Distributed Demand Side Management and Tariff Design in Distribution Networks

by

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Abstract

As a non-storable commodity with time-varying demand, the ‘logistics’ of electricity has always been challenging. To ensure reliability, electricity systems are required to be built with redundant generation, transmission and distribution capacity such that they are able to withstand the maximum demand under expected fault conditions. On the one hand, structuring the electricity system in such a way is not efficient. A significant portion of network capacity is under utilised or sitting idle for most of the time and the situation is getting worse over time as shown by increasing peak to average ratios. On the other hand, emerging technologies such as Electric Vehicles (EV), photovoltaic (PV) and storage devices will fundamentally change the existing paradigm of the existing electricity network. The potential peak surge caused by large numbers of EVs and possible over-supply from PVs may harm the networks but the large flexible battery capacity of EVs, inexpensive and clean energy from PVs together with storage devices could, on the other hand, greatly benefit the networks. This is especially the case with smart measuring devices being widely installed and affordable sensors being ubiquitous. Therefore, it is hugely important to implement the technologies appropriately and prepare the electricity networks for upcoming changes.

Demand Side Management (DSM) and tariff design are proposed, from two complementary directions, as promising concepts to increase the efficient use of electricity grid assets, to enable a smooth transition towards smarter grids and to promote fairness among network users. Among the rich literature on DSM algorithms, advanced electricity grid information and communication technology (ICT) is usually assumed to exist. In this thesis, however, the requirement of communication is relaxed and solutions for networks with different ICT infrastructures are developed and compared. It is shown that satisfactory DSM can be achieved even without communication via the stochastic voltage-demand model we develop. Network tariff design focusing not only on revenue recovery and social welfare maximisation, but also fairness among electricity users especially in the presence of PVs has also been studied. A utility based model to quantify the benefit and cost for an individual user using the network is proposed. The model is adopted to assess efficiency and fairness of existing tariff structures as well as designing optimal tariffs.
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Abbreviations

EV  electric vehicle
PV  photovoltaic
DSM  demand side management
ICT  information and communication technology
TSO  transmission system operator
DSO  distribution system operator
NEM  national electricity market
AEMO  Australian energy market operator
VIC  Victoria
TOU  time-of-use
DLC  direct load control
MPC  model predictive control
SOC  state of charge
AIMD  additive increase multiplicative decrease
TCP  transmission control protocol
PLL  phase lock loop
**NSW** New South Wales

**LV** low voltage

**AC** air conditioning

**PDF** probability distribution function

**CDF** cumulative distribution function
Chapter 1

Introduction

Electricity grids or power systems are the largest machines ever built by humans. They have been changing and evolving since the first complete system, Pearl Street Station, was built in New York City in the late 19th century. The station was designed by Thomas Edison and served a few hundred lamps in a local neighbourhood [48]. Then Alternating Current replaced Direct Current, isolated local networks expanded to huge interconnected networks and the mission to light lamps became to power billions of appliances for millions of people. We are now stepping into the era of the smart grid where the electricity networks will be integrated with distributed generation, electrified vehicles, communication technologies, advanced control devices, fairer tariffs and many more renewable energy technologies. This smart grid is much more environmentally friendly and economically efficient. However, such exciting opportunities do not exist without challenges and a tremendous effort is required to make this transition to a smart grid as smooth as possible.

In this chapter, we use the Australian power system as an example to explain some of the basic facts in regards to electricity network and market operations. Note that the electricity grids in different countries or regions, though sharing similarities, may operate differently. The Australian power grid is not one of the larger systems and it has its own specialities. However, an in-depth understanding of the Australian power grid would greatly facilitate the comprehension of other power systems. Meanwhile, we also point out the challenges faced by traditional power networks in the era of the
smart grid. Then, as possible remedies to the problems faced by electricity networks, DSM and tariff design will be discussed on a macro level. Later in this chapter, we review the relevant literature and point out our approaches and contributions. A list of our publications, upon which this thesis is based, is appended at the end of this chapter.

1.1 Background

In the context of the Australian grid, in this section, we introduce the basic components of an electricity network and the associated operators. We also explain how electrical energy is traded on a designated market including the bidding and dispatching process. Then DSM technologies and network tariffs are discussed with a listing of common practices.

1.1.1 Background on power systems

In terms of operation, an electricity grid or power system typically comprises three major components: generation, transmission and distribution. All these components together form a massive network which delivers electrical energy from generators to individual users instantaneously as shown in Figure 1-1. Generators or plants are owned and operated by generator companies who trade energy on a wholesale market which will be explained later in this section. In Australia, coal fired plants supply around 75% (as in 2014) of the total consumption while the renewable energy supply is growing steadily [12]. Electrical energy then flows through the transmission network which is owned and operated by transmission system operators (TSO) and supervised by network regulators. TSOs are monopolies by nature and they play a crucial role in interstate power imports and exports. When electricity arrives at the region of need, distribution networks take over and distribute energy to end users which include industrial, commercial and residential users. Distribution networks are owned and operated by distribution system operators (DSO). DSOs are also monopolies by geography and they are responsible for the power quality, metering and the
distribution assets.

Though both are called networks, there are many differences between a transmission network and a distribution network. A transmission network usually operates at high voltage and transmits electrical power over long distances. Such a network is usually equipped with dedicated and advanced measuring, communicating and controlling devices. At various points of a transmission network, the states are closely monitored and regulated. Distribution networks, especially the last mile, are usually considered as a mere disturbance of lesser interest in power system analysis and operation. Their attributes are not usually closely monitored or communicated for cost reasons as they are much less important than the transmission network. As a result, though power is measured, DSOs would not know the voltages, unbalances, current, or even outages at individual households.

As for the financial operation of electricity grids, energy is usually traded on wholesale markets. Most of the states in Australia trade electricity on the National Electricity Market (NEM), the largest wholesale electricity market in the nation, which is operated by the Australian Energy Market Operator (AEMO). The NEM spans the east and south east coast of Australia and serves 9 million customers from 5 of the most populated states [12]. In the NEM, generators sell electricity and retailers buy it to resell to consumers. Retailers normally do not own any network assets, they handle billing and customers services in the power system. The NEM has over 100 registered generators and retailers placing bids to sell and to buy respectively [12]. Since electricity is not easily stored, power supply and demand is matched instantaneously by controllers and regulators through some optimal dispatching algorithm [11]. The NEM effectively works as a spot market where electricity trading is settled. Generators submit supply bids every five minutes which are gathered by AEMO to calculate the most cost effective way of buying energy while meeting real time consumer demand. Though energy is dispatched in five minutes intervals, the successful bidding prices are averaged every half hour to determine the spot price for trading settlement [11].
1.1.2 Demand side management

Under the existing paradigm of the power system, supply matches demand or the system encounters problems, the consequences of which are seen in the frequency variations of the network. When switches are flicked at the consumers’ side, electricity will be delivered without any delay. DSM aims to alter the paradigm by enabling demand to coordinate with supply. Certain appliances such as pool pumps, pool heaters and EVs may choose to delay or even advance their demand instead of using electricity straight away. In such a way, stress of the grid can be relieved and utilisation of the entire infrastructure can be improved.

However, is it physically possible for power demand to be adapted to match supply? Arguably the answer is yes and we justify the assertion by explaining some of the relevant time constants in the grid. An overview of typical time scales is displayed in Figure 1.2 whose detailed explanation can be found in [69]. DSM operates in grid electrical steady state (frequency and voltage regulated) which can typically be considered on time scales of minutes and longer. This so called steady state is regulated within national operating guidelines such as [2,10,71]. As we have seen from the previous section, in the Australian electricity market, energy dispatch is or-
organised on a 5 minute interval basis, and real time trading is organised in 30 minute time slots. The implication from the numbers is that aggregated demand does not fluctuate significantly over half an hour scale. The DSM algorithms we propose work on time scales of 1 minute up to 15 minutes. It is not to let everyone bid for as much electricity as they need now, but to average the power over the energy needs on a longer time scale of half an hour or more.

The term DSM was proposed soon after the energy crisis in 1973 in the USA [30]. DSM is not precisely defined but it commonly refers to techniques deployed by DSOs and/or consumers to achieve certain management goals at the demand side [91]. Such goals include shaving peak demand, flattening consumption patterns and creating better billing schemes. DSM techniques range from energy efficient devices and demand controlling devices all the way to tariffs that encourage certain usage patterns especially the Time-of-Use (TOU) Tariff [77]. In fact, tariffs could not only encourage certain demand patterns, but also maximise social welfare and fairness if designed properly. Thus we discuss tariff design in addition to DSM technologies
In Chapter \[6\], in the next section, we briefly explain the background on DSM and tariff design.

### 1.1.3 Common DSM technologies

DSM technologies are those techniques deployed by consumers and/or DSOs that redistribute the demand in time without a major reduction of energy consumption unless there is a huge incentive for consumers. Typical and most commonly implemented DSM technologies can be classified as the following:

1. **Direct load control (DLC):** Utilities have free access to control consumers’ appliances or equipment and consumers are able to receive reduced electricity prices as a result \[91\]. Some common appliances that can be operated in this way are air-conditioners, water heaters and swimming pool pumps. In case of a supply shortage, a signal will be received by complying appliances which will then stop operating for a period of time. A problem with this approach is that direct access of appliances is not desirable from the consumers’ perspective. Furthermore, switching off air-conditioners or reducing their cooling output on the hottest summer day is likely to be frustrating for consumers. Note that some big industry consumers can also be subject to DLC.

2. **Commercial, industrial programs:** examples of such programs are energy saving systems for buildings (e.g. motion detectors to switch lights off) which are not the focus of this thesis. Another important type is the interruptible loads installed by commercial and industrial customers who in return get a significant price reduction on their electricity bill for this provision. Whenever necessary, these loads can forgo a certain amount of electricity which then flows to the consumers in need. These loads are virtually acting as reserve generators and the customers are paid for providing reserve services. The Western Australia Electricity Market is a typical example where this type of reserve service is provided \[53\].
3. Demand response programs: A promising DSM technique is the introduction of demand response programs in residential, industrial and commercial properties. Local controllers will adjust demand behaviour according to price or other information broadcast by DSOs whilst still meeting the energy needs (not the power needs) of the customer. Together with a well designed pricing structure, this could result in huge shifts of energy usage and benefit for both the network and the consumers [91].

1.1.4 Common network tariff structures

The billing system for electricity has been changing and reforming since the discovery of electricity. For every dollar a consumer spends on their electricity bill, about 51% goes to the network operators which eventually pays for the grid infrastructure such as poles and wires [12]. The costs for electricity for consumers are largely determined by network tariffs. Collaborative efforts by electrical engineers and economists (as summarized in [87]) have been made to design a pricing structure that on the one hand covers sunk infrastructure costs of electricity networks, and on the other hand, accommodates a non-storable commodity with time-varying demand. The following describe different tariffs.

1. Two-part tariff: a two-part tariff consisting of a fixed charge and a variable usage charge (at a fixed unit rate), was proposed [38, 59] and widely implemented. However, a fixed unit rate does not relieve peak demand which has become a major problem for most electricity networks;

2. Time-of-Use tariff: TOU charge was proposed [42, 52]. Under this system, electricity usage during peak hours is priced higher than during off-peak hours. The partitioning of periods can be finer to create shoulder hours and critical peak hours. Nevertheless, despite having taken peak usage into account, a TOU tariff does not align electricity billing with actual benefits and costs consumers get from networks.
3. Three-part tariff (demand tariff): a three-part tariff comprising a fixed charge (can be zero in some cases), a TOU charge and a maximum demand charge was then introduced \[50\] \[99\]. Despite the fact that more advanced pricing structures, including dynamic pricing \[22\], have been proposed more recently, the three-part tariff has an advantage in terms of implementability and optimality and it has been gradually adopted by network operators. In Australia, CitiPower offers three-part tariff to new low voltage customers with a large load while residential customers are still on a TOU tariff \[24\]. United Energy has a similar tariff for residential customers. However, for customers with a large load, daily standing charge is replaced by a more aggressive demand charge \[94\].

1.2 Motivation for DSM and tariff design

1.2.1 Infrastructure utilisation

The power grid is designed to generate, transmit and distribute the peak power required because there is no simple and cost-effective storage solution for electrical energy. In addition, infrastructure is required to provide for spare capacity in power transfer: a safety margin to cope with inevitable failures in infrastructure assets. As a consequence, the grid infrastructure is always under-utilised and the utilisation rate is especially low during periods other than peak periods. The transmission and distribution assets in particular form a significant part of the electricity price and under-utilised infrastructure eventually leads to higher electricity prices especially where network companies are promised a fixed return on their investments. This situation is getting worse over time due to surging peak demand and over designing. In the state of Victoria, Australia, about half of the electricity price goes towards network infrastructure \[12\]. The other half goes towards energy costs (30%) and retail margins (20%) \[12\]. Inefficient use of the grid ultimately leads to more costs for consumers. Using DSM, peak loads can be rescheduled to off-peak periods without sacrificing appliance performance \[91\]. Doing so, more total energy can be delivered
in the existing grid without causing outages and this brings reliability to the existing electricity grid. Moreover, by spending less money on infrastructure, more load can be supplied. This helps to address the demand peaking issue that some researchers have discussed with large scale electric vehicle integration. In addition, demand peaks usually create accumulated heating in cables and transformers which shortens equipment lifespan. In addition, equipment can be used for more than 100% of its design capacity (remains below the safety limit), and the more you go over this limit, the more you reduce the life span of the device. By alleviating peaks via DSM, ageing of equipment would be eased to a certain extent.

This argument also applies to generators. The nature of electricity demand is uncontrollable and varies with seasons, temperatures and time of day. On the other hand, service interruption of the electricity network can be very costly [91]. Therefore, in Australia and many other parts of the world, the grid is built with generation redundancy such that the highest possible demand can be matched whenever necessary. In order to do so, a certain level of power has to be made available immediately through spinning reserves and non-spinning reserves [11]. In Victoria, the installed generation capacity is almost twice that of average demand [11]. Such a generation setting is not efficient since a huge part of the generation is not used for most of the year and DSM might be better placed in terms of ensuring power system reliability and supply security. By predicting and directing demand patterns, managed demand can act as capacity reserve. Peak demand can be reduced and uncertainties limited. Therefore, by deploying DSM, generation investment efficiency can be improved.

In addition, as mentioned earlier, electricity is traded through a spot market where generators submit supply bids and AEMO gathers all the offers to calculate the most cost effective way of buying energy while meeting real time consumer demand. Due to the market mechanism and uncertainty of demand, generators are able to charge scarcity prices for electricity in certain periods when demand outruns supply. Figure 1-3 shows the 30 minute wholesale price of electricity on a very hot day in Victoria and South Australia, 2014. In the afternoon, the spot price jumps to several magnitudes
Such an incredibly high price threatens small retailers and a portion of the excessive cost will eventually be paid by consumers through increased electricity prices. However, by deploying DSM technology at a large scale, the market power of consumers can be exercised. Decision-making processes regarding the operation and future development of the grid will place consumers of electricity in the center. By doing so, power grid investment will be made more efficient. Regulators, DSOs and especially consumers will benefit from this shift.\textsuperscript{5 26}

\textsuperscript{1}average whole sale spot price in Victoria (VIC), Australia, Jan, 2013 is around $50/MWh according to AEMO website
1.2.2 Integrating emerging technologies

The electricity grid is evolving, many technologies have been developed or popularized in recent years. These technologies include electric vehicles and distributed generation and these have raised concerns about the stability and reliability of the electricity grid. Shifting from internal combustion engine vehicles to EVs (including plug-in hybrid EVs) can lead to benefits for the environment and enhance energy security. Many major automakers are investing significant resources into extending EV range and reducing the cost of production, especially of EV batteries. If both of these bottlenecks can be overcome, EVs will become very market competitive and it is therefore possible that a significant market penetration of EVs will be reached in the future. In fact, the sales of EVs are growing very fast in developed countries. In the U.S. alone, more than 50,000 EVs were sold in 2012 which was three times that in the previous year. Beside the United States, studies have shown that EV penetration may reach 30% in Belgium by 2030 [64]. However, one important factor to note about EVs is that they run on electricity and have to be charged. Currently, commercially available plug in electric vehicles are equipped with a battery with a storage capacity around 25kWh or more (Nissan Leaf\(^2\) 24kWh, Tesla model S\(^3\) 60kWh – 85kWh ). Consider normal urban users in Australia, and assume an average passenger vehicle travels nearly 75km per day. With a conversion rate of 0.2kWh per km \(^70\) (the actual value could be higher depending on road condition and drivers skill), this would result in a need for 15kWh of electricity. To compare, a typical Australian family, depending on the geographical location, uses 15kWh - 30kWh of electricity per day \(^28,70\). One EV would increase household electricity consumption by at least 50% and two EVs would double the current demand. The existing grid is not designed for the large uptake of EVs and the integration problem has raised many concerns. Studies have shown that even with a small EV penetration, losses in power are likely to increase and voltages could likely deviate from regulated levels \(^25\). However, DSM could play a very positive role in this situation and solve the problem with minimum

\(^3\)http://www.teslamotors.com/models
cost.

In a similar way to EV, solar energy, in the form of distributed photovoltaic (PV), is a major step towards a low emission society. The environmental benefits are clearly enormous. Most distributed PV systems in Australia are installed under the Commonwealth Government’s Renewable Energy Target scheme which commenced on 1 April 2001 [15]. After a period of steady low growth for almost 10 years, a period of extremely rapid growth occurred between 2010-2013. By August, 2015, the total installed PV capacity in Australia was around 4.5 GW (approximately 10% of total capacity) which contributes approximately 1% of the total energy generation [11]. The imbalance between the two percentages is caused by the variations in irradiation time. The distributed PV installation rate or penetration rate among dwellings varies depending on geographic locations. The highest penetration is approximately 27% in Queensland and South Australia. In New South Wales and Victoria, about 12 percent of households have PV systems installed [15].

A high penetration of distributed PV generation, however, may not be desirable for the network under the existing paradigm and the reasons are three-fold. Firstly, due to naturally inherent uncertainty, PV systems need to be backed up by reliable reserve generation and ancillary services which makes the overall paradigm rather inefficient [55]. Secondly, the electricity grid is designed for a one-way power flow. A high penetration of PV systems could lead to over-voltage at various points in the distribution network and reverse power flow (where an entire feeder line is over-supplied and power flows into the substation from downstream) [55]. Impacts of such phenomena include shortened lifespan of equipment, unstable power quality and difficulties in generation planning. Thirdly, the network tariff in the traditional structure is levied on a uniform basis assuming a similar demand pattern among users but the cost is actually largely determined by the peak demand. Due to the fact that PV peak does not coincide with peak demand, PV systems could well reduce average usage but not peaks. In [87] [100], the authors argue that solar PV systems reduce the average network demand but their impact on peak demand is insignificant or limited. As a result, the overall utilisation of the electricity infrastructure is reduced. Further-
more, because of an old fashioned tariff structure and various incentive programs for rooftop PV installers including feed-in tariffs, the authors show that households and businesses that have not installed solar rooftop PV systems are spending billions of dollars subsidising households that have in Australia. However, from a different perspective, it has been pointed out in [82] that the so called subsidising exists because existing electricity distribution charges are levied on a uniform basis, but the costs of the electricity distribution network depends mostly on peak demand. The problem has been there for a long time and PV systems are not completely responsible for it. Therefore, we propose a quantitative measurement from an engineering point of view such that fairness and social welfare can be design features of the network and its operation.

1.3 Related Work

There exists an extensive literature on modern DSM including technical papers and overview papers [5, 23, 27, 31, 41, 47, 60, 68, 72, 73, 77, 79, 91, 98]. The ones that are most relevant to this thesis will be briefly reviewed in this chapter. In addition, we also review the literature on demand management for EVs [9, 25, 32, 36, 37, 41, 44, 45, 62, 67, 83, 90]. EVs are loads with huge demand and storage capacity. The concepts and algorithms used for EV charging management can be easily migrated to other appliances.

Most modern, novel DSM technologies in the literature make use of communication infrastructure and information exchange to achieve better performance and optimal results. Depending on how information is exchanged and where the decision is made, the algorithms can be categorized into two categories: centralised and distributed. Centralised approaches solve the problem from top down where a master controller has access to all the key information relating to the grid and users. This information may include voltage, demand and frequency. Computation is done by the master controller and control signals are then broadcast to individual users. The solutions from centralised algorithms are generally accurate and optimal as long as
there is adequate information and computing power. However, there is always the need for additional communication facilities as well as a powerful central computing unit which could be too expensive and complicated to be installed especially for distributed networks. Scalability is always an issue with this category of algorithms. The computational complexity increases dramatically as the size of the network increases.

Alternatively, calculations can be done in a distributed way at the point of each load or household, only where there is access to the limited information broadcast by neighbours or nearby transformers. Compared to centralised algorithms, distributed algorithms generally require less information and are much easier to implement. However, the accuracy and optimality of distributed methods are generally not as desirable as those of centralised algorithms.

### 1.3.1 Centralised algorithms

Centralised charging algorithms usually assume the availability of a model of some sort. Most of such problems are formulated as optimisation problems with various cost functions to minimise (or objective functions to maximise) and with constraint sets. The cost functions may be power losses across the network, voltage deviations from an operating point or billable electricity cost. The objective functions may be total power delivered or social welfare. Apart from the differences in cost functions, the algorithms can also be categorized according to whether a prediction or forecast is required. Two common categories are discussed in this section.

**Centralised algorithms with prediction**

If each load is able to submit a demand schedule (time to start, time to finish and energy required) for the next period of time (typically a day), the central controller will be able to calculate the optimal demand profile for all the loads. The demand profiles are then broadcast to all users for execution. The whole process is then repeated after this period of time.

A possible cost function as shown in (1.1) which minimises power losses in the
whole network for an entire day is introduced in [25] where $R_l$ represents the line resistance of line $l$ and $I_{l,t}$ represents the current flowing on line $l$ at time $t$. The algorithm assumes a static model with a time step size of several minutes and the prediction horizon is 24 hours.

$$\min \sum_{t=1}^{t_{end}} \sum_{l=1}^{\text{lines}} R_l I_{l,t}^2$$  \hspace{1cm} (1.1)

Users are required to submit details regarding how much energy is needed in what time window. The central controller collects the information and solves (1.1) using quadratic programming. This also involves solving the load flow equations many times to calculate the line currents as inputs for the optimisation problem. The biggest issue with such centralised algorithms is the heavy dependence on the accuracy of forecast, especially load profiles which are hard to determine in advance with high accuracy. To relax the dependence, in [90], the authors argue that instead of knowing the exact demand profiles, a stochastic modelling of loads with expected values and variances can also be used to formulate the optimisation problem but with reduced optimality.

More sophisticated Model Predictive Control (MPC) approaches which also require more model information are introduced in [32] where realistic network constraints are considered. In [32], a standard objective function is proposed as shown in (1.2).

$$\max \sum_{i=1}^{N} \sum_{t=0}^{T-1} x_{i,t}$$  \hspace{1cm} (1.2)

The current used for EV $i$ at time $t$ is shown as $x_{i,t}$. Current is used instead of power in this objective for the reason that the EV charger standard J1772 is specified in terms of charging current. The algorithm assumes a static model with a time step size of 15 minutes. i.e. $t$ is 15 minutes apart and decisions are made every 15 minutes. Whenever a decision is made, charging profiles for a future window of length 8 hours are calculated. However, such profiles will only be executed for another 15 minutes when the central controller calculates the next horizon and makes the next decision. In this way, future events are taken into consideration and flexibility is also
guaranteed.

A similar algorithm with a different triggering mechanism is used in [37]. The algorithm in [32] is time triggered which means the calculation is repeated every certain time interval regardless of what is happening in the network. However, in [37], the user proposes an event triggering mechanism where charging control signals are updated when an ‘event’ takes place. Such an event can be a EV arriving at home or leaving home and sudden changes in non-EV demand.

However, for all the above algorithms, load flow equations need to be solved at each iteration which requires accurate knowledge of the network configuration. In other words, information like network topology and individual line impedances need to be known centrally which is not easy in practice. In addition, the fact that both methods require significant computation power (highly non-linear load flow computation) may not be favourable for immediate implementation.

Centralised algorithms without prediction

In order to develop algorithms that are more realistic and adaptive, on-line centralised algorithms are proposed where computation can also be performed in real time with less complexity but no prediction.

An optimisation technique that maximises the total amount of energy that can be delivered to EVs over a fixed charging duration while ensuring network limits are maintained is introduced in [83]. The basic objective function is modelled as follows:

$$\max \sum_{i=1}^{N} P_{EV_i} x_i$$  \hspace{1cm} (1.3)

In the function, $N$ is the total number of EVs that is registered at the central controller and whether or not they are connected is represented by $x_i$ being 0 or 1. $P_{EV_i}$ is the power delivered to EV $i$. Several constraints are considered in the formulation including the physical limit of charging power and the changing speed in charging power. The voltage level at each household and thermal loading of the network are also listed in the constraint set. In the above formulation, EVs at the far end of the
transformer are less favoured because of higher line losses while charging them. To deal with this issue, the objective function can be altered to prioritize EVs with a low battery state of charge (SOC) given that the central controller has access to all SOC levels in real time. The optimal solution to the problem is calculated using linear programming to reduce computational complexity and the process will be performed at each chosen time interval of several minutes. The performance of such an algorithm has been tested by simulations and it is shown that the power delivered to EVs is increased significantly and all grid limits outlined above are well maintained.

