Sub-Miniature Hot-Wire Anemometry for High Reynolds Number Turbulent Flows

by

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Declaration of Authorship

This is to certify that:

■ the thesis comprises only my original work towards the PhD,

■ due acknowledgement has been made in the text to all other material used,

■ the thesis is fewer than 100,000 words in length, exclusive of tables, maps, bibliographies and appendices.

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Abstract

High Reynolds number turbulent boundary layer flows are experimentally investigated in this study. Despite several decades of research in wall-bounded turbulence there is still controversy over the behaviour of streamwise turbulence intensities near the wall, especially at high Reynolds numbers. Much of it stems from the uncertainty in measurement due to finite spatial resolution. Conventional hot-wire anemometry is limited for high Reynolds number measurements due to limited spatial and temporal resolution issues that cause attenuation in the streamwise turbulence intensity profile near the wall. In an attempt to address this issue we use the NSTAP (nano-scale thermal anemometry probe) developed at Princeton University to conduct velocity measurements in high Reynolds number boundary layer facility at the University of Melbourne. NSTAP is almost one order of magnitude shorter than conventional hot-wires. The measurements cover a friction based Reynolds number range $Re_\tau = 6000-20000$ and the viscous-scaled sensor length is in the range 2.5-3.5 which is unique for these high values of Reynolds number in the boundary layer turbulent flows.

The study starts with addressing the challenges associated with the operation of sub-miniature hot-wires in constant temperature mode. An existing theoretical model is applied to optimise a Melbourne University Constant Temperature Anemometer and successful measurements are conducted in the high-$Re$ boundary layer facility as well as a channel flow facility.

NSTAP has been mainly used at Princeton University for wall-bounded turbulence measurements. We use nearly a dozen NSTAP probes together with conventional 2.5µm-diameter hot-wires to measure the velocity fluctuations in a turbulent boundary layer, in order to assess the NSTAP’s performance in an independent survey. Comparison of the mean and the variance of the streamwise velocity component reveals discrepancies
between some of the NSTAPs and the conventional hot-wires; the results of the “reliable NSTAPs” (those that compare well against the conventional hot-wires in the outer region of the turbulence intensity profile) are further analysed.

Results of the “reliable NSTAPs” show that in the near-wall region, the viscous-scaled streamwise turbulence intensity, $u^{+}$, increases with $Re_{\tau}$ in the Reynolds number range of the experiments, and the near-wall peak in $u^{+}$ follows a logarithmic relation with $Re_{\tau}$. Moreover, $u^{+}$ exhibits outer-scaling in the outer region following a logarithmic relation in the overlap region as predicted by the attached eddy hypothesis. The energy spectra in the near-wall region appear to show excellent viscous-scaling over the small to moderate wavelength range, followed by a large outer-scale influence that becomes increasingly noticeable with Reynolds number. In the outer region energy spectra exhibit excellent agreement with outer-scaling in the moderate to large wavelength range. Examination of spectrograms reveals that the energetic outer site that appears as an outer peak in the moderate Reynolds numbers $Re_{\tau} = 6000-14500$, grows into a small region of plateau at $Re_{\tau} = 20000$ and 25000, which signals emergence of the $k^{-1}$ region at Reynolds numbers higher than 25000.
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<td>Constant Temperature Anemometer</td>
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# Symbols

## Roman symbols

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<td>$A$</td>
<td>additive universal constant in the logarithmic law of the wall</td>
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<tr>
<td>$A'_1$</td>
<td>universal function in the quasi-logarithmic law for the streamwise turbulence intensity</td>
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<tr>
<td>$B_1$</td>
<td>additive characteristic constant in the logarithmic law for the streamwise turbulence intensity</td>
</tr>
<tr>
<td>$B'_1$</td>
<td>additive characteristic function in the quasi-logarithmic law for the streamwise turbulence intensity</td>
</tr>
<tr>
<td>$B_2$</td>
<td>additive characteristic constant in the logarithmic law for the spanwise turbulence intensity</td>
</tr>
<tr>
<td>$B_3$</td>
<td>additive characteristic constant in the logarithmic law for the wall-normal turbulence intensity</td>
</tr>
<tr>
<td>$c_w$</td>
<td>wire material specific heat</td>
</tr>
<tr>
<td>$D$</td>
<td>dissipation spectrum</td>
</tr>
<tr>
<td>$d$</td>
<td>diameter</td>
</tr>
</tbody>
</table>
Nomenclature

\( d_w \)  
hot-wire diameter

\( E_{qi} \)  
offset voltage

\( E_0 \)  
CTA’s static output voltage

\( E_w \)  
static voltage across hot-wire

\( \mathcal{E} \)  
error

\( e_0 \)  
CTA’s output voltage fluctuation

\( e_s \)  
voltage fluctuation injected to the offset voltage node

\( e_w \)  
fluctuating voltage across hot-wire

\( f \)  
frequency

\( f_c \)  
cut-off frequency

\( f_s \)  
sampling frequency

\( h \)  
channel half width

\( I \)  
static electric current

\( I_1 \)  
static electric current passing through the hot-wire in a CTA circuit

\( i' \)  
electric current fluctuation

\( K_0 \)  
Kolmogrov universal constant for the streamwise velocity component

\( K_a \)  
DC gain of the first stage amplifier in a CTA

\( K_b \)  
DC gain of the second stage amplifier in a CTA

\( K_u \)  
Kurtosis

\( k_g \)  
gas thermal conductivity

\( k_w \)  
wire material thermal conductivity

\( k_x \)  
streamwise wavenumber

\( l, l_w \)  
hot-wire length

\( \text{Nu} \)  
Nusselt number

\( n \)  
a constant representing asymmetry in the wire boundaries

\( P \)  
perimeter

\( p \)  
turbulence production

\( R_g \)  
resistance of the wire at the gas temperature

\( R_l \)  
sum of resistance of the stubs and wire connectors to the circuit
Nomenclature

\( R_w \)  
hot-wire’s resistance

\( Re \)  
Reynolds number

\( Re_\tau \)  
Reynolds number based on friction velocity, \( Re_\tau = u_\tau \delta / \nu \)

\( Re_w \)  
hot-wire Reynolds number, \( Re_w = Ud_w / \nu \)

\( S_u \)  
Skewness

\( s \)  
Laplace variable

\( T_w \)  
a time constant for hot-wire defined by \( \rho_w c_w^2 / k_w \)

\( U \)  
streamwise mean velocity

\( U_{cl} \)  
centreline velocity in channel or pipe

\( U_\infty \)  
freestream velocity

\( u, u' \)  
streamwise velocity fluctuation

\( u_\tau \)  
friction velocity

\( v \)  
spanwise velocity fluctuation

\( v_\eta \)  
Kolmogrov velocity scale

\( w \)  
wall-normal velocity fluctuation

\( w_c \)  
Coles wake function

\( x \)  
streamwise coordinate

\( x^* \)  
non-dimensional coordinate along the wire

\( y \)  
spanwise coordinate

\( Z_w \)  
hot-wire impedance

\( z \)  
wall-normal coordinate

Greek symbols

\( \alpha \)  
resistance temperature coefficient of the wire

\( \Delta \)  
characteristic height of an attached eddy

\( \delta \)  
boundary layer thickness

\( \delta^* \)  
displacement thickness

\( \epsilon \)  
turbulent energy dissipation

\( \eta \)  
Kolmogrov length scale
Nomenclature

\( \theta_s \) stub temperature
\( \theta_w \) hot-wire temperature
\( \kappa \) von Kármán constant in logarithmic law of the wall
\( \lambda_x \) streamwise wavelength
\( \phi_{uu} \) one-sided cross power spectral density of streamwise fluctuating component
\( \pi \) Coles wake strength
\( \nu \) kinematic viscosity
\( \nu_g \) gas kinematic viscosity
\( \rho \) density
\( \tau_w \) mean wall shear stress
\( \tau_\eta \) Kolmogrov time scale

**Superscript**

+ indicates viscose scaling using \( u_\tau \) and \( \nu \), also referred to as inner scaling.

**Subscript**

\( p \) prong
\( s \) stub
\( w \) wire
Chapter 1

Introduction

1.1 Motivation

Turbulent flows are ubiquitous in nature and technological applications. Common examples of turbulent flows are blood flow in arteries, oil transport in pipelines, atmosphere and ocean currents, and the flows over aircrafts and submarines. In many of these applications the Reynolds number is high ($O(10^5 \sim 10^8)$). For wall-bounded turbulent flows, the friction Reynolds number, $Re_\tau = u_\tau \delta / \nu$, is defined as the ratio of $\delta$ (the boundary layer thickness in boundary layer flows and pipe radius and channel half width in pipe and channel flows, respectively) to the viscous length scale $\nu / u_\tau$. Here $\nu$ is the kinematic viscosity and $u_\tau = \sqrt{\tau_w / \rho}$ is the friction velocity where $\tau_w$ is the wall shear stress. Therefore, increasing the Reynolds number results in an increase in scale separation. This large scale separation in high-$Re$ turbulent flows makes them complicated and difficult to study. A turbulent motion, described by the instantaneous Navier-Stokes (NS) equations, is naturally a multi-scale phenomenon where energy is extracted from the largest scales of the flow through velocity gradients and transported down to smaller scales until dissipating at the smallest viscous scales. This cascade process is highly time-dependent, hence, in order to study turbulence one needs to resolve all these spatial and temporal scales simultaneously, down to the very small dissipative scales. Ideally this should be done through Direct Numerical Simulation (DNS) where the Navier-Stokes
and continuity equations are solved simultaneously at grid points in a mesh. A reliable DNS needs to have a computational domain size large enough to capture the largest motions and its mesh grid size should be fine enough to resolve the smallest dissipative motions in the flow. In wall-bounded turbulent flows this requires computer resources that scale nominally as $Re^4$ (Piomelli and Balaras [118]). Considering that the highest Reynolds number DNS available currently is $Re_\tau \approx 5200$ in turbulent channel flow (Lee and Moser [67]) and $Re_\tau \approx 2500$ in turbulent boundary layer flow (Sillero et al. [128]), which are many orders of magnitude smaller than the Reynolds number in the industrial and environmental applications, one realises that it is unrealistic to expect that DNS can be used for high-$Re$ engineering applications in the near future.

A more common numerical approach, especially in engineering applications, is solving the Reynolds-averaged Navier-Stokes (RANS) equations, where time- or ensemble-averaged NS equations are solved, while the Reynolds stresses due to velocity fluctuations are calculated based on empirical models. Developing RANS models has been the subject of many studies, however no RANS model has been found that is reliably applicable to all turbulent flow geometries. This may be due to the fact that the large, energy-carrying eddies are greatly affected by the boundary conditions, and universal models that account for their dynamics may be impossible to develop [118].

A numerical approach that is not as computationally demanding as DNS, yet captures much of the unsteady and three dimensional motions in a turbulent flow is Large eddy simulation (LES). In LES the large, energy-containing motions are computed, while only the small, sub-grid scales of motion are modelled. LES is usually more accurate than the RANS approach because the small scales tend to be more isotropic and homogeneous than the large ones, and hence more compliant to universal modelling. However, in the presence of the wall, the grid-resolution requirements for resolving the flow are constrained by the near-wall region where the flow dynamics are dominated by the viscous-scaled motions. In fact, Reynolds [122] estimated that the computational costs scales as $Re_\tau^{0.5}$ for the outer layer and scales as $Re_\tau^{2.4}$ for the inner layer. Therefore, the only economical way to perform high-$Re$ LES is to compute the outer layer and
empirically model the inner layer using wall models [118]. Here insights from high-$Re$ experiments can be helpful [132].

The experimental study of high-Reynolds number turbulent flows has progressed rapidly over the past few decades thanks largely to the construction and development of high Reynolds number experimental facilities ([15, 96, 99, 106, 135, 149]). In conjunction with this, advances in sub-miniature hot-wire ($l < 100 \, \mu m$) anemometry have also been required in order to capture the smallest scales of motions in these flows. The friction Reynolds number $Re_\tau$ can be viewed as a measure of the ratio of the largest scale size to the smallest scale size. $Re_\tau$ can exceed $10^5$ in high-Reynolds-number flows which means that in a laboratory facility with fixed dimensions, the small-scale eddy size can be of the order of micrometers. Using sensors with sizes greater than the smallest scales of motion results in spatial filtering [47]. Conventional hot-wires with a sensing length of 500-1000 $\mu m$ are not usually suitable for high-Reynolds number flow studies in existing facilities due to spatial resolution issues. Temporal resolution is another limit in using hot-wires for measuring velocity fluctuation of high-Reynolds number flows. Hutchins et. al [47] showed that the maximum frequency content of a wall-bounded turbulent flow spectrum is equal to or greater than $u_\tau^2/3\nu$, meaning that if the experimental measuring system cannot resolve time scales down to $t = 3\nu/u_\tau^2$, there will be an excessive temporal filtering of the measured energy, in addition to the spatial attenuation. Therefore, there is a direct relationship between the Reynolds number and the highest frequency of a flow. Typically, hot-wire anemometers have been reported to have frequency responses in the range $30 < f < 100$ kHz which might be insufficient to fully resolve high-Reynolds number flows temporally [47].

A number of notable efforts have been made towards realising adequately small hot-wires, (see e.g. [6, 13, 51, 74]) one of the most important of them being the development of the Nano-Scale Thermal Anemometry Probe, NSTAP [6], which uses MEMS-based technology for the probe construction and results in sensor lengths of the order 30-60 $\mu m$. 
NSTAP has been used in the Superpipe and HRTF at Princeton to acquire unique high-$Re$ data \[44, 45, 124, 146, 147\]. Although, these data have yielded noticeable advances in the community’s understanding of high-$Re$ turbulence, they have caused controversies with regard to the quantitative and even qualitative $Re$ scaling of the streamwise variance profile, which are at odds with the scaling observed in DNS and lower Reynolds number experimental results \[21, 67, 103, 105\].

Given the significance of high-$Re$ turbulent flows and the incapability of numerical approaches to realize them in the near future, accurate high-$Re$ experimental studies are essential to improve our understanding of the underlying physics of these fluid flows. By using NSTAP probes for studying turbulent boundary layer flows in the High Reynolds Number Boundary Layer Wind Tunnel (HRNBLWT) at the University of Melbourne, we follow two primary goals: (i) scrutinizing the performance of the NSTAP in high-$Re$ wall-bounded turbulent flows, which might enable us to clarify some of the disputes over the NSTAP results in the Superpipe and HRTF by employing it in a different facility, and (ii) obtaining well-resolved measurements in turbulent boundary layer flows up to $Re_\tau = 20000$. These measurements result in unique datasets with the viscous-scaled sensor length $l^+ = l u_\tau / \nu < 3.5$. Here $l$ is the sensor length. Hopefully this study can help elucidate the underlying physics of near-wall, logarithmic and outer layers in high-$Re$ wall-bounded turbulent flows.

### 1.2 Thesis aims and outlines

The main goals of this study are:

(i) To address the challenges involved in the operation of sub-miniature hot-wires using available constant temperature anemometers with the aid of theoretical modelling.
(ii) To investigate the effect of relaxing the proposed length-to-diameter ratio required to avoid contamination of statistics and energy spectra due to the end conduction effect \((l/d \geq 200)\) in high Reynolds number wall-bounded turbulent flows.

(iii) To evaluate the performance of NSTAP operated with a modified custom-made Melbourne University Constant Temperature Anemometer (MU CTA) and a Dantec Streamline system in high Reynolds number turbulent boundary layer flows.

(iv) To investigate the scaling of turbulence intensity in the near-wall, intermediate, logarithmic, and outer regions using well-resolved experimental data obtained by the NSTAP in turbulent boundary layer flows.

(v) To characterise the streamwise energy spectra in various wall regions using the well-resolved data.

Chapter 2 provides a literature review of turbulent boundary layer flows focusing on high-\(Re\) studies and related problems.

Chapter 3 outlines the experimental apparatus and techniques employed.

Operation of NSTAP probes and other sub-miniature hot-wires in constant temperature mode, which are required for the study of high Reynolds number turbulent flows, is not straightforward. Currently NSTAP can be operated with the Dantec Streamline anemometer but only using external resistors \([145]\). That is, this anemometer is not designed for sensors with the high resistance as found with NSTAPs. Attempts to operate an NSTAP with other anemometers result in frequent sensor breakages. Moreover, the best possible square-wave test response of an NSTAP with the Dantec Streamline anemometer does not resemble a typical optimal square-wave test response of conventional wires as introduced by Freymuth \([34]\) (See ref. \([145]\) for NSTAP square-wave responses). The issue of appropriate operation of sub-miniature hot-wires, with regards to aim (i), is addressed in Chapter 4 by employing theoretical modelling to optimise an in-house CTA: the Melbourne University Constant Temperature Anemometer (MU CTA) for this purpose.
Chapter 1. *Introduction*

The effect of insufficient length-to-diameter ratio of hot-wires on the measured turbulence intensities and energy spectra is explored theoretically and experimentally in Chapter 5 addressing aim (ii). This problem is relevant since one may attempt to improve the spatial resolution associated with a hot-wire measurement by reducing the wire’s length without reducing its diameter proportionally as handling wires with diameters less than 2.5 µm is challenging due to breakage and drift issues (see e.g. Li et al. [71], Morrison et al. [96] and Vallikivi [144]).

Nearly a dozen NSTAP probes together with conventional 2.5 µm-diameter hot-wires are used to measure the velocity fluctuations in turbulent boundary layer flows at varying Reynolds numbers and results are compared in Chapter 6 towards fulfilling aim (iii).

In Chapter 7, well-resolved turbulence intensities and energy spectra are presented. Turbulence intensities are closely examined in the near-wall, intermediate, and logarithmic and outer regions to survey their scaling in various wall-distance regions in response to aim (iv). Furthermore, the viscous-scaling in the near-wall region and $z$- and $\delta$-scaling in the logarithmic and outer layers are examined for the energy spectra up to $Re_\tau = 20000$ in the near-wall region and $Re_\tau = 25000$ in the logarithmic and outer layers to address aim (v). Also presented in Chapter 7 are the examinations of Kolmogrov scaling of the energy spectra, higher order statistics, and turbulent energy dissipation rate and Kolmogrov scale characteristics using the well-resolved high-$Re$ data. Finally, the most important findings of this study are summarised in Chapter 8.
Chapter 2

Literature review

2.1 Boundary layer definition

A fluid flowing over a solid surface is affected by it within a thin layer in the immediate vicinity of the surface where fluid viscosity is significant. The flow speed is zero at the solid surface due to ‘no-slip’ condition and equals the free-stream condition at the upper edge of the thin layer resulting in a steep normal gradient in the flow velocity, which, in turn is responsible for shear forces within the layer. This layer was first introduced by Prandtl \cite{120} and was referred to as the boundary layer. The development of the boundary layer contributes to a considerable portion of the aerodynamic drag, hence is of great importance in many engineering applications. In the region beyond the boundary layer, flow is irrotational and viscosity effects can be neglected. Therefore, Prandtl’s boundary layer solved d’Alembert’s paradox of zero drag in inviscid theory.

The coordinates used throughout this thesis are $x$, $y$, and $z$ referring to the streamwise, spanwise, and wall normal directions respectively. $u$, $v$, and $w$ are the corresponding velocity fluctuations. Furthermore, capitalization and overbars indicate time-averaging.
2.2 Mean velocity

In this thesis our focus is on zero-pressure-gradient (ZPG) turbulent boundary layers. For such flows, at high Reynolds number, in the near the wall, streamwise mean velocity $U$ is governed by wall-normal distance $z$, kinematic viscosity $\nu$, and friction velocity defined as $u_\tau = \sqrt{\tau_w/\rho}$. Here $\tau_w$ is the mean wall shear stress and $\rho$ is the fluid density.

Applying the dimensional analysis yields

$$\frac{U}{u_\tau} = f(z^+),$$

for the near-wall known as the law of the wall following Prandtl [120]. Here ‘+’ denotes normalisation by viscous scales $u_\tau$ and $\nu$.

In the regions far from the wall important variables affecting $U$ are, $u_\tau$, $z$, free stream velocity $U_\infty$, and outer length scale $\delta$. Arguing that the relevant velocity scale in this region is $u_\tau$ Von Kármán [150], showed that in this region the mean flow velocity is governed by

$$\frac{U_\infty - U}{u_\tau} = F(z/\delta),$$

known as the defect law.

Millikan [88] argued that an overlap region exists where equations 2.1 and 2.2 are valid simultaneously, which yields

$$\frac{U}{u_\tau} = \frac{1}{\kappa} \log(z^+) + A$$

(2.3)

and

$$\frac{U_\infty - U}{u_\tau} = -\frac{1}{\kappa} \log\left(\frac{z}{\delta}\right) + B.$$
Here $\kappa$ (known as Kármán constant) and $A$ are universal constants, while $B$ is dependent on the flow geometry. The logarithmic laws for the mean flow may be developed through different approaches for instance see Prandtl [120], Townsend [140], Oberlack [102], and Klewicki et al. [60].

Coles [22] noted that in the outer layer, the mean velocity departs from the logarithmic law, and hence proposed an empirical law of the wall/law of the wake of the form

$$\frac{U}{u_*} = \frac{1}{\kappa} \log(z^+) + A + \frac{\pi}{\kappa} w_c(z) \left( \frac{z}{\delta} \right),$$  \hspace{1cm} (2.5)

where $\pi$ is the Coles wake strength factor being a function of the streamwise development and the pressure gradient, while $w_c$ which is Coles wake function is common to all two-dimensional turbulent boundary layer flows. In Coles’ [22] study, numerical values for the law of the wall were suggested to be $\kappa = 0.40$ and $A = 5.1$. Several extensions and alternative formulations for the logarithmic law have been proposed, but the main alternative is a power law representation. Barenblatt and co-workers in a series of controversial papers (Barenblatt [8], Barenblatt et al. [9, 10]) suggested that a power law describes the mean profile more accurately than the classical log law. Following Branblatt’s power law scaling, George et al. [35] argued that $U_\infty$ is the only theoretically acceptable velocity scale for the outer region leading to a power law representation for the mean flow. Jones et al. [54], however, showed that the friction velocity is equally acceptable in the asymptotic limit.

The debate over power law versus log law motivated researchers to start new sets of high Reynolds number measurements, most notable of which is Superpipe measurements in Princeton (see e.g. Zagarola and Smits [159] and Zagarola et al. [158]). Reporting various values for $\kappa$ ranging from 0.37 (Zanoun et al. [161]) to 0.44 (Zagarola and Smits [159]), these measurements among others cast doubt on the universality of $\kappa$. For a comprehensive review of the variations of $\kappa$ in the recent experimental studies see Marusic et al. [78]. Recently Marusic et al. [79], analysing experimental data in the Reynolds number range $2 \times 10^4 - 6 \times 10^5$ from boundary layers, pipe, and the atmospheric surface
layer showed that within experimental uncertainty, the data support the existence of a universal logarithmic region with the von Kármán constant $\kappa = 0.39$.

### 2.3 Streamwise turbulence intensity

Among all the three turbulence intensity components ($u^2$, $v^2$ and $w^2$), the streamwise normal Reynolds stress $u^2$ has received the most attention due to the experimental challenges associated with measuring the rest. In this thesis our focus is on the scaling of $u^2$ in different wall normal regions.

$u^2$ is most often scaled with the friction velocity squared $u_f^2$ and is shown as $u^{2+}$, however other inner-outer mixed scalings have been tried due to the failure of $u^{2+}$ to collapse. George et al. [35] based on asymptotic analysis of the Navier-Stokes equations concluded that the appropriate scale for $u^2$ is $U_{\infty}^2$. Zagarola and Smits [159] used $(U_{cl} - U)^2$ and showed that this scaling collapses $u^2$ better than $u_f^2$ in the outer region. De Graaff and Eaton [25] used mixed scale $u_f U_{\infty}$, and Monkewitz et al. [92] used $U_{\infty}^2$. Most recently Monkewitz and Nagib [91] used Taylor expansion of Navier-Stokes equations at large Reynolds number and concluded that these equations can only be balanced if the near-wall parts of $u^2$ and $w^2$ scale with $u_f^2$.

In the near wall region a maximum can be located for $u^{2+}$ referred to as the inner peak $u^{2+\text{max}}$ and is located at $z^+ \approx 15$, independent of Reynolds number. However, the Reynolds number scaling of the magnitude of the peak has been controversial. While most of the recent studies show that $u^{2+\text{max}}$ exhibits a weak $Re$ dependence (De Graaff and Eaton [25], Hutchins and Marusic [46], Klewicki and Falco [59], Lee and Moser [67], Metzger et al. [86], among others), some researchers have reported that it is invariant with $Re$ (Mochizuki and Nieuwstadt [89], Fernholz and Finleyt [29], Hultmark et al. [44, 45], Vallikivi et al. [147]). It should be noted here that although internal (channel and pipe) and external (boundary layers) flows are different in the outer region, the near-wall viscous-scaled structures remain largely unaffected (Monty et al. [93], Sillero et al. [128]). Hence, comparisons between these flows at matched $Re_\tau$ is expected to
yield at least similar trends. The experimental studies that supported the increase of the inner peak with \( Re_\tau \) took special care to ensure that spatial resolution issues did not influence the results, since several studies suggest that finite probe size leads to the attenuation of the turbulence intensity in the near-wall region (Johansson and Alfredsson [53], Willmarth and Sharma [154], Ligrani and Bradshaw [72], Hites [40], and Hutchins et al. [47]). Dependency of \( \bar{u}_r^2 \) on Reynolds number is attributed to the interaction of near-wall modes that scale on wall units with modes residing away from the wall that scale on the outer length scale \( \delta \) (Hoyas and Jiménez [41], Marusic et al. [77]).

In addition to the inner peak, appearance of a second outer peak or plateau has also been reported in high Reynolds number measurements in the streamwise turbulence intensity profile by Fernholz et al. [30], Morrison et al. [96], Metzger et al. [84], Hultmark et al. [44] and Vallikivi et al. [147]. The viscous-scaled wall-distance location and value of the this peak increase with \( Re_\tau \). Pullin et al. [121] presented an analysis that supports a logarithmic increase in the outer peak value with Reynolds number. Vassilicos et al. [148] investigated the intermediate region in the streamwise turbulence intensity in the pipe flow. They modified the spectral model of Perry et al. [115] by adding a new wavenumber range to the model at wavenumbers smaller than those with the \( k_x^{-1} \) scaling, and showed that the new model could predict an outer peak in the turbulence intensity profile as seen in the high Reynolds number pipe data of Hultmark et al. [44].

In the logarithmic region of wall-bounded flows, Townsend [140] proposed that the scaling from the wall can be associated with corresponding attached eddies. The geometric lengths of these eddies scale with \( z \), and have population densities per characteristic eddy height that scale inversely with \( z \). Townsend showed that at asymptotically high Reynolds number this argument leads to a logarithmic profile for the streamwise (and spanwise) turbulence intensities of the form

\[
\frac{\bar{u}_r}{\bar{u}_r^2} = B_1 - A_1 \log(z/\delta),
\] (2.6)
where $\delta$ is the boundary layer thickness (or pipe radius or channel half-height), $A_1$ is expected to asymptote to a universal value and $B_1$ is a large-scale characteristic constant. Perry et al. [115] presented scaling arguments developed for specific regions of the spectrum, where low-wavenumber motions were assumed to scale on outer-layer length scale ($\delta$), intermediate wave numbers were assumed to be related to attached eddies, therefore they scaled inversely with the distance from the wall ($z^{-1}$), and high wavenumbers were assumed to adopt Kolmogorov scaling. Overlap arguments then specified (for the spatial inertial sublayer) a region of $k_x^{-1}$ and $k_x^{-5/3}$ in the $u$-spectrum. By integrating the spectrum, scaling laws were derived. Perry and Li [110] then modified this analysis slightly and proposed streamwise turbulence intensity of the form

$$\frac{u'^2}{u_r^2} = B_1 - A_1 \log(z/\delta) - V(z^+), \quad (2.7)$$

for the spatial inertial sublayer (or logarithmic region). Here $V(z^+)$ accounts for the small-scale viscous dependent motions and is universal. According to this analysis $A_1$ is equal to a low-wavenumber universal plateau in the normalised premultiplied energy spectrum $k_x \phi_{uu}/u_r^2$. Marusic et al. [76] added a wake deviation function to account for the deviation in the outer part of the boundary layer due to wake effects. Marusic and Kunkel [75] then extended the previous works by considering a universal turbulence intensity profile in the near-wall region together with an outer flow influence felt all the way down to the viscous sublayer. It should be noted that in the early work of Perry et al. [115], from the experimental spectra in relatively low $Re_\tau$ ($Re_\tau < 4000$) in turbulent pipe flow $A_1$ was estimated to be 0.9. Perry and Li [110] estimated $A_1 = 1.03$ from experimental turbulent boundary layer data at slightly higher $Re$. Most recently very high Reynolds number ($Re_\tau \sim \mathcal{O}(10^5)$) experiments in turbulent boundary layer and pipe flow have shown that $A_1 = 1.26$ fits equation 2.6 to the data very well (Marusic et al. [79] and Örlü et al. [105]). It is not still clear if $A_1 = 1.26$ is the asymptotic value or it can increase further with $Re_\tau$ for higher Reynolds number than $Re_\tau \sim \mathcal{O}(10^5)$. Örlü et al. [104] used the diagnostic scaling concept of Alfredsson et al. [3] given by
\[ \frac{\sqrt{\overline{u'^2}}}{U} = F\left(\frac{U}{U_\infty}\right), \]  

(2.8)

to address this question. Here \( F \) is assumed to be a universal function. They performed Taylor-series expansion of equation 2.8 around any given value of \( U_0^+/U_\infty^+ \). Equating the result with equation 2.6, they concluded that \( A_1 \propto U_\infty^+ \), hence \( A_1 \) increases with \( Re \) unboundedly. This is in contrast with the attached-eddy hypothesis. It has been many years since Townsend proposed equation 2.6, however it is only recently that data have been available at very high Reynolds numbers in order to test the hypothesis properly.

Perry and coworkers (Perry and Abell [108], Perry et al. [115], Perry and Li [110] and Perry and Marusic [111]) reported an extensive series of experiments towards this goal, but it was clear that their data were not at a sufficiently high Reynolds number to establish unambiguously a logarithmic profile for \( \overline{u'^2} \) [79].

The scaling of the turbulence intensity profile in the outer region has received less attention. For this region, McKeon and Morrison [83] report that \( \overline{u'^2} \) profiles as functions of \( z/\delta \) collapse in pipe flow, but in different manners for high and low Reynolds numbers. They associate this behaviour to the relatively slow development of self-similarity of the spectrum. Fernholz and Finleyt [29] reported similar behaviour in ZPG turbulent boundary layer flow. Marusic et al. [80] report a very good collapse of \( \overline{u'^2} \) profiles in the outer scaling for the Reynolds number range \( 2800 \leq Re_\tau \leq 13400 \) in the turbulent boundary layer flow.

2.4 The energy cascade and Kolmogrov hypotheses

Consider very high Reynolds number turbulence with the characteristic length scale \( \mathcal{L} \), velocity scale \( \mathcal{U} \), and kinematic viscosity \( \nu \). Hence Reynolds number for this is \( Re_\mathcal{L} = \mathcal{U} \mathcal{L}/\nu \). This turbulence is composed of eddies of different sizes. Let the eddies in the largest range be characterised by the length scale \( L \), which is comparable to \( \mathcal{L} \); and their velocity scale \( U \) is on the order of the r.m.s turbulence intensity which is comparable to \( \mathcal{U} \). Therefore the Reynolds number associated with these eddies \( Re_L = U L/\nu \) is high
and the effect of viscosity is negligible. As a result, these eddies are unstable and break into eddies of smaller sizes, transferring their energy to the smaller eddies. The smaller eddies themselves are still unstable and break into eddies of smaller sizes transferring their energy to the next eddy level. This process of eddy breakup and energy transfer (‘energy cascade’) continues until the Reynolds number of the smallest eddies $Re_l = ul/\nu$ is sufficiently small that the eddies are stable. At this stage viscosity is dominant and the eddies’ energy is dissipated by molecular viscosity. This notion was first introduced by Richardson [123]. Since the rate of energy transfer at all scales in the cascade remains constant in this concept, the average rate of viscous energy dissipation per mass ($\epsilon$) is determined by the energy rate of the first process in the cascade, which is transfer of energy from the largest eddies. These eddies have energy per mass of order $U^2$ and timescale $L/U$, therefore energy transfer rate is $U^3/L$. Consequently this picture of the cascade indicates that $\epsilon$ scales as $U^3/L$ (Pope [119]).

A fundamental question that remains to be answered is that what are the scale characteristics of the smallest eddies. These questions can be answered by the theory developed by Kolmogorov [62]. Kolmogrov hypothesised that in an arbitrary turbulent flow with sufficiently high Reynolds number, not near the boundary of the flow or its other singularities, the smallscale turbulent motions are locally isotropic with good approximation (Kolmogrov’s hypothesis of local isotropy). Subsequently, he hypothesised that for the locally isotropic turbulence the statistics of the smallscale motions are uniquely determined by the quantities $\nu$ and $\epsilon$ (Kolmogrov’s first similarity hypothesis). This range of scales is known as the ‘universal equilibrium range’. Therefore, Kolmogrov lengthscale ($\eta$), velocity scale ($v_\eta$), and time scale ($\tau_\eta$) are defined as

\[
\eta = (\nu^3/\epsilon)^{1/4},
\]

\[
v_\eta = (\epsilon\nu)^{1/4},
\]

\[
\tau_\eta = (\nu/\epsilon)^{1/2}.
\] (2.9)

Moreover, Kolmogrov hypothesised that, there is a sub-domain inside the universal
equilibrium range referred to as the ‘inertial subrange’ where the turbulent statistics are uniquely determined by the quantity $\epsilon$ and do not depend on $\nu$ (Kolmogrov’s second similarity hypothesis). Consequently, it was shown that the second order longitudinal velocity structure function, $\langle (\Delta u)^2 \rangle = \langle (u(x + r) - u(x))^2 \rangle$, in the ‘inertial subrange’ is given by

$$\langle (\Delta u)^2 \rangle = C \epsilon^{2/3} r^{2/3}, \quad (2.10)$$

where $r$ is the distance of the points for which structure function is determined. Here the angle brackets $\langle \rangle$ denote ensemble averaging and $C$ is a universal constant.

One can show that the equivalent spectral form of equation (2.10) in the inertial subrange is given by

$$\phi_{uu} = K_0 \epsilon^{2/3} k_x^{-5/3}, \quad (2.11)$$

where $\phi_{uu}$ is the one-dimensional streamwise spectral density defined as $\overline{u'^2} = \int_0^\infty \phi_{uu}(k_x) dk_x$. Equation (2.11) is the famous Kolmogrov $-5/3$ spectrum, and $K_0$ is a universal constant for the longitudinal velocity component. Saddoughi and Veeravalli [125] tested the predictions of the Kolmogrov hypotheses in a turbulent boundary layer claimed to be the highest Reynolds number attained in a laboratory and found $K_0$ to be 0.49. Sreenivasan [133] compiled numerous experimental data including laboratory grid turbulence and shear flows and geophysical flows and concluded that $K_0$ is a universal constant equating 0.5.

### 2.5 Attached eddy hypothesis

Townsend [140] proposed a model for the structure of wall-bounded logarithmic region based on arrangements of large ‘eddies’. These eddies may be thought of as the velocity fields of some representative vortex structures (Nickels et al. [100]). Considering the
success of the log-law in describing the mean velocity profile, Townsend [140] states, ‘it is difficult to imagine how the presence of the wall could impose a dissipation length-scale proportional to distance from it unless the main eddies of the flow have diameters proportional to distance of their ‘centres’ from the wall, because their motion is directly influenced by its presence. In other words, the velocity fields of the main eddies, regarded as persistent, organized flow patterns, extend to the wall and, in a sense, they are attached to the wall’. In other words, any type of eddy whose size scales as its distance from the wall can be considered as an attached eddy and its velocity fields extend to the wall. This type of eddies form the basis of the ‘attached eddy hypothesis’. The attached eddy hypothesis is that the main energy-containing motion of a turbulent wall-bounded flow may be described by a random superposition of such eddies of different sizes, but with similar velocity distributions. In order for this model to lead to a constant Reynolds shear stress, population density of eddies must vary inversely with size and hence with distance from the wall. It should be noted that, the attached eddy hypothesis assumes the flow to be inviscid and therefore can only be used to describe the logarithmic region and above.

Although Townsend did not specify the shape of the eddies for the attached eddy model, Perry and Chong [109] suggested that a Λ-vortex, consistent with the flow visualisation of Head and Bandyopadhyay [37], is a suitable candidate for the model. Taking a random distribution of such eddies with a population density that varies inversely with size and hence with distance from the wall. It should be noted that, the attached eddy hypothesis assumes the flow to be inviscid and therefore can only be used to describe the logarithmic region and above.

Perry et al. [115] used Λ-vortices as shown in figure 2.1. In the figure three attached eddies of different scales are shown as well as a probe located at the distance $z$ from the wall. It can be seen that an eddy of the scale $\Delta_A = \mathcal{O}(\delta)$ contributes to the $u$ and $v$ fluctuations at the location of the probe and for $z \ll \delta$ these contributions are invariant with $z$. It is also clear that the eddy of the scale $\Delta_A = \mathcal{O}(\delta)$ contributes little to the $w$ fluctuation at the probe location for $z \ll \delta$. An eddy of scale $\Delta_B = \mathcal{O}(z)$ contributes to the $u$, $v$, and $w$ fluctuations strongly, therefore these motions scale on $z$. Eddies of
scale $\Delta_c \ll z$ do not contribute to any of the $u$, $v$ or $w$ fluctuations at $z$ because of the weak far-field effect above the the eddies as shown by Perry and Chong [109]. Therefore, only eddies of scale $\Delta = O(z)$ contribute to $w$-fluctuations and all the eddies of order $\Delta \geq O(z)$ contribute to $u$ and $v$ fluctuations at $z$.

