Voltage Stability Issues in Power Grids: Analysis and Solutions

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Submitted in partial fulfilment of the requirements of the degree of

Doctor of Philosophy

Department of Electrical and Electronic Engineering
THE UNIVERSITY OF MELBOURNE

December 2017
Abstract

Voltage Stability (VS) is gaining increasing significance in today’s power systems, which are undergoing sizeable growth in power consumption and higher integration of renewables. Economic and environmental barriers impede new investment on network infrastructure to keep up with the load growth and renewables’ intermittency. As a result, many of the power systems around the world are being operated close to their VS limits. This has made voltage instability an ever-existing operational problem for many power systems, and reveals the need for smarter and more efficient approaches to analyse and ensure VS. The significance of VS has been well demonstrated by many evidences of real-life incidents of power system instability which have been associated with VS.

From an analytical perspective, with the increasing variability of today’s power systems, with higher levels of intermittent renewables integrated into the grid, more frequent evaluation of power system’s VS condition is imperative. Hence, more efficient VS evaluation tools, in terms of speed, accuracy, and automated applicability are needed. Also, from a practical point of view, prohibitive cost of upgrading power systems’ infrastructure necessitates taking smarter, more efficient alternative approaches to ensure VS of power systems. This includes operating the existing power system components through intelligent, active network management (ANM) schemes.

Continuation power flow (CPF) is the conventional, mostly used approach of steady-state VS analysis. CPF algorithm and all its improved versions, however, suffer from high complexity and relatively long execution time. Considering the need for more frequent VS analysis in today’s renewable-rich power systems, in this thesis, a more efficient approach of plotting the P-V curves, and identifying VS limits, i.e. saddle-node bifurcation (SNB) and limit-induced bifurcation (LIB) points, of power systems is proposed. The method is based on standard Newton-Raphson power flow (NR-PF) algorithm and, thus, relaxes all the complexities of the existing CPF methods. It offers much reduced execution time, high accuracy, automated applicability, and ease of implemen-
tation and comprehension. Several novel, simple techniques are used in the proposed approach to identify both SNB and LIB points. The method is tested on several, including a large-scale, power systems and its performance is compared with some established CPF methods.

Modal Analysis (MA) is another commonly-used approach that can be used to identify the weak areas of a power system, from a VS viewpoint. This thesis proposes two improved MA methods, applicable to radial distribution systems. The proposed MA methods, unlike the original MA, do not ignore active power variation and allow taking into account any combination of active and reactive power variations. As a result, the proposed methods improve the accuracy of the original MA, in identifying the best buses to apply active or reactive compensation, with the aim of improving the distribution system’s voltage stability margin (VSM).

On the other hand, the ongoing technological advances in energy storage systems (ESSs) has made the grid integration of these devices technically and economically more viable. Accordingly, in this thesis, optimal placement and operation of ESSs in power systems with possible embedded wind farms, with a VSM improvement viewpoint, is carried out. The probabilistic nature of the wind is taken into account, through the probability density function (PDF) of the wind farm’s output power. A combination of MA and CPF is used to identify the best placement of ESS in the network. A new method of power sharing between the ESSs, based on their effect on system’s VSM, is proposed too. The required power injection of ESSs, at an optimal power factor (PF), to ensure a pre-specified minimum required VSM, is also calculated at all load-wind levels.

Furthermore, in this thesis, the problem of ESS placement is formulated as a probabilistic optimization framework, through which optimal placement, sizing, and operation of ESS devices in wind-embedded distribution systems are carried out. The main objective of the allocation problem is to minimize the required power and energy ratings of ESSs to be installed, such that a desired level of VSM is always ensured. The reactive power loss and reactive power import from the upstream network are also minimised through a multi-objective optimization framework. Wind uncertainty is accounted for through optimally generated wind power scenarios and using risk-based stochastic optimization approach. Besides, ANM tools, such as tap position of on-load tap changers (OLTCs), modelled by using a new method, and reactive power capabilities of both ESS devices and wind farms, are used as additional means to reduce the required ESS size.