Compared to algorithms with forecast, the real time algorithms work on the network states at a particular time instance without foreseeing the future and planning ahead. It is therefore difficult to provide guarantees regarding, for example, the final SOC levels of EVs.

1.3.2 Distributed algorithms

The reviewed centralised algorithms, though they differ from one another in terms of design goals and optimisation methodology, share the same weakness in that they all rely heavily on information and they all require significant computation power. For the centralised optimisation to work, the controller requires an accurate model of the network to perform power flow analysis \[25,32,36,37,83,90\]. Such a model is often not available and the power flow computation requires much processing power even when the network is small. Moreover, the optimisation itself requires quadratic or more complex programming which is resource-intensive especially when the network size increases. Distribution networks, commonly considered as the less significant part of the grid, are not very well equipped with varies monitoring and remote control devices. And the fact that computation power is costly may also limit the implementation of centralised algorithms.

Distributed algorithms, which may not be able to guarantee the same level of optimality as centralised algorithms do, have good scalability and are much easier to implement. Different distributed algorithms require different patterns of communication and different intensity of information exchange. Some algorithms use two-way
communication which means that consumers or users negotiate with DSOs to make charging decisions. Methods using two-way communication require the highest level of information exchange, therefore their level of distribution is low. Other algorithms use one-way communication which only requires users to listen to the information broadcast by the DSOs and no negotiation is required. In all the algorithms, apart from users and the DSOs, there could be regulators who oversee the entire operation and intervene if necessary. In this section, several state-of-the-art distributed charging management algorithms will be discussed according to their communication patterns and requirement on information exchange.

Two-way communication algorithms

Game theoretical algorithms are common in distributed DSM and information exchange is usually intensive. An example of an algorithm to schedule demand using game theoretical modelling is presented in [73]. The model has two major objectives: one is the peak-to-average ratio minimisation desired by DSOs; the other is the cost minimisation desired by end consumers. The authors show that by selecting the right pricing function, a Nash equilibrium can be established and both DSOs and consumers can win from the formulation. One issue about this algorithm is that consumers are required to inform DSOs of their demand patterns one day ahead in an asynchronous manner which is difficult to implement in practice. In fact, most of the decentralised algorithms that use two-way communication are designed in a day ahead manner [41, 44, 45, 67].

The key to shifting computation from the central controller to individual agents is using feedback signals. Price is used as the feedback signal in [67] where the overall goal is to flatten the demand profile, obtain a valley-filling effect to minimise the cost for generation and maximise the use of assets. In order to achieve the goal, the authors assume that the price (may not be the actual monetary price) is proportional to total demand at all times and the process works as follows repeatedly until convergence: before the start of a new day, the DSO broadcasts its prediction of the price profile to all the EVs. Each of the EVs proposes a charging profile that minimises its own
cost function with respect to the average price profile broadcast by the DSO. The objective function proposed in [67] is shown in (1.4):

\[
\min_{\Omega} \sum_{t=0}^{T} \left( p(\cdot)u^n_t + \sigma(u^n_t - u^{avg}_t)^2 \right)
\]  

where \(p(\cdot)\) is the price function which is continuously differentiable and strictly increasing. The example function used is a price proportional to total demand; \(u^n_t\) is the charging power at time \(t\) for EV \(n\); \(\sigma(u^n_t - u^{avg}_t)^2\) shows the deviation of individual charging profile from average behaviour. The DSO then collects all the individual optimal charging strategies and updates the price profile (proportional to aggregate demand) in line with the proposed charging strategies. This updated price is broadcast back to all EVs for distributed decision making. The whole process will be repeated until convergence. Then the resultant charging profiles will be executed by EVs for the day.

Inspired by this work, [44] provides a similar valley filling algorithm that makes less assumptions about the grid and base demand. The utility function used in [44] consists of both electricity cost (based on the most recent price broadcast by the DSO) and the cost of deviating from the previous charging profile. So far, only homogeneous agents have been considered, but it is also possible to individualize users as demonstrated in [41]. Willingness to pay is used as the way to distinguish users in [41] and randomness is used in [45] to break the symmetry.

Instead of the interaction between a single DSO and many customers, [68] proposes a framework to coordinate multiple DSOs and multiple consumers based on a Stackelberg game. The overall goal of this game theoretical algorithm is to maximise both the revenue of each DSO and the pay-off of each user. In order to do so, each user will have to set a monetary budget constraint and a utility function. Based on this information, the objective on the users’ side is to maximise the aggregated utility while keeping the cost within limits; the objective on the DSO side is to maximise revenue. The DSOs and users will jointly compute a equilibrium solution in advance which will be executed for the next time interval.
Compared to centralised algorithms, the method using two-way communication decomposes and distributes computation from a central processing unit to individual users. The DSO commonly uses a price alike signal to coordinate users and the signal is usually proportional to the total load in the network. By doing so, network information such as topology and phase allocations are no longer required. And users’ privacy is better protected as only the aggregated demand information is required. However, optimality of solutions is no longer guaranteed and network constraints are not as well monitored and maintained.

In [41, 44, 45, 67, 73] as discussed above, users are all required to know their demand profiles or network background demand profile for a future horizon in order to compute local optimums which may be impractical and inconvenient. Moreover, loads are not modelled in full details and they are often assumed to be resistive and interruptible. Considering that different appliances have distinct demand patterns, the algorithms may encounter difficulties during actual implementation.

One-way communication algorithms

It could be argued that the algorithms using two-way communication are not really distributed since they require each vehicle to communicate extensively with a central agent (e.g. the DSO). Some may also argue that most papers in the literature are prediction-based, which may lead to inaccuracy and robustness issues. Therefore, algorithms that are less dependent on information, less dependent on forecasting and do not require extensive two-way communication have been proposed. These algorithms usually use only one-way communication and some examples will be discussed in the following paragraphs.

One of the early approaches to optimally managing storage type demand in response to electricity spot price was proposed in 1989 [31]. The authors assume that electricity is priced such that the actual price reflects the marginal cost of generation. The spot price $P_k, k = 1, 2, 3, ...$ for period $k$ is updated after every interval and is kept constant for the following interval. Each household has access to the price information through communication channels. The overall cost to be minimised for
every moving time horizon $N$ is shown in (1.5) where $U_k$ denotes the consumption of electricity during period $k$.

$$\min_{U_k} \sum_{k=1}^{N} P_k \times U_k$$

(1.5)

The cost function together with constraint sets can be solved mathematically. One constraint that the authors emphasise is that the energy storage level at the end should be regulated. Otherwise the final energy storage level will be as small as possible. The authors test their algorithm on the data from an air compression company and show that a generation saving of 7.53% to 15.04% can be achieved. However, the authors assume that customers have access to spot prices in real time even for a period in the future. In Victoria, the spot price is determined every 5 minutes and fluctuates depending on actual demand. It is difficult to have a precise model in advance. Moreover, in Victoria, generation cost only counts for only a small portion of the retail price which makes the optimisation impractical. Many other papers also use total price as an objective. As an example, in [36] the authors proposed a novel load management solution for coordinating EV charging. The algorithm minimises the total cost of electricity generation plus the associated losses in the grid. It also incorporates the market electricity price to make the algorithm more realistic.

Computer networks and power networks share a lot in common especially as they are facing the same challenge where multiple agents share a common medium. Therefore, some protocols developed for computer networks can be applied to DSM problems. The principal of utility theory and congestion pricing (rather than actual price) in computer networks is applied to DSM in [41,60].

In [41], each user is modelled as a net utility problem where net utility for each household is the utilities of appliances minus the price (which may or may not be actual monetary price) they pay as in (1.6)

$$u(x(n)) - x(n)p(n)$$

(1.6)

where $u$ denotes the utility of a user and $x(n)$ is the power demand for discrete time
slot \( n \), \( p(n) \) indicates the congestion price at that point of time which goes up linearly with the total demand of all users. Instead of using prediction and optimising over a horizon, users maximise their utility function after every update interval for that time instance only.

Instead of a homogeneous utility function, one of the contributions of [60] is that appliances are modelled by different utility functions reflecting the nature of appliances. The prime focus of that paper is residential loads which are classified into four categories with distinct utility functions. The categories include temperature related loads such as air-conditioners and fridges whose objective is to keep the temperature within desired bounds and the utility is modelled as a function of the difference between actual temperature and a comfortable temperature range; fixed total energy loads such as EVs and poll pumps which need a certain amount of energy to complete their duties regardless of the distribution of the energy in time and their utilities are represented as a function of the total energy delivered to the loads; rigid loads such as lights which must be on for a continuous amount of time and their utility will be a function of both power and time; entertainment appliances such as a TV or Xbox whose utility will be similar to rigid loads except that users are more flexible about these appliances.

Having defined all the utility functions, the authors formulate the problem as a net utility optimisation problem. By modelling the price as a function of total demand, an equilibrium can be established among all users. However, the utility models of appliances are somewhat ideal. For example, interrupting a washing machine or dish washer during washing cycles may be problematic.

Similar approaches are adopted for EV charging management [3] and [61]. Instead of communicating with all vehicles, the DSO only collects information on the current charging power of each EV. Based on transformer capacity and historical data of household demand, the DSO calculates and broadcasts the average power reserve information to each EV. The vehicles only listen to the DSO passively to make charging decisions based on their SOC and the power reserve information. However, it is still necessary for the DSO to know the number of EVs that are plugged in at all times.
and the available power at the transformer.

In [9, 62] and [92], an additive-increase-multiplicative-decrease (AIMD) charging algorithm is introduced and studied. The concept of AIMD came from the Transmission Control Protocol (TCP) in the internet transport layer and is well known for its sawtooth behaviour. In the application of EV charging, AIMD simply means that each EV increases its charging power linearly until a feedback message (e.g. no more capacity in the transformer) is received, upon which, the charging power will be halved. Different analytical approaches are taken in the literature for the AIMD style algorithm. [62] and [92] use the traditional AIMD where the size of increment and decrement is fixed. In a different study, [9] developed a sophisticated control algorithm that changes the rate for increasing and decreasing the power for optimality and faster convergence.

Similar to algorithms using two-way communication, the ones using only one-way communication also make use of a price-like signal broadcast by the DSO to calculate a local strategy. And this signal is usually proportional to total demand in the network. One-way communication algorithms are usually implemented on-line with or without little forecast. They are less optimal but easier to implement.

Communication independent algorithms

So far, almost all the DSM algorithms discussed above involve a certain level of communication and information exchange. Sensing, monitoring and controlling facilities are not available in distributed networks in most countries. The question is how demand can be managed without these facilities? An interesting approach is proposed in [46] to schedule home appliances where the local voltage level is used as a feedback signal. It is argued in the paper that in a distribution network, load voltage will drop significantly if the total grid demand exceeds 60% – 70% of capacity. Therefore, appliances will run when local voltage is high and stop when local voltage is low [46] to protect the grid and make the most of spare capacity. Consumers will get a certain level of compensation for installing the device. However, the voltage model used by nPlug neglects the topologies of networks and control parameters need to be carefully
chosen for different networks to yield acceptable behaviours. Also, optimality, fairness and other global aspects of the emerging network behaviour under such decentralised and distributed demand management remain open areas of research. Nevertheless, this idea of using local voltage as a grid indicator inspires many algorithms that will be proposed in later chapters.

1.3.3 Related work on tariff design

As for network tariff design, we will give an introduction to common practices as well as the state-of-art approaches on optimal tariff design. At the time of this thesis, a debate on how PV systems are impacting network tariff structure is ongoing. Some researchers believe that domestic solar PV systems reduce the overall utilisation of the electricity infrastructure which may eventually lead to higher prices. In addition, distributed PV users with altered electricity consumption patterns may take advantage of the current pricing structure and eventually, receive hidden subsidies from users without distributed PV [87] [100]. Others point out that all the hidden subsidy arguments originate from the fact that existing electricity distribution charges are levied on a uniform basis, but the costs of the electricity distribution network depend mostly on peak demand [82]. The problem has been there for a long time and PV systems are not completely responsible for it. The viewpoint both parties share in common is that network tariffs are due for an upgrade. In this section, we will briefly look into the history of the pricing of electricity and how network operators charge for infrastructure.

Electricity tariffs have existed since the establishment of electricity networks in the late 19th century and it was first priced uniformly like other commodities according to demand and supply in the market [87]. However, it did not take long before the utility companies realised that the consumption of electricity is extremely unstable where peak load differs significantly from off-peak load. The resultant low utilisation of expensive infrastructure, which has to be constructed according to peak load, created an enormous challenge for network operators at that time and a profitable business was pointed towards bankruptcy [81]. The root of the problem came from the nature
of how electricity is consumed and a simple increase or decrease of unit price would not be the solution. One of the widely accepted remedies at that time was a tariff that consists of a variable energy charge (c/kWh) and a demand charge ($/kW), the latter can be calculated based on the user’s historical peak load. The tariff was first introduced by Prof. John Hopkinson and was adopted in a few countries [50]. The application was very limited because the meters were designed to record aggregated consumption only and were not able to track historical peak demand. However, there is no doubt that the pricing scheme is ingenious and one of the major contributions of this tariff structure is the identification of a sunk capital cost or infrastructure cost which is dominating. In a way, demand charge directly corresponds to sunk cost. Running cost or operation cost which is a relatively less important term corresponds to the energy charge. Almost concurrently, [38] introduced a fixed charge to further reduced the impact for short-using customers.

In the early 20th century, economists stepped in and worked with engineers to design more optimal tariff structures which also maximise welfare. The tariffs introduced previously try to set prices in order to recover overall costs. But economists believed that the marginal running cost should be the one to be recovered and a penalty charge should only be applied when the demand is approaching the limit of supply [51]. Such a pricing structure would very well recover the operational cost but not the sunk capital cost. The authors argue that the balance should be recovered through the taxation system rather than be uniformly levied on consumers. The paper should be credited for suggesting that electricity consumed at different marginal running costs should be charged accordingly which forms the basis for the TOU tariff.

Consolidating all the work to date, a well known and well implemented tariff structure was introduced in [20] where three network classes are identified: a) the overall network where prices are determined based on the marginal running cost according to the collective consumption and a TOU charge; b) individual component and the price is determined according to a personal peak as a fixed demand charge; c) intermediate network where total peak consumption is volatile and the pricing is also determined following the marginal cost principle as an additional peak charge.
This particular tariff structure was highly regarded and implemented.

Pricing of electricity was made more systematic in [21] where 10 crucial principles were introduced and followed by electrical engineers as well as economists. Among the principles, three primary ones are static efficiency, revenue adequacy and fair allocation of sunk cost. Following these guidelines, in the context of Australian networks faced by increasing distributed PV penetration, an optimal three-part tariff structure has been defined in [87] as follows: a) a fixed charge designed to cover the fixed operation cost; b) a TOU charge to cover nominal variable marginal costs; c) a demand charge to cover sunk infrastructure costs.

1.4 Our approach

In this thesis, we aim to re-engineer both DSM and network tariff for the distribution networks. The following technical goals or requirements were kept in mind during the engineering process.

**Minimal communication** Many countries are promoting a smart grid nowadays, however large scale ICT monitoring and controlling facilities are still not commonly available in distribution networks. Many existing DSM techniques rely on communication between DSOs and consumers. However, without the necessary infrastructure and protocols, such techniques may not be readily applicable. We aim to exploit the existing infrastructure and design solutions that use as little communication as possible. Also, from a philosophical point of view, the interference of a DSO or a regulator could cause anxiety among users. Hence the more that can be done from a user perspective, the more the user is in control, or feels in control, the better the acceptance of the technologies. Strategies with minimal communication naturally leave more decisions for users to make locally, which would lead to better understanding and acceptance.

**Scalability** We aim to design solutions that are easily scalable with respect to the numbers of users and the sizes of networks. The designs will be based on the
last mile of a distribution network, however, when scaled to the distribution network level, the cost of implementation should only increase linearly with respect to the size of the network.

**Utilisation** For a DSM solution, we want to maximise the utilisation of existing grid infrastructure such that an upgrade can be deferred and the overall cost can be reduced. For a tariff design, we want to increase the social welfare or the aggregated happiness of all users under the tariff.

**Compatibility** The sunk cost in the infrastructure of the grid is enormous, new technologies should be designed in such a way that existing infrastructure can be better utilized without any compatibility issues. Compatibility also means that possible future technologies should be taken into account in the design process to increase investment efficiency.

**Fairness** Fairness is often omitted in electricity network tariff design and the design of DSM. We first of all want to ensure that users, regardless of what appliances they have or how much energy they use, pay their fair share of the infrastructure cost. We also want to make sure that nobody is disadvantaged or advantaged under any DSM we propose.

### 1.5 Our contribution

A lot of effort has been made to develop efficient DSM solutions. However, one of the key questions remaining to be answered is how to design DSM solutions that are readily applicable and tailored for the last mile distribution network? In other words, the algorithms have to be communication independent and responsible for maintaining basic network constraints. Both of these aspects are not satisfactorily addressed in the literature. For tariff design, we aim to fill the gap and evaluate the efficiency and fairness of network tariffs in presence of domestic solar generation.

More specifically, our work complements the existing literature and answers the following high level questions:
• Is it possible to make reasonably good demand management decisions without using any explicit communication or using a minimal level of communication? What is the benefit achieved by communication, i.e. what is the cost-benefit analysis for communication? In the context of a distribution network, we propose a stochastic model that correlates household voltages and network demand. Based on such a model, we develop solutions for demand management (including distributed generation management) using only local measurements. We then compare it with solutions using different levels of communications. We study the performance of these solutions and quantify the benefit of information.

• If we have a variety of local ‘needs’ how can these be compatible with the overall ‘system needs’ (satisfying constraints, and minimising energy use)? What is the trade-off between individual freedom, social welfare and system constraints when it comes to electricity distribution? To answer this question, we model the last mile of a distribution network and the relevant constraints such as unbalance and voltage levels both in theory and in simulation. The constraints are taken into account while performing optimisation and the results are verified in simulations. The aim of this study is to provide users and operators with a way to maximise the utilisation of networks without violating the operation limits.

• How to quantify the efficiency and fairness of tariff? How do emerging technologies, especially PV systems, affect the efficiency and fairness of electricity network tariffs and can one design a better one? Through analysis of real demand and supply data at the household level from Australian networks, we propose a utility theory based framework to quantify the benefits a user gets from the network infrastructure. Based on such a model, we are able to then quantify the aggregated social welfare of a tariff and evaluate the fairness of it. In addition, we also form an optimisation objective to find a better tariff.
Further, there are a few extensions to the work performed in this thesis which are worth further study. First of all, we only study DSM for networks under normal operating conditions. Network dynamics and control under fault conditions, which are a crucial component to power system study, will be left for future investigation. Secondly, though the modelling is adopted from real networks and simulations are driven by real data, hardware experiments are not performed in this thesis.

1.6 List of publications

This thesis is based on the following conference and journal articles which are sorted chronologically.

Journals:


Conferences:


1.7 Chapter summary

In this chapter, we have first of all discussed the background of power systems. In the context of the Australian grid, which is operated in a similar way to many major grids around the globe, we have explained the physical load flow and governing bodies involved in network operation and electricity trading. We have also introduced the terminologies of DSM and tariff design followed by a listing of the common practices. Next we discussed the challenges and opportunities of existing networks in the era of the smart grid. This serves to motivate our work, and that of others which we reviewed briefly. From this review, a number of open questions transpired that we have addressed in this thesis. After an overview of state-of-art approaches and identification of their strengths and weaknesses, we highlight particular design requirements which will form the basis of our methodology and be followed throughout the engineering process. These requirements make the work in this thesis distinguishable and, to a certain extent, practical. We then explain our contributions more specifically in the context of our design milestones.
Chapter 2

Models

Many theoretical and simulation models have been developed and utilized in this thesis. The ones of particular importance which are crucial to the rest of the thesis are introduced in this chapter. These models include the distribution network, particularly the last mile level model and the voltage-demand relationship model. We explain how the models are constructed both in theory and in simulation.

2.1 Last mile network model

2.1.1 Overview

As illustrated earlier, the power grid usually consists of generation, transmission and distribution components as shown in Figure 1-1 where the last mile network is highlighted. Traditionally, power systems research has been largely concentrated at the generation and transmission level; with distribution networks considered generating slight disturbances. Remote sensing, monitoring and controlling devices are widely available at the transmission level. The distribution networks, especially the last mile are essentially operating ‘blindly’. In the new paradigm of the grid, this is changing: shiftable loads, distributed generation and the matching of demand to supply using novel control technologies mean that now distribution network modelling is an important part of power systems planning and one that is poorly understood because of
the lack of attention to measurements and modelling of the last mile in the network. If each and every last mile network can be properly managed, then the stress on the entire grid can be very much limited. In this thesis, we model in detail the last mile of a radial distribution network - a neighbourhood network. Note that the radial network is typical in Australia. There might be a few tie lines that can make a difference but by and large the distribution network is radial only. An example is presented in Figure 2-1 where 113 users are supplied by a single DSO via a transformer which steps from medium voltage to 230V. The transformer voltage is regulated in the medium voltage side substation which for this thesis is treated as a constant amplitude voltage source.

Since such last mile networks are usually not treated with significance, off-the-shelf simulation softwares such as OpenDSS and PSS_SINCAL which had been used by utility companies for analysis do not have detailed enough load model to facilitate DSM analysis. In addition, the variation in last mile implementation across the globe is large and there is no standardisation across the globe on last mile network modelling. For example, in the USA the low voltage networks have very few houses after a transformer because of the 110V outlet voltage, earthing, and neutral
wire connectivity. Most papers in the literature use an IEEE standard transmission network model and modify some parameters to approximate the last mile network. Such models are not capable of catching important network features such as phase allocation and power factors. In order to simulate DSM algorithms and understand the impact from emerging technologies, it is absolutely important to have an accurate model. We therefore construct a model ourselves which represents the state-of-art in last mile network modelling. Initial work on such models can be found in [32, 34]. Note that our models are for Australia, and not directly translatable to Europe, Asia or the USA.

This model is constructed based on a real network in Australia and data are collected from utility providers or government bodies. The network configuration including transformer specifications, backbone wires and service lines specifications, locations of poles, distances between houses and phase allocations which are based on real data. The number of houses on each of the three phases is different which introduces a certain level of unbalance. Inside a household, as shown in Figure 2-2, there are a variety of appliances such as air-conditioners, electric water heaters, etc. A household may also have an EV or may have a PV system installed. For most of the algorithms proposed in this thesis to function, a control unit is required to be installed on the piece of equipment that is to be managed. This control unit can simply be a plug-in device which goes on to normal power outlets as shown in Figure 2-2.

2.1.2 Simulation software and data

In order to verify the performance of our proposed algorithms, a sophisticated simulation model of the last mile network is constructed in MATLAB which captures as many real network properties as possible. A sample simulation network is shown in Figure 2-3 where a four wire three phase system is constructed. Each individual house in the network is connected to a single phase. MATLAB could only perform load flow analysis at a given point of time. In order to generate continuous time simulations and create appliances profiles, we use the POSSIM Simulator concurrently (POSSIM
Figure 2-2: Configuration of the connections within a household.

Figure 2-3: A model of a sample last mile network in MATLAB.
was developed by Julian de Hoog, and extended and customized by myself. More details can be found on www.possim.org).

POSSIM effectively acts as an interface between a profile library and MATLAB. The profile library keeps track of the internal states of all controllable and uncontrollable loads as time series values. These states include (if applicable) demand, supply, SOC, control strategy, etc. Some of these states are used as inputs to MATLAB for load flow analysis and the library can be updated based on the outputs from the load flow analysis.

Specifically, my contribution to the simulator include the following:

- Design and integration of distributed DSM algorithms which will be discussed in the rest of the thesis.
- Integration of domestic PV units and associated generation profiles.
- Design and integration of storable and shiftable loads.
- Simulator testing and verification.

### 2.1.3 Component modelling

In this section, we give more details on how the major components in our analysis are modelled in theory and in simulation.

**Modelling household loads**

Households may possess a variety of loads. Conventional lighting, various heaters (water, space), cooking appliances are mostly constant impedance loads. Devices that involve power electronics, such as computers, TVs, EV chargers and energy-efficient lighting, tend to behave as constant power. Motor loads, such as air-conditioner compressors, exhibit more complex behaviours.

As for the actual mathematical modelling in our analysis, we consider households as current loads varying over time and the currents are derived from overall consumption at transformer level through averaging or stochastic operations. This assumption
relaxes the condition that the consumption at a household depends on the voltage at that household and therefore makes the load flow problem linear.

We acknowledge that there is a gap between our model and the actual behaviour of the loads. However, the gap is relatively small and negligible in our case as our focus is residential networks which are typically highly linear and resistive.

The network total demand profile of a summer day in 2013 for the network shown in Figure 2-1 is presented in Figure 2-4. The data is obtained from a utility provider based in the state of Victoria. Demand could vary on a day to day basis but the evening peak and overnight demand valley is a universal phenomenon in residential networks. More on the demand of individual households and the demand distribution among a network will be introduced in the Section 2.2.

Modelling EVs

EVs form an important part of this thesis and similar to other household loads, they are modelled as varying impedances consuming only real power. However, EVs possess some features which need to be modelled separately in POSSIM. Firstly, EVs are all equipped with batteries each of which has an associated SOC at any point in time. Secondly, EVs also have travel profiles describing when they arrive and leave as well as how much distance they have travelled. We therefore create a profile in POSSIM
Figure 2-5: An average workday EV travelling profile generated from a sample of size 100 from a Victoria government survey [1].

