Robust Control of DC Microgrids

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Abstract

RAPID Rapid development in photovoltaic (PV) cells manufacturing technology and emerging environmental concerns have significantly increased photovoltaic penetration over the last decade. However, the inherent intermittency of solar radiation necessitates integration of other power generation sources to provide an uninterruptible, and reliable power supply that satisfies network requirements. Implementing energy storages can diminish the risk of power interruption due to PV system intermittency while ensuring demand satisfaction. To utilise PV and energy storage systems, a group of controllers should be implemented to control the PV systems to extract maximum power from each PV system, while the energy storage systems are employing as a secondary source of energy for a consistent operating of the whole network.

In this thesis, initially, we will present a robust control design based on linear matrix inequality (LMI) for a small scale islanded (off-grid) DC hybrid system. This hybrid system is consisting of a photovoltaic (PV) system and an energy storage system (ESS). A robust $H_{\infty}$ controller is designed to regulate PV system input voltage such that the PV array is operating at maximum power point. Also, energy storage system is controlling to act as a complementary source of energy to maintain DC bus voltage at a constant value; This has achieved by designing a robust controller based on LMI with maximising the region of stability and minimising the RMS gain for the energy storage system. The design method considers both converters non-linearities, modelled as a convex polytope, and achieves $H_{\infty}$ performance. Secondly, a robust coupled controller is designed by taking into account the interaction between the PV and energy storage system components. Consequently, the proposed coupled controller causes a significant improvement in the system transient performance and efficiency.
Furthermore, to expand the proposed approach for medium to large scale systems a new decentralised, autonomous power management structure for DC microgrids comprising of multi PV and energy storage units is proposed. Each unit is controlled by a robustly designed controller using its local measurements only. Thus, the requirements for communication links are omitted. Each control unit aims to deal with system parametric uncertainties and to guarantee a required disturbance rejection performance over both internal and external disturbances. Seven modes of operations are defined to cover all possible practical operating condition that can occur in a DC micro grid. The transition between the modes occurs autonomously and seamlessly for each control unit, and thus unit-to-unit communication is not required.

Finally, to enable grid connection capability for the considered DC micro grid a new current control approach for three-phase grid-connected voltage source inverter (VSI) is proposed. An observer-based state-feedback dynamic controller is implemented to control the VSI. Moreover, the controller and observer are robustly designed, to account for grid disturbances, and achieve a guaranteed $H_{\infty}$ performance level. The existence of parametric uncertainties caused by inexact knowledge of grid impedance and inductance are modelled as a convex polytope. Additionally, by imposing a set of constraints that prevent the VSI from operating in the nonlinear region, saturation of control input is avoided, through overmodulation.
Declaration

This is to certify that

1. the thesis comprises only my original work towards the PhD,

2. due acknowledgement has been made in the text to all other material used,

3. the thesis is less than 100,000 words in length, exclusive of tables, maps, bibliographies and appendices.

Majid Fard Nour Mohammad Nia, August 2017
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Preface

It is acknowledged that this research was funded by National ICT Australia, NICTA (Data61), and The University of Melbourne through Melbourne International Fee Remission Scholarship (MIFRS). This is acknowledged that this thesis comprises only original work of the author and where appropriate, references have been made to the original source when established techniques are used. The assistance of any third-party editorial has not been used in the preparation of this thesis. Unless otherwise stated below, the material presented in this thesis, submitted for the degree of Doctor of Philosophy, is a result of collaboration between my supervisors and myself during my postgraduate study undertaken at the University of Melbourne, Australia. The following is a list of publication done during the course of research.


• M Fard, M Aldeen, “Linear Quadratic Regulator Design for a Hybrid Photovoltaic-Battery System”, in Proceedings of the Australian Control Conference, New Castle, Australia, 2016, [3].


• M Fard, M Aldeen, “Robust Autonomous Decentralised Power Management for Hybrid PV-Battery DC Microgrids”, *IEEE Transactions on Smart Grid*, 2017, Under Revision.

The first author of all above-listed papers has been involved in writing, formulation, and simulation of each paper. The second author of all above papers has been involved in supervision and proof reading of the above-listed articles.
I would like to express my sincere appreciation and gratitude to my supervisor Associate Professor Dr Mohammad Aldeen who has provided me with guidance and insight during each step of my PhD. I would like to thank you for encouraging my research and for allowing me to grow as a research scientist. I have provided with tremendous support from you that was always motivating. Your advice on both research as well as on my career have been priceless. I wish to thank my committee chair, Professor Robin Evans, for his positive and encouraging feedbacks. I would like to take this opportunity to thank DATA61 (NICTA, Victoria laboratory) and The University of Melbourne for their financial supports. A sincere thank goes to Dr Adel Ahmadi, who like a good friend, is always willing to help and to give his best suggestions.

I would like to express my exceptional and deepest appreciation and gratefulness to my family for their immense courage, supports, and sacrifices. Words cannot express how grateful I am to my father, mother, and sister for all of the sacrifices that you have made on my behalf. Your prayer for me was what sustained me thus far, and without your encouragement and support, it was unimaginable to accomplish my goal.
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Dedicated to my beloved Father and Mother, who always believe in me.
# Contents

1 Introduction .................................................. 1
  1.1 Motivation ............................................... 1
  1.2 Literature Review ....................................... 6
    1.2.1 Review on DC Hybrid Systems Control .............. 6
    1.2.2 Review on DC Microgrids Power Management Scheme ... 9
    1.2.3 Review on Grid-connected VSI Control ............... 10
  1.3 Summary .................................................. 12
  1.4 Thesis Outline .......................................... 12
  1.5 Acronyms ................................................. 16

2 Decentralised Control for a DC Hybrid System ................. 17
  2.1 Introduction ............................................ 17
  2.2 Preliminaries ........................................... 18
    2.2.1 Photovoltaic Array Model ......................... 19
    2.2.2 Unidirectional Boost Converter Model ............ 22
    2.2.3 Maximum Power Point Tracker ...................... 26
    2.2.4 Energy Storage Model ............................. 28
    2.2.5 Bi-directional Buck-Boost Converter Model ....... 28
  2.3 DC Hybrid System Control Objectives .................... 30
    2.3.1 PV System Control Objective ....................... 31
    2.3.2 Energy Storage System Control Objective ........ 32
  2.4 Control Problem Formulation ............................ 33
    2.4.1 State-feedback Design .............................. 35
    2.4.2 Pole Placement .................................... 37
    2.4.3 $H_\infty$ Formulation ............................. 40
    2.4.4 Control Input Saturation ........................... 43
    2.4.5 Polytopic Parametric Uncertainties ................ 44
  2.5 Control Design .......................................... 47
    2.5.1 PV Controller ...................................... 47
    2.5.2 Energy Storage System Controller ................. 48
  2.6 Simulation and Results .................................. 49
    2.6.1 Designed Parameters ............................... 50
    2.6.2 Results ........................................... 54
  2.7 Summary .................................................. 64
6 Conclusion

6.1 Conclusion .................................................. 133
6.2 Future Research ............................................. 136
# List of Figures

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1</td>
<td>Schematic diagram of an islanded DC hybrid system.</td>
<td>19</td>
</tr>
<tr>
<td>2.2</td>
<td>Single-diode PV cell circuit.</td>
<td>20</td>
</tr>
<tr>
<td>2.3</td>
<td>Circuit schematic of unidirectional DC-DC boost converter for PV system.</td>
<td>23</td>
</tr>
<tr>
<td>2.4</td>
<td>Switching pulse for a boost converter with a single switch $S_1$.</td>
<td>24</td>
</tr>
<tr>
<td>2.5</td>
<td>Circuit schematic of bidirectional synchronous DC-DC buck-boost converter for ESS.</td>
<td>29</td>
</tr>
<tr>
<td>2.6</td>
<td>Switching pulse for a synchronous buck-boost converter with two switches $S_2$ and $S_3$.</td>
<td>29</td>
</tr>
<tr>
<td>2.7</td>
<td>State-feedback control diagram.</td>
<td>34</td>
</tr>
<tr>
<td>2.8</td>
<td>Diagram of PV system boost converter and its controller.</td>
<td>34</td>
</tr>
<tr>
<td>2.9</td>
<td>Diagram of ESS buck-boost converter and its controller.</td>
<td>35</td>
</tr>
<tr>
<td>2.10</td>
<td>PV system closed-loop poles’ region $S_p(\alpha_p, r_p, \theta_p)$.</td>
<td>38</td>
</tr>
<tr>
<td>2.11</td>
<td>Projection of function $f(D')$, green lines: original polytope, black lines: new tetrahedron.</td>
<td>45</td>
</tr>
<tr>
<td>2.12</td>
<td>Plot of polytopic area of uncertainties.</td>
<td>46</td>
</tr>
<tr>
<td>2.13</td>
<td>PV system output voltage for case I.</td>
<td>55</td>
</tr>
<tr>
<td>2.14</td>
<td>PV system output current for case I.</td>
<td>55</td>
</tr>
<tr>
<td>2.15</td>
<td>PV system output power injecting to DC bus for case I.</td>
<td>56</td>
</tr>
<tr>
<td>2.16</td>
<td>PV system output voltage in the presence of disturbance for case II.</td>
<td>56</td>
</tr>
<tr>
<td>2.17</td>
<td>PV system output current in the presence of disturbance for case II.</td>
<td>56</td>
</tr>
<tr>
<td>2.18</td>
<td>PV system output power in the presence of disturbance for case II.</td>
<td>57</td>
</tr>
<tr>
<td>2.19</td>
<td>PV system output power of direct and indirect control approaches.</td>
<td>57</td>
</tr>
<tr>
<td>2.20</td>
<td>PV system output current of direct and indirect control approaches.</td>
<td>58</td>
</tr>
<tr>
<td>2.21</td>
<td>Solar radiation on the PV system for case IV.</td>
<td>59</td>
</tr>
<tr>
<td>2.22</td>
<td>PV system generated output power for case IV.</td>
<td>59</td>
</tr>
<tr>
<td>2.23</td>
<td>Unregulated DC bus voltage for case IV.</td>
<td>60</td>
</tr>
<tr>
<td>2.24</td>
<td>Regulated DC bus voltage for case IV.</td>
<td>60</td>
</tr>
<tr>
<td>2.25</td>
<td>Energy storage state of charge for case IV.</td>
<td>61</td>
</tr>
<tr>
<td>2.26</td>
<td>Generated power by PV system for case V.</td>
<td>61</td>
</tr>
<tr>
<td>2.27</td>
<td>Loading condition at DC bus for case V.</td>
<td>62</td>
</tr>
<tr>
<td>2.28</td>
<td>Regulated DC bus voltage for case V.</td>
<td>62</td>
</tr>
<tr>
<td>2.29</td>
<td>Energy storage state of charge for case V.</td>
<td>62</td>
</tr>
<tr>
<td>2.30</td>
<td>DC bus regulated voltage at the presence of disturbance for case VI.</td>
<td>64</td>
</tr>
<tr>
<td>2.31</td>
<td>Normalised integral time absolute error (ITAE) performance index.</td>
<td>65</td>
</tr>
<tr>
<td>3.1</td>
<td>Schematic diagram of an islanded DC hybrid system.</td>
<td>69</td>
</tr>
</tbody>
</table>
List of Tables

1.1 Acronyms and their explanation ........................................... 16
2.1 Parameters of PV system and energy storage system. ............... 51
2.2 D-stability criteria parameters for PV system and energy storage system. 51
3.1 Switching operating modes of hybrid system. ....................... 69
3.2 DC load variation during simulation for Case II. ................... 83
4.1 Electrical parameters of tested DC MG. ............................... 103
5.1 System parameters. .............................................................. 126
5.2 Designed parameters and gains. .......................................... 126
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Chapter 1
Introduction

1.1 Motivation

By expeditious evolution in power electronics and semiconductor technology, a case can be made for future migration from conventional AC systems to DC systems. That is so because DC systems (e.g. DC hybrid systems, DC microgrids) offer higher efficiently obtained by elimination of unnecessary AC-DC and DC-AC conversion stages; as a matter of fact many renewables based distributed energy sources are inherently DC sources [5] (e.g. photovoltaic system, fuel cell system). Furthermore, DC systems regulation is much simpler and straightforward to achieve, due to the non-existence of reactive power and frequency control [6]. Additionally, in recent decades new DC loads, such as computers, plug-in hybrid electric vehicles, giant data servers as well as variable speed drives, have made DC systems more attractive.

Rapid development in photovoltaic (PV) cells manufacturing technology and emerging environmental concerns have significantly increased photovoltaic penetration over the last decade [7]. Thus, besides wind energy, photovoltaic systems have become another major provider of environmentally friendly electrical power energy [8]. Additionally, PV systems attract more attention for implementing as a primary source of energy in DC systems due to their inherent DC electricity generation, low required maintenance, and ease of implementation. A photovoltaic system can be implemented either as a grid-connected or as an islanded system (off-grid). The PV systems with small scale power generation capacity (e.g. 1-6 KW rooftop systems, telecommunication stations) adopt the islanded topology, though the systems with medium to large scale power genera-
tion capacity (e.g. DC microgrids, solar farms) implement grid-connected topology. In a grid-connected structure, photovoltaic systems operate at maximum power [9]. Thus, the surplus or deficit of power between the local demand and the extracted power is directly exported or imported into or from the grid, requiring regulation of DC bus voltage at the point of common coupling (PCC). In contrast, an islanded mode (off-grid) of operation requires power conditioning resources (e.g. battery, fuel cell), due to the innate intermittency of solar radiation, that keep power balance in the islanded system [10] and provide an uninterruptible and reliable power supply that satisfies network requirements. The challenge stems from the practical operation where power generated by PV panels varies with the weather conditions, and the load demands constantly change from hour to hour. One of many possible solutions to intermittency is to design a hybrid system comprising PV systems and energy storage units [5,11–17]. Therefore, a DC hybrid system comprising of photovoltaic systems and energy storage systems have increasingly used in the recent years. It follows that islanded hybrid systems are formed out of directly integrating different energy resources with each other. The aim is to supply local loads, connected to the DC bus at the point of common coupling (PCC), with the required power at the required bus voltage level irrespective of the changes in the operating conditions of the system.

By developing more efficient PV cells, adopting effective ways of controlling PV systems’ DC-DC converters for higher performs appears as a new challenge. The primary goal of any PV system is extracting maximum power, which can be done using maximum power point tracker (MPPT). The MPPT can be implemented to control DC-DC converter directly without any actual controller such as in [18,19]. However, in this approach current and voltage regulation may not achieve appropriately. Also, the converter is significantly subjected to increased switching stress and losses [20] in MPPT direct control approach. To improve the system performance and to reduce power dissipation, caused by the increased switching losses, the DC-DC converter should be regulated with the medium of an MPPT as well as a control loop (indirect control). Commonly, the dynamics of power converters are described by nonlinear models. In spite of nonlinearity, these power converters are usually driven utilising linear feedback controllers (e.g.
output feedback). However, linear controllers are typically designing based on the linearised model at a particular operating point known as an equilibrium point. PV cell’s voltage and generating power significantly depend on atmospheric conditions. Consequently, the maximum power voltage is varying time to time due to stochastic nature of PV cell. As a direct result, the controller must continually change the converter duty cycle to assure the MPP voltage is tracking. Additionally, load variation and its exposed disturbance can also affect the system performance and operating conditions. Hence, this justifies that parameters mentioned above can be treated as parametric uncertainties in nonlinear models; ignoring these uncertainties (e.g. in linear controllers) can result in large-signal transients that may make the system diverge from the desired operating point or deterioration of output signal. The solution lies in the design of robust controllers that can accommodate (reject) such variations and maintain system stability under various atmospheric conditions.

Regulating the DC bus voltage and extracting the maximum power at the same time, in islanded (off-grid) configuration, is challenging unless the maximum generated power is equal to the power demand. In practice, as PV power generation varies with weather conditions, the load voltage varies too, which is not permissible for the majority of loads. Therefore, energy storage systems (ESS), as a secondary power source, are required to maintain the DC bus voltage for consistent operation of the whole DC hybrid system. Implementing energy storages can diminish the risk of power interruption due to PV system intermittency while ensuring demand satisfaction [21]. This structure is based on a direct coupling of different energy sources to a common DC bus. Subsequently, DC bus voltage regulation can be achieved by operating the energy storage system through three different modes, charge, discharge and rest mode (idle mode). Surplus power reflects a decrease in the load demand, which leads to a temporary rise in the DC bus voltage. In this condition, the ESS operates in the charge mode and absorbs the surplus in power to restore the voltage at the DC bus to its nominal level. However, in the discharge mode, the ESS exports the stored power to meet the increase in the load demand, which causes a temporary decrease in the DC bus voltage, to regulate the DC bus voltage to its nominal level. A combination of these modes ensures a constant DC voltage within the DC
system. Power management in this scheme is widely accepted to provide better stability, accuracy and faster response [9]. Similar to PV system, designing of a robust controller for energy storage system is required to achieve switching between modes of operation and maintaining the DC bus voltage at the desired level such that the system stability maintains under various loading and power generation conditions. Hence, this requires considering the energy storage system parametric uncertainties as well as attenuation of possible disturbance caused by variation of connected DC loads within a wide range of operation. Consequently, proposing a systematic decentralised robust control design for an islanded small scale DC hybrid system, comprising of a photovoltaic system and an energy storage system, which extracts the maximum power from PV system and utilises the energy storage system to regulate the DC bus was the main motivation of chapter 2.

Decentralised control is the most predominated approach used to control both small and large scales DC hybrid systems. However, a parallel connection of DC-DC converters at the DC bus introduces an interaction between the PV system and the energy storage system control-loops, which is ignored in existing controller designs. Thus, this would obviously lead to suboptimal outcome compared to whole system approach, where the dynamics of the interaction is fully included. Furthermore, as existing control approaches do not consider the interaction within the control-loops, a desired transient behaviour might not be achievable. For small scale hybrid systems, that consumers are solely relying on the DC hybrid system, the system performance and efficiency are crucial. Although, considering the system interaction and designing a coupled control can improve the total system efficiency, but it also makes the system modelling and controller design more complicated. Therefore, in the medium to large scale DC systems (e.g. DC microgrids), that the system is comprising multi PV systems and multi energy storage systems, designing a coupled controller may not be simply possible. Therefore, increasing the small scale DC hybrid systems efficiency by proposing a systematic approach based on linear matrix inequity to design a robust coupled control, that extracts maximum power from PV system while regulates the DC bus voltage by means of controlling energy storage system was the core motivation behind chapter 3.

A combination of multi PV systems, energy storage units, and loads, supplied via
1.1 Motivation

A common DC bus, can form a DC microgrid (MG) [22] with a higher power generation capacity is compared to the initially considered DC hybrid system comprising a single set of PV system and an energy storage system. To maintain a continuous and smooth operation of the DC MG, adopting an efficient and autonomous power management scheme is essential. This power management scheme ensures each PV system is operating at its maximum power point, while the energy storage units switch between charge, discharge, and idle mode according to the amount and nature of change in the DC bus voltage. Therefore, a combination of different modes of operation guarantees the DC microgrid network requirement are satisfied. Thus, this has predominantly achieved through adopting a centralised power management schemes. For centralised control scheme, the existence of communication infrastructure between each terminal and central energy controller unit (CEC) is required. Thus, for example, a sensor fault in grid power controller at supervisory level might affect the normal operation of the whole system and in some cases destabilises it. Additionally, in centralised structure the system scalability is also constrained [23]. Hence, the centralised configuration might not be appropriate for microgrids with anticipated expansion. Apart from control structure, control design approach directly affects MG efficiency, performance, and transient response. In general, DC MGs are subject to a variety of internal and external disturbances [24]. These disturbances can affect the voltage level of the common DC bus, due to switching from islanded into grid-connected modes (external) [25], as well as from stepwise demand variations within the DC MG (internal). Therefore, a power scheme with a minimum requirement of the communication link must be designed in adjacent to design a robust controller for each power generation unit that has a robust disturbance rejection performance capability against DC MG operating point variations [26]. Therefore, this was taken as the main motivation for chapter 4.

DC MGs can be operated either in an islanded mode (off-grid) or a grid-connected mode (on-grid) through a DC-AC inverting stage [11]. A grid-connected DC microgrid can export its excessive generated power, due to its low demand, into the grid. For an islanded DC microgrid, the excess generated power should be either stored in the energy storage systems or power management scheme must shift PV systems maximum power
points to ensure the DC bus voltage will not rise above its upper boundary. Therefore, grid connection capability enables the DC microgrid to operate at both islanded, and grid-connected modes depend on the system conditions. Voltage source inverters (VSI) have predominantly adopted as an interface between AC systems (electrical grids) and DC power sources (RESs). In a grid-connected VSI, although the grid’s voltage dominates the output voltage of VSI, the output current can be regulated such that specific active and reactive powers can be injected into the grid. Therefore, a grid-connected VSI must be controlled by a current controller. However, to achieve a satisfactory performance, the existence of parametric uncertainties caused by inexact knowledge of grid impedance and inductance must be considered during the controller design. Subsequently, a systematic, robust control design is required to regulate the injecting active and reactive powers from DC microgrid into AC grid. Thus, the gap of existing a systematic approach to design such a robust controller that has low implementation cost with a high level of performance motivated chapter 5.

1.2 Literature Review

The literature covers plenty of works in the area of DC microgrids control and power management, though in this section the main emphasis is on DC microgrids comprising PV systems and energy storage systems. Therefore, first, a review on control of DC hybrid systems comprising single PV system and single energy storage system is provided. After that, control structure and power management scheme for islanded DC microgrids formed by multi PV systems and multi energy storage systems are described. Finally, a summary of some relevant recent works on the control of a grid-connected voltage source inverter as an interface between a DC microgrid and AC grid are elucidated.

1.2.1 Review on DC Hybrid Systems Control

The integration of renewable energy sources and energy storage systems has obtained a significant interest in the recent decays. Among many different types of renewable energy sources, photovoltaic systems have had the most implementation in comparison
with to the others. These PV systems are usually adopted either as a single-stage system [27–29] or two-stage system [30–32]. In the single-stage configuration, the PV system is directly integrated with an AC grid through a voltage source inverter. Therefore, in this topology, the system is unable to supply DC loads. In contrast, two-stage configuration requires a DC-DC converting stage which makes it able to supply DC loads, secondly by implementing an inverting stage the excess generated power can be exported to the AC grid.

The primary objective of a PV system is to extract the maximum power by controlling and regulating the PV system DC-DC converter through adjusting its duty cycle. Thus, to control and regulate maximum power extraction from PV systems, different control approaches have proposed since increasing the popularity of these systems. These approaches can be easily categorised into two distinct sub-categories, direct [18, 19] and indirect control [1,33]. The key element of maximum power extraction is implementing an MPPT algorithm. The MPPT algorithm is responsible for finding an operating point in which the PV system can be operated that leads to generating the maximum power. Therefore, there are wide variety of MPPT algorithms with different convergence and oscillation rates that have been reported [34–38]. However, the MPPT can be implemented without using any control loop refers as direct mode and in conjunction with a control-loop refers as a direct mode. Therefore, implementing a direct control approach provides a simplicity for controlling the DC-DC converter, but current, and voltage regulation may not achieve appropriately due to the absence of any control loop. Additionally, the converter is significantly subjected to increased switching stress and losses [20]. For the direct control approach classical output-feedback PI-control is the most linear controller adopted in literature. In fact, the simplicity of designing a PI-controller made it the widely used approach in comparison with other control techniques. This PI-controller can be designed either as a single loop controller or a cascade loop controller. The cascade control approach enables the designer to limit the current following through the DC-DC converter’s inductor due to the inner loop, while the outer loop regulated the PV system output voltage. For example, the cascade control approach is used in [39] such that the PV system operates at MPP. A similar approach can be used to control the energy
storage system that regulates the DC bus voltage. In [9, 15] and [13] a peak current mode PI-control approach is proposed to control both the PV system and the energy storage system through two cascaded control loops. Although the control approach in [15] is the same as [9] and [13], authors improved the method by using a complex power algorithm, but the improvement came at the cost of using an extra DC bus voltage regulator. However, in the approaches mentioned above the controllers are designed for the nominal system that were linearised around their steady state equilibrium points. Therefore, the designed controller are not able to deal with changes in the operating conditions, especially when such changes are large, which frequently occur under shading conditions for the PV system. In the last decade, Fuzzy logic based controller attracts a significant attention due to its non-requirement of mathematical modelling, see for example [40–42] in which the controller is designed through a table of linguistic roles despite a mathematical formulation. In such fuzzy logic, instead of mathematical models, is employed to control the hybrid system. Thus, the stability of these fuzzy systems can be easily questioned.

Predictive control is another control technique that is used as in [43], sliding mode [44] and passivity-based control [45] are the other recent methods that have been proposed to take into account the converters’ non-linearities. Transient prediction difficulty and complexity of these approaches can be considered as their main drawbacks yet. Additionally, the existence of chattering for sliding controllers caused by sliding over the surfaces can significantly reduce the system efficiency and cause sequential oscillation in MPPT algorithm. In all of the above-referenced approaches, controller design is carried out for the PV system and the energy storage system separately, without accounting for the interaction between the two loops. In fact, considering the interaction requires modeling both the PV system and the energy storage system as a single multi-input multi-output system. Therefore, the single multi-input multi-output model can account the iteration between the PV system and the energy storage system. Thus, it requires a new modelling approach for the switching power converters. Neglecting the interaction between the PV system and the energy storage system, which is widely reported, naturally leads to suboptimal outcome in comparison with the whole system approach, where the dynamics of the interaction is fully included. This interaction in PV-ESS hybrid system is
due to the parallel connection of DC-DC converters, which are coupled at the same PCC. As existing control approaches do not consider the interaction within the control-loops, a desired transient behaviour might not be achievable.

1.2.2 Review on DC Microgrids Power Management Scheme

Similar to its AC counterpart, DC microgrids can operate under either centralised [15–17] or decentralised [5, 11, 12] control structure. For centralised control scheme, the existence of communication infrastructure between each terminal and central energy controller unit (CEC) is required. Thus, for example, a sensor fault in grid power controller at supervisory level might affect the normal operation of the whole system and in some cases destabilises it. Although hierarchical control [46, 47] was proposed as an extension of a centralised control structure, to protect the MG from sequential faults, the MG control structure remains vulnerable to communication delays and failure between each layer of the control scheme. Therefore, centralised control schemes are strongly dependent on communication links and any failure in CEC could have a profound adverse impact on the operation of the MG [48]. In addition to system reliability and flexibility system scalability is also constrained [23] in centralised structures. Hence, the centralised configuration might not be appropriate for microgrids with anticipated expansion. In fact, any expansion in the microgrid controlled with a centralised structure requires total modification of the central control unit and the communication infrastructure.

The above issues, which could arise in centralised control structures, are addressed by propounding a decentralised control structure, in which each terminal makes local control decision based on its local information [49]. Autonomous and smooth mode transition capability is another key requirement in a microgrid to enhance its reliability. As an example, transition between different control modes illustrated in [14], may not be done seamlessly (e.g. battery charge and discharge) and autonomously (PV MPPT and constant voltage mode). Thus, an extra level of control is needed to manage the transition from one mode of operation to another, to address the power imbalance arising from changing operating conditions.

Similar to small scale DC hybrid systems, another challenge in MGs stems from
structural nonlinearities (e.g. semiconductor switches) and parametric uncertainties (e.g. duty cycles) in DC-DC converters. Predominately, either a classical single-loop output-feedback PID controller [12] or peak-current output-feedback (cascade) PID controller [17] are implemented to design local controllers for microgrids, and the design is commonly performed on a linearised model of the system around an operating point in the aforementioned previous works. Consequently, the designed controller might not be robust against plant uncertainties, caused by operating variations in the conditions of each converter. Despite implementation facileness, such controllers lack global stability over different equilibrium points, which is a requirement in complex, scalable MG [26]. Therefore, it is required to propose a decentralised power management scheme for DC MG which has robustly designed controllers that are accounted the system uncertainties. Furthermore, omitting or minimising communication links requirement between controllers should be considered to operate the DC MG more efficiently. Last but not least, shifting maximum power point of PV systems adaptively, must be considered for such a practical situation when all energy storage systems reached their maximum permissible state of charges, while the DC MG is operating in an islanded mode. The existence of the surplus of power in this situation can cause a voltage rise within the MG. Therefore, a mechanism requires to be implemented to minimised the amount of surplus of power by shifting each PV system maximum power point. However, such that mechanism has not addressed in the above-cited research.

