INTERNAL AND EXTERNAL REFERENCE EFFECTS IN A TWO-TIER SUPPLY CHAIN

Abstract

We study a two-tier supply chain with a newsvendor retailer that has an internal reference effect related to its own profit as well as an external reference effect centered on the profit of the supplier. We show that the internal and external reference effects in isolation can have opposite effects on the wholesale price set by a supplier who has full knowledge on the retailer’s ordering bias. The internal reference effect, which is driven by the relative disutility from ex-post inventory error, causes the supplier to increase the wholesale price, whereas the external effect, which results from disadvantageous profit inequity, causes the supplier to decrease the wholesale price. While the internal reference effect can achieve supply chain coordination for items with a low profit margin, the external reference effect improves efficiency for high profit margin products, but never achieves coordination. When the retailer’s ordering behavior is influenced by both reference effects, then the interaction can improve efficiency and lead to an equilibrium where the supply chain is coordinated. We find that the relative impact of each reference effect is highly dependent upon the profit margin. We study these equilibria with full information as well as for a supplier who is naïve to the retailer’s behavioral biases and compare these results to derive insights into the behavioral management of supply chains.

Keywords: Supply chain management, Inventory, Behavioral operations, Reference effects.

1. Introduction

Experimental research on behavioral newsvendors shows that inventory decisions in the presence of demand uncertainty tend to deviate from the profit maximizing values due to cognitive biases. Although a variety of models have been proposed to explain suboptimal ordering behavior such as mean-anchoring (Schweitzer and Cachon 2000), demand chasing (Bostian et al. 2008), and overconfidence (Ren andCroson 2013), application of these models to experimental results primarily focus on aggregate data for an entire sample. In the process, these analyses tend to obscure important differences in heterogeneous decision making between individuals.
An important advantage of the model proposed by Long and Nasiry (2015) is that they measure ordering bias through a reference effect based on whether the newsvendor perceive payoffs as gains or losses in relation to a reference profit that is unique to each individual. Unlike competing models, their reference effect model can explain heterogeneous ordering behavior between individual newsvendors in addition to aggregate trends.

There is a significant body of research in economics showing that individuals sacrifice profit due to inequity aversion (for a detailed review of the literature see Fehr and Schmidt 2006). Thus, in addition to cognitive biases, ordering behavior may also be influenced by social perceptions of the fairness of the split in profits across the supply chain. To account for fairness in supply chains, Cui et al. (2007) incorporate inequity aversion into a two-tier supply chain model with a single retailer and supplier. However, the model by Cui et al. (2007) utilizes a linear price-sensitive demand curve, which negates potential effects of demand uncertainty. Wu and Niederhoff (2014) address this additional complexity by generalizing the inequity aversion model of Cui et al. (2007) to the newsvendor setting.

The newsvendor’s utility function in Wu and Niederhoff (2014) balances profit maximization with the disutility from profit inequity. Disutility from inequity aversion can also be thought of as a reference effect, since the newsvendor compares its own profit to that of the supplier’s. We refer to this as an external reference effect, because the reference effect is centered on the supplier’s profit. This contrasts the reference effect from Long and Nasiry (2015), which we refer to as an internal reference effect, since the reference effect only depends on the newsvendor’s own profit. Thus, in a two-tier supply chain, a behavioral newsvendor’s ordering decision may be based on a utility function that encapsulates both cognitive biases from the internal reference effect as well as social factors from the external reference effect.

An interesting outcome of reference effects is that they may improve efficiency by facilitating supply chain coordination, overcoming the issue of double marginalization when channel members are strictly profit maximizing. For example, Cui et al. (2007) demonstrate that a wholesale price contract maximizing the manufacturer’s profit can achieve coordination if the retailer has a sufficient level of inequity aversion. In addition, the experiments by Loch and Wu (2008) show that relationships with suppliers can influence the ordering decisions of retailers.
in line with their perceptions of fairness. To the best of our knowledge, the literature has
yet to consider the combined impact and potential interaction between internal and external
reference effects on supply chain coordination. Thus, our research objectives are as follows:
(1) to explore the impact of a retailer’s internal reference effect on the equilibrium order quan-
tities and profit; (2) to understand how the internal reference effect differs from the external
reference effect in terms of its impact on the equilibrium; and (3) to examine the impact of
interactions between disutility from internal and external reference effects on supply chain
coordination.

To address our research objectives, we consider a Stackleberg game between a profit maxi-
mizing supplier and a utility maximizing retailer. We analyze the equilibria where the retailer’s
utility is influenced by an internal reference effect, an external reference effect, as well as both
reference effects. Our primary focus for each equilibrium analysis is to determine the con-
ditions where the supply chain achieves coordination due to a retailer’s reference effects. In
our analysis, we consider a “sophisticated” supplier, who has full information on the retailer’s
behavioral bias, as well as a “naïve” supplier who assumes the retailer is strictly profit maxi-
mizing. Contrasting these settings provides insights into the role of the supplier’s knowledge
of the retailer’s biases on improving supply chain efficiency.

1.1 Literature Review

Profit maximization creates incentive misalignment issues in supply chains, which result in
losses of efficiency. The most commonly cited issue is the failure of wholesale price contracts
to coordinate a supply chain due double marginalization (Spengler 1950). In the newsvendor
problem, double marginalization results in the newsvendor stocking less inventory relative
to the supply chain optimal order quantity (Lariviere and Porteus 2001).\footnote{Achieving coordination with a newsvendor retailer requires more sophisticated contracts, such as buyback con-
tracts (Pasternack 1985, Wu 2013, Zhao et al. 2014), revenue sharing contracts (Cachon and Lariviere 2005, Heese
and Kemahlıoğlu-Ziya 2016, Hu et al. 2016a), and quantity discount contracts (Tsay 1999, Zissis et al. 2015). In ad-
dition, suppliers can coordinate the supply chain by utilizing other operational strategies, such as sales rebates (Chiu
et al. 2011) and delays in payment (Chaharsooghi and Heydari 2010). For a comprehensive review on coordination,
we refer the reader to Cachon (2003) and Arshinder et al. (2011).} However, this
result relies on the assumption that newsvendors are profit maximizing. Indeed, experimental
evidence has consistently demonstrated that (1) inventory orders from human newsvendors differ significantly from the profit maximizing quantity and that (2) when human retailers interact with human suppliers they have concerns over the profit distribution in the supply chain. To this point, the literature on behavioral newsvendors and fairness in supply chains has been largely segregated. As a result, we review each of these streams of literature individually.

**Behavioral Newsvendors.** In their seminal paper, Schweitzer and Cachon (2000) explored the decision making of human newsvendors in a laboratory setting. They observed that the participants tended to place orders between the profit maximizing solution and the expected level of demand, a phenomenon commonly known as the pull-to-center effect. Schweitzer and Cachon (2000) proposed a number of utility models to try and explain the pull-to-center effect and concluded that mean-anchoring\(^2\) and disutility from ex-post inventory can capture newsvendor behavior. Although mean-anchoring predicts the pull-to-center effect, Lau et al. (2014) showed that many individuals order outside the pull-to-center zone (order quantities between the mean of demand and the optimal level), and that the pull-to-center effect only holds at the aggregate level. Thus, mean-anchoring by itself is not an appropriate model for modeling newsvendor behavior. On the other hand, Ho et al. (2010) extended the model of the disutility from ex-post inventory by differentiating the disutilities from stockout aversion (under supplying) and waste aversion (over supplying). They showed that their model is capable of predicting newsvendor behavior and found that people are more sensitive to left-over inventory rather than lost sales. The latter finding has gathered further support from the experiments of Ovchinnikov et al. (2015) and Feng and Zhang (2017).

Prospect Theory, developed by Kahneman and Tversky (1979), explains systematic violations of expected utility theory for choices given uncertainty. Prospect Theory, which states that individuals experience realized payoffs in terms of losses and gains in relation to a pre-existing reference point, can often explain systematic sub-optimal financial decision. As a result, Prospect Theory has been widely applied to behavioral decisions in finance and economics (see Barberis (2013) for a detailed review and discussion on these and other appli-

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\(^2\)Mean-anchoring and as well as overconfidence (see Ren and Croson 2013) dictate that the newsvendor’s order quantity is a weighted average between the optimal value and the mean, where a greater anchoring effect or greater overconfidence places more weight on the mean of demand.
Many studies in operations research and operations management also incorporate Prospect Theory into decision making. Examples of applications utilizing Prospect Theory in operations include pricing decisions (Nasiry and Popescu 2011, Hu et al. 2016b), preference learning (Bertsimas and O’Hair 2013), joint inventory and pricing management (Chen et al. 2016, Baucells et al. 2017, Mandal et al. 2018), lot sizing (Li and Li 2016), and emergency response (Liu et al. 2014). Despite its wide-ranging applicability, Schweitzer and Cachon (2000) dismissed Prospect Theory as an explanation of newsvendor behavior because the model did not predict the pull-to-centre effect when the product’s demand was sufficiently high such that all demand realizations resulted in profits exceeding the reference point. This result was later supported by Nagarajan and Shechter (2014), who incorporated other aspects of Prospect Theory, such as probability weighting, into the newsvendor model and still found that the theoretical predictions contradicted the experimental results.

Long and Nasiry (2015) showed that this contradictory result was due to Schweitzer and Cachon (2000) and Nagarajan and Shechter (2014) using the retailer’s status-quo wealth\(^3\) (assumed zero) as the reference point. Long and Nasiry (2015) proposed a reference point that is a weighted average of the maximum and minimum payoff, given the order quantity, and showed that the reference point can predict the pull-to-center effect irrespective of the profit margin and the range of the demand distribution. They refer to the weight as the newsvendor’s level of optimism; however, Kirshner and Ovchinnikov (2017) showed that the reference points of Long and Nasiry (2015) and Ho et al. (2010) are equivalent and that the weight should be interpreted as the retailer’s relative preference between stockout and waste aversion. Uppari and Hasija (2017) provided further justification for the use of Prospect Theory to model newsvendor behavior showing that prospect-theoretic models, such as Long and Nasiry (2015), satisfy two important criteria for being appropriate for modeling behavior. Firstly, models need to be able to predict orders that are inside and outside the pull-to-center zone. Secondly, models need to be able to predict that some individuals exhibit the pull-to-center effect under both low- and high-margin settings.

\(^3\)Despite the shortcomings of the status-quo reference point matching the experimental behavior, this model has been utilized to study loss aversion in various newsvendor problems (Wang and Webster 2009, Ma et al. 2012, Vipin and Amit 2017).
The model by Long and Nasiry (2015) was extended by Mandal et al. (2018) to analyze the impact of the reference effect on a joint newsvendor pricing and stocking decision. Kirshner and Ovchinnikov (2017) also extended the reference effect model by Long and Nasiry (2015) to competition, showing that the model explains multiple observed regularities from competitive experiments, such as a behavioral newsvendor ignoring its competitor’s decisions. Becker-Peth and Thonemann (2016) studied reference effects in the setting of revenue sharing contracts. They developed a model where the retailer’s reference point is based on an expectation of the offered revenue share. This differs from the model by Long and Nasiry (2015), since their reference point is exogenously specified based on pre-existing cognitive bias due to ex-post inventory error. Furthermore, unlike Becker-Peth and Thonemann (2016), the reference effect from Long and Nasiry (2015) is dependent on the retailer’s order quantity, which is important to capture the disutility from ex-post inventory error.

**Fairness in Supply Chains.** As previously mentioned, behavioral supply chain research has shown that an inequitable distribution of profit between the retailer and supplier can create fairness concerns, which impacts ordering. Cui et al. (2007) demonstrated that a wholesale price contract can induce coordination in a two-tier supply chain with deterministic demand when the retailer has (or both the retailer and supplier have) aversion to the inequitable distribution of profit. Loch and Wu (2008) experimentally investigated the role of inequity aversion in supply chains by weakly manipulating participants to act cooperatively by shaking hands before interacting in a laboratory supply chain game based on the setup of Cui et al. (2007). They found that the efficiency of the supply chain for participants primed for cooperation improved compared to the control group and the level predicted by profit maximizing behavior. In addition, Loch and Wu (2008) found that full coordination was achieved in nearly 25 percent of cases for the cooperative group.