Now consider the $u$-spectrum $\phi_{uu}$ in the wavenumber region that the motions are not dependent on viscosity. This covers most of the energy containing motions. In this region the involved variables are $u_\tau$, $k_x$, $z$ and $\delta$. Since contributions to the large-scale motions of $u$ at $z \ll \delta$ are dominated by eddies of scale $\Delta = O(\delta)$, an outer-flow law scaling is expected

$$ \frac{\phi_{uu}(k_x\delta)}{u_\tau^2} = g_1(k_x\delta) = \frac{\phi_{uu}(k_x)}{\delta u_\tau^2}. \quad (2.12) $$

Here $\phi_{uu}(k_x\delta)$ is the power-spectral density per unit non-dimensional wavenumber $k_x\delta$.

Eddies of scale $\Delta = O(z)$ contribute to the motions with moderate to high wavenumber, therefore a $z$-scaling for these wavenumbers is expected as

$$ \frac{\phi_{uu}(k_xz)}{u_\tau^2} = g_2(k_xz) = \frac{\phi_{uu}(k_x)}{zu_\tau^2}. \quad (2.13) $$

The very-high-wavenumber viscosity-dependent motions are expected to be locally isotropic, thus $u$-spectrum follows Kolmogrov scaling.
\[
\frac{\phi_{uu}(k_x \eta)}{v_{\eta}^2} = g_3(k_x \eta) = \frac{\phi_{uu}(k_x)}{\eta v_{\eta}^2},
\]
(2.14)

where \( \eta \) and \( v_{\eta} \) are Kolmogrov’s length and velocity scales given in equation 2.9. Perry et al. [115] assumed that in the logarithmic wall region turbulence energy production and dissipation are approximately in balance, which leads to the relation \( \epsilon = u_{\tau}^3/\kappa z \) for energy dissipation rate. This together with the equation 2.9 for \( \eta \) and \( v_{\eta} \) yields

\[
\eta = \left( \frac{\nu^3 \kappa z}{u_{\tau}^3} \right)^{1/4},
\]
(2.15)

\[
v_{\eta} = \left( \frac{\nu u_{\tau}^3}{\kappa z} \right)^{1/4}.
\]

They argued that two overlap regions are expected in the spectral regions of \( u \)-spectra as shown in figure 2.2 (a). In region of ‘overlap I’ equations 2.12 and 2.13 are simultaneously valid, leading to a power law region of the form

\[
\frac{\phi_{uu}(k_x \delta)}{u_{\tau}^2} = \frac{A_1}{k_x \delta} = g_1(k_x \delta),
\]
(2.16)

\[
\frac{\phi_{uu}(k_x z)}{u_{\tau}^2} = \frac{A_1}{k_x z} = g_2(k_x z),
\]

where \( A_1 \) is a universal constant. In the region of ‘overlap II’ equations 2.13 and 2.14 are simultaneously valid. Substituting \( \eta \) and \( v \) from 2.15 into equation 2.14 and comparing with equation 2.13 leads to

\[
\frac{\phi_{uu}(k_x z)}{u_{\tau}^2} = \frac{1}{\kappa^{2/3}} \frac{K_0}{(k_x z)^{5/3}} = g_2(k_x z),
\]
(2.17)

\[
\frac{\phi_{uu}(k_x \eta)}{v_{\eta}^2} = \frac{K_0}{(k_x \eta)^{5/3}} = g_3(k_x \eta).
\]

Sketches of the premultiplied power spectra as predicted by the attached eddy model for \( u \) and \( w \)-spectra are shown in figures 2.3 and 2.4. Integrating the \( u \)-spectra in different regions with the aid of equations 2.16 and 2.17 leads to
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(a) Motions that are independent of viscosity (i.e. Townsend’s Reynolds-number-similarity hypothesis)

\[ \frac{\phi_{uu}}{u^2} = g_1(k_x \delta) \]  
(Outer-flow scaling)

\[ \phi_{uu} = g_2(k_x \bar{z}) \]  
(z-scaling)

Viscosity-dependent motions

\[ \frac{\phi_{uu}}{u^2} = g_3(k_x \eta) \]  
(Kolmogrov scaling)

Overlap region I

Overlap region II

0 \[ k_x \delta = F \] \[ k_x \bar{z} = P \] \[ k_x \bar{z} = N \] \[ k_x \eta = M \] \[ k_x \]

(b) Motions that are independent of viscosity (i.e. Townsend’s Reynolds-number-similarity hypothesis)

\[ \frac{\phi_{ww}}{u^2} = h_1(k_x \delta) \]

(Outer-flow scaling)

\[ \phi_{ww} = h_2(k_x \bar{z}) \]  
(z-scaling)

Viscosity-dependent motions

\[ \frac{\phi_{ww}}{u^2} = h_3(k_x \eta) \]  
(Kolmogrov scaling)

Overlap region

0 \[ k_x \bar{z} = N \] \[ k_x \eta = M \] \[ k_x \]

Figure 2.2: Various spectral regions for velocity fluctuations in the logarithmic wall region. (a) \( u \)-spectra (spectral regions for \( v \)-spectra is similar) and (b) \( w \)-spectra. Adapted from Perry et al. [115].

\[ \frac{\bar{u}^2}{u^2} = B_1 - A_1 \log(z/\delta) - V(z^+) \].  
\[ (2.18) \]
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\[ k_x \phi_{uu} = \frac{M(z^+)^{3/4}}{\kappa^{1/4}} \]

\[ k_x \delta = A_1 \]

Figure 2.3: Sketches of the premultiplied \( u \)-spectra as predicted by the attached eddy hypothesis for different \( z \) locations within the logarithmic region (\( v \)-spectra are similar) (a) with \( z \)-scaling and (b) with outer-scaling. Adapted from Perry et al. [115].

Here \( A_1 \) is the universal constant introduced in equation 2.16 and shown as a plateau in figure 2.3, \( V(z^+) \) is a universal function describing the dissipative motions, and \( B_1 \) depends on the large-scale flow geometry. Similarly for the \( v \)-fluctuations,

\[ \frac{u^2}{u_\tau^2} = B_2 - A_2 \log(z/\delta) - V(z^+), \]  \tag{2.19}
where $A_2$ is a universal constant, while $B_2$ depends on the large-scale flow geometry. For the $w$-fluctuations,

$$\frac{\overline{w^2}}{u_T^2} = B_3 - V(z^+),$$

(2.20)

where $B_3$ depends on the large-scale flow geometry.

### 2.6 The $k_x^{-1}$ power scaling law

It was shown in section 2.5 that a dimensional analysis directed by the attached eddy hypothesis in a turbulent flow in the presence of a solid boundary, results in a $k_x^{-1}$ power law scaling for the $u$ and $v$-spectra occurring at low wavenumbers (Perry and Abell [108], Perry et al. [115] and Perry and Li [110]). Tchen [139] was the first to theoretically predict the $k_x^{-1}$ scaling in a shear flow turbulence via a spectral budget equation with the help of turbulent viscosity model of Heisenberg [38]. Kader and Yaglom [55] used dimensional analysis to arrive at the inverse power law for the three fluctuating components in turbulence. Nikora [101] proposed a simple phenomenological model explaining the $k_x^{-1}$ law in wall turbulence as a result of superposition of eddy cascades generated at all possible $z$. Consequently, the energy flux, $\epsilon_f$, follows the form $\epsilon_f(k) \sim u_3^3k_x$ for $1/\delta \leq k_x \leq 1/z$ and $\epsilon_f(k) \sim \epsilon$ for $k_x \geq 1/z$. Combining these relations with the relation of spectra in the inertial subrange, $\phi_{uu} \sim \epsilon^{2/3}k_x^{-5/3}$, yields $\phi_{uu} \sim u_3^2k_x^{-1}$ for $1/\delta \leq k_x \leq 1/z$ and $\phi_{uu} \sim \epsilon^{2/3}k_x^{-5/3}$ for $k_x \geq 1/z$. It should be noted that this model predicts a $k_x^{-1}$ power scaling law for all the velocity components in wall-bounded turbulent flows as opposed to the attached eddy model that doesn’t predict this scaling law for the $w$ fluctuating component. Katul et al. [57] presented a phenomenological spectral theory based on Heisenberg’s [38] eddy-viscosity approach that recovers Nikora’s [101] scaling arguments for infinite Reynolds number. They also explained the effect of intermittency in modifying the $k_x^{-1}$ power law and the expected role of coherent structures and very-large-scale motions.

Although existence of the $k_x^{-1}$ power law for the energy spectra has long been predicted, no strong experimental support has been found for it in the sense of Perry et al. [115]
(universal $A_1$ constant in $\phi_{uu}^+ = A_1 k_x^{-1}$) for Reynolds number up to $Re_\tau \sim 10^5$ in the laboratory (Vallikivi et al. [146] and Rosenberg et al. [124]). Nickels et al. [99] reported evidence for the $k_x^{-1}$ power law in boundary layer turbulence in a controlled laboratory measurement, however, the $A_1$ constant they found does not match with the slope of the streamwise turbulence intensity. The experimental evidence for the $k_x^{-1}$ behaviour of the $u$-fluctuations are primarily from the atmospheric surface layer measurements (see Katul and Chu [56] for a comprehensive review), where a linear relation between $\log \phi_{uu}$ and $\log k_x$ with a slope approximately equal to unity is accepted as the $k_x^{-1}$ behaviour, while a closer scrutiny of the spectra in the premultiplied form reveals large scatter about $\phi_{uu}^+ = A_1 k_x^{-1}$ (see e.g. Katul et al. [57] and Kunkel and Marusic [66]). Davidson et al. [24] have shown that the $k_x^{-1}$ is equivalent of a logarithmic law in streamwise velocity structure function in wall-layer turbulence. Arguing that the one dimensional spectra can be contaminated by aliasing, resulting in masking any probable $k_x^{-1}$ power law, they suggest that looking for a logarithmic law in the velocity structure function is a more effective approach. To avoid contamination of one-dimensional energy spectra due to aliasing, Chandran et al. [16] utilised two-dimensional energy spectra to investigate self-similarity in high Reynolds number turbulent boundary layer flows. Plateau values in the $u$-spectra versus streamwise and spanwise wavelengths were compared at various Reynolds numbers and wall distances, and it was concluded that one should not expect to see a clear $k^{-1}$ scaling in both one-dimensional streamwise and spanwise spectra for Reynolds numbers below $Re_\tau \approx 60000$.

### 2.7 Effect of spatial resolution in turbulent measurements

Turbulence is composed of fluctuating motions with varying range of scales. Using sensors with sizes greater than the smallest scales of motion present in the flow, results in spatial filtering. Filtering effect due to finite length of sensors was recognised early on, by Dryden et al. [27], Frenkiel [31], Frenkiel [32], Uberoi and Kovasznay [143] and Wyngaard [157]. These early works usually involved assumption of isotropy, which is
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invalid for wall-bounded turbulent flows in the near-wall region. This filtering effect in
the wall-bounded flow has been experimentally investigated by Johansson and Alfredsson [53],
Willmarth and Sharma [154], Ligrani and Bradshaw [72], Hutchins et al. [47],
and Ng et al. [97]. Ligrani and Bradshaw [72] used hot-wire sensors with lengths ranging
1-60 viscous length scales in a relatively low Reynolds number boundary layer flow to
investigate the effect of spatial resolution in turbulent wall-bounded flows. They used
sub-miniature hot-wires to achieve sensors with lengths smaller than Kolmogrov length
scale and showed that at the near-wall peak in the streamwise turbulence intensity, hot-
wires with sensing lengths of less than 20-25 wall units are required to avoid substantial
spatial filtering. Compiling a large number of data sets, Hutchins et al. [47] concluded
that recorded scatter in the measured near-wall peak in the inner normalised streamwise
turbulence intensity is due in large part to the simultaneous competing effects of the
Reynolds number and viscous-scaled wire length $l^+$, and presented an empirical expres-
sion to account for these effects. They showed that the spatial filtering due to the sensor
finite length can extend a considerable distance beyond the immediate near-wall region,
leading to appearance of a false second outer peak in the turbulence intensity profile.

The above-mentioned studies clearly show the importance of sufficient spatial resolution
(also known as point measurement) in order to avoid issues related to this effect in high
Reynolds number turbulent flows. One way to achieve point measurement is reducing
the sensor size. Development of Princeton Nano-Scale Thermal Anemometer (Bailey
et al. [6], Fan et al. [28]) has been towards this goal. Another alternative for point
measurement is to increase the length scales of the turbulence, while maintaining high
$Re$; this requires very large wind tunnels, such as the high Reynolds number boundary
layer wind tunnel at the University of Melbourne (Nickels et al. [99]) and the long pipe in
CICLoPE (Talamelli et al. [135]). Even with such small hot-wires and/or large facilities,
the problem of spatial resolution of sensors is unavoidable (see e.g. Hutchins et al. [47]).

Several correction schemes for spatial filtering of single component hot-wire measure-
ments have been proposed recently (Chin et al. [20], Monkewitz et al. [90], Philip et al.
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[117], Segalini et al. [127], Smits et al. [131]), and Miller et al. [87] compared these methods. Abundance of this type of study in literature highlights the importance of accurate near-wall data in high Reynolds number wall-bounded turbulent flows which is scarce.

2.8 Sub-miniature hot-wire anemometry

Studying high-Reynolds number boundary layer flows has improved extensively over the past few decades thanks largely to the construction and development of high Reynolds number experimental facilities (e.g. Morrison et al. [96], Österlund [106], Nickels et al. [99], Carlier and Stanislas [15], Vincenti et al. [149], Talamelli et al. [135]). In conjunction with this, advances in sub-miniature hot-wire anemometry have also been required in order to capture the smallest scales of motions in these flows. As mentioned in section 2.7 using sensors with sizes greater than the smallest scales of motion results in spatial filtering. Temporal resolution is another limit in using hot-wires for measuring velocity fluctuation of high-Reynolds number flows. Hutchins et al. [47] showed that the maximum frequency content of a wall-bounded turbulent flow spectrum is equal to or greater than $u^2/3\nu$, meaning that if the experimental measuring system cannot resolve time scales down to $t = 3\nu/u^2$, there will be an excessive temporal filtering of the measured energy, in addition to spatial attenuation. Therefore, there is a direct relationship between Reynolds number and highest frequency of a flow. Typically, hot-wire anemometers have been reported to have frequency responses in the range $30 < f < 100$ kHz which might be insufficient to fully resolve high-Reynolds number flows temporally [47].

A number of notable efforts have been made towards realising adequately small hot-wires, (see e.g. Jiang et al. [51], Marshall et al. [74], Borisenkov et al. [13] and Bailey et al. [6]) one of the most important of them being the development of the Nano-Scale Thermal Anemometry Probe, NSTAP [6], which uses MEMS-based technology for the probe construction and results in sensor lengths of order 30-60 µm. However operating NSTAP and other sub-miniature probes is not straightforward. Currently NSTAP can be operated with the Dantec Streamline anemometer but only using external resistors
That is, this anemometer is not designed for sensors with the high resistance as found with NSTAPs. Attempts to operate NSTAP with other anemometers result in frequent sensor breakages. Moreover, the best possible square-wave test response of NSTAP with the Dantec Streamline anemometer does not resemble a typical optimal square-wave test response of conventional wires as introduced by Freymuth [34] (See ref. [145] for NSTAP square-wave responses).

There are numerous analytical studies in the literature devoted to hot-wire anemometry, e.g. Freymuth [33], Freymuth [34], Perry and Morrison [113], Wood [155], Perry [116], Comte-Bellot [23], Smits et al. [130], Watmuff [151], Morris and Foss [95], Li [68], Li [69], Li [70]. Freymuth [33, 34] derived CTA governing equations in the time domain for a CTA with a single stage amplifier in the feedback circuit. After linearisation Freymuth showed that a well tuned CTA is governed by a third order ODE assuming that the time constant of the hot-wire is much larger than that of the amplifiers and also depicted the square-wave response for a well tuned CTA system. Perry [116] and Perry and Morrison [113] studied the CTA system theoretically and derived the governing equations in the frequency domain proposing a third order transfer function for the linearised case assuming again an idealised single-stage amplifier in the feedback circuit with infinitely flat broadband response. Wood [155] followed Perry and Morrison’s [113] approach but used a single stage amplifier with a frequency dependent transfer function and a lumped inductance in the Wheatstone bridge. This system was controlled by changing the amplifier gain and bridge compensation inductance rather than the amplifier offset voltage used by Perry and Morrison [113]. Wood [155] compared this theoretical model with a DISA 55D01 square-wave response and observed that the model could not accurately predict the anemometer’s square-wave response. Watmuff [151] expanded Perry’s model and analysed a general CTA in the frequency domain with multiple amplifier stages with frequency dependent transfer functions and also considered the general impedance for bridge components. The analysis of Watmuff shows that a minimum of two equivalent amplifier stages are required to properly account for the introduction of offset voltage perturbations. This is a key point, because the poles of the amplifier which precede the
offset voltage injection stage appear as zeros in the overall system transfer function for offset voltage fluctuations. Watmuff’s study was limited to analysis, and did not involve validation against real CTAs.

### 2.9 End-conduction effects in hot-wire anemometry of high Reynolds number turbulent flows

In high Reynolds number measurements using hot-wires where usually sensors with lengths shorter than 0.2 mm are required to avoid severe spatial filtering, wires with diameters less than 1 µm are needed to avoid errors associated with end conduction. These wires are fragile and subject to severe drift that significantly reduces the accuracy of the measurements. Hence, experimentalists are tempted to relax the commonly accepted length-to-diameter ratio criterion to achieve better spatial resolution in wall-bounded high Reynolds number flow measurements, which in turn might cause unwanted errors associated with the end conduction effects.

The problem of end conduction losses in constant temperature hot-wire anemometry has been investigated theoretically in a number of studies with different degrees of simplifications due to the complexity of the problem when conduction losses in the wire filament are taken into account. Smits [129] investigated the end conduction effect in the hot-wire-CTAs using a perturbation analysis. He assumed that the temperature of the junction of the unetched portion of the wire (stub) and the prong was fixed at the ambient temperature (therefore the effect of large time response of the prongs on the wire filament sensitivity to the velocity fluctuations was ignored) and derived the hot-wire filament voltage sensitivity to the velocity and current fluctuations and also the transfer function of the hot-wire-CTA system. Subsequently the modelled hot-wire filament was incorporated in a simple CTA model. The analysis revealed the presence of low frequency steps in the system transfer function, sizes and signs of which were not discussed further in detail due to the presence of several unknown parameters in the transfer function. He concluded that the dynamic sensitivity could be either more or
less than the static sensitivity, i.e. the low frequency steps could have either positive or negative signs based on the magnitude of the involved parameters. Perry et al. [114] modelled the problem of the effect of heat conduction on the transfer function of the hot-wire-CTA system with the focus on the effect of asymmetry in the wire filament temperature distribution and heat convection. Similar to Smits [129] they neglected the time response of the prongs. Their analysis showed low frequency, positive steps in the system transfer function that could be as large as $10^{-20}$. Li [68] studied the dynamic response of the hot-wire-CTA systems taking into account the conductive heat transfer to the wire supports using a perturbation analysis. In this analysis the temperature of the wire ends were fixed at ambient temperature and the effects of the stubs and the prongs were neglected. Dynamic sensitivity of the wire to the velocity fluctuations was related to the ratio of the fluctuations of the electric current passing through the hot-wire to the velocity fluctuations, and low frequency behaviours related to the end conduction losses to the supports were examined for the hot-wire without considering the CTA circuit. He observed attenuation in this dynamic sensitivity, the magnitude of which decreases with length-to-diameter ratio, Reynolds number, and overheat ratio. Morris and Foss [95] analysed the response of the hot-wire anemometer systems numerically for a particular hot-wire and Reynolds number. They neglected the effect of the prongs and concluded the presence of a relatively low frequency negative step in the system transfer function related to the velocity fluctuations. Li et al. [71] employed an indirect method to investigate the problem of the end conduction effect in the hot-wire-CTA systems at extremely high Reynolds numbers. Instead of performing a dynamic analysis, they simulated the steady-state heat balance for the wire and the stub numerically. They calculated fractional end-conduction losses, $\sigma$, which is the ratio of the conductive heat losses to the total heat losses in the hot-wire filament for different cases of stub-to-wire length and wire Reynolds numbers. They concluded that at very high wire Reynolds numbers, the wire length-to-diameter ratio can be reduced significantly, while keeping $\sigma < 0.07$ equivalent to accepted limits for wire Reynolds numbers common in laboratory turbulent flows performed in atmospheric pressure conditions. This analysis cannot determine whether the dynamic sensitivity of the hot-wire-CTA system is less or more
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than the static sensitivity. Hultmark et al. [43] proposed a more general parameter \( \Gamma \approx 1/\sigma \) (rather than \( l_w/d_w \)) to describe the significance of the end conduction effect taking into account the overheat ratio, Nusselt number, and wire material as well as the conventional parameter \( l_w/d_w \). Using numerical and experimental data they showed that \( \Gamma > 14 \) indeed satisfies \( \sigma < 0.07 \) for different cases. In this model it is presumed that end conduction always results in a negative step in the system transfer function.

Due to the engagement of a multitude of parameters and also experimental constraints, experimental studies addressing the effect of end conduction in the hot-wire anemometers are scarce. Ligrani and Bradshaw [73] used wires with different diameters to measure velocity fluctuations in a relatively low Reynolds number turbulent boundary layer flow. Their study was limited to the near wall peak of the streamwise turbulence intensity, and showed attenuation in the near wall peak for all the wires with \( l_w/d_w < 200 \). This attenuation was attributed to the end conduction effect. Unfortunately no comparison of the energy spectra was made for the wires with insufficient \( l_w/d_w \). Hutchins et al. [47] used three hot-wire probes with matched lengths and different length-to-diameters ratios to measure velocity fluctuations in a turbulent boundary layer flow at \( Re_\tau = 14000 \). Comparison of the broadband turbulence intensity profiles for these wires showed attenuation in the turbulence intensity for the wires with length-to-diameters ratios smaller than 200. These attenuations extend far beyond the near-wall region, affecting the recorded broadband intensities throughout the boundary layer. Again no comparison was made of the energy spectra that can offer useful information including frequency response of the wires and their effect on the high frequency end of the spectra especially for wires with larger diameters that are significantly prone to insufficient frequency response due to their larger mass. This insufficient frequency response for larger wires cannot be distinguished from the end conduction effect by comparing the broadband turbulence intensity profiles alone.
Chapter 3

Experimental setup

3.1 Flow facility

3.1.1 Boundary layer wind tunnel facility

Most of the measurements in this study were conducted in the High Reynolds Number boundary Layer Wind Tunnel (HRNBLWT) located in the Walter Basset Aerodynamics Laboratory at the University of Melbourne. A schematic diagram of the facility demonstrating its main features is shown in figure 3.1. This is an open return wind tunnel with a working section of $27 \times 2 \times 1$ m. The long working section provides the opportunity to achieve high Reynolds numbers with moderate velocities, which ensures the acquisition of data at reasonably good spatial and temporal resolutions with conventional measurement instruments (high Reynolds numbers are achieved through relatively large boundary layer thickness rather than high velocity). The inlet of the tunnel is fitted to a bell mouth in order to acquire a smooth transition, which reduces the likelihood of separation. The flow then travels through a heat exchanger that has been designed to maintain the temperature of the flow to within ±0.1°C using a feedback loop. The heat exchanger leads to a transition chamber that converts the cross section from a rectangle at the heat exchanger to a circle at the fan inlet. The fan is driven by a 200 kW DC motor controlled by a dedicated computer which can provide a maximum speed of 45 ms$^{-1}$.
at its full capacity. The speed of the DC motor can be controlled both manually and externally by a computer through a controller. This allows automation of the calibration procedure for the hot-wires, which requires incremental change of the motor speed. The fan is followed by a $180^\circ$ turn in two stages using turning vanes. These redirect the flow while minimising the pressure drop. This turn is followed by a settling chamber consisting of a honeycomb and six fine meshes to straighten the flow as well as reducing its non-uniformities. The settling chamber leads to a contraction with an area reduction ratio of 6.2:1. The contraction reduces the free stream turbulence intensity $\sqrt{u^2}/U_\infty$ and keeps it at less than 0.05% at the start of the working section and in the range of $0.15 - 0.2\%$ at $x = 18$ m from the start of the working section for the free stream flow range of 10-40 ms$^{-1}$. Here $u^2$ is the turbulence intensity and $U_\infty$ is the free stream velocity.

The flow is tripped at the entrance to the working section by a 35 mm wide P40 grit sand paper (with a grit size of 425-500 µm) to produce a canonical boundary layer. For further discussion on the behaviour of the flow in this facility with the present tripping refer to Marusic et al. [80]. Measurements are made on the floor at different stations in the working section. Four 6 m and one 3 m long polished aluminium plates with a root-mean-square surface roughness of no greater than 1.5 µm are flush mounted side by side to create the test section length of 27 m. Epoxy filler is used between the plates to ensure that there is no gap. A series of rectangular medium density fibre board (MDF) panels measuring $0.9 \times 1.2$ m are used to form the ceiling of the tunnel. The pressure gradient in the tunnel is controlled by air bleeding through spanwise slots at the ceiling that are produced by setting the gaps between the ceiling panels. At the centre line of the working section of the tunnel there is a streamwise gap of 35 mm in the ceiling to permit the movement of a computer-controlled traverse that holds and moves the measuring probe. The streamwise gap is sealed using brush seals to preserve the required pressure gradient along the working section of the tunnel.
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Figure 3.1: A schematic of the HRNBLWT flow facility. Adapted from Kulandaivelu [65].
The facility is normally used at zero pressure gradient. The pressure coefficient $C_p$ is constant to within ±0.87% (Marusic et al. [80]). Therefore, the flow can be considered zero-pressure-gradient for all the measurements in this study.

The measurements of the wind tunnel facility are performed at $U_\infty = 20, 30,$ and $40 \text{ ms}^{-1}$ in this study. The temperature of the working section increases due to the heat generated by the motor. The heat exchanger has been designed to keep the temperature inside the tunnel near constant by removing the excessive heat generated by the motor; however, in practice it adds more complications since during the calibration process the motor speed is changed multiple times in a relatively short interval. The response time of the heat exchanger is not fast enough to keep up with the temperature changes due to the changes in the motor speed. As a result, higher temperature variations have been observed when the heat exchanger has been employed (Baidya [5]).

To avoid these complications the heat exchanger was not employed for any of the measurements in the present study. Instead the tunnel was prewarmed so that the tunnel temperature stabilised before the measurements were taken. Regarding the temperature variations during the calibration, for the measurements conducted at the nominal velocity of $20 \text{ ms}^{-1}$ a maximum temperature variation of $0.5^\circ\text{C}$ was observed during calibration. According to Perry [116] if the temperature is within ±0.5$^\circ\text{C}$ during the calibration and measurement procedure, the temperature corrections are not necessary.

The effect of the temperature variations during the measurement and calibration on the results is inspected in more detail in appendix A.

### 3.1.1.1 Traverse system and wall normal positioning in the boundary layer flow facility

A schematic of the traverse system used in the HRNBLWT is shown in figure 3.2. This is the same traverse system used by Kulandaivelu [65] and allows automatic movement in streamwise and wall normal directions. The body of the traverse is similar to a NACA 0061 aerofoil with a chord length of 180 mm which ensures minimal blockage and vibrations due to vortex shedding.
Chapter 3. Experimental setup

3.3 Retractable pneumatic foot

Hot-wire arm

0.5 m

Roof

Pitot-static tube

0.55 m

θ

Pneumatic cylinder

Parallel rails

Ball screw

Gold strip

Encoder read head

Retractable pneumatic foot

Figure 3.2: A schematic of the hot-wire traverse system installed in the HRNBLWT. Adapted from Kulandaivelu [65].
A retractable pneumatic foot allows the traverse to be secured to the floor during measurements to further reduce its vibrations. The wall-normal traversing uses a servo-motor system with a RENISHAW RGH24-type linear optical encoder feedback (with a positional accuracy of \( \pm 0.5\mu m \)) and is fully computer controlled. A ball-screw driven plate allows the traverse to move in the wall normal direction with a positional accuracy of \( \pm 5\mu m \). Kulandaivelu [65] showed that the measurements made 0.55m upstream of the traverse body are not contaminated by the blockage effect. Hence a relatively long arm is used to hold the hot-wire during the measurement and can be adjusted to make two angles of 10° or 3° with the tunnel floor to investigate the effect of the angle on the thermal anemometry probes employed in this study. Hot-wire measurements up to the wall normal height of 0.5m are made in this study which ensures resolving the entire boundary layer since the boundary layer thickness in this facility is below 0.34m everywhere for free stream velocities of 20 ms\(^{-1}\) and more.

The sensor is positioned with respect to the wall before commencing a measurement using a depth measuring displacement microscope from Titan Tool Supply which is fitted to an Amoscope MU300 CCD camera through a video adapter to display the microscope images on a computer screen. The microscope has a depth of focus of 30 \( \mu m \) and is equipped with a digital indicator with a resolution of 1 \( \mu m \) to accurately show the displacement of the microscope when its focus location is changed. To position the sensor, first it is located approximately 1 mm above the wall. Then the microscope is focused on the wall surface beneath the sensor and is set to zero. Then we focus the microscope on the sensor. The digital indicator shows the accurate location of the sensor with respect to the wall. After this stage, the optical linear encoder is used to locate the sensor accurately during positioning and throughout the measurement. After the measurement was completed, comparing the inner normalised mean velocity profiles against a DNS profile at a lower Reynolds number around the meso-layer revealed that the wall normal position for the sensor needed to be shifted by a maximum of 50 \( \mu m \) towards the wall for a free stream velocity of 20 ms\(^{-1}\), and more for higher velocities, to obtain good agreement between the DNS and the experiment. Since this wall normal
location adjustment is always negative (towards the wall) and increases with the velocity of the flow, we assume that it is due to the aerodynamic loading on the hot-wire arm causing it to move towards the wall when the flow is on. It is noted that the sensor positioning is performed without the flow. All the profiles shown in this study have been adjusted in this manner.

3.1.1.2 Pitot-tube in the boundary layer flow facility

A 0.55 m long custom-made Pitot-static tube is mounted to the traverse body 0.5 m above the wall to measure the freestream velocity in the HRNBLWT. The pressure difference across the Pitot-static tube is measured with a 10 Torr 698A MKS Baratron differential pressure transducer with the aid of an MKS type 270 signal conditioner. The sensitivity of the pressure transducer can be computer controlled to increase it by factors of 10 and 100 for velocities below 13 and 4 ms\(^{-1}\) respectively during the calibration. Moreover, prior to the start of each measurement, the pressure transducer is shorted and the zero is adjusted, if required, to ensure that there is no zero shift on its output signal.

3.1.1.3 Ambient conditions in the boundary layer flow facility

Ambient pressure \(P_a\) and temperature \(T_a\) are measured using a Sensor Technique 144S-BARO digital barometer and an Omega OL series linear thermistor connected to a DP25 controller, respectively. The ambient pressure and temperature are used to calculate the air density using the perfect gas law. The dynamic viscosity of the air \(\mu\) is calculated using the Sutherland’s [134] correlation.

3.1.2 Channel flow facility

The channel flow facility used in this study is located in the Walter Basset Aerodynamics Laboratory at the University of Melbourne. A schematic diagram of this facility is shown
in figure 3.3. This is a blower type tunnel which uses a DC motor and a fan that are able to produce an air mass flow rate of 3.6 m$^3$s$^{-1}$ equivalent to a bulk velocity of approximately 30 ms$^{-1}$ inside the channel when running at full capacity. The fan is followed by a honeycomb screen before travelling through a diffuser, which is filled by a series of mesh screens to straighten the flow. The flow then enters a contraction with an area ratio of 9:1 to minimize the streamwise turbulence intensity. The contraction is followed by the working section which measures 22 m in length and has a cross section area of 1170 $\times$ 100 mm. The floor and the roof of the working section are constructed from MDF plates and supported by side walls made with aluminium C sections. Because of the large aspect ratio of the channel (11.7) the roof deflects at different velocities. In order to alleviate this problem braces made from MDF have been attached to the exterior side of the roof at 1 m intervals in the streamwise direction to improve the lateral stiffness. To further reinforce the roof of the channel, adjustable bracing was installed at 35 streamwise locations. The bracing can be adjusted to ensure the flatness of the channel roof for different Reynolds numbers.

The flow is tripped at the entrance to the working section with 80 grit sandpaper which surrounds the entire perimeter and is 100 mm wide in the flow direction. The channel flow measurements were conducted at $x = 205H$ downstream from the trip at the spanwise middle of the cross section. Here $H$ is the channel height. Monty [94] showed that the aspect ratio of 11.7 is enough to ensure the two dimensionality of the flow by demonstrating that at the channel exit, the ratio of the mean velocity to the centre line velocity $U/U_{cl}$ varied less than $\pm 0.5\%$ within $y \approx \pm 1.25H$ of the channel spanwise centre line.

### 3.1.2.1 Traverse system and wall normal positioning in the channel flow facility

The traverse system in the channel flow facility which is shown in figure 3.3-b consists of a stepper motor which rotates a threaded rod. A carriage is attached to the rod through a tapped hole. This allows the carriage to move in the wall normal direction carrying a
Figure 3.3: a) A schematic of the channel flow facility. b) A schematic of the external rig and traverse system in the channel facility. Adapted from Ng [98].
vertical sting, which penetrates the channel roof through a Perspex plug. Measurements are made on the floor of the channel. The hot-wire and the Pitot tube were held horizontally by an adapter which was attached to the vertical sting. The displacement resolution of the traverse is 3.125 µm and a RENISHAW linear optical encoder is employed to account for the errors associated with the backlash in the traverse system. The traverse was computer controlled with the aid of a custom-made controller.

The hot-wire probe and Pitot tube were aligned (to make sure the hot-wire was parallel to the channel floor) inside a mock-up channel, which is an external rig with a cross section resembling the working section of the channel with access to the front and rear of the hot-wire. The Pitot tube and the hot-wire were positioned at the same wall normal location and 1 cm apart in the spanwise direction. The hot-wire was positioned 0.5 mm above the wall in the mock-up channel and then traversed up to the centre of the channel. Subsequently, the traverse system was transferred to the channel facility and calibration was made at the centre, which has the least fluctuations. Then the hot-wire was traversed towards the wall where the location of the wire was known to be 0.5 mm above the wall. To acquire the location of the hot-wire more accurately, knowing that the near-wall peak in the turbulence intensity profile is fixed at 15 wall units, the variance of the voltage signals (sampled for short intervals of 10 s) was calculated and plotted while moving the wire in small steps towards the wall. Once a peak was observed in the voltage variance versus wall normal location plot followed by a steep change that persisted with further small steps towards the wall (10 µm), the inner normalised location of the peak was considered as \( z^+ = z u_\tau / \nu = 15 \) and was converted to physical units knowing \( u_\tau \) and \( \nu \). Here \( \nu \) is the kinematic viscosity of air and \( u_\tau \) is the friction velocity defined as \( u_\tau = \sqrt{\tau_w / \rho} \) where \( \tau_w \) is the wall shear stress and \( \rho \) is the air density. For the channel flow measurements \( u_\tau \) was calculated using the empirical correlation given by Monty [94] from data previously acquired in this facility. In this method it is assumed that peak of the voltage signal and velocity signal are located at the same wall normal location.
3.2 Anemometry probes

A Nano-Scale Thermal Anemometry Probe (NSTAP) manufactured at Princeton University along with three standard hot-wire probes were used throughout the measurements. These probes and their manufacturing processes are explained in the following sections.

3.2.1 Nanoscale thermal anemometry probe (NSTAP)

High Reynolds number wall-bounded turbulent flows have been an active area of research over the past two decades due to their applications in industry and ubiquity in nature \[78\]. Hot-wire anemometry is the preferred method for measuring the time resolved fluctuating signals in turbulent flows. However, conventional hot-wires are not suitable for high Reynolds number measurements in most of the existing facilities due to the issues related to spatial and temporal resolutions \[126\]. In an effort to significantly reduce the size of hot-wire sensors for high Reynolds number turbulent flows, the Nano-Scale Thermal Anemometry Probe (NSTAP) was manufactured at Princeton University \[145\]. The first successful version of NSTAP was introduced by Bailey et al. \[6\]. This sensor, which had a sensing element of platinum filament measuring 60 $\mu$m $\times$ 2 $\mu$m $\times$100 nm was machined using a pulsed UV laser with low repeatability and a fairly long manufacturing time \[28\]. Vallikivi \[144\] substantially improved the probe by using a deep reactive ion etching lag-based process for fabricating a 3D silicon support structure for the platinum sensing part of the probe. This method made possible high-yield, low-cost fabrication of durable sensors which were less bulky than those of Bailey et al. \[6\]. Freestanding platinum filaments of the NSTAPs made by Vallikivi \[144\] come in two sizes: 30 or 60 $\mu$m $\times$1 $\mu$m $\times$100 – 120 nm. In the present study the 60 $\mu$m long NSTAP is used. Design diagrams and SEM images of the above-mentioned versions of NSTAP are shown in figure 3.4. The NSTAP improves the spatial resolution of the conventional hot-wires by an order of magnitude. Moreover a square-wave test shows
an improvement of approximately one order of magnitude in the frequency response compared to a 2.5 μm-diameter hot-wire as shown in figure 3.5.

Because of its manufacturing process the NSTAP has a rectangular cross section, contrary to conventional hot-wires that have circular cross sections. Therefore, the criteria for the conventional wires to minimise the end conduction effect ($l_w/d_w > 200$) cannot
be used for the NSTAP. Hultmark et al. [43] proposed a new criterion for the end conduction effect. Vallikivi [144] refined this method to be used for sensors with arbitrary cross sections and showed that NSTAP could not fulfil the criteria as conventional hot-wires do. Comparing results from NSTAPs of two different lengths at the same measurement conditions Vallikivi [144]’s study showed that there were no errors that could be associated with the end conduction effect. Therefore, it was concluded that the threshold required for NSTAP to fulfil the criteria has to be less than that for conventional hot-wires.