Figure 2-6: Charging flexibility. [1][69].
for each EV which keeps track of the charging power, charging time, travel time and travel distances whenever applicable. The SOCs are then updated accordingly using a simple linear charging model. Battery chemistry and ageing issues are not modelled in our simulation. The EV travelling models are based on real data from a Victoria Government survey [1]. A typical average workday EV behaviour profile in Victoria is shown in Figure 2-5 and the actual travel profiles over 24 hours for a number of typical EVs are depicted in Figure 2-6. Each horizontal line represents a real individual 24-hour vehicle travel profile. Required charging time is based on distance travelled. The green area therefore represents charging flexibility. Only residential networks will be examined in this thesis where EVs can only be charged at home; and charging in the workplace is not considered. The distribution of daily travelled distances before arriving home are shown in Figure 2-7. An EV in our simulation would randomly pick a travel schedule as in Figure 2-6 and a distance as in Figure 2-7. EVs, currently with a small market penetration, are not a major risk to the grid. However, we will show in the next chapter how the uptake of EVs could put distribution networks in jeopardy when when the penetration level goes up.

We further make the following assumptions on our EV model:

- The maximum charging rate is 3.5 kW (default rate) and an EV is able to be charged at any rate between 0 kW and the maximum rate.

- If at home, EV charging can be interrupted and shifted.
• Battery chemistry is not considered and we assume SOC changes linearly with charge/discharge rate.

• Vehicle-to-Grid power transfer is not permitted.

Modelling a distributed PV system

Throughout this thesis, we assume PV systems to be voltage controlled current sources. In our simulation model, a phase locked loop (PLL) measures the grid voltage phase and frequency. The inverter injects current in phase with the grid voltage using the two pieces of information available locally, which are: (a) reference current information from the direct current voltage regulation, and (b) voltage and frequency information from the PLL. The power injected in this manner is purely real and the power factor associated is unity which is a widely used assumption in the literature [63]. One advantage of unity power factor injection is that it can be expressed as an absolute current supplied to a system at a given voltage. However, including all these individual entities into our theoretical model would lead to complex nonlinearities. To avoid this, for theoretical analysis, we treat PV systems as controllable current sources. This is a reasonable approximation (controllability is embedded in the power electronic systems) in the given context and is also preceded by assumptions made in section 2.1.3 [63].

As for the output of PV systems, users may install panels of different sizes and depending on the orientation, efficiency, cloud coverage, etc., output from panels may differ from one another. However, in the same area, the differences are rather insignificant. Therefore, we take an averaged approach and assume all the PV systems in a last mile network have the same behaviour which is depicted in Figure 2-8 as obtained from a utility company operating in NSW. The profile captures a long day of sunshine and cloud effects are attenuated through averaging.
Figure 2-8: Averaged rooftop PV generation profile during January in NSW.

2.2 The voltage-demand relationship model

Having the components modelling explained, we now introduce the most important model in this thesis: the voltage-demand relationship model. Most demand side management algorithms make use of remote sensing and controlling devices which are not widely available in distribution networks [9]. In order to design a solution that is readily applicable and easily implementable, we propose a voltage-demand relationship model to correlate the voltage measurements at individual households and the total demand on the feeder using stochastic modelling. Stochastic analysis is a popular tool in low voltage (LV) network demand forecasting and LV network demand modelling [49,71,76]. A beta distribution model is proposed in [49] to estimate household load profiles such that the peak demand can be estimated and transformers can be sized accordingly. Such an approach has been adopted in the electricity network design in South Africa [49]. More recently, a gamma distribution model has been shown to be more accurate in terms of capturing household half-hourly demand patterns via Monte Carlo simulations [71]. The authors have also shown that parameters of probability density functions can be extracted from known data-sets and the model is calibrated against varying temperature. Such stochastic modelling can be implemented on a finer level where load profiles of different appliances within a household are modelled as different random variables [76]. Instead of a single random variable, the demand profile of a household is now a combination of several random...
variables. The results have also been shown to match the actual demand profiles. In this thesis, we use a simple gamma distribution to model household demand patterns and the parameters for the distribution are extracted from actual smart meter data.

With this probability distribution of demand profiles, using classic circuit theory, we correlated demand and voltages in a distribution network such that the probability distribution of household voltages under a given network demand can also be derived. Intuitively, if the demand of a network is high, line voltage drops will increase as a result of increased line current. Using that information, the power-line itself can be viewed as a communication channel and household voltage measurements can be used as a signal for demand management coordination.

2.2.1 Theory assumptions

Based on the statistical evidence, we proceed by making the following assumptions which is for the ease of theoretical analysis:

- We assume that the transformer feeding the network is a constant voltage source and there is a capacity limit associated with it.

- We assume (as in residential networks) that the networks of interest are dominated by resistive loads.

- We assume that the backbone wires have a low reactance/resistance (X/R) ratio (< 1/5) which is again true for residential networks.

- We assume that the network is balanced on the three phases as required by relevant regulations though not enforced in reality.

- We assume that the distribution line impedances between two adjacent households on the same phase are identical.

- For theoretical analysis only, if there is distributed generation in the network, we take an averaging approach and assume the generation comes evenly from all households.
Figure 2-9: System diagram for a single branch network

Note that the assumptions made are for theoretical analysis only and there will certainly be inaccuracies. However, we will see later in the paper how these inaccuracies can be eliminated by feedback and how the model corrects itself. In the simulations, the actual network will be realistically modelled under its normal operating condition. The network will have reactive power, phase unbalance, source voltage disturbance, etc. all based on real network data. And as a result, most of the assumptions made above will be violated. The simulations can therefore be seen as a way of examining the validity of the theoretical model in practice.

2.2.2 Single branch network

We begin the analysis by extracting the fundamental components from the network model illustrated in Figure 2-1 and turning them into electrical circuits as explained above. We start by looking at networks having only one branch and then extend the approach to branched networks. Figure 2-9 shows the schematic of one of three phases, for a single branch. There are in total $n$ houses connected on this phase.

As explained above, households are current sinks where $d_x, x = 1, 2, ..., n$ depicts the demand of house $x$ in terms of current at a time instance; a household may be equipped with a rooftop solar system which is modelled as a current source where $s_x, x = 1, 2, ..., n$ depicts the amount of current injected. The internal structure diagram is shown in Figure 2-10. The net current $I_x, x = 1, 2, ..., n$ whose direction
is shown in [2-9] can be written as follows:

\[ I_x = d_x - s_x, \quad x = 1, 2, ..., n \]  \hspace{1cm} (2.1)

The total demand in the network at this time instance is also defined in terms of current, \( d_s = \sum_{x=1}^{n} d_x \), and the total supply from rooftop PV systems is \( s_s = \sum_{x=1}^{n} s_x \). The total net demand \( I_s \) which is defined as net current flowing out of the transformer is therefore given by:

\[ I_s = d_s - s_s \]  \hspace{1cm} (2.2)

Following the assumptions as in Section 2.2.1 that the voltage at the source side, \( V_s \), is a constant and the line impedances between any two adjacent houses are the same which is denoted by \( Z_L \) as shown in Figure 2-9, using circuit theory, the relationship between voltages and demand is expressed as (2.3).

\[
\begin{align*}
V_1 &= V_s - Z_L \sum_{i=1}^{n} I_i \\
V_2 &= V_1 - Z_L \sum_{i=2}^{n} I_i = V_s - Z_L \sum_{i=1}^{n} I_i - Z_L \sum_{i=2}^{n} I_i \\
V_3 &= V_s - Z_L (\sum_{i=1}^{n} I_i + \sum_{i=2}^{n} I_i + \sum_{i=3}^{n} I_i) \\
\vdots \\
V_n &= V_s - Z_L (\sum_{k=1}^{n} \sum_{i=k}^{n} I_i)
\end{align*}
\]

(2.3)

To model household demand, we have collected and analysed smart meter data samples, obtained from a number of utility companies, from multiple distribution networks in Australia with thousands of users. The dataset consists of half hourly sampled demand values from each household for an entire year. A histogram of household demand at an instance is shown in Figure 2-11. The distributions have tall heads and a long tails and can be fitted by gamma probability density functions.
Figure 2-10: The internal modelling of a household with solar PV.

Figure 2-11: Distribution of household demand at a time instance fitted by a gamma distribution
(shown as a continuous curve in Figure 2-11). Therefore, we proceed by assuming that the household demands \( d_x, \ x = 1, 2, ..., n \), at a time instance, are mutually independent variables \( D_x, \ x = 1, 2, ..., n \) which satisfy a gamma distribution that is described as follows:

\[
D_x \sim \Gamma(k, \theta) \tag{2.4}
\]

\[
E[D_x] = k\theta := \frac{d_s}{n} \tag{2.5}
\]

\[
Var[D_x] = k\theta^2 := \theta \frac{d_s}{n} \tag{2.6}
\]

where \( k \) is the shape parameter and \( \theta \) is the scale parameter for the distribution. The mean of the distribution is \( E[D_x] \) and the variance is \( Var[D_x] \). Note that the distribution is time varying and \( d_s \) is also time varying. The notation in this thesis captures the behaviour of the network at a point of time. Analysis using half-hourly demand data suggests that a constant scale parameter \( \theta \) can be used to describe the distributions at any particular time in a day or a year. To obtain the scale parameter \( \theta \), we perform gamma distribution fittings on the histograms from all the half-hourly instances over the entire year. The average value of scale parameters from all the fittings is used as the constant \( \theta \). The shape parameter \( k \) then changes according to the total demand \( d_s \) proportionally as per (2.5).

For domestic solar PV generation, despite the fact that depending on the geographic location, cloud coverage, installation capacity and orientation, the output of rooftop PV systems can vary, we nevertheless adopt an averaged approach for theoretical analysis. We assume that the total distributed generation is evenly supplied by all households in the network even though not every house has an PV system. In other words, for \( x = 1, 2, ..., n \) we have:

\[
s_x = \frac{s_s}{n} \tag{2.7}
\]

Now let \( U_x \) be the random variable corresponding to voltage \( V_x \) at house \( x \) at a time instance. After substituting the net current \( d_x - s_x \) with \( D_x - s_x \), (2.3) can be
rewritten as follows for user $x$, $x = 1, 2, ..., n$:

\[ U_x = V_s - Z_L B_x + Z_L J_x \]  \hspace{1cm} (2.8)

\[ B_x = x \sum_{i=x}^{n} D_i + \sum_{i=1}^{x-1} (iD_i) \]

\[ J_x = s_x \frac{(2n - x + 1)x}{2} \] \hspace{1cm} (2.9)

where $B_x$ is a random variable and $J_x$ is a deterministic value. Note that the nature of the distribution of $B_x$, which is a linear combination of independent gamma variables, is an open research question. However, a widely-used method, the Welch-Satterthwaite method [84], approximates $B_x$ using a gamma random variable $\hat{B}_x \sim \Gamma(k_{\hat{B}_x}, \theta_{\hat{B}_x})$ for $x = 1, 2, ..., n$ that can be written as follows:

\[ E[\hat{B}_x] = k_{\hat{B}_x} \frac{(2n - x + 1)x}{2} \] \hspace{1cm} (2.10)

\[ Var[\hat{B}_x] = k_{\hat{B}_x} \frac{-4x^3 + 6nx^2 + 3x^2 + x}{6} \] \hspace{1cm} (2.11)

\[ k_{\hat{B}_x} = \frac{E[\hat{B}_x]}{Var[\hat{B}_x]} \] \hspace{1cm} (2.12)

\[ \theta_{\hat{B}_x} = \frac{Var[\hat{B}_x]}{[\hat{B}_x]} \] \hspace{1cm} (2.13)

Then the values of $U_x$ in (2.8) can be evaluated approximately.

**2.2.3 Multiple-branch network**

We now extend this model to branched networks with multiple feeders. Figure 2-12 is a generic representation of one phase in a network with $m$ branches. This diagram applies to most topologies - a new branch can split off from anywhere on an existing branch, not just at the start. The problem can be solved in two stages.

**Stage 1**: Assuming that each branch is a single load, the problem is transformed into a single branch problem as in the previous subsection where loads have different characteristics. Assume that the number of houses on branch $1, 2, ..., m$ is $n_1, n_2, ..., n_m$, using (2.4) for the household demand, the aggregated demand on each
branch is a gamma random variable \( M_y \sim \Gamma(k_y, \theta), y = 1, 2, ..., m \) which can be described as follows:

\[
k_y = n_y k
\]

\[
E[M_y] = n_y k \theta
\]

\[
Var[M_y] = n_y k \theta^2
\]

Now let \( U_y^0 \) be the random voltage at the first node of branch \( y, y = 1, 2, ..., m \). Apply (2.8) and (2.9) here, we have:

\[
U_y^0 = V_s - Z_L B_y^0 + Z_L J_y^0
\]

\[
B_y^0 = y \sum_{i=y}^{m} M_i + \sum_{i=1}^{y-1} (i M_i)
\]

\[
J_y^0 = s_x (y \sum_{i=y}^{m} n_i + \sum_{i=1}^{y-1} (i n_i))
\]

Again, \( B_y^0 \) is a linear combination of random gamma variables which can be approximated using a gamma distributed random variable \( \hat{B}_y^0 \). The parameters of
\[ \hat{B}_y^0 \sim \Gamma(k_{\hat{B}_y^0}, \theta_{\hat{B}_y^0}) \] for \( y = 1, 2, \ldots, m \) are given as follows:

\[
E[\hat{B}_y^0] = k\theta(y \sum_{i=y}^{m} n_i + \sum_{i=1}^{y-1} (in_i))
\]

\[
Var[\hat{B}_y^0] = k\theta^2(y^2 \sum_{i=y}^{m} n_i + \sum_{i=1}^{y-1} (i^2n_i))
\]

\[
k_{\hat{B}_y^0} = E^2[\hat{B}_y^0] / Var[\hat{B}_y^0]
\]

\[
\theta_{\hat{B}_y^0} = Var[\hat{B}_y^0] / E[\hat{B}_y^0]
\]

**Stage 2:** If the voltage at the first node of each branch \((V_1^0, V_2^0, \ldots, V_m^0)\) is constant, the problem on each branch is identical to the single branch scenario. However, since the voltage at the first node of each branch is now a random variable itself \((U_1^0, U_2^0, \ldots, U_m^0)\), the problem is now slightly different. We can write the following for house \(x\) on branch \(y\):

\[
U_y^x = U_y^0 - Z_LB_y^x + Z_LJ_y^x
\]

\[
= V_s - Z_LB_y^0 - Z_LB_y^x + Z_LJ_y^0 + Z_LJ_y^x
\]  

(2.18)

Similar to (2.10) and (2.11), the random variable \(B_y^x\) can be approximated by a gamma random variable \(\hat{B}_y^x\) described as follows:

\[
E[\hat{B}_y^x] = k\theta \frac{(2n_y - x + 1)x}{2}
\]

\[
Var[\hat{B}_y^x] = k\theta^2 \frac{-4x^3 + 6n_yx^2 + 3x^2 + x}{6}
\]

And \(J_y^x\) is calculated as below:

\[
J_y^x = s_x \frac{(2n_y - x + 1)x}{2}
\]  

(2.19)

Using (2.18), define the voltage drop due to demand only (isolating the effect of
supply) at house $x$ on branch $y$ as:

$$\Delta W_y^x = V_s - U_y^x + Z_L J_y^0 + Z_L J_y^x = Z_L B_y^0 + Z_L B_y^x$$ \hspace{1cm} (2.20)

This can be approximated as $\Delta \hat{W}_y^x = Z_L \hat{B}_y^0 + Z_L \hat{B}_y^x$, and hence $\Delta \hat{W}_y^x \sim \Gamma(k_{\Delta \hat{W}_y^x}, \theta_{\Delta \hat{W}_y^x})$ can be described by the following parameters:

$$E[\Delta \hat{W}_y^x] = k Z_L \theta \left( \frac{2n_y - x + 1}{2} \right) + y \sum_{i=y}^{m} n_i + \sum_{i=1}^{y-1} (i n_i)$$ \hspace{1cm} (2.21)

$$Var[\Delta \hat{W}_y^x] = k Z_L^2 \theta^2 \left( \frac{-4x^3 + 6n_y x^2 + 3x^2 + x}{6} \right) + y^2 \sum_{i=y}^{m} n_i + \sum_{i=1}^{y-1} (i^2 n_i)$$ \hspace{1cm} (2.22)

$$k_{\Delta \hat{W}_y^x} = E[\Delta \hat{W}_y^x] / Var[\Delta \hat{W}_y^x]$$

$$\theta_{\Delta \hat{W}_y^x} = Var[\Delta \hat{W}_y^x] / [\Delta \hat{W}_y^x]$$

Then, using (2.5) and (2.6), we can simplify (2.21) and (2.22) as:

$$E[\Delta \hat{W}_y^x] = K_y^x Z_L d_s$$ \hspace{1cm} (2.23)

$$Var[\Delta \hat{W}_y^x] = \theta C_y^x Z_L^2 d_s$$ \hspace{1cm} (2.24)

where $K_y^x$ and $W_y^x$ are house specific constants depending purely on the topology of network and the household location. The constants $K_y^x$ and $W_y^x$ can be written as follows:

$$K_y^x = \frac{1}{n} \left( \frac{2n_y - x + 1}{2} \right) + y \sum_{i=y}^{m} n_i + \sum_{i=1}^{y-1} (i n_i)$$ \hspace{1cm} (2.25)

$$C_y^x = \frac{1}{n} \left( \frac{-4x^3 + 6n_y x^2 + 3x^2 + x}{6} \right) + y^2 \sum_{i=y}^{m} n_i + \sum_{i=1}^{y-1} (i^2 n_i)$$
Similar to (2.20), the final expression for voltage drop $\Delta U^x = V_s - U_y^x$ is:

$$E[\Delta \hat{U}^x_y] = K^x_y Z_L d_s - Z_L J^0_y - Z_L J^x_y$$
$$= K^x_y (Z_L d_s - Z_L s_s) \quad (2.26)$$

$$Var[\Delta \hat{U}^x_y] = \theta C^x_y Z^2_L d_s \quad (2.27)$$

$$E[\hat{U}^x_y] = V_s - K^x_y Z_L (d_s - s_s) \quad (2.28)$$

Therefore, we have obtained a linear expression as in (2.28) to relate the expected values of individual household voltages and total network demand. The variance of such an approximation is also linear to the total demand as shown in (2.27). We then naturally state the following theorem:

**Theorem 1** Subject to the assumptions in Section 2.2.1, for the last mile radial network whose topology matches the generic topology shown in Figure 2-12, the expected local voltage value for a household in the network can be expressed as a location specific linear function of the total net demand in the network as per (2.28).

Most distributed demand response algorithms require the total network demand to be broadcast either in its direct form or in the form of a dynamic price via a dedicated communication channel. The above analysis shows that the power line itself can be such a communication channel: household voltages averaged over certain interval can be used as the dynamic price signals to coordinate demand management of various users.

### 2.2.4 Verification

To further justify the model under actual operating conditions, using collected demand data and our simulation model, we plot the network total demand versus a randomly selected household’s voltage in Figure 2-13. Note that in practice we want to use averaged voltages over certain intervals to avoid noises in measurements. In simulation, the data we used was already averaged at collection. The curves span 36 hours where total network demand and local voltage are shown as a blue solid line.
and a green dotted line respectively. From the demand curve, the evening peak and overnight valley can be observed clearly. More importantly, when the demand is at maximum, the voltage measurement drops to its minimum and vice versa.

We then correlate these two curves and plotted the relationship between total load and household local voltages for three houses selected at random (one on each phase) in Figure 2-14. From the plot, we can see that the three phases are not very well balanced. For phase C, the total demand values are mainly distributed towards the lower end (0.2 p.u. - 0.4 p.u.). While for phases A and B, the total demand values are heavily distributed between 0.4 p.u. to 0.8 p.u. As a result, phase A and B are constantly more heavily loaded than phase C. This unbalance is not uncommon in reality and it will be observed in our simulations that are based on network measurements. Despite the unbalance, there is an approximate linear relationship between local voltage and total demand for each household. From observation, the approximation errors increase as total demand increases, which corresponds to our theoretical model. The parameters associated with the linear relationship may be different for different houses (which could be learnt from historical data), but an approximate linear relationship can be demonstrated for all houses in the network.

To verify the performance of our model in a network with PV systems, we fix the demand profiles from households and assign PV systems to users randomly. Figure 2-15 shows the voltage-demand relationship with 30% distributed PV penetration and and 2-16 shows the relationship with 50% distributed PV penetration. The scatter plots manifest clear linear trend and can be fitted with straight lines. The approximation errors in both plots increase as total current increases.

2.3 Chapter summary

In this chapter, we have first of all introduced the last mile network model used in this thesis. We have explained how the network and each component in the network is constructed in theory and in simulation. We have followed a data-driven approach in this thesis and the categories of data we collected and make use of have been intro-
Figure 2-13: Voltage profile of the house A with dotted line on right axis and corresponding total phase demand with solid line on left axis.

Figure 2-14: Plot of total demand of network for a given phase against voltage at a randomly selected house on the phase (effects of transformer tap changes inclusive)
Figure 2-15: Relationship between network net current consumed and local voltage of a house in the network. (30% penetration)

Figure 2-16: Relationship between network net current consumed and local voltage of a house in the network. (50% penetration)
duced. In addition, we have developed a stochastic model to describe the correlation between household voltages and the total demand in the last mile of LV networks which has been derived in theory and verified in simulation using real data. The linear relationship can be learnt from transformer and local data, but later will be used in a different manner, namely to use local voltage as a pseudo measurement for network demand.
Chapter 3

Distributed EV charging management

In this chapter, we concentrate on EV charging management which offers special opportunities for DSM. Their storage capacity and reasonable flexibility make EVs excellent agents for DSM. With minor modifications, most charging management algorithms for EVs can be applied to other home appliances. We start the chapter by giving some background information on EVs. Next we present two distributed DSM algorithms. The first algorithm is a heuristic greedy approach based on solutions widely applied in computer networks. We show that it achieves reasonably good results with significantly less requirement for information and communication than centralised algorithms. To gain a better understanding of convergence and to have a theoretical guarantee on performance, we then propose a game theoretical approach. Uniqueness, existence, convergence and stability properties of the proposed solution are established. The game theoretical algorithm is also shown to perform well with minimal requirement for information and communication.

3.1 Background on EVs

In the last few hundred years, two massive energy conversion systems have been developed [70]. One is the electric utility system which ranges from small home appliances
like a lamp to heavy industrial equipment mainly concerned with stationary power conversion applications (apart from some electrified public transport systems). The other is the internal combustion engine running on fossil fuels mainly concerned with mobile applications, and remote locations. Internal combustion engines have served society well but there are also issues people are concerned about. These issues include pollution, energy security, fuel price and carbon emissions [86]. A large scale transition to electric vehicles from conventional fossil-fuel powered vehicles would benefit both the environment and economy. However, the electricity grid does not currently have the capacity to support a significant uptake of EVs. Much research has been carried out on studying the impact of increased EV penetration [25,39,58], and it has been shown that even a 10% EV penetration could cause problems on the grid especially during peak periods. To get a better understanding of what would happen when the number of EVs increases, a series of simulations have been carried out based on the network model as shown in Figure 2-1 of Section 2.1.

Figure 3-1 shows the demand profile of the network shown in Figure 2-1 with 80% EV penetration. Even though 80% penetration is high and will probably not be reached in the immediate future, it shows the effect of additional EV demand very clearly and it helps to illustrate the efficiency of the charging management algorithms that will be proposed. The peak demand increases significantly which leads to a critical situation for the grid, i.e. it requires further investment in infrastructure, large conductors, and transformers.

The voltage profiles of all households in the same network is shown in Figure 3-2 and different colours stand for different houses. The network is not well balanced in that the load on Phase A is heavier than the other phases. For as long as there is ample spare capacity, the phase unbalance is of little concern. However, with the increased demand to meet the EV requirements, the phase unbalance leads to the electricity supply not meeting the Australian Distribution Code for about a third of all the houses on phase A for part of the day.

Figure 3-3 shows the SOC of all EVs in the network when there is no charging management. Horizontal lines indicate that the EVs are either away on the road or
Figure 3-1: Demand profile of a distribution network with 80% EV penetration.

parked but not being charged. Sudden drops indicate that the EV has arrived at home and is starting to charge. The increasing lines show the progress of charging. Most of EVs arrive during evening peak period and start to charge at full rate.

However, demand in the morning and overnight periods (commonly referred to as valley periods) in Figure 3-1 is significantly lower than the demand in the evening peak period. The day valley (8:00-16:00) corresponds to users being at work and if charging facilities are available at work (which is not considered in our model), these periods can be well utilized; the overnight valley (0:00 - 8:00), gives a lot of potential for residential network demand management. If the green area in Figure 3-1 can be shifted to fit into the overnight valley and if the light brown area in Figure 3-1 can be evenly spread across the time, demand requirements will be well maintained and the problem could be solved. In this chapter, two algorithms particular aiming at solving EV demand management problem (management of the green area) are discussed. How these results and conclusions can be extended to general demand management problems will be explored in the next chapter.
Figure 3-2: Voltage profile of each household in a distribution network with 80% EV penetration.
Figure 3-3: Battery SOC profiles of EVs in a distribution network with 80% EV penetration.