Finally, dynamic simulation is carried out to demonstrate the effectiveness of ESS devices
to dynamically improve VS of power systems. The effects of induction motor (IM) loads, fixed speed induction generator (FSIG)-based wind turbines (WTs), and over-excitation limiter (OEL) of synchronous generators (SGs), on the power system’s short term voltage stability (ST-VS) are evaluated. Then, the use of ESSs to provide dynamic voltage support (DVS) to power system during and after large disturbances, as a countermeasure against short term voltage instability, is investigated. In order to do so, systematic control of ESS, to inject any desired active and reactive powers into the system, is carried out. The effects of implementing fault ride through (FRT) and time-overload (TOL) capabilities of ESS, as well as the ESS’s PF, on ST-VS are also analysed.
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Declaration

This is to certify that

1. the thesis comprises only my original work towards the PhD,

2. due acknowledgement has been made in the text to all other material used,

3. the thesis is less than 100,000 words in length, exclusive of tables, maps, bibliographies and appendices.

________________________________________
Ahvand Jalali, December 2017
THESIS CONTRIBUTIONS

• Chapter 2
  – A new, efficient continuous power flow method is proposed. Several new, simple tech-
niques are introduced into the standard NR-PF algorithm to enable it to plot complete
P-V curves and identify both SNB and LIB points. The proposed method is fully au-
tomated and offers much reduced execution time and less complexity, compared to the
existing CPF-based methods.
  – Two new modal analysis (MA) approaches, applicable to distribution power systems,
are proposed which improve the accuracy of the original MA, in identifying the best
buses to apply compensation at. The proposed MA methods, unlike the original MA,
do not ignore the active power variation, and their efficacies are verified through com-
parison of the results with the VSM analysis results.

• Chapter 3
  – A new method for placement of ESS devices is proposed with the aim of improving the
system’s VSM in an optimal manner. MA results is used for clustering power system
buses, which lays the foundation for systematic ESS placement. The optimal PF of the
ESS devices are also computed to maximize the achieved VSM improvement.
  – A new method for placement and operation of ESS devices, using a combination of
CPF and MA, is proposed to optimally ensure a desired VSM for a wind-embedded
power system under all load and wind variations, and by taking into account the wind
uncertainty.

• Chapter 4
  – A VS-constrained, probabilistic optimal ESS allocation and operation method is pro-
posed, taking into account the joint PDF of the wind and loading level. The method
calculates the minimum required power rating of ESS devices to ensures a desired
level of VSM.
– A new risk-based stochastic placement, sizing and operation of ESSs is proposed. Minimum required power and energy ratings of ESS is computed to ensure a desired VSM, through inclusion of risk-based VS constraints in the problem formulation. Wind uncertainty is modelled through an optimal set of wind power scenarios. The effects of ANM tools, such as reactive power capabilities of ESSs and wind farms, as well as the action of OLTCs, modelled through a new method, are investigated too.

• Chapter 5

– A new method for dynamic procurement of a desired VSM for power system, using systematic control of ESS devices in decoupled d-q frame, is presented. The method accounts for N-1 contingency criterion and the effects of voltage-dependant and constant-power loads are also taken into account.

– A new method for ST-VS improvement of power systems, using DVS capability of ESS, is proposed. In order to do so, ESS is systematically controlled at P-Q operating mode, using a state-feedback approach. The effects of IM loads, FSIG-based WTs, and OEL of SGs on ST-VS are analysed using several illustrative examples and case studies. FRT and TOL capabilities of ESS’s inverter are used to improve the performance of ESS in assuring power system’s ST-VS. The effect of PF of ESS, when providing DVS to the system, is also investigated.
Preface

This thesis contains only the student’s original work towards his Ph.D. The thesis is mainly done by the student, who carried out the literature review, research gap identification, solution approach design, computer simulations, results analysis, and paper/thesis writing. The principle supervisor has also contributed in the thesis through supervision, technical advice/comments provision, and manuscripts proofreading, over regular meeting sessions.

Financial support provided by the University of Melbourne including Melbourne International Research Scholarship (MIRS) and Melbourne International Fee Remission Scholarship (MIFRS) are gratefully acknowledged. Also, the financial help by the Australian Research Council, through Discovery Project grant DP140102180, is highly appreciated.

The outcomes of this thesis are published or under review for publication in the following journals and conferences:

- Chapter 2

  The contribution of each author is as follows: First author: Finding the suitable approach to tackle the problem, writing parts of the paper, and performing simulations. Second author: Supervision, proofreading, writing parts of the paper, and providing technical comments.