### 3.2.2 Conventional hot-wires

Two standard hot-wire probe types were used throughout the experiments. In the majority of the measurements, a Dantec 55P15 boundary layer type probe was used, with prong spacing of 1.5 mm. Wollaston wires were soldered to the prong tips and etched to reveal a 2.5 μm-diameter, 0.5 mm long platinum-sensing element in the middle. The un-etched portions (stubs) are 0.5 mm in length and have an estimated diameter of 50 μm. The other conventional hot-wire probe was a Dantec 55P05 boundary layer type
with prong spacing of 3 mm. Wollastan wire with platinum core of 5 µm-diameter was used to produce it. The production process is identical to that of the 2.5 µm-diameter wire. The etched sensing length and the stubs are 1 mm for the 5 µm-diameter.

### 3.3 Constant temperature anemometers

Anemometry probes are operated with two constant temperature anemometers throughout the measurements. An in-house Melbourne University Constant Temperature Anemometer (MUCTA) is used to operate the conventional hot-wires. For operation of the NSTAP a modified version of MUCTA is used as well as a Dantec Streamline. The required modifications for MUCTA to operate NSTAP are explained in detail in chapter 4. Both MUCTA and Dantec Streamline use an internal square-wave circuit to test the frequency response of the combination of the probe and the constant temperature anemometer. Ideally a direct method should be used to measure the frequency response of the system in which the probe is exposed to a fast, known velocity perturbation. This requires special facilities, e.g., the one used by Khoo et al. [58]. Moreover, such facilities typically are not capable of producing very high frequency velocity perturbations ($O(100)$ kHz) as required for high Reynolds number turbulent flow measurements. Alternatively one can employ a square-wave test in which one point in the CTA circuit is perturbed with a square-wave perturbation and the output voltage of the anemometer circuit is analysed to measure the frequency response of the hot-wire-CTA system as shown by Perry [116]. Hutchins et al. [49] showed that the magnitude of the Fourier transform of the square-wave response of a hot-wire-CTA system (as opposed to the Fourier transform of the impulse response of the system) shows the true frequency response of that system to the velocity fluctuations. We employ Perry’s [116] method to estimate the hot-wire-CTA system frequency response.
3.4 Data acquisition

Hot-wire signals were sampled using an analogue to digital converter (DT9836 from Data Translation) with a resolution of 16 bit in the range of ±10 V. The sampling frequency was set to $f_s = 50$ kHz for the free stream velocity $U_\infty = 20$ ms$^{-1}$ and $f_s = 80$ kHz for $U_\infty = 30$ and 40 ms$^{-1}$. In order to avoid aliasing in the sampled signals, they were low pass filtered at $f_c = f_s/2$. This leads to a viscous-scaled filtering frequency of $f_c^+ = f_c \nu/u_\tau^2 \approx 0.9$ for $U_\infty = 20$ ms$^{-1}$, $f_c^+ \approx 0.7$ for $U_\infty = 30$ ms$^{-1}$ and $f_c^+ \approx 0.45$ for $U_\infty = 40$ ms$^{-1}$. Therefore, it was ensured that the entire energetic frequencies were resolved for all the measurements since Hutchins et al. [47] have shown that the maximum energetic frequencies in turbulent boundary layers across various $Re_\tau$ are no more than a fixed viscous-scaled frequency $f_{max}^+ \approx 0.33$. Here $\nu$ is the kinematic viscosity of air and $u_\tau$ is the friction velocity defined as $u_\tau = \sqrt{\tau_w/\rho}$ where $\tau_w$ is the wall shear stress and $\rho$ is the air density.

The total sampling time at each wall normal location $z$ is given by $T$ and is outer normalised to give the boundary layer turnover times $TU_\infty/\delta$ ($TU_{cl}/\delta$ for channel flow where $U_{cl}$ is the centre line velocity). In order to obtain converged statistics and partially converged spectra this number should be large to capture several hundreds of the largest structures past the probe. These large structures are in the order of $\delta$. In this study, the total sampling time is chosen such that the boundary layer turnover time is between 10000-18000 for all measurements.
Chapter 4

Sub-miniature hot-wire anemometry


4.1 Introduction

Studying high-Reynolds number boundary layer flows has improved extensively over the past few decades thanks largely to the construction and development of high Reynolds number experimental facilities. In conjunction with this, advances in sub-miniature hot-wire ($l < 100 \mu m$) anemometry have also been required in order to capture the smallest scales of motions in these flows. For wall-bounded turbulent flows, the Reynolds number based on friction velocity, $Re_\tau = u_\tau \delta/\nu$, represents the ratio of the largest scale size, which is proportional to the boundary layer thickness in turbulent boundary layer flows and pipe radius in pipe flows ($\sim \delta$), to the smallest scale, which is given by viscous length scale ($\sim \nu/u_\tau$). $Re_\tau$ can exceed $10^5$ in high-Reynolds-number flows which means
that in a laboratory facility with fixed dimensions, the small-scale eddy size can be of order of micrometers. Using sensors with sizes greater than the smallest scales of motion results in spatial filtering.

A number of notable efforts have been made towards realising adequately small hot-wires, (see e.g. \[6, 13, 51, 74\]) one of the most important of them being the development of the Nano-Scale Thermal Anemometry Probe, NSTAP \[6\], which uses MEMS-based technology for the probe construction and results in sensor lengths of order 30-60 µm. However operating NSTAP and other sub-miniature probes is not straightforward. Currently NSTAP can be operated with the Dantec Streamline anemometer but only using external resistors \[145\]. That is, this anemometer is not designed for sensors with the high resistance as found with NSTAPs. Attempts to operate NSTAP with other anemometers result in frequent sensor breakages. Moreover, the best possible square-wave test response of NSTAP with the Dantec Streamline anemometer does not resemble a typical optimal square-wave test response of conventional wires as introduced by Freymuth \[34\] (See ref. \[145\] for NSTAP square-wave responses). We address the issue of appropriate operation of sub-miniature hot-wires by employing theoretical modelling to optimise an in-house CTA: the Melbourne University Constant Temperature Anemometer (MUCTA) for this purpose.

As mentioned in section 2.8, there are numerous analytical studies in the literature devoted to hot-wire anemometry. Despite all those studies and the noted difficulties associated with operation of sub-miniature wires, interaction of these wires with the CTA has not been studied sufficiently to the best of the author’s knowledge, which is vital today due to the increasing need for employing smaller and smaller wires for studying high-Reynolds number flows. An exception is the study by Watmuff \[151\] in which he briefly discussed stability conditions of sub-miniature wires, showing that a higher cut-off frequency and gain of amplifiers is required for stable operation of sub-miniature wires than for larger wires. In the present chapter first Watmuff’s model is evaluated by comparing its results against experiment, then it is employed to investigate the operation of sub-miniature wires with CTAs more comprehensively.
4.2 Theory

In this section the theoretical model developed by Watmuff [151] is presented briefly.

4.2.1 Static analysis

Figure 4.1 shows a typical CTA circuit. It consists of a Wheatstone bridge in which the hot-wire \((Z_w)\) is one of the arms of the bridge, and impedances \(Z_a\), \(Z_b\), and \(Z_c\) form the other arms. Feedback includes two amplifiers with DC gains of \(K_a\) and \(K_b\) and an offset voltage \(E_{qi}\) applied to the second amplifier. The static electric current passing through the wire \(\bar{I}_1\) needs to be determined first since it is required in dynamic analysis. Only resistive component of the impedances need to be considered in the static analysis, i.e. \(Z_w = R_w\), \(Z_a = R_a\), \(Z_b = R_b\), and \(Z_c = R_c\). A static circuit analysis determines \(\bar{I}_1\) as follows.

\[
\bar{I}_1 = \frac{K_b E_{qi} (R_b + R_c)}{(R_b + R_c)(R_a + (R_w + R_l))} + \frac{K_a K_b \dot{R}}{R}
\]

(4.1)

where \(\dot{R} = [(R_w + R_l)R_c - R_a R_b]\), \(R_w\) (still unknown) is the hot-wire’s static operating resistance, and \(R_l\) is the sum of resistances of the stubs (un-etched part of the wire) and the cable connectors to the circuit.

The mean temperature of the wire (which is constant throughout the operation) is the result of balancing the Joule heating and heat convected by the flow over the wire (axial heat conduction is neglected here for simplicity). If the equation recommended by Kramers [64] is used for Nusselt number (assuming \(Pr = 0.7\) for air) another equation relating hot-wire current and resistance will be given as

\[
\bar{I}_1^2 = (1 - \frac{R_g}{R_w})(0.39 + 0.51 \sqrt{\frac{dU}{\nu_g}})
\]

(4.2)

In equation 4.2, \(R_g\) is the wire resistance at the gas temperature, \(k_g\) is the gas conductivity, \(l\) is the wire length, \(d\) is the wire diameter, \(\alpha\) is the resistance temperature coefficient of the wire, \(\nu_g\) is the gas viscosity, \(R_0\) is the wire resistance at the reference temperature, and \(U\) is the mean gas velocity past the wire. By solving equations 4.1...
and 4.2, $I_1$ and $R_w$ are obtained and the static operating point is determined. It is noteworthy that $R_w$ is not determined only by bridge resistors since the bridge is not usually in perfect balance, i.e. $(R_w + R_l)R_c - R_a R_b \neq 0$. It is clear from equations 4.1 and 4.2 that $K_b$, $K_a$, and $E_{qi}$ contribute to the operating point as well as $R_a$, $R_b$, and $R_c$. Hence by changing the offset voltage $E_{qi}$, the hot-wire resistance $R_w$ and overheat ratio $R = \frac{R_w}{R_g}$ vary. The overheat ratio dictates the static operating temperature of the wire.

4.2.2 Dynamic analysis

In order to analyse the dynamic behaviour of a CTA, a square-wave test is used to optimise its frequency response. A square-wave signal is applied to a specific point of the feedback circuit (second stage differential amplifier here) and by tuning the circuit, the best frequency response is obtained. It has been shown that a square-wave test, if appropriately interpreted, can be representative of the CTA response to the velocity fluctuations ([116],[151], [34],[70]). Hence, a voltage perturbation analysis is essential in order to gain a comprehensive understanding of the dynamic behaviour of CTA.

Figure 4.1: Model of CTA circuit.
Chapter 4. *sub-miniature hot-wire anemometry*  

The dynamic circuit equations of the circuit shown in figure 4.1 can be written as follows:

\[ e_o = Z_a i_1 + e_w \]  
\[ e_o = (Z_c + Z_b) i_2 \]  
\[ e_o = \left[ (Z_b i_2 - e_w) K_a \frac{A_a(s)}{B_a(s)} - e_s \right] K_b \frac{A_b(s)}{B_b(s)} \]  

where \( Z_a, Z_b, \) and \( Z_c \) are the Wheatstone bridge arm impedances, \( e_w \) is the wire’s voltage fluctuation, \( e_o \) is the output voltage fluctuation, \( i_1 \) and \( i_2 \) are fluctuations of the electric currents of Wheatstone bridge arms shown in figure 4.1, and \( e_s \) is voltage fluctuation injected to the offset voltage node, i.e. square-wave applied to the second amplifier. In our circuit analysis differential amplifiers are treated with the governing equation \( e_o = \frac{K_i A_i(s)}{B_i(s)} \Delta e_i, (i = a, b) \) where \( K_i \) is the amplifier’s DC gain, \( A_i(s) \) and \( B_i(s) \) are polynomials for the zeros and the poles in the transfer function of the amplifiers respectively, \( \Delta e_i \) is the amplifier’s input voltage difference, and \( e_{oi} \) is its output voltage. When a square-wave test is conducted, velocity fluctuations are absent and the wire voltage fluctuation is

\[ e_w = i_1 R_w + \bar{I}_1 r_w = i_1 Z_w \]  

where \( Z_w \) is the wire impedance and a lumped model is used, following Perry [116]. By combining equations 4.3 to 4.6 and after some manipulation, the voltage fluctuation transfer function of the CTA is obtained as

\[ \frac{e_o}{e_s} = \frac{B_a(s)}{K_a A_a(s) P(s)} \]  

where

\[ P(s) = \frac{Z_b}{Z_b + Z_c} - \frac{Z_w}{Z_w + Z_a} - \frac{B_a(s) B_b(s)}{K_a K_b A_a(s) A_b(s)} \]  

When velocity fluctuations are measured by the hot-wire, offset voltage fluctuations
are absent, i.e. $e_s = 0$. Perry [116] showed that voltage fluctuations of a constant
temperature wire exposed to velocity fluctuations to the linearised approximation is

$$
e_w = \frac{\partial E_w}{\partial U} u + \frac{\partial E_w}{\partial I} i \tag{4.9}
$$

where $u$ and $i$ are velocity and electric current fluctuations respectively, $\frac{\partial E_w}{\partial I} = Z_w$, and $\frac{\partial E_w}{\partial U}$ depends on the adopted wire heat transfer relation. In this study we use the
equation recommended by Kramers [64] for the wire heat transfer.

By combining equations 4.3-4.5 and equation 4.9, and remembering that $e_s = 0$, the
velocity fluctuations transfer function is determined as

$$
e_o u = \frac{Z_a}{Z_w + Z_a \frac{\partial E_w}{\partial U}} P(s) \tag{4.10}
$$

where $P(s)$ is introduced in equation 4.8.

If all the parameters are introduced in the polynomial form in equations 4.7, 4.8, and
4.10, after trivial algebra, transfer functions are obtained in rational form. (See ref.
[151].)

### 4.3 Experiments

A set of experiments was conducted to examine the accuracy of the analytical model
and also to test the possibility of using sub-miniature wires, which has been troublesome
due to breakage and also inefficient tuning issues.

Three standard hot-wire probe types along with the sub-miniature NSTAP probes were
used throughout the experiments described in this chapter. In the majority of measure-ments, a Dantec 55P05 boundary layer type probe was used, with prong spacing 3 mm.
Wollaston wires were soldered to the prong tips and etched to reveal a 5 $\mu$m-diameter
platinum-sensing element of length 1 mm in the middle. The un-etched portions (stubs)
are 1 mm in length and have an estimated diameter of 40 $\mu$m. Other hot-wire probe
Table 4.1: Dimensions of the probes used in this chapter. \( l_w \) is the sensing element length, \( d_w \) is the hot-wire sensing element diameter, \( l_s \) and \( d_s \) are the hot-wire stub length and diameter respectively, and \( w_w \) and \( t_w \) are the NSTAP probe sensing element width and thickness respectively.

<table>
<thead>
<tr>
<th></th>
<th>( l_w(\mu m) )</th>
<th>( d_w(\mu m) )</th>
<th>( l_s(\mu m) )</th>
<th>( d_s(\mu m) )</th>
<th>( w_w(\mu m) )</th>
<th>( t_w(\mu m) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>HW1</td>
<td>1000</td>
<td>5</td>
<td>1000</td>
<td>40</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>HW2</td>
<td>500</td>
<td>2.5</td>
<td>500</td>
<td>50</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>HW3</td>
<td>250</td>
<td>1</td>
<td>625</td>
<td>30</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>NSTAP</td>
<td>60</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>2</td>
<td>0.1</td>
</tr>
</tbody>
</table>

types were Dantec 55P15 boundary layer type with prong spacing 1.5 mm. Wollastan wires with platinum cores of 2.5 and 1 \( \mu \)m-diameter are used to produce them. The production process is identical to that of the 5 \( \mu \)m-diameter wires; the etched sensing length is 0.5 mm for 2.5 \( \mu \)m-diameter and 0.25 mm for 1 \( \mu \)m-diameter wire. The NSTAP probe had a sensing element consisting of a platinum (Pt) filament measuring 60 \( \mu \)m \( \times \) 2 \( \mu \)m \( \times \) 100 nm. The dimensions of all the probes used for the experiments in this chapter are summarised in table 4.1.

An in-house Melbourne University Constant Temperature Anemometer (MUCTA) designed based on references [116] and [113] was used in the experiments. In the MUCTA a 10:1 ratio Wheatstone bridge is used with \( R_a = 100 \ \Omega \), \( R_c = 1000 \ \Omega \), and variable \( R_b \) to set the desired overheat ratio. A high performance instrumentation amplifier (simple pole) with DC gain \( K_a = 100 \) and corner frequency \( f \approx 750 \ \text{kHz} \) is used as the first stage amplifier and a differential amplifier configuration as the second stage amplifier. By selecting appropriate resistors and capacitors, the desired corner frequency and DC gain of the second stage amplifier can be set.

Validation of the analytical model is presented in the following two sections: §4.3.1 square-wave test, and §4.3.2 velocity fluctuations.

4.3.1 Square-wave test validation

A 5 \( \mu \)m-diameter wire was utilized for this test and was located at the centre of a fully turbulent channel flow. A \( \sim 1 \) kHz square-wave with an amplitude of 50 mV was used.
Figure 4.2: Comparison between theory and experiment for the square-wave response of a 5 µm-diameter wire. A MUCTA is used with $R_a = 100 \, \Omega$, $R_b = 120 \, \Omega$, $R_c = 1000 \, \Omega$, and different offset voltages of (a): 1.43 V, (b): 0.74 V, (c): 0.64 V, (d): 0.54 V. The mean flow velocity over the hot-wire is $\dot{U} = 10.5 \, \text{ms}^{-1}$. experiment, theoretical model.

Since the core flow of the channel is fully turbulent, the velocity fluctuations, though small, were superimposed on the applied square-wave response. To isolate the square-wave response from the velocity fluctuations, the square wave response was sampled using an A/D board at 1 MHz for 60 sec and several thousand response signatures were ensemble averaged. To extract the square-wave response signature, the injected square-wave signal was sampled simultaneously and the risings of the sampled square-wave signal were used to phase average the CTA signal, and produce a conditionally averaged square-wave response.

Several flow velocities and anemometer adjustments were tested in different runs; four of them are presented here in figure 4.2. The experimentally measured square-wave responses are shown by the dashed lines, while the solid lines show the model prediction.
It can be seen that except for a slight difference in the first peak, which is at \( t \approx 2 \times 10^{-5} \) s (corresponding to a sinusoid function with frequency of \( f \approx 12.5 \) kHz), theory and experiment are in excellent agreement. To further investigate these differences one can explore the transfer functions corresponding to the transient responses shown in figure 4.2. The transfer function of a control system can be obtained experimentally, by taking the Fourier transform of the impulse response of it (which can be obtained by taking the derivative of the square-wave response of that system). By doing so, the transfer functions of the CTA corresponding to the tests shown in figure 4.2 are determined and depicted in figure 4.3. Theoretical and experimental transfer functions are normalized by their magnitude at \( f \approx 2.2 \) kHz to make comparison between theory and experiment feasible. It can be seen in figure 4.3 that the experimental transfer functions start to peel off from the theoretical ones at \( f \approx 15 \) kHz; but agree reasonably well up to \( f \approx 40 \) kHz, which is the upper bound of the frequency range in which the signal contains most of
its energy. The reasons for the discrepancy between the theoretical and experimental square-wave test transfer function observed here will be discussed in §4.5.

### 4.3.2 Validation of transfer function related to velocity fluctuations

Measuring the direct frequency response of a hot-wire-CTA system to velocity fluctuations is not as easy as the electronic square-wave test, especially up to very high frequencies.

Perry and Morrison [112] exposed a hot-wire to the periodic Karman Vortex streets behind a cylinder. Using this technique they were able to measure their system frequency response up to nearly 10 kHz. They observed that their in-house CTA exhibited a flat response to velocity fluctuations up to 10 kHz, while the tested DISA 55A01 anemometer exhibited a 3 dB attenuation at 6 kHz. Khoo et al [58] utilized a specially designed apparatus consisting of a top rotating disc with recesses cut on its surface and a bottom stationary disc for providing known velocity fluctuations up to 1600 Hz over the hot-wire. By using this apparatus they observed that a Dantec 56C01 anemometer showed a flat response to velocity fluctuations up to 1600 Hz, which is not unexpected at such a relatively low frequency. Hutchins et al [49] used a fully developed turbulent pipe flow to create the input velocity fluctuation to the hot-wire. They exploited the unique capabilities of the Princeton Superpipe, in which the kinematic viscosity of the air can be changed via pressurisation, to explore various turbulent pipe flows at matched Reynolds numbers, but with turbulent energy in different frequency ranges. They argued that based on Reynolds similarity, any difference among the normalised energy spectra is due to differences in frequency response of the CTA systems. They used three different anemometers and showed that the frequency response of under- or over-damped CTA systems is approximately flat up to only 5-7 kHz.

Considering special apparatus is required to carry out direct testing of CTA systems, the proposed velocity transfer function by Watmuff [151] and developed in §4.2 proves to be advantageous. However, the accuracy of this model should be assessed first. To
this end, a 5 µm-diameter hot-wire was placed at the centre of a turbulent flow channel. The MUCTA was used to operate the wire. Two different tuning conditions (one slightly under-damped and the other over-damped) were examined to measure velocity fluctuations of the flow at the same location. Figure 4.4 depicts square-wave test results for two tunings which are classically considered as slightly under-damped tuning (4.4(a)) and over-damped (4.4(b)). Figure 4.5(a) shows voltage spectra (premultiplied by frequency $f$, so that equal areas equate to equal energy densities on the semilogarithmic plot : $\overline{\varepsilon^2} = \int_0^{\infty} \phi df = \int_0^{\infty} f \phi d(logf)$ where $\overline{\varepsilon^2}$ is the variance of the voltage signals and $\phi$ is energy spectrum of the voltage fluctuations) as measured by the slightly under-damped

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{Figure4.4.png}
\caption{Square-wave test response of two tuning conditions used for validating analytical transfer function of velocity fluctuations: (a) slightly under-damped, (b) over-damped. --- experiment, --- theoretical model.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{Figure4.5.png}
\caption{(a) Pre-multiplied energy spectra of uncalibrated voltage fluctuations measured by slightly under-damped and over-damped hot-wire-CTA systems at the centre of channel. (b) Velocity fluctuation transfer function magnitude as predicted by the theoretical model for two mentioned tunings of the MUCTA used in experiments with pre-multiplied energy spectra shown in a. --- slightly under-damped condition, · · · : over-damped condition}
\end{figure}
and over-damped hot-wire systems. The corresponding velocity transfer function magnitudes for two different tunings as calculated by the theoretical model are depicted in Figure 4.5(b). One can see in both plots that the energy spectra and transfer function magnitudes of the two tunings start to diverge at $f \approx 200$ Hz. In order to compare the theoretical transfer function with the real transfer function of two measurement conditions we can use relative transfer functions since real absolute transfer functions cannot be realized without an accurate, known, high frequency velocity fluctuation. The relative experimental transfer function can be obtained by dividing the energy spectrum measured by the over-damped system by the one measured by the under-damped system. Likewise, the relative theoretical transfer function can be obtained by dividing the magnitude of the over-damped transfer function by the magnitude of the under-damped transfer function. Experimental and theoretical relative transfer functions are plotted in figure 4.6. In general the theoretical relative transfer function captures well the salient characteristics of the experimental system. However, it can be seen that experimental relative transfer function rolls off at a slightly lower frequency than the theoretical one. One reason for this difference might be the simple lumped model for the hot-wire sensor which has been chosen to make the analysis tractable. In the lumped model, conductive

\[ \frac{e_o}{u} \]

\[ f(\text{Hz}) \]

Figure 4.6: Comparison between relative theoretical and experimental velocity transfer functions. --- experimental, --- theoretical model.
heat transfer to the prongs (known as end conduction) and also vibrations of the wire sensor are neglected. Despite the observed discrepancy between theory and experiment for velocity transfer function, the model can be used as a quick reference for estimating the real frequency response of a CTA system in the absence of any other practical methods and also for studying the sensitivities of the velocity transfer function to different tuning settings such as offset voltage, bridge compensation inductance, and amplifier gain and corner frequency.

So far we have shown that the model is able to mimic the operation of conventional hot-wires \( l \approx 500-1000 \text{ µm} \) with good accuracy. In §4.4 we implement the model to demonstrate the behaviour of sub-miniature wires \( l < 100 \text{ µm} \) operated with the CTAs designed for conventional wires; and in §4.5 we address issues associated with operation of these wires such as frequent hot-wire breakages and stability problems.

### 4.4 Frequency response of sub-miniature hot-wires

Square-wave tests are used to optimise the frequency response of a CTA system and find its cut-off frequency. The dashed curves in figures 4.7(a) and 4.7(b) show the optimal frequency response of a conventional hot-wire and its corresponding velocity transfer function which are identical to those of an optimally tuned system introduced by Freymuth [34]. A circuit with the following components is modelled in this section: a 10:1 ratio Wheatstone bridge with \( R_a = 100 \Omega, R_c = 1000 \Omega, \) and variable \( R_b \) (and lumped inductance \( L_b \)) to adjust overheat ratio (and optimise frequency response) of the sensor. The first stage differential amplifier has a DC gain \( K_a = 100 \) and a single pole transfer function with cut-off frequency \( f_c = 400 \text{ kHz} \). The Second stage amplifier is a differential amplifier having DC gain \( K_b = 10 \) and single pole transfer function with cut-off frequency \( f_c = 15 \text{ kHz} \). The circuit is combined with a 2.5 µm-diameter (500 µm-long) wire. Different adjustments by changing the offset voltage \( E_{qi} \) are tested and the square-wave responses and the corresponding velocity transfer function magnitudes are shown in figure 4.7. The response curves are shifted in the ordinate axis for clarity. It can
be seen that by increasing the offset voltage, the response of the CTA system becomes increasingly damped. The same circuit is modelled to operate a 0.2 µm-diameter (30 µm-long) wire and the results are shown in figure 4.8. Contrary to the 2.5 µm-diameter wire, it is not possible to obtain a square-wave response that looks like a classic optimal square-wave response for the 0.2 µm-diameter wire by changing offset voltage with this circuit. If just the electrical response is considered, one could conclude from the square-wave response of figure 4.8(a) that by increasing the offset voltage, the frequency response of the system increases. However this is disapproved by the transfer functions shown in figure 4.8(b). This test demonstrates that square-wave tests can be misleading in operating a sub-miniature wire with a CTA designed for conventional hot-wires. In fact, in the transfer function of the CTA system in addition to the conjugate poles which are dominant in the conventional hot-wire case and create a second order peak in the transfer function, a simple pole is present which becomes significant for the 0.2 µm-diameter wire and becomes dominant by increasing offset voltage $E_{qi}$ and determines the real frequency response of the CTA system. This behaviour highlights the importance of analytical methods in finding the ideal response of sub-miniature hot-wires, which in turn requires sufficient information regarding the anemometer circuit and the hot-wire sensor heat transfer governing equation.
Chapter 4. *sub-miniature hot-wire anemometry*

![Diagram](image)

**Figure 4.8:** Variation of the frequency response of a modelled CTA system with offset voltage $E_{qi}$, operating a 0.2 µm-diameter (30 µm-long) wire. Numerical values of offset voltages are 0.8, 1.2, 2, 3, and 4 V and arrow direction indicates its increase. (a) Square-wave response. (b) Velocity transfer function magnitudes corresponding to the square-wave responses in a. Square-wave response curves have been shifted in the ordinate axis for clarity.

### 4.5 Operating sub-miniature wires: Instability and breakage issues

Studying high-Reynolds number flows experimentally, requires very small sensors to avoid spatial averaging. This requirement has encouraged researchers to build sub-miniature sensors like the Princeton NSTAP [6]. Decreasing the dimensions of the sensor, increases its frequency response drastically; therefore, a high frequency response anemometer is required to operate it. Currently NSTAP can be safely controlled only with the Dantec Streamline anemometer with an external resistor. In this section the feasibility of using an anemometer based on the circuit given in figure 4.1 to operate sub-miniature probes is investigated with the aid of the theoretical model presented in §4.2.

A MUCTA circuit with the following characteristics is simulated: a 10:1 ratio Wheatstone bridge with $R_a = 100 \, \Omega$, $R_c = 1000 \, \Omega$, and variable $R_b$ (and lumped inductance $L_b$) to adjust overheat ratio (and optimise frequency response). The first stage differential amplifier has a DC gain $K_a = 100$ and a single pole transfer function with cut-off frequency $f_c \approx 750 \, \text{kHz}$. The second stage amplifier is a differential amplifier having DC gain $K_b = 10$ and single pole transfer function with cut-off frequency $f_c \approx 15 \, \text{kHz}$. 
Perry [116] and Watmuff [151] showed that the transfer functions related to both velocity and the square-wave test have the same poles. Moreover, a square-wave test is the only practical way to tune a CTA system and it usually provides a good representation of the system response to the velocity fluctuations; hence, to study the behaviour of the CTA system, the square-wave test transfer function (equation 4.7) is investigated in this section.

Figure 4.9 shows the location of the dominant poles of the closed-loop transfer function related to the square-wave injection of the modelled CTA operating 5, 2.5, 1, 0.6, 0.3, and 0.12 \( \mu \text{m} \)-diameter wires on the s-plane. Lengths of the wires are 200 times their diameters, i.e. \( l = 200d \). All wires have overheat ratios of 1.6 except for the 0.12 \( \mu \text{m} \)-diameter wire which has an overheat ratio of 1.3. Simple poles for 5, 2.5, 1, and 0.6 \( \mu \text{m} \)-diameter wires are far from the origin and cannot be seen in the figure. It can be seen that for the mentioned circuit characteristics, theory does not predict instability even for a 0.12 \( \mu \text{m} \)-diameter wire, since the poles are located in the left-half of the s-plane and reasonably far from the imaginary axis. Figure 4.10 shows normalized transfer function
magnitudes related to the square-wave test for the wires of figure 4.9. One can see that by decreasing the hot-wire size, the peak in the transfer function that corresponds to the square-wave frequency response of the CTA system moves to higher frequencies as expected, and for wires with diameters smaller than 0.6 \( \mu \text{m} \), the frequency of the peak exceeds 300 kHz.

A 2.5 and 1 \( \mu \text{m} \)-diameter wire were tested with the MUCTA having circuit characteristics similar to the modelled anemometer to observe the system behaviour at higher frequencies more clearly. Their square-wave test transfer functions compared with model prediction are shown in figure 4.11. Like those of the 5 \( \mu \text{m} \)-diameter wire in figure 4.3, the experimental transfer functions deviate from theory at high frequencies for both the 2.5 and 1 \( \mu \text{m} \)-diameter wires. The magnitude of the transfer function at \( f = 350 \text{ kHz} \) is approximately twice the theoretical magnitude for both wires. While it is not possible to precisely predict the effect of this deviation from theory for much smaller sub-miniature wires, a synthetic square-wave test transfer function and its corresponding time response for a 0.3 \( \mu \text{m} \)-diameter wire operated by the MUCTA at its current condition might be
expected to resemble those shown in figure 4.12. The characteristic used to generate
the synthetic transfer function shown in figure 4.12 is the ratio of the magnitude of the
experimental transfer function to the magnitude of the theoretical transfer function at
the expected peak of the 0.3 µm-diameter wire which is equal to this ratio for the tested
2.5 µm-diameter wire at the same frequency (≈ 350 kHz). The resulting square-wave re-
sponse is seen to be under-damped and close to instability. A fragile sub-miniature wire
cannot survive such an oscillatory current passing through it and breaks immediately.

To find the reason for the discrepancy between theory and experiment, especially at high
frequencies, all parts of the MUCTA circuit were reviewed and the second stage amplifier
was found to be responsible for the near instability. Based on the theoretical model, it
was determined that both amplifiers utilized in practice should be differential amplifiers
in order for the circuit to comply with the theoretical analysis. The first amplifier should be an Instrumentation Amplifier since it amplifies very small differential signals of the Wheatstone bridge, and the second amplifier can be constructed by a configuration as shown in figure 4.13 so that its DC gain and cut-off frequency can be controlled by choosing proper resistors and capacitors. In our anemometer circuit (figure 4.1), \( e_1 \) is the node connected to the output of the first amplifier and \( e_2 \) is the node connected to the offset voltage \( E_{qi} \) (and square-wave test signal injection node). The transfer function for this configuration is given as

\[
e_o = \left( \frac{Z_1 + Z_3}{Z_2 + Z_4} \times \frac{Z_4}{Z_1} \right) e_2 - \left( \frac{Z_3}{Z_1} \right) e_1 \quad (4.11)
\]

If \( Z_1 = Z_2 \) and \( Z_3 = Z_4 \), equation 4.11 will reduce to

\[
e_o = \frac{Z_3}{Z_1} (e_2 - e_1) \quad (4.12)
\]

In practice it is not possible to have an infinitely flat amplifier response; therefore, the output of the second amplifier should be filtered at an appropriate frequency. To achieve...
this filtering, a capacitor $C$ is used in parallel with a resistor $R_3$ to form impedance $Z_3$. In that case and if $Z_1 = Z_2 = R_1$ and $Z_3 = Z_4$, the amplifier’s relation will be

$$e_o = \frac{R_3}{R_1} \times \frac{1}{R_3Cs + 1}(e_2 - e_1)$$  \hspace{1cm} (4.13)

If this capacitor is only applied in impedance $Z_3$, i.e. impedance $Z_4$ is just comprised of a resistor $R_4 = R_3$ (this is the case in the MUCTA circuit), the assumption that $Z_3 = Z_4$ will not be true for all frequencies and equation 4.12 or equivalently equation 4.13 cannot be used for the second amplifier. If we assume that resistors $R_1 = R_2$, $R_3 = R_4$ and capacitor $C$ are chosen such that $\frac{R_3}{R_1} = 10$ and $R_3C \approx 10^{-5}$ s (as in MUCTA), the resulting magnitudes of coefficients of $e_2$ in equations 4.11 and 4.12 can be calculated. For these values, the magnitudes of the coefficients with and without inclusion of the capacitor $C$ in $Z_4$ are shown in figure 4.14 as a function of frequency $f$.

It is interesting to notice that similar to figures 4.3 and 4.11, the curve corresponding to the amplifier without a capacitor in $Z_4$ peels off from the one corresponding to the amplifier with a capacitor in $Z_4$, and their difference accentuates as frequency increases.

It should be noted that the mentioned difference between MUCTA and the theoretical square-wave transfer function is small for frequencies less than 100 kHz, hence it does not affect conventional hot-wire operations; furthermore, this capacitor does not impact the velocity fluctuations response since impedance $Z_4$ merely appears in the transfer function of node $e_2$ (see equation 4.11) and during the velocity measurement $e_2 = 0$ since $e_2$ is connected to the square-wave test signal which is zero during the velocity measurement. For sub-miniature wires it will effect the square-wave response appearance (and anemometer tuning as a result) and also the stability of the square-wave test.

To solve the problem and force the MUCTA circuit to follow our theoretical model more faithfully, a capacitor equal to the one used in $Z_3$ was added to $Z_4$ (in parallel to $R_4$). The modified MUCTA was used to operate 2.5 and 1 μm-diameter wires with test conditions similar to those of figure 4.11 again. Transfer functions for sample square-wave tests are indicated in figure 4.15. By comparing figures 4.11 and 4.15 one realizes that, after
Figure 4.14: Magnitude of coefficient of $e_2$ (in equation 4.11) for the amplifier circuit shown in figure 4.13 with capacitor in impedance $Z_4$ and without capacitor in impedance $Z_4$.

Figure 4.15: Comparison of theoretical and experimental transfer functions for square-wave test of the CTA system after adding the capacitor $C$ to the impedance $Z_4$ in parallel to $R_4$. (a) 2.5 µm-diameter wire, (b) 1 µm-diameter wire. --- theory, --- experiment.

The modification, the match between MUCTA circuit and theory at high frequencies is significantly improved. However, the match is not still perfect which can be attributed to inaccurate capacitance measurement and/or the simple lumped model used for the wire.

The modified MUCTA circuit was used to operate NSTAP and a square-wave test was performed successfully to obtain an optimal square-wave response. However, frequent
breakages during measurements persisted. Considering the fact that a stable square-wave response was obtained, breakage could not be due to instabilities alone. Fan et al. [28] mention that excessive electric current can burn NSTAPs due to their small size compared with conventional wires (approximately 100 nm thickness compared to 5 µm diameter); therefore electric current fluctuations can damage them instantly. One way of dampening probable electric current fluctuations passing through NSTAP during operation is to increase the top-of-bridge resistors $R_a$ and $R_c$ [28]. MUCTA has been designed for conventional wires with a cold resistance of $R = O(10)$ Ω while the NSTAP cold resistance is $R = O(1000)$ Ω; therefore increasing the top-of-bridge resistors can help dampen electric current spikes which can burn-out the NSTAP. We tested this idea using the theoretical model by computing the current response of a 0.6 µm wire operated with simulated MUCTA to a 1 V step voltage signal injected to the offset voltage node. Two cases were simulated: (i) the CTA circuit with $R_a = 100$ Ω and $R_c = 1000$ Ω, (ii) the same circuit but with $R_a = 1000$ Ω and $R_c = 10000$ Ω. Results are depicted in figure 4.16. The peak electric current magnitude in the latter case is approximately 3 times smaller than the former case suggesting that for a given voltage fluctuation injected into the circuit, we can dampen electric current fluctuation magnitude by a factor of 3.
in the MUCTA circuit by increasing the resistance of the top-of-bridge resistors by a factor of 10. Noting that for sub-miniature wires the static current during operation is on the order of milliamperes, one realises the importance of this difference. Convinced by theory, we changed the top-of-bridge resistors to $R_a = 1000 \, \Omega$ and $R_c = 10000 \, \Omega$ in MUCTA and after this modification successfully operated NSTAP for greatly extended periods with a much lower occurrence of breakage. We also noticed that after this modification the signal-to-noise ratio increased by a factor of approximately 3, i.e. for a given flow speed range past the NSTAP, the output voltage range became 3 times larger. After applying both of the above-mentioned modifications, MUCTA was used to operate NSTAP at an overheat ratio $R = 1.3$ to measure streamwise velocity in the turbulent channel flow facility detailed in [94]. Mean velocity and associated broadband turbulence intensity profiles were measured using an NSTAP and a conventional 2.5 µm-diameter wire, and the results are presented in figure 4.17 where they are compared with DNS results from Hoyas and Jimenez [41] at comparable friction Reynolds number ($Re_\tau = \frac{u_\tau h}{\nu} \approx 950$); here $u_\tau$ is friction velocity, $h$ is channel half-width, and $\nu$ is kinematic viscosity. In figure 4.17, $u$ denotes the streamwise fluctuating velocity, $z$ is wall-normal distance, and the superscript ‘+’ indicates normalisation by inner variables. In this measurement the 2.5 µm-diameter wire has a non-dimensional length $l^+ = \frac{u_\tau l}{\nu} = 10$ where $l$ is active portion of the wire, while for NSTAP $l^+ = 1.2$. The mean velocity

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure4.17.png}
\caption{Comparison of the first and second order statistics, shown in the inner normalised coordinates. (a) Streamwise mean velocity ($U$) and (b) variance of streamwise velocity component ($\overline{u^2}$). $\longrightarrow$ DNS, $\circ$ : 2.5 µm-diameter wire results, and $\triangleright$ : NSTAP results.}
\end{figure}
profiles are all seen to be in very good agreement, while the variance of streamwise velocity exhibits some small variations. The NSTAP results are seen to have higher peak turbulence levels as compared to the DNS. This behaviour has been formerly observed in pipe flow measurements when compared to DNS at relatively low Reynolds number $Re_\tau \approx 3000 \ [2]$. This is likely because NSTAP is designed for high Reynolds number flow measurements, therefore its velocity sensitivity at low velocities is substantially less than conventional wires. Low sensitivity of NSTAP for very low velocities can be inferred from the calibration curve, where for velocities less than $2.5 \text{ ms}^{-1}$ the slope of the calibration curve flattens out [145].