3.2 A TCP-like algorithm for EV charging management

Many centralised and distributed DSM algorithms have been compared and discussed in Chapter 1.3. In general, distributed algorithms have better scalability and robustness over centralised solutions at the price of losing optimality. Among all the distributed charging solutions, those using only one-way communication require the least amount of additional infrastructure and are easiest to implement. However, even for the algorithms using one-way communication, some sort of infrastructure is required and information exchange is necessary. At the distribution network level, this is impractical for immediate deployment. In this section, we propose a distributed algorithm that is inspired by the TCP used in computer networks. However, compared to conventional solutions, this algorithm requires no communication facilities at all while achieving similar performances. The key to the solution is the use of the linear voltage-demand estimation model we have developed as in Theorem 1.
Table 3-1: List of Symbols for Chapter

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m$</td>
<td>electric vehicle $m$</td>
</tr>
<tr>
<td>$M(t)$</td>
<td>number of electric vehicles plugged in at time $t$</td>
</tr>
<tr>
<td>$t_m(s)$</td>
<td>charging start time of EV $m$</td>
</tr>
<tr>
<td>$t_m(e)$</td>
<td>charging finish time of EV $m$</td>
</tr>
<tr>
<td>$\tau$</td>
<td>length of each charging interval</td>
</tr>
<tr>
<td>$k$</td>
<td>constant relating to user preference</td>
</tr>
<tr>
<td>$V_m(t)$</td>
<td>local voltage at house $m$ (or EV $m$) at time $t$</td>
</tr>
<tr>
<td>$V_m(t)$</td>
<td>minimum local voltage at house $m$ at time $t$</td>
</tr>
<tr>
<td>$V_m(t)$</td>
<td>maximum local voltage at house $m$ at time $t$</td>
</tr>
<tr>
<td>$S_m(e)$</td>
<td>starting SOC level of EV $m$</td>
</tr>
<tr>
<td>$S_m(t)$</td>
<td>SOC level of EV $m$ at time $t$</td>
</tr>
<tr>
<td>$S_m(s)$</td>
<td>finish SOC level of EV $m$</td>
</tr>
<tr>
<td>$p_m$</td>
<td>maximum charging power for EV $m$</td>
</tr>
<tr>
<td>$e_m(t)$</td>
<td>travelling profile of electric vehicle $m$ at time $t$</td>
</tr>
</tbody>
</table>

A tant consequence of this model is that the local voltage measurement is an indicator for the loading of the network. Using a mere comparison of where the local voltage samples fits within the range of minimum to maximum local voltage it is possible to make a well informed DSM decision, regarding the consumption of electricity whilst the network is lightly loaded.

3.2.1 Preliminaries

It is assumed that each EV charger is connected to a controlling unit (or has a digital controller embedded) which is able to read local voltages, EV battery SOC, perform calculations and supply charge with discretionary current. This would be the only hardware required for the proposed algorithm to work on any existing electricity grid. The variables we use in this section are summarized in Table 3-1. In the following paragraphs, unless specified, we abuse the notation slightly and drop all the subscripts $m$ from Table 3-1. There are some inputs a controller requires from the users. The charging start time $t(s)$ is the time when the EV is plugged in. The user needs to input an expected finish time $t(e)$ which is when the EV needs to be ready for departure. It is assumed in this chapter that users behave rationally and do not cheat by setting a finish time that occurs long before they need to depart. The users’ behaviours can
be influenced by introducing price incentives.

The required SOC level \( S(e) \) is usually 100\% upon finishing but a user could overwrite such a value. The controller also records the initial SOC level \( S(s) \) when the vehicle is plugged in. The sensing of controllers is executed in a slotted manner with equal slots of several minutes. The slots for each EV are equal in length, but they are not synchronized across EVs. At the beginning of each slot, each controller monitors the SOC level \( S(t) \), the local voltage \( V(t) \) and calculates a charging level which is maintained for the duration of that slot.

### 3.2.2 TCP style algorithm

**Algorithm 1** Fair EV charging with local measurements

<table>
<thead>
<tr>
<th>Step</th>
<th>Condition</th>
<th>Action</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>( p(t) = 0 )</td>
<td>initialisation</td>
</tr>
<tr>
<td>2.</td>
<td>( S(\text{wanted}) = S(e) - S(s) ), ( T = t(e) - t(s) )</td>
<td>required charge and available time</td>
</tr>
<tr>
<td>3.</td>
<td>( B = S(\text{wanted})/T )</td>
<td>average charging rate, initialisation completed</td>
</tr>
<tr>
<td>4.</td>
<td>update ( V(t) ) and ( \overline{V}(t) )</td>
<td>main algorithm starts</td>
</tr>
<tr>
<td>5.</td>
<td>if ( S(e) - S(t) \leq 0 ) then</td>
<td>check if fully charged</td>
</tr>
<tr>
<td>6.</td>
<td>charge (OFF)</td>
<td></td>
</tr>
<tr>
<td>7.</td>
<td>else if ( V(t) - \overline{V}(t) \leq 0 ) then</td>
<td>network overloaded</td>
</tr>
<tr>
<td>8.</td>
<td>( p(t) = 0.5p(t - 1) )</td>
<td>drop to half</td>
</tr>
<tr>
<td>9.</td>
<td>charge (ON)</td>
<td></td>
</tr>
<tr>
<td>10.</td>
<td>else</td>
<td></td>
</tr>
<tr>
<td>11.</td>
<td>( A \leftarrow (S(e) - S(t))/(t(e) - t) )</td>
<td>required average rate to meet deadline</td>
</tr>
<tr>
<td>12.</td>
<td>( \Delta p(t) \leftarrow k \cdot \overline{p} \cdot (V(t) - \overline{V}(t))/(\overline{V}(t) - \overline{V}(t)) )</td>
<td>increment as per available capacity</td>
</tr>
<tr>
<td>13.</td>
<td>( p(t) \leftarrow (p(t - 1) + \Delta p(t)) \cdot (A/B) )</td>
<td>adjusted charging rate</td>
</tr>
<tr>
<td>14.</td>
<td>charge (ON)</td>
<td></td>
</tr>
<tr>
<td>15.</td>
<td>charge rate = ( \min{p(t),\overline{p}} )</td>
<td>maximum charging rate protection</td>
</tr>
<tr>
<td>16.</td>
<td>end if</td>
<td></td>
</tr>
<tr>
<td>17.</td>
<td>keep charging for ( (\tau) )</td>
<td>typically 5 to 15 minutes</td>
</tr>
<tr>
<td>18.</td>
<td>goto (step 4)</td>
<td></td>
</tr>
</tbody>
</table>

The proposed distributed EV charging algorithm for each individual EV is summarized in Algorithm 1. The discrete time algorithm is executed every few minutes and \( t \) denotes the current time. The charging start time \( t(s) \) is the time when an EV is plugged in and \( t(e) \) is an expected finish time. The parameter \( \tau \) de-
notes the time slot which can be a few minutes and the constant \( k \) controls the power increment step size (with a large \( k \), the charging profile will exhibit more spikes). The variable \( V(t) \) is the local voltage sensed at time \( t \). The parameters \( V(t) \) and \( V(t) \) are the threshold values on local voltages which will be used to regulate the total EV demand in the grid, i.e. when the total demand is too high, the local voltages will drop below the threshold and EVs will decrease their charging rate. \( V(t) \) and \( V(t) \) can be obtained via a number of ways. We determine the values from historical data by calculating rolling maximum and minimum over a horizon of length \( H \), i.e. \( V(t) = \max\{V(t - H), V(t - H + 1), ..., V(t)\} \) and \( V(t) = \min\{V(t - H), V(t - H + 1), ..., V(t)\} \). Note that both thresholds are then bounded by limits set by relevant distribution code. Also note that data on weekends, weekdays and holidays will be classified in different sets since the demand profiles are different. \( S(e) \) is the required SOC level, \( S(s) \) is the initial SOC level when a vehicle is plugged in and \( S(t) \) is the SOC measured at time \( t \). \( p \) denotes the physical maximum charging power. We assume a linear battery model such that \( dS(t)/dt = p(t)/c \) where \( c \) is a constant relating energy to battery percentage change.

The algorithm works unsynchronised. Each EV controller will independently execute the Algorithm 1 starting from step 1 as soon as an EV is plugged in.

While controlling EV charging from a distributed point of view, it is very difficult for individual EV controllers to coordinate or communicate with others without additional complexity and infrastructure. Therefore, it is natural for each EV to take power greedily if it is safe to do so. By greedily, it is meant that each EV tries to maximise the charging such that the set of all charging profiles at \( t \) is close to a solution of (3.1).

\[
\begin{align*}
\text{minimise} & \quad \left( C(t) - \sum_{m=1}^{M(t)} p_m(t) \right)^2 \\
\text{subject to} & \quad 0 \leq p_m(t) \leq e_m(t)\bar{p}_m(t), \quad m = 1, ..., M(t), \\
& \quad S_m(t) \leq S_m(e), \quad m = 1, ..., M(t).
\end{align*}
\]
This performance will be backed up via realistic simulation. The parameter $e_m(t)$ which is either 0 or 1 represents the travelling profile for EV $m$ at time $t$ denoting whether the EV is away or at home respectively. $p_m$ is the charging profile for vehicle $m$ which is the main dependent variable for the system. $S_m(e)$ is the intended SOC level upon finishing the charging for EV $m$. $\bar{p}$ is the maximum charging power limit for EV $m$. Also the total number of EVs plugged in at time $t$ is $M(t) = \sum_{m=1}^{M} e_m(t)$ where $M$ is the total number of houses with EVs. $T_m$ is the total charging time for EV $m$ and $C(t) = \max\{0, E(t) - D(t)\}$ is the spare capacity where $E(t)$ is the maximum level of power the transformer is willing to supply and $D(t)$ is the total non-EV demand in the distribution network.

In (3.1), $C(t)$ is not directly measurable and the equations rather represent an ideal case which one would like to realise. Each EV could estimate the spare capacity in the grid using local voltage as stated in Theorem 1 if the topology of the network were known. However, given the linear voltage-demand relationship, the minimum voltage threshold corresponds to the maximum level of power $E(t)$ the transformer is willing to supply. Nevertheless, without using the topology of the network, each EV could gradually increase its charging power until the local voltage threshold $V(t)$ is reached. The charging power increments may be determined proportionally to the spare capacity in the grid as follows:

$$\Delta p(t) = k_{\bar{p}} \frac{V(t) - V(t)}{V(t) - V(t)} \quad (3.2)$$

When the grid limit is about to be reached, the power increment $\Delta p(t)$ as in (3.2) for each EV decreases and avoids a sudden violation of grid constraints.

Fairness is important while developing EV charging algorithms. Due to location and load conditions, when using the greedy algorithm, it is always the case that some vehicles have charging advantages over the others. However, it is not acceptable to have any EVs unduly disadvantaged. Though it is not possible to communicate with other households, EV $m$ has an average charging rate, $(S_m(e) - S_m(s))/(t_m(e) - t_m(s))$, calculated from user settings which can be used as a benchmark charging speed. In
this thesis, it is assumed there is a price incentive such that users plug in their EVs as soon as they get home and set the finish time as late as possible to avoid paying extra. At time $t$, (3.3) will be applied to calculate a correction factor $p(t)$ which is used to adjust the current charging rate to keep it consistent with the benchmark charging speed.

$$p(t) = \frac{S_m(e) - S_m(t)}{t_m(e) - t} / \frac{S_m(e) - S_m(s)}{t_m(e) - t_m(s)}$$  \hspace{1cm} (3.3)

What (3.3) does for EV $m$ at time $t$ is to determine how much faster/slower it has to be charged after $t$ such that $S_m(e)$ can be achieved exactly at $t_m(e)$. These steps will be repeated until local voltage reaches the threshold value, in which case the power will be dropped to half to ensure safety and electricity quality across the grid.

### 3.2.3 Simulation results

Simulations have been run for the above algorithm using a real suburban distribution network with 113 households as shown in Figure 2-1. In order to test the capability and robustness of the algorithm, the simulations assume an EV penetration rate of 80% with realistic demand profile and EV travelling profiles.

To compare with the uncontrolled case presented in Section 3.1, two sets of simulations are performed in this section. The first set is an implementation of (3.1) where the central controller has perfect knowledge of how much spare capacity the grid has and how many EVs are plugged in. Therefore the controller is able to distribute spare capacity equally among all EVs. This algorithm is a centralised version of the greedy algorithm presented in [33]. The other set follows the instructions in Algorithm 1.

The discrete time step is 15 minutes across the simulations and a time window of 24 hours is used to calculate $V(t)$ and $\bar{V}(t)$ for each household.

Figures 3-4 and 3-5 show the aggregated demand of all EVs, aggregated household non-EV demand and their sum for centralised control and distributed control respectively. It can be seen from Figure 3-1 and 3-2 that without control, additional EV load will cause significant peak increase which affects not only electricity price but also electricity quality (e.g. voltage stability, power factor and phase unbalance).
Figure 3-4: Demand profile of the network under centralised control

Figure 3-5: Demand profile of the network under distributed control
Figure 3-6: Battery SOC level of all EVs in the network under centralised control.

Figure 3-7: Battery SOC level of all EVs in the network under distributed control.
Figure 3-8: The household local voltage levels with the centralised charging.
Figure 3-9: The household local voltage levels with distributed greedy fair charging.
The centralised algorithm makes very good use of spare capacity in the grid and the total demand when most EVs are plugged in oscillates between 100kW and 120kW. The oscillation is caused by the synchronous behaviour in the simulations. Due to computing power limitation, rather than making decisions in a fully unsynchronised way, users are divided into several groups and each group will make decisions together which introduces slight oscillation in total demand. The distributed fair charging algorithm also controls peak load well and moves the majority of EV demand to the overnight valley. Almost all EVs are charged to above 80% in this algorithm. Even though it is not as good as the centralised algorithm, the results for the distributed algorithm are still impressive given the difference in the amount of information used in the two strategies.

Figure 3-6 and 3-7 show how the SOC level of each EV changes over time for centralised control and distributed control respectively. During the middle of the day, most EVs are out and there is not much EV demand in the grid. Since controllers do not have access to SOC levels while EVs are travelling, the SOC levels are therefore shown as constants when EVs are away and adjusted as soon as the vehicles are plugged in. It is worth noting that in general, EVs with higher SOC levels are charged with smaller rates which are represented as flatter curves in the figures and those with low battery level tend to have a steep charging profile. This is essentially due to the fairness correction term as in (3.3).

Figure 3-8 and 3-9 show the local voltage on each phase for each house under the two algorithms respectively. Compared to the uncontrolled case where Phase A voltages drop below distribution code regulated level 216V, both the centralised and distributed algorithms keep the voltage within range. As we have explained above, though the algorithms are designed to be fully asynchronous in reality, due to computation power limitation, EVs are grouped in simulations and they update synchronously within each group. Therefore, some fluctuations in demand and voltage are observed. This is especially the case in voltage profiles simulation during overnight period as shown in Figure 3-9. When a group of EVs make charging decisions, they may shoot for more energy than intended since they are unaware of the
decisions the other EVs in the same group are making. Then because of the feedback mechanism, the overshoot might be over-corrected when the next group make decisions causing the oscillation in voltage. However, in reality, each EV will be making decisions asynchronously. And under fully asynchronous updates, such oscillation can be avoided.

3.3 Strategic game based model for EV charging management

In the previous section, we have proposed a TCP-like algorithm which achieves remarkable results given the fact that no communication is used. The greedy algorithm follows a heuristic approach. In this section, a more theoretically driven distributed control algorithm is proposed to manage the electrical power demand for the purpose of charging electric vehicles so that: (a) the overall power demand remains within the limitations of the distribution network; (b) each vehicle obtains a sufficiently charged battery at the end of the charging cycle and (c) the performance is theoretically derived and verified in simulations. The control problem is modelled as a non-cooperative game with weakly coupled cost functions for each vehicle. The cost function for each vehicle consists of an individual cost term and a group cost term. The group cost term expresses the aggregated demand of all vehicles and serves the purpose of ensuring that the infrastructure capacity constraints are respected. It is shown that this term can be estimated from local voltage measurements. The individual cost term reflects the need to achieve a desired charge level in the battery. Sufficient conditions for the overall system to admit a unique Nash Equilibrium are identified. Convergence and stability properties are also studied. To illustrate the efficiency of the proposed charging methodology, the algorithm is again simulated in the context of an Australian suburban low voltage electricity distribution network as in Figure 2-1.
3.3.1 Strategic game

Game theory was first introduced more than 60 years ago by John Nash. Game theory deals with strategic interactions among multiple agents. Each agent has their own preferences among many possibilities and the preferences are often captured by an objective function or cost functions. Generically, in a multiple-agents game, one player’s objective function is related to the behaviours of others. In other words, all players are coupled with each other through their objective function. Depending on the preferences of agents and constraints of the game, agents can either play a cooperative game where everyone chooses to work together to establish a win-win situation, or choose not to collaborate and play a non-cooperative game where no one can benefit from a unilateral move. Since communication is not an option for most existing distribution networks, the system is modelled here as a non-cooperative game which requires the least amount of information exchange.

The EV charging problem is formulated as an $M$-player strategic (non-cooperative) game $G = \langle M, P, J \rangle$. The player set $M := \{1, \ldots, M\}$ includes all houses with an EV in a low voltage network. The strategy set $P_m$ is the collection of all actions $p_m$ the player $m$ can take and $P$ is the union of all $P_m$. The set $J := \{J_1, \ldots, J_m\}$ is the set of cost functions to be minimised for all players.

The variables used in this section are summarized in Table 3-2. We first give the mathematical definition of Nash equilibrium in an $M$ player strategic game.

**Definition 1 Nash Equilibrium**[73] A Nash equilibrium of a strategic game $G = \langle M, P, J \rangle$ is a profile of strategies $p^*_m \subset P$, $m = 1, 2, \ldots, M$, with the property that for every player $m \subset M$ we have

$$J_m(p^*_m, p^*_m') \leq J_m(p_m, p^*_m) \text{ for all } p^*_m \subset P \quad (3.4)$$

In simple words, at a Nash equilibrium, no player can benefit by deviating from his equilibrium strategy if other players keep their equilibrium strategies unchanged.

The cost functions $J_m, m = 1, 2, \ldots, M$ of the game are chosen to be quadratic. Each cost function has two terms: a group cost and an individual cost. The group
The other cost to be minimised is the cost for deviating from local charging plans. When EV $m$ is plugged in, the user will set the total time $T_m$ available for recharging the EV. The ratio $\gamma_m = R_m/T_m$ gives an average charging power for the EV to be charged on time; where $R_m$ is the total energy needed to fully charge the battery of EV $m$. This average is a bottom line that the EV wants to keep track of. Without such a cost term, it is possible that some EVs are given much more charging power than others and fairness issues arise. It is important to note that it is assumed all
users leave EVs plugged in for as long as possible and input the data honestly. It is also assume that a tariff system is in place which encourages smaller average charging rate and honest behaviours.

The cost function is then defined as follows:

$$J_m(p_m, p_{-m}) = \left( k_m (\bar{E} - p_m - \bar{p}_{-m}) \right)^2 + \beta_m \left( \frac{p_m - \gamma_m}{\gamma_m} \right)^2$$  \hspace{1cm}(3.5)$$

where $k_m = 1/(\bar{D} - \bar{D})$ is the difference between upper bound and lower bound of demand. $\bar{E} = \bar{D} - H$ is the upper bound on total EV demand where $H$ is the household non-EV demand. $\bar{E}$ is essentially a time-varying quantity as household load varies. However, as explained in Section 1.1.2 and Figure 1-2, the variation is on a rather slow time scale of 30-minute or more which is sufficiently long for EVs to converge to their optimal charging power as we will show later. $\gamma_m$ is the average charging power from user setting and $\beta_m$ is a positive constant.

Since there is no direct access to total demand levels, EVs estimate such values using local voltage measurements according to Theorem 1 as follows:

$$\frac{\bar{D} - \bar{D}}{\bar{D} - \bar{D}} = \frac{V^m - \bar{V}^m}{\bar{V}^m - \bar{V}^m}.$$  \hspace{1cm}(3.6)$$

As a result, the goal is equivalent to the local voltage of agent $m$ being as close to $\bar{V}^m$ as possible.

Using (3.6), the cost function (3.5) is equivalent to the following:

$$J_m(p_m, p_{-m}) = \left( k_m (\bar{E} - p_m - \bar{p}_{-m}) \right)^2 + \beta_m \left( \frac{p_m - \gamma_m}{\gamma_m} \right)^2$$

$$= \left( \frac{\bar{D} - \bar{D}}{\bar{D} - \bar{D}} \right)^2 + \beta_m \left( \frac{p_m - \gamma_m}{\gamma_m} \right)^2$$

$$= J_m(p_m, V_m)$$

$$= \left( \frac{V_m(p_m, p_{-m}) - \bar{V}^m}{\bar{V}^m - \bar{V}^m} \right)^2 + \beta_m \left( \frac{p_m - \gamma_m}{\gamma_m} \right)^2$$  \hspace{1cm}(3.7)$$

The players minimise this cost function by adjusting their charging power $p_m$. 

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Because of the convexity of (3.5) with respect to \( p_m \), by taking its partial derivative with respect to \( p_m \) and letting it be zero, we obtain:

\[
\frac{\partial J_m}{\partial p_m} = 2k^2_m (p_m + \overline{p}_m - E) + \frac{2\beta_m}{\gamma_m^2}(p_m - \gamma_m) = 0 \quad (3.8)
\]

The solution to (3.8) is given as follows:

\[
p_m = \Phi_m(\overline{p}_m, \beta_m, k_m, \gamma_m)
= k_m^2 \gamma_m^2 \overline{E} - k_m^2 \gamma_m^2 \overline{p}_m + \beta_m \gamma_m
= -\overline{p}_m \frac{k_m^2 \gamma_m^2}{k_m^2 \gamma_m^2 + \beta_m} + \frac{k_m^2 \gamma_m^2 \overline{E} + \beta_m \gamma_m}{k_m^2 \gamma_m^2 + \beta_m} \quad (3.9)
\]

whose value is always non-negative since \( \overline{E} \geq \overline{p}_m \).

This is also the best response reaction function of EV \( m \) which is in a linear form with respect to \( \overline{p}_m \). Let \( \alpha_m = (k_m^2 \gamma_m^2 \overline{E} + \beta_m \gamma_m)/(k_m^2 \gamma_m^2 + \beta_m) \), we can write the equilibrium solution as:

\[
\begin{pmatrix}
1 & k_2^2 \gamma_1^2 & \cdots & k_m^2 \gamma_m^2 \\
k_1^2 \gamma_1^2 & 1 & \cdots & k_m^2 \gamma_m^2 \\
\vdots & \vdots & \ddots & \vdots \\
k_1^2 \gamma_1^2 & k_2^2 \gamma_1^2 & \cdots & 1
\end{pmatrix}
\begin{pmatrix}
p_1^* \\
p_2^* \\
\vdots \\
p_m^*
\end{pmatrix}
= \begin{pmatrix}
\alpha_1 \\
\alpha_2 \\
\vdots \\
\alpha_m
\end{pmatrix}
\quad (3.11)
\]

\[
\Leftrightarrow Ap^* = \alpha \quad (3.12)
\]

where the updated values are shown on the left of the equality sign. Then we introduce the following proposition:

**Proposition 1** There exists a unique Nash Equilibrium in the \( M \)-player noncooperative EV charging game \( G \).

Since all \( p_m \) values are non-negative, the proof of Proposition 1 is straightforward. It can be shown by contradiction that under generic conditions, matrix \( A \) is non-singular, thus invertible. Therefore, there exist a unique solution to (3.12) which can
be computed as:

\[ p^* = A^{-1} \alpha \Leftrightarrow \]

\[
\begin{pmatrix}
    p_1^* \\
    p_2^* \\
    \vdots \\
    p_m^*
\end{pmatrix} = \sum_{i=1}^{m} \alpha_i
\begin{pmatrix}
    k_1^2 \gamma_1^2 \\
    k_2^2 \gamma_2^2 \\
    \vdots \\
    k_m^2 \gamma_m^2
\end{pmatrix} - \begin{pmatrix}
    \alpha_1 k_1^2 \gamma_1^2 - \alpha_1 \\
    \alpha_2 k_2^2 \gamma_2^2 - \alpha_2 \\
    \vdots \\
    \alpha_m k_m^2 \gamma_m^2 - \alpha_m
\end{pmatrix} \tag{3.14}
\]

**Lemma 1** If all EVs have the same setting of \( \beta, \gamma \) and \( k \), from (3.9), then for any EV \( m \), the charging power at equilibrium can be calculated as follows:

\[ p_m = \frac{k^2 \gamma^2 E + \beta \gamma}{k^2 \gamma^2 M + \beta} \tag{3.15} \]

To prove the lemma, the following equation holds at equilibrium:

\[ p_m = \frac{k^2 \gamma^2 E - k^2 \gamma^2 (M - 1)p_m + \beta \gamma}{k^2 \gamma^2 + \beta} \tag{3.16} \]

The solution to equation (3.16) is given by (3.15).

**Remark 1** The reaction function defined in (3.9) represents the optimal response of EV \( m \) to the charging rate of all other users. And this charging rate of all other users is only determined through the sum. Again, in practice, there is no access to the value of the aggregated charging rate from each house’s point of view. However, (3.9) can be approximated as follows using (3.7):

\[ p_m = \frac{k_m \gamma_m^2 (V_m - V_m^m)/(V_m^m - V_m) + \beta_m \gamma_m}{k_m^2 \gamma_m^2 + \beta_m} \tag{3.17} \]

**Remark 2** The value \( \gamma_m \) is considered a constant for each EV in this thesis. However, in reality, \( \gamma_m \) can be used as a pricing parameter and associated with a tariff. The users can choose to have their vehicles charged faster by paying more and \( \gamma_m \) can be adjusted to a higher value to meet the requirement. Customers who do not need
to use their vehicle in the next day can choose a very small $\gamma_m$ and pay much less than normal electricity price (making benefit in some sense). More research will be carried out to study the effect of $\gamma_m$ and how to set the value based on different user requirement.

