while remaining natural. The $v_2 > 0$ models require $\tan\beta \gtrsim 4$; we found 10^3 GeV ≲ $M_{N_1}$ ≲ few × 10^7 GeV was viable for Type I ν2HDMs, and 10^3 GeV ≲ $M_{N_1}$ ≲ few × 10^7 GeV for all other types if they are to remain perturbative. The interesting areas of parameter space are well summarized in Figs. 5 and 6.

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**APPENDIX A: SQUARED SCALAR MASSES**

AT $\mathcal{O}(m_{12}^4/m_{22}^4)$

To order $m_{12}^4/m_{22}^4$, the scalar masses Eqs. (14) are given by

\[ m_{12}^2 = 0 \text{ and } \lambda_5 \neq 0, \text{ we find viable parameter space for } \text{10}^3 \text{ GeV} \lesssim M_{N_1} \lesssim 10^8 \text{ GeV and } 10^{-5} \lesssim \lambda_5 \lesssim 0.5. \]

The lifetimes of the heavier scalar states are governed by mass splittings $\Delta$ via $\Gamma \sim G_F^2 \Delta^3/(16\pi^3)$. In the parameter space of interest, one can check that $\Delta$ is typically already large enough at tree level so that lifetimes remain well below $\mathcal{O}(1 \text{ s})$ and therefore do not disturb big bang nucleosynthesis.
Following Casas–Ibarra [71], it is possible to write
\[
y_v = \frac{\sqrt{2}}{v} U^\dagger D_{M} D_{f(M)} y_v^T,
\]
where \( R \) is a (possibly complex) orthogonal (\( RR^T = R^T R = I \)) matrix. The Vissani bound on each right-handed neutrino mass becomes [17]
\[
1 \frac{2}{4\pi^2 v^2} \sum_{i,j} |m_{ij}|^2 < 1 \text{ TeV}^2 \\
\Rightarrow M_{N_i} \lesssim 3 \times 10^7 \text{ GeV} \times f(M_{N_i})^\frac{1}{2}.
\]


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