There have been several extensions to the model of Cui et al. (2007). Katok et al. (2014) extended the model to determine the impact of private information on fairness concerns and found that even under information asymmetry, a wholesale price contract can still coordinate the supply chain provided that the retailer has a sufficient level of inequity aversion. In a related analysis, Pavlov and Katok (2011) studied the role of fairness, incomplete information,
and bounded rationality in predicting contract rejections by the retailers. They showed that the supplier’s incomplete information on the retailer’s level of fairness concerns is an important determinant for the rejection of contract offers. To test the extent that fairness, incomplete information, and bounded rationality impact supply chain performance, Katok and Pavlov (2013) extended their work to the laboratory setting. They found that inequity aversion is the primary driving factor for deviations from profit maximizing ordering behavior on the part of retailers. By comparison, they found that suppliers exhibit little inequity aversion, but their inability to coordinate the supply chain is primarily due to incomplete information regarding the extent to which the retailer’s fairness concerns impact ordering. Qin et al. (2016) continued this line of research by comparing the impacts of fairness concerns and bounded rationality when the supplier’s production cost is private information. In this setup, fairness concerns also had a significant impact on improving the supply chain’s efficiency and the profit distribution between the supplier and retailer. They also showed that the retailer’s fairness concerns can be suppressed when the supplier withholds information on their costs thus preventing the retailer from being knowledgeable on distribution of profit.

Ho et al. (2014) extended the supply chain model of Cui et al. (2007) to include two retailers to study the impact of peer induced fairness between the retailers in addition to distributional fairness between the retailers and the supplier. In their game, the supplier sequentially offers a wholesale price contract to each retailer. The second retailer observes a signal corresponding to the contract offered to the first retailer, and compares its potential payoff to the supplier as well as peer retailer, creating two reference effects. Their analytical and experimental results showed that both types of fairness significantly impact the retailer’s ordering behavior.

In their extension to the newsvendor setting, Wu and Niederhoff (2014) found that retailer’s inequity aversion causes the supplier to prioritize supply chain profits, thereby improving supply chain efficiency. Similar to Katok et al. (2014), they found that degrees of fairness concerns that elicit coordination are beyond levels commonly observed in practice. Avcı et al. (2014) also considered fairness effects in the newsvendor problem, but studied social comparison effects between two competing newsvendors. They found that inequity aversion leads to herding decisions between the competing newsvendors irrespective of the correlation structure of the
retailers’ demand realizations. Finally, Cui and Mallucci (2016) measured the magnitude of fairness concerns in supply chains and found that (1) the impact of fairness concerns on decisions is comparable to the impact of maximizing profit and (2) MBA students have greater fairness concerns compared to undergraduates, providing further support to the robustness of fairness concerns in supply chains.

This paper contributes to the literature on behavioral operations by considering both internal and external reference effects in a two-tier supply chain. Given the prevalence of fairness concerns due to profit inequity and internal behavioral biases due to uncertain demand, it is important to theoretically explore the joint impact of the two reference effects on the equilibrium order quantity and supply chain profit. From a practical perspective, understanding the interaction of the two reference effects has the potential to help large organizations and policymakers designing intervention aimed at improving efficiency.

1.2 Organization

In our most general setup, the retailer’s utility function consists of an internal reference effect based on the disutility from ex-post inventory error as well as an external reference effect based on the disutility from having a lower profit relative to the supplier. The model is fully described in §2. In §3 we consider the special case of a retailer that is only influenced by the internal reference point, while in §4 we study the special case of the external reference point. In §5 we consider the general utility structure and how the relative disutility from the two types of ex-post inventory error interact with the level of aversion to profit inequity. In §6 we summarize our contributions and discuss the managerial implications of the model. Finally, we conclude the paper and outline avenues for future research in §7.

2. Model

Consider a two-tier supply chain with a retailer and a supplier operating in a single period newsvendor environment. The supplier has a production cost $c$ and the retailer sells the product at price $p$. The random demand $D$ follows a distribution with c.d.f. $F(x)$ and p.d.f.
f(x) over the interval [x, \bar{x}]. Following Lariviere and Porteus (2001) we assume the demand distribution has an increasing generalized failure rate, i.e. \( \frac{x f(x)}{1 - F(x)} \) increases in x. The supplier offers the retailer a contract specified by a wholesale price w and the retailer responds by placing an order quantity q. For demand realization x, define the retailer’s profit as \( \pi_r(q, x) = p(q \land x) - wq \) where \( q \land x \) is the minimum of q and x. The retailer and supplier expected profits are \( \pi_r(q) = \mathbb{E}[p(q \land D)] - wq \) and \( \pi_s(q) = wq - cq \), respectively, where the expectation is taken over D. The total expected profit of the supply chain is \( \Pi(q) = \pi_s(q) + \pi_r(q) = \mathbb{E}[p(q \land D)] - cq \).

The supplier’s objective is to set w to maximize \( \pi_s(q) \).

Departing from the traditional supply chain literature, the retailer’s ordering decision is potentially impacted by two reference effects: \( r_I(q, x) \), an internal reference point, and \( r_E(q, x) \) an external reference. We adopt internal reference effect developed by Long and Nasiry (2015), where the reference point is a weighted average of the retailer’s maximum and minimum potential profit levels. Thus, given an order quantity of q, the reference point is \( \beta \pi_r(q, q) + (1 - \beta) \pi_r(q, x) \), where \( \beta \in [0, 1] \) is the retailer’s relative disutility between over and under ordering. As \( \beta \) increases (decreases) towards one (zero), the retailer experiences greater disutility from having excess (insufficient) inventory. The retailer’s ex-post utility from the internal reference effect for demand realization x is \( r_I(q, x) = \eta(\pi_r(q, x) - (\beta \pi_r(q, q) + (1 - \beta) \pi_r(q, x))) \),\(^4\) where \( \eta > 0 \) measures the strength of the internal reference effect.

To model the external reference effect, we draw on the literature of inequality aversion in supply chains and assume that the retailer experiences disutility when \( \pi_r < \pi_s \) due to disadvantageous inequity. Thus, the external reference point is \( r_E(q, x) = -\alpha (\pi_s(q, x) - \pi_r(q, x))^{+} \), where \( \alpha \geq 0 \) is the strength of the external reference effect regarding the disadvantageous inequity in profit compared to the supplier. Combining the profit function with the two reference effects implies that the retailer’s ex-post utility is \( u_r(q, x) = \pi_r(q, x) + r_I(q, x) + r_E(q, x) \). The objective of the retailer is to select an order quantity q which maximizes the expected utility \( u_r(q) \equiv \mathbb{E}[u_r(q, D)] \). Table 1 provides a summary of the notation used in the paper.

\(^4\)To focus on understanding the influence of ex-post inventory bias on responses to wholesale contracts, we omit loss aversion. Kirshner and Ovchinnikov (2017) showed that assuming loss-neutrality leads to the equivalence between the models of Long and Nasiry (2015) and Ho et al. (2010), which permits \( \beta \) being interpreted as the relative disutility of stockout aversion. In addition, we omit shortage costs and salvage values in the model to simplify the analysis. If these factors were included in the model, the reference point would adjust to \( \beta \pi(q, q) + (1 - \beta) (\pi(q, \bar{x}) \land \pi(q, x)) \).
### Table 1: Notation list

<table>
<thead>
<tr>
<th>Notation</th>
<th>Definition</th>
</tr>
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<tbody>
<tr>
<td>$c, p$</td>
<td>Supplier’s per-unit cost and retail price</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Product’s cost-to-price ratio</td>
</tr>
<tr>
<td>$F, f, \underline{x}, \overline{x}, D$</td>
<td>Demand c.d.f., p.d.f, lower bound, upper bound, and expectation</td>
</tr>
<tr>
<td>$\Pi, \pi_s, \pi_r$</td>
<td>Expected supply chain, supplier, and retailer profit</td>
</tr>
<tr>
<td>$u_r$</td>
<td>Retailer’s expected utility</td>
</tr>
<tr>
<td>$I, E, B$</td>
<td>Subscripts for internal, external, and both reference effects</td>
</tr>
<tr>
<td>$r_I, r_E$</td>
<td>Retailer’s internal and external reference effects</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Retailer’s relative disutility from ex-post inventory error</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Retailer’s internal reference effect strength</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Retailer’s disadvantageous inequity parameter</td>
</tr>
<tr>
<td>$q^*(w)$</td>
<td>Retailer’s unbiased best response functions</td>
</tr>
<tr>
<td>$q_i(w)$</td>
<td>Retailer’s behavioral best response functions for $i \in {I, E, B}$</td>
</tr>
<tr>
<td>$\hat{q}_i(w)$</td>
<td>Retailer’s response function to achieve equity profits for $i \in {E, B}$</td>
</tr>
<tr>
<td>$\tilde{q}_i(w)$</td>
<td>Retailer’s response function given inequity in profits for $i \in {E, B}$</td>
</tr>
<tr>
<td>$w^*$</td>
<td>Supplier’s optimal wholesale price for an unbiased retailer</td>
</tr>
<tr>
<td>$w_i$</td>
<td>Supplier’s optimal wholesale price for a retailer with $i \in {I, E, B}$</td>
</tr>
<tr>
<td>$w_o, q_o$</td>
<td>Supply chain coordinating wholesale price and order quantity</td>
</tr>
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</table>

### 3. Internal Reference Effect

We start the analysis with the case where the retailer is only influenced by the internal reference effect, i.e. $\alpha = 0$. For ease of exposition, we derive all of our results for the uniform demand distribution with $\underline{x} = 0$ and $\overline{x} = 1$. Irrespective of the magnitude of the internal reference effect, the supply chain obtains the first best solution $q_o$ if and only if $q_o = 1 - \rho$, where $\rho \equiv \frac{c}{p}$ is the supply chain’s cost-to-price ratio. If the retailer is a strictly profit maximizing agent ($\eta = \alpha = 0$), then its best response function given a wholesale price of $w$ is $q^*(w) = 1 - \frac{w}{p}$. The wholesale price $w^* = \frac{1}{2}(c + p)$, which maximizes the supplier’s profit, leads to the equilibrium order quantity $q^*(w^*) = \frac{1}{2}(1 - \rho)$. Consequently, a wholesale price contract cannot achieve the supply chain coordinating order quantity $q_o = 1 - \rho$ (for $c < 1$) when there are no reference effects present. When the retailer’s decision is affected by a reference effect ($\eta > 0$), the utility maximizing ordering quantity $q_I(w)$ deviates from the optimal unbiased quantity $q^*(w)$.

**Lemma 1.** Given the supplier’s wholesale price $w$, the best response function of the retailer in the presence of the internal reference effect is $q_I(w) = 1 - \frac{w/p + \eta \beta}{1 + \eta}$.
The best response function $q_I(w)$ is decreasing in both the wholesale price and the inventory bias parameter $\beta$. To understand why $q_I(w)$ is decreasing in $\beta$, consider a retailer who plans to order $q$ units. An increase in $\beta$ causes an increase in its internal reference point $r_I(q)$, which decreases its utility, since more demand realizations are considered losses. Thus, the retailer responds to an increase in $\beta$ by lowering the order quantity below $q$.

The best response function in Lemma 1 indicates that the retailer will over (under) order relative to $q^*(w)$ when $\beta$ is less (greater) than $w/p$. The analysis in Long and Nasiry (2015) focused on the relationship between $\beta$ and $w/p$, since it determined the newsvendor’s pull-to-center effect for an exogenous wholesale price $w$. However, in the context of a two-tier supply chain, the supplier is a Stackelberg leader and considers the unit production cost (as well as the retailer’s best response function) when setting the wholesale price. We find that the retailer’s value of $\beta$ needs to be compared to the production cost-to-price ratio, $\rho$, to determine whether the retailer will over or under order with respect to $q^*(w^*)$.

**Proposition 1.** In a wholesale price contract game with the internal reference effect, the supplier selects the wholesale price $w_I = w^* + \frac{\eta p}{2}(1 - \beta)$. The retailer’s corresponding behavioral equilibrium order quantity is $q_I(w_I) = q^*(w^*) - \frac{\eta}{2(1+\eta)}(\beta - \rho)$.