**Remark 3** In terms of how to determine parameters when the algorithm is embedded in hardware, it is suggested that simulations should be carried out for a system where everyone has the same setting. First of all, (3.15) can be written as follows:

$$p_m = \frac{E/M + \beta \gamma/(k^2\gamma^2M)}{1 + \beta/(k^2\gamma^2M)}$$

(3.18)

There are two terms in the numerator; the first term indicates the average spare capacity that each household could have and the second term shows the willingness to pay in exchange for faster charging speed. $\beta$ is a weighting factor and the two terms can be equally weighted by setting $\beta = k^2\gamma^2M$.

### 3.3.2 Update scheme and stability analysis

The Nash Equilibrium solution derived in the previous section can be computed in a decentralised manner where each agent executes the best-response algorithm using only local information. There are a variety of distributed update schemes such as asynchronous, parallel and round robin [17]. In this section, the stability property is investigated under an asynchronous random update algorithm.

It is assumed that time is discretized. At the beginning of interval $i+1$, the player $m$ updates with a non-zero probability $\pi_m(i + 1)$ based on the residual information from the last interval and does not change the present policy with probability

$$p_m^{(i+1)} = \begin{cases} 
\Phi_m(p_{-m}^{(i)}), & \text{with probability } \pi_m(i + 1) \\
p_m^{(i)}, & \text{with probability } 1 - \pi_m(i + 1)
\end{cases}$$

(3.19)

The reason for adopting an asynchronous and random update model is that if all agents sense the grid and make decisions at the same time, the demand will fluctuate
Figure 3-10: Numerical demonstration of the algorithm for 10 homogeneous players among 91 from cold start.

a lot. In reality, people do not use electricity synchronously, despite similarity in behaviour.

Also note that the update scheme in (3.19) is very general due to the update probability being dependent on time. It can be seen that random round robin and parallel update are special cases by using appropriately defined probability functions.

Figure 3-10 shows numerically the asynchronous behaviour and the convergence of this update algorithm as per (3.19) for 10 homogeneous players among 91. Distribution network constraints are not considered in this numerical example and players update their charging power with probability of one tenth at each interval. It is observed that it only takes a very short time for the game to converge in this ideal case. Stability properties of the algorithm will be establish in Theorem 2.

**Theorem 2** Under update scheme (3.19), the system asymptotically converges in the mean to the unique Nash equilibrium from any starting point if \( k_m, \beta_m \) and \( \gamma_m \) are chosen such that the following condition is satisfied:

\[
M \leq 2 + \frac{\beta_m}{k_m^2 \gamma_m^2}, \quad \forall m.
\]  

(3.20)
Note that this sufficient condition is easy to satisfy since $k^2 m^2$ is a very small unit free value in most cases.

**Proof** The proof here is based on [19] and the proof of Theorem 4.1 in [7]. At the unique Nash Equilibrium:

$$p_m^* = \frac{k^2 m^2 E - k^2 m^2 p_m^* + \beta_m \gamma_m}{k^2 m^2 + \beta_m} \quad (3.21)$$

Let the difference between the $m^{th}$ user’s charging power and equilibrium power level at interval $i$ be $\Delta p_m^{(i)} = p_m^{(i)} - p_m^*$ where $p_m^*$ is always positive. It will be shown that the update function (3.19) generates a contraction mapping. From (3.19), no matter what the equilibrium value $p_m^*$ for user $m$ is, the following holds:

$$E|\Delta p_m^{(i+1)}| = E|\Delta p_m^{(i)}|\pi_m + E|\Delta p_m^{(i)}|(1 - \pi_m)$$

$$= \frac{k^2 m^2 \gamma_m^2 \pi_m}{k^2 m^2 + \beta_m} \sum_{n \neq m} E|\Delta p_n^{(i)}|$$

$$+ E|\Delta p_m^{(i)}|(1 - \pi_m), \quad (3.22)$$

where $E$ denotes expected value.

Now let the infinity norm of the vector $(\Delta p_1, \Delta p_2, ..., \Delta p_M)^T$ be $\|\Delta p\|_\infty$ which is the maximum entry in the vectors, the inequality becomes:

$$E\|\Delta p^{(i+1)}\|_\infty \leq \max_m \left( \frac{k^2 m^2 \gamma_m^2 \pi_m (M - 1)}{k^2 m^2 + \beta_m} \right)$$

$$+ (1 - \pi_m)) E\|\Delta p^{(i)}\|_\infty \quad (3.23)$$

Therefore, it is sufficient for the right hand side of (3.22) to be a contraction mapping if the condition in Theorem [2] holds.
Figure 3-11: POSSIM generated demand profile of the Melbourne network using the distributed charging algorithm based on game theory under 80% EV penetration.

Figure 3-12: POSSIM generated battery state of charge (SOC) level changes of 10 EVs from 91 (113 houses with 80% EV penetration). Each curve represents a vehicle. Dashed lines indicate vehicles being away, as opposed to solid lines. Gradients of curves denote the charging rate.
Table 3-3: Performance comparisons under no EVs, uncontrolled charging and distributed charging.

<table>
<thead>
<tr>
<th>Algorithm (50% EV)</th>
<th>no EVs</th>
<th>uncontrolled</th>
<th>distributed</th>
</tr>
</thead>
<tbody>
<tr>
<td>voltage outliers%</td>
<td>0</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>average charging rate</td>
<td>n/a</td>
<td>3.45kW</td>
<td>0.89kW</td>
</tr>
<tr>
<td>peak vs valley demand</td>
<td>3.63</td>
<td>5.68</td>
<td>3.67</td>
</tr>
<tr>
<td>cost for charging</td>
<td>n/a</td>
<td>$15.86</td>
<td>$14.13</td>
</tr>
<tr>
<td>unbalance time%</td>
<td>0.05</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>adequately charged%</td>
<td>n/a</td>
<td>100</td>
<td>97.55</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Algorithm (80% EV)</th>
<th>no EVs</th>
<th>uncontrolled</th>
<th>distributed</th>
</tr>
</thead>
<tbody>
<tr>
<td>voltage outliers%</td>
<td>0</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>average charging rate</td>
<td>n/a</td>
<td>3.45kW</td>
<td>0.61kW</td>
</tr>
<tr>
<td>peak vs valley demand</td>
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<td>6.19</td>
<td>3.47</td>
</tr>
<tr>
<td>cost for charging</td>
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<td>$25.34</td>
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<td>unbalance time%</td>
<td>0.05</td>
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<tr>
<td>adequately charged%</td>
<td>n/a</td>
<td>100</td>
<td>95.16</td>
</tr>
</tbody>
</table>

### 3.3.3 Simulations

In order to verify the actual performance of this algorithm, simulations are run again on the real Australian suburban three-phase distribution network with 113 households as in Figure 2-1. We examine EV penetration rates of 50% and 80% with real demand profiles (provided by Australian distributors) and EV travelling profiles. It should be noted that there is no distributed generation such as domestic solar panels in the network. The demand level upper bound is set to be similar to the existing peak demand value without EVs, which is very low considering the additional demand from EVs, to test the performance of this algorithm under extreme conditions. Details on network modelling and simulation platforms can be found in Chapter 2. The data for simulations in this section was obtained from a utility company based in Australia.

Figure 3-1 shows the demand profile of the network with 80% EV penetration without any management. Figure 3-11 shows the performance of the distributed algorithm in the network on a typical working day under 80% EV penetration. It is clear from the figure that demand peak in the evening decreased and additional demand is distributed to the overnight demand valley. From Figure 3-12, it is apparent that the charge rates (solid lines) are steeper during the overnight period than in the
evening peak. In the morning, all cars are sufficiently charged for the day and most of them are fully charged. In reality, the demand level upper bound can be set to higher values such that EVs can be charged more and faster than shown in the simulations. One fact to note about the EV travelling profiles is that most vehicles are out for work during the middle of the day and only residential charging is considered at this stage.

Table 3-3 presents some key performance parameters of the system in 24 hours averaged over several typical winter working days under three conditions: no EVs, uncontrolled charging and this distributed charging. The percentage of voltage outliers indicates the percentage of time that the system is violating the voltage requirements (below 216 V) according to distribution code. The cost of charging calculates the generation price of all electricity used within that 24 hours subject to a typical spot price. The unbalance time shows the duration that the system is experiencing 3% or more phase unbalance per house over 24 hours. An EV is called adequately charged if it is charged above 80% before 8am in the morning. Note that 80% is often much more than what a car needs for the following day. It can be seen that without any control, all EVs will start to charge as soon as they arrive at home and this increases the peak demand significantly. Using this distributed charging control, the peak demand is decreased significantly without violating any constraints of the network and the electricity price paid for charging is significantly lower. More importantly, this algorithm requires no communication infrastructure, therefore, no update of the existing facilities and it is ready to be used.

### 3.4 Chapter summary

In this chapter, a distributed greedy fair charging algorithm based on the feature of AIMD in TCP protocols is proposed for EVs in a smart grid. Unlike all other algorithms, this method is based totally on local information. By simulating an actual distribution network, it is shown that, even with a high 80% penetration rate, the distributed greedy fair algorithm successfully mitigates peak demand, ensures battery
level and fairness without breaking any grid constraints. Even though the performance is not as efficient as a centralised solution, given the amount of information used in the above algorithm, the result is remarkable. Moreover, the solution can be readily implemented in the network and requires no upgrade of the current grids’ infrastructure.

Then a noncooperative game framework for the EV charging problem is established based also only on local information. The existence and uniqueness of a Nash Equilibrium is proved as well as the stability of an asynchronous update scheme under a mild sufficient condition. Simulations conducted using realistic data show that even with a high (80%) EV penetration rate, the distributed algorithm successfully mitigates demand peaks and ensures satisfactory battery levels. Most importantly, it requires no additional infrastructure and works only with local information.
In the previous chapters, we have proposed several DSM algorithms which operate purely on local measurements without requiring explicit communication or information exchange. We have shown that the algorithms have the potential to reduce peak demand, improve grid utilisation (improving the peak-to-base ratio) and increase users’ benefit. However, an important question remains to be answered: what is the trade-off between network physical constraints and users’ benefit? In other words, how can we maximise grid utilisation without violating network constraints? To answer this question, network properties such as voltage levels, unbalance, current ratings have to be modelled and analysed. The demand management problem in this chapter is formulated as a constrained optimisation problem to meet end-user energy demand subject to the physical limits of the electrical network. The adopted model focuses on the last mile of distribution networks and accounts for transformer loading, line loading, voltage levels and phase unbalance limitations. Note that only normal operating conditions are considered in this thesis. Faults and operating under faults is not part of this study.

The optimisation problem is decomposed and solved via two different mechanisms. In the first mechanism, the behaviour of all users is coordinated through a time-varying price signal (a virtual price) that reflects how much power can be distributed
given the network constraints. This virtual price signal is broadcast to all end-users by the DSO through a one-way communication channel. The second mechanism is a relaxation of the first. An algorithm, local to the user, approximates the state of congestion in the network using the history of local voltage measurements. This method does not require any communication infrastructure. The performance of both methods is compared and their characteristics are illustrated using realistic simulations.

4.1 Network model and constraints

4.1.1 Network model

In this section, we build upon the model proposed in Figure 2-1 from Chapter 2. A user may own several appliances with flexible demand patterns (e.g. an EV, a water heater or an air conditioning (AC) unit). We assume that the DSO has the incentive to flatten total demand profiles and limit peak demand for asset protection and profit maximisation. Such goals can be achieved by appropriately incentivising consumers [40,88] and it is assumed that such incentives are place.

We consider two network scenarios with different communication infrastructures. In the first scenario, we assume that uni-directional communication is available in the grid such that customers are able to receive virtual price signals from the DSO and are able to make control actions accordingly. Bi-directional information infrastructure and protocols are not required. Figure 4-1 (left) shows the information and load flow schematics of this scenario. In the other scenario, we assume that no communication is available. Households make demand management decisions only based on local measurements as in Figure 4-1 (right).

4.1.2 Network constraints

Three phase, low voltage networks have many constraints, most of which are imposed by the electricity distribution codes. There are penalties for not abiding by the code
or not delivering expectations placed on the DSOs by the regulator.

We first analyse the constraint sets for each of the three phases. Note that, residential networks usually have high power factors (we will further demonstrate this in Section 4.5) such that the real component of current is dominant. For simplicity of analysis, and presentation, we present the network constraints under the assumption that the network, from the transformer down, is resistive as the power factor in last mile networks is close to unity. The transformer itself is treated as a constant voltage source. This is for the purpose of analysis only. In the simulation and testing of the algorithms this assumption is removed and realistic network and network constraints are used. We also represent most of the quantities in terms of current rather than power or energy. However, under a constant voltage assumption and unity power factor current and power can be interchanged and that leads to a connection to energy through integration over time.

1) Transformer capacity rating constraint: The first constraint relates to the capacity of the distribution transformer. The aggregated power drawn at any time in the distribution network should not exceed the transformer rating. Continuous overloading of a transformer will shorten equipment lifespan and increase losses. This constraint is represented as follows:

$$\sum_{m \in M} x_m \leq I_{\text{rating}}, \quad (4.1)$$

where $I_{\text{rating}}$ is the transformer rating in terms of current and $x_m$ is the demand

Figure 4-1: Information and power flow schematic of the algorithm using virtual price signals from the DSO (left) and the algorithm using only local measurements (right).
defined in terms of current of household $m$ among a total of $M$ households at a given point of time. Note that the real rating is time varying, i.e. for short periods of time, we are allowed to go over the limit \[97].

2) **Real line current rating constraint:** Power lines in the network have current ratings that should not be exceeded. Let $I_{\phi}$ be the rating of phase $\phi$. This constraint is represented as follows for phase 1, 2 and 3:

$$\sum_{m \in M_{\phi}} x_m \leq I_{\phi}, \ \phi \in \{1, 2, 3\},$$

(4.2)

where $M_{\phi}$ represents set of the house numbers $m$ on phase $\phi$.

3) **Phase unbalance constraint:** Phase unbalance in a network leads to increased current in the neutral and creates additional losses (as in principle, in a balanced network, the neutral conductor is current free). In addition, phase unbalance also leads to pulsating torque on motors and overheating of motors. Phase unbalance is usually defined in terms of voltages. A common definition is the maximum deviation of phase to neutral voltage from the average voltage normalized by the average voltage \[80]. In our case, assuming unity power factor, phase unbalance comes from uneven active loads across the three phases. And we define the phase unbalance constraint as a linear expression as shown in (4.3) where the maximum deviation of individual phase load from the average load is capped.

$$\left| \frac{\sum_{m \in M_{\phi}} x_m - \frac{1}{3} \sum_{m \in M} x_m}{\frac{1}{3} \sum_{m \in M} x_m} \right| < k, \ \phi \in \{1, 2, 3\}$$

(4.3)

where $k$ is a given constant. Note that unbalance levels at various points in the network would be different and ideally they should all be monitored. This imbalance constraint we set is for the transformer node only, where the largest imbalance mainly occurs. However, we acknowledge that it really can occur anywhere in the network depending on the demand profile along the lines.

4) **Artificial peak demand constraint:** For the purpose of reducing peaks, it is important that the variation in aggregated demand is minimised. Hence, we apply an artificial constraint to the system which may supersede the transformer capacity rat-
ing constraint. We assume that as part of the normal operation of the network, the DSO predicts the mean power and sets a total current demand threshold $\mathcal{T}$ for users of all phases in the network as in (4.4).

$$\sum_{m \in M} x_m \leq \mathcal{T} \tag{4.4}$$

$\mathcal{T}$ is time varying, and it is larger than the expected average load. In the simulations, we assume this value to be 30% larger than the expected average load.

All the constraints are represented as linear inequalities in terms of aggregated current consumption and they can be combined as in (4.5):

$$AX \leq B, \tag{4.5}$$

where $A$ is a sparse matrix, $X$ is the vector of all $x_m$ and $B$ is a vector of constants.

4.2 Problem formulation

4.2.1 Load and preference analysis

The preference levels of energy consumption for households are modelled using utility functions as commonly used by economists [43]. For a consumer, the utility is a function representing the satisfaction level or benefit received from the amount of current consumed. In practice, each home appliance may have a different preference in the way it consumes energy and the utility function may be a non-continuous function. However, it is reasonable to approximate the aggregated utilities of all appliances in one household with a differentiable function. The following assumption is introduced.

**Assumption 1** The aggregated utilities of all appliances in a household can be approximated by an increasing, twice continuously differentiable and concave function whose curvature is bounded away from zero. In the context of this thesis, we will
represent household utilities using a logarithmic function by the following:

\[ U_m(x_m) = \alpha \log \left( \frac{x_m}{x_m^{\text{max}}} \right), \quad x_m > 0 \]  \hspace{1cm} (4.6)

where \( \alpha \) is a scaling factor that is the same for all households in the entire network. \( U_m(x_m) \) is the utility of house \( m \) at a point of time and \( x_m^{\text{max}} \) is the circuit breaker current rating of house \( m \).

Note that the knowledge of \( \alpha \) and \( x_m^{\text{max}} \) has to be known separately as \( \alpha \) is a global parameter and \( x_m^{\text{max}} \) is a local one. The household utilities are relative measures and the signs of such values do not matter.

Logarithmic utility functions have been studied for congestion control in communication networks due to their proportional fairness \[57\]. We conjecture that they may also be applicable to demand management \[9\]. We justify the use of a normalised logarithmic function with the following arguments:

\textbf{a) Prioritizing loads:} The first reason behind choosing a logarithmic function is that it is well suited for prioritizing loads within a household. The decreasing slope of a logarithmic function could well represent the priority of loads. To better understand what happens within a household, we categorize household loads into three categories.

\textbf{1) Storable demand loads:} The first category of loads includes those appliances that are relatively robust to variations in power input. Some typical examples include AC units, fridges, space heaters, electrical water heaters and EVs. For AC units and fridges, the objective is to keep interior temperature within a desired range. Therefore, an interruption for a period of time (minutes to hours, depending on thermal mass, and acceptable temperature range) followed by a higher cooling output or vice versa can still sustain desired temperature range and provide scheduling flexibility. For EVs and water heaters, the flexibility is even higher. Such appliances can be interrupted for hours without affecting the quality of service.

\textbf{2) Shiftable demand loads:} The second category of loads includes those whose demand can be shifted but not easily interrupted. Examples of such loads include washing machines, driers and dish washers. These appliances can be delayed until
there is increased capacity in the network before they are turned on. However, once these appliances are turned on, they should not be interrupted until their desired functional cycles finish.

3) **Rigid loads**: The third category of loads includes those whose demand cannot be easily shifted or interrupted. Examples of such loads includes lights, most kitchen appliances and entertainment systems. These appliances consume energy as soon as they are switched on, cannot be shifted to a later time, and are therefore assigned higher priority in terms of energy supply. These loads will be called rigid loads or inflexible loads.

The first two categories of loads are also called flexible loads. In this chapter, we assign lower priority to flexible loads. It is assumed that there is a local controller at each house which controls the energy consumption of all loads within that house. This controller follows the logarithmic utility function and checks the virtual cost for electricity. It then calculates an optimal consumption level and allocates power to loads according to their priorities. As a result, flexible loads are likely to be curtailed or delayed during peak hours.

**b) Geometric mean maximisation (Fairness among users)**: Having the consumers’ utility defined as logarithmic functions, the objective function for the network operator, which takes the form of maximizing \( \sum_{m \in M} U_m(x_m) \), can be interpreted as the maximisation of the geometric mean of all users’ demand. Traditionally, from the operators point of view, the more power it sells without breaking network constraints, the more profit it generates. Therefore, a traditional utility function of a operator will be to maximise the sum of all households demand. However, this may result in unbalanced power distribution among users. Geometric mean maximisation equalizes the portion allocated to each user as well as maximizing total demand. We assume the cost to be constant and do not model the term explicitly in the objective function.
4.2.2 System objectives

As described above, in order to maximise the benefits to all users and the DSO while ensuring fairness, the following system objective function is proposed

$$\max_X \sum_{m \in M} U_m(x_m)$$

subject to $0 < x_m \leq x_m^{\max}$ $\forall m \in M$ \hspace{1cm} (4.7)

$$AX \leq B$$

where $A$, $B$ and $X$ are as described in (4.5). The Lagrangian function of the optimisation formulation is as follows:

$$L(X; \lambda) = \max_{0 < x_m \leq x_m^{\max}} \left\{ \sum_{m \in M} U_m(x_m) + \lambda^T (B - AX) \right\}$$

where $\lambda$ is the Lagrangian multiplier. The corresponding dual problem \[18\] is

$$\min_{\lambda} \max_{0 < x_m \leq x_m^{\max}} \left\{ \lambda^T B + \left( \sum_{m \in M} U_m(x_m) - \lambda^T AX \right) \right\}$$ \hspace{1cm} (4.9)

4.2.3 User objectives

The dual problem in (4.9) can be solved by the DSO and the users iteratively according to \[78\]. The first part, which is for the DSO to optimize and compute prices, will be introduced in Sections 4.3 and 4.4. The second part is for each user $m$ to locally optimize and it is given as:

$$\max_{x_m} U_m(x_m) - \lambda^T A_m x_m$$

subject to $0 < x_m \leq x_m^{\max}$ \hspace{1cm} (4.10)

where $A_m$ is the $m^{th}$ column of matrix $A$. $\lambda^T A_m$ reflects how congested the distribution route to user $m$ is and how the constraints on this route are satisfied. Substituting $U_m$ with (4.6) and letting the virtual price per unit signal be $p_m = \lambda^T A_m$, (4.10) can
be written as:

$$\begin{align*}
\text{maximise} & \quad \alpha \log \left( \frac{x_m}{x_{m}^{\text{max}}} \right) - p_m x_m \\
\text{subject to} & \quad 0 < x_m \leq x_m^{\text{max}}.
\end{align*}$$

(4.11)

The first term in (4.10) is a unitless term. The second term, virtual price per unit times the number of units gives the total virtual price which has no unit either.

Having the load preferences, utility functions and system objectives defined, we will propose in the next two sections distributed algorithms for all users and the DSO to jointly compute the optimal demand strategy iteratively.

## 4.3 Algorithm using virtual pricing signals from the DSO

**Algorithm 2** Update the algorithm using pricing signals from the DSO

*For the DSO*

1. if $t \in T_{DSO}, T_{DSO} = \{0, \tau, 2\tau, 3\tau \ldots\}$ then
2. \hspace{1em} $\lambda(t + 1) = \max \{0, \lambda(t) - \delta(B - AX(t))\}$
3. else
4. \hspace{1em} $\lambda(t + 1) = \lambda(t)$.
5. \hspace{1em} $p_m(t + 1) = \lambda(t + 1)^T A_m$
6. \hspace{1em} Broadcast $p_m$
7. end if

*For each user*

1. if $p_m$ received then
2. \hspace{1em} $x_m(t + 1) = \min \left\{ \max \left\{ 0, \frac{\alpha}{p_m(t)} \right\}, x_m^{\text{max}} \right\}$
3. end if

One way to solve the problem defined in Section 4.2 is via a price signal from the DSO. Assume that there is a uni-directional communication channel is available in the network. The DSO sends a discrete time-varying virtual price signal which is updated every $\tau$ seconds or minutes to coordinate users’ behaviours. Note that the DSO price signal is a virtual price for the sake of management, and not a real price. The problem can then be solved using Algorithm 2 where $\delta$ is the step size vector for virtual price update.
Theorem 3 If the update step size $\delta$ satisfies $0 < \delta < 2\alpha/(x_s^2 M)$ where $x_s := \max\{x_m^{max}, m \in M\}$, then starting from any initial current $0 < x \leq x_{max}$ and constraints price vector $\lambda \geq 0$, the Algorithm 2 will converge to an equilibrium which is primal-dual optimal.

The proof is similar to the proof of Theorem 1 in [66]. Our application is a special example where there is only one link in the system and there are multiple linear constraints.

To understand how fast the desired value is reached, we have simulated the algorithm in the network model proposed under sudden changes of total demand in the network starting from random initial values. The resultant behaviours of 10 randomly selected houses are shown in Figure 4-2. At iteration 0, total network rigid demand increases suddenly and it takes two iterations for households to adapt to the new equilibrium. At iteration 5, total network rigid demand decreases suddenly and it only takes one iteration before steady state. In the simulations for these extreme cases, convergence is reached after a maximum of two iterations. In our application, dynamics mainly result from the change in network demand which is on a slow time scale as explained in Section 1.1.2. Therefore, the proposed algorithm is fast enough to respond to changes in demand.