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  - A. Jalali and M. Aldeen “Modified Modal Analysis Approach for Distribution Power
Systems”, Accepted in proceedings 2017 IEEE-PES Innovative Smart Grid Technologies (ISGT-Europe), Turin, Italy, October 2017.

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• Chapter 3


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Chapter 5


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Acknowledgements

This thesis owes its existence to the help, support and inspiration by many people. First and foremost, I would like to express my genuine gratitude to my supervisor A/Professor Mohammad Aldeen for his continuous support, unceasing patience, and invaluable constructive criticism. He made my Ph.D. a pleasant, rewarding experience via his friendly and tactful supervision, and by giving me intellectual freedom, supporting my attendance at several conferences, and demanding high quality research throughout my Ph.D.

I appreciate the financial support provided by the University of Melbourne. I also would like to thank my awesome fellow lab-mates Amin, Majid, Sasan, Saeed, Adel, and Narx for providing me with technical and intellectual support and advice during my Ph.D. Most importantly, none of these would have been possible without the love and patience of my family for providing me with unfailing support and continuous encouragement. And finally, my deep gratitude for my beloved wife, Sheler, who has always been an amazing friend and a main reason I managed through the ups and downs of my Ph.D. journey.
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To my beloved parents, Abdolkhalegh and Fatemeh, darling wife, Sheler, and dearest sisters, Ashti, Azhin, and Awaz.
## Contents

1 Introduction .......................... 1
   1.1 Motivation .......................... 1
   1.2 Literature Review .................. 6
   1.3 Thesis Outline ..................... 11

2 Improved Continuous Power Flow and Modal Analysis Approaches .... 15
   2.1 Introduction ....................... 15
   2.2 Efficient Continuous Power Flow to Identify SNBP and LIBP ..... 17
      2.2.1 Fundamental Definitions .......... 18
      2.2.2 Calculation of SNBP and complete P-V curves ........ 20
      2.2.3 Identification of LISB Point ........ 26
   2.3 Simulation Results of the Proposed Continuous Power Flow .... 29
      2.3.1 Case 1 - Ignoring the $Q_G$-Limits of Generators .......... 30
      2.3.2 Case 2 - Considering the $Q_G$-Limits of Generators ...... 32
      2.3.3 Case 3 - Identifying LISBP ........ 33
   2.4 Modified Modal Analysis for Distribution Power Systems ....... 35
      2.4.1 Original Modal Analysis [9] ........ 36
      2.4.2 Proposed Improved MA Methods .......... 37
      2.4.3 Validation of the Proposed MA methods .......... 40
   2.5 Simulation results of the proposed MA Methods ............... 41
      2.5.1 Results of $MA^P$ ................. 43
      2.5.2 Results of $MA^{PQ}$ ............... 43
      2.5.3 Overall Comparison of All Methods ........ 44
   2.6 Summary ................................ 46

3 Placement and Operation of ESS to Improve Voltage Stability Margin of Power Systems 49
   3.1 Introduction ........................ 49
   3.2 ESS Placement To Improve Power System’s VSM ................ 51
      3.2.1 Modal Analysis .................... 51
      3.2.2 Clustering the Power System Buses ............ 51
      3.2.3 ESS Placement .................... 53
      3.2.4 Optimising Power Factors of ESS ............ 53
   3.3 Simulation Results for the Optimal ESS Placement .............. 56
      3.3.1 Case A. Ignoring Generators Reactive Power Limits ........ 56
3.3.2 Case B. Considering Generators’ Reactive Power Limits .......................... 57
3.4 ESS Placement and Operation Considering Wind’s Uncertainty .................. 60
3.4.1 VS Issue in Highly Compensated Power Systems ................................. 61
3.4.2 PDF of the Output Power of Wind Farm ............................................. 62
3.4.3 VSM-Oriented ESS Placement ............................................................. 63
3.4.4 Power Sharing Between the ESSs ......................................................... 64
3.4.5 Operation of the ESSs ................................................................. 65
3.5 Simulation Results for ESS Placement Under Wind’s Uncertainty ............... 66
3.5.1 ESS Placement for VSM Improvement ................................................. 67
3.5.2 Power Sharing Scheme of the ESSs ...................................................... 68
3.5.3 Power Injection Level of ESSs .......................................................... 69
3.6 Summary ............................................................................................. 72