### 4.6 Summary and conclusions

An existing theoretical framework for CTAs that takes into account the finite frequency response of amplifiers in the feedback circuit was applied to analyse anemometry of sub-miniature hot-wires ($l < 100 \ \mu m$). Such probes are increasingly required for high Reynolds number turbulent flow measurements. Two transfer functions are developed by the theoretical analysis; one related to a circuit square-wave test and the other for velocity fluctuations. The accuracy of the model was assessed by modelling a MUCTA circuit. A 5 $\mu m$-diameter hot-wire was used for the assessment tests. The theoretical model compared well for the electrical fluctuations transfer function and fairly well for the velocity fluctuations transfer function when compared to experiment. A relative transfer function for velocity fluctuations was introduced since determining absolute experimental velocity transfer function up to high frequencies is not possible. Comparing theoretical relative velocity transfer functions to the experimental counterparts shows good qualitative agreement, although the model overestimates the cut-off frequency of the CTA, which is probably due to the simplified heat transfer model used for analysis of the hot-wire filament. Furthermore, the model was used to analyse operation of a sub-miniature hot-wire with a standard CTA designed to operate conventional hot-wires.
and revealed that the square-wave test could be misleading in showing true frequency response of the hot-wire-CTA system for sub-miniature wires.

A MUCTA circuit was further modelled to investigate the possibility of operating sub-
miniature wires. With the aid of the model it was found that proper filtering of the second stage amplifier is required in order for MCUTA to perform the square-wave test with a sub-miniature wire, and the top-of-bridge resistors values, which are ideal for conventional wires, were found to be too small for NSTAP as even small electric current fluctuations can damage the sensor. By changing those resistors to 10 times the original values the fluctuations were dampened by a factor of 3 (as predicted by the theoretical model) and NSTAP wires were successfully operated by MUCTA. Using the modified MUCTA, NSTAP was used to measure velocity and turbulence profiles in a fully turbulent channel and these were compared with a 2.5 µm-diameter wire and DNS with very good observed agreement.
Chapter 5

End conduction effects in constant temperature hot-wire anemometry

5.1 Introduction

Constant temperature hot-wire anemometers (CTAs) are widely used for measuring velocity fluctuations in turbulent flows and are used and calibrated based on the assumption that the heat generated in the hot-wire filament by Joule heating is balanced by the convective heat transfer. It is assumed that the dynamic and static sensitivities are equal for the system (i.e. the velocity transfer function for the system is flat up to the cut-off frequency of the hot-wire-CTA system), and the static calibration is used to convert the voltage signals to the velocity time series for the entire range of turbulence frequencies. However, this assumption might not always be acceptable for hot-wires with finite lengths and can lead to associated errors. It is easy to show that if the heat conduction to the supports for the hot-wire filament is neglected, a single pole frequency response for the filament will be obtained. Then can then be incorporated in a CTA model to show that the transfer function of the system is flat up to a relatively large
cut-off frequency in the range of 10-50 kHz for conventional hot-wires [116]. This assumption in practice requires very large values for the hot-wire length-to-diameter ratio \( l_w/d_w \). \( (l_w/d_w > 200 \) is the generally accepted value following the experimental study of Ligrani and Bradshaw [73].) On the other hand, in wall bounded turbulent flows it is known that using sensors longer than several Kolmogrov length scales to measure the fluctuating velocity causes attenuation in energy spectra and consequently turbulence intensity due to spatial averaging. Hence, experimentalists are tempted to relax the commonly accepted length-to-diameter ratio criterion to achieve better spatial resolution in wall-bounded high Reynolds number flow measurements, which in turn might cause unwanted errors associated with the end conduction effects.

Despite decades of employing hot-wire anemometry in turbulence measurement and the many theoretical studies devoted to them, the consequences of the conduction losses to the wire supports on the dynamic sensitivity of the hot-wire-CTA systems is not clear. Looking back at theoretical studies one notes that conclusions on the form of the system transfer function are sometimes conflicting. Due to the complexity of the problem most of the analytical studies are limited to the indirect, static analysis of the wire with the presumption of the attenuation of the energy in the system transfer function (which is in contrast with the conclusion of Perry et al. [114]). Moreover, most of the studies are limited to the wire filaments with circular cross section forms and the effect of the prongs is usually ignored.

We address this problem to help elucidate the transfer function of the hot-wire-CTA system directly with the least possible simplifications in this chapter. In §5.2 perturbation analysis is employed to find the sensitivity of the wire with arbitrary cross section to velocity and current fluctuations taking into account the wire filament, the stubs, and the prongs, and also asymmetry in the wire filament. Wire sensitivities are incorporated in the CTA model previously developed by Watmuff [151] and experimentally verified by Samie et al. [126]. Furthermore, experiments in the turbulent boundary layer flow using various wire diameters and lengths are conducted to assess the theoretical model predictions. To this end, measured broadband turbulence intensity profiles and energy
spectra are carefully examined to discuss the effect of end conduction for wires with different length-to-diameter ratios.

5.2 Theory

In this section a hot-wire filament is modelled as a distributed system with axial conduction and a general geometry for the filament; i.e. we are not limited to only circular wires. Stubs and prongs are both treated as lumped systems and boundary conditions for the hot-wire filament are assumed to be asymmetric. The temperature at the far end of the prongs where they are in contact with the holder are fixed at the ambient gas temperature and solutions to the hot-wire filament heat balance differential equations are completed by matching the boundary conditions of the hot-wire filament, stub, and prong at their junctions.

5.2.1 Hot-wire filament with axial conduction

It can be shown that by considering Joule heating, heat convection, heat conduction, and accumulated heat in a wire filament with a uniform cross section area as shown in figure 5.1 and assuming temperature variation only along the wire length, a differential equation for the instantaneous temperature relative to the ambient temperature along the wire $\theta_w$ can be written as ([116])

$$\frac{\partial^2 \theta_w}{\partial x^* \partial t} - (\xi - q) \theta_w - T_w \frac{\partial \theta_w}{\partial t} = -\frac{q}{\alpha}$$  \hfill (5.1)

with the boundary conditions

$$\theta_w(x^* = \frac{1}{2}, t) = \theta_{s1}(t)$$

$$\theta_w(x^* = -\frac{1}{2}, t) = \theta_{s2}(t) = n\theta_{s1}(t)$$  \hfill (5.2)
where $n$ is a constant accounting for the asymmetry of the wire boundaries ($n$ is explained in more detail in section 5.2.3), $x^* = \frac{x}{l_w}$ is the non-dimensional coordinate along the wire with the origin at the midpoint, $\theta_{s1}$ and $\theta_{s2}$ are the instantaneous temperatures of the stubs at their junctions with the wire at $x^* = \frac{1}{2}$ and $x^* = -\frac{1}{2}$ respectively, and

$$\mathcal{T}_w = \frac{\rho_w c_w l_w^2}{k_w}$$

(5.3)

in which $\rho_w$, $c_w$, and $k_w$ are the wire material density, specific heat, and thermal conductivity respectively. $t$ is time, $\alpha$ is the wire material temperature coefficient of resistance, and $\xi$ is a non-dimensional number defined by

$$\xi = Nu \frac{k_f}{k_w} \frac{P l_w^2}{A_{w} w}$$

(5.4)

where $Nu$ is the wire spatially averaged Nusselt number, $k_f$ is the fluid thermal conductivity evaluated at the spatial and temporal mean film temperature, $P$ is the wire perimeter, $A_w$ is the wire cross-section area, and $w$ is a relevant length scale which is
the diameter for a wire with a circular cross section form and width for a wire with a rectangular cross section form. \( q \) is defined as

\[
q = \frac{I^2 R_g l_w \alpha}{k_w A_w}
\]  

(5.5)
in which \( I \) is the wire electric current and \( R_g \) is the wire resistance at the ambient gas temperature.

Equation 5.1 can be written as a time averaged equation and a small perturbation equation. The time averaged equation is

\[
\frac{\partial^2 \overline{\theta_w}}{\partial x^*^2} - \overline{\Pi} \overline{\theta_w} = -\frac{\overline{q}}{\alpha}
\]  

(5.6)

with the boundary conditions

\[
\overline{\theta_w}(x^* = \frac{1}{2}) = \overline{\theta}_{s1}
\]

\[
\overline{\theta_w}(x^* = -\frac{1}{2}) = \overline{\theta}_{s2} = n\overline{\theta}_{s1}
\]

(5.7)

where \( \Pi = \xi - q \), \( \overline{\Pi} = \overline{\xi} - \overline{q} \) and the overbar denotes temporal averaging.

Static temperature which is the solution to equation 5.6 with the boundary conditions of equation 5.7 is

\[
\overline{\theta_w}(x^*) = A \sinh \sqrt{\overline{\Pi}} x^* + B \cosh \sqrt{\overline{\Pi}} x^* + \frac{\overline{q}}{\alpha \overline{\Pi}}
\]

\[
A = \left( 1 - \frac{n}{2} \right) \frac{\overline{\theta}_{s1}}{\sinh \frac{\sqrt{\overline{\Pi}}}{2}} \quad B = \left( 1 + \frac{n}{2} \right) \frac{\overline{\theta}_{s1} - \frac{\overline{q}}{\alpha \overline{\Pi}}}{\cosh \frac{\sqrt{\overline{\Pi}}}{2}}
\]

(5.8)

Therefore spatially and temporally mean temperature of the wire is obtained as

\[
\langle \overline{\theta_w} \rangle = \int_{-\frac{1}{2}}^{\frac{1}{2}} \overline{\theta_w}(x^*) dx^* = \frac{2B}{\sqrt{\overline{\Pi}}} \sinh \frac{\sqrt{\overline{\Pi}}}{2} + \frac{\overline{q}}{\alpha \overline{\Pi}}
\]

(5.9)
and the static resistance of the wire is

\[ \overline{R}_w = R_g (1 + \alpha \langle \theta_w \rangle) \] (5.10)

The fluctuating part of equation 5.1 in Laplace transform is

\[ \frac{\partial^2 \theta'_w}{\partial x'^2} - \Pi_s \theta'_w = (\xi' - q') \overline{\theta}_w(x^*) - \frac{q'}{\alpha} \] (5.11)

with the boundary conditions

\[ \theta'_w(x^* = 1/2) = \theta'_{s1} \] \[ \theta'_w(x^* = -1/2) = \theta'_{s2} = n\theta'_{s1} \] (5.12)

where \( \Pi_s = \Pi (1 + \frac{T_w}{\Pi s}) \) and \( s \) is the Laplace variable. \( \xi' \) and \( q' \) may be written as

\[ \xi' = \frac{d\xi}{dU} u' \] \[ q' = \frac{dq}{dI} i' = 2 \frac{\bar{q}}{\bar{I}} i' \] (5.13)

where \( u' \) and \( i' \) are velocity and electric current fluctuations respectively. Substituting equations 5.8 and 5.13 into equation 5.11 gives

\[ \frac{\partial^2 \theta'_w}{\partial x'^2} - \Pi_s \theta'_w = (C_u \sinh \frac{\sqrt{\Pi}}{2} x^* + D_u \cosh \frac{\sqrt{\Pi}}{2} x^* + E_u)u' + \] \[ (C_i \sinh \frac{\sqrt{\Pi}}{2} x^* + D_i \cosh \frac{\sqrt{\Pi}}{2} x^* + E_i)i' \] (5.14)

where

\[ C_u = \frac{d\xi}{dU} A, \quad D_u = \frac{d\xi}{dU} B, \quad E_u = \frac{d\xi}{dU} \frac{\bar{q}}{\alpha \Pi} \] \[ C_i = -2 \frac{\bar{q}}{\bar{I}} A, \quad D_i = -2 \frac{\bar{q}}{\bar{I}} B, \quad E_i = -2 \frac{\bar{q}}{\bar{I}} \frac{\frac{\bar{q}}{\alpha \Pi} + \frac{1}{\alpha}} \] (5.15)
The solution to equation 5.14 and the boundary conditions introduced in equation 5.12 is

\[
\left[ \frac{\theta'}{u'} \right]_{s=0} = \frac{C_i}{\Pi - \Pi_s} \left( \sinh \sqrt{\Pi} x^* - \frac{\sinh \frac{\Pi_s}{2}}{\sinh \frac{\Pi}{2}} \sinh \sqrt{\Pi_s} x^* \right) + 
\]

\[
\frac{D_u}{\Pi - \Pi_s} \left( \cosh \sqrt{\Pi} x^* - \frac{\cosh \frac{\Pi_s}{2}}{\cosh \frac{\Pi}{2}} \cosh \sqrt{\Pi_s} x^* \right) + \frac{E_u}{\Pi_s} \left( 1 - \frac{\cosh \frac{\Pi_s}{2}}{\cosh \frac{\Pi}{2}} \right) + 
\]

\[
\left( \frac{1}{2} \sinh \frac{\sqrt{\Pi} s}{2} + \frac{1}{2} \cosh \sqrt{\Pi} x^* \right) \left[ \frac{\theta'}{u'} \right]_{s=0}, \quad (5.16)
\]

\[
\left[ \frac{\theta'}{\bar{u'}} \right]_{s=0} = \frac{C_i}{\Pi - \Pi_s} \left( \sinh \sqrt{\Pi} x^* - \frac{\sinh \frac{\Pi_s}{2}}{\sinh \frac{\Pi}{2}} \sinh \sqrt{\Pi_s} x^* \right) + 
\]

\[
\frac{D_i}{\Pi - \Pi_s} \left( \cosh \sqrt{\Pi} x^* - \frac{\cosh \frac{\Pi_s}{2}}{\cosh \frac{\Pi}{2}} \cosh \sqrt{\Pi_s} x^* \right) + \frac{E_i}{\Pi_s} \left( 1 - \frac{\cosh \frac{\Pi_s}{2}}{\cosh \frac{\Pi}{2}} \right) + 
\]

\[
\left( \frac{1}{2} \sinh \frac{\sqrt{\Pi} s}{2} + \frac{1}{2} \cosh \sqrt{\Pi} x^* \right) \left[ \frac{\theta'}{\bar{u'}} \right]_{s=0}, \quad (5.16)
\]

and the spatially averaged fluctuating temperature of the wire \( \langle \theta'_w \rangle \) with respect to velocity fluctuations \( u' \) and electric current fluctuations \( i' \) can be written as

\[
\left[ \frac{\langle \theta'_w \rangle}{u'} \right]_{s=0} = \int_{-\frac{1}{2}}^{\frac{1}{2}} \left[ \frac{\theta'_w(x^*)}{u'} \right]_{s=0} \, dx^* = \frac{D_u \cosh \frac{\sqrt{\Pi}}{2}}{\Pi - \Pi_s} \left( \frac{2 \tanh \frac{\sqrt{\Pi}}{2}}{\sqrt{\Pi}} - \frac{2 \tanh \frac{\sqrt{\Pi_s}}{2}}{\sqrt{\Pi_s}} \right) + 
\]

\[
\frac{E_u}{\sqrt{\Pi}} \left( 1 - \frac{2 \tanh \frac{\sqrt{\Pi_s}}{2}}{\sqrt{\Pi_s}} \right) + \frac{1 + n}{2} \frac{2 \tanh \frac{\sqrt{\Pi_s}}{2}}{\sqrt{\Pi_s}} \left[ \frac{\theta'_s}{\bar{u'}} \right]_{s=0}, \quad (5.17)
\]

\[
\left[ \frac{\langle \theta'_w \rangle}{i'} \right]_{s=0} = \int_{-\frac{1}{2}}^{\frac{1}{2}} \left[ \frac{\theta'_w(x^*)}{i'} \right]_{s=0} \, dx^* = \frac{D_i \cosh \frac{\sqrt{\Pi}}{2}}{\Pi - \Pi_s} \left( \frac{2 \tanh \frac{\sqrt{\Pi}}{2}}{\sqrt{\Pi}} - \frac{2 \tanh \frac{\sqrt{\Pi_s}}{2}}{\sqrt{\Pi_s}} \right) + 
\]

\[
\frac{E_i}{\sqrt{\Pi}} \left( 1 - \frac{2 \tanh \frac{\sqrt{\Pi_s}}{2}}{\sqrt{\Pi_s}} \right) + \frac{1 + n}{2} \frac{2 \tanh \frac{\sqrt{\Pi_s}}{2}}{\sqrt{\Pi_s}} \left[ \frac{\theta'_s}{\bar{i'}} \right]_{s=0}. \quad (5.17)
\]
5.2.2 Stubs and prongs

So far we have obtained the hot-wire time averaged temperature $\overline{\theta}_w$ (equation 5.8) and temperature perturbations $\theta'_w$ (equation 5.16) as functions of the wire characteristics and wire-stub junction average and fluctuating temperature $\overline{\theta}_{s1}$ and $\theta'_{s1}$. $\overline{\theta}_{s1}$ and $\theta'_{s1}$ have yet to be determined. Consider the right side stub and prong. Let $\theta_p$ be the instantaneous temperature of the stub-prong junction with respect to the ambient gas temperature. If the prong and the stub are treated as lumped systems and convective heat transfer is ignored for simplicity a heat balance for the prong yields

$$k_s A_s \frac{\theta_{s1} - \theta_p}{l_s} = k_p A_p \frac{\theta_p}{l_p} + \rho_p A_p l_p c_p \frac{d \theta_p}{dt} \frac{1}{2}$$  \hspace{1cm} (5.18)

where $l_s$, $A_s$ and $k_s$ are the stub length, cross-section area, and material thermal conductivity respectively and $l_p$, $A_p$, $\rho_p$, $k_p$, and $c_p$ are the prong length, cross-section area, material density, thermal conductivity, and specific heat respectively.

Under steady conditions equation 5.18 reduces to

$$\overline{\theta}_p = \frac{\overline{\theta}_{s1}}{K_1 + 1}$$  \hspace{1cm} (5.19)

where $K_1 = \frac{k_p A_p l_s}{k_s A_s l_p}$. Under dynamic conditions a perturbation analysis of equation 5.18 after applying Laplace transform yields

$$\theta'_p = \frac{\theta'_{s1}}{K_1(1 + \frac{1}{K_1} + \mathcal{T}_p s)}$$  \hspace{1cm} (5.20)

where $\mathcal{T}_p = \frac{\rho_p c_p l_p^2}{2k_p}$.

Similarly for the stub a heat balance yields

$$-\frac{k_w A_w}{l_w} \left[ \frac{d\theta_w}{dx^*} \right]_{x^*=\frac{1}{2}} = k_s A_s \frac{\theta_{s1} - \theta_p}{l_s} + \rho_s A_s l_s c_s \frac{d}{dt} \left( \frac{\theta_{s1} + \theta_p}{2} \right)$$  \hspace{1cm} (5.21)
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Under steady conditions with the aid of equation 5.19, equation 5.21 reduces to

\[
\left[ \frac{d\theta_w}{dx^*} \right]_{x^*=\frac{1}{2}} = -K_2 \bar{\theta}_{s1} \left( 1 - \frac{1}{1 + K_1} \right) \tag{5.22}
\]

where \( K_2 = \frac{k_s A_s l_w}{k_w A_w l_s} \).

And under dynamic conditions a perturbation analysis of equation 5.21 with the aid of equation 5.20 in Laplace transform is

\[
\left[ \frac{d\theta_w'}{dx^*} \right]_{x^*=\frac{1}{2}} = -K_2(1 + T_s s) \left( 1 - \frac{1 - T_s s}{1 + T_s s} \times \frac{1}{K_1\left( \frac{1}{K_1} + 1 + T_p s \right)} \right) \theta_{s1}' \tag{5.23}
\]

where \( T_s = \frac{\rho_s c_s l_s^2}{2k_s} \).

Now we are ready to eliminate \( \bar{\theta}_{s1} \) and \( \theta_{s1}' \) in equations 5.8 and 5.16 respectively. One may take the following steps to eliminate \( \bar{\theta}_{s1} \).

From equation 5.8 at \( x^* = \frac{1}{2} \) one has

\[
\bar{\theta}_{s1} = A \sinh \frac{\sqrt{\Pi}}{2} + B \cosh \frac{\sqrt{\Pi}}{2} + \frac{\bar{q}}{\alpha \Pi} \tag{5.24}
\]

From equation 5.9, \( B \) can be written as a function of the static wire resistance \( \overline{R_w} \), parameters \( \Pi, \bar{q} \) (effectively steady electric current \( \bar{I} \) and wire average Nusselt number \( \overline{Nu} \)) and wire properties as

\[
B = \left( \frac{\overline{R_w} - R_g}{R_g \alpha} - \frac{\bar{q}}{\alpha \Pi} \right) \frac{\sqrt{\Pi}}{2 \sinh \frac{\sqrt{\Pi}}{2}} \tag{5.25}
\]

From equation 5.8 by eliminating \( \bar{\theta}_{s1} \) between \( A \) and \( B \) and introducing \( B \) from equation 5.25 one has

\[
A = \frac{1 + \alpha}{1 + \alpha} \left[ \frac{\overline{R_w} - R_g}{R_g \alpha} - \frac{\bar{q}}{\alpha \Pi} \right] \frac{\sqrt{\Pi}}{2 \sinh \frac{\sqrt{\Pi}}{2}} \coth \frac{\sqrt{\Pi}}{2} + \frac{\bar{q}}{\alpha \Pi \sinh \frac{\sqrt{\Pi}}{2}} \tag{5.26}
\]
Using equation 5.8 to find \[
\frac{d\theta_w}{dx^*} \bigg|_{x^* = \frac{1}{2}}
\] and equation 5.24 for \(\bar{\theta}_{s1}\) and introducing them into equation 5.22 one obtains

\[
A \sqrt{\Pi} \cosh \frac{\sqrt{\Pi}}{2} + B \sqrt{\Pi} \sinh \frac{\sqrt{\Pi}}{2} = -K_2 \left( A \sinh \frac{\sqrt{\Pi}}{2} + B \cosh \frac{\sqrt{\Pi}}{2} + \bar{q} \alpha \Pi \right) \left( 1 - \frac{1}{1 + K_1} \right)
\]

(5.27)

where \(A\) and \(B\) are introduced from equations 5.26 and 5.25.

Equation 5.27 relates the static operating point of the CTA hot-wire system (static wire resistance \(R_w\) and electric current \(I\)) to the flow velocity past the wire and properties of the wire, stubs, and prongs.

In the dynamic equations 5.16 and 5.17 \(\theta'_{s1}\) is still unknown. In order to find it we can use equations 5.16 and 5.23 which yield

\[
\begin{bmatrix}
\theta'_{s1} \\
u'
\end{bmatrix}
\bigg|_{u' = 0} = \frac{N_u(s)}{D(s)}
\]

(5.28)

\[
\begin{bmatrix}
\theta'_{s1} \\
u'
\end{bmatrix}
\bigg|_{u' = 0} = \frac{N_i(s)}{D(s)}
\]

(5.29)

where

\[
N_u(s) = \frac{C_u}{\Pi - \Pi_s} \sinh \frac{\sqrt{\Pi}}{2} \left( \sqrt{\Pi} \coth \frac{\sqrt{\Pi}}{2} - \sqrt{\Pi_s} \coth \frac{\sqrt{\Pi_s}}{2} \right) + \frac{D_u}{\Pi - \Pi_s} \cosh \frac{\sqrt{\Pi}}{2} \left( \sqrt{\Pi} \tanh \frac{\sqrt{\Pi}}{2} - \sqrt{\Pi_s} \tanh \frac{\sqrt{\Pi_s}}{2} \right) + \frac{E_u}{\sqrt{\Pi_s}} \tanh \frac{\sqrt{\Pi_s}}{2}
\]

\[
N_i(s) = \frac{C_i}{\Pi - \Pi_s} \sinh \frac{\sqrt{\Pi}}{2} \left( \sqrt{\Pi} \coth \frac{\sqrt{\Pi}}{2} - \sqrt{\Pi_s} \coth \frac{\sqrt{\Pi_s}}{2} \right) + \frac{D_i}{\Pi - \Pi_s} \cosh \frac{\sqrt{\Pi}}{2} \left( \sqrt{\Pi} \tanh \frac{\sqrt{\Pi}}{2} - \sqrt{\Pi_s} \tanh \frac{\sqrt{\Pi_s}}{2} \right) + \frac{E_i}{\sqrt{\Pi_s}} \tanh \frac{\sqrt{\Pi_s}}{2}
\]

(5.29)

\[
D(s) = -K_2 (1 + T_\alpha s) \left[ 1 - \frac{1 - \frac{1}{1 + T_\alpha s}}{K_1 \left( \frac{1}{K_1} + \frac{1}{T_\alpha s} \right)} \right]
\]

\[
- \frac{1 - n}{2} \sqrt{\Pi_s} \coth \frac{\sqrt{\Pi_s}}{2} - \frac{1 + n}{2} \sqrt{\Pi_s} \tanh \frac{\sqrt{\Pi_s}}{2}
\]

(5.29)

Now from equations 5.28, 5.17 and 5.29 the spatially averaged temperature fluctuations of the hot-wire due to velocity and electric current fluctuations are fully determined.
5.2.3 Asymmetry: $n$

It is probable for a hot-wire probe that the boundaries of the wire filament are not similar. That is, one stub has different dimensions than the other one. We have simulated this asymmetry by fixing the temperature at one boundary $n$ times the other one in the hot-wire filament boundary conditions in equation 5.2. Assume that the radius of one stub is two times that of the other stub (cross section area is larger by a factor of four). One can roughly estimate that the temperature at the junction of the wire with the smaller stub is four times the temperature at the junction with the larger stub due to greater conductive heat transfer through the larger stub.

5.2.4 Hot-wire voltage perturbation

Let $E_w$ and $e'_w$ be the voltage drop across a constant temperature hot-wire and its perturbation relatively. If the electric current $I$ and velocity of the flow over the hot-wire $U$ are fluctuating about an operating point, to the linearised approximation

$$e'_w = \left[ \frac{\partial E_w}{\partial U} \right]_{u'=0} u' + \left[ \frac{\partial E_w}{\partial I} \right]_{u'=0} i'$$

where $u'$ and $i'$ are velocity and electric current perturbations.

Since $E_w = R_w I$ and as a result $e'_w = \overline{R_w i'} + \overline{T r'_w}$ we can use

$$\left[ \frac{\partial E_w}{\partial U} \right]_{u'=0} = I \left[ \frac{\langle r'_w \rangle}{u'} \right]_{u'=0} = I R_g \alpha \left[ \frac{\langle \theta'_w \rangle}{u'} \right]_{u'=0}$$

$$\left[ \frac{\partial E_w}{\partial I} \right]_{u'=0} = Z_w = \overline{R_w} + I \left[ \frac{\langle r'_w \rangle}{i'} \right]_{u'=0} = \overline{R_w} + I R_g \alpha \left[ \frac{\langle \theta'_w \rangle}{i'} \right]_{u'=0}$$

(5.31)

to find the sensitivity of the hot-wire filament to the velocity fluctuations and impedance of the hot-wire in constant temperature mode $Z_w$ (which are required for the CTA-hot-wire system transfer function) where $R_w$ and $\overline{I}$ are obtained from the steady operating point of the CTA hot-wire system and $\left[ \frac{\theta'_w}{u'} \right]_{u'=0}$ and $\left[ \frac{\theta'_w}{i'} \right]_{u'=0}$ are given in equation 5.17 with $\left[ \frac{\theta'_w}{u'} \right]_{u'=0}$ and $\left[ \frac{\theta'_w}{i'} \right]_{u'=0}$ given in equations 5.28 and 5.29.
It is noted that the analysis presented in § 5.2.1-5.2.4 is similar to that by Perry [116] with the following differences: here we consider an asymmetric filament with an arbitrary cross section form while Perry [116] analysed a circular filament with symmetric boundaries. Furthermore, in § 5.2.5 we incorporate a hot-wire with axial conduction in a CTA model, which has not been performed by Perry [116].

### 5.2.5 Hot-wire with a constant temperature anemometer

Assume that a hot-wire is operated by a CTA with the circuit shown in figure 5.2. $Z_a$, $Z_b$, and $Z_c$ are impedances of the Wheatstone bridge, and $Z_w$ is the impedance of the hot-wire given in equation 5.31. $K_a$ and $K_b$ are DC gains of the amplifiers used in the feedback loop and $E_{qi}$ is an offset voltage. A static analysis in which only static components of the impedances are considered ($R_a$, $R_b$, $R_c$, and $\overline{R}_w$) yields

$$\bar{I} = \frac{K_b E_{qi} (R_b + R_c)}{(R_b + R_c) R_a + \bar{R}_w + R_l} + K_a K_b \hat{R}$$

(5.32)

where $\hat{R} = [(\overline{R}_w + R_l) R_c - R_a R_b]$ and $R_l$ is the resistance of the cables connecting the wire to the CTA. Solving equations 5.27 and 5.32 together using a computing program such as MATLAB determines $\overline{R}_w$ and $\bar{I}$ and consequently $\bar{\xi}$, $\bar{q}$, and $\bar{\Pi}$, which are required for the subsequent dynamic analysis.

Watmuff [151] assumed the rational transfer functions for the electronic components of the CTA and the hot-wire impedance in Laplace transforms as

$$Z_a(s) = \frac{Z_{aN}(s)}{Z_{aD}(s)}, \quad Z_b(s) = \frac{Z_{bN}(s)}{Z_{bD}(s)},$$

$$Z_c(s) = \frac{Z_{cN}(s)}{Z_{cD}(s)}, \quad Z_w(s) = \frac{Z_{wN}(s)}{Z_{wD}(s)}$$

(5.33)

and treated the amplifiers with the governing equation $e_{oi} = K_i \frac{A_i(s)}{B_i(s)} \Delta e_i, (i = a, b)$ ($K_i$ is amplifier’s DC gain, $A_i(s)$ and $B_i(s)$ are polynomials for the zeros and the poles in the transfer function of the amplifiers respectively, $\Delta e_i$ is the amplifier’s input voltage difference and $e_{oi}$ is its output voltage) and derived the transfer function related to the
electric test signal for the hot-wire CTA system as

\[
e_o = \frac{K_b A_b(s) B_a(s) Z_1(s)}{K_a K_b A_a(s) A_b(s) Z_2(s) - B_a(s) B_b(s) Z_1(s)} (5.34)
\]

and the transfer function related to the velocity fluctuations for the hot-wire CTA system as

\[
\frac{e_o}{u'} = \frac{K_a K_b A_a(s) A_b(s) Z_{aN}(s) Z_{WD}(s) [Z_{bN}(s) Z_{cD}(s) + Z_{cN}(s) Z_{bD}(s)]}{K_a K_b A_a(s) A_b(s) Z_2(s) - B_a(s) B_b(s) Z_1(s)} \times \left[ \frac{\partial E_w}{\partial U} \right]_{_{t'=0}} (5.35)
\]

where

\[
Z_1(s) = [Z_{cN}(s) Z_{bD}(s) + Z_{bN}(s) Z_{cD}(s)] [Z_{aN}(s) Z_{WD}(s) + Z_{aW}(s) Z_{aD}(s)]
\]

\[
Z_2(s) = Z_{aN}(s) Z_{bN}(s) Z_{cD}(s) Z_{WD}(s) - Z_{aW}(s) Z_{cN}(s) Z_{bD}(s) Z_{aD}(s) - Z_{bN}(s) Z_{bD}(s) (5.36)
\]

In the equations 5.34-5.36, \([\frac{\partial E_w}{\partial U}]_{_{t'=0}}\), \(Z_{WN}(s)\), and \(Z_{WD}(s)\) are required which are obtained from equation 5.31 by fitting rational form transfer functions to the transcendental functions obtained for a distributed hot-wire filament.
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5.3 Experiments

5.3.1 Flow facility

Measurements were performed in the High-Reynolds-Number Boundary Layer Wind Tunnel located at the University of Melbourne which has a working section of $27 \times 2 \times 1$ m. Measurements are made on the tunnel floor at $x = 12.5$ m downstream from the tripped inlet at the nominal free-stream velocity $U_\infty = 20$ ms$^{-1}$ and friction Reynolds number $Re_\tau = 10000$. The pressure coefficient $C_p$ is constant to within $\pm 0.87\%$; therefore, the flow can be considered zero-pressure-gradient. Further details of the facility are available in §3.1.1.

5.3.2 Probes

Five hot-wire probes were used in the measurements fabricated to Dantec 55P05 boundary-layer-type probe with prong spacing of $\sim 3$ mm and Dantec 55P15 boundary-layer-type probe with prong spacing of $\sim 1.5$ mm. Silver coated platinum Wollaston wires were soldered to the prong tips and etched to reveal the desired length of the platinum-sensing element in the middle. The prongs are made from stainless steel passing through a ceramic holder. The probe/sensor specifications of all five probes used for the experiments presented in this chapter are summarised in table 5.1. The length of the platinum-sensing element is denoted by $l_w$ and $l_w^+ (= l u_\tau/\nu)$ in physical and viscous-scaled units respectively; its diameter is given by $d_w$ and its length-to-diameter ratio by $l_w/d_w$. The length and diameter of the unetched part of the Wollaston wire (stub) are given by $l_s$ and $d_s$ respectively; and prong length and diameter are given by $l_p$ and $d_p$ respectively. The cut-off frequency of the hot-wire-CTA system as introduced by Freymuth [34] is given by $f_c$. 
Table 5.1: Specifications of the probes used in the experiments. $l_w$ and $d_w$ are the hot-wire sensing element length and diameter respectively, $l_s$ and $d_s$ are the hot-wire stub length and diameter respectively, and $l_p$ and $d_p$ are the prong length and diameter respectively. $f_c$ is the cut-off frequency of the hot-wire-CTA system.

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<th>$l_w$ (µm)</th>
<th>$l_w^+$</th>
<th>$d_w$ (µm)</th>
<th>$l_w/d_w$</th>
<th>$l_s$ (µm)</th>
<th>$d_s$ (µm)</th>
<th>$l_p$ (µm)</th>
<th>$d_p$ (µm)</th>
<th>$f_c$ (kHz)</th>
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</tbody>
</table>

Table 5.2: Dimensions of the probes used in the simulations. Wire sensing element material is platinum, stub material is silver, and prongs are steel.

<table>
<thead>
<tr>
<th></th>
<th>$l_w$ (µm)</th>
<th>$d_w$ (µm)</th>
<th>$l_w/d_w$</th>
<th>$l_s$ (µm)</th>
<th>$d_s$ (µm)</th>
<th>$l_p$ (µm)</th>
<th>$d_p$ (µm)</th>
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<td>400</td>
<td>1000</td>
<td>50</td>
<td>8000</td>
<td>300</td>
</tr>
<tr>
<td>SHW2</td>
<td>500</td>
<td>2.5</td>
<td>200</td>
<td>500</td>
<td>50</td>
<td>8000</td>
<td>300</td>
</tr>
<tr>
<td>SHW3</td>
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<td>100</td>
<td>625</td>
<td>50</td>
<td>8000</td>
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</tr>
<tr>
<td>SHW4</td>
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<td>690</td>
<td>50</td>
<td>8000</td>
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</tr>
<tr>
<td>SHW5</td>
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<td>500</td>
<td>40</td>
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<tr>
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<td>40</td>
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<tr>
<td>SHW7</td>
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<tr>
<td>SHW8</td>
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<td>5</td>
<td>50</td>
<td>625</td>
<td>40</td>
<td>8000</td>
<td>300</td>
</tr>
</tbody>
</table>
5.3.3 Constant-temperature anemometry

An in-house Melbourne University Constant Temperature Anemometer (MUCTA) designed based on references [116] and [113] was used to operate the hot-wires with an overheat ratio of 1.65. In the MUCTA circuit a 10:1 ratio Wheatstone bridge is used with $R_a = 100 \, \Omega$, $R_c = 1000 \, \Omega$, and variable $R_b$ to set the desired overheat ratio. A high performance instrumentation amplifier (simple pole) with a DC gain $K_a = 100$ and corner frequency $f \approx 750 \, \text{kHz}$ is used as the first stage amplifier and a differential amplifier configuration as the second stage amplifier. By selecting appropriate resistors and capacitors, the desired corner frequency and DC gain of the second stage amplifier can be achieved. An electronic square-wave signal injected to the second stage amplifier is utilized to ensure a second-order system response, and damping of the system is controlled by changing the offset voltage. The CTA-hot-wire system was set slightly overdamped for all the hot-wires to ensure their stability throughout the boundary layer. The square-wave responses (normalised by their peak response value) for HW1 and HW3 at free-stream velocity are shown in figure 5.3 to demonstrate the degree of damping for all the hot-wires.

