Computational Models of Human Motor Movement and Learning and their Application to Neurorehabilitation

by

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Doctor of Philosophy (PhD) Thesis

October 2017

in the
School of Engineering
Department of Mechanical Engineering
THE UNIVERSITY OF MELBOURNE

A thesis submitted in total fulfillment for the degree of Doctor of Philosophy
Abstract

Neurological impairment, commonly the result of stroke or traumatic brain injury, can have a devastating effect on lives of those who survive. This impairment can affect many capabilities including speech and reasoning, vision and movement. With 470,000 people in Australia currently living with the effects of stroke, and the expectation that this will rise due to our aging population, improving the efficiency and efficacy of neurorehabilitation is becoming more and more important.

Of interest in this work is the rehabilitation of motor control - that is, the recovery of basic motor function to allow the survivor to perform everyday activities such as eating, bathing and dressing; to return to work; or simply to reduce their reliance on carers and other services. The improvement of motor recovery can be aided through the investigation of models for motor control and learning, which may be leveraged to assess the presenting impairments, target specific impairments, or to develop rehabilitation exercises. Despite this promise, limited work has been done on the translation of existing models of movement for healthy persons, to those with neurological impairments.

This work therefore explores how computational models for motor control and learning may be developed and used for neurorehabilitation by:

1. Proposing a framework under which motor control, learning and recovery may be considered for neurologically impaired individuals

2. Proposing a model for motor adaptation applicable to neurologically-impaired individuals (including the proposal of appropriate algorithm which finds an iterative solution to the Finite Horizon Linear Quadratic Regulator problem)

3. Experimentally evaluating the model
Declaration

I, Justin Chun Mun Fong, declare that the content presented in this thesis titled, “Computational Models of Human Motor Movement and Learning and their Application to Neurorehabilitation” is my own, and that:

- The thesis contains only original work towards the degree of Doctor of Philosophy
- Appropriate acknowledgement has been made in the text to all other material used
- The thesis is fewer than 100,000 words in length, exclusive of tables, maps, bibliographies and appendices, as required by the University of Melbourne’s Research Higher Degrees Committee

Justin Chun Mun Fong
Preface

The work contained within this thesis has been completed with the support of Australia Research Council Discovery Projects DP160104018 and DP130100849.
Acknowledgements

The work within this thesis would not have been possible without the support and assistance of many.

First, my supervisors Denny Oetomo and Ying Tan — your countless hours of guidance, support and counsel have been extremely appreciated. Thanks also to Vincent Crocher for valuable input into the direction this work. I’ve been lucky to have such an approachable group of advisors for this PhD.

Secondly, members past and present of the Robotics Lab — Shou Han, Hadi, Ben, Rina, Demian, Michael, Jonathan, Gijo, Alireza, Wences, Florence and Ricardo. I’ve enjoyed our many discussions, both those related to research and those which have not. You have made the last four years much more enjoyable than they could have been.

I am also extremely grateful to Marlena and Bec who have always been willing to lend me their time to answer my questions around neurorehabilitation and patient presentations, given me invaluable practical insight into the treatment of neurologically impaired patients, and for their seemingly unlimited optimism.

Finally, my family. Mum, Dad, Brendan and Calvin — each of you inspire, love and support me in different ways, and I count myself fortunate for that. And Tania — thank you for joining me on my adventures through life, and sharing yours with me. Having you by my side makes everything so much better.
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Chapter 1

Introduction

In human motor control, redundancy exists at many levels, including task, joint, muscle and neural. Computational models for the human motor control system are developed to improve our understanding of how human resolve these different levels of redundancy when attempting to complete tasks — a problem first proposed by Bernstein in 1967 (Bernstein, 1967). These models include both those that apply to motor control, and those that apply to motor learning. Computational models of motor control can be used to evaluate and understand how people move given a specific task and — in applications such as assistive robotics — to model and/or predict movement patterns to assist in producing better robotic control performance. Computational models for human motor learning are commonly employed to describe how humans adapt to changes in the environment or learn a new task. Both these classes of computational model have potential application to neurorehabilitation of individuals who have experienced a neurological injury with motor control consequences. For example, computational models for motor control can be used to understand the manner in which the neurological injury is affecting the motor control system, or how interactions with a particular device will affect the movements of a patient. A computational model of learning or recovery may also be used to decide how best to treat a patient, or how best to assist or hinder a patient with a robotic device, in order to accelerate their recovery (Reinkensmeyer et al., 2016).

These potentials have not been realised, as existing computational models of human motor control and learning have yet to be extended to the neurorehabilitation scope. Individuals with neurological injury often have impaired capabilities in movement, often include spasticity (Thibaut et al., 2013), increased neuromotor
noise (McCrea and Eng, 2005), muscle weakness due to lack of use, and problems forming and executing motor plans (Kitago and Krakauer, 2013). These impairments mean that assumptions typically made when modelling motor control and adaptation are no longer valid.

This thesis therefore aims to address this, by investigating the differences between the motor control systems of healthy subjects and patients with neurological injury from this computational model perspective. Specifically, the work aims to:

- Propose a computational framework from within which computational modelling of motor control, learning and rehabilitation can be considered
- Propose a method of modelling motor adaptation (a subset of motor learning), which can be applied to neurologically impaired patients

1.1 Motivation and Scope

The rate of neurological injury, particularly stroke, is rising and is expected to continue to rise as life expectancy increases (Deloitte Access Economics, 2013). With the ever-increasing development of new sensing and actuation technology, significant potential exists to incorporate technology into the rehabilitation schedule, with the aim of reducing costs and providing easy-to-replicate, systematic rehabilitation. Neurological injury can affect a different areas of a human’s brain, which in turn can impair speech, vision, memory, cognition and motor control, each in varying degrees depending on the location and severity of the incident. This neurological injury generally affects motor control of one side of the body (hemiparesis), and can affect an individual’s ability to perform simple activities of daily living such as eating, washing or bathing, drastically affecting their independence and quality of life. Despite this, rehabilitation of motor control has been identified as an area which can be improved by new sensing and actuation technologies (Zhou et al., 2016a) — suggesting the application of engineering concepts to the field is promising.

Existing rehabilitation processes include both assessment and interventions. In the realm of assessments of motor capability, methods of assessment for stroke patients, are often coarse-scaled (on a scale of 0 to 3 or 0 to 5), and observation-based. Examples of these include the Wolf Motor Function Test (Wolf et al., 1989) and the Fugl-Meyer Assessment (Fugl-Meyer et al., 1974). Further assessments are used to
address different types of impairment, such as the Modified Ashworth Scale (Bo-
hannon and Smith, 1987) for muscle spasticity or Albert’s Test (Fullerton et al.,
1986) for unilateral spatial neglect. Interventions are also similarly varied, rang-
ing from simple movement exercises (such as asking a patient to reach forward)
to constraint induced therapy (Taub et al., 1993), in which the unimpaired arm is
constrained for portions of the day to encourage use of the impaired arm. Trans-
lating this — or parts of this — highly variable and experience-reliant therapy
to technology presents many challenges, some of which can be addressed with a
greater understanding of human motor control and learning which may be obtained
through the development and study of computational models. Furthermore, the
existence of such a model may also be applied more directly to neurorehabil-
iation. For example, a parameterised computational model of motor control may
be compared to patients’ movement to identify the type and severity of the motor
impairment — that is, such models may be used to assess patients. Furthermore,
incorporating recovery (through modelling of neurorehabilitation techniques) into
such a model may also be used to decide how to best treat a patient, or how best
to design control algorithms for neurorehabilitative robotic devices.

However, it is noted that humans, including the human motor control system, are
extremely complex. As such, the aim of this thesis is to present only a small piece
of the puzzle of towards a large and complicated task. The initial part of this
thesis therefore aims to provide groundwork towards this lofty goal, providing def-
initions and a framework from which neurological impairment, rehabilitation and
recovery techniques, can be considered from a computational perspective. This
work will define many areas of investigation which may be studied if these compu-
tational models are used within this neurorehabilitation context. The second part
of this thesis will develop a specific computational model for one of these areas —
motor adaptation to changes in dynamics. This choice is primarily due to its
relevance to neurorehabilitation, as the use of rehabilitation robots can introduce
new dynamics, often unintentionally (Fong et al., 2015a,b).

1.2 Contributions

This thesis provides three major contributions. First, a structured discussion of
neurological impairment and rehabilitation from a computational model perspec-
tive is presented. The second contribution is the proposal of an iterative algorithm
for solving an optimisation problem — with the view that this algorithm reflects
certain characteristics of motor adaptation. Finally, a computational model for motor control and adaptation is proposed, and applied to healthy subjects and neurologically-impaired individuals.

The first contribution is a proposal of how motor control, learning, impairment and rehabilitation can be considered from a computational model perspective. Within the literature, there are a number of existing computational models of motor control and learning, which are discussed in Chapter 2. Existing computational models of motor control have rarely been extended to deal with patients with neurological impairments. Such impairments are varied in both their effects and magnitude of effects, and drastically affect movement. Furthermore, in a neurological impairment and rehabilitation context, changes in motor control strategies and performance are not limited to traditionally-studied motor learning (that is, skill acquisition and motor adaptation). Changes over time also occur due to spontaneous recovery and in response to rehabilitation techniques. Such changes are not compatible with existing models of motor learning. As such, this thesis attempts to structure discussions for these differences. First, a systems model framework of motor control (Muratori et al., 2013) is presented, drawing inspiration from Shadmehr and Krakauer (2008). From this framework, features of motor control and learning are highlighted, based on optimal feedback control theory of motor control. Features of neurological impairment and rehabilitation are then identified from within the context of this framework. The goal of this contribution is to provide a framework from which features of neurological impairment and recovery can be clearly discussed and considered within computational models for motor control and learning.

The second contribution relates to the proposal of an algorithm which can be used to model motor adaptation. The algorithm developed here is proposed as an iterative solution to the Continuous Time Finite Horizon Linear Quadratic Regulator (FH LQR) problem, which does not require explicit knowledge of the dynamics of the system but instead relies on measurements to understand an improve performance. The FHLQR problem is a commonly-studied optimal control problem, where the optimal control scheme is derived for a system with linear dynamics, for a cost function which is quadratic according to some association with state and input effort. The solution proposed here is an adaptive dynamic programming (ADP) technique, in which estimates of the value (or cost-to-go) function are used to improve performance. However, the solution proposed here is novel, in that ADP solutions to this continuous time problem have not been considered in the literature. This algorithm is proposed an appropriate model of motor adaptation
because of the following features. First, the motor control and execution is generally accepted to be the result of an optimisation of goal achievement and effort or energy utilised to achieve it (Nakano et al., 1999, Guigon et al., 2007, Kang et al., 2005, Uno et al., 1989). Secondly, humans learn through a feedback mechanism in which they attempt tasks and adjust their strategy with the knowledge of their performance (i.e. they learn through practice) (Krakauer, 2006). Thirdly, humans are able to generalise a task - for example, learning a task in a given environment can translate to performance of similar tasks in the same environment (Shadmehr and Mussa-Ivaldi, 1994). Finally, humans are capable of adapting to environments with unknown dynamics (often investigated in through the implementation of a force field in reaching tasks) (Burdet et al., 2006, Zhou et al., 2012). As such, although it is not claimed nor suggested that humans perform motor adaptation through the implementation of this algorithm, it is proposed as a method which is capable of reflecting these important characteristics of motor adaptation.

The final major contribution of this project is the implementation of a computational model for motor adaptation, based on the aforementioned framework. A number of computational models for motor adaptation exist in literature. The model proposed here differs due to its application to a number of distinct purposes. First, the computational model applies to adapting to different dynamics in a family of tasks, rather only a single movement. Secondly, the computational model describes an adaptation process in which the adapted trajectory is unknown and different from the initial trajectory, unlike other models proposed in literature (Zhou et al., 2012, Bhushan and Shadmehr, 1999), which is more consistent with compensatory movements observed in neurologically impaired individuals. Finally, this computational model is constructed under the consideration of individuals with neurological impairments, addressing features of their adaptation which may differ from those observed in healthy humans. As with existing computational models, the computational model proposed here applies to a specific experimental method, which is designed here to demonstrate the adaptation features as mentioned, but also be accessible to patients with neurological impairments.

1.3 Thesis Structure

The structure of the remainder of this thesis is presented here, including a brief summary of the content presented in each chapter.
Chapter 2 presents a summary of existing related literature which is divided into two areas. First, existing computational models of motor control and learning, primarily for healthy subjects are presented. This is followed by a review of computational models of motor movement and learning for neurologically-impaired patients. Conclusions are drawn with respect to how the computational models of healthy subjects can be extended to neurologically impaired patients, and how models of motor learning cannot be directly applied to these neurologically impaired patients, particularly when considering rehabilitation.

Based on this, Chapter 3 outlines a framework for considering motor control, learning, recovery and rehabilitation from a computational model perspective. Within this chapter, subsystems considered within computational models of motor control are outlined, based on those seen within the literature review. Following this, definitions for skill acquisition, motor adaptation, recovery and rehabilitation are proposed. Finally, examples of specific impairments and rehabilitation techniques are discussed with respect to the framework, and subsystems of interest for each of these are identified.

The remainder of the thesis focuses on the development of a computational model for motor adaptation. Chapter 4 presents the development and mathematical analysis of an algorithm which can be applied as a model of motor adaptation. Chapter 5 presents the development of an experiment and a model to investigate motor adaptation. Chapter 6 presents results of the experiment on a limited number of healthy and patient subjects, and evaluates the computational model in terms of its applicability to modelling both motor control and motor adaptation.

Finally, Chapter 7 summarises the content of the thesis, and suggests avenues for future research and applications within this area.
Chapter 2

Background

Motor control and learning is often studied from a neuroscience perspective. In such work, hypotheses are made with respect to the mechanisms of motor movements or learning, and experiments are constructed to validate or reject these hypotheses. Such studies have led to an increase in understanding of the mechanisms under which humans learn. Based on these studies, computational models of these concepts have also been developed. Such models attempt to explain these concepts in a mathematical formulation, and can be used to predict and further understand behaviour.

Within this chapter, a number of topics are explored. First, an overview of human motor control systems is presented, particularly the problem that the human motor control system solves, and what is involved in doing so. This is followed by a review of existing computational models of motor control for healthy people, focusing on the optimality principle in motor control, and motor learning. Finally, a background in the motor control system of neurologically-impaired individuals is presented, including a discussion of existing computational models designed to model motor control, learning and rehabilitation in such individuals. These features are discussed with relation to movements of the upper limb — that is, the arms — although this discussion is presented with the view that parallels can be draw to movements of other parts of the body. However, a particular emphasis is placed on the arms in this thesis, due to their importance in activities of daily living, such as eating, bathing and dressing.
2.1 Existing Computational Models of the Human Motor System

The human body, in terms of motion, possesses a high number of degrees of freedom and thus a high degree of redundancy. Within the context of the human sensorimotor system, this redundancy exists at many levels, which the human brain is capable of resolving. For example, for any given reaching task, in which the objective is to place the hand on some object infront of the body, there are infinitely many hand trajectories which achieve the task, infinitely many joint trajectories for any given hand trajectory, and infinitely many muscle activation patterns for any given joint trajectory (see Figure 2.1). It is noted that goal-oriented movements of the hand are difficult to model for this reason — the movements are not cyclic, as they are in walking and gait; and they are inherently extremely variable, depending on the goal of the movement and the environment in which the movement is conducted in.

To produce effective movement, the human motor control system must be able to resolve this redundancy, a problem studied by Bernstein in the 1950s and 1960s:

\[
\text{The coordination of a movement is the process of mastering redundant degrees of freedom of the moving organ, in other words its conversion to a controllable system — N. Bernstein (Bernstein, 1957, page 127)}
\]

This approach, where the human body is considered a system to be controlled, translates the study of the biological system of the human body into a problem which can be (and often is) considered by engineers and control theorists. Within this thesis, resolution of redundancy in two scenarios is of interest — the resolution of redundancy in well-practised tasks — Motor Control —, and the resolution of redundancy in unfamiliar environments or for unfamiliar tasks — Motor Learning.

From an engineering perspective, the human motor control system can be considered a physical system (the musculoskeletal system), which is controlled by a controller in the central nervous system (CNS), see Figure 2.2. Based on some initial intention or task, the brain produces an actuation or control signal (a motor command), which is passed into the dynamics of the musculoskeletal system causing movement. At the same time, the brain observes the impact of its control signal on the physical system through sensory feedback mechanisms including visual, auditory and proprioperceptive. This feedback allows it to understand and
Task Level — For the task of reaching to the star, there are infinitely many possible paths to take.

Muscle Level — For any required joint torque, there are infinitely many possible muscle force combinations (diagram indicative only, does not represent actual muscle configurations).

Joint Level — For any given task space position, there are infinitely many possible joint configurations.

Figure 2.1: Examples of levels of redundancy in a simple reaching task. Note that the degree of redundancy is further increased when considering three-dimensional movements, as well as additional degrees of freedom such as that of the scapula and torso.

Each component within the system dynamics is complex. The musculoskeletal dynamics represent features of the spinal cord, muscles, muscle fibres and tendons and joints, most of which are nonlinear. Furthermore, the sensory feedback provided to the controller is noisy and delayed. This makes precise modelling of the system difficult or even infeasible. Instead, models of human motor control are of the computational nature — that is, simplified models with an emphasis on input/output characteristics. Depending on the context and motivation for the model, different simplifications are made for different models.
For the ease of discussion within this summary, a simple notation is now introduced (and shown in Figure 2.2). In accordance with control theory convention, the input to the system is denoted $u \in \mathbb{R}^m$, where $m$ is the number of inputs into the system. The state of the system is represented by state vector $x \in \mathbb{R}^n$, where $n$ is the (potentially large) number of states in the system. The feedback from the system is through measurements $s \in \mathbb{R}^p$, which represent the feedback derived from the senses, including visual and proprioceptive feedback. The dynamics are then represented by the general nonlinear function $\dot{x} = f(x, u, t)$, which is potentially (but not necessarily) time varying. The dynamics of the senses based on the state but including the addition of delay or noise, are represented by the function $s = g(x)$. The task-dependent controller takes the information from the feedback and (based on the task), constructs the input to the system, as described by the function $u = C(s, t)$. It is noted that the exact structure of the function $C$ is not necessarily known, and, for example, may also include some time dependence.

The remainder of this section discusses computational models which have been developed to understand how redundancy is resolved for motor control and learning.

### 2.1.1 Models for Human Motor Control

The redundancy in human motor control is commonly thought to be resolved through an optimisation process — that is, movements are performed to be optimal with respect to some cost function. Many computational models have been developed to attempt to model this unknown cost function, with weighting often given to achievement of objective, and the effort expended to achieve it (Flash and Hogan, 1985, Uno et al., 1989, Nakano et al., 1999, Kang et al., 2005, Guigon et al., 2007, Liu and Todorov, 2007, Kim et al., 2011, Jiang and Jiang, 2014). The
idea of optimality in motor control is also intuitive — minimising energy consumption whilst attempting to move has an obvious evolutionary benefit. Anecdotal evidence for an optimisation process can also be observed in movement patterns for any given individual. For example, placing a heavy cast iron saucepan into a cupboard involves more torso movement than placing an empty mug in the same cupboard. Whilst the requirement of the movement is the same, the action taken by the human changes according to the requirements of the task.

Optimality is well-studied in engineering literature as a method of resolving redundancy in engineered systems, and can be described mathematically as follows. Given the task to be achieved, an optimal movement is one in which some task-specific cost function $L(x, u, t)$ is minimised. It is noted here that for the sake of example, the optimisation is presented as being finite-time ($t_0 \leq t \leq t_f$) and time varying. However, other formulations (time-invariant, infinite horizon) can and have been used in the literature.

With this formulation, the optimal movement trajectory is denoted $x^*$ with associated optimal input $u^*$, which is defined by:

$$u^*(t) = \arg \min_{u(t)} L(x(t), u(t), t), \quad t_0 \leq t \leq t_f$$

(2.1)

and where $x^*(t)$ is the resulting trajectory subject to the dynamics:

$$\dot{x}^*(t) = f(x^*(t), u^*(t), t), \quad x^*(t_0) \in R^n, t_0 \leq t \leq t_f$$

(2.2)

Within the literature, computational models for human motor control differ through the following aspects. First, the description of the plant dynamics ($f(x, u, t)$) differ in both their states ($x$), the input signals ($u$) and the dynamics themselves. Different simplifications are employed depending on the context and purpose of the models. Secondly, different cost functions $L(x, u, t)$ are used to describe the objective of the movements, and the resulting redundancy resolution. Thirdly, different controller structures are utilised. These variations are generally motivated by the desire to model different tasks or aspects of human motor control. Within this subsection, we focus our discussion on the latter two of these aspects, with
the understanding that the representation of the dynamics emerges as the result of these choices.

### 2.1.1.1 Cost Function

A number of different cost functions have been proposed over the last few decades. A more detailed discussion of the different types of cost functions, and their differences and similarities follows. These cost functions have been classified into three groups — Those based on kinematic parameters; mechanical dynamic parameters (based on dynamic parameters associated with the mechanical properties of the model); and neurological dynamic parameters (those associated with hypothetical signals in the brain). A brief summary of the discussed works, including their cost functions and their optimisation objectives is presented in Table 2.1.

#### Table 2.1: Summary of Cost Functions in Literature

<table>
<thead>
<tr>
<th>Reference</th>
<th>Parameters in Cost Function</th>
<th>Optimisation Class</th>
<th>Resolved Redundancies*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flash and Hogan (1985)</td>
<td>Jerk of Hand Trajectory</td>
<td>Kinematic</td>
<td>Task</td>
</tr>
<tr>
<td>Kim et al. (2012)</td>
<td>Swivel Angle</td>
<td>Kinematic</td>
<td>Joint</td>
</tr>
<tr>
<td>Qian et al. (2013)</td>
<td>Motor Signal</td>
<td>Neur. Dynamic</td>
<td>Task, Time</td>
</tr>
</tbody>
</table>

*The optimisations here are all constrained by initial conditions. However, many are also constrained by final condition (or state) as well. The works for which the end state is a result of the optimisation are identified by specifying ‘Endpoint’ here.

Flash and Hogan’s (Flash and Hogan, 1985) early work proposed a cost function based on minimising jerk (the third derivative of position). It was proposed that reaching movements in a horizontal plane are performed with a velocity profile consistent with minimising the time integral of the square of the magnitude of
jerk:

\[ \mathcal{L}(x) = \frac{1}{2} \int_{t_0}^{t_f} \dot{x}^2(\tau) + \dot{y}^2(\tau) d\tau \]  

(2.3)

where \( x \) and \( y \) reflected the position of the hand in those coordinates. This optimisation results in approximately straight paths for unconstrained point-to-point movements, and that constrained movements (such as those requiring obstacle avoidance) are also performed with a view for minimising (2.3). In this work, no cost is associated with the input, and thus the optimisation is entirely kinematic-based. Furthermore, this optimisation provides only resolution of the task trajectory, given that the start and end points are constrained, and provides no information about the resolution of redundancy at the joint level. More recently, another kinematic optimisation criteria for three dimensional tasks based on an evolutionary trait — that arms are used to eat (Kim et al., 2012). In that work, the authors proposed there that the resolution of arm redundancy is completed such that the arm is configured in a manner which makes it easiest to move the hand towards the mouth. This is achieved by modelling the arm as a two-link seven degree-of-freedom manipulator, taking the plane formed by the humerus (upper arm — link 1) and the lower arm (link 2), and ensuring that the mouth remains in this plane. Again, the trajectory of the task is resolved by this optimisation, but also the redundancy in joint configuration.

Kinematic optimisations, however, cannot account for all features observed in human movement. For example, throwing a tennis ball has an obviously different movement pattern to throwing a heavy shot put. Optimisations based on dynamic parameters are able to address these observations, and have been developed. These include optimisations based on mechanical dynamics (utilising forces and torques), as well as neurological dynamics (based on modelled signals such as motor command signals originating in the brain).

Among mechanical dynamic optimisations, cost functions have been proposed which penalise work done by the joints (Kang et al., 2005) and minimise torque change (Uno et al., 1989, Nakano et al., 1999). The minimal work discussion in Kang et al. (2005) utilises a 4 degree of freedom model, with a spherical joint representing the shoulder, and a revolute joint representing the elbow. Based on this, utilising a pre-recording trajectory of the wrist, the swivel angle (Tolani and Badler, 1996) is the only redundant parameters in this optimisation. As such, this mechanical dynamic optimisation resolves only a single degree of freedom — it does not resolve for the redundancy in the hand or task space trajectory. The
minimum torque change optimisation (Uno et al., 1989, Nakano et al., 1999) does resolve the redundancy in task space also, providing a model utilising 6 muscles in a two link manipulation. It is noted that the formulation was corrected from its initial form (Uno et al., 1989) to a revised form (Nakano et al., 1999) in which revised parameters were used. It is noted in these revised computations that the formulation does not model well all arm movements in the horizontal plane. Thus, an updated minimum \textit{commanded} torque change model is also proposed in the same paper (Nakano et al., 1999). This model takes into account the viscous properties of the muscle which the commanded torque must overcome in order to move the limb.

The final class of optimisation are those based on neurological signals — that is, signals which are representative of parameters in the brain, but cannot be directly measured. This includes a minimum ‘effort’ optimisation (Guigon et al., 2007), which models the muscle dynamics with a second-order low pass filter model, and equates effort to a ‘neural control signal’. The optimisation proposed was then used to describe a number of reaching behaviours across different activities, through the utilisations of three different models of the arm — a 2 degree of freedom planar model, a 4 degree of freedom (spherical-revolute) model, and a 7 degree of freedom (spherical-revolute-spherical) model. The introduction of the final neurological dynamic optimisation brings about another interesting property — that of endpoint posture redundancy. In the previously-discussed optimisations, the optimisation problems are constrained in final posture, or the final state is known. However, as can be observed in Figure 2.1c, redundancy also exists at the end point. As such, humans are capable of exploiting this redundancy through their movements, and utilise this redundancy to ensure completion of tasks in the presence of disturbances. Such redundancy has been modelled in Todorov and Jordan (2002) where a general ‘motor command signal’ is penalised, but the analysis is only performed in task space. This is also consistent with Harris and Wolpert (1998) — which, on the assumption that noise in the control signal is proportional to the magnitude of the signal, means that variance can be reduced with respect to a reduction in control signal. Furthermore, this allows the explanation of the ‘Uncontrolled Manifold’ statement (Latash et al., 2002) in which disturbances in areas which do not affect task completion are not corrected for.

It is also noted here that under optimisations with unconstrained time, another variable is possible — the time allowed for the completion of the movement — observed and modelled (empirically) under Fitts’ Law (Fitts, 1954). An optimisation which produces these results is presented in Qian et al. (2013), in which
movement duration is not explicitly stated, but resultant under the propose of an infinite-horizon neurological dynamic-based optimisation.

As is evident from this discussion, a large number of computational models of varying complexities have been proposed in the literature. Although the models introduced here are all constructed for reaching movements, the number of different cost functions suggest that this is still an open problem. However, it is also noted that some cost functions can be related to each other — for example, Todorov’s model (Todorov and Jordan, 2002) minimises a ‘neural control signal’, which, given the model utilised, can be made equivalent to minimising jerk with appropriate scaling factors. However, minimising a ‘neural signal’ predicts differing behaviours under different conditions (for example, if redundancy in the task is given). To provide another example, the effort principle in (Guigon et al., 2007), is related to the change in muscle force, which is related to the commanded torque change. It is possible that such differences can be related by appropriate parameter selection, linearisation or other appropriate approximation. Furthermore, cost functions are likely to differ depending on context. For example, catching a ball requires just that the hand is in the correct position when the ball arrives at that point. In contrast, hitting a ball requires some hand velocity at the time of contact — the maximisation of this hand velocity may therefore also form part of the cost function. As such, these different tasks are likely to have different cost functions in accordance with these differences.

**2.1.1.2 Controller Structure**

A second attribute of computational models of the motor control system is how the controller is structured. In this section, two commonly-utilised structures are presented — trajectory tracking of pre-calculated optimal trajectory, and optimal control, in which the control policy is optimal for any given state (often termed ‘Optimal Feedback Control’ in the human motor control literature). These two structures are capable of modelling different aspects of motor control, and are also sometimes presented together in combination.

The key feature of the trajectory tracking formulation is that a reference trajectory is calculated offline — that is, a desired trajectory is determined before the movement is commenced, and input signal generated by the controller includes a feedforward component (to follow this trajectory) as well as a feedback component to reject any disturbances. This trajectory is planned in state space (task, joint
or muscle) before the movement starts, and is based on the optimisation of a cost function (as discussed in Section 2.1.1.1). The structure of such a controller can be seen in Figure 2.3. A simplified example feedback control policy which fits within this framework may be of the form:

\[
    u(t) = C_{fb}(x_{ref}(t), \dot{x}(t), t) = u^*(t) + K(\dot{x}(t) - x_{ref}(t))
\]  

(2.4)

where \( K \in \mathbb{R}^{m \times n} \) is a feedback gain, designed to stably bring the trajectory back to \( x_{ref} \) should the system be affected by disturbances, \( u^* \) is defined as in (2.1), and \( x_{ref} = x^* \) is the optimal trajectory as defined in (2.2). This approach is commonly used in engineered systems, and notably separates the planning and execution stages. This sequential, trajectory generation to trajectory tracking approach has been the accepted structure for modelling human motor control with the planning and execution stages commonly discussed as the ‘motor planning’ and ‘motor execution’ stages of motor control respectively.

The Equilibrium Point Hypothesis (EPH), or Threshold Control Theory (TCT) (Feldman and Levin, 2009) is a variant, in which the control signal specifies only some reference ‘state’ of the system, and the properties of the muscles and dynamics produces the resulting motion and steady state position. This is achieved through the specification of the input as affecting the threshold of alpha-motoneurons — which are related to muscle length. The muscle then attempts to achieve this length, with the actual equilibrium configuration dependent on the combination of input signals, and external forces.

However, this model does not explain well how the body responds in the presence of disturbances. It has been observed that disturbances in directions which do not
Affect the completion of a goal are not compensated for. This has been observed in a number of different tasks. For example, in the sit to stand task (Scholz and Schöner, 1999), small movements in the sagittal (left/right) plane are not corrected for, as the base over which the centre of gravity must remain for stability is wider in the left/right direction than in the forward/back direction. Similarly in a pistol shooting task (Scholz et al., 2000), movements in the forward/back directions are not as important for hitting the target as movements in the left/right and up/down directions. The disturbances which are not compensated for are a manifestation of the Uncontrolled Manifold (Scholz and Schöner, 1999) — a manifold of effects of disturbances which do not affect the completion of the task. The presence of this uncontrolled manifold is inconsistent with the nature of this trajectory tracking controller structure — the definition of a trajectory in state space (and a feedback controller guiding towards it) does not allow the use of the redundancy to accommodate for the disturbances in the uncontrolled manifold.

As such, a second control structure is proposed for the controller structure — optimal control. In this instance, a mapping between the sensory input and control signal is produced, which is optimal given that sensory information at that point in time. In contrast to the trajectory tracking approach, no reference trajectory is calculated. This controller structure is outlined in Figure 2.4.

Mathematically, the optimal controller can be described as:

$$u(\hat{x}, t) = C_{op}(\hat{x}, t) = \arg \min_{\tau} \mathcal{L}(\hat{x}, u, t)$$  \hspace{1cm} (2.5)
subject to the dynamics ($\dot{x} = f(x, u, t)$), state and input constraints. That is, using an estimate of the state, the controller applies the control input which is optimal from that point on — regardless of what the previous state was. Mathematically, solving for this optimal control policy can be extremely difficult, and is, in engineering cases, generally solved through computationally-intensive searches completed offline. Furthermore, the solutions are generally unlikely to have a closed form solution, meaning that application of the controller for complex systems is commonly reduced to a lookup table. Due to these complexities, applications to computational models of motor control are therefore based on simplified dynamics and demonstrative examples.