4 Risk-Based Stochastic Allocation of ESS to Improve Voltage Stability of Power Systems ........................................ 73
4.1 Introduction ......................................................................................... 73
4.2 Probabilistic ESS Allocation to Ensure VSM ......................................... 76
4.2.1 PDFs of the Wind Power and Loading Level ..................................... 76
4.2.2 Candidate Buses for ESS Placement ............................................... 76
4.2.3 Problem Formulation ...................................................................... 77
4.3 Simulation Results of the Probabilistic ESS Allocation ............................ 82
4.4 Risk-Based Stochastic ESS Allocation ................................................... 86
4.4.1 Wind Power Uncertainty Modelling Through Scenarios ..................... 87
4.4.2 Modelling the OLTCs’ Action ............................................................. 89
4.4.3 Stochastic Optimization Problem for ESS Allocation ....................... 90
4.5 Simulation Results of the Risk-Based Stochastic ESS Allocation ............ 97
4.5.1 Case Study ..................................................................................... 97
4.5.2 Numerical Results ....................................................................... 98
4.6 Summary .......................................................................................... 107

5 Short-Term Voltage Stability Improvement Using Dynamic Control of Storage Devices ........................................ 109
5.1 Introduction ....................................................................................... 109
5.2 Dynamic VSM Procurement Using ESS Devices ................................... 112
5.2.1 Control of ESS Device ................................................................. 112
5.2.2 Case Study and VSM Criterion .................................................... 115
5.2.3 The Effect of Voltage-Dependent Loads ........................................ 115
5.2.4 Generation of $P^*$ and $Q^*$ From VS Viewpoint ......................... 117
5.3 Simulation Results of Dynamic VSM Procurement .............................. 120
5.3.1 Modelling of Loads .................................................................... 120
5.3.2 Results for Voltage-Dependant Loads ........................................... 121
5.3.3 Results for Constant-Power Loads .............................................. 122
5.4 ST-VS Improvement Using DVS Capability of ESSs ........................... 124
5.4.1 ESS Control Using State-Feedback Approach ................................. 124
5.4.2 Induction Motor Model and Effect on ST-VS ................................. 128
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List of Figures

