Using Online Collaborative Learning Spaces in Primary Mathematics Education

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Abstract

Research has found that use of digital technologies in Australian primary mathematics classrooms is often superficial and focuses on the lower-order drilling of algorithms and basic facts (Day, 2013). The current Australian Curriculum mandates that technology be utilised to support students to “investigate, create and communicate mathematical ideas and concepts” (ACARA, 2014). The lack of alignment between research-based evidence and these curriculum requirements suggests a disconnect between the intended and expected use of technology within primary mathematics.

In this study 54 year 5 students participated in online collaborative mathematical problem solving over a period of 9 weeks. The resultant text-based student-discussion and software derived artefacts (MS Word, Excel files etc) were investigated in an effort to understand how this approach aligns with Australian Curriculum expectations. Next, a Bakhtinian lens is applied to dialogue from the online student discussion as a means to understand the way that students construct and develop their ability to communicate mathematically. An examination of the frequency/density of technical mathematical vocabulary use and identified examples of Mercer and Wegerif’s (1999) Talk Types is then used as a means to understand how often students within the online environment are likely to be engaged in work that might be considered productive within their learning. Perkins and Murphy’s (2006) Clarification, Inference, Assessment, Strategies (CAIS) model is utilized to gain an understanding of the types of higher critical thinking students engaged in when working in the online space. Finally, synthesis and analysis of semi-structured student interviews is offered as a means to understand how the students perceived collaborative mathematical online problem solving.

Little research exists investigating primary school students use of online mathematical collaborative problem solving. This study shows that not only is this approach to utilising digital technologies in primary mathematics possible, but it offers opportunities for student problem solving, reasoning, critical thinking and mathematical language development.
Declaration of Originality

This is to certify that:

1. this thesis comprises only my original work;

2. due acknowledgement has been made in the text to all material used; and

3. the thesis is less than 55,000 words in length, exclusive of tables, figures, bibliographies and appendices.

Signature:

Duncan Symons
## List of Acronyms

<table>
<thead>
<tr>
<th>Acronym</th>
<th>Description</th>
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<tbody>
<tr>
<td>ACARA</td>
<td>Australian Curriculum Assessment and Reporting Authority</td>
</tr>
<tr>
<td>CSCL</td>
<td>Computer Supported Collaborative Learning</td>
</tr>
<tr>
<td>CSFII</td>
<td>Curriculum and Standards Framework II</td>
</tr>
<tr>
<td>FMR</td>
<td>Formal Mathematics Register</td>
</tr>
<tr>
<td>KBN</td>
<td>Knowledge Building Network</td>
</tr>
<tr>
<td>MPS</td>
<td>mathematical problem solving</td>
</tr>
<tr>
<td>ICSEA</td>
<td>Index of Community Socio-Educational Advantage</td>
</tr>
<tr>
<td>IMR</td>
<td>Informal Mathematics Register</td>
</tr>
<tr>
<td>ICT</td>
<td>Information and Communications Technologies</td>
</tr>
<tr>
<td>IWB</td>
<td>Interactive Whiteboard</td>
</tr>
<tr>
<td>TML</td>
<td>Transitional mathematical Language</td>
</tr>
<tr>
<td>TMR</td>
<td>Transitional mathematical Register</td>
</tr>
<tr>
<td>VELS</td>
<td>Victorian Essential Learning Standards</td>
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Acknowledgments

This thesis was undertaken under the expert supervision of Associate Professor Robyn Pierce, without whom it would certainly never have been possible. Robyn took a chance on me after facilitating a subject in my Masters course considering the use of technology in mathematics education. It was during this subject that the seeds for this thesis germinated. During my study within Teaching Mathematics with Technology I became increasingly aware of the lack of thought that often goes into the use of technology within mathematics teaching (from a primary school perspective). I was also led to the belief that where technology is used in primary mathematics it is most regularly used for the drilling of facts. Therefore, the focus is normally on the development of fluency. It was this thought process that led to an effort to trial an approach to technology integration that could help students develop problem solving, reasoning, and have them engage in Higher-Order Thinking.

Mention also should be made of Robyn’s patience. During my candidature, amongst a number of obstacles, I have sustained broken limbs and the birth of children. Despite this Robyn has always remained enthusiastic about my/our work and reassuringly positive about an eventual submission and successful examination of this work.

Dr. Max Stephens is also owed a great debt. He graciously stepped in, towards the end of the thesis and undertook the significant work of reviewing a complete draft. At different times the story of his teaching my father secondary school mathematics is told. I find it fitting, that a man that had such a long lasting and important impact on my father, has now contributed similarly to my own educational trajectory.

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Chapter 1: Introduction

1.1 Introduction

The first iPhone was born in July 2007, as were many of the students in this study. Not surprisingly the Australian Curriculum (2014) recognizes the importance of technology in young people’s lives and expects that it be embedded across all areas of teaching and learning. There is an expectation by the curriculum developers that technology will be used to support pedagogy across the range of mathematical proficiencies: Fluency, Understanding, problem solving and reasoning. However, as will be discussed below, in mathematics at the primary school level, technology has typically only been used to encourage and support Fluency.

In this project an asynchronous, online, Computer Supported Collaborative Learning (CSCL) environment was developed. Within this environment upper primary students participated in mathematical problem solving tasks. In a nine-week period, groups of three to five students engaged in discussion, responded and provided feedback to each other, and ultimately provided their solution to one mathematical problem solving task per week. This study was designed to analyze the impact that this approach to technology integration may have on mathematical language and vocabulary development, the development of critical and Higher Order Thinking (HOT). Finally, it was conceived as an approach to investigate what other opportunities and limitations mathematical problem solving may provide in a technologically rich context.

1.2 Rationale for Research Project

Having worked in a Victorian State primary school as a teacher for a number of years, I have had the opportunity to observe the delivery of curriculum in the area of mathematics on a daily basis. Throughout this period, it has been my observation that the common approach adopted by teachers across the whole of primary school for teaching mathematics mirrors the teaching of literacy in the early years. The ‘Early Years Literacy’ model (Department of Education, 1997) was first introduced in Victoria in the mid to late 1990’s. This model promoted a ‘whole-group-whole’ approach to the
structure of literacy lessons, whereby a whole group explicit introduction would be conducted by the teacher addressing the area of focus for the day’s lesson, students would then be partitioned into groups (or learning centers). Students would then participate in a range of activities supporting the focus of the lesson. Commonly, one of these activities would involve the use of technology. Following group work, students would then be brought together as a whole group and reflections would be shared. This lesson structure has now been widely adopted within primary mathematics. In parallel with their approach to literacy, teachers allocate one of the ‘learning centre’ activities as an ICT (Information Communication Technologies) group. As reported by Day (2013), this use of ICT can be of questionable benefit.

More recently a study was conducted in the USA investigating how students use computers in year 8 mathematics instruction (Progress, 2015). Across, the USA it was found that 74% of students never or hardly ever used computers to research a mathematics topic. Another 20% only using their computers once or twice a month. Only 6% of students either used their computers everyday or almost everyday or once or twice a week in their mathematics instruction (see Figure 1.1).

![Computer Use](image.png)

*Figure 1.1: Student use of computers to research mathematics (Unattributed, 2017)*
It is not my intention to explore the ideal structure of mathematics delivery in the primary curriculum. Instead, the intention of this study is to explore one approach to the integration of technology in the upper primary context, which has, up to this point, not received any significant attention from researchers.

Recent research in primary schools in Western Australia (Day, 2013) reveals that although there is a perceived integration of technology in the mathematics classroom, the number of teachers who are regularly incorporating technology is potentially less than school administrators and departments of education report. The chart on the left of Figure 1.2 shows school Principals’ perception of the integration of technology in the mathematics classroom. The chart on the right shows the perceived integration of technology in mathematics by pre-service teachers observing teaching practices in the same schools (as the Principals). Evidently, there is a disparity between the beliefs that Principals hold about technology integration within the mathematics classroom and the observations recorded by pre-service teachers.

Figure 1.2: Perception of ICT use as perceived by school principals and PSTs (Day, 2013, p. 17)
Day (2013) provides data relating to the ways in which ICT is integrated within mathematics. Figure 1.3 shows that by far the most common method of ICT integration involves use of the ‘internet’. This is described as typically involving students playing procedurally-focused games that encourage students to practice routine procedures. Although Day’s (2013) research shows that teachers, school administrators and pre-service teachers all believe that ICT integration has the potential to lead to conceptual knowledge development, the evidence presented suggests that the resources most commonly used do not target this goal.

Figure 1.3: ICT Resources to Teach Mathematics (Day, 2013, p. 19)
Day’s (2013) findings support the earlier claims of Herrington & Kervin (2007) who state that, “Technology use in classrooms is often employed for all the wrong reasons—such as convenience, pressure from school administrators, the belief that students need to be entertained, and so on” (p. 12). This claim sums up my own impressions gained as a primary school teacher. When students are asked to independently participate in a ‘computer rotation’, this is achieved typically by accessing a variety of mathematics games based web sites. Often, these internet-based activities constitute games that have been colloquially labelled ‘skill and drill’. They may drill students in the areas of multiplication facts, simple addition, subtraction, basic fractions or decimals. Often these games may provide the option of students selecting the correct answer to a series of basic mathematical multiple-choice questions, or they may allow students to succeed through repeatedly hitting buttons in a process of trial and error. Wachira and Keengwe (2011) state that such approaches to the integration of technology in mathematics education potentially lead to superficial learning that will, at best, provide some degree of procedural knowledge without allowing students to gain a deeper conceptual knowledge that will provide them the opportunity to apply their understanding to broader contexts.

The findings of this study should be of value to mathematics teacher educators as well as pre-service and practising teachers. The conclusions of earlier research on the integration of technology within the mathematics classroom, such as those referred to above, suggest that a paradigm shift is required in the enacted curriculum for more effective use of ICT in primary mathematics classrooms. An approach that does allow for deeper conceptual understanding of mathematical subject matter is necessary. My research, therefore, aims to inform this new paradigm by addressing the questions stated in the following section.

1.3 The Research Questions and Key Assumptions

The aim of this study was firstly to develop and trial an approach to the integration of ICT within the primary mathematics classroom that promotes and fosters a conceptual understanding of mathematics. Research aims were to understand how this approach might align with the Australian Curriculum problem solving and reasoning proficiency strands (ACARA, 2014), explore mathematical language and vocabulary development,
the development of HOT and critical thinking and also to investigate the opportunities this approach might present for researchers, teachers and students. It is anticipated that the findings of this study will be of value to the school community and will be suitable for sharing through professional as well as research platforms. It is anticipated that the outcomes of this project may provide evidence-based advice and examples for those teachers who do not feel confident to use technology within the mathematics classroom for uses other than drill and practice. With these aims in mind the following research questions were investigated:

- RQ1 How does student engagement with online mathematical problem solving align with the Australian Curriculum?
- RQ2 How is language used to communicate mathematical meaning when year 5 students work in an online CSCL environment?
- RQ3 What evidence is there of critical thinking when students engage in ‘talk’ within online mathematical problem solving?
- RQ4 How do Year 5 students respond to the experience of working within an online environment to collaboratively solve mathematical problems?

1.4 Context of Research Project

In Victorian schools the use of ICT has been mandated for many years, spanning a series of curricula. Currently, the Australian Curriculum (ACARA, 2014) uses the Melbourne Declaration (Barr et al., 2008) as a basis from which to justify this position:

The Melbourne Declaration on the Educational Goals for Young Australians recognises that in a digital age, and with rapid and continuing changes in the ways that people share, use, develop and communicate with ICT, young people need to be highly skilled in its use. To participate in a knowledge-based economy and to be empowered within a technologically sophisticated society now and into the future, students need the knowledge, skills and confidence to make ICT work for them at school, at home, at work and in their communities. (ACARA, 2014)

An emphasis here is on young people needing to be highly skilled in the use of technology. They should be able to negotiate their way through our ‘technologically sophisticated society’. Having provided us with this broad statement, the curriculum
document refers to a series of specific activities that should take place in a mathematics classroom where ICT is successfully integrated:

Students develop ICT capability when they investigate, create and communicate mathematical ideas and concepts using fast, automated, interactive and multimodal technologies. They employ their ICT capability to perform calculations, draw graphs, collect, manage, analyse and interpret data; share and exchange information and ideas and investigate and model concepts and relationships. (ACARA, 2014)

It is important to note that the ACARA (2014) document refers specifically to ICT being used to help communicate mathematical ideas and concepts (ACARA, 2014). Its emphasis is on promoting and communicating broad, context driven and conceptual understandings of mathematics, with no reference to the drilling of procedures.

This statement is supported by research. Hong and Jacob (2012) describe the importance of communication in the mathematics classroom (particularly when engaged in mathematical problem solving): “Communication is an important process in learning mathematics allowing for sharing of ideas and clarifying of understanding. There is a need for communities of mathematical inquiry, where students learn to speak and act mathematically through participating in mathematical discussions and solving new or unfamiliar problems” (p. 17).

The Victorian Essential Learning Standards (VCAA, 2007) were the previous curriculum documents used in Victorian schools. They emphasized the interdisciplinary nature of ICT:

ICT, an interdisciplinary domain, focuses on providing students with the tools to transform their learning and to enrich their learning environment. The knowledge, skills and behaviours identified for this domain enable students to:

- develop new thinking and learning skills that produce creative and innovative insights
- develop more productive ways of working and solving problems individually and collaboratively
- create information products that demonstrate their understanding of concepts, issues, relationships and processes
- express themselves in contemporary and socially relevant ways
• communicate locally and globally to solve problems and to share knowledge
• understand the implications of the use of ICT and their social and ethical responsibilities as users of ICT. (VCAA, 2007)

Earlier still the Curriculum Standards and Framework II (CSFII) (VCAA, 2002) provided the following list (only two examples are provided) of ICT based activities considered suitable for inclusion in mathematics in the upper primary years:

• creates templates/ worksheets to calculate and chart information, eg. calculates average height of class from data list and produces a chart of results (chance and data)
• structures fields, enters data and prints sorted reports, eg. creates a file and enters data on Olympic records for a given event over the past ten Olympic Games (measurement) (VCAA, 2002)

An emphasis within all three of these curriculum documents is on students using the technology as a means to create and investigate within mathematics. The latter two documents (VELS and the Australian Curriculum) also place emphasis on using technology to communicate and share mathematical ideas and concepts. ‘Communication’ only appeared in the latter documents presumably because prior to this, the technological demands of this made it prohibitive. Additionally, it was after 2002 that web 2.0 platforms (for example wikis, blogs and message boards) were adopted on a mass scale. These observations, together with the findings reported in the literature, lead to two observations providing context for my study. Firstly, for many years there has been an understanding by school administrators and researchers of how technology is best integrated and it has been directly stated in our curriculum documents. Secondly, despite this understanding, as indicated in the work of Day (2013), Herrington and Kervin (2007), Wachira and Keengwe (2011), technology integration within the mathematics classroom is rarely incorporated for the purposes of improving communication and conceptual understanding.

This project responds to the need, identified by researchers to design, conduct and carefully research appropriate and pedagogically-sound methods of ICT integration in the mathematics classroom. Scott, Downton, Gronn, and Staples (2008) state:
Findings from studies reviewed indicate that some experienced teachers find it challenging to integrate ICT into lessons and deepen mathematical understandings. Hence, it seems important for teacher educators to investigate this issue (p. 439).

1.5 Rationale for the Mathematical Problems chosen for the Study

A choice was deliberately made to use a combination of problems from each of the three content strands of the Australian Curriculum: Mathematics (2014) (Number and Algebra, Statistics and Probability, and Measurement and Geometry). Consideration was given to focusing on only one of these three areas throughout the intervention. However, it was decided that the mathematical focus for the students was developing their generic proficiency in the areas of problem solving and reasoning (two of the four proficiency strands described in the Australian Curriculum).

1.6 Limitations of the Study

Some limitations of this study are inherent in the study design. First, in this study, students engaged only in asynchronous online mathematical problem solving. No concurrent traditionally run (in class) mathematical problem solving approach was facilitated as a point of comparison. This decision was made because my intention within this study was not to test the hypothesis that online collaborative problem solving can be more effective or superior in some way to classroom-based problem solving. Instead, I was interested in gaining a general understanding of what would occur when students were asked to participate in this way (RQ1).

Stahl (2005, 2009, 2011) has written extensively on the use of Computer Supported Collaborative Learning (CSCL) in mathematical problem solving at the upper secondary and tertiary level in mathematics. Additionally, there are (limited) examples, see for example, Monaghan (2005), where student use and development of mathematical language has been a focus when working in an ICT based primary mathematics setting. However, whilst gathering research to inform this study, I was unable to identify any studies in which primary age students were engaged in asynchronous online mathematical problem solving. I hypothesise that this could be largely a result of a belief that students at this age are not capable of combining the great deal of understandings and skills associated with language use, mathematics, and
ICT that is required of working in such a manner. As such, it was important for me as researcher to provide evidence for what students are capable of achieving, as well as showing further possibilities for researchers and practicing teachers.

Further to this, I became interested in levels of participation exhibited by the various (teacher assigned) ability groups in the online space (RQ5). Whilst no in-class problem solving occurred (as a point of comparison), a series of semi-structured interviews were included in the study design. Therefore, as a ‘proxy’ for a comparison between participation in online vs offline problem solving, I used the levels of participation in semi-structured interview to help provide further insight.

Second, the study was constrained by the software available within the school at the time when the project was undertaken. Technology is forever evolving. Since the planning and data collection associated with this study occurred, advances in software that allow for online collaboration have occurred. For example, the Google suite of applications (including Google Docs, Google Sheets and Google Drawings) would allow for all students to concurrently create, edit and modify a single document or artefact at a given time. It would now be possible to see which student contributed to each part of the document.

Whilst, an early version of the Google Suite was considered, for the purposes of this study Edmodo, designed specifically for use in schools, was selected. The ease of use, in combination with lack of need for students to have an e-mail address, along with the small group function, made Edmodo more practical for the primary level.

1.7 Situating the Research and the Researcher

A pivotal component of this study was the classroom-based one-hour discussions that occurred on a weekly basis prior to students entering and contributing to the online space throughout the remainder of the week. Since I facilitated and scaffolded the discussions that took place in the classroom, my influence was certainly a factor in the research. More detail of classroom discussion is described in Chapter 3.

Acknowledging that the discussion that was led by me would have an impact on the discussion that took place within the online space, I was interested to consider how perceptible this might be. I used a Bakhtinian (Bakhtin, 1981) approach to assessing
language use, to help provide some insight into this. Bakhtin’s dialogic view of
language, that all utterances made in the present are in dialogue, with those made in the
past resonated with this idea. I looked for evidence of the co-negotiation of definitions
and concepts from the physical classroom having impact on the students’ discussion in
the online space.

1.8 Thesis Structure & Methodology

A qualitative methodological approach was taken to the analysis of data collected as a
result of the teaching intervention. Each research question required a different lens on
the data. To achieve this data was re-coded and re-analysed, using a different
appropriate theoretical framework for each research question.

Following this introductory chapter, a literature review is provided in Chapter 2
followed by details of the setting of the study and details of the teaching intervention
in Chapter 3. The research questions within the broader thesis are addressed in
individual chapters. Each of Chapters 4 - 7 comprise a discussion of the methodological
or theoretical framework applied, implications for coding and analysis of data and
results with discussion and conclusions. The final chapter, Chapter 8, draws together
conclusions, notes implications and makes recommendations for teaching and research.
Chapter 2: Literature Review

2.1 Introduction

This literature review is being written for the purposes of informing research concerning upper primary school students, engaged in mathematical problem solving, using online collaborative learning environments. Firstly, I will provide a general overview of literature describing some of the history of ICT in Education. Secondly, I will provide an overview of relevant research that has been undertaken in the area of Online Collaborative Learning. The literature I have chosen to review was selected because it is connected with either the primary school setting or it involves the use of online collaborative learning within the context of mathematics education. When searching for material to include within the review the key words ‘Online collaborative learning’ or ‘Computer Supported Collaborative Learning’ (henceforth CSCL) were attached to either the words ‘in mathematics’, ‘in mathematical problem solving’, ‘in primary schools’ and/or ‘in elementary schools’. A great deal of literature was identified and considered during the planning for this study. My preference was to focus on the literature that related as closely as possible to this study. At times, this has proved problematic. Whilst quite a deal has been written about the use of CSCL in mathematics, very little literature reported the specific use of online collaborative learning spaces to teach or deliver mathematical problem solving in a primary school setting. Illustrative of this is Lipponen, Rahikainen, Lallimo, and Hakkarainen’s (2003) reflection that “previous studies on CSCL and participation have mainly been conducted at the university or high school level, and there are few extensive descriptions of how elementary students participate in CSCL.” (p. 2) Whilst this observation was made some time ago, it seems that little research investigating primary student online mathematical problem solving has occurred since then. A conclusion to be drawn from the latter observation is that the issues in focus for this study have not yet been researched and hence represent a ‘gap’ in the literature.

The third major focus of this review is mathematical problem solving (henceforth MPS). The review of literature related to this field will be based on the structure provided by Lesh and Zawojewski (2007). In their article problem solving and Modeling, Lesh and Zawojewski provide a summary of research conducted in the area
in the 50 years since 1957 when Polya (1957) first described the range of heuristics students utilize when problem solving. Given the large amount of research literature about problem solving some boundaries were required to narrow the scope of the review. Firstly, only research that involved mathematical problem solving has been considered. Secondly, I privilege material where the setting of the research was a primary school. Whilst ‘Higher Order’ Thinking (HOT) exists outside of the realm of MPS, I have also chosen to include this area as a subsection within MPS. Definitions and beliefs about HOT within the area of mathematics education (particularly within MPS) are well defined. HOT is not explored in its broader sense as this includes further characteristics beyond those involved in MPS.

Attention will be paid to research in the area of mathematical vocabulary development. The research that I have considered in this sub-section has not been narrowed as greatly as previous sections. Nevertheless, I have targeted literature that uses research founded in a primary school setting as a basis.

The concluding sub-section of this review will discuss connections between CSCL, MPS and mathematical language development.

2.2 ICT and Education

Since the proliferation of the Personal Computer, much has been made of the potential for technology to transform teaching and learning within primary and secondary education. Governments and Departments of Education commonly use the mass purchase of ICT for use by students as key policies for Education. These initiatives are considered popular amongst the electorate and therefore ‘sure fire’ approaches to improving approval ratings. In 2007, former Prime Minister of Australia, Kevin Rudd, promised that should his party, the Australian Labor Party, win government at the then forthcoming election all secondary students in Australia would be furnished with a new, lap top computer (Gibson, 2008). Perhaps this promise did contribute, in part, to a landslide victory for Mr. Rudd and his party.

Whilst students certainly need convenient access to technology, if it is to be a vital and powerful tool within the classroom, most researchers suggest placing a higher initial priority on ‘up skilling’ teachers with the facility to make good use of (possibly already
existing) ICT, rather than large scale purchasing of new technology. When describing priorities in teacher education Niess (2005) argued that:

for technology to become an integral component or tool for learning, science and mathematics preservice teachers must also develop an overarching conception of their subject matter with respect to technology and what it means to teach with technology—a technology PCK (TPCK) Pedagogical Content Knowledge, Technological Pedagogical Content Knowledge] (p. 510).

This statement indicates that, for technology integration to be effective in the classroom, teachers need significant professional development in the area of ICT. More importantly a specific subject area (e.g. mathematics) requires its own distinct set of principles and considerations, which again require focused development through teacher professional development and education.

2.2.1 Procedural and Conceptual Learning in ICT embedded Mathematics Education

Zbiek, Heid, Blume, and Dick (2007) draw attention to a research driven, historical, dichotomy between approaches to technology integration in the mathematics classroom. One approach develops technical (or skill and procedure focused) proficiencies while the other promotes conceptual development (for example finding and describing patterns, defining, conjecturing, generalizing, abstracting, connecting representations, predicting, testing, proving and refuting).

These authors do, however, contend that viewing technology purely through the lens of this dichotomy is an oversimplification of what actually might occur. They suggest that while technology may ‘free’ students from laborious computation, thus allowing them to arrive at a deeper, more conceptual level of thinking, a great deal of conceptual understanding is required of the student in order to decide upon the particular operation(s) they require the technology to compute. Thus, conceptual understanding and development may be implicit when students utilize technology for routine computation. In many cases, students must have deeper understanding of the mathematics in order to successfully operate the computer/technology to execute the required operation.
Much of the work of Zbeik et al. (2007) refers to the use of sophisticated computer algebra systems and hence this dichotomy allows us only limited understanding within the primary school Mathematics setting. As Day (2013), Kuiper and de Pater-Sneep (2014), Turvey (2006) have reported, a very large proportion of time spent on technology integration within the Primary mathematics classroom is often assigned to drill-and-practice mathematics software that is conveniently and freely available on the Internet. In fact, Day’s (2013) study of 118 Government, Independent and Catholic primary schools throughout Western Australia found that 80% to 90% of the time ‘the Internet’ was the ICT resource reportedly used to teach mathematics when ICT was being integrated. The value of these types of applications is discussed below.

Researchers such as Goldenberg (2000), Herrington and Kervin (2007), Way and Webb (2006) have attempted to define principles by which the success of technology integration within mathematics education can be judged. These principles highlight the previously described dual purposes of technology integration.

Goldenberg’s (2000) paper may be considered dated given it sits within the field of technology education. However, based on the literature search conducted, it remains seminal in its approach to clearly identifying and justifying principles for technology integration within mathematics teaching. Additionally, while technology has certainly changed/advanced since 2000, the principles outlined in the article still resonate today. In the article Goldenberg (2000) suggests the following 6 principles by which teachers of mathematics (primary and secondary) may plan and think about integration of technology within their teaching:

- **Genre Principle:** Think about your goal. Choose the technology that will help you further your goal.
- **Purpose Principle:** Allow calculator (or computer) use when computational labour can get in the way of the purpose of the lesson.
- **Answer vs Analysis Principle:** Sometimes students’ understanding of a problem will benefit from going through the intermediate steps of computation. At such times, a technology that obscures the details and skips directly to the answer is no help.
• **Who does the thinking Principle**: Good use of technology depends on consciously understanding what technology is doing for us.

• **Change Content Carefully Principle**: Deleting ‘obsolete’ content from curriculum must be done very carefully taking into account what students need to be able to do- especially how they need to be able to reason.

• **Fluent Tool use Principle**: Rather than teach a broad range of technology tools superficially, we are better to teach a narrow range so that students may use them “knowledgably, intelligently, mathematically, confidently, and appropriately.” (p.3)

These principles take a predominantly technical (or skill centred) approach. The author is chiefly concerned with how ICT is utilised to aid the gaining of skills in addition to providing efficient access to required solutions. This is seen in his ‘Purpose Principal’ in addition to his ‘Answer Vs Analysis Principle’. He refers to easing the student’s ‘computational labour’ in an effort to allow them access to the purpose of the lesson. It appears that he sees the integration of ICT as less concerned with the acquisition of broader concepts, understandings and associated vocabulary and more of a tool that can aid in the consolidation of processes.

In contrast to the principles that Goldenberg (2000) provides, the ten principles of authentic learning with technology presented by Herrington and Kervin (2007) are more focused on developing conceptual understanding through using technology as a conduit for the purposes of communication and making connections to the world outside of the classroom. It is also worth noting that whilst the former author places a great emphasis on calculators as ‘technology’, Herrington and Kervin (2007) investigate the potential of online simulated environments, online discussion forums, web resources, Microsoft Office applications and digital cameras to produce the following principles:

• Provide authentic contexts that reflect the way the knowledge will be used in real life.

• Provide authentic activities.

• Provide access to expert performances and the modelling of processes.

• Provide multiple roles and perspectives.
• Support collaborative construction of knowledge.
• Promote reflection to enable abstractions to be formed.
• Promote articulation to enable tacit knowledge to be made explicit.
• Provide coaching by the teacher at critical times, and scaffolding and fading of teacher support.
• Provide for authentic, integrated assessment of learning within the tasks (p.223)

The principles outlined by the latter authors have been written from a cross curricular perspective (inclusive of mathematics). Therefore, their preference for an approach to technology integration that highlights a desire for HOT and conceptual understanding may be partially a product of this additional factor.

Despite the latter set of criteria not being framed specifically with the mathematics classroom in mind, these ‘ideals’ suggest further possibilities for what may be possible through the integration of ICT, including the possibility of the establishment of online communities of practice (Lave and Wenger, 1991) leading to the formation of the evolutionary notion of the Knowledge Building Network (Scardamalia and Bereiter, 2002). These concepts will be discussed later in this review.

Lastly, Way and Webb (2006) describe the potential ways in which ICT can be best utilized in the mathematics classroom as:

• a shift from “instructivist” to constructivist education philosophies;
• a move from teacher-centred to student-centred learning activities;
• a shift from a focus on local resources to global resources;
• and an increased complexity of tasks and use of multi-modal information. (p.21)

Whilst these principles hint at the concept of collaboration, they focus more upon the “heutagological” (Hase & Kenyon, 2007) notion that technology should allow students to self-direct and determine their own learning. Heutagogy is “concerned with learner-centred learning that sees the learner as the major agent in their own learning, which occurs as a result of personal experiences” (Hase and Kenyon, 2007). Again, this concept will be dealt with in greater detail later in this review.
An aim of this study is to offer a mode of technology integration within the mathematics classroom that offers the potential of HOT, a Knowledge Building Network and the affordance of learner-centred learning (Heutagogy).

2.2.1.1 Purported Benefits of ICT in Mathematics Education

A study by Chrysanthou (2008) cited a number of benefits associated with the integration of ICT in primary school mathematics. This study has been apportioned a greater volume of attention because of its direct relevance to this thesis.

Firstly, Chrysanthou’s (2008) study showed that the ‘break with routine’ that ICT provides in the mathematics classroom allowed students to maintain their engagement with material being learnt and therefore a greater volume of ‘time on task’ was evident.

Secondly, the author cites the benefit of ICT to assist with mathematical processing more quickly, reliably, and with greater ease. Ease, efficiency and accuracy of computation are amongst the most commonly cited examples of benefits of ICT within mathematics education. Goldenberg (2000) and Herrington and Kervin (2007), for example, also claimed this as a major potential benefit.

A third benefit reported by Chrysanthou (2008) was a boost in classroom productivity. Her study utilized Geogebra, a common piece of educational software designed to allow secondary and primary students to interactively explore a great variety of geometric concepts. In Chrysanthou’s study, teachers reported that simple physical activities like creating shapes with certain dimensions could be achieved virtually using Geogebra in a fraction of the time. As such, over the course of single lesson, students were exposed to and able to produce a far greater number of mathematical meaning making objects and thus their opportunity for learning was significantly increased.

Chrysanthou (2008) echoes the view of Way and Webb (2006) that ICT in the mathematics classroom should allow for a repositioning (within the classroom) from being teacher-centred to becoming more student-centred. They claim that greater student ownership of learning leads to deeper, more conceptually based learning. Brousseau’s (2006) notion of the frame, ‘didactic contract’ as utilized by Pierce, Stacey, and Wander (2010) is also helpful in understanding the impact technology may play in mediating responsibility for learning. It is possible that teachers may view technology
as relieving them of some responsibility for student learning, thus considerably impacting the didactic contract. Chrysanthou (2008) observed that, as students became more familiar with a software based environment, they became less likely to use their teacher as a resource for clarification and assistance and more likely to use their peers as a resource in this way.

The facility of ICT to extenuate students’ weaknesses was also noted by Chrysanthou (2008). For example, in the case of many common activities and experiences undertaken to support the learning of geometric concepts, precision, accuracy and high levels of control over fine motor skills (dexterity) are required. When completed virtually, these requirements are mitigated and therefore students who may previously have been disadvantaged are given the opportunity to participate. The author states, “It seems that the use of Geogebra in mathematics lessons transformed some students from inactive observers to energetic members of the classroom” (p. 56).

Students were also reported to fear mistakes to a lesser extent because of the facility, when using ICT, to quickly erase or edit their work. The speed and ease associated with this also meant that students were prepared to take greater numbers of risks, safe in the knowledge that they would be able to quickly revert to their earlier stage of work if their trial was unsuccessful.

The dynamic interactions with geometric figures including enlargement, reduction or rotation allowed students to deduce mathematical concepts and relationships. For example, students could dynamically enlarge or reduce a triangle and see that whilst the triangle could ‘grow’ or ‘shrink’ the internal angles would remain the same. This could help them form a conception of the mathematical notion of similarity.

ICT as an approach to ‘establishing ideas’ is promoted as a benefit of its integration within the mathematics classroom. Chrysanthou (2008) describes a group of students experimenting by taking the circumference of various circles that they have created using Geogebra and then dividing the various circumferences by their diameter and finding that each time they do this the result is always the same (an approximation of \( \pi \)). The ICT in this case has allowed students to quickly and efficiently experiment with applying operations to a geometric figure thus allowing them to discover an important mathematical relationship.
2.2.1.2 Criticisms of ICT in Mathematics Education

ICT integration can lead to a level of inequity in the classroom (whether focused on mathematics or otherwise). Livingstone (2012) raises the concern that unless access to both computers and the Internet (if required) are universal a proportion of students may be unfairly disadvantaged. This is a common criticism levelled at many forms of blended or online learning. For example, in her critique of the Flipped Classroom approach (a recent innovation fitting within the broader umbrella concept of blended learning), Nielsen (2012) states her position, that student access to computers, iPads, tablets or other devices, and the Internet is currently unequal and therefore potentially discriminatory.

In the context of this study, the question of inequity as a result of lack of access is mitigated, in part by the previously discussed mandate that all Australian school students should have one-to-one access to a laptop or tablet. It does however retain some relevance to this study because, whilst the majority of students (participants) had home access to the internet, a number of students did not. Thus, lack of access to hardware could not be said to represent a level of disadvantage but lack of home access to internet did cause a level of inequity that teachers sought to address by providing greater access at school.

More general criticisms of ICT integration have included a lack of teacher confidence in their ability to utilize it meaningfully, inappropriate or low levels of training for teachers in how to effectively utilize ICT, a lack of teacher time to fully understand and plan for the use of ICT, technical faults with ICT resulting in a reluctance by teachers to use the technology in future lessons, resistance to change pedagogical approaches in a manner that would make the use of ICT possible and effective and a reported greater willingness of male teachers’ integration of ICT over females (this is a particular problem in the primary setting, given the imbalance of male to female teacher ratio) (Agyei & Voogt, 2011; BECTA, 2004; Dawes, 2001).

2.2.2 Current Conceptions of Technology Integration

As outlined in 2.2.1, research (Day, 2013; Kuiper & de Pater-Sneep, 2014; Turvey, 2006) suggests that technology integration within primary school setting is often
superficial and unidirectional. That is, students passively consume information rather than author new models, representations and engage with the ICT interactively. As reported, the most common methods for integration are represented by ‘drill and practice’ style games and applets commonly available on the Internet.

The second most commonly reported use of technology is the use of Interactive Whiteboards (IWB) and the software that is often provided with the devices (Day, 2013). Again, reservations such as those by Zevenbergen and Lerman (2007) raise questions about how these tools are utilized within the classroom. It appears that in Australian schools these devices have quickly been embraced, often without significant, appropriate pedagogical development occurring. Zevenbergen & Lerman (2007) point to teachers’ reliance on pre-prepared packages and lessons for the IWB, and adherence to these lessons causing teachers to react less often and fail to adapt to students’ needs throughout their lessons. They suggest that the ‘seductive’ quality of the device can capture students’ attention, but may not lead to improved learning.