Despite this, this structure of motor control is currently a well-discussed theory and well-regarded as a suitable model for motor control (Diedrichsen et al., 2010b, Shadmehr, 2009, Scott, 2004). In the absence of disturbances, a model based on this structure of controller generates optimal movements, according to the cost function specified. Furthermore, Todorov’s works (Todorov and Jordan, 2002, Todorov, 2004, Liu and Todorov, 2007) have detailed how the movements resulting from the use of optimal control can be consistent with the uncontrolled manifold.

The two controller structures presented here have been discussed thoroughly in the literature. Like the discussion around cost function, consensus has not been reached regarding a structure for a human’s motor control system. Similar to the cost functions, the appropriate structure depends on the intended behaviour which is to be modelled — some behaviours can be modelled adequately by either or both of these controller structures. Finally, combinations of these structures may also be considered, where optimality includes some notion of a reference trajectory as part of the cost.

### 2.1.2 Models for Human Motor Learning

Models for motor motor control attempt to describe how a task is executed when it is well known. In contrast, models for *motor learning* attempt to describe how the human motor system develops this control approach. More generally, motor learning can be described as how the motor controller changes over time, with respect to either a new task or a change in the task dynamics. For example, learning to ride a motorcycle, and having to simultaneously balance and steer at the same time is a form of motor learning — skill acquisition. Another form of learning is motor adaptation, which can be illustrated by learning to ride a lighter
scooter after having learnt how to ride the heavier motorcycle. The motor control system adapts to the lower inertia of the scooter, altering how one adjusts one’s weight whilst turning or riding on uneven surfaces.

From the computational model perspective, based on Figure 2.2, motor learning can be seen as how $C(s, t)$ changes with respect to being presented with a new, novel or modified task or dynamics $f(x, u, t)$. In accordance to the principle of optimality, this may also be seen as improving one’s performance of a task for a given cost function. It is noted here that that switches between known tasks are not included under this categorisation of motor learning.

Motor learning plays an essential part in the development and evolution of human motor capabilities — including learning to walk and reach as children, to learning to play new sports, utilise equipment, and even learn musical instruments. It is also well accepted (Krakauer, 2006, Kitago and Krakauer, 2013, Huang and Krakauer, 2009) that motor learning plays an important part in rehabilitation from neurological injury. However, although the development of human brain imaging technologies has allowed exploration as to the parts of the brain which are active during motor learning (Hikosaka et al., 2002), the mechanisms behind motor learning (both skill acquisition and motor adaptation) are not well understood. As such, within this section, a review of computational models of both skill acquisition and motor adaptation is provided. Given the focus of the latter part of this thesis, a more detailed discussion of existing models of motor adaptation is given here.

2.1.2.1 Models of Motor Skill Acquisition

Motor skill acquisition is the development of competency at a new motor task. Methods of acquiring and training motor skills has been a topic of research for over 100 years (Stratton et al., 2007) — unsurprising given its application to optimisation in human-run factory lines. In this section, a brief discussion is presented on existing computational models developed to model skill acquisition.

The majority of models described as “Computational Models for Motor Skill Acquisition” are based on modelling the progress of performance of tasks. These models characterise the curve relating a performance index, such as how quickly the task is complete, and the number of times that the task has been attempted. This includes the classic 1926 work of Snoddy (1926), which modelled the increase in performance of a tracing task (indicated by tracing speed and number of errors) as a power law. Other similar models exist modelling the change in performance
over time, utilising different mathematical relationships (Mazur and Hastie, 1978, Welford, 1987), and more complex nonlinear frameworks to reconcile the existence of these different models (Newell et al., 2001). Whilst these models provide a mechanism for describing the evolution of performance under the experimental conditions described, and have scope to evaluate and predict performance in learning of a particular task, such models do not well address how different factors may influence the learning of the task.

A more detailed framework based on neurological aspects has been developed (Doya, 2000). In this review work, Doya provides evidence suggesting that learning is driven by both reward and error, and attributes these types of learning to the basal ganglia and cerebellum (areas of the brain) respectively. This construction can be related to a well-visited discussion regarding knowledge of result (being told a ‘score’ — performance based) and knowledge of performance (intrinsic feedback about how the task was performed — error based) which is reviewed in (Salmoni et al., 1984). Although Doya’s work proposes only a framework, the idea is of interest to this thesis due to the clarification of what information is used to update the motor control strategy. In particular, it suggests that models of skill acquisition should adapt based on information about how well a subject performs a task, as well as how well the subject ‘thinks’ they performed the task. This work has inspired a range of control algorithms which will be discussed in more detail in Chapter 4.

2.1.2.2 Computational Models of Adaptation

Motor adaptation can be seen as how motor control changes in response to disturbances external or internal to the body. Adaptation is most commonly studied through experiments which involve a reaching movement in the horizontal plane whilst holding a robotic manipulandum. In these experiments, a change in the dynamics of the movement are affected by the apparatus through the unexpected introduction of a visual disturbance — a rotation of the visual interface — or a dynamic disturbance — force fields actuated by the robot. These changes induce a difference in trajectories predicted by the subject, and the trajectories perceived by the subject. It is believed that the difference in these trajectories drives changes to the motor control system — both in the generation of the control signal, the estimation of the states —, which is termed motor adaptation. It is suggested that these changes occur without subject’s awareness (Kitago and Krakauer, 2013). By
this nature, in general, the differences in dynamics are generally considered quite small (although noticeable by the subjects).

A first class of models for motor adaptation are state-space based models (Cheng and Sabes, 2006, Thoroughman et al., 2007, Ueyama, 2017). These models represent parameters involved in motor control as a ‘state’, and sensory information about the performance of a task as an ‘input’. Dynamics are then used to relate the update of the ‘state’ to the ‘input’. As such, the choice of ‘state’ and ‘input’ for these models is specific to the control type of that particular movement, and do not easily generalise. For this reason, this review does not present these models in detail, but focuses on models for motor adaptation which explicitly model the mechanics behind each individual movement. Specifically, three such models are presented.

The first is the work published by Bhushan and Shadmehr (1999), which models a reaching experiment where the subject held onto a planar robot in the horizontal plane, and were asked to move to one of targets 10 centimetres from the initial location in 8 different directions. The robot was initially passive, causing no external force to be exerted on the subject’s hand. Following this, two curl (rotational) force fields \( F_{1a}, F_{1b} \) were applied of the form:

\[
F_{1a} = \begin{bmatrix} 0 & 13 \\ -13 & 0 \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} \tag{2.6}
\]

\[
F_{1b} = -F_{1} \tag{2.7}
\]

where \( \mathbf{x} = [x, y] \in \mathbb{R}^2 \) is the position of the hand. These two force fields produced a force proportional to the velocity of the hand, in a different direction to the velocity. For example, movement in the \( y \) direction caused a force in the \( x \) direction, and movement in the \( x \) direction caused a force in the negative \( y \) direction.

Bhushan and Shadmehr (1999) utilised a trajectory-tracking controller structure, in which adaptation occurred through updating two internal model within the motor controller — an inverse model (which predicted a control signal based on a desired trajectory), and a forward model (which estimated the state based on sensory-information). If the forward and inverse model match the actual forward and inverse dynamics, the trajectory is executed perfectly. However, if there is a difference between the two (such as when the subject expects one field and is presented with another), another trajectory results. Adaptation is modelled
utilising a simple update law:

\[
FM(n) = FM(0) + \Delta FM (1 - e^{-nr_{fm}}) \tag{2.8}
\]

\[
IM(n) = IM(0) + \Delta IM (1 - e^{-nr_{im}}) \tag{2.9}
\]

where \(FM(n), IM(n)\) represent the estimated forward and inverse models utilised at the \(n^{th}\) attempt at the movement, \(\Delta FM, \Delta IM\) represent the difference between \(FM(0), IM(0)\) and the actual forward and inverse models, and \(r_{fm}, r_{im}\) represent the learning rates of the forward and inverse models.

There are a number of important characteristics of the model. First, it utilises a trajectory tracking approach, where the aim of the controller is to follow some offline-generated reference trajectory. Secondly, it is noted that no information from the trajectory is utilised to improve the performance. Instead, it is assumed that the internal models converge to the new actual model exponentially, as the rate of model change is proportional to the difference between the estimated model and the actual model at any iteration.

The second work discussed here is the model proposed by Zhou et al. (2012, 2016b). The model addresses only a single task — subjects were asked to reach forward in a straight line 25 centimetres away, again whilst holding onto a planar robot manipulandum. Two different experimental studies are presented. The first (Zhou et al., 2012) again involves two force fields — a curl force field \(F_{2a}\), and a divergent force field \(F_{2b}\) of the forms:

\[
F_{2a} = -\begin{bmatrix} 13 & -18 \\ 18 & 13 \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} \tag{2.10}
\]

\[
F_{2b} = -\begin{bmatrix} 450 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \tag{2.11}
\]

where again \([x, y]\) represents the hand position, where \(y\) was the desired direction of the reach. Within \(F_{2a}\) the force was again proportional to the velocity of the hand — movement in each direction resulted in both resistance in that same direction, as well as a push in the perpendicular direction. \(F_{2b}\) was only proportional to the position in the \(x\) axis — in a perfect straight line movement to the target, no forces were applied, however, if the reach strayed off this straight line path, the hand was pushed even further away.

The controller in this model attempts to track the states of a reference model (reflecting a reference output trajectory). Updates to the control law over iteration
include updates to both components, based on an update law proportional to the
difference in the actual motion and the state of the reference model. As such, here
the errors explicitly drive the change in control structure.

Within this particular experiment, the experimental results demonstrated that the
subjects reject the disturbances caused by the force field, and return to the original
reference trajectory. As such, the goal trajectory is modelled as agnostic to the
disturbances. Furthermore, the computational model implies this goal trajectory
is known. It also requires the development of a single controller for each unique
task — the controller rejects the disturbance to that particular trajectory, and
does not learn the overall effect of the field, if, for example, a task was presenting
requiring a reach to a different location.

A second work by the same authors (Zhou et al., 2016b) involved an experiment
utilising similar apparatus, investigating the role of feedback on learning patterns.
In this experiment, subjects were asked to reach a target infront of them within
550 ms, where vision of the hand position was blocked by a screen. The location of
the hand was revealed to the subject only at the end point, along with information
about the speed of the movement (whether they had reached the target too fast,
at the right time, or slowly). It is noted that no consideration is given to other
forms of feedback, such as proprioperceptive. The work in this paper utilised a
feed forward controller only (as no online information is available for feedback).
However, the objective for the update law was to minimise the endpoint error,
but a secondary objective is also introduced, which translates to a minimum jerk
requirement as proposed by Flash and Hogan (1985). As such, some non-explicit
cost is presented as having a contribution to the motor controller. The update law
presented here, then, was of a gradient descent method, in order to reduce both
the endpoint error, and the jerk of the movement.

The models presented by Zhou et al. explicitly utilised the error introduced by the
disturbances, and attempt to reject them. In the first work, an explicit reference
trajectory was constructed to be followed, whereas in the second, the trajectory is
shaped by a cost function. However, the adaptation laws here are presented only
for a single task, and they do not generalise to others — even something as simple
as reaching to a slightly different point in the same force field.

A more recent work to this field is that proposed in Jiang and Jiang (2014). The
work in this publication utilised existing results within the literature, including the
force field reaching experiments in the works of Zhou and Bhushan. The controller
proposed within Jiang’s work presents all tasks as feedback stabilisation tasks in
a Linear Quadratic Regulator (LQR) formulation. The controller falls within the optimal control framework, and as such no reference trajectory is designed. Instead, the optimal trajectory naturally evolves as a result of the application of this controller. This is consistent with the work by Izawa et al. (2008), which describes adaptation as “a process of reoptimisation” (although no model for this reoptimisation process is presented).

The resulting proposed adaptation model updates the control structure based on the measurements obtained during the movements. It then adjusts its control strategy towards reducing the cost for subsequent movements — no comparison to a reference trajectory is required for the adaptation process, and a systematic way of improving the performance is presented. This model, however, requires the use of unique cost functions for each movement, which changes between dynamic conditions. As such, it is difficult to use this model to predict how movements would be adapted, as knowledge of a new cost function is required, which differs after the introduction of the new force field dynamics.

Each of these models of motor adaptation illustrate how the motor control changes over time due to changes in the environment, through updating aspects of the controller — either the basic control strategy, or through internal models used to implement this control strategy. Due to the inherent variability in human subjects, exact quantification of the mechanisms behind motor adaptation are difficult to determine. However, each of these models portrays one of two main ideas. Either humans attempt to follow some reference trajectory and when presented with a new environment attempt to reject the disturbances, or humans adapt by re-optimising some cost function.

Parallels can be drawn to the controller structures discussed in Section 2.1.1.2. It should be noted, however, an optimal control structure can produce similar results to a trajectory tracking example, simply by including a reference tracking cost into the cost function. Nevertheless, due to the uncertainties and complexities of the system to be modelled, computational models of human motor control and learning should be assessed based only on the application that they were designed for. Therefore, although the existing models of learning and adaptation provide ideas and inspiration for future models, a discussion is now presented based on the goals of the computational models proposed in this thesis — application of motor control and learning models to those with motor deficits caused by neuroimpairment.
2.2 Models of Motor Control in Neurorehabilitation

Neurological injuries can happen through a variety of incidents, including a stroke, brain injury through trauma, or through the presence or growth of a tumour in the brain. Neurological injuries can cause a large variety of impairments, including problems with cognition, problems with sensory processing, problems with communication, and behavioural issues, each of which may have a lasting and large impact on a patient’s quality of life. The focus within this thesis, however, are problems with motor ability, which in turn can also be affected through a number of different ways, and to various degrees. Stroke is the most commonly-discussed form of neurological injury, due to its status as the leading cause of neurological injury, and an expected increase in rate due to the world’s ageing population. In Australia, approximately 420 000 people were living with the effects of stroke in 2013, which is predicted to reach 709 000 or 2.4 % of the population by 2032 (Deloitte Access Economics, 2013). Up to 77 % of stroke patients suffer from upper limb impairment (Lawrence et al., 2001). As such, improving understanding and efficiency of the neurorehabilitation of these motor abilities therefore has potential to have a significant impact on the quality of life of stroke survivors, and a significant affect on large number of people.

One of the motivating factors behind the development of motor control and learning models has often been its relevance to neurorehabilitation – the possibility that such models may be used to inform and improve the process. There are many ways that this can be done, including using them for measurement and for improving the control of robotic devices (Zhou et al., 2016a). Despite this potential, limited work has been conducted in the construction and application of models to neurorehabilitation.

Within this section, two key issues are discussed. First, a brief background to neuroimpairment is given, and a discussion about the different ways that such impairment can affect movement. Secondly, a discussion on existing computational models related to neurorehabilitation, and the reasons that such models are limited.
2.2.1 The Complexity of Neurological Injury and its Impact on the Human Motor System

Neurological injuries are complex in their cause, physiological effects, and manifesting impact. Each neurological injury can have an unique impact on a patient’s motor control capabilities, depending on the location and size of the damage. This has an obvious impact in the capabilities of producing computational models — the models should be complex enough to capture the impact of the impairment, but simple enough to be useful.

Physiologically, different areas of the brain contribute to different components of motor control. This has been investigated through examination of the physical structure of the brain (identifying which areas connect with which) as well as experiments with both humans and animals (usually primates). For example, the cerebellum is thought to be involved in developing forward models — used to predict the effect of one’s own actions. In a weight-catching experiment (Nowak et al., 2007), subjects were asked to hold an object with their hand with their eyes closed, and keep hold of it whilst an experimenter dropped a ball into a receptacle connected to the object. As a result of this, the participants increased their grip strength when they detected this impact, measured by force sensors on the object. The experiment was conducted again, except this time the participant him/herself was responsible for dropping the object. The healthy participants increased their grip strength when they were about to drop the ball, in anticipation of the force and weight of the ball caused by its landing into the receptacle. However, a patient without a cerebellum (caused by cerebellar agenesis — a brain development condition) did not change her response — she responded in the same way regardless of whether it was an experimenter or herself dropping the ball. In an animal-experiment example, activity of the parietal cortex was measured using electrodes during a reaching task (Ferraina et al., 1997). In the task, the monkey had to first look at and then move their hand to a central target, before one of 8 other targets was illuminated. At this point, the monkey subject was required to look at and move their hand to this new target in order to receive a reward. The results of the study suggested that the parietal cortex is utilised to identify the location of the hand combining the information from vision and other senses.

Given these examples, it can be observed that the types of studies in this area are quite coarse — they can only hypothesise and conclude whether certain areas of the brain are involved in the completion of a particular task. Whilst this suggests
Chapter 2 Background

that a modular approach can be utilised to model the brain movement, it does not provide details as to how the modules of the brain perform these specific tasks. Furthermore, these studies do not investigate how these capabilities are affected under neurological injury. Indeed, the effects of neurological injury can vary. Finally, they do not investigate how the resulting movement may change under damage due to neurological injury. A neurological injury has a physical location in the brain, which can differ from patient to patient. Therefore, it is unsurprising that the magnitude and type of impact an injury has on motor function is dependent on the magnitude and location of the neurological injury (Kunesch et al., 1995).

This unknown presents a clear difficulty in modelling — for example, if a patient is unable to accurately estimate the state of the system (i.e. the state of his or her own body), what effect does this have in the resulting movement? Will the patient accommodate for this and thus make slower movements, will the patient’s movements become less accurate, or will a systematic error in the estimate cause a systematic error across movement types? Given the large variability in patients, patient impairment types, and patient impairment levels, such changes are difficult to model.

Regardless of the cause and location of the neurological impairment, there are a number of impairments which occur with a high frequency amongst neurologically impaired patients. These include spasticity (Thibaut et al., 2013), a ‘stiffness’ in muscles which resists change in muscle length; increased neuromotor noise (McCrea and Eng, 2005) causing ‘shaky’ movements; muscle weakness, often caused by lack of use; and problems forming and executing motor plans (Kitago and Krakauer, 2013). Although difficult to model, such patterns suggest representations of such impairments can be constructed within a computational model, and parameterised to account for varying degrees of effect.

Due to the large number of symptom and possible combinations of symptoms, this thesis does not attempt to completely model the motor control system of neurologically impaired patients. Instead, it does attempt to construct a framework for doing so. Such a framework must include scope to model various impairments, and present ideas as to how such impairments affect the motor capabilities of that person. A more detailed discussion about different impairments, how they are defined and may be represented is presented in Chapter 3. However, in the next section, computational models of neurologically-impaired individuals’ motor control systems currently within the literature are first discussed.
2.2.2 Computational Models of the Human Motor System for Neurologically Impaired Individuals

Within this section, a discussion of computational models of the human motor system are presented. First, computational models of human motor control for patients with impairment are presented, and secondly, computational models related to motor learning, recovery and rehabilitation.

Existing computational models of human motor control in application to neurologically impaired individuals are limited. Only one model has been found in literature (Zadravec and Matjačić, 2013). This study investigated point-to-point planar reaching movements in the horizontal plane. A two-link model was again utilised for the arm, with a simplified muscle model (2 for each joint, and 2 biarticular muscles — those which cover both joints). The muscles were modelled as having both passive and active components. For healthy subjects, an optimisation minimising the torque contribution of the active portion of the muscles was utilised with constrained end points. These simulations produced movement paths and velocity profiles similar to experimental data. Neurologically-impaired individuals with muscle stiffness were then simulated by increasing the passive muscle forces within the model, resulting in different movement trajectories. Although this particular study did not make comparisons with patient data, the movement patterns when compared to the simulations of healthy subjects produced velocity profiles which were significantly more curved, and — for some movements — produced two-segment velocity profiles. Such traits in movements are observed in patients with neurological injuries, and have been measured through metrics in robotic devices designed for neurorehabilitation (Nordin et al., 2014). Although the work is preliminary, the approach is of interest to this work — that is, a parameterisation of the human motor system which represents the impairment to be modelled is derived, and the principle of optimality is applied to predict how the movement patterns are changed by this change in dynamics. The challenge therefore, is determining how different impairments should be parameterised, and the construction of a model which allows this.

The second area of interest is how the motor control system changes during the recovery process. This is obviously related to motor learning, according to the definition presented in section 2.1.2. However, the link between computational models of human motor learning and recovery and rehabilitation of neurologically-impaired patients is not clear. In his investigation into the role of motor learning
in neurorehabilitation from a neuroscience perspective (Krakauer, 2006), Krakauer comments “Although some aspects of brain reorganization are probably unique to brain injury, there are large overlaps with development and motor learning”. As such, work in this area must carefully consider which elements in the change of the motor controller can be modelled in a similar manner to motor learning, and which cannot.

Two reviews of existing work in an adjacent field — computational models for neurorehabilitation — has recently been conducted in Reinkensmeyer et al. (2016) and Casadio et al. (2013). These existing reviews discuss models of recovery — defining computational models of neurorehabilitation as those which utilise as inputs descriptions of sensorimotor activity, model mechanisms of plasticity, and have as output variables which relate to functional outcomes. Such models include models which attempt to model neural activation patterns in movement tasks (Han et al., 2008, Reinkensmeyer et al., 2012). Utilising a model for the neurons based on reinforcement learning — where activation patterns are updated based on a ‘reward’ function depending on the outcome of the action, these models demonstrate how different therapy treatment plans can affect the strategy taken to complete certain tasks. Other models do not attempt to model neural patterns specifically, but instead how the capabilities of a patient evolve under certain training regimes (Casadio and Sanguineti, 2012, Hidaka et al., 2012). For example, one model (Casadio and Sanguineti, 2012) presents a linear model of recovery, designed for application in robotic rehabilitation. In this model, the state \( x \) represents the capabilities of the patient. This \( x \) is updated at each attempt at a task, with parameters relating to the effort the robot provides to the task, as well as conditions of the task being performed (for example, whether the subjects had vision of the hand). Although no strong conclusions are presented, the model was reported as being able to reflect some characteristics of patient capability, and general trends of patient performance. Such models, due to their focus on modelling the learning process, do not reconcile directly with motor control, and do not predict movements. Instead, they can be considered ‘macro’ models of motor control — such as those previously discussed for skill acquisition and the ‘state space’ models for motor adaptation — which attempt to model recovery. Within the present work, a focus is placed on computational models of the human motor system of neurologically-impaired patients, through parameterisation of impairments. As such, the models discussed within the Reinkensmeyer et al. (2012) and Casadio et al. (2013) reviews can be considered as models of parameter evolution, and thus
are auxiliary models to the computational models of the human motor system considered within the present work.

The complexity of the human motor control system and the variability within the patient population poses a challenge when attempting to produce computational models for human motor control amongst the neurologically impaired population. Some preliminary studies have been conducted in this area, however, application and confirmation of such models is limited.

2.3 Summary

The study of human motor control and learning has a long history, however its application to neurological patients and their recovery is very limited.

It is clear that motor control adheres to the principle of optimality — where the redundancy of the movement is resolved through the minimisation of some cost — and that the optimal control structure is capable of modelling much of the behaviour observed in experimental studies. Furthermore, although motor skill acquisition, motor adaptation and recovery of motor function after neurological impairments all involve changes to the controller in the human motor system, it is not clear how these changes are related. However, the applications for such a model are clear, and thus these models are of interest.

As such, the present work will explore the extension of optimal motor control to neurologically impaired patients, with the suggestion that the principle of optimality is maintained in such patients. Furthermore, a discussion of how motor learning and recovery can be related will be presented under this framework, to facilitate the development of models for these purposes. Finally, the development of one such model — a model of motor adaptation — will be presented and evaluated against experimental data.
Chapter 3

Recovery and Neurorehabilitation from a Computational Model Perspective

The overall goal of this thesis is to contribute to the use of computational models of the human motor system in neurorehabilitation. As discussed in the previous chapter, although many hypotheses exist regarding impairments, and their causes from within the human motor system, a systematic computational framework does not yet exist. As such, this chapter defines a framework for the human motor system, and uses it to capture the different types of changes which can occur within this human motor system. The goal is to present clear definitions as to what is and is not modelled in any particular computational model, how various impairments may be modelled, and how models for motor skill learning and adaptation may be transferred to recovery and neurorehabilitation.

The chapter is structured as follows. Firstly, an overview for the human motor control system is defined, including the subsystems which are to be considered within this framework. Secondly, with regards to the principle of optimality in motor control, a discussion about cost function and its form is presented. Thirdly, based on the structure and cost function, definitions for motor learning (adaptation and skill acquisition), and recovery are presented. Fourthly, common impairments as observed in neurologically-impaired individuals are categorised in terms of their cause and effect on the subsystems. Finally, a discussion about recovery and rehabilitation techniques is presented.
3.1 Model of the Human Motor System

Within this section, a descriptive framework for human motor system is proposed and justified. The model proposed and utilised within this work is a systems model (Muratori et al., 2013) for human motor control system, and is inspired by existing models in the literature (Shadmehr and Krakauer, 2008).

As discussed in Chapter 2, the motor control system can be modelled as a controller coupled to dynamics. Presented here is a discussion of the features and functions of the components of the sensorimotor system and biomechanical dynamics of the system, followed by a division of these components between the ‘controller’ and ‘dynamics’.

There are many areas of the central nervous system (brain and spinal cord) involved in sensorimotor control. There are many ways in which regions of the brain can be divided and described. Within this thesis, we focus on areas of the brain which are involved in the sensorimotor loop. Figure 3.1 identifies the key areas of interest, which are thought to have the following functions:

- The Motor Cortex: Motor Planning and Feedback Control
- The Parietal Cortex: Sensory Processing
- Cerebellum: Coordination and Prediction of Resultant Movements
- Brainstem and Spinal Cord: Reflexes and Activation Patterns

Each of these functions is discussed later in this chapter. It is also noted that many of the areas of the brain (such as those responsible for vision, logic, reasoning and speech) are not included within this discussion, however, can also have an impact on an individual’s motor behaviour and capability after neurological injury. Coupled with the central nervous system is the musculoskeletal system. The musculoskeletal system includes the bones, muscles, ligaments and other connective tissue within the body. This system features mechanical properties, which respond to the muscle activations signals generated by the Central Nervous System. However, it is noted that the changes in the characteristics of the musculoskeletal system vary in their nature and rate of change. For example, loss of muscle mass or weight gain occur extremely slowly with respect to the execution of a single task.
On the other hand, fatigue can occur faster. However, within the scope of activities of daily living, for a given task completed in a short period, the properties of the musculoskeletal system can be assumed to remain approximately constant.

In the construction of the controller-dynamics model initially discussed in Chapter 2 and illustrated in Figure 2.2, a division must be made between the features of the Central Nervous System, and those of the dynamics. Within this framework, it is proposed that features that change slowly (with respect to the time taken to complete the task) are considered as ‘dynamics’, and those which may potentially change quickly are considered as the ‘motor controller’, for example, if someone unexpectedly steps onto a slippery surface, their approach to walking immediately changes. This change of approach originates in the ‘motor controller’. With this guideline in mind, the entire musculoskeletal system is considered the dynamics, as well as the Spinal Cord, responsible for the reflexes and muscle activation patterns (Shadmehr and Wise, 2005). The controller therefore consists of the Motor Cortex, Cerebellum and the Parietal Cortex. This framework can be seen in Figure 3.2. The remainder of this section will discuss elements of these systems in more detail, and divide them further into subsystems.

### 3.1.1 Dynamics

Whilst an overview of the dynamics is presented here, dynamics in computational models are often represented by simplified models involving point masses and forces in task space (Zhou et al., 2012, Todorov and Jordan, 2002, Todorov, 2004) or are not considered at all motor control (such as in neural models (Han et al., 2008)).
At the very least, all models are simplified to consider only areas of interest — seated reaching movements do not consider the dynamic properties of the legs, as they are not relevant. In most cases, the result of these simplifications is often not significant, as the dynamics are assumed to not change significantly throughout the course of the experiments, and assumptions can be made that the outcomes of a motor command are performed exactly as intended, due to the unimpaired nature of the modelled subjects. However, within application to the neurologically impaired, changes in the dynamics are often significant, and as such areas in the model which can be affected by neurological conditions are presented here.

For ease of reference, Table 3.1 details symbols used within this model. Importantly, the input to the dynamics $u$ is defined as a motor command signal, which is an internal representation of the desired movement. This defines the boundary between the controller and the dynamics. How the effects of this input signal propagate through the dynamics to affect the system are presented next, starting with how the task is described, and tracing back to the brain signal ($u$).

### 3.1.1.1 Task Mapping

The Task Mapping is the mapping from the body or joint configuration to task relevant parameters $x$, which can be considered the state of the system. These parameters will vary depending on task. For example, if the task is to reach and grasp an object, the task-relevant parameters likely include the position of the hand, the orientation of the hand, as well as the velocity of the hand (as it must be zero at that point). Alternatively, if the task is to maintain a constant force on a plate, these parameters relate to the force applied by the hand. Furthermore,
Table 3.1: Parameters Utilised in Framework

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td>States</td>
<td>Parameters relevant to the cost (includes both task and internal states).</td>
</tr>
<tr>
<td>u</td>
<td>Input</td>
<td>Input states (motor signal)</td>
</tr>
<tr>
<td>v</td>
<td>Muscle Activation Signal</td>
<td>Electrical activation signal to the muscles.</td>
</tr>
<tr>
<td>m</td>
<td>Muscle and Tendon States</td>
<td>Parameters related to the current muscle state.</td>
</tr>
<tr>
<td>f</td>
<td>Muscle Force</td>
<td>Forces generated by each muscle, a function of muscle state.</td>
</tr>
<tr>
<td>θ</td>
<td>Joint Angles</td>
<td>Parameters related to the joint configuration — commonly, in the case of arm reaching tasks, the joint angles of the arm.</td>
</tr>
<tr>
<td>s</td>
<td>Sensory Information</td>
<td>Information about the state of the system as perceived by the senses.</td>
</tr>
<tr>
<td>t</td>
<td>Time</td>
<td>Allows the representation of time varying dynamics.</td>
</tr>
<tr>
<td>p</td>
<td>Task-dependent Parameters</td>
<td>Unknown parameters which apply only to the specific task modelled.</td>
</tr>
</tbody>
</table>

NOTE: All parameters are vectors of varying lengths and may include states of derivatives (for example, a joint angle and a joint angular velocity)

these parameters also include other parameters which may not be directly relevant to the task, but also those which are important in execution in the task — that is, relating back to the principle of optimality, those which are in the cost function. This will be discussed in more detail in Section 3.1.2.

The expression for deriving these task-relevant parameters are represented with the following function.

\[ x = T(\theta, t, p) \]  (3.1)

It is noted that other parameters may be required in the context of determining these task-relevant parameters. As such, these unknown parameters are represented with \( p \).
3.1.1.2 Task Dynamics

The Task Dynamics describe how the generalised forces generated by the muscles and the environment affect the motion of the arm.

The joint dynamics relate the two sets of generalised forces and torques — those generated by the muscles ($\tau_m$) and those generated by the environment ($\tau_e$) — to the motion of the system:

$$\dot{\theta} = J_d(\tau_e, \tau_m, \theta, t)$$  \hspace{1cm} (3.2)

where the derivative term indicates the change in the joint configuration as a result of the forces applied. These dynamics can be expressed through the standard equations of motion:

$$M(\theta)\ddot{\theta} + C(\theta, \dot{\theta})\dot{\theta} + G(\theta) = \tau_m + \tau_e$$  \hspace{1cm} (3.3)

where $M(\theta), C(\theta, \dot{\theta})$ and $G(\theta)$ refer to the arm inertia matrix, Coriolis matrix and gravitational effects on the equations of motion respectively. These dynamics can commonly be derived through an analysis of the kinematic chains which make up the body, and analysing these with respect to the muscle attachment points (Lau et al., 2015, Anderson and Pandy, 2001).