1.2.3 Review on Grid-connected VSI Control

Voltage source inverter (VSI) has predominantly adopted as an interface between AC systems (electrical grids) and DC power sources (RESs). In a grid-connected VSI, although the grid’s voltage dominates the output voltage of VSI, the output current can be regulated such that specified active and reactive powers can be injected into the grid [50]. Therefore, a grid-connected VSI must be controlled by a current controller. Due to switching nature of VSIs, the output current and voltage spectrums might contain high-frequency harmonics. These undesired harmonics must be limited to below 5% to satisfy the grid-codes stated in IEEE 1547 standard [51]. Thus, this should be achieved
by implementing an LCL filter at the VSI output [52, 53]. A properly designed LCL filter can eliminate high-frequency harmonics while its reactive power consumption remains low. Among many proposed VSI current controllers in literature, one can cite classical PI-feedback controller as one of the earliest control approaches [50, 54–56]. These controllers have been used both as single-loop and cascade-loop controllers. Although PI-feedback controller benefits from the simplicity of design, it has a limited stability margin and it is sensitive to system parametric uncertainties. Furthermore, it provides a very limited disturbance rejection capability for the voltage source inverters. However, its ease of implementation and design make this type of controllers the most implemented controllers in the industry. One of the main challenges to control VSIs is the existing of sinusoidal output at the grid side of VSI which requires tracking the reference values generated by the power management scheme or the grid operator. Therefore, the VSI sinusoidal tracking problem must be transferred to a fixed (stationary) frame tracking problem [57]. Thus, proportional-resonant PI current controller, based on internal mode control principal, has been adopted in [33, 58–60] in the form of tracking problem in a stationary frame. This controller, however, is sensitive to system uncertainties and thus should be designed carefully to avoid an adverse impact on system’s bandwidth and phase margin. In [61, 62] repetitive control technique is proposed to control the VSI. Hysteresis current control and sliding mode control are other techniques that have developed in [63–66]. The existence of chattering phenomena and high current ripple led the designers to use higher LCL filter values, but this leads to an increase in the reactive power loss. Predictive control is proposed in [67–69], where in [69] plant delay is addressed; it is well known that plant delay is a conventional drawback in predictive control approaches. It is also well acknowledged that the existence of parametric uncertainties and disturbances, imposed by the grid, makes the VSI current control design a challenging problem [70]. Therefore, robust control approaches have been employed to control the VSI in the presence of uncertainties and disturbances. Robust pole placement is reported in [71, 72], robust optimal digital linear quadratic regulator (DLQR) is reported in [73], robust adaptive current control is addressed in [74], and robust predictive control is considered in [75, 76]. Similar to DC-DC converters, VSI control input saturation must be avoided. Overmodulation caused
by control input saturation can increase both switching stress and introduce low-order harmonics in the VSI output voltage spectrum [57, 77]. A controller without an upper bound on its output may also destabilise the VSI. Thus, it is essential to maintain the norm of the control input signal below an upper limit. However, none of the above-cited research has been avoided overmodulation occurrence within their control design.

1.3 Summary

In this chapter, first, a motivation behind the research summarised by this thesis is explained; This illustrated the importance of the carried out research on this thesis. Then, this is followed by surveying the existing research works on controlling DC microgrids and its power management. After that, the thesis outline based on each chapter is described.

1.4 Thesis Outline

Chapter 2: Decentralised Control for a DC Hybrid System. This chapter embarks with an introduction to small scale DC hybrid systems comprising a PV system and an energy storage system, which is integrated at a common DC bus. After that, preliminaries information that is required to develop a mathematical model of PV system, unidirectional DC-DC boost converter, energy storage unit, and bidirectional DC-DC buck-boost converter are provided. It is followed by descriptions of control objectives for both PV system as well as energy storage system. Subsequently, the control problem formulation is developed based on linear matrix inequity (LMI) by providing necessary and sufficient conditions for stabilisation of the considered linear time invariant systems. Afterwards, the proposed, designed method is presented using optimisation problems to achieve an $H_{\infty}$ performance for each system. Finally, several case studies are used to confirm the results of this chapter. The main contributions of chapter 2 are reported in [1] and [2] that can be summarised as:

- Proposing a robust control design based on linear matrix inequality to extract maximum power from a PV system. The resulting control system shows better perfor-
formance, in terms of less power dissipation inside the inverter and damped ripples in its output voltage in comparison with existing approaches.

- Avoiding possible system destabilisation due to control input saturation, by imposing constraint on the level of the inverter control signal in the controller problem formulation.

- Proposing a robust control design for an energy storage system to maintain a constant DC voltage and at the same time facilitates seamless battery charge and discharge mode switching.

- Considering converters nonlinearities in the controller design, modelled as a convex polytope so that controller performs satisfactorily over an extended operation range.

Chapter 3: Coupled Control for a DC Hybrid System. This chapter is devoted to design a robust coupled multi-input multi-output controller for the considered DC hybrid system expressed in the previous chapter. An introduction is provided to elaborate the significant of considering system interaction in comparison to the proposed approach in chapter 2 as well as existing literatures. Thereafter, description of a MIMO state-space representation for the DC hybrid system is covered. Subsequently, the problem formulation based on linear matrix inequity (LMI) is extensively explained. Afterwards, the proposed controller design approach is explained. Subsequently, performance of the designed coupled controller is investigated by means of various case studies. Finally, significance of considering system interaction, and its detrimental effect caused by neglecting the interaction on the performance of the system, is verified by establishing a comparison between coupled and decoupled controllers. The main contributions of chapter 3 are reported in [3] and [4] that can be summarised as:

- Proposing a systematic approach to design a robust controller for a hybrid DC-microgrid system comprising a PV solar unit and Battery unit.

- Including the interaction between the PV and the energy storage units in the controller design to improve the overall system transient performance.
• Modelling the PV system and battery inverters duty cycles variations as parametric uncertainties.

Chapter 4: Autonomous Decentralised Power Management for DC Microgrid. This chapter provides a robust decentralised control solution for a DC microgrid comprising of multi PV systems and multi energy storage systems. First of all, an introduction on a power management in DC microgrids is provided. After that, the considered DC microgrid description and topology are explained. It is followed by a brief explanation of the proposed power management scheme and each mode of operation that is considered in this scheme. Furthermore, the control structure for multi PV systems and multi energy storage systems implemented in this DC microgrid is covered. Finally, it is concluded by providing simulation results under different scenarios that verify the effectiveness of the proposed design approach as well as the proposed power management scheme. The main contributions of chapter 4 can be summarised as:

• Proposing a decentralised and autonomous power management scheme for a DC microgrid comprising multi PV units and multi energy storage devices (batteries), where only local measurements are required.

• Design of a decentralised, instead of centralised, power management system, and thus avoiding the need for communication links between the local controllers and a central processing unit (control centre).

• Using droop control to automatically shift the maximum power point of the PV-system to achieve power balance between generation and load demand.

Outcome of this chapter is submitted with under revision status as “M Fard, M Aldeen, “Robust Autonomous Decentralised Power Management for Hybrid PV-Battery DC Microgrids”, IEEE Transactions on Smart Grid, 2017”

Chapter 5: Robust Observer-based Controller for Three-phase Grid-connected Voltage Source Inverters. This chapter provides a solution to connect the considered DC microgrid in the previous chapter into an AC grid. Thus, it is started with an introduction on a grid-connected voltage source inverter. Thereafter, a mathematical model
of the considered grid-connected voltage source inverter and its LCL filter is provided. It is followed by a description on the VSI control objective. Afterwards, an observer-based controller design is extensively explained by providing necessary and sufficient condition for stabilisation of the voltage source inverter, and to satisfy the required active and reactive powers injection. Subsequently, avoiding control input saturation and improving reference tracking performance are addressed based on linear matrix inequality (LMI) formulation. Then, grid uncertainties are covered by introducing a convex polytope. It is followed by explaining the control design through an optimisation problem. Finally, extensive simulation results provided under different scenarios verify the accuracy and effectiveness of the proposed design approach. The main contributions of chapter 5 can be summarised as:

- Proposing an observer-based dynamic state-feedback controller that assures a required performance. This is achieved by minimising the effect of grid voltage variation, modelled as an input disturbance, on the active and reactive powers injected by voltage source inverter into the main grid.

- Designing a robust observer-based controller to deal with uncertainties in the grid, modelled by a lumped parameter inductance, resistance and a constant voltage source.

- Formulating the controller design problem to avoid the voltage source inverter operating in the non-linear region in case of PWM over modulation caused by control input saturation.

Outcome of this chapter is submitted with under revision status as ”M Fard, M Aldeen, ”Robust Observer-based Controller for Three-phase Grid-connected Voltage Source Inverter”, IEEE Transactions on Power Electronics, 2017”

**Chapter 6: Conclusion.** This chapter summarises the key aspects and contributions provided by this thesis. Thereafter, possible extensions of the presented research in this thesis are provided for future research directions.
Table 1.1: Acronyms and their explanation

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>AC</td>
<td>Alternative Current</td>
</tr>
<tr>
<td>CEC</td>
<td>Central Energy Controller</td>
</tr>
<tr>
<td>DC</td>
<td>Direct Current</td>
</tr>
<tr>
<td>DLQR</td>
<td>Digital Linear Quadratic Regulator</td>
</tr>
<tr>
<td>d-q frame</td>
<td>Direct and Quadratic frame</td>
</tr>
<tr>
<td>ESU</td>
<td>Energy Storage Unit</td>
</tr>
<tr>
<td>ESS</td>
<td>Energy Storage System</td>
</tr>
<tr>
<td>ITAE</td>
<td>Integral Time Absolute Error</td>
</tr>
<tr>
<td>LHP</td>
<td>Left Hand Plane</td>
</tr>
<tr>
<td>LMI</td>
<td>Linear Matrix Inequality</td>
</tr>
<tr>
<td>LQR</td>
<td>Linear Quadratic Regulator</td>
</tr>
<tr>
<td>MIMO</td>
<td>Multi-Input Multi Output</td>
</tr>
<tr>
<td>MG</td>
<td>Microgrid</td>
</tr>
<tr>
<td>MPP</td>
<td>Maximum Power Point</td>
</tr>
<tr>
<td>MPPT</td>
<td>Maximum Power Point Tracking</td>
</tr>
<tr>
<td>NOCT</td>
<td>Nominal Operating Cell Temperature</td>
</tr>
<tr>
<td>PCC</td>
<td>Point of Common Coupling</td>
</tr>
<tr>
<td>PID</td>
<td>Proportional-Integral-Derivative</td>
</tr>
<tr>
<td>PLL</td>
<td>Phase-Locked Loop</td>
</tr>
<tr>
<td>PV</td>
<td>Photovoltaic</td>
</tr>
<tr>
<td>PV-ESS</td>
<td>Photovoltaic-Energy Storage System</td>
</tr>
<tr>
<td>PWM</td>
<td>Pulse Width Modulation</td>
</tr>
<tr>
<td>RES</td>
<td>Renewable Energy Sources</td>
</tr>
<tr>
<td>RMS</td>
<td>Root Mean Square</td>
</tr>
<tr>
<td>SOC</td>
<td>State of Charge</td>
</tr>
<tr>
<td>STC</td>
<td>Standard Testing Condition</td>
</tr>
<tr>
<td>VSI</td>
<td>Voltage Source Inverter</td>
</tr>
</tbody>
</table>

1.5 Acronyms

Table 1.1 illustrates all acronyms that are used in this thesis.
Chapter 2
Decentralised Control for a DC Hybrid System

In this chapter, a robust control design based on linear matrix inequality (LMI) is presented to control a DC hybrid system comprising a Photovoltaic (PV) system and an energy storage system (ESS). A robust $H_\infty$ control design is proposed for PV system such that the maximum power can be extracted from the PV system, while the ESS is robustly controlled to regulate the hybrid system DC bus voltage. The designed method considers converters’ nonlinearities, modelled as a convex polytope, and achieves $H_\infty$ performance, regarding load disturbance rejection, as well as system stability. Control input saturation constraint is also considered during control problem formulation to avoid excessive switching stress and system destabilisation. The proposed controllers are operating independently with a decentralised control structure. The results show that the proposed approach achieves faster tracking of reference maximum power voltage provided by MPPT with less oscillation for PV system with compared with existing approaches in literature, while the DC bus voltage remains constant under all operating conditions and disturbances.

2.1 Introduction

DC hybrid systems comprising photovoltaic systems and energy storage systems have increasingly used in recent years. A DC hybrid system enables the integration of distributed photovoltaic solar energy units with energy storage units. A DC hybrid system can be one of two topologies, namely grid-connected (on-grid) and islanded (off-grid). In a grid-connected topology, a photovoltaic system operates at maximum power point [9]. Thus, the extracted power is directly exported into the grid. Therefore, the grid is implemented as a backbone system to support the balance of power and con-
sequently regulates the DC bus voltage. In contrast, lack of a dominant source (grid) in an islanded system requires power conditioning system (e.g. battery, fuel cell) to ensure power balance within the DC hybrid system. Though grid is the major energy source in grid-connected configuration, PV system is the main source of energy in islanded topology [10]. Regulating the DC bus voltage and extracting the maximum power at the same time, in islanded configuration, is challenging unless the maximum generated power is equal to the power demand. In practice, as PV power generation varies with weather conditions, the load voltage varies too, which is not permissible for the majority of loads. Therefore, energy storage systems (ESS), as a secondary power source, are required for consistent operation of the whole hybrid system. This structure is based on a direct coupling of different energy sources to a common DC bus. Power management in this scheme is widely accepted to provide better stability, accuracy and faster response [9].

This chapter provides designing of two robust model based state-feedback controllers through implementing linear matrix inequality (LMI). The aim of this approach is to utilise PV system at its maximum power point while regulating the hybrid system DC bus voltage using an energy storage system such that the hybrid system voltage maintains at the desired level. Figure 2.1 shows a diagram of considered DC hybrid system in this chapter. The formulation is developed such that control input saturation is avoided for both PV and energy storage systems. Thus, this is crucial from realistic view to prevent converter performance degradation and possible destabilisation for both PV system with unidirectional converter and energy storage system with a bidirectional converter. The LMI problem incorporates a polytope of uncertainties and loads disturbance rejection is accomplished through using $H_{\infty}$ bounds.

2.2 Preliminaries

In this section, the essential preliminaries to design and simulate the proposed DC hybrid system is provided. First, a modified mathematical model of the photovoltaic array is described. After that, a linearized state-space model of the unidirectional DC-DC boost converter is explained. Later on, this linearized model is used to design a robust control-
The concept of maximum power point tracking (MPPT) is briefly explained to extract maximum power from PV arrays. Subsequently, a mathematical model of the energy storage unit and its synchronous bidirectional DC-DC buck-boost converter are provided.

2.2.1 Photovoltaic Array Model

A PV array is constructed by a set of PV panels, and each PV panel is comprising of several PV modules in which each has made by a series connected PV cells. To simulate the behaviour of a PV system a mathematical model of PV array needs to be developed and employed in the simulation. Therefore, in this section, a general mathematical description of a single-diode model with series and parallel resistance for a photovoltaic panel will be described. The presented model here is a modified model of general, existing models in the literature that has previously published in [78]. A single PV cell can be represented by a mathematical model in which the radiance level and the ambient temperature are the inputs and the current being the output. Figure 2.2 shows a schematic of PV cell represented by a single-diode with series and parallel resistances, where $I_{pv}$ is the PV cell output current for a given radiance and ambient temperature levels. The effect of the depletion region is taken into account by using a Shockley diode in parallel with a current source. $R_p$ is used to model leakage current in $p-n$ junction of Shockley diode. Also, structural resistance is modelled by a series resistor ($R_s$) that value of which...
depends on manufacturing technology and process. By using the Kirchhoff’s current law, PV cell circuit in Fig. 2.2 can be described as

\[ I = I_{pv} - I_d - \left( \frac{V + IR_s}{R_p} \right). \] (2.1)

where \( I \) and \( V \) are PV cell output current and voltage, respectively. \( I_d \) is used as depletion region current leakage (diode). The Shockley diode equation relates the diode current to its voltage. This relationship is called as the diode \( I-V \) characteristic. This characteristic can be expressed by (2.2).

\[ I_d = I_o \left( e^{\frac{V_d}{aVT}} - 1 \right). \] (2.2)

Where \( I_o \) is reverse bias saturation current, \( V_d \) is denoted as the voltage across the diode, \( V_t \) as the thermal voltage which is usually about 26 mV at normal temperatures. \( a \) is used as diode ideally factor that is typically between 1 and 2 for silicon based diodes. The thermal voltage can be given as following

\[ V_t = \frac{k \times T \times N_s}{q}. \] (2.3)

Where \( q \) is equal to 1.60217648740 \( \times 10^{19} \) coulombs (amp-sec) charge on the electron, \( k \) is 1.380650424 \( \times 10^{23} \) J/K (watt-sec/K) as Boltzmann’s constant and \( T \) denoted as diode temperature in Kelvin. \( N_s \) is the number of elementary series cells of any individual photovoltaic module. Reverse bias saturation current which is usually a small value can be described by (2.4).

\[ I_o = \frac{I_{SC,H} + K_I \Delta T}{e^{\frac{V_{oc,H} + K_I \Delta T}{a \times V_t}} - 1}. \] (2.4)
2.2 Preliminaries

In this equation, $I_{sc,n}$ is used as nominal short circuit current and $V_{oc,n}$ is used as nominal open circuit voltage. $K_I$ and $K_V$ are current and voltage coefficients which are temperature dependent. The temperature difference $\Delta T$ is derived from the difference of cell temperature and standard temperature.

$$\Delta T = T_c - T_n.$$  \hspace{1cm} (2.5)

In (2.5), $T_n$ is considered as standard temperature (298.15 Kelvin), and $T_c$ is used as cell temperature in Kelvin which is almost considered equal to the ambient temperature in literatures [79]. However, it cannot be an accurate approximation when PV is exposed to wide range of temperature variations. Hence, cell temperature is implemented based on Nominal Operation Cell Temperature (NOCT) data. Therefore, cell temperature can be given by (2.6) as by Evan’s equation.

$$T_c = T_a + (219 + 832K_T)\left(\frac{NOCT - 20}{800}\right).$$ \hspace{1cm} (2.6)

where $K_T$ is a regional coefficient that indicates the monthly clearness index which normally has a range of 0.2 to 0.8 based on geographical location of PV system. The Clearness Index ($K_T$) is defined as the ratio of the horizontal global radiance to the corresponding radiance available out of the atmosphere. By substituting (2.4) into (2.2) ones can obtain:

$$I_d = \left[\frac{I_{sc,n} + K_I\Delta T}{\left(e^{\frac{V_{oc,n}+K_V\Delta T}{V_{oc,n}-V_{oc,n}}} - 1\right)}\right].$$ \hspace{1cm} (2.7)

Current in a PV cell generates due to encountering of photons to semiconductor junction which is modelled as an independent current source $I_{pv}$. Generating current in $p-n$ junction of the semiconductor is proportion to solar radiation ($W/m^2$) which is radiating on the surface of the cell as well as cell temperature (Kelvin). Increasing in solar irradiation leads to generating current amplification in the cell while it has an inverse relation with cell’s temperature. Equation (2.8) describes mathematical relation of $I_{pv}$ with solar...
radiation and cell’s temperature.

\[ I_{pv} = (I_{pv,n} + K_I \Delta T) \frac{G}{G_n}. \]  

(2.8)

In (2.8) \( G \) is used as solar radiation \( (W/m^2) \), \( G_n \) standard solar radiation equal to 1000 \( W/m^2 \), \( I_{pv,n} \) used as nominal photovoltaic current and \( K_I \) indicates the impact of temperature on current. By considering current division formula \( I_{pv,n} \) can be written as:

\[ I_{pv,n} = \left( \frac{R_p + R_s}{R_p} \right) I_{sc,n}. \]  

(2.9)

which is almost neglected in available mathematical models. Finally, by substituting (2.7) and (2.8) into (2.1) generating current of a PV cell can be expressed as:

\[ I = (I_{pv,n} + K_I \Delta T) \frac{G}{G_n} - \left[ \frac{I_{sc,n} + K_I \Delta T}{(e^{(\frac{V + IR_s}{N_{ser}V_n})} - 1)} \right] - \left( \frac{V + IR_s}{R_p} \right). \]  

(2.10)

Equation (2.10) shows output current of a single PV panel with \( N_s \) series connected PV cells. A PV array is consisting of series and parallel connected panels. Based on Ohm’s law, series panels have a direct effect on the output voltage, and parallel panels have a direct effect on output current. By considering both series and parallel effect of connected panels, a PV panel equation can be modified and represented as an equation that shows a PV array output as

\[ I = I_{pv,n} N_{par} - I_0 N_{par} \left( e^{(\frac{V + IR_s}{N_{ser}V_n})} - 1 \right) - \left( \frac{V + IR_s}{R_p \left( \frac{N_{ser}}{N_{par}} \right)} \right). \]  

(2.11)

In (2.11), \( N_{ser} \) is number of series panel in an array and \( N_{par} \) is the number of parallel panels. Therefore, (2.11) can be used as final equation of a PV array.

### 2.2.2 Unidirectional Boost Converter Model

A unidirectional DC-DC boost converter is used as an interface to deliver the PV array power to the DC bus. Thus, extracting power from PV system can be controlled through
this interface. Hence, this requires developing a set of mathematical dynamic equations that express the boost converter behaviour. These dynamic equations can be expressed in a canonical state-space representation that later on can be used for designing a proper controller to regulate maximum power extraction from the PV system. The PV array can be modelled by a Thevenin equivalent circuit of a voltage source, $V_{eq}$ in series with a resistor, $R_{eq}$ to simplify the converter modelling; however, the simulation is carried out based on the actual PV array model explained in the previous section. The output voltage of the PV panel, $V_{pv}$ is the input to the DC-DC converter and $V_{C_{op}}$ is the converter output voltage, as shown in Fig.2.3. The converter is operating within two subintervals. The first sub-interval is when the switch ($S_1$) is on and the second one is when it is off. Each subinterval can be defined by a duty ratio which indicates activating time for that subinterval. Therefore, two different state-space models can be obtained for the converter each for one subinterval. The first sub-interval is activated for $D_p T_s$ seconds where $D_p$ is the duty ratio between 0 to 1 and $T_s$ is the total time for one complete interval that is shown in Fig.2.4. The state-space model of the boost converter for the first sub-interval can be expressed as

\[
\dot{x}(t) = A_{on}x(t) + B_{on}u(t). \tag{2.12}
\]

\[
y(t) = C_{on}x(t) + E_{on}u(t). \tag{2.13}
\]
Figure 2.4: Switching pulse for a boost converter with a single switch $S_1$.

\[
\frac{d}{dt}\begin{bmatrix}
i_{Lp}(t) \\
v_{C_{op}}(t) \\
v_{pv}(t)
\end{bmatrix} = \begin{bmatrix}
-(r_{Lp}+r_{onp}) \frac{1}{L_p} & 0 & \frac{1}{L_p} \\
0 & -\frac{1}{RC_{op}} & 0 \\
\frac{1}{C_{po}} & 0 & -\frac{1}{RC_{po}}
\end{bmatrix}\begin{bmatrix}
i_{Lp}(t) \\
v_{C_{op}}(t) \\
v_{pv}(t)
\end{bmatrix} + \begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
\frac{1}{R_{eq}C_{po}} & 0 & 0
\end{bmatrix}\begin{bmatrix}
V_{eq}(t) \\
V_{D}
\end{bmatrix}.
\]

(2.14)

\[
y(t) = \begin{bmatrix}
0 & 0 & 1
\end{bmatrix}\begin{bmatrix}
i_{Lp}(t) \\
v_{C_{op}}(t) \\
v_{pv}(t)
\end{bmatrix} + \begin{bmatrix}
0 & 0 & 0
\end{bmatrix}\begin{bmatrix}
V_{eq}(t) \\
V_{D}
\end{bmatrix}.
\]

(2.15)

where the boost converter inputs are Thevenin equivalent of input source $V_{eq}$ and $V_D$ is voltage across the diode. These two variables have no connection to the average duty cycle, $D$. Subsequently, the second subinterval is activated for $D_p' T_s$ where $D_p' = 1 - D_p$ is the difference between the total time of one interval and the first subinterval time. For the second subinterval, the dynamic equations of converter can be described by

\[
\dot{x}(t) = A_{off}x(t) + B_{off}u(t).
\]

(2.16)

\[
y(t) = C_{off}x(t) + E_{off}u(t).
\]

(2.17)

\[
\frac{d}{dt}\begin{bmatrix}
i_{Lp}(t) \\
v_{C_{op}}(t) \\
v_{pv}(t)
\end{bmatrix} = \begin{bmatrix}
-(r_{Lp}+r_{onp}) \frac{1}{L_p} & \frac{1}{C_{po}} & \frac{-1}{C_{po}} \\
\frac{1}{C_{op}} & 0 & \frac{1}{C_{op}} \\
\frac{-1}{C_{po}} & 0 & -\frac{1}{RC_{po}}
\end{bmatrix}\begin{bmatrix}
i_{Lp}(t) \\
v_{C_{op}}(t) \\
v_{pv}(t)
\end{bmatrix} + \begin{bmatrix}
0 & \frac{-1}{L_p} & 0 \\
0 & 0 & 0 \\
\frac{1}{R_{eq}C_{po}} & 0 & 0
\end{bmatrix}\begin{bmatrix}
V_{eq}(t) \\
V_{D}
\end{bmatrix}.
\]

(2.18)
2.2 Preliminaries

\[
y(t) = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} i_{Lp}(t) \\ v_{C_{op}}(t) \\ v_{pv}(t) \end{bmatrix} + \begin{bmatrix} 0 & 0 & V_{eq}(t) \\ 0 & 0 & V_{D} \end{bmatrix}.
\] (2.19)

A nonlinear average model of the boost converter can be developed by averaging over one interval comprising two subintervals. Thus, the average state-space model of the converter can be expressed as

\[
A_{avg} = A_{on}D_p + A_{off}(1 - D_p).
\] (2.20)

\[
B_{avg} = B_{on}D_p + B_{off}(1 - D_p).
\] (2.21)

\[
C_{avg} = C_{on}D_p + C_{off}(1 - D_p).
\] (2.22)

\[
E_{avg} = E_{on}D_p + E_{off}(1 - D_p).
\] (2.23)

Subsequently, by using small signal approximation, a linearized state-space model of the boost converter can be expressed as

\[
\dot{x}_p(t) = A_p x_p(t) + B_{up} u_p(t).
\] (2.24)

\[
y_p(t) = C_p x_p(t).
\] (2.25)

where superscript \( p \) indicates PV system and \( x_p(t) = [\dot{i}_{Lp}(t) \ \dot{v}_{C_{op}}(t) \ \dot{v}_{pv}(t)]^T \), \( u_p(t) = d_p(t) \), \( y_p(t) = \dot{v}_{pv}(t) \), and the parameter matrices \( A_p \), \( B_{up} \), and \( C_p \) are define as:

\[
A_p = \begin{bmatrix}
-\frac{r_{onp}D_p - r_{Dp}D'_p}{L_p} & \frac{-D'_p}{L_p} & \frac{1}{L_p} \\
\frac{D'_p}{C_{op}} & -\frac{1}{RC_{op}} & 0 \\
\frac{-1}{C_{pv}} & 0 & -\frac{1}{R_{eq}C_{pv}}
\end{bmatrix}.
\] (2.26)

\[
B_{up} = \begin{bmatrix}
\frac{(V_{eq} - R_{eq}I_{eq})}{D^2_{eq}} + \frac{(V_{eq} - R_{eq}I_{eq})}{D_{eq}} + V_{D} \\
\frac{-L_p}{D^2_{eq}RC_{op}} \\
0
\end{bmatrix}.
\] (2.27)

\[
C_p = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}.
\] (2.28)
where $i_{Lp}$, $v_{op}$, and $v_{pv}$ are boost converter small signal inductor current, output voltage and its input voltage respectively that are the states of the model. Parameters $r_{Lp}$, $r_{onp}$, and $r_{Dp}$ are parasitic elements for converter’s inductor, semiconductor switch and diode. The PV array steady-state Thevenin equivalent voltage and resistance are denoted by $V_{eq}$, $R_{eq}$ and its output current is $I_{eq}$. $L_p$ is the boost converter inductor. $C_{pv}$ and $C_{op}$ are input and output filter capacitors of the boost converter. The parameter $R$ is the resistive load connected to the DC bus. Steady state duty cycle of converter is $D_p$ while $D'_p$ is equal to $1 - D_p$. The steady state parameters or duty cycle is defined to obtain a specified output voltage for a given input voltage. Thus, for a given system with a constant voltage source, $D$ is adjusted so that the output voltage is within the operational requirement of the system. The parameter $\dot{d}_p(t)$ is the incremental change in the duty cycle around $D_p$. To account for the disturbance (ripples), at the point of common coupling caused by hybrid system load variation, a current source is connected with reverse polarity in parallel to the load [80]. Therefore, this can characterise the output impedance of the converter by describing the behaviour of converter output voltage while the output current is changing [81]. Thus, the state-space model (2.24) and (2.25) can be described by

$$\dot{x}_p(t) = A_p x_p(t) + B_{up} u_p(t) + B_{wp} w(t).$$  \hfill (2.29)

$$y_p(t) = C_p x_p(t).$$  \hfill (2.30)

where $A_p$, $B_{up}$, $C_p$ are as in (2.26)-(2.28) and

$$B_{wp} = \begin{bmatrix} 0 & 0 & -\frac{1}{C_{op}} \end{bmatrix}^T.$$  \hfill (2.31)

$$w(t) = i_{dis}.$$  \hfill (2.32)

where $i_{dis}$ is the disturbance current at the output of the converter.