Proposition 1 characterizes the equilibrium wholesale price and order quantity in the presence of the internal reference effect. It shows that the equilibrium wholesale price always exceeds the wholesale price set by the supplier when the retailer is strictly profit maximizing.

The retailer’s utility function is comprised of the profit and the internal reference effect. The internal reference effect $\eta(\pi_r(q,x) - r(q))$ reduces to $\eta(p(q \land x) - \beta pq)$, since there is no uncertainty in the retailer’s cost. As a result, the reference effect decreases the retailer’s sensitivity to changes in the supplier’s wholesale price. This reasoning is observable from the retailer’s best response function from Lemma 1. Since $\frac{\partial q_I(w)}{\partial w} = \frac{1}{p(1+\eta)}$, the retailer is most sensitive to change in the wholesale price when $\eta = 0$, and decreases its sensitivity to changes in $w$ as $\eta$ increases. Thus, if $\eta > 0$, the supplier always sets $w_I \geq w^*$, irrespective of the level of $\beta$.

Interestingly, the increase in the wholesale price, due to the presence of the reference effect, does not necessarily lead to a uniform decrease in the equilibrium order quantity compared to $q^*(w^*)$. Indeed, the relationship between $\beta$ and $\rho$ determines whether the order size increases
or decreases with respect to the profit maximizing equilibrium solution. Specifically, if $\beta \geq \rho$ such that the retailer has a sufficiently high internal reference point, then $q_I(w_I) \leq q^*(w^*)$. On the other hand, if $\beta < \rho$, then $q_I(w_I) > q^*(w^*)$ and the retailer will over order. This resembles the results of Long and Nasiry (2015), except that the relationship determining whether the reference effect increases or decreases the order quantity depends on $\beta$ and $\rho$. In the appendix, we show that the above result carries over to the case where the demand distribution is characterized by an increasing generalized failure rate.

The impact of the internal reference effect on the equilibrium order quantity has important implications for coordinating the supply chain. When there is no reference effect (i.e., $\eta = 0$), the only cost-to-price ratio where the supply chain is coordinated occurs when $\rho = 1$, which leads to no sales because the retailer does not place any orders. For $\rho < 1$, the supply chain fails to coordinate because the supplier always engages in the practice of double marginalization. However, the retailer’s over ordering behavior due to a low $\beta$ can produce an order quantity exceeding the case where both agents are profit maximizers. Indeed, it is possible for a wholesale price contract to coordinate the supply chain.

For $\eta > 0$, the supply chain will be coordinated in equilibrium if and only if $\rho = \frac{1+\eta+\beta\eta}{1+2\eta}$. The value $\bar{\rho} = \frac{1+\eta}{1+2\eta}$ provides a lower bound on the feasible cost-to-price ratios where the supply chain optimal result can be achieved. Thus, for $\rho \in [\bar{\rho}, 1]$, there exists a unique pair $(\rho, \beta)$ that coordinates the supply chain. The range of feasible cost-to-price ratios where $\beta$ can coordinate the supply chain increases as the reference strength $\eta$ increases, since the derivative of $\bar{\rho}$ with respect to $\eta$ is $-\frac{1}{(1+2\eta)^2}$. As $\eta \to \infty$, $\bar{\rho} = \frac{1}{2}$, which implies that $\bar{\rho}$ is bounded below by $\frac{1}{2}$. Although a retailer’s decision is unlikely to be dominated by its reference point, this result emphasizes that wholesale contracts can achieve coordination for low-margin products.

**Internal Reference Effect with a Naïve Supplier.** Thus far, the supplier is assumed to be sophisticated and knows the retailer’s behavioral bias. If the supplier is naïve to the retailer’s reference effect, then the supplier assumes that the retailer is profit maximizing and offers the wholesale price contract $w^*$. The wholesale price $w^*$ is less than $w_I$ (as per Proposition 1), which implies that $q_I(w^*) > q_I(w_I)$. Clearly, the supplier becomes worse off when not accounting for the retailer’s reference effect, i.e. $\pi_s(q_I(w^*)) \leq \pi_s(q_I(w_I))$. Conversely,
Figure 1: Equilibrium order quantities and supply chain profit across $\rho$ for $\beta = 0.5$ and $\eta = 1$.

The retailer benefits from the lower wholesale price, and $\pi_r(q_I(w^*)) \geq \pi_r(q_I(w_I))$.

For $\eta > 0$ and wholesale contract $w^*$, the supply chain will be coordinated in equilibrium if and only if $\rho = \frac{1 + 2\beta\eta}{1 + 2\eta}$. The value $\bar{\rho} = \frac{1}{1 + 2\eta}$ provides a lower bound on the cost-to-price ratio that can coordinate the supply chain when the supplier neglects the retailer’s reference effect. Since $\rho \leq \bar{\rho}$, neglecting the behavioral bias leads to a greater set of feasible cost-to-price ratios that result in coordination. Furthermore, for a fixed value of $\beta$, the value of $\rho$ that coordinates the supply chain in the na"ive equilibrium will always be lower than the value of $\rho$ that coordinates the behavioral equilibrium.

To illustrate the relationships among the supply chain optimal, profit maximizing, and the two behavioral equilibria order quantities, consider the case of $\beta = 0.5$ and $\eta = 1$, i.e. a retailer who is equally sensitive to over and under ordering with a moderate internal reference strength. Figure 1 (a) shows the supply chain optimal order quantity $q_o$, the profit maximizing equilibrium quantity $q^*(w^*)$, the behavioral equilibrium quantity $q_I(w_I)$, and the na"ive equilibrium quantity $q_I(w^*)$ across $\rho$. The figure demonstrates the results that $q_I(w_I) = q^*(w^*)$ at $\rho = \beta$ and that the value of $\rho$ where $q_I(w^*) = q_o$ is less than the value of $\rho$ where $q_I(w_I) = q_o$. Moreover, the figure shows that the quantity ordered in the na"ive equilibrium exceeds the profit maximizing order quantity for a large range of values of $\rho$. The impact of the behavioral retailer on profit can be seen in Figure 1 (b), which plots the supply chain profit for each order.
quantity. In the high-margin setting, underordering due to the internal reference effect causes a significant decrease (up to 18.75%) in the supply chain’s profit. However, a naïve supplier improves the profitability of the supply chain beyond the level of the profit maximizing outcome. For lower profit margins, \( \rho \in [0.425, 0.765] \), a naïve supplier enables profit to reach at least 90% of the first best. Thus, the internal reference effect can have both a positive and negative consequences, depending on the profit margin and sophistication of the supplier.

4. External Reference Effect

In this section, we analyze the model for the case where the retailer is subject to external reference effect only (i.e. \( \eta = 0 \)). Observe that the retailer’s expected utility function \( u_r(q) = \pi_r(q) - \alpha(\pi_s(q) - \pi_r(q))^+ \) is piecewise nonlinear. Thus, the retailer may experience disutility given the supplier’s choice of the wholesale price \( w \). Define \( \hat{q}_E(w) \) as the order quantity such that \( \pi_r(\hat{q}_E(w)) = \pi_s(\hat{q}_E(w)) \). For a wholesale price \( w < \frac{1}{2}(p + c) \), i.e. a wholesale price that is sufficiently low such that the retailer orders form the supplier, the cutoff order quantity is \( \hat{q}_E(w) = \frac{2(p+c-2w)}{p} \). Therefore, if \( q < \hat{q}_E(w) \) (\( q > \hat{q}_E(w) \)), the retailer makes a higher (lower) profit than the supplier. The retailer’s expected utility function is given by

\[
u_r(q) = \begin{cases} \pi_r(q) & \text{if } q < \hat{q}_E(w) \\ \pi_r(q) - \alpha(\pi_s(q) - \pi_r(q)) & \text{if } q \geq \hat{q}_E(w). \end{cases}
\]

Since \( u_r \) is a piecewise function of \( q \), we can solve the retailer’s optimization problem by examining the cases \( q \leq \hat{q}_E(w) \) and \( q > \hat{q}_E(w) \).

**Lemma 2.** Given the supplier’s wholesale price \( w \), the best response function of the retailer in the presence of the external reference effect is

\[
q_E(w) = \begin{cases} q^*(w) & \text{if } w < w_{2E} \\ \hat{q}_E(w) & \text{if } w_{2E} \leq w < w_{1E} \\ \tilde{q}_E(w) & \text{if } w_{1E} \leq w, \end{cases}
\]

where \( \tilde{q}_E(w) = q^*(w) - \frac{\alpha(w-c)}{(1+\alpha)p} \), \( w_{1E} = \frac{p+2c+\alpha(p+c)}{2\alpha+3} \), and \( w_{2E} = \frac{p+2c}{3} \).
Lemma 2 shows that the retailer’s optimal order quantity is a piecewise function of \( w \). The retailer orders the profit maximizing order quantity for a sufficiently small wholesale price (i.e. \( w < w_{2E} \)), since it does not experience disutility from the external reference effect. For a moderate wholesale price (i.e. \( w_{2E} \leq w < w_{1E} \)), the retailer orders \( \hat{q}_E(w) \) so that the retailer and the supplier make the same profit. When the wholesale price is large (i.e. \( w_{1E} \leq w \)), the retailer experiences disutility from the external reference effect and responds by ordering \( \tilde{q}_E \). Clearly, \( \tilde{q}_E(w) < q^*(w) \); thus, the retailer responds to disadvantageous inequity by lowering its order quantity from the profit maximizing value. Intuitively, the magnitude of the decrease in the order quantity is increasing in the magnitude of \( \alpha \) and the difference between the wholesale price and cost, as can be seen from the expression of \( \tilde{q}_E(w) \).

The one-to-one mapping between wholesale price \( w \) and the retailer’s order quantity \( q_E \) implies that we can identify the supplier’s optimal wholesale price \( w \) using the retailer’s order quantity. Defining \( w_E(q) \) as the inverse function of \( q_E(w) \), the function \( w_E(q) \) is the inverse demand function faced by the supplier. The inverse demand function \( w_E(q) \) is also piecewise, given the piecewise nature of \( q_E(w) \). Define \( q_{1E} \) and \( q_{2E} \) as the retailer’s corresponding order quantities to the threshold wholesale prices \( w_{1E} \) and \( w_{2E} \), respectively. Specifically, \( q_{1E} = \frac{2(p-c)}{(2\alpha+3)p} \) and \( q_{2E} = \frac{2(p-c)}{3p} \), since \( q_{1E} \) corresponds to \( q^*(w_{1E}) = \hat{q}_E(w_{1E}) \) and \( q_{2E} \) corresponds to \( \hat{q}(w_{2E}) = \tilde{q}_E(w_{2E}) \). Observe that \( q_{2E} > q_{1E} \) for \( \alpha > 0 \). Using Lemma 2 we obtain

\[
\begin{align*}
    w_E(q) &= \begin{cases} 
        \frac{\alpha c + (\alpha + 1)(1-q)p}{2\alpha + 1} & \text{if } q \leq q_{1E} \\
        \frac{2c - pq + 2p}{4} & \text{if } q_{1E} < q \leq q_{2E} \\
        p(1-q) & \text{if } q_{2E} < q
    \end{cases}
\end{align*}
\]

and reformulate the supplier’s problem to \( \max q \pi_s = (w_E(q) - c)q \).

**Proposition 2.** In the presence of external reference effect, if \( \alpha < 1/7 \), the supplier induces the behavioral equilibrium order quantity \( q_E = q^*(w^*) \) by setting the wholesale price to \( w_E = \frac{p(\alpha + \alpha + \alpha + 1)}{2(1+2\alpha)} \); if \( \alpha \geq 1/7 \), the supplier induces the behavioral equilibrium order quantity \( q_E = \hat{q}_E \) by setting the wholesale price to \( w_E = w_{2E} \).