Included in the description of the task dynamics are the forces and torques applied onto the human body by the conditions of the environment and task itself. For example, the weight of an object while lifting it produces and additional force on the arm. The effect of these environmental conditions are written with respect to the arm configuration also, along with potentially time-varying aspects of the task and environment, as:

$$\tau_e = F_e(\theta, t)$$  \hspace{1cm} (3.4)

If the environment causes forces to be applied to the arm, this can be written as $\tau_e = J(\theta)^T F_{ext}$ where $J(\theta)$ is the Jacobian relating the joint kinematics to the position of the force.
3.1.1.3 Joint Mapping

The generalised forces and torques generated by the muscles, $\tau_m$, are dependent on the configuration of the joints $\theta$, and the forces generated by the muscles themselves, $f$:

$$\tau_m = \mathcal{F}_m(f, \theta)$$  \hspace{1cm} (3.5)

This can be written less generically as $\tau_m = J_m(\theta)^T f$, where $J_m(\theta)$ is the joint configuration-dependent Jacobian relating the muscle space to the joint space.

3.1.1.4 Muscles and Tendons

Muscles and tendons produce force through multiple mechanisms, and have complex properties with respect to their response. Muscle fibres contract in response to the muscle activation signal, which produces a force. However, the force is also affected by the mechanical response of the tendons, and other factors such as muscle fatigue. Within this work, however, a generic model of the muscles and tendons is presented. The muscle activation signals $v$ (commonly measured using Electromyography — EMG) activate the muscles, producing forces $f$, which are also a function of muscle and tendon state ($m$):

$$f = M(v, m)$$  \hspace{1cm} (3.6)

where the dynamics of the muscle themselves representing how the muscle state changes over time are represented by:

$$\dot{m} = M_d(v, m)$$  \hspace{1cm} (3.7)

No specific model is suggested here — many different models have been developed in the biomechanics field (for example Hill (1938), Zajac (1989)) — however, it is noted that muscles are complex, and each muscle in the body will have different characteristics, depending on their construction, their length, and mass.

3.1.1.5 Brain Activity (Brainstem and Spinal Cord)

Given a set of motor commands $u$, a set of motor neurons are fired, which in turn generates muscle activation signals $v$. Furthermore the brainstem and spinal cord
are also involved in reflexes — automatic responses to stimuli. As such, the muscle activation signal \( v \) is generated in response to the motor command signal \( u \), and in response to sensory stimuli (such as touch), \( s \). Therefore, the mapping between the motor commands and muscle activation signal is given the notation \( B(\circ) \) as follows:

\[
v = B(u, s, t)
\]  

(3.8)

where \( t \) is included to highlight the time dependence of these signals. Whilst this mapping occurs within the brain (in the Corticospinal Tract), they are considered here part of the dynamics of the system, as this mapping does not change quickly, and cannot be quickly and consciously changed. It is also noted that the input signal \( u \) is affected by noise (neuromotor noise), which is included within the description of the brain activity mapping, \( B(\circ) \).

### 3.1.1.6 Sensory Feedback

Feedback about the performance of the task is obtained through Sensory Feedback. Sensory feedback includes information from all stages of the dynamics, and can include proprioceptive feedback, as well as that through other senses such as vision or sound. Within this framework, the sensory feedback is written generically as:

\[
s = S(v, f, \tau_m, \theta, t)
\]  

(3.9)

It is noted that this feedback is noisy and potentially includes other sources of error. This, combined with the input signal noise means that, for successful movement, this noise and error must be accounted for within the motor control.

### 3.1.2 Motor Control

The discussion now turns to how the motor control signal \( u \) is formed — that is, modelling motor control. As discussed, motor control within the sensorimotor system is performed by subsystems within the central nervous system. The role of the motor controller is to generate a motor command \( u \) based on the definition of the task, and sensory input. The structure of this controller proposed within this work is based on the areas of the brain involved in motor control identified in Figure 3.1. These components include the Feedback Controller, the Forward Model, and
the Estimator. These components and their roles will be presented here, with small justification of their location within the brain. More comprehensive neurological discussions can be observed in Shadmehr and Krakauer (2008), Frith et al. (2000).

First, further parameters are defined. In addition to those parameters defined in Table 3.1, \( \hat{x} \) is defined as the best estimate of the current state, based on all available information. In addition, \( \tilde{x} \) is defined as an intermediate estimate, based on the expected outcome of the motor signal \( u \), which does not immediately utilise sensory information.

### 3.1.2.1 Forward Model (Cerebellum)

Mathematically, the Forward Model produces an estimate of the state parameters (\( \tilde{x} \)), given the motor command \( u \), and the current estimate of the state \( \hat{x} \). This can be represented in the following form:

\[
\tilde{x} = F(u, \hat{x}, t) \tag{3.10}
\]

The existence of such a model has been justified as a means to explain humans’ abilities to accentuate unexpected, external stimuli — to use the classic example, it is not possible to tickle oneself, even though the physical sensation of ‘tickling oneself’ is identical to having an external tickler. The argument presented, therefore, is that the forward model is used to predict the consequences of ones own actions, and thus heightened awareness is given to the unexpected sensation\(^1\). Its location in the cerebellum has been investigated utilising brain imaging, in an experiment where cerebellum activity was correlated with temporal difference between a movement and a robotically-translated movement (Blakemore et al., 2001). Further studies suggest that the cerebellum produces a forward model, based on activity levels in that area of the brain when new tools are used (Stein, 2009) and experimental evidence of humans without a cerebellum (Nowak et al., 2007).

From a learning perspective, if a forward model is accurate, the estimated states should match the actual states when the motor command is estimated. If it is inaccurate, the forward model may produce conflicting estimates which conflict with sensory information, which would provide incentive to change the forward model to match.

\(^1\)This has been highlighted by an interesting robotic experiment (Blakemore et al., 1999)
3.1.2.2 Estimator (Parietal Cortex)

The role of the estimator is to combine the information from $\hat{x}$ with the sensory information $(s)$ to produce an updated and more accurate estimate of the task-relevant parameters, $\hat{x}$.

$$\hat{x} = \mathcal{E}(s, \hat{x}, t) \quad (3.11)$$

This estimator may also be considered a ‘state observer’, as its purpose is to estimate a value for the states based on the measured (sensory) information. The difference between the predicted state $\hat{x}$ and the actual state $x$ is used to drive a change in the control strategies — that is, it motivates a form of motor learning.

State estimation is thought to exist within the Parietal Cortex, whose role involves the representation of the state of the system in different coordinate systems, and integrating or ‘remapping’ between such coordinate systems (Frith et al., 2000). Further conclusions can be drawn from a series of experiments on monkeys (Rushworth et al., 1997). In these experiments, monkeys were trained to make two types of movements — movements to a target in the light and in darkness. A lesion was made in one of two locations in the parietal cortex for each animal. The results demonstrated that each lesion location resulted in errors in movements in either the light or dark, but not both. This suggests that these different areas corresponded to processing sensory information from different senses.

3.1.2.3 Feedback Controller (Motor Cortex)

The role of the feedback controller is to generate the motor commands $u$. The construction of this command dependent on the task, and the estimate of the state of the system, i.e.:

$$u = \mathcal{C}_{\text{task}}(\hat{x}, t) \quad (3.12)$$

This is generated through control law $\mathcal{C}_{\text{task}}(\hat{x})$. This feedback controller exists in the motor cortex, which sits adjacent to the parietal cortex, and generates the neural impulses to the brainstem and spinal cord. The role of the motor cortex has been studied in experiments utilising brain imaging and stimulation techniques (Shibasaki et al., 1993, Gerloff et al., 1998).
3.2 Modelling Strategies of Motor Control

It is clear that computational models of movement aim to model an extremely complex system, in which many interconnected subsystems combine to produce an overall action (movement) in response to some motivation or given task. It is also obvious that changes (either through neurological damage or externally-generated disturbances) to any particular subsystem can result in changes to the overall movement.

As discussed in depth in Section 2.1.1, motor control is based on the idea of an optimal control scheme with respect to a cost function. Previous investigations have focussed on producing general models of human motor control, studying general movement patterns of healthy subjects, and proposing cost functions based on this. Within this framework, the cost is represented by two components, as follows:

\[ \mathcal{L}(x, u, t) = \mathcal{L}_{\text{task}}^i(x, u, t) + \mathcal{L}^p(x, u, t) \]  

(3.13)

where, \( \mathcal{L}_{\text{task}}^i(x, u, t) \) is a task-dependent intrinsic cost function, which, in reaching actions, is as simple as producing a movement in which the hand reaches a target location and/or orientation. \( \mathcal{L}^p(x, u, t) \) represents a task-independent, but individual-dependent personal cost, which represents a personal preference for certain movement patterns.

The reason for this breakdown is to allow goals for a particular movement to be explicitly stated (\( \mathcal{L}_{\text{task}}^i(x, u, t) \)), whilst maintaining the idea of an overarching optimisation in resolving the redundancy, characterised by \( \mathcal{L}^p(x, u, t) \). For healthy people, the personal cost function is similar between subjects, a trait which is captured in existing computational models for movement. However, variations in this \( \mathcal{L}^p(x, u, t) \) can account for differences in movement patterns, such as different running styles or tennis swings.

In most tasks, the intrinsic cost function is significantly more highly weighted than the personal — the achievement of a task goal is paramount. As such, in some cases it can be treated as a constraint, however, this is not always the case, especially in the case where two competing objectives are relevant.
Whilst no claim to the actual values or forms of these cost function is presented in this framework, such a structure provides a mechanism for discussing the approach taken to completing any particular movement. Further discussion about the Personal and Intrinsic Costs, as defined here, is now presented.

### 3.2.1 Personal Cost

The personal cost is most commonly associated with the studies in literature, which are discussed in detail in Section 2.1.1.1. Examples include kinematic and dynamic cost functions such as minimum jerk or minimum commanded torque change. The main goal of this personal cost function is to represent costs which are not associated with the explicit achievement of the task. As such, the personal cost can be thought of as the cost associated with the resolution of redundancy — it dominates how redundancy is resolved when the task is easily achieved.

For example, a person who has pain in their hip may have a modified personal cost adopted to avoid pain, and thus walk with an unnatural gait (i.e. a limp). Alternatively, someone who has cut their index finger will avoid utilising that index finger if possible to avoid pain. Otherwise, within the scope of neurological injury, the personal cost is also associated with the preference of certain movement patterns.

### 3.2.2 Intrinsic Cost

On the other hand, the intrinsic cost includes information regarding the performance of the task attempted. In many reaching experiments, the intrinsic cost is self-explanatory — for example, in point to point reaching tasks, the intrinsic cost is simply to have the hand at that particular location in Cartesian space. In timed tasks (Burdet et al., 2001, Zhou et al., 2012) the cost may also include some indication of required timing. Furthermore, intrinsic cost may also include some redundancy, where two costs associated with a task are competing — in the swinging of a baseball bat, both accuracy and power are desired, however, can be considered completing objectives. The intrinsic cost function therefore provides the mechanisms for balancing these objectives.

It is important to note that the cost function dependent significantly on the task at hand, and, in experimental conditions, may also be influenced by a subject’s
understanding of the task. For example, in a reaching task, if asked to only reach to a certain point, a subject may also include a cost to reach in a straight line, which may not be explicitly stated within the task. Determining the exact form of this intrinsic cost, however, is therefore again difficult.

### 3.3 Motor Learning Definitions and Models

With a general framework of the motor control system complete, this section now presents some definitions within the context of this framework, setting the boundaries of investigations relating to the field. As such, this section defines motor learning, and discusses how existing models of motor learning can be classified within this definition. The literature of motor learning highlights two independent but related aspects — motor adaptation and skill acquisition (Kitago and Krakauer, 2013). As reviewed in Section 2.1.2, motor adaptation occurs due to changes in the environment or task, such that the execution of a task does not match the expected execution. On the other hand, skill acquisition is harder to define, and therefore study. Classic definitions of skill acquisition from psychology suggest that the a motor skill is an action in which the movement and outcome of the action are emphasised (Newell, 1991). Therefore, acquiring a skill is the process of learning to complete that action to a suitable action outcome.

#### 3.3.1 Skill Acquisition

Motor skill acquisition has, in the past, been noted as being difficult to define (Kitago and Krakauer, 2013). Newell (1991) defines a motor skill as an action in which the outcome of action is important. A skill is therefore acquired if satisfactory performance of the outcome is achieved.

As such, skill *acquisition* is defined as the process of identifying the Task-intrinsic cost $L_{\text{task}}^i(x, u, t)$, and identifying a strategy for minimising it. Beyond this acquisition stage, the performance of the personal cost function $L^p(x, u, t)$ can be further optimised, during a period of motor adaptation, which will be defined in the sequel.

This definition is consistent with the stages of skill acquisition discussed in Muratori et al. (2013). During the initial stage of skill acquisition, a basic movement pattern is learned, and the components of environment important to the task are
identified. This stage is relevant when presented with a new task and/or cost function, and sets the local region for the optimality of the cost. This is the identification of the cost function $\mathcal{L}_{task}^i(x, u, t)$, as well as the exploration which defines the initial condition for the secondary stage. In this secondary stage, adaptation occurs, as the personal cost is minimised. It is noted that this is akin to smaller searches within the local area to improve the performance with respect to the cost function. This is an ‘automatic’ adaptation process.

### 3.3.2 Motor Adaptation

Motor adaptation is the process of changing the control scheme in response to perturbations or disturbances which affect the movement. In these cases, a discrepancy between the predicted motion (of the forward model) and the actual (estimated) motion (Krakauer, 2006) occurs, which drives this adaptation. Adaptation is therefore commonly investigated within experimental studies. In such studies, visual distortions or force fields (Shadmehr and Mussa-Ivaldi, 1994, Izawa et al., 2008, Bhushan and Shadmehr, 1999, Zhou et al., 2012) are applied to the hand during movements, and the adaptation process is described.

Within this work, we propose that motor adaptation be defined as the process of finding the optimal controller for a given task (and therefore cost function). As such, $\mathcal{C}(\hat{x}, t)$, $\mathcal{E}(s, u, t)$ and $\mathcal{F}(u, \dot{x}, t)$ are updated iteratively, driven by the difference in predicted versus actual output. This reflects the commonly-posed belief that “Learning is error driven”.

Experiments exploring motor learning are usually structured to present the same task in an initial environment, followed by a sudden switch in the dynamics. After this switch, it is expected that the subjects improve their performance of that task, given some criteria involving both completion of the task (if the change in dynamics has made the task infeasible) and also potentially some secondary criteria (Zhou et al., 2013). Due to the focus on motor adaptation in the remainder of this thesis, this section now revisits existing computational models of adaptation, and a discussion as to how they can be included within this framework and definition is presented.

Zhou et al. (2012) present an iterative learning control (ILC) formulation to modelling adjustments in movement trajectories due to changes in dynamics. In this case a controller of the form $\mathcal{C}(x, t) = K_{1i}(t)z_i(t) + K_{2i}(t)r(t) - K_{3i}(t)1_{m \times 1}$, where
x consists of \( z_i(t) \), representing the trajectory of the task-relevant parameter \( z \) in the \( i^{th} \) iteration; and \( r(t) \) representing the desired reference trajectory for this task-relevant parameter. \( K_{ji}(t) \) for \( j = 1, 2, 3 \) are controller gains which are utilised during the \( i^{th} \) iteration. The adaptation is then modelled through the update:

\[
K_{ji}(t) = K_{ji}(i-1)(t) - C\beta_j e_i \phi_{j,i}^T, \quad j = 1, 2, 3
\]  

(3.14)

where \( e_i \) is the difference between the actual and reference trajectories in the \( i^{th} \) iteration, \( C \) is the mapping between the state and the output; \( \beta_j \) is a constant update matrix for each gain; and \( \phi_{j,i} \) is a vector containing \( z_i \) and \( r \). The estimator and forward model are assumed to be modelled accurately, but compensated for with a feedback linearised term in the controller, such that the dynamics can be represented as a simple double integrator. It is inherent within this computational model for adaptation that the cost function (be it personal or intrinsic) contains terms relating to the magnitude of \( e_i \) — that is, the difference between the reference trajectory and the actual trajectory. The controller does not stop changing (and thus Motor Adaptation does not stop) until this error is equal to zero. This convergence is proven within Zhou et al. (2012).

The second model investigates an experiment in which participants reached multiple directions, under the effects of no field, and then a velocity-dependent curling force field (Bhushan and Shadmehr, 1999). In this model, the muscle, arm, and spinal dynamics are modelled, along with delays associated with sensory feedback. To place the motor controller within the framework proposed in this thesis, the three components of motor control — Forward Model, Estimator and Feedback Controller — can be identified. The controller, \( C_{\text{task}}(\hat{x}, t) \), is modelled with a feedback controller as well as a combined estimator and forward model in the following form:

\[
u = \mathcal{I}M \left( K_p(\hat{x} - x_d) + K_v(\dot{\hat{x}} - \dot{x}_d) \right)
\]  

(3.15)

where \( \mathcal{I}M(\cdot) \) is an inverse model, which converts the PD controller (in task space) to an appropriate motor command signal (which affects the system in joint space). The estimator, \( E(s, \hat{x}, t) \), and forward model \( F(u, \hat{x}, t) \) are represented as a combined unit, in the form:

\[
\dot{x} = \mathcal{F}M(s, u)
\]  

(3.16)

where \( \mathcal{F}M(s, u) \) is termed ‘Forward Model’, but serves as a combined function of
the Forward Model and Estimator by the definitions in this proposed framework — that it, an estimate of the state is created using information from the sensory information, as well as the constructed control signal.

Adaptation in (Bhushan and Shadmehr, 1999) is then modelled as changes in the forward and inverse models of the controller. As described in Section 2.1.2.2, but repeated here with slightly different notation for convenience, adaptation is modelled as:

\[
\mathcal{FM}_i(\circ) = \mathcal{FM}_0(\circ) + (1 - e^{-ir_{fm}})(\mathcal{FM}^*(\circ) - \mathcal{FM}_0(\circ)) \tag{3.17}
\]

\[
\mathcal{IM}_i(\circ) = \mathcal{IM}_0(\circ) + (1 - e^{-ir_{im}})(\mathcal{IM}^*(\circ) - \mathcal{IM}_0(\circ)) \tag{3.18}
\]

where:

- \(\mathcal{FM}^*(\circ)\) and \(\mathcal{IM}^*(\circ)\) represent the ‘correct’ forward and inverse models according to the dynamics of the system
- \(\mathcal{FM}_i(\circ)\) and \(\mathcal{IM}_i(\circ)\) represent the forward and inverse models utilised in the controller at the \(i^{th}\) iteration (where \(i = 0\) is a special case equal to the models of the previously-experienced dynamics)
- \(r_{fm}\) and \(r_{im}\) are the learning adaptation rates
- \(\circ\) is used to represent the respective parameters of the models

Such a strategy involves an explicit update in the adaptation to the force field, and assumes an exponential rate of adaptation in both the forward and inverse models. Whilst using slightly different terminology, and (like the previous example) combining aspects of the model for motor control, the model for motor adaptation in Bhushan and Shadmehr (1999) does conform to the proposed framework.

The third computational model for adaptation utilising an Adaptive Dynamic Programming (ADP) algorithm has also been proposed in Jiang and Jiang (2014), which employs the dynamics from Liu and Todorov (2007), combining the dynamic subsystems into simplified linear dynamics. The controller representation can be summarised as having a feedback controller of the form \(C(x) = -Kx\), and the forward model and estimators which are perfect (that is, the human is capable of estimating the results of their movement, and interpreting the sensory feedback). The adaptation model involves an update in \(K\) matrix of the controller, based on
feedback from previous attempts of the task, in the form:

$$ K_{i+1} = K_{\text{update}}(x_i, u_i) \quad (3.19) $$

The update here is based on the performance of the task, and attempts to search for an optimal feedback control strategy. In this case, the model can be fit within the framework in multiple ways. For example, instead of perfect estimator and forward model implementations, the formulation $u = -Kx$ may be considered the entire controller structure (feedback controller, forward model and estimator combined). Such a representation intuitively allows there to be a difference between the predicted output and sensory feedback, which is noted to drive adaptation.

The models presented here are capable of modelling the phenomena observed in their respective experiments. However, it is important to consider the non-convex nature of the optimisations which must be performed within adaptation. Although an older work, Newell (1991) identifies that many existing studies of skill acquisition and adaptation are reported to be linear. However, Newell (1991) suggests that this may be due to the simple experiments performed in their investigation, leading to little acquisition or learning to be done. This plays an important part in neurorehabilitation — if the initial acquisition is structured towards a locally optimal (but not globally optimal) location, the adaptation stage of acquisition will not converge to the globally optimal control strategy. Furthermore, emphasis on these moments promotes use-based learning, which can therefore influence the performance of other tasks as well.

### 3.3.3 Other Changes in the Motor Control System

In addition to the changes caused by changes in environment or task, there are a number of other changes which occur within the motor control system. These include changes to the dynamics associated with the body, as well as personal cost functions over time. Such changes can be externally observed through changes in the movement patterns, and can be internally justified and represented within the model.

For example, changes in the dynamics of the system can occur over time also. Exercise (or lack therefore) can cause changes in the muscle mass, therefore changing the mass of the system, as well as the muscle activation signal to muscle force function — i.e. the functions $M(v, m)$ and $M_d(v, m)$. This function may also be
affected by aging, which can change the elasticity, ligaments and joints of the person. Similarly, weight gain cause changes the dynamic response of the limbs of the body to a given joint torques — that is, a change in $J_d(\tau_f, \tau_m, \theta, t)$. These changes to the dynamics of the system can affect the optimal (and observed) movement patterns.

Changes can also occur to the personal cost function. For example, use-dependent learning causes more-utilised movement patterns to be preferred (Diedrichsen et al., 2010a). This has an impact not just on the present task, but on other unrelated tasks. Within the framework proposed here, this is represented as a change in the personal cost function $L_p(x, u, t)$, which therefore affects the way other tasks are performed — that is, certain movement patterns are preferred over others, irrespective of the task. This has an important impact on rehabilitation and training, as will be discussed later. Pain can also be a factor in the personal cost function, as certain movement patterns may aggravate an injury, causing pain. This, therefore, naturally causes people to avoid employing that particular movement patterns.

The changes to the motor system discussed here have so far been within the scope of healthy people — that is, changes that occur to the general population. In the next sections, a discussion of the framework and how it can be used to describe features of neurological impairment will be presented.

### 3.4 Neurological Impairment

Neurological impairments are varied in their cause — strokes, traumatic brain injury, brain cancers or other neural diseases. These neurological injuries can affect different areas of the brain, which, in turn may cause different impairments, which manifest themselves in the motions utilised by neurologically-impaired individuals.

Impairments due to neurological injury can either occur as a direct result of a brain injury, or developed over time after the injury. Within this work, impairments are classified in this way, and are termed either primary or secondary impairments. Primary impairments are defined as impairments which are a direct result of the neurological event. On the other hand, secondary impairments are caused by actions which develop as a result of the primary impairment, which may occur within numerous subsystems both within the central nervous system, but also within the musculoskeletal system.
Within this section, a discussion is presented on commonly-observed impairments in neurologically impaired individuals, their origins, the resulting effects on the individuals’ movements, and where the impairments can be modelled within the motor control framework.

### 3.4.1 Primary Impairments

Neurological events, by definition, occur in the central nervous system and, in particular, the brain. As such, the primary impairments affect subsystems physiologically residing in the brain. Hemiparesis — weakness in movement in one side of the body — is commonly observed in individuals due to lesions the motor cortex and/or the corticospinal tract — which lie in different subsystems in the proposed framework.

Damage to the corticospinal tract causes change in the generation of muscle commands, due to the damage to the corticospinal neurons. Such a change affects the way that an individual performs a movement — certain motor commands are ‘forgotten’ and thus need to be recovered. Thus, many of the symptoms of neurological injury can be modelled as changes in the mapping between motor command and muscle command, i.e. a change in \( B(u, s, t) \), particularly, it is suggested that this change is a change in the relationship of \( u \) to its output.

Spasticity is caused by damage to the corticospinal system (Thibaut et al., 2013), and can include spastic dystonia, spastic co-contraction, extensor or flexor spasms, clonus and exaggerated deep tendon reflexes. These issues can be modelled as a change in \( v = B(u, s, t) \), particularly with respect to the \( s \) parameters. For example, dystonia, muscle constriction in the absence of voluntary movement, can be modelled by including a constant term within \( B(u, s, t) \) resulting in some degree of muscle activation \( v \) unrelated to the motor command \( u \). A similar model can be used to model spasms or clonus. Unnatural pathological synergies — where individual joint movements (such as between the elbow and the shoulder) cannot be isolated Levin (1996) — can also be modelled through a modification of \( B(u, s, t) \), by modifying the mapping between motor commands \( u \) and activations \( v \) of these muscles in such a way that they cannot be separated. Excessive co-contraction, in which groups of agonist and antagonist simultaneously contract, can be modelled in a similar fashion.

Neurologically-impaired individuals can also suffer from increased neuromotor noise (McCrea and Eng, 2005), which can also be modelled as an increase in
noise within $B(u, s, t)$. The results of this increased neuromotor noise are that the individual’s movements have more deviation and variation, are slower (a result of the optimal control scheme to deal with the increased noise), and are more segmented (as there is a higher reliance on feedback control).

Damage associated with the control areas of the brain is harder to model. However, as discussed in Section 3.1, different areas of the brain are associated with different functionality, and thus a correlation can be made between the location of the neurological incident, the functionality associated with this location, and the resulting motor control limitations. For example, ideomotor apraxia is associated with difficulty in planning and executing motor tasks. This can include problems identifying joint angles, and difficulty in defining motor plans (Sathian et al., 2011). At the same time, ideomotor apraxia is associated with damage to the parietal cortex or premotor cortex (part of the motor cortex), which correspond to the estimator $E(s, \bar{x}, t)$ and feedback controller $C(\hat{x}, t)$ respectively. It is obvious that damage to these subsystems can affect movement in the ways associated with ideomotor apraxia. Damage to the parietal cortex can also cause hemispatial neglect (in which the individual behaves as if the left or right half of the visual plane or body do not exist). Again, it is straightforward to see how problems with sensory input to state estimation can affect this.

Similarly, ataxia is the occurrence of poor coordination, inaccurate and variable movements, dysmetria and intention tremor in neurologically-impaired individuals (Sathian et al., 2011). It is associated with damage to the cerebellum, responsible for the forward model $F(u, \hat{x})$ in the framework. Associated with these changes, it is hypothesised that with inaccurate or absent estimates of the forward model, the estimator must rely more heavily on the sensory information, resulting in more corrections in movement patterns being required, and therefore leading to these inaccuracies and tremor.

### 3.4.2 Secondary Impairments

Secondary impairments are defined as impairments developed over the time after the neurological event, generally as a result of the primary impairments. For example, in the same way that strength is gained through additional muscle mass when weight-training, the human motor system also changes as a result of the use (or disuse) of certain motor patterns as a result of the primary impairments.
Chapter 3 Recovery and Neurorehabilitation from a Computational Model Perspective

One such commonly-observed secondary impairment is shoulder subluxation. Subluxation can be caused by spasticity — which causes the muscles to pull the humerus out of the shoulder socket. However, it can also be caused by muscle weakness, which can be caused by lack of use. Subluxation can occur when the muscles are no longer strong enough to hold the humerus in the shoulder socket, and is correlated with increased chance of shoulder pain (Paci et al., 2007). This, in turn, can lead to changes in movement patterns due to changes in the personal cost function due to the pain.

Additionally, a preference for unnatural motor patterns can also lead to increased unnatural synergies, or unnatural compensatory strategies. In the days and weeks immediately following the event, spontaneous change occurs in the brain, allowing the recruitment of muscle patterns. Those that are recovered first are then utilised more thoroughly, and thus become more favoured in the optimisation used to determine the motor commands in the first place, due to use-dependent learning. This, in turn, causes them to be used more, creating a positive feedback loop in which the more favoured motor commands are utilised more and more, and leading to increased preference for compensatory strategies (Levin, 1996). Such a phenomena can be observed with individuals who engage these compensatory strategies even after having recovered more functionality later.

Muscle weakness in general is also a form of secondary impairment, leading to deficits in movement. A muscle which is under-utilised requires more ‘effort’ to produce similar force. Thus the utilisation of other, stronger muscles may become more optimal, weakening the muscle further, again compounding the problem.

These suggestions for modelling within the are based on both neurological location of injury, and observed behaviours resulting from such an impairment. As such, although no explicit changes to computational models for these impairments are presented here, it is suggested that the proposed structure may be used for computational models of movement for individuals with neurological impairment.

3.4.3 Optimality in Neurologically-impaired Movement

With these changes in the dynamics, its is argued that neurologically-impaired individuals still behave optimally — this is human nature — however, when inflicted with these impairments as a result of their neurological incident, their dynamic and control subsystems change according to the primary and secondary impairments. As such, the resulting movement from the optimisations also changes.
Evidence for this can be seen in a reaching study, in which neurologically-impaired individuals were able to recruit additional degrees of freedom of the body to achieve goals, given limitations in some degrees of freedom — in this case reaching forward with limited elbow extension by using their torso (Cirstea and Levin, 2000). This suggests that neurologically-impaired individuals are able to search for strategies for completion of a task, and thus can adjust accordingly. Furthermore, it provides some clues for the development of rehabilitation techniques — if the dynamics or the cost can be changed such that more ideal movements become more optimal, the individuals will naturally move towards performing these more ideal movements. Such an approach, and how it can be discussed within the context of this framework, is now discussed.

3.5 Relating Recovery and Rehabilitation to Motor Learning

Given a description and method of modelling the impairments, this section now attempts to model mechanisms of the reduction of these impairments — termed recovery. This section defines ‘recovery’ more precisely with respect to the other definitions within this framework, and then how common rehabilitation techniques can be considered to promote recovery.

3.5.1 Defining Recovery and Rehabilitation

The goals of rehabilitation therapists can be competing. The role of the therapist is to ensure their patients are capable of independently performing their Activities of Daily Living (ADLs), which — perhaps counter intuitively — can involve teaching the patient compensatory strategies. However, at the same time, most compensatory strategies are not ideal, and can result in further complications related to pain and development of secondary impairments as discussed in Section 3.4.

Although utilising compensatory strategies are useful for allowing a patient to return to their daily lives, here, due to the potential downfalls in learning these compensatory strategies, this is not considered rehabilitation. Instead, true recovery is defined as a change in the motor control system, such that the movement
pattern performed by a patient for a given action is more similar to a nominal trajectory as would be performed by a healthy person (the ‘ideal’ trajectory). As such, recovery can occur due to changes in the dynamics (such as recovery of muscle strength), but also changes to the voluntary movement patterns performed by the patient. This can be either through a modification of the cost function (and therefore control policy) used by the patient to complete the task (particularly the personal cost function), or through a change in dynamics making particular movement patterns more optimal.

Rehabilitation therefore, is the act of attempting to shift the patient’s movement patterns towards nominal trajectory through change of the dynamics. This can occur through a variety of mechanisms, which can be seen through the different therapy approaches. Within the remainder of this section, a discussion is presented with respect to rehabilitation techniques and by what mechanism they attempt to obtain true recovery.

### 3.5.2 Rehabilitation Techniques

Therapists use a wide variety of rehabilitation techniques in order to help the patient recover. Although many such techniques have been developed through therapists’ intuition and experience, here a discussion is presented on how these techniques can be fit within the scope of this framework, irrespective of the initial motivation behind the development of the techniques. With a model of the changes induced by these techniques, the effectiveness of each of these techniques can be assessed, and further development of the techniques may be possible.