In the almost 10 years since their article was published one might wonder if a possible ‘novelty effect’ may have now worn off, allowing for the true pedagogical potential of the tool to be realized. Unfortunately, Sheffield (2015) still reports a reluctance of primary/elementary school teachers to move beyond the unidirectional use of an IWB as purely a data projector. Therefore, no use is made of their interactivity, arguably the IWB’s most potentially transformative capability.

In Chapter 1, in Figure 1.2, ten categories are listed for methods of technology integration (Day, 2013). The use of CSCL or similar was not represented. Based on the research of Day (2013); Lipponen et al. (2003) it seems that technology integration through the use of CSCL, within mathematics education at a primary school level is yet to be researched. The findings of this study will make a new contribution to our understanding of technology use, specifically CSCL in Primary mathematics.

2.3 Online Collaborative Learning

Dillenbourg (1999) reflects on how difficult the task of defining collaborative learning is. He described how the term is used in a variety of ways, giving it a variety of definitions. He notes the wide array of differing opinions as to the definition of the
concept. Hesse (2014) compares collaboration with cooperation. In doing so he illustrates how defining collaboration can be problematic. He states, “while cooperative learners might coordinate at some points of their activity, they often work in parallel”. This is contrasted with collaborative learning, where “learners jointly orchestrate their activities in order to address a particular task or particular problem. The activities from learners are inextricably intertwined, contributions by learners mutually build upon each other, and one learner’s actions might be taken up or completed by another learner.” (p.2)

In this study, I will rely on the latter definition of collaborative learning, provided by Hesse (2014). The students in this study must work together to share, test and critique each other’s ideas in order to move forward and have success with their online mathematical problem solving.

2.3.1 CSCL as Web 2.0

Tim Berners-Lee first successfully implemented communication between a Hypertext Transfer Protocol (HTTP) client and a server via the Internet in 1990 after having made his seminal proposal in 1989 (Berners-Lee, 1989). This is broadly considered the birth of the World Wide Web (WWW). Throughout the 1990’s the WWW became an important resource for obtaining information and data. The term coined for the facility available in this period was Web 1.0. The term ‘read-only web’ aptly encompasses the potential affordance of the platform throughout the period.

Following Web 1.0, the term Web 2.0 was first coined by Darcy DiNucci (1999) and then popularized in 2004 by Tim O’Reilly (2005). Pifarré and Staarman (2011) describe web 2.0 as follows:

Collaboration is a central tenet of the new Social Web. In Web 2.0 technologies, users are active participants who dynamically and collaboratively create new content (Luo 2010).

Online content generation and sharing tools, such as blog writing tools (Blogger, GoogleBlog), wiki software (Wikipedia, WikiSpaces) and photo sharing software (Flickr, Picasa) are used by millions (p.2).

When juxtaposed, we see that whilst the former iteration was ‘unidirectional’, the latter provided us with the ability to engage in discourse and participate in online activities. A parallel can be drawn between the evolutions of pedagogy, from outmoded...
transmission based approaches, to teaching and learning founded on the social (more recently translated) constructivist theory of Vygotsky (1987) and the move from an iteration of the WWW that positioned the user as a passive consumer of information to a more active participant in the construction of knowledge and ideas.

Through understanding the connection between the affordances of Web 2.0 and social constructivism, a justification for teaching and learning that takes advantage of this connection is made apparent.

2.3.2 Asynchronous and Synchronous Online Collaboration

Fleming (2008) describes ‘asynchronous’ and ‘synchronous’ online discussion. Asynchronous discussion occurs at a student’s leisure and does not depend on anyone else being online at the same time. This may take the form of a wiki, blog or message board. Kung-Ming and Khoon-Seng (2005) cite additional (and less common) examples of tools that also involve asynchronous interaction, including CD-ROMS, E-Mails (the most common example of asynchronous interaction) and Fax machines. In a standard example of asynchronous interaction in an education context, a student will post a statement or question in a forum and at a later time another student may log in and respond to the original post. In the case of the CSCL environment used in this study a problem was posted, students solved the problem within a forum and this allowed them the opportunity to subsequently post responses, discuss, clarify and question each other’s methods and potential solutions. “Synchronous” online discussion, on the other hand, occurs in real time and may take the form of text ‘chat’. Programs such as ‘Skype’ and ‘MSN messenger’ provide this in addition to audio and video options.

Whilst the original intention for this study was to take a solely asynchronous based approach to interactions, as will be reported in the discussion section, at times students interacted with each other in real time (much like online ‘chat’) on the message board, thus combinations of the two types of interaction (synchronous and asynchronous) were ultimately incorporated.

Fleming (2008); Kung-Ming and Khoon-Seng (2005) cite the following advantages of asynchronous interaction:

- Flexibility;
• Time to reflect;
• No need to provide immediate response;
• Anonymity or pseudonymity;
• No time zone constraints;
• Situated learning and
• Cost-effectiveness.

On the other hand advantages of synchronous interaction are reported as:

• Stimulate motivation;
• Interactive participation;
• User-Friendly and
• Cost and time saving.

2.3.3 Discourse and Language Use in CSCL

Lemke (2003) argues that:

formal and social semiotic perspectives are used to show how natural language, mathematics, and visual representations form a single unified system for meaning-making. In this system, mathematics extends the typological resources of natural language to enable it to connect to the more topological meanings made with visual representations (p.1).

The complex system of meaning-making that combines and relies on connections between, and the interface of, graphical and visual representations and ‘natural’ language is clearly evident when students are engaged in mathematical discussions and construction of visual representations recorded within a CSCL environment.

Lemke (2003) describes the difficulty some students experience trying to communicate their mathematical meaning (this might relate to describing a pattern or relationship). Often students are limited to communicating this through ‘natural’ language.

Nason and Woodruff (2005) suggest that the multiple representations (some of which include the ability to dynamically manipulate representations) accessible to students working on mathematics in a CSCL environment (Nason & Woodruff, 2005). “enable young children to communicate meaning via showing and telling rather than by merely telling” (p.119).

In addition to students having the ability to express their mathematical meaning in multiple modes in the CSCL environment, researchers also recommend that students
be coached in developing skills in the use of talk to problem solve and reason with each other (Alexander, 2006; Mercer & Sams, 2006; Mercer & Wegerif, 1999; Wegerif, 2007).

2.3.4 Quality and Types of Talk in CSCL

A great deal of discussion focuses on the quality and types of talk promoted in CSCL environments. Researchers point to the low level, superficial talk that can be a result of lack of structure and lack of instruction with regards to what productive talk might constitute (Hong & Jacob, 2012; Mercer & Wegerif, 1999; Pifarré & Staarman, 2011). These authors, equate productive talk with aspects of higher order thinking, such as critical thinking, reflective thinking and creativity. They believe that the dialogues that are possible through embracing and utilizing student online interaction within a Web 2.0 environment make CSCL ideally placed as an opportunity to positively impact current pedagogies (Pifarré & Staarman, 2011; Wegerif, 2007). A dislike of rigid structure and a preference for more freedom and (student) control within CSCL environments allows for greater creativity (Kuiper & de Pater-Sneep, 2014).

Additionally, Turvey (2006) comments that the freedom of students to create their own online presence through open ended facilities, where they are able to communicate via virtual exchanges and share their ideas, creates an opportunity for creativity and learning to be observed. In this study through selection of the online environment (Edmodo), choice of open-ended, contextualised problems and through electing to make problem solving and reasoning the focus of learning rather than specific content areas of Number & Algebra, Statistics & Probability and Measurement & Geometry, I hoped to promote and focus directly on the freedom, creativity, sharing of ideas and higher order thinking described by Turvey (2006).

2.3.5 CSCL and Bakhtinian Dialogism

As stated, the second research question in this study is the following:

*How is language used to communicate mathematical meaning when year 5 students work in an online CSCL environment?*
A lens was sought through which to understand the interplay and interactions that were evident in student use of language within the online environment. As the online discussions were analysed it became evident that the way students were using language drew on discussions from the classroom, with each other, and with themselves. These observations are closely aligned with the work of Mikhail Bakhtin (1981), a Russian philosopher of language and literary critic who lived between 1895 – 1975. His work is important to any review of literature and framing of theory when considering CSCL. The dialogic approach to teaching and learning (within CSCL environments) that has now been made prominent by such researchers as Alexander (2006), Mercer, Dawes, and Staarman (2009) and Wegerif (2007) is grounded in his dialogic theory of language.

Wegerif (2007) has identified four approaches to the utilization of the term ‘dialogic’. These are briefly summarized as follows:

2.3.5.1 Dialogic as ‘Pertaining to Dialogue’

This is the most literal interpretation of the term. This definition emphasizes the notion of dialogue as shared inquiry. Wegerif (2007) writes that this allows us to differentiate non-productive ‘social conversations’ and individually spoken monologues from discussion where multiple participants engage with, internalize and respond to other participants within a group. This ability to differentiate modes of communication can be helpful, however, this may be seen as an oversimplification of Bakhtin’s (1981) work. A more nuanced understanding of Bakhtin’s work can be found in Wegerif’s (2007) three remaining interpretations.

2.3.5.2 Dialogic Texts as opposed to Monologic Texts

In the context of education, Wegerif (2007) suggests this interpretation as a juxtaposition between a teacher as ‘authoritative voice’ and the teacher as ‘persuasive voice’. A dialogic approach through this interpretation would involve the teacher taking a more ‘persuasive’ approach whereby student and teacher are positioned as equal participants within the classroom. The teacher would facilitate discussions with students in the hope of empowering them to engage in rich, meaning making, dialogue. On the other hand, a more traditional hierarchical approach to teaching would involve
the teacher transmitting information via a monologic approach. A continuum between monologic and dialogic is proposed as a means to examine dialogicality in CSCL.

2.3.5.3 Dialogic as an Epistemological Paradigm

Wegerif (2007) describes the opportunity for students to engage in productive or Exploratory Talk (Mercer & Wegerif, 1999) as opening up a ‘dialogic space’. They see the opening up of this space as allowing participants to take on and trial each other’s voices. As this process plays out, collaborative and social construction of concepts and understanding takes place. This utilization of each other’s voices and language is suggestive of Bakhtin’s (1981) notion that all language is in dialogue and that language in use always relies on its previous application.

Bakhtin (1981) refers to unified or unitary languages upon which centripetal forces act. These can be viewed as monologic. He contrasts unitary languages with a multi-faceted form of language in which many voices co-exist, known as ‘heteroglossia’. This representation corresponds with the continuum suggested in the previous interpretation. Whilst broadly providing a polarising juxtaposition of languages, as being either unified (monologic) or heteroglossic (multifaceted, ever changing, amorphous, with infinite paths), Bakhtin (1981) does suggest, that theoretically, in fact, all language is dialogic.

2.3.5.4 Dialogic as Social Ontology

In Wegerif’s (2007) interpretation of Bakhtin’s (1981) work the question of what it is to ‘be’ and the notion of ‘self’ is addressed. Rather than viewing identity as a static position, only inhabited by our ‘self’, Bakhtin views identity itself as ‘dialogic’. Namely, dialogue within ourselves and outside of ourselves informs our identity. In the context of the CSCL environment students inhabit each other’s perspectives whilst engaged in dialogue. The shifting of perspectives in this way informs their identity. The notion of ‘intersubjectivity’ (later described in section 2.3.9) provides a tangible example of how this interpretation of Bakhtin’s work might be explored.

2.3.6 Knowledge Building Networks in CSCL

Nason and Woodruff (2005) describe Scardamalia and Bereiter’s (1996) notion of Knowledge Building Networks (KBNs) as “one of the most promising pedagogical
advances for online collaborative learning that has emerged in recent years” (p. 104). They list a number of activities expected of students working within a KBN, including students being engaged with production of conceptual artefacts. These should be discussed, tested, compared, edited, improved and modified by other participants within the KBN.

Bereiter (2005) argues that theories of learning that rely on the mind as a vessel to be filled with new knowledge, do not allow for the creation of new knowledge. That is, if knowledge were only to be acquired through transmission from one individual to another, we would not have the capacity to create new ideas and understandings. Bereiter (2005) offers the notion of Knowledge Building as a means to describe the collective and collaborative production of new knowledge, as a means to describe the process by which groups of learners engage. Lipponen, Hakkarainen, and Paavola (2004) refer to the same idea as the “advancement and elaboration of conceptual artefacts” (p. 5).

The description of the activities that participants within a KBN engage in, mirror closely the actions of students within this study and therefore the CSCL environment students worked within, may be considered an example of a KBN.

Importantly, Nason and Woodruff (2005) compare the many successes teachers and researchers have had in implementing KBNs (within CSCL) in the social sciences as opposed to mathematics. One can imagine that it might be more natural to facilitate rich discussions and debates online and in subjects where this occurs more frequently within the classroom. However, if students have received a traditional form of mathematics instruction whereby, for example, an approach to solving an algorithm is explained by the teacher in front of a board and then students spend most of the remainder of the session independently solving similar problems, they may find it very difficult to debate, question, conjecture and modify each other’s mathematical thinking in the online environment.

Nason and Woodruff (2005) identify two main reasons why KBNs (within CSCL) in the area of mathematics instruction have been largely unsuccessful to this point:

1. Inability of most “textbook” math problems to elicit ongoing discourse and other knowledge-building activity either during or after the process of problem solving.
2. Limitations inherent in most CSCL environments’ math representational tools and their failure to promote constructive discourse or other mathematical knowledge-building activities (p.105).

Nason and Woodruff (2005) argue that for KBNs to be successful in the mathematical educational context they must involve ‘authentic’ mathematical problems that promote student discussion, analysis and the capability of editing/improvement.

Two major aspects of ‘authenticity’ as promoted by Herrington and Kervin (2007) include:

1. Authentic contexts in the classroom are more than simple examples from real-world practice that act as illustrations of a concept being taught. The context needs to be all-embracing, to provide the purpose and motivation for learning, and to provide a sustained and complex learning environment that can be explored at length. It needs to reflect the way the knowledge will ultimately be used, so it presents the whole environment first, rather than introducing elements one by one. Through the use of technology, it is possible to bring a range of authentic contexts into the classroom.

2. Authentic activities or tasks reflect the kind of activities that people do in the real world, that are completed over a sustained period of time, rather than a series of shorter disconnected examples. They are generally ill-defined—that is, students find as well as solve the problems. Many classroom activities are so structured that they fail to account for the nature of real-world problem-solving. An authentic approach would have learners exploring a resource with all the complexity and uncertainty of the real world (p. 223).

Additionally, Nason and Woodruff (2005) see a need for the inclusion of, not only an area for discussion within the CSCL environment, but also a capability for students to model their mathematical representations in ways that can be shared, thus facilitating online student to student ‘hypermedia-mediated’ discourse (2005).

One aim of this study is to develop a CSCL environment that allowed for ‘authenticity’ in addition to the facility for students to represent mathematical models in ways that could then be described, discussed and critiqued in the discussion area of the space, thereby adhering to stated tenets of KBNs.
2.3.7 Identity Building in CSCL

Lipponen et al. (2003) found that the patterns of participation in CSCL environments when compared with patterns in classroom learning can be quite different. They emphasise that some students tend to isolate themselves whilst others become central figures within these spaces. Whilst the Lipponen et al. study within this quickly evolving field can be considered quite old, they also suggest that students’ position (e.g. more assertive, less confident etc.) is an area where further research should occur.

Additionally, the issue of identity and identity formation in the CSCL environment can be addressed by drawing attention to Bakhtin’s (1981) notion, as highlighted by Wegerif (2007), of dialogism as social ontology. The notion that participants are required to take on the perspective of each other, whilst making clear their own perspective occurs through dialogue within the space. The continual interplay of perspective taking informs and transforms the identities of individuals within the space. This process is further explored in the following sub-section.

2.3.8 Intersubjectivity

Pifarré and Staarman (2011) describe ‘intersubjectivity’ as a concept that is important to develop within CSCL environments if true collaboration is to be achieved. It involves the ability of participants within spaces to both take the perspective of others and also to set or make clear their own perspective, such that other participants are able to gain an understanding of their perspective.

Matusov (2001) argues, that in addition to perspective taking and setting, intersubjectivity also provides students within a community of learners the tools with which to identify, manage and deal with conflict or disagreement in a constructive manner. This is possible because through their ability to take the perspective of other students learners are able to understand that their opinion and ideas represent only one element of a discussion. As a result, where intersubjectivity has been fostered within a CSCL environment, participants will behave in a more flexible and respectful manner.
2.3.9 Social cohesion in CSCL

Tolmie et al. (2010) found that collaborative learning in the primary school context not only has positive impacts on learning on student academic outcomes, but also this approach to learning can substantially positively impact on students’ abilities to interact in social situations and aid their abilities to form positive social relationships. Further, they found that students’ academic growth was ‘interlinked’ with the social aspects of learning. They recommend any such program of learning should be preceded by a period of preparatory activity devoted to the building up of social skills. This informed the approach initially taken in the face-to-face classroom sessions conducted as part of the study reported in this thesis, where during the initial classroom discussion some time was spent talking with students about appropriate and respectful approaches to communicating in the online environment.

2.4 Mathematical Problem Solving

Whilst many definitions exist for mathematical problem solving (MPS), see for example, Kahney (1993), Newell and Simon (1972) and Skinner (1984), there is some disagreement. Lester and Kehle (2003) believe that:

Successful problem solving involves coordinating previous experiences, knowledge, familiar representations and patterns of inference, and intuition in an effort to generate new representations and related patterns of inference that resolve some tension or ambiguity (i.e., lack of meaningful representations and supporting inferential moves) that prompted the original problem-solving activity (p. 510).

This definition supports a view of MPS as providing a bridge or a pathway to deep conceptual understanding.

On the other hand, Lester (2013), who wrote a comprehensive review and critique of research in the area of MPS, defines problem solving as “an activity requiring an individual or group to engage in a variety of cognitive actions, each of which requires some knowledge and skill, and some of which are not routine.” (p. 4) For this study, Lester’s definition is helpful as it emphasises that MPS is not something that must only occur individually but may be part of a small group collaborative learning environment.
For the purposes of this study I rely on Lester and Kehle’s (2003) and Lester’s (2013) definition to support my focus on the capacity for MPS to allow the development of deep conceptual understanding through engagement in collaborative online small group based learning.

In Jaworski’s (1996) seminal work on mathematical investigations she defined mathematical investigations as “contextualised problem-solving tasks that foster learning potential through open ended inquiry” (p. 1). This definition is useful also for the purposed of this study because whilst it is possible to claim that the tasks students completed in the study were mathematical investigations, this definition allows us to see that students are still engaging in MPS whether or not the problems are considered ‘investigations.

### 2.4.1 Relationships between Mathematical Problem Solving and Conceptual Understanding

Conceptual understanding of mathematics has been described by Kilpatrick, Swafford, and Findell (2001) as follows:

> Conceptual understanding refers to an integrated and functional grasp of mathematical ideas. Students with conceptual understanding know more than isolated facts and methods. They understand why a mathematical idea is important and the kinds of contexts in which is it useful. They have organized their knowledge into a coherent whole, which enables them to learn new ideas by connecting those ideas to what they already know (2001, p. 118).

This definition of conceptual understanding has evident links to Lester and Kehle’s (2003) earlier definition of MPS. Both emphasise the importance of linking old and new ideas into a coherent framework and being able to apply knowledge and skills in a systematic way to non-routine situations.

The relationship between MPS and conceptual understanding has been reported by many researchers, for example, Charles (2003), Lester and Kehle (2003), Lester (2013) and Silver (1985). Lesh and Zawojewski (2007) reinforce this when stating that MPS should be treated “as important to developing an understanding of any given mathematical concept or process” (p. 764).
It has also been recommended by Lesh & Zawojewski (2007) that future research in the area of HOT in the area of problem solving “needs to be based on the assumption that mathematical ideas (concepts) and HOT develop interactively” (p. 803). The remainder of discussion of MPS will be focussed within the area of development of HOT.

2.4.2 Higher Order Thinking

For the purposes of my study ‘critical thinking’ is highlighted as a key component of HOT. Therefore, throughout this section connections and parallels will be drawn between the two terms. Facione (2013, p. 26) defines critical thinking as “purposeful, self-regulatory judgment which results in interpretation, analysis, evaluation, and inference, as well as explanation of the evidential, conceptual, methodological, criteriological, or contextual considerations upon which that judgment is based” (p. 26).

Lesh and Zawojewski (2007) propose ‘Higher-Order Thinking’ as an umbrella term incorporating 3 specific components; ‘Metacognition’, ‘Beliefs and Dispositions’ and ‘Habits of Mind’. I will adopt their three sub-topics in my discussion of HOT. As can be seen from this discussion, many characteristics attributed to both critical thinking and HOT are shared; therefore in the following chapters where approaches to analysis of student critical thinking are utilised, the aim is to gain insight into one aspect of their HOT.

2.4.2.1 Metacognition

Flavell (1976) is first associated with defining ‘metacognition’. A definition he proposed is as follows:

Metacognition refers to one's knowledge concerning one's own cognitive processes or anything related to them, e.g. the learning-relevant properties of information or data. For example, I am engaging in metacognition... if I notice that I am having more trouble learning A than B; if it strikes me that I should double-check C before accepting it as a fact; if it occurs to me that I should scrutinize each and every alternative in a multiple-choice task before deciding which is the best one.... Metacognition refers, among other
Facione’s (2013) description of critical thinking involving purposeful self-regulatory judgments overlaps with the definition of metacognition provided above. Within this study through analysis of online discourse, an insight into students’ metacognitive processes has been made possible. Through utilisation of the Clarification, Assessment, Inference, Strategies framework proposed by Perkins and Murphy (2006) in addition to mapping of student discussion to the Australian Curriculum: Mathematics proficiency strands of problem solving and reasoning (ACARA, 2014), this study will investigate metacognition occurring within critical thinking. In Lester’s (2013) summary of the research, he highlights student development of metacognition as an under researched aspect of MPS.

Schoenfeld (1992) describes the relationship between self-regulation and metacognition; “the issue is one of resource allocation during cognitive activity and problem solving” (p. 57). Lesh and Zawojewski (2007) describe self-regulation as “the mechanism that enables problem solvers to break up larger and more complex problems into subtasks, prioritize the subtasks, efficiently sequence each subtask, and finally do each subtask” (p. 767). Student online discussion in this study offers insights into how upper primary school students who are engaged in collaborative MPS approach this.

2.4.2.2 Beliefs and Dispositions

In addition to seeing self-regulation within the area of metacognition as being essential to HOT, Lesh and Zawojewski (2007) also identify beliefs and dispositions as a key component of this area. Schoenfeld (1992) further delineates ‘beliefs’ as ‘student beliefs’, ‘teacher beliefs’ and ‘societal beliefs’. In this review, I will focus only on ‘student beliefs’ as these will be particularly relevant to my study.

Student beliefs (in the context of mathematical problem-solving) are extrinsically mediated through the process of adoption of various classroom-based cultures within their educational experience. Lampert (1990) describes the process of admission into this culture and how;
"cultural assumptions are shaped by school experience, in which doing mathematics means following the rules laid down by the teacher; knowing mathematics means remembering and applying the correct rule when the teacher asks a question; and mathematical truth is determined when the answer is ratified by the teacher. Beliefs about how to do mathematics and what it means to know it in school are acquired through years of watching, listening, and practicing” (p. 32)

There are some clear parallels between the implicit rules of the mathematics classroom described by Lampert (1990) and the ‘didactic contract’ as interpreted by Pierce et al. (2010).

Schoenfeld (1992) cites the example of a secondary mathematics teacher who encourages his students to complete their set tasks as quickly as possible; "you'll have to know all your constructions cold so you don't spend a lot of time thinking about them" (p. 70). This example is proffered as emblematic of a culture promoted within schools demanding that students solve problems expeditiously. Unfortunately, this results in the student belief that if a problem can not be solved within a short time, it is a reasonable course of action to ‘give up’. Prevalent student beliefs within the broader theme of metacognition are examined in my study within the discourse analysis of the online asynchronous learning environment. Schoenfeld (1992) lists the following beliefs as potentially having a negative impact on mathematical thinking:

- **Mathematics problems have one and only one right answer.**

- **There is only one correct way to solve any mathematics problem -- usually the rule the teacher has most recently demonstrated to the class.**

- **Ordinary students cannot expect to understand mathematics; they expect simply to memorize it, and apply what they have learned mechanically and without understanding.**

- **Mathematics is a solitary activity, done by individuals in isolation.**

- **Students who have understood the mathematics they have studied will be able to solve any assigned problem in five minutes or less.**

- **The mathematics learned in school has little or nothing to do with the real world.**

- **Formal proof is irrelevant to processes of discovery or invention (p. 69)**
During later discussion regarding student online interactions, there are examples of comments that students have made within the online asynchronous learning environment that suggest beliefs similar to those recorded above. It is suggested that these comments may impact on willingness to contribute within the online space and thus provide evidence of critical thinking.

2.4.2.3 Habits of Mind

Cuoco, Goldenberg, and Mark (1996) lament that “it has been shown possible for students to learn the facts and techniques that mathematicians (historians, auto diagnosticians, etc.) have developed without ever understanding how mathematicians (or these others) think” (p. 377). This statement makes reference to the issue already recounted, of the development of procedurally driven skills in mathematics at the expense of a broad conceptual understanding of mathematical processes.

Cuoco et al. (1996) suggest that high school students must develop a range of general habits of mind in order to counter this disconnected understanding of mathematics. They describe habits of mind as "mental habits that allow students to develop a repertoire of general heuristics and approaches that can be applied in many different situations" (p. 378).

The authors then proceed to outline the following list of mental habits’ that they see as leading to the development of an approach to solving mathematical problems in a manner similar to that of a 'mathematician':

- Students should be pattern sniffers
- Students should be experimenters
- Students should be describers
- Students should be tinkerers
- Students should be inventors
- Students should be visualisers
- Students should be conjecturers
More recently, the same authors have published work suggesting that the same principles should be applied to an upper elementary (primary) school setting. They state, “developing mathematical habits of mind in the middle grades is essential for students who are making the critical transition from arithmetic to algebra” (Mark, Cuoco, Goldenberg, & Sword, 2010, p. 505). Their approach to the development of higher order mathematical thinking has relevance to my study given that it also occurred in an upper primary classroom. Whilst, analysing discourse within the online learning environment discussion will occur as to the development of ‘habits of mind’.

2.5 Mathematical Vocabulary Development

Mercer and Sams (2006) cite two ways in which language can promote mathematical development. The first is teacher led interaction with pupils. This approach emphasises the guiding role of the more knowledgeable other (the teacher) within the discourse. They describe this approach being at its most effective when integrating what has been labelled by Alexander (2006) as ‘dialogic teaching’. ‘Dialogic teaching’ can be identified by the following characteristics:

- Questions are structured so as to provoke thoughtful answers [
- Answers provoke further questions and are seen as the building blocks of dialogue rather than its terminal point;
- Individual teacher-pupil and pupil-pupil exchanges are chained into coherent lines of enquiry rather than left stranded and disconnected (p. 32)

The second of Mercer and Sam’s (2006) contexts in which language can promote mathematical development is labelled peer to peer interaction. They define this as:

Working in pairs or groups, children are involved in interactions which are more ‘symmetrical’ than those of teacher-pupil discourse... In maths education, such collaboration can be focused on solving problems or practical investigations, which also have potential value for helping children to relate their developing understanding of mathematical ideas to the everyday world. (p. 510)

The second context that is described has specific relevance to my study because
students will be placed in a collaborative working group focused on mathematical problem solving. Within this study I examine how peer-to-peer interaction within an online collaborative learning environment impacts on mathematical language development.

Interestingly, the authors note that research performed in a physical classroom-based mathematical problem solving context regularly results in talk that is described as “uncooperative, off-task, inequitable and ultimately unproductive” (Mercer & Sams, 2006, p. 510). This has implications for my study in that the asynchronous online environment may discourage students from deviating from discussion involving the goal of obtaining a solution to the mathematical problem that they have been set.

2.5.1 Transitional Mathematical Language

The acquisition of appropriate language is a part of learning mathematics. Doerr and Lerman (2010) describe communication as the driving force behind all learning. Their four-year study provides insights into the role of speaking, writing and reading within mathematics teaching and learning. They draw on the concept of a Transitional mathematical Language (TML) (Herbel-Eisenmann, 2002) when referring to teacher or student-developed idiosyncratic language and note the need, in time, for the use of official mathematical words. They emphasise that, if mathematical language is to be appropriated it is important for students to have opportunities to discuss mathematics with peers, to connect words to each other and to contexts meaningful to them. In this study, I will examine and investigate the way that language is used, drawing on Halliday’s (1978) notion of an informal and formal mathematical register. I will attempt to determine how students’ preferences impact on different types of mathematical language.

2.6 Conclusions

In this literature review some background as to the historical difficulties that have been faced in using ICT within mathematics education have been offered. Discussion has been provided about student use of language and mathematical problem solving. Within the following conclusion a brief statement will be provided highlighting how each research question has been addressed.
2.6.1 Research Question 1: Alignment with the Curriculum

Problem solving is a proficiency strand in the Australian Curriculum (ACARA, 2014) where students are expected to engage in problem solving across the mathematical content areas. Schoenfeld (1992) describes several unhelpful or destructive approaches to thinking about mathematical thinking. These include belief that mathematics is best approached in isolation; that there can only ever be one ‘correct’ answer; and that mathematics learnt at school has nothing at all to do with the real world. Within the approach to mathematical problem solving that is adopted in this study, whilst aligning with Australian Curriculum (ACARA, 2014) mandates, I have also attempted to negate these destructive beliefs.

2.6.2 Research Question 2 and 3: Critical Thinking, Language, Talk Types and Vocabulary in CSCL

The design of this study was carefully planned to develop a ‘dialogic’ environment as advocated by Alexander (2006). I wanted to foster an environment where discussion involved the ‘symmetrical’ quality that Mercer and Sams (2006) describe as being a product of peer-to-peer discussion. The decision to include no ‘teacher facilitator’ within the CSCL environment was taken for this reason. Additionally, literature from the CSCL field (Andresen, 2009; Dennen, 2007; Turvey, 2006) suggests that removing the ‘symmetrical’ nature of discussion referred to by Mercer and Sams (2006) through the addition of a teacher facilitator can negatively influence student learning, motivation, communication and effort within CSCL.

2.6.3 Research Question 4: Student Perceptions of working in CSCL Environments

Little research literature was found examining student attitudes towards engaging in online collaborative mathematical problem solving. Research in CSCL more commonly reports rates of participation and interaction within associated learning environments (Fleming, 2008; Jazby & Symons, 2015; Lipponen et al., 2003). Thus, the reporting on and analysis of student discussion taken from semi-structured interviews relating to their perceptions of the approach that is utilised in this study appears to highlight an underexplored area of research into CSCL.
2.6.4 Gaps in the Literature

In preparing this literature review, an extensive search for relevant literature occurred. Various combinations of search terms were used in addition to a variety of search tools. For example, I utilised The University of Melbourne library online search facility in addition to Google Scholar. Little literature was found dealing specifically with primary level students working in an online CSCL environment to engage in mathematical problem solving. Therefore, rather than discussing and synthesising literature providing previous examples of the approach taken in this study, I have sought to divide the review into component parts of the greater study, for example CSCL, mathematical problem solving etc. The limited amount of literature appropriating the research design utilised in this study points to the need for research in the area.

2.6.5 Directions for Subsequent Chapters

The literature review will be used as a basis to inform subsequent chapter of this thesis. After providing discussion of the approach taken to data collection and analysis in Chapter 3, Chapter 4, will investigate how the approach taken aligns with the Australian Curriculum. Chapter 5 investigates the use of language by students in the online space, drawing on the work of a range of literature discussed in this review in the area of dialogic teaching and learning, for example, Alexander (2006), Bakhtin (1981), Barwell (2012), Mercer et al. (2009), Pifarré and Staarman (2011) and Wegerif (2007). Chapter 6 will then utilise the notion of Talk Types (Mercer & Wegerif, 1999) as an additional approach to understanding how student mathematical vocabulary changed and developed throughout the nine-week data collection period. In Chapter 7 student rates of participation and perceptions of working in the online environment are discussed. Studies by, for example, Dillenbourg (1999), Lipponen et al. (2004) and Stahl (2011) are drawn on as points for comparison and synthesis of generated themes. Finally, literature from the fields of CSCL and mathematical problem solving are used to frame the conclusions and implications in the final chapter.
Chapter 3: Setting, Intervention & Approach to Research

3.1 Introduction

This chapter provides details of the setting for the teaching intervention and research. Following this, information regarding the weekly activities (the mathematical problems set for the students to solve collaboratively in the online environment) is provided. For each problem, the actual question and information provided to the students is included. This is followed by a statement regarding the mathematical purpose of the problem in terms of the Australian Curriculum, then the technological affordances of each problem solution; and finally, to set the scene for readers of this thesis, there is a brief overview of students’ responses.

The final section of this chapter introduces the approaches taken to the research, which are expanded on in chapters 4 to 7.

3.2 The Setting

The intervention and study were conducted in a middle-outer suburban state primary school in the north of Melbourne. The school has an Index of Community Socio-Educational Advantage (ICSEA) value of 1017. This value allows insight into the school’s average level of socio-educational advantage when compared against the State average (1000). A value of 1017 suggests that the school in which this study took place aligns closely with the state average. Sixty-four percent of students at the school fall within the 2nd (33%) and 3rd (31%) quartiles, again indicating that more than two-thirds of students at the school have socio-economic backgrounds mirroring the state average. Few students have a high level of socio-economic advantage. Few students are considerably disadvantaged.

Across the school, 43% of students are from a language background other than English (LBOTE), while in the sample of students participating in the study 40% were LBOTE. This indicates that the sample were, to an extent, representative of the school’s student population.
3.3 The Participants

Participants in the project were 54 Year 5 students (ranging in age between 10 and 12 years old). There were 26 boys and 28 girls across two classes. The 54 students were allocated to 10 mixed ability groups within the online space. These groups were created on the basis of teacher judgment, since students were already classified as either below level, at level or above level in mathematics. Teachers had classified students on the basis of a series of tests they had conducted, assessing the students’ level of procedural and algorithmic fluency and general understanding across key areas of mathematics.

3.4 The Researcher as Teacher

Whilst I, the researcher undertaking this study, was not one of the students’ regular teachers, I did facilitate the in-class discussion within this study. In addition to setting up and monitoring, but not entering, the online environment, I conducted one hour face-to-face sessions with the students each week. These face-to-face sessions were conducted with all 54 students present.

These face-to-face sessions had several goals, one of which was to introduce some new mathematical language as needed for each new weekly topic since problem solving was not included in the current topics in their regular mathematics classes. It was anticipated that LBOTE students may find aspects of the approach taken in this study difficult. It was expected that the language and literacy based demands of expressing mathematical thinking via asynchronous text-based communication could be challenging for all students and that it may prove to be especially challenging for the LBOTE students.