Proposition 2 shows that there is a critical value in \( \alpha \) determining the equilibrium outcome of the Stackelberg game. When the retailer’s aversion to disadvantageous inequity is sufficiently
low (i.e. $\alpha < 1/7$), then the supplier induces the profit maximizing order quantity. In the special case where $\alpha = 0$, observe that $w_E = w^*$. As $\alpha$ increases (towards $1/7$), the supplier decreases the wholesale price to maintain the profit maximizing order quantity $q^*$. At $\alpha = 1/7$, the supplier’s profit from inducing $q^*(w^*)$ is equal to the profit from setting $q_E = q_{2E}$, which corresponds to the wholesale price to $w_E = w_{2E}$. From Lemma 2, setting the wholesale price at this boundary point leads to the best response order quantity $q_E = \hat{q}_E$. Thus, if $\alpha$ exceeds $1/7$, then continuing to lower the wholesale price becomes sub-optimal, and the supplier prefers to set the wholesale price such that both supply chain members earn the same profit. In this case, the retailer does not experience disadvantageous inequity in equilibrium.

A consequence of the external reference effect for supply chain coordination is that $q_E < q_o$, since for low levels of $\alpha$ the retailer orders $q^*(w^*) < q_o$ and at higher levels of $\alpha$ the retailer orders $\hat{q}_E \leq q^*(w^*)$. Thus, if the retailer is influenced by disadvantageous inequity, then the supply chain can never be coordinated using wholesale price contracts. Nevertheless, observing that $w_{2E}$ and $\hat{q}_E$ are independent of $\alpha$, it is easy to verify that the increased order quantity due to fairness leads to a constant improvement in supply chain profit, which is also independent of the profit margin $\rho$. For $\alpha \geq 1/7$, fairness concerns results in the supply chain achieving 88.88% of the first best. This result sharply contrast with the internal reference, which can decrease profits as well as achieve coordination.

**External Reference Effect with a Naïve Supplier.** If the supplier is naïve, then it offers a profit maximizing wholesale price $w^*$. Proposition 2 shows that the retailer’s optimal wholesale price is decreasing in $\alpha$, thus $w^* > w_E$ for $\alpha > 0$. Consequently, if the supplier is naïve and the retailer has an external reference effect, the retailer will place an order quantity that is smaller than the profit maximizing quantity. Therefore, unlike the internal reference effect, if the retailer has an external reference effect, then a naïve supplier can only decrease the efficiency of the supply chain. This effect, which harms supply chain profit, is linearly increasing in the retailer’s sensitivity to disadvantageous inequity.

Figure 2 (a) plots the behavioral equilibrium order quantity of a retailer with an external reference effect of $\alpha = 0.5$ for the case of a sophisticated supplier as well as a naïve supplier with the supply chain optimal and the profit maximizing quantity across $\rho$. The figure illustrates
Figure 2: Equilibrium order quantities for the external reference effect across $\rho$ with $\alpha = 0.5$.

(a) Comparison with $q_o$ and $q^*(w^*)$

(b) Comparison with $q_I(w_I)$ and $q_I^*(w^*)$

that $q_o \geq q_E(w_E) \geq q^*(w^*) \geq q_E(w^*)$ independent of the cost-to-price ratio. The figure also shows that the impact of the supplier being sophisticated or naïve is greater at lower costs. To compare the impacts of the internal and external reference effect on the equilibrium order quantities, consider a retailer with $\beta = 0.5$ and $\eta = 1$, which are the same internal reference parameters as in Figure 1. Figure 2 (b) shows the behavioral equilibrium order quantities for both the internal and external reference effects with a sophisticated and naïve supplier. Although the supply chain can not be coordinated when the retailer has an external reference effect, the performance at low $\rho$ exceeds the performance of the internal reference effect. Thus, if the retailer only has one reference effect, an external reference effect is beneficial in high-margin situations, whereas an internal reference effect can be more beneficial, and even coordinate a supply chain, in low-margin situations.

5. Internal and External Reference Effects

This section presents the general model where the retailer is subject to both internal and external reference effects. Similar to the case of the external reference effect only, $\hat{q}_B(w) = \frac{2(c+p-2w)}{p}$ is the threshold order quantity such that $\pi_r(\hat{q}_B(w)) = \pi_s(\hat{q}_E(w))$. Although the order quantities $\hat{q}_B(w)$ and $\hat{q}_E(w)$ are equivalent, the wholesale price that induces the retailer
to order this quantity may differ due to the presence of the internal reference effect. The 
retailer’s expected utility function in the presence of internal and external reference effects is 
given by

\[
u_r(q) = \begin{cases} 
\pi_r(q) + r_I(q) & \text{if } q < \hat{q}_B(w) \\
\pi_r(q) + r_I(q) - \alpha(\pi_s(q) - \pi_r(q)) & \text{if } q \geq \hat{q}_B(w).
\end{cases}
\]

The retailer chooses an optimal order quantity to maximize its expected utility \(u_r(q)\).

**Lemma 3.** In the presence of both the internal and external reference effects, the retailer’s op-
timal order quantity depends on \(\beta\). Define \(\beta_1 = \rho\), \(\beta_2 = \frac{(2\rho-1)(1+\eta)}{\eta}\), and \(\beta_3 = \rho + \frac{(\rho-1)(\alpha+\eta+1)}{\eta}\), where \(\beta_3 < \beta_2 < \beta_1\). Given \(w\), the retailer’s optimal order quantity is as follows:

(i) if \(\beta \leq \beta_3\), then \(q_B(w) = \tilde{q}_B(w)\),

(ii) if \(\beta_3 < \beta \leq \beta_2\), then

\[
q_B(w) = \begin{cases} 
\hat{q}_B(w) & \text{if } w \leq w_1 \\
\tilde{q}_B(w) & \text{if } w_1 \leq w \leq w_2
\end{cases}
\]

(iii) if \(\beta_2 < \beta \leq \beta_1\), then

\[
q_B(w) = \begin{cases} 
q_I(w) & \text{if } w \leq w_2 \\
\hat{q}_B(w) & \text{if } w_2 \leq w \leq w_1 \\
\tilde{q}_B(w) & \text{if } w_1 \leq w
\end{cases}
\]

(iv) if \(\beta > \beta_1\), then

\[
q_B(w) = \begin{cases} 
\tilde{q}_B(w) & \text{if } w \leq w_3 \\
q_I(w) & \text{if } w_3 \leq w \leq w_2 \\
\hat{q}_B(w) & \text{if } w_2 \leq w \leq w_1 \\
\tilde{q}_B(w) & \text{if } w_1 \leq w
\end{cases}
\]

where \(\tilde{q}_B(w) = 1 - \frac{w + \beta \eta p + \alpha(2w - c)}{p(\alpha + \eta + 1)}\), \(w_1 = \frac{c(\alpha + 2\eta + 2) + p(\alpha + \beta \eta + \eta + 1)}{2\alpha + 4\eta + 3}\), \(w_2 = \frac{c(2\eta + \eta + 1)p}{4\eta + 3}\), and \(w_3 = \frac{c(1+\eta) + \beta \eta p}{2\eta + 1}\).

The lemma highlights the interaction between internal and external reference effects in
determining the retailer’s order quantity. When \( \beta \) is extremely small (i.e. \( \beta \leq \beta_3 \)), the retailer orders the amount \( \tilde{q}_B(w) \). The order quantity \( \beta_3 \) is only positive (and thus relevant) when costs are high (equivalently, \( \rho \) is large) and when the retailer is sufficiently impacted more by the external reference effect compared to the internal reference effect (i.e. \( \alpha \) is large relative to \( \eta \)). In these cases, a low \( \beta \) leads to over ordering, which leads to the supplier having a greater profit, irrespective of wholesale price. Thus, the retailer experiences disutility from both reference effects and orders \( \tilde{q}_B(w) \), which accounts for disadvantageous inequity. When \( \beta \) is between \( \beta_3 \) and \( \beta_2 \), then the behavior for a sufficiently high wholesale price is similar to the case where \( \beta < \beta_3 \). However, if the wholesale price decreases below the threshold value \( w_{1B} \), then the retailer maximizes utility by ignoring its internal reference effect, and orders \( \hat{q}_B(w) \).

The case for \( \beta > \beta_2 \) is similar to the case of \( \beta_3 < \beta \leq \beta_1 \), except that the retailer orders based on its internal reference effect if the wholesale price is below the threshold value \( w_{2B} \). For \( w < w_{2B} \), the wholesale price is sufficiently low such that ordering at \( \hat{q}_B(w) \) creates disutility associated with waste aversion, causing the retailer to account for \( \beta \) in its order quantity. For \( w < w_{2B} \), as \( \beta \) increases, the retailer decreases its order quantity, decreasing profit, which negates the benefit of the low wholesale price. Interestingly, if \( \beta \) increases beyond \( \beta_1 \) and if \( w < w_{3B} \), then the retailer will decrease the order quantity to the point where there is profit inequity, and thus orders at \( \tilde{q}_B(w) \).

Similar to the case where the retailer only has an external reference effect, there is a one-to-one mapping between the wholesale price and order quantity. Consequently, the inverse demand function faced by the supplier is piecewise linear. For brevity, we defer the details of the derivation of the inverse demand function with both reference effects to the appendix.

The suppliers optimization problem is then to select the retailer’s order quantity, which can be induced by the corresponding wholesale price \( w_B \).

**Proposition 3.** In the presence of both internal and external reference effects, the supplier sets the wholesale price \( w_B \) to induce the behavioral equilibrium order quantities as follows:

(i) if \( \beta < \beta_2 \), then \( q_B(w_B) > q_o \),

(ii) if \( \beta_2 \leq \beta < \beta_1 \), then \( \frac{2(1-\rho)(2\eta+1)}{4\eta+3} \leq q_B(w_B) \leq q_o \),

(iii) if \( \beta \geq \beta_1 \), then \( q_B(w_B) \leq \frac{2(1-\rho)(2\eta+1)}{4\eta+3} \).
Proposition 3 characterizes the equilibrium for the contract design game when the retailer is subject to both the internal and external effects. The proposition provides a lower bound on the equilibrium order quantity for $\beta < \beta_2$, an upper bound on the equilibrium order quantity for $\beta \geq \beta_1$, and shows that these values bound the possible equilibrium order quantities for $\beta_2 \leq \beta < \beta_1$. The proposition shows that if $\beta$ is sufficiently low (i.e. less than $\beta_2$), then the supply chain will be “over coordinated”, since the equilibrium order quantity exceeds the first best quantity $q_o$. Similarly, for $\beta > \beta_1$ the supply chain cannot be coordinated, since the equilibrium order quantity is less than $\frac{2(1-\rho)(2\eta+1)}{4\eta+3}$, which is less than the first best.

For ease of exposition, we have limited Proposition 3 to analyze regions where supply chain coordination is feasible. The supplier has a different profit function for each of the four cases in Lemma 3 creating different equilibrium order quantities that depend on $\beta$. Furthermore, the closed-form expression for the optimal induced order quantity varies for $\beta > \beta_2$ due to the interactions among $\alpha$, $\eta$, and $\beta$. In the appendix, we provide a more detailed characterization of the equilibrium, providing the order quantity $q_B$ for each value of $\beta$ and all possible subcases.

To illustrate the relationship between the supply chain optimal and equilibrium order quantities and profits when a retailer has both reference effects, Figure 3 explores the impact of $\alpha$ and $\beta$ across $\rho$. The figure shows that the equilibrium order quantities can be piecewise linear. Therefore, unlike in the cases where there is only an internal or external reference effect, there is potential for supply chain coordination at multiple values of $\rho$ for a fixed set of behavioral parameters. If $\rho$ is low and $\alpha$ is sufficiently high, then the order quantity increases. However, at low $\rho$, a stronger external reference effect, i.e. a higher $\alpha$, does not further increase the order quantity; rather, it extends the value of $\rho$ where the retailer maintains a high order quantity, potentially reaching the first best solution. Comparing Figure 3 (a) and (b) shows that for low-mid $\rho$, the supply chain gets greater benefits from a higher $\alpha$ when $\beta$ is low. For example, at $\rho = 0.4$, the order quantity is only high with $\alpha = 0.1$ at $\beta = 0.25$, whereas, the order quantity is high for both $\alpha = 0.5$ and $\alpha = 1$ at $\beta = 0.5$. Thus, if the retailers have moderate $\eta$, then it may be only important to foster fairness concerns, when the retailer is more sensitive to disutilities from insufficient rather than excess inventory levels.