Rehabilitation techniques will be classified into a number of categories. First, those that attempt to improve the dynamics of the system. Second, those that attempt to change the personal cost function by introducing new tasks. Third, those which attempt to change the cost function by changing the dynamics of the system by introducing constraints. Additional categories are presented based on the goal of improving a patient’s Estimator and Forward Model. It is also noted that some exercises can be placed into multiple categories. A summary of these categories is presented in Table 3.2.
Table 3.2: Classifications of Rehabilitation Techniques

<table>
<thead>
<tr>
<th>Category</th>
<th>Purpose</th>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>Improving Dynamics</td>
<td>Improve the dynamics of the human motor system, affecting the optimisation performed by the patient</td>
<td>Stretching, Transcutaneous Electrical Nerve Stimulation (TENS), Resistance Training</td>
</tr>
<tr>
<td>Changing Personal Cost via Dynamics</td>
<td>Change the personal cost function through use-dependent learning through restricting how certain tasks can be performed.</td>
<td>Constraint Induced Movement Therapy (CIMT), Robotic Devices, Functional Electrical Stimulation (FES)</td>
</tr>
<tr>
<td>Changing Personal Cost via Novel Tasks</td>
<td>Change the personal cost function through use-dependent learning through the definition of specific exercises</td>
<td>Functional and activity-based exercises at home as defined by therapy</td>
</tr>
<tr>
<td>Improving the Estimator</td>
<td>Improving the performance of the estimator through direct stimulation of senses (usually without active movement)</td>
<td>Passive joint mobilisation, Functional Electrical Stimulation (FES)</td>
</tr>
</tbody>
</table>

3.5.2.1 Improving Dynamics

The motivation behind improving the dynamics of the system is clear — if both the dynamics and cost function are the same as that of a healthy person, a patient will move in an ideal manner. The dynamics is therefore one of the two core components. Strategies for improving the dynamics are used for treating spasticity and for muscle weakness.

Spasticity can be seen as a change in the brain and spinal cord dynamics $B(u, s, t)$, which causes hyperactivation of the muscles (Bhakta, 2000). Many treatments for spasticity do not attempt to change these dynamics, but instead aim to change the dynamic properties of the motor system. For example, stretching and relaxation are common physical therapies for spasticity. The goal of stretches is to “improve the viscoelastic properties of the muscle-tendon unit and to increase its extensibility” (Thibaut et al., 2013). This is an explicit attempt to modify the muscle dynamics $\mathcal{M}(v, m)$ and $\mathcal{M}_d(v, m)$ in response to the increased muscle activation signal. Another treatment approach is the use of Transcutaneous Electrical Nerve Stimulation (TENS), which utilises electrical stimulations at the areas of spasticity. This treatment has also been shown to reduce spasticity, due to the production
of β-endorphins, which lower the excitability of the motor neurons — a modification of the spinal cord dynamics included in $B(u, s, t)$. It has also been noted that it may also facilitate cortical synaptic reorganisation (Thibaut et al., 2013), which is also a change in $B(u, s, t)$. Finally, other pharmacological treatments have also been used to treat spasticity, in which the mechanisms behind muscle activation in both the muscle and/or the spinal cord are modified (see (Thibaut et al., 2013) for a discussion on the variants).

Treatments intended to change muscle and brain dynamics are also utilised for treatment of muscle weakness. In contrast to the treatments for spasticity, however, such treatments look to increase the response of the muscle to input. The goal of strength training is to increase the strength of the muscles through repetitive effortful muscle contractions. Strength training can include progressive resistance training, but also include treatments such as electrical stimulation and mental practice (Ada et al., 2006). Resistance training obviously changes the muscle properties $M(v, m)$ such that greater force can be achieved with a given muscle (in the same way that weightlifters increase their strength). However, other treatments, such as mental practice aim to change the brain activation patterns $B(u, s, t)$ through use-based learning.

### 3.5.2.2 Changing the Personal Cost Function by Changing the Dynamics of Tasks

Neurologically impaired individuals often employ compensatory movement patterns when faced with limitations on joint range — for example, they utilise torso or trunk movements to compensate for a lack of elbow movement (Cirstea and Levin, 2000). However, such preferences may carry over even if the patient is regains this movement capability. In a study investigating the use of a trunk restraint, it was found that while the impaired individuals utilised less elbow movement than healthy subjects, these individuals utilised more elbow movement within a trunk restraint than without (Michaelsen et al., 2001). This suggests that the movement pattern (the reduction in elbow movement and increase in torso movement) is learned and preferred rather than essential due to a lack of capability.

This same study introduces a method of attempting to correct for this — utilising a trunk restraint changes the dynamics of the task, such that the preferred method
of completing the task is not possible. The patient must then recruit other degrees of freedom and therefore movement patterns to perform the task. Through use-dependent learning, the use of these new movement patterns become more preferred, modelled in this framework as a change in the personal cost function. Such a principle is more commonly applied in Constraint Induce Motion Therapy (CIMT) (Taub et al., 1993). This technique encourages use of the impaired hand by placing the nonimpaired hand in a sling during waking hours (or some portion of waking hours), preventing its movement.

Furthermore, robotic devices may also be used to enforce certain movement patterns. The development of some cost functions for reaching actions has been for the development of robotic devices (for example, Kim et al. (2012)). It is interesting to note however, that the use of an exoskeleton may encourage ‘normal’ kinematic movement patterns, but may not necessarily encourage ‘normal’ muscle activation patterns due to the change in dynamic environment enforced by the exoskeleton only.

In a similar manner to robotic devices, Functional Electrical Stimulation (FES) particularly when activated utilising Electromyography (EMG) can also be used affect the personal cost function by attempting to amplify the effects of those muscle activation patterns. This increase in effect may lead to such activation patterns becoming more preferred.

3.5.2.3 Changing the Personal Cost Function by Introducing Novel Tasks

A second methodology for encouraging use-dependent learning is by introducing novel exercises or tasks for the patient to perform. Such exercises can be constrained by description — ‘normal’ movement patterns are enforced due to the definition of the task. For example, a therapist may ask a patient to reach and pick up a cup many times a day, but with the provision that the patient should actively attempt to not move the torso when doing so. This encourages a more ‘correct’ movement pattern, which becomes favoured due to use-dependent learning. These exercises also have secondary effects — the strengthening of muscle groups utilised in these preferred movement patterns. This, in turn, makes these movements easier, creating a positive feedback loop of recovery. Alternatively, the cost function of the task (and therefore definition of the task) may also be changed through the use of non-physical feedback. For example, during a reaching task, the
therapist may observe the patient, and provide verbal feedback that the patient is utilizing too much torso or shoulder movement. The goal of this ‘online’ feedback is to again change how the movement is performed.

Attempting to modify the personal cost function by introducing novel tasks also may have an advantage over those which change the dynamics — that is, if the patient practises the correct task, they are practising the ‘correct’ muscle activation pattern. This principle has also led to the development of devices and methods to automate the clinician feedback (Thielman, 2010, Crocher et al., 2014).

Mental practice, in which the patient is asked to imagine moving their arm has been shown to be an effective form of therapy (Page et al., 2007), which results in an increase use of the impaired arm. This practice activates the same areas of the brain as actual practice, and thus it may be concluded that rehearsing a movement using the impaired arm mentally promotes use of that arm — that is, reduces the cost associated with using that impaired arm when the task is to be completed physically.

### 3.5.2.4 Improving the Estimator

Certain techniques are also targeted at improving the estimator — used to estimate the state of the body based on sensory information. Such techniques stimulate sensory inputs, which thus engage the estimator within the proposed framework. Whilst all movement exercises stimulate the senses to some extent — through visual, proprioceptive and sometimes touch feedback — certain rehabilitation techniques provide only sensory feedback, without any active movement from the patient. Furthermore, the mechanism for this improvement is not clear, other than the belief that the estimator improves with use. Regardless, there are a number of rehabilitation techniques which explicitly engage the estimator.

The simplest form of sensory stimulation is simply tapping or stroking the hand or arm. The signal associated with this touch propagates back to the brain through the central nervous system to, in our model, the estimator, forcing it to become engaged. Passive joint mobilisation, in which the therapist or a robotic device holds the arm and moves it through movement patterns, is another such example. Although the patient may be unable to complete this movement through active control, the estimator is exposed to sensory input, which can be associated with knowledge about the movements completed within the task. Another commonly-employed example is the use of Functional Electrical Stimulation (FES).
FES directly uses electrical stimulation to engage muscles. The sensory feedback associated with the muscles engaging, as well as the mechanical result of this engagement, is affected by the electrical stimulation.

### 3.5.2.5 Improving the Forward Model

The role of the forward model is to predict the results of motor commands. As such, the forward model is used in every attempt at a motor task, and therefore its accuracy should be improved with use, which occurs every time a patient moves. As such, any exercise is likely to have an impact on the forward model, if the patient is able to view and understand the results of the movement, for example, functional reaching exercises in which the patient is able to see and understand the movement of their arms.

### 3.6 Summary

This chapter has proposed a modelling framework for the motor control system based on optimal control, and presented a discussion on how motor skill learning and adaptation can be placed within this framework. Motor impairment due to neurological injury, and how this can be reconciled within the framework is also discussed, including potential methods for modelling impairments commonly observed within those with neurological injury. Finally, a discussion is presented defining the distinction between rehabilitation, recovery, and motor learning, as well as how common rehabilitation techniques fit within this framework.

Although this chapter does not present any detailed models for evaluating rehabilitation, it defines a framework from which rehabilitation can be discussed. Furthermore, by defining how the rehabilitation techniques target certain areas of the motor system, a methodology for assessing the effectiveness of certain techniques may be developed in the future. For example, the success of techniques which improve the estimator can be evaluated through a test which assesses the performance of the estimator alone, rather than general tests which assess general motor function (such as the Wolf Motor Function Test or the Fugl Meyer Assessment). In addition to this application, the framework is also useful for providing a basis for modelling motor control and learning for those with motor impairments.
With this framework in place, the remainder of this thesis address focuses on one component — how motor adaptation can be modelled within the optimal control framework, for both healthy subjects and those with motor impairment.
Chaprer 4

An Algorithm for Modelling Motor Adaptation through FHLQR

Motor adaptation is commonly-studied amongst healthy subjects, but these studies have not yet been translated to patients with motor impairment, caused by neurological injury. However, the translation and application of motor adaptation studies within this field are not necessarily straightforward — a neurologically-impaired individual’s movement capabilities are less than that of a healthy individual, and the desired characteristics of a model for adaptation differ between these populations. In the previous chapter, an overall framework for modelling motor control and learning was proposed. This chapter introduces a specific implementation based on that framework — a model for motor adaptation, through the proposal of an iterative algorithm for the Finite Horizon Linear Quadratic Regulator (FHLQR) formulation. The algorithm uses an iterative approach to produce an optimal control policy, based on the information of the previous trajectories — it therefore can be classed as an adaptive optimal control algorithm.

This class of algorithm is considered relevant to the application of motor adaptation due to a number of reasons. First, it provides a solution to the optimal control problem, thus adhering to the theory of optimality in motor control. Secondly, adaptive algorithms estimate the optimal control policy through iterative attempts — that is, an initial guess is used to first define the policy, and subsequent guesses are used to improve this estimate. Finally, these iterative attempts at improving the control policy are made based on information gained from previous attempts.
at the task, not the explicit knowledge of the dynamics. This has its own parallels to motor adaptation, in which the dynamics are not necessarily known or explicitly calculated, but instead learned through experience.

The choice to study motor adaptation specifically was based on two main reasons. The first was related to the existing models in literature. As discussed in previous chapters, existing models in literature for motor adaptation have two undesirable characteristics — either they require an optimal reference trajectory, or they require changing cost functions for each new dynamic condition. The requirement of an optimal trajectory suggests that the subjects attempt to return to the original trajectory under the new dynamic conditions — this is at odds with observations with different dynamic conditions — for example, one’s gait when walking uphill is different to when walking on flat ground. Similarly, placing a mug on a shelf utilises different activation patterns than placing a heavy saucepan in a cupboard. The use of a new cost function with each new dynamic condition is also undesirable — under the definitions proposed in Chapter 3, this is no longer considered adaptation. Secondly, adaptation to change in dynamics is also considered due to its particular relevance in rehabilitation robotics. Rehabilitation robotics has developed immensely over the last two decades, with a number of devices having being built to assist with therapy (Hogan et al., 1992, Burgar et al., 2000, Nef and Riener, 2005, Garrec et al., 2008). The implementation of these robots allows the introduction of novel dynamics onto the patient. Furthermore, some devices may introduce undesired forces onto the patient (Jarrassé et al., 2010, Fong et al., 2015a,b), which can (and does) result in changes to their movements. As such, an understanding of how patients adapt to these changes in dynamics, may be of use to determine the best control strategies in the control of the robotic devices, and how a patient’s movement may be expected to change once placed within a robotic device.

This chapter is organised as follows. First, the use of the Finite Horizon Linear Quadratic Formulation is discussed and justified. Secondly, a review of existing adaptive algorithms in literature is presented and notation for the remainder of the chapter introduced. Following this, some mathematical preliminaries are presented, and the algorithm and proposed and convergence properties proven. Finally, general simulation results are presented, demonstrating the convergence of the algorithm, and a discussion regarding the performance characteristics of the algorithm is presented.
Chapter 4 An Algorithm for Modelling Motor Adaptation through FHLQR

It is noted that the work within this chapter has been submitted to ‘Systems and Control Letters’ and as of October 2017, has been provisionally accepted.

4.1 The Application of the FH LQR Formulation for Modelling Motor Control

The construction of a model for motor adaptation is inherently tied to the selection of model for motor control. Within the remainder of this work, the Finite Horizon Linear Quadratic Regulator as a basis for the models of motor control. Within this section, a discussion of the reasons for the choice of the FH LQR formulation are presented, as well as some limitations associated with this approach.

The choice of this formulation is based on three main reasons. First, it is an optimal control formulation. Secondly, it is a formulation which applies to a continuum of tasks, rather than a single action. Thirdly, it provides a method for application to finite time tasks.

The desire for a formulation which is an optimal control formulation for this motor control framework is obvious — it is consistent with the theory of optimality for motor control. As discussed in Chapter 3, this theory of optimality applies to both healthy and neurologically-impaired individuals. In addition, this formulation allows the definition of a model for motor adaptation which does not require the use of a reference trajectory — in particular, one in which motor adaptation does attempt to move back towards a reference trajectory, as discussed earlier. Instead, this formulation allows for the description of adaptation as a “process of reoptimisation” as described by Izawa et al. (2008).

The application to a continuum of tasks is important, particularly with respect to modelling motor adaptation for neurologically impaired patients, especially the neurorehabilitation process. Some existing models for motor adaptation model a single task. However, with respect to application to neurorehabilitation, a more general formulation is preferred for two reasons. First, neurorehabilitation exercises often a variety of related movements — for example, moving blocks to different locations on a table. As such, a single movement trajectory is not desired. Secondly, generalisation itself is an important goal of rehabilitation — that is, the goal is not that the patient learns a single movement, but rather a family of movements. Therefore, a more general formulation is preferable.
Finally, the use of finite time tasks is preferred. The goal of most movements includes the idea that some finite time limit is required for the success of the movement. This is also applicable in neurorehabilitation exercises, particularly on robotic devices, which have traditionally been set up with a finite ‘goal’ time, as are many human actions in general.

Although the choice of the FH LQR formulation is justified through by the characteristics of the formulation, there are some characteristics which present some limitations in which behaviours can be modelled. In particular, the formulation presents some limitations on the dynamics and the cost function. The nonlinearities of the dynamics of the human motor system cannot be represented by the linear time varying dynamics of the FH LQR formulation. However, the dynamics can be linearised in a local region in which the model can be applied. Similarly, the requirement of a quadratic cost function means that more complicated cost functions cannot be represented, however, most forms can be also locally approximated as quadratic. As such, the proposed problem formulation can be considered relatively general, and thus be used in this application. Another limitation of this formulation is the assumption of perfect state information. Within the context of the framework presented in this work, this is equivalent to having a perfect forward model and estimator. This is not likely in the context of human motor control, however, it is suggested that with a sufficient close state estimate, the resolution of redundancy can be modelled in this manner.

With this choice of formulation, an appropriate algorithm is proposed for modelling motor adaptation. However, first, a survey of the control theory literature around existing Adaptive Optimal Control Algorithms is presented next.

4.2 A Review of Adaptive Optimal Control Algorithms

The optimal control problem is commonly-considered in the control field. The foundation of many solutions to this problem is Bellman’s Optimality Principle (Bellman, 1957), and the solution of the Hamilton-Jacobi-Bellman (HJB) Equation. Solving the HJB Equation, however, generally requires precise knowledge of the dynamics, and often does not have an analytic solution. Thus, numerical solutions are normally employed. Additionally, in real world systems exact knowledge
of the dynamics is often not available. Furthermore, the dynamics can also undergo changes over time, and these changes are more pronounced in some systems than in others.

The Continuous Time Finite Horizon Linear Quadratic Regulator problem is well studied. Many engineered systems are often posed in this formulation, where the objective is to minimise a cost function quadratic in both error in state and control effort, over a given finite time period. The finite duration of time given in many of the practical specified tasks lends itself to the finite horizon of the controller, which provides an explicit mechanism to trade off the accuracy of task completion with the effort we are willing to spend to achieve it. If the dynamics are known, the optimal control scheme for this problem can be calculated using the Differential Riccati Equation (DRE). This cannot be calculated if the dynamics are unknown. Furthermore, if an inaccurate model is used, or if the dynamics changes between iterations (either slowly, for example due to wear and tear, or suddenly due to a part failing), the control scheme becomes suboptimal. Such inaccurate or unknown dynamics can also be found in the examples of engineered systems; and is exceptionally pronounced among complex biological systems, such as the motor control system. As such, the use of adaptive optimal control algorithms — those that construct an optimal (or close to optimal) control policy based on ‘experience’ are of interest.

Some solutions to similar problems have been proposed in the literature. In Jiang and Jiang (2012), a linear time invariant (LTI) Infinite Horizon Linear Quadratic Regulator (IHLQR) problem was investigated. The present work takes a similar approach, with the major difference being that Jiang and Jiang (2012) considers infinite horizon, time invariant dynamics and no terminal cost. The technique is an Adaptive Dynamic Programming (ADP) technique, which utilise successive estimates of the value function to estimate the optimal control law. Lewis and Vrabie (2009) and Wang et al. (2009) provide good reviews of existing ADP techniques. Other approaches also exist for the FHLQR problem, but with discrete time dynamics — (Zhao et al., 2014) proposes an adaptive algorithm for the Discrete Time FHLQR problem with constant dynamics while (Frihauf et al., 2013) uses an extremum-seeking iterative approach to find an open-loop control sequence for the discrete time FHLQR problem with time varying dynamics.

Similar problems exist in Iterative Learning Control, such as the Linear Quadratic Optimal Learning Control (Frueh and Phan, 2000) and the norm-optimal iterative learning control (Gunnarsson and Norrlöf, 2001, Lee et al., 2000), where optimal
performance is sought over a finite horizon of each iteration. However, these algorithms seek an optimal control trajectory for the given task, as opposed to an optimal control law for a family of tasks characterised by the given cost function. Fundamentally, they require an identical initial conditions for each iteration, and only work when the optimal trajectory is identical for all iterations.

Therefore, the algorithm proposed here — although motivated by the construction of a model for motor adaptation — is designed to be general. Specifically, it finds the optimal control gain for the FH LQR problem without requiring the knowledge of the dynamics, regardless of what those dynamics represent. The proposed method utilises an iterative process to compute the optimal gain matrix using measured state trajectories, using the results of an iterative solution to the DRE proposed in Kleinman (1967). By not assuming the knowledge of the system dynamics, the proposed algorithm can therefore re-identify an optimal control strategy should the plant dynamics change.

4.3 Problem Formulation

The systems of interest take the following form:

\[ \dot{x}(t) = A(t)x(t) + B(t)u(t), \quad x(t_0) = x_0 \]  

(4.1)

where \( x(t) \in \mathbb{R}^n \), \( u(t) \in \mathbb{R}^m \) and dynamics matrices \( A(\cdot) \in \mathcal{C}^{n \times n}[t_0, t_f] \) and \( B(\cdot) \in \mathcal{C}^{n \times m}[t_0, t_f] \).

The objective of the Finite Horizon (FH) Linear Quadratic Regulation (LQR) problem is to minimise the following cost function, subject to the dynamic system (4.1):

\[
J(u(\cdot)) = x^T(t_f)\Phi_f x(t_f)
\]

\[ + \int_{t_0}^{t_f} (x^T(t)Q(t)x(t) + u^T(t)R(t)u(t)) \, dt \quad (4.2) \]

where \( x \) is the resulting trajectory from the dynamics (4.1), \( \Phi_f \in \mathbb{R}^{n \times n} \) is symmetric positive semidefinite (\( \Phi_f = \Phi_f^T \geq 0 \)), \( Q(\cdot) \in \mathcal{C}^{n \times n}[t_0, t_f] \) is also symmetric positive semidefinite, and \( R(\cdot) \in \mathcal{C}^{m \times m}[t_0, t_f] \) is symmetric positive definite (\( R(t) = R(t)^T > 0 \)).
For the cost function (4.2) subject to (4.1) the optimal control law is a time-varying feedback control scheme of the following form:

\[ u(t) = -K^*(t)x(t), \quad \forall t \in [t_0, t_f], \]  

where \( K^*(t) \in C^{m \times n}[t_0, t_f] \) satisfies

\[ K^*(t) = R^{-1}B^T(t)P(t), \]  

here \( P(t) \in C^{n \times n}[t_0, t_f] \) is the solution of the following Differential Riccati Equation (DRE) (Reid, 1972):

\[ \dot{P}(t) = -A(t)^T P(t) - P(t)A(t) - Q(t) + P(t)^T B(t)R^{-1}(t)B(t)^T P(t) \]  

subject to \( P(t_f) = \Phi_f \).

The problem formulation is a standard LQR problem with the assumption that all state signals are measurable. If some state cannot be not measured, an appropriate observer can be designed to estimate the state.

### 4.4 Preliminaries

Kleinman (Kleinman, 1967, Theorem 8, page 53) proposed a method of iteratively solving the DRE offline, using a dynamic programming approach to iteratively solve the DRE. This section first introduces notation used within this work, before introducing Kleinman’s algorithm and identifying two properties of this algorithm to be later used in the analysis.

#### 4.4.1 Notation

For any \( x \in \mathbb{R}^n, |x| = \sqrt{x^T x} \). For any \( A \in \mathbb{R}^{n \times m}, |A| \) is its induced matrix norm. The set consisting of all continuous functions defined over \( \mathbb{R}^{n \times m} \) over \([t_0, t_f]\) for any \( n, m \in \mathbb{N} \) is denoted \( C^{n \times m}[t_0, t_f] \). For any \( A(\cdot) \in C^{n \times m}[t_0, t_f], \|A\|^{n \times m} = \max_{t_0 \leq t \leq t_f} |A(t)| \).
For a given $x \in \mathbb{R}^n$, and a given $V = V^T \in \mathbb{R}^{n \times n}$, $x^TVx$ can be written as $\bar{x}^T\bar{v}$ with $\bar{x} \in \mathbb{R}^{\frac{n(n+1)}{2}}$ and $\bar{v} \in \mathbb{R}^{\frac{n(n+1)}{2}}$, where:

$$\bar{v} = [V_{11}, 2V_{12}, ..., 2V_{1n}, V_{22}, 2V_{23}, ..., 2V_{2n}, ..., V_{n-1,n-1}, 2V_{n-1,n}, V_{nn}]^T$$

(4.6)

$$\bar{x} = [x_1^2, x_1x_2, ..., x_1x_n, x_2^2, x_2x_3, ..., x_2x_n, ..., x_{n-1}^2, x_{n-1}x_n, x_n^2]^T$$

(4.7)

where $V_{ij}$ represents the element of $V \in \mathbb{R}^{n \times n}$ in the $i^{th}$ row and $j^{th}$ column and $x_i$ represents the $i^{th}$ element in vector $x$.

Furthermore, $y^TKx$, where $x \in \mathbb{R}^m$ and $y \in \mathbb{R}^n$, and $K \in \mathbb{R}^{n \times m}$ can be written as $(x \otimes y)^k$ with $x \otimes y \in \mathbb{R}^{nm}$ where:

$$x \otimes y = [x_1y_1, x_1y_2, ..., x_1y_m, x_2y_1, ..., x_2y_m, ..., x_ny_1, ..., x_ny_m]^T$$

(4.8)

$$k = [K_{11}, K_{21}, ..., K_{n1}, K_{12}, K_{22}, ..., K_{n2}, ..., K_{1m}, ..., K_{nm}]^T$$

(4.9)

### 4.4.2 Kleinman’s Iterative Solution to the FH LQR Problem

Analysis and description of the algorithm requires the definition of the Cost-to-go Matrix, $V_k(t)$, for a given control gain $K_k(t)$ as the solution to:

$$\dot{V}_k(t) = -[A(t) - B(t)K_k(t)]^TV_k(t) - V_k(t)[A(t) - B(t)K_k(t)]$$ $- Q(t) - K_k^T(t)R(t)K_k(t)$$

(4.10)

with final condition $V_k(t_f) = \Phi_f$. Utilising this definition, the following was proposed in Kleinman (1967).

**Proposition 4.1** (Kleinman’s Algorithm (Kleinman, 1967)). For the FH LQR problem, with system dynamics (4.1) and cost function (4.2), if the following algorithm is followed:

1. Define arbitrary $K_0(t)$, set $k = 0$

2. Solve for $V_k(t)$ using Equation (4.10)
3. Update $K_{k+1}(t)$ through:

$$K_{k+1}(t) = R^{-1}(t)B^T(t)V_k(t) \quad (4.11)$$

4. Set $k = k + 1$ and go to 2.

Then:

1. $P(t) \leq V_{k+1}(t) \leq V_k(t)$, for all $k \in \mathbb{N}_{\geq 0}$

2. $V_k(t)$ converges to $P(t)$ uniformly\(^1\)

3. $K_k(t)$ converges to $K^*(t)$ uniformly.

where $K^*(t)$ is as defined in (4.4). The algorithm will monotonically converge to the value function associated with the optimal control gain. \(\diamondsuit\)

The proof of Proposition 4.1 is given in Kleinman (1967). This is an iterative process, in which successive estimates of the optimal control gain are made. The algorithm provides a less computationally expensive method of solving the DRE, as the algorithm involves iteratively solving a linear matrix equation (4.10), rather than the quadratic DRE (4.5). However, the complete knowledge of the dynamics of the system ($A(t)$ and $B(t)$) is required to solve (4.10). In the sequel, we explore some properties of Kleinman’s algorithm.

### 4.4.3 A Converse Theorem for Kleinman’s Algorithm

The first property generalises the Converse Theorem proposed in Jiang and Wang (2002) to characterize the convergence properties of $V_k(t)$ in the algorithm.

It is noted that $V_k(t)$ is directly related to $K_k(t)$ through (4.10). Therefore, for any $K_k(t)$, we can write $V_k(t)$ as $V(t, K_k(t), A(t), B(t), Q(t), R(t))$. A new variable $\Sigma_k(t) = [V(t, K_k(t)) - P(t)] \in \mathbb{C}^{n \times n}[t_0, t_f]$ is now introduced. Using the results listed in Proposition 4.1, the update laws (4.10) and (4.11) can be rewritten as:

$$\Sigma_{k+1}(t) = f(\Sigma_k(t), A(t), B(t), Q(t), R(t)) \quad (4.12)$$

\(^1\)The sequence $V_k(t)$ converges to $P(t)$ uniformly indicates that $\lim_{k \to \infty} \|V_k - P\|_{n \times n} = 0$.\)
with $\Sigma_k(t_f) = 0_{n \times n}$ for any $k \in \mathbb{N}_{\geq 0}$. Here the mapping $f$ satisfies $\mathcal{C}^{n \times n}[t_0, t_f] \times \mathcal{C}^{n \times n}[t_0, t_f] \times \mathcal{C}^{n \times n}[t_0, t_f] \times \mathcal{C}^{m \times m}[t_0, t_f] \to \mathcal{C}^{n \times n}[t_0, t_f]$.

Proposition 4.1 indicates that the system (4.12) is uniformly globally asymptotically stable (UGAS) with respect to the set $\mathcal{C}^{n \times n}[t_0, t_f]$ (see (Jiang and Wang, 2002, Definition 2.1) for the definition of UGAS for discrete-time nonlinear systems — the same definition can be extended to system (4.12)). By using Converse Theorem (Jiang and Wang, 2002, Theorem 1 & Lemma 2.8), and noting importantly that the convergence has no dependence on iteration number $k$, the following property holds:

**Property 1.** Let $V_k$ and $K_k$ be defined as per Kleinman’s algorithm, leading to the dynamic system (4.12) and $\alpha_l$, $l = 1, 2, 3$ be class-$\mathcal{K}_\infty$ functions. Then there exists a continuous function $W : \mathbb{R}^{n \times n} \to \mathbb{R}_{\geq 0}$ such that for any $\Sigma \in \mathcal{C}^{n \times n}[t_0, t_f]$:

1. $W(\Sigma)$ is bounded by:
   \[
   \alpha_1 (|\Sigma|) \leq W(\Sigma) \leq \alpha_2 (|\Sigma|),
   \]
   (4.13)

2. The update law (4.12) satisfies:
   \[
   W(f(\Sigma, A, B, Q, R)) - W(\Sigma) \leq -\alpha_3 (|\Sigma|)
   \]
   (4.14)

**Remark 4.2.** Jiang and Wang (2002) defines a function $W : \mathbb{N}_{\geq 0} \times \mathcal{C}^{n \times n}[t_0, t_f] \to \mathbb{R}_{\geq 0}$, however, in Kleinman’s algorithm, the convergence has no dependence on iteration number $k$. Therefore, the converse function defined for this problem is independent of $k$.

### 4.4.4 Property of the Cost-to-go Matrix

A property of the Cost-to-go Matrix $V_k(t)$ is constructed utilising a system input which is perturbed by an excitation signal $w(t)$, i.e. when considering the dynamics using the $k^{th}$ estimate of $K^*(t)$:

\[
\mathbf{u}(t) = -K_k(t)x(t) + w(t),
\]
(4.15)

where $w \in \mathbb{R}^m$.

---

\[\text{A function } \gamma : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0} \text{ is a } \mathcal{K}_\infty \text{ function if it is continuous, strictly increasing, } \gamma(0) = 0 \text{ and } \gamma(s) \to \infty \text{ as } s \to \infty.\]
Property 2. Let $t_0 \leq t_a < t_b \leq t_f$ and $x(t)$ be a trajectory obtained from applying (4.15) to (4.1). Then the following equality holds

$$x^T(t_b)V_k(t_b)x(t_b) - x^T(t_a)V_k(t_a)x(t_a) = \int_{t_a}^{t_b} \left[ 2w^T(t)R(t)K_{k+1}(t)x(t) - x^T(t)(Q(t) + K_k^T(t)R(t)K_k(t))x(t) \right] dt \quad (4.16)$$

Proof: Using the dynamics of the system with control law (4.15) and the derivative of the Cost-to-go matrix (4.10), an expression for $\frac{d}{dt} (x^T(t)V_k(t)x(t))$ can be found. Integrating over $t = t_a$ to $t = t_b$, and substituting $B^T(t)V_k(t) = R(t)K_{k+1}(t)$ produces the above property.

4.5 Proposed Algorithm

This section presents an algorithm which iteratively solves the Continuous Time FHLQR problem without requiring the explicit knowledge of the dynamics of the system. A high level overview of the proposed algorithm is first presented, followed by more a detailed analysis of each iteration of the outer loop, and finally the convergence of the algorithm to the optimal control gain is shown.

4.5.1 High Level Overview

The overall structure of the proposed algorithm is shown in Figure 4.1. The nested structure contains two loops (both in the iteration domain), and allows two objectives to be fulfilled. The objective of the Outer Loop (index $k$) is to iterate over estimates of $K^*(t)$, to eventually converge to a region around $K^*(t)$, along the solutions of Klienman’s Algorithm. Within the Inner Loop (index $j$), the computed feedback gain ($K_k(t)$) is applied along with sufficient excitation (in the $j$ domain) to generate an appropriate number of online measurements of state trajectories, which are then used to estimate $K_{k+1}(t)$ and $V_k(t)$ at given sampling points.
4.5.1.1 Notation for the Algorithm

A new set of notation is defined in Table 4.1 for the remainder of this work, based on the notation used in Section 4.4 with modifications to cater for the features in the proposed algorithm.