5.3.4 Wire calibration and data acquisition

The hot-wires were statistically calibrated in situ against a Pitot-static tube in the free-stream before and after each boundary layer traverse; and to further reduce the errors associated with calibration, the procedure proposed by Talluru et al. [137] was applied. Both third-order and forth-order polynomials were used to fit to the calibration data and no appreciable difference was observed in the first and second order statistics.

The hot-wire signals were sampled using a Data Translations DT9836 analogue-digital converter at $f_s = 50 \, \text{kHz}$ resulting in a viscous-scale normalised sampling frequency of $f_s^+ = f_s u_r/\nu = 1.77$. To ensure the aliasing issue is avoided, the signals were low-pass filtered using an analogue filter at $f_{lp} = 25 \, \text{kHz}$ (therefore $f_{lp}^+ = f_{lp} u_r/\nu = 0.89$). Hutchins et al. [47] have shown that there is negligible energy content in wall-bounded
Figure 5.3: System square-wave response (normalised) for HW1 (——) and HW3 (---) at freestream velocity $U_\infty = 20 \text{ ms}^{-1}$. Both systems are optimally damped as suggested by Freymuth [34].

turbulent flows for $f^+ > 0.33$. The total sampling time $T$ must be long enough for acquiring converged statistics and energy spectra. Here $TU_\infty/\delta = 15000$.

Each measurement was made at least twice to ensure that the results were repeatable and if the mean and/or the broadband turbulence intensity did not match to within $\pm 0.5\%$ the dataset was discarded.

5.4 Results and discussion

In order to investigate the effects of the end conduction losses on the hot-wire-CTA system transfer function, the model developed in §5.2 is employed. The effects of various parameters on the dynamic response of hot-wire-CTA systems as predicted by the model are demonstrated in §5.4.1. For the purpose of modelling, the MUCTA circuit as described in §5.3.3 and 8 hot-wire probes with the dimensions given in table 5.2 are used. The hot-wire filament, stubs, and prongs material are similar to those described in §5.3.2. In §5.4.2 experimental results from the turbulent boundary layer flow measurements with different hot-wire probes (introduced in table 5.1) are presented to further discuss the validity of the theoretical model’s predictions.
5.4.1 Theoretical model results

Hot-wire-CTA systems are usually calibrated statically, therefore their dynamic sensitivity against static sensitivity is of great importance since it can affect the measured statistics that are comprised of fluctuations with a wide range of frequencies. The transfer function of a system illustrates its dynamic sensitivity. Here we first consider SHW1-SHW4 operated with the MUCTA with an overheat ratio of \( \sim 1.7 \) in a flow with a mean velocity \( \bar{U} = 10 \text{ ms}^{-1} \). The magnitude of the sensitivity to the velocity fluctuations for the hot-wire filaments \( \left| \frac{\partial E_w}{\partial U} \right| \) together with the sensitivity for an ideal wire without end conduction losses are shown in figure 5.4(a). The model shows negative low frequency steps (one at \( f < 1 \text{ Hz} \) and the other at \( f \sim 100 \text{ Hz} \); the second step is evident for SHW4 only) in the wire filament sensitivity, the sizes of which increase...
with a decrease in the length-to-diameter ratio. The location of the first step (lower frequency) is associated with the time response of the prongs, and that of the second step is associated with the time response of the stubs. This behaviour is similar to the cold-wire behaviour that has been theoretically and experimentally investigated thoroughly (see e.g. references [4, 107, 142, 156]). However, behaviour of the hot-wire-CTA system transfer function $\frac{\phi}{\phi}$ seems to be different as shown in figures 5.4(b) and 5.4(c). The low frequency steps are positive for SHW1-SHW3 and the step size increases with a decrease in the length-to-diameter ratio; however, a further decrease in length-to-diameter ratio results in a flip in the system’s behaviour and SHW4 with $l_w/d_w = 50$ has negative steps. Although the step sizes are not large in the plots of figure 5.4 (0.5 – 2%), they can increase with changes to some of the parameters which vary from one wire to another.

In order to better appreciate the effect of dynamic sensitivity of the hot-wire-CTA system on the turbulence intensity measurements, an energy spectrum measured in the turbulent boundary layer flow by a hot-wire with $l_w/d_w = 400$ located at $z^+ \approx 700$ is used as an input to the modelled hot-wire-CTA systems to predict their simulated measured energy spectra. Let $\phi_{uu_i}(f)$ be the input energy spectrum and $\phi_{uu_m}(f)$ be the simulated measured energy spectrum. These two and the hot-wire-CTA transfer function are related through

$$\phi_{uu_m}(f) = \frac{\alpha_o}{u'} \phi_{uu_i}(f)$$  (5.37)

Since $u^2 = \int_0^\infty \phi_{uu} df$, the error in the measured turbulence intensity can be predicted as

$$\mathcal{E}(\%) = \frac{u'^2_m - u'^2_i}{u'^2_i} \times 100 = \frac{\int_0^\infty (|\phi_{uu}| - 1)\phi_{uu_i} df}{\int_0^\infty \phi_{uu_i} df} \times 100$$  (5.38)

Here $\frac{\phi_{uu}}{u'}$ is calculated by the theoretical model (equation 5.35) to predict the error due to inequality of dynamic and static sensitivities (end conduction effect).

We first discuss the effect of damping of the hot-wire-CTA system on the error in the measured turbulence intensity due to the end conduction. In the CTA circuit shown in figure 5.2 damping is controlled by the offset voltage $E_{qi}$. A square-wave signal is
injected to the offset voltage node to test the damping of the system. Square-wave responses for the CTA with a 5 \( \mu \)m-diameter hot-wire with different damping values \( \zeta \) as predicted by equation 5.34 are shown in figure 5.5. Error in the measured turbulence intensity as a function of damping as predicted by the model is shown in figure 5.6. It can be seen that the errors from the hot-wires with \( l_w/d_w = 400 \) and 200 are relatively insensitive to the damping. However, the errors for the hot-wires with \( l_w/d_w = 100 \) and 50 vary significantly with variations in the damping. Specifically one can see that for the hot-wires with \( l_w/d_w = 100 \) the model predicts positive errors for low damping values \( (\zeta < 1.5) \) and negative errors for high damping values \( (\zeta > 1.5) \). For the theoretical model results presented hereafter a damping of \( \zeta \approx 0.6 \) is used.

Conduction constants \( K_1 = \frac{k_p A_p l_s}{k_s A_s l_p} \) and \( K_2 = \frac{k_s A_s l_w}{k_w A_w l_s} \) play an important role in the transfer function and subsequently the measured turbulence intensity since they determine the end conduction losses to the supports and the effect of the stubs and prongs on the system transfer function. Perry, Smits, and Chong [114] noted that these constants should be used with caution because of the presence of tapers and dissimilarities in the metallic interface of the stubs that can change the properties of materials. We introduce
Figure 5.6: Error in the measured turbulence intensity $E$ at $z^+ \approx 700$ as predicted by the model as a function of damping $\zeta$; $\bar{U} = 10\text{ms}^{-1}$ and $0.7 < R < 0.75$ ($R$ is Overheat ratio) for all the wires. a SHW1-SHW4. b SHW5-SHW8.

A constant $\gamma$ so that

$$K_2 = \frac{\gamma k_s A_s l_w}{k_{iw} A_{iw} T_s}$$  \hspace{1cm} (5.39)

to take into account these dissimilarities and also to investigate the effect of $K_2$ on the transfer function of different hot-wire-CTA systems. The error in the measured turbulence intensity $E$ as a function of $\gamma$ for SHW1-SHW8 is shown in figure 5.7. When $\gamma = 0$ the problem reduces to a wire without end conduction losses and $E = 0$. Another important prediction is that by increasing $\gamma$ (effectively $K_2$), the error magnitude increases and reaches a maximum and with a further increase in $\gamma$ the error magnitude drops monotonically. Since the conduction coefficient $K_2 = 0$ cannot be realised in practice these results suggest that in order to reduce the errors due to conduction effects, $K_2$ should be increased, which may be achieved by increasing the stub thermal conductivity and cross-section area or by decreasing its length. (Geometrical changes should be considered with caution since these might cause aerodynamic effects from the stubs or prongs.) Moreover one can see that for both 2.5 and 5 $\mu$m-diameter wires when $l_w/d_w = 50$, the error is negative, i.e. the dynamic sensitivity is less than the static sensitivity, and for the other cases examined here it is positive.

Figure 5.8 shows the effect of asymmetry of the hot-wire filament given by $n$ (in equation 5.2) on the system transfer function and as a result on $E$. One can see that error magnitude increases with $n$ for all the probes while the sign of the error is negative when $l_w/d_w = 50$ and positive for all other probes tested here. Also from the slopes
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![Diagram](image1)

**Figure 5.7:** Error in the measured turbulence intensity $E$ at $z^+ \approx 700$ as predicted by the theoretical model as a function of $\gamma$ (introduced in equation 5.39); $\bar{U} = 10\text{ms}^{-1}$ and $1.7 < R < 1.75$ for all the wires. (a) SHW1-SHW4. (b) SHW5-SHW8.

![Diagram](image2)

**Figure 5.8:** Error in the measured turbulence intensity $E$ at $z^+ \approx 700$ as predicted by the theoretical model as a function of the asymmetry present in the hot-wire filament represented by $n$; $\bar{U} = 10\text{ms}^{-1}$ and $1.7 < R < 1.75$ for all the wires. (a) SHW1-SHW4 ($\gamma = 0.06$ is used). (b) SHW5-SHW8 ($\gamma = 0.1$ is used).

![Diagram](image3)

**Figure 5.9:** Error in the measured turbulence intensity $E$ at $z^+ \approx 700$ as predicted by the theoretical model as a function of wire Reynolds number $Re_w = \frac{U d}{\nu}$; $\bar{U} = 10\text{ms}^{-1}$ and $1.7 < R < 1.75$ for all the wires. (a) SHW1-SHW3 ($\gamma = 0.06$ is used). (b) SHW5-SHW7 ($\gamma = 0.1$ is used).

of the curves it can be seen that in general the shorter wires are more sensitive to asymmetry than the longer wires, and the error magnitude can reach up to 6% for the
Figure 5.10: Error in the measured turbulence intensity $\mathcal{E}$ at $z^+ \approx 700$ as predicted by the theoretical model as a function of $l_w/d_w$ for $n = 1$ (---) and $n = 4$ (--.--); $\bar{U} = 10\text{ms}^{-1}$ and $R \approx 1.8$.

wires with a length-to-diameter ratio of 50 and 100 in extreme asymmetry cases.

Figure 5.9 demonstrates the effect of wire Reynolds number $Re_w = \frac{\bar{U}d_w}{\nu}$ on the error in the measured turbulence intensity (for the range $0 < Re_w < 35$ equivalent to $0 < \bar{U} < 100 \text{ms}^{-1}$ for the 5 $\mu$m-diameter wires and $0 < \bar{U} < 200 \text{ms}^{-1}$ for the 2.5 $\mu$m-diameter wires). Since an increase in Reynolds number gives rise to an increase in the convective heat transfer, measurement at higher Reynolds numbers is expected to reduce the errors associated with conduction losses, which is in accordance with our results. It is noteworthy, however, that Reynolds number is more effective for the wires with a negative error ($l_w/d_w = 50$) than the ones with a positive error.

It was seen in all of the above-mentioned cases (for $\zeta = 0.6$) that error is positive for $l_w/d_w = 100$, 200 and 400, while it is negative for $l_w/d_w = 50$. Figure 5.10 shows the error in the measured turbulence intensity as a function of $l_w/d_w$ for a 2.5 $\mu$m-diameter hot-wire for $n = 1$ and $n = 4$ when other parameters are kept constant. It appears that for very small $l_w/d_w$ values ($l_w/d_w < 60$) $\mathcal{E}$ is negative; by increasing the $l_w/d_w$, $\mathcal{E}$ crosses zero at $l_w/d_w \approx 60$ and reaches a (positive) peak at $l_w/d_w \approx 100$. However, the positive peak is usually small unless for a hot-wire with large asymmetry coefficient $n$. This probably is part of the reason why most of the experimental studies with conventional
hot-wires have not reported amplification in the turbulence intensity when $l_w/d_w$ is not large enough ($l_w/d_w < 200$).

5.4.2 Experimental results

It is challenging to directly determine the hot-wire-CTA transfer function experimentally with high accuracy since a known, accurate, high frequency velocity perturbation input is needed, which in turn requires special apparatus. A review of notable attempts at direct methods can be found in references [126] and [49]. In the present study according to our need to investigate the effect of hot-wire length-to-diameter ratio on the turbulent measurements in wall-bounded flows we use the boundary layer flow as the input to the hot-wire-CTA system. Five hot-wire probes with different length-to-diameter ratios with the specifications summarised in table 5.1 are employed to measure velocity fluctuations across the turbulent boundary layer. By comparing turbulence intensity and spectral content of the velocity signal at each wall-normal location from different hot-wires we can explore the low frequency effect of length-to-diameter ratio on the system transfer function.

Figure 5.11(a) shows streamwise mean velocity and figure 5.11(b) shows associated broadband turbulence intensity profiles measured by HW1, HW2, HW3, and HW5 normalised in wall units at friction Reynolds number $Re_{\tau} = 10000$ (HW4 will be analysed separately later). Here $u$ denotes the streamwise fluctuating velocity, $z$ is wall-normal distance, and the superscript + indicates normalisation by inner variables. Mean velocity profiles are all in very good agreement, while variations are observed in the streamwise intensity profiles. Since HW1 has the highest length-to-diameter ratio (and therefore the least likelihood of experiencing the end conduction effect) and cut-off frequency simultaneously, we adopt it as the reference probe and compare results from other probes with those from HW1. The wires’ lengths are not equal; as a result, observed differences in the turbulence intensity profiles might be due to the combination of spatial resolution and end conduction effects. Therefore the effect of end conduction on the system transfer function cannot be fully comprehended by examining the broadband
Figure 5.11: Comparison of first and second order statistics measured by HW1 (---), HW2 (----), HW3 (○), and HW5 (-----) normalised in wall units. (a) Streamwise mean velocity ($U$) and (b) streamwise turbulence intensity $u^2$.

Figure 5.12: Comparison of premultiplied energy spectra against frequency at sample $z$ locations measured by HW1, HW2, HW3, and HW5. (a) $z^+ = 17$, (b) $z^+ = 67$, (c) $z^+ = 190$, (d) $z^+ = 720$, (e) $z^+ = 1600$, (f) $z^+ = 4600$. Symbols are as in figure 5.11.
turbulence intensity profiles alone. In this case comparing energy spectra at different wall normal locations can offer useful insight into the differences between spatial resolution and end conduction effects and reveal further information about the influence of insufficient $l_w/d_w$ on the system transfer function. Energy spectra at sample wall normal locations associated with the turbulence intensity profiles of figure 5.11(b) are shown in figure 5.12 in a premultiplied form so that on the semilogarithmic plot, the area under the curve is equal to $u'^2$ at each wall normal location. The following points can be deduced from comparing the spectra in the figure:

- When the length-to-diameter ratio is not large enough (HW2 and HW5), attenuation or amplification in energy is observed starting at a relatively low frequency ($\sim 10$ Hz), which is associated with the relatively large time response of the stubs and/or prongs. This effect extends to higher wall normal locations than the spatial resolution effect does since the former is related to the transfer function of the hot-wire-CTA system and changes relatively slowly with the mean velocity while the latter is associated with the Kolmogrov length scale, which increases more rapidly with the distance from the wall [131].

- When the length-to-diameter ratio is insufficient both amplification and attenuation of energy are possible. Here we see amplification for HW2 with $l_w/d_w = 104$ and attenuation for HW5 with $l_w/d_w = 44$, which is in accordance with the theoretical model’s predictions. This low frequency effect is observed up to wall normal position $z^+ = 4600$ for HW2 and $z^+ = 700$ for HW5. This is in contrast with the conclusions made by Hutchins et al. [47] and Ligrani and Bradshaw [72] who reported attenuation of energy for all the wires with $l_w/d_w < 200$. Both studies used wires with different diameters to keep the spatial resolution constant while changing the length-to-diameter ratios. This adversely affects the system frequency response for wires with larger diameters and might result in attenuation of energy in the high frequency end of the spectra. The only way to confidently comment on this discrepancy would be to examine the energy spectra from those studies, which are currently unavailable.
• Attenuation due to the poorer spatial resolution effect seen in the results of HW3 compared with HW1 starts from medium frequencies and extends to very high frequencies in the spectra. It is also observed that these attenuations are alleviated more rapidly compared with those caused by the end conduction effect seen in the HW5 results with an increase in $z^+$. Therefore although HW3 and HW5 might show similar behaviour in the streamwise turbulence intensity profiles, one can distinguish them by examining the energy spectra.

• Higher energy levels measured by HW2 than those of HW1 throughout the boundary layer cannot be attributed to its better spatial resolution alone (especially far from the near wall peak) because the extra energy in the broadband turbulence intensity profile extends to a very high wall normal location, and the deviation from the reference in the energy spectra starts from relatively low frequencies (for all wall normal locations), which is a sign of the end conduction effect caused by wire filament-supports interaction.

Figures 5.13(a) and 5.13(b) show the streamwise mean velocity and associated broadband turbulence intensity profiles, respectively, normalised in wall units at friction Reynolds number $Re_\tau = 10000$ measured by HW1 and HW4. In this comparison wire lengths are equal having a matched spatial resolution while the length-to-diameter ratio is different. This case is similar to the Hutchins et al. [47] and Ligrani and Bradshaw [72] studies of the $l_w/d_w$ effect. Streamwise turbulence intensity measured by HW4 is seen to be larger than that measured by HW1 for $z^+ < 25$, nominally equal in the region $25 < z^+ < 60$, and less in the region $60 < z^+ < 700$. Spectra for measurements of figure 5.13 at sample wall normal locations are shown in figure 5.14. It can be seen that at $z^+ = 13$ and 23 a region of energy amplification starting at the relatively low frequency of 80 Hz exists. This region is followed by a region of energy attenuation starting at the relatively high frequency of 1000 Hz. Since the first amplification region starts at a low frequency, this ought to be related to the end conduction effects; however, the relatively high frequency attenuation is most likely related to the lower frequency response of the HW4-CTA system which is a natural characteristic of larger wires. Moving further away
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**Figure 5.13:** Comparison of first and second order statistics measured by HW1 (---) and HW4 (-----) normalised in wall units. (a) Streamwise mean velocity ($U$) and (b) streamwise turbulence intensity $u^2$.

**Figure 5.14:** Comparison of premultiplied energy spectra against frequency at sample $z$ locations measured by HW1 and HW4. (a) $z^+ = 13$, (b) $z^+ = 23$, (c) $z^+ = 150$, (d) $z^+ = 550$, (e) $z^+ = 1600$, (f) $z^+ = 6000$. Symbols are as in figure 5.13.
from the wall the amplification region disappears while the high frequency attenuation region persists. We suspect that this high frequency attenuation, which is related to the lower temporal resolution of larger diameter wires, is part of the reason for the observations in Hutchins et al. [47] and Ligiani and Bradshaw [72] where only attenuation was observed with decreasing $l_w/d_w$. The frequency response of each hot-wire-CTA system was not systematically checked by carefully examining the square-wave test response in those studies. This has been done here and the square-wave responses of HW1 and HW3 are shown in figure 5.3 as examples. Moving further away from the wall as the mean velocity of the flow increases the frequency response of the HW4-CTA system increases (the system transfer function becomes less damped) and the high frequency attenuation gradually diminishes.

5.5 Summary and conclusions

A theoretical model for the hot-wire-CTA system transfer function was presented that takes into account the axial heat conduction for the hot-wire filament, the stub’s and prong’s effects, and the asymmetry in the boundary conditions of the wire filament. In this model the hot-wire filament is a cylinder with an arbitrary cross-section shape making it suitable for modelling the dynamic response of ribbon-shaped sensors constructed by MEMS-based technology for high Reynolds number turbulence measurements (e.g. [6, 13]) as well as for conventional Wollaston wires used in constant temperature anemometry.

For wires with insufficient length-to-diameter ratio ($l_w/d_w < 200$), the model predicts both positive and negative steps in the overall transfer function of the system to the velocity fluctuations with different combinations of probe geometry and conductivity parameters. That is, depending on the probe’s characteristics, a hot-wire-CTA system can amplify (positive steps) or attenuate (negative steps) the turbulence energy while measuring the correct mean velocity if the wire is not long enough. It was observed that for the length-to-diameter ratios of 100 and 200, positive steps are more likely to exist.
in the system transfer function while for the length-to-diameter ratio of 50, negative steps are present. The step size generally increases with the decrease in the length-to-diameter ratio. The model predicts that increasing the conductivity ratio between the wire filament and the stubs decreases the size of the steps resulting in smaller associated errors in the turbulence intensity measurements. This can be achieved by increasing the stubs’ thermal conductivity and cross-section area or decreasing its length for a fixed hot-wire filament geometry and material. (Geometrical changes to the stubs must be considered with caution to avoid unwanted aerodynamic effects.) The model also shows that more asymmetric sensors cause larger errors associated with the low frequency end conduction effect, which can be as large as 6% in the cases of extreme asymmetry.

A set of experiments was conducted in the turbulent boundary layer flow at the relatively high friction Reynolds number \( Re_\tau = 10000 \) using five hot-wires with different length-to-diameter ratios and lengths. The hot-wire with the largest length-to-diameter ratio, frequency response, and shortest length (HW1) was chosen as the reference wire. Results from other wires were compared with those from HW1. Experimental results were in accordance with the theoretical model predictions confirming that both positive and negative low frequency steps associated with the end conduction effect were possible for hot-wires of different length-to-diameter ratios. These effects extend further away from the wall compared to those seen in spatial resolution effects and start from lower frequencies in the energy spectra.

The results of this chapter show that the effect of end conduction on the hot-wire results can be less predictable than was traditionally thought. With the increasing need for shorter wires for studying very high Reynolds number flows, the idea of relaxing the length-to-diameter ratio is tempting. However, this may cause significant errors in the measured turbulence intensity even at high Reynolds numbers. These errors can extend a considerable distance beyond the near wall region and influence the concluded similarity laws based on such results. It seems that the only practical way to check the presence of the end conduction related steps in the system transfer function is to use different wires.
for a measurement and compare the spectra to observe possible low frequency steps in the hot-wire-CTA system’s transfer functions.
Chapter 6

Assessment of the NSTAP

In this chapter results of several measurements in the HRNBLWT facility at various Reynolds numbers are presented. These measurements are conducted as a collaborative project between the Fluid Mechanics group from the University of Melbourne and the Gas Dynamics Lab group from Princeton University directed by Professor Alexander Smits. Several NSTAP probes together with 2.5 µm-diameter hot-wires are used for the measurements. NSTAPs are operated with a modified Melbourne University Constant Temperature Anemometer (MUCTA) as well as a Dantec Streamline anemometer. The conventional hot-wires are operated with the MUCTA. Mean velocity $U$ and streamwise turbulence intensity $u'^2$ profiles as measured by the NSTAPs are compared to those measured by the hot-wires. Among all the tested NSTAPs most of them are in agreement with the 2.5 µm-diameter hot-wires in the outer region of the turbulence intensity profile while some of them are not. Probable reasons for the conflicting behaviours of some of the NSTAPs are discussed briefly, although proof of these reasons remain an open topic for further studies. Comparison of the turbulence intensity profiles as measured by the NSTAPs against those of 2.5 µm-diameter hot-wires enables us to identify the reliable NSTAPs. NSTAPs that are in agreement with the 2.5 µm-diameter hot-wire in the outer region are noted as reliable. In chapter 7 results from these reliable NSTAPs will be discussed in more detail.
The friction velocity $u_\tau$ is determined using the Coles-Fernholz relation \cite{29}

$$\frac{U_\infty}{u_\tau} = \frac{1}{\kappa} \ln(\frac{\delta^* U_\infty}{\nu}) + C \tag{6.1}$$

where $\kappa=0.384$ and $C=3.3$ are taken from Monkewitz et al. \cite{92} who arrived at equation 6.1 for the composite profile. $\delta^*$ is the displacement thickness and $\nu$ is the kinematic viscosity of the gas. The composite profile is developed using an inner and an outer expression for the mean velocity. The inner expression describes the profile in the sublayer and the logarithmic law of the wall. The outer expression is an exponential function, which is added to the inner expansion. For more detail on the construction of the composite profile for the mean velocity see Chauhan et al. \cite{17}. The measured mean velocity $U$ is fitted to the composite profile of Chauhan et al. \cite{18} to determine the boundary layer thickness $\delta$. The log-law constants in the composite profile are $\kappa=0.384$ and $B=4.17$. Reynolds number $Re_\tau$ and boundary layer thickness $\delta$ listed in table 6.1 are calculated using the fitted composite profile.

### 6.1 Operation with the MUCTA

In this section, results of three NSTAP probes, namely NSTAP1, NSTAP2, and NSTAP3, operated with the modified MUCTA (modifications detailed in Chapter 4) are presented. Along with the NSTAPs, 2.5 $\mu$m-diameter hot-wires are employed as reference for the measurements at each Reynolds number. Overheat ratio defined as $R/R_g$ was 1.25-1.35 for the NSTAP measurements and 1.8 for the hot-wire measurements. Here $R$ is the hot resistance of the sensor during the operation and $R_g$ is its resistance at the ambient gas temperature. Measurements are made in the HRNBLWT at $x = 10$ and 21.5 m, details of which are given in table 6.1.
Table 6.1: Summary of the experimental conditions presented in this chapter.

<table>
<thead>
<tr>
<th>$Re_\tau$</th>
<th>$x$ (m)</th>
<th>$U_\infty$ (ms$^{-1}$)</th>
<th>$\nu/u_\tau$ (μm)</th>
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Inner-normalised streamwise mean velocity against wall height normalised in wall units for \( Re_\tau = 10000 \) and \( 14500 \) are shown in figure 6.1. The mean velocity profiles are seen to be in very good agreement for all the probes shown in the figure. Inner-normalised streamwise broadband turbulence intensity profiles associated with the measurements of figure 6.1 are shown in figure 6.2. For the purpose of comparison of the turbulence intensity profiles measured by different probes we divide the turbulence intensity profiles into two regions of near-wall region \( (z^+ < 100) \) and outer region \( (z^+ > 100) \). In the near-wall region, turbulence intensity as measured by the NSTAP probes is higher than those measured by the hot-wire at both Reynolds numbers. This is due to an extensively improved spatial resolution of NSTAP compared to the conventional 2.5 \( \mu \)m-diameter hot-wire. Hutchins et al. [47] showed that spatial resolution of the probes (with \( l^+ \leq 153 \)) does not affect the turbulence intensity profiles in the outer region \( (z^+ \geq 200) \). Therefore we are able to compare the NSTAP and hot-wire results in that region without being concerned about their different spatial resolutions. It can be seen in figure 6.2 that NSTAP1 and the 2.5 \( \mu \)m-diameter hot-wire are in very good agreement in the outer region while NSTAP2 and NSTAP3 show greater values at both Reynolds numbers. Moreover, NSTAP2 and NSTAP3 appear to give different results when compared to each other as well. We calculate the error in the inner-normalised turbulence intensity
Figure 6.2: Inner normalised streamwise turbulence intensity $u_2^+$ against inner normalised wall distance $z^+$ measured with probes of figure 6.1 at (a) $U_\infty = 20$ m/s, $x = 13$ m, $Re_\tau = 10000$ (b) $U_\infty = 20$ m/s, $x = 21$ m, $Re_\tau = 14500$. : 2.5 μm-diameter HW, : NSTAP1, + : NSTAP2, △ : NSTAP3. When an NSTAP probe is used more than once colour shading is used.

Figure 6.3: Error in the streamwise turbulence intensity as measured by the NSTAP probes shown in figure 6.2 against wall distance at (a) $U_\infty = 20$ m/s, $x = 13$ m, $Re_\tau = 10000$ (b) $U_\infty = 20$ m/s, $x = 21$ m, $Re_\tau = 14500$. : NSTAP1, + : NSTAP2, △ : NSTAP3.

profiles as measured by NSTAP probes using

$$E(\%) = 100 \times \frac{(u_2^+)_{NSTAP} - (u_2^+)_{HW}}{(u_2^+)_{HW}},$$

and the results are shown in figure 6.3. Focusing on the outer region it is evident that error associated with the measurements of NSTAP1 is less than 4%, which is within the accepted experimental error for the turbulence intensity measured by single wires. However, errors associated with NSTAP2 and NSTAP3 are around 10-30% in the outer
For further investigation of the NSTAP results, premultiplied streamwise energy spectra at sample wall normal locations for the measurements at \( Re_\tau = 10000 \) (figure 6.2(a)) and \( Re_\tau = 14500 \) (figure 6.2(b)) are shown in figures 6.4 and 6.5, respectively. Streamwise energy spectrum \( \phi_{uu} \) is defined as \( \int_0^\infty \phi_{uu} df = \overline{u'^2} \) in which \( f \) is the frequency. \( \phi_{uu}(f) \) quantifies contribution to turbulence energy from various scales having various frequencies. Premultiplied spectra are used so that equal areas equate to equal energy densities on the semilogarithmic plot: \( \int_0^\infty \phi_{uu} df = \int_0^\infty f \phi_{uu} d(\log f) \). At \( z^+ = 20 \) the superior spatial resolution of NSTAPs (\( l^+ \approx 2.5 \) compared to \( l^+ \approx 21 \)) leads to resolving all the small-scale fluctuations contributing to around 15% more energy in the turbulence intensity profile by NSTAP1. NSTAP2 and NSTAP3 exhibit larger differences which are likely amplifying the correct turbulence energy since in the outer region they amplify

\begin{figure}[h]
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\includegraphics[width=\textwidth]{figure6_4.png}
\caption{Comparison of the inner-normalised premultiplied energy spectra as measured by different probes for the measurements shown in figure 6.2(a) at \( Re_\tau = 10000 \). (a) \( z^+ = 20 \) (b) \( z^+ = 130 \) (c) \( z^+ = 800 \) (d) \( z^+ = 2800 \). \( \black\) : 2.5 \( \mu \)m-diameter HW, \( \green\) : NSTAP1, \( \red\) : NSTAP2, \( \blue\) : NSTAP3.}
\end{figure}
energy as well compared to NSTAP1 and the hot-wire. At z locations of more than 100 wall units (which is of concern here) it is evident at both Reynolds numbers that NSTAP2 and NSTAP3 amplify energy. This energy amplification starts from a relatively low frequency of $f = \mathcal{O}(10 \text{ Hz})$ and continues all the way up to high frequencies. Considering that the mean velocity profiles are in very good agreement, the discrepancies in the turbulence intensity profiles are likely to be due to dynamic issues rather than static ones.

A premultiplied streamwise spectrogram at $Re_\tau = 10000$ as measured by the hot-wire is shown in figure 6.6 and the excess energy $\Delta(f^+ \phi_{uu}^+)$ seen by NSTAP1, NSTAP2, and NSTAP3 are shown in figure 6.7. As seen previously, NSTAP1 measures minimal excess energy at $z^+ > 100$ while NSTAP2 and NSTAP3 show different levels of excess energy in the outer region which start at frequencies of $f = \mathcal{O}(10 \text{ Hz})$ around $z^+ = 100$ and
Figure 6.6: Premultiplied streamwise spectrogram as measured by the 2.5 µm-diameter HW for the measurement of figure 6.2(a).

gradually move to lower frequencies of \( f = O(1 \text{ Hz}) \) with the increase in the \( z^+ \). The same plots are shown for the measurements at \( Re_\tau = 14500 \) in figures 6.8 and 6.9 which indicate similar behaviours from NSTAP2 and NSTAP3 at this Reynolds number.

Although the modified MUCTA was able to operate the NSTAP probes and several of them were used to make measurements in the HRNBLWT and also the channel facility, the occurrences of probe breakage during operation was still excessive and the number of available NSTAP probes were limited. Therefore, a Dantec Streamline that has proved to operate NSTAP with less breakage occurrences at Princeton University, was employed to operate more NSTAP probes, results of which are presented in §6.2.
Figure 6.7: Excess energy content seen in pre-multiplied spectrogram of the streamwise velocity for the measurements of figure 6.2(a) (when compared to the 2.5 \( \mu \)m-diameter HW) as measured by (a) NSTAP1, (b) NSTAP2 and (c) NSTAP3.
Figure 6.8: Premultiplied streamwise spectrogram as measured by the 2.5 μm-diameter HW for the measurement of figure 6.2(b).

Figure 6.9: Excess energy content seen in pre-multiplied spectrogram of the streamwise velocity for the measurements of figure 6.2(b) (when compared to the 2.5 μm-diameter HW) as measured by (a) NSTAP2 and (b) NSTAP3.
6.2 Operation with Dantec Streamline system

In this section, results of several NSTAP measurements in the HRNBLWT at four different Reynolds numbers (summary of which are given in table 6.1) are presented. Seven NSTAP probes from a new batch (NSTAP4-NSTAP10) were operated with a Dantec Streamline in the constant temperature mode with a 1:1 circuit and an external resistance which was used to set the overheat ratio of the probe. For this batch of NSTAPs the unusually high overheat ratio of 1.45-1.55 was required to start the operation of the probes. With smaller overheat ratios it was found that the NSTAPs were not sensitive to velocity, i.e. were not operating. The initiation of operation of the NSTAPs was checked by conducting the square-wave test. When the overheat ratio was not sufficiently high, switching on the square wave signal didn’t yield a response from the NSTAP-CTA system. It is also noted that, contrary to the MUCTA, when the Dantec Streamline system is used to operate the NSTAPs, there is no control on the damping of the system and we have to proceed with any degree of damping that the system has. It was observed that usually the square-wave test showed an underdamped system.

Inner-normalised streamwise mean velocity profiles for these measurements are shown in figure 6.10. Similar to the measurements with the MUCTA, mean velocity profiles are all seen to be in very good agreement. The associated inner-normalised streamwise turbulence intensity profiles are shown in figure 6.11. The agreement in the turbulence intensity profiles between the NSTAP probes and the hot-wire in the outer region \((z^+ > 100)\) is significantly improved compared to the measurements made with the modified MUCTA; however, since a new batch of NSTAPs is used here a direct comparison between the anemometers is not possible at this stage. At \(Re_\tau = 10000\) and 14500 all NSTAPs follow the 2.5 µm-diameter hot-wire in the outer region and display resolving small-scale fluctuations in the near-wall region. However still some discrepancies are observed; namely NSTAP9 at \(Re_\tau = 6000\), and NSTAP5 at \(Re_\tau = 20000\) do not follow the other NSTAPs and the 2.5 µm-diameter hot-wire in the outer region. In the following these discrepancies will be discussed in more detail.
Chapter 6. Assessment of the NSTAP

Figure 6.10: Inner normalised streamwise mean velocity $U^+$ against inner normalised wall distance $z^+$ measured with different probes (NSTAP probes operated with the Dantec Streamline) at (a) $U_\infty = 20$ ms$^{-1}$, $x = 6$ m, $Re_\tau = 6000$, (b) $U_\infty = 20$ ms$^{-1}$, $x = 13$ m, $Re_\tau = 10000$, (c) $U_\infty = 20$ ms$^{-1}$, $x = 21$ m, $Re_\tau = 14500$, (d) $U_\infty = 30$ ms$^{-1}$, $x = 21$ m, $Re_\tau = 20000$. : 2.5 μm-diameter hot-wire, : NSTAP4, : NSTAP5, : NSTAP6, : NSTAP7, : NSTAP8, : NSTAP9, : NSTAP10. When an NSTAP probe is used more than once colour shading is used.
Figure 6.11: Inner normalised streamwise broadband turbulence intensity $u'^2$ against inner normalised wall distance $z^+$ measured with different probes (NSTAP probes operated with the Dantec Streamline) at (a) $U_\infty = 20\text{ ms}^{-1}$, $x = 6\text{ m}$, $Re_\tau = 6000$, (b) $U_\infty = 20\text{ ms}^{-1}$, $x = 13\text{ m}$, $Re_\tau = 10000$, (c) $U_\infty = 20\text{ ms}^{-1}$, $x = 21\text{ m}$, $Re_\tau = 14500$, (d) $U_\infty = 30\text{ ms}^{-1}$, $x = 21\text{ m}$, $Re_\tau = 20000$. ——: 2.5 μm-diameter hot-wire, ⋆: NSTAP4, ∧: NSTAP5, ★: NSTAP6, ▽:NSTAP7, ★★: NSTAP8, ◊: NSTAP9, ●: NSTAP10. When an NSTAP probe is used more than once colour shading is used.
Firstly, it is crucial to examine the repeatability of the measurements to ensure that the observed spreads in figure 6.11 are not related to precision error. To this end, results from experiment repeats with NSTAP5 and NSTAP10 at \( Re \tau = 6000 \) and NSTAP5 and NSTAP9 at \( Re \tau = 20000 \) are shown in figures 6.12 and 6.13, respectively. One can see that each NSTAP probe yields repeatable results demonstrating that the observed variations in the turbulence intensity profiles is between different NSTAP probes rather than different runs with the same probes. These results also demonstrate the quality of the experimental techniques employed to acquire the datasets.
It was previously mentioned that NSTAP5 and NSTAP9 exhibit differences in the turbulence intensity profiles in the outer region \((z^+ > 100)\). We focus on these probes in figure 6.14 where turbulence intensity profiles as measured by these two probes are compared with each other and a conventional hot-wire at \(Re\tau = 6000\) and \(20000\). It was shown that different repeats of each NSTAP probe were in very good agreement, therefore one run is used to represent each probe at each \(Re\tau\) in figure 6.14. Note that all the NSTAPs resolve the entire range of small-scale fluctuations that highly contribute to the intensity in the near-wall region leading to greater intensity levels in that region compared to the hot-wire. This is because of superior spatial resolution of the NSTAPs. In the outer region we can see that NSTAP9 exhibits attenuation of energy up to 5% at \(Re\tau = 6000\) and less than 4% across the majority of the boundary layer at \(Re\tau = 20000\), while NSTAP5 similar to NSTAP2 and NSTAP3 amplifies energy in this region. This
amplification leads to nearly 4% excess energy at $Re_\tau = 6000$ and 10% excess energy at $Re_\tau = 20000$. These results suggest that we can consider NSTAP5 results at $Re_\tau = 6000$ and NSTAP9 results at $Re_\tau = 20000$ acceptable since error is within ±4% which is regarded normal for intensity measured by the single hot-wires (see e.g. [97]). Note that all other NSTAP probes operated with the Dantec Streamline display very good agreement with the 2.5 $\mu$m-diameter hot-wire in the outer region. We will use this factor later to select the reliable NSTAPs and use their results for further discussions.