### 2.2.3 Maximum Power Point Tracker

MPPT is an algorithm based controller that calculates the value of maximum voltage ($V_{mpp}$) in which PV delivers maximum power ($P_{mpp}$). The inputs of MPPT are PV array
2.2 Preliminaries

voltage \((V_{pv})\) and current \((I_{pv})\), while its output is PV MPP voltage \((V_{mpp,ref})\) which maximum power generates at that voltage by PV arrays. It can be shown by multiplying both sides of (2.10) with \(V_{mpp}\) and replace PV current with its respective maximum current \((I_{mpp})\)

\[
P_{mpp} = V_{mpp} I_{mpp} = V_{mpp} \left\{ N_p(I_{pv,n} + K_I \Delta T) \frac{G}{G_n} - \left[ N_p(I_{sc,n} + K_I \Delta T) \left( e^{\left( \frac{V_{mpp} + I_{mpp} R_s N_s}{N_p} \right)} - 1 \right) \right] - \left( \frac{V_{mpp} + I_{mpp} R_s N_s}{R_p \frac{N_s}{N_p}} \right) \right\}.
\]

(2.33)

Therefore, if a set of PV array operates at its maximum power point voltage \((V_{mpp})\) the output power from PV array is the maximum extractable power. There are several proposed algorithms to calculate the \(V_{mpp}\) with different oscillation and convergence rate. One of the most predominate algorithms is incremental conductance proposed in [82] that has used here. This algorithm can be explained as following

\[
\frac{dP}{dV} = \frac{d(VI)}{dV} = I \frac{dV}{dV} + V \frac{dI}{dV} = I + V \frac{dI}{dV},
\]

(2.34)

\[
I + V \frac{dI}{dV} = 0. \tag{2.35}
\]

\[
- \frac{I}{V} = \frac{dI}{dV}. \tag{2.36}
\]

The left-hand side of (2.36) represents the instantaneous conductance, and the right-hand side represents PV incremental conductance. Therefore, analysis of (2.36) can show the PV maximum power point. The incremental conductance algorithm can be improved by adding an epsilon value to (2.36) because condition \(\frac{dP}{dV} = 0\) is seldom obtainable because of approximation made in differentiates of voltage and current. Then, (2.36) can be rewritten as:

\[
\frac{dP}{dV} = \pm \epsilon. \tag{2.37}
\]

The MPPT is used to provide a reference voltage for PV system controller that results in maximum power extraction from PV system.
2.2.4 Energy Storage Model

Some of the reported battery models have proposed in [83–86] with different accuracy and complexity, though as the main focus here was to design robust controller, a mathematical model of a lead acid battery proposed in [85] is implemented to simulate the behaviour of energy storage battery banks. A battery bank is formed by the connection of series and parallel connected batteries with the same electrical characteristic. Battery charge and discharge terminal voltage can be obtained from

\[ V_{b_{\text{dis}}} = E_0 - R_b i_b - K \frac{Q}{Q - it} (it + i_b^*) + \text{Exp}(t). \]  

(2.38)

\[ V_{b_{\text{ch}}} = E_0 - R_b i_b - K \frac{Q}{it - 0.1Q} i_b^* - K \frac{Q}{Q - it} it + \text{Exp}(t). \]  

(2.39)

where \( V_{b_{\text{dis}}} \) and \( V_{b_{\text{ch}}} \) are battery voltage in discharge and charge mode respectively. \( R_b \) is indicated as the internal resistance of battery. \( K, Q \) and \( \text{Exp}(t) \) are battery polarisation constant (\( V/(\text{Ah}) \)), capacity (\( \text{Ah} \)) and exponential zone voltage (\( V \)) successively. The battery actual charge (\( \text{Ah} \)), \( it \), can be defined with respect to the current (\( i_b \)) flowing in or out of the battery unit during operation (\( t \)).

\[ it = \int_0^t i_b dt. \]  

(2.40)

2.2.5 Bi-directional Buck-Boost Converter Model

A bidirectional synchronous DC-DC buck-boost converter is used to establish an interface between energy storage unit and the DC bus. Figure 2.5 shows the adopted buck-boost converter for the energy storage system here. The converter acts as a buck converter during charge mode because the battery voltage is lower than the DC bus voltage. However, during discharge mode, it is acting as a boost converter to step up the battery voltage to the DC bus voltage level. Due to synchronous nature of the converter, as it is shown in Fig.2.6, whenever switch \( S_2 \) is on the second switch \( S_3 \) is off and vice versa. Therefore, there are only two subintervals for the bidirectional buck-boost converter. Consequently, initially, the state-space of each subinterval is obtained and then an average model of
the converter is developed. Subsequently, by using the small signal approximation, the nonlinear average model is linearized around its equilibrium point. Finally, a linearized state-space model of the bidirectional buck-boost converter can be obtained by (2.41) and (2.42) successively.

\[
\dot{x}_b(t) = A_b x_b(t) + B_{ub} u_b(t) + B_{wb} w(t). \tag{2.41}
\]

\[
y_b(t) = C_b x_b(t). \tag{2.42}
\]
where \( x_{\text{b}}(t) = [\hat{v}_{C_{\text{ob}}}(t) \ \ \hat{v}_{C_{\text{ib}}}(t) \ \ \hat{i}_{L_{\text{b}}}(t)]^T \) is system’s state vector, \( u_{\text{b}}(t) = \hat{d}_{\text{b}}(t) \), \( y_{\text{b}}(t) = \hat{v}_{C_{\text{ob}}}(t) \), and the matrices \( A_{\text{b}} \), \( B_{\text{ub}} \), and \( C_{\text{zb}} \) are defined as

\[
A_{\text{b}} = \begin{bmatrix}
0 & 0 & \frac{(1-D_{\text{b}})}{C_{\text{ob}}} \\
0 & 0 & -\frac{R_{\text{b}}}{C_{\text{ob}(R_{\text{b}}+r_{C_{\text{ib}}})}} \\
-\frac{(1-D_{\text{b}})}{L_{\text{b}}} & \frac{R_{\text{b}}}{L_{\text{b}(R_{\text{b}}+r_{C_{\text{ib}}})}} & -\frac{a_1+a_2}{L_{\text{b}(R_{\text{b}}+r_{C_{\text{ib}}})}} \\
\end{bmatrix}. \tag{2.43}
\]

\[
B_{\text{ub}} = \begin{bmatrix}
-\frac{L_{\text{b}}}{L_{\text{b}(1-D_{\text{b}})}} \\
0 \\
\frac{V_{\text{b}}}{L_{\text{b}(1-D_{\text{b}})}} - \frac{L_{\text{b}}}{L_{\text{b}}}(b_1 + b_2 - r_{C_{\text{ob}}}) \\
\end{bmatrix}. \tag{2.44}
\]

\[
C_{\text{zb}} = \begin{bmatrix}
1 & 0 & 0 \\
\end{bmatrix}. \tag{2.45}
\]

\[
B_{\text{wb}} = \begin{bmatrix}
-\frac{1}{C_{\text{ob}}} & 0 & 0 \\
\end{bmatrix}^T. \tag{2.46}
\]

\[
a_1 = R_{\text{b}}r_{C_{\text{ib}}} + R_{\text{b}}r_{L_{\text{b}}} + r_{L_{\text{b}}}r_{C_{\text{ib}}}. \tag{2.47}
\]

\[
a_2 = (R_{\text{b}}r_{C_{\text{ob}}} + r_{C_{\text{ob}}}r_{C_{\text{ib}}})(1 - D_{\text{b}}). \tag{2.48}
\]

\[
b_1 = \frac{R_{\text{b}} + r_{L_{\text{b}}} + r_{C_{\text{ib}}}(1 - D_{\text{b}})}{(1 - D_{\text{b}})^2}. \tag{2.49}
\]

\[
b_2 = \frac{R_{\text{b}}r_{C_{\text{ob}}} + r_{C_{\text{ib}}}r_{C_{\text{ob}}}}{(R_{\text{b}} + r_{C_{\text{ib}}})(1 - D_{\text{b}})}. \tag{2.50}
\]

The state variables of the model are bidirectional converter’s output voltage, input voltage and inductor current \((\hat{v}_{C_{\text{ob}}} \ \ \hat{v}_{C_{\text{ib}}} \ \ \hat{i}_{L_{\text{b}}})\) respectively. Parasitic parameters are taken into account by \(r_{C_{\text{ib}}} \ \ r_{L_{\text{b}}} \ \ r_{C_{\text{ob}}} \). \(L_{\text{b}}\) is the buck-boost converter inductor. \(C_{\text{ib}}\) and \(C_{\text{ob}}\) are input and output filter capacitors of the converter. Complementary steady state duty cycle of converter is denoted by \(D'_{\text{b}}\). Both \(V_{\text{b}}\) and \(I_{\text{ob}}\) are used as steady state values of battery voltage and current while \(R_{\text{b}}\) indicates the battery internal impedance.

### 2.3 DC Hybrid System Control Objectives

The considered DC hybrid system shown in Fig.2.1 is comprising of a PV system and an energy storage system. As the DC hybrid system is considered in islanded mode, the
PV system is considered as the primary source of power. Thus, the energy storage system is used to compensate the inherent intermittency of generated power by PV system. Implementing energy storages can diminish the risk of power interruption due to PV system intermittency while ensuring demand satisfaction [21]. In the following, control objectives of both the PV system controller and the energy storage system controller are separately described.

2.3.1 PV System Control Objective

The main goal of any PV system is extracting maximum power. Each PV cell (panel) is characterised by its MPP, which occurs at the specified output voltage and current levels for a given radiance intensity. To track this point a maximum power point tracking (MPPT) algorithm needs to be used. The inputs to the MPPT algorithm are the actual measured values of the PV output voltage ($V_{pv}$) and current ($I_{pv}$). The output of the MPPT algorithm is a reference input voltage ($V_{mpp}$) of which the PV system generates the maximum power. The DC-DC boost converter must be utilised such that its input voltage is equal to the $V_{mpp}$. This can be achieved by continuously adjusting the duty cycle of the DC-DC converter. At any time the value of duty cycle defines the impedance of the DC-DC converter that can be seen by PV arrays. Hence, the maximum power extraction from PV system can be assured. The MPPT can be implemented to control and adjust the DC-DC converter duty cycle directly without any actual controller such as in [18, 19]. However, in this approach current and voltage regulation may not achieve appropriately. Also, the converter is significantly subjected to increased switching stress and losses [20] in MPPT direct control approach. To improve the system performance and reduce power dissipation the duty cycle of DC-DC converter should be regulated with the medium of an MPPT complemented with a control loop. This approach has widely used in literature and known as an indirect control technique. Thus, this means that MPPT calculates the PV voltage simultaneously such that in that voltage the maximum power can be generated by PV arrays. This reference value should be implemented by the PV bus voltage controller to adjust the duty cycle of the DC-DC converter and thus the PV output voltage. To summarise, the PV system control objective is to regulate the duty cycle
of the PV system DC-DC converter such that the PV system output voltage maintains at maximum power point voltage ($V_{mpp}$) obtained by MPPT. Thus, the tracking error should become zero when time goes to infinity for satisfactory control performance.

### 2.3.2 Energy Storage System Control Objective

In practice, as PV power generation varies with weather conditions, the load voltage varies too, which is not permissible for the majority of loads. Therefore, energy storage systems (ESS), as a secondary power source, are required for consistent operation of the whole hybrid system. The ESS operation is consisting of three modes, charge, discharge and idle mode. The charging mode is initiated when there is a surplus of power ($P_{pv} > P_{load}$), while discharge mode is launched when there is a lack of power ($P_{pv} < P_{load}$). Idle mode is under operation when the following condition is achieved,

$$|P_{pv} - P_{load}| < \delta_p.$$  \hfill (2.51)

The power imbalance $\delta_p$ is defined such that the DC bus voltage remains within an upper limit and a lower limit for the DC bus voltage that is commonly refers as permissible operating range (0.90 to 1.1 p.u) based on AS61000 grid code [87]. The upper threshold of the DC bus voltage is limited to $+10\%$ of the nominal voltage and the lower limit is set as $-10\%$ of the nominal voltage. A combination of charge, discharge, and idle modes ensures the DC bus voltage remains within the defined threshold. The direction and magnitude of power flowing in or out of the energy storage system depends on the working condition of the bidirectional DC-DC buck-boost converter that defines by continuously adjusting the converter’s duty cycle. Therefore, to regulated the DC bus voltage by the means of the energy storage system a control loop requires to be adopted such that the control loop continuously adjusts the converter’s duty cycle. To summarise, the control objective of the energy storage system is to maintain the DC bus voltage at a desired level by the means of adjusting the energy storage DC-DC converter duty cycle.
2.4 Control Problem Formulation

Since all states of both systems (PV and ESS) are measurable two state-feedback controllers, \( u(t) = Kx(t) \), can be implemented to stabilise the PV system and the energy storage system separately by using an appropriate control strategy. An integrator needs to be inserted in a feedforward path of each controller to obtain zero steady-state error for both controllers, i.e. reference input tracking, that results in a new state-space representation for each system. Therefore, this can be shown through a general state-space representation in the following

\[
\dot{x}(t) = Ax(t) + Bu(t) + Bw(t).
\]

\( y(t) = Cx(t) \).  

\( u(t) = -KPx(t) + KI\xi(t) \).

\( \dot{\xi}(t) = R_{ref} - y(t) = V_{ref} - Cx(t) \). 

where \( \xi \) is the output of integrator and \( KI \) is the integrator gain. \( R_{ref} \) is indicated as the input reference value corresponding to the considered system (PV system or ESS). Subsequently, the \( \dot{x}(t) \) and \( \dot{\xi}(t) \) can be augmented and create a new state-space representation.

\[
\begin{bmatrix}
\dot{x}(t) \\
\dot{\xi}(t)
\end{bmatrix} =
\begin{bmatrix}
A & 0 \\
-C & 0
\end{bmatrix}
\begin{bmatrix}
x(t) \\
\xi(t)
\end{bmatrix} +
\begin{bmatrix}
B_u \\
0
\end{bmatrix}
\begin{bmatrix}
u(t) \\
0
\end{bmatrix} +
\begin{bmatrix}
B_w \\
0
\end{bmatrix}
\begin{bmatrix}
w(t) \\
1
\end{bmatrix}
R_{ref}.
\]

Figure 2.7 shows the general block diagram of the proposed full state feedback. This can be implemented in the design process by augmenting the state equations of (2.29), (2.30) for the PV system and (2.41), (2.42) for the energy storage system with the integrator dynamic as shown below

\[
\dot{x}_p(t) = \bar{A}_p\dot{x}_p(t) + \bar{B}_{up}u_p(t) + \bar{B}_{wp}w(t).
\]

\( u_p(t) = -K_p\dot{x}_p(t) \).
34 Decentralised Control for a DC Hybrid System

\[ K_p = \begin{bmatrix} K_{p} & -K_{I} \end{bmatrix}. \]  

(2.59)

\[ \dot{x}_b(t) = \tilde{A}_b \hat{x}_b(t) + \tilde{B}_{ub} u_b(t) + \tilde{B}_{wb} w(t). \]  

(2.60)

\[ u_b(t) = -K_b \hat{x}_b(t). \]  

(2.61)

\[ K_b = \begin{bmatrix} K_{p} & -K_{I} \end{bmatrix}. \]  

(2.62)

Figures 2.8 and 2.9 show the adopted full state-feedback control structure for the PV system and the energy storage system respectively.
2.4 Control Problem Formulation

2.4.1 State-feedback Design

Consider a homogeneous linear time invariant system (LTI) (2.63),

\[ \dot{x}(t) = Ax(t). \]  

(2.63)

One can write a quadratic Lyapunov function for system (2.63) such that

\[ V(x) = x^T(t)Px(t), \quad \forall x(t) \neq 0. \]  

(2.64)

where \( P \) is a symmetric positive definite matrix. If matrix \( P \) can be find such that, \( V(x) \) is positive definite, and the time derivative of \( V(x) \) is always negative for all \( x \) (sufficient and necessary condition), the system (2.63) is globally asymptotically stable. This implies that all trajectories of (2.63) converge to zero as \( t \to 0 \). Now, consider system (2.57) when \( w(t) = 0 \), by substituting the control law (2.58) into the system (2.57), the closed-loop PV system can be given by

\[ \tilde{\dot{x}}_p(t) = (\tilde{A}_p - \tilde{B}_upK_p)\tilde{x}_p(t). \]  

(2.65)

Time derivative of Lyapunov function in (2.68) can be modified such that a minimum decay rate of \( \alpha_p \) is ensured,

\[ \dot{V}(\tilde{x}_p(t)) = \tilde{x}_p^T(t)P\tilde{x}_p(t) + \tilde{x}_p^T(t)P\tilde{x}_p(t). \]  

(2.66)
By substituting (2.65), into (2.66)

\[
\dot{V}(\tilde{x}_p(t)) = \dot{\tilde{x}}_p^T(t)(\tilde{A}_p - \tilde{B}_{up}K_p)\tilde{x}_p(t) + \dot{\tilde{x}}_p^T(t)P(\tilde{A}_p - \tilde{B}_{up}K_p)\dot{\tilde{x}}_p(t). \tag{2.67}
\]

\[
\dot{V}(\tilde{x}_p(t)) = \dot{\tilde{x}}_p^T(t) \left[ (\tilde{A}_p - \tilde{B}_{up}K_p)^T P + P(\tilde{A}_p - \tilde{B}_{up}K_p) \right] \tilde{x}_p(t). \tag{2.68}
\]

Therefore, the system (2.65) can be globally asymptotically stable by control law (2.58), if \( \dot{V}(\tilde{x}_p(t)) < 0 \), which implies \( \Omega < 0 \) for all \( \tilde{x}_p(t) \).

\[
(\tilde{A}_p - \tilde{B}_{up}K_p)^T P + P(\tilde{A}_p - \tilde{B}_{up}K_p) < 0. \tag{2.69}
\]

The above relationship implies that a stabilising PI-controller exists if a positive definite symmetric matrix \( P \) can be found to satisfy the inequality (2.69). Equation (2.69) can be expressed as

\[
\tilde{A}_p^T P - K_p^T \tilde{B}_{up}^T P + P\tilde{A}_p - P\tilde{B}_{up}K_p < 0. \tag{2.70}
\]

By pre- and post-multiplying (2.70) with \( P^{-1} \), it can be described as

\[
\tilde{p}^{-1} \tilde{A}_p^T - P^{-1} K_p^T \tilde{B}_{up}^T + \tilde{A}_p P^{-1} - \tilde{B}_{up} K_p P^{-1} < 0. \tag{2.71}
\]

Defining \( P^{-1} = W_p \) results to

\[
\tilde{A}_p W_p + W_p \tilde{A}_p - \tilde{B}_{up} K_p W_p - W_p K_p^T \tilde{B}_{up} < 0. \tag{2.72}
\]

Therefore, a sufficient and a necessary condition to stabilised the system (2.57) can be summarised by the following theorem.

**Theorem 2.1.** The system (2.57) is stabilisable by the means of control law (2.58) if and only if there exist a symmetric positive definite matrix \( W_p \in \mathbb{R}^{n \times n} \) and a matrix \( Y_p \in \mathbb{R}^{m \times n} \) such that the following linear matrix inequalities hold,

\[
\tilde{A}_p W_p + W_p \tilde{A}_p^T + \tilde{B}_{up} Y_p + Y_p^T \tilde{B}_{up}^T < 0. \tag{2.73}
\]

\[
W_p > 0. \tag{2.74}
\]
where

\[
K_p = -Y_p W_p^{-1} = \begin{bmatrix} KP_p & -KI_p \end{bmatrix}.
\]  

(2.75)

The same procedure that concludes to Theorem 2.1, can also be used to provide a sufficient and a necessary condition to stabilise the system (2.60).

**Corollary 2.1.** The system (2.60) is stabilisable by the means of control law (2.61) if and only if there exist a symmetric positive definite matrix \( W_b \in \mathbb{R}^{n \times n} \) and a matrix \( Y_b \in \mathbb{R}^{m \times n} \) such that the following linear matrix inequalities hold,

\[
\tilde{A}_b W_b + W_b \tilde{A}_b^T + \tilde{B}_b Y_b + Y_b^T \tilde{B}_b^T < 0. \quad (2.76)
\]

\[
W_b > 0. \quad (2.77)
\]

where

\[
K_b = -Y_b W_b^{-1} = \begin{bmatrix} KP_b & -KI_b \end{bmatrix}.
\]  

(2.78)

### 2.4.2 Pole Placement

The LMI problem formulated by theorem 2.1 for the PV system returns a stabilising controller, however, its performance level is not guaranteed. To guarantee the performance level, the closed-loop poles must be placed in appropriate locations in the left-half of the s-plane (LHP). A common practice is using D-stability criteria [88], where the dominant closed-loop poles are placed in a region bounded lines in the LHP representing required minimum real parts of closed-loop eigenvalues and damping ratio. Such requirement can be incorporated in the LMI problem through the addition of bounded constraints. The bounded region that satisfies the above constraints can be a set \( S_p(\alpha_p, r_p, \theta_p) \) of complex poles \( x_p \pm j y_p \) for the PV system as

\[
-x_p < \alpha_p < 0, \quad \forall \alpha_p < 0. \quad (2.79)
\]
This set region is illustrated in Fig. 2.10. The shaded area is the permissible location that the PV system closed-loop poles can be located by theorem 2.1 in the LHP. To assure a minimum decay rate of \( \alpha_p \), time derivative of Lyapunov function in (2.68) can be modified to

\[
\begin{align*}
(\hat{A}_p - \hat{B}_{up}K_p + \alpha_p I)^T P + P(\hat{A}_p - \hat{B}_{up}K_p + \alpha_p I) &< 0. \\
(\hat{A}_p W_p + W_p \hat{A}_p^T + \hat{B}_{up}Y_p + Y_p^T \hat{B}_{up}^T + 2\alpha_p W_p &< 0.
\end{align*}
\] (2.82)

Subsequently, theorem 2.1 can be modified based on (2.82).

where \( \alpha_p \in \mathbb{R} \) is a scalar. Equation (2.83) ensures that the real part of closed-loop poles designed for the system (2.57), are smaller than \( \alpha_p \), and consequently the minimum required decay rate \( \alpha_p \) is achieved. It is required to limit the left-hand side of s-plane by locating the closed-loop poles within a valid frequency range; This can be achieved by defining a circle centred at the centre of s-plane. Thus, the radius of the circle defines the left-hand side boundary of permissible closed-loop poles location. This can be achieved
through a modification of a proposed theorem in [88] such that, the eigenvalues of the closed-loop system, 
\( A_{clp} = \tilde{A}_p - \tilde{B}_{up}K_p \), in (2.73) lies in a disk of radius \( r_p \) with centre at \((0,0)\) if and only if there exists a matrix \( W_p > 0 \) such that

\[
\begin{bmatrix}
-r_p W_p & A_{clp} W_p \\
* & -r_p W_p \\
\end{bmatrix} < 0.
\]

(2.84)

Therefore, the radius of disk in a form of an LMI constraint for the system (2.73) can be defined as

\[
\begin{bmatrix}
-r_p W_p & \tilde{A}_p W_p + \tilde{B}_{up} Y_p \\
* & -r_p W_p \\
\end{bmatrix} < 0.
\]

(2.85)

Finally, to obtain the required minimum damping ratio the imagery part of the closed-loop poles needs to be limited to a restricted triangle expressed by

\[
f_p(A_{clp}) = \begin{bmatrix}
\sin \theta_p (A_{clp} P + PA_{clp}^T) & \cos \theta_p (A_{clp} P - PA_{clp}^T) \\
\cos \theta_p (PA_{clp}^T - A_{clp} P) & \sin \theta_p (A_{clp} P + PA_{clp}^T) \\
\end{bmatrix} < 0.
\]

(2.86)

where \( P \) is the same symmetric positive definite matrix as in (2.69). After that, by substituting the closed-loop matrix \( (A_{clp}) \), it can be written as

\[
\begin{bmatrix}
\sin \theta_p (\tilde{A}_p W_p + W_p \tilde{A}_p^T + \tilde{B}_{up} Y_p + Y_p^T \tilde{B}_{up}^T) & \cos \theta_p (\tilde{A}_p W_p - W_p \tilde{A}_p^T + \tilde{B}_{up} Y_p - Y_p^T \tilde{B}_{up}^T) \\
\cos \theta_p (-\tilde{A}_p W_p + W_p \tilde{A}_p^T - \tilde{B}_{up} Y_p + Y_p^T \tilde{B}_{up}^T) & \sin \theta_p (\tilde{A}_p W_p + W_p \tilde{A}_p^T + \tilde{B}_{up} Y_p + Y_p^T \tilde{B}_{up}^T) \\
\end{bmatrix} < 0.
\]

(2.87)

Intersection of (2.83) for \( S_p(\alpha_p, \infty, 90) \), (2.85) for \( S_p(0, r_p, 90) \), and (2.87) for \( S_p(0, \infty, \theta_p) \) defines the set region of \( S_p(\alpha_p, r_p, \theta_p) \). This set region can be expressed by the following theorem.

**Theorem 2.2.** The system (2.57) is stabilisable by the means of control law (2.58) if and only if there exist a symmetric positive definite matrix \( W_p \in \mathbb{R}^{n \times n} \) and a matrix \( Y_p \in \mathbb{R}^{m \times n} \) such that the linear matrix inequality (2.73) holds, and the system (2.57) closed-loop poles are only located in a bounded region \( S_p(\alpha_p, r_p, \theta_p) \), such that \( \alpha_p \) defines the system minimum decay rate, \( \theta_p \) defines the minimum damping ratio if and only if the following linear matrix inequalities hold.

\[
\tilde{A}_p W_p + W_p \tilde{A}_p^T + \tilde{B}_{up} Y_p + Y_p^T \tilde{B}_{up}^T + 2\alpha_p W_p < 0.
\]

(2.88)
Decentralised Control for a DC Hybrid System

\[
\begin{bmatrix}
-r_p W_p & \tilde{A}_p W_p + \tilde{B}_{up} Y_p \\
* & -r_p W_p
\end{bmatrix} < 0. \quad (2.89)
\]

\[
\begin{bmatrix}
\sin \theta_p (\tilde{A}_p W_p + W_p \tilde{A}_p^T + \tilde{B}_{up} Y_p + Y_p^T \tilde{B}_{up}^T) & \cos \theta_p (\tilde{A}_p W_p - W_p \tilde{A}_p^T + \tilde{B}_{up} Y_p - Y_p^T \tilde{B}_{up}^T) \\
\cos \theta_p (-\tilde{A}_p W_p + W_p \tilde{A}_p^T - \tilde{B}_{up} Y_p + Y_p^T \tilde{B}_{up}^T) & \sin \theta_p (\tilde{A}_p W_p + W_p \tilde{A}_p^T + \tilde{B}_{up} Y_p + Y_p^T \tilde{B}_{up}^T)
\end{bmatrix} < 0. \quad (2.90)
\]

A bounded region \( S_b(\alpha_b, r_b, \theta_b) \) for the energy storage system expressed by (2.60) can also be defined for the corollary 2.1 in order to achieve a required performance for the system (2.60) stabilised by control law (2.60).