<table>
<thead>
<tr>
<th>$K(t)$</th>
<th>Description</th>
<th>$V(t)$</th>
<th>$\Sigma(t)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K^*(t)$</td>
<td>The optimal control gain</td>
<td>$P(t)$</td>
<td>$0$</td>
</tr>
<tr>
<td>$K_k(t)$</td>
<td>The control gain applied at the $k^{th}$ iteration</td>
<td>$V_k(t)$</td>
<td>$\Sigma_k(t)$</td>
</tr>
<tr>
<td>$\hat{K}_{k+1}(t)$</td>
<td>The next estimate of the Optimal Control Gain as defined by Klienman’s Algorithm, where the previous point is $K_k(t)$</td>
<td>$\hat{V}_{k+1}(t)$</td>
<td>$\hat{\Sigma}_{k+1}(t)$</td>
</tr>
<tr>
<td>$K^*_{k+1}(t)$</td>
<td>A piecewise approximation of $K^<em>_{k+1}(t)$, defined: $\tilde{K}^</em><em>{k+1}(t) = K^*</em>{k+1}(t_i), \forall t \in [t_i, t_{i+1}), i = 0, ..., N - 1$</td>
<td>unused</td>
<td>$\Sigma^*_{k+1}(t)$</td>
</tr>
<tr>
<td>$\tilde{K}^*_{k+1}(t)$</td>
<td>A least squares estimate of $K^*_{k+1}(t)$</td>
<td>unused</td>
<td>$\tilde{\Sigma}^*_{k+1}(t)$</td>
</tr>
</tbody>
</table>

It is important to note that $\tilde{K}_{k}(t)$ is the gain applied at the $k^{th}$ iteration of the outer loop. This has the associated cost-to-go matrix $V_k(t)$. This is not the same as the gain as calculated by Klienman’s algorithm. For this, $K^*_{k+1}(t)$ is
used. That is, given $K_k(t)$ with associated cost-to-go matrix, $V_k(t)$, $K_{k+1}^*(t) = R^{-1}(t)B^T(t)V_{k-1}(t)$. The reader is also reminded that each of these matrices can be represented in a vector form as discussed in Section 4.4.1.

### 4.5.2 Outer Loop

Given any arbitrary $K_k(t) \in C^{m \times n}[t_0, t_f]$, $K_{k+1}^*(t)$ can be computed based on the knowledge of $A(t), B(t)$ using Kleinman’s Algorithm. Without the knowledge of $A(t), B(t)$, Property 2 and Least Square Estimation can be used to estimate $K_{k+1}^*(t)$ and $V_k(t)$. As continuous functions are difficult to identify explicitly, these two functions are discretised over $[t_0, t_f]$ so that parameter identification methods can be applied. A sampling period $T$ is therefore selected such that there exist $N = \frac{T-t_0}{T}+1$ sampling instants: $t_i = t_0+iT, i = 0, \ldots, N$. The proposed algorithm attempts to find a piecewise constant approximation of $K_{k+1}^*(t)$ at time instants $t_j, j \in [0, \ldots, N-1]$. This discretisation, of course, introduces an error, however, it will be shown that this error can be bounded.

#### 4.5.2.1 Least Square Estimation of $K_{k+1}^*(t)$ and $V_k(t)$ at Each Sampling Instant

With the given $K_k(t)$, the system has the following controller and the corresponding closed-loop dynamics:

$$
\begin{align*}
    u_k(t) &= -K_k(t)x_k(t) + w_k(t) \\
    x_k(t) &= (A(t) - B(t)K_k(t))x_k(t) + B(t)w_k(t), x_k(0) \in \mathbb{R}^n. 
\end{align*}
$$

(4.17)

By applying Property 2 in any interval $[t_i, t_{i+1}]$, $i = 0, \ldots, N-1$, it follows that

$$
\begin{align*}
    x_k^T(t_{i+1})V_k(t_{i+1})x_k(t_{i+1}) - x_k^T(t_i)V_k(t_i)x_k(t_i) \\
    &= \int_{t_i}^{t_{i+1}} \left[ 2w_k^T(t)R(t)K_{k+1}^*(t)x_k(t) \right] dt \\
    &\quad - \int_{t_i}^{t_{i+1}} \left[ x_k^T(t) (Q(t) + K_k^T(t)R(t)K_k) x_k(t) \right] dt. 
\end{align*}
$$

(4.18)
With the introduction of $\tilde{K}_{k+1}(t) = K_{k+1}(t), \forall t \in [t_i, t_{i+1})$, using the notation from Section 4.4.1, and introducing:

$$
\delta_i^T(x_k, w_k) := 2 \int_{t_i}^{t_{i+1}} (x_k(t) \otimes R^T(t)w_k(t))^T dt
$$

(4.19)

$$
\gamma_i(x_k, K_k) := -\int_{t_i}^{t_{i+1}} [x_k^T(t)(Q(t) + K_k^T(t)R(t)K_k(t))x_k(t)] dt
$$

(4.20)

$$
\rho_i(x_k, w_k) := 2 \int_{t_i}^{t_{i+1}} (x_0(t) \otimes R^T(t)w_k(t))^T (\tilde{K}_{k+1}(t) - \tilde{K}_{k+1}(t_i)) dt
$$

(4.21)

(4.18) can be written as:

$$(\bar{x}_k)^T(t_i+1)\bar{v}_k(t_{i+1}) - (\bar{x}_k)^T(t_i)\bar{v}_k(t_i)$$

$$= \delta_i^T(x_k, w_k) \cdot \tilde{K}_{k+1}^*(t_i) + \gamma_i(x_k, K_k) + \rho_i(x_k, w_k)
$$

(4.22)

with $V_k(t_N) = \Phi_f$, and where $\tilde{K}_{k+1}^*(t_i)$ is the vector form of $\tilde{K}_{k+1}^*(t)$ at $t \in [t_i, t_{i+1})$, and $\bar{v}_k(t_{i+1})$ is the vector form of $\bar{v}_k(t_i)$.

Assuming knowledge of $\bar{v}_k(t_{i+1})$, this equation can be used as the basis to estimate $\tilde{v}_k(t_i)$ and $\tilde{K}_{k+1}^*(t_i)$. However, (4.22) is a scalar equation, whereas $\bar{v}_k(t_i) \in \mathbb{R}^{n(n+1)/2}$ and $\tilde{K}_{k+1}^*(t_i) \in \mathbb{R}^{nm}$ are not scalar. Therefore, to estimate $\tilde{v}_k(t_i)$ and $\tilde{K}_{k+1}^*(t_i)$, more information is required. This is addressed using a Least Squares Estimation (LSE). In order to use a LSE, a sequence of dither signals $w_{k,j}(t), j = 1, 2, \ldots, \ell$ are required to generate sufficient excitation along the inner loop ($j$) iteration domain. For a sufficiently large $\ell$, and with sufficient variation in the excitation signals, it is possible to estimate $\tilde{v}_k(t_i)$ and $\tilde{K}_{k+1}^*(t_i)$.

More precisely, at each inner loop iteration $j = 1, \ldots, \ell$, dynamics are of the form:

$$
u_{k,j}(t) = -K_k(t)x_{k,j}(t) + w_{k,j}(t)
$$

$$x_{k,j}(t) = (A(t) - B(t)K_k(t))x_{k,j}(t) + B(t)w_{k,j}(t)
$$

(4.23)
where \( x_k(0) \in \mathbb{R}^n \). Utilising this information, at time \( t_i \), \( \ell \) equations in the form of (4.22) can be obtained. By using the following notation

\[
X_{k,i} = \begin{bmatrix}
\bar{x}_k^T(t_i) \\
\bar{x}_k^T(t_i) \\
\vdots \\
\bar{x}_k^T(t_i)
\end{bmatrix},
\Delta_{k,i} = \begin{bmatrix}
\delta_{k,i}(x_{k,1}, w_{k,1}) \\
\delta_{k,i}(x_{k,2}, w_{k,2}) \\
\vdots \\
\delta_{k,i}(x_{k,\ell}, w_{k,\ell})
\end{bmatrix},
\mathbf{c}_{k,i} = \begin{bmatrix}
\gamma_i(x_{k,1}, K_k) \\
\gamma_i(x_{k,2}, K_k) \\
\vdots \\
\gamma_i(x_{k,\ell}, K_k)
\end{bmatrix},
\epsilon_{0,i} = \begin{bmatrix}
\rho_i(x_{k,1}, w_{k,1}) \\
\rho_i(x_{k,2}, w_{k,2}) \\
\vdots \\
\rho_i(x_{k,\ell}, w_{k,\ell})
\end{bmatrix}
\]

this leads to the following matrix equation:

\[
\begin{bmatrix}
X_{k,i} & \Delta_{k,i} & -X_{k,i+1}
\end{bmatrix}
\begin{bmatrix}
\bar{v}_k(t_i) \\
\hat{\bar{v}}_{k+1}(t_i) \\
\bar{v}_k(t_{i+1})
\end{bmatrix}
= -\mathbf{c}_{k,i} - \epsilon_{k,i}
\] (4.25)

At \( i = N - 1 \), \( \bar{v}_k(t_{i+1}) = \bar{v}_k(t_N) \). Furthermore, at all iterations, the value \( \epsilon_{k,j} \) is unknown, however, can be made small with small sampling period \( T \).

Consider now the least squares problem, in which all time steps are augmented into a single estimation. With the following definitions:

\[
\Phi_k = \begin{bmatrix}
X_{k,0} & \Delta_{k,0} & -X_{k,1} & 0 & \ldots & 0 & 0 \\
0 & 0 & X_{k,1} & \Delta_{k,1} & \ldots & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & 0 & 0 & \ldots & X_{k,N-1} & \Delta_{k,N-1}
\end{bmatrix},
\mathbf{c}_k = [-\mathbf{c}_{k,0}, -\mathbf{c}_{k,1}, \ldots, -\mathbf{c}_{k,N} + X_{k,N} \bar{v}_k(t_N)]^T,
\mathbf{e}_k = [\mathbf{e}_{k,0}, \mathbf{e}_{k,1}, \ldots, \mathbf{e}_{k,N}]^T,
\mathbf{\xi}_k = [\bar{v}_k(t_0), \hat{\bar{v}}_{k+1}(t_0), \bar{v}_k(t_1), \hat{\bar{v}}_{k+1}(t_1), \ldots, \bar{v}_k(t_{N-1}), \hat{\bar{v}}_{k+1}(t_{N-1})]^T
\]

(4.26)

where \( \Phi_k \in \mathbb{R}^{N \ell \times N\left(\frac{n(n+1)}{2}+nm\right)} \), \( \mathbf{c}_k, \mathbf{e}_k \in \mathbb{R}^{N \ell} \) and \( \mathbf{\xi}_k \in \mathbb{R}^{N\left(\frac{n(n+1)}{2}+nm\right)} \), and \( \hat{\bar{v}}_{k+1}(t_i) \) is an estimate of \( \bar{v}_{k+1}(t_i) \), the following least squares problem can be posed:

\[
\Phi_k \mathbf{\xi}_k = \mathbf{c}_k
\] (4.27)
It is noted that \( \xi_k \) contains estimates for \( \bar{v}_k(t) \) and \( \tilde{k}_{k+1}^*(t) \) (rather than \( k_{k+1}^*(t) \)) at each sampling instant \( t_i \), and thus \( \epsilon_k \) can be considered a disturbance to the solution, and the error can be bounded by:

\[
\delta \xi_k = \kappa(\Phi_k) \| \epsilon_k \| \tag{4.28}
\]

where \( \kappa(\Phi_k) \) is the condition number of the matrix \( \Phi_k \) (see Björck (1996) for complete definition).

Utilising the result of this least squares problem, \( \hat{k}_{k+1}^*(t_i) \) for \( i = 0, \ldots, N - 1 \) can be used as a feedback control gain \( K_{k+1}(t) \) in the piecewise constant form:

\[
K_{k+1}(t) \equiv \hat{K}_{k+1}^*(t_i), \forall t \in [t_i, t_{i+1}), i = [0, \ldots, N - 1] \tag{4.29}
\]

The rest of this section will show that \( K_{k+1}(t) \equiv \hat{K}_{k+1}^*(t) \) can be made arbitrarily close to \( K_{k+1}^*(t) \). This can be shown by demonstrating two facts:

1. The discretisation error \( |k_{k+1}^*(t_i) - \hat{k}_{k+1}^*(t_i)| \) can be made arbitrarily small.
2. The estimation error \( |\hat{k}_{k+1}^*(t_i) - \tilde{k}_{k+1}^*(t_i)| \) can be made arbitrarily small.

The following fact comes from the uniform continuity of the \( k_{k+1}^*(t) \) over a compact time interval \([0, T]\).

**Fact 1.** Let \( (\nu_1, \Delta_1) \) be an arbitrary positive pair. There exists a sufficiently small \( T_1^* \), such that for any continuous function \( k_{k+1}^*(t) \) satisfying \( \|k_{k+1}^*(t)\|_a \leq \Delta_1 \) and any \( T \leq T_1^* \) and \( N_1 = \frac{T - t_0}{T} \) is an integer such that at each sampling instant \( t_j, j = 1, \ldots, N_1 \), the following inequality holds

\[
\max \left\{ |k_{k+1}^*(t_i) - \hat{k}_{k+1}^*(t_i)| \right\} \leq \nu_1 \\
\forall t \in [t_i, t_{i+1}], \forall i = 0, \ldots, N - 1 \tag{4.30}
\]

Before the introduction of the second fact conditioning techniques (Björck, 1996) for the matrix are now introduced. For the purposes of this algorithm, it is obvious that the estimate of \( \hat{k}_{k+1}^*(t_i) \) is of greater interest, whereas \( \bar{v}_k(t_i) \) has less importance. As such, to minimise the impact of the error in the estimate of \( \hat{k}_{k+1}^*(t_i) \) the least squares problem (4.27) is conditioned as:

\[
\Phi_k D_{k}^{-1} D_k \xi_k = c_k, \tag{4.31}
\]

\[
\Phi_k D_{k}^{-1} D_k \xi_k = c_k, \tag{4.31}
\]
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where $D_k \in \mathbb{R}^{N(\frac{n(n+1)}{2}+nm) \times N(\frac{n(n+1)}{2}+nm)}$ is a diagonal conditioning matrix. This matrix serves as a scaling factor in estimating $\hat{v}_k(t_i)$ and $\hat{k}_{k+1}(t_i)$ terms. In particular, $D_k$ is chosen such that the elements corresponding to $\hat{k}_{k+1}(t_i)$ are 1, i.e. it is of the form:

$$D_k = diag(s_0, 1_{nm \times 1}, s_1, 1_{nm \times 1}, \ldots, s_{N-1}, 1_{nm \times 1}),$$  \hspace{1cm} (4.32)

where $s_i \in \mathbb{R}^{\frac{n(n+1)}{2}}$ for $i = [0, \ldots, N-1]$, and $1_{nm \times 1}$ is a $R^{nm}$ vector of ones.

The least squares can then be solved in a two-step process:

$$D_k \xi_k = (\Phi_k D_k^{-1})^\dagger c_k$$  \hspace{1cm} (4.33)

$$\xi_k = D_k^{-1} (\Phi_k D_k^{-1})^\dagger c_k$$  \hspace{1cm} (4.34)

where $(\Phi_k D_k^{-1})^\dagger$ is the pseudoinverse of $(\Phi_k D_k^{-1})$.

Based on this, the error in the estimate of $(D_k \xi_k)$ is bounded by:

$$\delta(D_k \xi_k) \leq \kappa(\Phi_k D_k^{-1}) |\epsilon_k|$$  \hspace{1cm} (4.35)

As $D_k$ is a diagonal matrix, $D_k^{-1}$ is simply:

$$D_k^{-1} = diag(s_0^{-1}, 1_{nm \times 1}, s_1^{-1}, 1_{nm \times 1}, \ldots, s_{N-1}^{-1}, 1_{nm \times 1})$$  \hspace{1cm} (4.36)

where $s_i^{-1} \in \mathbb{R}^{\frac{n(n+1)}{2}}$ is a vector in which each element is the reciprocal of the corresponding element in $s_i$. Therefore, the error in the estimate of the elements in $\tilde{k}_{k+1}^*(t_i)$ is bounded by:

$$\left|\tilde{k}_{k+1}^*(t_i) - \tilde{k}_{k+1}^*(t_i)\right| \leq \kappa(\Phi_k D_k^{-1}) |\epsilon_k|$$  \hspace{1cm} (4.37)

The excitation signals need to be well-selected in order to ensure that the estimation error is still bounded. The next assumption assumes that by using enough excitation signals $w_{k,j}, j = 1, \ldots, \ell$, for any iteration $k$, the solution of the LSE is always bounded. It is worthwhile to highlight that the tuning parameters in the design are $\{\ell, T\}$.

The following assumption is thus needed:

Assumption 1. For any given discretisation step size $T = t_{i+1} - t_i, i = [0, \ldots, N-1]$, $\lambda > 1$, $k \in N_{\geq 0}$, and bound on magnitude of excitation signal $b_w > 0$, there exists a positive integer $\ell^*$ such that for any $\ell \geq \ell^*$, there are some appropriate set of
discretisation signals \( w_{k,j}, j = 1, \ldots, \ell \), with \( \| w_{k,j} \| \leq b_w \) and some scaling matrix in the form of (4.32) such that \( \kappa(\Phi_k D_k^{-1}) \leq \lambda \).

**Remark 4.3.** This assumption is similar to a persistent excitation condition needed for LSE. Improving the condition number can be done both by increasing variance in the \( w_{k,j} \) signals and using the \( D_k \) matrix. The largest differences in relative magnitudes in \( \Phi_k \) are due to the difference between \( \zeta_{k,i} \) components (proportional to the magnitude of the state \( x_{k,i} \) only), and the components of \( \mu_{k,i} \) (proportional to the state, input weighting matrix \( R(t) \), excitation signals \( w_{k,j}(t) \) and, importantly, discretisation step size \( T \)). The \( D_k \) matrix can be used to bring these elements to a similar orders of magnitude once \( \Phi_k \) has been computed. The variance in the set of \( w_{k,j}(t) \) signals can then be used to ensure suitable inter-row independence.

It is also noted that \( \ell^* \) is also highly dependent on the dimension of the system. With a larger number of states and control inputs, the number of parameters to be identified increases, and thus more iterations are required to achieve sufficient excitation.

**Remark 4.4.** It is worthwhile to highlight that the tuning parameters are selected sequentially. The sampling interval \( T \) and bound on the magnitude of the excitation signal \( b_w \) can be first selected, creating a bound on the discretisation error. Then a family of the dither signals satisfying the bound (including the number of inner loop iterations \( \ell \)) can be selected.

**Fact 2.** Given Assumption 1 and \((\nu_2, b_w, \lambda, \ell^*)\), there exists a sufficiently small \( T_2^* \), such that for any \( T < T_2^* \) and \( N_2 = \frac{t_f - t_0}{T} \) is an integer, such that at each sampling instant \( t_j, j = 1, \ldots, N_2 \), the following inequality holds

\[
\max \left\{ \left| \hat{k}_{k+1}^*(t_i) - \hat{k}_{k+1}^*(t_i) \right| \right\} \leq \nu_2 \\
\forall t \in [t_i, t_{i+1}], \quad \forall i = 0, \ldots, N - 1
\]  

(4.38)

**Proof:** Noting that \( \epsilon_k \) is expressed as

\[
\epsilon_{k,i} = \begin{bmatrix}
2 \int_{t_i}^{t_{i+1}} q_{k,1}(k_{k+1}^*(t) - \hat{k}_{k+1}^*(t)) dt \\
2 \int_{t_i}^{t_{i+1}} q_{k,2}(k_{k+1}^*(t) - \hat{k}_{k+1}^*(t)) dt \\
\vdots \\
2 \int_{t_i}^{t_{i+1}} q_{k,\ell}(k_{k+1}^*(t) - \hat{k}_{k+1}^*(t)) dt
\end{bmatrix}
\]  

(4.39)

where \( q_{k,i} = (x_{k,i}(t) \otimes R^T(t) w_{k,i}(t))^T \). Therefore, the bound of \( |\epsilon_0| \) is proportional to the size of sampling \( T \). The proof follows by using (4.37) and Assumption 1. ■
4.5.2.2 Convergence Towards $K^*(t)$

To prove convergence towards $K^*(t)$, the converse theorem in Property 1 is used. $W(\Sigma)$ is bounded by two $K_\infty$ functions of $|\Sigma|$. Therefore, $W(\Sigma)$ can be used as a measure of the optimality of the solution. That is, a larger value indicates a less optimal solution, and a value of 0 indicates the optimal solution. As such, if it can be shown that the change in $W(\Sigma)$ is negative, $\hat{K}_{k+1}(t)$ is more optimal than $K_k(t)$.

This is formalised in the following theorem, which also utilises the fact that $\hat{K}_{k+1}(t)$ can be made arbitrarily close to $K^*_{k+1}(t)$ with sufficiently small $T$.

**Theorem 4.5.** Let $(\Delta, \nu)$ be a positive pair. For any $K_k(t)$ satisfying $|\Sigma_k(t)| \leq \Delta$ for any $t \in [t_0, t_f]$, there exists some $T^*$ such that for any $T < T^*$, there exists a $b_w$, some positive integer $\ell^*$ (by Assumption 1), and $\rho < 1$ such that given $\ell \geq \ell^*$, appropriately constructed excitation signals $w_{k,j}, j \in [1, ..., \ell]$ satisfying $\|w_{k,j}\|_s \leq b_w$ can be constructed such that $K_{k+1}(t) \equiv \hat{K}_{k+1}^*(t)$ computed utilising the least squares estimation (4.31), satisfies

$$W(\hat{\Sigma}_{k+1}) \equiv W(\hat{\Sigma}_{k+1}^*) \leq \rho W(\Sigma_k) + \nu,$$

(4.40)

**Proof:** It is noted that $\Sigma = [V(t, K(t)) - P(t)], W(\Sigma)$ is continuous in $\Sigma$, and $V(t)$ is also continuous in $K(t)$. Therefore, $W(\Sigma)$ varies continuously with $K(t)$. As such, $W(\Sigma(\hat{K}_{k+1}(t)))$ can be arbitrarily close to $W(\Sigma(K_{k+1}(t)))$ by selecting sufficiently small sampling utilising Fact 1 and Fact 2. From Property 1, there exists some $K_\infty$ function $\alpha_3(o)$ such that:

$$W(\Sigma_{k+1}^*) - W(\Sigma_k) \leq -\alpha_3(|\Sigma_k|)$$

(4.41)

Therefore,

$$W(\hat{\Sigma}_{k+1}^*) - W(\Sigma_k) = W(\Sigma_{k+1}^*) - W(\Sigma_k) - W(\Sigma_{k+1}^*) + W(\hat{\Sigma}_{k+1}^*)$$

$$\leq -\alpha_3(|\Sigma_k|) - \left|W(\Sigma_{k+1}^*) - W(\hat{\Sigma}_{k+1}^*)\right|$$

(4.42)

Two cases are considered:
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Case 1: $|\Sigma_k| \leq \alpha_2^{-1} \left( \frac{\nu}{2} \right)$. Here $\alpha_2$ comes from Property 1. It is possible to select a sufficiently small $T^*$ such that for any $T \in (0, T^*)$

$$\left| W(\hat{\Sigma}_{k+1}^*) - W(\Sigma_{k+1}) \right| \leq \frac{\nu}{2}. \quad (4.43)$$

Consequently, it follows that

$$W(\hat{\Sigma}_{k+1}^*) \leq W(\Sigma_k) + \frac{\nu}{2} \leq \alpha_2 \left( \alpha_2^{-1} \left( \frac{\nu}{2} \right) \right) + \frac{\nu}{2} \leq \nu. \quad (4.44)$$

Case 2: $\alpha_2^{-1} \left( \frac{\nu}{2} \right) \leq |\Sigma_k| \leq \Delta$. Under such a situation, by choosing a sufficiently small $T^*$ such that for any $T \in (0, T^*)$, we have

$$W(\Sigma_{k+1}^*) - W(\hat{\Sigma}_{k+1}^*) \leq \frac{\alpha_3(|\Sigma_k|)}{2} \quad (4.45)$$

$$W(\hat{\Sigma}_{k+1}^*) - W(\Sigma_k) \leq -\frac{\alpha_3(|\Sigma_k|)}{2} \leq -\alpha_3 \circ \alpha_1^{-1} \left( W(\Sigma_k) \right)$$

$$\Rightarrow W(\hat{\Sigma}_{k+1}^*) \leq W(\Sigma_k) - \frac{\alpha_3 \circ \alpha_1^{-1} \left( W(\Sigma_k) \right)}{2} \quad (4.46)$$

This indicates that there exists $\rho \in (0, 1)$ such that $W(\Sigma_k) - \frac{\alpha_3 \circ \alpha_1^{-1} \left( W(\Sigma_k) \right)}{2} \leq \rho W(\Sigma_k)$ for any $\alpha_2^{-1} \left( \frac{\nu}{2} \right) \leq |\Sigma_k| \leq \Delta$. Consequently, it has

$$W(\hat{\Sigma}_{k+1}^*) \leq \rho W(\Sigma_k). \quad (4.47)$$

Combining the two cases, we have

$$W(\hat{\Sigma}_{k+1}^*) \leq \max \{ \rho W(\Sigma_k), \nu \} \leq \rho W(\Sigma_k) + \nu, \forall |\Sigma_k| \leq \Delta \quad (4.48)$$

This completes the proof.

Remark 4.6. It is also noted that presence of measurement error can also introduce an error into $\hat{K}_{k+1}$, due to the introduction of errors into the matrices used in the least squares operation. However, estimation errors associated with this measurement error may be mitigated with appropriate filtering of the measurements. Zero-mean noise in the dynamics of the system would not have as significant an effect, as the effects of this noise on the trajectory average out over sampling periods and iterations.
4.5.3 Main Result

The main result of this chapter is stated in Theorem 4.7. It shows that for any desired accuracy there exists some discretisation step size $T^*$ such that when the algorithm is applied with $T < T^*$, it will converge on a set of gains close enough to optimal.

**Theorem 4.7.** Let $(\Delta, \nu)$ be a positive pair. For any $K_0(t)$ satisfying $|\Sigma_0(t)| \leq \Delta$, there exists some $T^* > 0$, such that if $T < T^*$ there exists some $b_w$ such that Assumption 1 holds, and some associated positive integer $\ell^*$, $\ell \geq \ell^*$, such that, with appropriately constructed excitation signals $w_{k,j}, j \in [1, ..., \ell]$ satisfying $\|w_{k,j}\|_s \leq b_w$, there exists a class-$K_\infty$ function $\alpha_4$ such that the proposed algorithm in Figure 4.1 converges to a cost close to optimal, i.e:

$$\limsup_{k \to \infty} |\Sigma_k| \leq \alpha_4(\nu)$$  \hspace{1cm} (4.49)

**Proof:** The proof comes directly from repeating Theorem 1 as

$$W(\hat{\Sigma}_k) = W(\Sigma_k) \leq \rho W(\Sigma_{k-1}) + \nu$$

$$= \rho W(\Sigma_{k-1}) + \nu$$

$$\leq \rho^2 W(\Sigma_{k-2}) + \rho \nu + \nu$$

$$\vdots$$

$$\leq \rho^k W(\Sigma_0) + \sum_{s=0}^{k} \rho^{k-s} \nu$$  \hspace{1cm} (4.50)

This completes the proof. $\blacksquare$

The convergence of the algorithm can be illustrated by the diagram in Figure 4.2. It can be observed at each iteration, Klienman’s algorithm proposes some new gain $K^*_{k+1}(t)$ which is more optimal than $K_k(t)$. The errors in approximating $K^*_{k+1}(t)$ can be bounded with the appropriate choice of tuning parameters, such that $\hat{K}_{k+1}(t)$ is close to $K^*_{k+1}(t)$ in optimality. Therefore, the algorithm converges to a region close to optimal as $k \to \infty$. 
4.6 Simulations

In this section, the performance of the algorithm is shown through a number of simulations. In particular, the simulations will illustrate (1) the convergence of the algorithm to near optimal; (2) potential difficulties in achieving excitation when smaller $T$ is chosen; and (3) the convergence of the algorithm to closer to optimal when a smaller $T$ is chosen, however, at increased computational cost.

4.6.1 System Under Investigation

A simple system is presented in order to demonstrate the practicalities involved in implementing this algorithm. However, the algorithm can be applied in the same manner to more complex systems, such as those with time-varying dynamics, or higher order systems. The system considered here is the simple double integrator:

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$$  \hspace{1cm} (4.51)
For the purpose of this simulation, the following finite horizon task is used. The cost matrices (which were chosen arbitrarily):

\[
Q(t) = \begin{bmatrix} 0.4t + 2 & 0 \\ 0 & 0.4t + 2 \end{bmatrix}; \quad \Phi_f = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}
\]

\[
R(t) = \begin{bmatrix} 2.5 - 0.3t & 0 \\ 0 & 2.5 - 0.3t \end{bmatrix}
\]

with \( t_0 = 0 \) and \( t_f = 5 \).

4.6.2 Parameter Selection

In order to simplify the simulations, this work presents changes in \( T \), and \( \ell \) for the purposes of achieving sufficient conditioning of the \( \Phi_k \) matrices. The choice of other parameters is outlined in this section. Other choices can be made for any of these parameters.

The set of excitation signals for the simulations, given the bound \( w_b = 0.1 \), were set as arbitrary values in the range \([-w_b, w_b]\), constant for each \( t \in [t_i, t_{i+1}) \).

Each element in the scaling matrix was calculated as:

\[
s_{k,i} = \begin{bmatrix} \left| X_{k,i,0} \right| & \left| X_{k,i,1} \right| & \cdots & \left| X_{k,i,n(\ell+1)} \right| \\ \Delta_{k,i} & \Delta_{k,i} & & \Delta_{k,i} \end{bmatrix}
\]

where \( X_{k,i,h} \) is the \( h^{th} \) column of \( X_{k,i} \). This scales elements in the \( X_{k,i} \) matrix to be a comparable order of magnitude to the \( \Delta_{k,i} \) matrix.

Finally, the initial value for the gain matrix was selected as \( K_0(t) = \begin{bmatrix} 0 & 0 \end{bmatrix} \). This again was an arbitrary choice, but selected for ease of repeatability.

4.6.3 Results

4.6.3.1 Convergence of the Algorithm

The first simulation is taken with \( T = 0.5s \) and \( \ell = 6 \). The elements of the cost-to-go matrix \( V_k(t) \) are shown in Figure 4.3. It can be seen that the estimates of
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$K^*(t)$ in the algorithm iteratively converge to optimal cost $P(t)$ — that is, the cost of the control scheme as $k$ increases approaches optimal.

![Figure 4.3: Elements of $V_k(t)$ for Double Integrator System with time step of 0.5 seconds.](image)

4.6.3.2 Compromise Between Computational Cost and Optimality

In the next set of simulations, different values of $T$ are compared, to demonstrate the trade-off between computational cost and optimality. In this case, the $T = 2.5$, $T = 0.5$ and $T = 0.05$ cases are compared, setting $\ell = 10$.

These cases are compared on two accounts. First, the optimality of the solution is measured through computation of $\|V_{10} - P\|_s$. Secondly, the average computational time for each outer loop iteration $k$ is computed. The simulations were
performed in Matlab 2015a on computer with a Intel i5-4670K (3.4GHz) Processor and 8.00 GB RAM. The results can be seen in Table 4.2.