The major focus of the 1 hour sessions was to ensure that all students had a very clear understanding of the problem for the week and then also to give the students a chance to begin discussing possible strategies that could be utilised when working in the online environment. Throughout the course of the nine-week intervention, the classroom based sessions largely took the format of a short review and discussion of the online interaction that occurred in the previous week followed by a discussion of the new problem to be solved throughout the following week, and then a short discussion of any additional technical information that may have been helpful for the students in terms of them understanding how to use the software (Edmodo, MS Excel, MS Word etc). Early
on, considerable time was spent outlining appropriate online behaviours and utilising the work of Alexander (2006) to facilitate productive dialogic interactions as opposed to discussion that might be of a more superficial nature. Generally, the level of support offered to students throughout these discussions diminished as students became more confident working in the online environment and engaging in MPS in this way.

The final two weeks of the intervention (weeks 8 and 9) were approached slightly differently. Whilst the support offered to students was very much student led discussion of the problems, I was interested to know how students would engage in the online space with no classroom-based support provided. I hypothesised that after working in the described way for some time, students may have developed the range of necessary skills to allow them to work with less facilitator or teacher based support. Therefore, in weeks 8 and 9 of the intervention, instead of the usual class-based discussion preceding their online MPS, I used the hour of class time to have the students immediately start their work in the online space. As a result, any data collected from weeks 8 and 9 were largely free of influence from the researcher (facilitator).

3.5 The Weekly Activities

Each week during the intervention the students were set a problem to solve. Informed by the work of researchers such as, Charles (2003), Lester and Kehle (2003) and Lester (2013), I aimed to choose problems that were authentic for 10-12-year-old students. These were either real world problems or ones which would appeal to their imagination. Problems were adapted from Holton and Symons (2015). No expectations were placed on students in terms of the amount of time they should spend working on the activities. The incorporation of technology was designed with Goldenberg’s (2000) principles in mind. Below connections are drawn to the most relevant of Goldenberg’s principles:

**Genre Principle**: Think about your goal. Choose the technology that will help you further your goal.

Whereas in a previous pilot study I had focussed on the development of procedures and algorithms, the open-endedness of the MPS in this context suited the technological affordance of students engaging in asynchronous text based interaction.
**Purpose Principle:** Allow calculator (or computer) use when computational labour can get in the way of the purpose of the lesson.

Given the overall focus on MPS generally, I wished to avoid situations where students would spend considerable time and effort labouring over algorithmic computation. Therefore, generally students were encouraged to use the technology (most often MS Excel) to quickly and efficiently perform calculations.

**Who does the thinking Principle:** Good use of technology depends on consciously understanding what technology is doing for us.

The focus of this study was for students to develop critical thinking in the areas of problem solving and reasoning, therefore the ‘technology’ in this case cannot ‘do the thinking’. It is incumbent upon the groups to discuss, reason, argue and provide evidence for their points of view.

**Fluent Tool use Principle:** Rather than teach a broad range of technology tools superficially. We are better to teach a narrow range so that students may use them “knowledgably, intelligently, mathematically, confidently, and appropriately.”

I utilised a very narrow range of technology in this study. The four pieces of software utilised were Edmodo, MS Paint, MS Excel and MS Word. Of these four pieces of software there were only two for which students required considerable instruction (MS Excel, and Edmodo).

Table 3.1 provides details about the mathematical content areas focussed on each week throughout the period of data collection. As can be seen the mathematical content focus for each week varied. All focus areas (the areas of mathematical Purpose) link explicitly with the Australian Curriculum: Mathematics (ACARA, 2014). As the aim of the study was on the development of problem solving, reasoning and critical thinking it was not important to maintain a single, consistent mathematical content area in the study. I chose to give students experience in problem solving across the curriculum content.
Table 3.1: mathematical Content Area in Weeks of Data Collection

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<thead>
<tr>
<th>Week</th>
<th>mathematical Content Area of Focus</th>
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<tbody>
<tr>
<td>Week 1</td>
<td>Number &amp; Algebra</td>
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<td>Week 2</td>
<td>Measurement &amp; Geometry</td>
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<td>Week 3</td>
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<td>Statistics &amp; Probability</td>
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<td>Week 5</td>
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<td>Week 7</td>
<td>Statistics &amp; Probability</td>
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<tr>
<td>Week 8</td>
<td>Number &amp; Algebra</td>
</tr>
<tr>
<td>Week 9</td>
<td>Number &amp; Algebra</td>
</tr>
</tbody>
</table>

3.5.1 Week 1 – Toilet Rolls

3.5.1.1 Problem:
Mr Mac was sitting on the toilet one beautiful sunny day. As he sat he thought about all sorts of strange and wonderful things. He thought about last September, when his mighty Hawks won the premiership. He sighed as he thought about Buddy going to Sydney. He also thought about other important questions like shall I have soup for lunch or a sandwich?

Suddenly, in a moment of mathematical beauty, as he peered around his temple of ablutions he wondered the following:

I wonder how long a brand new toilet roll is?

Your task is to answer Mr Mac’s question.

Find a 'brand new' toilet roll in your house (you may need to ask your parent's permission for this).

You may not unroll the toilet roll.

The following questions may help you to solve this:

- What do you need to measure?
- What are you allowed to measure?
- What will allow you to measure this?

Present your solution in any manner you wish.

IMPORTANT: We are not looking for an exact answer to this question. We are simply looking for an accurate 'estimation'. Therefore, if you have taken a logical and systematic approach you will have success, however you solve the problem.

* Please do not just 'google' the answer to the question. As this would:

  a) Not demonstrate your amazing ability to think mathematically and

  b) Ruin the whole activity.
3.5.1.2 *mathematical Purpose:*

The mathematical purpose of this question was to allow students to develop and represent basic multiplicative thinking.

3.5.1.3 *Technological Affordances:*

- Some students used the Internet as a resource to obtain the average number of sheets of toilet paper in a toilet roll.
- The online record of student’s responses provides formative and summative assessment for the teacher.
- Students may have used the Microsoft Windows based calculator to multiply/ add.

3.5.1.4 *Summary of Responses/ Observations/ Reflections:*

As this was the first of the problems attempted by students, the level of technical demand involved in using new software was high for them despite having had some (limited) experience working with Edmodo in the previous year. Based on the semi-structured interviews conducted as part of this study it seems that the students’ prior experience was largely superficial and they had not hitherto been required to use the technology when completing any tasks that demanded higher levels of cognition. As such, whilst the students had been asked to post their comments within their ‘small group’ message board, the majority of students posted their comments to the ‘whole class’ board. This meant that there was some confusion about who was communicating with whom and so on. This also meant that my intention for the small groups to work purely with their own group (and not receive assistance from others) did not succeed in this first week.

Despite these organizational issues, some students/ groups did provide mathematically appropriate, well-considered and articulated answers and explanations. Of the students who answered the question most used a similar strategy. The strategy involved them discovering the number of sheets of toilet paper in a toilet roll (either through reading packaging or searching the Internet), measuring the length of a sheet of toilet paper and
then multiplying the length of a sheet by the number of sheets. Interestingly, one student made the following suggestion:

*I think we should go onto Google and searching up how many squares there are then measuring the square of how many cm or mm there are in a square and just add them all up... does anyone agree with me?*

Whilst this strategy is similar to the approach that I have recounted, it appears that this student is not yet completely comfortable in her use of multiplicative thinking and therefore suggests an additive approach.
3.5.2 Week 2 – Working Days

3.5.2.1 Problem:

The United Nations has issued a decree that from now on, everywhere on Earth, there will be 10 hours a day instead of the current 24 and every hour will be made up of 100 minutes. Your job is to describe the new system and how it will work.

Create a new school timetable based on this system.
When will school start? When will you have recess/ lunch? When will school end?
You might need to create some sort of diagram/ table for this. Which program would allow you to do this? Once you have done this you can save the file and upload it into the folder associated with week 2. Make sure you label the file with your group number.
Gather data about people’s working lives. (you will need your netbook to help you find out about the average person’s working week)
Create a timetable for the working day. You may do this with word or excel and upload to this space.
What time should people start work?
For example. How long should they have for lunch? How long morning break?
As a result of an hour being much longer pay rates will need to change. Research the average hourly pay rate and convert this rate to the new system.

Extension (only do this if you really feel like a challenge):

How long would it take to fly to Sydney to London using the new system?
What would the speedometer on a car look like (how might it change)?

How would daylight savings work?

3.5.2.2 mathematical Purpose:

A number of mathematical purposes existed for this problem including:
• The development of a deeper understanding of how our system of time operates
• The development of proportional thinking
• Application of fractional/decimal understanding

3.5.2.3 Technological Affordances:

• Students used the Internet as a resource to research a range of issues. E.g. Average number of hours in working week, average salary etc.
• Students used Microsoft Word to help create a diagram of a school day representing the two time systems in parallel.
• Students may have used the Microsoft Windows based calculator for various operations.

3.5.2.4 Summary of Responses/Observations/Reflections:

The students found this problem difficult. The mathematical demands of the problem were very high for this age group and, given that it was still only the second week of the project, students were still working out how to negotiate the Edmodo platform. As Year 5 students, they had not yet developed a sophisticated understanding of proportional reasoning and their various understandings of fractions/decimals were diverse and often poor. In order to have success with the problem students would have required extensive time and support in the form of explicit instruction in these mathematical areas.
3.5.3 Week 3 – Wallpaper Symmetry

3.5.3.1 Problem:

Wallpaper patterns are an example of symmetry in our everyday lives.

Record your group's understanding of the different types of symmetry in this space (You may need to research this using Google - DO NOT JUST CUT AND PASTE - USE YOUR OWN WORDS)

You can find a very good explanation of these in the folder attached to week 3.

Now your group will design a wallpaper ‘block’. You will need to decide how to incorporate symmetry into your block. Your group will create (and upload to this space) one sheet of wallpaper using Microsoft Word. An example of this has been uploaded to this space for your benefit.

Most importantly… Your group will write a summary of which types of symmetry you have used and how you have incorporated these into your wallpaper designs. These will be posted within this forum.

3.5.3.2 mathematical Purpose:

The mathematical purpose of this problem was to facilitate the development of student understanding in the areas of line (mirror) symmetry, rotational symmetry and translational symmetry.

3.5.3.3 Technological Affordances:

- Students were required to develop a range of new skills and competencies in their use of Microsoft Word in this problem (For example, creating, rotating and reflecting shapes).
• Some students chose to use Microsoft Paint for this task. These students needed to develop the same skills listed.

3.5.3.4 Summary of Responses/ Observations/ Reflections:

Overall, students had more success with this task as compared to Task 2. It appeared that students’ apparent familiarity with the concept of ‘symmetry’ gave them the confidence to instigate work on the problem. Two students held the following discussion:

Student 1:

*I think symmetry is the exact the same thing on both sides. What do you think?*

Student 2:

*That’s what I think, I remember learning about it in grade 3*

Having discussed the students’ current level of ability with their teachers, this confirmed my expectation that most students had prior conceptions/ learning in the area of mirror/ line symmetry; however the majority had not explored rotational symmetry (specifically thinking about angles and orders of rotation) or translational symmetry. This problem gave them the opportunity to explore these new areas. After the class-based discussion was held it was pleasing to see some students trial and experiment with language associated with the broader range of symmetry types. The following discussion is evidence of this:

Student 1:

*Hey guys. This is everything in this folder. I have in this folder my understanding of symmetry, the background and the explanation on what I did. These diamonds have 4th rotation.docx*

*DOCX File*

Student 2:

*Nice 4th rotation symmetry!*
3.5.4 Week 4 – How Big is a Dog?

3.5.4.1 Problem:

What is the biggest breed of dog?

- Research a variety of dogs using your netbook.
- Decide what ‘biggest’ means. Provide a definition. Your group will have to decide whether they think ‘biggest’ means heaviest, tallest, longest etc.
- How do breeders measure this?
- Create a graph in Excel representing the data you have found.
- Horizontal axis (x axis) should be breed of dog and vertical axis (y axis) should be height/ weight/ length etc.
- Upload the graph that you have made to this message board.
- Which dog according to your definition is the 'biggest'?
- Can you discuss any other facts that you can 'read' from the graph that your group has created?
- Now think about another measurement you can use to define 'biggest'. E.g. If you defined 'biggest' as height of the dog last time, you might like to use weight this time.
- Create a new graph.

Extension 1

You may repeat the process with breeds of cats if you wish.

Extension 2

Take a selection of animals from the zoo and arrange them in order of height on a graph. Compare their heights with their weights.

Extension 3

Can you create a graph of animals based on the speed that they can run?
3.5.4.2 **mathematical Purpose:**

The mathematical purpose of this task was to develop students’ abilities to select and collect appropriate numerical data to investigate an issue and represent this data using digital technologies.

3.5.4.3 **Technological Affordances:**

- Students were introduced to Microsoft Excel and began experimenting with its basic features.
- Students used the Internet as a source for data.

3.5.4.4 **Summary of Responses/ Observations/ Reflections:**

Generally, groups had a high level of success with this problem. They were able to decide on their definition of biggest (this varied between, height, weight and length), collect the relevant data, synthesize their data into a single graph and explain their thinking and draw conclusions.

Some students found it difficult to understand that the variable chosen to determine ‘biggest’ needed to be consistent within the group in order to synthesize the data into one representative graph. This is represented by the following discussion:

Student 1:

*Student 2 I can't include your work because everyone is using the weight.*

Student 1:

*You've used the height or length either one.*

Student 2:

*But I did the task?*

Student 1:

*But you did it without looking at anything we said which was about weight.*
3.5.5 Week 5 – Animal Ages

3.5.5.1 Problem:

It is sometimes said that each year a human lives is equivalent to 5 cat years. What does this mean? How old is your cat in human years (If you don’t have one pretend!)?

- Create a ‘cat age’ table in Excel. This should include columns for a cat’s age and a column for the equivalent human age (up to 100 years).
- Create a line graph in Excel representing these data up to 100 human years.
- Like last week you may copy both the table and graphs that you have created into word. In word summarise what the graph shows. You may also include images.
- Every year a human lives, is said to be the same as 2 years for a Camel. Repeat the above process for Camels (add to your chart). How old are you in Camel years?
- Some lorikeets live to be 25 years old. Add a column to your table that makes it easy to convert between lorikeet and human ages.
- Use the table to tell how old the members of your family would be if they were lorikeets.
- Create a 'pictograph' using Excel displaying the various animal data up to 10 'human' years.
- Why in your opinion would Mr Symons have asked you to make this chart only up to 10 human years?

Figure 3.5: Week 5 - Animal Ages

3.5.5.2 mathematical Purpose:

The purpose of this problem was to help students develop an understanding of proportional reasoning. An additional development that emerged from students interacting with the problem and each other was a foundational ability to create
algebraic expressions and evaluate them by substituting a given value for variables using digital technologies.

3.5.5.3 Technological Affordances:

- Using the fill down function of Microsoft Excel (aiding in the development of pattern creation/ recognition and understanding of multiples)
- Experimentation with formulas in Microsoft Excel
- Creating basic graphs (E.g. bar graphs) using Microsoft Excel
- Using advanced features of Microsoft Excel to create proportional pictograms. (E.g. 1 dog = 7 dog years)

3.5.5.4 Summary of Responses/ Observations/ Reflections:

Students had various levels of success with this problem. At the simplest level students were able to identify and record the growing patterns (or recursive equations). For example, they knew that each successive dog year would be 7 human years more than the previous dog year. Some students laboriously entered each multiple of 7 into each of the subsequent cells, whilst others realized the opportunity to use Excel’s ‘fill down’ feature, which would recognize the ‘recursive expression’ and pre-fill subsequent cells with these data.

Students working at a higher level of sophistication were able to create a series of ‘explicit equations’ allowing subsequent terms to be obtained. For example, they understood that multiplying the given number of human years by 5, 2 and 4 would provide the number of cat, camel and lorikeet years. They were then able to transfer this understanding into basic algebraic formulae (e.g. cat years = B7 x 5, Camel Years = B7 x 2 and Lorikeet Years = B7 x 4). This understanding, represents a significant level of achievement for students of this age/ year level. For example, The Australian Curriculum (ACARA, 2014) anticipates that within ‘Patterns and Algebra’ at level 7 students should Create algebraic expressions and evaluate them by substituting a given value for each variable

Students’ graphical representations of the various scenarios also represented a dichotomy. Students working at a lower level of ability represented the linear growth
of the different animal ages using basic line graphs or histograms. Students with more sophisticated understanding (in addition to creating the more ‘basic’ graph types) were also able to create pictograms in Excel where images of animals were proportionally representative (although the majority of students used one image to represent one year for the different animals).
3.5.6 Week 6 – Shapes

3.5.6.1 Problem:

How many different shapes can you make with MS Paint that have four straight sides?

- Provide an appropriate name for each of your shapes.
- Are there two or more of these shapes that are, in some sense, the same?
- Your group will need to agree on a definition for each of your shapes.
- You will prove or disprove that your shape is what you have called it based on your definition.
- E.g. I know that the shape I have made is a square because it fits my definition - A square has four equal side lengths, and four 90 degree (right) angles).
- You can make a table in word with a diagram of your shape, a label (name) for your shape, a definition, also include a column for 'attributes'. You may also like to include an additional column showing real world examples of your shape taken from the internet.
- Create a Venn Diagram or a Tri-Venn that allows you to classify your shape based on particular attributes. E.g. You may group all of your shapes that have equal side lengths together.
- Ensure that you provide a written explanation of how you have classified your shapes in addition to the Venn diagram.

Additional problem:

Kwa has a square piece of toast every morning for breakfast. He cuts it with one straight slice. What shapes can he make this way?

3.5.6.2 mathematical Purpose:

The mathematical purpose of this problem is to provide an opportunity for students to develop an understanding of the role of specific definitions for basic two-dimensional shapes (in this case quadrilaterals). This problem allows students to understand that
subtle changes to definitions can result in new shapes. An additional purpose is to raise the awareness of students that shapes can be represented in a hierarchical framework. For example, a square is a rectangle, but a rectangle isn’t necessarily a square.

3.5.6.3 **Technological Affordances:**

- Students used a range of software (E.g. Microsoft Word or Microsoft Paint) to create Venn Diagrams representing their thinking
- Students used ‘auto shapes’ or ‘shape tools’ within this software to show thinking
- Students researched a range of ‘real world objects’ using the internet representative of a range of common quadrilaterals
- Students created tables in Microsoft Word demonstrating their thinking

3.5.6.4 **Summary of Responses/ Observations/ Reflections:**

Students again varied in the degree to which they had success in this problem. Whether through carelessness or through difficulties in the area of reading comprehension, despite the instructions stating that shapes should have 4 straight sides a number of students included a variety of shapes that did not fall within these guidelines. Pleasingly though, this gave students the opportunity to critique each other’s work and give each other feedback. The following discussion is an example of this:

Student 1:

*This is the table that he [Mr. Symons] showed us how to do it. If you [other group members] have any comments please, I actually advise you to reply or comment on this. I have not finished it but if you have anything you would like me to change please reply. Haven't finished but please get some more shapes so I can finish it off. This is not my computer so please say anything if there is something wrong.*

*Thanks Group 9,*

table_for_edmodo_about_shapes.docx

*DOCX File*
Student 2:

With the table that you have done it was good didn't it also have to only be with straight sides ?? Like you added a circle but it was still really good.

Student 1:

Don't know, but I'll add another bar so it will make for that one if we were not supposed to do it.
3.5.7 Week 7 – Pet Names

3.5.7.1 Problem:

“My cat has a really funny name!”, Aadala exclaimed cheekily.

“Yes and a very unkind one!” Nusrat replied. “Fancy calling a cat ‘Dog’!”

“Really, do you think I have been cruel?” asked Aadala, who was feeling a bit bad about herself.

“Yes the poor animal will be scarred for life thinking of having to go on a lead and beg.”

“But I’m not the only one who has named their pet something strange! A friend of mine called her cat Indigo Blue Joy”, said Aadala, who was now feeling like she was being treated unfairly.

“That seems a bit long to me, certainly longer than Dog, but just as strange!”

“I wonder how long cat’s names usually are”, mused Aadala.

In your small group discuss the conversation that talks about the lengths of the two cats’ names that we know of. Make sure that everyone in the group understands the discussion that has taken place.

- How can your group find out how long cats’ names usually are?
- You need to represent this data. Given that we have used excel already a number of times you will probably want to represent your data using Excel.
- Decide in your group which excel chart you want to use to represent the data, and provide a good reason for why you chose the specific graph or chart you chose (e.g. did you choose a line graph, bar chart, pie chart? Why?)
- Compare the lengths of cats’ and dogs’ names. In your small groups make a 'conjecture' about whether cats' names or dogs' names are generally longer, shorter or of an equivalent length.
- Now use a similar approach to your initial method to show whether you were correct.
- Through going through these processes what interesting conclusions did you reach?
- What further questions did doing this investigation make you wonder?
- How could you answer those questions? (Feel free to go ahead and answer the questions that you wondered and provide your responses on this space).

Figure 3.7: Week 7 - Pet Names
3.5.7.2 **Mathematical Purpose:**

The mathematical purpose of this activity is for students to begin to develop a basic understanding of and an ability to represent descriptive statistics. The problem requires students to retrieve, organize and represent numerical data using digital technologies.

3.5.7.3 **Technological Affordances:**

- Students used the Internet to research common cat and dog names
- Students used Microsoft Excel to organise and represent the collected data
- Students used the sort function of Microsoft Excel to order their data in ascending order.
- Some students utilised formulae (in Microsoft Excel), allowing them to calculate the mean of their data set.

3.5.7.4 **Summary of Responses/ Observations/ Reflections:**

Again, the students’ responses showed differing levels of mathematical and technical sophistication. At the simpler levels students entered or copied the data they found on the Internet into Microsoft Excel organized under appropriate column headings. The majority of students also used the ‘sort’ function of Excel to arrange their data in ascending numerical order. The students working at a more sophisticated mathematical level provided the mean, median and mode of their data set. In this way they were able to provide a number of letters that represented the average length of cat and dog names. Students who provided this additional information utilized formulae allowing them to calculate the mean number of letters in both cat and dog names. Figure 3.8 shows a response featuring a formula providing the mean number of letters in the 20 most common Australian male cat names. Also, notice that the student represents his/ her conceptual understanding of the ‘median’ by highlighting the median length of Cat names in green. The student has highlighted the middle name but there are several names with equal letter frequency – these are then ordered alphabetically. The highlighted single name is not the median, rather the median is 5 letters. However, such a response could open up a rich teacher facilitated discussion. It is likely that it would be too much to expect Year 5 students to pick up on this subtlety. After all, it is not until level 7 that within Statistics and Probability the Australian Curriculum (ACARA,
2014) anticipates students will *Calculate mean, median, mode and range for sets of data. Interpret these statistics in the context of data.*

The teacher led discussion should also focus on which of mean, median and mode is the most useful piece of information.

![Figure 3.8 Screenshot of Higher Level Response to 'Pet Names'](image)

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>20 Most Common Girl Cat names in Australia</td>
<td>20 Most Common boy cat names in Australia</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Cleo</td>
<td>Max</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>Coco</td>
<td>Sam</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>Lucy</td>
<td>Jack</td>
<td>4</td>
</tr>
<tr>
<td>5</td>
<td>Puss</td>
<td>Milo</td>
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</tr>
<tr>
<td>6</td>
<td>Bella</td>
<td>Puss</td>
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</tr>
<tr>
<td>7</td>
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<td>Toby</td>
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</tr>
<tr>
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<td>Daisy</td>
<td>Felix</td>
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</tr>
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<td>9</td>
<td>Kitty</td>
<td>Kitty</td>
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<td>Lucky</td>
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<td>Missy</td>
<td>Mommy</td>
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</tr>
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<td>Molly</td>
<td>Simba</td>
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<td>Tiger</td>
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<td>Tiger</td>
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<td>Thomas</td>
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</tr>
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<td>Sophie</td>
<td>Tigger</td>
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<td>5.15</td>
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</table>
### 3.5.8 Week 8 – Mr. Mac’s iPhone

#### 3.5.8.1 Problem:

You are all very familiar now with Microsoft Excel. Use this program to help you demonstrate what is occurring in each of the following situations:

- Mr Mac just bought a new IPhone 5. Unfortunately, the battery is faulty. When he bought it, it had a battery life of 64 hours. Each time he charges the IPhone, it loses half of its original capacity. How many charges will it be before it only has a one hour battery life?
- Imagine that the IPhone started with a battery life of 1024 hours. How many charges will it take now to have only a one hour capacity?
- The IPhone is still starting with an original charge of 1024 hours; however now it is losing only a quarter of its capacity each charge. Now, how long will it take to only have one hour left?
- Mr Mac has decided to hit the phone with a hammer in frustration. Strangely this has reversed the malfunction. Each time he charges it now the capacity doubles instead of halves. How long will it be until the IPhone has 16,384 hours of charge?

**Figure 3.9: Week 8 - Mr Mac’s IPhone**

#### 3.5.8.2 mathematical Purpose:

The mathematical purpose of this problem is to explore division and multiplication of common, simple fractional quantities. Students are also given the opportunity to explore the inverse relationship that exists between the two operations.

#### 3.5.8.3 Technological Affordances:

- Students used Microsoft Excel to record the change in battery life of the mobile phone.
- Students used formulae in Microsoft Excel to model this change.
- Students used fill down function of Microsoft Excel.
- Students used graphical representation of change in battery life in Microsoft Excel.
3.5.8.4 Summary of Responses/ Observations/ Reflections:

Students were exposed to the idea that multiplying by $\frac{1}{2}$ is equivalent to dividing by 2. Thus, the possibility of rearranging equations in different but equivalent ways was introduced (providing a demonstration of the inverse nature of division and multiplication). An example of students’ emergent thinking in this area is as follows:

Student 1:

I'm working on an Excel document. All you have to do is half 64 then half the number that you came up with and just keep doing that till you get to one.

The following is a subtly different response:

Student 2:

I think you have to charge it 6 times till it gets to 1-hour charge.

I used the calculator and did:

\[
\begin{align*}
64/2 &= 32 \\
32/2 &= 16 \\
16/2 &= 8 \\
8/2 &= 4 \\
4/2 &= 2 \\
2/2 &= 1
\end{align*}
\]

Whilst student 1 appears to be using a strategy involving multiplication of fractions, student 2 divides by 2 (a whole number). Both strategies are suitable and by articulating these responses students demonstrate the equivalence of the two equations.

Levels of student thinking again emerged. Whilst, some students were able to divide and multiply by whole numbers to obtain the next term in the series, very few were able to demonstrate that they understood that for example, $1024 \times \frac{3}{4} = 1024 \div 1 \frac{1}{3}$. Whilst I
would not have anticipated students of this age displaying this level of understanding, given previous sophisticated thinking displayed by some students, I considered it a possibility. Further, with additional teacher support/ facilitated discussion, I am confident that students could have achieved this understanding.
### 3.5.9 Week 9 – Geese

#### 3.5.9.1 Problem:

In various places around the world (for example Canada) if you look up into the sky at different times of the year (Autumn in Canada) you will see a skein of geese in a curious symmetrical formation.

- Research the interesting pattern geese make when flying
- Create word document and describe this formation. Include a picture from the internet.
- You will now create an excel table showing the way this pattern grows. Your table will show 100 rows of geese.
- The first group of geese in the pattern will be 3, and the pattern will grow from there.
- How many geese will be in the 10th term of the pattern?
- How many geese will be in the 50th term of the pattern?
- How many geese will be in the 100th term of the pattern?
- You will need to create two columns in your table. One will describe the 'term' within the pattern and the second column will show the number of geese within the corresponding 'term'.
- Can you work out a rule for the pattern that is being created.
- Can you use your rule to create a formula in excel?
- Go back to the term where there are 11 geese in the pattern. So far we have assumed the pattern that the geese make is always symmetrical. This is not always the case! How many possible patterns will there be if the formation (with 11 geese) is allowed to be asymmetrical?
- Create a diagram of all of the different combinations (patterns).

---

Figure 3.10: Week 9 - Geese
3.5.9.2 **mathematical Purpose:**

The mathematical purpose of this problem was to initiate an emergent understanding of recursive and explicit (algebraic) equations.

3.5.9.3 **Technological Affordances:**

- Students used Microsoft Excel to model the growing ‘V’ pattern that geese make when flying.
- Some students utilized formulae (within Excel) to describe the growing pattern.
- Students used the Internet as a research tool allowing them to identify the pattern that a skein of geese makes.

3.5.9.4 **Summary of Responses/ Observations/ Reflections:**

Students demonstrated significant growth in both their technical ability to utilize digital technologies when working mathematically and also in their growing understanding and development in basic algebraic thinking in this problem. Varying levels of conceptual development were again evident. Some students could detect and utilize recursive ‘growing’ patterns. This is evident in the following statement:

Student 1:

*I think the rule is you count by 2's and it helps you if you know your odd numbers.*

This student is aware that each subsequent term in the series will be 2 more than the previous term. This is helpful, but does not allow us to answer questions like, ‘how many geese will there be in the 2345th term?’ In order to answer this question we require an explicit equation. The following student statement is an example of this:

Student 2:

*RULE 1*

*You double the number like 5000 and you’ll get 10000 and then you add a 1 to the number and you do the same thing to all the numbers.*
Whilst, the student has not yet described this equation using symbolic algebraic notation, they are still able to describe the process and make it clear that this process is repeated.

### 3.6 Student Access to Hardware

The school in which the study took place operated a one-to-one Laptop program. The program did not allow for ‘bringing your own device’. This means that all students were equipped with exactly the same netbook. All netbooks were kept in good working order by the schools’ Information Communication and Technology manager. However, the majority of work students completed in the CSCL environment occurred at home. This meant that students required a home Internet connection that they could connect to wirelessly. This could have caused issues for some students in that either they had no access to the Internet at home or their connection was insufficient. To allow for this students’ classroom teachers made classrooms available throughout the week during break times for students to complete their work. In this way, the process was fair and equitable.

### 3.7 Student Access to Software

Software was also managed by the school’s Information and Communication and Technology manager. All netbooks were preloaded with an identical image; including Microsoft Windows, various web browsers (including Google Chrome, and Mozilla Firefox) and the Microsoft Office suite of applications. Throughout the period of the teaching intervention students used any of the listed web browsers to access Edmodo and used Microsoft Excel, Microsoft Word and Microsoft Paint to create artefacts allowing them to answer the problem solving questions. I was conscious of Goldenberg’s (2000) Fluent Tool Use Principle, when deciding on an approach that would largely build on students’ understanding of software that they were already familiar with. Students initially uploaded these artefacts to folders within the Edmodo platform, but quickly realized that these files were more easily accessible by uploading the files directly to the message board to which they were they were contributing. The latter option also allowed them to upload the file within a part of a discussion where it was directly relevant, thus making the reasoning behind their response clearer to their fellow groups members.
3.7.1 Edmodo

The CSCL environment was developed within the Edmodo (Edmodo, 2014) online ‘social learning platform’. A number of other Internet based platforms had been trialed for this study, including Wikispaces (Wikispaces, 2014) and (the now decommissioned) Ultranet (DEECD, 2010); however the Edmodo platform was chosen for a number of reasons. Firstly, despite not having a great deal of experience with the platform, the participants had at least used it to a limited extent in a previous year. Secondly, the Edmodo platform allows for the allocation of participants to ‘small groups’ required for collaborative problem solving. Many researchers advocate the use of small collaborative groups within mathematical problem solving (Charles, 2003; Lester & Kehle, 2003; Lester, 2013; Schoenfeld, 1992). Thirdly, the Edmodo platform allows for various student created artefacts to be uploaded (Excel spreadsheets, Word Documents, images etc). Lastly, the Edmodo platform is relatively simple and intuitive to negotiate. This was very important given the age and lack of experience of the participants.

3.7.2 Microsoft Excel

Programs within the Microsoft Office suite were relied on in this study. Their general availability makes them an ideal choice for use in CSCL based mathematical learning. Microsoft Excel in particular was fundamental to students’ exploration and mathematical meaning making. Excel provided students:

- A repository to store data being investigated
- The ability to efficiently organize their data in a way that made it easily understood
- The ability to quickly and easily sort their data
- The ability to easily complete complicated computation (involving addition, subtraction, multiplication and division)
- An ability to apply formulae to a data set

The incorporation of Microsoft Excel adhered to Goldenberg’s (2000) principles of technology integration in the mathematics classroom (described in 2.2.1). In his Purpose Principle, he states that we should allow technology use “when computational labour can get in the way of the purpose of the lesson” (Goldenberg, 2000, p. 3).
Certainly, by this point students should have gained the necessary skills to complete all four operations. Whilst an important aspect of these tasks was development of mathematical thinking in the content areas of Number and Algebra, Statistics and Probability, and Measurement and Geometry, of great importance was having students’ develop the ability to model, investigate, communicate, analyze, evaluate, explain, infer, justify and generalize. I believed that by removing some of the computational labour associated with solving basic arithmetic and sorting and ordering sets of numbers etc. students would be less distracted from the greater task and therefore, more able to have success in collaboratively solving the problems.

3.7.3 Microsoft Word

Symons (2011) described the benefits of Microsoft Word as software in the primary mathematics classroom. Because of its ‘omnipresence’ as software that we can expect to find on almost all computers that students are likely to encounter at school, it is important to investigate Word as a tool for generalizable mathematics teaching and learning. The range of tasks and activities that can be completed with Word also make it ideally placed to incorporate into CSCL. Students have the ability to manipulate and interact with 2D shapes (giving them the opportunity to explore congruence, similarity, transformations and symmetry etc.). They can also explore the area of regular and irregular shapes, as described in Symons (2011). Word also provides students with the tools to organize their thinking through the use of tables and the ability to create graphic-organizers (for example Venn diagrams).

3.8 Approaches to Research

When first conceptualizing this study, an overall theoretical or methodological framework was sought. As various theoretical lenses were applied to the primary source of data (the online mathematical problem solving based discussion within small groups), I became aware that each lens offered a different perspective and different understandings of students’ mathematical development and engagement within the online space. No one approach allowed the research questions to be answered. However, independently of each other, the various approaches build to form a coherent and complete picture of how the students interacted in this innovative environment.
As such, an extensive discussion of the research project methodology is not provided in this section. Instead, a discussion of each theoretical lens is included separately in chapters 4, 5, 6 and 7 alongside a discussion of the research question addressed.

In Chapter 4 we investigate Online Mathematical Problem Solving and the Australian Curriculum. Themes are derived from the Australian Curriculum. These are coded for in online discussion and analyzed in an attempt to understand how the approach taken in this study allows students to develop in the areas of problem solving, reasoning and ICT proficiency within mathematics.

In Chapter 5 Bakhtin’s dialogic theory of language (Bakhtin, 1981) is utilized as a means to examine the interface between formal and informal mathematical vocabulary use in the online environment. This allows the reader insight into how students made use of language to develop and construct mathematical meaning, thus addressing the associated research question.

In Chapter 6 I examine the relationship between the densities of technical mathematical vocabulary use and the (teacher perceived) ability level of the students. I also aimed to test whether, over the duration of the intervention, the density of technical mathematical vocabulary use would increase. I adopted the three tier model of Beck, McKeown, and Kucan (2002) to allow coding for technical mathematical (tier 3) vocabulary. I examined the density of this vocabulary use in the various described contexts. The Clarification, Assessment, Inference, Strategies (CAIS) framework developed by Perkins and Murphy (2006) was utilized to understand the variety and modes of critical thinking occurring in the online environment. I was able to test if there was a relationship between the (teacher perceived) ability groups to which students were allocated and the preference towards a particular type of thinking. I was able to investigate whether the approaches to communicating their thinking changed over the course of the intervention.