**Corollary 2.2.** The system (2.60) is stabilisable by the means of control law (2.61) if and only if there exist a symmetric positive definite matrix \( W_b \in \mathbb{R}^{n \times n} \) and a matrix \( Y_b \in \mathbb{R}^{m \times n} \) such that the linear matrix inequality (2.76) holds, and the system (2.60) closed-loop poles are only located in a bounded region \( S_b(\alpha_b, r_b, \theta_b) \), such that \( \alpha_b \) defines the system minimum decay rate, \( \theta_b \) defines the minimum damping ratio if and only if the following linear matrix inequalities hold.

\[
\tilde{A}_b W_b + W_b \tilde{A}_b^T + \tilde{B}_{ub} Y_b + Y_b^T \tilde{B}_{ub}^T + 2 \alpha_b W_b < 0 \quad (2.91)
\]

\[
\begin{bmatrix}
-r_b W_b & \tilde{A}_b W_b + \tilde{B}_{ub} Y_b \\
* & -r_b W_b
\end{bmatrix} < 0 \quad (2.92)
\]

\[
\begin{bmatrix}
\sin \theta_b (\tilde{A}_b W_b + W_b \tilde{A}_b^T + \tilde{B}_{ub} Y_b + Y_b^T \tilde{B}_{ub}^T) & \cos \theta_b (\tilde{A}_b W_b - W_b \tilde{A}_b^T + \tilde{B}_{ub} Y_b - Y_b^T \tilde{B}_{ub}^T) \\
\cos \theta_b (-\tilde{A}_b W_b + W_b \tilde{A}_b^T - \tilde{B}_{ub} Y_b + Y_b^T \tilde{B}_{ub}^T) & \sin \theta_b (\tilde{A}_b W_b + W_b \tilde{A}_b^T + \tilde{B}_{ub} Y_b + Y_b^T \tilde{B}_{ub}^T)
\end{bmatrix} < 0 \quad (2.93)
\]

### 2.4.3 \( H_\infty \) Formulation

Load variation in the DC hybrid system can be considered as an input disturbance, \( w(t) \), for the PV system and the energy storage system. Therefore, both systems’ controllers are required to be robustly designed in term of input disturbance. The importance of minimising the disturbance effect can be described clearly through the RMS gain of each
system. For any system, the RMS gain from input disturbance to the system desired output, \( z(t) \) in which it can be the same as a system actual output \( y(t) \) or different, can be described by an infinity norm such that

\[
\| G_{zw}(s) \|_\infty = \frac{\| z \|_2}{\| w \|_2} < \gamma, \quad w(t) \neq 0.
\] (2.94)

where \( \| \cdot \|_\infty \) and \( \| \cdot \|_2 \) are infinity norm and Euclidean norm. By taking a supremum over the infinity norm of \( G_{zw}(s) \) ones can guarantee the RMS gain in (2.94) is always less than \( \gamma \). Thus, this can be expressed by

\[
\| G_{zw}(s) \|_\infty = \sup_{0 < \| w \|_2 < \infty} \frac{\| z \|_2}{\| w \|_2} < \gamma.
\] (2.95)

Therefore, in order to bound the infinity norm (2.95) for the PV system (2.57), ones can define function \( J(\tilde{x}_p(t), w(t)) \) such that

\[
J(\tilde{x}_p(t), w(t)) = \dot{V}(\tilde{x}_p(t)) + z_p^T(t)z_p(t) - \gamma_p^2 w^T(t)w(t) < 0.
\] (2.96)

\[
J(\tilde{x}_p(t), w(t)) = \dot{\tilde{x}}_p^T(t)P\tilde{x}_p(t) + \tilde{x}_p^T(t)P\dot{\tilde{x}}_p(t) + z_p^T(t)z_p(t) - \gamma_p^2 w^T(t)w(t) < 0.
\] (2.97)

for all \( \tilde{x}_p(t) \) and \( w(t) \). If a symmetric positive matrix \( P \in \mathbb{R}^{n \times n} \) can be found such that the inequality (2.97) holds, it can be guaranteed that the infinity norm of the system (2.57) is less than \( \gamma_p \). Here we aim to minimise the effect of input disturbance to plant output signal. Therefore,

\[
z_p(t) = \tilde{y}_p(t) = \tilde{C}_p \tilde{x}_p(t).
\] (2.98)

\[
\tilde{C}_p = \begin{bmatrix} C_p & 0 \end{bmatrix}.
\] (2.99)

Thus, by substituting \( z_p(t) \) in (2.97), we have

\[
J(\tilde{x}_p(t), w(t)) = \dot{V}(\tilde{x}_p(t)) + \tilde{x}_p^T(t)\tilde{C}_p^T \tilde{C}_p \tilde{x}_p(t) - \gamma_p^2 w^T(t)w(t) < 0.
\] (2.100)
Equation (2.100) can be rearranged in a form of linear matrix inequality as

\[
J(\tilde{x}_p(t), w(t)) = \begin{bmatrix} \tilde{x}_p(t) \\ w(t) \end{bmatrix}^T \begin{bmatrix} \hat{A}_p W_p + W_p \hat{A}_p^T + \hat{B}_{up} Y_p + Y_p^T \hat{B}_{up}^T + W_p \hat{C}_p^T \hat{C}_p W_p & B_{wp} \\ * & -\gamma_p^2 I \end{bmatrix} \begin{bmatrix} \tilde{x}_p(t) \\ w(t) \end{bmatrix} < 0. \tag{2.101}
\]

By using Schur complement, \( \Lambda \) in (2.101) can be written as

\[
\begin{bmatrix}
\hat{A}_p W_p + W_p \hat{A}_p^T + \hat{B}_{up} Y_p + Y_p^T \hat{B}_{up}^T & B_{wp} & W_p \hat{C}_p^T \\
* & -\gamma_p^2 I & 0 \\
* & 0 & -I
\end{bmatrix}
\tag{2.102}
\]

**Theorem 2.3.** The system (2.57) can be stabilised by (2.58) and \( \|z_p\|_2 / \|w\|_2 < \gamma_p \) if and only if there exist a symmetric positive definite matrix \( W_p \in \mathbb{R}^{n \times n} \) and a matrix \( Y_p \in \mathbb{R}^{m \times n} \) such that the following linear matrix inequality holds

\[
\begin{bmatrix}
\hat{A}_p W_p + W_p \hat{A}_p^T + \hat{B}_{up} Y_p + Y_p^T \hat{B}_{up}^T & B_{wp} & W_p \hat{C}_p^T \\
* & -\gamma_p^2 I & 0 \\
* & 0 & -I
\end{bmatrix} < 0. \tag{2.103}
\]

The same approach can also be applied to the energy storage system to minimise the effect of load variation at the DC bus (input disturbance) to its output.

**Corollary 2.3.** The system (2.60) can be stabilised by (2.61) and \( \|z_p\|_2 / \|w\|_2 < \gamma_p \) if and only if there exist a symmetric positive definite matrix \( W_b \in \mathbb{R}^{n \times n} \) and a matrix \( Y_b \in \mathbb{R}^{m \times n} \) such that the following linear matrix inequality holds

\[
\begin{bmatrix}
\hat{A}_b W_b + W_b \hat{A}_b^T + \hat{B}_{ub} Y_b + Y_b^T \hat{B}_{ub}^T & B_{wb} & W_b \hat{C}_b^T \\
* & -\gamma_b^2 I & 0 \\
* & 0 & -I
\end{bmatrix} < 0. \tag{2.104}
\]
2.4 Control Problem Formulation

2.4.4 Control Input Saturation

In DC-DC converters duty cycle saturation can cause performance degradation. This also may destabilise the whole system [89]. To avoid this from occurring, a constraint must be included in the formulation of LMI problem such that the control input signal (duty cycle) is maintained within an upper bound. Practically, the duty cycle that is using as control input signal is bounded within a certain range, \( D \in [D_{\min}, D_{\max}] \subset [0,1] \). The upper limit and lower limit of the duty cycle is defined by physical constant of pulse width modulation (PWM) and solid-state switching devices. This boundary ensures the saturation of solid state switching devices is avoided. The switching pulses generated by PWM must be either 0 or 1 when it is applied to any solid-state device to make a switching device off or on continuously. Therefore, this corresponds to \( D_{\min} - D_0 \leq u_e \leq D_{\max} - D_0 \) where \( D_0 \) is quiescent value of duty cycle. If one defines \( u_L = D_{\min} - D_0 \) and \( u_H = D_{\max} - D_0 \), a saturation function can be introduced for control input [90] such that,

\[
\text{sat}(u_p) = \begin{cases} 
  u_{pH}, & u_p > u_H. \\
  u_p, & |u_p| \in [u_L, u_H]. \\
  u_{pL}, & u_p < u_L.
\end{cases} 
\]  

(2.105)

Accordingly, the input signals in (2.57), \( u_p(t) \) can be replaced by saturation function \( u_p = \text{sat}(K\hat{x}_p) \). Therefore, control input bound, \(||u_p(t)|| < u_{p\text{max}}\), will not be breached, if for any initial condition \( \hat{x}_p(0) \), matrices \( W_p \in \mathbb{R}^{n \times n} \) and \( Y_p \in \mathbb{R}^{m \times n} \) can satisfy following inequalities [91].

\[
\begin{bmatrix} 
  W_p & Y_p^T \\
  * & u_{p\text{max}}^2
\end{bmatrix} > 0. 
\]  

(2.106)

\[
\begin{bmatrix} 
  W_p & I \\
  * & \mu_p
\end{bmatrix} > 0. 
\]  

(2.107)

where \( u_{p\text{max}} \) is the maximum admissible value for the control input signal, and \( \mu_p \) defines the region of stability. Thus, this can be similarly applied to the system (2.60) to specify an upper limit bound for its control input signal generated by (2.61).
2.4.5 Polytopic Parametric Uncertainties

As discussed earlier the PV output is stochastic. Thus, this is because the PV output is a function of solar radiation ($G$) and ambient temperature ($T$), which are weather dependent and change frequently. Furthermore, to guarantee maximum power extraction the unidirectional boost converter’s duty cycle ($D_p$) needs to be changed depends on the PV array output voltage. Thus, it results in deviation of duty cycle from its quiescent value simultaneously. The duty cycle is appeared in matrix $\tilde{A}_p$ as a linear form and as a nonlinear form in matrix $\tilde{B}_{up}$ of unidirectional converter state-space model (2.57). As the PV system is considered as the main power source, the variation of DC loads connected at the hybrid system’s DC bus needs to be taken into account for controller design of the PV system. To summarise, a vector of five uncertainties mentioned above, $\nu_p$, can be constructed for PV system.

$$\nu_p = \begin{bmatrix} V_{eq} \ D_p' \ \frac{1}{D_p'} \ \frac{1}{R} \end{bmatrix}. \quad (2.108)$$

Battery output voltage significantly depends on the current that is exporting or importing from the battery. Higher current causes quicker voltage drop (discharge mode) or rise (charge mode) in the battery. As a consequence, bidirectional converter’s duty cycle ($D_b$) needs to be varied on either required DC bus voltage or battery voltage (depends on control objective). Hence, the duty cycle may not remain at its quiescent value all the times. Similar to the unidirectional converter, in bidirectional converter duty cycle is appeared with the same character in both matrices $\tilde{A}_b$ and $\tilde{B}_{ub}$ in the system model (2.60). These imply that battery output voltage and duty cycle can be considered as uncertain parameters in the state-space model of the bidirectional converter expressed in (2.60). Hence, a vector of four uncertainties, $\nu_b$, can be constructed for the ESS.

$$\nu_b = \begin{bmatrix} V_b \ D_b' \ \frac{1}{D_b'} \ \frac{1}{V_b} \end{bmatrix}. \quad (2.109)$$

Both uncertainties vectors in (2.108) and (2.109) form a convex polytope for each system that can be defined by their vertices $\{\nu_1, ..., \nu_N\}$ for $N = 2^n$ where $n$ is the number of uncertainties in each system. The first polytopes containing all possible values of $\tilde{A}_p$ and
Figure 2.11: Projection of function $f(D')$, green lines: original polytope, black lines: new tetrahedron.

$\hat{B}_{up}$ for the system (2.57) can be expressed as

$$
\begin{bmatrix}
\hat{A}_p(v_p) & B_{up}(v_p)
\end{bmatrix} \in C_0\{v_1, ..., v_N\} := \left\{ \sum_{i=1}^{N} \lambda_i v_i, \lambda_i \geq 0, \sum_{i=1}^{N} \lambda_i = 1 \right\}, \quad \forall N = 1, ..., 5.
$$

(2.110)

where $N$ is the number of vertices for the system described in (2.57). Subsequently, a polytope can be constructed for the energy storage system described by (2.60).

$$
\begin{bmatrix}
\hat{A}_b(v_b) & \hat{B}_{ub}(v_b)
\end{bmatrix} \in C_0\{\sigma_1, ..., \sigma_M\} := \left\{ \sum_{i=1}^{M} \delta_i \sigma_i, \delta_i \geq 0, \sum_{i=1}^{M} \delta_i = 1 \right\}, \quad \forall M = 1, ..., 4.
$$

(2.111)

where $M$ is the number of vertices for the system (2.60). A nonlinear function, $f(D')$, can be written for three independent uncertain parameters of each converter introduced by its respective duty cycle $(D', \frac{1}{D'}, \frac{1}{D'^2})$.

$$
f(D') = \left\{ \left( D', \frac{1}{D'}, \frac{1}{D'^2} \right) : D' \in [D', \bar{D}'] \right\}.
$$

(2.112)

One can write a function of a cube described by eight vertices that covers (2.112). This cube is shown in Fig.2.11 with green lines. The projections of $f(D')$ onto the three planes are shown in figures 2.12a-2.12d. The number of vertices representing the nonlinear function (2.112) is $2^3 = 8$, and thus can be represented by a cuboid. Note that a cuboid with eight vertices can contain the $f(D')$ in a convex region, as any two points inside
it can be connected with an affine line within it. The dash lines in figures 2.12a-2.12d show the tangent lines at the two extreme points in each plane. As the extremes of figures (2.12a)-(2.12d) are all matched in a same Cartesian position, two vertices of the new convex polytopic can be obtained easily. The other two vertices are obtained from the intersection of tangent lines. Note that as four vertices cover smaller space compared with 8, it is evident that the new polytopic is less conservative [92]. This approach has identified by polytopic function covering by Amato in [93]. Hence, this makes the convex set less conservative and reduces the number of vertices from 8 to 4; This is possible because in practice the duty cycle of any converter is limited to avoid entering the converter to the high-nonlinear region. Figure 2.11 shows the new tetrahedron covering the function \( f(D') \) with black lines. Now, the PV system polytope can be expressed as a polytope with eight vertices, 4 for the duty cycle function and 4 for the PV voltage and the load resistance. Similarly, the energy storage system uncertainties can be defined by a polytope.
with six vertices, 4 for the duty cycle function and 2 for the battery voltage.

2.5 Control Design

In this section, the control design for the PV system, as well as the energy storage system, are explained. As it was mentioned in section 2.4.3, apart from the main control objective described in 2.3 the effect of input disturbances on each plant’s output must be minimised. Thus, to achieve this minimization, the formulated $H_\infty$ problem is expressed in the form of an optimisation problem with a minimising objective function. Hence, a convex optimisation problem can be solved such that the required linear matrix inequalities are satisfied while the $H_\infty$ (RMS gain) is minimised.

2.5.1 PV Controller

The controller design problem for the PV system can be stated by implementing theorems 2.2, 2.3, with control input saturation constraint. Thus, the PV system controller design is stated as following: Design a state-feedback controller (2.58) to stabilise the system (2.57) in the presence of parametric uncertainties. This can be solved by finding LMI variable matrices $W_p$ and $Y_p$ that minimise the $H_\infty$ norm $\gamma_p$ for all vertices of the PV polytope ($N = 8$).
\[
\begin{align*}
\min_{W_p, Y_p} & \gamma_p \\
\text{Subject To} & \\
\bar{A}_p W_p + W_p \bar{A}_p^T + \bar{B}_{up} Y_p + Y_p^T \bar{B}_{up}^T + 2x_p W_p < 0, \\
\begin{bmatrix} -r_p W_p & \bar{A}_p W_p + \bar{B}_{up} Y_p \\ * & -r_p W_p \end{bmatrix} < 0, \\
\begin{bmatrix} \sin \theta_p (\bar{A}_p W_p + W_p \bar{A}_p^T + \bar{B}_{up} Y_p + Y_p^T \bar{B}_{up}^T) & \cos \theta_p (\bar{A}_p W_p - W_p \bar{A}_p^T + \bar{B}_{up} Y_p - Y_p^T \bar{B}_{up}^T) \\ \cos \theta_p (-\bar{A}_p W_p + W_p \bar{A}_p^T - \bar{B}_{up} Y_p + Y_p^T \bar{B}_{up}^T) & \sin \theta_p (\bar{A}_p W_p + W_p \bar{A}_p^T + \bar{B}_{up} Y_p + Y_p^T \bar{B}_{up}^T) \end{bmatrix} < 0, \\
\begin{bmatrix} \bar{A}_p W_p + W_p \bar{A}_p^T + \bar{B}_{up} Y_p + Y_p^T \bar{B}_{up}^T & B_{wp} & W_p C_p^T \\ * & -\gamma_p^2 I & 0 \\ * & 0 & -I \end{bmatrix} < 0, \\
\begin{bmatrix} W_p \\ * & u_{p max}^2 \end{bmatrix} > 0, \\
\begin{bmatrix} W_p \\ * & \mu_p \end{bmatrix} > 0, \\
\forall \{v_i\}, i = 1, ..., N.
\end{align*}
\]

Because the PV system is considered as the main source of power, the disturbance rejection performance is the top priority. Therefore, the objective function is defined to minimise the RMS gain while the region of stability, \(\mu_p\), is assumed constant which requires to be specified by the designer.

### 2.5.2 Energy Storage System Controller

The energy storage system is constantly switching from one mode to another to regulated the DC bus voltage therefore, maximising region of stability of the energy storage controller becomes more crucial. Therefore, the objective function is defined as minimising \(H_\infty\) norm \(\gamma_b\), as well as maximising region of stability defined in (2.107) by \(\sqrt{1/\mu_b}\) which correlated by minimising \(\mu_b\). Therefore, the energy storage system controller, design can
be described as: Design a state-feedback controller (2.61) to stabilise the system (2.60) in the presence of parametric uncertainties, by finding \( W_b, Y_b \) matrices that minimise the \( H_\infty \) norm \( \gamma_b \) as well as \( \mu_b \) for all vertices of the energy storage system convex polytope \((M = 4)\).

\[
\begin{align*}
\min_{W_b, Y_b} (\gamma_b + \mu_b) \\
\text{Subject To} \\
\begin{bmatrix}
- r_b W_b & \tilde{A}_b W_b + \tilde{B}_{ub} Y_b \\
* & - r_b W_b
\end{bmatrix} < 0, \\
\begin{bmatrix}
\sin \theta_b (\tilde{A}_b W_b + \tilde{W}_b \tilde{A}_b^T + \tilde{B}_{ub} Y_b + Y_b^T \tilde{B}_{ub}^T) & \cos \theta_b (\tilde{A}_p W_p - W_p \tilde{A}_p^T + \tilde{B}_{up} Y_p - Y_p^T \tilde{B}_{up}^T) \\
\cos \theta_b (-\tilde{A}_b W_b + W_b \tilde{A}_b^T - \tilde{B}_{ub} Y_b + Y_b^T \tilde{B}_{ub}^T) & \sin \theta_b (\tilde{A}_b W_b + W_b \tilde{A}_b^T + \tilde{B}_{ub} Y_b + Y_b^T \tilde{B}_{ub}^T)
\end{bmatrix} < 0,
\begin{bmatrix}
\tilde{A}_b W_b + W_b \tilde{A}_b^T + \tilde{B}_{ub} Y_b + Y_b^T \tilde{B}_{ub}^T & B_{wb} & W_b \tilde{C}_b^T \\
* & - \gamma_b^2 I & 0 \\
* & 0 & -I
\end{bmatrix} < 0,
\begin{bmatrix}
W_b & Y_b^T \\
* & \mu_b^2_{\max}
\end{bmatrix} > 0,
\begin{bmatrix}
W_b & I \\
I & \mu_b
\end{bmatrix} > 0,
\forall \{ \sigma_i \}, i = 1, ..., M.
\end{align*}
\]

\[(2.114)\]

2.6 Simulation and Results

In this section, the effectiveness of the proposed, designed controllers for the PV system and the energy storage system is demonstrated. The described DC hybrid system in section 2.1 comprising a PV system and an energy storage system is modelled and simulated to show the control objectives are achieved. The simulation has been done in Matlab/Simulink [94]. Additionally, the systems’ parameters and the designed gains are
provided. Subsequently, an LQR controller and an output-feedback PI controller are designed with the same $D$-stability restrictions to establish a comparison with the proposed approach. Furthermore, comparison of the proposed model including existing parametric uncertainties with the proposed model excluding these uncertainties is provided to show the effectiveness of this consideration. Several case studies are defined to investigate the PV system controller performance as well as the energy storage system. It is shown that while the PV system is extracting the maximum power, the energy storage system can maintain the DC bus voltage at its desired voltage.

### 2.6.1 Designed Parameters

To verify the proposed control approach, a 6kW PV system consisting of 2 parallel arrays that each constructed by 15 series connected PV panel is used as a benchmark. A 2.5 Ah energy storage system consisting of 25 series connected lead acid battery is also used as the energy storage system connected to the DC bus. A total load of 4kW DC load connected to the DC hybrid system is considered, and the system adopted topology is islanded (off-grid). Table 2.1 shows the parameters that have been used during simulation. The $D$-stability criteria that have been utilised for the PV system and the energy storage system are illustrated in table 2.2. The value of $\alpha_p$ is chosen based on the fastest possible response that does not cause infeasible solution for PV system. However, $\alpha_b$ is chosen such that the energy storage system has an acceptable range for its RMS gain. As a rule of thumb both $r_p$ and $r_b$ are set to 1/10 of each converter’s switching frequency, to locate closed-loop poles inside a valid frequency range for each converter [81]. To guarantee minimum damping ratio of 0.4 both $\theta_p$ and $\theta_b$ has been set to 65º, obtained by

$$\theta = \cos^{-1}(\zeta).$$

(2.115)

where $\zeta$ is the required damping ration. To prevent saturation converters’ control input signals are assumed to be bounded by 0.8 ($\mu_p, \mu_b$), which represents the maximum value of the duty cycle for each system. The nominal voltage of the hybrid system DC bus ($V_{pcc}$) is 600 volts. Solving the optimisation problem (2.113) for the PV system and (2.114)
Table 2.1: Parameters of PV system and energy storage system.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_{mpp}$</td>
<td>7.61 A</td>
<td>$I_{p,mpp}$</td>
<td>15.22 A</td>
</tr>
<tr>
<td>$V_{mpp}$</td>
<td>26.3 V</td>
<td>$V_{p,mpp}$</td>
<td>394.50 V</td>
</tr>
<tr>
<td>$P_{mpp}$</td>
<td>200.134 W</td>
<td>$R_{eq}$</td>
<td>24.9816 Ω</td>
</tr>
<tr>
<td>$V_{oc}$</td>
<td>32.9 V</td>
<td>$V_{eq}$</td>
<td>774.7194 V</td>
</tr>
<tr>
<td>$I_{o,n}$</td>
<td>$9.825 \times 10^{-8}$ A</td>
<td>$L_{pv}$</td>
<td>34 mH</td>
</tr>
<tr>
<td>$I_{pv}$</td>
<td>8.214 A</td>
<td>$r_{Lp}$</td>
<td>0.001 Ω</td>
</tr>
<tr>
<td>$a$</td>
<td>1.3</td>
<td>$C_{opv}$</td>
<td>2.52 $\times 10^{-5}$ F</td>
</tr>
<tr>
<td>$R_p$</td>
<td>415.405 Ω</td>
<td>$C_{pv}$</td>
<td>0.1 $\times 10^{-6}$ F</td>
</tr>
<tr>
<td>$R_s$</td>
<td>0.221 Ω</td>
<td>$V_{Dpv}$</td>
<td>0.8 V</td>
</tr>
<tr>
<td>$r_{Dpv}$</td>
<td>$0.1 \times 10^{-5}$ Ω</td>
<td>$r_{on}$</td>
<td>0.1 $\times 10^{-6}$ Ω</td>
</tr>
<tr>
<td>$f_{spv}$</td>
<td>50 kHz</td>
<td>$f_{sb}$</td>
<td>20 kHz</td>
</tr>
<tr>
<td>$V_b$</td>
<td>300 V</td>
<td>$V_{PCC}$</td>
<td>600 V</td>
</tr>
<tr>
<td>$R_b$</td>
<td>1.2 Ω</td>
<td>$r_{Lb}$</td>
<td>9.6 $\times 10^{-3}$ Ω</td>
</tr>
<tr>
<td>$L_b$</td>
<td>0.0375 H</td>
<td>$C_{ob}$</td>
<td>1.3889 $\times 10^{-4}$ F</td>
</tr>
<tr>
<td>$r_{Cob}$</td>
<td>$5 \times 10^{-4}$ Ω</td>
<td>$C_{ib}$</td>
<td>2.0833 $\times 10^{-4}$ F</td>
</tr>
</tbody>
</table>

†: single PV panel   §: total PV system   ‡: total ESS

Table 2.2: D-stability criteria parameters for PV system and energy storage system.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_p$</td>
<td>137</td>
<td>$\alpha_b$</td>
<td>30</td>
</tr>
<tr>
<td>$r_p$</td>
<td>$\frac{2\pi}{10f_{spv}}$</td>
<td>$r_b$</td>
<td>$\frac{2\pi}{10f_{sb}}$</td>
</tr>
<tr>
<td>$\theta_p$</td>
<td>65°</td>
<td>$\theta_b$</td>
<td>65°</td>
</tr>
</tbody>
</table>
Decentralised Control for a DC Hybrid System

for the energy storage system using Matlab’s standard LMI toolbox [95], result state-feedback controller gains $K_p$ and $K_b$

$$K_p = \begin{bmatrix} -95.1 & 1.5 & 5.4 \\ -2075.5 \end{bmatrix} \times 10^{-3}. \quad (2.116)$$

$$K_b = \begin{bmatrix} 6.1 & 1.2 & 181.1 \\ -586.1 \end{bmatrix} \times 10^{-3}. \quad (2.117)$$

with a guaranteed $H_\infty$ bound for input disturbance attenuation $\gamma_p$ of 5.97 for the PV system. The simulation carried on such that the PV system is delivering maximum possible power at any instance to the DC bus while the energy storage system as the secondary power source is responsible for maintaining the microgrid DC bus voltage at its desired level (600V).

Linear Quadratic Regulator (LQR) for PV System

To compare the performance of the proposed method an LQR design technique is used for the same system such that the control law (2.58) minimises the performance index (2.118) for (2.57).

$$J = \int_0^\infty (e^T Q_p e + u_r^T R_p u_r) dt. \quad (2.118)$$

where $Q_p$ is a positive definite (semi-positive definite) matrix and $R_p$ is a positive definite matrix. By using the trace operator $\text{Tr}(.)$ properties, (2.118) can be written in the form of LMI [96]. Later on, theorem 2.2 is considered to satisfy the $D$-stability constraints for the LQR optimisation to restrict the poles locations. In addition to pole location restriction, the polytopic uncertainties constraint (2.110) is also considered as another constraint in the LQR optimisation problem (2.119). Thus, this makes LQR optimisation problem able to consider system uncertainties which, is not possible in classical LQR optimisation problem. Subsequently, Inequalities (2.106) and (2.107) are also considered as additional constraints for the LQR optimisation problem in order to limit the control input signal. Finally, the LQR optimisation problem can be written in a form of linear matrix inequali-
ties as

\[
\min_{W_p, Y_p, X_p} \text{Tr}(Q_p W_p) + \text{Tr}(X_p)
\]

Subject To

\[
\begin{bmatrix}
\tilde{A}_p W_p + W_p \tilde{A}_p^T + \tilde{B}_u Y_p + Y_p^T \tilde{B}_u^T + I < 0, \\
X_p & R_p^\dagger Y_p \\
* & W_p
\end{bmatrix} > 0,
\]

(2.88), (2.89), (2.90), (2.106), (2.107), and (2.110).

where \(W_p \in \mathbb{R}^{n \times n}\), \(Y_p \in \mathbb{R}^{m \times n}\) and \(X_p \in \mathbb{R}^{m \times m}\) and gain \(K_{LQR} = -Y_p W_p^{-1}\). Both weighing matrices \((Q_p, R_p)\) are chosen based on the proposed method in [97] in order to obtain optimal weighting matrices. Solving the optimisation problem (2.119) using Matlab’s standard LMI toolbox, results a state-feedback controller \(K_{LQR}\) for the PV system as following

\[
K_{LQR} = \begin{bmatrix}
-0.2393 & 0.0044 & 0.0088 & -3.9145
\end{bmatrix}.
\]  

(2.120)

### Output PI-controller for PV System

An output PI-controller as a conventional control technique is designed to compare the performance of the proposed method in this chapter. The PI gains are designed such that the controller satisfies the same \(D\)-stability criteria as in table 2.2.