<table>
<thead>
<tr>
<th>$T$</th>
<th>$|V_{10} - P]|_s$</th>
<th>Computational Time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.5</td>
<td>0.7</td>
<td>0.00032</td>
</tr>
<tr>
<td>0.5</td>
<td>0.17</td>
<td>0.00051</td>
</tr>
<tr>
<td>0.05</td>
<td>0.013</td>
<td>0.0029</td>
</tr>
</tbody>
</table>

*Computational time is average time taken for each iteration

It is apparent that reducing $T$ produces a closer estimate of the optimal gain — due to a smaller discretisation error, however at a greater computational cost. Therefore such a tradeoff should be considered in any implementation of this algorithm. It is also noted that this matrix is a sparse matrix, and there are a number of more efficient solvers available (such as Fong and Saunders (2011)), which can improve the computation times of the algorithm.

### 4.6.3.3 Achieving Sufficient Excitation

Excitation in this algorithm is required to ensure the condition number of the inverted matrix in the least squares estimate is sufficiently small, and therefore that the algorithm converges. Given any $T$ (set to bound the discretisation error) and $b_w$ (set to bound the disturbance in the least squares estimate), the condition number can be affected by tuning the number of inner loop iterations, $\ell$, selection of the scaling matrices $D_k$, and selection of excitation signals $w_{k,j}(t)$. As demonstrated in Section 4.6.3.2, a smaller $T$ results in a more optimal solution as $k \to \infty$. However, due to the smaller $T$, changes in $x$ between samples are small, and thus this makes attaining sufficient excitation more difficult. Tuning of the additional parameters can resolve this. As an example, Figure 4.4 illustrates the convergence of the algorithm with $T = 0.05s$, $\ell = 20$, whereas with $\ell = 6$, the algorithm quickly diverged.

### 4.7 Summary

This chapter presents an algorithm which addresses the Finite Horizon Linear Quadratic Regulator problem, without the need for explicit knowledge of the dynamics of the system. This algorithm utilises a two-loop structure, in which the
inner loop (index \( j \)) is used to gather information about the system, and the outer loop (index \( k \)) is used to make successive approximations of the optimal control gain. This structure may also potentially be used in applications in which the dynamics are slowly varying over iteration, or which are mostly iteration-invariant but may change suddenly at a particular iteration, to ensure that a control scheme which is close to optimal is maintained.

Although the algorithm presented is limited in application to linear time varying systems, extension to nonlinear systems is of interest. If applied directly to a nonlinear system, the performance of the algorithm would be affected. The addition of this nonlinearity will add additional errors to the estimates, and thus increase
the region around the optimal solution that the algorithm converges to. A for-
mal analysis around how significantly nonlinearity affects the performance of the
algorithm has been left as future work.

The application of this algorithm to modelling motor adaptation is considered in
the following chapter. The algorithm has been structured such that it is capable
of modelling the desired characteristics, including its applicability to the FHLQR
problem; its capability in handling multiple initial conditions (and thus be able to
complete a family of tasks); and its approach of not explicitly using knowledge of
the dynamics to calculate the optimal policy, but instead using previous attempts
at the trajectory. The next chapter introduces the experimental methods used to
evaluate the performance of the algorithm.
Chapter 5

An Experiment for Investigating Motor Control and Adaptation

With the proposal of a model for motor adaptation, the next goal of this thesis is to evaluate its performance experimentally. This validation involves the construction of an experimental task, and comparison of experimental results to model-based simulations. This chapter therefore details the design of the experiment, and the construction of a model for motor control and adaptation of this task.

This chapter is structured as follows. First, the experimental procedure is detailed, presenting a description of the task, the tools and the protocol used. Secondly, a detailed model for motor control is presented, including both the dynamics and control subsystems, which are related to the proposed framework.

5.1 Experimental Methods

This section details the experiment designed as a demonstration of this work. First we present the objectives and motivation. Following this, a description of the task chosen for the experiment, and the protocol observed are detailed.

5.1.1 Objectives and Requirements

The goal of the experiment were to investigate features of motor control and motor adaptation. In particular, the experiment was designed to explore:
• Whether movement patterns differ over a ‘family’ of tasks
• Whether movement patterns differ under different dynamic conditions
• Whether a proposed model can be used to predict these differences in movement patterns in the ‘family’ of tasks and under these different dynamic conditions
• How movements of healthy subjects and neurologically-impaired individuals differ
• How movement patterns change when transition between different dynamic conditions (motor adaptation)

As such, the experiment, and the task used in the experimental, was designed with the following desired characteristics:

• Simple to understand and complete but novel
• Task must require the resolution of redundancy at joint and task level (including at the endpoint)
• To utilise equipment safe and approved for patient use
• Capable of being modelled using linear dynamics and a quadratic cost function

These are be discussed in more detail in the following paragraphs.

First, the experimental task and protocol was designed to be simple to understand and complete, but novel. The simplicity of the task was important to ensure that it could be completed by neurologically-impaired subjects. However, at the same time, the task was designed to ensure that it was not well practised — to attempt to promote adaptation as the task was learnt.

Secondly, to investigate motor control and the models of motor control, redundancy must exist in the task to differentiate the movements. The task was designed such that redundancy exists at both through the trajectory and at the end posture. Furthermore, for simplicity of example and modelling, the task was limited to only two degrees of freedom, thus the task was designed to utilise only a single output variable as the combination of these two degrees of freedom.
Thirdly, in order to investigate both healthy subjects and neurologically impaired individuals, the experiment was designed such that it utilised equipment which was approved and safe for both healthy subjects and patients. In particular, the ArmeoPower (Hocoma, Switzerland) (see Figure 5.1), an exoskeleton designed for rehabilitation was used.

Finally, whilst a general model of arm movements is desirable, the overall purpose is to demonstrate the viability of the proposed model in motor control and adaptation tasks. As such, it was desired that the talk be modellable within the framework posed by the FH LQR problem. Therefore, it was desired that the task be designed in a manner such that effects of linearising the model could be minimised.

\subsection*{5.1.2 Task Description}

The task is the discrete action which is required to be completed by the subject during the experiment. Throughout the course of the experiment, the task is completed multiple times, where each attempt is termed a ‘trial’. Through multiple trials the ‘normal’ trajectory can be investigated, and, when the dynamics change, the motor adaptation can be investigated.

In this experiment, a single task was used. Within this task, the subjects’ arms were constrained to allow movement only in the horizontal plane level with the
shoulder, and joints were constrained to only allow horizontal adduction/abduction of the shoulder — measured as $\theta_1$, and elbow flexion/extension, measured as $\theta_2$ (see Figure 5.2).

![Figure 5.2: Experimental Setup (Top View). $\theta_1$ is related to the horizontal abduction/adduction movement of the shoulder. $\theta_2$ is related to the elbow flexion/extension movement.](image)

Within the task, the two joints were mapped to a single output variable, $y$, through the relationship $y = a_1\theta_1 + a_2\theta_2$, where $a_1$ and $a_2$ were scaling constants. To begin the task, the subjects were placed to an initial posture, $[\theta_1, \theta_2] = [\theta_{1,\text{init}}, \theta_{2,\text{init}}]$. From this initial position, at $t = t_0$, the subject was asked to move to such that $y = y_{\text{targ}}$ at time $t = t_f$.

The choice of $a_1, a_2$ and $y_{\text{targ}}$ were such centre of the range of motion of $\theta_1$ and $\theta_2$ satisfied the target goal, and scaled accordingly to the range of motion of the subject.

5.1.2.1 Redundancy

The task has two degrees of freedom mapped to a single output. Therefore, there are infinitely many combinations of $\theta_1$ and $\theta_2$ exist for any given $y$, and thus the task is redundant (see Figure 5.3). This provides redundancy which is not present in two-dimensional point-reaching tasks, as the end point posture is not fixed.
5.1.2.2 A Family of Tasks — Different Initial Conditions

The task was completed from six different initial conditions, which corresponded to both internal and external rotations. Thus, the model of motor control and the resolution of redundancy could be studied over a greater span. Additionally, the model for adaptation could also be evaluated over a family of tasks.

All initial conditions were equidistant to the goal in task space, according to (5.1).

\[ \alpha_1 \theta_1,_{\text{init}} + \alpha_2 \theta_2,_{\text{init}} - y_{\text{targ}} = |y_{\text{init}}| \] (5.1)

In addition, the value of \(|y_{\text{init}}|\) was set such that the initial posture was close to the upper and lower limits of the shoulder joint’s range of motion, but also that the redundancy could be resolved entirely through the movement of a single joint, if the subject desired.

5.1.2.3 Changes in Dynamics

Changes in dynamics were investigated by performing the experiment in three different dynamic conditions. The first was in the robotic device itself, without any additional weight. This condition is termed the ‘No Weight’ condition. The second was with a 5 kg mass, which was placed on the elbow. This is termed the ‘Elbow Weight’ condition. The third was with a 5 kg mass placed on the hand. This is termed the ‘Hand Weight’ condition.

It is noted that the robotic device only allowed movement in joints with a vertical axis of rotation. As such, the presence of the additional weights did not cause the device to move down with gravity — the weights simply added to the inertia of
Chapter 5 An Experiment for Investigating Motor Control and Adaptation

the relevant arm segment. Furthermore, it is noted that links of the robotic device itself also had some mass, which is not present during properly ‘free’ movements. An illustration of this can be seen in Figure 5.4.

![Figure 5.4: The masses on the arm. $m_{r,fa}$, $m_{r,ua}$, $m_h$, $m_e$ represent the mass of the robot moving with the forearm, the mass of the robot moving with the upper arm, the mass placed at the hand, and the mass placed at the elbow respectively.](image)

5.1.2.4 Robotic Device

The ArmeoPower (see Figure 5.1) was utilised within this experiment, due to patient familiarity, its ability to prevent movements of certain joints, and its safety capabilities. The ArmeoPower is a commercially-available robotic exoskeleton with assistive capabilities, which patients use as part of their rehabilitation. The device has six degrees of freedom — three positioned at the shoulder, one at the elbow, and two at the wrist (allowing for pronation/supination and wrist flexion/extension). Also included is a handle which can measure the grip pressure of the patient. The user interface for the device is a computer monitor, which can be used to display games and graphics related to the movements of the robot.

The ArmeoPower also has a number of characteristics which make it useful for this study. First, the exoskeleton design provides the ability to control each joint individually. As such, the subjects’ movements can be limited to a small number of joints. The structure also allows the movements to be constrained to a horizontal plane, limiting the requirement to move against gravity and the associated muscle strength (for patients) and nonlinearities associated with it. Secondly, although when used for its primary purpose it provides mechanical assistance to the patient
in his or her movements, it is also capable of being programmed to a ‘free movement’ mode, which does not provide any assistance to the subject. Thirdly, the software also allows recording of each of the joint angles at approximately 80Hz, which is sufficient for analysis of movement. Fourthly, the ArmeoPower can be programmed to bring the subjects to any particular joint configuration, which is useful in the initialisation of the tasks. Finally, this commercially-available device has been approved by the Therapeutic Goods Administration (TGA), the authority for regulating therapeutic goods in Australia. This allowed the inclusion of patient subjects in this experimental protocol.

However, due to its exoskeleton design, the ArmeoPower also has large masses on each of its kinematic links, resulting in a non-negligible amount of inertia which is not compensated by the control strategy, therefore the device does not feel weightless. As such, this mass must be considered within the model used in this experiment.

5.1.2.5 User Interface and Trial Procedure

Graphically, this task was represented by the interface which can be seen in Figure 5.5. The horizontal position of the cursor (smiley face) was mapped to the value $y = a_1\theta_1 + a_2\theta_2$ — this meant that there was no direct representation of the individual joint angles available to the subjects. The vertical position of the cursor was used to represent the elapsed time for each task. At $t = 0$, the cursor was positioned at the green line at the top of the screen, and at $t = t_f$, the cursor as at the red line at the bottom of the screen. The vertical position of the cursor changed at a constant rate. The vertical dotted grey line represented the target position $a_1\theta_1 + a_2\theta_2 = y_{\text{targ}}$. The subjects were told that the cursor would move left or right with either elbow or shoulder movements, and that their goal was to reach and stop at the grey line when the cursor reached the bottom of the screen. They were told that they could not influence the vertical position of the cursor.

Before each trial, the robot moved the subject’s arm to the required posture, which was reflected on the screen as a large black dot. Once the subject’s arm had reached this position, a “3...2...1...GO!” countdown was presented to indicate the start of the movement to allow the subjects to prepare. Feedback to the subject was given in the form of a black trajectory which followed the path of the cursor as it dropped, and a score indicating their distance to the target line. The path persisted at the end of each trial for approximately 2 seconds. Each trial was
Figure 5.5: The user interface presented to the participants. The cursor (smiley face) represents the position given by $y = a_1\theta_1 + a_2\theta_2$ and is dropped at a constant speed. A black line shows the path of the cursor during the movement, which persists for a short period of time after the trial is complete. The subject aims to move the cursor to the dotted target line such that it is at that location when the cursor reaches the red finish line.

scored depending on how close the cursor was to the target line at $t = t_f$, and this information displayed to the subject for motivation.

This interface was designed such that it was simple to understand without requiring the presentation of the mathematical complexities of the task; and to be scalable to the different capabilities between subjects (through the scaling factors $a_1, a_2$).

5.1.3 Experimental Protocol

Four healthy subjects and three patients were selected for this study. These experiments were performed under Melbourne Health Human Research Ethics Committee Application 2013.144.

Healthy subjects were asked to complete 3 sessions — one in each dynamic condition. Initial conditions for each trial were completed in blocks of 6, however, the order in which each initial condition was presented in each block was randomised. Due to their more limited capabilities and high susceptibility to fatigue, patients were only asked to complete two sessions — one in the No Weight Condition, and
one in the Hand Weight condition (see Figure 5.6). These two conditions were chosen due to the higher variability between movements in dynamic conditions, as observed in the results of the healthy subjects. Each session last 15 blocks for the healthy subjects. Patient subjects were limited by time, according to the capabilities of the patients.

- **Healthy Subject**
  - S1: No Weight
  - S2: Elbow Weight
  - S3: Hand Weight

- **Patient Subject**
  - S1: No Weight
  - S2: Hand Weight

**Session Outline**

```
Block 1  IC 4,1,5,2,6,3
Block 2  IC 6,1,2,4,3,5
...      ...
Block N  IC 1,4,5,2,3,6
```

Figure 5.6: The protocol for healthy subjects and patients. Initial conditions (IC) in each block were randomised.

With this description of the experiment, a model of human motor control and adaptation for this particular experiment will be presented next.

### 5.2 Model for Motor Control System

The analysis presented here comprises of a model of motor control for both the healthy subjects and patients. The dynamics and control aspects are presented in that order. The dynamics of the subject-robot system of the aforementioned experiment are assumed to be invariant during the experiment, with the exception of the enforced dynamic changes under the Hand Weight and Elbow Weight conditions. The model for motor control, based on the principle of optimality, is also presented.

A number of simplifications are made. The dynamics of the sensors (including delays and noise) are not considered within this model - it is assumed that all knowledge of the movement is known instantaneously. In addition, as will be
discussed throughout this section, the model is linearised to adhere to the FH-LQR formulation. Finally, it is noted that the values for the parameters utilised within this model vary from person to person. As such, for readability and ease of reference, these values are presented only in Chapter 6.

5.2.1 Model Dynamics

Within the framework proposed in Chapter 3, there are a number of subsystems within the dynamics. Within this work, it is convenient to represent subsystems as combined subsystems. Figure 5.7 illustrates the combined subsystems of interest, including the arm, task and robot (environment) dynamics; muscle and joint dynamics; and corticospinal tract and spinal cord (brain) dynamics. Based on this description of the dynamics, a discussion is then presented on how various impairments can be described within the context of this model.

It is also noted that no noise is present in the sensor dynamics, and, that combined with a perfect Estimator and Forward Model, it is assumed that all subjects had perfect knowledge of the state. This is not an unrealistic assumption for healthy subjects.

5.2.1.1 Arm, Task and Robot (Environment) Dynamics

Due to the arrangement of the experiment, the dynamics of the arm can be modelled as a two-link planar manipulator, characterised by two revolute joints with
vertical axes of rotation. Each arm segment (upper arm, forearm) are approximated by cylinders of appropriate length, the mass of the robot is approximated by a point mass, and the hand and elbow weights (when they exist) are also approximated by point masses. This arrangement can be seen in Figure 5.8.

Parameters utilised include mass, inertia, position of the centre of mass and length of the upper arm (link 1) and the lower arm (link 2), which are given parameter names as $m_1, I_1, r_1, l_1$ and $m_2, I_2, r_2, l_2$ respectively. The mass of the robot moving with the upper arm is notated $m_{r,ua}$, and is modelled as a point mass positioned at $r_{r,ua}$ along from the shoulder. The mass of the robot at the forearm is notated $m_{r,fa}$ and is at $r_{r,fa}$ along the forearm away from the elbow. Finally, the mass of the weight at the elbow (if present) is notated $m_{we}$ and is positioned at $r_{we}$ from the elbow, and the mass of the weight at the hand (if present) is notated $m_{wh}$ and is positioned at $r_{wh}$ along the forearm away from the elbow.
For a two-link manipulator, the dynamics can be derived utilizing a standard Newton Euler Method. That is, the dynamics are represented by:

\[
M(\theta)\ddot{\theta} + C(\theta, \dot{\theta}) + G(\theta) = \tau \tag{5.2}
\]

\[
\dot{\theta} = M(\theta)^{-1}\tau - M(\theta)^{-1}C(\theta, \dot{\theta}) - M(\theta)^{-1}G(\theta) \tag{5.3}
\]

where \( \theta = [\theta_1, \theta_2]^T \) are the two angles at the shoulder and the elbow, \( \tau = [\tau_1, \tau_2]^T \) are the torques applied at the shoulder and elbow respectively, and:

\[
M(\theta) = \begin{bmatrix}
M_{11}(\theta) & M_{12}(\theta) \\
M_{21}(\theta) & M_{22}(\theta)
\end{bmatrix} \tag{5.4}
\]

\[
M_{11}(\theta) = I_1 + I_2 + m_1 r_1^2 + m_2(l_1^2 + r_2^2 + 2l_1r_2\cos(\theta_2)) \tag{5.5}
\]

\[
M_{12}(\theta) = I_2 + m_2(r_2^2 + l_1r_2\cos(\theta_2)) \tag{5.6}
\]

\[
M_{12}(\theta) = M_{12}(\theta) \tag{5.7}
\]

\[
M_{22}(\theta) = I_2 + m_2 r_2^2 \tag{5.8}
\]

\[
C(\theta, \dot{\theta}) = \begin{bmatrix}
  m_2 l_1 r_2 \sin(\theta_2) \dot{\theta}_2 & -m_2 l_1 r_2 \sin(\theta_2) (\dot{\theta}_1 + \dot{\theta}_2) \\
m_2 l_1 r_2 \sin(\theta_2) \dot{\theta}_1 & 0
\end{bmatrix} \tag{5.9}
\]

\[
G(\theta) = 0 \tag{5.10}
\]

where \( m_1, m_2 \) represent the mass of the upper and lower arms respectively, \( l_1, l_2 \) represent the lengths of the upper and lower arms, \( r_1, r_2 \) represent the distance to the centre of mass of the upper and lower arms, and \( I_1, I_2 \), represent the moments of inertia about the centre of mass of the upper and lower arms.

Due to the masses of the robotic device and the weights located at the elbow and hand, parameters in the model are modified as:

\[
\bar{r}_1 = \frac{m_1 r_1 + m_{we} r_{we} + m_{r,ua} r_{m,ua}}{m_1 + m_{we} + m_{r,ua}} \tag{5.11}
\]

\[
\bar{I}_1 = I_1 + m_1(\bar{r}_1 - r_1)^2 + m_{we}(r_{we} - \bar{r}_1)^2 + m_{r,ua}(r_{r,ua} - \bar{r}_1)^2 \tag{5.12}
\]

\[
m_1 = m_1 + m_{we} + m_{r,ua} \tag{5.13}
\]

\[
\bar{r}_2 = \frac{m_2 r_2 + m_{wh} r_{wh} + m_{r,fa} r_{r,fa}}{m_2 + m_{wh} + m_{r,fa}} \tag{5.14}
\]

\[
\bar{I}_2 = I_2 + m_2(\bar{r}_2 - r_2)^2 + m_{wh}(r_{wh} - \bar{r}_2)^2 + m_{r,fa}(r_{r,fa} - \bar{r}_2)^2 \tag{5.15}
\]

\[
m_2 = m_2 + m_{wh} + m_{r,fa} \tag{5.16}
\]
5.2.1.2 Muscle and Joint Dynamics

The dynamics of the muscles are derived here. This model of the muscle dynamics is similar to a previous work (Katayama and Kawato, 1993), but is repeated here for ease of presentation and clarity.

Each individual muscle is modelled as an elastic element and a viscous element in parallel — known as the Kelvin-Voigt Model. Specifically, the \( i \)th muscle is modelled as having muscle force \( f_i \):

\[
f_i(l_i, \dot{l}_i, v_i) = k_i(v_i)(l_{r,i}(v_i) - l_i) - b_i(v_i)\dot{l}_i
\]

(5.17)

where \( k_i(v_i), b_i(v_i) \) and \( l_{r,i}(v_i) \) represent the muscle stiffness, muscle viscosity and rest length of the muscle, which all vary with muscle activation \( v_i \). Noting that the length of the muscle can be written as a function of the joint angles \( \theta \), and similarly that the rate of change in length of the muscle can be written as a function of the rate of change in joint angle, this can also be written as:

\[
f_i(\theta, \dot{\theta}, v_i) = k_i(v_i)(l_{r,i}(v_i) - l_i(\theta)) - b_i(v_i)\dot{l}_i(\dot{\theta})
\]

(5.18)

Each muscle length is approximated as a linear function of the joint angles, based on a linearisation about some point \( \bar{\theta} \):

\[
l_i(\bar{\theta}) = \bar{l}_i + \Gamma_i \dot{\bar{\theta}}
\]

(5.19)

\[
\dot{l}_i(\dot{\bar{\theta}}) = \Gamma_i \ddot{\bar{\theta}}
\]

(5.20)

where \( \bar{\theta} = \theta - \tilde{\theta} \), \( \Gamma_i \in \mathbb{R}^{1 \times 2} \) indicates the influence of each angle \( \theta_1, \theta_2 \) on the length of muscle \( i \), and \( \bar{l}_i \) is the length of the muscle at the linearisation point \( \bar{\theta} \).

Substituting (5.20) into (5.18), produces:

\[
f_i(\theta, \dot{\theta}, v_i) = k_i(v_i)(l_{r,i}(v_i) - l_i(\theta)) - b_i(v_i)\dot{l}_i(\dot{\theta})
\]

(5.21)

\[
= k_i(v_i)(l_{r,i}(v_i) - \bar{l}_i) - \left[ \begin{array}{c} k_i(v_i) \\ b_i(v_i) \end{array} \right] \Gamma_i \left[ \begin{array}{c} \dot{\theta} \\ \ddot{\theta} \end{array} \right]
\]

(5.22)

Another linearisation approximation is made about \( \bar{v}_i \), where we have:

\[
l_{r,i}(v_i) = \bar{l}_{r,i} + \lambda_i \bar{v}_i
\]

(5.23)
where again, $\tilde{v}_i = v_i - \bar{v}_i$, and $l_{r,i}$ is value of $l_{r,i}(v_i)$ when $v_i = \bar{v}_i$. Substituting this into \(5.22\) produces:

$$f_i(\hat{\theta}, \dot{\theta}, \tilde{v}_i) = k_i(v_i)(\bar{l}_{r,i} + \lambda_i \tilde{v}_i - \bar{L}_i) - \left[ k_i(v_i) \ b_i(v_i) \right] \Gamma_i \begin{bmatrix} \hat{\theta} \\ \dot{\theta} \end{bmatrix}$$  \hspace{1cm} (5.24)$$

$$= k_i(v_i)(\bar{l}_{r,i} - \bar{L}_i) + k_i(v_i)\lambda_i \tilde{v}_i - \left[ k_i(v_i) \ b_i(v_i) \right] \Gamma_i \begin{bmatrix} \hat{\theta} \\ \dot{\theta} \end{bmatrix}$$  \hspace{1cm} (5.25)$$

Thus, a vector of all \(N\) muscle forces can be written as:

$$f(\hat{\theta}, \dot{\theta}, v) = \begin{bmatrix} k_1(v_1)(\bar{l}_{r,1} - \bar{L}_1) \\ \vdots \\ k_N(v_N)(\bar{l}_{r,N} - \bar{L}_N) \end{bmatrix} - \begin{bmatrix} k_1(v_1) \Gamma_1 \\ \vdots \\ k_N(v_N) \Gamma_N \end{bmatrix} \begin{bmatrix} \hat{\theta} \\ \dot{\theta} \end{bmatrix}$$

$$+ \begin{bmatrix} k_1(v_1)\lambda_1 & \ldots & 0 \\ 0 & \ddots & 0 \\ 0 & \ldots & k_N(v_N)\lambda_N \end{bmatrix} \tilde{v}$$  \hspace{1cm} (5.26)$$

Choosing $\tilde{v}$ such that $\bar{l}_{r,i} = \bar{L}_i$ for $i = 1, \ldots, N$, the first term is equal to zero.

Finally, the torque generated by all of these muscle forces on the two joints of interest, $\theta_1, \theta_2$ is calculated through the torques generated as these joints as:

$$\tau = \begin{bmatrix} r_{1,1}(\theta) & r_{1,2}(\theta) & \ldots & r_{1,N}(\theta) \\ r_{2,1}(\theta) & r_{2,2}(\theta) & \ldots & r_{2,N}(\theta) \end{bmatrix} f(\theta, \dot{\theta}, v)$$

$$= R(\theta)f(\hat{\theta}, \dot{\hat{\theta}}, v)$$  \hspace{1cm} (5.27)$$

$$= R(\theta)\begin{bmatrix} f_1(\hat{\theta}, \dot{\theta}, v), \ldots, f_N(\hat{\theta}, \dot{\theta}, v) \end{bmatrix}^T$$  \hspace{1cm} (5.28)$$

where $r_{i,j}(\theta)$ is the (angle-dependent) moment arm of the $j^{th}$ muscle on the $i^{th}$ joint, some of which will be equal to zero if that muscle has no influence over that joint, and $f(\hat{\theta}, \dot{\theta}, v) = \begin{bmatrix} f_1(\hat{\theta}, \dot{\theta}, v), \ldots, f_N(\hat{\theta}, \dot{\theta}, v) \end{bmatrix}^T$.

Taking \(5.28\), and substituting \(5.26\), we can write:

$$\tau = T\tilde{v} - S(\theta)\hat{\theta} - D\dot{\theta}$$  \hspace{1cm} (5.29)$$
where

\[
S(\theta, v) = R(\theta) \begin{bmatrix}
    k_1(v_1)\Gamma_1 \\
    \vdots \\
    k_N(v_N)\Gamma_N
\end{bmatrix} \in \mathbb{R}^{2 \times 2} \quad (5.30)
\]

\[
D(\theta, v) = R(\theta) \begin{bmatrix}
    b_1(v_1)\Gamma_1 \\
    \vdots \\
    b_N(v_N)\Gamma_N
\end{bmatrix} \in \mathbb{R}^{2 \times 2} \quad (5.31)
\]

\[
T(\theta, v) = R(\theta) \begin{bmatrix}
    k_1(v_1)\lambda_1 & \ldots & 0 \\
    0 & \ddots & 0 \\
    0 & \ldots & k_N(v_N)\lambda_N
\end{bmatrix} \in \mathbb{R}^{2 \times N} \quad (5.32)
\]

There are a total of 15 muscles in the human arm, which contribute to motion of the shoulder and elbow, each of which have their own muscle characteristics, which makes modelling the upper arm difficult. As such, a model based on experimental data is utilised, with the following assumptions:

- The moment arms are approximately constant \((R(\theta) = R)\)
- Each muscle has constant stiffness and viscosity \((k_i(v_i) = k_i, b_i(v_i) = b_i)\)

These assumptions also result in \(S(\theta, v), D(\theta, v)\) and \(T(\theta, v)\) also being constant.

### 5.2.1.3 Brain Dynamics

A simple representation of the brain dynamics and motor command signal is utilised here. The definition of motor command is somewhat arbitrary in this case — it represents the intention of the movement, which physiologically exists as electrical impulses within neurons of the brain, and cannot be measured or understood with current technology. However, for the purposes of modelling it is often described as a command which directly correlates to task-relevant parameters. This will be the case with healthy subjects within this model as well.

As such, here the motor command is chosen as:

\[
u = \frac{d}{dt} \{ (S^T S)^{-1} S^T T \dot{v} \} \quad (5.33)
\]

\[
u = (S^T S)^{-1} S^T T \ddot{v} \quad (5.34)
\]
which again relies on the assumption that the muscle moment arms, stiffness and viscosity are constant, and that $S^T S$ is invertible, which corresponds to the fact that the shoulder and elbow can be actuated independently — this is not unreasonable for healthy subjects.

This choice of motor command is motivated by its physiological representation. The relationship between $u$ and $\tilde{v}$ in (5.34) indicates that the input is proportional to the rate in change in $v$. Revisiting (5.29), $T \tilde{v}$ can be considered a reference position, which determines the equilibrium position of the joint. As such, $u$ can be treated as a ‘change in reference position’ command. Intuitively, it suggests that the motor command represents ‘move this joint, at this speed’ in this particular task. This can also related to the equilibrium point hypothesis (Feldman and Levin, 2009), in that a reference position for the joint angle can be constructed. Furthermore, minimising $u$ also leads to a minimisation of command torque change, which is used as the basis of the cost function described in Uno et al. (1989), Nakano et al. (1999).

### 5.2.1.4 State Space Representation

Based on the dynamics of the combined subsystems discussed in the previous sections — including the combined joint, task and robot dynamics; the muscle and joint dynamics; and the brain dynamics — the overall dynamics of the system can be represented in a state space form with linear dynamics.

Utilising a state vector $x$ and motor command input $u$ as:

$$x = [\tilde{\theta}_1, \tilde{\theta}_2, \dot{\tilde{\theta}}_1, \dot{\tilde{\theta}}_2, \tilde{\theta}_{r,1}, \tilde{\theta}_{r,2}]^T$$

$$u = [u_1, u_2]^T$$

where $\tilde{\theta}_{r,i}$ represents the current desired reference posture, which is an internal state. As such, the dynamics can be written as:

$$\dot{x} = \begin{bmatrix}
0_{2\times2} & I_{2\times2} & 0_{2\times2} \\
-M(\theta)^{-1}S & -M(\theta)^{-1}D & M(\theta)^{-1}S \\
0_{2\times2} & 0_{2\times2} & 0_{2\times2}
\end{bmatrix} x + \begin{bmatrix}
0_{2\times2} \\
0_{2\times2} \\
I_{2\times2}
\end{bmatrix} u$$

(5.37)

it is noted that for this formulation to be consistent, $\tilde{\theta} = \tilde{\theta}_r$, which can be considered the ‘at rest’ pose.
5.2.1.5 Modelling Impairments

To this point, the model has been written under the assumption that the subjects are healthy. However, the overall goal is to apply this computational model to individuals with a neurological impairment. As discussed in Chapter 3, impairments can cause changes in the dynamics at various subsystems within the model, including all components of the controller (Estimator, Forward Model and Feedback Controller), but, more commonly, in the dynamics of the brain (the Corticospinal Tract and Spinal Cord).

It is also noted that there is a large variability associated with the effect and magnitude of effect of each individual patient’s condition. This has two consequences. First, a large number of different impairments results in a number of different types of changes to the model, which adds a large deal of complexity to the model. Secondly, to fit the model to each patient requires tuning of each parameter, to account for its level of impact. As such, this can make the model prone to overfitting. This work attempts to demonstrate the impact of only a small number of parameters describing the more common impairments observed in neurologically impaired individuals, with the view that more sophisticated tests and models may be developed along the lines of this principle.