Based on our discussions so far, we can consider all the results of NSTAP1, NSTAP4, NSTAP6, NSTAP7, NSTAP8 and NSTAP10 and results of NSTAP5 at $Re_\tau = 6000$ and results of NSTAP9 at $Re_\tau = 20000$ as reliable.

We can further scrutinise the different behaviours of NSTAP5 and NSTAP9 shown in figure 6.14, by investigating their corresponding premultiplied energy spectra. Energy
spectra at sample $z$ locations associated with the broadband turbulence intensity of figure 6.14(a) is shown in figure 6.15. Unsurprisingly, in the near-wall region ($z^+ = 13$), improved spatial resolution of NSTAPs compared to the hot-wire, results in an improved resolving of the fluctuating energy due to the near-wall cycle, hence giving higher intensity level. However, in the outer region NSTAP5 exhibits amplification and NSTAP9 exhibits attenuation of energy compared to the hot-wire. These start at frequencies of $f = \mathcal{O}(10\ Hz)$ and extend to high frequencies.

Premultiplied energy spectra at sample locations for $Re_\tau = 20000$ associated with the turbulence intensity profiles of figure 6.14(b) are shown in figure 6.16. Similar to the spectra measured by NSTAP5 and NSTAP9 at $Re_\tau = 6000$, NSTAP5 demonstrates amplified energy and NSTAP9 demonstrates attenuated energy, both of which start at frequencies of $f = \mathcal{O}(10\ Hz)$. These low frequency behaviours are similar to the
low frequency behaviours observed for HW4 and HW5 demonstrated in §5.4.2 where conventional hot-wires were intentionally under-etched to investigate the effect of end-conduction on the hot-wires with different length-to-diameter ratios. It was observed for hot-wires, that depending on the length-to-diameter ratio, both attenuation and amplification of energy in the energy spectra is possible which usually started at relatively low frequencies \( f = \mathcal{O}(10 \, \text{Hz}) \).

In order to perceive the differences of NSTAP5 and NSTAP9 with the 2.5 µm-diameter hot-wire at all \( z \) locations we can use premultiplied spectrograms. The premultiplied energy spectrogram as measured by the 2.5 µm-diameter hot-wire at \( Re_\tau = 6000 \), associated with the turbulence intensity profiles in figure 6.14(a), is shown in figure 6.17 and the excess energy seen by NSTAP5 and NSTAP9 are shown in figure 6.18. In accordance with the observations of figure 6.15 it is evident that NSTAP5 amplifies and NSTAP9 attenuates energy in the outer region where according to Hutchins et al. [47] the 2.5 µm-diameter hot-wire with the length equating 21 wall units must be able to resolve all the turbulence energy content. Moreover, the positive and negative excess energy seen by NSTAP5 and NSTAP9 start from very low frequencies and hence cannot be attributed to the spatial resolution differences.

The spectrograms associated with the 2.5 µm-diameter hot-wire measurements and the excess energy measured by NSTAP5 and NSTAP9 at \( Re_\tau = 20000 \) are shown in figures 6.19 and 6.20, respectively. Again, similar to the results at \( Re_\tau = 6000 \), low frequency \( (f = \mathcal{O}(10 \, \text{Hz})) \) amplification and attenuation (by NSTAP5 and NSTAP9) is observed in the outer region where all probes used here are expected to measure identical energy.

At this stage, one might ask why some NSTAPs tend to display large variations in the turbulence intensity profiles and energy spectra? It was seen that among all 10 NSTAP probes presented in this chapter, NSTAP2, NSTAP3, and NSTAP5 showed amplified energy while NSTAP9 attenuated energy (at \( Re_\tau = 6000 \)). In §6.3 the probable reasons for these anomalous behaviours are discussed.
Figure 6.17: Premultiplied streamwise spectrogram as measured by the 2.5 μm-diameter hot-wire for the measurement of figure 6.14(a).

Figure 6.18: Excess energy content seen in pre-multiplied spectrogram of the streamwise velocity for the measurements of figure 6.14(a) (when compared to the 2.5 μm-diameter hot-wire) as measured by (a) NSTAP5 and (b) NSTAP9.

6.3 Why do some NSTAPs deviate from conventional hot-wires in the outer region?

It was shown in the previous sections that most of the NSTAP probes exhibited very good agreement with the conventional hot-wires in the turbulent boundary layer flow.
measurements at various Reynolds numbers. However, some of them displayed deviations from the rest in the broadband turbulence intensity profiles. These deviations are not likely to be attributed to the spatial resolution effects and cannot be considered legitimate because:
1. Different NSTAP probes exhibit different results while they have identical lengths; namely NSTAP2 and NSTAP3, both of which were operated with the modified MUCTA, exhibited different levels of magnification in the outer region of the broadband intensity profile. Also, NSTAP5 and NSTAP9 were both operated with the Dantec Streamline and showed dissimilarities in the intensity profiles.

2. Both magnification and attenuation of energy were observed in the outer region of the turbulence intensity profiles as measured by different NSTAP probes when compared to the longer 2.5 µm-diameter hot-wires. However, due to the improved spatial resolution of the NSTAPs compared to the hot-wires, the only probable difference in the turbulence intensity profile (regardless of the z locations) would be amplification of energy due to resolving small-scaled energetic fluctuations that might not be achievable by the longer wires.

3. The difference of concern in the turbulence intensity profile is located in the outer region where the contribution of small-scaled energetic fluctuations is known to be minimal. Hutchins et al. [47] have shown that effect of spatial filtering due to the sensor length on the turbulence intensity profiles does not exceed $z^+ = 200$ when hot-wires with the lengths of 22, 79, and 153 wall units are used to measure the streamwise velocity fluctuations.

In the following sections possible reasons for the anomalies observed in some of the NSTAP probes are proposed and discussed.

### 6.3.1 Constant temperature anemometers

Two CTAs were used to operate the NSTAPs: a modified MUCTA and a Dantec Streamline. Measurements with both anemometers contained reliable and unreliable results. NSTAP1 operated with the modified MUCTA, agreed with the hot-wire in the outer region of intensity profile while NSTAP2 and NSTAP3 amplified energy in this region. Similarly, NSTAP4, NSTAP6, NSTAP7, NSTAP8, and NSTAP10 that were operated
with the Dantec Streamline showed very good agreement with the hot-wires in the outer region of the turbulence intensity profiles, while NSTAP5 and NSTAP9 exhibited rather large variations.

The other fact that needs to be noted is that in the regions where unwanted amplification and attenuation of energy are noticed, the deviations from premultiplied spectra for conventional hot-wires start at relatively low frequencies of $f = \mathcal{O}(10 \text{ Hz})$, whereas the frequency response of the feedback loop in the anemometers is of $f = \mathcal{O}(100 \text{ kHz})$ [68], [152].

Considering the above-mentioned points, it is unlikely that anemometers are responsible for the inconsistencies seen in some NSTAPs’ results. It should be noted that Dantec Streamline does outperform the MCUTA in operation of NSTAPs in the sense that a lower occurrence of breakage was observed using the Dantec compared to the MUCTA. This provided the opportunity of testing more NSTAP probes with the Dantec Streamline.

### 6.3.2 Effect of pitch angle

The NSTAP sensor filament is a ribbon with a high aspect ratio, rather than the circular sensor as used in a hot-wire probe. As a result, high pitch angles have been seen to adversely impact the effective measured velocities by the NSTAPs compared to conventional hot-wires [145]. In the HRNBLWT facility, the probe holder is tilted at a shallow angle $\theta$ to allow for the sensor tip to approach the wall more easily. $\theta = 10^\circ$ is used for conventional hot-wires. This angle might be too large for the NSTAP since it is more sensitive to the large pitch angles. We investigated the effect of the probe holder angle with the tunnel floor by running a measurement with two probe holder angles $\theta = 10^\circ$ and $3^\circ$. Results of these measurements are shown in figure 6.21 and depict no effect on the mean velocity profiles. However, the turbulence intensity profile appears to be attenuated in the near-wall region when measurement is made with the larger probe holder angle. This comparison also reveals no difference in the outer region of the
Chapter 6. Assessment of the NSTAP

6.3.3 Calibration curves

One of the major sources of uncertainty in hot-wire anemometry is the error in the calibration, since any errors in calibration give rise to greater levels of uncertainty in the hot-wire measured velocity [137]. Hence, comparing the calibration curves of the inconsistent NSTAPs might be elucidating. Pre- and post-calibration curves of NSTAP5, NSTAP8, and NSTAP9 associated with the measurements of figure 6.11(d) are shown in figure 6.22. These specific probes were selected for this comparison since NSTAP8 appeared to be consistent with the conventional hot-wire in the outer region of the intensity profile while NSTAP5 and NSTAP9 exhibited deviations. Calibration drift appears to be negligible for all three probes and the 4th order polynomial curve fit passes through all the points with very good accuracy. Furthermore, mean velocity profiles measured with these three probes are in very good agreement with each other and the conventional hot-wire. Therefore, it appears implausible that calibration can be the source of the discrepancies observed in the turbulence intensity profiles.
6.3.4 Effect of temperature variations

When the ambient fluid temperature varies between the calibration and the measurements a correction procedure is required [14]. According to Perry [116], as a rule of thumb, if the temperature is within ±0.5° throughout the measurement and the calibrations, temperature corrections are unnecessary; otherwise a correction scheme is required. Since the temperature variations in the HRNBLWT exceed this rule of thumb bound at high speeds ($U_\infty > 20$ ms$^{-1}$) we checked the effect of temperature correction scheme on the NSTAP results in Appendix A. A temperature correction scheme proposed by Hultmark and Smits [42] was applied and a comparison of the results with and without the temperature correction did not reveal any significant effects on the NSTAP results.

6.3.5 End conduction effect

Consistency of the mean velocity profiles, as well as the calibration curves, indicate that the variations in the turbulence intensity profiles produced with some of the NSTAP probes is unlikely to be a static complication. Examination of the premultiplied energy spectra revealed deviations in the energy spectra measured by the unreliable NSTAPs
that initiated from low frequencies \(f = \mathcal{O}(10 \text{ Hz})\). Furthermore, it was argued that the observed variations could not be related to the spatial resolution effects. Therefore, these deviations must be rooted in a dynamic issue, i.e. transfer function of the anemometry system. In fact, the behaviours seen in this chapter from the deviating NSTAPs are very similar to the results of HW2 and HW5 presented in §5.4.2. NSTAPs are manufactured using MEMS-based technology which often suffers from lack of precision. For most MEMS machining processes, the dimensional control deteriorates as the size of the critical elements decreases and therefore the precision worsens\[36\]. This results in relatively large variations in the probe’s dimensions which, as shown in Chapter 5, can adversely impact the anemometry system transfer function if the aspect ratio of the sensor is not sufficiently large by producing low frequency steps in the transfer function of the probe-CTA system. It was also shown that the step could be positive or negative depending on the precise characteristics of the probe.

### 6.4 Reliable NSTAPs

So far NSTAPs that were conflicting with the conventional hot-wires in the outer region of the turbulence intensity profile were examined thoroughly and different aspects of their behaviours were investigated. Most of the NSTAP probes exhibited good agreement with the hot-wires in the outer region, where the improved spatial resolution of the NSTAPs is expected to have minimal influence on the measured energy. In this section we select the reliable NSTAPs through comparing them with the 2.5 \(\mu\text{m}\)-hot-wire results. Among the 10 NSTAP probes compared throughout the measurements, the reliable ones are: NSTAP1, NSTAP4, NSTAP5 (at \(Re_\tau = 6000\)), NSTAP6, NSTAP7, NSTAP8, NATAP9 (at \(Re_\tau = 20000\)) and NSTAP10. Results from these reliable NSTAPs are averaged to give smooth mean and turbulence intensity profiles at various Reynolds numbers and are shown in figures 6.23 and 6.24. The average of repeat of NSTAP measurements shown with blue lines exhibit very good agreement in the outer region of the turbulence intensity profile with the 2.5 \(\mu\text{m}\)-diameter hot-wire.
Chapter 6. Assessment of the NSTAP

Figure 6.23: Inner normalised streamwise mean velocity $U^+$ against inner normalised wall distance $z^+$ at (a) $U_\infty = 20$ ms$^{-1}$, $x = 6$ m, $Re_\tau = 6000$, (b) $U_\infty = 20$ ms$^{-1}$, $x = 13$ m, $Re_\tau = 10000$, (c) $U_\infty = 20$ ms$^{-1}$, $x = 21$ m, $Re_\tau = 14500$, (d) $U_\infty = 30$ ms$^{-1}$, $x = 21$ m, $Re_\tau = 20000$. —: 2.5 µm-diameter hot-wire, ---: reliable NSTAP measurements, ——: average of reliable NSTAP measurements. Colour shading is used for the average of NSTAPs at various Reynolds numbers.
Figure 6.24: Inner normalised streamwise broadband turbulence intensity $\frac{u'^2}{\langle u'^2 \rangle}$ against inner normalised wall distance $z^+$ at (a) $U_\infty = 20 \text{ ms}^{-1}$, $x = 6 \text{ m}$, $Re_\tau = 6000$, (b) $U_\infty = 20 \text{ ms}^{-1}$, $x = 13 \text{ m}$, $Re_\tau = 10000$, (c) $U_\infty = 20 \text{ ms}^{-1}$, $x = 21 \text{ m}$, $Re_\tau = 14500$, (d) $U_\infty = 30 \text{ ms}^{-1}$, $x = 21 \text{ m}$, $Re_\tau = 20000$. ---: 2.5 μm-diameter hot-wire, ---: reliable NSTAP measurements, ---: average of reliable NSTAP measurements. Colour shading is used for the average of NSTAPs at various Reynolds numbers.
6.5 Summary and conclusions

10 NSTAP probes together with the 2.5 µm-diameter hot-wires were used to measure streamwise velocity fluctuations in the HRNBLWT facility at various Reynolds numbers. Of these, NSTAP1, NSTAP2, and NSTAP3 were operated with a modified MUCTA and the rest (NSTAPs 4-10) were operated with a Dantec Streamline anemometer. Comparison of the mean velocity profiles as measured by the NSTAPs and the hot-wire exhibited very good agreement between all the NSTAPs and the hot-wire. However comparison of the turbulence intensity profiles revealed some discrepancies between some of the NSTAPs and the hot-wires in the logarithmic and outer region of the turbulence intensity profiles. Based on the study of Hutchins et al. [47] in this region minimal difference in resolved energy for wires of different $l^+$ is expected since small-scaled energetic fluctuations (that cannot be fully resolved with the longer hot-wires) are not dominant in the outer region. Moreover, the observed variations between NSTAPs and the hot-wires were not consistent, neither in size nor in sign. In fact both magnification and attenuation of turbulence intensity were observed for different NSTAP probes when compared to the hot-wires. An investigation of the premultiplied energy spectra revealed that the energy deviation in those NSTAPs, which disagreed with the hot-wires in the outer region of the turbulence intensity profiles, initiated at very low frequencies ($\sim 10$ Hz).

Measurements with both the MUCTA and the Dantec Streamline contained reliable NSTAPs and unreliable NSTAPs. This rules out the effect of the anemometer in the conflicting behaviours of some NSTAPs. Reliable NSTAPs refers to the NSTAP probes that agree with the 2.5 µm-diameter hot-wire in the outer region of the turbulence intensity profiles, with the remaining NSTAP probes referred to as unreliable.

The effect of pitch angle was investigated on the NSTAPs by conducting measurements with two probe holder angles $\theta = 3^\circ$ and $10^\circ$. These measurements revealed attenuation in the turbulence intensity in the near wall region for $\theta = 10^\circ$. No differences were observed in the outer region of the intensity profiles by changing $\theta$, eliminating a role
of the probe holder angle in the discrepancies between different NSTAPs and hot-wires observed here.

The effect of the ambient temperature variations during the measurements and quality of the calibration curves were discussed as well. It was shown in Appendix A that the ambient temperature variations have minimal effects on the results of the NSTAPs. Also, comparing calibration curves of three NSTAP probes revealed no differences that could explain the observed discrepancies in their measured energy. All the NSTAPs that we started the measurements with had similar physical characteristics, namely similar appearances and electrical resistances. The reason for the anomalous behaviour of some of the NSTAPs remains an open topic for further studies.
Chapter 7

Well-resolved measurements in turbulent boundary layer flows

Nano-Scale Thermal Anemometry Probes (NSTAP) [6] together with conventional hot-wire probes are used to measure streamwise velocity fluctuations in turbulent boundary layer flow at \( Re_\tau = 6000, 10000, 14500, 20000 \) and 25000. A summary of the measurement conditions are shown in table 7.1. Multiple measurement repeats were carried out at each Reynolds number (number of these repeats is given by \( N \) in table 7.1) and reliable NSTAPs have been selected through analysis detailed in chapter 6. These \( N \) repeats are averaged to obtain the averaged NSTAP at each \( Re_\tau \). In this chapter results from the reliable NSTAPs are reported in six sections:

- § 7.1 Streamwise mean velocity
- § 7.2 Streamwise turbulence intensity
- § 7.3 Spectra of the streamwise velocity fluctuations
- § 7.4 Spectrograms
- § 7.5 Skewness and kurtosis
- § 7.6 Turbulence energy dissipation rate and Kolmogrov scale characteristics
Table 7.1: Summary of the experimental conditions presented in this chapter.

<table>
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<th>$Re_{\tau}$</th>
<th>$x$ (m)</th>
<th>$U_\infty$ (ms$^{-1}$)</th>
<th>$\nu/u_\tau$ (µm)</th>
<th>$u_\tau$ (ms$^{-1}$)</th>
<th>$\delta$ (m)</th>
<th>$TU_\infty/\delta$</th>
<th>$\Delta t^+$</th>
<th>$l^+$</th>
<th>Probes</th>
<th>$N$</th>
<th>Symbols/lines</th>
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Chapter 7. Well-resolved measurements in turbulent boundary layer flows

7.1 Streamwise mean velocity

The streamwise mean velocity profile has been extensively studied in turbulent boundary layer flows (e.g. Österlund [106]), channel flows (e.g. Monty [94]), and pipe flows (e.g. Zagarola and Smits [159]). The studies support the existence of a logarithmic relation in the overlap region known as the law of the wall

\[ \frac{U}{u_r} = \frac{1}{\kappa} \log\left(\frac{z}{\delta}\right) + A \]  

(7.1)

where \( \kappa \) and \( A \) are universal constants in smooth wall turbulent flows. The universality of these constants have been questioned recently, with a wide range of \( \kappa \) reported for different types of wall-bounded turbulent flows (0.37-0.436 [78]). However, using experimental data in the Reynolds number range \( 2 \times 10^4 < Re < 6 \times 10^5 \) for boundary layers, pipe flow and the atmospheric surface layer, Marusic et al. [79] showed within experimental uncertainty, the data support the existence of a universal logarithmic region.

Figure 7.1(a) and (b) show the mean velocity profiles as measured by the NSTAP probes and conventional 2.5\( \mu \)m-diameter hot-wires together with the DNS data from Sillero et al. [128] in inner normalised and velocity defect form, respectively. Note that there is excellent agreement between the NSTAP and the hot-wire where both are available.

It should also be noted that the boundary layer thickness has been calculated by fitting the composite profile of Chauhan et al. [18]. Sillero et al. [128] used \( \delta_{99} \) to determine the boundary layer thickness yielding \( Re_\tau = \delta_{99}^+ \approx 1990 \), while if the composite profile fit is used to estimate the boundary layer thickness \( Re_\tau = \delta_{com}^+ \approx 2500 \) is obtained.

It is evident that all the profiles, when shown in the inner normalised form, follow the equation 7.1 with \( \kappa = 0.384 \) and \( A = 4.17 \) in the overlap region. The profiles in the velocity defect form in the overlap region follow

\[ \frac{U_\infty - U}{u_r} = B - \frac{1}{\kappa} \log\left(\frac{z}{\delta}\right) \]  

(7.2)
with $B = 2.3$.

**Figure 7.1:** (a) Inner normalised mean velocity $U^+$ against inner normalised wall distance $z^+$. Straight dot-dashed line indicates the log-law $U^+ = 1/0.384 \ln(z^+) + 4.17$. (b) Velocity defect law against outer normalised wall distance $z/\delta$. Straight dot-dashed line corresponds to the relation $U^+ = 2.3 - 1/0.384 \ln(z/\delta)$. \(\rightarrow\) NSTAP-$Re_{\tau} = 6000$, \(\rightarrow\) NSTAP-$Re_{\tau} = 10000$, \(\rightarrow\) NSTAP-$Re_{\tau} = 14500$, \(\rightarrow\) NSTAP-$Re_{\tau} = 20000$, \(\rightarrow\) hot-wire at the $Re_{\tau}$ in which NSTAP is used, \(\rightarrow\) hot-wire data at $Re_{\tau} = 25000$, \(\rightarrow\) DNS at $\delta_{99} = 1990$ (Sillero et al. [128]).
7.2 Streamwise turbulence intensity

While the mean flow field has received the majority of attention over the past few decades, the streamwise turbulence intensity ($\overline{u^2}$) has also been the subject of a considerable number of studies [78].

A profile of the streamwise turbulence intensity scaled on $u_\tau$ ($\overline{u^2+/}$) can be divided into four main regions within the boundary layer. The region closest to the wall ($0 \leq z^+ \leq 100$) is referred to as the near-wall region, which is followed by the intermediate region in the range $100 \leq z^+ \leq 3.9Re_\tau^{1/2}$. The inertial sublayer (or logarithmic region) is located in the range $3.9Re_\tau^{1/2} \leq z^+ \leq 0.15Re$. Finally we have the outer region in the range $0.15Re \leq z^+ \leq Re_\tau$. These regions will be discussed in detail in the following sections.

7.2.1 The near-wall region in the streamwise turbulence intensity

In the near-wall region a maximum appears in the streamwise turbulence intensity profile, known as the inner peak. The inner normalised wall normal location of this peak is commonly accepted to be independent of the Reynolds number and fixed at $z^+ \approx 15$. However, the Reynolds number scaling of the magnitude of the inner peak has been controversial. While most of the recent studies show that the inner peak exhibits a weak $Re$ dependence (De Graaff and Eaton [25], Hutchins and Marusic [46], Klewicket and Falco [59], Lee and Moser [67], Metzger et al. [86], among others), Superpipe and HRTF data (Hultmark et al. [44, 45], Vallikivi et al. [147]) show that this peak is invariant with $Re$. It is however pointed out that experimental uncertainty could be masking any weak $Re$ dependence. The studies that support the growth of the inner peak with $Re$ took special care to ensure that spatial resolution issues did not influence the results, since several studies suggest that finite probe size leads to the attenuation of the turbulence intensity in the near-wall region (Johansson and Alfredsson [53], Willmarth and Sharma [154], Ligrani and Bradshaw [72], Hites [40], and Hutchins et al. [47]).
Figure 7.2(a) and (b) show the turbulence intensity profiles scaled with $u_\tau$ at various $Re_\tau$ against inner and outer normalised wall distance respectively. The NSTAP and the conventional hot-wire exhibit very good agreement in the outer region where spatial resolution influence is minimal. While the hot-wire results do not show a clear $Re_\tau$ trend for the inner peak (because of spatial resolution effects), NSTAP measurements exhibit a growth of the inner peak with increasing $Re_\tau$. As $l^+ < 3.5$ for all the NSTAP measurements, we can compare our results with the DNS from Sillero et al. [128] to see the low Reynolds number effect on the inner peak trend. The inner peak values scaled on $u_\tau$ are plotted against $Re_\tau$ in figure 7.3 for the data from the current study as well as for DNS of boundary layer [128] and channel flow [41]. Also inner peaks measured in the Superpipe and HRTF are shown at the Reynolds numbers where the peak has been resolved. It is evident that our experimental results extends the logarithmic trend of $u_{2+\text{max}}$ previously seen in the DNS data. Dependence of the peak in $u_{2+}$ on $Re_\tau$ was fitted to the boundary layer DNS and our higher $Re$ boundary layer data to obtain

$$u_{2+\text{max}} = 3.54 + 0.646 \log(Re_\tau).$$

(7.3)

This relation is shown in figure 7.3 together with the correlations proposed by Lee and Moser [67] ($u_{2+\text{max}} = 3.66 + 0.646 \log(Re_\tau)$) and Monkewitz and Nagib [91] (their equation 4.1). While proximity of the two former equations is evident, one can see that the latter underpredicts $u_{2+\text{max}}$ for $Re_\tau < 1000$ and overpredicts it for $Re_\tau > 1000$ (at least in the domain shown here).

Another way of understanding the near wall behaviour of the turbulence intensity is decomposing $u_{2+}$ into small-scale $u_{s+}^2$ and large-scale $u_{l+}^2$ components such that $u_{2+} = u_{s+}^2 + u_{l+}^2$ [77]. In order to compute these components, the fluctuating $u^+$ signal was separated into small-scale $u_s^+$ ($\lambda_x < 10000$) and large-scale $u_l^+$ ($\lambda_x > 10000$) components using a sharp cut-off spectral filter. Then small-scale and large-scale variances ($u_{s+}^2$ and $u_{l+}^2$) were computed separately from these high and low pass filtered components. The choice of the cut-off wavelength was based on the information found
in the energy spectra presented in §7.3.1. It will be shown that energy spectra in the near wall region collapse in the domain $\lambda_x < 10000$ for the $Re_\tau$ range of this study. The results of this decomposition are shown in figure 7.4. It can be seen that the small-scale

![Graph](image)

**Figure 7.2:** Inner normalised turbulence intensity profile $u^+$ against (a) inner normalised wall distance $z^+$; (b) outer normalised wall distance $z/\delta$. Dotted line corresponds to $u^+_{max} = 5.4 - 0.642 \log(z_{max}/\delta)$. Dot-dashed line corresponds to $u^+ = 1.95 - 1.26 \log(z/\delta)$. : NSTAP-$Re_\tau = 6000$, : NSTAP-$Re_\tau = 10000$, : NSTAP-$Re_\tau = 14500$, : NSTAP-$Re_\tau = 20000$, : hot-wire at the $Re_\tau$ in which NSTAP is used (grey color is used for $z^+ < Re_\tau^{1/2}$ for clarity), : hot-wire data at $Re_\tau = 25000$, : DNS at $\delta_{99} = 1990$ (Sillero et al. [128]).
components are invariant with $Re_\tau$ while the large-scale components increase with $Re_\tau$ leading to the increasing trend of the $u_+^{2+}$ in the near-wall region. Large-scale components exhibit a peak shown with diamonds in figure 7.4. Mathis et al. [82] used the Reynolds number trend of these peaks compared to the trend of the large-scale components at the location of the inner peak ($z^+ \approx 15$) to argue that the outer peak of the total intensity surpasses the inner peak of the total intensity at extremely high Reynolds numbers. This approach might result in a misleading conclusion, since while the small-scale component is invariant with $Re_\tau$ at the location of the inner peak, it decreases with the $Re_\tau$ at the location of the outer peak. Moreover, the location and the intensity of the outer peak is subject to experimental errors around the outer peak. In §7.2.3 a new method to find an upper limit for the outer peak will be proposed.

![Figure 7.3: Dependence of maximum of $u_+^{2+}$ on $Re_\tau$.](image_url)

- : averaged NSTAP, : NSTAP measurement repeats, : Superpipe data [45], : HRTF data [147], : boundary layer DNS [128], : channel DNS [41], $u_+^{2+}_{max} = 3.54 + 0.646 \log(Re_\tau)$, $u_+^{2+}_{max} = 3.66 + 0.642 \log(Re_\tau)$ from Lee and Moser [67], equation (4.1) in Monkewitz and Nagib [91].
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7.2.2 The inertial sublayer and the outer region in the streamwise turbulence intensity

Recent turbulence experiments provide convincing evidence that at sufficiently high $Re_{\tau}$, streamwise turbulence intensity follows a logarithmic decay before the onset of the wake region (see e.g. Hultmark et al. [44, 45], Marusic et al. [79, 80], Örlü et al. [105]). Townsend [140] proposed that the scaling from the wall can be associated with corresponding attached eddies. The geometric lengths of these eddies scale with $z$, and have population densities per characteristic eddy height that scale inversely with $z$. Townsend showed that this argument leads to a logarithmic profile for the streamwise (and spanwise) turbulence intensities of the form

$$
\overline{u^2} = B_1 - A_1 \log(z/\delta),
$$

(7.4)
where $\delta$ is the boundary layer thickness (or pipe radius or channel half-height). It can be seen in figure 7.2(b) that our experimental results follow this equation reasonably well with $A_1 = 1.26$ and $B_1 = 1.95$ as previously given by Marusic et al. [79] by analysing high $Re_\tau$ data obtained in different facilities. The upper bound of the logarithmic region appears to be $z/\delta = 0.15$. With increasing $Re_\tau$, the region for which equation 7.4 is valid, expands towards the wall. That is, the lower bound of this logarithmic region in viscous scaling is believed to be proportional to $Re_\tau^{1/2} (z^+ \propto Re_\tau^{1/2})$ so that $z/\delta \propto Re_\tau^{-1/2}$. For the DNS data at $Re_\tau \approx 2500$ no clear logarithmic region is observed although its $\overline{u'^2}$ profile against $z/\delta$ follows the profiles corresponding to higher Reynolds numbers in the outer region ($z/\delta > 0.15$). The lack of logarithmic region for the DNS data can be attributed to the insufficient scale separation as the $Re_\tau$ is relatively low.

The scaling of the turbulence intensity profile in the outer region has received less attention. For this region, McKeon and Morrison [83] report that $\overline{u'^2}$ profiles as functions of $z/\delta$ collapse in pipe flow, but in different manners for high and low Reynolds numbers. They associate this behaviour to the relatively slow development of self-similarity of the spectrum. Marusic et al. [80] report a very good collapse of $\overline{u'^2}$ profiles in the outer scaling for the Reynolds number range $2800 \leq Re_\tau \leq 13400$ in the turbulent boundary layer flow. The results of the current study also show very good collapse of the intensity profiles in outer region with outer scaling for the Reynolds number range $2500 \leq Re_\tau \leq 25000$ as seen in figure 7.2.

7.2.3 The intermediate region in the streamwise turbulence intensity

Thanks to the uniquely high Reynolds number data of the Superpipe, many new aspects of turbulence have been revealed, one of which is the possibility of the emergence of a second outer peak in the streamwise turbulence intensity profile, of which both the location and intensity may depend on $Re_\tau$. The outer peak has been reported for $Re_\tau \geq 20000$ (Hultmark et al. [44] and Hultmark et al. [45]). However, the presence of this peak and as to whether it is a peak or a plateau is not still clear since spatial resolution of the Superpipe measurements degrades for the very high $Re_\tau$ measurements where the
outer peak is noticeable. According to Hutchins et al. [47] the spatial resolution filtering extends to wall distances higher than the immediate near-wall region and may produce a delusive outer peak. The prediction of a second outer peak arising at high Reynolds number would be notable as it may indicate the presence of new outer phenomena [44, 78].

We address this problem in this section since spatial filtering is not an issue in our measurements up to \( Re_\tau = 20000 \) (obtained with the NSTAP). Our Reynolds numbers are not as high as those in the studies of Hultmark et al. [44, 45], however, our highest well-resolved \( Re_\tau \) case is equal to the lowest \( Re_\tau \) case in their study where the outer peak was reported. In fact, our results exhibit the presence of plateaus for \( Re_\tau = 14500 \) and 20000, and it can be seen that locating the outer peak accurately is challenging due to the lack of smoothness of the intensity profiles. This issue can be remedied by trying to find an upper bound for the outer peak instead of the peak itself. To this end, a logarithmic line, tangent to the intermediate part of the intensity profile at the inflection point of the profile is drawn as shown in figure 7.5. The inflection point in the intermediate region is associated with the point where \( d^2(u^2+)/d(\log z^+)^2 \) changes sign; and the tangent line is fitted to totally four to five points in the profile around the inflection point. The intersection of this tangent line with the logarithmic equation 7.4 gives the upper bound for the outer peak. The tangent lines show emergence of an outer peak which becomes more evident with increasing the \( Re_\tau \). Figure 7.6(a) and (b) show the \( Re_\tau \) dependency of the intensity (shown by \( u_p^{2+}_{\text{int}} \)) and location (shown by \( (z/\delta)_{\text{pint}} \)) of the intersections respectively. Dependency of the slope of the tangent logarithmic line \( S_{\text{int}} \) is shown in figure 7.6(c). These points can be fitted using appropriate functions to obtain

\[
\begin{align}
    u_p^{2+}_{\text{int}} &= -2.99 + 0.98 \log(Re_\tau), \\
    (z/\delta)_{\text{pint}} &= 32.66 Re_\tau^{-0.73}, \\
    S_{\text{int}} &= -2.42 + 0.25 \log(Re_\tau).
\end{align}
\]

(7.5a) (7.5b) (7.5c)
Marusic et al. [79] used $z/\delta = 3Re_\tau^{-1/2}$ as the lower bound of the logarithmic region and Mathis et al. [82] used $z/\delta = 3.9Re_\tau^{-1/2}$ as the location of the outer peak. Therefore, an exponential function is used here to find the correlation for the Reynolds number trend of $(z/\delta)_{pint}$ since it is analogous to the outer peak location. Also shown in figure 7.6(a) and (b) (by diamonds) are the total intensity (total of large-scale and small-scale intensity) and $z/\delta$ location of the large-scale component peak as shown in figure 7.4, together with the curve fits used by Mathis et al. [82] ($u_{p}^{2+} = -2.13 + 0.84\log(Re_\tau)$ and $z/\delta = 3.9Re_\tau^{-1/2}$). Very good agreement is seen between our correlation for the wall normal-location of the intersection ($(z/\delta)_{pint} = 32.66Re_\tau^{-0.73}$) and $z/\delta = 3.9Re_\tau^{-1/2}$ for high $Re_\tau$. While, as expected, our curve fit for the intensity of the intersection $u_{p}^{2+}_{int}$ over predicts the outer peak magnitude since it is introduced as an upper bound.
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\[ u^2 = u^2_{p\text{ int}} + S_{int} \log \left( \frac{z/\delta}{(z/\delta)_\text{int}} \right). \]  

Equations 7.5 and 7.6 are used to predict the intermediate region in the turbulence intensity profile for \( Re_\tau = 40000 \) and \( 1 \times 10^6 \). This region together with the logarithmic
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Figure 7.7: Comparison of \( u'^2 \) for the SLTEST surface layer and CICLoPE pipe experiments with the logarithmic model of equation 7.4 (---) and intermediate region correlation of equation 7.6 (----). ○ : SLTEST hot-wire data of Metzger et al. [84], □ : SLTEST sonic data of Metzger et al. [84], △ : SLTEST sonic data of Hutchins et al. [48], • : CICLoPE pipe PIV data of Willert et al. [153] at \( Re_\tau = 40000 \). The friction Reynolds numbers for the data from Metzger et al. [84] and Hutchins et al. [48], are estimated as \( Re_\tau \approx 7.7 \times 10^5 \) and \( Re_\tau \approx 7.8 \times 10^5 \), respectively.

Equations 7.3 and 7.5a can be used to predict whether at sufficiently high \( Re_\tau \) the outer peak overcomes the inner peak. The extrapolated result in figure 7.8 shows that such behaviour happens around \( Re_\tau = 10^5 \). Note that since we are using an upper bound for the outer peak (equation 7.5a), this is a lower bound for the predicted \( Re_\tau \). In fact, if we use the correlation proposed by Mathis et al. [82] for \( Re_\tau \) dependence of the outer peak intensity (0.84\log(Re_\tau) − 2.13), the predicted \( Re_\tau \) at which the outer peak overcomes the inner peak exceeds \( 10^{12} \). Also, one should acknowledge that these predictions are valid if the inner and the outer peak trends remain unchanged up to these extreme
values for \( Re_{\tau} \). Such questions remain controversial \([44, 77, 84, 96]\) since accurate near-wall measurements up to high \( Re_{\tau} \) face significant challenges. This makes any prospect of this phenomenon tentative. However, according to our data and the atmospheric surface layer data \([85]\) it is unlikely that this phenomenon happens at laboratory scale or terrestrial \( Re_{\tau} \) (if it ever happens).

### 7.2.4 Spatial resolution filtering in near-wall region

Several correction schemes for spatial filtering of single component hot-wire measurements have been proposed recently (Chin et al. \([20]\), Monkewitz et al. \([90]\), Philip et al. \([117]\), Segalini et al. \([127]\), Smits et al. \([131]\)). Comparison of all these corrections is out of the scope of this thesis. For a full assessment and comparison of these methods refer to Miller et al. \([87]\). The abundance of this type of study in literature highlights the importance of accurate near-wall data in wall-bounded turbulent flows up to high Reynolds number, which is scarce. Therefore, spatial correction schemes are usually used to deduce the correct turbulence intensity from spatially filtered hot-wire data at high \( Re_{\tau} \). In this section we employ our well-resolved data to assess the correction scheme.
of Smits et al. [131] which is the most highly cited, and also the scheme used to correct the NSTAP data in the Superpipe and the HRTF.