4.4 Algorithm using local measurements only

In most parts of the world, low voltage distribution networks are not equipped with any information exchange infrastructure. We therefore propose an asynchronous distributed algorithm that works using local measurements only to solve the problem in Section 4.2. In radial LV networks, voltages are a good indication of network demand and (4.12) presents a reasonable approximation for a voltage to current demand relationship, which is in general non-linear and multidimensional. The general form of (4.12) follows from Theorem 1.
Algorithm response of 10 houses to a sudden increase and a sudden decrease in total demand

<table>
<thead>
<tr>
<th>Iteration number</th>
<th>Current consumption $x_m$ (A)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>0.5</td>
<td>1.5</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>1.5</td>
<td>2.5</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>2.5</td>
<td>0.5</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>0.5</td>
<td>1.5</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
</tr>
</tbody>
</table>

Figure 4-2: Algorithm response of 10 randomly selected houses to a sudden increase in demand at iteration 0 and a decrease in demand at iteration 5

\[ V_m(t) - V_m^* = \frac{1}{c_m} (I_\phi^* - \sum_{n \in M_\phi} x_n) \] (4.12)

where $V_m(t)$ is the voltage of house $m$ at a particular time, $V_m^*$ is a known benchmark local voltage value which corresponds to a known total demand $I_\phi^*$ on the same phase. $c_m$ is a constant associated with the slope which can be learnt from historical data sets. Generally, high demand in a distribution network leads to high voltage drops along the distribution lines which will result in lower voltages at each household [108].

In addition, voltage is also an accurate indicator of potential faults in a network and making demand decisions based on voltage measurements ensures reliability in a network though in most cases, faults would be cleared much faster than demand response actions. Note that the linear model is not an error-free model. The linear approximation may over-estimate or under-estimate true demand value. However the model does not need to be perfect to allow for good control. Indeed in the algorithm to be presented, the model (4.12) is used to inform a decision about increasing or decreasing further demand. It can be seen that the modelling errors are compensated through feedback control.

For implementation of this algorithm, some modifications have to be made to the constraint set as described below. Since there is no communication channel, each
household will use local voltage measurements to approximate how much load there is on the corresponding phase. Unbalance constraint \([4.3]\) is taken out of consideration since it is not possible to estimate it from the perspective of a single house. Constraints \([4.1]\) \([4.2]\) \([4.4]\) can be grouped and written as in \((4.13)\).

\[
\sum_{m \in M_\phi} x_m \leq I_\phi, \quad \phi \in \{1, 2, 3\}. \tag{4.13}
\]

where \(I_\phi\) represents the combined requirements in \((4.1)\) \((4.2)\) and \((4.4)\). We therefore propose the following algorithm:

**Algorithm 3** Update algorithm using local measurements

*For users only*

1. if \(t \in T_m, T_m = \{0, \tau_m, 2\tau_m, 3\tau_m \ldots\}\) then
2. \(p_m(t + 1) = \max\left\{0, p_m(t) - \delta c_m(V_m(t) - V_{m}^{\min})\right\}\)
3. \(x_m(t + 1) = \min\left\{\max\left\{0, \frac{\alpha}{p_m(t+1)}\right\}, x_{m}^{\max}\right\}\)
4. else
5. \(x_m(t + 1) = x_m(t)\).
6. end if

where \(\delta\) is the step size which is a small number. The update interval \(\tau_m\) is usually of a minute up to 15 minutes and \(\alpha\) is the scaling factor.

\(V_{m}^{\min}\) is a lower bound for our algorithm. It could be anywhere between the mean and the minimum over a historic observation window depending on the design goals. Fine tuning of this parameter is not within the scope of our study and for our simulation, we set \(V_{m}^{\min}\) to be one standard deviation lower the mean.

### 4.5 Simulations and discussion

#### 4.5.1 Simulation setup

To illustrate the performance of our proposed algorithms under realistic operating conditions, we simulate the algorithms above on validated models of various Australian networks. In this thesis, we present a model of a real Australian suburban three-phase residential distribution network using January demand data (the hottest
Figure 4-3: Demand profile where half of the demand (showing in pink) has flexibility and can be shifted.

Figure 4-4: The current profiles of three phases and neutral.

Figure 4-5: Phase unbalance averaged over all poles.
Figure 4-6: Averaged power factor of all phases for 24 hours under normal operating condition.

month of the year) as measured at the transformer of this network. The chosen model is highly unbalanced and January is the month where maximum demand occurs. As a result, this is a particular challenging scenario which helps to understand the tolerance of our algorithms. The network has 113 households. The phase allocation is shown in Figure 2-1. Note that the network is modelled with line losses, reactive power and phase unbalance. Therefore the simulations extend the theoretical analysis and verify the performance of our algorithms.

It has been indicated in [96] that in an EV enabled society, up to half of domestic loads have a certain level of energy storage capacity (thermal, chemical, etc.). We therefore assume that half of the total demand (approximately 24kWh in our simulation) for each house is flexible and half of the demand is rigid. Rigid loads cannot be interrupted and they will remain the same throughout the simulations. Flexible loads can consume up to 3kWh of energy (of a total of around 12kWh) up to 6 hours before or after the original time of operation in any one day. In other words, flexible loads could distribute 3kWh of energy in a 12-hour time window (or opportunity window) according to certain rules. While this assumptions on loads may not capture the actual situation precisely, the purpose of this load model is to illustrate and compare the effectiveness of two proposed algorithms. To this end, the model is of reasonable resolution especially with EVs and energy storage devices becoming popular. The
utilitarian formulation implies that there will be unsatisfied demand because of the network constraints. In the simulations, we assume that unsatisfied demand will be left unsatisfied until the next opportunity window commences. However, for flexible loads, it is the energy rather than demand that needs to be satisfied. And due to the huge amount of spare network capacity released by the proposed algorithms in off-peak periods, the situations where households do not get enough energy rarely occur. We also try to design the constraints in a way such that demand can be satisfied as much as possible without violating network constraints.

Figures 4.3 - 4.6 show the network profiles under normal operation with no DSM. Values are recorded at the end of every 5 minutes update interval. From Figure 4.6 we observe that the average power factors on all phases are close to unity. Therefore, our assumption of high power factor holds and the theoretical performances of the proposed algorithms are expected to be achieved in this network. It is also clear, from Figure 4.3, that the demand varies significantly during the day. The demand difference between peak hours and valley hours is large, but realistically so. In addition, this transformer is operating close to its capacity and may need to be upgraded in the near future if demand increases. Figure 4.4 shows the amount of current flowing on three phases and neutral. Because the network is not well balanced, the current flowing in the neutral is not negligible. The unbalance level is calculated at each pole in the network using (4.3), and Figure 4.5 shows the average unbalance of all poles.

4.5.2 DSM using virtual price signals

In this section, we assume half of the loads are controlled in response to a discrete virtual price signal from the DSO. The time between each price signal update, $\tau$, is set at 5 minutes. A shorter update interval would result in faster convergence. However, it might be operationally infeasible for loads to change demand patterns on a millisecond or second scale in distribution networks. The choice of step size $\delta$ can be important. A small $\delta$ may leads to slow response and large $\delta$ may cause fluctuating demand profiles. In the simulations, following the guidance of Theorem 3, we set the value to be $\alpha/(x_s^2 M)$ which yields fastest response without causing
Figure 4-7: Demand profile for the algorithm using virtual price signal.

Figure 4-8: Current profiles of three phases and neutral. for the algorithm using virtual price signal.

Figure 4-9: Phase unbalance averaged over all poles for the algorithm using virtual price signal.
significant oscillation. As we can see from Figure 4-7 and comparing with Figure 4-3, peak demand is brought down from around 160\(\text{kW}\) to around 100\(\text{kW}\) and the valleys are well filled. In reality, what happens is that flexible loads will greedily consume energy when the virtual price is low (e.g. EVs will charge as much as possible when the virtual price is low) such that the peak demand is automatically reduced. At the same time, line currents as in Figure 4-8 and phase unbalance as in Figure 4-9 are well reduced which will result in fewer losses in the system. Figure 4-13 shows the broadcast virtual price for three users on different phases. Since the network was originally heavily unbalanced, consumers on different phases do receive different virtual prices. To encourage consumers to participate in DSM, the DSO might have to compensate consumers on heavily loaded phases or rebalance the network.

4.5.3 DSM using local measurements

When a communication facility is not available, local voltage measurements are used to make DSM decisions. We set the update interval for each house to be 5 minutes. Houses update asynchronously (which is the natural mode of operation in the distribution grid), thereby avoiding instability issues related to sudden jumps or drops in network demand. The peak demand is also successfully brought down to around 120\(\text{kW}\) and current variation is well reduced as shown in Figure 4-10 and 4-11. Phase unbalance as in Figure 4-11 is not sufficiently reduced as we have taken it out of the constraints set as in (4.13). In reality, to effectively encourage DSM and avoid fairness issues, the DSOs need to better balance the networks. As for the individually approximated virtual prices, we show the virtual price approximation of one house from each phase (same selection as in Section 4.5.2) in Figure 4-14. Compared to the price profile in Figure 4-13, both algorithms capture the evening peak and increase price accordingly which encourages less usage. For off-peak periods, the algorithm uses a broadcast signal that captures more details on demand and it is able to perform minor adjustments on prices.
Figure 4-10: Demand profile for the algorithm using local measurements.

Figure 4-11: Current profiles of three phases and neutral for the algorithm using local measurements.

Figure 4-12: Phase unbalance averaged over all poles for the algorithm using local measurements.
Figure 4-13: The virtual price of three users on different phases broadcast by the DSO.

Figure 4-14: Locally approximated prices of three users on different phases.
Table 4-1: Performance comparisons of the same network under no DSM, Algorithm 1 and Algorithm 2 with 50% flexible loads

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>no DSM</th>
<th>Algorithm 1</th>
<th>Algorithm 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>maximum total active power demand</td>
<td>164kW</td>
<td>113kW</td>
<td>120kW</td>
</tr>
<tr>
<td>maximum line RMS current magnitude</td>
<td>391A</td>
<td>225A</td>
<td>267A</td>
</tr>
<tr>
<td>maximum average pole unbalance</td>
<td>1.83</td>
<td>1.24</td>
<td>1.61</td>
</tr>
<tr>
<td>peak to average ratio</td>
<td>2.06</td>
<td>1.15</td>
<td>1.26</td>
</tr>
</tbody>
</table>

### 4.5.4 Comparison

The differences in key performance parameters between the above two algorithms are summarised in Table 4-1. In terms of peak shaving, both algorithms achieve the design goal. The algorithm that uses explicit virtual price signal (Algorithm 1) has a better regulated total demand throughout the day while the algorithm that only uses local information (Algorithm 2) has a more fluctuating behaviour. As for unbalance, again Algorithm 1 has a more stable behaviour due to global communication. However, the installation and maintenance cost associated with communication channels are also a factor to be considered when making the decision. More importantly, despite the fact that there are some mismatches between the approximated virtual prices from Algorithm 2 and the virtual prices from Algorithm 1, they both indicate the commencement and intensity of peak demand. Given the fact that Algorithm 2 does this using only local measurements, it is arguable that Algorithm 2 is a good alternative for networks requiring less expensive, effective and readily applicable DSM solutions.

To understand the effect of load composition on the performance of the algorithms, we have simulated the cases with different flexible load penetration from 0% to 50%. The results on peak to average ratio are presented in Figure 4-15. A key observation is that the amount of flexible load affects the performance of proposed algorithms significantly. The major performance indicator, the peak to average ratio, drops approximately linearly with the amount of flexible load. In general, Algorithm 1 performs better than Algorithm 2 as the level of flexible load increases.
<table>
<thead>
<tr>
<th>Shiftable Penetration</th>
<th>Algorithm 1</th>
<th>Algorithm 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>0%</td>
<td>2.5</td>
<td>2.5</td>
</tr>
<tr>
<td>10%</td>
<td>2</td>
<td>1.5</td>
</tr>
<tr>
<td>20%</td>
<td>1.75</td>
<td>1.5</td>
</tr>
<tr>
<td>30%</td>
<td>1.5</td>
<td>1.25</td>
</tr>
<tr>
<td>40%</td>
<td>1.25</td>
<td>1.1</td>
</tr>
<tr>
<td>50%</td>
<td>1</td>
<td>0.9</td>
</tr>
</tbody>
</table>

Figure 4-15: Peak to average ratio under different levels of flexible load penetration

### 4.6 Chapter summary

In previous chapters, models and algorithms have been proposed to manage demand in distribution networks. However, we have not taken network constraints into consideration apart from aggregated total demand. In this chapter, we built upon the models and algorithms proposed in earlier chapters and explicitly include network physical constraints in the theoretical formulation. The DSM problem has been modelled as a centralised constrained optimisation which maximises aggregated utilities of all users and then decomposed for distributed implementation. The utility function we designed takes into account marginal fairness and has the ability to prioritize loads. We have proposed two implementations for the optimisation problem. In the first implementation, using a uni-directional (DSO to consumer) communication infrastructure, the DSO uses a time varying virtual price to coordinate users’ behaviours. In the other implementation, users approximate the virtual constraint prices via local voltage measurements. We have shown theoretically and in simulations that both algorithms achieve demand management targets and successfully shave peaks. Generally, the algorithm using price signal performs better but it requires a certain level of additional infrastructure. On the other hand, the algorithm using only local measurements could be a good solution for networks where communication infrastructure is not available.
Chapter 5

DSM and PV curtailment in networks with domestic distributed generation

In the previous chapters, we have studied DSM algorithms that can be applied in networks with no distributed solar PV generation. We move the focus to networks with domestic solar PV systems in this chapter. The contribution is twofold. We first illustrate the impact of domestic solar PV systems have on distribution networks and the subsequent risks. The impacts are illustrated via network data analysis as well as realistic simulations.

Then, we study the compatibility of DSM in networks with PVs. In the previous chapters, we started with a heuristic DSM approach and then established several theoretical DSM models. We followed a data-driven approach such that all the proposed algorithms are verified in realistic network models with real network data. Domestic PVs bring some fundamental changes to our models and algorithms. One of the major changes is that power is now flowing in both upstream and downstream directions. All the previous algorithm and simulation models need to be updated to cater for such changes. Instead of reproducing all the algorithms and associated theories, we aim to give insights on how DSM can also be applied in networks with PVs and how PVs can be managed in a similar way. Therefore we propose two heuristic al-
gorithms for demand management and PV curtailment respectively. The algorithms are implemented in the simulator and the performance is supported by simulation results.

5.1 Residential PV systems in distribution networks

Renewable power such as derived from rooftop solar PV systems is believed to be a major player in any future grid. In Australia, several subsidy schemes have stimulated the uptake of solar PV systems in the domestic market. Australia has indeed significant solar power potential, and its low density housing is well suited to domestic solar PVs. Even so, there are technical issues associated with solar PV in the distribution network that are not fully understood. Solutions presented at present do not make full use of the potential of PV, nor are they integrated into the grid with the same level of rigour that is demanded from the normal grid operations, but rather solar PV are treated as negative demand. This is safe from a grid-operational point of view, but undervalues the PV potential. In this section, we examine some of the major effects rooftop solar PV systems bring to the electricity grid.

5.1.1 Capacity utilisation

Rooftop solar PV systems act as distributed generation sources which offset households’ electricity consumption from the grid. However, this reduction in consumption may not always be favourable for networks whose sizes are determined by maximum possible demand. In particular, notice that the generation peak of PV systems in general does not coincide with the peak in residential demand. As a consequence the network infrastructure, which must cater for peak demand, cannot be reduced in size, but at the same time is used less as total demand is reduced. This observation also goes a long way to explaining the consideration of PV supply as negative demand.

The distribution assets form a significant part of the electricity price and under-
utilised infrastructure eventually leads to higher electricity prices (because less energy must pay for the same cost of the infrastructure, price per unit electricity must go up), especially where network companies are promised a fixed return on their investments.

Figure 5-1 shows the change of National Electricity Market peak demand and average demand from 1999 to 2013. The average demand curve shows a clear decrease in demand in recent years compared to a more fluctuating behaviour of peak demand. The fluctuation could be caused by different weather conditions as the main sources of electricity peak consumption, space heating and cooling, are weather dependent. Figure 5-2 plots the ratio between peak demand and average demand. Though varying, an increasing trend is evident for such a ratio over the period of time, especially in recent years. Distributed domestic solar PV generation is considered to be one of the causes of this increase. Generally, because of the extra energy solar panels generate, the average amount of energy supplied through the network is reduced. However, given the fact that solar generation usually occurs during the day while residential peak demand occurs in the evening, PV systems are not effective in reducing peak demand.

To further verify the change in capacity utilisation of networks, we approach this problem from a different scale and simulate the demand profile of households in a NSW distribution network. For this network of thousands of customers, we have half-hourly demand and supply data for 537 houses which are randomly located. Due to such randomness, we argue that an analysis of these 537 houses is scalable to the entire network. More details about the assumptions on demand profiles will be introduced in later sections and Table 6-1. Here, we assume that the individual demand stays unchanged but that the number of households having rooftop PV changes. Figure 5-3 shows how the peak to average ratio evolves with distributed PV penetration from 0% to 75%. With this network, it can be clearly seen that as the number of PVs increases, the peak to average ratio increases, too. This is intuitively reasonable as PVs generate energy mainly during non-peak periods which lowers the average consumption. However, the contribution to peak demand reduction is limited.
Figure 5-1: Annual peak and average demand of NEM from 1999 to 2013 [13]

Figure 5-2: Peak to average demand ratio of NEM from 1999 to 2013 fitted by a linear and a quadratic function
5.1.2 Power quality issues reflected in simulations

To understand the physical effects of distributed generation on the electricity grid and therefore the network capital expenditure, we have chosen a typical Australian suburban network with 113 houses for a case study. This type of network is the last mile of the electricity grid and the root of possible problems caused by domestic PV systems. The schematic diagram in Figure 2-1 shows the network configuration which is based on an actual network in an Australian suburb. The simulation setup is also explained in Chapter 2. The network is supplied by a single transformer and is configured radially. The network also accurately represents phase connections with 48 houses on phase A, 28 houses on phase B and 37 houses on phase C. More details on the modelling and model validation can be found in [33, 105, 108, 109].

Figure 5-4 shows the aggregated demand profile of the network under normal operating conditions with no PV. The demand data is based on an actual January (the hottest month of the year) usage as measured at the transformer of this network (supplied by the network operator). The chosen model is highly unbalanced and January is the month where maximum demand occurs.

Figure 5-5 shows the power factor on each of the three phases at the transformer. This network has very high power factor as expected.
Figure 5-4: Demand of the neighbourhood network with no distributed PV on a typical January day (peak to average ratio 2.06).

Figure 5-5: Corresponding power factor at transformer of the neighbourhood network with no distributed PV.
Voltage viability

Voltage plays a very important role in power quality. All electrical appliances need an appropriate voltage to operate; over/under voltage could lead to shortened lifespan of appliances and unsatisfactory performance.

In the simulation study, we maintain the consumption profiles of all households but increase the number of PV systems in the network. The locations of the PV systems are randomly selected, and uniformly distributed over the 3 phases. Three cases are examined in this thesis, with PV penetrations of 30%, 50% and 100% of total number of houses on each phase (the penetration levels are high to better illustrate impacts and performance of possible remedies). The PV profiles are based on averaged real generation data in January NSW as introduced in Section 2.1.3 Figure 2-8

Figures 5-6 - 5-8 show gross demand, as well as net demand which is gross demand minus distributed PV generation of the given network. The figures indicate a significant discrepancy between the PV generation peak and electricity consumption peak which further confirms our previous demonstrations about the increased peak-to-average ratio.

In the simulation model, we are able to monitor the voltage level at each household and the levels at individual households are also recorded and presented (we present phase C as an example) for the three scenarios as shown in Figures 5-9 - 5-11. In Australia, voltages at each point of connection must be maintained at 230 V, +10% to -6%, i.e. in the range 216 V to 253 V [9], which are denoted as the dotted (red) lines in the figure. With penetration smaller than 50%, the impact on voltage levels are insignificant. However, with greater penetration, it can be observed that the PV systems have imposed over-voltage risks to the network during the daytime. However, the risks are not as significant as one may anticipate. This is because that injected power of individual PVs are locked to the phase of the associated households’ voltages which would be different from houses to houses. As a result, some of the voltage rises cancel out. The level of such over-voltage increases as the number of PV systems goes up. For readers’ information, though utilities in Australia have
specified that inverters should disconnect from the grid if local voltage exceeds 253 V, in practice, many inverters are not configured this way and inverter connection point voltages of up to 270 V which is the maximum allowed according to AS 4777 have been observed [95].

Power factor

Power factor measures the ratio between active power being consumed and apparent power. The Distribution Code in Australia requires a 0.8 or higher power factor for most installations and residential networks are known to have a high power factor [97]. However, the introduction of PV systems has changed the picture completely. Without reactive power compensation embedded in PV systems, the power supplied by PV systems will reduce the total active power demand from the transformer significantly. However, the transformer still supplies the reactive power which has not been changed much. As a result, the power factor at the transformer will be distorted significantly. This would potentially affect the billing system as well as the efficiency of the entire grid.

Figures 5-12 - 5-14 show the power factor value at the transformer level for individual phases. In the morning, when PV systems start to generate, the transformer starts to supply less and less active power but the amount of reactive power remains the same. As a result, the power factor keeps decreasing. When the supply from PV systems is high enough, active power starts to reversely flow into the transformer and it can be observed that the power factor goes up again. However, since active power and reactive power supplied by the transformer are flowing in opposite directions, the power factor goes from lagging to leading. This situation is very undesirable from the operators’ point of view. This phenomenon will be reversed in the afternoon when the generation from PV systems reduces and the figures clearly illustrate the point of change.
Figure 5-6: Gross and net demand of the neighbourhood network with 30% distributed PV penetration.

Figure 5-7: Gross and net demand of the neighbourhood network with 50% distributed PV penetration.

Figure 5-8: Gross and net demand of the neighbourhood network with 100% distributed PV penetration.
Figure 5-9: Household voltages of all houses on phase C in the neighbourhood network with 30% distributed PV penetration.

Figure 5-10: Household voltages of all houses on phase C in the neighbourhood network with 50% distributed PV penetration.

Figure 5-11: Household voltages of all houses on phase C in the neighbourhood network with 100% distributed PV penetration.
Figure 5-12: Power factor at transformer of the neighbourhood network with 30% distributed PV penetration.

Figure 5-13: Power factor at transformer of the neighbourhood network with 50% distributed PV penetration.

Figure 5-14: Power factor at transformer of the neighbourhood network with 100% distributed PV penetration.
Figure 5-15: Averaged and worst voltage unbalance values from all poles in the neighbourhood network with no distributed PV penetration.

Figure 5-16: Averaged and worst voltage unbalance values from all poles in the neighbourhood network with 50% distributed PV penetration evenly distributed on all phases.

Figure 5-17: Averaged and worst voltage unbalance values from all poles in the neighbourhood network with 50% distributed PV penetration which is distributed on a single phase.
Phase unbalance

Phase voltage unbalance in a network leads to increased current in the neutral, creates losses and affects the functioning of three phase connected motors. Therefore it is important to limit the unbalance level. The Distribution Code requires voltage unbalance to be maintained within 5% for the network voltage under consideration. We assume that the real component in voltage is dominant, the angles between any two phases are kept close to 120 degrees and phase shifts are negligible. Therefore the unbalance is determined by the magnitude of the differences of the voltages in the three phases. Phase voltage unbalance is expressed in terms of the ratio between positive sequence voltage and negative sequence voltage. It could also be approximated using (4.3).

Note that the voltage unbalance level will be different across the network and in the simulation model, we measure unbalance at the poles whose positions are shown in Figure 2-1. To illustrate the potential impact of distributed PV systems on the unbalance level of a network, we first simulate the network where there is no distributed PV at all as a benchmark scenario. The unbalance level averaged over all poles is shown in Figure 5-15. We then simulate two scenarios with the same degree of distributed PV penetration (50%) and electricity consumption patterns but different locations for PV systems. In the first scenario, we randomly allocate the domestic solar PV systems to households on all three phases. In other words, each phase gets a share of the 34 domestic solar PV systems roughly proportional to the total number of households on that phase. The resulting unbalance averaged over all poles as well as the value on the worst pole are shown in Figure 5-16. An interesting observation is that the added domestic solar PV systems have reduced the average voltage unbalance level in the network which is of benefit to the network. It is also worth mentioning that close to sunset, the small amount of output from PVs, which is in the same phase as household voltage, is actually helping power factor. And therefore the worst pole is less unbalanced than average. In the second scenario, all domestic solar panels are located on a single phase. The result is shown in Figure 5-17.
where a significant spike can be seen. The peak unbalance value is more than 500% the value when PV systems are evenly distributed across the phases. To conclude, if domestic solar PV systems are distributed properly, it could be beneficial to a given network in terms of improving voltage unbalance level and reducing losses. However, if the installation of PV systems is skewed toward any particular phase, it could bring a massive negative impact to the network by exacerbating the level of unbalance.

5.2 Distributed algorithms

Based on the voltage-demand relationship model introduced in Chapter 2, we present in this section two sets of illustrative examples of possible distributed algorithms. The first set aims to vary or shift demand which has enough flexibility such that overall demand can be flattened. The other set of algorithms curtails distributed PV generation when necessary to regulate voltages and prevent reverse power flow.

5.2.1 Feasibility of demand management algorithms in the presence of PV

The DSM algorithms proposed in Chapter 3 and 4 can be applied to loads here with some modifications. In the following sections, we use the same simple proportional control logic for both loads and PV systems. Rather than developing optimal algorithms, we are more interested in the validity of our linear voltage-demand model in a network with PV systems. Therefore, some heuristic algorithms, as defined below, are more suitable for the purpose.

With respect to a single appliance, we define the following variables. Note that the algorithm works asynchronously and all the variables are user specific. We abuse the notation by not putting a user specific subscript for every variable. Let $d_{\text{rated}}$ represent the current rating of the appliance. Let $V(t_s)$ be the voltage measured at $t_s \in T_s$ where $T_s = \{0, \tau_s, 2\tau_s, \ldots\}$ is the set of sampling times.