Within the experiment proposed here, it is assumed that no recovery occurs, and thus impairments are non-changing differences in the dynamics, which vary depending on impairments specific to that patient. With the proposed model, and with reference to Section 3.4, a discussion about how different impairments can be constructed is now presented. Three commonly-observed impairments are discussed and parameterised in the model — a lack of motor control over certain joints, coactivation and increased spasticity of muscles, and increased neuromotor noise.

Many impairments can be related to the inability to control muscles correctly. In very simple cases, this can be modelled via an ‘impairment matrix’ in the brain dynamics, each reflecting the impairments. These include dense hemiparesis, in which certain muscles in the arm cannot be controlled; coactivation, in which muscles are unintentionally activated together; and spasticity causing hyperactivation, in which muscles are unintentionally activated with too large a magnitude. This ‘impairment matrix’ can take the form of $E = HC$, where
\( E, H, C \in \mathbb{R}^{N \times N} \), and affect the dynamics through multiplying the brain dynamics: \( B_{imp}(u, m, s, t) = EB(u, m, s, t) \). The values of these matrices will be discussed now.

Dense hemiparesis and spasticity can be modelled by changes in the brain dynamics, with relation to the muscles which are used. For a patient who has difficulties in controlling particular muscles, the scaling matrix \( H \) of the form:

\[
H = \begin{bmatrix}
\alpha_{h1} & 0 & 0 & \ldots & 0 & 0 \\
0 & \alpha_{h2} & 0 & \ldots & 0 & 0 \\
\vdots & \ddots & \ddots & \ddots & \vdots & \vdots \\
0 & 0 & 0 & \ldots & \alpha_{h(N-1)} & 0 \\
0 & 0 & 0 & \ldots & 0 & \alpha_{hN}
\end{bmatrix}
\]

(5.38)

can be used, where \( 0 \leq \alpha_{hi} \leq 1, \forall i = 1, \ldots, N \) if a particular muscle is difficult to control, and \( 1 \leq \alpha_{hi} \) for \( i = 1, \ldots, N \) if a particular muscle suffers from hyperactivation.

Secondly, coactivation of muscles may be represented by a matrix of the form:

\[
C = \begin{bmatrix}
1 & \alpha_{c,1,2} & \alpha_{c,1,3} & \ldots & \alpha_{c,1,N-1} & \alpha_{c,1,N} \\
\alpha_{c,2,1} & 1 & \alpha_{c,2,3} & \ldots & \alpha_{c,2,N-1} & \alpha_{c,2,N} \\
\vdots & \ddots & \ddots & \ddots & \vdots & \vdots \\
\alpha_{c,N-1,1} & \alpha_{c,N-1,2} & \alpha_{c,N-1,3} & \ldots & 1 & \alpha_{c,N-1,N} \\
\alpha_{c,N,1} & \alpha_{c,N,2} & \alpha_{c,N,3} & \ldots & \alpha_{c,N,N-1} & 1
\end{bmatrix}
\]

(5.39)

where \( \alpha_{c,j,i} \) represents the coactivation of the \( j^{th} \) muscle with respect to the \( i^{th} \) muscle.

Increased neuromotor noise may be modelled through the addition of random motor noise to the brain dynamics. Neuromotor noise is thought to be proportional to magnitude of movement (Fitts, 1954), however, it can be increased with patients with neurological impairments. This can be simply modelled as:

\[
B_{noise}(u, s, t) = B + \Psi
\]

(5.40)
where $\Psi$ is:

$$\Psi = \begin{bmatrix}
\eta_1 & 0 & \ldots & 0 \\
0 & \eta_2 & \ldots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \ldots & \eta_N
\end{bmatrix} \quad (5.41)$$

where $\eta_i$ are functions which generate noise which are increased with a patient with neurological impairment, and assumed zero for healthy subjects.

It is noted that propagating this through the dynamics results in non-linear dynamics. As such, here a simplified representation is utilised, of the form:

$$v = B(u) = \begin{bmatrix}
\alpha_{h1} & \alpha_{c1} \\
\alpha_{c2} & \alpha_{h2}
\end{bmatrix} \int u dt \quad (5.42)$$

This creates a relationship between an intent to extend the elbow and the shoulder. This can generate abnormal muscle synergies (Levin, 1996), and lead to patterns such as elbow extension being coupled with shoulder extension, adduction and internal rotation. Such coactivations may also be modelled with a more complete model of the arm (with more joints). It is noted that this simplifications removes the capability of the model to model joint stiffness due to coactivation of agonist and antagonist muscles about the same joint.

5.2.1.6 Summary

This section has proposed a simplified dynamic model to represent the dynamics within a computational of human motor control. These dynamics have as input a motor command $u$, representing intended movement of the joints, and include states related to joint angle and desired joint angle. Suggestions for how this model may be adapted to represent a small number of possible neurological impairments have been made. The next section discusses the model utilised for the controller subsystems, which is based on optimal control for this simplified subsystem.

5.2.2 Model of Controller

The role of the model of the controller is to generate the input signal $u$ based on some estimate of the state $x$, which is derived using information from sensory
feedback. Although there are many different possible choices which can be used to construct this controller, within this model implementation, a simple model is utilised which relies on the Finite Horizon Linear Quadratic Regulator (FHLQR) formulation. This is due to a number of reasons, including its applicability to the model of linear dynamics, application to the finite time problem, and known closed form solution. These reasons have been discussed in more detail in Section 4.1.

This choice of formulation requires a cost function to be minimised of the form:

\[ J(x, u, t) = x^T(t_f)\Phi x(t_f) + \int_{t_0}^{t_f} x^T(\tau)Q(\tau)x(\tau) + u^T(\tau)R(\tau)u(\tau) \]  

(5.43)

where \( Q(t) \) is symmetric positive semidefinite, \( R(t) \) is symmetric positive definite, both are defined over \( t \in [t_0, t_f] \) and \( \Phi \) is symmetric positive semidefinite.

This choice of formulation also describes the form of the controller for these dynamics — the solution to the FH LQR problem is a control law of the form:

\[ u(t) = -K^*(t)x(t) \]  

(5.44)

where \( K^*(t), t \in [t_0, t_f] \) is a function which can be easily calculated utilising the Differential Riccati Equation.

5.2.2.1 Reconciliation with Proposed Framework

As discussed in in Section 3.1.2, the overall framework proposed within this work includes a forward model, an estimator and a feedback controller within the overall controller structure. However, within this implementation, this is simplified by assuming that the Forward Model and Estimator are perfect — resulting in a perfect estimation of the state. Furthermore, the dynamics of the senses are ignored — delays and sensory noise is not considered.

This assumption can be considered realistic within the healthy subject population. With respect to neurological impairments, this assumption also means that impairments related to problems with the estimator and forward model cannot be modelled if these impairments produce errors in the state estimate for this particular task. As such, patients who exhibit such impairments were not studied. However, as the task is not very complex, it is assumed that accurately estimating the state does not present a large difficulty for many patients.
5.2.2.2 Construction of Cost Function

For the FH LQR formulation, the cost function must be of the form defined in (5.43). Furthermore, as discussed in Section 3.2, the cost function is composed of two parts — the task intrinsic function $L_i(x, u, t)$, and the personal cost function $L_p(x, u, t)$.

Explicitly, given the task to be achieved, $a_1\dot{\theta}_1 + a_2\dot{\theta}_2 = 0$ at $t = t_f$, the task-intrinsic cost can be represented in the following form:

$$L_i(x, u, t) = \gamma_1 x^T(t_f) \begin{bmatrix} a_1^2 & a_1a_2 & 0 & 0 & 0 & 0 \\ a_1a_2 & a_2^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} x(t_f) \quad (5.45)$$

where $\gamma_1$ is a weighting relative to the rest of the cost function. The personal cost function involves additional components related to the different aspects of the task that a subject may consider. We present the personal cost function in the following form:

$$L_p(x, u, t) = \int_{t_0}^{t_f} \gamma_2(a_1\dot{\theta}_1 + a_2\dot{\theta}_2) + \gamma_3\dot{\theta}_1^2(\tau) + \gamma_4\dot{\theta}_2^2(\tau) + \gamma_5\ddot{\theta}_r,1(\tau) + \gamma_6\ddot{\theta}_r,2(\tau) d\tau$$

$$+ \int_{t_0}^{t_f} \gamma_7u_1^2(\tau) + \gamma_8u_2^2(\tau) d\tau \quad (5.46)$$

where $\gamma_2$ represents a preference to being near the goal, $\gamma_3, \gamma_4$ represent the preference for the subject to be moving slowly, $\gamma_5, \gamma_6$ represent the preference for the subject to be in a ‘neutral’ position, and $\gamma_7, \gamma_8$ represent a weight on the amount of effort.
This can be written in the form:

\[
\mathcal{L}^p(x, u) = \int_{t_0}^{t_f} x^T(\tau) \begin{bmatrix}
\gamma_2 a_1^2 & \gamma_2 a_1 a_2 & 0 & 0 & 0 & 0 \\
\gamma_2 a_1 a_2 & \gamma_2 a_2^2 & 0 & 0 & 0 & 0 \\
0 & 0 & \gamma_3 & 0 & 0 & 0 \\
0 & 0 & 0 & \gamma_4 & 0 & 0 \\
0 & 0 & 0 & 0 & \gamma_5 & 0 \\
0 & 0 & 0 & 0 & 0 & \gamma_6
\end{bmatrix} x(\tau) + u^T(\tau) \begin{bmatrix}
\gamma_7 \\
0 \\
\gamma_8
\end{bmatrix} u(\tau) d\tau
\]  

(5.47)

Which satisfies the requirements of the standard FH LQR cost function (5.43).

### 5.2.3 Modelling Motor Adaptation

The construction of the model adheres to the FH LQR formulation. As such, the algorithm developed in Chapter 4 can be directly applied as a model for motor adaptation, with appropriate choice of algorithmic parameters.

In particular, the excitation noise parameter must be tuned for stability, but also to be representative of the ‘exploration’ used by the subject — this may be investigated through the variability of the movements. Additionally, the choice of parameter \( \ell \) may be related to the rate of adaptation, however, there is a upper limit to this rate as required for sufficient excitation.

The model and simulations for motor adaptation were conducted as follows. First, a model for motor control was applied first in the ‘No Weight’ condition. This model established the values for the parameters — specifically for the cost function — as well as a ‘No Weight’ optimal control gain. Following this, the dynamics of the system were changed to the ‘Elbow Weight’ condition. The ‘No Weight’ optimal control gain was used as the initial control gain in the Elbow Weight condition, and the other parameters (including that of the cost function) was kept the same. Simulations were conducted where the initial condition of each trial was in the same order as that presented in the experiments. This process was repeated for the ‘Hand Weight’ condition.

The next chapter presents and evaluates the results of this experiment.
Chapter 6

Experimental Results in Motor Control and Adaptation

The purpose of this chapter is to evaluate the effectiveness of the proposed model of motor adaptation. This is achieved through a comparison between experimental results and simulation of the experimental protocol discussed in Chapter 5. Two aspects are considered. First, an analysis of the suitability of the model to represent motor control is presented, including the application to both healthy subjects and patient subjects. Secondly, based on the results of the first investigation, an analysis of the application of the model to motor adaptation is presented.

6.1 Motor Control

This work has defined a task and experiment for investigating motor adaptation. However, a necessary requirement that the model is appropriate to model motor adaptation, is that the model for motor control sufficiently models the characteristics of attempts of the task within the experiment.

There are many different modes of analysis which can be used to evaluate the applicability of the model for motor control, as such, this section first introduces these modes. Secondly, experimental and simulation results are presented for the healthy subjects, and the suitability of this model for this purpose is discussed. Finally, this is extended to neurologically impaired individuals, where further discussions regarding the applicability of this model are presented.
6.1.1 Modes of Analysis

The joint angle trajectories are utilised for the analysis. The raw data captured from the ArmeoPower is resampled at 20Hz ($T = 0.05\text{s}$), such that $N = \frac{t_f - t_0}{T}$ datapoints are utilised for the analysis. The raw data is offset such that the point $[\bar{\theta}_1, \bar{\theta}_2] = [0, 0]$ represents a position in which the shoulder is 55% externally rotated (with respect to the available range of motion in the robotic device), and the resulting elbow position is such that the task is satisfied. This position is taken to be a ‘comfortable position’ for the subject, and is also the position about which the model is linearised.

Based on this processed data, a number of methods of analysis of the data are presented. These include those which investigate the overall movement patterns and the variance in the movements. Specifically, the goal of this section on motor control is to investigate:

- The changes in movement patterns in different dynamic conditions
- The changes in movement patterns in different initial conditions
- Whether these changes movement patterns can be captured adequately by the model for motor control
- The differences in movements between healthy subjects and those with neurological injury

6.1.1.1 Movement Patterns

The first mode of analysis is a presentation of the overall joint paths. The joint paths are presented by plotting the shoulder angle $\bar{\theta}_1$ against the elbow angle $\bar{\theta}_2$. The shape of these movements under different dynamic and initial conditions illustrates the movement pattern utilised for the trials.

To simplify the analysis, the joint utilisation figures (see Figure 6.5) plot two points for the average trajectory for each initial and dynamic condition. A vector represents the difference in joint position at the final time $(\bar{\theta}(t_f) - \bar{\theta}(t_0))$. A cross represents the difference in joint position halfway through the time period $(\bar{\theta}\left(\frac{t_f - t_m}{2}\right) - \bar{\theta}(t_0))$. Initial conditions are represented with different colours, with dynamic conditions represented on different sets of axes. This figure demonstrates how the redundancy is resolved by each subject under the different initial and
dynamic conditions, as well as provides a simplified representation of the temporal aspects of the movement — through the inclusion of the initial direction (in joint space) of the movement.

These modes of analysis related to the resolution of redundancy are primarily considered for comparison between experimental and simulation results. Particularly, they are used to discuss which features of the movement the simulated model captures.

In addition to the Vectorised Joint Utilisation, the Final Error is also considered. This is defined as:

\[
Error = \left| a_1 \bar{\theta}_1(t_f) + a_2 \bar{\theta}_2(t_f) - y_{targ} \right|
\] (6.1)

where \(a_1, a_2, y_{targ}\) are as defined in Section 5.1, and \(\bar{\theta}_1(t_f), \bar{\theta}_2(t_f)\) represent the respective angles at the end of the movement. This error indicates whether the subject was successful in completing the task, which is true when \(Error = 0\).

It is again calculated as an average for each combination of initial and dynamic conditions. It is noted that this metric is scaled with respect to the magnitude of movement required (through scaling factors \(a_1, a_2\)), normalising the metric against different magnitudes of range of motion.

This Final Error metric is included for comparisons between healthy subjects and patients. Particularly, it is used to determine the success of the movements, and the related performance between the healthy subjects and patients.

### 6.1.1.2 Variance in Movement

This variance can be observed through calculation of the variance along the trajectory under each set of conditions. This is calculated as:

\[
var = \frac{1}{\left\| \bar{\theta}_{av}(t_f) - \theta_{av}(t_0) \right\|} \frac{1}{t_f - t_0} \sum_{i=1}^{N} \sum_{j=1}^{M} \left\| \bar{\theta}_j(t_i) - \bar{\theta}_{av}(t_i) \right\|^2
\] (6.2)

where \(N\) is the number of sample points, \(M\) is the number of trials for each target, \(\bar{\theta}_{av}(t)\) represents the average trajectory for each target. Additionally, this variance is normalised against the distance (in joint space) of the resulting movement. This metric is also used to provide another mode of comparison between movements of healthy subjects and movements of patients.
Chapter 6 Experimental Results in Motor Control and Adaptation

6.1.2 Healthy Subjects

This section presents the results of the experiment for the healthy subjects, and accompanying simulation results. A discussion of the experimental results is first presented in isolation, identifying the features and differences between the movements under the different conditions. The simulations are then presented, including a discussion of the selection of parameters, and a comparison is made to the experimental results.

6.1.2.1 Experimental Results

Figures 6.1 to 6.4 illustrate the paths for each of the four healthy subjects within this experiment.

Each of these healthy subjects were capable of completing the task — the average trajectories plotted here all result in endpoints on the target line. However, importantly, it is noted that the movement patterns used to complete the task vary with a number of parameters. First, different dynamic conditions result in different movement patterns. Secondly, movement patterns also vary with initial condition. Thirdly, variations in approach also occurs from subject to subject.
Chapter 6 Experimental Results in Motor Control and Adaptation

Joint Path - Subject 2

Figure 6.2: Steady State Experimental Results for Subject 2

Joint Path - Subject 3

Figure 6.3: Steady State Experimental Results for Subject 3
The change in movement pattern associated with dynamic condition can be observed from all subjects — indicated by a divergence in the paths of each dynamic condition from each initial condition. The way that this occurred differed from subject to subject. In the No Weight to Elbow Weight to Hand Weight transition, internal movements (movements starting above the target line) tended to involve more initial movement of the shoulder away from the target, thus requiring more elbow movement to complete the goal — this was obvious in subjects 1, 2 and 3, but the opposite occurred with subject 4. A similar trend can be observed for subjects 1 and 4 external movements, however, for the other subjects it is not clear.

Similarly, changes in initial condition also resulted in changes in movement patterns. For internal movements, initial conditions in which the shoulder ($\theta_1$) had a more positive angle resulted in increased use of the shoulder to complete the task for subjects 1 to 3, whereas again, subject 4 was the opposite. For external movements, subjects 1 and 4 utilised more shoulder movement when the shoulder was more internally rotated, but this was the opposite for subjects 2 and 3.

It is noted that these joint paths are not simply the shortest straight-line distance between the initial condition and the target line. This suggests that the costs
associated with moving the shoulder and elbow are not equal, and the changes in dynamics also change this distribution.

These changes can be further studied in the joint utilisation, in Figure 6.5. It is obvious from this graph that (reading within each subfigure), the change associated with each initial condition has a similar trend across subjects and initial conditions. However, the trend across changes in dynamic condition is not as similar across subject, as previously discussed.

Figure 6.5 also demonstrates the directions of the initial movements. Within each subject, the direction of initial movement relative to the final joint utilisation is relatively close to consistent across the different dynamic conditions — for example, for Subject 1, Initial Condition 1, the redundancy is resolved mostly through use of shoulder, however, the initial movement involves more elbow. This movement pattern is reflected across No Weight, Elbow Weight and Hand Weight initial conditions. Furthermore, it is interesting to note that similar relative movement patterns exist across subjects — even though the overall redundancy (at $t = t_f$) differs from subject to subject, the relative initial movement is similar.

It is obvious in the above descriptions of the results that the subjects’ movements were different from person to person, with another possible distinction being drawn with respect to the difference between external and internal movements. However, some similarities can be drawn between the resolution of redundancy between the subjects.

The variance in each of the trajectories can be seen in Table 6.1. This indicates a relatively constant variance across dynamic conditions, but a higher difference between initial conditions.

Similarly, Table 6.2 lists the Final Error for each subject. It can be seen that most subjects (with the exception of Subject 3) achieved an error of less than 0.1 across the dynamic conditions, and across initial conditions. This will be compared to patient results in the next section.

6.1.2.2 Simulations

Simulations constructed here are with respect to the steady state behaviour of the experiment of the healthy subjects. Here, the results simulations based on the model proposed in Section 5.2 are presented, with the aim of demonstrating that
Figure 6.5: Vectors of Joint Utilisation. Changes with respect to each initial condition have similar trends amongst all subjects. Changes with respect to dynamic condition vary from subject to subject.
### Table 6.1: Normalised Variance in Trajectories for Healthy Subjects

<table>
<thead>
<tr>
<th>Subject 1</th>
<th></th>
<th>Subject 2</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Init Cond</td>
<td>NW</td>
<td>EW</td>
<td>HW</td>
</tr>
<tr>
<td>1</td>
<td>0.684</td>
<td>0.942</td>
<td>0.457</td>
</tr>
<tr>
<td>2</td>
<td>0.794</td>
<td>0.491</td>
<td>0.583</td>
</tr>
<tr>
<td>3</td>
<td>1.120</td>
<td>0.662</td>
<td>0.495</td>
</tr>
<tr>
<td>4</td>
<td>0.903</td>
<td>1.309</td>
<td>0.565</td>
</tr>
<tr>
<td>5</td>
<td>0.695</td>
<td>0.913</td>
<td>0.791</td>
</tr>
<tr>
<td>6</td>
<td>0.352</td>
<td>0.666</td>
<td>0.293</td>
</tr>
<tr>
<td>Mean</td>
<td>0.758</td>
<td>0.831</td>
<td>0.531</td>
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<th>Subject 4</th>
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<td>0.533</td>
<td>0.754</td>
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<td>Mean</td>
<td>0.592</td>
<td>0.808</td>
<td>0.745</td>
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### Table 6.2: Final Error for each Healthy Subject (rad)

<table>
<thead>
<tr>
<th>Subject 1</th>
<th></th>
<th>Subject 2</th>
<th></th>
</tr>
</thead>
<tbody>
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<td>EW</td>
<td>HW</td>
</tr>
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</tr>
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</tr>
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<td>0.058</td>
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</tr>
<tr>
<td>5</td>
<td>0.134</td>
<td>0.110</td>
<td>0.145</td>
</tr>
<tr>
<td>6</td>
<td>0.095</td>
<td>0.118</td>
<td>0.141</td>
</tr>
<tr>
<td>Mean</td>
<td>0.114</td>
<td>0.110</td>
<td>0.133</td>
</tr>
</tbody>
</table>
such a computational model can be used to represent the movements of this task in healthy subjects.

6.1.2.3 Simulation Parameter Selection

Healthy subjects within this work were of similar age (26-32), and of similar weight (60-80kg). As such, a common set of dynamic parameters were utilised across all simulations. This is further justified due to the high mass of the ArmeoPower’s links, which result in the variations having a net effect of less than 10% in the total mass of the system. The parameters taken for dynamics in the presented simulations were selected as indicated in Table 6.3. These values were taken as the same as those published in Katayama and Kawato (1993). It is noted that the Muscle Stiffness and Damping Matrices were derived in Katayama and Kawato (1993) through their simplified model, which were coherent with individual experiments regarding individual joint stiffness.

<table>
<thead>
<tr>
<th>Parameter Name</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass of Upper Arm</td>
<td>$m_1$</td>
<td>1.59 kg</td>
</tr>
<tr>
<td>Mass of Forearm</td>
<td>$m_2$</td>
<td>1.44 kg</td>
</tr>
<tr>
<td>Length of Upper Arm</td>
<td>$l_1$</td>
<td>0.3 m</td>
</tr>
<tr>
<td>Length of Forearm</td>
<td>$l_2$</td>
<td>0.35 m</td>
</tr>
<tr>
<td>Distance to CoM (Upper Arm)</td>
<td>$r_1$</td>
<td>0.165 m</td>
</tr>
<tr>
<td>Distance to CoM (Forearm)</td>
<td>$r_2$</td>
<td>0.2 m</td>
</tr>
<tr>
<td>Moment of Inertia (Upper Arm)</td>
<td>$I_1$</td>
<td>0.0477 kg.m^2</td>
</tr>
<tr>
<td>Moment of Inertia (Forearm)</td>
<td>$I_2$</td>
<td>0.0588 kg.m^2</td>
</tr>
<tr>
<td>Muscle Stiffness Matrix</td>
<td>$S$</td>
<td>3.9 1.6</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.6 3.0</td>
</tr>
<tr>
<td>Muscle Damping Matrix</td>
<td>$D$</td>
<td>0.26 0.11</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.11 0.20</td>
</tr>
</tbody>
</table>

In contrast to the dynamics, the cost function utilised here is individualised, due to the contribution of the individual’s personal cost. As such, the cost function is calculated here based on the following:

- The form discussed in section 5.2.2.2 is taken (that is, a quadratic form with weights associated with the states and input, and a form which is consistent with completing the task)
- The cost is taken as the best fit for each individual on the No Weight condition only using data from all the initial conditions
As such, changes in the movements to the dynamic change are a result of the construction of the model, and not due to data-matching approach here.

Specifically, with the parameterised cost function constructed in 5.2.2.2, the Matlab function \texttt{lsqnonlin} was utilised to find the parameters $\gamma_i$, $i = 1, \ldots, 8$ which best matched the average trajectory for the No Weight condition. The original estimates given were $[10000, 6000, 1000, 2000, 10000, 10000, 0.5, 1]$ (derived using a trial and error approach) and the algorithm was left to converge.

### 6.1.2.4 Simulation Results

Figures 6.6 to 6.9 shows the joint paths both from the data and the simulations. Figure 6.10 illustrates the evolution of movement patterns across the different conditions. Similar to the experimental results, the simulation results demonstrate that movements for external initial conditions utilise more shoulder movement to complete the task. This is consistent with the experimental results. With regards to the change in dynamic conditions, the results did not necessarily follow the same pattern. In the experimental results, some subjects utilised more shoulder
Chapter 6 Experimental Results in Motor Control and Adaptation

Figure 6.7: Comparison of Simulation and Data Paths - Subject 2 (Solid - Data, Dashed - Simulation)

Figure 6.8: Comparison of Simulation and Data Paths - Subject 3 (Solid - Data, Dashed - Simulation)
as the weight was added to the elbow and then more again to the hand. This is similar to the simulation results. However, some subjects utilised less shoulder through this process, which is not reflected in the simulations. Reasons for this difference will be discussed in the following section.

With respect to the initial movements — both the Joint Path and Joint Utilisation figures indicate that the initial direction of the movement is different to the final direction of the movement (i.e. the movements are not made in the same direction in joint space). However, the Joint Utilisation figures demonstrate that the simulation movements are slower than those seen in the experiments.

Finally, as a mode of comparison, the average error in the simulation results was calculated as:

$$SimError = \frac{1}{N} \sum_{i=0,...,N} \sqrt{(\theta_{1,av}(t_i) - \theta_{1,sim}(t_i))^2 + (\theta_{2,av}(t_i) - \theta_{2,sim}(t_i))^2}$$  (6.3)

Where $t_i = 0.05i$, $N = 20$ representing sample points along the average and simulated trajectories respectively. This provides a measure of how well the simulations match the movements.
Joint Utilisation - All Subjects, Simulation Results

Figure 6.10: Vectors of Joint Utilisation for Simulations of Healthy Subjects
The results of the average error for the simulations can be observed in Table 6.4. It can be seen that, despite being ‘trained’ only on the No Weight condition data, the average error across all condition is similar. Furthermore, it is noted that the movements here required a total of approximately 1 rad of movement (as a sum of $\theta_1$ and $\theta_2$). As such, a model predicting no movement would read an average error of approximately 0.5 rad. The simulation errors therefore represent less than 10% of this upper bound.

### 6.1.2.5 Limitations in the Model

It is noted that with only four healthy subjects, any conclusions drawn from this experiment are far from conclusive. However, in addition to this, two main limitations of this model are discussed here. First, the difference in behaviour for internal and external movements is not considered. Secondly, the model is incapable of modelling changes in approach between dynamic conditions.

The difference in internal and external movements is likely due to muscle dynamics which are simplified in this model. In particular, muscles contributing to internal movements (particularly the pectoral muscles in the shoulder) are stronger than those contributing to external movements. This asymmetry is removed in the
model due to the linearisations, which leads to a simpler optimisation process at the cost of fidelity in the model. This may be changed through attempting to make the model less general through separate models for internal and external movements.

Finally, changes in approach by the subjects cannot be considered by this model. Although attempts were made to keep the subjects relaxed between dynamic conditions, it is noted that with the addition of the weights made the task more difficult. As such, some subjects were observed to have changed their focus when this was the case, to achieve a better score. As such, their movement patterns may have been affected. This may be handled within the model by changing the cost function, however, this was not attempted within this work.

6.1.3 Patient Subjects

The movement capability of individuals with neurological impairment are significantly less understood. Within the proposed model, therefore, these patients are presented as modifications to the model presented for healthy subjects. The modifications presented add further complexities and model inaccuracies, therefore, a detailed discussion is presented regarding the suitability and applicability of this and other models of patient movement.

The protocol involving the patient subjects differed from the healthy subjects in three ways. First, only two dynamic conditions were used — the No Weight and Hand Weight conditions. These two conditions were chosen due to the larger differences in movement patterns between these two dynamic conditions, as identified in the results of the healthy subjects. Secondly, the duration of the movement was lengthened from 1 second to 1.4 seconds (that is, \( t_f = 1.4 \)). This allowed the patients more time to complete the task, again due to their reduced movement capabilities. Finally, the time spent in each dynamic condition was limited by time duration, rather than number of trials. This was due to consciousness of each patient’s fatigue levels.

Three patients participated in the experiment. Characteristics of the patient are identified in Table 6.5. Patients were clinically assessed by a trained occupational therapist using a modified version of the Wolf Motor Function Test (WMFT). The tasks included were tasks 3 (Hand to table — Front), 4 (Hand to box — Front), 10 (Reach-retrieve object) 11 (Lift can to mouth), 12 (Lift pencil) and 19 (Fold
Table 6.5: Characteristics of the Patient Subjects

<table>
<thead>
<tr>
<th>Patient</th>
<th>Gender</th>
<th>Modified WMFT*</th>
<th>Characteristics of Movement</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>F</td>
<td>18/30</td>
<td>Gross motor movement possible. Fine motor control lacking. Overactive Shoulder.</td>
</tr>
<tr>
<td>2</td>
<td>M</td>
<td>13/30</td>
<td>Limited shoulder movement. Lack of fine motor control.</td>
</tr>
<tr>
<td>3</td>
<td>F</td>
<td>11/30</td>
<td>Extremely limited movement. Flacid arm with only some shoulder activation.</td>
</tr>
</tbody>
</table>

Table 6.6: Number of Trials for Each Patient for each Initial Condition

<table>
<thead>
<tr>
<th>Init Cond</th>
<th>Patient 1 NW</th>
<th>Patient 1 HW</th>
<th>Patient 2 NW</th>
<th>Patient 2 HW</th>
<th>Patient 3 NW</th>
<th>Patient 3 HW</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6</td>
<td>6</td>
<td>3</td>
<td>3</td>
<td>6</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
<td>6</td>
<td>3</td>
<td>3</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
<td>6</td>
<td>2</td>
<td>3</td>
<td>3</td>
<td>7</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>5</td>
<td>4</td>
<td>3</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>5</td>
<td>6</td>
<td>4</td>
<td>3</td>
<td>3</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>6</td>
<td>4</td>
<td>6</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>6</td>
</tr>
</tbody>
</table>

towel). The patients were scored according to the standard WMFT criteria (Wolf et al., 1989), thus a total score of 30 was achievable. All patients were chronic stroke patients (more than 6 months since their incident), did not have perceptual problems, and were capable of understanding the instructions of the task and experiment.

In addition to the time limitations, some movements resulted in collisions with the joint limits of the robotic device — these trials were excluded, as the defining characteristic of the movements were not the result of voluntary patient movement. As a result limited trials under each dynamic condition and each initial condition were available for each patient, this is summarised in Table 6.6.

Due to the limited number of trials, and observed higher variability between trials, these results are presented in a different way to the results of the healthy subjects. Specifically, joint paths are presented for each recorded trial. However, the same modes of analysis were as those used for the healthy subjects.
6.1.3.1 Experimental Results

The results of the experimental protocol with the patients are now presented. For the analysis related to movement patterns, the shoulder-elbow angle paths of all trials can be seen in Figures 6.11 - 6.13.

From these figures, a number of general observations can be made with reference to the movements of the healthy subjects. First, unlike the trials for the healthy subjects, not all movements are successful — suggesting that under some conditions, the task was outside the capabilities of the patients. Secondly, the differences between internal and external movements were much more pronounced — internal movements were of higher speed and more uncontrolled, whereas external movements were difficult to perform. Thirdly, a much higher variance in movements was observed, suggesting that the patients were unable to produce movements with regularity. Furthermore, due to the variability between the movements of each patient, a number of observations can be made with respect to the performance of each patient specifically.