Figure 7.9 shows $u'^2$ as measured with the NSTAP and the 2.5 μm-diameter hot-wire together with the Smits et al. [131] corrected hot-wire profiles for various $Re_\tau$. It is noted

---

**Figure 7.9:** Streamwise turbulence intensity measured with the hot-wires and the NSTAP probes at various $Re_\tau$ where _—_ : uncorrected hot-wires' data, _—-—_ : corrected hot-wires' data for spatial resolution using the scheme proposed by Smits et al. [131], _—_ : NSTAP-$Re_\tau = 6000$, _—_ : NSTAP-$Re_\tau = 10000$, _—_ : NSTAP-$Re_\tau = 14500$ and _—_ : NSTAP-$Re_\tau = 20000$. (a) $Re_\tau = 6000$, (b) $Re_\tau = 10000$, (c) $Re_\tau = 14500$, (d) $Re_\tau = 20000$. (e) $Re_\tau = 25000$. $l_{HW}$
that the NSTAP results are not corrected for spatial resolution since their associated $l^+$ is small enough to ensure that all the small-scale turbulence energy content is resolved. It can be seen that the correction works reasonably well for measurements at $Re_\tau = 6000$, 10000, and 14500 where inner normalised hot-wire length $l^+_{HW} \approx 20$. However, for $Re_\tau = 20000$ with $l^+_{HW} = 29$, its performance degrades resulting in an underestimation of the turbulence intensity around the inner peak ($z^+ = 15$). This may indicate that performance of this correction scheme for $Re_\tau$ out of the range that it has been validated in the original paper ($Re_\tau \geq 10000$) might not be as accurate as previously thought.

7.3 Spectra of the streamwise velocity fluctuations

A wide range of eddy sizes are present in the turbulent motions. Quantifying the contribution of various scales to the turbulent kinetic energy improves our understanding of turbulence. This can be done through use of Fourier decomposition. In this method the turbulence signal is decomposed into a spectrum of wave-numbers to find the one sided spectral power density $\phi_{ij}$ at each wave number. Unfortunately, experimentally measuring the full three dimensional energy spectrum is extremely difficult, as one needs fully resolved three dimensional velocity data. Instead, we can often only examine the power spectral density of the streamwise velocity fluctuations $\phi_{uu}$ as a function of the streamwise wave-numbers $k_x$. In order to deduce spatial spectra from the time series data obtained from the stationary hot-wire, Taylor’s frozen turbulence hypothesis (Taylor [138]) is employed. It can be shown that the integral of $\phi_{uu}$ over $k_x$ is equal to the streamwise turbulent stress

$$\int_0^\infty \phi_{uu} dk_x = \bar{u}^2. \quad (7.7)$$

In this section energy spectra of the well-resolved measurements in the turbulent boundary layer flow are examined in two different subsections: (i) the near-wall region, (ii) the inertial sublayer.
7.3.1 Energy spectra in the near-wall region

We start this section with a dimensional analysis for the streamwise energy spectrum \( \phi_{uu} \). The spectrum can be separated into three wave-number domains:

1. **Small wave-number motions**: In this domain, although near the wall, the boundary layer thickness is important since the contributing large-scale motions are of the order of the boundary layer thickness, \( \delta \). Hence, the relevant variables are \( \phi_{uu} \), \( k_x \), friction velocity \( u_\tau \), kinematic viscosity \( \nu \), wall-distance \( z \), and boundary layer thickness \( \delta \). A dimensional analysis leads to

\[
\frac{k_x \phi_{uu}}{u_\tau^2} = f_1(k_x z, z^+/\delta) = f_1(k_x^+ z^+, z^+/\delta^+).
\]  
(7.8)

Here ‘+’ indicates inner normalisation, i.e. normalisation using friction velocity \( u_\tau \) and kinematic viscosity \( \nu \). Equation 7.8 implies that in the near wall region over the small wave-number domain, \( k_x \phi_{uu}/u_\tau^2 \) is dependant on \( Re_\tau \) as well as \( k_x^+ \) and \( z^+ \).

2. **Moderate to high wave-number motions**: In this wave-number domain in the near-wall region, the relevant variables are \( \phi_{uu} \), \( k_x \), \( u_\tau \), \( z \), and \( \nu \). Here, since \( z/\delta \ll 1 \) for sufficiently high \( Re_\tau \), \( \delta \) is not important. Therefore, a dimensional analysis yields

\[
\frac{k_x \phi_{uu}}{u_\tau^2} = f_2(k_x z, z^+) = f_2(k_x^+ z^+, z^+).
\]  
(7.9)

Therefore, in this domain \( k_x \phi_{uu}/u_\tau^2 \) can be expressed in terms of \( k^+ \) and \( z^+ \) only.

3. **Very high wave-number motions**: In this range of motions, \( \phi_{uu} \) is dependent on wave-number \( k_x \), Kolmogrov’s length scale \( \eta \) and Kolmogrov’s velocity scale \( v_\eta \).

Here, \( v_\eta = (\nu \epsilon)^{1/4} \) and \( \eta = (\nu^3/\epsilon)^{1/4} \) where \( \nu \) is kinematic viscosity and \( \epsilon \) is the dissipation rate. With these variables two non-dimensional parameters can be considered such that
\[ \frac{k_x \phi_{uu}}{v_0^+} = f_3(k_x \eta) = f_3(k_x \eta^+). \] (7.10)

It will be shown in §7.6 that in the near-wall region and the inertial sublayer, \( \eta^+ \) and \( v_0^+ \) can be expressed as functions of inner normalised wall-distance \( z^+ \), independent of \( Re_x \), i.e. \( \eta^+ = h_\eta(z^+) \) and \( v_0^+ = h_v(z^+) \). Therefore, equation 7.10 can be rewritten as

\[ \frac{k_x \phi_{uu}}{u_r^2} = [h_v(z^+)]^2 f_3(k_x h_\eta(z^+)) \] (7.11)

which shows that in this wave-number domain, similar to **moderate to high wave-number motions**, \( k_x \phi_{uu}/u_r^2 \) is a function of \( k^+ \) and \( z^+ \) and independent of \( Re_x \).

The above arguments indicate that at a fixed \( z^+ \) in the near-wall (sufficiently small \( z/\delta \)), \( k_x \phi_{uu}(k^+)/u_r^2 \) (or equivalently \( k_x \phi_{uu}(\lambda_x^+)/u_r^2 \) where \( \lambda_x = 2\pi/k_x \) is the wave-length) curves should collapse over medium to very high wave-numbers (medium to very low wave-lengths) when various \( Re_x \) are compared, and an \( Re_x \) dependency is expected in the low wave-number (high wave-length) end of the spectra only. Since small-scale energy can be significantly affected by spatial resolution (as documented by Hutchins et al. \[47\] and Chin et al. \[19\]), the data here may be helpful to clarify the small-scale behaviour in the near-wall. The inner-scaled, pre-multiplied energy spectra \( k_x \phi_{uu}/u_r^2 \) is plotted against streamwise wavelength, \( \lambda_x^+ \), in figure 7.10 at the peak turbulence intensity location, \( z^+ \approx 15 \) for the experimental data (\( Re_x = 6000, 10000, 14500, \) and \( 20000 \)) as well as channel DNS data of Hoyas and Jiménez \[41\] (\( Re_x = 550, 2000 \)) and boundary layer DNS data of Sillero et al. \[128\] (\( \delta_{99}^+ = 1990 \)). Here, spectra are plotted in premultiplied form so that equal areas equate to equal contributions to the turbulence intensities on the semilogarithmic plot: \[ \int_0^\infty \phi_{uu} dk_x = \int_0^\infty k_x \phi_{uu} d(log k_x) = \int_0^\infty k_x \phi_{uu} d(log \lambda_x) = \overline{u'^2}. \]

All the spectra appear to collapse well for the wave-lengths below \( \lambda_x^+ \approx 1500 \) and the experimental spectra (\( Re_x \geq 6000 \)) collapse well for the wave-lengths below \( \lambda_x^+ \approx 15000 \).
Moreover, in the high wave-length end of the spectra an increasing trend with $Re_\tau$ is evident. These observations are consistent with the experimental study of Hutchins et al. [47] in the boundary layer flow (up to $Re_\tau \approx 19000$) where hot-wire sensors with lengths of 22 wall units were used, and for the lower Reynolds number DNS studies of Hoyas and Jiménez [41] (up to $Re_\tau \approx 2000$) in the channel flow, and Chin et al. [21] (up to $Re_\tau \approx 2000$) in the pipe flow. The peak of the innernormalised premultiplied energy spectra is seen to remain constant at $k_x \phi_{uu}/u^2_\tau \approx 2.2$ at the innernormalised wave-length $\lambda^+_x \approx 850$. It should also be noted that the small-scale energy is not only invariant with $Re_\tau$, but independent of the flow geometry since channel and boundary layer flow spectra in the small-scale region appear to collapse well. Figure 7.11 shows the same plots for the experimental data at other innernormalised wall-distance locations.

It can be seen that the $k_x \phi_{uu}/u^2_\tau$ curves collapse for wave-lengths below $\lambda^+_x \approx 15000$ up to $z^+ = 280$ for the Reynolds number range $6000 \leq Re_\tau \leq 20000$. Only at $z^+ = 670$ the $Re_\tau = 6000$ spectrum starts to deviate from the spectra of other Reynolds numbers in the small-scale region. This behaviour in the spectra explains the increasing trend with $Re_\tau$ for the peak $u^2$ in the near-wall region (There is an increasing amount of
large-scale superimposed energy in the near-wall region as $Re_\tau$ increases). This scaling law in the near-wall region is in close accordance with the “spectral analogue of the law of the wall” by Zamalloa et al. [160].

### 7.3.2 Energy spectra in the inertial sublayer and the outer region

Let’s consider the distribution of the $u$-power spectra $\phi_{uu}$ in the wall-distance region where the energy-containing motions are independent of viscosity, i.e. $\nu/u_\tau \ll z$. The relevant variables to express $\phi_{uu}$ in the range of energy-containing motions are $k_x$, $u_\tau$, $z$,
and $\delta$. According to the Buckingham’s II-theorem, three dimensionless parameters are sufficient to express the solution, however, five dimensionless parameters are possible: $\phi_{uu}/\delta u^2$, $\phi_{uu}/z u^2$, $k_x \delta$, $k_x z$, $z/\delta$. We separate the viscous-independent energy-containing spectral region into two separate regions to take into account all the above dimensionless parameter.

1. **Small wave-number motions ($k_x \sim 1/\delta$):** At these wave-numbers $k_x$ scales as $1/\delta$.

   Therefore, we can normalise $k_x$ with $\delta$ and expect a $\delta$-scaling law of the form

   $$\frac{\phi_{uu}(k_x \delta, z/\delta)}{u_x^2} = g_1(k_x \delta, z/\delta) = \frac{\phi_{uu}(k_x \delta, z/\delta)}{\delta u_x^2}. \quad (7.12)$$

   Here $\phi_{uu}(k_x \delta, z/\delta)$ is the power-spectral density per unit non-dimensional wave-number $k_x \delta$ at outer-normalised wall-distance $z/\delta$. This scaling is demonstrated in figure 7.12 for the experimental results for various Reynolds numbers in the logarithmic region ($3Re_\tau^{0.5} \leq z^+ \leq 0.15Re_\tau$). We can see that plotting the energy spectra against $k_x \delta$ does not collapse the spectra around the small wavenumber peaks/plateaus. This shows the dependence of the spectra on $z/\delta$ as well as $k_x \delta$ suggesting that even in the logarithmic region $z$ cannot be ignored as a relevant variable in this range of motions. However, it appears that for the higher $Re_\tau$ cases (figures 7.12(d),(e)) where logarithmic region extends to relatively small $z/\delta$ values towards the wall ($z/\delta \sim 2 \times 10^{-2}$) we start to see that the dependence on $z/\delta$ vanishes for a small wall-distance region located at the start of the logarithmic region where the magnitudes of the peaks/plateaus remain almost constant with decreasing $z/\delta$. In this region $\nu/\delta u_\tau \ll z \ll \delta$ is valid and equation 7.12 approximately reduces to

   $$\frac{\phi_{uu}(k_x \delta)}{u_x^2} = g_1(k_x \delta) = \frac{\phi_{uu}(k_x)}{\delta u_x^2}, \quad (7.13)$$

   as shown by Perry et al. [115] in the context of Townsend’s attached eddy hypothesis [140]. It is expected that with increasing $Re_\tau$ this $z/\delta$-independent region extends towards the wall.
Figure 7.12: Premultiplied energy spectra in $\delta$-scaling at various distances from the wall in the inertial sublayer at (a) $Re_\tau = 6000$, (b) $Re_\tau = 10000$, (c) $Re_\tau = 14500$, (d) $Re_\tau = 20000$, (e) $Re_\tau = 25000$. Solid lines correspond to the region $3\delta^{+1/2} < z^+ < 0.15\delta^+$, while dashed lines and dotted lines correspond to $z^+ < 3\delta^{+1/2}$ and $z^+ > 0.15\delta^+$, respectively. Arrow indicates the increase in $z$. 
2. **Moderate to high wave-number motions** ($k_x \sim 1/z$): At these wave-numbers $k_x$ scales as $1/z$. Therefore, we can normalise $k_x$ with $z$ and expect a $z$-scaling law of the form

$$\frac{\varphi_{uu}(k_x z, z/\delta)}{u_f^2} = g_2(k_x z, z/\delta) = \frac{\varphi_{uu}(k_x, z/\delta)}{z u_f^2}.$$ \hspace{1cm} (7.14)

This $z$-scaling law is shown in figure 7.13 for our experimental data at various $Re_\tau$. Again the effect of $z/\delta$ term is evident. This shows that $\delta$ cannot be neglected for the moderate-to-high wave-number region of the spectra, even in the inertial sublayer. Similar to the $\delta$-scaling when $z/\delta \sim 2 \times 10^{-2}$, while still in the logarithmic region (which is true only for the higher $Re_\tau$ cases of figures 7.13(d),(e) in a small portion of the start of the logarithmic layer where the magnitudes of the peaks/plateaus remain almost constant with decreasing $z/\delta$), the $z/\delta$ effect appears to start diminishing and in that region equation 7.14 reduces to

$$\frac{\varphi_{uu}(k_x z)}{u_f^2} = g_2(k_x z) = \frac{\varphi_{uu}(k_x)}{z u_f^2},$$ \hspace{1cm} (7.15)

3. **Very high wave-number motions** ($k_x \sim 1/\eta$): In the inertial sublayer and the outer layer, there is still a range of very-high-wave-number motions that are dependent on viscosity. Similar to the near-wall region the variables involved in the spectral analysis are, $k_x$, $\eta$, and $v_\eta$ since we expect the classic Kolmogrov’s scaling to hold. Therefore one has

$$\frac{\varphi_{uu}(k_x \eta)}{v_\eta^2} = g_3(k_x \eta) = \frac{\varphi_{uu}(k_x)}{\eta v_\eta^2}.$$ \hspace{1cm} (7.16)

Equation 7.16 implies that $k_x \varphi_{uu}(k_x)/v_\eta^2$ curves must collapse in the very high wave-number region regardless of the wall-distance and $Re_\tau$. This is demonstrated through experimental results in figure 7.14. Note that for $Re_\tau = 25000$ results, poor spatial resolution of the sensor affects the high-wave number end of the spectra.
Figure 7.13: Same as figure 7.12, but abscissa is normalised by $z$ now. The dot-dashed line correspond to the relation given for $g_2(k_xz)$ in equation 7.18.
Figure 7.14: Same as figure 7.12, but abscissa and ordinate are normalised by Kolmogrov length scale $\eta$ and Kolmogrov velocity scale $v_\eta$, respectively here. The solid straight line corresponds to $k_x \eta \phi_{uu}(k_x \eta) / v_\eta^2 = -6.8 \log_{10}(k_x \eta) - 4.5$. The dot-dashed line correspond to the relation given for $g_3(k_x \eta)$ in equation 7.18.
A useful way of inspecting the proposed scaling laws is shown in figure 7.15, where pre-multiplied spectra associated with various $Re_\tau$ are plotted in $\delta$-scaling at various $z/\delta$. At each fixed $z/\delta$, spectra corresponding to five Reynolds numbers are compared, while at $z/\delta \approx 0.335$, spectra from the DNS study of Sillero et al. [128] are also added to the comparison. It appears that for the largest $z/\delta$, all the spectra collapse reasonably well in the spectral range $k_x\delta \leq 10$. By decreasing $z/\delta$, spectra corresponding to the lower Reynolds numbers start to deviate from those corresponding to the higher Reynolds numbers. In general, a similarity among spectra (at different outer-scaled wall-distances, $z/\delta$) corresponding to different $Re_\tau$ for which $z/\delta > 4Re_\tau^{-1/2}$ is observed. In other words, similarity is expected only in the inertial sublayer and outer region wherein viscosity is not an effective factor.

Now two spectral regions of overlap are expected. In the region of ‘overlap I’ equations 7.12 and 7.14 are simultaneously valid. Therefore, $k_x\phi_{uu}(k_x, z/\delta)/u_\tau^2 = k_x\delta g_1(k_x\delta, z/\delta) = k_x z g_2(k_x z, z/\delta)$. Introducing functions $g'_1(k_x\delta, z/\delta) = k_x\delta g_1(k_x\delta, z/\delta)$ and $g'_2(k_x z, z/\delta) = k_x z g_2(k_x z, z/\delta)$, the above equality can be rewritten as $k_x\phi_{uu}(k_x, z/\delta)/u_\tau^2 = g'_1(k_x\delta, z/\delta) = g'_2(k_x z, z/\delta) = A^*_1(z/\delta)$, where $A^*_1(z/\delta)$ (in analogy to the $A_1$ introduced by Perry et al. [115]) is a universal function at least in the turbulent boundary layer. Note that since $g'_1$ is a function of $k_x\delta$ and $z/\delta$, and $g'_2$ is a function of $k_x z$ and $z/\delta$, equality is possible if they both equate to a function of $z/\delta$ only, which is given as $A^*_1(z/\delta)$ here. Therefore, in region of overlap I

\[
\frac{\phi_{uu}(k_x\delta, z/\delta)}{u_\tau^2} = \frac{A^*_1(z/\delta)}{k_x\delta} = g_1(k_x\delta, z/\delta),
\]

\[
\frac{\phi_{uu}(k_x z, z/\delta)}{u_\tau^2} = \frac{A^*_1(z/\delta)}{k_x z} = g_2(k_x z, z/\delta).
\]

$A^*_1(z/\delta)$ can be considered as the magnitude of a small-wave-number plateau or peak in $k_x\phi_{uu}/u_\tau$ which varies with $z/\delta$. These peaks/plateaus can be seen in figure 7.15 that are functions of $z/\delta$ (and independent of $Re_\tau$ since at each fixed $z/\delta$, spectra with different $Re_\tau$ have the same peak/plateau magnitudes as long as the spectra are located in the logarithmic wall region).
In the region of ‘overlap II’, equations 7.14 and 7.16 are valid simultaneously. We assume that in the region of overlap II, \( \delta \) is not important and equation 7.15 is valid (so that we can proceed with our analysis). Therefore, similar to Perry et al. [115], in the region of overlap II we have
\[
\left\{ \begin{align*}
\frac{\phi_{uu}(kxz)}{u^2_\tau} &= \frac{1}{\kappa^{2/3}} \frac{K_0}{(kxz)^{5/3}} = g_2(kxz), \\
\frac{\phi_{uu}(kx\eta)}{v^2_\eta} &= \frac{K_0}{(kx\eta)^{5/3}} = g_3(kx\eta),
\end{align*} \right. \tag{7.18}
\]

where \( K_0 \) is the Kolmogrov’s constant. In developing equation 7.18, one is required to assume that in the wall region, turbulence energy production and dissipation are approximately in balance, which leads to the relation \( \epsilon = u^3_\tau / \kappa z \) for the dissipation. We will explore the validity of this assumption for our experimental data later in §7.6. The relations given in equation 7.18 are depicted in figures 7.13 and 7.14. It appears that these functional forms are consistent only with a limited portion of the spectra in the logarithmic wall region.

### 7.3.3 Existence of the \( k^{-1} \) power scaling law

The \( k^{-1} \) power scaling law in wall-bounded turbulence refers to a spectral wave-number region in which the streamwise spectra decay as \( \phi_{uu} \sim k^{-1}_x \). Perry et al. [115] showed that \( \phi_{uu}/u^2_\tau = A_1 k^{-1}_x \) in the energy-containing part of the spectra in the logarithmic wall region and deduced a logarithmic law for the streamwise turbulence intensity in wall-bounded flows. Here \( A_1 \) is a universal constant. Although the existence of the \( k^{-1} \) law in the energy spectra has long been predicted (see e.g. Tchen [139], Katul and Chu [56], Nikora [101], Banerjee and Katul [7], Perry et al. [115]), no strong experimental support has been found for it in the sense of Perry et al. [115] for Reynolds numbers up to \( Re_\tau \sim 10^5 \) (Vallikivi et al. [146] and Rosenberg et al. [124]). In this section we address this problem using the dimensional analysis presented in §7.3.2 and our experimental data as well as atmospheric surface layer data of Kunkel and Marusic [66]. It was stated that in the wave-number region of overlap I in the streamwise power spectrum, a peak/plateau is anticipated which is a function of \( z/\delta \). Moreover, it was shown in figure 7.15 that the magnitude of the peak increases with the decrease in \( z/\delta \) in the range \( 0.01 \leq z/\delta < 1 \) available in our experiment. We examine atmospheric surface layer power spectra seeking the \( k^{-1} \) law for smaller \( z/\delta \) values while viscosity is not
still effective thanks to higher $Re_\tau$. Figure 7.16 shows premultiplied spectra against $z$-scaled wave-number in the atmospheric surface layer at the estimated Reynolds number $Re_\tau = 3.8 \times 10^6$ at outer-scaled wall-distances $z/\delta = 9.9 \times 10^{-4}$, $1.67 \times 10^{-3}$, $4.19 \times 10^{-3}$, $5.43 \times 10^{-3}$ and $1.01 \times 10^{-2}$. A plateau with the magnitude of $1.4 \pm 0.1$ is recognised for the three closest points to the wall. Figure 7.17 shows the magnitude of the peak/plateau $A_1^*$ (in analogy to Perry’s $A_1$) as a function of $z/\delta$ for our laboratory data and the atmospheric surface layer data. Least square fitting a third order polynomial function to the data returns

$$A_1^* = -29.3(z/\delta)^3 + 20.5(z/\delta)^2 - 5.4(z/\delta) + 1.425. \quad (7.19)$$

It appears that $A_1^*$ increases with the decrease in $z/\delta$ and asymptotes to $\sim 1.4 \pm 0.1$ as $z/\delta$ approaches 0. This observation is consistent with our dimensional analysis and contradicts the prediction of Örlü et al. [104] who based on the diagnostic plot concluded that $A_1^*$ (in Perry et al. [115] formulation of streamwise turbulence intensity) increases

![Figure 7.16: Atmospheric surface layer premultiplied energy spectra in the z-scaled form at the estimated outer scaled wall distances of $z/\delta = 9.9 \times 10^{-4}$, $1.67 \times 10^{-3}$, $4.19 \times 10^{-3}$, $5.43 \times 10^{-3}$ and $1.01 \times 10^{-2}$. Data are from Kunkel and Marusic [66] at $Re_\tau \approx 3.8 \times 10^6$. Dashed line corresponds to the plateau value of 1.4 and the arrow indicates the increase in the wall distance.](image-url)
with $Re$ unboundedly. In fact, for $z/\delta$ effect on $A^*_1$ to diminish, a sufficiently small $z/\delta$ is required in theory. According to our experimental data, the wall region for which equations 7.13 and 7.15 and consequently the $k_x^{-1}$ law as proposed by Perry et al. [115] in the power spectra hold is $z/\delta \leq 0.02$. Therefore, to obtain a decade of $z/\delta$ over which the $k_x^{-1}$ region is present, assuming the lower bound of the logarithmic region to be $z^+ = 2Re^{1/2}_\tau$, a minimum $Re_\tau = 10^6$ is required. However, such high $Re_\tau$ is currently unattainable in the laboratory, and obtaining converged spectra in the atmospheric surface layer experiments in the small-wave-number end of the spectra where the $k_x^{-1}$ law is anticipated is challenging, as this requires hours of sampling [66]. The preceding analysis explains why clear experimental data in support of the $k_x^{-1}$ law have remained elusive.

### 7.3.4 The $k_x^{-5/3}$ power scaling law

Power spectra in Kolmogrov scaling for $0.01 \leq z/\delta \leq 0.4$ at various $Re_\tau$ are shown in figure 7.18. It can be seen that for the normalised wave-number range $k_x\eta > 0.02$
the spectra collapse well, as was seen in figure 7.14. As \( z/\delta \) increases, a \( k_x^{-5/3} \) power law scaling range appears. To attain a better view of this scaling, the compensated Kolmogrov-scaled spectra are shown in figure 7.19. It appears that the \( k_x^{-5/3} \) scaling only holds for \( z/\delta > 0.15 \), i.e. in the wake region, for all the Reynolds numbers of this study. This is not consistent with the arguments of Perry et al. \[115\] and those presented in §7.3.2 (equation 7.18) for the wave-number region of overlap II, where a \( k_x^{-5/3} \) functional form was anticipated in the logarithmic wall region. This might be due to the breakdown of the isotropic assumption, which is used to calculate the dissipation rate, \( \epsilon \) in the logarithmic wall region (using \( \epsilon = 15 \nu \int_0^{\infty} k_x^2 \phi_{uu} dk \)). Further to this, in developing the functional form of equation 7.18, we have assumed that turbulence energy production and dissipation are in balance to estimate the dissipation rate as \( \epsilon = u_t^3 / \kappa z \).

We will see in §7.6 that these two estimates do not agree across most of the boundary layer. However, the fact that the \( k_x^{-5/3} \) law sits in the wake region agrees well with inertial subrange scaling arguments, which suggest that the \( k_x^{-5/3} \) region emerges when the separation between large energetic scales and small dissipative scales is large and the flow can be considered nearly isotropic. Figure 7.19 also shows that the plateau, which equates to the Kolmogrov’s constant \( K_0 \), changes with the \( Re_\tau \) from 0.52 for \( Re_\tau = 6000 \) to 0.48 for \( Re_\tau = 20000 \) (The plateau values are determined by fitting a horizontal line to the flat portions of the compensated spectra of figure 7.19 at \( z/\delta = 0.4 \)). This is probably because in the lower \( Re_\tau \) cases, the separation between large energetic scales and small dissipative scales is insufficient. Our Kolmogrov’s constant in the highest \( Re_\tau \) (\( K_0 = 0.48 \)) is slightly lower than that reported by Saddoughi and Veeravalli \[125\] (\( K_0 = 0.49 \)) in their extremely high \( Re_\tau \) experiment.

We can also examine the inertial subrange behaviour with changing Reynolds number. This is shown in figure 7.20. It appears that the \( k_x^{-5/3} \) range extends to smaller \( k_x \eta \) values with increasing \( Re_\tau \). The compensated Kolmogrov scaled spectra are shown in figure 7.21. It is evident that the \( k_x^{-5/3} \) scaling is not existent for \( z/\delta < 0.4 \). We can see almost a decade of the \( k_x^{-5/3} \) power law scaling at \( Re_\tau = 20000 \) for \( z/\delta = 0.4 \).
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Figure 7.18: Kolmogrov’s universal scaling for the streamwise energy spectra at outer-scaled wall normal locations of $z/\delta \approx 0.01, 0.05, 0.1, 0.15, 0.2, 0.4$. (a) $Re_\tau = 6000$, (b) $Re_\tau = 10000$, (c) $Re_\tau = 14500$, (d) $Re_\tau = 20000$. Arrow indicates the increase in $z/\delta$.

Figure 7.19: Premultiplied form of figure 7.18.
10^{-6} 10^{-5} 10^{-4} 10^{-3} 10^{-2} 10^{-1} 10^0

\frac{\phi_{uu}}{\left(\epsilon \nu^5\right)^{1/4}}

z/\delta = 0.05

\frac{\phi_{uu}}{\left(\epsilon \nu^5\right)^{1/4}}

z/\delta = 0.1

\frac{\phi_{uu}}{\left(\epsilon \nu^5\right)^{1/4}}

z/\delta = 0.15

\frac{\phi_{uu}}{\left(\epsilon \nu^5\right)^{1/4}}

z/\delta = 0.4

**Figure 7.20:** Kolmogrov’s universal scaling for the streamwise energy spectra at $Re_\tau = 6000, 10000, 14500, 20000$ at (a) $z/\delta \approx 0.05$, (b) $z/\delta \approx 0.1$, (c) $z/\delta \approx 0.15$, (d) $z/\delta \approx 0.4$. Arrow indicates the increase in $Re_\tau$.

\begin{align*}
\epsilon^{-2/3} k_{x}^{5/3} \phi_{uu} & \\
0.7 & \\
0.6 & \\
0.5 & \\
0.4 & \\
0.3 & \\
0.2 & \\
0.1 & \\
0 & \\
10^{-5} & 10^{-4} 10^{-3} 10^{-2} 10^{-1} 10^0
\end{align*}

**Figure 7.21:** Premultiplied form of figure 7.20. Saddoughi and Veeravalli [125] found the Kolmogrov constant to be $K_0 = 0.49$ which is slightly higher than our result which is $K_0 = 0.48$. Arrow indicates the increase in $Re_\tau$. 
7.3.5 Modelling the streamwise turbulence intensity in the inertial sublayer

We can now calculate the streamwise turbulence intensity in the inertial sublayer by integrating the energy spectra similar to Perry et al. [115] and Vallikivi et al. [146]. It should be noted that the difference between our analysis and the above-mentioned studies is that spectra are considered functions of $z/\delta$ in the small and medium wavenumber regions here, while Perry et al. [115] and Vallikivi et al. [146] ignored this dependency.

A summary of various spectral regions for the $u$-spectra are shown in figure 7.22 based on the scaling arguments presented in §7.3.2. Sketches of the pre-multiplied $u$-spectra in $\delta$- and $z$-scaling at fixed $Re$ for varying $z/\delta$ values are shown in figure 7.23. We can integrate the spectra in different regions to obtain the turbulence intensity

$$\overline{\frac{u''^2}{u_\tau^2}} = \int_{0}^{F(z/\delta)} \frac{\phi_{uu}(k_x \delta, z/\delta)}{u_x^2} \, d(k_x \delta) + \int_{F(z/\delta)}^{(\delta/z) \cdot P(z/\delta)} \frac{\phi_{uu}(k_x \delta, z/\delta)}{u_x^2} \, d(k_x \delta) + \int_{N}^{M} \frac{\phi_{uu}(k_x \delta, z/\delta)}{u_x^2} \, d(k_x \delta) + \int_{M}^{\infty} \frac{\phi_{uu}(k_x \delta, z/\delta)}{u_x^2} \, d(k_x \delta).$$

(7.20)

![Figure 7.22: A summary of various spectral regions for the u-spectra.](image)
'I' can be simplified as

\[
\int_0^{F(z/\delta)} g_1(k_x \delta, z/\delta) d(k_x \delta) = G_1(F(z/\delta), z/\delta) - G_1(0, z/\delta),
\]  

where \( G_1(k_x \delta, z/\delta) = \int g_1(k_x \delta, z/\delta) d(k_x \delta). \)

'II' covers the region of overlap I. Using 7.17, this can be simplified as

\[
\int_{F(z/\delta)}^{(\delta/z) P(z/\delta)} g_1(k_x \delta, z/\delta) d(k_x \delta) = \int_{F(z/\delta)}^{(\delta/z) P(z/\delta)} \frac{A_1^*(z/\delta)}{k_x \delta} d(k_x \delta)
\]

\[
= A_1^*(z/\delta) \left[ -\log(z/\delta) + \log\left(\frac{P(z/\delta)}{F(z/\delta)}\right) \right].
\]  

(7.22)

For 'III' one has

\[
\int_N^P g_2(k_x z, z/\delta) d(k_x z) = G_2(N, z/\delta) - G_2(P(z/\delta), z/\delta),
\]  

where \( G_2(k_x z, z/\delta) = \int g_2(k_x z, z/\delta) d(k_x z). \)

'IV' covers the region of overlap II, therefore \( \phi_{uu}/v_\eta^2 \) follows equation 7.18 and, 'IV' simplifies to a universal constant.
\[
\frac{v^2}{u^2} \int_{M'/z}^{M'/z} g_3(k_x \eta) \, d(k_x \eta) = (\kappa z^+)^{-1/2} \int_{N/\eta}^{M'/\eta} \frac{K_0}{(k_x \eta)^{5/3}} \, d(k_x \eta) \\
= \frac{3}{2} K_0 (N^{-2/3} - M'^{-2/3}) \kappa^{-2/3},
\]

(7.24)

where \( N \) and \( M' \) are universal constants, \( K_0 \) is the Kolmogrov constant, and \( \kappa \) is the Kármán constant. Here we have used \( v_\eta/u_\tau = (\kappa z^+)^{-1/4} \).

‘V’ covers the very-high-wavenumber region that follows Kolmogrov scaling and is simplified as

\[
\frac{v^2}{u^2} \int_{M'/\eta}^{\infty} g_3(k_x \eta) \, d(k_x \eta) = C(z^+).
\]

(7.25)

We will later determine the exact functional form of \( C(z^+) \) using the experimental data.

Introducing equations 7.21-7.25 into equation 7.20 yields

\[
\frac{u^2}{u^2} = B_1^*(z/\delta) - A_1^*(z/\delta) \log (z/\delta) + C(z^+),
\]

(7.26)

where \( A_1^* \) is plotted against \( z/\delta \) in figure 7.17. Equation 7.26 is analogous to the logarithmic equation 7.4 derived by Perry et al. [115] and Townsend [140], while here the constants have been replaced by functions that asymptote to finite constants when \( z/\delta \rightarrow 0 \).

For further examination of equation 7.26 we need to determine \( C(z^+) \) first. Figure 7.24 shows the premultiplied \( u \)-spectra at \( Re_\tau = 20000 \) scaled with Kolmogrov velocity and length scales. Also shown in the figure are, the formulation for the region of overlap II given by \( \phi_{uu}/v_\eta^2 = K_0/(k_x \eta)^{5/3} \), a curve fit of the form \( \phi_{uu}/v_\eta^2 = -6.8 \log_{10}(k_x \eta) - 4.5/k_x \eta \) and a curve computed using Kovasznay’s [63] spectral formula. It is evident that \( \phi_{uu}/v_\eta^2 = K_0/(k_x \eta)^{5/3} \) is not accurate anywhere in the region \( k_x \eta > 0.02 \), \( \phi_{uu}/v_\eta^2 = [-6.8 \log_{10}(k_x \eta) - 4.5]/k_x \eta \) follows the experimental spectra faithfully in the domain \( 0.02 < k_x \eta < 0.15 \), and \( \phi_{uu}(k_x \eta) = 0.5(k_x \eta)^{-5/3} - 6.04(k_x \eta)^{-1/3} - 1.8(k_x \eta)^{1/3} + 0.3(k_x \eta)^2 + \)
Figure 7.24: Premultiplied $u$-spectra scaled with Kolmogrov velocity and length scales at $Re_τ = 20000$ at selective wall heights. 

\[ k_x η \phi_{uu}(k_x η)/v_η^2 = K_0/(k_x η)^{2/3} \] 

\[ k_x η \phi_{uu}(k_x η)/v_η^2 = -6.8 \log_{10}(k_x η) - 4.5 \] 

\[ k_x η \phi_{uu}(k_x η)/v_η^2 = 0.5(k_x η)^{-2/3} - 6.04(k_x η)^{2/3} - 1.8(k_x η)^2 + 0.3(k_x η)^3 + 7.03(k_x η) \] [63]. The arrow indicates the increase in $z/δ$.

7.03 is accurate for $0.15 < k_x η < 1$. Therefore, $g_3(k_x η) = \phi_{uu}/v_η^2$ in ‘V’ can be described accurately by

\[
    g_3(k_x η) = \begin{cases} 
    -6.8 \log_{10}(k_x η) - 4.5 & \text{if } 0.02 < k_x η < 0.15 \\
    0.5(k_x η)^{-5/3} - 6.04(k_x η)^{-1/3} - 1.8(k_x η)^2 + 0.3(k_x η)^3 + 7.03 & \text{if } 0.15 < k_x η < 1 
    \end{cases}
\]

(7.27)

to evaluate $C(z^+)$. Figure 7.25(a) shows the ratio of $C$ (calculated using equation 7.27) to the total turbulence intensity, $C/u^{+2}$, against $z/δ$ for various Reynolds numbers in the logarithmic region. It can be seen that the very-high-wave-number motions contribute to more than 10% of the total turbulence energy near the outer bound of the logarithmic region and their contribution drops to less than 4% near the inner bound of the logarithmic region. In figure 7.25(b) $u^{+2}$ and $u^{+2} - C$ against $z/δ$ are compared.

Although $C$ is not a function of $z/δ$ it can be seen that both $u^{+2}$ and $u^{+2} - C$ exhibit reasonable collapses in the outer scaling. Therefore, it is convenient to treat $C$ as a constant and combine it with $B_1^*(z/δ)$ so that
\frac{\overline{u'^2}}{u'^2} = B_1(z/\delta) - A_1(z/\delta) \log(z/\delta). \quad (7.28)

$A_1$ and $B_1$ in equation 7.4 can be viewed as the spatial averages of $A_1^*(z/\delta)$ and $B_1^*(z/\delta)$ over the logarithmic region, respectively. Note that $A_1^* = 1.33$ at $z/\delta = 0.02$ and decreases monotonically to $A_1^* = 0.98$ at $z/\delta = 0.15$ for the Reynolds number range $Re_\tau = 6000 - 25000$, and $A_1 = 1.26$.