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1</td>
<td>P-V curves before and after $Q_G$-limit: (a) LIDB, and (b) LISB</td>
<td>18</td>
</tr>
<tr>
<td>2.2</td>
<td>Curve angle ($\Phi$) definition</td>
<td>19</td>
</tr>
<tr>
<td>2.3</td>
<td>(a) Diagram of $tan(\Phi)$ and step-size for the IEEE 30-bus system, (b) Step size with and without considering $Q_G$-limits</td>
<td>21</td>
</tr>
<tr>
<td>2.4</td>
<td>The flow chart of the proposed method</td>
<td>24</td>
</tr>
<tr>
<td>2.5</td>
<td>(a) P-V curves around the SNBP for CPF and proposed method, (b) Symmetry of P-V curves in close neighbourhood of SNBP</td>
<td>24</td>
</tr>
<tr>
<td>2.6</td>
<td>The real part of the critical eigenvalue of $J_R$ for (a) IEEE 30-bus system with LIDBP, (b) Modified IEEE 30-bus with both LIDBP and LISBP</td>
<td>28</td>
</tr>
<tr>
<td>2.7</td>
<td>(a) LISBP identification with $\Delta \lambda^{\text{min}}$ precision, (b) Complete P-V curve including LISBP for modified IEEE 30-bus test system</td>
<td>29</td>
</tr>
<tr>
<td>2.8</td>
<td>P-V curves for all test systems (a) case1 (b) case2</td>
<td>32</td>
</tr>
<tr>
<td>2.9</td>
<td>(a) P-V curve including LISBP, and (b) Sample P-V curves of IEEE 118-bus system</td>
<td>34</td>
</tr>
<tr>
<td>2.10</td>
<td>The effect of compensation on the system’s VSM</td>
<td>41</td>
</tr>
<tr>
<td>2.11</td>
<td>Single-line diagram of 33-bus radial distribution test system</td>
<td>42</td>
</tr>
<tr>
<td>2.12</td>
<td>Voltage Profile at the nominal and critical loading conditions</td>
<td>42</td>
</tr>
<tr>
<td>2.13</td>
<td>Diagram of $\Delta VSM$ for 0.1 MVar reactive power compensation applied at all buses</td>
<td>43</td>
</tr>
<tr>
<td>2.14</td>
<td>$\Delta VSM$ values for 0.1 MW active power compensation</td>
<td>44</td>
</tr>
<tr>
<td>2.15</td>
<td>$\Delta VSM$ values for 0.1 MVA (PF=0.7071) active and reactive power compensation</td>
<td>45</td>
</tr>
<tr>
<td>2.16</td>
<td>Correlation Coefficients between the results of L-index, MA approaches and $\Delta VSM$ calculations</td>
<td>45</td>
</tr>
<tr>
<td>3.1</td>
<td>$\Delta VSM$ values for 60 MW/MVar power injection at weak buses (case A)</td>
<td>52</td>
</tr>
<tr>
<td>3.2</td>
<td>Voltage profile at the SNBP ($\lambda = 3.3152$) with the weakest modes identified</td>
<td>54</td>
</tr>
<tr>
<td>3.3</td>
<td>VSM surface for ESSs at buses 2 and 3</td>
<td>55</td>
</tr>
<tr>
<td>3.4</td>
<td>Voltage profile at $\lambda = 3.3152$ after each round of ESS installation (case A)</td>
<td>57</td>
</tr>
<tr>
<td>3.5</td>
<td>P-V curves for bus 2 after each round (case A)</td>
<td>58</td>
</tr>
<tr>
<td>3.6</td>
<td>$\Delta VSM$ values for 60 MW/MVar power injection at the weak buses (case B)</td>
<td>59</td>
</tr>
<tr>
<td>3.7</td>
<td>Voltage profile at $\lambda = 2.6025$ after each round of ESS installation (case B)</td>
<td>60</td>
</tr>
<tr>
<td>3.8</td>
<td>P-V curves for bus 33 after each round (case B)</td>
<td>60</td>
</tr>
<tr>
<td>3.9</td>
<td>PV curves of bus 30 for IEEE 30-bus power system for different compensation levels</td>
<td>61</td>
</tr>
<tr>
<td>3.10</td>
<td>(a) Weibull distribution function, (b) Sample $P_w - \nu$ characteristic of a wind turbine</td>
<td>62</td>
</tr>
<tr>
<td>3.11</td>
<td>Obtained PDF of the wind farm’s output power</td>
<td>63</td>
</tr>
<tr>
<td>3.12</td>
<td>The proposed method of assuring power system’s VSM</td>
<td>66</td>
</tr>
</tbody>
</table>
3.13 Sensitivity of VSM to 5 MW/MVar power injections at weak buses and two wind power output levels

3.14 $\Delta V_{SM}$ values for buses 30 and 7, and sharing coefficients $(\alpha_k^{P_w})$ at different wind power output levels

3.15 Existing and desired VSM surfaces of the system

3.16 $P_{\lambda}^{\lambda, P_w}$ and $Q_{\lambda}^{\lambda, P_w}$ surfaces for ESS at buses 30 and 7

3.17 The initial and final obtained VSM surfaces

4.1 (a) PDF of the wind farm’s output power, (b) PDF of the system’s loading level

4.2 Joint PDF of the wind farm’s output power and loading level

4.3 Existing maximum loadability ($\lambda^{\max}$) without ESS, versus the desired loadability ($\lambda^* = 1.1 \times \lambda$)

4.4 Active power injection by ESS at bus 30 for case 1

4.5 Active power injection by ESS at bus 30 for case 5

4.6 The Proposed Method of OLTC Modelling

4.7 41-bus rural distribution system under study

4.8 The 6 selected scenarios for the wind power for a summer day

4.9 Normalized load profile and wind power, and system’s existing VSM for the 6th scenario of the considered summer day, before ESS installation

4.10 Simulation results of case 1 for the 6th scenario of the summer day

4.11 Simulation results of case 1 for all considered days

4.12 Simulation results of case 2, 6th scenario

4.13 Achieved VSM ($\alpha^{\max}$), case 3, 6th scenario of the summer day

4.14 Simulation results of case 4, 6th scenario

4.15 Simulation results of case 6, (a) current flow of branch 1-2 in all scenarios, (b) expected violation of thermal limit in line 1-2