Chapter 7 allowed me to investigate how the student participants within the study perceived the process of collaborative online mathematical problem solving. In this chapter, relative levels of student participation in the online environment were investigated in contrast with face-to-face interactions.
Prior to the commencement of data collection ethics approval was obtained from the University of Melbourne (HREC 1441426.1) and permission to undertake research in government schools was approved by the Victorian Department of Education and Early Childhood Development (2014_002258). In addition, I obtained written consent from the school’s Principal to proceed with the research plan outlined in the ethics documents. Letters, including HREC approved Plain Language Statements, were sent to all parents and guardians of students in the target classes seeking their consent for their child’s participation in the study. These documents explained to students and their parents that allowing or not allowing the students’ work to be used as research data would in no way affect their learning program. It was explained that problem solving and the use of the CSCL were being trialled as part of the students’ normal work in mathematics. All 54 students within the year 5 class in which the study took place returned their signed consent forms.

All methods of coding and analysis presented in the following chapters are founded in well established, research based, approaches. At least 10% of raw data was independently coded by my supervisor using each of the described approaches. At a minimum, 90% Inter-rater reliability was achieved for each approach. Where there was disagreement, discussion between coders provided agreement.

Examples of discussion data from the online CSCL environment are included within Chapters 4, 5 and 6. Detailed descriptions of exactly how decisions were made, within these chapters, when coding, provides an ‘audit trail’ of my research, ensuring a high level of dependability.
Chapter 4: Online Mathematical Problem Solving and the Australian Curriculum Proficiencies

In the project reported in this thesis, a key aim of the teaching and learning intervention was to develop an approach to integrating technology for the upper-primary mathematics classroom that was authentic, purposeful and most importantly would provide students the opportunity to engage in deep level, higher order and critical thinking. As reported, there is a general sense that within Australian primary schools, whilst technology is integrated in the primary mathematics classroom, the impact and quality of the learning taking place is often marginal and superficial at best (Day, 2013).

With the Australian context in mind, in this chapter I aim to address the following question:

*How does student engagement with online mathematical problem solving align with the Australian Curriculum?*

A desire to utilize technology in a manner that would promote and develop higher order thinking and reasoning led to the development of an approach that would require students to be able to communicate with each other. Therefore, a decision was made to utilize CSCL environments where students could use written communication and representations in order to talk, debate, reason and argue with each other about various mathematical issues.

As a result of the pilot project experience described in sub-section 3.7.2, the focus for the current intervention, instead of being founded in one content area, was instead focused within the Australian Curriculum proficiency strands of problem solving and reasoning (ACARA, 2014). It should be noted that whilst problem solving was the focus across the entire intervention, as each weekly problem required students to engage with one or more of the Australian Curriculum content strands (Number & Algebra, Statistics & Probability, Measurement & Geometry) (ACARA, 2014).
4.1 The Australian Curriculum as Prescribed Curriculum

The mathematical proficiency strand of problem solving as prescribed by the Australian Curriculum: Mathematics (ACARA, 2014) states:

*Students develop the ability to make choices, interpret, formulate, model and investigate problem situations, and communicate solutions effectively. Students formulate and solve problems when they use mathematics to represent unfamiliar or meaningful situations, when they design investigations and plan their approaches, when they apply their existing strategies to seek solutions, and when they verify that their answers are reasonable.*

(ACARA, 2014)

This statement was then summarised as the framework set out in Figure 4.1 and used to analyse the data.

<table>
<thead>
<tr>
<th>Mathematics</th>
<th>problem solving: Students Developing ability to:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Make Choices</td>
</tr>
<tr>
<td></td>
<td>Interpret (understand the problem)</td>
</tr>
<tr>
<td></td>
<td>Formulate (Use Procedure)</td>
</tr>
<tr>
<td></td>
<td>Model and Investigate Problem Situations</td>
</tr>
</tbody>
</table>

*Figure 4.1: Australian Curriculum Framework for Analysis of problem solving*

In addition, approaches to technology integration, within the area of Information Communication Technology (ICT): Mathematics, are described in the Australian Curriculum (ACARA, 2014). The following is stated:

*In the Australian Curriculum: Mathematics, students develop ICT capability when they investigate, create and communicate mathematical ideas and concepts using fast, automated, interactive and multimodal technologies. They use their ICT capability to perform calculations; draw graphs; collect, manage, analyse and interpret data; share and exchange information and ideas; and investigate and model concepts and relationships.*

(ACARA, 2014)

This statement was again used as a basis for a framework to be drawn on when coding all discussion generated throughout the intervention. This is represented in Figure 4.2.
It is worth noting that the curriculum recommends ICT (within the area of mathematics) be used to help students develop the ability to ‘investigate’, ‘create and communicate’, ‘share and exchange information and ideas’ etc. Day (2013) describes ICT use in Primary Mathematics as currently largely involving Internet derived drill and practice games. These are activities that arguably will only aid in the development of lower level skills and concepts. Therefore, a valid argument for the investigation of approaches to ICT integration within mathematics that potentially offer some of the curriculum defined outcomes is founded.

Figure 4.3 summarises themes extracted from the following description of the reasoning proficiency of the Australian Curriculum: Mathematics (2014).

Students develop an increasingly sophisticated capacity for logical thought and actions, such as analysing, proving, evaluating, explaining, inferring, justifying and generalising.

Students are reasoning mathematically when they explain their thinking, when they deduce and justify strategies used and conclusions reached, when they adapt the known to the unknown, when they transfer learning from one context to another, when they prove that something is true or false, and when they compare and contrast related ideas and explain their choices.
4.2 Drawing Together Online Discussion and Student Artefacts

The purpose of using the Australian Curriculum descriptors as a source for coding and analysis of the discussion data was twofold. Approaching the data in this way relies on evidence of students engaging in collaborative online mathematical problem solving in the manner described. This demonstrates the skills associated with the proficiencies of mathematical problem solving and reasoning and also ICT use within mathematics as recommended in the curriculum. Analysis of the students’ use of language (Chapters 5 and 6) is based on data derived from their online discussions without consideration of the artefacts (graphs, diagrams, tables) they have uploaded. It has been seen that both of these sources had much to offer; yet I struggled to find an approach that would allow them to be analyzed and interpreted together. The research offered in this chapter takes into account all of the information that the students shared via Edmodo: comprising the texts of their online discussion and artefacts (graphs, diagrams, tables, pictures) uploaded as separate attached files.

4.3 Coding & Analysis

As described in 4.1 three frameworks based on the Australian curriculum documents were used as a basis for coding and analyzing data. This analysis was supported by the use of the qualitative analysis software NVIVO (International, 2015). The various framework themes were designated as nodes in NVIVO (see appendix 4 for an example of coding).

Within the online discussion, identifiers were allocated indicating the presence of an uploaded student file. For the purposes of analysis within this chapter, whenever one of the uploaded files was detected the relevant file was opened and coded according to the three frameworks. These codes were attached to the indicators embedded within the
record of online discussion. All other online discussion was also coded using these Australian Curriculum based frameworks. As a result, I was able to investigate both the online discussion and the artefacts in a way that allowed them to be treated as two parts of a whole rather than independent entities.

An additional procedure undertaken was to separately code each of the indicators of uploaded files as either Word files, Excel files, or Paint files. This would allow us to gain a sense of where (or if) the students were more likely to engage in skill or conceptual development during their online discussions or whilst constructing their supporting artefacts.

4.3.1 Defining and Describing the Codes

In order to ensure consistency, reliability and validity (Guba, 1981; Shento, 2004) in the coding process, I have allocated each of the codes a definition and provided examples of how the codes were allocated.

A purposeful sample of 20% of the data was coded independently by my doctoral supervisor. This sample deliberately included sections of text that related specifically to the mathematical problems rather than chat like “is anybody there?” We achieved a 94% inter-rater reliability, discussed discrepancies and came to an agreement on appropriate codes. This audit supported the reliability and validity of the data analysis process.

4.3.1.1 Mathematics: problem solving Framework

Diezmann, Watters, and English (2001) provide discussion of problem solving and investigative approaches including descriptions of the different categories of mathematical problem solving. Their discussion was used to help inform the definitions provided in Figure 4.4. Four principal categories were employed. These are provided, along with definitions of these categories and an example of how each category was coded for.
<table>
<thead>
<tr>
<th>Category</th>
<th>Definition</th>
<th>Evidence</th>
</tr>
</thead>
<tbody>
<tr>
<td>Formulate (Use Procedure)</td>
<td>• Description of an approach to solving the problem</td>
<td>• An example of Formulation from the data would be a student suggesting the use of an algorithm to convert between 24 hour and 10 hour time (week 2)</td>
</tr>
<tr>
<td></td>
<td>• Description of a mathematical procedure was provided</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Demonstration of a procedure used (This may have occurred within a spreadsheet or other uploaded artefact.)</td>
<td></td>
</tr>
<tr>
<td>Interpret (Understand the Problem)</td>
<td>• The student was attempting to understand the problem</td>
<td>• An example of Interpreting from the data would be a student a student providing a definition of ‘rotational symmetry’ within the ‘symmetry’ problem</td>
</tr>
<tr>
<td></td>
<td>• The student understood the problem</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• The student was attempting to understand a comment made by another student</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• The student understood a comment made by another student</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• An artefact that suggests the student understood the problem</td>
<td></td>
</tr>
<tr>
<td>Make Choice</td>
<td>• The student states an opinion/preference when engaged in discussion/peers</td>
<td>• An example would be a student expressing that they believe ‘biggest’ should mean ‘longest’ in the ‘Biggest Dog’ problem</td>
</tr>
<tr>
<td></td>
<td>• The student makes decisions in the construction of uploaded artefacts</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• The choices by the student do not necessarily lead to accurate results</td>
<td></td>
</tr>
<tr>
<td></td>
<td>They simply need to be present</td>
<td></td>
</tr>
<tr>
<td>Model and Investigate Problem Situations</td>
<td>• Evidence of students attempting to make sense of ‘interdisciplinary’ nature of problem</td>
<td>• An example would be students providing additional contextual information about Canadian Geese in V patterns problem</td>
</tr>
<tr>
<td></td>
<td>• Uploaded Artefact allows creation of a model (mathematization) of the context-based scenario</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Exploration of related non-mathematical concepts to help move understanding of the context forward. A holistic approach is taken</td>
<td></td>
</tr>
</tbody>
</table>

Figure 4.4: mathematical problem solving Framework

4.3.1.2 Mathematics reasoning Framework (Logical Thought and Actions)

Seven principle categories were employed for the Mathematics reasoning Framework as displayed in Figure 4.5 (below).
<table>
<thead>
<tr>
<th>Category</th>
<th>Definition</th>
<th>Evidence</th>
</tr>
</thead>
<tbody>
<tr>
<td>Analysing</td>
<td>Evidence that student has gathered information/data and reorganised or reinterpreted this information or data. Often the product of this process is realised in an uploaded Artefact.</td>
<td>An example would be students gaining ‘height data’ of animals and sorting this in ascending/descending order within Excel.</td>
</tr>
<tr>
<td>Evaluating</td>
<td>Evidence of evaluation of data/information presented. Evidence of evaluation of ideas and thoughts provided by peers in online discussion. Evidence of the evaluation and modification of peers’ uploaded artefacts.</td>
<td>Example would be students referring to the criteria for the inclusion of certain shapes (in the ‘shapes’ investigation) when evaluating each other’s work.</td>
</tr>
<tr>
<td>Explaining</td>
<td>Students explain a concept or approach to solving the problem. The explanation may or may not be mathematically accurate. An explanation may be implied through the inclusion of indicators of mathematical thinking within uploaded artefacts.</td>
<td>An example could include a student providing discussion that, ‘biggest’ could be defined as ‘heaviest’, ‘longest’ or ‘tallest’ in the ‘biggest dog’ investigation.</td>
</tr>
<tr>
<td>Generalizing (Use of Explicit Equation/Formula)</td>
<td>Students providing explicit equations allowing them to generalize a rule that they have found. This may occur within uploaded artefacts or in online discussion.</td>
<td>Students using a formula in Excel to calculate the number of dog/camel/chameleon years given any number of human years in the ‘animal ages’ investigation would be evidence of this.</td>
</tr>
<tr>
<td>Inferring</td>
<td>This is evident when students make deductions based on other student comments, their own mathematical thinking or contextual clues contained within the problem. Inferences may be evident when students place their mathematical.</td>
<td>After sorting data related to the size of a dog students inferred which dog breed was the ‘biggest’.</td>
</tr>
<tr>
<td><strong>Justifying</strong></td>
<td><strong>Proving</strong></td>
<td></td>
</tr>
<tr>
<td>----------------</td>
<td>-------------</td>
<td></td>
</tr>
<tr>
<td>• Students provide more than explanation of approach taken/argument/opinion</td>
<td>• Students using criteria in ‘shapes’ investigation to support their opinion provides evidence of this</td>
<td></td>
</tr>
<tr>
<td>• Students reinforce statements with evidence or rationale</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Proving</strong></th>
<th><strong>Justifying</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>• In the context of year 5 mathematical problem solving, proof was considered evident where a generalizable rule was established</td>
<td>• Students generating an explicit equation to describe the V formation of geese in the ‘geese investigation’ was evidence of proof (at an age appropriate level)</td>
</tr>
<tr>
<td>• Students were able to utilize an explicit equation or statement that was true for all cases</td>
<td></td>
</tr>
</tbody>
</table>

**Figure 4.5: Mathematics reasoning Framework (Logical Thought and Action)**
4.3.1.3 *Mathematics Embedded ICT Framework*

Seven principle categories were employed for the Mathematics embedded ICT framework as displayed in Figure 4.6 (below).

<table>
<thead>
<tr>
<th>Category</th>
<th>Definition</th>
<th>Evidence</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Create and Communicate mathematical Ideas</strong></td>
<td>• Evident when students make statements or upload artefacts either explicitly or implicitly expressing mathematical thinking</td>
<td>• Students uploading a spreadsheet representing the growing pattern in the Geese problem would be an example of this</td>
</tr>
<tr>
<td><strong>Investigate</strong></td>
<td>• Students exploring concepts within the domain of mathematics, ICT or concepts that related to the contextual nature of a problem evidenced investigation here</td>
<td>• For example, students may have investigated why geese fly in ‘v’ formations</td>
</tr>
<tr>
<td><strong>Collect, Manage, Analyze and Interpret Data</strong></td>
<td>• Evidence is presented of students having collected, managed analysed or interpreted data. This may have occurred in online discussions or may be evident in uploaded artefacts</td>
<td>• An example would be students gathering data about different dog breed weights to support their investigation of the ‘biggest dog’</td>
</tr>
<tr>
<td><strong>Draw Graphs</strong></td>
<td>• Evident when students have uploaded an Excel or Word file containing a graph</td>
<td>• An example would be ‘pictographs’ created by students in Excel representing ratios between different animal ages</td>
</tr>
<tr>
<td><strong>Investigate and Model Concepts and Relationships</strong></td>
<td>• Evident where students have made visible connections and relationships they have developed</td>
<td>• Examples of students discussing approaches to developing a schema allowing them to convert between 24 hour and 10 hour time provide evidence of this</td>
</tr>
<tr>
<td></td>
<td>• Students may have provided discussion of their schema or they may have uploaded an artefact making their developing</td>
<td></td>
</tr>
</tbody>
</table>

82
Perform Calculations

- Students describe their calculations either in online discussion or provide them in an uploaded artefact
- Uploaded Excel files showing the use of an algorithm to generate the mean of a data set provides an example of this

Share and Exchange Information and Ideas

- Students provide discussion of their ideas and suggestions to help solve the problem in the online discussion
- Students make effort to represent their thinking in uploaded artefact
- An example of this would be a student disagreeing and pointing out an error in another student’s thinking in the ‘shapes’ investigation

Table 4.1 shows that when students used Excel, the files that they created and uploaded almost all involved formulation (the use of procedures), interpretation (or showed evidence that the students either fully understood or partly understood the problem), making choices and modelling and/ or investigation of some aspect of the problem.
situation. Of the 97 Excel files uploaded 95 of them displayed evidence of the development of these themes.

Table 4.1: Frequency of examples of Mathematics: problem solving Categories evident in uploaded artefacts

<table>
<thead>
<tr>
<th>Category</th>
<th>Excel Artefacts</th>
<th>Paint Artefacts</th>
<th>Word Artefacts</th>
</tr>
</thead>
<tbody>
<tr>
<td>Formulate (Use Procedure)</td>
<td>96</td>
<td>3</td>
<td>59</td>
</tr>
<tr>
<td>Interpret (Understand the Problem)</td>
<td>95</td>
<td>3</td>
<td>59</td>
</tr>
<tr>
<td>Make Choices</td>
<td>96</td>
<td>3</td>
<td>59</td>
</tr>
<tr>
<td>Model and Investigate Problem Situations</td>
<td>96</td>
<td>3</td>
<td>60</td>
</tr>
</tbody>
</table>

Of the 66 Word Files created and uploaded, 60 showed evidence of at least one of the four mathematical problem solving categories. In the 6 files where these themes were not detected typically these students were providing general (non-mathematical) background information about the problem. For example, a number of Word files that did not include evidence of the four mathematical problem-solving themes were detected in the Week 9 – ‘Geese’ problem. Some students elected to create Word files including general background information about geese and why they fly in a V formation. Whilst this information was interesting and added to the authenticity of the problem by providing general context that allowed the students to gain a clearer understanding of the problem, this was not coded as one of the problem solving themes, because a clear link to mathematical process/concept was not evident.

Table 4.2 provides an indication of differences between how students used the various software platforms in different problems. Whilst, across the nine weeks the overall mathematical focus was on problem solving, as indicated in the table, each week the problems drew on a particular mathematical content area specified by the Australian Curriculum.
Table 4.2: Comparison of Student use of Software Across Problems

<table>
<thead>
<tr>
<th></th>
<th>Excel Artefact</th>
<th>Paint Artefact</th>
<th>Word Artefact</th>
</tr>
</thead>
</table>
| Week 1 - Toilet Roll Length  
(Measurement & Geometry)  | 0              | 0              | 0             |
| Week 2 - 10 Hour Day  
(Number & Algebra)       | 0              | 0              | 1             |
| Week 3 – Symmetry  
(Measurement & Geometry) | 0              | 0              | 30            |
| Week 4 - Biggest Dog  
(Statistics & Probability) | 26             | 0              | 8             |
| Week 5 - Animal Ages  
(Number & Algebra)      | 30             | 0              | 5             |
| Week 6 – Shapes  
(Measurement & Geometry) | 0              | 3              | 15            |
| Week 7 - Pet Names  
(Statistics & Probability) | 10             | 0              | 0             |
| Week 8 - Mr Mac’s iPhone  
(Number & Algebra)  | 27             | 0              | 0             |
| Week 9 – Geese  
(Number & Algebra)      | 3              | 0              | 6             |

Students were encouraged to use any software platform throughout, (although at times they were encouraged to use a particular software) depending on which they believed would offer them the greatest opportunity to support their thinking and communicate their ideas.

Despite being led to preference a particular software platform in Weeks 5 and 8, most of the time students had free choice; therefore it is interesting to reflect on the preferences students have displayed depending on the mathematical content area being explored. It is apparent that when students developed and communicated their thinking in a problem where measurement & geometry was a focus, they preferred to use Microsoft Word. This is unsurprising given the lack of utility Excel represents for creating and manipulating shapes. Word by comparison includes a range of capabilities allowing students to create, enlarge, reduce, rotate, translate and reflect shapes. Additionally, having worked with Word many times prior to the study, students were relatively comfortable working with the software. This aligns with Goldenberg’s (2000) fluent use principle. In week 3, students were asked to create a panel of wallpaper, representing their understanding of symmetry (See Figure 4.7). Of the 30 artefacts
uploaded all utilized Microsoft Word for this problem. When students were asked to investigate four sided shapes in week 6 again most chose to use Microsoft Word. In this problem, whilst using the auto-shapes function within Word was heavily employed, an additional support mechanism for students was the ability to structure, organize and represent their thinking using tables in word (See Figure 4.8).

![Figure 4.7: Example of Student’s Investigation of Symmetry](image-url)
Table 4.3 (below) represents how students engaged in the four categories of MPS in online discussion compared to their level of engagement with these categories when constructing artefacts. It is evident from the table that students more commonly showed evidence of engaging with these important categories when representing their mathematical thinking through creating a representation within their uploaded artefacts. It is interesting that in the ‘Interpret’ category the distribution is more evenly shared between artefacts and online discussion. This indicates that students valued and benefited from unpacking and discussing their ideas with each other in order to ensure they fully understood the problem.
Table 4.3: Comparison of Development of problem solving Concepts in Artefacts Vs Online Discussion

<table>
<thead>
<tr>
<th>Category</th>
<th>Examples within all Uploaded Artefacts</th>
<th>Examples within Online Discussion</th>
<th>% of Developing Concepts in Uploaded Artefacts</th>
<th>% of Developing Concepts in Online Discussion (Excluding Artefacts)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Formulate (Use Procedure)</td>
<td>158</td>
<td>54</td>
<td>75</td>
<td>25</td>
</tr>
<tr>
<td>Interpret (Understand the Problem)</td>
<td>157</td>
<td>109</td>
<td>59</td>
<td>41</td>
</tr>
<tr>
<td>Make Choices</td>
<td>158</td>
<td>103</td>
<td>61</td>
<td>39</td>
</tr>
<tr>
<td>Model and Investigate Problem Situations</td>
<td>159</td>
<td>67</td>
<td>70</td>
<td>30</td>
</tr>
</tbody>
</table>

4.4.2 Student Development of Australian Curriculum Mathematics: reasoning Categories (Logical Thought and Actions)

Analysis of data taken from online discussion and uploaded artefacts shows that the types of reasoning engaged in over the period of the intervention changed from week to week. Analysis of data also shows that while students from the same in-class, teacher-designated, ability groups engaged in reasoning in some ways that were similar to each other, they also differed in subtle ways. Differences were found in the way boys and girls engaged in reasoning in the online environment. Table 4.4 (below) shows that there was a general increase in student reasoning across the nine weeks of the intervention. It appears that students were able to engage in analysis, evaluation and explanation throughout, however it is not until week four that consistent evidence of students engaging in Generalizing, Inferring and Proving occurs. Unlike, the analysis undertaken in this thesis of mathematical vocabulary development (See Chapter 6), the change of mathematical content area from week to week should not impact the results here. The evidence here suggests that students’ development of reasoning skills progressed over the course of the intervention. In weeks eight and nine students did not have the benefit of classroom discussion and support prior to engaging in collaborative problem solving. Therefore, the fact that students were able to show evidence of all areas of reasoning in these weeks was revealing. We can hypothesize that students’
development in the area of reasoning was not solely a representation of their ability to extrapolate what they had previously discussed in the classroom, but instead was a product of a growth in their collective ability to reason.

Table 4.4: reasoning used Across Nine Weeks

<table>
<thead>
<tr>
<th></th>
<th>Analysing</th>
<th>Evaluating</th>
<th>Explaining</th>
<th>Generalising</th>
<th>Inferring</th>
<th>Justifying</th>
<th>Proving</th>
</tr>
</thead>
<tbody>
<tr>
<td>Week 1 - Toilet Roll Length</td>
<td>1</td>
<td>3</td>
<td>22</td>
<td>0</td>
<td>0</td>
<td>13</td>
<td>0</td>
</tr>
<tr>
<td>Week 2 - 10 Hour Day</td>
<td>3</td>
<td>4</td>
<td>6</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Week 3 - Symmetry</td>
<td>1</td>
<td>4</td>
<td>35</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>Week 4 - Biggest Dog</td>
<td>32</td>
<td>43</td>
<td>46</td>
<td>0</td>
<td>2</td>
<td>6</td>
<td>0</td>
</tr>
<tr>
<td>Week 5 - Animal Ages</td>
<td>23</td>
<td>26</td>
<td>37</td>
<td>3</td>
<td>7</td>
<td>8</td>
<td>6</td>
</tr>
<tr>
<td>Week 6 - Shapes</td>
<td>12</td>
<td>15</td>
<td>17</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Week 7 - Pet Names</td>
<td>9</td>
<td>14</td>
<td>18</td>
<td>1</td>
<td>0</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>Week 8 - Mr Mac’s iPhone</td>
<td>25</td>
<td>31</td>
<td>50</td>
<td>3</td>
<td>5</td>
<td>17</td>
<td>5</td>
</tr>
<tr>
<td>Week 9 - Geese</td>
<td>11</td>
<td>17</td>
<td>21</td>
<td>5</td>
<td>2</td>
<td>8</td>
<td>3</td>
</tr>
</tbody>
</table>
Table 4.5 (below) shows the degree to which students who were assessed (by their teacher) as below level, at level and above level, in mathematics, engaged in reasoning throughout the intervention. Across the three groups there was a fairly consistent high level of Analysis, Evaluation and Explaining occurring. This is interesting because it might be assumed that the students who had been described as below level (by their teachers) would show less ability to engage in all areas of reasoning. Additionally, the lower ability students’ engagement with the remaining concepts (with the exception of ‘Generalizing’) was of a similar level to that of their peers. All students demonstrated fewer instances of Generalizing, Inferring and Proving when collaboratively problem solving. This is not unexpected given that these aspects of reasoning are considered to involve more sophisticated processes. The different problems themselves may have provided fewer or greater opportunities for students to demonstrate HOT.

<table>
<thead>
<tr>
<th>Reasoning Category</th>
<th>Below Level</th>
<th>At Level</th>
<th>Above Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>Analysing</td>
<td>36</td>
<td>51</td>
<td>43</td>
</tr>
<tr>
<td>Evaluating</td>
<td>48</td>
<td>72</td>
<td>51</td>
</tr>
<tr>
<td>Explaining</td>
<td>80</td>
<td>116</td>
<td>93</td>
</tr>
<tr>
<td>Generalising</td>
<td>0</td>
<td>4</td>
<td>8</td>
</tr>
<tr>
<td>Inferring</td>
<td>3</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>Justifying</td>
<td>18</td>
<td>26</td>
<td>18</td>
</tr>
<tr>
<td>Proving</td>
<td>4</td>
<td>3</td>
<td>8</td>
</tr>
</tbody>
</table>
Table 4.6 (below) represents how girls and boys, who had been allocated to ability groups, within the study engaged with the various categories of mathematical reasoning in the online learning environment. There is evidence that the ability of boys to Reason increased according to the ability group they had been assigned. For example, the Below Level Boys exhibited 8 instances of Analysing, the At Level Boys exhibited 13 instances of Analysing and the Above Level Boys exhibited 32 examples of Analysing. It is worth noting here that students had been evenly distributed across the three ability classifications. Thus, the tendency of this pattern to be replicated across the various reasoning categories is important. The reasoning of girls did not follow the pattern of increasing according to teacher assigned ability group. At Level and Below level girls showed evidence of a greater volume and variety of approaches to reasoning. This may indicate that the pre-post tests conducted for the purpose of allocating ability groups may not provide teachers with adequate information about their students’ ability to engage in mathematical reasoning.

Table 4.6: Comparison of reasoning between Genders

<table>
<thead>
<tr>
<th></th>
<th>Below Level Boys</th>
<th>Below Level Girls</th>
<th>At Level Boys</th>
<th>At Level Girls</th>
<th>Above Level Boys</th>
<th>Above Level Girls</th>
</tr>
</thead>
<tbody>
<tr>
<td>Analysing</td>
<td>8</td>
<td>28</td>
<td>13</td>
<td>38</td>
<td>32</td>
<td>11</td>
</tr>
<tr>
<td>Evaluating</td>
<td>9</td>
<td>39</td>
<td>20</td>
<td>52</td>
<td>39</td>
<td>12</td>
</tr>
<tr>
<td>Explaining</td>
<td>16</td>
<td>64</td>
<td>29</td>
<td>87</td>
<td>67</td>
<td>26</td>
</tr>
<tr>
<td>Generalising</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>3</td>
<td>7</td>
<td>1</td>
</tr>
<tr>
<td>Inferring</td>
<td>0</td>
<td>3</td>
<td>4</td>
<td>3</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>Justifying</td>
<td>3</td>
<td>15</td>
<td>11</td>
<td>15</td>
<td>13</td>
<td>5</td>
</tr>
<tr>
<td>Proving</td>
<td>1</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>7</td>
<td>1</td>
</tr>
</tbody>
</table>
4.4.3 Student Development of Australian Curriculum Mathematics Embedded ICT Categories

The following results indicate that the opportunities students have to develop mathematical ICT proficiency vary depending on the collaborative activity they are engaged in.

Table 4.7 (below) highlights both the number of uploaded artefacts produced by students over the course of the intervention, in addition to providing some indication of how the 3 software programs (outside of the base environment of Edmodo) differed in terms of opportunities for mathematical ICT concept development.

<table>
<thead>
<tr>
<th>Mathematics Concepts Embedded within Uploaded Artefacts</th>
<th>Excel Artefacts</th>
<th>Paint Artefacts</th>
<th>Word Artefacts</th>
</tr>
</thead>
<tbody>
<tr>
<td>Create and Communicate Mathematical Ideas</td>
<td>94</td>
<td>3</td>
<td>53</td>
</tr>
<tr>
<td>Investigate</td>
<td>88</td>
<td>1</td>
<td>41</td>
</tr>
<tr>
<td>Collect, manage, analyse and Interpret Data</td>
<td>87</td>
<td>0</td>
<td>28</td>
</tr>
<tr>
<td>Draw Graphs</td>
<td>92</td>
<td>0</td>
<td>16</td>
</tr>
<tr>
<td>Investigate and Model Concepts and Relationships</td>
<td>92</td>
<td>2</td>
<td>48</td>
</tr>
<tr>
<td>Perform Calculations</td>
<td>75</td>
<td>0</td>
<td>11</td>
</tr>
<tr>
<td>Share and Exchange Information and Ideas</td>
<td>94</td>
<td>3</td>
<td>57</td>
</tr>
</tbody>
</table>

Generally, Microsoft Excel appears to have offered the greatest number of opportunities for development in the categories. It is evident that there is a low level of student engagement in the concepts of, Collect, manage, analyse and Interpret Data, Draw Graphs and Perform Calculations whilst using Microsoft Word. It should be noted that 16 examples of students including graphs in their work have been included within the Word Artefacts, however these graphs had been created in Excel and then exported to Word. These are examples of the Draw Graphs category defined in Figure 4.6. It is not surprising that students used Excel to perform calculations because of the intrinsic opportunity Excel provides to enter and manipulate data. The findings here indicate that these upper primary students are very capable of utilizing many of the core functions
of spreadsheets (Excel). Students, were able to recognise the limitations and opportunities that the different pieces of software represented in terms of constructing mathematical representations. Whilst, at times students were given advice on which software might be more helpful for some of the problems, ultimately, they were always free to choose. The results indicate that they generally made sound choices when selecting a piece of software for a mathematical purpose. Interestingly, conversations with the students’ teachers prior to the study taking place, indicated a lack of confidence that the students would be able to make effective use of Excel.

Many of the students reported this intervention had been their first experience of using Microsoft Excel. Therefore, one might have expected students to lack confidence in using the software, perhaps preferring to perform calculations using hand held calculators or in workbooks. Instead we see that students generally embraced Excel. The findings in this study point to it being their preferred software for supporting and communicating their mathematical thinking and ideas.

Table 4.8 (below) provides a comparison between the level to which students developed and explored mathematical ICT concepts in online discussion and the level to which they engaged in this development when constructing their various uploaded artefacts. The total number of coded examples from each category was calculated. A percentage was then calculated and provided in Table 4.8, showing the percentage of instances where:

- artefacts were shown to allow students to engage in ICT based mathematical development and
- online discussion was shown to allow students to engage in ICT based mathematical development.

These percentages are represented in the latter two columns of Table 4.8.
Students created and communicated their mathematical ideas, investigated and modelled concepts and relationships both when creating artefacts and when discussing these ideas with their peers. Table 4.8 shows that students demonstrated these two categories at an almost equivalent rate in artefacts and in online discussion.

As shown in Table 4.8, the major areas where students were able to make use of creating artefacts were collecting, managing, analyzing and interpreting data, drawing graphs and performing calculations. When we consider that Excel was utilized more than any other piece of software it is unsurprising that these were the areas where software was most helpful. Important though is the implication that upper primary students in this study were very capable of making use of the software in a manner that moved their problem solving forward.

There were areas where the software appeared to be less helpful. The sharing and exchanging of information and ideas, both mathematical and organizational, happened more often in online discussion than through the additional layer of the uploaded artefact. An implication from this is that for online collaborative mathematical problem solving to promote the development of these concepts, the facility for both online

Table 4.8: Comparison of ICT: Mathematics Development in Online Discussion Vs Uploaded Artefacts

<table>
<thead>
<tr>
<th>Examples Uploaded Artefacts (no.)</th>
<th>Examples Online Discussion (no.)</th>
<th>Developing Concepts in Artefacts (%)</th>
<th>Developing Concepts in Online Discussion (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Create and Communicate mathematical Ideas</td>
<td>150</td>
<td>176</td>
<td>46</td>
</tr>
<tr>
<td>Investigate</td>
<td>130</td>
<td>114</td>
<td>53</td>
</tr>
<tr>
<td>Collect, manage, analyse and Interpret Data</td>
<td>115</td>
<td>9</td>
<td>93</td>
</tr>
<tr>
<td>Draw Graphs</td>
<td>108</td>
<td>0</td>
<td>100</td>
</tr>
<tr>
<td>Investigate and Model Concepts and Relationships</td>
<td>142</td>
<td>123</td>
<td>54</td>
</tr>
<tr>
<td>Perform Calculations</td>
<td>86</td>
<td>56</td>
<td>61</td>
</tr>
<tr>
<td>Share and Exchange Information and Ideas</td>
<td>154</td>
<td>248</td>
<td>38</td>
</tr>
</tbody>
</table>
discussion and mathematical software that allows for their thinking ideas to be communicated and developed is necessary. The artefacts provided a basis for the online discussion and the online discussion often led to the creation and the improvement of the mathematical meaning making representation contained within the artefact.

4.5 Summary

An aim of this chapter was to address the following question:

*How does student engagement with online mathematical problem solving align with the Australian Curriculum?*

With this in mind the following quote (describing anticipated approaches students should take to interacting with ICT in mathematics) taken from the curriculum shaped our analysis and discussion:

*Students develop ICT capability when they investigate, create and communicate mathematical ideas and concepts using fast, automated, interactive and multimodal technologies. They employ their ICT capability to perform calculations, draw graphs, collect, manage, analyse and interpret data; share and exchange information and ideas and investigate and model concepts and relationships. (ACARA, 2014)*

In the introductory chapter to this thesis we showed that whilst there have been intentions for the integration of ICT within Mathematics instruction to allow for the communication, representation and investigation of mathematical ideas and concepts over many curricula, for many years, as Day (2013) has reported this is rarely achieved in Australian Primary Mathematics classrooms.