- \(KP = -0.699 \times 10^{-3}\).  

(2.121)
- \(KI = -0.486\).  

(2.122)

### Nominal PV system Controller

To evaluate the effectiveness of considering uncertainties the optimisation problem of (2.113) is resolved without counting the present parametric uncertainties. Therefore, the designed controller has not examined the possible uncertainties within the PV system. Solving this optimisation problem by using Matlab’s standard LMI toolbox, results in a
state-feedback controller $K_{nom}$ for the PV system is given as

$$K_{nom} = \begin{bmatrix} -0.2927 & 0.0072 & 0.0032 \\ -2.2283 \end{bmatrix}. \quad (2.123)$$

### 2.6.2 Results

In this section by using various case studies, performance of the PV system and the energy storage system controllers are investigated. These case studies illustrate and conclude the effectiveness of the proposed control design approach for the PV system and the energy storage system which are the main components of a DC hybrid system. The simulation has been done by using Matlab/Simulink [94]. It is worth to note that the standard testing condition (STC) in the following studies refers to a condition that the PV arrays are exposed to solar radiation of 1000 $W/m^2$ with ambient temperature of 25 Celsius.

#### Case Study I: maximum power reference tracking

In this case, the performance of PV system in transition from nominal condition to the non-nominal condition is evaluated. We start operating the PV system under nominal standard test condition (STC) which consider as 1000 $W/m^2$ for solar radiation, and the load is kept unchanged, 42 Ohms. At 0.05 second the solar radiation, exposed to PV panels, is dropped by 200 $W/m^2$. At 0.1 second, it is followed by another plunge of the same step while the solar radiation stabilises at 600 $W/m^2$. However, at 0.15-second the solar radiation is increased to its STC value (1000 $W/m^2$) again. Figure 2.13 shows the PV output voltage regulated by the proposed method including uncertainties, the proposed method excluding uncertainties, the LQR LMI based approach and the output feedback PI-controller respectively. The proposed method here shows better performance with lower over and under shoot as well as less oscillation. In Figure 2.14 the converter output current is depicted to illustrate that, less current is dissipated by proposed controller. In Figure 2.15 the extracted power from the PV array is shown. It is clear that the proposed method has dissipated less power and as a result has extracted more power in compari-
Case Study II: PV system disturbance rejection

In this case, performance of the PV system is analysed in a presence of disturbance exposed at the converter output. During this case, the PV system is operating at its nominal condition, and it is assumed that the solar radiation is constant. At 0.1 second a 2 Amps step current which is equal to 25% of the DC hybrid system nominal load current appears at the converter output. This disturbance remains in the system for the rest of simulation. As a result, the PV system controller is responsible for rejecting the disturbance occurred and remained in the system. Figures 2.16 and 2.17 show the effect of disturbance appearance on PV output voltage and current respectively. Despite this effect, the PV system controller is responsible for rejecting or minimising this disturbance effect. In comparison with LQR technique, the proposed method here with uncertainties has superior
performance to reject this effect in shorter time with a lower peak. It is obvious, shown in Fig. 2.18, that the proposed approach achieved a better performance and lower power dissipation in compared to the other approaches.
Case Study III: Direct versus indirect control of PV system

As it was mentioned in the introduction, direct control of boost converter with MPPT elevates switching stress and losses [20]. These can reduce the system performance. Figures 2.19 and 2.20 shown a comparison of PV system output power and converter’s inductance current for the same system with direct duty cycle control approach against the proposed method in this chapter. As in direct control method converter losses is increased, less power can be delivered to the load by %16 at low sun radiation (Fig.2.19). It is also evident that the direct control method has more fluctuation in both PV output power and converter current.

Case Study IV: Variable solar radiation

In this case, the effect of variable power generation by the PV system, while the demand on DC bus remains constant, is investigated. As mentioned in section 2.3.2, the energy storage system is implemented to maintain the DC bus voltage within its upper and
lower limits, as specified by the IEEE standards. Therefore, as long as the DC bus voltage remains within the limit \((V_L\) and \(V_H)\), the energy storage system operates in idle mode, i.e. no charge or discharge occurs. However, if the amount of power generated by the PV system is more than consumed power by the load that causes the imbalance of power exceeds \(\delta_p\) in (2.51), which reflects a voltage rise above the upper limit on the DC bus, the energy storage system starts to operate in charge mode to absorb the surplus of power. As a result, the DC bus voltage is controlled to remain at its upper limit \((V_H)\).

Nonetheless, when the power generated by the PV system is lower than the demand (load), i.e. when the condition in (2.51) is violated, a voltage dip below the lower limit of the DC bus occurs. This will initiate the energy storage system to operate in discharge mode. In other words, the power shortage is provided by the energy storage system to ensure that the DC bus voltage remains at its lower permissible limit \((V_L)\). The operation of the energy storage system in its different modes, and the transition between the modes, is now illustrated through the following case study (scenario) carried out on the system of Fig.2.1.

**Scenario (Figure 2.21):** From time 0.0 to 0.4 seconds the system is assumed to operate at equilibrium, i.e. the solar radiation of 800\(W/m^2\) to generate the required load demand of 4,000W. Then, the radiation level is suddenly dropped by 300\(W/m^2\) (to become 500\(W/m^2\)), and remains so until 0.8 seconds. Next, the solar radiation level is increased back to 800\(W/m^2\), and remains constant between 0.8 to 1.2 seconds. From 1.2 to 1.6 seconds the solar radiation is increased to its maximum value of 1000\(W/m^2\). Finally, from 1.6 to 2.0 seconds, the radiation level drops by 500\(W/m^2\), and remained steady at
As a result of the solar radiation changes shown in Fig. 2.21, the generated power by the PV system experiences corresponding changes as shown in Fig. 2.22, ignoring the negligible transient periods. For the above scenario two cases were considered: (i) without an energy storage system being connected to the DC bus of the hybrid system, and (ii) with an appropriate energy storage system connected as per Figure (2.1) to the DC bus.

**Without Energy Storage:** This case demonstrates that the absence of an energy storage system, the DC bus voltage will fluctuate (uncontrolled increase or decrease) and may shoot beyond the upper and/or lower permissible limits, as shown in Fig. 2.23. The figure demonstrates that from 0.4 to 0.8 seconds the DC bus voltage decreases to below the lower threshold \( V_L \), in response to the reduction in the radiation level from 800 \( W/m^2 \) to 500 \( W/m^2 \). This voltage dip is clearly caused by insufficient amount of power being generated by the PV system. In addition, between 1.2 to 1.6 seconds, the PV system generates more power due to a rise in the radiation level, reflected by the increase in the DC bus voltage to above its upper limit \( V_L \).
With Energy Storage: This case illustrates that the presence of an appropriate energy storage system can maintain the DC bus voltage within its permissible limits, as shown in Fig. 2.24. The figure clearly shows that the DC bus voltage is being regulated by interfacing the energy storage system with the PV system in the presence of the changes in the solar radiation levels depicted in Fig. 2.21.

Energy Storage Charge and Discharge: Figure 2.25 illustrates the reasons behind the hybrid system being able to keep the voltage levels within its permissible voltage limits, which for the system of Fig. 2.1 are 1.1pu and 0.90pu. The figure shows that the state of charge of the energy system at equilibrium is 77.8%. However, when the radiation level changes from 800W/m² to 500W/m² at 0.4 sec, an imbalance in the amount of power generated and the demanded occurs, which results in a drop in the DC bus voltage to below its lower limits as shown in Fig. 2.23. This imbalance in the power will be compensated by the energy storage system, which turns to the discharge mode, as shown in Fig. 2.25. The discharge stops when the power balance is restored, which occurs at 0.8 seconds.

As the radiation level is restored to 800W/m², the DC bus voltage level returns back
to within the permissible limits and consequently the energy storage system remains in the idle mode. As the radiation level changes again from $800 \, \text{W/m}^2$ to $1000 \, \text{W/m}^2$ at 1.2 seconds, the voltage level increases to beyond its upper limit, due to the excess power being generated by the PV system. In order to bring the voltage level back to within the limits, the energy system starts to charge (absorb) until all of the excess power is stored, and a state of power balance occurs again.

**Case Study V: Variable loading condition**

In this case, the power generated by the PV system is kept unchanged, as shown in Fig.2.26, assuming that the radiation level remains constant at $800 \, \text{W/m}^2$, however the loading condition is changed as depicted in Fig.2.27. From time 0.0 to 0.4 seconds, the load demand is assumed to be at its nominal value (4000W). Then, at 0.4 seconds the demand increased by 30% (i.e. the demand is increased to 5,200W) and remains steady until 0.8 seconds. Then, from 0.8 to 1.2 seconds the demand is reduced to its nominal value. Following that from 1.2 seconds until 1.6, the demand increased by 45%. Finally,
the demand drops by 58% to be 3500W. Figure 2.28 shows the regulated DC bus voltage, and Fig.2.29 demonstrates the state of charge of the energy storage system, in response to the loading conditions described above.

**Energy Storage Charge and Discharge:** Figure 2.29 shows that the state of charge of the energy system at equilibrium is 77.8%. However, when the load demand changes from 4000W to 5200W at 0.4 seconds, an imbalance in the amount of power generated and the demanded occurs, which results in a drop in the DC bus voltage to below its
lower limits as shown in Fig. 2.28. This imbalance in the power will be compensated by
the energy storage system, which turns to the discharge mode, as shown in Fig. 2.29. The
discharge stops when the power balance is restored, which occurs at 0.8 seconds.

As the load demand is restored to 4000W, the DC bus voltage level returns back to
within the permissible limits and consequently the energy storage system remains in the
idle mode. As the load demand changes again from 4000W to 5800W at 1.2 seconds,
which results in a drop in the DC bus voltage to below its lower limits as shown in Fig.
2.28. Once again this imbalance in the power will be compensated by the energy storage
system, which turns to the discharge mode. At 1.6 seconds the load demand changes
from 5800W to 3500W, which results to increase in the DC bus voltage above its upper
limit, as shown in Fig. 2.28, due to the excess power being generated by the PV system.
In order the bring the voltage level back to within the limits, the energy system starts to
charge (absorb) until all of the excess power is stored, and a state of power balance occurs
again.

**Case Study VI: Disturbance rejection performance**

The last case investigates the robustness of the proposed control approach for the energy
storage system against an exposed disturbance caused by the load variation at the DC
bus. Therefore, the solar radiation is considered constant while a disturbance equal to
50% of load’s nominal current is applied at the DC bus from 0.5 seconds to 0.7 seconds
which reflects the effect of 50% load shedding in the MG. The energy storage controller
was able to reject the effect of the experienced disturbance in a short period shown in
Fig. 2.30.

**Performance Index**

To provide a quantitative evaluation on the performance of the proposed PV system con-
troller against the conventional one, an integral time absolute error (ITAE) has been used
as a performance index. This index is commonly used to assess the transient response of
systems. The value of ITAE is expressed in equation (2.124), where \( t \) is time and \( E(t) \) is
Figure 2.30: DC bus regulated voltage at the presence of disturbance for case VI.

The results of tracking performance and disturbance rejection performance are illustrated in Fig.2.31. It is evident that the proposed state-feedback controller has significantly superior performance index in compared with the same controller that ignores the parametric uncertainties (nominal), and the designed linear quadratic regulator (LQR).

2.7 Summary

In this chapter, a robust control approach by an implementation of linear matrix inequality technique (LMI) is proposed for a DC hybrid system with an islanded (off-grid) topology. The control objective is defined for PV system such that the maximum power extraction is ensured while the energy storage system is responsible for regulating the hybrid system DC bus voltage. Both nonlinearities and uncertainties for the PV system and the energy storage system converters are taken into account. Comparison of the proposed method including existing parametric uncertainties with the proposed method excluding these uncertainties shows the effectiveness of this consideration. Also, the proposed method shows a superior performance in compared with the LQR controller and the output feedback PI controller with the same $D$-stability restrictions. The results demonstrated that the proposed method is efficiently able to stabilise the system with less power loss for the PV system in compared with the other methods. The $H_\infty$ designed
controllers assure better performance for each controller at the presence of possible disturbances caused by load variation. The control approach satisfied the required transient performance during both nominal and non-nominal condition despite the presence of parametric uncertainties within the hybrid system. The highlighted advantage of this approach is that the designed controllers based on state-feedback technique can be synthesised automatically.
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Chapter 3
Coupled Control of a DC Hybrid System

In this chapter, a robust controller design based on linear matrix inequality (LMI) for a hybrid system is presented. The hybrid system consists of a photovoltaic (PV) system, and an energy storage system (ESS) integrated at the DC bus. The control design takes into account the interaction between the PV and ESS components, which is commonly neglected in existing literature. Coupled and decoupled controller designs are carried out for the hybrid system with the objective of controlling both the PV maximum power extraction and DC bus voltage. The results of both controller designs are compared against each other and show that ignoring the interaction has a significant detrimental effect on the performance of the system. In fact, the results indicate that coupled control improves system performance up to sixfold in comparison with existing decoupled approaches.

3.1 Introduction

HYBRID DC systems comprising a PV system and an energy storage system are predominantly used to provide electricity for small scale consumers. These systems can be utilised from hundred watts up to 10 kilowatts. They can be implemented either as a grid-connected system or islanded system. For the grid-connected systems, the hybrid system is used as the secondary power source, though in the islanded mode the hybrid system is adopted as the primary source of energy. Remote and rural areas which are considered as small scale consumers are not reliably connected to national grids. Therefore, hybrid systems in these locations are usually considered as the primary source of electricity due to their islanded topology. In chapter 2, the proposed control approaches were described to control such a hybrid system, of which the PV system with unidirec-
onal boost converter and energy storage system with bidirectional buck-boost converter were connected in parallel. The parallel connection of switching converters introduces an interaction between the PV system and the energy storage system. Accounting this interaction is important in systems comprising two or more sub-systems linked and interact together. The current published works deal with and design controller for each sub-system separately, and thus this interaction has been ignored. This is due to the fact that the controller designs were carried out for the PV system and energy storage system separately that does not account the interaction between the two control loops. Therefore, this naturally leads to a suboptimal outcome in comparison with any other approaches that considered the interaction through designing a single control system where the dynamics of the interaction is fully included. Additionally, the existing control approaches which does not consider the interaction within the control-loops, are not able to achieve a desired transient behaviour. For small scale hybrid systems such as the one discussed in Fig. 3.1, which consumers are solely relying on the hybrid system, the system performance and efficiency are crucial. These improvements may extend the energy storage unit (e.g. lead acid batteries) operating life time. Thus, proposing a coupled control method for a small scale DC hybrid system that considers the system interaction was the main motivation of this chapter. In this chapter, the PV-ESS is modelled as one system and a controller is designed for it, thus taking into account the interaction. The obtained results shown an improvement in the system transient performance as well as the system efficiency.

3.2 Description of State-space Model

In the previous chapter, the state-space model of PV system with unidirectional boost converter and energy storage system with bidirectional buck-boost converter were separately derived while these systems were connected in parallel. However, the parallel connection of DC-DC converters at DC bus introduces an interaction between the PV system and energy storage system control-loops, which is ignored in existing controller designs. The existence of this interaction between the two control loops profoundly affects
3.2 Description of State-space Model

The transient performance of the whole system if it is neglected. Therefore, in response to this problem this section dedicates to introduce a new model that considers the interaction, in controller design problem, between PV system and energy system connected in parallel with a common DC bus that forms a hybrid system. Thus, in the following the hybrid system shown in Fig. 3.1, comprising a PV system and an energy storage system, is modelled as a multi-input-multi-output (MIMO) system. It is worth to note that the input, output, and performance measures for the modelled hybrid system in this chapter are similar to those described in chapter 2, preliminary section. This in fact is because the same systems are used in this chapter though, with different modelling approach.

A fifth-order average state-space model of the hybrid system, shown in Fig.3.1, can be developed by considering each switching state \( S_i \) such that \( S_i \in [0, 1] \) for \( i = 1, 2, 3 \) and both \( S_2 \) and \( S_3 \) switches are operating synchronously. This introduces four different operating modes, as illustrated in table 3.1. Figure 3.2 shows the hybrid system switching states at each operating mode. An average state-space model for the hybrid system can

Table 3.1: switching operating modes of hybrid system.

<table>
<thead>
<tr>
<th>State</th>
<th>( S_1 )</th>
<th>( S_2 )</th>
<th>( S_3 )</th>
<th>Average Duty</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>ON</td>
<td>ON</td>
<td>OFF</td>
<td>( D_p(1 - D_b) )</td>
</tr>
<tr>
<td>2</td>
<td>ON</td>
<td>OFF</td>
<td>ON</td>
<td>( D_pD_b )</td>
</tr>
<tr>
<td>3</td>
<td>OFF</td>
<td>ON</td>
<td>OFF</td>
<td>( (1 - D_p)(1 - D_b) )</td>
</tr>
<tr>
<td>4</td>
<td>OFF</td>
<td>OFF</td>
<td>OFF</td>
<td>( (1 - D_p)D_b )</td>
</tr>
</tbody>
</table>

Figure 3.1: Schematic diagram of an islanded DC hybrid system.
Coupled Control of a DC Hybrid System

Figure 3.2: Circuit diagram of the DC hybrid system at each operating state.
be obtained by averaging over four different operating modes illustrated in Fig.3.2. This average state-space model is obtained over one switching period, $T_s$, for each converter. Therefore, the average MIMO state-space model of the hybrid system in Fig.3.1 can be expressed as

\[
\dot{x}(t) = A_{avg} x(t) + B_{avg} u(t). 
\] (3.1)
\[
y(t) = C_{avg} x(t) + E_{avg} u(t). 
\] (3.2)

where the system matrices in (3.1) and (3.2) are given as

\[
A_{avg} = \begin{bmatrix}
-\frac{(r_{lp}-r_{dp})}{L_p} & \frac{1}{L_p} & -\frac{(1-D_p)}{L_p} & 0 & 0 \\
-\frac{1}{C_{eq}C_{po}} & 0 & 0 & 0 & 0 \\
\frac{(1-D_p)}{C_{op}} & 0 & -\frac{1}{R_{eq}C_{op}} & 0 & \frac{(1-D_b)}{C_{ob}} \\
0 & 0 & 0 & \frac{1}{L_b} & -\frac{1}{L_b} \\
0 & 0 & 0 & -\frac{(1-D_b)}{L_b} & \frac{1}{L_b} - \frac{(1-D_b)r_{on2}+r_{on1}+r_{on2}D_b}{L_b}
\end{bmatrix}. 
\] (3.3)

\[
B_{avg} = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}. 
\] (3.4)

\[
C_{avg} = \begin{bmatrix}
0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 \\
\end{bmatrix}. 
\] (3.5)

After that, by using small signal approximation the average state-space model of (3.1)-(3.2) is linearised around the system’s equilibrium point. A linearised multi-input multi-output state-space model of the hybrid system has derived as follows

\[
\dot{x}(t) = Ax(t) + Bu(t) + B_w w(t). 
\] (3.6)
\[
y(t) = Cx(t). 
\] (3.7)
where

\[
A = \begin{bmatrix}
-\frac{(r_{lp}-r_{Dp})+(r_{onp}-r_{Dp})D_p}{L_p} & \frac{1}{L_p} & -\frac{(1-D_p)}{L_p} & 0 & 0 \\
-\frac{1}{C_{pp}} & -\frac{1}{R_{pp}C_{pp}} & 0 & 0 & 0 \\
(1-D_p) & 0 & -\frac{1}{R_{cp}C_{op}} & 0 & \frac{(1-D_b)}{C_{op}} \\
0 & 0 & 0 & -\frac{1}{R_{cb}C_{ob}} & -\frac{1}{C_{ob}} \\
0 & 0 & -\frac{(1-D_b)}{L_b} & \frac{1}{L_b} & -(1-D_b)(r_{on3}+r_{on2}+r_{on2}D_b)
\end{bmatrix}.
\] (3.8)

\[
B = \begin{bmatrix}
\frac{(-r_{onp}+r_{Dp})I_{lp}+V_{dc}}{L_p} & 0 \\
0 & 0 \\
-\frac{I_{lp}}{C_{pp}} & -\frac{I_{lp}}{C_{op}} \\
0 & 0 \\
0 & \frac{(-r_{on2}+r_{on3})I_{lb}+V_{dc}}{L_b}
\end{bmatrix}.
\] (3.9)

\[
C = \begin{bmatrix}
0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0
\end{bmatrix}.
\] (3.10)

\[
B_w = \begin{bmatrix}
0 & 0 & -\frac{1}{C_o} & 0 & 0 & 0 & 0
\end{bmatrix}^T.
\] (3.11)

### 3.3 State-feedback Design

Since all states are measurable a state-feedback controller, \( u(t) = -Kx(t) \), is feasible and can be implemented to regulate and stabilise the hybrid system. To attain zero steady-state error for both of the system’s outputs, two integrators need to be inserted in feed-forward paths. The two integrators can be incorporated into the design process by augmenting the state equation (3.6) with the two integrators to yields

\[
\dot{x}(t) = \dot{A}x(t) + \dot{B}_w u(t).
\] (3.12)

\[
u(t) = -Kx(t).
\] (3.13)
3.3 State-feedback Design

where

\[
\mathbf{\dot{x}}(t) = \begin{bmatrix} i_{Lp} & v_{pv} & x_3(t) & v_{dc} & v_b & i_b & x_7(t) \end{bmatrix}^T. 
\] (3.14)

\[
\mathbf{\check{A}} = \begin{bmatrix} A & 0_{5 \times 5} \\ -C & 0_{2 \times 2} \end{bmatrix}. 
\] (3.15)

\[
\mathbf{\hat{B}}_u = \begin{bmatrix} B \\ 0_{2 \times 2} \end{bmatrix}. 
\] (3.16)

where \(x_3(t)\) and \(x_7(t)\) are the states of integrators.

3.3.1 Problem Formulation

In the proposed controller design, the following considerations are addressed: (i) attainment of desired transient performance of the hybrid system; This can be achieved by placing the closed-loop poles at certain locations in the s-plane; (ii) avoidance of control inputs saturation in both of the PV and energy storage converters; (iii) smooth transition from one operating condition to another due to sudden changes in the reference inputs; and (iv) adequate level of disturbance rejection capability, i.e. robustness in the controller design. In the following, the above considerations are incorporated in the formulation of an LMI-based robust control design problem.

D-stability Formulation

D-stability criteria can be used to place the closed-loop poles in the desired region in the complex-plane to achieve a desired transient response (decay rate). In chapter 2, pole placement assignment was defined by using theorem 2.2. In this chapter theorem 2.2 is modified to define the D-stability region, \(\Psi\), for any pair of complex pole, \(x \pm jy\), in the complex-plane set \(S\) such that

\[
\Psi = \{x \pm jy \in S : -\alpha_1 < x < -\alpha_2 < 0\}. 
\] (3.17)
The permitted pole placement location by (3.17) for the system (3.12) is illustrated by Fig.3.3. Therefore, to assign the closed-loop poles of the system (3.12) within the shaded area shown in Fig.3.3, the modified theorem 2.2 can be stated as the following.

**Theorem 3.1.** The system (3.12) is stabilisable by the means of control law (3.13) if and only if there exist a symmetric positive definite matrix \( W \in \mathbb{R}^{n \times n} \) and a matrix \( Y \in \mathbb{R}^{m \times n} \) such that the closed-loop poles are only located in a bounded region \( \Psi \) expressed by (3.17), such that \( \alpha_2 \) defines the system minimum decay rate if and only if the following linear matrix inequalities hold.

\[
\dot{A}W + WA^T + \bar{B}_uY + YT\bar{B}_u^T + 2\alpha_1W > 0, \quad (3.18)
\]

\[
\dot{A}W + WA^T + \bar{B}_uY + YT\bar{B}_u^T + 2\alpha_2W < 0, \quad (3.19)
\]

\[
\begin{bmatrix}
\sin \theta(\dot{A}W + WA^T + \bar{B}_uY + YT\bar{B}_u^T) \\
\cos \theta(\dot{A}W + WA^T + \bar{B}_uY - YT\bar{B}_u^T)
\end{bmatrix} < 0. \quad (3.20)
\]

The choice of \( \alpha_1 \) and \( \alpha_2 \) determine the setting time and the choice of theta determines the level of oscillation (overshoot) required for the control system to achieve. In real design environment, these are chosen according controller design transient and static specifications, which are mainly settling time and overshoot (level of allowable oscillations). In the design procedure \( \alpha_1 \) is commonly chosen to be 10 times the switching frequency, hence \( \alpha_1 = 10 \times f_s \). However \( \alpha_2 \) was determined after several simulations trials to obtain the fastest transient decay rate within the prescribed region.
Control Input Saturation

As it was explained in the previous chapter, duty cycle saturation can cause performance degradation in the performance of DC-DC converters and also destabilise the whole system. Hence, control input saturation needs to be addressed in the controller design. Control input saturation for both energy storage system and PV systems can be avoided if following inequalities are satisfied [91].

\[
\begin{bmatrix}
W & Y^T \\
Y & \min(u_i^2)
\end{bmatrix} \geq 0, \quad \forall i \in [1,2]. \quad (3.21)
\]

\[
W - \mu^{-1} \geq 0.
\quad (3.22)
\]

where \(u_i\) is maximum admissible value for each control input signals, and \(\mu\) defines the region of stability. It is evident that by maximising \(\mu\) the stability region is maximised and the control input saturation for both PV and energy storage system is avoided.

Control Output Signal Overshoot

To achieve a smooth the transient response of the hybrid system a bound on the maximal output signals can be implemented in the form of linear matrix inequality.

\[
\begin{bmatrix}
\gamma_i & C_i W \\
WC_i^T & W
\end{bmatrix} \geq 0, \quad \forall i \in [1,2]. \quad (3.23)
\]

where \(y_1(t) = C_1x(t)\) and \(y_2(t) = C_2x(t)\) are the hybrid system outputs, while \(\gamma_i\) is the respective bounded value for each output. It is to be noted that minimising \(\gamma_i\) guarantees reduction in both overshoots and undershoots of each output signal.

Uncertainty

The state-space model of equation (3.6) is linearized around a given converters nominal equilibrium point, where the duty cycles are fixed. But in reality, the duty cycles are continuously adjusted by the controller to extract maximum power out of the PV system.
as well as regulating DC bus voltage by adjusting energy storage DC-DC converter duty cycle. As a result, the values of $D_p$ and $D_b$ are constantly changing but within a boundary. These parametric uncertainties appear in matrix $\tilde{A}$ shown by (3.15). Then, a vector $p$ consisting of each duty cycle’s lower and upper limit can be constructed as

$$p = [\bar{D}_p, \bar{D}_p, \bar{D}_b, \bar{D}_b].$$

(3.24)

The over-bar line indicates upper limit value and under-bar indicates lower limit value. Vector $p$ forms a convex polytopic that can be defined by its vertices $\{v_1, ..., v_N\}$ for $N = 2^n$ where $n$ is the number of uncertainties and $N$ is the number of vertices.

$$[\bar{\tilde{A}}(p), \bar{B}_u(p)] \in \text{Co}\{v_1, ..., v_N\} := \left\{ \sum_{i=1}^{N} \lambda_i v_i, \lambda_i \geq 0, \sum_{i=1}^{N} \lambda_i = 1 \right\}.$$  

(3.25)

Thus, the constructed polytope (3.25) should be considered as a constraint during problem formulation.