We can also estimate $A_1$ at infinitely high $Re$. Suppose that

$$A_1 = \frac{\int_{UB}^{LB} A_1^*(z/\delta)d(z/\delta)}{\int_{LB}^{UB} d(z/\delta)}. \quad (7.29)$$

For $Re_\tau = 25000$, which is the highest Reynolds number in the current study, $LB = 0.02$ and assuming that $UB = 0.05$, equation 7.29 with the aid of equation 7.19 yields $A_1 = 1.26$. Note that $UB = 0.05$ was chosen deliberately (rather than $UB = 0.15$) so that $A_1 = 1.26$ is obtained. Now for infinitely high Reynolds number $LB=0$ and keeping $UB = 0.05$ yields $A_1 = 1.3$. Therefore, this simple analysis shows that the $A_1$ obtained from experimental data up to $Re_\tau = 25000$ will not change significantly, even for extremely high Reynolds numbers.

**Figure 7.25:** (a) Ratio of $C$ to the total turbulence intensity, $C/\overline{u'^2}$ and (b) $\overline{u'^2}$ and $\overline{u'^2} - C$ against $z/\delta$. $\therefore: Re_\tau = 6000$, $\therefore: Re_\tau = 10000$, $\therefore: Re_\tau = 14500$, $\therefore: Re_\tau = 20000$, $\therefore: Re_\tau = 25000$. 
7.4 Spectrograms

In an attempt to provide an overview of the energy distribution across various wave-numbers and wall-distances, and also the effect of \( \text{Re}_\tau \) on the energy distribution, contour maps of the premultiplied energy spectra \( k_x \phi_{uu}/u^2_\tau \) are plotted against inner-scaled wall distance \( (z^+) \) and inner-scaled streamwise wavelength \( (\lambda^+_x = 2\pi/k^+_x) \) in figure 7.26 for \( \text{Re}_\tau = 6000 - 20000 \). Here the magnitude of \( k_x \phi_{uu}/u^2_\tau \) is indicated using colour variations. This type of presentation of the energy spectra was introduced by del Álamo and Jiménez [26].

These contour maps show a distinct near-wall peak in the spectrogram for all Reynolds numbers (indicated with white ‘+’ symbols) known as the ‘inner energy site’, which appears to occur at a fixed inner-scaled wall distance \( z^+ \approx 13 \) and inner-scaled wavelength \( \lambda^+_x \approx 1000 \) at a fixed level \( k_x \phi_{uu}/u^2_\tau = 2.2 \). This wall distance corresponds to approximately the peak in the broadband turbulence intensity. This inner site in the spectrogram is related to the near-wall energetic cycle of streaks and quasi-streamwise vortices (Hutchins and Marusic [46], Jiménez and Pinelli [52], Kline et al. [61]). Kline et al. [61] state that the inner peak in the broadband turbulence intensity is the energetic signature due to the viscous-scaled near-wall structure of elongated high- and low-speed regions.

In order to acquire better comprehension of the \( \text{Re}_\tau \) effect on the near-wall energy spectral density, iso-contours of premultiplied energy for the spectral surfaces shown previously, are now shown in figure 7.27 for various \( \text{Re}_\tau \). Four contour levels corresponding to \( k_x \phi_{uu}/u^2_\tau = 0.47, 0.96, 1.4 \) and 1.9 are shown and colour gradients are used to indicate \( \text{Re}_\tau \) variations. The four sets of contours (four Reynolds numbers) show reasonable collapse on the left hand side of the straight dashed line. However, if we look at the right hand side of the dashed line, it is evident that there is an increasing amount of high-wavelength energy with \( \text{Re}_\tau \) extending across all wall heights. This high-wavelength energy in the near-wall region is footprint of an emergent ‘outer energy site’, growth of which with \( \text{Re}_\tau \) is evident in figure 7.26. The outer energy site is known
to be related to the ‘superstructures’ reported first by Hutchins and Marusic [46] for turbulent boundary layer flows. They reported the outer peak location as \( z/\delta \approx 0.06 \) and \( \lambda_x/\delta \approx 6 \). Using measurement data at various \( Re_\tau \), Mathis et al. [81] later refined the location of the peak to be \( Re_\tau \)-dependent as \( z^+ \approx \sqrt{15 Re_\tau} \), which coincides with the geometric centre of the logarithmic region. Since these superstructures scale on \( \delta \) it is instructive to look at the spectrograms with outer-scaled axes as shown in figure 7.28 and 7.29. For the lower Reynolds number cases (\( Re_\tau = 6000, \ 10000 \)), consistent with Mathis et al. [81], we see outer peaks at \( z/\delta \approx \sqrt{15/Re_\tau} \). However, at higher \( Re_\tau \) cases, it is not possible to locate a single peak in the outer region. In fact, the peak starts to grow into a region of plateau, approximate zone of which is demarcated by triangles in the figures. This region of plateau is associated with the peaks/plateaus observed in the one-dimensional spectra of figures 7.12(d),(e) and 7.13(d),(e) where near the lower bound of the logarithmic region, the magnitude of the peaks/plateaus remain nearly constant with decreasing \( z/\delta \). The right vertex of the triangle appears to be fixed at \( z/\delta \approx 0.03 \) and its left edge creeps towards the wall with the increase in \( Re_\tau \) leading to an increased plateau area at higher \( Re_\tau \). Although this large-wavelength region with the plateau is not sufficiently wide, it signals the emergence of the \( k_x^{-1} \) region at higher Reynolds numbers as was discussed in §7.3.3. Moreover, it appears that the lower bound of \( \lambda_x/\delta \) range, over which the premultiplied spectra plateau, decreases with the decrease in \( z/\delta \), while the upper bound is relatively fixed.

In order to gain better comprehension of the \( Re_\tau \) effect on the outer-region in the energy spectral density, iso-contours of premultiplied energy for the spectral surfaces shown previously, are now shown in figure 7.30 in outer scaling for Reynolds number range \( Re_\tau = 6000 - 20000 \). Four contour levels corresponding to \( k_x \phi_{uu}/u_\tau^2 = 0.47, 0.96, 1.4 \) and 1.9 are shown and colour gradients are used to indicate \( Re_\tau \) variations. It is evident that the spectra collapse reasonably well in the outer region (showing existence of an outer-scaling similarity) and gradually separate at lower \( z/\delta \) and \( \lambda_x/\delta \).
Figure 7.26: Premultiplied spectra for the streamwise velocity $k_z \phi_{uu}/u_\tau$ against inner scaled wavelength $\lambda^+_{zz}$ and wall distance $z^+$ at (a) $Re_\tau = 6000$, (b) $Re_\tau = 10000$, (c) $Re_\tau = 14500$ and (d) $Re_\tau = 20000$. ‘+’ indicates inner energy site.
Figure 7.27: Iso-contours of $k_z \phi_{uu}/u_*^2$ against inner scaled wave length $\lambda_z^+$ and wall distance $z^+$ at the contour levels of 1.9, 1.4, 0.96, and 0.47 for various $Re_\tau$. Reynolds numbers are 6000 (---), 10000 (--), 14500 (--), and 20000 (---). The arrow indicates increase in the Reynolds number.
Figure 7.28: Same as figure 7.26, while ordinate and abscissa are normalised by $\delta$ here. ‘x’ indicates the outer energy peak in (a) and (b) while the triangles demarcate region of plateau in the outer region in (c) and (d).
Figure 7.29: Premultiplied spectra for the streamwise velocity, $k_x \phi_{uu}/u'^2$, against outer scaled wavelength $\lambda_x/\delta$ and wall-distance $z/\delta$ at $Re_{\tau} = 25000$. The triangle demarcate the region of plateau. Energy level is not shown in the near-wall region since poor spatial resolution affects the energy in that region.

Figure 7.30: Same as figure 7.27, whereas here ordinate and abscissa are normalised by $\delta$. 
7.5 Skewness and Kurtosis

The third and forth standardized moments of a random variable are called skewness \( S_u \) and kurtosis \( K_u \), respectively. Skewness is a measure of the symmetry and kurtosis is a measure of the tailedness of the probability distribution of a signal and are defined as

\[
S_u = \frac{u'^3}{u'^2^{3/2}},
\]

\[
K_u = \frac{u'^4}{u'^2^2}.
\]  

Figure 7.31(a) and (b) show skewness and kurtosis profiles of the streamwise velocity signals against inner-scaled wall distance \( z^+ \). In these figures, skewness and kurtosis as measured by the NSTAP and the 2.5 \( \mu \text{m} \)-diameter conventional hot-wire for the Reynolds number range \( 6000 < Re_\tau < 20000 \) are compared. It appears that spatial resolution affects both \( S_u \) and \( K_u \) in the near-wall region up to \( z^+ \approx 100 \) but beyond this \( (z^+ > 100) \) good agreement is observed between the NSTAP and the conventional hot-wire. These results are consistent with the observations of Talamelli et al. [136] about the effect of spatial filtering on the skewness and kurtosis in turbulent wall-bounded flows. It should also be noted that in the skewness profiles, a near-wall negative region is seen.
in the NSTAP results for all the Reynolds numbers, while the conventional hot-wire wires cannot resolve this due to spatial filtering.

Figure 7.32(a) and (b) show the evolution of the skewness factor with $Re_\tau$ against inner-scaled and outer-scaled wall-distance, respectively. Similar plots are shown in figure 7.33(a) and (b) for the kurtosis. In the near-wall region a very weak dependency on $Re_\tau$ in the Reynolds number range of this study is observed especially in the skewness profiles. Metzger and Klewicki [85] and Inoue et al. [50] have reported a similar trend over a much larger Reynolds number range. Metzger and Klewicki [85] related this trend to the growing influence of the large-scales with $Re_\tau$. They also reported that the negative peak present in the skewness profiles of low $Re_\tau$ number data is not present for the high $Re_\tau$ data of atmospheric surface layer. Considering the weak dependence of the near-wall trend seen here and also noting that the spatial filtering and $Re_\tau$ exhibit similar near wall effects in the skewness and kurtosis profiles, care must be taken about the spatial resolution when studying such $Re_\tau$ trends. Further away from the wall for $z^+ > 200$ a reasonable collapse can be seen with inner scaling for both skewness and kurtosis before the wake region, where a clear $Re_\tau$ trend occurs. In contrast, with the outer-scaling no clear collapse is seen for the skewness profiles in the Reynolds number range $Re_\tau = 6000 - 20000$ while kurtosis profiles show some degree of collapse in the range $0.03 < z/\delta < 1$.

The non-Gaussian characteristics of the turbulent boundary layer are demonstrated in figures 7.32 and 7.33 especially in the near-wall region. In the logarithmic region, skewness appears to asymptote to -0.1 which is near 0 (= Gaussian standard for skewness), and kurtosis maintains a magnitude of $\sim 2.7$ which is near 3 (=Gaussian standard for kurtosis). The drastic deviation from Gaussian standard of the skewness and kurtosis profiles near the edge of the boundary layer is due to the frequent low speed fluctuations associated with the intermittent turbulent/non-turbulent interface.
Figure 7.32: Skewness profiles for various Reynolds numbers against (a) inner-scaled wall-distance $z^+$ and (b) outer-scaled wall-distance $z/\delta$. The inset in (a) depicts an enlarged version of skewness for $4 < z^+ < 500$. Colour code corresponds to those shown in figure 7.31. Arrows’ direction indicates increase in the $Re_\tau$.

Figure 7.33: Kurtosis profiles for various Reynolds numbers against (a) inner-scaled wall-distance $z^+$ and (b) outer-scaled wall-distance $z/\delta$. The inset in (a) depicts an enlarged version of skewness for $4 < z^+ < 600$. Colour code corresponds to those shown in figure 7.31. Arrows’ direction indicates increase in the $Re_\tau$. 
7.6 Turbulent energy dissipation rate and Kolmogrov scale characteristics

The smallest, dissipative eddies in turbulence are characterised by Kolmogrov scales as seen in §7.3.2. These scales depend only on the turbulent energy dissipation rate ($\epsilon$) and viscosity ($\nu$). Kolmogrov length scale ($\eta$) and velocity scale ($v_\eta$) depend on $\epsilon$ and $\nu$ as

$$\eta = \left(\frac{\nu^3}{\epsilon}\right)^{1/4},$$

(7.31)

and

$$v_\eta = (\nu \epsilon)^{1/4}.$$  

(7.32)

Here we attempt to analyse $\eta$ and $v_\eta$ in turbulent boundary layers using our well-resolved single velocity component measurements. To estimate these, an estimate of $\epsilon$ must first be found. One approach is to assume local isotropy which allows $\epsilon$ to be determined from integration of the one-dimensional dissipation spectrum $D(k_x)$ (Townsend [140])

$$\epsilon = 15 \nu \int_0^\infty D(k_x)dk_x = 15 \nu \int_0^\infty k_x^2 \phi_{uu}dk_x.$$  

(7.33)

Using equation 7.33 leads to large errors if the velocity signals are contaminated with high frequency noise, or if spatial and/or temporal resolution are limiting factors since the multiplication of $k_x^2$ amplifies the high frequency end of the spectra, which do not amount to much energy in the turbulence intensity yet contribute to a significant portion of the energy dissipation rate. Figure 7.34(a) shows the inner-scaled premultiplied one-dimensional dissipation spectra against inner-scaled wave-number at $z^+ = 15$ for the experimental data as well as DNS data at various Reynolds numbers, in the range $Re_\tau = 550 - 5200$ for DNS and $Re_\tau = 6000 - 20000$ for the experiment. Dissipation spectra are premultiplied to allow for visual integration in the semi-logarithmic plot. The
Chapter 7. Well-resolved measurements in turbulent boundary layer flows

Figure 7.34: Inner normalised premultiplied dissipation spectra $\nu^2 k_x D/\nu^+ x$ against (a) inner normalised streamwise wave-number $k_x^+$ and (b) frequency (for the experimental data only). — NSTAP-$Re_\tau = 6000$, — NSTAP-$Re_\tau = 10000$, — NSTAP-$Re_\tau = 14500$, — NSTAP-$Re_\tau = 20000$, — DNS channel-$Re_\tau = 5200$ [67], — DNS boundary layer $\delta_g^+ = 1990$ [128], — DNS channel-$Re_\tau \approx 2000$ [41], — DNS channel-$Re_\tau \approx 550$ [41]. The arrow indicates increase in the $Re_\tau$.

Experimental spectra appear to be noise free and the majority of the motions contributing to the dissipation rate are resolved.

An obvious increasing trend with $Re_\tau$ in the high wave-number end of the DNS spectra is seen. The experimental data however do not exhibit a clear trend. It should be noted that these spectra represent very high frequency phenomena, in the range $1 < f < 30$ kHz in our measurements (figure 7.34(b)), wherein anemometry sensors are known to amplify or attenuate energy [49]. Although owing to the very high frequency response of the NSTAP compared to the conventional hot-wires ($\sim 200$ kHz compared to $\sim 50$ kHz based on the electronic testing) this temporal effect is expected to be minimal in our measurements, careful assessment of NSTAP results in this frequency range is required, which is out of the scope of this study. Therefore, we cannot rely on the $Re_\tau$ trend seen in the experimental dissipation spectra of figure 7.34.

Figure 7.35 shows the inner-scaled energy dissipation rate $\epsilon^+$ against inner-scaled wall-distance $z^+$ for the experiments obtained using equation 7.33. The inner-scaling collapses the profiles in the near-wall and log regions, while a $Re_\tau$ dependence is seen in the
outer region. It should be noted that the logarithmic scaling of the ordinate artificially improves the collapse in the near-wall region. Also shown in the figure is the relation

$$\epsilon^+ = \frac{1}{\kappa z^+}. \quad (7.34)$$

Equation 7.34 follows the assumption that at sufficiently high Reynolds number, there is an intermediate region between inner and outer layers where the transport terms in the kinetic energy equation are small compared with production, so that in this region production and dissipation are approximately in balance, \( p \approx \epsilon \) (Hinze [39]). Using \( p = -\overline{uw} \frac{dU}{dz} \approx u''_t \frac{dU}{dz} \), \( U/u_\tau = 1/\kappa \log(zu_\tau/\nu) + B \), and \( p \approx \epsilon \) leads to equation 7.34. It is noted that this estimate is expected to hold only for the logarithmic region based on the mentioned assumptions. However, it can be seen that the agreement between our experimental data and equation 7.34 in the logarithmic region is not particularly convincing. This might stem from the assumption of isotropy in the logarithmic region, or the balanced production and dissipation.

![Figure 7.35: Inner-scaled dissipation rate \( \epsilon^+ \) against inner-scaled wall-distance \( z^+ \) for various \( Re_\tau \). The arrow indicates increase in \( Re_\tau \). Dashed line corresponds to equation 7.34.](image-url)
Using equations 7.31, 7.32, and 7.34 yields

$$\eta^+ = (\kappa z^+)^{1/4}, \tag{7.35}$$

and

$$v_n^+ = (\kappa z^+)^{-1/4}, \tag{7.36}$$

for Kolmogrov’s length and velocity scales.

Figure 7.36 and 7.37 show the inner-scaled Kolmogrov’s length and velocity scales obtained using equations 7.33, 7.31, and 7.32 across the boundary layer for the experimental data. The inner-scaling leads to reasonable collapse in the near-wall and logarithmic regions, while the profiles exhibit \( Re \tau \) trends in the outer region. Similar to the dissipation
rate, the approximate power law relations do not show convincing agreement with the experiments in the logarithmic region.

7.7 Summary and conclusions

A Nano-Scale Thermal Anemometry Probe (NSTAP) was used to measure the streamwise velocity fluctuations in turbulent boundary layer flows in the Reynolds number range $Re_\tau = 6000 - 20000$ with a unique inner-scaled sensor length range $l^+ = 2.4 - 3.5$. First and second order turbulence statistics where compared to those measured by conventional hot-wires and very good agreement was seen in the logarithmic and outer regions where the effect of sensor spatial resolution is minimal. Furthermore, in the near-wall region where conventional hot-wires fail to resolve the smallest scales of motion, the NSTAP results provide useful insight. It was shown that in the near-wall region the streamwise turbulence intensity normalised with $u_\tau (u^2)$ increases monotonically with $Re_\tau$ in the Reynolds number range of the experiments, and the near-wall peak

Figure 7.37: Inner-scaled Kolmogrov’s velocity scale $v_\eta^+$ against inner-scaled wall-distance $z^+$ for various $Re_\tau$. The arrow indicates increase in $Re_\tau$. Dashed line corresponds to equation 7.36.
in $\overline{u'^2}$ was seen to follow a logarithmic relation with $Re_\tau$. Moreover, profiles of $\overline{u'^2}$ against outer-scaled wall-distance $z/\delta$ in the inertial sublayer and outer layer collapsed well, following a logarithmic decay of the form $\overline{u'^2} = B_1 - A_1 \log(z/\delta)$, with $A_1 = 1.26$ and $B_1 = 1.95$ in the inertial sublayer. A logarithmic curve fit was proposed for the intermediate region of the turbulence intensity profile. Extrapolating the fit, showed a clear outer peak at Reynolds number of $O(10^6)$ exhibiting a very good agreement against atmospheric surface layer data.

Streamwise energy spectra were investigated in the near-wall region, inertial sublayer, and outer region. When spectra of various Reynolds numbers were compared at fixed viscous-scaled wall distances, in the near-wall region ($z^+ \leq 300$) inner-scaling collapsed the spectra for small and moderate wavelengths (large and moderate wavenumbers), while the spectra exhibited an increasing trend with $Re_\tau$ for large wavelengths. These large-scale energy contributions in the $u$-spectra, which increase with $Re_\tau$, are attributed to the footprint of large-scale features in the inertial sublayer, and are responsible for the growth of the turbulence intensity inner peak with $Re_\tau$. In the inertial sublayer, energy spectra scale both with $z$ and $\delta$, therefore exhibiting a mixed scaling. A low wavenumber region of overlap in the spectra was found as a function of $z/\delta$ over which spectra have a quasi-plateau. It was shown that the magnitude of this quasi-plateau (named $A^*_1$ in analogy to Perry’s $A_1$) asymptotes to $1.4 \pm 0.1$ when $z/\delta \to 0$ using our NSTAP data together with previously available atmospheric surface layer data. Moreover, the energy spectra were integrated and a relation of the form $\overline{u'^2} = B^*_1(z/\delta) - A^*_1(z/\delta) \log(z/\delta)$ for the streamwise turbulence intensity was derived suggesting that the Perry’s constant $A_1$ is a function of $z/\delta$ given here as $A^*_1(z/\delta)$. Spectrograms were examined and it was shown that the energetic outer site that appears as an outer peak in the moderate Reynolds numbers $Re_\tau = 6000 - 14500$ grows into a small region of plateau at $Re_\tau = 20000$ and 25000, which signals emergence of the $k^{-1}_x$ region at Reynolds numbers higher than 25000. It is noted that given the observed outer-scaling similarity, it should not be expected to see a $k^{-1}_x$ region anywhere in the boundary layer above outer-scaled wall-distance $z/\delta \approx 0.01$ even at a very high Reynolds number.
We used a local isotropy assumption to determine energy dissipation rate from integration of the one-dimensional dissipation spectra across the boundary layer, which allowed us to calculate Kolmogrov’s length and velocity scales. Good collapse of all the above parameters was seen with inner-scaling in the logarithmic region. Using two assumptions of local isotropy and balance of turbulence energy production and dissipation to calculate the dissipation rate did not seem to give a convincing agreement in the logarithmic region, which might be due to invalidity of either of these assumptions. However, drawing conclusion for turbulent energy dissipation based on one-dimensional data might not be correct.
Chapter 8

Conclusions and future work

8.1 Conclusions

The application of sub-miniature hot-wires in high Reynolds number turbulent wall-bounded flows is thoroughly examined in this study. Such probes are increasingly required for high Reynolds number turbulent flow measurements. Challenges regarding the operation of these hot-wires in constant temperature mode is investigated in Chapter 4. Moreover, in high Reynolds number turbulent flows, researchers are inclined to relax the accepted length-to-diameter ratio (required to avoid end-conduction effects) in order to achieve better spatial resolution (see e.g. Morrison et al. [96], Li et al. [71] and Vallikivi [144]). Therefore, the effect of end-conduction in constant temperature hot-wire anemometry is revisited in Chapter 5.

A Nano-Scale Thermal Anemometry Probe (NSTAP) with a sensing length \( l = 60\mu\text{m} \) is used to measure velocity fluctuations in turbulent boundary layers in the Reynolds number range \( Re_\tau = 6000 - 20000 \) with the probe viscous-scaled length of the range \( l^+ = 2.4 - 3.5 \). A modified Melbourne University Constant Temperature Anemometer (MUCTA) as well as a Dantec Streamline system are used to operate nearly a dozen NSTAP probes at various Reynolds numbers, and their results are compared with those of conventional 2.5\( \mu \text{m} \)-diameter hot-wires in Chapter 6. The NSTAPs that compare
favourably against conventional hot-wires in the outer region of the turbulence intensity profiles are noted as reliable and their data are used for further analysis of high Reynolds number turbulent flows with a unique spatial resolution range of \( l^+ = 2.4 - 3.5 \) in Chapter 7.

The most important findings of this study are:

- Proper filtering of the second stage amplifier is required in order for MUCTA to perform the square-wave test with a sub-miniature wire without damaging the wire due to instability.

- Cold resistance of sub-miniature wires are usually one order of magnitude greater than the conventional wires (\( O(100) \) against \( O(10) \)). Top-of-bridge resistors’ values in MUCTA, which are ideal for conventional wires, were found to be too small for NSTAP as even small electric current fluctuations can damage the sensor. By changing those resistors to 10 times the original values, the electrical fluctuations were dampened by a factor of 3 (as predicted by a theoretical model) and the NSTAP probes were successfully operated by the MUCTA.

- In Chapter 5, a theoretical model for the hot-wire-CTA system transfer function was developed that takes into account the axial heat conduction for the hot-wire filament, the stub’s and prong’s effects, and the asymmetry in the boundary conditions of the wire filament. The model predicts both positive and negative steps in the overall transfer function of the system to the velocity fluctuations with different combinations of probe geometry and conductivity parameters. It should be noted that the presence of positive steps in the transfer function of the hot-wire-CTA system leads to amplification of the measured energy and turbulence intensity, while negative steps result in attenuation of energy meaning that both amplification and attenuation of energy due to end conduction errors are possible. It was observed that for the length-to-diameter ratios of 100 and 200, positive steps are more likely to exist in the system transfer function, while for the length-to-diameter ratio of 50, negative steps are dominant.
Experiments (using conventional hot-wires) were conducted to verify the model with regards to the end-conduction effect, and results were in accordance with the theoretical model predictions confirming that both positive and negative low frequency steps were possible for hot-wires of different length-to-diameter ratios. These effects extend further away from the wall compared to those seen in spatial resolution effects and start from lower frequencies (\( \sim 10 \) Hz) in the energy spectra.

Nearly a dozen NSTAP probes together with a 2.5 µm-diameter hot-wire were used to measure streamwise velocity fluctuations in the HRNBLWT facility at various Reynolds numbers. Comparison of the mean velocity profiles as measured by the NSTAPs and the hot-wire exhibited very good agreement between all the NSTAPs and the hot-wire. However, comparison of the turbulence intensity profiles revealed discrepancies between some of the NSTAPs and the hot-wires in the logarithmic and outer region of the turbulence intensity profiles, where based on the study of Hutchins et al. [47], minimal difference between wires of different lengths is expected. Moreover, the observed variations between NSTAPs and the hot-wires were not consistent neither in size nor in sign. In fact both magnification and attenuation of turbulence intensity were observed in different NSTAP probes when compared to the hot-wires. An investigation of the premultiplied energy spectra revealed that the energy deviation in the NSTAPs, which were in conflict with the hot-wires in the outer region of the turbulence intensity, initiated at very low frequencies (\( \sim 10 \) Hz).

Measurements with both the MUCTA and the Dantec Streamline contained reliable NSTAPs and unreliable NSTAPs. This rules out the effect of the anemometer in the conflicting behaviours of some NSTAPs. Reliable NSTAP is referred to an NSTAP probe that agrees with the 2.5 µm-diameter hot-wire in the logarithmic and outer region of the turbulence intensity profile and the rest of the NSTAP probes are referred to as unreliable NSTAPs. The mean velocity and the turbulence intensity profiles measured by the reliable NSTAPs were averaged to obtain smooth, averaged NSTAP results, which compared well against hot-wires.
Chapter 8. Conclusions and future work

- Results of the reliable NSTAPs showed that in the near-wall region the streamwise turbulence intensity normalised with $u_\tau$ ($u'^2$) increases monotonically with $Re_\tau$ in the Reynolds number range of the experiments, and the near-wall peak in $u'^2$ was seen to follow a logarithmic relation with $Re_\tau$.

- Profiles of $u'^2$ against outer-scaled wall-distance $z/\delta$ in the inertial sublayer (or logarithmic layer) and outer layer collapsed well, following a logarithmic decay of the form $u'^2 = B_1 - A_1 \log(z/\delta)$, with $A_1 = 1.26$ and $B_1 = 1.95$ in the inertial sublayer.

- A logarithmic curve fit was proposed for the intermediate region of the turbulence intensity profile ($100 \leq z^+ \leq 3Re_\tau^{1/2}$). Extrapolating the fit to higher Reynolds number showed a clear outer peak at Reynolds number $Re_\tau \sim O(10^6)$ exhibiting a very good agreement against the atmospheric surface layer data of Metzger et al. [84].

- When energy spectra at various Reynolds numbers and fixed viscous-scaled wall distances were compared in the near-wall region ($z^+ \leq 100$), inner-scaling collapsed the spectra for small and moderate wavelengths (large and moderate wavenumbers), while the spectra exhibited an increasing trend with $Re_\tau$ for large wavelengths. These large-scale energy contributions in the $u$-spectra, which increase with $Re_\tau$, are attributed to the footprint of large-scale features in the inertial sublayer, and are responsible for the growth of the turbulence intensity inner peak with $Re_\tau$.

- In the spatial inertial sublayer, energy spectra scale both with $z$ and $\delta$, therefore exhibiting a mixed scaling. A low wavenumber region of overlap in the spectra was found as a function of $z/\delta$ over which spectra have a quasi-plateau. The magnitude of this quasi-plateau asymptotes to $1.4 \pm 0.1$ when $z/\delta \to 0$ according to our NSTAP data and previously available atmospheric surface layer data. Because of this asymptotic behaviour it is expected that a low wavenumber $k_z^{-1}$ region
appears at extremely high $Re$ in which the logarithmic region extends towards the wall for asymptotically small $z/\delta$.

- Examination of spectrograms revealed that the energetic outer site that appears as an outer peak in the moderate Reynolds numbers $Re_\tau = 6000 - 14500$, grows into a small region of plateau at $Re_\tau = 20000$ and 25000, which signals the emergence of the $k_x^{-1}$ region at Reynolds numbers higher than 25000. This plateau region in the spectrograms is associated with the low wavenumber quasi-plateau region described in the one-dimensional energy spectra.

8.2 Future work

Although most of the NSTAP probes proved to be reliable, some of them yielded inconsistent turbulent energy results, while their mean velocity results were in excellent agreement with conventional hot-wires. Hence, these malfunctions are considered as dynamic rather than static. We were able to identify the reliable NSTAPs by comparing them against the results of conventional hot-wires. However, discovering a procedure to pinpoint the reliable NSTAPs as part of the manufacturing process can be valuable. To this end, one needs to find the reason(s) for these anomalous behaviours. NSTAPs are manufactured using MEMS fabrication techniques, the accuracy of which is known to deteriorate as the dimensions decrease [36]. Therefore, we speculate that the observed inconsistent results could be due to inconsistency in the sensor’s dimensions. A systematic study of the physical properties of each NSTAP probe, i.e. all dimensions of the sensing part, stubs, and prongs as well as the cold resistance of the probes, before using them in high Reynolds number turbulence measurements, can enable us to discover a correlation between the probe’s principal dimensions and the errors in the results. It is noted that the thickness of the sensor as well as its length and width are important, and given the minuscule size and fragility of the NSTAPs, an accurate measurement of all the dimensions is challenging. Finding such a correlation provides a screening scheme
by which the reliable NSTAPs can be identified as part of the manufacturing process, and also helps to improve the probe design by modifying the dimensions accordingly.

Since the problems found with some of the NSTAP probes are dynamic, they might be related to the forming of hot spots on the sensors during the constant temperature operation as reported by Perry et al. [114]. They showed that hot spots can affect the dynamic properties of hot-wires, leading to large measurement errors. To determine whether hot spots are causing the above-mentioned problems, nano-scale thermal imaging may be exploited to acquire temperature distribution of the probes during the operation, which in turn can provide valuable information with regards to the axial conduction related errors discussed in Chapter 5.

Thanks to their smaller size compared to conventional hot-wires, NSTAPs are suitable for the study of high Reynolds number wall-bounded turbulence in the near-wall region. They have been utilized in the Superpipe and HRTF at Princeton University and the HRNBLWT at the University of Melbourne, among other facilities. Recently, construction of a long pipe in the framework of the Center for International Cooperation in Long Pipe Experiments (CICLoPE) at the University of Bologna has been finalized. Although, the main purpose of the construction of the CICLoPE was the viability of resolving the smallest scales using conventional measurement tools, recent published data from this facility revealed spatial resolution issues in the near-wall region at Reynolds numbers higher than $Re_\tau \approx 5000$ [105]. Utilizing NSTAP allows us to overcome this spatial resolution issue up to the highest Reynolds number attainable in the CICLoPE. As a screening scheme for identifying the reliable NSTAPs is not available yet, in order to avoid publishing discrepant data, carrying out multiple measurements at each Reynolds number using various NSTAP probes is required, similar to those taken in the course of this thesis and elaborated in Chapter 6.

Despite its importance to wall-bounded turbulence, there is currently a lack of high Reynolds number data of two-component turbulence measurements. Baidya [5] utilized custom-made miniature cross-wire sensors in boundary layer turbulent flows. However,
this study was limited to Reynolds numbers below $Re_{\tau} = 10000$ to avoid spatial resolution filtering. In the present study we were limited to using single component NSTAP probes to measure the streamwise velocity fluctuating component. Recently, improvements have been made towards the development of x-NSTAPs [28]. x-NSTAPs, similar to conventional cross-wire sensors, are capable of measuring two velocity components in the sensor’s plane with nearly one order of magnitude smaller size in every dimension. Therefore, they can be used in high Reynolds number facilities such as HRNBLWT and CICLoPE to measure all components of turbulence intensity as well as Reynolds shear stress up to very high Reynolds numbers with exceptional spatial resolution.
Appendix A

Temperature correction

The governing equations for a hot-wire-CTA system (e.g. equation 2.36a in Brunn [14]) show that the output voltage of the system not only is a function of the flow velocity past the wire but also the ambient temperature. Therefore, if the temperature of the flow changes significantly during the calibration or the measurement, the regular calibration where the voltage of the system is calibrated against the flow velocity measured by a Pitot-static tube might lead to errors in the results. Perry [116] suggests that if the temperature is within ±0.5°C throughout the calibration and measuring procedure, temperature corrections are not necessary. This is fulfilled for the measurements at $U_\infty = 20 \text{ ms}^{-1}$ in the HRNBLWT, however for higher velocities the temperature variation during the calibration is higher. This is shown in figure A.1 where the flow temperature is demonstrated versus velocity for pre- and post-calibrations for the measurements made at free stream velocities 20, 30, and 40 ms$^{-1}$. Two types of temperature variations are recognised for all the cases:

1. Temperature variation during each calibration which is $\Delta T \approx 0.5^\circ\text{C}$, $\Delta T \approx 1.5^\circ\text{C}$, and $\Delta T \approx 3^\circ\text{C}$ for $U_\infty = 20 \text{ ms}^{-1}$, $U_\infty = 30 \text{ ms}^{-1}$, and $U_\infty = 40 \text{ ms}^{-1}$, respectively.

2. Temperature variation between pre- and post-calibrations when same velocities are compared.
Appendix A. Temperature correction

The latter is dealt with by using the correction scheme proposed by Talluru et al. [137]. However a temperature correction scheme might be required for the cases where temperature variations during the calibrations are higher than 0.5°C since the calibration curves might need to be dewarped for temperature changes.

Several temperature correction methods have been proposed in the literature (see e.g. Abdel-Rahman et al. [1], Bearman [11], Benjamin and Roberts [12], Brunn [14], Hultmark and Smits [42], Tropea et al. [141]). Here we first use the correction by Hultmark and Smits [42]. In this method, similar to other analytical correction techniques, the raw voltage signal from the hot-wire-CTA system (without any signal conditioning) is required. This is available with the Dantec Streamline anemometer where the gain and offset in the signal conditioning stage are known. However the numerical values for the
gain and offset are unknown for the MUCTA. Therefore using this method is not viable for the data obtained using the MUCTA. Figure A.2 shows the corrected and uncorrected data acquired by NSTAP operated by the Dantec Streamline CTA in the HRNBLWT at free stream velocity $U_\infty = 20$ m s$^{-1}$. As expected the effect of the correction on the results is negligible at this velocity. This correction was applied to another dataset acquired at $U_\infty = 30$ m s$^{-1}$ in the HRNBLWT with the same anemometry system and the results are shown in figure A.3. It can be seen that at $U_\infty = 30$ m s$^{-1}$ the effect of temperature correction is negligible as well. Hence we conclude that no temperature
Appendix A. Temperature correction

Figure A.4: (a) Hot-wire-CTA voltage versus the ambient temperature. (b) Hotwire voltage versus the free stream velocity measured by a Pitot-static tube during the measurement as part of the correction scheme proposed by Talluru et al. [137].

correction is required for the data obtained at $U_\infty = 20$ and $30$ ms$^{-1}$.

Measurements conducted at $U_\infty = 40$ ms$^{-1}$ in the HRNBLWT in this study were made with conventional 2.5 µm-diameter hot-wires operated with the MUCTA. Therefore using the above mentioned correction scheme was not possible for these measurements as the raw voltage signal of the hot-wire-CTA system was not retrievable. To overcome this problem we use the trace of the system voltage (conditioned signal) in the free stream that is obtained as part of the correction procedure proposed by Talluru et al. [137]. In this procedure the hot-wire is traversed into the free stream at regular intervals during the course of the measurement and single recalibration points are obtained. If we assume that the temperature drift is the only reason for the drift of the hot-wire-CTA system voltage, we can find a correlation between the voltage and the ambient temperature. This is shown in figure A.4(a) which shows a near linear relation between the hot-wire-CTA system voltage and the ambient temperature. At this stage one might suspect that the voltage drift can occur due to slight changes in the speed of the tunnel fan. This is examined by plotting the hot-wire-CTA system voltage versus the velocity measured by the Pitot-static tube placed in the free stream as shown in figure A.4(b).

It can be seen that there is no correlation between the free stream velocity and the hot-wire-CTA system voltage. Therefore one can assume that the drift between the pre- and post-calibration voltages is due to the temperature drift only and to a linear approximation use $dE/dT = (E_{\text{post}} - E_{\text{pre}})/(T_{\text{post}} - T_{\text{pre}})$ for each calibration point.
(speed) in both pre- and post-calibration curves. Then we use these voltage sensitivities (obtained for all the calibration points) to correct the voltages in the calibration curves using a linear relation as

\[ E_{\text{corr}} = E + \frac{dE}{dT}(T - T_{\text{meas}}) \]  

(A.1)

where \( E \) and \( T \) are the hot-wire-CTA system voltage and the ambient temperature for each calibration point, respectively. \( T_{\text{meas}} \) is the average of temperature during the course of the actual measurement. Using this procedure one can correct the calibration curves for temperature drifts. As the reference temperature is considered to be the (averaged) temperature of the actual measurement, the voltages of the measurement are not required to be corrected. This correction was implemented for the data obtained by a 2.5 \( \mu \)m-diameter hot-wire operated with the MUCTA in the HRNBLWT (for which the hot-wire voltage trace at the free stream was shown in figure A.4). The streamwise mean velocity (\( U \)) profile and the variance of the streamwise velocity component (\( u^2 \)) before and after this correction are shown in figure A.5. It can be seen that the correction leads to larger differences in both mean velocity and turbulence intensity profiles compared to the measurements made at \( U_\infty = 20 \text{ ms}^{-1} \) and \( 30 \text{ ms}^{-1} \) (after using the temperature correction) shown in figures A.2 and A.3, respectively.
Bibliography


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