Through examining the Australian Curriculum: Mathematics (ACARA, 2014) and delineating three frameworks identifying student use of problem solving, reasoning and ICT embedded Mathematics, an approach that offers possibilities for achieving the goals set by the curriculum has been shown.
Chapter 5: Online Mathematical Problem solving and Communicating Mathematical Meaning

The role of language as social semiotic has been extensively acknowledged within teaching and learning research. See for example, Austin and Howson (1979); Mercer and Sams (2006); Morgan (2005); Morgan, Craig, Schuette, and Wagner (2014). The social construction of knowledge takes place in a variety of contexts. It may occur in the physical classroom or, as is the case in this study, may occur online in the CSCL environment.

In this chapter, we aim to address the following research question:

How is language used to communicate mathematical meaning when Year 5 students work in an online CSCL environment?

This chapter is a more comprehensive adaptation of Symons and Pierce (2017). The structure will firstly involve providing a theoretical framework relying on the work of Bakhtin (1981) and an interpretation of his work by Barwell (2012). We then describe an approach to coding and analysis of the online discussion data with this theoretical framework as a basis. One rich excerpt of discussion from each of the nine problems is then analysed according to the framework and then finally some conclusions and implications are offered.

I will explore the research question by investigating how informal mathematical registers (IMR) and formal mathematical registers (FMR) (Halliday, 1978) are used to develop and communicate mathematical meaning when working in these environments. I will consider if and how language is used and developed in the space between the FMR and IMR. The notion of the Transitional mathematical Language suggested by Herbel-Eisenmann (2002) will be adapted to include the Bakhtinian (1981) notion of dialogism. Through appropriating these perspectives, I will offer the term Transitional mathematical Register. These are issues worthy of consideration for mathematics educators because it is commonly believed that it is important to assist students to transition from the use of an IMR to a formal register (Barwell, 2012). In the current Australian Curriculum (ACARA, 2014), little mention is made of students using correct
mathematical language (the FMR) to express their mathematical thinking in the Primary Years. However, prescriptions that students should be, for example, “using the properties of similarity and ratio, and correct mathematical notation and language, to solve problems involving enlargement” (ACARA, 2014) become common place in the upper secondary curriculum.

5.1 Use of Registers in Mathematics in Online Mathematical Problem Solving

Halliday (1978, p. 195) describes registers as “a set of meanings that is appropriate to a particular function of language, together with the words and structures which express these meanings”. He lists various ways in which words come to be accepted, and their meaning agreed on, for usage within a given register. In this section of the study the ‘mathematics register’ is considered, Halliday provides the following ways that words and phrases come to be accepted within a given register:

- Reinterpreting existing words. E.g. row, weight, random
- Creating new words out of native word stock. E.g. Clockwise, output
- Borrowing words from another language. E.g. Degree, infinite, probable
- ‘Calquing’: creating new words in imitation of another language. E.g. almighty calqued on omnipotens
- Inventing totally new words. E.g. gas
- Creating ‘locutions’. E.g. right-angled triangle, lowest common multiple
- Creating new words out of non-native word stock. E.g. parabola, denominator

(Halliday, 1978, p. 195)

The primary school students who participated in this study were gradually being introduced to the language of the FMR. I was particularly interested in the words that they chose to use in their online discussions. I was interested in whether the words and structures would be used in unusual or atypical ways. It was anticipated that the CSCL environment may encourage students to trial and experiment with new and emerging understandings of specific mathematical vocabulary. This was anticipated because students were given a great deal of freedom within the space and were encouraged to take risks with their thinking and discussion. Additionally, no facilitating moderator
was present in the online space. This decision was deliberately conceived, to allow the discussion to not be directed or biased in any way. For young students, the IMR encompasses everyday words such as “going” or “pointy,” while “multiply”, “equation,” and “median” lie in the FMR.

The acquisition of appropriate language is a part of learning mathematics. Doerr and Lerman (2010) describe communication as the driving force behind all learning. Their four-year study provides insights into the role of speaking, writing and reading within mathematics teaching and learning. They draw on Herbel-Eisenmann’s (2002) concept of a Transitional mathematical Language (TML) when referring to teacher or student developed idiosyncratic language specific to a particular classroom and note the need, in time, for the use of official mathematical words. They emphasise that, if mathematical language is to be appropriated it is important for students to have opportunities to discuss mathematics with peers, to connect words to each other and to contexts meaningful to them.

5.2 Language use in CSCL

Working in a CSCL environment requires the use of appropriate language that carries and communicates the meaning of the author of the post. Turvey (2006) notes that when students share their ideas via virtual exchanges this creates an opportunity for learning to be observed. Sharing mathematics asynchronously online removes communication through facial expressions and gesture but enhances the opportunities for visual, graphic and tabular representations. Lemke (2003) describes the difficulty some students experience trying to communicate their mathematical meaning when describing a pattern or relationship. Nason and Woodruff (2005) suggest that the use of multiple representations, some offering dynamic manipulation, “enable young children to communicate meaning via showing and telling rather than by merely telling” (p. 119).

However researchers point to superficial online talk that can be a result of lack of structure and lack of instruction with regards to what productive talk might constitute (Hong & Jacob, 2012; Mercer & Wegerif, 1999; Pifarré & Staarman, 2011). These authors equate productive talk with critical thinking, reflective thinking and creativity. They believe well prepared student online interaction, with access to multiple
Online Mathematical Problem solving and Communicating Mathematical Meaning

representations, makes CSCL ideally placed to positively impact current pedagogies (Pifarré & Staarman, 2011; Wegerif, 2007).

5.3 Bhaktinian Perspectives

This section of the study draws on Barwell’s (2012) application of the Bhaktinian dialogic perspective as a means to expose the tensions that exist between informal and formal mathematical language. He demonstrates that the FMR is privileged throughout international curricula by pointing to a need for these documents to require relatively simple and informal mathematical language to describe mathematical ideas in the earlier years of schooling, whilst working towards embracing the FMR in later years. He argues that informal and formal registers are always required and always in tension.

Barwell (2012) suggests that privileging the FMR within the curriculum is not ideal because it places greater importance on the ‘correct’ use of mathematical language at the potential expense of meaning making. Bakhtin’s (1981) view of language was that it is situated, dynamic and dialogic. He sees languages as being either unified (unitary) or, to use his term, in a state of ‘heteroglossia’. The theoretically complete FMR can be seen as a unified language. The tensions and centripetal forces that exist within curricula, our schools and educational institutions and society around us, mandate and expect a single, agreed upon language and register for the teaching and learning of mathematics. Employing the centripetal force of the unitary language may inhibit students’ experimentation and trialling of new and unfamiliar language. As a theoretical construct, there is a place for the FMR; however experience suggests that, in the reality of a classroom, it seldom exists.

In the analysis that follows students’ use of language in the CSCL environment is examined for evidence of the four tenets that Barwell (2012) identifies in Bakhtin’s work. These four tenets (referred to in this chapter as B1, B2, B3 and B4) will frame subsequent discussion and language use in mathematics. These tenets both inform each other and conform to each other.

B1. Language is dialogic.

Language use in the physical mathematics classroom and online environment is dialogic. A most obvious example of this occurs when students engage in mathematical
conversations in small or large groups. The less obvious examples of this, as Bakhtin sees it, are utterances of the past informing all present and future utterances. Bakhtin suggests that any utterance made in the present is in dialogue with those made in the past. He provides only one exception to this, illustrating the all-encompassing nature of this tenet:

Only the mythical Adam, who approached a virginal and as yet verbally unqualified world with the first word, could really have escaped from start to finish this dialogic inter-orientation with the alien word that occurs in the object. Concrete historical human discourse does not have this privilege: it can deviate from such inter-orientation only on a conditional basis and only to a certain degree (Bakhtin, 1981, p. 279).

B2. Language precedes us.

All language used in the physical or online mathematical classroom context is in dialogue with that of the past. This applies to all language, whether taken from the mathematical register or other languages or registers.

B3. Tensions exist between the unitary language and heteroglossia.

For any given moment, a range of alternative modes of communication are possible. These various registers or languages compete with each other. The adoption, selection and use of these languages causes tension. Whilst we place a major focus on the tensions that exist between the privileging of the FMR and the IMR, we will also consider more literal tensions that build as a by-product of the approach students take to discourse and use of language within the online space.

B4. Language is not unidirectional.

Bakhtin emphasises the great variety of routes that discourse and language take. Barwell (2012) suggests that the various imprecise ways in which language may be used in the mathematics classroom contribute to meaning making. This dialogic perspective is in conflict with views on mathematical language acquisition promoting the movement from informal to formal language use as unidirectional.
5.4 Coding & Analysis

A frame of Barwell’s (2012) tenets, as informed by Bakhtin (1981) was applied to dialogue generated within the CSCL environment, with the intention of highlighting forces that may inhibit or promote mathematical language development. The term ‘utterance’ will be used to refer to one ‘turn’ taken, as part of discussion in the online environment. Whilst this may be an atypical use of the word, we choose to use it because of its association with a range of social theorists inclusive of Bakhtin.

Language that represents students’ mathematical thinking has been bolded. This includes language that might usually be attributed to a FMR or more informal language that has been used to convey mathematical meaning. This action was taken to allow a careful examination of the dialogic nature of language-based mathematical interactions within the online space used in this study and to establish how the interactions correspond/align with Barwell’s (2012) four tenets.

The following excerpts of online discussion have not been edited for punctuation or spelling. The online discussion is provided in this way in order to preserve a sense of the students’ level of development in their written expression and communication. The posts within each excerpt all occurred within one week of each other. At times students posted immediately, one after another, thus making the process ‘synchronous’. However, more commonly students would log in, in their own time and post some time after the previous post (asynchronously). A clear ‘log’ of exactly when each post occurred, is not available (as this was not a feature available within Edmodo at the time). Again, coding was cross-moderated with my principal supervisor in order to ensure validity and reliability.

5.5 Results and Discussion

The following data are excerpts of discussion from each of the nine problems solved by the students over the course of data collection. A single representative excerpt of discussion for each problem has been selected for the purpose of balance, ensuring rigor and some degree of efficiency of space within the write up of this study. Excerpts selected were those that I along with my supervisor felt best represented student
development of a ‘dialogic’ learning space. Typically, the excerpts provided display more than one Bakhtinian tenet.

Throughout each of the nine excerpts students’ attempts at conveying mathematical ideas have been bolded, highlighting use of FMR or IMR.

The excerpts and analysis are provided in the order in which they occurred over the course of the intervention.

5.5.1 Toilet paper

In this task (see Figure 3.1) students were required to estimate/ calculate the length of a toilet roll without unrolling it. This was given in the first week of data collection and as can be seen from Figure 5.1 in this preliminary stage of the study, a dialogic approach was limited within student utterances. In this initial investigation, it appears that rather than collaborating, the students intended to independently state their responses to the problem and included explanation of how they came to their answer.

As noted in B1 and B2 (see section 5.3), Bakhtin views all language as dialogic, that is, all language used in the present is also informed by, and in dialogue with, that of the past. Illustrative of this is the likelihood that the contributions in Figure 5.1 were influenced by the discussion led by me (the author of this study) in the classroom prior to the students sharing their thoughts in the online space. During the classroom discussion, the words ‘measurement’, ‘estimation’, ‘multiply’ and ‘multiplication’ were used frequently. We see these words in the discussion excerpt, thus providing evidence of the ongoing nature of the ‘dialog’.

Sally: Hi my answer is 9,112cm. I got this by measuring 1 sheet and i searched how many sheets are on a new toilet paper roll. I multiplied the sheets measurement with the number of sheets are on a new toilet paper roll. This is how i got 9,112cm.

Jessica: My answer was 2,090cm. All i had to do was look on the paket with the toilet rolls in it and on the paket it said how many sheets were on each roll and how long each sheet was. When i found how long and how many sheets there were i just multiplied and then i got my answer 2,090!

Nathan: Hi my answer is 9,110 cm. I got this answer because I searched up in google how long one piece/block is. Then I estimeted it is all together .

Krystal: My answer is 17,100 cm. I found this out because it said on the packet of toilet rolls.
All I had to do multiply 180 x 95 = 17,100 cm.

Figure 5.1: Discussion Excerpt from Toilet Paper Problem – Week 1

5.5.2 Working Day

In this task students were asked to decide and communicate how our system of time would work if we had 10 hours per day, sometimes referred to as ‘decimal time’, rather than the current 24 hours (See Figure 3.2). They were then asked to consider what a school timetable might look like based on the ‘new’ system. There were a number of extension questions (e.g. what would the speedometer on a car look like and how might it change?), however no groups were able to get to that point.

Isla: So who will be the leader? And I have an idea. What if we get the 24 hrs and the 10 hrs and divide the ten by 24 to get a clearer idea
Josie: Yeah I'll try
Josie: You cant divide ten by twenty four it has to be twenty four divided by ten.
Josie: The answer to it is 2.4
Isla: So 2 hrs and 40 min would be one hour 24 hr time??
Josie: Yeah I divided twenty-four by ten and 2.4 was my answer
Isla: Can I please be the leader??

Figure 5.2: Discussion Excerpt from Working Day Problem – Week 2

As noted, in the previous week (see Figure 5.1) utterances were posted to the online space, however the students appeared to work in isolation, without collaboration. In Figure 5.2 we see the beginnings of a more collaborative ‘dialogic’ approach emerge between Josie and Isla. The following interaction between reflects this:

Isla: So who will be the leader? And I have an idea. What if we get the 24 hrs and the 10 hrs and divide the ten by 24 to get a clearer idea
Josie: Yeah I'll try
Josie: You cant divide ten by twenty four it has to be twenty four divided by ten.

We see the two students negotiating meaning through dialogue in this example. Isla offers her suggested strategy, which is quickly refined by Josie to reflect a solution that may help them move forward with the problem. Josie’s response to Isla’s original utterance reflects B3 (Tensions exist between the unitary language and heteroglossia). Her strongly proposed response: You cant divide ten by twenty four it has to be twenty
four divided by ten, indicates her impatience or incredulity towards Isla using mathematical language in a way that she believes is not in keeping with the FMR (that should constitute the ultimate goal). Josie accurately indicates Isla’s slightly erroneous thinking, through pointing out that the required calculation is 24 divided by 10 (and not the reverse). However, both girls fail to realise that 2.4 represents 2 hours and .4 of an hour, rather than 2 hours and 40 minutes. Neither girl has accounted completely for the conversion from the current modified sexigesimal (base 60) system of measuring time to a theoretical metric/decimal system.

Whilst students may not be consciously aware of the curricula imperative moving them towards using the FMR, even at this early stage of their schooling they may have internalized a need to express themselves in this way. A desire to communicate their mathematical meaning is to some extent in conflict with a desire to express themselves using the language of mathematics, the FMR.

5.5.3 Wallpaper Symmetry

In this problem/ investigation students were required to demonstrate their understanding of symmetry by creating a piece of ‘digital wallpaper’. Students were asked to represent line/mirror symmetry, rotational symmetry and translational symmetry and then describe how symmetries were evident in their wallpaper in their online discussion (see Figure 3.3).
Olivia: I think to **work this out** we would need to **choose a shape** with the **pointy sides** (don't really know how to say it) so it would be easier with for us to do it does anyone agree with me?

Olivia: When I mean the **point sides** something like the example Mr. Symons showed us a **shape** similar to that.

Chris: What **shape** is everyone deciding on. I was thinking of a **hecsigon**

Olivia: I'm thinking of an **shape** that has a **pointy side**. It also would be much easier if we do a **shape** that has a **pointy side** in my opinion. Does anyone agree with me?

I'm also thinking about doing little key box and say what you did and also saying what we used and explain how we done it? And why we **came up with the shape** we are going to use? Anyone agree with me?

Olivia: I changed it I have done a triangle I created something like a fan so when it spins you could see the pattern and also it would never changes I have uploaded mine to edmodo.

Zander: **Mirror/Line Symmetry**- **line symmetry** means when you have a **shape** or anything, and you cut it in half, it looks exactly the same size and lining on every single thing as the other side.

**Rotation Symmetry**- rotational symetry means, depending on how many pointy sides they have, say for example, i had a plus sign +, it has 4 pointy sides. So then, after you move it 4 times, it goes back to the same spot.

**Reflection Symmetry**- reflection symmetry means if you have a picture of your face, you keep drawing that, making look the same height, the same length, and etc.

Olivia: week_3_investigation_symmetry_2.docx

Igor: week_3_investigations_homework.doc

Zander: Good job. But there was a slight mistake.

Zander: This is Week 3 homework. It isn't really my work. I just edited Isaac's so it's better and it has more symmetry.

week_3_investigations_homework.doc

Zander: I edited Olivia's because she made a slight mistake with her fan picture on the right.

week_3_investigation_symmetry_2.docx

Olivia: Oh thanks..

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**Figure 5.3: Discussion from Excerpt of Wallpaper Symmetry Problem – Week 3**

In Figure 5.3 we see a number of examples of students wanting to express their mathematical thinking, but doing so with some reluctance. This may be a result of an inner tension (B3) caused by their desire to express their mathematical thinking in the FMR and their understanding that they lack the words to do so. An example of this occurs in Olivia’s opening utterance:

*Olivia: I think to **work this out** we would need to **choose a shape** with the **pointy sides***
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(dont really know how to say it) so it would be easier with for us to do it does anyone agree with me?

Here Olivia has stated that she does not have the specific formal mathematical vocabulary to describe the geometric feature to which she refers.

She states:

Olivia: i changed it i have done a triangle i created something like a fan so when it spins you could see the pattern and also it would never changes i have uploaded mine to edmodo.

Olivia’s use of language may be referred to as disjointed (certainly far removed from the FMR), underlying this however is evidence of her gradual development of an important mathematical concept.

When Olivia’s comments are read carefully, we realize that Olivia is developing an understanding that when considering rotational symmetry, a shape will look identical when rotated on its axis the number of degrees corresponding to its order of rotation. She does not have the words to describe this accurately, however she is able to make her meaning known through language that is neither entirely informal language nor part of the FMR. This is evidence of what we describe as the Transitional mathematical Register (TMR). This category is closely related to Herbel-Eisenmann’s (2002) Transitional mathematical Language. We have used a different term because the phenomenon we describe identifies language that can evolve, take any number of routes and is suggestive of Bakhtin’s (1981) notion of heteroglossia. This was not an explicit focus of Herbel-Eisenmann (2002).

It is interesting to note that Olivia’s use of language evolves from a transitional mathematical register to represent mathematical thinking and becomes a shared language through dialogue with other students in her group. Zander states:

Rotation Symmetry- rotational symetry means, depending on how many pointy sides they have, say for example, i had a plus sign +, it has 4 pointy sides. So then, after you move it 4 times, it goes back to the same spot.

It appears that Zander has observed Olivia’s use of the phrase ‘pointy sides’ as a form of mathematical meaning making and is using it in a, mathematically accurate similar
way to Olivia who first introduced it within the discussion. This development of a shared language, and through this a shared approach to meaning making exhibits aspects of B1, B2 and B4.

Worth also noting is Zander’s statement, after you move it 4 times, it goes back to the same spot. If read literally, this statement would be quite difficult to make sense of, but, given the context, we can see that Zander is making the accurate assertion that, when rotated $4 \times 90$ degrees, the cross will return to its original position. Again, his adoption of language from a ‘transitional mathematical register’ allows him to express his mathematical thinking.

5.5.4 How Big is a Dog?

This problem firstly involved students collaboratively deciding on an agreed definition of ‘biggest’ dog. The word ‘biggest’ was used intentionally in the problem because of its ambiguous meaning. As can be seen in Figure 5.4, the students spent time discussing the various possibilities for ‘biggest’ (e.g. longest, heaviest, tallest etc).

Secondly, students searched for numerical data allowing them to represent graphically the results of their investigation based on their definition of ‘biggest’ (see Figure 3.4).
Donna: Hi everyone,

Firstly, I’d like to know if I can be group leader for this activity?

Secondly, everyone needs to pick 5 breeds of dogs to research on. (please reply your breeds)

Thirdly, I will make an excel sheet including my breeds of dogs and their weight and leave it open for others to edit so you can put your breeds and info into excel and when finished anyone is free to make it into a table.

Donna: Can I please do the Border Collie, Old English Sheepdog, Kelpie, Rotweiler, Weimaranier

Larry: Ok

Donna: Hi, this is a table that includes work from everyone. All about the dogs!

biggest_dog.xlsx

Opal: done the task

opal_edmodo_week_4.xlsx

Opal: but the bars won’t work

Donna: Opal I can’t include your work because everyone is using the weight!

You’ve used the height or length either one.

Opal: but I did the task?

Donna: but you did it without looking at anything we said which was about weight.

Larry: this is my dog breed table

dog_breeds_edmodo.xlsx

Donna: I am getting everyone’s work and copying the info into one document I will present the table and graph when I get everyone’s info.

Chris: the great dane is the biggest dog and its about 100 centimeters (40 inches). It weighs up to about 54 kilograms (120 pounds)

Chris: week 4 - how big is a dog

Larry: we’re doing the weight!

The discussion in Figure 5.4 shows a struggle between students to develop a shared definition for ‘biggest’. Initially, Donna assumes an authoritative role within the discourse:

Donna: Hi everyone,

Firstly, I’d like to know if I can be group leader for this activity?
Secondly, everyone needs to pick 5 breeds of dogs to research on. (please reply your breeds)

Thirdly, I will make an excel sheet including my breeds of dogs and their weight and leave it open for others to edit so you can put your breeds and info into excel and when finished anyone is free to make it into a table.

Initially, Donna asks the group whether she can leads the group for this task. However, without waiting for a response she begins to provide a sequence of instructions for the group to follow.

We then see her assertion that ‘biggest’ should be defined by the weight of the various dogs.

The following discussion then occurs between Opal and Donna:

*Opal: but the bars won’t work*

*Donna: Opal I can’t include your work because everyone is using the weight!*

*You’ve used the height or length either one.*

*Opal: but I did the task?*

*Donna: but you did it without looking at anything we said which was about weight*

Here Donna’s earlier assertion that the group should use weight as the variable to determine ‘biggest’ ‘appears to be in conflict with Opal’s contributions to the group activity. She has identified the height or length of the various dog breeds as her approach to classifying them. However, Donna points out that Opal’s work cannot be assimilated into the group’s spreadsheet because she has used different variables. Opal misunderstands the issue being relayed when she replies, but I did the task? Donna attempts clarification by stating in a matter of fact manner, but you did it without looking at anything we said which was about weight.

Chris and Larry conclude this excerpt from the discussion articulating similar difficulties to those of Opal and Donna. Again, Chris provides data about the height of a dog (a great Dane) and is then reminded by Larry that weight is the variable being used to support their ideas and work on the problem.
In a very literal sense, tensions are evident between Opal and Donna as they searched for shared understanding of their mathematical meaning. Clearly there is a range of different approaches (variables) the group can choose from in order to move forward with the problem. Different alternatives are selected by group members and this is a cause of some tension. Unclear definitions prompt examples of B3. *Tensions exist between the unitary language and heteroglossia.*

### 5.5.5 Animal Ages

Students were asked to consider the commonly stated claim that ‘each year a human lives is equivalent to 5 cat years’. They were then asked to think about how this might be represented mathematically (see Figure 3.5). They considered and compared the relative rates of aging of camels, dogs and lorikeets. Students represented their thinking using tables and graphical representations predominantly in Excel.

---

**Olivia:** what does it mean by saying **how old**? Could someone please help me?

**Zander:** It means **what age** is your cat.

**Chris:** we should all so put more animals like eliphants and girrafs

**Olivia:** yeah but first we need to do the cats and the other animals that mr symons has given us so we should be talking about that first and then do the other animals in a different excel document. so how would we be doing the cat one first? lets discuss the cat one first.

**Olivia:** i still kinda dont get it when it says by **human years** could anymore help me please?

**Zander:** What do you mean. You just go up by one. I'm confused. And what do you mean by anymore.

**Olivia:** So what we should do in my opinion is for the cat we could work it out with the strategies that Mr. Symons showed us and like for the camel as well and then who should put all of them together?

**Zander:** I am **20 years old in camel years**.

**Olivia:** Yea same

**Zander:** I'm going to do the section where you have to show **how old your family is** if they were Lorikeets.

**Olivia:** I'm not really sure about that section but soon I would be doing the **cat years** it goes by 5s yeah?

**Olivia:** **So like 5 10 15**

And so on. What does it need to go up to again?

---

*Figure 5.5: Discussion of Animal Ages Problem*
Olivia, Zander and Chris are involved in the initial discussion as depicted in Figure 5.5:

*Olivia:* what does it mean by saying **how old**? Could someone please help me?

*Zander:* It means **what age** is your cat.

*Chris:* we should all so put more animals like eliphants and girrafs

*Olivia:* yeah but first we need to do the cats and the other animals that mr symons has given us so we should be talking about that first and then do the other animals in a diffrent excel documant. so how would we be doing the cat one first? lets discuss the cat one first.

*Olivia:* i still kinda dont get it when it says by **human years** could anymore help me please?

*Zander:* What do you mean. You just **go up by one**. I'm confused. And what do you mean by anymore.

Olivia, Zander and Chris initially take a collaborative approach to their discussion. We see Olivia, appealing for support and Zander offering some advice in response to her question.

Chris takes the opportunity to elaborate the problem stating, **we should all so put more animals like eliphants and girrafs**. This suggests his confidence and willingness to think beyond the instructions provided and take advantage of both the open ended nature of the problem and also the many possibilities for research that the online environment allows.

Again, Olivia asks for further support, specifically asking for clarification about the term ‘human years’. Upon her second request for support Zander seems to become irritated stating, *What do you mean. You just go up by one. I'm confused. And what do you mean by anymore.*

B3 and B4 seem to be represented within this excerpt of the discussion. Tensions evidently develop between Zander and Olivia through language based choices and how they choose to engage with one another. Olivia appears to be asking questions and seeking clarification as an important part of the mathematical meaning making process. This is understandable as students were encouraged to engage in the process in a way similar to this in the classroom discussions. We can theorise that Olivia is in dialogue with the discussion previously occurring in her classroom (B1, B2 and B3). Zander, on
the other hand, appears to think his explanations should be clear to others. He seems to view the questions Olivia poses as distracting, trivial and unhelpful to moving the group forward in the greater task.

Chris’ statement, *we should all so put more animals like elipphants and girrafs* appears disconnected from the discussion taking place between Zander and Olivia either side of it. However, Chris’ intervention highlights the great variety of routes that discourse and language can take. That is a feature of B4; *Language is not unidirectional*. The, at times, arbitrary way in which language is used to connect with and move forward mathematical thinking is in conflict with views of mathematical language acquisition as a linear and systematic unidirectional process.

The discussion continues:

*Olivia:* So what we should do in my opinion is for the cat we could work it out with the strategies that Mr. Symons showed us and like for the camel as well and then who should put all of them together?

*Zander:* I am 20 years old in camel years.

*Olivia:* Yea same

*Zander:* I'm going to do the section where you have to show how old your family is if they were Lorikeets.

*Olivia:* I'm not really sure about that section but soon I would be doing the cat years and for the cat years it goes by 5s yeah?

*Olivia:* So like 5 10 15

And so on. What does it need to go up to again?

As the discussion between Olivia and Zander continues, Olivia continues to take a more collaborative approach by making suggestions, asking for clarification and seeking the other group members’ thoughts. However, we see the approach taken by Zander change subtly. When he states, *I am 20 years old in camel years*, Zander seems to be deliberately ignoring the contribution offered by Olivia. Olivia again responds in a way that seems focused on promoting group success when she states, *Yea same*. She responds to Zander’s contribution despite his seemingly wilful efforts to exclude her
from discussion. Again, literal ‘tensions’ (contributing to B3) are perceptible here, despite Olivia’s best efforts to retain harmony and a sense of collaboration within the group.

5.5.6 Shapes

In ‘Shapes’ students investigated alternative shapes that could be made with four straight sides. They then provided discussion about the identity of the shapes they represented based on a definition that they developed and provided (see Figure 3.6).

Zander: This is the table that he showed us how to do it. If you have any comments please, I actually advise you to reply or comment on this. I have not finished it but if you have anything you would like me to change please reply. Haven't finished but please get some more shapes so I can finish it off. This is not my computer so please say anything if there is something wrong.

Thanks Group 9,

Yours Sincerely, Zander.

table_for_edmodo_about_shapes.docx

venn_diagram_edmodo_zander_k.docx

Olivia: I'm think of a square shape that have straight sides and a rectangle anyone else have a idea ?!

And they also have a straight length for the sides

And the square is 90 degree

And also the spyware is a right angle anyone agree with me?

Zander: Sorry something going on with me and my dad important. So i have read your replies, I think we should do what you said. I agree with you.

Olivia: With the table what shape are you planning to do on I know that a square and rectangle could be one what else do you agree???

Olivia: With the table that you have done it was good didn't it also have to only be with straight sides ?? Like you added a circle but it was still really good

Zander: Don't know, but I'll add another bar so it will make for that one if we were not supposed to do it.

Olivia: Okay that is a good idea

Figure 5.6: Discussion of Shapes Problem
Figure 5.6 depicts further discussion between Olivia and Zander. Viewing conversation between these two students in subsequent tasks gives some indication of how their language use changed over time within the online space:

Zander: This is the table that he showed us how to do it. If you have any comments please, I actually advise you to reply or comment on this. I have not finished it but if you have anything you would like me to change please reply. Haven't finished but please get some more shapes so I can finish it off. This is not my computer so please say anything if there is something wrong.

Thanks Group 9.

Yours Sincerely, Zander.

table_for_edmodo_about_shapes.docx

venn_dia_gram_edmodozander_k.docx

Olivia: I'm think of a square shape that have straight sides and a rectangle anyone else have a idea?!

And they also have a straight length for the sides

And the square is 90 degree

And also the spyware is a right angle anyone agree with me?

Zander initiates the discussion by stating This is the table that he showed us how to do it. This is evidence of B2. Students working in the online environment drew on the utterances and discussion that occurred previously within the classroom in order to move discussion and thinking forward.

Zander takes the role of leader within the group through his choice of language: If you have any comments please, I actually advise you to reply or comment on this. His use of the words actually advise you develops a sense that he is in an authoritative position; a position that grants him the right within the group, to make decisions and influence the group more so than other group members.

Olivia responds by making her suggestions to move the problem on. She states: I'm think of a square shape that have straight sides and a rectangle anyone else have a idea?!

It is interesting to note here that whilst her use of grammar lacks accuracy,
generally her use of mathematical language is sound. We may infer that Olivia understands that both squares and rectangles represent examples of four sided shapes. The difficulty she has communicating her thinking is evidence of tension (B3). A desire to communicate her ideas is in tension with her understanding that her use of language is disjointed and may cause difficulties for her peers’ understanding of her contributions.

Olivia continues; *And they also have a straight length for the sides… And the square is 90 degree.* Her language use here appears to fit somewhere between an informal and formal mathematics register. When Olivia refers to *straight length for the sides*, she is attempting to convey her understanding that squares have *equal* side lengths. She uses the word *straight* in the place of the word *equal*. From her statement; *the square is 90 degree* she is aware that the four interior angles of a square are each 90 degrees. She may not yet have the FMR that allows her to express her understanding without interpretation; however by expressing herself in this way she does show understanding and she continues to develop her mathematical understanding. The way she uses language sits in between an informal and FMR. This state of mathematical language development can be described as a transitional mathematical register. The transitional state of mathematical language development described here is representative of B4 (language is not unidirectional). The various imprecise ways in which language is used in the online space to share mathematical meaning.

The discussion between Zander and Olivia continues:

*Olivia: With the table what shape are you planning to do on I know that a square and rectangle could be one what else do you agree???

*Olivia: With the table that you have done it was good didn't it also have to only be with straight sides ?? Like you added a circle but it was still really good

*Zander: Don't know, but I'll add another bar so it will make for that one if we were not supposed to do it.

*Olivia: Okay that is a good idea*
Here is confirmation from Olivia that she understands that a rectangle and a square both fit the guidelines of the task (i.e. the identification of shapes that have four straight sides).

She then takes the opportunity to provide feedback to Zander when she states: “With the table that you have done it was good didn't it also have to only be with straight sides?? Like you added a circle but it was still really good.” Olivia, correctly advises Zander that the inclusion of a circle in his table does not satisfy the requirements of the task. She exemplifies her understanding by highlighting a criterion that a circle fails to meet; the shape must only have straight sides. A level of apprehension is evident here which is a product of Olivia’s understanding that Zander’s work contains a ‘mistake’. This contrasts with an understanding that by highlighting this within the group some degree of embarrassment may be experienced by Zander. The resultant tension is compounded by the previously identified tensions that have occurred during discussion between Olivia and Zander. Olivia attempts to mediate some of the potential for embarrassment by phrasing her feedback as a question and also by adding that it was still really good. Olivia’s collaboration is evidence of the dialogic nature of language (B1).

Lastly, tension is largely averted as Zander states: Don't know, but I'll add another bar so it will make for that one if we were not supposed to do it. Hence, whilst not explicitly taking ownership of his error, Zander acknowledges that it may not have been in accordance with the problem’s guidelines and therefore he will update the work to reflect Olivia’s observations.

5.5.7 Pet names

Students undertook a statistical investigation in this problem whereby a scenario was suggested of two children having a disagreement about the average length of pets’ names. Students were asked to resolve the dilemma that the two children were having by conducting some research about cat and dog names and then representing their thinking within the online discussion and in an online artefact (see Figure 3.7).
Sally: Week 7-Pet Names

Sam: Guys I think that cat names are normally 5 because in my graph it shows in the bar graph that it has more with five letters

Please reply

Sam: Guy I am going to start doing cat and dogs names and compare them both ok

I think dogs and cats are the same number of letters because in my graph it came up with 8 fives and eight fives each.

So that my Information

Please reply

Thanks guys

Sally: Hi Sam,

where is your graph?

I have done the excel spread sheet and the names that I have got are female and male. I am nearly completed.

Holly: Hey guys what do you do after you have written down all the names and numbers?

Sally: hi everyone.

what are the three words that we have to do. they are the M words. What are they?

Sam: Same I forgot about those m words I think one was maintain.

I am not sure about if it is right.

Please reply under

Thanks guys

😄😄😄

Sam: Any one on Edmodo?

Guys I know the m words they are mean, mode and median.

I just remembered today.

I hope this helps you in your bar graph, column graph and lastly line graph etc.

Please reply if you are on Edmodo

Thanks guys

And see you tommorow

Figure 5.7: Discussion of Pet Names Problem
Here, we see Sam, Sally and Holly attempt to assimilate language associated with measures of central tendency with conceptual understanding of this concept.

Initially, Sam makes the comment: *Guys I think that cat names are normally 5 because in my graph it shows in the bar graph that it has more with five letters.* Sam is making the correct observation here that through the graphical representation of the data that he has collected he is able to observe that more cat names have five letters than any other letter. He has clearly and accurately described his observation of the mode of the data set.

He then follows up his initial observation by stating the following:

_Sam: Guy I am going to start doing cat and dogs names and compare them both ok_

_I think dogs and cats are the same number of letters because in my graph it came up with 8 fives and eight fives each._

_So that my Information_

_Please reply_

_Thanks guys_

This confirms his conceptual understanding of the mode. He is able to demonstrate his understanding, based on the data sets that he has collected for both cat names and dog names, that the mode length of both cat and dog names is five letters.