5.1 Schematic diagram of the overall ESS control approach

5.2 Decoupled current control for $d$ axis

5.3 The decoupled control of ESS

5.4 The decoupled control of ESS

5.5 The P-V curves of the test system (bus 30) for normal and worst contingency condition

5.6 The existing VSM for the most severe contingency and different load and wind levels

5.7 The effect of PF of the ESS on system’s VSM

5.8 The required apparent power injection by the ESS to ensure desired VSM criteria for the system

5.9 Simulation results for voltage-dependent load, without ESS

5.10 Simulation results for voltage-dependent load, with ESS

5.11 (a) voltage and (b) load variations, for different types of load

5.12 Simulation results for constant-power load, without ESS

5.13 Simulation results for constant-power load, with ESS

5.14 The effect of FCT on system’s VS, with and without ESS

5.15 State feedback control approach of ESS

5.16 Torque, current, and power factor characteristics of a typical IM
<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.17</td>
<td>Schematic representation of the used IM model</td>
<td>129</td>
</tr>
<tr>
<td>5.18</td>
<td>Test circuit: effect of fault clearing time on IM’s behaviour</td>
<td>130</td>
</tr>
<tr>
<td>5.19</td>
<td>Effect of fault clearing time on ST-VS involving IM</td>
<td>131</td>
</tr>
<tr>
<td>5.20</td>
<td>Effect of fault clearing time on ST-VS involving FSIG</td>
<td>132</td>
</tr>
<tr>
<td>5.21</td>
<td>The block diagram of transient OEL</td>
<td>133</td>
</tr>
<tr>
<td>5.22</td>
<td>(a) Time-overload capability of ESS’s inverter [103], (b) The applied function $f(\Delta V)$</td>
<td>134</td>
</tr>
<tr>
<td>5.23</td>
<td>The effect of FSIG on ST-VS</td>
<td>137</td>
</tr>
<tr>
<td>5.24</td>
<td>The effect of OEL on ST-VS</td>
<td>138</td>
</tr>
<tr>
<td>5.25</td>
<td>Effect of ESS’s DVS, FRT, and TOL capabilities</td>
<td>140</td>
</tr>
<tr>
<td>5.26</td>
<td>The effect of FCT on ST-VS, case9 (CCT=0.131s)</td>
<td>142</td>
</tr>
<tr>
<td>5.27</td>
<td>The effect of ESS’s PF on $\Delta V$ at bus 30</td>
<td>143</td>
</tr>
</tbody>
</table>
This page intentionally left blank.
List of Tables

2.1 Weakest buses at the SNBP using Modal Analysis .......................... 30
2.2 SNBP coordinate and other parameters of Case1 .......................... 31
2.3 Required time for finding the proper value of $\alpha$ ......................... 33
2.4 Required time to find the proper value of $\eta$ ................................. 33
2.5 SNBP coordinate and other parameters of Case2 ............................ 34
2.6 LISBP coordinate and execution time for both tested systems .............. 35
2.7 The required additional time for LISBP identification ......................... 35
2.8 Original Modal Analysis and L-index results ................................... 42
2.9 Original MA and L-index results for the first 20 weak buses ................ 43
2.10 $MA^P$ results for the first 20 weak buses .................................... 43
2.11 $MA^{PQ}$ results for the first 20 weak buses .................................. 45
3.1 Modal Analysis Results for the Base Case ...................................... 53
3.2 Results of ESS Placement Algorithm for Case A ............................... 57
3.3 Modal Analysis Results for Case B .................................................. 58
3.4 Results of ESS Placement Algorithm for Case B ............................... 59
3.5 The Most Effective Buses for VSM Improvement at Different Wind Power Levels 68
3.6 Optimal PFs of ESSs at each wind power level identified by PSO ......... 69
4.1 MA results (weak buses) at different wind-load levels ......................... 82
4.2 ESS allocation results for different cases ........................................ 85
4.3 ESS Placement Results for Different Case Studies ............................ 101
5.1 Applied load and wind profiles and required ESS injections ................. 121
5.2 CCT (sec.) for different cases and different PFs of ESS ..................... 141
### ABBREVIATIONS