### 3.3.2 Controller Design

The control objective is defined by a constrained weighted-sum objective function such that (i) the boost converter input voltage is regulated to extract maximum power from PV system, and the DC bus voltage is regulated at a constant value, (ii) control input saturation is avoided, and (iii) maximal region of stability is achieved. The optimisation
problem is stated as follows:

$$\inf_{W>0, Y, \mu, \gamma_i} f(\mu, \gamma_i) \quad \forall i \in [1, 2]$$

Subject To

$$\tilde{A}W + W\tilde{A}^T + \tilde{B}_u Y + Y^T \tilde{B}_u^T + 2\alpha_1 W > 0,$$

$$\tilde{A}W + W\tilde{A}^T + \tilde{B}_u Y + Y^T \tilde{B}_u^T + 2\alpha_2 W < 0,$$

$$\begin{bmatrix}
\sin \theta(\tilde{A}W + W\tilde{A}^T + \tilde{B}_u Y + Y^T \tilde{B}_u^T) & \cos \theta(\tilde{A}W - W\tilde{A}^T + \tilde{B}_u Y - Y^T \tilde{B}_u^T) \\
\cos \theta(-\tilde{A}W + W\tilde{A}^T - \tilde{B}_u Y + Y^T \tilde{B}_u^T) & \sin \theta(\tilde{A}W + W\tilde{A}^T + \tilde{B}_u Y + Y^T \tilde{B}_u^T)
\end{bmatrix} < 0,$$

$$\begin{bmatrix}
W & Y^T \\
* & \min(u_i^2)
\end{bmatrix} \geq 0, \quad \forall i = 1, 2,$$

$$W - \mu^{-1} \geq 0,$$

$$\begin{bmatrix}
\gamma_i & C_i W \\
* & W
\end{bmatrix} \geq 0,$$

$$\forall \{\upsilon_i\}, i = 1, \ldots, N.$$

(3.26)

where

$$f(\mu, \gamma_i) = w_1 \mu + w_2 \gamma_1 + w_3 \gamma_2.$$ 

(3.27)

Note that minimising $\mu$ is identical to maximising $\mu^{-1}$ which translates to maximising the region of stability. In equation (3.27), $w_i, \forall i = 1, 2, 3$ are weighting gains that reflect the importance of each parameter in optimisation cost function. The values of $\alpha_1$ and $\alpha_2$ are chosen before solving the optimisation problem (3.26) in order to satisfy the transient response requirement.

### 3.4 Simulation and Results

To illustrate the advantages of the proposed control approach in this chapter over the proposed approach in chapter 2, that neglects the existence of interaction between two systems, the same hybrid system as in chapter 2 is adopted. The optimisation problem (3.26) is solved by using standard LMI Matlab toolbox while the simulation carried out
in Matlab/Simulink.

The values of \( \alpha_1 \) and \( \alpha_2 \) are chosen according to controller design transient and static specifications, which are mainly settling time and overshoot (level of allowable oscillations). Therefore, the left hand side, \( \alpha_1 \), of set \( \Psi \) in (3.17) is defined such that closed-loop poles are located within the valid frequency range \( (\alpha_1 = 10 \times f_s) \) [81]. However, \( \alpha_2 \) in (3.19) that determines the settling time was concluded after several simulations trials to obtain the fastest transient decay rate within the feasible region \( (\alpha_2 = 130) \); This ensures that the optimisation problem 3.26 has a feasible solution. additionally, the value \( \theta \) in (3.26) is considered as 65° in order to obtain minimum damping ratio of 0.8. The modelling of the PV system for the propose of computer simulation follows the PV cell model described in chapter 2, section 2.2.1.

3.4.1 Designed Parameters

First, a coupled state-feedback controller, proposed in this chapter, is designed for the hybrid system described in (3.6) by solving the optimisation problem (3.26), and the designed state-feedback gain matrix is presented in (3.28). Secondly, the contraction terms of both matrices \( \tilde{A} \) and \( \tilde{B}_u \) in (3.6) are considered zero. Therefore, the obtain state-feedback gains obtained from (3.26) resembles the conventional design approach such as the one proposed in the previous chapter. The computed decoupled state-feedback gain is presented by (3.29).

\[
K_{CP} = \begin{bmatrix}
0.0321 & -0.0172 & 0.0001 & -0.0016 & 2.3933 & -0.0229 & -2.8396 \\
0.0118 & 0.0008 & 0.0002 & 0.0068 & -0.0454 & 0.0268 & -3.6644
\end{bmatrix}.
\] (3.28)

\[
K_{DCP} = \begin{bmatrix}
0.0221 & -0.01 & 1.5859 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0.0099 & 0.0004 & 0.0148 & -3.7971
\end{bmatrix}.
\] (3.29)

An LMI-based LQR state-feedback controller, described in chapter 2, is also designed for the system described by (3.6) subject to the same constraints as in (3.26). The obtained
LQR gain is presented by (3.30).

\[
K_{LQR} = \begin{bmatrix}
0.2533 & -0.2642 & 0.0651 & 0.0106 & -0.0034 & 28.6581 & 6.6053 \\
0.1687 & 0.0007 & 0.1248 & 0.0174 & 0.0841 & 1.9892 & -13.7624
\end{bmatrix}.
\] (3.30)

The optimal choice of weighting matrices for the designed linear quadratic regulator is described in the following.

**Choice of Matrix Q**

The choice of state weighting matrix, \(Q\), in the linear quadratic regulator is depending on physical characteristic as well as the practicality of the studied system. In a multi-variable dynamic system, this choice remains difficult due to the interactive nature of the system’s states and control signals. Therefore, to choose an optimal state weighting matrix, the proposed systematic method in [98] is adopted here. The proposed method transforms the original system into a balanced system. Thus, the most controllable and observable states appear as the first element in the state vector of the balanced system if both controllability, \(W_c\), and observability, \(W_o\), gramians of the balanced system obtained from (3.31) and (3.32) are equal and diagonal such that \(\sigma_i \geq \sigma_{i+1} \geq 0\).

\[
AW_c + W_c A^T + BB^T = 0. \quad (3.31)
\]

\[
AW_o + W_o A^T + CC^T = 0. \quad (3.32)
\]

\[
W_c = W_o = diag(\sigma_1, ..., \sigma_m, \sigma_{m+1}, ..., \sigma_n). \quad (3.33)
\]

Hence, the participation of weak states, \(n - m\), can be neglected because they need more energy effort to be controlled. Consequently, the weighting of weak states should be considered as zero. Thus the balanced state weighting matrix can be obtained as follow,

\[
Q_b = diag(1, \frac{\sigma_1}{\sigma_2}, \frac{\sigma_1}{\sigma_3}, ..., \frac{\sigma_1}{\sigma_m}, 0, 0, 0). \quad (3.34)
\]
Now, by transforming the balanced state weighting matrix into the standard system, one can get the state weighting matrix to solve LQR problem.

\[ Q = M^T Q_b M. \]  

(3.35)

where \( M \) is the transformation matrix.

**Choice of Matrix R**

The choice of control weighting matrix here is followed the procedure proposed in [99]. The system in (3.6) is partitioned as follows

\[
\dot{x}(t) = Ax(t) + \sum_{i=1}^{r} b_i u_i(t) + B w(t).
\]

\[ y(t) = C x(t). \]  

(3.36)

where \( r \) is the number of single inputs of the derived model in (3.6). After that, each single-input multi-output system is transformed into a balanced form and the contribution of each input, \( w_i \), is evaluated as

\[ w_i = Tr(W_{oi}) = Tr(W_{ci}). \]  

(3.37)

Subsequently, the diagonal input weighting matrix can be constructed as

\[ R = diag(\gamma, \frac{w_2}{w_1}, \frac{w_3}{w_1}, ..., \frac{w_r}{w_1}). \]  

(3.38)

\[ \gamma = \frac{w_1}{\sigma_1}. \]  

(3.39)

where \( \gamma \) is a positive scalar constant that determines the tightness of the control action.

**3.4.2 Results**

In this section, a performance comparison and evaluation of aforementioned designed controllers are established by defining two different scenarios: (i) constant load with variable PV power and (ii) variable load with constant PV power. Finally, a performance
index is introduced to compare the coupled and decoupled controllers’ performances.

**Constant load with variable PV power**

Due to continuously changing atmospheric conditions, photovoltaic systems are weather dependent by nature. Consequently, the maximum extracted power from the PV array varies with variations in the radiance levels resulting from changes in the weather conditions. As a consequence, the DC voltage level at the PCC varies. As explained in in chapter 2, energy storage system regulates the imbalance between power generation and demand and thus can maintain the voltage at the PCC constant in steady-state. To simulate this case study, a constant 4.2 kW DC load with 600V rated voltage is connected at PCC. A variable solar radiation, Fig.3.4, is used as the input to the PV system model to simulate both surplus and deficit in the hybrid system. Figure 3.5 shows the results (i) Coupled controller proposed in this chapter, (ii) decoupled controller with interaction neglected, and (iii) the coupled linear quadratic regulator controller. It is evident that discarding the interaction leads to significantly higher over/undershoot in voltage magnitude at PCC. It is apparent from this figure that the controllers track the input signal, but with tracking achieved by the proposed method is more efficient regarding overshoot and settling time. The regulated PV output voltage extracted from PV array is shown in Fig.3.6. Figures 3.7 and 3.8 illustrate the battery voltage and current. The positive (negative) value of batteries current indicates the battery is in the state of discharging (charging).
Figure 3.5: Point of common coupling (PCC) voltage for constant load.

Figure 3.6: PV array output voltage.

Figure 3.7: Energy storage voltage.

Figure 3.8: ESS current, positive for discharging and negative for charging.
3.4 Simulation and Results

Figure 3.9: Point of common coupling (PCC) voltage for variable load.

Table 3.2: DC load variation during simulation for Case II.

<table>
<thead>
<tr>
<th>Load demand (kW)</th>
<th>Time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.2</td>
<td>0.0-0.2</td>
</tr>
<tr>
<td>6.4</td>
<td>0.2-0.5</td>
</tr>
<tr>
<td>10.0</td>
<td>0.5-0.8</td>
</tr>
<tr>
<td>8.0</td>
<td>0.8-1.0</td>
</tr>
</tbody>
</table>

Variable load with constant PV power

In the second case study, the performance of each controller about load variation is investigated. Thus, solar radiation is used and kept constant at 800 \( \text{W/m}^2 \), while the loading condition is varied. Table 3.2 illustrates the load variation and its duration for this case. As shown in Fig.3.9, decoupled controller with discarded interaction exhibits higher undershoot in comparison to the other coupled controllers. Consequently, it verifies the significant of accounting for the interaction in controller design process.

Performance Index

To provide a quantitative evaluation of the performance of proposed coupled controllers against the conventional one, integral time absolute error (ITAE) has used as a performance index. The index is commonly used to assess the transient response of systems. The value of ITAE is expressed in equation (3.40), where \( t \) is time and \( E(t) \) is the controller error.

\[
ITAE = \int_0^\infty t |E(t)| \, dt \tag{3.40}
\]
The results of performance index obtained for both cases are illustrated in Fig. 3.10. It is evident that the proposed coupled state-feedback controller has significantly superior performance index, more than six times, in comparison to decoupled controller that ignores the interaction.

3.5 Summary

Decoupled classical output-feedback control is the most predominated approach used to control PV-ESS hybrid power systems. However, parallel connection of DC-DC converters at DC bus introduces an interaction between the PV and energy storage system control-loops, which is ignored in existing controller designs. In response to this problem, this chapter introduces a new model that considers the interaction in controller design problem. An LMI based coupled and decoupled state-feedback, as well as coupled LQR controllers are designed. The problem formulation considers the control input saturation while the controller gains are designed to reduce the under/overshoot of output signals. Also, the design method accounts for uncertainties in the system’s input and
loading conditions. A comparison of coupled and decoupled controllers of proposed approach shows that the proposed approach results in a far better performance compared with the existing approaches. Hence, it is verified by using ITAE as a performance index. The index demonstrates that the proposed coupled controller improves the performance by approximately sixfold.
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Chapter 4
Autonomous Decentralised Power Management for DC Microgrid

This chapter propounds a new decentralised, autonomous power management structure for DC microgrids comprising of multi PV and energy storage systems. Each unit is controlled by a robustly designed controller, described in chapter 2, using its local measurements only. Unlike existing approaches, the requirements for communication links are omitted. Seven modes of operations are defined to cover all possible practical operating conditions of a DC microgrid. The transition between the modes occurs autonomously and seamlessly for each control unit, and thus unit-to-unit communication is not required. The efficacy of the proposed structure is tested through various case studies on a DC microgrid comprising of different sources of power generation and multi storage units with different capacity and state of charge.

4.1 Introduction

In the previous chapters, a hybrid system composed of a PV system and an energy storage system was considered. This type of hybrid systems is used for small scale systems (e.g., residential buildings, villages, telecommunication stations). In chapter 2, robust control approaches were implemented to control a PV system and an energy storage system separately. Subsequently, chapter 3 proposed a centralised, robust control approach that enables ones to increase the system efficiency due to a better control performance in comparison to a decoupled approach that neglects the system interaction. The proposed coupled approach can not be easily expanded for a large scale system comprising multi PV systems and energy storage systems due to the complexity in the modelling of switching devices. By increasing the number of DC-DC power converters, and conse-
quentley switching elements, obtaining a single state-space model consisting of each sys-

tem’s states vector become extremely difficult. In fact, the proposed approach in chapter

3 can be a suitable solution for residential hybrid systems, with a limited number of swit-
ching converters, to increase the system efficiency where system performance and effi-
ciency play important roles to motivate residential consumers to implement a renewable
energy based hybrid systems in their buildings.

A medium to a large scale hybrid system is formed by interconnection of multiple
small scale hybrid systems. In other words, a large scale hybrid system comprising mul-
tiple PV systems and energy storage systems can establish an independent DC microgrid.
Thus, to maintain the DC microgrid stability, each power generating unit and storage unit
should be able to operate under a larger frame. Traditionally, this larger frame is formed
by providing a unit-to-unit communication link as well as a supervisory system with a
communication link to each unit. The oversight system evaluates the microgrid require-
ments at each time while receiving information from each unit. After that, by processing
this information, the supervisory system sends a command to each PV system and energy
storage system. As a result, the aggregation of generating powers can respond the cur-
rent demand in the DC microgrid. However, a failure in communication links can be a
fatal failure for the whole system. Therefore, in this chapter a new decentralised power
management scheme is proposed which is independent of unit-to-unit communication
link and avoids any fatal failure previously caused by lose of unit-to-unit communication
links. Despite conventional control approaches for DC microgrids, the proposed robust
control approaches in chapter 2 is implemented to introduce a robust autonomous de-
centralised power management scheme for a DC microgrid. Additionally, the proposed
power management scheme is flexible and scalable for possible future expansion because
of requirement of only local measurements and the proposed droop control mechanism
for the PV systems.
4.2 DC Microgrid Description

A DC MG depicted in Fig. 4.1 comprising multiple PV systems, storage units, and DC loads is studied in this chapter. Each power source is complimented with a local controller to facilitate the smooth transition between modes of operation and to implement an efficient power management scheme proposed in this chapter.

4.2.1 PV Systems

Each PV system considered in this chapter consists of 15 series connected PV panels (KC200GT) and two parallel PV arrays, with a total of 6kW output power. The modelling of the PV systems for the propose of computer simulation follows the PV cell model described in chapter 2, section 2.2.1. A DC-DC unidirectional boost converter is implemented as an interface to integrate the PV arrays with the DC bus. The boost converter is controlled by a robust state-feedback controller explained and designed in chapter 2. For the considered DC microgrid depicted in Fig. 4.1, three PV systems with the same power generation capacity are studied.

4.2.2 Energy Storage Systems

Two energy storage units are integrated to DC MG through a bidirectional DC-DC buck-boost converter to compensate for the intermittency nature of the PV systems. Each energy storage unit consists of a bank of 25 lead-acid batteries connected in series, and the output voltage of each battery is 12 volts. Thus, each energy storage unit provides a DC output voltage of 300 volts with 2kW capacity. A robust state-feedback controller is designed to control each energy storage unit for both charge and discharge operations. The structure of the implemented controller is explained extensively in chapter 2.

4.3 Power Management Structure

This section provides a detailed description of the proposed power management scheme for the multi unit PV systems and multi unit energy storage system DC microgrid of
Fig. 4.1. Initially, a general overview of the operating states considered in this chapter is given. Subsequently, it is followed by a brief description of each mode of operation.

4.3.1 Operating States

This section provides a detail description of the proposed power management scheme comprising five operational modes and two none operational modes. Each controller objective is designed to switch from any current state to another operating state such that a smooth operation of the DC MG can be achieved. Activation of each operating state depends on the amount of the aggregated power generated by the PV systems, reserved capacity for charge or discharge of the storage units, and current load demands and their priorities. Under the proposed power management scheme, the energy storage units switch between charge, discharge, and idle mode according to the amount and nature of change in the DC bus voltage. The charge mode is activated in the case of power surplus, i.e. when electricity generation exceeds energy demand. Conversely, the discharge mode is enabled whenever the total power generation is insufficient to meet the total load demand. In real systems, a steady state is almost never achieved due to continuous changes in the loading condition of power systems. Instead power systems operate in a quasi-steady-state. In this situation the energy storage system operates in idle mode. The energy management system introduced in this thesis is designed to switch between all possible modes of operation in a seamless manner, according to the amount and the
nature of power imbalance in the DC bus, reflected in continuous changes in the voltage profile of the system. The idle mode is there to avoid excessive charge and discharge cycles of the battery, which would reduce its life. Additionally, the PV systems can be operated either as current sources at their MPP or as constant power sources (modes I-VI, or VII, respectively). Subsequently, integrated loads are prioritised with high, medium, and low priority for principal load shedding, occurring in mode VI. It is beneficial to mention that the DC bus voltage should be maintained within a required lower ($V_L$) and upper ($V_H$) thresholds due to local standards. Thus, during system operation the DC bus voltage must remain within this threshold. This threshold is defined such that the lower boundary ($V_L$), 10% below nominal DC bus voltage (700 volts), and the upper boundary ($V_H$), 10% above 700 volts, as described in table 4.1. As demand increases the DC bus voltage drops. As long as the voltage drop does not cause violation of the lower limit, the system keeps operating in the same mode. When, however, the DC bus voltage falls beyond the minimum permissible value (lower threshold, $V_L$) the mode of operation changes. In this case, the battery system moves from Idle to discharge mode to compensate for deficit in the power generated by the PV system, which was the cause of the voltage drop. Similarly, decreasing in demand leads the DC bus voltage to rise, due to the surplus in the amount of power being generated. As soon as the DC bus voltage rises above the upper limit ($V_H$) the operating mode changes to that where the battery system starts charging again to absorb the surplus in the power. The transition between these modes ensures the DC bus voltage always remains within its lower ($V_L$) and upper ($V_H$) thresholds. Additionally, the hysteresis effect caused by imposing the lower and upper boundaries on the DC bus voltage reduces the number of required switching, and consequently results in increasing the battery system life time. The state of charge (SOC) indicates the amount of available charge (energy) that can be utilised from a battery unit. Theoretically, the state of charge can be between 0% and 100%. However, in practice, and to avoid over-charging or under-discharging, an operating constraint is imposed on the battery system. The lower boundary ($SOC^{\text{min}}$) ensures the avoidance of under-charging, while the upper boundary ($SOC^{\text{max}}$) ensures the avoidance of over-charging. In this thesis, we follow the industry convention of the lower boundary being 55% of
the total charge and the upper boundary being 95% of the total charge, as indicated in table 4.1. Thus, the battery unit cannot be discharged below 55% of its total capacity, and subsequently it cannot be charged more than 95% of its total capacity. Minimising this boundary can be theoretically done, but it would cause more frequent switching between charge and discharge modes, as the upper and lower boundaries of the SOC can be reached in a shorter period of time. As a result, the effective life time of the battery will be reduced significantly, as it is directly related to the total number of charge and discharge cycles. In addition frequent switching will inject harmonic in the system, and thus reduces the quality of supply. Consequently, the switching regime demonstrated in Fig 4.2 can be summarised as:

- When the voltage is below $V_L$ and the SOC is below $SOC^{\text{min}}$, the MPPT extracts maximum power and the battery is in idle mode. Thus, load must be shed to achieve power balance, this is Mode VI.

- When the voltage is below $V_L$, and the SOC is higher than $SOC^{\text{min}}$, the MPPT extracts maximum power and the battery is in the discharge mode, as the power generated is less than the load demand, i.e. deficit in power exists, this is Mode III.

- When the voltage is higher than $V_H$, and the SOC is below $SOC^{\text{max}}$, the MPPT extracts maximum power and the battery is in the charge mode, as the power generated is more than the load demand, i.e. surplus in power exists, this is Mode II.

- When the voltage is higher than $V_H$, and the SOC is higher than $SOC^{\text{max}}$, the MPPT must operate in the constant power mode and the battery is in the idle mode (because it is fully charged), i.e. the generated power is more than the load demand, and thus surplus in power exists, this is Mode VII.

- Modes IV and V are in fact never operational (none operational modes) for the simple reason that the SOC is lower than $SOC^{\text{min}}$ and higher than $SOC^{\text{max}}$, respectively.

Throughout this thesis, positive power for the battery system refers to storing energy which occurs whenever the battery system is in the charge mode, negative power refers
to realising energy occurs during discharge mode, and idle mode refers when the battery system neither absorbs power nor exports power. The operating modes mentioned above are briefly explained in the following.

**Mode I, Maximum power point for the PV system and idle mode for the battery:**

In this mode of operation, the DC bus voltage is within its permissible threshold (lower boundary $V_L$ and higher boundary $V_H$), as illustrated in Fig.4.2. Therefore, the battery systems operate in idle mode while PV systems operate at their respective MPPs. The MPPT unit for each PV array provides a reference voltage, as illustrated in Fig.4.6. The controller issues a signal to the PMW to regulate the duty cycle of the DC-DC converter; such that the input voltage is at its maximum power point. This mode operates as long as the DC bus remains within its upper and lower boundaries. If the DC bus voltage falls beyond its lower boundary the operating mode transfers to mode III and operates the battery systems in discharging mode to compensate the voltage dip at the DC bus. However, if the DC bus voltage rises more than the allowable upper limit the operating mode transfers to mode II and operates the battery systems in charging mode. As it is illustrated in Fig.4.2, mode I is considered as an operational mode caused by satisfying the lower threshold $SOC_{min}$ and the upper threshold $SOC_{max}$ for the state of charges as
well as by satisfying DC bus voltage threshold.

In this thesis, whenever the DC bus voltage is within its threshold (upper and lower boundaries) the battery operates in the idle mode. This avoid excessive and unnecessary usage of the battery because the DC bus voltage is within its allowable threshold specified by standards, such as IEEE standards (AS61000 grid code). However, we have suggested for any interested reader, that another approach can be adopted such as implementing an optimisation algorithm to operate and switch the battery unit between charge and discharge modes even when the DC bus voltage is within the allowable threshold to tighten the DC bus voltage variation more than permissible standard threshold, e.g. such as in case of supplying sensitive loads. However, it needs to be stressed that adopting such that approach results in increasing charge and discharge cycles which significantly reduces the battery life time. That was the main reason which in this thesis the battery unit is remained in the idle mode whenever the DC bus voltage is within its permissible lower and upper boundaries specified in IEEE standard (AS61000 grid code).

Mode II, Maximum power point for the PV system and charging mode for the battery:

This mode is activated during off-peak times when the total amount of generated power ($P_G$) is greater than the total energy demand ($P_D$). This case may lead to a temporary rise in the DC bus voltage above its upper limit, which would subsequently require the energy storage units to absorb the surplus of power ($+P_B$) by moving to the charge mode and maintain the DC bus voltage below its upper ($V_H$) limit.

$$P_B = P_D - P_G = \sum_{j=1}^{m} P_{L_j} - \sum_{i=1}^{n} P_{pvi}, \quad P_B < 0. \quad (4.1)$$

As long as the state of charge (SOC) of the storage units are below $SOC^{max}$, the PV systems in this mode are allowed to operate at their maximum power points. In this case, the PV systems act as a current source and inject the maximum harvested power, while the energy storages serve as a voltage regulator for the DC bus. Accordingly, the surplus of power is shared among the active (non-idle due to the violation of $SOC^{max}$) energy storage units in proportion to their state of charges. In practice, the SOC of each energy
storage unit does not remain constant, due to the self-discharge characteristic of the unit (e.g. 15-30% per month for Ni-MH battery and 3-5% for Li-ion) [100]. To achieve power sharing that depends on each energy storage state of charge, an adaptive double-quadrant droop function proposed in [101] is implemented. The characteristic of this adaptive droop function is given as

\[ V_{ref_i} = V_{DC}^* - m_i P_{Bi}. \] (4.2)

where \( V_{ref_i} \) and \( P_{Bi} \) are the \( i \)-th energy storage controller’s reference voltage and actual output power, respectively, \( m_i \) is the droop coefficient of \( i \)-th energy storage unit and \( V_{DC}^* \) is the desired MG DC bus voltage. The droop coefficient of each energy storage unit in the charge mode can be defined in proportion to its SOC. Thus, the requirement of SOC balancing can be achieved by requiring the energy storage unit with lower SOC to absorb more of the surplus power than the unit with higher SOC, until balancing in SOCs is obtained.

\[ m_i = m_{ci} SOC_i. \] (4.3)

where \( m_{ci} \) is the charging droop coefficient of \( i \)-th energy storage unit when its \( SOC_i \) is equal to %100.

**Mode III, Maximum power point for the PV system and discharging mode for the battery:**

When the total generated power by the PV systems is less than the load demand, this mode is activated to supply the power deficit to the load from the energy storage units; this avoids the DC bus voltage drops below the lower limit. Hence, the energy storage units are shifted into the discharge mode. However, this mode should be only activated if SOC of energy storages is higher than the minimum state of charge (\( SOC_{min} \)) constraint to protect the storage from degradation due to under discharging. Similar to the modes I and II, the PV systems in this mode operate as current sources, and the storage units are
employed as voltage regulators which can be expressed as

\[
P_B = P_D - P_G = \sum_{j=1}^{m} P_{L_j} - \sum_{i=1}^{n} P_{pv_i}, \quad P_B > 0. \quad (4.4)
\]

The adaptive droop function (4.2) is also used in this mode to satisfy the SOC balancing requirement. However, in this mode, the energy storage with the higher SOC contributes more than the energy storage with the lower SOC until their SOCs are equal. Therefore, the discharge droop coefficient is modified such that it has an inverse relation with SOC [101]; this is expressed by equation (4.5).

\[
m_i = \frac{m_{di}}{SOC_i}. \quad (4.5)
\]

where \( m_{di} \) is the discharge droop coefficient of \( i \)-th energy storage unit when its \( SOC_i \) is equal to %100.

**Mode IV and V, None operational modes:**

Modes V and VI are considered as none operational modes, and illustrated in Fig. 4.2 in shaded areas. The fact that these modes are non-operational is because of the imposed constraint on the state of charge of the battery system. Thus, the battery unit is not utilised and remains idle. For the sake of completeness the none operational modes V and VI are depicted in Fig. 4.2 as shaded areas.

**Mode VI, Maximum power point for the PV system, idle mode for the battery, with active load shedding:**

This mode occurs when the total power generation by PV systems is less than load demand. Additionally, in this mode, the SOCs of all of the energy storage units are beyond their permissible lower limits. Therefore, utilisation of the energy storage units is not permitted, and consequently, they are set to idle mode. This situation arises if this mode is immediately activated after one of the modes I, III where total depletion of energy storage units could happen. Thus, load shedding would be the only possible practical
solution to maintain the DC voltage at the required level. Hence, a portion of loads must be disconnected. As a result, reduction in the power demand ($P_{sh}$) allows the DC MG to maintain the DC bus voltage above its lower limit.

$$P_G = P_D - P_{sh}. \quad (4.6)$$

$$P_{sh} = \sum_{j=1}^{m} P_{L_{kj}} - \sum_{j=1}^{k} P_{L_{kj}}. \quad (4.7)$$

Three different subsets of the total load $L \in \mathbb{R}^m$ are introduced to prioritise load shedding. These are low priority load set (LP), medium priority load set (MP), and high priority (uninterruptible) load set (HP), which is expressed in (4.8). The portion of the load to be shed, $SH \in \mathbb{R}^k$ must be comprised of both low and medium priority load sets only.

$$L = LP \cup MP \cup HP. \quad (4.8)$$

$$SH \subseteq L = LP \cup MP. \quad (4.9)$$

Figure 4.3 illustrates how load shedding requirement should be checked at each time step.
Figure 4.4: Shifting PV maximum power point into a constant power point.