Sam does not however, have the formal mathematical language to describe this. Sally facilitates evidence of this by asking:

_Sally: hi everyone.

what are the three words that we have to do. they are the M words. What are they?_

To which Sam responds:

_Sam: Same I forgot about those m words I think one was maintain._

_I am not sure about if it is right._

_Please reply under_
Thanks guys

This is an interesting piece of discussion because Sam’s comments show a very clear awareness of the mathematical concept, the mode of a data set. Following this, evidence that the students are aware that a mathematical term exists to describe this concept however no one in the group at that moment can provide the mathematically correct label for the concept.

After some time, Sam recalls three terms for central measures stating; *Guys I know the m words they are mean, mode and median... I just remembered today.* This short interaction exhibits all four tenets. Through engaging in a dialogic interaction finally Sam is able to give the concepts he describes a label (B1). The language that has previously been used within classroom discussion precedes and informs this interaction (B2). There is some level of tension associated with the frustration that arises from being unable to find the formal language to describe the concept (B3). Finally, the discussion has taken alternate ‘routes’ as a conclusion is eventually reached. Sam statement: *Same I forgot about those m words I think one was maintain* is one potential approach to using language before later deciding on an alternative.

### 5.5.8 Mr Mac’s iPhone

Figure 5.8 shows an excerpt from discussion involving the problem of dealing with a phone that exhibits a number of malfunctions. Each time the phone is charged its capacity either improves or is reduced. Initially the phone starts with a 64-hour battery life and loses half of its capacity each charge. Students are asked to determine how many charges before it only has a one-hour battery life. The phone then continues to behave in other strange ways that the students are asked to model mathematically (see Figure 3.9). This problem was undertaken the week prior to the Geese Problem. Online discussion from the same students has been provided for this week and the following week as a means of comparison.
Maddie: Do you know how to **take away half of an odd number**, I don’t know? e.g. 3 take away a half of it.

Indigo: Hold on what is the **formula** for 1024 which is in **square a1** divided by 16824 or what the **number is at the bottom**?

Jemima: I know I said I would do the first but I also did the second sorry

Indigo: **16384**

Indigo: No because Edwin is doing that one

Jemima: **One quarter of 1024 is 256**

Indigo: I need help on the **formula** help me please

Indigo: no I need **one half.... I am doing the last one**

Indigo: Somebody please help me nobody is.

Maddie: If I was to **put my data in a graph**, what would the **graph** be?

Indigo: OK so I **found the proper formula** which is \(a^2/2\) for my case. I hope this can help some of you who are stuck

<table>
<thead>
<tr>
<th>Figure 5.8: Discussion of Mr Mac’s iPhone Problem</th>
</tr>
</thead>
</table>

In this problem we see a greater level of tension being exhibited in the use of language. Maddie, Jemima and Indigo appear more unsure and reluctant in their use of language, especially in their regular use of questions and the desire for clarification. Changes in the density of mathematical language use (every 3-4 words in the first week, and every 2-3 words in the second) were noted by the researchers. Additionally, there is a greater variety of mathematical vocabulary appearing in the following week. For example, the words ‘rule’, ‘equation’ and ‘pattern’ only appear in the Geese problem (5.5.9), when they could just as easily be applied to the Phone problem. The students are in dialogue with each other in the moment. They are also in dialogue with their online communication from previous weeks. Additionally, they are in dialogue with their previous classroom interactions (B1). These dialogic opportunities support the emergence of a ‘transitional mathematical register’. These are illustrations of subtle differences that become evident between the two weeks.
5.5.9 Geese

*Geese* is an adaptation of a problem from the Modelling Middle school Mathematics website (2014). In the problem, students explore the ‘V pattern’ made by geese as they fly (see Figure 3.10).

<table>
<thead>
<tr>
<th>Indigo: The <strong>rule</strong> is it is <strong>going up by twos</strong> as an <strong>odd number</strong> so instead of the simple 2 4 6 8 it is 1 3 5 7 9 etc</th>
</tr>
</thead>
<tbody>
<tr>
<td>Indigo: and my <strong>formula</strong> I am still <strong>working out</strong></td>
</tr>
<tr>
<td>Indigo: OK I have <strong>found a formula</strong>! What I did was (say the square was _b_2 and the <strong>number</strong> in it was three) I did = _b_2+2 because 3 <strong>plus</strong> 2 is five which is the next equation in the pattern. It <strong>goes up by two every time</strong> so that would be the <strong>formula</strong>.</td>
</tr>
<tr>
<td>Maddie: Do you know how to <strong>drag down the numbers</strong> so you can go to <strong>100</strong>?</td>
</tr>
<tr>
<td>Maddie: That is very good Indigo. I liked how you explained the <strong>formation</strong>.</td>
</tr>
<tr>
<td>Indigo: thank you Maddie</td>
</tr>
</tbody>
</table>

**Figure 5.9: Discussion of Geese Problem**

In Figure 5.9 we see Indigo experimenting with the identification of growing patterns. She is able to establish that each successive term changes by the same amount as the preceding term.

Indigo has formed a recursive expression when attempting to generalise. She has realised that if you know one term (in this case the term that is equal to 3), the value of the next term will be 2 more (thus, she quite rightly states that this value is equal to 5). Indigo is not yet able to develop an explicit equation, allowing her to calculate the number of geese given the ‘geese v-term’. An understanding that if we double the ‘v-term’ and then add 1 we can form a helpful multiplicative relationship is not present. If ‘n’ is the number of geese and ‘v’, the ‘v-term’, this could be symbolically represented as n=2v+1.

Maddie shows her willingness to contribute, despite a lack of vocabulary from the FMR. She demonstrates her understanding of the pattern as a recursive expression also. She knows that if the same operation is applied to each preceding term, the subsequent term will then represent the desired result. She communicates her meaning, by referring to the Microsoft Excel ‘fill down’ function. The students use the formal mathematical language they have acquired, mixed with relevant but informal language. The
Online Mathematical Problem solving and Communicating Mathematical Meaning

combination of some non-mathematical terms (e.g. ‘drag down’) with mathematical terms (e.g. ‘numbers’) is evidence of the tension experienced by Maddie between using the unified FMR and the heteroglossic IMR (B3).

Indigo’s ability to communicate mathematically is not yet developed enough to express herself using language from the FMR. However, she is able to establish meaning through her utterances. In other examples of online discussion within this study (see Figure 5.5 for example) students are seen using the everyday phrase ‘going up by’. This phrase, whilst not a part of the FMR, is helpful in the meaning making process. A tension (B3) is perceptible in the way Indigo chooses to express herself. Whilst she seems to have a desire to express herself in a manner that conforms and adheres to the FMR, she undoubtedly also has a desire to communicate her mathematical meaning to the members of her group. These aims are to some extent at variance with each other. The laborious and time-consuming efforts required by Indigo to articulate her thoughts using the FMR compounded by the high-level literacy based demands make communicating mathematical thinking difficult. The stress of wanting to communicate her thinking, whilst doing it in a mathematically ‘accurate’ way causes tension. Indigo’s discussion falls between an informal and FMR. Indigo’s use of language here is further evidence of the TMR.

The use of certain examples of technical mathematical vocabulary that was previously uncommon for these students is evident in this excerpt. Use of the word ‘formula’ does not appear in any of the group discussions until the fifth week of data collection when there were two references to this word. However, in the eighth week, nine references are made and in the ninth week (the final week of data collection and also the week from which this dialogue is taken) we see twelve uses of this term. This is evidence that the discussion and co-negotiation of definitions that occurred in the classroom is in ‘dialogue’ with that of the online space (B1). It is also evidence that the students have shared and developed aspects of mathematical language from and with each other. This suggests that students may now feel comfortable using and taking risks with language that they previously did not attempt (B4). In order for students to make these connections it was necessary that the language preceded them (B2).
5.6 Summary

The constraints of the CSCL discussion created a need for students to use mathematical words to describe their solution processes. The dialogue was not simply with each other (B1) in the online space but also with participants from the earlier classroom discussions (B2). The students showed benefit from being in dialogue with the language and understanding of the class-based facilitator, who in turn had acquired this knowledge through dialogue with their past peers and teachers (B2). Students’ written discussion, required in the CSCL environment exposed their struggles to use new terms along with common language (B3) allowing one to observe the to-and-fro between the IMR, TMR and FMR (B4).

Analysis of data from this study illustrates that Year 5 is a ‘bridging’ point in student mathematical language development. No longer are they required to develop and utilise the language of basic place value and the four operations. They must begin, at this juncture, to develop an understanding of more sophisticated concepts; for example, those that require algebraic, proportional and relational thinking. These concepts require the acquisition of aspects of the more unified, formal language of mathematics. However, students can only make sense of their newly emerging understanding of this language through appropriating familiar informal language in combination with the newly discovered formal vocabulary. Together, the informal and formal use of language allows students to reason and communicate their emerging understandings. While the goal is their use of formal mathematical language, students make sense of these new concepts through appropriating familiar language in combination with the new formal vocabulary. This hybrid language, or TMR, allows students to reason and communicate their emerging understandings.

The dialogic nature of language has great significance for this study where students who have been exposed to and contributing to the co-definition of various mathematical terms use this mathematical vocabulary within the CSCL environment. The language is used with various degrees of precision. This study shows evidence that may support Barwell (2012)’s view that curricula-based and other societal tensions placed on students to use only the FMR are counterproductive. I see the use of the transitional mathematical register informed by prior discussions as (i) being important to the
meaning making required in the problem solving process and (ii), a necessary intermediate step for some students to improve the precision and accuracy with which mathematical language is used.
Chapter 6: Online Mathematical Problem Solving and Evidence of Critical Thinking

In this chapter, an overall aim will be to address Research Question 3:

*Is there evidence of critical thinking when students engage in ‘talk’ within online mathematical problem solving (MPS)?*

This chapter draws on work initially disseminated in Symons and Pierce (2015) and Jazby and Symons (2015). Firstly, some discussion of the importance that the Australian Curriculum places on critical thinking will be provided. Following this, a framework developed by Perkins and Murphy (2006) will be used to explore whether different aspects of critical thinking were evident in student online discussion. Following use of this tool, student discussion is analysed through the lens of ‘Talk Types’ (Mercer & Wegerif, 1999) with the aim of showing that where *Exploratory talk* was evident, student critical thinking also occurred. It is hypothesised that critical thinking may be evident through the presence of a greater density of specific technical mathematical vocabulary use in identified examples of *Exploratory talk.*

Figure 6.1 (below) shows the cyclical approach taken to investigating student critical thinking in this chapter. The lens through which critical thinking is initially examined occurs at a macro level. After providing evidence that critical thinking is now a fundamental aspect of the Australian Curriculum, the Clarification, Assessment, Inference, Strategies (CAIS) framework (Perkins & Murphy, 2006) provides a macro level understanding of how one group of students within the study engaged in different aspects of critical thinking. The framework allows inferences to be made about how various members of the group position themselves, according to which of the categories of critical thinking they display more or less of. The analysis of student online discussion then allows a micro level understanding of the types of talk that were more likely to be productive. Whether a higher density of technical mathematical vocabulary would be observed where productive talk was observed was also an area for investigation. A final aim is to align the identification of Exploratory Talk and a higher density of mathematical vocabulary use back to the prevalence of the Australian...
Curriculum prescribed general capability of Critical and Creative thinking (ACARA, 2017).

As previously described (see 1.2), there is strongly held opinion that much technology integration in primary level mathematics, at best emphasizes Fluency rather than Understanding, reasoning and problem solving. At worst, it is simply an exercise in keeping students busy with little emphasis on mathematical concept development.

The aim of the teaching and learning intervention developed for this study was to focus on an approach to technology integration for primary mathematics where problem solving, reasoning and critical thinking were outcomes. Consequently, an approach to assessing the types of Higher Order Thinking (specifically critical thinking in this case) that was occurring and developing was sought. A range of research was considered, for example that of Facione (2013) and Fleming (2008) for the purpose of informing the approach taken to coding and analysis of data in order to answer research question three. However, this research did not provide clearly articulated approaches to assessing the different components of critical thinking occurring within individual student online discussion. Perkins and Murphy (2006) provided a clear and practical tool to achieve this goal.
6.1 Critical Thinking in the Australian Curriculum

The Australian Curriculum lists seven ‘general capabilities’. When describing the capabilities ACARA (2017) states:

*General capabilities are identified where they are developed or applied in the content descriptions. They are also identified where they offer opportunities to add depth and richness to student learning via the content elaborations, which are provided to give teachers ideas about how they might teach the content. Teachers are expected to teach and assess general capabilities to the extent that they are incorporated within learning area content (ACARA, 2017).*

Teachers are expected to embed and assess development of the general capabilities throughout their teaching of all content areas, including mathematics.

‘Critical and Creative Thinking’ is one of the listed general capabilities. The Australian Curriculum provides the following broad discussion of Critical and Creative Thinking within the content area of Mathematics:

*In the Australian Curriculum: Mathematics, students develop critical and creative thinking as they learn to generate and evaluate knowledge, ideas and possibilities, and use them when seeking solutions. Engaging students in reasoning and thinking about solutions to problems and the strategies needed to find these solutions are core parts of the Australian Curriculum: Mathematics.*

*Students are encouraged to be critical thinkers when justifying their choice of a calculation strategy or identifying relevant questions during a statistical investigation. They are encouraged to look for alternative ways to approach mathematical problems; for example, identifying when a problem is similar to a previous one, drawing diagrams or simplifying a problem to control some variables (ACARA, 2017).*

The above statement provides further rationale or justification for the approach taken to mathematical problem solving adopted in this study. This included a major emphasis on encouraging students to justify their choices and to ask relevant and appropriate questions of themselves and each other throughout the process.

In addition to the broad statement of Critical and Creative thinking within mathematics, the Australian Curriculum provides the following delineated expectations (see Figure
In the following subsection, a discussion of Perkins and Murphy’s (2006) CAIS framework will be provided. When closely analysed, it is evident that the indicators of
critical thinking embedded within the Australian Curriculum (ACARA, 2017) content descriptions (in Figure 6.2) are represented in the CAIS framework. Therefore, the CAIS framework is used as a method of aligning and assessing whether or not, and also how, students engaged with the Australian Curriculum expectation of critical thinking.

6.2 Clarification Assessment Inference Strategies

The CAIS model developed by Perkins and Murphy (2006) (see Table 6.1 below) is a practical model for identifying engagement in critical thinking. They describe the process of constructing this tool for data analysis as involving a rigorous review of literature surrounding indicators of critical thinking. They examined the essential features of the concept within the literature and synthesized a number of critical indicators as the basis of their tool.

They identified the increasing use of CSCL (in the form of asynchronous message boards) across a range of educational contexts. This did not include the primary school setting. As such the framework will be applied in a new context for the purpose of this study. They describe a generally held belief that having students engage in discussion in these online environments may lead to development of HOT, specifically in the area of critical thinking. Perkins and Murphy (2006) developed the CAIS model as an approach to help test this conjecture.

As can be seen in Table 6.1 below, there is a great similarity between the indicators of critical thinking identified by Perkins and Murphy (2006) and the delineated developmental continuum (of Critical and Creative Thinking) as provided by ACARA (2017). Both emphasise student clarification and organisation of concepts and ideas, the generation of different strategies ideas and approaches, reflecting on and analysing and evaluating decisions taken and strategies utilised. The practical nature of Perkins and Murphy’s (2006) framework meant that there was high reliability in classification of student online discussion between coding undertaken by the author of this study and his supervisor. The high level of inter-rater reliability in addition to the manner in which it mirrored the Australian Curriculum focus areas gave confidence that it was an appropriate tool to investigate student thinking in the CSCL environment.
Table 6.1: Perkins & Murphy’s Clarification, Assessment, Inference, Strategies Model (2006, p. 301)

<table>
<thead>
<tr>
<th>CLARIFICATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proposes an issue for debate.</td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>ASSESSMENT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Provides or asks for reasons that proffered evidence is valid.</td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>INFERENCE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>STRATEGIES</th>
</tr>
</thead>
<tbody>
<tr>
<td>Takes action.</td>
</tr>
</tbody>
</table>

6.3 Data Collection, Coding & Analysis

The analysis of critical thinking using CAIS was collected from one representative group of four students who participated in the CSCL environment. This group was chosen as ‘representative’ of the whole group of students through discussion with my principal supervisor. The group included a mix of genders; students from each of the three teacher assigned ability groups and students from both Language Backgrounds other than English and English-speaking backgrounds. The choice was made to investigate this one group in order to provide a more fine-grained analysis. Perkins and Murphy’s (2006) CAIS framework was used to code students’ online posts, where each post is treated as an utterance. Coded utterances can be classified as relating to critical
thinking and have then broken down into the four categories used by Perkins and Murphy. Uncoded utterances are those utterances which could not be related to critical thinking. These uncoded utterances typically related to organisational or conversation queries such as: “Please reply here” or questions such as, “Is anybody online?” The assessment framework was applied to the records of eight weeks of online discussion (see appendix 4 for an example of coding for CAIS). The first week of the nine-week intervention was not analysed as the learning objective for the week had focused on familiarising students with the Edmodo platform used rather than improving mathematical problem solving.

6.4 Student Focus of Critical Thinking

Table 6.2: Assessment of students' critical thinking evident in students' utterances

<table>
<thead>
<tr>
<th>Student</th>
<th>Aspect of critical thinking</th>
<th>Coded utterances (N=128)</th>
<th>Uncoded utterances (N=104)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Clarification (%)</td>
<td>Assessment (%)</td>
<td>Inference (%)</td>
</tr>
<tr>
<td>Chaz</td>
<td>8</td>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>Igor</td>
<td>3</td>
<td>8</td>
<td>7</td>
</tr>
<tr>
<td>Olive</td>
<td>73</td>
<td>46</td>
<td>29</td>
</tr>
<tr>
<td>Zaid</td>
<td>16</td>
<td>42</td>
<td>64</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
</tbody>
</table>

Table 6.2 summarises the percentage of utterances coded as *Clarification, Assessment, Inference and Strategies* posted by students within this group. The total number of coded and uncoded utterances for each student is also provided in the last two columns.

Two students, Chaz and Igor, made the smallest number of posts and contributed less than 10% to the group’s posts exhibiting critical thinking. As a consequence, it is difficult to identify which of the four critical thinking categories they engaged most or least with. The majority of the group’s critical thinking posts were made by Zaid and Olive. Zaid and Olive contributed an almost equal proportion of *Assessment* related posts (42% and 46% respectively). Olive contributed 73% of the group’s *Clarification* posts while Zaid posted 64% of the *Inference* and 63% *Strategies* posts. This indicates the prominent role that two students took within their group. Olive’s high level of
Clarification based posts indicates that she was more likely to question the assumptions made by others within the group, seek clarity around shared definitions and to draw together important relationships (mathematical or otherwise). Zaid more commonly made utterances aligning with the Inference and Strategies classifications. Zaid was more likely to pose questions and help the group co-construct definitions. His role within the group was to take the evidence and data provided by himself and others within the group and draw conclusions, make deductions and generalise ideas and concepts. He would also be more likely to either suggest or take action within the group. This may suggest Zaid is positioning himself as a leader or authoritative figure within the group.

6.5 Implications of Analysis

The aim of applying the analytical lens of CAIS was to demonstrate that critical thinking was evident within student interactions in the CSCL environment. By using this tool to examine one group’s online collaborative MPS, each of the indicators of critical thinking (Clarification, Analysis, Inference and Strategies) can be observed. It is also possible to understand the position each of the participants took within the space. Whilst there was too little data to obtain this kind of fine grained analysis for Chaz and Igor it was also possible to infer the positions taken by Zaid and Olive based on how their codable utterances were weighted.

Interestingly, Olive had been ‘streamed’ by her teacher into the lowest achieving mathematics group for her daily mathematics instruction. Tests assessing her ability to perform routine mathematical skills and procedures were used to inform this placement. Yet, in the CSCL environment, Olive’s contribution to online learning is significant, making up almost 40% of the group’s discussion.

The use of the framework not only provided summative data about the group’s general critical thinking based participation within the group, but also provided valuable information about those aspects of critical thinking that each student exhibited to a greater or lesser extent. This analysis provides us with the ability to consider the Australian Curriculum (2017) general capability of Critical and Creative thinking (see Figure 6.2) and ensure, firstly that critical thinking, as prescribed by the curriculum, is evident in this online approach to MPS and secondly to understand how the individual
Online Mathematical Problem Solving and Evidence of Critical Thinking

members of a small group focus their attention on different aspects of critical thinking thereby ensuring an effective collaborate approach.

6.6 Talk Types

Whilst the CAIS framework made visible the fact that students were engaging in critical thinking, and gave us a global understanding of the ways in which some students positioned themselves within their groups, it did not allow us to understand the way they used ‘talk’ (online discussion in this case) to construct their learning. The following sub-sections will examine how the types of talk that students engaged in within the space, allow us to understand which Talk Types could be seen to be more productive and thus more likely to lead to critical thinking.

In this section of the study students’ use and development of mathematical vocabulary in the context of the CSCL environment will be examined. It is hypothesized that student use of ‘Talk Types’ (Mercer & Wegerif, 1999) will become evident during student online discussion. The three types of special interest are Mercer and Wegerif’s (1999) Disputational Talk, Cumulative Talk, and Exploratory Talk. After having established that it is possible to observe the three distinct Talk Types within student online discussion, I will measure the density of technical mathematical vocabulary use within the identified examples to gain an understanding of how productive students were when engaged in each variety of talk.

In this section, we also explore whether there is a relationship between students’ (teacher identified) mathematical ability and their use of tier-three mathematical vocabulary. Finally, I aim to investigate whether the density of tier-three mathematical vocabulary use changed throughout the intervention and also if there were any ability groups (below level, at level or above level) where changes were more obvious.

The theoretical framework utilized for this sub-section relies on Mercer and Wegerif’s (1999) Talk Types. Three broad Talk Types were identified when these authors analysed many hours of videotaped discussion amongst British primary age students: Disputational Talk, Cumulative Talk and Exploratory Talk. I rely on their definitions of these terms in this study:

*Disputational talk, which is characterised by disagreement and individualised decision*
making. There are few attempts to pool resources, or to offer constructive criticism of suggestions. Disputational talk also has some characteristic discourse features – short exchanges consisting of assertions and challenges or counter-assertions.

**Cumulative talk**, in which speakers build positively but uncritically on what the other has said. Partners use talk to construct a ‘common knowledge’ by accumulation. Cumulative discourse is characterised by repetitions, confirmations and elaborations.

**Exploratory talk**, in which partners engage critically but constructively with each other’s ideas. Statements and suggestions are offered for joint consideration. These may be challenged and counter-challenged, but challenges are justified and alternative hypotheses are offered. Compared with the other two types, in exploratory talk knowledge is made more publicly accountable and reasoning is more visible in the talk (p. 85).

Unlike the study by Mercer and Wegerif (1999), the discourse analysed in this thesis occurred in an online environment, and occurred asynchronously in a discussion board. However, I continue to feel justified to use the term ‘utterances’ when referring to students’ statements. Like Mercer and Wegerif (1999), I am interested in gaining an understanding of how students might jointly construct knowledge as they work to solve a series of mathematical problems. I hypothesise that given that **Exploratory Talk** is represented by talk where public accountability is evident, in addition to reasoning being visible, a greater density of technical mathematical vocabulary is likely to be present when students engage in this talk type.

### 6.7 Vocabulary- The Three-Tier Framework

Beck et al. (2002) established a basic system for the classification of vocabulary. In their system, vocabulary is classified as tier-one, tier-two and tier-three. They established these terms as a means to frame teaching and learning in the area of vocabulary development. Their framework has since been appropriated by various researchers for the purposes of understanding aspects of mathematical language development. The following is my paraphrasing of the three tiers:

Tier-one vocabulary encompasses everyday language. These words are the most basic and are used with a high degree of frequency, particularly in spoken language. Tier-one vocabulary includes such words as ‘warm’, ‘cold’, ‘talk’, ‘cat’, ‘dog’, etc.
Tier-two vocabulary represents words that are primarily used in written language. They are words with a very high degree of utility. These words are generally utilised by more mature users of language. As a result of their usage, primarily in written language, they can be more difficult for students to learn independently. Examples of tier-two vocabulary include, ‘precede’, ‘following’, ‘retrospect’, ‘contradictory’ etc.

Tier-three vocabulary includes words with a technical or domain-specific usage. Generally, these words are of a specified and limited usage. However, in this study we see them occurring more frequently because of the mathematical context of the study. They are generally the most difficult words for students to acquire because of the very limited opportunities students have to experiment with them. In the context of mathematics, Tier–three vocabulary would include such terms as: ‘formula’, ‘equation’, ‘symmetry’, ‘median’.

6.8 Data Collection, Coding & Analysis

Whilst, I have primarily used the definitions provided in sub-section 6.6 as a basis for the coding of data, in terms of Talk Types, taken from the CSCL environment, a number of specific adaptations were made based on the context of the setting. Talk Types were coded using two approaches.

I first adopted a course-grained approach to the coding of data, whereby a general code was allocated for each small group discussion, for each problem. Given that students completed 9 problems and there were 10 small groups, this meant that there were approximately 90 individual passages coded using this approach. Despite many of the passages containing discourse to exemplify two or even three of the different Talk Types, I selected the talk type that was most evident in the passage. Coding in this way provided a general sense of the more and less prevalent Talk Types occurring within the space.

The second approach to coding was more fine grained. Each individual ‘turn’ taken or ‘utterance’ made by participants was coded using this approach. This method provided a great deal of information that was not available using the first approach. For instance, Disputational Talk was not detected when taking the course-grained approach, however a substantial number of examples of this talk type were identified when coding using a
more ‘fine grained’ method. This showed that whilst Disputational Talk was unlikely to represent the dominant talk type within a passage, consistently short bursts of this talk type would occur within a passage that was more broadly characterised as either Cumulative or Exploratory Talk. Additionally, since all participants’ utterances have been allocated a unique identifier, in addition to various demographic information, such as their gender, mathematics ability level and language background, the fine-grained method allowed us to investigate whether a particular talk type was more prevalent amongst particular demographic groups (see appendix 4 for an example of coding for Talk Types).

In addition to adhering to the definitions provided by Mercer and Wegerif (1999), the following complementary criteria were used for the (fine grained coding) of Talk Types in this context:

**Disputational talk:**

- Students becoming frustrated and/or annoyed.
- Students deliberately engaging in antagonism
- Some degree of conflict, generally not of a ‘constructive’ nature

**Cumulative talk:**

- Conversation is of a more general nature. Questions are equally non-specific. E.g. “Can you help me? I don’t get it.”
- Does not involve the operations, digits, numbers or mathematical representations.
- Students provide an answer only. No explanation of thinking is provided.
- Seemingly, polite, non-specific feedback. E.g. “That’s great work”, “Fantastic!”
- Talk is vaguely (but not specifically) related to the task.

**Exploratory talk:**

- Critiques are constructive and contain logical arguments
- Feedback may be polite, however includes specific observation. E.g. “I like how you included 3rd order rotational symmetry in your wallpaper”
- Questions are not of a general nature. Questioner asks questions indicating a specific need that requires addressing. E.g. when trying to find out the labels for
mean median and mode a participant might ask, “I can’t remember the three m words. What were they again?”

- Answer to problems includes some description of thinking behind contribution.

This study only coded for tier-three vocabulary. Tier-one and tier-two vocabularies were not central to the question being investigated. I was interested in the level to which students would integrate these ‘technical’ terms within their online discussion after having been introduced to them and having been given the opportunity to co-negotiate definitions, trial and experiment with the terms during whole class discussions. It was also hypothesized that generally a greater density of tier-three mathematical vocabulary would be present when students were engaged in Exploratory talk.

In addition to the provided description of tier three vocabulary, I also used the following guidelines when coding for tier-three ‘mathematical’ vocabulary:

- Of interest was only tier three mathematical vocabulary; therefore, any terms that whilst representing tier-three vocabulary that were not mathematical in nature were not counted
- Misspelt words were included
- Seemingly ‘simple’ words with specific mathematical meaning were included. E.g. ‘shape’, ‘pattern’, ‘height’, ‘centimetre’.
- Homonyms, where the meaning conveys mathematical meaning were included. E.g. “I have used the animal names as headings for my table.”

6.9 Results and Discussion

Figure 6.3 shows an example illustrative of discussion from the online message board coded as exploratory talk with tier-three mathematical vocabulary bolded. The discussion in Figure 6.3 (below) is provided verbatim from the online space. In this example students worked on the problem (see Figure 3.7) where they were required to make a conjecture, gather evidence and support a claim about whether cats’ names or dogs’ names are generally longer. The students researched a number of the most common cats’ names and dogs’ names, calculated the mean, median and mode of these data, and graphed their results using Microsoft Excel and discussed their results.
In this example of *Exploratory Talk*, we see the students attempting to decide on appropriate mathematical vocabulary to describe the three common measures of central tendency. One student offers the word ‘maintain’ as a possibility to describe one of the measures. Eventually though, they are able to arrive at the conclusion that the words ‘mean’, ‘median’ and ‘mode’ are the words that they have been seeking to find. This suggests that students’ vocabulary may benefit from the co-negotiation of definitions, trialling and experimentation with new terms that the context of this setting allows. The two transcripts that follow are provided verbatim and in an unedited format.

| Sally  | *I think dogs and cats are the same number of letters because in my graph it came up with 8 fives and eight fives each.*  
|        | *So that my Information*  
|        | *Please reply*  
|        | *Thanks guys*  
| Sally  | *Hi Sunny, where is your graph?*  
| Sally  | *Hi everyone, i have done the excel spreadsheet and the names that i have got are female and male. i am really compleated.*  
| Holly  | *Hey guys what do you do after you have written down all the names and numbers?*  
| Sally  | *Hi everyone. What are the three words that we have to do. They are the M words. What are they?*  
| Sam    | *Same I forgot about those m words I think one was maintain. I am not sure about if it is right. Please reply under*  
|        | *Thanks guys*  
| Sam    | *Guys I know the m words they are mean, mode and median.*  
|        | *I just remembered today. I hope this helps you in you **bar graph**, column graph and lastly line graph etc. Please reply if you are on edmodo.*  
|        | *Thanks guys*  
|        | *And see you tommorow*  

*Figure 6.3: Example of Discussion Coded as Exploratory Talk*
Figure 6.4 (below) shows an example from the online discussion where *Cumulative Talk* is apparent. This discussion is taken from a different small group of students attempting to solve collaboratively the ‘Pet Names’ problem. In this example, Annie positions herself as leader within the group. She repeatedly rephrases her desire for suggestions or agreements related to whether she should provide information about the various pet names. No constructive criticism is present; however eventually we see some common knowledge emerging. This excerpt of discussion shows an absence of tier-three mathematical vocabulary. The group did not present any analysis of the length of their pet names.

| Annie | hey guys do we need to do male and female cats names if you do please post |
| Kevin | do you what me to rshoq [means ‘research’] |
| Annie | what is rshoq? |
| Kevin | that mans resuch |
| Annie | no i was thinking i have already done the female and the male cat names and they are french names is that alright with you guys |
| Annie | and you spell research like this. |
| Sheldon | hi |
| Annie | hi just tell me if you guys want to know the names and i will tell you |
| Annie | i will tell you anyway the female names are: Sassy Misty Princess Samantha Kitty Puss Fluffy Molly Daisy Ginger Midnight Precious Maggie Lucy Cleo Whiskers Chloe Sophie Lily Coco |
| Annie | And my male names are: Max Sam Tigger Sooty Smokey Lucky Patch Simba Smudge Oreo Milo Oscar Oliver Buddy Boots Harley Gizmo Charlie Toby |

| Table 6.3 (below) shows the number of examples of Talk Types identified, from the coarse grained analysis and the number of examples of tier-three mathematical vocabulary. The dominant talk type throughout all discussion during the nine weeks of data collection was *Cumulative Talk* as shown in Table 6.3. Forty-nine examples of *Cumulative Talk* were identified, whilst only 27 examples of *Exploratory Talk* were identified. Across the data we see an average of between seven and eight mathematical tier-three words used during examples of *Cumulative Talk* discussions, whilst there are
between ten and eleven examples of this type of vocabulary used in examples of *Exploratory Talk*. This shows that students engaged in *Exploratory Talk* were more likely to use tier-three vocabulary than when they were engaged in *Cumulative Talk*.

Table 6.3: Frequency of Tier 3 Vocabulary use in Cumulative and Exploratory talk

<table>
<thead>
<tr>
<th>Talk Type</th>
<th>Identified examples of Talk Type</th>
<th>Average No. Tier-three Vocabulary per Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cumulative Talk</td>
<td>361</td>
<td>7.4</td>
</tr>
<tr>
<td>Exploratory Talk</td>
<td>284</td>
<td>10.5</td>
</tr>
</tbody>
</table>

Table 6.4 shows the density of tier-three mathematical vocabulary use by differing ability levels of students. With the possible exception of the *at level* boys, there is an association between the density of mathematical tier-three vocabulary use and the student ability level. One possible explanation for the lower than expected use of tier-three vocabulary use in this group is that 7 of the 11 students in this group had a Language Background Other than English compared with 17 out of 54 overall. Statistical inference techniques have not been applied since the nature of the sample means that the results are not generalizable but do point to some associations that may warrant further investigation.

Table 6.4: Density of Tier three Vocabulary use in Student Utterances

<table>
<thead>
<tr>
<th></th>
<th>No. Of Students</th>
<th>Tier-three Vocabulary Use</th>
<th>Total No. of Utterances</th>
<th>Tier-three Vocabulary Use per Utterance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Above Level Boys</td>
<td>5</td>
<td>129</td>
<td>198</td>
<td>0.65</td>
</tr>
<tr>
<td>Above Level Girls</td>
<td>3</td>
<td>78</td>
<td>96</td>
<td>0.81</td>
</tr>
<tr>
<td>At Level Boys</td>
<td>11</td>
<td>77</td>
<td>301</td>
<td>0.26</td>
</tr>
<tr>
<td>At Level Girls</td>
<td>12</td>
<td>183</td>
<td>350</td>
<td>0.52</td>
</tr>
<tr>
<td>Below Level Boys</td>
<td>7</td>
<td>35</td>
<td>96</td>
<td>0.42</td>
</tr>
<tr>
<td>Below level Girls</td>
<td>9</td>
<td>128</td>
<td>308</td>
<td>0.42</td>
</tr>
</tbody>
</table>

Table 6.5 (below) shows the density of mathematical tier-three vocabulary use throughout the study. There does not appear to be any clear evidence of progression in growth of students’ use of mathematical tier-three vocabulary throughout the period. However, it can be argued that each problem offered different opportunities. Also shown are rates of online participation of the ten small groups throughout the period.
In weeks 2 and weeks 7, for example, the fewest number of groups participated in online discussion. These weeks also correspond with the lowest average number of tier-three mathematical terms used per participating group. It is possible to conjecture that in these two weeks students found it more difficult to engage with the tasks. Even though there was a classroom introduction, including explicit discussion of the required mathematical language, the mathematical content required was new and also difficult for some students. For example, in week 7, when students undertook the Pet Names problem, they were required to calculate a central measure (mean, median and mode). The development of skills and understanding in this area of statistics does not appear in the Australian Curriculum (ACARA, 2014) until Year 7.