Patient 1 was capable of producing a large range of movements. In Figure 6.11, there is a large variation in the directions of the movements, even those from
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Target HW
Target NW
Hand Weight
No Weight

\[ \bar{\theta}_2 \text{ (rad)} \]
\[ \bar{\theta}_1 \text{ (rad)} \]

Joint Path - Patient 2

Figure 6.12: Experimental Paths for Patient 2

Note: Two different target lines were used due to slight changes in the range of motion during the second session

the same initial condition. This suggests that although the patient had sufficient strength and control authority to successfully complete the movement, they were unable to sufficiently and consistently control their movements. It can also be observed that there was a large difference between movements from initial conditions requiring internal movement, compared with those which required external movement. Internal movements (those originating from above and to the right of the target line) utilised a lot of shoulder movement to complete the task. On the other hand, external movements were completed with less shoulder and more elbow. Furthermore, the changes in movement patterns due to changes in dynamic conditions is different. When in the hand weight condition, the bias was even larger — even more shoulder was used for internal movements, and more elbow was used for external movements.

Patient 2 produced more erratic movements. It can be seen that in the No Weight condition, the patient utilised almost exclusively shoulder movement to complete the task. In contrast, under the Hand Weight condition, Patient 2 utilised mostly elbow for the internal movements, and shoulder for the external movements. It is also noted for the external movements, the elbow moved in a manner which was counter-productive to the end goal — i.e. the elbow moved in an internal direction.
when external movement was required. It is possible (and perhaps likely) that the patient changed his/her approach in between conditions. In fact, observations during the experiments themselves indicated that the patient appeared more motivated during the Hand Weight condition trials. However, it is noted that the movement patterns utilised during these trials are still significantly different to those observed with the healthy subjects.

It is also noted here that some movements for Patient 2 involve shoulder movement or elbow movement only (indicated by straight horizontal or vertical paths). This is likely to be due to the influence of the robotic device, in which friction in one joint was not overcome by the patient’s actuation.

Patient 3 demonstrated very little movement capabilities — particularly in the elbow. In almost all cases, the movements involved only the shoulder, with a limited amount of elbow movement in the No Weight Condition. This suggests again that the patient did not have sufficient control over their elbow to produce enough torque to overcome the friction in the elbow joint of the robotic device.

As such, it can be seen that each patient behaved differently in these experimental conditions, and that the joint paths themselves provide some indication and
reflection of the patient’s capabilities. However, it is noted that, due to the variability, much more data is required to establish the trends in movement. However, within this work, this was not possible, due to the more limited capabilities of the patients.

Nevertheless, the average joint utilisations are presented in Figure 6.14. As expected, it is noted that there is a much less clear pattern than was established with the healthy subjects. Patient 1 shows significantly different movement patterns for the external initial conditions, however, the internal initial conditions demonstrate similar patterns to those observed in the healthy subjects. Patient 2 moved much more with the Hand Weight, whereas Patient 3 demonstrated limited movement under the No Weight condition, and even less with the Hand Weight.

The normalised variance of the patient movements is quantified in Table 6.7. Compared to the normalised variance of the healthy subjects, it is noted that Patients 1 and 2 have much higher variances under the No Weight condition, Patient 2 has a much lower variance in the HW condition, and all others are comparable to those observed with the healthy subjects. It is noted, however, that Patient 2 had very limited trials (which reduces the reliability of the variance as a measure), and Patient 3 had very little movement, which also limits its effectiveness.

The final error for the patients can be seen in Table 6.8. It can be seen that these values are higher than that for the healthy subjects. Additionally, Patients 1 and 2 both have approximately equal errors across conditions, however, Patient 3 has a significantly larger error in the hand weight condition compared with the no weight.

Importantly, the vast differences in the patients’ capabilities highlight the difference in capability between this small subsample of neurologically impaired patients, and emphasises the need to consider each patient independently in any
Figure 6.14: Vectors of Joint Utilisation for Patient Subjects. Changes with respect to each initial condition have similar trends amongst all subjects. Changes with respect to dynamic condition vary from subject to subject.
Table 6.8: Final Error for Each Patient (rad)

<table>
<thead>
<tr>
<th>Init Cond</th>
<th>Patient 1</th>
<th>Patient 2</th>
<th>Patient 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>NW</td>
<td>0.100</td>
<td>0.081</td>
<td>0.198</td>
</tr>
<tr>
<td>HW</td>
<td>0.047</td>
<td>0.113</td>
<td>0.458</td>
</tr>
<tr>
<td>NW</td>
<td>0.153</td>
<td>0.165</td>
<td>0.255</td>
</tr>
<tr>
<td>HW</td>
<td>0.223</td>
<td>0.151</td>
<td>0.517</td>
</tr>
<tr>
<td>NW</td>
<td>0.168</td>
<td>0.237</td>
<td>0.254</td>
</tr>
<tr>
<td>HW</td>
<td>0.349</td>
<td>0.175</td>
<td>0.579</td>
</tr>
<tr>
<td>NW</td>
<td>0.097</td>
<td>0.382</td>
<td>0.013</td>
</tr>
<tr>
<td>HW</td>
<td>0.165</td>
<td>0.086</td>
<td>0.549</td>
</tr>
<tr>
<td>NW</td>
<td>0.146</td>
<td>0.228</td>
<td>0.093</td>
</tr>
<tr>
<td>HW</td>
<td>0.150</td>
<td>0.071</td>
<td>0.451</td>
</tr>
<tr>
<td>NW</td>
<td>0.132</td>
<td>0.106</td>
<td>0.110</td>
</tr>
<tr>
<td>HW</td>
<td>0.057</td>
<td>0.155</td>
<td>0.306</td>
</tr>
<tr>
<td>Mean</td>
<td>0.133</td>
<td>0.165</td>
<td>0.200</td>
</tr>
<tr>
<td></td>
<td>0.125</td>
<td>0.154</td>
<td>0.477</td>
</tr>
</tbody>
</table>

efforts to model them.

6.1.3.2 Model Parameter Identification

The experimental results relating to the patients in the previous section indicate that there are a number of potential challenges when attempting to model the movements of the patient subjects within this experiment. In this section, the existing model is modified to represent the individual impairments of the patients, however, it is accepted that certain aspects of the movements will be not be captured with this simplified model.

The model constructed for each patient varied according to their capabilities. Although many of these movement characteristics may be inferred from the data, the aim was to represent and understand how the evaluated impairments propagate themselves through the simulation model. Simulations presented here thus aim to illustrate that a parameterised representation of the impairments can be used to represent some of the movements.

As such, the simulation model is constructed differently to the approach taken for the healthy subjects. First, due to the obvious non-symmetric movements of the patients, internal and external movements are modelled separately. Secondly, no ‘best fit’ is sought. Instead, impairments are represented through a parameterisation, with values of these parameters hand-tuned to produce a coarse approximation. A discussion is then presented based on how the different impairments and levels of impairment change the simulated movements.

For simplicity, only limited parameters related to the impairments of the patient subjects were modelled. These are:
- Weakness in moving the shoulder and elbow joints due to corticospinal tract damage ($\alpha_{ws}, \alpha_{we}$) modelled in the brain dynamics

- Co-contraction — agonist and antagonist muscles of the elbow are consistently activated due to spasticity ($\alpha_c$)

These are common primary impairments associated with neurological injury. Such impairments are associated with the brain dynamics, as noted in Section 5.2.1.5. From this section, an impairment matrix of the following form can be used to model the weakness and coactivation impairments:

$$E = \begin{bmatrix} \alpha_{ws} & 0 \\ 0 & \alpha_{we} \end{bmatrix} \quad (6.4)$$

It is noted that the shoulder weakness can also manifest itself as a lack of control — a healthy subject will nominally have $\alpha_{ws} = \alpha_{es} = 1$. For a patient, a value less than one indicates weakness in that respective joint, whereas a value of greater than one indicates an overexcitation of that muscle.

Secondly, although the cause of co-contraction is within the brain dynamics, its effects cannot be modelled within the $E$ matrix of the simplified model here. Instead, co-contraction has the effect of increasing the muscle stiffness matrix $S(\theta, v)$, which is constant as a result of the linearisation in this work (i.e. $S(\theta, v) = S$). As such, the dynamics here are modified to include co-contraction through modification of this stiffness matrix:

$$S = S_{healthy} + \begin{bmatrix} 0 & 0 \\ 0 & \alpha_c \end{bmatrix} \quad (6.5)$$

With this parametrisation, a ‘healthy’ person would have a $\alpha_c$ value of 0. This modification is an indication that the ‘average’ muscle activation over the duration of the movement is higher, and as a result, the stiffness around a particular joint is higher. Within the patients who participated within this experiment, such co-contraction was observed only around the elbow joint, and thus the parameterisation reflects this.

The remainder of the parameters were taken to be the same as those utilised for the healthy subjects, and a representative cost function of the same form as that used for the healthy subjects, with representative parameters of: $\gamma_i, i = 1, ..., 8$ set to values of [10000, 6000, 10000, 10000, 10000, 10000, 0.5, 1]. These were similar to those used for the healthy subjects, however, with a higher preference
Table 6.9: Impairment Parameters for Patient 1

<table>
<thead>
<tr>
<th>Parameter</th>
<th>External Movement</th>
<th>Internal Movements</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_{ws}$</td>
<td>0.5</td>
<td>2</td>
</tr>
<tr>
<td>$\alpha_{we}$</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>$\alpha_c$</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

on moving the arm to a ‘comfortable posture’ (with both joints in the middle of their respective range of motions).

In the evaluation of these simulations, a simple visual comparison is made with the average path figures.

**Patient 1**  As has been discussed, Patient 1 was able to move both joints of interest. A weakness was evident in her shoulder extensors, making external movements difficult to achieve. On the contrary, overactive shoulder contractors were observed. Weakness was observed in both elbow extensors and flexors, and only minor coactivation was present. Therefore, the parameters within our simulations are set as in Table 6.9.

The results of this simulation can be seen in Figure 6.15. As can be observed, the simulations for the external movements produced paths which have some characteristics of the experimental movements — that is, an initial elbow extension, followed by a shoulder extension. Furthermore, under the Hand Weight conditions, more elbow and less shoulder movement was observed. However, these simulation results are not similar to those observed for the internal movements. The experimental results suggest a large change in the approach taken — from almost exclusively elbow movement to almost exclusively shoulder movement. Within the context of the proposed model, this is likely to be a result of a complete change in the personal cost function, which is not modelled here.

**Patient 2**  For external movements, Patient 2 had limited elbow extension, however, was capable of external shoulder movement. For internal movements, Patient 2 had limited control over his internal shoulder movements, and a preference for using his elbow. As such, the parameters chosen for Patient 2 were as shown in Table 6.10.

These produced simulations which can be observed in Figure 6.16. Similar to the results of Patient 1, it was noted that these simulations do not well represent the
Chapter 6 Experimental Results in Motor Control and Adaptation

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**Figure 6.15:** Simulations and average paths of Patient 1.

**Table 6.10:** Impairment Parameters for Patient 2

<table>
<thead>
<tr>
<th>Parameter</th>
<th>External Movement</th>
<th>Internal Movements</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha_{ws} )</td>
<td>0.1</td>
<td>0.01</td>
</tr>
<tr>
<td>( \alpha_{we} )</td>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>( \alpha_c )</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

movements of the patient. Grossly, the external movements involved more of the shoulder, and the internal movements utilised more elbow. However, in general these results suggest an extremely poor fit, likely due to unmodelled dynamics which will be discussed in more detail in Section 6.1.3.3.

It was also noted that Patient 2 also had an extremely large difference in movements between movements in the different dynamic conditions. In particular, during the Hand Weight condition, it appears that Patient 2 attempted to move the same final posture, regardless of initial condition. This is a behaviour which is extremely atypical — it was not observed in any of the other healthy or patient subjects. This, however, may be again represented with a change to the personal cost function, placing a large importance on the final posture. It is noted that Patient 2 was a former competitive sportsperson, which may have led to this analysis and approach to the task in this final condition.
Patient 3  Patient 3 had extremely limited movement, with extremely limited capabilities in terms of movements. Within the robotic device, movement suggested overactive pectoralis muscles (suggesting ease of internal movements of the shoulder), but very little other movement. As such, the impairment parameters for Patient 3 were set as in Table 6.11

Figure 6.17 displays the average paths for Patient 3, and the result of the simulation above. These simulations match the experimental movements quite well. However, given the extremely limited movement in external movements, and shoulder-only movements in internal movements, this does not necessarily produce great confidence in the simulations presented here.
6.1.3.3 Limitations and Applications with Respect to Patient Subjects

Whilst the simulations constructed to model the movements of the healthy subjects were capable of representing some features of the experimental data, the patient simulations were less capable. There are a number of reasons for this. First, the natural variability in the patients’ movements make it difficult to establish a trend without large amounts of data. Secondly, the unmodelled components of the dynamics appear to have significantly more impact when applied to patient movements, compared with healthy subjects.

As discussed previously, the patient subjects within this experiment had a much higher variability in their movements. This can be seen through calculation of the variance, as well as simple visual inspection of the movement paths, and (to some extent) the average paths, which are not smooth. To fit a model of movement for these patients, this variability requires more data to estimate and/or understand the underlying trends in the movements. This correlates to many attempts at the same task, which again poses a problem — patients often become fatigued faster than healthy subjects.
Secondly, the results suggest that there are characteristics of the patients’ motor systems which were not included in the model used within this work, which were less prominent in the movements of the healthy subjects. This includes the assumption that the patients’ were perfect, nonlinearities in the dynamics, and potential changes in the personal cost function. A major assumption made with the construction of this model is that the patients were perfectly capable of estimating their state. As discussed in Chapter 3, an impaired estimator is a common result of neurological injury, and this can cause erratic movement patterns resulting from the feedback controller utilising incorrect state information. With respect to the nonlinearities, a large number of the impairments associated with neurological injury introduce inherently nonlinear dynamics. For example, muscle co-contraction can be considered an input mapping problem — a single input results in activations of muscles which cause movement in two opposing directions (co-contraction). This cannot be modelled within the given linear structure. Similar problems occur if, for example, flexor muscles are overactive, whereas extensor muscles have weakness — this assymmetric actuation property results again in nonlinear dynamics which cannot be well-modelled within the proposed model and problem formulation. Whilst these assymmetries and irregularities do exist within healthy subjects, they did not pose an issue in the completion of this task. As such, the linearised dynamics were adequate. This was not the case for the patient subjects. Furthermore, these nonlinearities may also explain the approach that the patients took when attempting to complete the task — in this case, the differences in initial and dynamic conditions caused a different approach to be taken, either a change in personal cost function, or due to the unmodelled differences in the dynamics themselves. Each of these unmodelled components have not been included within this particular model, however, can be included in other models which utilise the framework proposed in Chapter 3.

Despite these problems with the proposed model, explorations into simplified models deserve merit — more complicated models require more data to fit, and produce results which are potentially less useful in real-world applications. Furthermore, it is also noted that the trends in the parameters chosen were roughly consistent with the Modified WMFT scores — that is, the patients who scored higher on the WMFT had parameters which were closer to that of a healthy person. Although the fitting is extremely rough, and only three patient subjects were included as part of this study, this suggests that there may be some merit in producing simple parameterised models, and utilising the parameters as a method for assessing patient capabilities. However, it is acknowledged that this requires significant further
investigation.

6.1.4 Summary

Although a number of conclusions can be drawn from these experimental and simulated results, one key feature is highlighted here. That is, under different dynamic and initial conditions, the movement patterns differ. This is observed in both the patient and the healthy subjects, and is inconsistent with some other models of motor adaptation, which require the movements to return to a reference trajectory. Additionally, the results of the simulations also reflect this — the model of motor control based on the optimal control formulation also predicts different movement patterns across the varying conditions. As such, the motor adaptation observed during the experiment is discussed next, and the model, which updates the control policy towards optimal via the optimal control formulation, is evaluated.

6.2 Motor Adaptation

As has been proposed in previous chapters, the process of motor adaptation revolves around the idea that movements change when presented with new environments or conditions. Within this case, adaptation has been induced in human subjects through the changing of the dynamics of the task. Specifically, in the arm movement task utilised within this work, a weight is added to two points of the arm independently. As explored in Section 6.1, this change in dynamics results in a change in the movement patterns of the patients.

As such, this section discusses the experimental and simulation results regarding motor adaptation. Due to the extremely large variability, and the poor performance of the model for motor control on the patient subjects, this was only performed for the healthy subjects.

This section first introduces the metrics used to analyse the motor adaptation. This is followed by a presentation of the experimental results, the construction and parameters used for the simulations, and then the simulation results themselves. Finally, a discussion is presented highlighting the features of this simulation.
6.2.1 Modes of Analysis for Motor Adaptation

Motor adaptation is analysed in multiple ways. First, an overview of the adaptation process is conveyed through the plotting of path trajectories of each of the first 5 trials. This provides a qualitative summary of the adaptation process. These paths (shoulder angle, $\bar{\theta}_1$ against elbow angle $\bar{\theta}_2$) demonstrate how the movement patterns evolve between each subsequent trial.

Two metrics are also used to quantitatively evaluate the adaptation process. First, the average difference for each individual trial from the steady state movement — termed the individual trial error (ITE) — is used. Secondly, a measure of the variance in the movements is used — this is presented as an indication of when a subject is exploring (i.e. changing their movement patterns significantly between trials), and when they have settled on a strategy.

Individual-trial error (ITE) is calculated as the difference from the steady state trajectory for each trial. This is considered an error, due to its difference from the nominally-optimal trajectory. The ITE in the $j^{th}$ trial at the $i^{th}$ initial condition is calculated as:

$$ITE_{i,j} = \frac{1}{N} \sum_{k=1}^{N} \sqrt{(\theta_{i,j}(t_k) - \theta_{i,ss}(t_k))^T(\theta_{i,j}(t_k) - \theta_{i,ss}(t_k))}$$ (6.6)

where $\theta_{1,i,j}(t_k) = [\theta_{1,i,j}(t_k), \theta_{2,i,j}(t_k)]^T$ are the shoulder and elbow angles at $t = t_k$ of the $j^{th}$ attempt at the $i^{th}$ initial condition respectively, and the respective ss values are the steady state equivalents (that is, the average trajectories over trial 6 onwards). Therefore, this represents the sum of the average error in both angles throughout the trajectory for each individual trial.

The second measure is the trend in variability. The variability over a block of $B$ trials starting at the $b^{th}$ is calculated as:

$$Var_{b+B} = \sum_{i=1}^{6} Var_{i,b+B}$$ (6.7)

where

$$Var_{i,b+B} = \frac{1}{B-1} \sum_{k=0}^{N} \sum_{j=b}^{b+(B-1)} (\theta_{i,j}(t_k) - \theta_{i,av(b+B)}(t_k))^T(\theta_{i,j}(t_k) - \theta_{i,av(b+B)}(t_k))$$ (6.8)
where $\theta_{i,\text{av}(b+B)}(t_k)$ is the value of the average trajectory from the $i^{th}$ initial condition at $t_k$ of the $B$ trials from $b$ to $b+B-1$. This provides a measure of how varied the trajectories are between that set of trials. It is noted that this variation is related to the exploration strategies used by the subjects during the adaptation process — an increased variance indicates the presence of natural variation or noise between trials, but also potentially that the subject has tried different strategies during this block of trials.

These methods are used to evaluate the adaptation in this experiment, and provide a mode of comparison to the simulated adaptation process.

### 6.2.2 Experimental Results

The experimental results suggest that very little adaptation occurs when presented with the new dynamics in this experiment. Figures 6.20 and 6.21 illustrate the change in movement paths during the adaptation process of each of the subjects in each of the dynamic conditions. As is evident, there does not appear to be a clear evolution, with the differences in each movement not suggesting a structured attempt at each changing attempt.

The lack of change in adaptation can also be seen in the progress of individual trial errors. Figure 6.18 demonstrates the trend of error progression across the individual trials to each target, as well as an average amongst each successive trial over each target. In some subjects and conditions, for example Subject 1 NW, Subject 1 HW, and Subject 2 NW, a downwards trend in this error is observed, suggesting that the approach taken to complete the task changed over the iterations. However, for most subjects, no clear trends are visible in this data, with most average errors staying relatively constant.

Finally, Figure 6.19 illustrates the trend in the variance over the trials. It is noted that again no clear trend is observed. This suggests that either is no separate ‘exploration’ and ‘steady state’ stage within the observed data, suggesting that the subjects are either still changing during the entire trial, or that the variability can be associated primarily with the natural trial-to-trial variability. Such results, in turn, may also suggest that the adaptation occurs as a result of exploration due to this noise. However, it is not necessarily clear within this data. This suggests that the task and the differences in dynamic conditions may have been too simple to investigate motor adaptation.
Figure 6.18: Trend in Errors (colours represent initial condition)
Figure 6.19: Trend in Variance by Blocks with $B = 4$
Figure 6.20: Adaptation of Subjects 1 and 2. Dark Blue represents average paths at steady state.
Figure 6.21: Adaptation of Subjects 3 and 4. Dark Blue represents average paths at steady state.
6.2.3 Simulation Construction

Simulations for this work were constructed as follows. The algorithm proposed in Chapter 4 was utilised with the dynamics constructed in Chapter 5. The cost function parameters were the same as those used to initialise the search for the data matching in Section 6.1, and thus was representative of the cost functions identified in of the healthy subjects. Specifically, the parameter values of $[\gamma_1, ..., \gamma_8]$ were set to $[10000, 6000, 1000, 1000, 2000, 10000, 10000, 0.5, 1]$.

6.2.4 Algorithm Parameter Selection

In addition to the parameters selected to represent the dynamics and cost function, a number of additional parameters directly related to the algorithm are selected and discussed here.

The sample time $T$ was chosen as $0.2s$. The impact of this selection was not significant, with the exception of ensuring that the simulations converged.

The number of trials per update $\ell$ was set to 80. This parameter is related to the adaptation rate, with smaller $\ell$ suggesting that less time is required to adapt, and a larger $\ell$ simulating a slower adaptation rate. However, due to the properties of the learning algorithm (the number of unknowns to be solved for using the least squares method) a lower bound exists on this parameter, which does impact its application to motor adaptation.

The initial gain $K_0$ for the No Weight was set to something close to optimal given the original dynamics — this represents the fact that the initial task was relatively well understood. The initial gain for the Elbow Weight and Hand Weight conditions were set to the final gains for the previous dynamics (i.e. the first trial of the Elbow Weight condition was set to the gain for the last trial of the No Weight condition).

The exploration noise level was set to 0.5, to provide sufficient excitation to identify the parameters. It is noted that this is a key parameter with respect to the metrics identified.
6.2.5 Simulation Results

The paths for the simulations can be observed in Figure 6.24. Note that for clarity, one every 10 trials only is plotted. It can be noted that similar to the experimental results, there is a lack of clear trend in the movements — any underlying trends are obscured by the exploration noise. It is however, noted that the contributing noise in experimental results are likely to be due to both exploration and control signal noise.

The trend in ITE can be observed in 6.22. It can be seen that the initial movements are further away from the steady state movements, however, this error decreases after a few trials. It is also noted that this change is not uniform across all initial conditions, with some initial conditions presenting a lower initial error.

Figure 6.23 illustrates the trends in variability. In this case, there is a higher variability in the first block of trials in all dynamic conditions, which drops in the later trials. This suggests that larger changes are made in the first trials, and that, in these simulations, the exploration noise does not dominate the variability.

6.2.6 Discussion

The results here demonstrate that the proposed model is capable of representing some characteristics of the motor adaptation process in humans. Specifically, it is
Trends in Variability - Simulation

\[ \text{Variance (rad)} \]

\[ \begin{array}{c}
\text{HWEWNW} \\
0, 5, 100, 5, 100, 5, 10
\end{array} \]

\[ \begin{array}{c}
0.05, 0.1, 0.15, 0.2, 0.25, 0.3, 0.35
\end{array} \]

\[ \begin{array}{c}
0, 0.05, 0.1, 0.15, 0.2, 0.25
\end{array} \]

\[ \begin{array}{c}
0, 0.05, 0.1, 0.15, 0.2, 0.25
\end{array} \]

Figure 6.23: Progression of Error - Simulation
Figure 6.24: Joint Paths for Simulation. Dark Blue represents average paths at steady state.
demonstrated that the model indicates that movements under different dynamic conditions reach a steady state trajectories (in the iteration domain); that this steady state trajectory can be defined by the optimal control formulation; and that this optimal control function can be found using information from previous trials, and does not require knowledge of the dynamics. It is, however, acknowledged that given the low signal to noise ratio with respect to the characteristics of motor adaptation in this experiment, and the fact that there were only four healthy subjects included within this experiment, strong conclusions cannot be drawn.

It is clear that the algorithm suggests that the movement converges towards the optimal solution over iterations — that is, when the first movement in a new dynamic condition is not optimal, subsequent movements are more optimal in that new environment. Although the experimental data did suggest a change in movement paths between dynamic conditions, the initial attempt in each new dynamic condition was not necessarily further away from the steady state trajectory of that dynamic condition than the final attempts. This is potentially due to the fact that the change in dynamic condition was not hidden from the subjects — they were aware of the weight being placed on their arm (although they did not have to carry this weight against gravity). As such, it is possible that the subjects changed their approach (i.e. gain $K_i$) instinctively, rather than using the gain from the previous dynamic condition, as simulated in the algorithm.

It is noted that the variance in movement is much more consistent across the experimental results, compared with the simulation results, in which the variability drops after a number of trials. The lack of change in variability suggests either that the exploration had not completed by the end of the trial, or that the variability measured here is dominated by non-exploratory noise (that is, noise which is not deliberately used to explore more optimal solutions). This non-exploratory noise is not modelled in the simulations. This may be reflected in the variability measured in the simulations, which are of a magnitude which would not be observable if a similar level of non-exploratory noise were added to it. However, it is noted that both the Individual Trial Error, and the Variability are greatly influenced by both the exploratory noise, and non-exploratory noise. Although not performed here, arbitrary tuning of this noise level in simulation may be used to provide a better fit for the data.

Additionally, there are a number of properties of the algorithm which affect its ability to represent the adaptation process in this task. Firstly, the rate of convergence (and particularly how the controller is updated) is not easily tunable in
the algorithm. Secondly, the dynamics are assumed to be time varying.

With regards to modelling different rates of adaptation — the only method of adjusting the adaptation rate within the algorithm is to change the number of iterations before an update, $\ell$. This artificially changes the update (and thus adaptation) rate of the algorithm, and also has the effect of increasing the condition number of the matrix (thus improving the accuracy of the update). However, each successive update of the control gain is governed by Klienman’s original algorithm. Generally, this approach is not necessarily a property of human motor adaptation — it is more natural to assume that small, natural updates to the control law are made at each attempt, rather than block of trials with identical control laws followed by a single update. This potentially leads to a much higher rate of convergence than what is possible with the algorithm.

This characteristic is also related to the second property of interest — the assumption that the dynamics are time varying within a trial. In the modelled example the dynamics of the task do not vary with time. However, the algorithm assumes that the dynamics may be time varying, which leads to the requirement of using many trials to ‘understand’ the dynamics. It is noted that as the dynamics do not change, adaptation may occur on a faster time scale — something which is not captured by this particular model.

### 6.3 Conclusion

The proposed model for motor adaptation is capable of representing some of the characteristics of adaptation in this task. It has been established that the proposed model for motor control was capable of representing the changes in movement patterns between initial and dynamic conditions, through its optimal feedback control formulation — thus it was capable of application to a family of finite time tasks, and to resolve redundancy both in the trajectory and end posture spaces. Furthermore, the model for motor adaptation was capable of representing a converge to the steady state trajectories, established by this optimal control formulation, and achieved without requiring explicit knowledge of the dynamics.

However, evaluation this model of motor adaptation was difficult to evaluate against experimental results, due to the limited adaptation observed in the data. In addition, this algorithm for adaptation does not apply to models which have nonlinear dynamics, such as those with less simplifications, those which model
more complex tasks, the involvement of more degrees of freedom, or which utilise more complex cost functions. It is noted that a more advanced model may be constructed based on existing algorithms within the Adaptive Dynamic Programming, Neurodynamic Programming or purely adaptive control fields, which may be applied to those tasks.

Nevertheless, this work describes one such model for motor adaptation, and indicates that some characteristics of motor adaptation can be modelled. Such models may be applied to neurorehabilitation — for example, the dynamics of a task may be changed such that the adaptation process utilises movements involving useful or ‘healthy’ movement patterns. Alternatively, if a certain movement pattern is to be encouraged for a particular movement, given a nonlinear search space with respect to a given cost function, dynamics can be chosen to move the search space past local minima, towards a desired, more global minimum.
Chapter 7

Conclusions and Future Work

Neurological injury has a large impact on a person’s life after such an event through impairing their memory, speech, reasoning, senses and movement. Each of these can significantly limit the quality of life of that person. With an ageing population, the incidence of such injuries are expected to increase, thus making advances in the treatment and rehabilitation of such conditions critical.

7.1 Contributions

With this in mind, this goal of this thesis was to investigate models of motor control and learning for those with neurological impairment. Three steps were taken to achieve this: the definition of a framework, the implementation of a model for motor movement and adaptation, and the definition of a suitable algorithm for modelling this adaptation.

First, an overarching framework for modelling motor movement was constructed, which outlined in a systems engineering methodology, subsystems important for the act of motor control and learning. Furthermore, a discussion was presented around which subsystems are most affected by neurological impairment, and how methods of rehabilitation attempt to change the subsystems towards recovery of motor function.

Secondly, a model was presented utilising this framework to model motor adaptation based on the Finite Horizon Linear Quadratic Regulator Problem. This model involved the proposal of an algorithm which had characteristics appropriate for motor adaptation in neurorehabilitation — it utilised an optimal control
formulation, it applied to a family of finite time tasks, and it utilised information from previous attempts at a task to produce better and better estimates of the optimal control policy, rather than knowledge of the dynamics.

Finally, an experiment was constructed to evaluate the performance of this model. The experiment involved redundancy in both the trajectory as well as the endpoint, and we performed by both healthy subjects and individuals with neurological impairment. It was observed that features of the resolution of redundancy were represented by the model, and that a simple parameterised model may be used to evaluate the capabilities of the patients. However, the results regarding motor adaptation were less definitive, due mainly to the lack of motor adaptation observed in the data.

7.2 Future Work

This thesis presented some initial findings in the modelling of patients with neurological injury. However, there are a number of avenues of future directions in this space. Furthermore, and perhaps more importantly, another avenue of opportunity exists — the translation of the knowledge we have gained so far into rehabilitation techniques.

The key feature regarding the application of these models to patients with neurological injury is that the simplified model presented here is insufficient to describe these movements. There are a number of potential improvements to this. Primarily, there are a number of nonlinearities which have been linearised as part of this model. Many impairments associated with stroke accentuate these nonlinearities. As such, the model loses fidelity much more quickly, due to the increased nonlinearity in the modelled system. As such, utilising a nonlinear model will potentially improve the accuracy of the model. However, it is noted that the additional complexity may result in additional parameters which may be difficult to identify.

Finally, the application of models of human movement to rehabilitation has been limited so far. Many rehabilitation techniques have been developed as a result of intuition and experience of talented therapists. However, with the development of robotic devices, a unique opportunity exists to apply knowledge from this model in more systematic ways to control these devices to promote rehabilitation. This may be through introducing forces or penalties to change the cost of a movement
in order to force adaptation, or simplify identifying potential impairments. The use of these devices may also lead to improved efficiencies and accessibility to rehabilitation. Therefore, it is in this area which there is significant potential to have a impact on the lives of those who have motor impairment in the near future.
Bibliography


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Title: Computational models of human motor movement and learning and their application to neurorehabilitation

Date: 2017

Persistent Link: http://hdl.handle.net/11343/192998

File Description: Computational Models of Human Motor Movement and Learning and their Application to Neurorehabilitation

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