Figure 3 (c) and (d) shows that the reference effects have a substantial impact on profit.
Figure 3: Equilibrium order quantities and profit across $\rho$ for $\eta = 0.5$ and various levels of $\alpha$.

For example, at $\beta = 0.25$, the profit is 90% of the supply chain optimal for the entire high margin region, while for $\beta = 0.5$ the profit is 90% of the first best for $\rho \in [0.3, 0.66]$. Indeed, the two reference effects create a substantial profit improvement compared to profit maximizing solution, with up to a 33% improvement for $\alpha = 1$ and $\beta = 0.25$. Comparing Figure 3 (c) and (d) shows that the supply chain benefits from greater fairness concerns at low $\beta$. This results from the fact that retailers have a higher disutility for stockout aversion and suppliers can take advantage of this by increasing the wholesale price. Increasing fairness concerns causes the supplier to lower $w$, which then leverages the retailer’s high relative disutility towards stockouts to improve profit and efficiency.
Figure 4: Equilibrium order quantities across $\rho$ for $\alpha = 1$ and various levels of $\eta$.

Figure 4, which plots the equilibrium order quantities for a fixed level of $\alpha$ and various levels of $\eta$ and $\beta$, also shows that the supply chain can be coordinated for multiple values of $\rho$ given a fixed set of behavioral parameters. For $\beta = 0.25$, Figure 4 (a) demonstrates that at low $\rho$, the supply chain is most efficient when the retailer has a strong internal reference effect. However, as $\rho$ increases, there is a point ($\rho \approx 0.2$) where an internal reference strength of $\eta = 1$ leads to the least efficient order quantity, and $\eta = 0.5$ becomes the most efficient. This drop in efficiency occurs because the retailer’s propensity for over ordering due to strong stockout aversion leads to a large discrepancy in profits at a higher level of $\rho$, which triggers the retailer to order based on disadvantageous inequity aversion. A similar pattern occurs at $\rho \approx 0.5$, where at $\eta = 0.5$, the retailer substantially drops its order quantity due to profit inequity. Observe for $\rho > 0.5$ that the equilibrium quantity at $\eta = 1$ is greater than at $\eta = 0.5$, due to the greater internal reference effect. Figure 4 (b) shows that this behavior also occurs for a higher $\beta$, except that it starts after $\rho \approx 0.2$. For $\rho < 0.2$, the cost is sufficiently low, such that the inequity aversion is at odds with the sensitivity to waste aversion. Thus, at $\beta = 0.5$, for low $\rho$, supply chain efficiency is improved when the retailer is only impacted by the external reference strength, and for high $\rho$, the supply chain benefits from the internal reference effects.

**Combined Reference Effect with a Naïve Supplier.** Proposition 3 as well as the previous discussion shows that the optimal quantity to induce depends on the value $\beta$ as well
as the relationships between $\alpha$ and $\eta$. However, if the supplier is naïve and offers the wholesale price $w^*$, then the results are similar to the case of only the external reference effect. Observe that $w^* - w_{1B} = \frac{2p\eta(1-\beta)+p-c}{4\alpha+8\eta+6}$, which is positive, since $c < p$ and $\beta \leq 1$. Therefore, from Lemma 3, irrespective of $\beta$, $q_B = \tilde{q}_B$, and the retailer orders based on inequity aversion. As a result, higher values of $\eta$ and lower values of $\alpha$, i.e. greater influence of the internal versus external reference effect, will increase the order quantity. The impact of $\eta$ is illustrated in Figure 5, which shows the equilibrium order quantities with a naïve supplier. The figure shows that the positive impacts of the internal reference effect are greater at lower $\beta$, with profits exceeding the unbiased equilibrium for the majority of the values of $\rho$. Comparing Figures 4

Figure 5: Equilibrium order quantities and profit with a naïve supplier across $\rho$ for $\alpha = 0.5$ and various levels of $\eta$. 

(a) $q_B$ at $\beta = 0.25$

(b) $q_B$ at $\beta = 0.5$

(c) $\Pi$ at $\beta = 0.25$

(d) $\Pi$ at $\beta = 0.5$
and 5 shows that the exact value of $\rho$ as well as the behavioral parameters determine whether a naïve or sophisticated supplier benefits the supply chain for a low-margin product. On the other hand, for a high-margin product, the supply chain is generally more efficient with a sophisticated supplier.

6. Discussion

Although a wholesale price contract should theoretically be based only on the production cost and retail price, understanding a retailer’s behavioral biases is critical for designing a profit maximizing contract. Loch and Wu (2008) show that the established relationships between suppliers and retailers can impact whether participants have fairness concerns. If the retailer does not have an existing relationship with the supplier, then the retailer’s concerns and perceptions of fairness tend to be limited. When retailer ordering behavior is only influenced by the internal reference point, we find that the wholesale price should always be higher than the prescribed value from the unbiased model. This result is driven by the retailer’s decreased sensitivity to cost due to the internal bias related to ex-post inventory error. Although the internal reference effect in this model is based on the disutility from inventory error, this result holds for other biases such as mean-anchoring and overconfidence, since ordering behavior is still being influenced by factors which are unrelated to cost. Our results also suggest that managers in charge of coordinating supply chains may not want to take counter measures to offset retailers’ biases when retailers are either balanced in their disutility from ex-post inventory errors or have greater aversion to stockouts, since the internal reference effect can enable coordination. Furthermore, the supply chain is likely to benefit from a naïve supplier when the profit margin is high due to the lower wholesale price. Thus, in large vertically integrated supply chains where different organizational units have autonomous decision making, it is better for operations managers to remain naïve towards potential biases of the retailers.

If there is an established relationship between supply chain partners, then this is likely to give rise to an external reference effect. Suppliers need to understand the extent of the retailer’s level of fairness to avoid setting the wholesale price either too low, over accommodating for
fairness concerns, or too high to avoid the retailer lowering its order quantity due to spite. If the firms are a part of a larger centralized supply chain, then unlike the internal reference effect, naïveté is harmful to supply chain efficiency. Thus, the efficiency of the supply chain benefits from suppliers having full knowledge on their retailers’ behavioral biases.

Our research has important insights for supply chain management to counter ordering biases that lead to inefficiencies. Retailers with greater sensitivity to stockout aversion positively influence efficiency, particularly for high-margin products. Therefore, supply chain managers may want to create incentives based on achieving high service levels to encourage retailers to hold higher inventory levels. This could counteract retailers’ aversion towards surplus inventory by increasing their level of $\beta$, and, in turn, raising the retailer’s order quantity. Alternatively, if suppliers have little control over influencing retailers’ internal reference point, research shows that the supplier can significantly influence the extent of retailers’ fairness concerns through greater transparency, such as disclosing their production costs (Qin et al. 2016). Therefore, managers can use the model’s results to determine when it is advantageous for suppliers to disclose or withhold cost information to increase or decrease fairness concerns.

Although the application of the model is dependent on the supplier being knowledgeable about the retailer’s internal bias, this is not an unrealistic expectation in practice. In absence of a policy of full disclosure, there will generally be asymmetric information on the downstream demand. In most situations a supplier will still have sufficient information on consumer demand to gain considerable insight into a retailer’s ordering bias. In fact, depending upon the relative size and resources of the supplier versus the retailer, in many situations, the supplier may have better information on product demand compared to the retailer, particularly in situations where inventory is perishable due to changing trends or fashions (Gal-Or et al. 2008). Thus, the insights provided by the model should be applicable to managing supply chains.

7. Conclusion

This paper draws on the behavioral newsvendor and behavioral supply chain literature to explore the impact of reference effects on supply chains. In the behavioral newsvendor problem,
the retailer’s reference point is based on its own profit. In behavioral supply chains, a retailer may care about profit disparity creating a reference point based on its supplier’s profit. Differentiating these reference effects as internal and external, we study the individual and combined impacts of these effects on supply chain efficiency given a wholesale price contract.

We first study the equilibrium for a retailer who has an internal reference based on the disutility from ex-post inventory error and a supplier who has full information on the retailer’s bias. Theoretically, wholesale price contracts fail to coordinate the supply chain due to double marginalization when the retailer is strictly profit maximizing. However, we show that the retailer’s reference effect can lead to improvements in the performance of the supply chain. The uncertainty of the retailer’s profits is entirely driven by revenue uncertainty, since the cost of items is fixed under a wholesale contract. Thus, increases in the strength of the internal reference effect make the retailer more concerned with placing an order quantity that balances its relative disutility from over and under supplying the market demand. As a result, the behavioral bias diminishes the retailer’s sensitivity to the order cost, creating an opportunity for an informed supplier to increase the wholesale price. Indeed, when retailer ordering behavior is influenced by the reference point, the wholesale price should always be higher than the prescribed value from the unbiased model.

Despite the higher wholesale price, greater disutility from waste-aversion causes the retailer to experience more realizations of demand as gains rather than losses, leading to a high order quantity. Consequently, in these cases, the retailer’s reference point can coordinate the supply chain. However, these parameter regions where a wholesale price contract coordinates the supply chain are restricted to retailers with low reference points and products with low profit margins. Nevertheless, in situations where the the supplier assumes that the retailer is strictly profit maximizing, i.e. supplier is naïve to the behavioral bias, the internal reference effect can increase supply chain performance for higher margin products. This results from the fact that a naïve supplier does not increase the wholesale price to the profit maximizing value, enabling the retailer to order closer to the supply chain optimal quantity.

Next, we consider a retailer with an external reference effect based on disadvantageous inequity to contrast the influence of the external and internal reference effects on the equilib-
rium. There is a threshold value on the strength of the inequity parameter that determines if the retailer offers the unbiased wholesale price or lowers the wholesale price to avoid the retailer lowering their order quantity due to unfairness. The threshold value is sufficiently low that even a low-moderate effect will cause the supplier to lower the offered wholesale price. Thus, the external reference effect has the opposite directional impact on the wholesale price in comparison to the internal reference. Furthermore, contrary to the internal reference effect, the supply chain cannot be coordinated for a retailer that experiences disutility from having a lower profit compared to the supplier. However, the external reference effect has a consistent effect of raising the supply chain’s profit to 88.88% of the coordinated profit. Thus, for products with a low-cost to price ratio, the external reference effect can create a more efficient supply chain compared to the internal reference effect, provided that the supplier is sophisticated. If the supplier is naïve, then not accounting for profit equity causes the retailer to engage in spiteful behavior, lowering the equilibrium order quantity and harming efficiency. This also contrasts the internal case, where a naïve supplier increases efficiency.

When the retailer is only impacted by one of the reference effects, the order quantity is linear in the supplier costs. However, when the retailer is impacted by both reference effects, the interaction causes the equilibrium to have a piecewise linear ordering behavior with respect to the cost to price ratio. This leads to multiple costs for a fixed price where coordination can be achieved for a sufficiently high inequity averse retailer. Furthermore, our results show that the magnitude of the strength of the internal reference effect leading to the most efficient supply chain varies with the cost-to-price ratio when the supplier is sophisticated. However, when the supplier is naïve, disadvantageous inequity will primarily drive the retailers decision, making the equilibrium order quantity linear. In this case, a higher magnitude of the internal reference effect will improve efficiency for most cost-to-price ratios by diminishing the retailer’s sensitivity to inequity.

There are several avenues for future research. Firstly, our research assumes that the external reference effect is based on disadvantageous inequity. However, it is feasible that retailers could experience disutility from having a greater profit compared to the retailer, i.e. they experience advantageous inequity. The model could be extended to study the impacts of advantageous
inequity on wholesale price and coordination for retailers influenced by an internal reference effect. Secondly, we assume that the supplier has full information on the retailers bias, i.e. the supplier is sophisticated, or the supplier naively assumes that the retailer is profit maximizing. In reality, the supplier may have a prior belief on the retailer’s behavioral bias. Thus a potential extension would be to consider a distribution on the potential bias of the retailer, and to study screening models that would lead to the most efficient outcomes. Thirdly, the model assumes that the retailer’s reference point is exogenous. Since Becker-Peth et al. (2013) and Becker-Peth and Thonemann (2016) find that the contract parameters can influence the behavior of retailers given a fixed profit margin, future research could analyze the influence of different contract parameters on fairness and disutility from ex-post inventory error. Fourth, the research focuses on wholesale price contracts. However, in practice suppliers utilize a variety of contracts including revenue sharing, buyback, and quantity discount contracts. Future research could analyze the impact of internal and external reference effects on these contract design settings. Finally, our model assumes that the supplier is strictly profit maximizing. However, there has been recent evidence that suppliers may also exhibit behavioral regularities (Zhang et al. 2015, Niederhoff and Kouvelis 2016). Thus, an interesting extension to this model would be to examine how the interaction of internal and external reference effects between a behavioral retailer and a behavioral supplier influences the equilibrium and supply chain coordination.