It is also worth considering the change in pedagogical approach that took place in the final two weeks of the intervention. As mentioned previously, classroom-based support as provided by the facilitator over the first seven weeks was withdrawn in the final two weeks. Taking this into account, the average number of tier-three mathematical terms used per group (with weeks 2 and 7 removed) in the period with a classroom introduction was 9.9 and in the final weeks without this support it was 7.4. As there was no specific mathematical content focus over the period of the intervention, each week a new and different set of vocabulary was required of the students. When classroom support was taken away, students had fewer opportunities to familiarise themselves and become somewhat comfortable with specific vocabulary that might be used by them in the online space in that week.
Table 6.5: Density of Tier three Vocabulary use throughout Intervention

<table>
<thead>
<tr>
<th>Week</th>
<th>Examples of Tier-Three Vocabulary</th>
<th>Number of Groups Participating in Online Discussion</th>
<th>Average Number of Tier-Three words per (participating) group</th>
</tr>
</thead>
<tbody>
<tr>
<td>Week 1</td>
<td>68</td>
<td>9</td>
<td>7.6</td>
</tr>
<tr>
<td>Week 2</td>
<td>19</td>
<td>6</td>
<td>3.2</td>
</tr>
<tr>
<td>Week 3</td>
<td>93</td>
<td>9</td>
<td>10.3</td>
</tr>
<tr>
<td>Week 4</td>
<td>129</td>
<td>9</td>
<td>14.3</td>
</tr>
<tr>
<td>Week 5</td>
<td>108</td>
<td>10</td>
<td>10.8</td>
</tr>
<tr>
<td>Week 6</td>
<td>64</td>
<td>10</td>
<td>6.4</td>
</tr>
<tr>
<td>Week 7</td>
<td>25</td>
<td>7</td>
<td>3.6</td>
</tr>
<tr>
<td>Week 8</td>
<td>85</td>
<td>10</td>
<td>8.5</td>
</tr>
<tr>
<td>Week 9</td>
<td>57</td>
<td>9</td>
<td>6.3</td>
</tr>
</tbody>
</table>

6.10 Aligning Critical Thinking, CAIS, Talk Types and Vocabulary

Table 6.6 (below) provides evidence for the drawing together of the frameworks (see Figure 6.1) underpinning this chapter. Perkins and Murphy’s (2006) CAIS framework was initially utilised as a means to provide evidence of student critical thinking within the CSCL environment. I have shown that it can also provide insight into how students focussed the critical thinking they communicated within their small groups. Mercer and Wegerif’s (1999) Talk Types were then used as a means to examine the discourse that students engaged in. The density of technical mathematical vocabulary (Beck et al., 2002) use within each of the three Talk Types was examined. A greater density of technical mathematical vocabulary was found in identified examples of Exploratory Talk, reinforcing Mercer and Wegerif’s (1999) suggestion that this Talk Type is generally more productive.

Table 6.6 (below) represents coding of Talk Types using the fine grained ‘utterance’ level analysis (described in section 6.8). It shows that of the three Talk Types, there is generally a greater incidence of all four CAIS categories of critical thinking where Exploratory Talk is present. Thus, our data indicate that where Exploratory Talk is evident students will engage in greater levels of critical thinking. Whilst each of the four categories were more evident where Exploratory Talk was evident, the CAIS (Perkins & Murphy, 2006) category of Inference was the most obvious example of this. Table 6.6 shows that of the identified examples of Inference, 84% occurred when
students were engaged in *Exploratory Talk*. It is likely that the more collaborative, dialogic nature of *Exploratory Talk* provided students the opportunity to engage in the important work of deducing appropriately, inferring appropriately, arriving at conclusions, making generalizations and finding relationships amongst concepts and ideas.

<table>
<thead>
<tr>
<th>CAIS Category</th>
<th>Cumulative Talk</th>
<th>Disputational Talk</th>
<th>Exploratory Talk</th>
<th>Exploratory Talk (% of Total)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Assessment</td>
<td>28</td>
<td>8</td>
<td>65</td>
<td>64</td>
</tr>
<tr>
<td>Clarification</td>
<td>59</td>
<td>7</td>
<td>131</td>
<td>66</td>
</tr>
<tr>
<td>Inference</td>
<td>8</td>
<td>0</td>
<td>42</td>
<td>84</td>
</tr>
<tr>
<td>Strategies</td>
<td>111</td>
<td>1</td>
<td>161</td>
<td>59</td>
</tr>
</tbody>
</table>

### 6.11 Summary

Data in this study appears to suggest that a positive relationship exists between student levels of procedural mathematical achievement and the density of tier-three mathematical vocabulary use. The data shows that students classified, as *below level* less regularly attempted the use of this type of vocabulary than their peers classified as *above level*. Furthermore, data suggest that LBOTE students are less likely to attempt high level vocabulary. Further research is required to test any hypothesis that a targeted approach to the teaching of tier-three mathematical vocabulary may lead to improved results in procedural assessments of mathematical ability.

The data suggest that students will use tier-three mathematical vocabulary more regularly when engaged in *Exploratory Talk* than when engaged in *Cumulative Talk*. It has also been shown that *Cumulative Talk* is likely to be the dominant talk type, given the conditions described. I suggest that it may be beneficial to specifically encourage the engagement of students in *Exploratory Talk* for two reasons. Firstly, in order to prompt students to more regularly experiment with newly acquired vocabulary. Secondly, as a means to encourage critical thinking within their collaborative MPS. Explicitly teaching students about the three Talk Types and discussing their various attributes and characteristics, including why *Exploratory Talk* might be the most
productive talk type, may promote this. Such teaching would include an explanation of the importance of building a repertoire of technical mathematical vocabulary. Approaches like this may result in groups ‘self-regulating’ their discussion and being aware of when talk had become less productive.

*Exploratory Talk* as discussed by Mercer and Wegerif (1999) appears to be closely linked to critical thinking. The facets of critical thinking, as indicated by both the CAIS framework (Perkins & Murphy, 2006) and the indicators of Critical and Creative Thinking as provided by the Australian Curriculum (ACARA, 2017) are represented when students engage in discussions involving critiquing, justifying, clarifying, analysing that *Exploratory Talk* describes. Having established that *Exploratory Talk* is likely to exhibit a greater density of technical mathematical language use, we can also make the claim that it is likely that when students utilise a greater density of this type of vocabulary that they are also more likely to engage in critical thinking. This is supported by the greater prevalence of the CAIS categories identified in *Exploratory Talk*. 


Chapter 7: Online Mathematical Problem Solving: The Students’ Response

This chapter considers how students viewed the experience of engaging in online collaborative mathematical problem solving.

Specifically, the aim of this chapter is to address the following research question:

*How do year 5 students respond to the experience of working within an online environment to collaboratively solve mathematical problems?*

1,411 posts were made by the 10 groups over nine weeks. Given the constraints of this thesis, as a case study, in this chapter I have looked in detail at the responses of the same group of students examined in section 6.4. Student participation has been analysed in terms of number of posts, percentage of the group’s overall posts and average length of posts (in terms of number of words). From this one can infer that students perceived that CSCL provided them an opportunity to participate in collaborative MPS.

Students’ ability to engage in face-to-face, small-group collaborative work will be described using a proxy based on data gathered in post-intervention semi-structured interviews, and by discussing students’ behaviour with their classroom teacher. These post-intervention student group interviews provide data concerning students’ patterns of interaction in face-to-face, small-group discussion. Individual students’ patterns of interaction in this interview were discussed with the class’s teacher to confirm whether the patterns observed in interviews were representative of a student’s general classroom behaviour in other face-to-face collaborative group work.

In order to investigate whether CSCL provided different students with different opportunities to participate in collaborative MPS, participation in CSCL interaction was contrasted to the proxy of students’ participation in face-to-face, small-group collaborative work.

### 7.1 Levels of Participation

Table 7.1 (below) summarises student participation in the CSCL and face-to-face environments respectively. Individual students displayed different patterns of
participation in each environment. Zaid led participation in both environments but the other three students’ level of participation varied. Chaz and Igor made fewer online contributions than others in their group (Chaz 6%, Igor 9%) but participated more when face-to-face (Chaz 13%, Igor 38%). Olive made a significant number of contributions in the CSCL environment (39% of the group’s posts), but participated less when face-to-face, both in terms of percentage of contributions (19% of utterances face-to-face) and average length of utterance. These results suggest that individual students may perform differently in different environments, with only Zaid having a similar pattern of performance across both environments. The variation in participation between environments of Olive, Chaz and Igor indicates that these students utilised different opportunities for action in the two environments. While Chaz and Igor participated more in face-to-face interaction, Olive participated more fully in the CSCL environment.

Discussions with the students’ classroom teacher confirmed the picture painted by this data. Olive was seen by the classroom teacher to be a shy, quiet girl who has been streamed into a below level grouping based on her performance in mathematics assessments. Zaid has been streamed into a above level group and was seen to participate regularly in classroom discussions and group work. Chaz and Igor are referred to as at level by their teacher.
Table 7.1: Student participation in online and face-to-face environments

<table>
<thead>
<tr>
<th>Student</th>
<th>Online contribution</th>
<th>Face-to-face contribution</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Number of Posts</td>
<td>% Total Posts (n=132)</td>
</tr>
<tr>
<td>Zaid</td>
<td>107</td>
<td>46</td>
</tr>
<tr>
<td>Olive</td>
<td>90</td>
<td>39</td>
</tr>
<tr>
<td>Chaz</td>
<td>15</td>
<td>6</td>
</tr>
<tr>
<td>Igor</td>
<td>20</td>
<td>9</td>
</tr>
</tbody>
</table>

7.2 Implications of Levels of Participation

The task that students were engaged in within the online and face-to-face environment was different and there was an adult facilitating the interview. While it cannot be concluded that variation in student participation is solely the product of a different environment, it seems plausible that this was a contributing factor. While Zaid’s contribution remained consistent across both environments (he makes numerous utterances in both environments), the other three students appeared to make use of opportunities for action in one environment more than the other. Chaz and Igor did not make use of opportunities to collaborate online but Igor, in particular made many contributions in the face-to-face environment (37% of utterances). One possible interpretation of this data is that Igor has the capacity to recognise and utilise affordances in the face-to-face, group environment which he does not recognise nor utilise in the CSCL environment. For example, Igor may find it easier to talk about his ideas than to write about them or he may like to be seen to have an opinion or he may respond well to non-verbal signals (a glance or hand movement) by the facilitator of the interview.

In contrast, Olive may have the capacity to recognise and utilise opportunities provided by CSCL which are in contrast to the opportunities she recognises and utilises in face-to-face environments. Although she did not contribute the fewest number of utterances in the face-to-face group discussion, the utterances Olive made are generally shorter than those of the other students. When this is taken into consideration her overall contribution within the face-to-face environment of the semi-structured interview is the
smallest of the group. In the audio recordings of the interview, it is also difficult to hear her voice. This is consistent with the picture that her classroom teacher has of her; that she is shy and quiet, and only reluctantly contributes to class discussion and requires prompting by her teachers to do so. Additionally, as discussed above, Olive has been ‘streamed’ into the below level mathematics group for her daily mathematics instruction. Yet, in the CSCL environment, we can see that Olive’s contribution to online learning is significant, making up almost 40% of the group’s discussion. Using the previously articulated CAIS framework (see Table 6.1), of her posts, 64% indicate critical thinking and she makes the most Clarification type posts (73% of all Clarification posts). Olive, a shy, soft-spoken girl working with a group of vocal boys, may be inhibited in face-to-face collaboration. The CSCL environment however, offered Olive opportunities for collaborative mathematical problem solving which do not exist for her in a face-to-face interaction. The asynchronous CSCL environment gave Olive more time to consider her responses before she posted. The interpersonal interaction was also different because she did not immediately see, or hear the tone of, the reactions of others in the group. Olive appeared to find her voice in the CSCL environment and made significant and valuable contributions to her group’s work which appeared at odds with her classification as a ‘low mathematical ability’ student within the class.

7.3 Student Perceptions of Working in the CSCL Environment

Whilst the research focus study related to student development of mathematical language, Higher Order Thinking and these facets of learning were made possible through engaging in mathematical problem solving in the online environment an important consideration was how the students perceived working in this manner.

Accordingly, a series of semi-structured interviews were conducted with 6 of the 10 small groups within the larger project. During these interviews, students were asked the following questions:

- Has the collaborative learning space helped you with your learning in mathematics? If so, how?
- How do you normally feel about maths? Is there any difference in the collaborative learning space?
• Do you feel you participate more or less in the collaborative learning space compared to your actual class? Why/ why not?
• Has the collaborative learning space had an impact on your vocabulary?
• Is there anything that the collaborative learning space provides, that isn’t available in normal class time?
• What have you found difficult about the collaborative learning space?
• In what other ways are computers (or other technology) used in maths? How helpful are those uses to your learning?

7.4 Coding and Analysis of Semi-Structured Interviews

Only 6 of the 10 small groups were interviewed because of limited opportunities within the data collection period for these interviews to occur. The groups that were interviewed were selected by their teachers. They were chosen for their interviews at times that would limit potential detrimental impact to their school work.

The interviews were audio recorded and transcribed verbatim. The transcripts were then uploaded to the NVIVO (International, 2015) qualitative analysis software. The transcripts were initially read and re-read to identify apparent themes. Themes were then coded for using the qualitative analysis software. The subsequent sub-sections within this chapter represent the discussion and analysis of the themes that emerged in the semi-structured interviews. Each sub-section includes exemplary excerpts of the interviews and relevant discussion and analysis, representative of the responses to interview questions. The interviews were not premeditated and were at the convenience of the supervising teachers. Participants in the interview are referred to as Student 1, Student 2 etc. because of difficulty distinguishing individual student voices within the audio file. Additionally, excerpts have been numbered indicating when utterances have occurred either within a single conversation, thus belonging to a single excerpt, or whether utterances are part of separate conversations. In this case they have been attributed separate excerpt numbers.
7.5 Problem Solving Requires Integration of Mathematical Concepts

Students indicated that unlike a great deal of the work that they do in their class based mathematics instruction, the problem solving tasks they undertook in the online environment required a breadth of mathematical procedures and thinking.

Student 1 in excerpt 2 indicates a need to identify the types of mathematics required to assist with developing a solution to the problem and then a need to decide how to apply the selected mathematics to assist with solving the problem.

1. Student 1: Yeah. Because it’s more so about multiplication and everything, but then, Edmodo can still be about that, but it’s fun and different.

2. Student 1: It’s different from all the other maths like multiplication. It doesn’t have – It has all of them in one and then you have to figure the question out.

7.6 Student Perceived Advantages of the CSCL Environment

Some general benefits associated with the asynchronous nature of the online learning environment were described.

Excerpt 1 provides an indication that students appreciated the opportunity to be able to review previous comments made by students in addition to reviewing the problem guidelines at their convenience. This helps them to get organised as they may be likely to lose a handout with any written instruction or guidelines, therefore the always available nature of the documentation in the online environment is of benefit for students.

1. Student1: I think it’s easy that you can just scroll down, have a look at the learning task, in case you forget, sort of happening – It's not like on paper so you don't have to find the sheet. So it's easy that you can just look at it by scrolling.

Discussion in excerpts 2 and 3 shows that a problem with previous face to face collaborative learning experiences has been a necessity, when working outside of school hours, for students to organise to meet physically. For example, collaborative project work often necessitates this. The students considered this a difficulty and point out that working in the online environment is more convenient.
2. **Student 1:** I think it’s very difficult having to – If it’s not on the learning space, you’ve gotta get a date that you can go to the friend’s house that you’re working with.

**Student 2:** But when with your – With the computer or technology, you could just go on rather than going to someone’s place.

3. **Student 1:** Well, with the online one you can just go on to the online space, but if you’re working on a project, say you’re at home, you’ve gotta organise a day that you can meet up and do it.

### 7.7 Student Perceived Disadvantages of the CSCL Environment

Students described a range of disadvantages associated with the asynchronous nature of the environment.

In excerpt 1 the students express a belief that purely text-based communication limited their ability to express themselves fully. In day-to-day, face-to-face conversations they rely on gesture, expression and tone of voice to fully convey meaning. The student then explains that, in the online, text-based environment, it is important to be as ‘specific’ as possible.

1. **Student 1:** Well, sometimes when you are talking to them[ other students], you use your hands to motion things. Sometimes you do not know you are doing it but you do and then –and also you can show people or use expressions – yeah your expressions. Sometimes when you say things, it depends in what way you’re saying it but that’s why expressions are different. So, you got to be really specific.

**Student 2:** Yeah the tone of voice.

In excerpt 2 the student describes a peer’s inability to participate in the sequence of learning because of a lack of access to Internet. Concerns associated with access (or lack of access) to required technology have been consistently highlighted as issues of equity in research involving blended education (Nielsen, 2012).

2. **Student 1:** And also on the computer, we were writing texts and then Tara (pseudonym) didn’t even know we were writing so she couldn’t respond to us and she hasn’t got Internet at home so she hasn’t been on. And she’s in our group and like on the Wednesday, she has someone stay at home to help to get it done and so she just – like I was typing to her and everything and then she didn’t know we
were typing.

The students were aware in excerpts 3 and 4 that more than simply being misinterpreted in the text-based online environment, they may inadvertently cause offence to other participants through the use of purely text-based communication. In excerpt 4, the student specifically refers to the case of providing feedback to peers. Students are aware that feedback can sometimes be received with a degree of hostility. They make the point that in the face-to-face environment it can be offered in a casual way. An implication is that this should lessen opposition towards it. However, the student is aware that without the ability to utilise tone of voice, gesture or expression, a message may be received in an unfiltered manner, with greater potential to cause offence.

3. **Student 1:** Also like, if you were like talking face to face, you have – like you can usually say if you were – like you say it in a way like in a different tone of voice and if you write it on Edmodo and you write something differently, they might not know if you mean it like a way of learning stuff or the way like if you wrote something wrong or something like that so yeah. But if you were saying like, if you were talking face to face, you can – you have your expressions and stuff to say if its like a way of saying you did something wrong or its in an angry way.

4. **Student 1:** If you were just giving feedback, but they might think – they might not take it the way – so you just said it. If you were saying it face to face with somebody you would say it just in a casual way but they might not think that.

The students in excerpts 5 and 6, indicate that some students saw the asynchronous nature of the online environment as being a disadvantage. Students describe their annoyance at difficulties they experienced as a result of their peers not being online all at the same time. They indicate a preference to have an immediate response from their peers to their comments and questions, rather than having to wait until some later time for a response.

5. **Student 1:** Probably, everyone not being on at the same time.

**Student 2:** Yeah. Finding a good time that everyone can get on. It’s pretty annoying sometimes.

6. **Student 1:** I was gonna say the same thing, but if you write something, like you’re writing and the person’s not on, you don’t know what you – You’re writing a question, but the person’s not on to answer it. But when you’ve – When you speak
to that person, you know they’re gonna answer back or do something back to help you.

In excerpt 7, the two students appear to suggest that functionality associated with interactions that often occur via SMS might enhance the process. They refer to knowing when their peers are typing as an attractive feature. There is a suggestion that whilst a peer is typing a response another peer who has been waiting may elect to log off their computer and thus not see the contribution. However, if the peer who was waiting was aware that a response was likely to arrive soon (via the indication that they are typing), they may choose to stay online and thus continue the discussion.

7. Student 1: ‘Cause you will know when they’re typing. You don’t know – If you’ve been waiting a while and they’re typing something and you shut it down that you – They will just do a post and they – And you won’t even know they did it and you would be off and they’ll be saying, “Where have they gone?” Yeah.

Student 2: If it says when they’re typing, you know that somebody is going to help you and you won’t like go off and be –

Excerpt 8 indicates students might prefer synchronous communication, rather than the asynchronous approach taken in this study. This student emphasises that eliminating the pressure to wait for a response would improve the process and this could be achieved by making the environment synchronous.

8. Student 1: If you have a question, it’ll be easier to collaborate ‘cause the messages would be quicker because sometimes with Edmodo, people are waiting a long time so to get all around and they wanna just go off, wait – They don’t wanna wait.

The students in excerpt 9, make reference to positive features available when using mobile phones. They suggest that it would be helpful to be alerted by a device when another student has posted a message. Some students also referred to making their contributions to the online space using their mobile device whilst engaged in online communication. For example, one student stated, I am doing this from my tablet whilst another stated I mean yes, sorry, silly auto correct on my phone. Whilst it was not anticipated that students would work on the tasks using Internet connected mobile
devices, this does suggest that further exploration of mathematical problem solving in upper primary CSCL environments by teachers or researchers could possibly involve significant student utilisation of mobile devices.

9. **Student 1**: It should make like a noise. You know how on phones?

   **Student 2**: Yeah. It’ll probably be easier just go, “Ding! Ding!” And then you go, “Oh. There’s a message.”

It is evident from the below excerpt (10) that students perceived a degree of inconvenience in using computer based typing as the mode of communication. These students indicate that it was possibly more difficult to state their ideas and opinions in a natural and immediate manner. They indicate that in the process of communicating through typing they may have other ideas and thoughts (that cannot be easily expressed), leading to some confusion within the group.

10. **Student 1**: It’s more easy to talk verbally.

   **Student 2**: ’Cause with – On a computer, it’s a bit harder ‘cause you always have to type it in and might type something else in that you were gonna say before you. And then you might get a question that you could just ask quicker and it’s easier.

### 7.8 Changes in ICT Proficiency

Students commented on their changing proficiency in using software applications. It appears from the following excerpts taken from interview data that the most noteworthy improvement for most students related to their ability to use Microsoft Excel to represent their mathematical thinking.

*In excerpt 1*, a student initially describes growth in their general familiarisation with Excel. The student then indicates that online discussion alone may limit their ability to communicate their mathematical thinking. However, the statement appears to suggest that the addition of Excel allowed them to ‘really show’ their thinking.

1. **Student 1**: Like Excel how you make – just getting to know it so like how do you form formulas and everything on it and like how to just – because I think when you talk to people on Edmodo or something, you have got to be really specific because you are not talking to any person, you can’t really show them or anything.
Excerpts 2 and 3 show students perceiving that their ability to complete procedures within Word and Excel has improved. They have also learnt how to upload various files to an online database/website. We see mention of a new ability to use formulae. Whilst this has implications in terms of improved ICT proficiency, this equally suggests that students’ mathematical understanding of basic algebraic thinking was developed through the process. The formulae that some students used in Excel indicated a developing understanding of recursive and explicit equations.

2. Student 1: Pretty good ‘cause I've learnt new things about Word and Microsoft applications.

3. Student 1: I was confused and I found difficult when we first started doing this and we had to find folders, uploading Excel sort of thing.

Student 2: I’ve improved on posting stuff on Edmodo with Excel and using Excel as well. Like graphing and adding up numbers with formulas.

Excerpt 4 provides a timely reminder that teachers may have, at times, assumed technical competence in using various software (in this case Excel). The discussion between the two students indicates that prior to the work completed in the study, whilst at times some mention may have been made of Excel and an expectation was that students would know how to use it, in fact students had little idea of what it was or its various functions.

4. Student 1: I didn’t even know what the hell Excel was.

Student 2: Me either. I didn’t even know – So whenever – Til this year teachers would say, “Okay. Do you have the spreadsheet, the Microsoft Excel spreadsheet?” I will be like, “What’s he speaking about,” ‘cause I wouldn’t know. I wouldn’t even know – I never knew what that was. I only started using Excel this year.

Through gaining an understanding of how to use and utilise Excel (particularly for graphing), students developed a more sophisticated ability to interpret spreadsheets and graphical representations of mathematics. This is represented in excerpt 5. The student’s statement, What can you see on this? imitates the voice of a teacher questioning their students about possible meaning that can be taken from a graph. The student then continues, “And it would help you get to that point when you can actually
say this and that”. This statement suggests that by going through the process of collaboratively discussing and then constructing their graphs the students felt confident to be able to accurately interpret similar graphs beyond the context of the work completed in this study.

5. Student 1: It helps – Graphs. Bar graphs and it helps you when you make a graph. ‘Cause in Microsoft Word, you can’t do these things and you can’t get it done. ‘Cause it tells you how it is more than in words.

Student 2: Yeah. I was gonna say – Yeah – It helps you. So say, you do a graph or just even a spreadsheet, it would help you with your maths knowledge and it would help you speak more kind of like maths-like. So instead of saying – If somebody said, “Okay. What does – What can you see on this?” And it would help you get to that point when you can actually say this and that.

7.9 Comparison of Previous Experiences of ICT use and Current Experience

Almost all students confirmed their previous experience in using ICT in mathematics as either very limited (or none at all) or comprising the use of ‘drill and practice’ style online games. Commonly students suggested that these games included ‘multiplication facts’ as core content.

Excerpt 1, suggests that use of ICT in mathematics is limited. It implies that although occasionally there is an intention to incorporate ICT (reference is made to online resource ‘Maths 300’), plans associated with integration of ICT are not always realised. This is exemplified by the student’s statement, “we had this Maths 300 and we were going to use it but we still haven’t used it. So – yeah.”

1. Student 1: We don’t really use our- well we do but we haven’t really used computers for maths because we go to math group – we signed on math groups and we only – we take our computers but sometimes, we don’t really use it because we have the interactive white board and all that but last term, we had this Maths 300 and we were going to use it but we still haven’t used it. So – yeah.

Excerpt 2, indicates that the student sees limited value for their mathematics learning in the drill and practice multiplication games that they are regularly asked to play during their mathematics instruction. The student’s observation, they don’t really teach you
anything because all you just do is just click, click, click, is powerful. It suggests that the student is disengaged from the process and that little or no mathematical meaning or thinking is derived from the activity.

2. **Student 1:** The multiplication games are more of like games but they don’t really teach you anything because all you just do is just click, click, click. With the games – with the typing ones and not really anything because you’re just seven, enter.

*Excerpt 3*, provides insight into how the student contrasted the use of ICT within this study and their previous experiences of ICT within mathematics. It outlines the student’s belief that during (ICT based) small group work, in the past, students were more likely to perceive an opportunity to relax and engage in activities of a non-mathematical nature. Whilst the student does not state it directly here, it might be supposed that during these times students might also engage in non-mathematical conversation. Interestingly, the student refers to this process as *secretly just doing really nothing*. This implies, that the students believe that they are able to engage in non-mathematical activities without their teacher becoming aware of what they are really doing.

3. **Student 1:** Yeah. Edmodo, you use your brain. ‘Cause I remember last year, kids were just going, “Yes, we got computers in our math groups.” ‘Cause they thought it was really kind of a – Hour off – just like, secretly just doing really nothing.

### 7.10 Student Perceptions of Collaboration in Online Environment

It was important to understand how students perceived their experience of collaboration when working in the online environment. How did students perceive the level of peer support? How did students feel about organizational aspects of collaborating with their peers in the online environment? Did students perceive any impact on their collaboration outside of the online environment?

The dialogue in *excerpt 1* suggests that as with collaboration in the classroom, whilst students can provide the appearance of contributing, sometimes these contributions may be supportive but somewhat trivial.

1. **Student 1:** Sometimes [in class] they'll just say like, "Oh. Yes. I think that's a good
idea.” But they’re not really adding anything on, like piggybacking and that doesn’t help.

Student 2: Yeah. They are. They’re just not trying very hard.

Excerpt 2 shows that the shared responsibility of working collaboratively in the online environment is perceived by some students as an attractive facet of the process. The feeling that they do not have to carry the whole load means that some students feel a greater sense of confidence and security.

2. Student 1: It [group work] makes me feel like I still give it my hundred percent, but you don’t have to just be stressing that you’ve got do it all yourself.

Student 2: Yeah. It makes you feel like you’ve got support.

Excerpt 3 shows that Student 1, having previously completed work in the Edmodo environment, feels that collaboration is only possible if the tasks provided are conducive to this approach to working.

3. Student 1: Last year, my teacher would post our homework on there, but when we finished, we’ll just have to reply to the person, do what we like then. Yeah. Not really collaborate.

The student in excerpt 4 shared the view that working collaboratively in the online environment allowed them to consider tasks positively, not as overwhelming or burdensome because of the shared nature of the approach.

4. Student 1: Yeah. I enjoyed the one on the computers and you could actually do the work with all the people and collaborate with. Like you could do half of the work then when you next saw it, it’s already done. So then you – it’s like you’re helping them finish the work.

The student in excerpt 5, indicates that in the previous years’ attempts at using Edmodo for online learning the same level of value and importance was not placed on students’ participation. As such, many students did not participate, making it difficult for collaboration to occur.

5. Student 1: So like this year and last year, comparing. Last year was bad because they gave us homework but then you don’t know what to do so then you go home, you say, “Okay. I’ll go on Edmodo and see who’s on and I will ask if anyone
knows what to do.” And you go, and because last year, not that much people went on because it wasn’t like you have to go. It wasn’t strict so – now this year, you had like – there’s something on there you have to do so then you can – you do it and then there’s people – there’s usually people on there to help you or at least answer your question or something. So yeah, it’s better.

Students 1 and 2 indicate some unexpected results of their collaboration in the online environment (excerpt 6). Some time was taken in the classroom discussing a preference for students to communicate in as clear and comprehensible way as possible when collaborating with their peers. The first student says, that her efforts to write in a more precise and articulate way in this study have transferred to the communication that she engages in within social media. She also suggests that her efforts to communicate as clearly as possible in the online environment may have transferred, to some extent, to her face-to-face communication, impacting on her levels of confidence and willingness to partake in everyday conversations.

These sentiments are reflected without the same degree of clarity and specificity, by the second student.

6. Student 1: We’ve – I’ve definitely improved in my speaking, my social media speaking because I had even in just real life, it’s changed that ‘cause I used to have problems speaking to people, like if I met somebody I haven’t seen in ages, I go I go shy. I don’t remember them. I hide behind my mum and maybe cry, because I do not remember. But now, I’m getting a bit better and it’s helped me speak socially –

Student 2: I’ve actually got better at collaborating ‘cause last year, I was – I hardly even posted. I was one of those kids that – Wasn’t really what to say and all that and like, would normally keep at home and never did anything. But then this year, I’m everywhere. I’m talking to xxxx, going on to xxxx, posting more and I can’t believe it.

7.11 Class Based Support

In the interviews, students contrasted and compared their experiences of working in an online collaborative learning environment in the previous year, when they were in Year 4, using Edmodo primarily to submit homework, and the experience that they had
during the data collection phase of this study. The following excerpts provide an understanding of how they contrasted the experiences.

The student in excerpt 1, believes that their understanding of how to utilise and interact within the online environment has been strengthened through clear teacher guidance and support and also indicates that in previous experience working in a similar online fashion adequate support was not provided and this made it difficult for learning to occur.

1. **Student 1:** Yeah, ‘cause they didn’t really explain it good to us but you do, you take time to explain it to us and show us what to do but as last year, they’d show your work and this is what you do and then that’s it.

In the statement below the student points to benefits from the social construction of concepts and ideas that is a product of whole group class discussion.

2. **Student 1:** Like you go through it and then we ask questions and then you answer the questions clearly and then we can just – then we go home and then we just do it and then we know what to do.

In excerpt 4, the student emphasizes the need for adequate discussion of how to go about working in the online environment when in the classroom. The student suggests that if the material is dealt with somewhat superficially in the classroom (by the teacher), it is likely that when students begin working on their computers online without support, that they will find it difficult getting started and having success with the process. It is probable that students in upper primary school would find a wholly online approach to collaborative mathematical problem solving very difficult. Students at this stage of development require classroom based structured discussion/ facilitation. This approach can be considered a form of blended learning.

3. **Student 1:** So it’s like sometimes like you read it and all that but when you get back home, like you say, “Oh I forgot.” Then you go through it again but because – if it’s not explained properly, then you say, “But what about this? How’d you do this and all that?” But if they [the teacher] explained properly, you can read it and you understand what you’re doing again.

The student indicates in excerpt 4 that having a clear description of the task embedded within the online environment was also valuable. If the requirements of the task were
not supported by guidelines being made available in the online environment this student suggests that it would be likely that some students may either forget what the requirements of the task were or that there would be a degree of lack of uncertainty which could produce a sense of unease.

4. **Student 1:** I know last year, we kind – like the teachers they gave it to us but they gave – so they said it in class but then people can forget and with this year, it’s on Edmodo, it’s online, the questions and what we had to do.

The excerpt below reiterates that students value ownership of the process after the initial support received through clear and targeted discussion in the classroom followed by independent practice. The students experienced success through knowing what the requirements of the tasks were and then being able to implement their approaches to solving the problems.

5. **Student 1:** Yeah. But now I can do it myself since you’ve explained it to me.

### 7.12 Student Perceptions of Grouping

Students identified their belief that small collaborative groups of three to five students supported them in the process.

*Excerpt 1* highlights a perceived difference between last year, when they also did some work involving an online discussion board, and the work that the students completed as part of this study. This student expresses their belief that ‘small groups’ make it easier for students to work together online when compared to their previous experience of having a whole class (up to 30 students) all in one online discussion board.

1. **Student 1:** Last year when we didn’t really have groups and it was a bit difficult but this year, you put us into groups. Yeah ’cause it’s a bit easier with the groups.

It appears from *excerpt 2*, that in the preceding year students had difficulty finding contributions by other students to either respond to or support them with their own thinking, when they were all placed in a single online discussion board. This supports the choice taken to move away from a whole group message board, which was an approach used in a pilot version of this study, to the alternative used in this study.

2. **Student 1:** Yeah because if you have everyone altogether, it’s just like too much
and you wanna look at somebody’s because they have a nice detailed one and you just wanna just read it. You go down and you just can’t find it so it’s really hard.

Evidently, some students believed that being placed in small groups allowed them to facilitate times for all members of the group to be online to collaborate on the mathematical problems concurrently. Students perceived a benefit of being online at the same time. This provided students the facility to work ‘synchronously’ despite no original intention for students to work in this way.

3. It’s good that you have a group. It’s not by yourself and you have certain times. So we will say, “Okay. What about we go – When we go home, we can have a bit of a break until four o’clock and go to five o’clock or so.” So you have that hour to do Edmodo. And that’s easy, and then it’s easy because we go in a group and then you can easily chat with people instead of being by yourself and you have to – Yeah.
Chapter 8: Conclusions, Implications, Recommendations and Limitations

This chapter will proceed in three parts. Firstly, a number of sub-sections will provide a general discussion of conclusions, along with any associated implications or recommendations. In the second part, four sub-sections address each of the four research questions outlined in the introductory chapter of the thesis. In the last part I will describe the value and originality of this research.

8.1 CSCL as an Approach to Assessment of Mathematical Problem Solving, Reasoning and Critical Thinking

One unanticipated implication that has stemmed from this study has been that the approach to collaborative mathematical problem solving described offers significant opportunities for student assessment.

Across the world, curricula are now requiring teachers to facilitate and assess problem solving, reasoning and critical thinking. It is no longer enough to require students to ‘do mathematics’ rather, there is an increasing demand that students should be able to demonstrate an ability think, behave and communicate mathematically (Boaler, 2008). In the USA, The Common Core Standards for mathematics, released in 2010 (NGA Center, 2012) for the first time included Standards for mathematical Practice. These detailed the level to which students should be able make sense of, persevere with, reason, argue and critique, model and choose the appropriate tools and strategies when engaging in mathematical activity. In Australia, a national curriculum released in 2013 also placed a new emphasis on problem solving, reasoning and Communicating mathematics. In the Australian Curriculum: Mathematics (ACARA, 2014), this takes the form of the ‘Proficiency Strands’. Within problem solving and reasoning, as has been discussed earlier, students are required to model, investigate, communicate, analyze, evaluate, explain, infer, justify and generalize.