Mode VII, Constant power for the PV system and idle mode for the battery:

This mode is initiated when the energy storage units are beyond their upper charging limit. In fact, this mode is designed to protect the energy storage units from over charging. Because the total power generation by PV systems is more than the required load demand, and the energy storage is set in idle mode, the PV systems mode of operation is switched from being current sources to constant power sources. Therefore, the DC bus voltage remains below its upper limit. It is made possible through the maximum power point (MPP) shifting controller, integrated with the local controller of each PV system. The controller shifts each PV system MPP by $\Delta P_{pv}$ (new power operating point, $P_n$) such that the generated PV power is equal to the total demand as illustrated in Fig. 4.4 and expressed in equation (4.10)

$$|P_D - P_G| = \left| \sum_{j=1}^{m} P_{L_j} - \sum_{i=1}^{n} P_{P_i} \right| = \left| \sum_{j=1}^{m} P_{L_j} - \sum_{i=1}^{n} (P_{pv_i} - \Delta P_{pv_i}) \right| < \delta_P. \tag{4.10}$$

where $\delta_P$ ensures the DC bus voltage remains within the lower ($V_L$) and upper ($V_H$) thresholds for the DC bus voltage illustrated in Fig.4.2. Therefore, to satisfied (4.10) it is required to move the PV operating point to the voltage source region that operates the PV array in a constant power mode, shaded by blue in Fig. 4.4. Therefore, to be able to shift the operating point of each PV array autonomously an adaptive droop function is
Figure 4.5: Shifting PV maximum power points by implementing an adaptive droop control (4.11).

proposed as follows

\[ v_{ni} = v_{mppi} + d_{i}i_{pv}(k_{P} + k_{I}\frac{1}{s})(V_{DC}^{*} - V_{DC}) \]  

(4.11)

where \( v_{ni} \) and \( i_{pv} \) are new operating power point and output current of \( i \)-th PV system, respectively. \( V_{DC}^{*} \) is the DC bus desired voltage, and \( V_{DC} \) is the measured DC bus voltage, successively. Both proportional and integral gains \((k_{P} \text{ and } k_{I})\) are designed to obtain the desired time response. As it is evident in (4.11), the PV system with the higher output power contributes more in power shifting than a PV system with lower injecting power at the DC bus. The droop coefficient \( d_{i} \) is defined as

\[ d_{i} = \frac{v_{mppi} - v_{oc i}}{P_{mppi}}. \]  

(4.12)

where \( v_{mppi}, v_{oc i}, \) and \( P_{mppi} \) are \( i \)-th PV system rated MPP voltage, open circuit voltage, and nominal maximum output power, respectively. As a result, the new operating power points will have less Euclidean distance from each other (Fig. 4.5), and consequently, the PV systems can be operated with higher efficiency; It can be mathematically expressed as

\[ \min_{i_{pv}, v_{pv}} \sum_{i=1}^{n-1} ||(P_{pv,i+1} - P_{pv}) + (v_{pv,i+1} - v_{pv})||_{2}. \]  

(4.13)

where \( P_{pv} \) and \( v_{pv} \) are PV array output power and voltage respectively.
4.4 Control Scheme

The robust control designed approach for both PV systems and energy storage units used in this chapter are extensively explained and compared with conventional control approaches in chapter 2. However, in the following, a brief explanation of the control structure for multi unit PV system and energy storage system are provided.

4.4.1 PV System Control Scheme

In Fig. 4.6 the control scheme for each PV system is depicted. As shown, each PV system is equipped with a maximum power point tracker (MPPT), an adaptive droop controller, and a local controller. An incremental conductance based MPPT is implemented to calculate the maximum power point voltage ($v_{mpp}$) for each PV array. The description of MPPT function and its mathematical expression are provided in chapter 2, section 2.2.3 though for clarity MPPT is an algorithm based controller that calculates the value of maximum voltage in which at that voltage PV delivers maximum power ($P_{mpp}$). Therefore, it is desirable to operate a PV system at the maximum power voltage which leads to extract maximum possible power and the higher efficiency.

The proposed adaptive droop controller that was introduced in the previous section is employed to shift the maximum power point of the PV systems when the energy storage units are unable to operate in the charging mode (mode VII); a brief description of this controller is provided in section 4.3. The input reference voltage to the local controller is generated by (i) the MPPT when the state of charge of the storage units allow them to operate in the discharge mode, or (ii) MPPT/droop controller when the state of charge of the storage units is insufficient to supply the power deficiency. The local controller regulates the converter duty cycle through the PWM, such that the output voltage of the PV array remains constant in the steady state. Therefore, PV controller generates the required duty cycle and feeds it into PWM to operate the converter such that $v_{mpp}$ is obtained at DC-DC converter input ports. Hence, this assures that maximum power is extracted from PV array. If the maximum power extracted from PV systems is higher than the load demand, the controller ensures that the surplus is stored in the storage units. The
controller is designed such that modelling error and parameter uncertainties, such as converter’s duty cycle, DC load, and PV array voltage are considered and modelled as a convex polytope in control design briefly described in chapter 2. The designed controller guarantees a global stability within a wide-range of operating equilibrium points. Additionally, a required level of disturbance rejection performance is granted through $H_\infty$ design. As a state-of-art controller is robustly designed to account for both internal and external applying disturbances that can be experienced by the PV systems.

### 4.4.2 Energy Storage Unit Control Scheme

To charge and discharge the energy storage units (through bidirectional power flow) a robust seamless controller formulated in chapter 2 is designed for each bidirectional asynchronous buck-boost converter. The designed controller facilitates the seamless and autonomous transition between charge and discharge mode. In the charge mode, the DC-DC converter operates as a buck converter to step down the DC bus voltage, and conversely, during discharge mode, it functions as a boost converter because DC bus voltage is higher than ESU’s charging voltage. Thus, the designed robust state-feedback controller generates the required duty cycle for the buck-boost converter to regulate DC bus voltage.

Similar to PV system, the storage controller is intended to minimise the effect of both internal and external disturbances at DC bus through $H_\infty$ analysis and design, additionally because of seamless control structure the region of stability for energy storage controller becomes critical. Therefore, apart from disturbance rejection capability and
parametric uncertainties consideration, the region of stability for the designed controller is considered to be maximised within problem formulation [2]. Figure 4.7 shows the control structure for storage units. Both PV and energy storage controllers are designed such that control input saturation is avoided. Although the control input saturation in DC-DC converters can destabilise converter and accelerate its degradation due to increasing switching stress, it has not addressed in adopted control approaches in previous works.

4.5 Results and Discussions

In this section, the obtained results of the proposed power management scheme are discussed. An MG comprising three 6 kW PV systems and two energy storage units is used to verify the proposed approach. Table 4.1 provides the electrical parameters of the PV systems and energy storage units (ESU). To demonstrate the effectiveness of the proposed controllers and power management scheme, different modes of operations, and the transition from one to another mode are investigated through the following case studies.

4.5.1 Case I

In this experiment, the performance of the adaptive droop controller is evaluated while the energy storage units are considered fully charge. Therefore, none of energy storage system can participate to absorb any surplus of power limited by their state of charge upper limits ($SOC^{max}$). In this situation the only practical solution to maintain the DC bus voltage below its upper limit is to shift maximum power point of PV system. To
4.5 Results and Discussions

Table 4.1: Electrical parameters of tested DC MG.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>PV array MPP voltage</td>
<td>$v_{mpp}$</td>
<td>394.5 V</td>
</tr>
<tr>
<td>PV array MPP current</td>
<td>$i_{mpp}$</td>
<td>15.22 A</td>
</tr>
<tr>
<td>PV array maximum Power</td>
<td>$P_{mpp}$</td>
<td>6 kW</td>
</tr>
<tr>
<td>PV array open circuit voltage</td>
<td>$v_{oc}$</td>
<td>32.9 V</td>
</tr>
<tr>
<td>PV converter inductance resistance</td>
<td>$r_{L_{pv}}$</td>
<td>0.1714 Ω</td>
</tr>
<tr>
<td>PV converter inductance</td>
<td>$L_{pv}$</td>
<td>1.1 mH</td>
</tr>
<tr>
<td>PV converter output filter</td>
<td>$C_{of}$</td>
<td>$1.47 \times 10^{-4}$ F</td>
</tr>
<tr>
<td>PV converter input filter</td>
<td>$C_{if}$</td>
<td>$30 \times 10^{-5}$ F</td>
</tr>
<tr>
<td>PV droop coefficient</td>
<td>$d$</td>
<td>0.0165</td>
</tr>
<tr>
<td>MPP droop propositional gain</td>
<td>$k_{p}$</td>
<td>2.5 S$^{-1}$</td>
</tr>
<tr>
<td>MPP droop integral gain</td>
<td>$k_{I}$</td>
<td>10.5 S$^{-1}$</td>
</tr>
<tr>
<td>Energy storage voltage</td>
<td>$V_{b}$</td>
<td>300 V</td>
</tr>
<tr>
<td>ES internal resistance</td>
<td>$R_{b}$</td>
<td>1.2 Ω</td>
</tr>
<tr>
<td>ESU converter resistance</td>
<td>$r_{L_{b}}$</td>
<td>$9.6 \times 10^{-3}$ Ω</td>
</tr>
<tr>
<td>ESU converter inductance</td>
<td>$L_{b}$</td>
<td>1 mH</td>
</tr>
<tr>
<td>ESU converter output filter</td>
<td>$C_{bf}$</td>
<td>$1.77 \times 10^{-4}$ F</td>
</tr>
<tr>
<td>ESU charge droop coefficient</td>
<td>$m_{c}$</td>
<td>$6 \times 10^{-4}$</td>
</tr>
<tr>
<td>ESU discharge droop coefficient</td>
<td>$m_{d}$</td>
<td>$5 \times 10^{-6}$</td>
</tr>
<tr>
<td>State of charge lower limit</td>
<td>$SOC_{\text{min}}$</td>
<td>55%</td>
</tr>
<tr>
<td>State of charge upper limit</td>
<td>$SOC_{\text{max}}$</td>
<td>95%</td>
</tr>
<tr>
<td>DC bus desired voltage</td>
<td>$V_{DC}^*$</td>
<td>700 V</td>
</tr>
<tr>
<td>DC bus upper limit</td>
<td>$V_L$</td>
<td>$700 + 10% V_{DC}^*$</td>
</tr>
<tr>
<td>DC bus lower limit</td>
<td>$V_H$</td>
<td>$700 - 10% V_{DC}^*$</td>
</tr>
</tbody>
</table>
verify the versatility of the proposed power management scheme the first and the second PV systems are equipped with the droop controllers, while the third PV system operates without droop control capability. Both ESUs are assumed to have 85% initial SOC, and all the PV systems generate an equal amount of power (2.5kW). Figure 4.9 shows the DC bus voltage and Fig. 4.8 illustrates the results where the dotted lines represent output power of the PV systems with droop controllers, and the solid lines demonstrate output power of the PV system without droop control capability. From t=0 to t=50 seconds the DC bus voltage remains within its lower and upper boundaries. Thus, ESUs are operating in the idle mode (mode I). At t=50s solar radiation increased by 50% and 40% for the first and the second PV systems, and 30% for the third PV system occurs. Therefore, the DC bus voltage rose above its upper threshold. Therefore, to maintain the DC bus voltage below its upper limit, the first and the second PV systems are required to operate as constant power sources (mode VII), which is achieved through the action of the droop controllers. As the first PV system generates more power during the period t=50-100 seconds than the second PV system, its MPP is shifted more in comparison with the second PV system. From t=100-150 seconds, the solar radiation drops for all PV systems. As a result, the DC bus remains within its lower and upper thresholds (mode I) and the PV systems operate as current sources (maximum power). Subsequently, at t=150 seconds, the solar radiation for the first two PV systems is increased by 15% while for the third PV system it is increased by 25%. As the ESUs are fully charged and operates in idle mode, the first two PV systems are required to operate as constant power sources (mode VII) once again. As both PV systems are generating an equal amount of power (4.5kW) at t=150 seconds, both PV systems MPPs are equally shifted until the DC bus voltage is below its upper limit. It should be noted that all of the above transitions between the modes occurs autonomously without any communication link between PV systems and the storage units.

4.5.2 Case II

In this experiment, the performance of PV systems, as well as energy storage units, are analysed, while the transition between the modes of operation is highlighted. The simulation study has been carried out for 200 seconds, in which four mode transitions occur.
Figure 4.8: PV outputs and transition to mode VII controlled by the adaptive droop controller shown by dotted lines and without the droop controller depicted by solid lines (case I).

Figure 4.9: DC bus voltage for case I.
Figure 4.10: Results of DC MG comprising of three PV systems and two energy storage units (ESU) in different PV generation conditions.

The first and second ESUs are assumed to have initial SOC of 90% and 80%, respectively. During this experiment, both the load demand and solar radiation vary in time to affect transitions between different modes of operation. Figure 4.10 shows the power generated by each PV system, energy storage output power, energy storage output current and energy storage state of charge. Figure 4.11 illustrates the DC bus voltage in this case which always remained within its upper and lower threshold. In the following analysis, each 50-second interval is briefly explained.

Figure 4.11: DC bus voltage for case II.
0-50 Seconds, Mode III

From t=0 to t=50 seconds the total generated power by the PV systems \( (P_G) \) is less than the load demand \( (P_L) \), while the SOC of both energy storage units (ESU) are above their lower limits \( (SOC^{\text{min}}) \). Thus, during this period the DC MG operates in mode III. The initial state of charge of ESU 1 is considered to be 90%, and for the second unit, it is 80%. For this reason, the ESU with the higher state of charge injects more power in comparison with the ESU with the lower SOC. Subsequently, it continues until the SOC of the two units reaches the same value at steady state. The ESU with higher SOC has the higher negative slope in comparison to the ESU with the lower SOC, shown in Fig. 4.10. In fact, the slope of discharge for each ESU is calculated by adjusting the discharging droop coefficient in (4.5).

50-100 Seconds, Mode II

From t=50 to t=100 seconds the load demand \( (P_L) \) is assumed to be less than the total generated power by PV systems \( (P_G) \). However, the state of charge of both ESU is below their upper limits \( (SOC^{\text{max}}) \), 83% and 78%, respectively. Consequently, the ESUs operate in the charge mode to absorb the excess power generated by the PV systems. The ESU droop controller sets the slope of the ESU with lower SOC (ESU 2) to be higher than the ESU with a higher SOC (ESU 1); This is achieved by adjusting charging droop coefficient (4.3) automatically.

100-150 Seconds, Mode I

For the interval between 100 to 150 seconds, the DC bus voltage remains within its upper and lower boundaries which implies that the system is operating in mode I. Therefore, both ESUs are transitioned to the idle mode. Subsequently, both ESUs SOCs remain unchanged.
4.6 Control Performance

Figure 4.12 illustrates the control performance of each robust controller during the transition from mode II to mode I at $t=2s$. In fact, the transition between modes moves each DC-DC converter from one equilibrium point to another. In chapter 2, the performance of the proposed controller was compared with classical PI, LQR; the results demonstrated in figures 2.13-2.18, shown that the proposed control approach has significantly less oscillation in comparison with the methods as mentioned earlier while the converter moves into a new equilibrium point. In fact, duty cycle variation in converters has the same effect as a parametric uncertainty which must be accounted for in the controller design. If this form of uncertainty is ignored, then the designed controller might not maintain system’s stability while it is not operated in the vicinity of its linearized model. Figure 4.12 shows the performance response of all three PV control systems during a mode transition. It is evident that both controllers (PV and ESU) can transit each unit from one mode to another mode smoothly within the necessary transition time.
4.7 Summary

A decentralised autonomous power management scheme for a DC microgrid is proposed in this chapter. Unlike the previous control techniques existing in the literature which was discussed briefly in chapter 2, robust control approach suggested in chapter 2 was implemented to design the required controllers such that both converter’s parametric uncertainties and disturbance rejection performance of controllers are addressed. In the proposed power management scheme each controller performs different control actions, based on information contained in the local measurements. Therefore, the requirements for having communication links between each unit is omitted. Through various case studies, it is shown that the system is flexible and scalable for possible future expansion. In the proposed method, the DC MG operates in seven different modes, depending on the amount of power generated by PV systems, the state of charge of the energy storage units, and power consuming by the load. An adaptive droop function is used for each PV system to operate it as a constant power source when the generated power is higher than the absorbing capability of the energy storage units and power demand. Shifting each PV system’s power point is carried out such that the total Euclidean distance between the MPPs of different units is minimised. Several investigations were conducted through various case studies to evaluate the performance of each controller and the proposed power management scheme. The provided results show that the proposed method can successfully adopt the right mode of operating.
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Chapter 5
Robust Observer-based Controller for Three-phase Grid-connected Voltage Source Inverters

This chapter provides a new current control approach for three-phase grid-connected voltage source inverters (VSI). An observer-based state-feedback dynamic controller is implemented to control the VSI to reduce the number of required measurements. Moreover, the dynamic controller and observer are robustly designed, to account for grid disturbances, and achieve a guaranteed $H_\infty$ performance level. The existence of parametric uncertainties caused by inexact knowledge of the grid impedance and inductance are modelled as a convex polytope. Additionally, saturation of control input is avoided by imposing a set of constraints that prevent the VSI from operating in the nonlinear region caused by overmodulation. The effectiveness of the proposed technique is analysed through various case studies. It is shown that the proposed control approach provides stability over a wide range of grid bounded uncertainties and disturbances, with improved transient performance and low implementation cost compared to other control schemes.

5.1 Introduction

DC microgrids can be operated either in an islanded-mode (off-grid) or a grid-connected mode (on-grid). In the previous chapter, the considered DC microgrid was implemented as an islanded network without any interconnection with an AC grid. Therefore, the generated power of the considered microgrid in chapter 4, should be either consumed by DC loads or be stored in the energy storage systems. As it was explained in chapter 4, if the energy storage systems are fully charged, and the power generated by PV systems exceeded the demand the proposed power management scheme must shift
PV systems’ MPPs (Mode VII) to ensure the DC bus voltage does not rise above its upper boundary. As a result, the generated power by each PV system is enforced to be below its maximum power capability. If a DC microgrid operates in a grid-connected mode, the excess generated power by PV systems can be exported to the grid. Thus, this enables the DC microgrid to operate at both islanded, and grid-connected modes depending on the system conditions. Therefore in this chapter, an inverting DC-AC stage is implemented to establish an interface between a DC microgrid and an AC grid.

Pulse-width modulated (PWM) voltage source inverters (VSI) are widely used as an interface between DC and AC networks to convert DC to AC voltage. In a grid-connected VSI, although the grid’s voltage dominates the output voltage of VSI, the output current can be regulated to inject active and reactive powers to the grid. Therefore, a current controller requires to control the grid-connected VSI. In this chapter, a robust control approach is implemented to control the VSI in the presence of uncertainties and disturbances. The proposed control technique, in this chapter, enables the considered DC microgrid in chapter 4 to inject and regulated both active and reactive powers to an AC grid.

5.2 VSI with LCL Filter Modelling

To design a robust observer-based controller for the VSI depicted in Fig.5.1, a dynamic state-space model of the grid-connect VSI is required. This dynamic model should be given in a stationary $dq$-frame to enable transfer of the VSI sinusoidal tracking problem to an equivalent DC tracking problem of two decoupled subsystems [57]. Thus, initially, a three-phase dynamic model in the phasor space is developed. After that, an equivalent model in the stationary $dq$-frame is presented.

5.2.1 Space Phasor Model

As it is shown in Fig.5.1 the voltage source inverter is connected to a three-phase grid using an LCL filter. It is obvious that the dynamic response of LCL filter is much slower than the VSI switching frequency ($f_s$), thus, without loss of generality, the switching dynamics of the VSI can be neglected. A simplified single-phase diagram of a VSI with an
LCL filter is shown in Fig. 5.2 to simplify the representation. By using Kirchhoff voltage and current laws, the LCL filter dynamic equations in the state-space is derived from a reference phase, say phase $a$, by

$$L_{fc} \frac{d\vec{i}_c(t)}{dt} = \vec{u}_c(t) - r_f \vec{i}_c(t) - \vec{v}_c(t). \quad (5.1)$$

$$C_f \frac{d\vec{v}_c(t)}{dt} = \vec{i}_c(t) - \vec{i}_g(t). \quad (5.2)$$

$$L_G \frac{d\vec{v}_g(t)}{dt} = \vec{v}_c(t) - r_g \vec{i}_g(t) - \vec{v}_g(t). \quad (5.3)$$

$$L_G = L_2 + L_g. \quad (5.4)$$

where $i_c$ and $u_c$ are space phasor quantities of the VSI output current and voltage, respectively. $r_f$, $L_{fc}$, and $v_c$ are the LCL filter input resistance, inductance, and capacitor voltage. $L_2$ is the output inductance of the filter. The nominal values of the grid resistance and inductance are indicated by $r_g$ and $L_g$, respectively. $v_g$ denotes the grid voltage.
modelled as an input disturbance to the VSI.

### 5.2.2 Stationary dq-frame Model

In order to transform the state equation from the abc frame to dq-frame, each state in (5.1)-(5.3) is substituted with

\[ \vec{x}(t) = x_{dq}e^{\int w(t)dt}. \]  

(5.5)

where \( w \) is the grid angular velocity. Equation (5.5) corresponds to a three-phase signal that is transformed into two stationary DC quantities in dq-frame. Therefore, by using (5.5), equations (5.1)-(5.3) can be transformed to dq-frame as follow

\[
L_f \frac{di_{cd}}{dt} = u_{cd}(t) - r_f i_{cd} - v_{cd}(t) + L_f w_{iq}(t). \]  

(5.6)

\[
L_f \frac{di_{cq}}{dt} = u_{cq}(t) - r_f i_{cq} - v_{cq}(t) - L_f w_{iq}(t). \]  

(5.7)

\[
C_f \frac{dv_{cd}}{dt} = i_{cd}(t) - i_{gd}(t) + C_f w_{cq}(t). \]  

(5.8)

\[
C_f \frac{dv_{cq}}{dt} = i_{cq}(t) - i_{gq}(t) - C_f w_{cd}(t). \]  

(5.9)

\[
L_G \frac{di_{gd}}{dt} = v_{cd}(t) - r_g i_{gd}(t) - v_{gd}(t) + L_G w_{igq}(t). \]  

(5.10)

\[
L_G \frac{di_{gq}}{dt} = v_{cq}(t) - r_g i_{gq}(t) - v_{gq}(t) - L_G w_{igd}(t). \]  

(5.11)

A canonical state-space representation of the VSI dynamic equations given by (5.6)-(5.11) can be expressed in dq-frame as

\[ \dot{x}(t) = Ax(t) + Bu(t) + B_w d(t). \]  

(5.12)

\[ y(t) = Cx(t). \]  

(5.13)

\[ z(t) = C_z x(t). \]  

(5.14)

where \( x(t) \), \( u(t) \), and \( d(t) \) are the state vector, control input, and input disturbance of the VSI. The system constant matrices in (5.12)-(5.14) are \( A \in \mathbb{R}^{n \times n} \) for state matrix, \( B \in \mathbb{R}^{n \times m} \) for input matrix, \( B_w \in \mathbb{R}^{n \times q} \) for input disturbance matrix, \( C \in \mathbb{R}^{m \times n} \) for output matrix, and \( C_z \in \mathbb{R}^{m \times n} \) which is equal to matrix \( C \).
5.3 Control Objective

The state variables in \( x(t) \) are defined as the VSI output current \( i_{cdq} \), LCL filter capacitor voltage \( v_{cdq} \), and grid injected current \( i_{gdq} \).

\[
A = \begin{bmatrix}
-\frac{R_f}{L_{fc}} & w & -\frac{1}{L_{fc}} & 0 & 0 & 0 \\
-w & -\frac{R_f}{L_{fc}} & 0 & -\frac{1}{L_{fc}} & 0 & 0 \\
\frac{1}{L_f} & 0 & 0 & w & -\frac{1}{L_f} & 0 \\
0 & \frac{1}{L_f} & -w & 0 & 0 & -\frac{1}{L_f} \\
0 & 0 & \frac{1}{L_c} & 0 & -\frac{R_g}{L_c} & w \\
0 & 0 & 0 & \frac{1}{L_c} & -w & -\frac{R_g}{L_c}
\end{bmatrix}.
\]

(5.15)

\[
B = \begin{bmatrix}
\frac{1}{L_{fc}} & 0 \\
0 & \frac{1}{L_{fc}} \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & \frac{1}{L_c} \\
0 & \frac{1}{L_c}
\end{bmatrix},
B_w = \begin{bmatrix}
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 1
\end{bmatrix},
C = \begin{bmatrix}
0 & 0 \\
0 & 0 \\
0 & 1
\end{bmatrix}^T.
\]

(5.16)

5.3 Control Objective

The VSI illustrated in Fig.5.1 is implemented as a grid-connected VSI. Thus, its output voltage and frequency are regulated by the grid. In order to follow the grid frequency a phase locked loop (PLL) controller is used to calculate \( w(t) \) in (5.5) in \( dq \)-frame. The active and reactive power in \( dq \)-frame are expressed by

\[
P(t) = \frac{3}{2} [v_{gd}(t)i_{gd}(t) + v_{gq}(t)i_{gq}(t)].
\]

(5.17)

\[
Q(t) = \frac{3}{2} [v_{gq}(t)i_{gd}(t) - v_{gd}(t)i_{gq}(t)].
\]

(5.18)

As \( v_{gd} \) and \( v_{gq} \) are regulated by the grid, a current tracking controller is adopted in this chapter to regulate the VSI grid injected currents, \( i_{gd} \) and \( i_{gq} \). By controlling the VSI output current, it is possible to regulate both active and reactive powers injected into the
grid. When the PLL is in steady state, $v_{gq}$ is zero and $v_{gd}$ is equal to the grid peak voltage [50]. For that reason, the reference current for the controller can be expressed by

$$i_{gd}^* = \frac{2}{3v_{gd}} P^*(t). \quad (5.19)$$

$$i_{gq}^* = -\frac{2}{3v_{gd}} Q^*(t). \quad (5.20)$$

where the quantities with asterisk superscript denote reference values. The reference currents in (5.19) and (5.20) are the reference inputs to the proposed observer-based controller. In Fig.5.3 a general overview of the VSI with its control loop is given. Figure 5.4 shows the implementation of the proposed controller, where $\Delta = diag(L_f, C_f, L_G)$ and $w_0$ is the grid nominal angular velocity. In fact, by decoupling the $dq$-axis, both $d$- and $q$-axis control loops are identical and consequently, the corresponding controller gains are also identical [57].
5.4 Observer-based Controller Design

In this section, a description of the proposed observer-based controller depicted in Fig. 5.3 is explained. The proposed approach enables the design of a controller and an observer such that a required disturbance rejection performance is guaranteed. To control the VSI and to achieve the control objectives explained in section 5.3 with a guaranteed disturbance rejection performance, a robust state-feedback controller is adopted here. However, implementing a state-feedback requires measurement of all states in (5.12). Therefore, to minimise the required number of measurements and associated installed sensors, a robust observer-based controller (5.21)-(5.22) with a state-feedback control law (5.23) is employed.

\[
\dot{x}(t) = A\dot{x}(t) + Bu(t) + L[y(t) - \hat{y}(t)]. \quad (5.21)
\]

\[
\hat{y}(t) = C\dot{x}(t). \quad (5.22)
\]

\[
u(t) = -K\dot{x}(t). \quad (5.23)
\]

where \(L, K,\) and \(\dot{x}\) are observer gain, state-feedback gain, and system’s states estimations. The following conditions are required to be satisfied to stabilise the system described in (5.12), using the control law (5.23),
Robust Observer-based Controller for Three-phase Grid-connected Voltage Source Inverters

(A1) Matrix $C$ has full row rank.

(A2) The pair $(A, C)$ is detectable.

Both controller and observer gains can be systematically designed as per the following new theorem.

**Theorem 5.1.** The system (5.12) is asymptotically stabilisable by (5.23), where the state estimations are generated by observer (5.21)-(5.22), with a guaranteed disturbance attenuation level $\gamma$ and a decay rates $\alpha$ for the controller and $\beta$ for the observer, if conditions A1 and A2 are satisfied, and there exist symmetric matrices $W_1 \in \mathbb{R}^{n \times n}$, $W_2 \in \mathbb{R}^{n \times n}$ and matrices $Y \in \mathbb{R}^{n \times n}$, $X \in \mathbb{R}^{n \times m}$, and $F \in \mathbb{R}^{n \times m}$ such that:

$$ \begin{bmatrix} \varphi_{11} & \varphi_{12} & 0 & W_1 C_T z^* \\ * & \varphi_{22} & B_w & W_2 C_T z^* \\ * & * & -\gamma^2 I & 0 \\ * & * & 0 & -I \end{bmatrix} < 0. \quad (5.24) $$

$$ CW_2 = FC. \quad (5.25) $$

$$ \varphi_{11} = AW_1 + W_1 A^T - BY - Y^T B^T + 2I \alpha W_1. \quad (5.26) $$

$$ \varphi_{12} = XC. \quad (5.27) $$

$$ \varphi_{22} = AW_2 + W_2 A^T - XC - C^T X^T + 2I \beta W_2. \quad (5.28) $$

where $K = -YW_1^{-1}$, $L = XF^{-1}$, $W_1 > 0$, and $W_2 > 0$, $\alpha$ and $\beta$ are positive scalars and $\beta > \alpha$ .