Technical Appendix

Proof of Lemma 1

The expected utility of the retailer under a wholesale price contract is $u_r(q) = pS(q)(1 + \eta) - \eta(\beta pq + (1 - \beta)px) - wq$, where $S(q) \equiv \mathbb{E}[q \wedge x] = q - \int_x^q F(x)dx$ is the expected level of sales. Since $\frac{\partial^2}{\partial q^2} u_r(q) = -(1 + \eta)p f(q) < 0$, the expected utility of the retailer is concave. The first order condition is $\frac{\partial}{\partial q} u_r(q) = (1 + \eta)p \bar{F}(q) - \eta \beta p - w = 0$. Thus, for a given $w$, the best response of the retailer is $\bar{q}(w) = F^{-1}(1 - \frac{w/p + \eta \beta}{1 + \eta})$. Noting that $q^*(w) = F^{-1}(1 - \frac{w}{p})$, it is straightforward to show that when $\beta < \frac{w}{p}$ ($\beta > \frac{w}{p}$), the retailer will over (under) order. For uniform demand between 0 and 1, the optimal order quantity reduces to $\bar{q}(w) = 1 - (\frac{w/p + \eta \beta}{1 + \eta})$. 

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Proof of Proposition 1

For a fixed $w$, the retailer’s optimal order quantity is $\hat{q}(w) = F^{-1}(1 - \frac{w/p + \eta \beta}{1 + \eta})$ (Lemma 1). The supplier’s problem is to maximize $\pi_s(w) = (w - c)\hat{q}(w)$ over $w$. Since $F$ is strictly increasing and continuous, there is a one-to-one mapping between $w$ and $\hat{q}$, and the wholesale price that induces the retailer to order $\hat{q}$ is $\hat{w}(q) = p((1 + \eta)\hat{F}(q) - \eta \beta)$. Thus, the supplier’s problem is equivalent to maximizing $\pi_s(q) = (\hat{w}(q) - c)q$ over $q$. The first order condition of $\pi_s(q)$ is $\frac{\partial}{\partial q} \pi_s(q) = p((1 + \eta)\hat{F}(q)(1 - \frac{qf(q)}{F(q)}) - \eta \beta) - c$. With the increasing generalized failure rate assumption, we can show there exists a unique maximizer, which is obtained by solving the first order condition. Therefore, the optimal quantity for the supplier to induce is the solution to $(1 + \eta)\hat{F}(\tilde{q})(1 - \frac{\tilde{q}f(\tilde{q})}{\hat{F}(\tilde{q})}) - \eta \beta = \rho$. Plugging $\tilde{q}$ into $\hat{w}(q)$ gives the supplier’s optimal wholesale price $\hat{w}(\tilde{q})$. Since the solution $\tilde{q}$ is obtained by taking into account the retailer’s best response, $\tilde{q}$ is also the equilibrium order quantity. When demand is uniformly distributed between $[0, 1]$, the optimal quantity for the supplier to induce is given by $\tilde{q} = \frac{1}{2}(1 - \frac{\rho + \eta \beta}{1 + \eta}) = q^*(w^*) - \frac{\eta}{2(1 + \eta)}(\beta - \rho)$. The corresponding wholesale price is $\hat{w} = w^* + \frac{\eta \beta}{2}(1 - \beta)$.

The result that the relation between $\beta$ and $\rho$ determines whether the order size increases or decreases carries over to the general demand distribution case. As shown earlier, the optimal solution for $\tilde{q}$ is obtained by solving $(1 + \eta)\hat{F}(\tilde{q})(1 - \frac{\tilde{q}f(\tilde{q})}{\hat{F}(\tilde{q})}) - \eta \beta = \rho$. Note that when there is no bias, the optimal quantity $q^*$ is given by $\hat{F}(q^*)(1 - \frac{q^*f(q^*)}{\hat{F}(q^*)}) = \rho$. Rearranging the first equation we obtain $(1 + \eta)\hat{F}(\tilde{q})(1 - \frac{\tilde{q}f(\tilde{q})}{\hat{F}(\tilde{q})}) = \rho + \eta \beta$. So if $\beta > \rho$, then $(1 + \eta)\hat{F}(\tilde{q})(1 - \frac{\tilde{q}f(\tilde{q})}{\hat{F}(\tilde{q})}) = \rho + \eta \beta > (1 + \eta)\rho$, which implies that $\hat{F}(\tilde{q}) (1 - \frac{\tilde{q}f(\tilde{q})}{\hat{F}(\tilde{q})}) > \rho$. Using the increasing generalized failure assumption, we can conclude that $\tilde{q} < q^*$. Similarly we can prove $\tilde{q} > q^*$ if $\beta < \rho$.

Proof of Lemma 2

We solve the retailer’s piecewise optimization problem by examining the cases $q \leq \hat{q}_E(w)$ and $q \geq \hat{q}_E(w)$. Case 1: $q \leq \hat{q}_E(w)$. In this case, $u_r(q) = \pi_r(q)$. The retailer’s problem is to maximize $u_r(q)$, subject to the constraint $q \leq \hat{q}_E(w)$. We have shown that $\pi_r(q)$ is concave in $q$. Let $q^*$ be the maximizer of $u_r(q)$, so from the first order condition, we obtain $q^*(w) = \frac{p-w}{p}$. Therefore, the optimal solution is $\min(q^*(w), \hat{q}_E(w))$. Case 2: $q \geq \hat{q}_E(w)$. In this case,
subject to the constraint \( q \geq \hat{q}_E(w) \). Since \( \pi_r(q) \) is concave in \( q \) and \( \pi_s(q) \) is linear in \( q \), \( u_r \) must also be concave. Let \( \tilde{q}_E(w) \) be the maximizer of \( u_r(q) \). Then from the first order condition, we obtain \( \tilde{q}_E(w) = \frac{\alpha c + \alpha p + p - 2aw - w}{(\alpha + 1)p} = q^*(w) - \frac{\alpha(w-c)}{(1+\alpha)p} \). With the lower bound \( \tilde{q}_E(w) \), the optimal solution is \( \max(\tilde{q}_E(w), \hat{q}_E(w)) \).

Next we compare the above two cases to obtain the global optimal solution. The global optimal solution is the second largest of \( q^* \), \( \tilde{q}_E \) and \( \hat{q}_E \). First, \( q^*(w) - \tilde{q}_E(w) = \frac{\alpha(w-c)}{(\alpha+1)p} > 0 \), thus \( q^*(w) > \tilde{q}_E(w) \). Second, \( \frac{\partial q^*(w) - \tilde{q}_E(w)}{\partial w} = 3/p > 0 \), \( q^*(w) - \tilde{q}_E(w) \rvert_{w=c} = -\frac{p-c}{p} < 0 \), and \( q^*(w) - \tilde{q}_E(w) \rvert_{w=p} = \frac{2(p-c)}{p} > 0 \). Therefore, there must exist a unique threshold, \( w_{2E} = \frac{2c+p}{3} \), such that \( q^*(w_{2E}) \geq \hat{q}_E(w_{2E}) \) if \( w \geq w_{2E} \) and \( q^*(w_{2E}) \leq \hat{q}_E(w_{2E}) \) if \( w \leq w_{2E} \). Similarly, we can show that there exists a threshold \( w_{1E} = \frac{\alpha c + 2c + \alpha p + p}{2\alpha + 3} \) such that \( \hat{q}_E(w_{1E}) \geq \hat{q}_E(w_{1E}) \) if \( w \geq w_{1E} \) and \( \hat{q}_E(w_{1E}) \leq \hat{q}_E(w_{1E}) \) if \( w \leq w_{1E} \). Furthermore, \( w_{2E} - w_{1E} = \frac{\alpha(c-p)}{60 + 9} < 0 \), so \( w_{2E} < w_{1E} \). Therefore, if \( w \leq w_{2E} \), then \( q^*(w_{2E}) \leq \hat{q}_E(w_{2E}), \hat{q}_E(w_{2E}) \leq \hat{q}_E(w_{2E}) \) and \( q^*(w) > \hat{q}_E(w) \). This implies that \( q^*(w) \) is the second largest, so \( q_E = q^* \); if \( w_{2E} \leq w \leq w_{1E} \), \( \hat{q}_E(w) \) is the second largest; and if \( w \geq w_{1E}, \hat{q}_E(w) \) is the second largest.

**Proof of Proposition 2**

To solve the supplier’s piecewise optimization problem, we analyze the three cases, depending on the order quantities. Case 1: \( q \leq q_{1E} \). In this case, the supplier solves a constrained optimization problem: \( \max_q \pi_s = (w_E(q) - c)q \), subject to \( q \leq q_{1E} \). We can show that \( \pi_s \) is concave in \( q \). Letting \( \bar{q}^u_E \) be the maximizer of the unconstrained optimization problem \( \pi_s \), we obtain \( \bar{q}^u_E = \frac{p-c}{2p} = q^* \). With the upper bound \( q_{1E} \), the optimal solution is \( \min(q_{1E}, \bar{q}^u_E) \). Case 2: \( q_{1E} \leq q \leq q_{2E} \). In this case, the supplier solves the following problem: \( \max_q \pi_s = (w_E(q) - c)q \), subject to the constraints \( q_{1E} \leq q \leq q_{2E} \). Let \( \bar{q}^u_E \) be the maximizer of \( \pi_s \). Then we obtain \( \bar{q}^u_E = \frac{p-c}{p} \). It can be readily shown that \( \bar{q}^u_E > q_{2E} > q_{1E} \). Therefore, the optimal solution is \( q_{2E} \).

Case 3: \( q \geq q_{2E} \). In this case, the supplier solves the following problem: \( \max_q \pi_s = (w_E(q) - c)q \), subject to the constraint \( q \geq q_{2E} \). Let \( q^u \) be the maximizer of \( \pi_s \). We obtain \( q^u = \frac{p-c}{2p} \). We can show that \( q^u < q_{2E} \). Therefore, the optimal solution is \( q_{2E} \).

We compare the three cases to obtain the global optimal solution. First, the sign of
\[ q_E^*-q_{1E} = \frac{(c-p)(1-2\alpha)}{4\alpha p+4p} \] is indeterminate. There are two cases: (i) If \( \alpha < 1/2 \), then \( q_E^* < q_{1E} \) and the global optimal solution is either \( q_E^* \) or \( q_{2E} \). We can further show that \( s(q_{1E}) = \frac{\alpha + 1)(c-p)^2}{4(2\alpha p+p)} \) and \( s(q_{2E}) = \frac{2(p-c)^2}{9} \); thus, we obtain \( \pi_s(q_{1E}) - \pi_s(q_{2E}) = \frac{1-7\alpha}{20(2\alpha+1)} (p-c)^2 \). Therefore, if \( \alpha \geq 1/7 \) the global optimal order quantity is \( q_{2E} \), and if \( \alpha < 1/7 \), the global optimal solution is \( q_{1E} \). (ii) If \( \alpha > 1/2 \), then \( q_E^* > q_{1E} \), since the range of the second case contains \( q_{1E} \), the global optimal order quantity is \( q_{2E} \). Combining the above two cases, we find the optimal order quantity is \( q_{2E} \) if \( \alpha \geq 1/7 \), and the optimal order quantity is \( q_{1E} \) if \( \alpha < 1/7 \). The equilibrium wholesale price can be obtained by substituting the order quantity into the inverse demand function, that is, \( w_E = p(1-q_{2E}) = \frac{p+2c}{3} \) if \( \alpha \geq 1/7 \), and \( w_E = \frac{3ac+c+p+3}{4\alpha+2} \) if \( \alpha < 1/7 \).