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>ANM</td>
<td>Active Network Management</td>
</tr>
<tr>
<td>ARIMA</td>
<td>Auto Regressive Integrated Moving Average</td>
</tr>
<tr>
<td>BPF</td>
<td>Bus Participation Factor</td>
</tr>
<tr>
<td>CCT</td>
<td>Critical Clearing Time</td>
</tr>
<tr>
<td>CPF</td>
<td>Continuation Power Flow</td>
</tr>
<tr>
<td>DFIG</td>
<td>Doubly Fed Induction generator</td>
</tr>
<tr>
<td>DG</td>
<td>Distributed Generator</td>
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<tr>
<td>DVS</td>
<td>Dynamic Voltage Support</td>
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<tr>
<td>EIC</td>
<td>Extra Inverter Capability</td>
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<tr>
<td>ESS</td>
<td>Energy Storage System</td>
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<tr>
<td>FCT</td>
<td>Fault Clearing Time</td>
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<tr>
<td>FDPF</td>
<td>Fast Decoupled Power Flow</td>
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<td>FRT</td>
<td>Fault Ride Through</td>
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<tr>
<td>FSIG</td>
<td>Fixed Speed Induction Generator</td>
</tr>
<tr>
<td>IM</td>
<td>Induction Motor</td>
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<td>JM</td>
<td>Jacobian Matrix</td>
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<td>Limit Induced Bifurcation Point</td>
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<tr>
<td>LIDB</td>
<td>Limit Induced Dynamic Bifurcation</td>
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<td>LIDBP</td>
<td>Limit Induced Dynamic Bifurcation Point</td>
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<td>Limit Induced Static Bifurcation</td>
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<tr>
<td>LISBP</td>
<td>Limit Induced Static Bifurcation Point</td>
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<tr>
<td>Abbreviation</td>
<td>Description</td>
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<tr>
<td>MA</td>
<td>Modal Analysis</td>
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<td>MLP</td>
<td>Maximum Loading Point</td>
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<td>NR-PF</td>
<td>Newton-Raphson Power Flow</td>
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<td>OEL</td>
<td>Over Excitation Limiter</td>
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<td>OLTC</td>
<td>On-load Tap Changer</td>
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<tr>
<td>OPF</td>
<td>Optimal Power Flow</td>
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<td>Point of Common Coupling</td>
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<td>Probability Density Function</td>
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<td>PF</td>
<td>Power Factor</td>
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<td>RPC</td>
<td>Reactive Power Capability</td>
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<td>Synchronous Generator</td>
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<td>Saddle-Node Bifurcation</td>
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<td>SNBP</td>
<td>Saddle-Node Bifurcation Point</td>
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<td>ST-VI</td>
<td>Short Term Voltage Instability</td>
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<td>ST-VS</td>
<td>Short Term Voltage Stability</td>
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<td>TOL</td>
<td>Time-Overload</td>
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<td>VS</td>
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<td>Voltage Stability Limit</td>
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<td>WT</td>
<td>Wind Turbine</td>
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xxvii
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Chapter 1
Introduction

1.1 Motivation

Voltage stability (VS) is referred to as the power system’s ability to maintain acceptable voltages throughout the system at normal operating conditions and after being subject to disturbances [1]. Abnormal events such as unsustainable increase in load demand, outage of generation or transmission elements, or faults can lead to unacceptable voltage drops across the system. Depending on the power system’s operating condition, this may lead to an immediate voltage collapse, or activate protective devices, which may initiate a cascading tripping of part(s) of the network, which may consequently lead to a partial or global voltage collapse. Voltage instability has been identified to be the root cause for many of the real-life instability incidents in power systems throughout the world.

Modern power systems are subject to growing risk of voltage instability, as a result of sizeable growth in power consumption and higher integration of renewables. Economic and environmental barriers impede new investment on network infrastructure to keep up with the load growth and renewables’ intermittency. This has made many power systems be operated close to their VS limits, and reveals the need for a smarter and more efficient VS analysis approaches. On the other hand, the increasing integration of intermittent distributed generators (DGs) into the power systems has introduced higher levels of uncertainty in the operating, including VS, condition of the grid. The ubiquity of the inverter-interfaced DGs and energy storage systems (ESSs), with considerable reactive power capabilities, can make the voltage instability problem even more challenging. It is known that while reactive power compensation can improve the voltage profile across the grid, it masks the voltage instability risk by increasing the voltage level at which voltage collapse occurs.
Introduction

These concerns, along with the low voltage stability margin (VSM) of today’s power systems, necessitates adopting proper approaches to secure a safe distance from voltage instability points, regardless of the variations of the load and generated power by renewables.