Let $t \in T$ denote the time when a control action is taken where $T = \{0, n\tau_s, 2n\tau_s, \ldots\}$
$(n \in \mathbb{Z})$ is the set of control times. Note that $\mathcal{T} \subset \mathcal{T}_c$. $V(t)$ then denotes the household voltage based on which a control action is taken at time $t$, and it is an average of measured voltages between two control actions: $V(t) = \sum_{t-n\tau_s}^{t} V(t_s)/(n\tau_s)$.

The measurement sampling time can be on a faster time scale of several seconds up to a minute and the control actions could happen on a much slower time scale of several minutes up to 15 minutes as explained in Section 1.1.2.

Let $d(t)$ denote the current consumption for a given appliance at time $t$ and let $p(t)$ denote the probability of $d(t)$, for a shiftable load; let $V_{min}, V_{max}$ be the minimum and maximum threshold voltages of a household which can be obtained from historical measurements. The control interval length $\tau_n = n\tau_s$ between iterations should be carefully chosen for the algorithm to obtain the desired outcome.

Algorithms 4 and 5 below are proposals for managing storable loads and shiftable loads respectively. Algorithm 4 is a rate-based controller which controls the current flowing into an appliance. Current increases or decreases linearly according to the household voltage measurements. Algorithm 5 is a probability-based controller that controls the probability for an appliance to be switched on or off. Once switched on, the appliance will complete its full duty cycle without interruption.

Algorithm 4 Storable loads management
\begin{algorithm}
\textbf{Require:} $V(t), V_{min}, V_{max}, \tau_n$
\textbf{Ensure:} $d(t)$
1: \textbf{while} Required \textbf{do}
2: \hspace{1em} $d(t) \leftarrow d_{\text{rated}} \ast (V(t) - V_{min})/(V_{max} - V_{min})$
3: \hspace{1em} $d(t) \leftarrow \min\{d(t), d_{\text{rated}}\}$
4: \hspace{1em} $d(t) \leftarrow \max\{d(t), 0\}$
5: \hspace{1em} wait for $\tau_n$
6: \textbf{end while}
\end{algorithm}
Algorithm 5 Shiftable loads management

Require: $V(t), V_{\text{min}}, V_{\text{max}}, \tau_n$

Ensure: $d(t)$

1: while Required do
2: \hspace{1em} $p(t) \leftarrow (V(t) - V_{\text{min}})/(V_{\text{max}} - V_{\text{min}})$
3: \hspace{1em} $p(t) \leftarrow \min\{p(t), 1\}$
4: \hspace{1em} $p(t) \leftarrow \max\{p(t), 0\}$
5: \hspace{1em} if $\text{rand}(0, 1) < p(t)$ then
6: \hspace{2em} $d(t) = d_{\text{rated}}$
7: \hspace{2em} run full duty cycle
8: \hspace{2em} end cycle
9: \hspace{1em} else
10: \hspace{2em} $d(t) = 0$
11: \hspace{2em} wait for $(\tau_n)$
12: \hspace{1em} end if
13: end while

5.2.2 Rooftop PV curtailment algorithm

A local controller that makes decisions only from voltage measurements can be installed on rooftop PV units such that the output can be limited to prevent reverse flow of power. An example algorithm for the controller is shown in Algorithm 6 where $s_g(t)$ represent the current supplied by the rooftop PV at time $t$ without control.
Algorithm 6 PV curtailment

Require: $V(t)$, $V_{\text{min}}$, $V_{\text{max}}$, $s_g(t)$, $\tau_n$

Ensure: $s(t)$

1: while Required do
2: \hspace{1em} $s(t) \leftarrow s_g(t) \ast (V_{\text{max}} - V(t))/(V_{\text{max}} - V_{\text{min}})$
3: \hspace{1em} $s(t) \leftarrow \text{min}\{s(t), s_g(t)\}$
4: \hspace{1em} $s(t) \leftarrow \text{max}\{s(t), 0\}$
5: \hspace{1em} wait for $(\tau_n)$
6: end while

5.3 Simulations

To illustrate the performance of our proposed model and algorithms under real operating conditions, we build a Simulink model based on the actual network, shown in Figure 2-1. We then simulated the algorithms using real demand data collected in this network. Based on the estimation in [70], we assume that 70\% of the load in the network is inflexible base load which cannot be managed, 15\% of the load is shiftable and 15\% is storable. In other words, 30\% of the load is flexible. We also assume that all of the households have PV systems installed and the generation profile is shown in Figure 2-8. Again, this is an exaggerated assumption for better presentation of the effectiveness of to-be-proposed algorithms.

Figure 5-18 shows the total demand on Phase C before any management. At noon, when distributed PV generation is at a maximum, supply outruns demand and energy starts to flow in reverse into the distribution transformer. Such a situation is extremely undesirable for operators as it distorts the power factor, voltage and other network physical constraints significantly. The black solid line in Figure 5-20 shows how the average power factor at the transformer is distorted. When power is flowing in reverse, the power factor goes from lagging to leading. In the evening, just as domestic solar generation dies out, demand peak starts to occur and this peak is significantly higher than average demand. During the overnight period, demand
Figure 5-18: Demand profile of the network (Phase C) without management (100% PV penetration).

Figure 5-19: Demand profile of the network (Phase C) with management (100% PV penetration).
Figure 5-20: Average Power factor at transformer before and after control (reverse current flow is shown as negative power factor).

decreases dramatically reaching a minimum at about 4:00 am. Figure 5-19 shows the demand profiles after applying the proposed algorithms. At noon, the most obvious difference is that distributed PV generation is curtailed to strictly prevent reverse flow of energy. In addition, a portion of peak demand is moved to the morning which is depicted by the increased width of the orange flexible demand area. During the peak demand period, which is around 8 pm at night, because the demand is already high, most flexible loads are shifted to other times so that total demand is reduced significantly. A large part of this shifted demand then occurs during the overnight demand valley, where again the orange area is much larger than in the original. The red dotted line in Figure 5-20 shows that the power factor is better maintained compared to the case without any management.

Figures 5-21 and 5-22 show the voltage profiles for all houses on Phase C in this network before and after management. Depending on the actual location of households and the distance from the transformer, the voltage profiles could vary. However, it is generally true that when total demand in the network increases, the voltages at individual houses will drop. Before management, voltage levels are pushed up in the morning and many of them are worryingly low in the evening. The overall pattern fluctuates a lot during one day. This situation is improved dramatically after
the control is in place. The overall pattern is much flatter and all voltages are within limits.

To further verify the voltage-demand relationship model we have derived in Section 2.2 and the performance of our proposed algorithms, we plot the network’s total net demand versus an individual household’s voltage before and after control as in Figure 5-23. The red dots are measurements before any management of loads or PV. Black solid dots are measurements after control. The scatter plots are squeezed to a middle area which can be viewed as a set-point. Two lines which are slightly different from each other can be fitted for the two scatter sets respectively. The norm of residuals for the two fittings are 0.67 and 0.52 respectively which means the scatters after control
Figure 5-23: Relationship between network net demand and local voltage of a house in the network before and after control.

are around 20% more linear than before control.

The differences in the lines and the improvement of linearity suggest two facts: a) the model is not absolutely accurate and it has noise; b) this open loop noise is well compensated through closed loop control. In other word, the controller makes good decisions within the noisy model and reduces modelling errors through feedback.

## 5.4 Chapter summary

In this chapter, we have extended the findings in previous chapters and looked at networks penetrated by rooftop solar panels. We first of all gave an overview of the impact of domestic solar generation in distribution networks, including financial aspects and physical aspects. The physical impacts include, but are not limited to, a lower network utilisation rate or higher peak-to-average ratio, voltage level distortion and possible increased phase unbalance. These quantities have been expressed as time series of physical values. Then, we have proposed a novel distributed approach to regulate demand and distributed supply in the last mile of distribution networks which requires no explicit communication and no upgrading of grid infrastructure. One of the key components of the approach is the relationship modelling of voltage and demand we have developed in Chapter 2. Based on such a model, we have proposed...
heuristic distributed demand and supply management algorithms targeted at flexible
demand and distributed supply respectively. Via simulations on a real Australian
suburban network, using real demand and supply profiles, the algorithms have been
shown to be effective for demand peak shifting, supply peak shaving and the flattening
of entire net demand profiles of the networks of interest. Most importantly, the fact
that such algorithms use only local information for decision making and do not require
any additional communication infrastructure makes them readily implementable.
Chapter 6

Fair network tariff design

The study of impacts of PV systems on distribution networks consist of two complementary components or directions. In Chapter 5, we have examined the first component, the impact of PV systems on the physical properties of a distribution network and the utilisation of grid infrastructure. We have also proposed a control algorithm to curtail the supply for network protection and infrastructure utilisation improvement. In this chapter, we continue the study on PV systems by looking at the other component, which is a focus on individual customers’ welfare. Given known network infrastructure expenditure, we are interested in what each consumer’s fair share of this expenditure is, and how do PV systems affect this share with respect to the cost paid under network tariffs? To answer this question, we propose a utility-theory-based approach to quantify individual customers’ benefits of using network infrastructure. Such benefits minus the billing amount capture the net utility or net infrastructure benefit for a given customer. The net benefits are then used to assess the fairness of various tariff structures, as well as to design optimal tariffs.
6.1 A Utility model for sharing network infrastructure and fairness

Although network tariffs have existed for a long time, there is no clear definition of the benefits that each customer gets from the network. In the public utility, the idea of a tariff is simply to provide a means for society as a whole to pay for the infrastructure and the energy use, with the firm belief that big users are better placed to pay more. Hence a very small flag fall cost, a proportional cost for energy use and a clear subsidy for those who can afford less form the basis of traditional electricity tariff design.

Current network tariff structure commonly consist of daily standing charge, demand charge and TOU charge. United energy, as an example, has specific tariffs for customer groups of different voltage and demand requirements as listed in [94]. Though there are many entries, all the tariffs can be explained using the three-part structure. Low voltage residential users have a demand charge of zero and depending on their metre type, they can be either on a higher standing charge (lower off-peak charge) or a lower standing charge (no off-peak) tariff. Low voltage industrial and commercial users are shifting from a zero demand charge tariff (higher standing charge and TOU) to a zero standing charge tariff (higher demand charge, lower TOU) reflecting distributor’s attention of peak demand problem.

Despite the constantly change tariff, the fairness among different customers is not well understood. In this section, we propose a rational economic pay structure where users pay proportionally to benefit derived, not necessarily taking means into account, or a society benefit of having electricity supply. We study the usage patterns of different groups of customers and propose a novel utility model for sharing network infrastructure. The model clearly quantifies the benefits and costs of households in the network for using electricity. We then use the model to study the fairness and efficiency of different tariff structures.
6.1.1 Network capital cost and operational cost

We first look at the costs from the provider’s point of view. In order to be able to deliver electricity energy to consumers, utility providers need to invest in infrastructure. They need to build, maintain and grow the network as per need basis, always catering for the worst possible load, under faulty conditions (as the network is never without faults essentially). A common approach for network expenditure analysis is to break the cost into capital cost and operational cost. The capital cost includes the sunk cost of initial infrastructure and ongoing infrastructure development. The operational cost includes maintenance costs, performance monitoring, billing. Note that these incurred costs are for the network operators since they are the assets’ owners.

However, from a customer’s point of view, their usage of the infrastructure, which is at the expense of the operators, can be considered as the customer’s benefit. Therefore, we use each customer’s share in the overall network expenditure to quantify that customer’s benefit of using the infrastructure. Nevertheless, the benefit comes at a cost which in reality is reflected as the amount on electricity bills.

6.1.2 Utility model

Let $\mathcal{M}$ be the set of $M$ customers and let the demand of customer $m \in \mathcal{M}$ at time $t \in \mathcal{T}$ be $x_m(t)$ where $\mathcal{T} = \{1, 2, ..., T\}$ is the set of sampling times. In this thesis, we use a half-hourly sampling rate and consider one year, i.e $T = 365 \times 48 = 17,520$. The demand profile of customer $m$ over the entire sampling period $\mathcal{T}$ is denoted as the set $x_m = \{x_m(1), x_m(2), ..., x_m(T)\}$. The annual average demand of $m$ is defined as $x_m^a = \frac{\sum_{t \in \mathcal{T}} x_m(t)}{T}$. The aggregated demand profile of all customers can then be denoted as set $x = \{\sum_{m=1}^{M} x_m(1), \sum_{m=1}^{M} x_m(2), ..., \sum_{m=1}^{M} x_m(T)\}$. Assume that the maximum total demand occurs at time $t_p \in \mathcal{T}$ such that $x(t_p) = \max(x)$. We then define the annual peak demand contribution for customer $m$ as $x_m^p = x_m(t_p)$. The annual network charge is a function of $x_m$ defined as $C_m = f_k(x_m)$ under a given tariff structure $k$, which will be explained in detail in Equation \[6.2\]. The gross benefit for customer $m$ is then defined as follows:
Table 6-1: Demand and supply profiles of sample households over a year. Houses from the same category are assumed to have identical profiles

<table>
<thead>
<tr>
<th></th>
<th>Category A Has PV</th>
<th>Category B No PV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of houses</td>
<td>131</td>
<td>406</td>
</tr>
<tr>
<td>Peak demand contribution $x_{pm}^p$ (kW/house)</td>
<td>1.4</td>
<td>1.4</td>
</tr>
<tr>
<td>Annual mean net demand $x_{am}^a$ (kW/house)</td>
<td>0.44</td>
<td>0.60</td>
</tr>
<tr>
<td>Weighted critical peak $\text{MEAN}(x_{pm}^p) + \text{STD}(x_{pm}^p)$ (kW/house)</td>
<td>1.025</td>
<td>1.027</td>
</tr>
<tr>
<td>Critical peak usage $X_{pm}^p$ (kWh/year/house)</td>
<td>87.6</td>
<td>89.6</td>
</tr>
<tr>
<td>Peak usage $X_{pm}^p$ (kWh/year/house)</td>
<td>707.8</td>
<td>725.0</td>
</tr>
<tr>
<td>Shoulder usage $X_{am}^a$ (kWh/year/house)</td>
<td>1496.4</td>
<td>2850.7</td>
</tr>
<tr>
<td>Off-peak usage $X_{am}^a$ (kWh/year/house)</td>
<td>1513.3</td>
<td>1551.2</td>
</tr>
<tr>
<td>Solar generation (kWh/year/house)</td>
<td>1855.4</td>
<td>0</td>
</tr>
<tr>
<td>Solar consumed (kWh/year/house)</td>
<td>1411.5</td>
<td>0</td>
</tr>
<tr>
<td>Solar exported (kWh/year/house)</td>
<td>443.9</td>
<td>0</td>
</tr>
<tr>
<td>Gross consumption (kWh/year/house)</td>
<td>5216.6</td>
<td>5216.6</td>
</tr>
<tr>
<td>Net consumption (kWh/year/house)</td>
<td>3805.1</td>
<td>5216.6</td>
</tr>
</tbody>
</table>

\[ B_m = CX \times \frac{x_{pm}^p}{\sum_{m=1}^{M} x_{pm}^p} + OX \times \frac{x_{am}^a}{\sum_{m=1}^{M} x_{am}^a} \]

where $CX$ is the capital cost of the network and $OX$ is the operational cost of the network, both averaged on a per annum basis. $M$ is the total number of households on the network. We define the net utility or net infrastructure benefit of customer $m$ as the following normalised equation:

\[ u_m(x_m, t_p) = \frac{B_m - C_m + c}{\frac{1}{M} \sum_{m=1}^{M} C_m} = \frac{CX \times \frac{x_{pm}^p}{\sum_{m=1}^{M} x_{pm}^p} + OX \times \frac{x_{am}^a}{\sum_{m=1}^{M} x_{am}^a} - f(x_m) + c}{\frac{1}{M} \sum_{m=1}^{M} f(x_m)} \quad (6.1) \]

where $c$ is a constant added such that each net utility $u_m \ (m \in \mathcal{M})$ is greater than 0. Note that the net utility is a relative measure and adding a constant will not change the difference in utility benefits between two customers. The constant $c$ can be interpreted as an insurance policy or mechanism provided by regulators and operators.

### 6.1.3 Load and tariff model

For the tariff illustration in this section, we use the New South Wales network data (half hourly sampled demand and supply) comprising 537 randomly selected households. Based on the formulas and regulations provided in [14] and the figures in [54],

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we assume the entire infrastructure cost per annum is $566,448 for the 537 houses. We further assume that the cost can be broken down into 50% capital expenditure and 50% operational expenditure.

To study the general cases, instead of using the profiles individually, we create a hypothetical load profile by averaging relative groups. With this averaged approach, although our analysis is performed on a relatively small sample size, we argue that the results are scalable to most distribution networks. Among the 537 houses, 131 houses have domestic solar PV systems installed and the remaining 406 houses do not. The half-hourly domestic solar supply profiles for all the 131 houses are averaged to create a general supply profile. The half-hourly gross demand profiles (exclude distributed PV) for all 537 houses are averaged to create a general demand profile which is also the profile for houses without PV systems. By subtracting the general demand profile by the general supply profile, we obtain the profile for houses with PV systems.

One important fact about this network is that most of the distributed PV units were installed early which implies that they are smaller in size than some of the later ones. Also, most of the houses may not have air-conditioners so the peak consumption and average consumption are lower than some of the larger users.

The key data are summarised in Table 6-1. In this table: Solar generation refers to the sum of domestic solar energy consumed and exported; Gross demand refers to the sum of domestic solar energy consumed and net demand; Peak demand contribution and annual mean net demand are defined in Section 6.1.2. The rest of the entries will be explained in the following paragraphs.

In line with the most common tariff structures, we assume that there are up to 4 types of billing periods. Let $I_{cp} \subset T$ be the set of ‘critical-peak’ periods from 4pm to 8pm on 12 days declared by network operators several hours or days in prior. For the subsequent simulation and analysis $I_{cp}$ is assumed to be known. Let $I_o \subset T$ be the set of ‘off-peak’ periods from 10pm to 7am the next morning on all days. Let $I_s \subset T$ be the set of ‘shoulder’ periods from 8pm to 10pm on all days, 7am to 4pm on all days, and 4pm to 8pm on non-work days. Let $I_p \subset T$ be the set of ‘peak’ periods from 4pm to 8pm on all working weekdays excluding $I_{cp}$.
Let the set of demand of customer \( m \) during critical peak period be \( x_{m}^{I_{cp}} = \{ x_{m}(t) \mid t \in I_{cp} \} \); the set of demand of customer \( m \) during peak period be \( x_{m}^{I_{p}} = \{ x_{m}(t) \mid t \in I_{p} \} \); the set of demand of customer \( m \) during shoulder period be \( x_{m}^{I_{s}} = \{ x_{m}(t) \mid t \in I_{s} \} \); the set of demand of customer \( m \) for off-peak period be \( x_{m}^{I_{o}} = \{ x_{m}(t) \mid t \in I_{o} \} \). By adding the mean and one standard deviation of the distribution \( x_{m}^{I_{cp}} \), we obtain the so-called weighted critical peak as in Table 6-1.

Assuming half-hourly sampling rate, the total consumption of customer \( m \) over a year during critical peak period is \( X_{m}^{I_{cp}} = \sum_{t \in I_{cp}} x_{m}(t) / 2 \). Note that \( x_{m} \) is measured in KW and \( X_{m}^{I_{cp}} \) in measured in KWh. The equation is to be interpreted as just representing the numerical values, not taking units into account; the total consumption of customer \( m \) over a year during peak period is \( X_{m}^{I_{p}} = \sum_{t \in I_{p}} x_{m}(t) / 2 \); the total consumption of customer \( m \) over a year during shoulder period is \( X_{m}^{I_{s}} = \sum_{t \in I_{s}} x_{m}(t) / 2 \); the total consumption of customer \( m \) over a year during off-peak period is \( X_{m}^{I_{o}} = \sum_{t \in I_{o}} x_{m}(t) / 2 \). The values for total consumptions are shown in Table 6-1.

Though consumers may usually deal with retailers rather than with network operators directly, we assume that the network cost component of their electricity bills is charged according to network tariffs. Table 6-2 shows the details of the different tariff structures. We introduce 6 parameters, \( T_{f}, T_{d}, T_{cp}, T_{p}, T_{s}, T_{o} \), corresponding to fixed daily standing charge, demand charge (weighted critical peak charge), critical peak usage charge, peak usage charge, shoulder usage charge and off-peak usage charge respectively. For household \( m \), under tariff structure \( k \), the annual (assume 365 days) cost for network usage \( f_{k}(x_{m}) \), \( k = 1, 2, 3 \) is defined as follows.

\[
\begin{align*}
 f_{1}(x_{m}) &= 365 \times T_{f} + \left( X_{m}^{I_{p}} + X_{m}^{I_{s}} + X_{m}^{I_{o}} + X_{m}^{I_{cp}} \right) \times T_{p} \\
 f_{2}(x_{m}) &= 365 \times T_{f} + \left( X_{m}^{I_{p}} + X_{m}^{I_{cp}} \right) \times T_{p} + X_{m}^{I_{s}} \times T_{s} + X_{m}^{I_{o}} \times T_{o} \\
 f_{3}(x_{m}) &= 365 \times T_{f} + X_{m}^{I_{p}} \times T_{p} + X_{m}^{I_{s}} \times T_{s} + X_{m}^{I_{o}} \times T_{o} \\
 &\quad + X_{m}^{I_{cp}} \times T_{cp} + (MEAN(x_{m}^{I_{cp}}) + STD(x_{m}^{I_{cp}})) \times T_{d} \\
\end{align*}
\]

(6.2)

Tariff structure 1 has only a daily standing charge and a flat usage charge; Tariff
<table>
<thead>
<tr>
<th>Number</th>
<th>Tariff Structure</th>
<th>Standing charge(c/day)</th>
<th>Demand charge($/kW)</th>
<th>TOU charge(c/kWh)</th>
</tr>
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<tr>
<td>1</td>
<td>Two-Part Tariff</td>
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<td>N/A</td>
<td>12.64</td>
</tr>
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<td>2</td>
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<td></td>
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<td></td>
<td></td>
<td>Shoulder 11.38</td>
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<td>Off-Peak 6.95</td>
</tr>
<tr>
<td>3</td>
<td>Three-Part Tariff</td>
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<td>315</td>
<td>Critical 11.59</td>
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<td>Shoulder 6.51</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Off-Peak 2.07</td>
</tr>
</tbody>
</table>

Table 6-2: Tariff options

2 has a daily standing charge and a TOU charge (such tariffs structure are termed TOU tariffs); Tariff 3 has a daily standing charge, a TOU charge and a peak demand charge (such a tariff structure is usually termed a demand tariff). An example of each tariff structure is shown in Table 6-2 as Tariff 1-3 respectively and these examples are consistent with the case studies in [87].

6.1.4 Tariff analysis and fairness criteria

In this section, we use the utility model developed earlier to evaluate consumers’ net benefit from the network, which leads to an understanding of the fairness of a given tariff. Tariff 1-3 in Figure 6-1 shows the net utility or net infrastructure benefit as per Equation (6.1), of two groups of customers, whose behaviours are summarised in Table 6-1 under the corresponding tariffs described in Table 6-2.

In the figure, the heights of the bars indicate the net benefit for the different groups of customers. In general, the greater the difference in heights between the two bars, the more unfair the tariff is. For this dataset, under a traditional two-part tariff (Tariff 1), people without PV systems get less net benefit from the network than customers with PV systems. This creates a ‘hidden subsidy’ or ‘wealth transfer’ among the customers. Such a finding coincides with the more general arguments in [87]. For
Figure 6-1: Net utility of customers under tariff structures 1-5

Tariff 2, we can see that by adding TOU components to the tariff, distributed PV customers’ advantage over customers without distributed PV is smaller (in fact, in our example, the bar for users without PV is larger indicating that PV customers are actually at a slight disadvantage), resulting in a fairer tariff. Tariff 3, though it improves overall welfare, favours customers with no distributed PV from a fairness point of view.

Now, we introduce several definitions which eventually lead to the concept of fairness.

**Definition 2 Demand elasticity** If the demand of a customer does not respond to changes in TOU prices, we call the demand price inelastic. Otherwise, if the demand of a customer does change with respect to changes in TOU prices, we call the demand price elastic.

**Definition 3 Tariff feasibility** Let $U$ be the set of all $u_m$, $m \in \mathcal{M}$ as defined in (6.1), $U = \{u_1, u_2, ..., u_M\}$. For a given $k \in \{1, 2, 3, ...\}$, we refer to any tariff structure satisfying $\sum_{m=1}^{M} f_k(x_m) = \text{Revenue}$ (with non-negative unit prices) as a feasible tariff and the resulting $U$ as a feasible net utility set.

Note that the ‘Revenue’ refers to the target revenue determined by network operators.
under relevant regulation.

**Definition 4 Max-min fairness** A set of net utilities $U$ is max-min fair if it is feasible and if for each $m \in \mathcal{M}$, $u_m$ cannot be increased without decreasing $u_n$ where $u_n < u_m$ [56].

For max-min fairness, absolute priority is granted for smaller elements in a set and any increase in bigger elements resulting from a decrease in smaller elements is not allowed. However, sometimes a tiny decrease in a smaller element could result in huge improvements in other elements (bigger and/or smaller). Such a situation could be desirable and an alternative fairness criterion namely *proportional fairness* is therefore proposed as follows:

**Definition 5 Proportional fairness** [56] A set of net utilities $U$ is proportionally fair if it is feasible and if for every other feasible set $V$, the aggregated proportional changes are non-positive:

$$\sum_{m \in \mathcal{M}} \frac{v_m - u_m}{u_m} \leq 0.$$  \hspace{1cm} (6.3)

From the perspective of the entire network as a whole where the demand of the users can be very different, we adopt the criterion of proportional fairness from here on. Note that proportional fairness means that we are not trying to equalize everyone’s benefit but to create maximum possible benefit for the group.