**Proof of Lemma 3**

We solve the retailer’s piecewise optimization problem by examining the following two cases, depending on the order quantity. Case 1: \( q \leq \hat{q}_B(w) \). In this case, the retailer maximizes its utility \( u_r(q) = \pi_r(q) + r_I(q) \), subject to the constraint \( q \leq \hat{q}_B(w) \). The first order condition of \( u_r(q) \) with respect to \( q \) yields the optimal order quantity \( q_I = \frac{-\beta np+np+p-w}{(q+1)p} \). With the upper bound \( \hat{q}_B(w) \), the optimal solution is \( \min(\hat{q}_B(w), q_I(w)) \). Case 2: \( q \geq \hat{q}_B(w) \). In this case, the retailer solves the following constrained optimization problem: \( \max_q u_r(q) = \pi_r(q) + r_I(q) - \alpha(\pi_s(q) - \pi_r(q)) \), subject to the constraint \( q \geq \hat{q}_B(w) \). The first order condition yields the optimal order quantity \( \tilde{q}_B = \frac{ac+op-\beta np+np+p-2\alpha w-w}{p(\alpha+\eta+1)} \). With the lower bound \( q_0(w) \), the optimal solution is \( \max(\tilde{q}_B(w), \hat{q}_B(w)) \).

Next we compare the two cases, and obtain the global optimal solution. The global optimal solution depends on the order of \( q_I, \hat{q}_B \) and \( \hat{q}_B \). Thus, we characterize the conditions under which different orders of the three quantities may occur. First, compare \( q_I \) and \( \tilde{q}_B \). We obtain \( \frac{\partial(q_I-\tilde{q}_B)}{\partial w} = \frac{\alpha(2\eta+1)}{(q+1)p(\alpha+\eta+1)} > 0 \). Define \( w_{3B} \) such that \( q_I(w_{3B}) = \tilde{q}_B(w_{3B}) \), and we obtain \( w_{3B} = \frac{c\eta+e+\beta np}{2\eta+1} \). Thus, if \( w > w_{3B} \), then \( q_I > \tilde{q}_B \); and if \( w < w_{3B} \), then \( q_I < \tilde{q}_B \).

Second, compare \( q_I \) and \( \hat{q}_B(w) \). We obtain \( \frac{\partial(q_I-\hat{q}_B)}{\partial w} = \frac{4\eta+3}{\eta p+3} > 0 \). Define \( w_{2B} \) such that \( q_I(w_{2B}) = \hat{q}_B(w_{2B}) \), and we obtain \( w_{2B} = \frac{2c\eta+2\epsilon+\beta np+p}{4\eta+3} \). Thus, if \( w > w_{2B} \), then \( q_I > \hat{q}_B \); and if \( w < w_{2B} \), then \( q_I < \hat{q}_B \). Third, compare \( \tilde{q}_B \) and \( \hat{q}_B \). We obtain \( \frac{\partial(\tilde{q}_B-\hat{q}_B)}{\partial w} = \frac{2\alpha+4\eta+3}{4\alpha+4\eta+3} > 0 \). Define \( w_{1B} \) such that \( \tilde{q}_B(w_{1B}) = \hat{q}_B(w_{1B}) \), and we obtain \( w_{1B} = \frac{c(\alpha+2\eta+2)+p(\alpha+\beta p+p)}{2\alpha+4\eta+3} \). Thus,
if $w > w_{1B}$, then $\tilde{q}_B > \hat{q}_B$; and if $w < w_{1B}$, then $\tilde{q}_B < \hat{q}_B$.

We can show that $w_{3B} - w_{2B} = \frac{(\gamma + 1)(c + p(2(\beta - 1)\eta - 1))}{(2\eta + 1)(4\eta + 3)} < 0$, and $w_{2B} - w_{1B} = \frac{\alpha(c + p(2(\beta - 1)\eta - 1))}{(4\eta + 3)(2\alpha + 4\eta + 3)} < 0$. Therefore, $w_{3B} < w_{2B} < w_{1B}$. We also note that the three wholesale prices are all smaller than $p$, but may be greater than or smaller than $c$, depending on the value of $\beta$.

Therefore, we also define the following thresholds on $\beta$: $\beta_1 = c/p$, $\beta_2 = \frac{2c\eta + c - \eta p - p}{\eta p}$, and $\beta_3 = \frac{c(\alpha + 2\eta + 1) - p(\alpha + \eta + 1)}{\eta p}$. It follows that $\beta_3 < \beta_2 < \beta_1$. We find: (i) if $\beta < \beta_3$, then $w_{3B} < w_{2B} < w_{1B} \leq c$. Thus, $q_I > \tilde{q}_B > q_0$, and the optimal order quantity is $q_B(w) = \tilde{q}_B(w)$; (ii) if $\beta_3 < \beta \leq \beta_2$, then $w_{3B} < w_{2B} \leq c < w_{1B}$. Thus, $q_I > \tilde{q}_B$; $q_I > \hat{q}_B$, and $\tilde{q}_B > \hat{q}_B$ if and only if $w > w_{1B}$. Therefore, the optimal order quantity is: $q_B(w) = \tilde{q}_B(w)$ if $w \leq w_{1B}$ and $q_B(w) = \tilde{q}_B(w)$ if $w > w_{1B}$; (iii) if $\beta_2 < \beta \leq \beta_1$, then $w_{3B} \leq c < w_{2B} < w_{1B}$. Thus, $q_I > \tilde{q}_B$, $q_I > \hat{q}_B$ if and only if $w > w_{2B}$, and $\tilde{q}_B > \hat{q}_B$ if and only if $w > w_{1B}$. Therefore, the optimal order quantity is: $q_B(w) = q_I(w)$ if $w \leq w_{2B}$, $q_B(w) = \tilde{q}_B(w)$ if $w_{2B} \leq w \leq w_{1B}$, and $q_B(w) = \tilde{q}_B(w)$ if $w \geq w_{1B}$; (iv) if $\beta > \beta_1$, then $c < w_{3B} < w_{2B} < w_{1B}$. Thus, $q_I > \tilde{q}_B$ if and only if $w > w_{3B}$, $q_I > \hat{q}_B$ if and only if $w > w_{2B}$, and $\tilde{q}_B > \hat{q}_B$ if and only if $w > w_{1B}$. Therefore, the optimal order quantity is: $q_B(w) = \tilde{q}_B(w)$ if $w \leq w_{3B}$, $q_B(w) = q_I(w)$ if $w_{3B} \leq w \leq w_{2B}$, $q_B(w) = \tilde{q}_B(w)$ if $w_{2B} \leq w \leq w_{1B}$, and $q_B(w) = \tilde{q}_B(w)$ if $w \geq w_{1B}$. This establishes the results required.

**Preliminary Results for the Proof of Proposition 3**

As in the case of a retailer with an external reference effect, there is one-to-one mapping between wholesale price and order quantity. Since $q_B(w)$ is a piecewise function of $w$, the inverse demand function $w(q)$ faced by the supplier is also piecewise. We define the corresponding order quantities to $w_{2B}$, $w_{1B}$ and $w_{3B}$ as follows:

$$
q_1 = -\frac{2(c + p(2(\beta - 1)\eta - 1))}{p(2\alpha + 4\eta + 3)}
$$

$$
q_2 = -\frac{2(c + p(2(\beta - 1)\eta - 1))}{(4\eta + 3)p}
$$

$$
q_3 = -\frac{c - 2\beta \eta p + 2\eta p + p}{2\eta p + p}.
$$
It is straightforward to show that \( q_3 > q_2 > q_1 \). For the case of \( \beta > \beta_1 \), using Lemma 2 we obtain the inverse demand function as follows:

\[
  w(q) = \begin{cases} 
    w_s(q) & \text{if } q \leq q_1 \\
    w_o(q) & \text{if } q_1 \leq w \leq q_2 \\
    w_g(q) & \text{if } q_2 \leq q \leq q_3 \\
    w_s(q) & \text{if } q \geq q_3,
  \end{cases}
\]

where

\[
  w_s(q) = \frac{\alpha c - p(-\alpha + \beta \eta - \eta + q(\alpha + \eta + 1) - 1)}{2\alpha + 1} \\
  w_g(q) = -p(\beta \eta - \eta + \eta q + q - 1) \\
  w_o(q) = \frac{2c - pq + 2p}{4}.
\]

Note that it is straightforward to obtain the corresponding inverse demand functions for the other cases of Lemma 3. With the inverse demand function, we can reformulate the supplier’s problem as max \( q \pi_s(q) = (w(q) - c)q \). The supplier’s problem is to maximize its profit over the order quantity. From Lemma 3 we know there are four cases for the retailer’s order quantity, depending on the value of \( \beta \). In the following analysis, we examine each case.

**Proposition 4.** If \( \beta \leq \beta_2 \), then the optimal order quantity is \( q^* = \tilde{q}_B^u = \frac{p(\alpha - \beta \eta + \eta + 1) - (\alpha + 1)c}{2p(\alpha + \eta + 1)} \), and the optimal wholesale price is \( w^* = \frac{3ac + c + p(\alpha - \beta \eta + \eta + 1)}{4\alpha + 2} \).

**Proof.** Note that \( \beta_2 > \beta_3 \). Given the results of Lemma 3, we consider two cases: \( \beta \leq \beta_3 \) and \( \beta_3 < \beta \leq \beta_2 \). First, if \( \beta \leq \beta_3 \), then from Lemma 3, we know the retailer’s optimal order quantity is \( \tilde{q}_B \). The supplier solves the following problem: max \( q \pi_s(q) = (w_s(q) - c)q \). Since \( \pi_s \) is concave in \( q \), we can use the first order condition to derive the optimal order quantity. The optimal order quantity is \( \tilde{q}_B^u = \frac{p(\alpha - \beta \eta + \eta + 1) - (\alpha + 1)c}{2p(\alpha + \eta + 1)} \). Substituting \( \tilde{q}_B^u \) into the function \( w_s \), we obtain the optimal wholesale price, which is given by \( w^* = w^u_s = \frac{3ac + c + p(\alpha - \beta \eta + \eta + 1)}{4\alpha + 2} \).

Second, if \( \beta_3 < \beta \leq \beta_2 \), then the supplier faces a piecewise optimization model with two sub-problems: (1) the supplier solves the problem: max \( q \pi_s(q) = (w_s(q) - c)q \), subject to \( q \leq q_1 \). The optimal solution is \( q_1 > \tilde{q}_B^u \) for \( \beta < \beta_2 \), thus the optimal solution
is $\tilde{q}_B^u$: (2) the supplier solves the problem: $\max_q \pi_s = (w_o(q) - c) q$, subject to the constraint $q \geq q_1$. The first order condition yields the maximizer of $\pi_s$, $q_o = \frac{p-c}{p}$. Therefore, the optimal solution is $q_o$, since $q_o > q_1$ for $\beta > \beta_3$. The global optimal solution is $q^* = \tilde{q}_B^u$ or $q^* = q_o$.

We can show that $\pi_s(\tilde{q}_B^u) = \frac{(\alpha+1)(c-p(\alpha-\beta_3+\eta+1))^2}{4(2\alpha+1)(\alpha+\eta+1)}$ and $\pi_s(q_o) = \frac{(c-p)^2}{4p}$. Thus, $\frac{\partial \pi_s(\tilde{q}_B^u) - \pi_s(q_o)}{\partial \beta} = \frac{\eta((\alpha+1)(c-p(\alpha-\beta_3+\eta+1)) + \beta-1)mp}{2(2\alpha+1)(\alpha+\eta+1)} < 0$ and $\pi_s(\tilde{q}_B^u) - \pi_s(q_o)|_{\beta=\beta_2} = -\frac{(\alpha^2-\alpha(2\eta+1)-4\eta^2-7\eta-3)(c-p)^2}{4(2\alpha+1)(\alpha+\eta+1)} > 0$. Therefore, $\pi_s(\tilde{q}_B^u) - \pi_s(q_o) > 0$ always holds for $\beta_3 < \beta < \beta_2$, and the global optimal order quantity is $q^* = \tilde{q}_B^u$, which corresponds to $w^* = \frac{3\alpha c + p(\alpha-\beta_3+\eta+1)}{4\alpha+2}$.