**Proof:** By defining $e(t) = x(t) - \hat{x}(t)$ as error between measured and estimated states, (5.12) and (5.21) with $d(t) \neq 0$ can be expressed as

$$ \begin{bmatrix} \dot{\hat{x}}(t) \\ \dot{\hat{e}}(t) \end{bmatrix} = \begin{bmatrix} A - BK & LC \\ 0 & A - LC \end{bmatrix} \begin{bmatrix} \hat{x}(t) \\ e(t) \end{bmatrix} + \begin{bmatrix} 0 \\ B_w \end{bmatrix} d(t). \quad (5.29) $$

In order to obtain the desired decay rates for both controller and observer, a $D$-stability criteria can be implemented such that the controller closed-loop poles $(\chi + j\gamma)$ can be
restricted to region $D_1$, and observer closed-loop poles ($\bar{x} + j\bar{y}$) can be located within region $D_2$ expressed as following:

$$D_1 = \{x + jy \in \mathbb{C} : x < -\alpha \}. \quad (5.30)$$

$$D_2 = \{\bar{x} + j\bar{y} \in \mathbb{C} : \bar{x} < -\beta \}. \quad (5.31)$$

where $\alpha$ and $\beta$ define the decay rates of controller and observer, successively. It should be noted that the observer decay rate needs to be faster than controller decay rate. Subsequently, equation (5.29 can be modified as

$$\begin{bmatrix} \dot{\hat{x}}(t) \\ \dot{e}(t) \end{bmatrix} = \begin{bmatrix} A - BK + \alpha I & LC \\ 0 & A - LC + \beta I \end{bmatrix} \begin{bmatrix} \hat{x}(t) \\ e(t) \end{bmatrix} + \begin{bmatrix} 0 \\ B_w \end{bmatrix} d(t). \quad (5.32)$$

Now, one can define a Lyapunov function such that

$$V(\hat{x}(t), e(t)) = \hat{x}^T(t)P_1\hat{x}(t) + e^T(t)P_2e(t). \quad (5.33)$$

where $P_1$ and $P_2$ are symmetric positive definite matrices. The time derivative of (5.33), along the trajectories of (5.32), is given by

$$\dot{V}(\hat{x}(t), e(t)) = 2\hat{e}^T(t)P_2B_wd(t) +$$

$$\begin{bmatrix} \hat{x}(t) \\ e(t) \end{bmatrix}^T \begin{bmatrix} \Psi_1 & 0 \\ C^TLTP_1 & \Psi_2 \end{bmatrix} \begin{bmatrix} \hat{x}(t) \\ e(t) \end{bmatrix} + \begin{bmatrix} \hat{x}(t) \\ e(t) \end{bmatrix}^T \begin{bmatrix} \Psi_3 & P_1LC \\ 0 & \Psi_4 \end{bmatrix} \begin{bmatrix} \hat{x}(t) \\ e(t) \end{bmatrix}. \quad (5.34)$$

where

$$\Psi_1 = A^TP_1 - K^TB^TP_1 + \alpha IP_1. \quad (5.35)$$

$$\Psi_2 = A^TP_2 - C^TL^TP_2 + \beta IP_2. \quad (5.36)$$

$$\Psi_3 = P_1A - P_1BK + P_1\alpha I. \quad (5.37)$$

$$\Psi_4 = P_2A - P_2LC + P_2\beta I. \quad (5.38)$$
Equation (5.34) can be summarised as

\[
\dot{V}(\hat{x}(t), e(t)) = \begin{bmatrix}
\dot{x}(t) \\
e(t) \\
d(t)
\end{bmatrix}^T \begin{bmatrix}
\Xi_{11} & \Xi_{12} & 0 \\
* & \Xi_{22} & P_2 B_w \\
* & * & 0
\end{bmatrix} \begin{bmatrix}
\dot{x}(t) \\
e(t) \\
d(t)
\end{bmatrix}.
\] (5.39)

where

\[
\Xi_{11} = P_1 A + A^T P_1 - P_1 B K - K^T B^T P_1 + 2\alpha I P_1. 
\] (5.40)

\[
\Xi_{12} = P_1 L C. 
\] (5.41)

\[
\Xi_{22} = P_2 A + A^T P_2 - P_2 L C - C^T L^T P_2 + 2\beta I P_2. 
\] (5.42)

If \(\dot{V}(\hat{x}(t), e(t)) < 0\), and subsequently \(\Omega < 0\) for all \(x(t), e(t),\) and \(d(t)\) all trajectories are bounded, and consequently the system in (5.12) is globally asymptotically stable by the means of control law (5.23). As the aim of this chapter is to design a robust observer-based controller that attenuates the effect of grid disturbance \(v_g\), the infinity norm (RMS gain) of input disturbance \(d(t)\) to \(z(t)\) should be bounded by \(\gamma\) which can be expressed by

\[
||G_{zd}||_{\infty} = \sup_{d(t) \neq 0} \frac{||z(t)||_2}{||d(t)||_2} < \gamma.
\] (5.43)

where \(||.||_{\infty}\) is infinity norm, and \(||.||_2\) is euclidean norm. Therefore, in order to bound the infinity norm (5.43) the time derivative of Lyapunov function in (5.33) should satisfy

\[
\dot{V}(\hat{x}(t), e(t)) + z^T(t)z(t) - \gamma^2 d^T(t)d(t) < 0.
\] (5.44)

for all \(\hat{x}(t), e(t),\) and \(d(t)\). Equation (5.44) guarantees the infinity norm of the system (5.29) is less than \(\gamma\). Thus, if (5.44) holds the system (5.12) is asymptotically stable by control law (5.23) with a guaranteed disturbance attenuation \(\gamma\). By substituting (5.39)
into (5.44), it can be expressed by

\[
\begin{bmatrix}
\Lambda_{11} & \Lambda_{12} & 0 \\
* & \Lambda_{22} & P_2B_w \\
* & * & -\gamma^2 I
\end{bmatrix} < 0. \tag{5.45}
\]

where

\[
\Lambda_{11} = \Xi_{11} + C_2^TC_2. \tag{5.46}
\]
\[
\Lambda_{12} = \Xi_{12} + C_2^TC_2. \tag{5.47}
\]
\[
\Lambda_{22} = \Xi_{22} + C_2^TC_2. \tag{5.48}
\]

By pre- and post multiplying (5.45) by

\[
\begin{bmatrix}
W_1 & 0 & 0 \\
0 & W_2 & 0 \\
0 & 0 & I
\end{bmatrix} \tag{5.49}
\]

where \(W_1 = P_1^{-1}\) and \(W_2 = P_2^{-1}\), equation (5.50) is obtained.

\[
\begin{bmatrix}
\Gamma_{11} & \Gamma_{12} & 0 \\
* & \Gamma_{22} & B_w \\
* & * & -\gamma^2 I
\end{bmatrix} < 0. \tag{5.50}
\]

\[
\Gamma_{11} = AW_1 + W_1A^T - BY - Y^TB^T + W_1C_2^TC_2W_1 + 2\alpha IW_1. \tag{5.51}
\]
\[
\Gamma_{12} = LCW_2 + W_1C_2^TC_2W_2. \tag{5.52}
\]
\[
\Gamma_{22} = AW_2 + W_2A^T - LCW_2 - W_2C^TL^T + W_2C_2^TC_2W_2 + 2\beta IW_2. \tag{5.53}
\]
where \( Y = K W_1 \). After that, by using Schur Complement (5.50) can be written as

\[
\begin{bmatrix}
\Phi_{11} & \Phi_{12} & 0 & W_1 C_z^T \\
\ast & \Phi_{22} & B_w & W_2 C_z^T \\
\ast & \ast & -\gamma^2 I & 0 \\
\ast & \ast & 0 & -I
\end{bmatrix} < 0. \tag{5.54}
\]

\[
\Phi_{11} = A W_1 + W_1 A^T - B Y - Y^T B^T + 2 I \alpha W_1. \tag{5.55}
\]

\[
\Phi_{12} = L C W_2. \tag{5.56}
\]

\[
\Phi_{22} = A W_2 + W_2 A^T - L C W_2 - W_2 C^T L^T + 2 I \beta W_2. \tag{5.57}
\]

As it is evident, (5.56) is a non-convex quadratic term. Thus, we defined two new variable matrices \( F \), and \( X \) such that

\[
L C W_2 = L F C = X C. \tag{5.58}
\]

By substituting (5.58) to (5.54) the stated theorem 5.1 can be obtained. ■

### 5.5 Control Input Saturation

Overmodulation caused by control input saturation can increase both switching stress and introduce low-order harmonics in the VSI output voltage spectrum [57, 77]. A controller without an upper bound on its output may also destabilise the VSI. Thus, it is essential to maintain the norm of the control input signal below an upper limit. Practically, under normal modulation, the VSI modulation signal \( \mu(t) \) is a three-phase sinusoidal periodic signal bounded between -1 and 1. This bound can be maintained by imposing a further constraint to the original control problem expressed in theorem 5.1. A constraint on the norm of the control input signal with an upper bound, \( ||\mu_c(t)|| \leq \mu \), can be enforced for all times \( t \geq 0 \) if the following LMI holds [91].

\[
\begin{bmatrix}
W_1 & Y^T \\
Y & \mu^2 I
\end{bmatrix} \geq 0. \tag{5.59}
\]
5.6 Reference Tracking

The observer-based controller (5.23) can be designed to stabilise the VSI, using an appropriate control strategy. However, to attain zero steady-state error, i.e. reference input tracking, an integrator needs to be inserted in the feedforward path between the error comparator and the VSI. The dynamic of this integrator can be defined to be

\[
\dot{\xi}(t) = r(t) - Cx(t).
\]

\[
\xi = \int_0^t (r(t) - Cx(t))dt.
\]

where \( r(t) \) is the reference vector involving \( i_{gd}, i_{gq} \), respectively. Therefore, the control law (5.23) can be modified as,

\[
u(t) = -K\hat{x}(t) + K_i \int_0^t (r(t) - Cx(t))dt.
\]

In order to design both the proportional controller gain, \( K \), and integrator gain, \( K_i \), simultaneously, equation (5.26) in theorem 5.1 needs to be augmented with the integrator dynamic equation. Hence, equation (5.26) can be modified as

\[
\phi_{11} = AW_1 + W_1A^T - BY - YTBT + BH + HTBT + 2I_\alpha W_1.
\]

where \( K_i = HW_1^{-1} \).

5.7 Polytopic Uncertainty

Three-phase grids are predominantly inductive. However, protection apparatus (e.g. relays, circuit breakers) have internal resistance. Therefore, the three-phase grid described in Fig.5.1 is modelled by a voltage source in series with an inductance \( (L_g) \) and resistance
Robust Observer-based Controller for Three-phase Grid-connected Voltage Source Inverters

$(r_g)$ per phase. The value of $L_g$ depends on the operating condition of the grid at the point of common coupling (PCC). Although both grid inductance and resistance may be estimated from the point of PCC, they are not precisely known [73]. However, both $L_g$ and $r_g$ are bounded within upper and lower bounds that can be selected. Thus

$$L_g \in \left[ L_{g_{\text{min}}} , L_{g_{\text{max}}} \right]. \quad (5.65)$$

$$r_g \in \left[ r_{g_{\text{min}}} , r_{g_{\text{max}}} \right]. \quad (5.66)$$

Accordingly, the grid resistance and inductance can be considered as parametric uncertainties in the system (5.12). Therefore, a vector of 4 uncertainties, $p$, can now be constructed as

$$p = \left[ L_{g_{\text{min}}} , L_{g_{\text{max}}} , r_{g_{\text{min}}} , r_{g_{\text{max}}} \right]. \quad (5.67)$$

As a result, the system (5.12) can be described by a 4-vertex convex polytope (5.68) containing all possible values of matrices $A(p)$ and $B_w(p)$ in which the grid inductance and resistance appear.

$$\left[ A(p) , B_w(p) \right] \in \text{Co}\{v_i , \ldots , v_j\} : = \{ \sum_{i=1}^{j} \lambda_i v_i \geq 0 , \sum_{i=1}^{j} \lambda_i = 1 \}. \quad (5.68)$$

As a result, the system representation in (5.12) can now be expressed by a convex polytope as

$$\dot{x}(t) = A(p)x(t) + Bu(t) + B_w(p)d(t). \quad (5.69)$$

### 5.8 Control Design

The controller design problem is now stated as follows: Design an observer-based controller (5.21) to stabilise the system (5.69) by the means of the control law (5.23) in the presence of the uncertainties (5.65) and (5.66). For the design to be completed, the inequalities (5.24), (5.59), (5.60), and equality (5.25) need to be solved to return $W_1$, $W_2$, $X$, $Y$, and $F$ matrices that minimise the $H_\infty$ norm $\gamma$ for all vertices of the polytopic model.
Thus, the following optimisation problem is stated.

\[
\min_{W_1, W_2, X, Y, F} \gamma \\
\text{subject to} \quad (5.24), (5.25), (5.59), (5.60), \forall v_i, i = 1, ..., N.
\]

### 5.9 Results and Discussion

In this section, the results of various case studies, carried out on the proposed observer-based controller, are discussed. In table 5.1, the parameters of VSI and LCL filter used in the studies are shown. To solve the proposed control problem, expressed in (5.70), and associated LMIs-LME, Matlab’s YALMIP toolbox is used [102]. Different case studies are examined to demonstrate the effectiveness of the proposed observer-based controller. Through these case studies, the observer performance, controller tracking performance, control input saturation, and disturbance rejection performance are demonstrated. Except for Case II, which shows the tracking performance of the proposed controller, the VSI injects 20kW active power at unity power factor \(Q(t) = 0\) during the entire simulation. Zero-initial condition applies to the observer only and only when it is switched on. Once the observer converges to the actual state of the system, two will behave in unison. The values of \(\alpha\) and \(\beta\) given in table 5.2 have been determined through simulations, such that both controller and observer show an acceptable performance, though it is important to consider that, as a rule of thumb, the observer requires faster dynamics than the controller. The results of controller and observer gains, and the design parameters are shown in table 5.2.

As shown in Fig. 5.3, the grid voltage measurement \(v_{\text{gabc}}\) is given to the PLL in order to determine the grid angular velocity \(w(t)\), which is required for \(abc - dq\) and \(dq - abc\) transformations (5.5). Subsequently, the control input signal \(m_{dq}\) is transformed back into \(abc\) sinusoidal modulation signal \(m_{\text{abc}}\) to generate the switching pulses, by means of pulse width modulation (PWM). The LCL filter output current \(i_{\text{gabc}}\) is measured and
Table 5.1: System parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>VSI nominal power</td>
<td>$P_n$</td>
<td>25 kW</td>
</tr>
<tr>
<td>VSI switching frequency</td>
<td>$f_s$</td>
<td>15 kHz</td>
</tr>
<tr>
<td>Grid voltage</td>
<td>$v_g$</td>
<td>220 V</td>
</tr>
<tr>
<td>Grid frequency</td>
<td>$f_g$</td>
<td>60 Hz</td>
</tr>
<tr>
<td>DC bus voltage</td>
<td>$V_{DC}$</td>
<td>600 V</td>
</tr>
<tr>
<td>Grid inductance</td>
<td>$L_g$</td>
<td>±20% 1 mH</td>
</tr>
<tr>
<td>Grid resistance</td>
<td>$r_g$</td>
<td>±40% 0.4 Ω</td>
</tr>
<tr>
<td>LCL Filter</td>
<td>$L_{fc}$</td>
<td>0.45 mH</td>
</tr>
<tr>
<td></td>
<td>$r_f$</td>
<td>0.4 Ω</td>
</tr>
<tr>
<td></td>
<td>$C_f$</td>
<td>62 uF</td>
</tr>
<tr>
<td></td>
<td>$L_2$</td>
<td>0.05 mH</td>
</tr>
</tbody>
</table>

Table 5.2: Designed parameters and gains.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>80</td>
</tr>
<tr>
<td>$\beta$</td>
<td>300</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>2.96</td>
</tr>
<tr>
<td>$K$</td>
<td>$\begin{bmatrix} 31.26 &amp; 0 &amp; 1.63 &amp; 0 &amp; -31.97 &amp; 0 \ 0 &amp; 31.26 &amp; 0 &amp; 1.63 &amp; 0 &amp; -31.97 \end{bmatrix} \times 10^{-3}$</td>
</tr>
<tr>
<td>$L$</td>
<td>$\begin{bmatrix} 1.84 &amp; 0 &amp; 1.86 &amp; 0 &amp; 293 &amp; 0 \ 0 &amp; 1.84 &amp; 0 &amp; 1.86 &amp; 0 &amp; 293 \end{bmatrix}^T \times 10^2$</td>
</tr>
<tr>
<td>$K_I$</td>
<td>$\begin{bmatrix} -149.9 &amp; 0 \ 0 &amp; -149.9 \end{bmatrix}$</td>
</tr>
</tbody>
</table>

transformed into $dq$ DC signals, and then used as an input signal to the observer. The $dq$ current reference values ($i_{gd}^*$ and $i_{gq}^*$), that ensure delivery of the required active and reactive powers ($P^*$ and $Q^*$) into the grid, are obtained from (5.19) and (5.20).

5.9.1 Case I

This preliminary experiment demonstrates the performance of the designed observer for the system (5.12). As depicted in Fig.5.3, the observe receives the system output (5.13) in $dq$ frame ($i_{gdq}$), as well as the system control input expressed in (5.63). Figure 5.5 illustrates the actual (measured), $x$, and estimated, $\hat{x}$, states of the system (5.12). It is
evident that the estimation error \( e(t) = x(t) - \hat{x}(t) \) converges to zero within 10mSec.

5.9.2 Case II

In this case study, the proposed observer-based controller performance is investigated in the absence of grid parameter uncertainties as well as changes in the loading conditions for different reference input signals through the following scenario: at time=0, the VSI starts injecting 20kW active power at unity power factor \( (Q(t) = 0) \) into the grid. After that, at time=50mSec, the reactive power reference is changed from zero to 15kVAR. Subsequently, at time=100mSec the active power reference is dropped from 20kW to 10kW. Afterwards, the reactive power reference is reduced from 15kVAR to 5kVAR at time=150mSec. Finally, the active power reference is changed to 25kW at time=200mSec.

Figure 5.6 shows the injected active and reactive powers into the grid by the VSI. It also shows that output powers of the VSI track the command signals and converge within 15ms, on all occasions. Figure 5.7 showed the corresponding three-phase VSI output current \( (i_{labc}) \) regulated by the controller and injected into the grid. As can be seen, the magnitude of the injected current changes with the required amount of active and reactive powers to be injected into the grid, as per the reference inputs. Figure 5.8 demonstrates the control input signal generated by the observer-based controller to control the duty cycle of the VSI through the PWM. Here again, the magnitude of the duty cycle is regulated according to the changes in the reference value of both active and reactive powers. The modulation signal remains within its physical limits \( (m_{abc} \in [-1, 1]) \), and thus saturation is avoided on all occasions. Therefore, the overmodulation that might have destabilised the VSI is avoided.

5.9.3 Case III

This experiment is performed to verify the disturbance rejection performance of the designed observer-based controller. As mentioned in section 5.2, the grid voltage in equations (5.10)-(5.11) is considered as an input disturbance. This disturbance could be caused by changes in the grid power balance, due to large sudden changes in either generation
Figure 5.5: Observer states estimation response versus measured states in Case I.
5.9 Results and Discussion

Figure 5.6: VSI active and reactive powers delivered to the grid for Case II.

Figure 5.7: The VSI output current ($i_{abc}$) for Case II.

Figure 5.8: Control input signal, $m_{abc}$ modulation signal for Case II.
or load demand, as well as faults. The effects of such variations must be attenuated at the VSI output to achieve smooth control action. To examine the disturbance rejection performance of the proposed controller the following scenario is considered: at time $= 50 \text{mSec}$, the grid voltage decreased by 15% (0.85 pu). After that, at time $= 80 \text{mSec}$ the grid voltage is returned to its nominal voltage (1 pu). In Fig.5.9 the output active and reactive powers, as well as the VSI injected current in the $dq$-frame are, for the aforementioned scenario, shown. As illustrated in Fig.5.9, although the disturbance causes initial oscillations in the output of both power and current, these oscillations die out in a short period, within $8.8 \text{mSec}$. After that, the output follows the reference values without any distortion. This has been made possible by the optimisation design process where an attenuation level of 2.96 (Table 5.2), is obtained by solving the optimisation problem (5.70).

5.9.4 Case IV

In this experiment, the robustness of the designed controller, regarding grid uncertainties is investigated. This is done through the following scenario: at time $= 40 \text{mSec}$, the grid resistance $(r_g)$ is decreased by 15%, followed by an increase in the grid inductance $(l_g)$ by 15% at time $= 70 \text{mSec}$. As shown in Fig.5.10, it is evident that, despite existence of parametric uncertainties $(r_g$ and $L_g$), the system remains stable. Consequently, This verifies that the designed observer-based controller can deal with grid inductance and resistance uncertainties as well.

5.10 Summary

This chapter presents a systematic approach to design an observer-based controller for a grid-connected voltage source inverter (VSI). The controller is designed for a grid connected VSI with an LCL filter. The system parametric uncertainties, due to inaccurate knowledge of grid inductance and impedance, are considered in the control problem formulation by implementing a convex polytope representing the system uncertainties. Overmodulation that might occur due to control input saturation is avoided, through a proper control problem formulation. The proposed observer-based controller is robustly
5.10 Summary

Figure 5.9: VSI active power, reactive powers, and injected current for Case III.

Figure 5.10: Active and reactive powers for the grid uncertainties variation in Case IV.
designed to accommodate grid voltage disturbances with a guaranteed $H_\infty$ performance. As the controller used observer generated states, the implementation cost is significantly decreased by reducing the number of required measurements. The effectiveness of the proposed control approach is verified through different case studies. The provided results demonstrate quite clearly that the observer performance, controller tracking performance, control input saturation avoidance, and disturbance rejection performance of the proposed controller all satisfy the design requirements.
Chapter 6

Conclusion

6.1 Conclusion

In this thesis, a robust control design and a power management scheme for a DC microgrid with grid connection capability comprising multiple PV systems and energy storage systems are proposed. Chapter 2 provides a robust $H_\infty$ control design based on linear matrix inequality (LMI) for a PV system and an energy storage system, that are integrated at a DC bus. The considered topology in this chapter is known as an islanded DC hybrid system, in which the PV system and the energy storage system controllers are operating independently with a decentralised control structure. The designed PV system controller ensures maximum power extraction from the PV system by regulating a unidirectional DC-DC boost converter through adjusting its duty cycle. Subsequently, the energy storage system is integrated to the DC bus using a bidirectional DC-DC buck-boost converter, that is equipped with a robustly designed controller to maintain the DC bus voltage at its desired value. The proposed design approaches in chapter 2 consider converters nonlinearities due to their switching elements, modelled as a convex polytope, and achieves $H_\infty$ performance that ensures satisfying the required disturbance rejection performance. Additionally, control input saturation is avoided within the problem formulation to prevent excessive switching stress and system destabilisation. The results show that the proposed approach achieves faster maximum power point reference tracking with less oscillation for the PV system in comparison with existing methods in literature such as classical control techniques, and direct control approaches. Additionally, it is shown that the DC bus voltage remains constant under all operating conditions and
disturbances despite neglecting the systems interaction in this chapter. Therefore, the proposed approaches in chapter 2 can be used to increase DC hybrid systems efficiency and reliability which is crucial for small scale hybrid systems that are predominantly implemented in rural areas.

On the other hand, chapter 3 provides a multi-input multi-output robust controller design for the DC hybrid system considered in the previous chapter, while the interaction between the PV system and the energy storage system is taken into account in this chapter. Subsequently, a coupled controller that considers the system interaction was compared with a decoupled controller with neglected system interaction to investigate the effect of system interaction. Both controllers were designed to achieve the same control objectives as described in chapter 2 with considering the system parametric uncertainties. The provided results shown that ignoring the interaction has a significant detrimental effect on the performance of the system. In fact, the results indicate that coupled control improves system performance up to sixfold in comparison with existing decoupled approaches. Consequently, the proposed control approach in this chapter can be more favourable in comparison with the proposed approach in chapter 2 for small scale DC hybrid systems. In small scale DC hybrid systems increasing the system efficiency and performance can motivate the implementation of these systems for villages and small colonies that suffer from lack of electricity due to the non-existence of a grid connection.

In chapter 4, a DC microgrid comprising multi DC hybrid systems is considered. Therefore, a decentralised autonomous power management structure for DC microgrids consisting of multi PV and energy storage systems was proposed. The proposed design approach in this chapter can be implemented for large scale DC hybrid systems with high capacity (DC microgrids). The adopted topology of the considered microgrid in this chapter is islanded (off-grid) connection, therefore the PV systems are considered as the primary source of power for the microgrid. Each PV system and the energy storage system within the DC microgrid is equipped with a robustly designed controller that is using its local measurements only. Thus, the requirements for unit-to-unit communication links are omitted. The proposed power management scheme defines five operational and two none operational modes that cover all possible practical operating conditions of
6.1 Conclusion

a DC microgrid. Additionally, mode transition occurs autonomously and smoothly from one operating mode to another mode. In the case of excessive power being generated by the PV systems, the maximum power point (MPP) of each PV system must be shifted in proportion to the rated power of the PV system. This ensures that the generated power by the multi-unit PV systems is regulated to match the load demand. This is achieved through a droop control mechanism where the design controller ensures that the PV system with a higher generated power is shifted more than the PV system with a lower power generation. This mode of operation is commonly referred to as "constant power region" of operation. In this region any increase on the DC bus voltage that may be caused by excessive power being generated by the PV units can be avoided. The efficacy of the proposed power management scheme is tested in chapter 4 through various case studies applied to a DC microgrid comprising multi PV and storage units with different capacities and states of charge.

Chapter 5 provides a solution for the considered DC microgrid in chapter 4 in order to operate as a grid-connected DC microgrid. Therefore, a robust observer-based controller is designed for a grid-connected voltage source inverter (VSI). The VSI enables the DC microgrid to export the excess generated power by PV systems into the AC grid, and subsequently minimising the number of required maximum power point shifting proposed in chapter 4. Additionally, the robust observer-based controller is designed to minimise implementation cost by reducing the number of required measurements and consequently required sensors. A current control approach for VSI is adopted to regulate the active and reactive powers injecting to the grid, in this chapter. The dynamic controller and observer are robustly designed to account for grid disturbances while a guaranteed $H_{\infty}$ performance level is ensured. The designed approach considered the inaccurate information of grid resistance and inductance at the point of common coupling. Moreover, the overmodulation in the VSI is avoided through the problem formulation. The provided results show the effectiveness of the proposed approach and its stability over a wide range of grid uncertainties and disturbances.

In conclusion, this thesis provides a comprehensive control and power management solution for both small scale DC hybrid systems as well as large scale DC microgrids.
The proposed approaches can potentially increase the efficiency and performance of the systems above while assuring the systems’ stability maintained over a wide range of operations. As a result, the main motivation of the carried out research, presented in this thesis, is fulfilled by providing renewable energy based DC power systems with a higher level of reliability that can address the shortage of electricity for the remote and rural areas.

6.2 Future Research

The work in this thesis can be extended in several directions. A few of them are given below,

1. In the proposed topology, the flow of power was considered from the DC micro-grid to the AC grid (unidirectional). Therefore, the control objective of the voltage source inverter is defined to regulate the injecting surplus of active and reactive powers into the AC grid. One of the possible extensions that can be considered is to implement a bidirectional voltage source inverter that enables energy storage charging through the AC grid during off-peak load of the AC grid. Thus, the control objective and control-loop require to be modified to control the flow of power in both directions.

2. Unregulated output voltage and frequency through a control loop for the implemented voltage source inverter make it unable to provide power for locally connected AC loads. Therefore in the future, the designed controller can be extended to regulate the VSI’s output voltage and frequency for the islanded mode such that the locally connected AC demands can be supplied. It can be achieved by using two cascade control loops. The first cascade loop controls the voltage through a current controller in which the inner loop controls the current and the outer loop controls the voltage. The second cascade loop can be used with a droop controller in order to regulated VSI output frequency.

3. By increasing the number of integrated PV systems into the DC microgrid, and
subsequently the exporting capacity of the DC microgrid, it might be required to employ multiple voltage source inverters. Consequently, operating and coordination of multi voltage source inverters connected in parallel can be another challenge that requires to be addressed.

4. Least but not last, the concept of coupled microgrids can be introduced to address the intermittency of PV systems and load variations for each microgrid. The interconnection of multiple microgrids can reduce the generation shortage by exporting the surplus of power from one microgrid into another one, which is experiencing a deficit in power generation. Hence, this requires to introduce bidirectional power flow capability for the voltage source inverters, and power management scheme modification.
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