### 6.1.5 Optimal tariff design

As we have seen so far, Tariff 1-3 presented in Table 6-2 either favour customers with PV systems or customers without PV systems. In addition, the tariffs do not necessarily maximise the overall welfare of all customers as observed in Figure 6-1. In this section, we aim to develop a framework to maximise the aggregated net utilities of all customers and, more importantly, guarantee proportional fairness.

Consider a demand tariff with the same structure as Tariff 3 where $f_3(x_m)$ is parametrised by $T_f, T_d, T_{cp}, T_p, T_s, T_o$ as in Equation (6.2). The aggregated net
utility maximisation problem of all network customers can be formulated as follows:

$$\max_{T_f, T_d, T_c p, T_p, T_s, T_o} \sum_{m=1}^{M} \log(u_m)$$ \hspace{1cm} (6.4)$$

s.t. $$\sum_{m=1}^{M} f(x_m) = \text{Revenue}$$ \hspace{1cm} (6.5)$$

$$T_f, T_d, T_c p, T_p, T_s, T_o \geq 0$$ \hspace{1cm} (6.6)$$

By expanding the relative terms, (6.4), (6.5) and (6.6) can be written as the following:

$$\max_{T_f, T_d, T_c p, T_p, T_s, T_o} \sum_{m=1}^{M} \log(C X \times \frac{x^p_m}{\sum_{m=1}^{M} x^p_m} + O X \times \frac{x^a_m}{\sum_{m=1}^{M} x^a_m}$$

$$- (365 \times T_f + X^I_m \times T_p + X^I_s \times T_s + X^I_o \times T_o + X^I_{cp} \times T_{cp}$$

$$+ (M E A N(x^I_{cp}) + S T D(x^I_{cp})) \times T_d) + c)$$

$$- \log(\frac{1}{M} \sum_{m=1}^{M} (365 \times T_f + X^I_m \times T_p + X^I_s \times T_s + X^I_o \times T_o + X^I_{cp} \times T_{cp}$$

$$+ (M E A N(x^I_{cp}) + S T D(x^I_{cp})) \times T_d))$$ \hspace{1cm} (6.7)$$

s.t. $$\sum_{m=1}^{M} (365 \times T_f + X^I_m \times T_p + X^I_s \times T_s + X^I_o \times T_o + X^I_{cp} \times T_{cp}$$

$$+ (M E A N(x^I_{cp}) + S T D(x^I_{cp})) \times T_d) = \text{Revenue}$$ \hspace{1cm} (6.8)$$

$$T_f, T_d, T_c p, T_p, T_s, T_o \geq 0$$ \hspace{1cm} (6.9)$$

In other words, we are trying to maximise the aggregated net benefits of all customers in a network while ensuring no one is disadvantaged. Meanwhile, we want to ensure the network operator meet the revenue target which is regulated by regulators. For inelastic customers whose demand does not change with prices, all the demand and usage terms can be treated as constants.

The problem defined by (6.4), (6.5) and (6.6) is therefore a convex optimisation problem with linear constraints.

We propose the following two theorems regarding the existence, uniqueness, optimality and fairness of possible solutions.
Theorem 4 If the optimisation problem defined by (6.7), (6.8) and (6.9) is feasible, then there exists a unique solution that is globally optimal.

The detailed proof would follow the guidelines in [18]. The objective function defined by (6.7) is strictly concave and differentiable. The feasible region defined by (6.8) and (6.9) is compact. Therefore, a unique global optimum for $U$ which is associated with a unique tariff can be found.

Theorem 5 A set of positive net utilities $U$ solves the problem defined by (6.7), (6.8) and (6.9) if and only if it is proportionally fair.

Assume $U$ solves the problem and assume $v_m$ is a perturbed signal for $u_m$ such that $v_m = u_m + \delta u_m$ for all $m \in M$. The objective function (6.4) can only be increased if $\sum_{m=1}^{M} \frac{d \log(u_m)}{du_m} \delta u_m > 0$. Such a condition implies that $\sum_{m=1}^{M} (v_m - u_m)/u_m > 0$ and vice versa. This proves the theorem.

The problem is formulated and solved using the MATLAB Optimisation toolbox and the result is shown as Tariff 4 in Table 6-2. Interestingly, the tariff suggests a heavier charge during critical periods and shoulder periods and a lower daily standing charge. From Figure 6-1, we can see the net benefits of different groups of customers under this tariff. The differences between bars are much smaller than the other tariffs.

6.1.6 Optimal tariff design with elastic demand

The optimal tariff we have designed gives a valuable insight on how the behaviours of customers affect a tariff and how fairness should be taken into consideration. However, we have assumed thus far that consumers’ behaviours are price-inelastic which means that they do not respond to price changes and their demand always remains the same. In reality, demand will show a certain level of elasticity to price changes. Generally, a higher price would trigger a decrease in demand and vice versa. In that case, the cost function would no longer be linear and the optimisation problem will have to be modified. We adopt a simple yet illustrative linear elasticity model where demand decreases proportionally to the increase in price. We assume that every 1% increase in
price corresponds to a $E\%$ drop in usage. Here we take the elasticity constant $E = 0.1$.

Assume that the initial demand sets at time $I_{cp}, I_p, I_s, I_o$ are $x_{m0}^{I_{cp}}, x_{m0}^{I_p}, x_{m0}^{I_s}, x_{m0}^{I_o}$ with corresponding initial total consumptions as $X_{m0}^{I_{cp}}, X_{m0}^{I_p}, X_{m0}^{I_s}, X_{m0}^{I_o}$. We use the values in Table 6-1 as initial gross demand. Assume the initial charges for the corresponding periods are $T_{cp0}, T_{p0}, T_{s0}, T_{o0}$ and we use Tariff 3 in Table 6-2 as the initial tariff. In this case, demand profiles of customers are no longer fixed. During the process of finding optimal prices, consumptions during each interval are calculated on the fly as follows:

\[
X_{m}^{I_p} = X_{m0}^{I_p} \times (1 - E \times (\frac{T_p}{T_{p0}} - 1)) \times T_p
\]

\[
X_{m}^{I_s} = X_{m0}^{I_s} \times (1 - E \times (\frac{T_s}{T_{s0}} - 1)) \times T_s
\]

\[
X_{m}^{I_o} = X_{m0}^{I_o} \times (1 - E \times (\frac{T_o}{T_{o0}} - 1)) \times T_o
\]

\[
X_{m}^{I_{cp}} = X_{m0}^{I_{cp}} \times (1 - E \times (\frac{T_{cp}}{T_{cp0}} - 1)) \times T_{cp}
\]

and the new optimisation problem is formulated accordingly as follows:

\[
\max_{T_f, T_d, T_{cp}, T_p, T_s, T_o} \sum_{m=1}^{M} \log(CX \times f_p(T_{cp}, T_p, T_s, T_o) + OX \times \frac{X_{m}^{I_p} + X_{m}^{I_s} + X_{m}^{I_o} + X_{m}^{I_{cp}}}{\sum_{m=1}^{M}(X_{m}^{I_p} + X_{m}^{I_s} + X_{m}^{I_o} + X_{m}^{I_{cp}})}) - (365 \times T_f + X_{m}^{I_p} \times T_p + X_{m}^{I_s} \times T_s + X_{m}^{I_o} \times T_o + X_{m}^{I_{cp}} \times T_{cp}) + (\text{MEAN}(x_{m}^{I_{cp}}) + \text{STD}(x_{m}^{I_{cp}}) \times T_d) + c - \log(\frac{1}{M} \times \text{Revenue}) \tag{6.10}
\]

\[
\text{s.t. } \sum_{m=1}^{M} (365 \times T_f + X_{m}^{I_p} \times T_p + X_{m}^{I_s} \times T_s + X_{m}^{I_o} \times T_o + X_{m}^{I_{cp}} \times T_{cp}) + (\text{MEAN}(x_{m}^{I_{cp}}) + \text{STD}(x_{m}^{I_{cp}}) \times T_d)) = \text{Revenue} \tag{6.11}
\]

\[
T_f, T_d, T_{cp}, T_p, T_s, T_o \geq 0 \tag{6.12}
\]

Note that to simplify the presentation, we have written the share in peak demand $x_{m}^{p} / \sum_{m=1}^{M} x_{m}^{p}$ as $f_p(T_{cp}, T_p, T_s, T_o)$, which is a non-linear equation, following the definitions in Section 6.1.2. The new problem is a non-linear optimisation with non-linear
Figure 6-2: Total aggregated logarithmic network utility under different tariff structures

constraints. We do not draw conclusions on the existence or uniqueness of optimal solutions theoretically. However, through observing the results generated using MATLAB Optimisation Toolbox, we found that under general conditions, a solution can be found. The resulting tariff is presented as Tariff 5 in Table 6-2, and the result on net utility is presented in Figure 6-1 as Tariff 5 (Elastic). The new demand patterns for consumers are summarized in Table 6-3. Note that in Figure 6-1, Tariff 3-5 recover the same amount of revenue which is slightly higher than what is the case for Tariff 1 and Tariff 2. Figure 6-2 shows the total aggregated network utilities under different tariffs. After comparing the results from Tariff 5 (Elastic) with that from Tariff 4, we found that both tariffs have good performance in terms of the net utility maximisation of each group of customers and fairness assurance compared to Tariff 1-3. However, judging from the differences between bars and total net utility, Tariff 5 does perform slightly worse than Tariff 4 which can be seen as a price for elasticity. Note that under Tariff 5 (Elastic), the total demand of both groups of customers has been reduced by approximately 7%.

Through the above analyses on existing tariffs and optimal tariffs, we found that by appropriately designing the daily standing charge component, the TOU component and the demand charge component, a much fairer tariff can be created while still
### Table 6-3: Demand and supply profiles of sample households over a year considering demand elasticity. Houses from the same category share the same profile

- **Number of houses**
  - Category A Has PV: 131
  - Category A No PV: 406
  - Category B Has PV: 131
  - Category B No PV: 406

- **Peak demand contribution $x_{pm}^p$ (kW/house)**
  - Category A Has PV: 1.34
  - Category A No PV: 1.34
  - Category B Has PV: 1.34
  - Category B No PV: 1.34

- **Annual mean net demand $x_{nm}^a$ (kW/house)**
  - Category A Has PV: 0.40
  - Category A No PV: 0.54
  - Category B Has PV: 0.40
  - Category B No PV: 0.54

- **Weighted critical peak $ \text{MEAN}(x_{xm}^I) + \text{STD}(x_{xm}^I)$ (kW/house)**
  - Category A Has PV: 1.02
  - Category A No PV: 1.02
  - Category B Has PV: 1.02
  - Category B No PV: 1.02

- **Critical peak usage $X_{im}^p$ (kWh/year/house)**
  - Category A Has PV: 87.0
  - Category A No PV: 89.0
  - Category B Has PV: 87.0
  - Category B No PV: 89.0

- **Peak usage $X_{im}^p$ (kWh/year/house)**
  - Category A Has PV: 705.7
  - Category A No PV: 722.9
  - Category B Has PV: 705.7
  - Category B No PV: 722.9

- **Shoulder usage $X_{im}^s$ (kWh/year/house)**
  - Category A Has PV: 1253.6
  - Category A No PV: 2450.0
  - Category B Has PV: 1253.6
  - Category B No PV: 2450.0

- **Off-peak usage $X_{im}^o$ (kWh/year/house)**
  - Category A Has PV: 1499.7
  - Category A No PV: 1537.7
  - Category B Has PV: 1499.7
  - Category B No PV: 1537.7

- **Solar generation (kWh/year/house)**
  - Category A Has PV: 1855.4
  - Category A No PV: 0
  - Category B Has PV: 1855.4
  - Category B No PV: 0

- **Solar consumed (kWh/year/house)**
  - Category A Has PV: 1298.5
  - Category A No PV: 0
  - Category B Has PV: 1298.5
  - Category B No PV: 0

- **Solar exported (kWh/year/house)**
  - Category A Has PV: 556.89
  - Category A No PV: 0
  - Category B Has PV: 556.89
  - Category B No PV: 0

- **Gross demand (kWh/year/house)**
  - Category A Has PV: 4844.5
  - Category A No PV: 4844.5
  - Category B Has PV: 4844.5
  - Category B No PV: 4844.5

- **Net demand (kWh/year/house)**
  - Category A Has PV: 3546.0
  - Category A No PV: 4844.5
  - Category B Has PV: 3546.0
  - Category B No PV: 4844.5

### 6.2 Unaveraged load profiles

In the previous section, we used an average profile to represent two categories of customers and showed their net infrastructure benefits under different tariff structures. Customers from the same category shared the same demand and supply profile. We argued that findings under such an approach are scalable to larger networks. In this section, we use the original profiles of the 537 houses for analysis. The demand and supply profiles will be different for each customer and the findings may be subject

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<table>
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<tr>
<th></th>
<th>Category A Has PV</th>
<th>Category B No PV</th>
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<tr>
<td>Number of houses</td>
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<td>406</td>
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<td>Average annual mean net demand $x^a_m$ (kW/house)</td>
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<td>0.68</td>
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<td>Average weighted critical peak $MEAN(x^I_m) + STD(x^S_m)$ (kW/house)</td>
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<td>Average critical peak usage $X^c_m$ (kWh/year/house)</td>
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Table 6-4: Average demand and supply profiles of sample households over a year to the specific dataset. However, some hidden properties of such a network may be revealed.

The average of demand and supply profiles is shown in Table 6-4. These averaged values are presented to give readers an overview of the consumption/generation patterns and they will not be used in later analysis. There are two key differences between Table 6-4 and Table 6-1. In Table 6-1, customers from the same category have the same profiles while they are all different in Table 6-4. In Table 6-1, gross consumption is averaged over all customers while in Table 6-4, a separate average is calculated for each category. In other words, we assume all customers have a similar demand pattern regardless of whether they have PV systems or not in Table 6-1 but we examine the individual differences in Table 6-4.

After a comparison between Table 6-4 and Table 6-1, we found that the rooftop PV customers in this network, even without the inputs from PV systems, use much less electricity in all periods than the customers who do not have PV systems. There are two possible explanations: one is that customers with PV systems are in general more conscious about electricity usage and the environment for which reason they have more energy efficient appliances or gas powered appliances; the other is that there are, in terms of percentage, more large consumers among the group without PV systems which increases the average. Such a feature of this dataset may affect the
numerical results but the methods and methodologies are applicable to any residential networks. The averaged net benefits of customers are shown in Figure 6-3 for comparison. Because of the huge differences in usage, all existing tariffs (Tariff 1-3) appear to favour customers without PV systems (as shown in Figure 6-3) and this is caused, to a large extent, by the daily standing charge. However, this difference should really be interpreted as: customers with heavier loads and higher consumptions are benefiting relatively more from the infrastructure compared to smaller customers in general. The unaveraged individual net benefits are plotted as histograms in Figures 6-4 to 6-8. The net benefits are also represented using probability density functions (PDF) and cumulative distribution functions (CDF) in Figures 6-9 and 6-10 by fitting the histograms.

Note that for other networks where rooftop PV customers use a lot of energy, the results would be different following the same methodology which is compatible with any residential network.

6.2.1 Tariff design and analysis

For individual demand profiles, we perform the same optimisation as in the previous section for the case with inelastic demand and the case with elastic demand. The
Figure 6-4: Histogram of individual customers’ net utilities under Tariff 1

Figure 6-5: Histogram of individual customers’ net utilities under Tariff 2

Figure 6-6: Histogram of individual customers’ net utilities under Tariff 3
Figure 6-7: Histogram of individual customers’ net utilities under Tariff 6

Figure 6-8: Histogram of individual customers’ net utilities under Tariff 7
results are shown as Tariff 6 and Tariff 7 respectively in Table 6-6 and the comparison of aggregated net logarithmic utilities for all tariffs is shown in Figure 6-11.

For the averaged profile, the optimiser tries to make two generic groups even. However, for unaveraged profiles, 537 individuals need to be levelled. Due to the diversity in demand (e.g. some customers may barely use any electricity at all), the optimal tariff (Tariff 6) suggests no charges for daily standing part and maximum demand. Instead, TOU charges are significantly increased to make up for the revenue. This is intuitively comprehensible since high daily standing charges would disadvantage a customer who barely uses any electricity at all. Nevertheless, this approach to bring fairness to every single individual may cause tariff robustness issues as well as scalability problems which is undesirable. As a result, we argue that using the average profiles may be more suitable for tariff design.

To complete the analysis, for the unaveraged profile, with elasticity, the new demand profile (averaged) under Tariff 6 is shown in Table 6-5. Due to elasticity, the overall consumptions for both groups of customers have been reduced by up to 45%. This is a drastic reduction as a consequence of the linear elastic model, which has really no limit to how much more or less energy will be consumed. The model aims to give insights on how elasticity changes tariff design. Future work would involve using a more sophisticated elasticity model and putting boundaries on how much demand changes with respect to price changes.

However, the tariff manages to maintain a high overall aggregated net utility. Obviously, this tariff finds its optimum by charging much more than Tariff 6 in all parts except for off-peak periods and forces customers to use less electricity.

The PDF and CDF for Tariffs 6 and 7 are shown in Figure 6-9 and Figure 6-10 by fitting histograms. In Figure 6-9, Tariff 6 has the narrowest envelope suggesting that the benefits of all houses are forced to an average position. Tariff 7, given that total consumption is significantly less than what it is under Tariff 6, produces the second best result. In Figure 6-10, it is more obvious that Tariff 6 and 7 have the steepest increase suggesting that most houses are having a similar amount of net benefits which can be interpreted as being more fair. The total aggregated logarithmic network
Table 6-5: Average demand and supply profiles of sample households over a year

<table>
<thead>
<tr>
<th>Number</th>
<th>Tariff Structure</th>
<th>Standing charge(c/day)</th>
<th>Demand charge($/kW)</th>
<th>TOU charge(c/kWh)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>Optimal Tariff</td>
<td>0.00</td>
<td>Demand 0.00</td>
<td>Critical 136.14</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Peak 22.56</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Shoulder 13.44</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Off-Peak 13.57</td>
</tr>
<tr>
<td>7</td>
<td>Elastic Tariff</td>
<td>84.98</td>
<td>Demand 102.34</td>
<td>Critical 66.05</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Peak 35.33</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Shoulder 15.32</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Off-Peak 9.16</td>
</tr>
</tbody>
</table>

Table 6-6: Tariff options

utility under different tariff structures, for unaveraged demand profiles, is shown in Figure 6-11.

6.3 Chapter summary

In this chapter, we looked at the network tariff fairness problem in distribution networks where there are PV customers. We defined the benefits as well as costs for each customer of using the network infrastructure, based on which we formulated a utility theoretical model for electricity usage where the net benefits of using the network infrastructure can be quantified. Using such a model, we analysed three existing different tariffs with distinct structures. The tariffs have been shown to favour either customers with PV systems or customers without PV systems. We then formulated an optimisation problem aiming to find a tariff that maximises the aggregated net
Figure 6-9: PDF (shown as sample numbers) obtained using kernel density estimation on histograms

Figure 6-10: Empirical CDF obtained using MATLAB
utility of all customers while ensuring fairness. Using a generic demand profile, unique solutions can be found to the problem formulated and proportional fairness can be established. We also extended the case to elastic loads for which the problem is more difficult to solve. However, local optimal solutions can be found using standard optimisation tools. The analysis is repeated using realistic demand profiles with more diverse demand/supply patterns and similar observations are made.

The overall analysis and the optimal tariffs suggest that PV systems do not simply bring ‘unfairness’ to a network and rooftop PV customers do not necessarily take advantage of the network under given tariffs. It is the overall demand patterns of customers that determine whether they are paying for their fair share of the network. However, it is true that PV systems contribute much more in shoulder and off-peak periods rather than during peaks, and when applied to average load, this has the effect of reducing non-PV customers’ welfare. The current two-part tariff comprising fixed ($/day) and variable charges (c/kWh) has historically served the industry reasonably well given metering and computing constraints. However, to remedy the substantial deviations brought about by PV systems, tariff reform has to be carefully planned and implemented. With the rising uptake of smart meters and the availability of half hourly data, the methodology introduced in this thesis could be of use in this
smart grid era. Due to inherent features of the sample dataset, such as small rooftop
PV units and low consumption from distributed PV customers, the numerical results
may not be universally applicable. However, the methods and methodologies are
compatible with most residential networks. As for future work, one direction is to
perform analysis on networks with different geographic locations and demographies
such as the mix of users and the penetration of PVs and EVs. Another important
question is to plan for demand changes because of the variations in household number
and technology changes.
Chapter 7

Conclusion and future work

In this final chapter, we will review the earlier chapters, outline the contributions made and point to useful future work.

7.1 Summary of contributions

In this thesis, we looked at the problems and potential challenges associated with the electricity distribution network from two complementary directions: power delivery and billing.

In Chapter 1, we have described in detail the two major concerns our distribution networks are facing. Firstly, the low and declining asset utilisation. Secondly, the disruption from emerging technologies. Later in the chapter, we have reviewed the state-of-art approaches of DSM and tariff design which aim to address the concerns about distribution networks. Built on the existing work, we then propose our own approach which focuses on the root of the cause, the last mile in the network.

Since distribution networks, in particular the last mile, are usually not the primary focus of electrical engineers, there is no satisfactory off-the-shelf simulation model available. We therefore built our own simulation network in MATLAB and POSSIM capturing all practical features of a real network in Australian conditions in Chapter 2. These features, which are not modelled in most other work, include distribution line impedances, network topologies, three phase unbalances, reactive loads and household
voltage levels. Despite various assumptions we make in our theoretical formulations as in Section 2.2.1, the simulation model is always as accurate and as detailed as our data allows it to be. The simulation models serve to test and illustrate the validity of the theoretical work against available network data/behaviour. In the second half of Chapter 2 we have proposed a voltage-demand approximation model tailored for another important feature of distribution networks where no remote sensing or controlling devices are available. The approximation model is local to each household and it is able to estimate network load from domestic voltage measurements. This model is used throughout the thesis for controller design because of its simplicity, and the insight it provides.

In Chapter 3 we addressed EV charging management problems. EVs could cause increased peaks and associated power quality problems even with a small uptake. Using real travel profiles and based on the voltage-demand model, we have proposed distribution control algorithms that efficiently shift EV loads and shave peaks even when we consider a very high EV penetration. The algorithms use only local information and expected performance under realistic network conditions are underscored through simulations.

In Chapter 4 we incorporated network constraints in the problem formulation stage and not only in the simulations. The DSM problem is proposed as a constrained optimisation problem which aims to maximise aggregated users’ welfare fairly without violating network limits. The centralised optimisation problem is then decomposed to distributed sub-problems which can be solved locally via price signals sent by the DSO. Building on the essence of previous chapters, we further approximate the algorithm and eliminate all communication requirements by replacing the DSO signals with signals derived from local measurements. Through simulations we illustrate that the sub-optimal decentralised communication free algorithms perform remarkably well, and approximate optimal behaviour under realistic network conditions. We conclude that the cost of real time communication and central coordination cannot be justified in this context.

In Chapter 5 we have extended the models and algorithms developed earlier to
networks with decentralised domestic PV systems. The same methodology is applied to not only demand shifting, but also rooftop PV supply curtailment. With simple control instructions, we have shown that the local algorithms successfully prevent current reverse flowing during the day and flatten evening demand peaks. Network properties such as voltage levels and power factors are maintained inside electricity regulations. As always, the algorithms require merely local measurements without any communication with neighbours or central controllers.

Chapter 3, 4 and 5 consider mitigating the impact of emerging demand side technologies on the grid, in particular the last mile, from a technical point of view. In Chapter 6 we approach the problem from a different yet complementary economic perspective: network tariff and associated fairness issues. We have analysed usage and billing data for a real distribution network in Australia to address the question as to whether distributed PV customers receive hidden subsidies from their non-PV neighbours under current network tariff structures. Our analysis showed that customers with high peaks yet low average usage do take advantage (unintentionally so) of the existing network tariff structure. Presently available data indicates that domestic PV systems could lead to demand profiles with average usage reduced but peaks unchanged. However, their impact on tariff fairness is to be assessed taking all other appliances and consumers usage pattern into account.

7.2 Future work

Thus far, we have analysed power systems under normal static operation. Network dynamics and control under fault conditions are not considered in this thesis. Fault conditions occur all the time and this is a important component of modern power system design and operation. DSM design and implementation under fault conditions are of significance and this is left for future investigation.

We have proposed several models and algorithms for DSM and rooftop PV curtailment in this thesis. Some of which, in theory, are based on assumptions like balanced networks and active power loads as in Section 2.2.1. However, all the models and
assumptions were tested on simulation networks which are almost as detailed and practical as a real network. And the results from simulations, to a large extent, reflect how the models and algorithms would behave in reality. The next step would be hardware implementation of the algorithms in a mini network. If the tests are successful, the hardware could be installed in real networks and their behaviours could be monitored.

The pricing structures we propose are based on the simple assumption that the users and their appliances are fixed which may not be realistic. Ideally, we would like to investigate tariff optimality and fairness under demand change scenarios, that include growing demand (EV) and decreasing demand (PV) and a different fault tolerance system (allowing reverse currents).
Bibliography