Traditional modes of assessment that privilege summative above formative approaches and focus on a student’s ability to perform procedures, for example, through the use of regular mathematical content driven pre- and post-tests, provide students, teachers and
parents with limited information about the student’s ability to engage mathematically. These traditional methods of assessment may provide the teacher with a skewed understanding of the range of abilities and levels of understandings in their classrooms. Students who may be very confident in completing basic computation and applying procedures, sometimes beyond the level expected of them, may struggle to apply these skills to problem based contexts. They may not have the ability to think creatively given the context of an unfamiliar problem. Whilst displaying logical procedural thought they may be largely unable to explain and communicate their mathematical thinking.

Conversely, some students who may not perform as well on the narrowly targeted pre-and post-tests may demonstrate a more refined and nuanced ability to engage in the mathematical communication, investigation and reasoning associated with collaborative problem solving. The approach taken to problem solving in this study has provided an exemplification of a new approach to assessment of a broader range of mathematical competencies. In Chapters 6 and 7 of this study, the work of a female student perceived by her teacher to be below level, was analysed as she worked in an online environment. Not only was she found to contribute a greater volume of discussion within the space, but through analyzing her contributions using Perkins and Murphy’s (2006) CAIS framework, it was possible to show that 64% of her posts pertained to critical thinking. Alternative assessment strategies may provide her teachers with a more accurate view of her abilities and how best to guide her learning.

Unfortunately, in the everyday congestion and frenzy of noise and movement that can often occur when collaborative mathematical problem solving takes place in a primary school context, it can be very difficult for the teacher to keep a clear record of the conversations, types of thinking and quality of work being produced from individuals and groups of students. Whilst taking notes, ticking off checklists and so on, the teacher may be distracted from the primary goal of providing carefully targeted feedback and support to their students; using questioning and challenging prompts that have been judiciously selected after deliberate and sustained observation of the interaction that is occurring.

An online environment can circumvent many of these issues. The online provisions utilised in this study provide a detailed record of interactions and the production of
meaning making artefacts that are necessary for teachers to reliably make judgments about students' abilities in these areas.

At best, a teacher within a classroom will have the opportunity to observe each group for a few moments within a session. In the CSCL environment the teacher can review all interaction and discussion. All discussion and interaction becomes available. This also means that the teacher has easy access to data related to the amount of time each group remained ‘on task’. Again, in a traditional classroom in an environment in which discussion is encouraged, it can be very difficult for the teacher to know that the discussion that is taking place in each small group is productive and related to the mathematical problem being investigated.

8.2 CSCL as an Approach to Assessment of Mathematical Language Development

In Chapter 6, the density of technical mathematical vocabulary use was identified and analyzed. Only generally does a student’s perceived level of ability indicate their use of technical mathematical vocabulary.

Producing and analyzing data in this way was a simple and not overly time-consuming process. The Introductory chapter to this thesis described the emphasis that the Australian Curriculum: Mathematics (ACARA, 2014) now places on communication of mathematical ideas through the use of digital technologies. This is an important component of reasoning, one of the four mathematical proficiency strands described in the curriculum. One benefit that practising teachers could take from adopting an online approach to mathematical problem solving similar to this study would be to enhance their ability to monitor the density of technical mathematical vocabulary use by their students in the space.

Throughout the nine weeks that the study took place, one question was whether the density of technical mathematical language use would increase. It was difficult to find definitive evidence that this was the case. There were several possible reasons for this. Firstly, the mathematical focus of the nine weeks of problem solving encompassed all content strands: Number & Algebra, Measurement & Geometry and Statistics & Probability. Each of these content strands could be viewed independently of one
another in terms of language. Whilst some technical mathematical language is shared amongst all areas of mathematics, much of the language and vocabulary used is unique to the individual strands. Therefore, the process of moving from one strand of mathematics to another on a weekly basis involved the students developing and utilizing a new technical register each week. If this study was replicated, it would be useful to maintain the focus of mathematical problems within the one mathematical content strand. This should lead to a perceptible increase in the density of technical mathematical vocabulary over the period of the study. But this question remains to be investigated.

8.3 Choice of Mathematical Tasks/Problems in the CSCL Unit of Work

This study can in some ways be considered exploratory or a pilot in that it appears to be the first that has required primary age students to engage in collaborative mathematical problem solving in a CSCL environment. Given the absence of other researchers’ findings about the structure, content and type of problems used, here problems were created and adapted based on the following tenets:

- Problems should be ‘open-ended’
  
  Given the goal of promoting discussion and collaboration within the online space, it was desirable to avoid closed problems with a single solution. The problems that were selected required a lack of closure. An example of this was the problem ‘How big is a dog?’ The somewhat ambiguous term ‘big’ was used in the hope of stimulating students to put forth an argument as to whether they believed ‘big’ should be based on ‘height’, ‘weight’, ‘length’ etc. After putting forth their argument it was hoped that they would then justify their stance.

- Problems should contain enough detail and support, such that the students are able to make progress without an online facilitator

  As previously described, the choice was made not to embed an online ‘facilitator’ within the space. Research by Andresen (2009) has shown that the instructor’s presence within such a space could significantly alter the natural flow of dialogue between participants. As such the problems themselves contained some hints and
Conclusions, Implications, Recommendations and Limitations

clues about appropriate approaches to solving the problems. Still there was a risk that without providing assistance some students might not be able to make a start and only limited collaboration would occur. This risk proved to be unfounded

- Problems should contain mathematical foci from each of the content strands of the relevant curriculum

It is possible that some mathematical content areas might be better suited to working in this way. Whilst this was part of the research design, a repeat of the study might benefit from a focus on one content area throughout. If students had worked on one mathematical area for the entirety of the period, a clearer increase in technical mathematical language might have become evident.

- There should be a sequence of increased but logical technological demands

Students were largely unfamiliar with ‘Edmodo’ in which the CSCL environment was situated. They had also had almost no experience with Microsoft Excel. This was also the first time the researcher has used Edmodo as a platform for CSCL. In addition to the students learning about how to interact with and use this particular platform, the researcher/facilitator was also on a ‘steep’ learning curve. However, the technological demands placed on students earlier on were minimal. There was no suggestion that the students might make use of formulae or other advanced functions of Excel and so on. Students gradually became more comfortable working in the environment. It is recommended that when coordinating an online CSCL environment, such as this, teachers should carefully monitor their students’ participation and tailor subsequent week’s activities based on the levels of success/competence displayed in previous weeks.

- Opportunities should be provided for students to represent their mathematical meaning making using various common applications (e.g. Microsoft Excel, Microsoft Word, Microsoft Paint)

Unlike the classroom where teachers explicitly embed tools/manipulatives (e.g. rulers, protractors, counters) within their lesson planning, the ability to physically interact with ‘meaning making’ representations may be limited. Students needed to be able to represent their thinking in ways beyond simply describing their thinking in the dialogue within the space. Students were encouraged and shown how to use software (such as Microsoft Excel, Microsoft Word, and Microsoft Paint) to
represent, develop, communicate and articulate their thinking. Files that were created were then uploaded to Edmodo.

8.4 Alignment of Online Mathematical Problem Solving and the Australian Curriculum

*RQ1*  *How does student engagement with online mathematical problem solving align with the Australian Curriculum?*

An important rationale for the thesis was to provide evidence that the approach taken to integration of technology within mathematics was consistent with the expectations of the Australian Curriculum: Mathematics (ACARA, 2014).

To this end, in Chapter 4 the mathematical proficiencies of problem solving and reasoning in addition to Australian Curriculum statements about technology embedded mathematics (ACARA, 2014) were analysed and used to code the online discussion students engaged in. This allowed understanding of whether the approach taken would assist students to develop in these areas and also provided insight into how different sub-groups within the greater study would respond in these areas.

It was found that small collaborative groups engaging in online discussion of the mathematical problems in combination with their capacity to develop and upload meaning-making artefacts did support the students in this study to utilise technology within their mathematics learning in manner consistent with the ambitious goals of the current Australian Curriculum (ACARA, 2014).

8.5 Communication of Mathematical Meaning through use of Language in the CSCL Environment

*RQ2*  *How is language used to communicate mathematical meaning when Year 5 students work in an online CSCL environment?*

Of special interest to this study was the way in which language would be used, and how it would develop in a CSCL environment. Little consideration had been given to other modes of mathematical meaning making. It was assumed that as students became more
confident working in the environment the volume and quality of discussion would increase.

However, as shown in Chapter 4, this was not conclusive. As students became more confident working in the CSCL environment and in their abilities to manipulate their mathematical ideas and represent their thinking in software (particularly Microsoft Excel), they were likely to embed this meaning in the artefacts that they were creating. They seemed to rely heavily on these artefacts to carry and communicate their mathematical ideas in combination with the use of text based language.

Chapter 4 showed that, as the weeks progressed, students tended to embed the categories of problem solving and reasoning regularly in their uploaded artefacts. In addition, Chapter 5 allowed investigation of how the students communicated their mathematical meaning when engaged in online discussion. In the Bakhtinian (1981) analysis central to this chapter, the dialogic nature of the context was important to understanding the way language was fundamental to making mathematical meaning throughout the nine weeks. The hour of classroom discussion facilitated by the researcher each week prior to work in the online space was vital for students. This provided students with the language to discuss and test their ideas online. This environment allowed students to be in dialogue with the researcher/facilitator who was not physically present. At times, within their online discussion, it was possible to see students borrow and utilise other students’ personalised terms to express their mathematical thinking. This is further evidence of the dialogic nature of the environment. In this study, the dialogic emergence of language being used between the informal and formal mathematical register (Halliday, 1978) has come to be known as the Transitional mathematical Register.

8.6 Evidence of Critical Thinking in Online Mathematical Problem Solving

RQ3 What evidence is there of critical thinking when students engage in ‘talk’ within online mathematical problem solving?

Data gathered and analysed in Chapter 6, indicates that the presence of Exploratory Talk can be used to reveal critical thinking. This study found that there was a
relationship between the density of technical mathematical vocabulary use and the use of *Exploratory Talk* as discussed by Mercer and Wegerif (1999). Generally, where *Exploratory Talk* was present, there was a greater density of technical mathematical vocabulary. As *Exploratory Talk* was found to be a more productive talk type, when compared with *Disputational* or *Cumulative Talk*, online mathematical problem solving can provide a practicing teacher with an objective measure of the quality and nature of mathematical meaning making dialogue. All four of Perkins and Murphy’s (2006) CAIS categories were in greater evidence where *Exploratory Talk* was identified. Where a higher density of technical mathematical vocabulary, which was common in students *Exploratory Talk*, is present it is more likely that critical thinking will also be present.

8.7 Students Responses to Working in the Online Mathematical Problem-Solving Environment

*RQ4*  *How do Year 5 students respond to the experience of working within an online environment to collaboratively solve mathematical problems?*

8.7.1 Student Levels of Participation

This study did provide findings that were not anticipated, but are worthy of reporting on. Observations and reflections on student levels of engagement in the online space are an example of this. In the study, explicit steps were taken to ensure that small groups had an even distribution of students from each ability level, thus creating mixed ability groups. Having this data provided insight into levels of participation according to assigned ability ranking. For example, would students who had been classified as *higher ability* contribute more than students classified as *lower ability*?
Table 8.1 Levels of Student Engagement with the Space

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<th>Average no. utterances per student</th>
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</tbody>
</table>

As seen in the above table, this largely eventuated, with one notable exception. The *below level* girls’ level of participation was the second highest of the six groups. Having established this, a further question arose as to whether this higher level of participation might be present in face-to-face discussions. Since a class-based mathematical problem solving group had not been employed for the purposes of comparison the semi structured interviews were used as a proxy, based on an assumption that levels of participation in these discussions would likely mirror levels of participation in class based problem solving. Not all groups were interviewed as a part of this process. However, one group, Group 9, was interviewed. This group was made up of two boys who were deemed *at level* one boy deemed *above level* and one girl deemed *below level*.

As reported the girl had been the second most active participant within the online environment. However, she was the second least active in discussions held during interviews. Additionally, each of her contributions (utterances) were the shortest length (number of words) in the group. Whilst it is difficult to draw strong conclusions based on one small focus group, an implication for further research would entail setting up a larger study involving groups of students who participated in both online collaborative problem solving and in class problem solving. All group discussion could be recorded and transcribed and then a comparison could be made between students’ rates of participation within the online environment and within class-based work.

A final observation to take from Table 8.1 is the very low relative level of engagement with the process of the *Below Level Boys*. It is concerning that these students were
somewhat disconnected from the process. It would be important in future research to investigate if this reoccurred, and if so to understand the likely cause.

### 8.7.2 Student Perceptions of Working in the Online Environment

The major themes that emerged from semi-structured interviews with students about their perceptions of working within the online environment were as follows:

- Students generally perceived that their technical proficiency improved throughout the nine weeks. For instance, a number of students made mention of their understanding of how to make productive use of Excel to represent data with graphs and to process basic calculations.
- Students compared their experience of working with Edmodo in the previous year and the experience they had as part of this study. Many concluded that the additional hour of classroom discussion was very important for them to understand what was required and the steps they needed to go through to have success.
- Students were aware when the quality of their discussion was less productive. They made comments about the difference between comments that moved the group forward in their thinking and comments that whilst positive in nature were not helpful to progress the current problem. This suggests that students at this level would be receptive to explicit teaching of how to engage in Exploratory Talk (productive talk) rather than other less productive talk types.
- Students contrasted the use of technology in mathematics with previous experiences of technology use in mathematics instruction. They concluded that where previously drill and practice activities did not require high level critical thinking, problem solving and reasoning, these areas were more clearly focussed on and developed through the approach taken in this study.
- Some students suggested a preference for a synchronous platform. They suggest that a ‘chat’ feature would have allowed for a more cohesive flow within their discussion.
8.8 Originality and Value of Research

Whilst other researchers, for example Stahl (2009), have utilised CSCL environments to study MPS in Secondary school and Tertiary contexts; and others have analysed innovative approaches to the integration of technology in primary school contexts, such as Chrysanthou (2008), it is difficult to identify examples of where researchers have investigated primary school students engaged in MPS in a collaborative, online asynchronous environments. This thesis makes an important contribution for researchers to consider because as outlined earlier, the context provides teachers the opportunity to foster a range of proficiencies and capabilities that are currently required by curricula around the world.

8.8.1 Contribution to New Knowledge

As a result of this study teachers and researchers can be confident that, with appropriately selected problems, mixed ability classes of Year 5 students can communicate mathematically through online asynchronous learning environments. At this level students can utilise a range of software platforms to support them in their investigations. They will communicate using aspects of informal and formal mathematical language, at times using a transitional mathematical register. Through approaching mathematical problem solving in this way, students will engage in and further develop problem solving, reasoning and critical thinking skills.

8.8.2 Opportunities for Researchers

No concurrent, conventional, classroom-based mathematical problem solving occurred as a point of comparison in this study. It would be of interest to researchers to facilitate a side-by-side comparison of an experimental group approaching data collection in a similar manner to that described in this study and a comparison group engaging in a conventionally led MPS activity. This would allow some analysis of how language, problem solving, reasoning and critical thinking all developed and evolved in two different contexts.

Whilst, there was evidence of growth in mathematical reasoning throughout the nine-week data collection period, there was no evidence of growth in density of technical mathematical language. As stated, the nine-week sequence of problems were not
focussed in any one of the three content domains of mathematics. An opportunity for research would therefore be to adapt or replicate the study in such a way that the mathematical content focus remained consistent across the period.

### 8.8.3 Opportunities for Teachers

It is important that the final statements contained within this thesis relate to what it has to offer for teachers and leaders within the field of mathematics education. This is a thesis written with the aim of fulfilling the requirements of the degree of Doctor of Education which is designed for experienced educators to engage in research which has direct relevance to the teaching profession. I made the choice to study and work towards this particular degree because it was my preference and goal to be able to conclude this journey with practical and tangible opportunities and advice for professionals in the field.

As such, when disseminated in seminars and professional publications teachers will identify opportunities for the assessment of problem solving, reasoning and critical thinking within the contents of this thesis. Given the current curriculum-based imperatives and educational climate this is a pressing need for educators.

This study also offers teachers a more productive approach to the integration of technology in Primary Mathematics. There is too much evidence that currently technology is predominantly used within Primary Mathematics to drill fluency. The approach provided in this study allows for the development of further mathematical proficiencies.

This study also shows how a comprehensive transcript of all student discussion can be made available to teachers when mathematical problem solving occurs online. This allows teachers to monitor student levels of participation and identify areas of student mathematical strength that might otherwise remain unnoticed.

Many researchers, for example, Boaler, Wiliam, and Brown (2000), Clarke and Clarke (2008) andForgasz (2010) suggest that grouping students according to ability in mathematics instruction can have many unintended negative consequences. Day to day mathematics instruction in the classroom where the study took place was organised according to teacher allocated ability groups. One third of students were placed in a
Below Level group for their mathematics instruction, another third were placed in an At Level group for their mathematics classes and the final group were placed in an Above Level grouping. The approach taken in this study showed that students who were placed in these groups could all succeed. In some cases, students who were placed in the Below Level group would outperform those in other groups. Thus, a final recommendation is that the approach taken in this study offers teachers a more equitable approach to Primary Mathematics teaching.
References


Appendices

Appendix 1: Plain Language Statement

"Using Online Collaborative Learning Spaces in Mathematics Education: What impact does Problem Solving in an online collaborative learning environment have on the promotion of authentic learning?"

Your child is invited to participate in the above research project, which is being conducted by Associate Professor Robyn Pierce (supervisor) and Mr. Duncan Synons (Doctoral Candidate) of the Melbourne Graduate School of Education at The University of Melbourne. Your child is in the year 5 classroom chosen to investigate how online resources (in this case "Wiki") can be used to help teach problem solving in mathematics. This project will form part of Mr. Synons' Doctoral thesis, and has been approved by the Human Research Ethics Committee.

The aim of this study is to investigate how students can benefit from engaging in mathematical problem solving collaboratively in an online environment. The work that your child participates in, within this study, will supplement their everyday mathematics instruction. All students will participate in this, however only those that return permission forms will be used for data collection and analysis.

Should you agree to your child's participation, your child would be asked to contribute to this in two ways.

- First, we would ask your child to discuss and solve one mathematical problem-solving question each week throughout term two. This would form part of their everyday work in their mathematics-based studies.
- Second, we would ask your child to participate in brief interviews to a total of 45 minutes, so that we can get a more detailed picture of your child’s thoughts, reactions and feedback surrounding this approach to mathematics instruction. With your permission, the interviews would be audio-recorded so that we can ensure that we make an accurate record of what your child has said.

We intend to protect your child's anonymity and the confidentiality of his/her responses to the fullest possible extent, within the limits of the law. Your child's name and contact details will be kept in a password-protected computer file separate from any data that is supplied. In the final report, your child will be referred to by a pseudonym. We will remove any references to personal information that might allow someone to guess your child’s identity, however, you should note that as the number of students we seek to interview is small, it is possible that someone may still be able to identify your child.

A brief summary of the findings will be available to you on application at the Melbourne Graduate School of Education. It is also possible that the results will be published in journal articles and presented at academic conferences. The data will be kept securely in the Melbourne Graduate School of Education for five years from the date of publication, before being destroyed.

Please be advised that your child's participation in this study is completely voluntary. Should you wish to withdraw your child at any stage, or to withdraw any unprocessed data supplied, you are free to do so without prejudice.

If you would like to participate, please indicate that you have read and understood this information by signing the accompanying consent form and returning it in the envelope provided.

Should you require any further information, or have any concerns, please do not hesitate to contact either of the researchers; Associate Professor Robyn Pierce: 8344 8610, Mr. Duncan Synons: 9376 4423. Should you have any concerns about the conduct of the project, you are welcome to contact the Executive Officer, Human Research Ethics, The University of Melbourne, on ph: 8344 2070, or fax: 9347 0729.
Appendix 2: Consent Forms

“Using Online Collaborative Learning Spaces in Mathematics Education: What impact does Problem Solving in an online collaborative learning environment have on the promotion of authentic learning?”

Your child is invited to participate in the above research project, which is being conducted by Associate Professor Robyn Pierce (supervisor) and Mr. Duncan Symons (Doctoral Candidate) of the Melbourne Graduate School of Education at The University of Melbourne. Your child is in the year 5 classroom chosen to investigate how online resources (in this case ‘Wiki’) can be used to help teach problem solving in mathematics. This project will form part of Mr. Symons’ Doctoral thesis, and has been approved by the Human Research Ethics Committee.

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We intend to protect your child’s anonymity and the confidentiality of their responses to the fullest possible extent, within the limits of the law. Your child’s name and contact details will be kept in a password-protected computer file separate from any data that is supplied. In the final report, your child will be referred to by a pseudonym. We will remove any references to personal information that might allow someone to guess your child’s identity. However, you should note that as the number of students we seek to interview is small, it is possible that someone may still be able to identify your child.

A brief summary of the findings will be available to you upon application at the Melbourne Graduate School of Education. It is also possible that the results will be published in journal articles and presented at academic conferences. The data will be kept securely in the Melbourne Graduate School of Education for five years from the date of publication, before being destroyed.

Please be advised that your child’s participation in this study is completely voluntary. Should you wish to withdraw your child at any stage, or to withdraw any unprocessed data supplied, you are free to do so without prejudice.

If you would like to participate, please indicate that you have read and understood this information by signing the accompanying consent form and returning it in the envelope provided.

Should you require any further information, or have any concerns, please do not hesitate to contact either of the researchers, Associate Professor Robyn Pierce: 8344 8519, Mr. Duncan Symons: 9635 4423. Should you have any concerns about the conduct of the project, you are welcome to contact the Executive Officer, Human Research Ethics, The University of Melbourne, on ph. 8344 2073, or fax: 9347 6739.
Melbourne Graduate School of Education

Consent form for persons participating in a research project

PROJECT TITLE: “Using Online Collaborative Learning Spaces in Mathematics Education: What impact does Problem Solving in an online collaborative learning environment have on the promotion of authentic learning?”

Name of participant:

Name of investigator(s): Associate Professor Robyn Pierce, Dr Caroline Bardini & Mr. Duncan Symons

1. I consent to my child’s participation in this project, the details of which have been explained to me in a written plain language statement to keep.

2. I understand that after I sign and return this consent form it will be retained by the researcher.

3. I understand that my child’s participation will involve interviews and observation of online collaboration and I agree that the researcher may use the results as described in the plain language statement.

4. I acknowledge that:
   a) the possible effects of participating in interviews and observation have been explained to my satisfaction;
   b) I have been informed that I am free to withdraw my child from the project at any time without explanation or prejudice and to withdraw any unprocessed data that has been provided;
   c) the project is for the purpose of research;
   d) I have been informed that the confidentiality of the information provided will be safeguarded subject to any legal requirements;
   e) I have been informed that with my consent interviews will be audio-taped and I understand that audio-tapes will be stored at University of Melbourne and will be destroyed after five years;
   f) my child’s name will be referred to by a pseudonym in any publications arising from the research;
   g) I have been informed that a copy of the research findings will be made available to me, should I agree to this.
   h) I have been informed that removal of any references to personal information that might allow someone to guess my child’s identity will occur, however, as the number of students being interviewed is small, it is possible that someone may still be able to identify my child.

   □ yes □ no (please tick)

I consent to interviews being audio-taped

I wish to receive a copy of the summary project report on research findings

Participant signature (child): Date:

Participant signature (Parent/Guardian): Date:
Appendix 3: Ethics Approvals

Department of Education and Early Childhood Development
Strategy and Review Group

2 Treasury Place
East Melbourne, Victoria 3002
Telephone: +61 3 9637 3000
DX 21033
GPO Box 4367
Melbourne, Victoria 3001

2014_002258

Mr Duncan Symons
Graduate School of Education
The University of Melbourne
234 Queensberry Street
MELBOURNE 3010

Dear Mr Symons

Thank you for your application of 17 January 2014 in which you request permission to conduct research in Victorian government schools and/or early childhood settings titled Using Online Collaborative Learning Spaces in Mathematics Education. What impact does Problem Solving in an online collaborative learning environment have on the promotion of authentic learning?

I am pleased to advise that on the basis of the information you have provided your research proposal is approved in principle subject to the conditions detailed below.

1. The research is conducted in accordance with the final documentation you provided to the Department of Education and Early Childhood Development.

2. Separate approval for the research needs to be sought from school principals and/or centre directors. This is to be supported by the DEECD approved documentation and, if applicable, the letter of approval from a relevant and formally constituted Human Research Ethics Committee.

3. The project is commenced within 12 months of this approval letter and any extensions or variations to your study, including those requested by an ethics committee must be submitted to the Department of Education and Early Childhood Development for its consideration before you proceed.

4. As a matter of courtesy, you advise the relevant Regional Director of the schools or governing body of the early childhood settings that you intend to approach. An outline of your research and a copy of this letter should be provided to the Regional Director or governing body.

5. You acknowledge the support of the Department of Education and Early Childhood Development in any publications arising from the research.

6. The Research Agreement conditions, which include the reporting requirements at the conclusion of your study, are upheld. A reminder will be sent for reports not submitted by the study’s indicative completion date.
24 April 2014

A/Prof. Robyn Pierce
Melbourne Graduate School of Education
The University of Melbourne

Dear A/Prof. Pierce,

I am pleased to advise that the Melbourne Graduate School of Education Human Ethics Advisory Group (MGE HEAG) has approved the following Minimal Risk application:

Project title: Using Online Collaborative Learning Spaces in Mathematics Education: What impact does Problem Solving in an online collaborative learning environment have on the promotion of authentic learning?
Researchers: Robyn Pierce, Duncan Symons and Caroline Bardi

Ethics ID: 1444126
MGE HEAG ID: 30/14

The project has been approved for the period: 24 April 2014 to 31 December 2014.

It is your responsibility to ensure that all people associated with the Project are made aware of what has actually been approved.

Research projects are normally approved to 31 December of the year of approval. Projects may be renewed yearly for up to a total of five years upon receipt of a satisfactory annual report. If a project is to continue beyond five years a new application will normally need to be submitted.

Please note that the following conditions apply to your approval. Failure to abide by these conditions may result in suspension or discontinuation of approval and/or disciplinary action:

(a) Limit of Approval: Approval is limited strictly to the research as submitted in your Project application.

(b) Amendments to Projects: Any subsequent variations or modifications you might wish to make to the Project must be notified formally to the Human Ethics Advisory Group for further consideration and approval before the revised Project can commence. If the Human Ethics Advisory Group considers that the proposed amendments are significant, you may be required to submit a new application for approval of the revised Project.

(c) Incidents or adverse effects: Researchers must report immediately to the Advisory Group and the relevant Sub-Committee any incident which might affect the ethical acceptance of the protocol including adverse effects on participants or unforeseen events that might affect continued ethical acceptability of the Project. Failure to do so may result in suspension or cancellation of approval.

(d) Monitoring: All projects are subject to monitoring at any time by the Human Research Ethics Committee.

(e) Annual Reports: Please be aware that the Human Research Ethics Committee requires that researchers submit an annual report on each of their projects at the end of the year, or at the conclusion of a project if it continues for less than this time. Failure to submit an annual report will mean that ethics approval will lapse.

(f) Auditing: All projects may be subject to audit by members of the Sub-Committee.

Please quote the Ethics registration number and the name of the Project in any future correspondence.

On behalf of the Ethics Committee I wish you well in your research.

Yours sincerely

[Signature]

Associate Professor Dianne Vale-Brodrick
Chairperson, Melbourne Graduate School of Education Human Ethics Advisory Group
Phone: 83444254, Email: dianne.vale-brodrick@unimelb.edu.au

cc: Duncan Symons, Caroline Bardi, and Human Research Ethics Committee, Melbourne Research Office.

Melbourne Education Research Institute (MERRI)
Melbourne Graduate School of Education
Level 9, 301 Alexandra Street | The University of Melbourne, Victoria 3010 | Australia
T: +61 3 8344 8220 F: +61 3 8344 8223 W: www.education.unimelb.edu.au/research
Appendix 4: Examples of Coding

Coding Example – Talk Types

Z. • May 24, 2014
I think the tallest dog in our case is the Blue Great Dane, or the other two options could be the Anatolian Shepherd, or the English Mastiff.

O. • May 24, 2014
Okay then we would just do the tallest. Mine cause it could be said. In 2 ways and for our group it could be tallest

O. • May 24, 2014
Witch one would you prefer the really first one or last?

Z. • May 25, 2014
I think for the second biggest meaning, we should make it mean the weight. What do you think?

O. • May 25, 2014
Yeah I agree but then Also I’m thinking other groups could be doing weight maybe most. Maybe let’s try doing something different so maybe tallest?? What do you think?

O. • May 25, 2014
I just thought once again and I decided on tallest

Z. • May 25, 2014
Do you have to do Extension 17?

O. • May 25, 2014
What do you mean??

O. • May 26, 2014
remember everyone needs to hop on edmodo more like issac and cooper that havent been on its only you and I that are discussing this homework task.

Z. • May 26, 2014
I know!

Z. to Group 9 (SKC and SBM - Collaborative Problem Solving)
This is my excel document. The one I worked out how to do the graph.
edmodo.xlsx
XLSX File

Z. to Group 9 (SKC and SBM - Collaborative Problem Solving)
This is my word document. that has the graph by it self clearly.
Coding Example – CAIS

Z. • May 24, 2014
I think the tallest dog in our case is the Blue Great Dane, or the other two options could be
the Amstollin Shepherd, or the English Mastiff.

O. • May 24, 2014
Okay then we would just do the tallest. Mine cause it could be said. In 2 ways and for our
group it could be tallest

O. • May 24, 2014
Which one would you prefer the really first one or last?

Z. • May 25, 2014
I think for the second biggest meaning, we should make it mean the weight. What do you
think?

O. • May 25, 2014
Yeah I agree but then also I'm thinking other groups could be doing weight maybe most.
Maybe let's try doing something different so maybe tallest?? What do you think?

O. • May 25, 2014
I just thought once again and I decided on tallest

Z. • May 25, 2014
Do you have to do Extension 1?

O. • May 25, 2014
What do you mean??

O. • May 26, 2014
remember everyone needs to hop on edmodo more like issue and cooper that haven't been on its
only you and i that are discussing this homework task.

Z. • May 26, 2014
I know!

Z. to Group 9 (5KC and 5BM - Collaborative Problem Solving)
This is my excel document. The one I worked out how to do the graph,
edmodo.xlsx
XLSX File

Z. to Group 9 (5KC and 5BM - Collaborative Problem Solving)
This is my word document. that has the result by it self clearly.
Coding Example – Australian Curriculum – Problem Solving/Reasoning

Z. • May 24, 2014
I think the tallest dog in our case is the Blue Great Dane, or the other two options could be the Anatolian Shepherd, or the English Mastiff.

O. • May 24, 2014
Okay then we would just do the tallest, mine cause it could be said, in 2 ways and for our group it could be tallest.

O. • May 24, 2014
Which one would you prefer the really first one or last?

Z. • May 25, 2014
I think for the second biggest meaning, we should make it mean the weight. What do you think?

O. • May 25, 2014
Yeah I agree but then also I'm thinking other groups could be doing weight maybe most. Maybe let's try doing something different so maybe tallest?? What do you think?

O. • May 25, 2014
I just thought once again and I decided on tallest.

Z. • May 25, 2014
Do you have to do Extension 1?

O. • May 25, 2014
What do you mean??

O. • May 26, 2014
Remember everyone needs to hop on edmodo more like issac and cooper that haven't been on its only you and I that are discussing this homework task.

Z. • May 26, 2014
I know!

Z. to Group 9 (SKC and 3BM - Collaborative Problem Solving)
This is my excel document. The one I worked out how to do the graph.
edmodo.xlsx
XLWX File

Z. to Group 9 (SKC and 3BM - Collaborative Problem Solving)
This is my word document, that has the graph by itself clearly.
Appendix 5: Examples of Student Artefacts from Online Space (Taken from Group 9)

<table>
<thead>
<tr>
<th>Week 2</th>
<th>Week 3</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Thursday</strong></td>
<td><strong>Week 3 investigations homework</strong></td>
</tr>
<tr>
<td><strong>Mathematics - 10 Hour a Day Investigation</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Legend</strong></td>
<td><strong>Legend</strong></td>
</tr>
<tr>
<td>- Study</td>
<td>- Get up. Eat breakfast.</td>
</tr>
<tr>
<td>- Work on or finish writing</td>
<td>- Get changed. Get your bag ready.</td>
</tr>
<tr>
<td>- Lunch.</td>
<td>- Leave home. School starts.</td>
</tr>
<tr>
<td>- Homework.</td>
<td>- Morning.</td>
</tr>
<tr>
<td>- Work out of school</td>
<td>- Work at school.</td>
</tr>
<tr>
<td>- Go home.</td>
<td>- Go home. Plan homework.</td>
</tr>
<tr>
<td>- Eat dinner.</td>
<td>- Eat dinner.</td>
</tr>
<tr>
<td>- Eat dinner.</td>
<td>- Eat dinner.</td>
</tr>
<tr>
<td><strong>Legend</strong></td>
<td><strong>Legend</strong></td>
</tr>
<tr>
<td>- Math investigation</td>
<td>- Get up.</td>
</tr>
<tr>
<td>- Writing</td>
<td>- Go to school.</td>
</tr>
<tr>
<td>- Lunch</td>
<td>- Math investigation</td>
</tr>
<tr>
<td>- Math homework.</td>
<td>- Lunch.</td>
</tr>
<tr>
<td>- Get changed. Get your bag ready.</td>
<td>- Go home.</td>
</tr>
<tr>
<td>- Leave home. School starts.</td>
<td>- Leave home.</td>
</tr>
<tr>
<td>- Homework.</td>
<td>- Homework.</td>
</tr>
<tr>
<td>- Eat dinner.</td>
<td>- Eat dinner.</td>
</tr>
<tr>
<td>- Eat dinner.</td>
<td>- Eat dinner.</td>
</tr>
</tbody>
</table>

*Source: Taken from Group 9*
Appendices
### Week 6

<table>
<thead>
<tr>
<th>Shape Name</th>
<th>Definition</th>
<th>Attributes</th>
<th>Real World Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>Circle</td>
<td>A circle has no sides or any angles, all it has is curves and turns.</td>
<td>A circle is a shape that has no angles or sides, it is just a circular outline that we call a circle.</td>
<td></td>
</tr>
<tr>
<td>Square</td>
<td>A square has 4 sides and 4 angles. And it has 1 face! Of course.</td>
<td>A square is a shape that has 4 angles and 4 sides that are the same size, and also has a 90 degrees angle on every corner.</td>
<td></td>
</tr>
</tbody>
</table>

### Week 7

[Table or chart related to Week 7 content]
*No Artefacts made in Week 1
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Author/s:
Symons, Duncan

Title:
Using online collaborative learning spaces in primary mathematics education

Date:
2017

Persistent Link:
http://hdl.handle.net/11343/194884

File Description:
Using Online Collaborative Learning Spaces in Primary Mathematics Education

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