**Proposition 5.** If $\beta_2 < \beta < \beta_1$, then the optimal order quantity is as follows:

(i) if $q_o \leq q_1$ and $q_1 > \tilde{q}_B^u$, then $q_B = \tilde{q}_B^u$;

(ii) if $q_1 < q_o \leq q_2$ and $q_1 \leq \tilde{q}_B^u$, then $q_B = q_o$;

(iii) if $q_1 < q_o \leq q_2$ and $q_1 > \tilde{q}_B^u$, then $q_B = q_o$ or $q_B = \tilde{q}_B^u$;

(iv) if $q_o > q_2$ and $q_1 \leq \tilde{q}_B^u$, then $q_B = q_2$;

(v) $q_o > q_2$ and $q_1 > \tilde{q}_B^u$, then $q_B = q_2$ or $q_B = \tilde{q}_B^u$.

**Proof of Proposition 5.** If $\beta_2 < \beta < \beta_1$, then we solve the following three subproblems: (1) the supplier solves the problem: $\max_q \pi_s = (w_s(q) - c) q$, subject to the constraint $q \leq q_1$. The optimal solution is $\min(q_1, \tilde{q}_B^u)$. (2) the supplier solves the problem: $\max_q \pi_o = (w_o(q) - c) q$, subject to the constraint $q_1 \leq q \leq q_2$. The first order condition yields the maximizer of $\pi_s$, $q_o = \frac{p-c}{p}$. Therefore, if $q_o \leq q_1$, the optimal solution is $q_1$; if $q_1 < q_o \leq q_2$, the optimal solution is $q_o$; if $q_o > q_2$, then the optimal solution is $q_2$. (3) the supplier solves the problem: $\max_q \pi_s = (w_g(q) - c) q$, subject to the constraint $q \geq q_2$. The first order condition yields the maximizer of $\pi_s$, $q_g^u = \frac{c-\beta_3 q_g + mp + p}{2(\eta+1) p}$. We can show that $q_g^u < q_2$, so the optimal solution is $q_2$.

Next we compare the three cases to obtain the global optimal solution. First we show $\partial q_B^u - q_o/\partial \beta = -\frac{n}{2(\alpha+\eta+1)} < 0$. Moreover, $\tilde{q}_B^u - q_o|_{\beta=\beta_2} = \frac{\alpha(p-c)}{2p(\alpha+\eta+1)} < 0$. Therefore, $\tilde{q}_B^u < q_o$ always holds for $\beta \in [\beta_2, \beta_1]$. Second we have shown that $q_2 > q_1$. Therefore, the global optimal solution is as follows: (i) if $q_o \leq q_1$ and $q_1 > \tilde{q}_B^u$, then the optimal solutions to the above three subproblems are $\tilde{q}_B^u$, $q_1$ and $q_2$. Observing that the interval of the first subproblem contains $q_1$ and that of the second subproblem contains $q_2$, we know that the global optimal solution is $q_B = \tilde{q}_B^u$; (ii) if $q_1 < q_o \leq q_2$ and $q_1 \leq \tilde{q}_B^u$, then similarly we can show that the optimal solution is $q_B = q_o$; (iii) if $q_1 < q_o \leq q_2$ and $q_1 > \tilde{q}_B^u$, then the optimal solution is
Proposition 6. If $\beta > \beta_1$, then the optimal order quantity is as follows:

(i) if $q_3 \geq \tilde{q}_B^u > q_1$ and $q_I^u \leq q_2$, then $q_B = q_2$;

(ii) if $q_3 \geq \tilde{q}_B^u > q_1$ and $q_3 \geq q_I^u > q_2$, then $q_B = q_I^u$;

(iii) if $\tilde{q}_B^u > q_3 > q_1$ and $q_I^u \leq q_2$, then $q_B = q_I^u$ or $q_B = q_2$;

(iv) if $\tilde{q}_B^u > q_3 > q_1$ and $q_3 \geq q_I^u > q_2$, then $q_B = q_I^u$ or $q_B = q_B^u$;

(v) if $\tilde{q}_B^u > q_3 > q_1$ and $q_I^u > q_3$, then $q_B = q_I^u$;

(vi) if $\tilde{q}_B^u > q_1 > q_I^u$ and $q_I^u \leq q_2$, then $q_B = q_2$ or $q_B = q_B^u$.

Proof of Proposition 6. If $\beta > \beta_1$, then we have the following four subcases: (1): if $q \leq q_1$, then the supplier solves a constrained optimization problem: $\max_q \pi_s = (w_s(q) - c)q$, subject to the constraint $q \leq q_1$. So the optimal order quantity is $min(q_1, \tilde{q}_B^u)$. (2) the supplier solves a constrained problem: $\max_q \pi_s = (w_o(q) - c)q$, subject to the constraint $q \in [q_1, q_2]$. The optimal order quantity is $q_2$. (3) the supplier solves a constrained optimization problem: $\max_q \pi_s = (w_s(q) - c)q$, subject to the constraint $q \in [q_2, q_3]$. The first order condition yields the maximizer for $\pi_s$, $q_I^u = \frac{-c-\beta p+\eta p+\nu}{2(\eta+1)p}$. So the optimal order quantity is one of the three quantities: if $q_I^u \leq q_2$, the optimal solution is $q_2$; if $q_2 < q_I^u \leq q_3$, the optimal solution is $q_I^u$; if $q_I^u > q_3$, then the optimal solution is $q_3$. (4) the supplier solves a constrained optimization problem: $\max_q \pi_s = (w_o(q) - c)q$, subject to the constraint $q \geq q_3$. The first order condition yields the maximizer for $\pi_s$, $\tilde{q}_B^u = \frac{\nu(\alpha-\beta p+\eta p+\nu)}{\nu-\nu(\alpha+\eta+1)}$. So the optimal order quantity is $\max(q_3, \tilde{q}_B^u)$.

Next we compare the four cases to obtain the global optimal solution. We know that $\tilde{q}_B^u > q_I^u$ for $\beta > \beta_1$, and also $q_3 > q_2 > q_1$. The global optimal solution is as follows: (i) if $q_3 \geq \tilde{q}_B^u > q_1$ and $q_I^u \leq q_2$, then the global optimal solution is $q_B = q_2$; (ii) if $q_3 \geq \tilde{q}_B^u > q_1$ and $q_3 \geq q_I^u > q_2$, then the global optimal solution is $q_B = q_I^u$; (iii) if $\tilde{q}_B^u > q_3 > q_1$ and $q_I^u \leq q_2$, then the optimal solution is $q_B = \tilde{q}_B^u$ or $q_B = q_2$; (iv) if $\tilde{q}_B^u > q_3 > q_1$ and $q_3 \geq q_I^u > q_2$, then the optimal solution is $q_B = q_2$ or $q_B = \tilde{q}_B^u$; (v) if $\tilde{q}_B^u > q_3 > q_1$ and $q_I^u > q_3$, then the global optimal solution is $q_B = \tilde{q}_B^u$; and (vi) if $q_3 > q_1 > \tilde{q}_B^u$ and $q_I^u \leq q_2$, then the global optimal solution is $q_B = q_2$ or $q_B = \tilde{q}_B^u$. \[ \square \]
Proposition 4 shows the equilibrium wholesale price and order quantity for the case of \( \beta \leq \beta_3 \). Note that \( \beta_2 \geq 0 \) is equivalent to \( c/p \geq \frac{1+n}{1+\alpha+2\eta} \). Thus, this case vanishes if \( c/p < \frac{1+n}{1+\alpha+2\eta} \).

If \( c/p \geq \frac{1+n}{1+\alpha+2\eta} \), then \( q^* = \tilde{q}_B^u \geq \frac{p-c}{p} \), suggesting that a small \( \beta \) may lead to an order quantity exceeding the first best when the profit margin is small, over-coordinating the supply chain.

Proposition 5 shows that the optimal order quantity is \( q_2, \tilde{q}_B^u, \) or \( q_o \). This implies that under some situations, the equilibrium order quantity is the first best and the supply chain can be coordinated. Proposition 6 shows that the optimal order quantity is one of the three order quantities, \( q_2, \tilde{q}_B^u \) and \( \tilde{q}_B^n \). We can show that all of these order quantities are smaller than the first best quantity \( q_o \). Thus, when \( \beta > \beta_1 = c/p \), the supply chain cannot be coordinated.

**Proof of Proposition 3**

We consider three cases individually. Case 1: \( \beta < \beta_2 \). From Proposition 4, it follows that if \( \beta < \beta_2 \) then \( q_B = \tilde{q}_B^u \), which is strictly decreasing in \( \beta \). Therefore, a lower bound on the order quantity for \( \beta < \beta_2 \) is \( \tilde{q}_B^u|_{\beta=\beta_2} = 1 - \rho \). Therefore, \( q_B < q_o \).

Case 2: \( \beta_2 \leq \beta < \beta_1 \). From the proof of Proposition 5, \( q_2 > q_1 \) and \( q_o > \tilde{q}_B^u \). Observe that the optimal behavioral order quantity is either \( q_2, \tilde{q}_B^u, \) or \( q_o \), and that both \( q_2 \) and \( \tilde{q}_B^u \) (as well as \( q_1 \)) are decreasing in \( \beta \), whereas \( q_o \) is constant and thus independent of \( \beta \). At \( \beta = \beta_2 \), \( q_2|_{\beta=\beta_2} = 2(1-\rho) > q_o \), thus from cases (i)-(iii) in Proposition 5, the optimal order quantity is \( q_o \) or \( \tilde{q}_B^u|_{\beta=\beta_2} = q_o \). Now consider \( \beta = \beta_1 = \rho \). Observe that \( \tilde{q}_2|_{\beta=\beta_1} = \frac{2(1-\rho)(1+2\eta)}{4\eta+3} < 1 - \rho = q_o \).

From cases (iv)-(v) in Proposition 5, the optimal order quantity at \( \beta = \beta_1 \) is either \( q_2|_{\beta=\beta_1} \) or \( \tilde{q}_B^u|_{\beta=\beta_1} \), where \( \tilde{q}_B^u|_{\beta=\beta_1} = \frac{1}{2}(1-\rho) = \frac{2(1-\rho)(2\eta+1)}{4(2\eta+1)} < \frac{2(1-\rho)(2\eta+1)}{4\eta+3} = q_2|_{\beta=\beta_1} \). From case (iv) the optimal solution is \( q_2 \). From case (v) the optimal solution is either \( q_2 \) or \( \tilde{q}_B^u \), depending on the value of \( \beta \). At \( \beta = \beta_1 \), the profit function \( \pi_s(\tilde{q}_B^u|_{\beta=\beta_1}) = (w_s(\tilde{q}_B^u|_{\beta=\beta_1}) - c)\tilde{q}_B^u|_{\beta=\beta_1} \) is decreasing in \( \alpha \), since \( \partial \pi_s(\tilde{q}_B^u|_{\beta=\beta_1})/\partial \alpha = -\frac{(2\eta+1)(1-\rho)}{2(\alpha+1)^2} \). Furthermore at \( \alpha = 0 \), \( w_s(q) = w_g(q) \), and therefore \( \pi_s(\tilde{q}_B^u|_{\beta=\beta_1}) < \pi_d(q_2|_{\beta=\beta_1}) \). Thus, for case (v) at \( \beta = \beta_1 \) it holds that the equilibrium solution is \( q_2|_{\beta=\beta_1} \), thus the equilibrium order quantity \( q_B \) is greater than \( q_2|_{\beta=\beta_1} \).

Case 3: \( \beta \geq \beta_1 \). From the proof of Proposition 6, \( \tilde{q}_B^u > q_1 \) and \( q_3 > q_2 > q_1 \). Evaluating \( \tilde{q}_B^u|_{\beta=\beta_1} = \frac{1}{2}(1-\rho) = \tilde{q}_B^u \). From Case 2, we know that \( \tilde{q}_2|_{\beta=\beta_1} > \frac{1}{2}(1-\rho) \), which provides an upper bound on the order quantity.
References


Chaharsooghi, S. K. and Heydari, J. (2010). Supply chain coordination for the joint deter-