A common approach to quantitatively ascertain voltage instability risk in a power system is to identify the proximity of an operating point to the voltage collapse (or bifurcation) point, at which the two equilibrium points on the upper and lower parts of the power-voltage (P-V) curve merge. Two common bifurcation points in power systems, which are directly related to the VS problem, are saddle-node bifurcation (SNB) and limit-induced bifurcation (LIB) points [2]. Continuation power flow (CPF), first introduced in reference [3], has conventionally been used to construct the P-V curves of power systems and identify the SNB and LIB points. However, despite being robust and well-designed, CPF is somehow complex to implement and time-consuming. More recently, optimization approaches have been proposed in references [2, 4] for identification of power system’s VS limits. Despite the computational efficiency of these methods, one cannot guarantee that an obtained solution satisfies the so-called optimality assumptions [2]. If those assumptions do not hold, the solution cannot be confidently categorized as a VS bifurcation point [2]. As a result, CPF-based methods are still broadly being used in power network control centres worldwide as a reliable means of power systems’ VS monitoring [5]. Due to the growing variability of power generation in renewable-rich power networks, it will become imperative to assess the system’s VS even more frequently. As a result, research on faster identification of VS limits, through improving the computational attributes of CPF, is still ongoing. Cases in point can be found in references [5–7]. Accordingly, in the second chapter of this thesis, a simple, efficient method of plotting complete P-V curves and identifying SNB and LIB points of power systems is proposed. The method avoids the complexities associated with the original CPF method and its improved versions, and requires much less execution time compared to the existing methods.

Another commonly used method for evaluating power system’s steady-state VS is Modal Analysis (MA). With the recent global trend for integration of DGs and ESSs into the distribution power systems, the placement of these devices is becoming more important. Improving the system’s VSM has been considered as a criterion for the mentioned problem [8]. Accordingly, MA has been used as a tool to identify the best buses to apply compensation, from a VS viewpoint, through eigenvalue and eigenvector analysis of the reduced Jacobian matrix [9]. The original MA,
1.1 Motivation

however, has the important drawback of ignoring the active power variation ($\Delta P$) at the system buses. Although implementing MA for a transmission network requires, and may justify, neglecting $\Delta P$ [9], this is not the case for most of the existing distribution systems. First, distribution systems are typically characterized with high $R/X$ ratio of the lines. As a result, the effect of active power variation on bus voltages can be significantly high. Hence, ignoring $\Delta P$ may lead to inaccurate identification of the most effective buses to apply compensation. Besides, most of the distribution systems have only one voltage regulated bus (i.e. the bus at which the voltage is normally kept constant), which is the connection point to the upstream network [10]. This fact, as will be more clarified in Chapter 2 (Section 2.4.2), eliminates the need for ignoring the active power variation in implementation of MA for distribution power systems. Therefore, in the second part of Chapter 2, two modified MA approaches, applicable to distribution power systems, are proposed. The proposed MA methods eliminate the drawbacks of the original MA and improve the accuracy of identifying the best buses to apply compensation at, to improve the distribution system’s VSM.

On the other hand, the ongoing technological advances in power electronics and storage devices, have made grid integration of large-scale ESSs more prevalent. Examples in this regard are the projects across U.S. to construct battery storage devices of up to 100 MW with four-hour duration capability (400 MWh), to handle the variability of the renewable energy resources [11]. Such ESSs can be used at the transmission and distribution levels for applications such as power quality assurance, load levelling, grid integration of intermittent renewables, power grid’s security improvement, etc. The ability of ESSs to independently control active and reactive powers, provides higher levels of flexibility and controllability and makes these devices highly suitable to improve the voltage stability and controllability of power systems too.

Notwithstanding the relatively extensive research carried out on ESS applications in power systems, the use of ESSs for VS improvement of power system, from both steady-state and dynamic viewpoints, has not received due attention. From a steady-state standpoint, ESSs can be effectively placed and operated in power systems, such that a desired VSM is always ensured, despite the variations of the loads and renewable energy resources. This normally leads to the common cost-effective practice of charging ESSs over low-load (and low electricity price) periods, where VSM is large, and discharging them over high-load (and high electricity price) periods,
where VSM is small. The relatively high cost of ESSs, necessitate efficient and optimal sizing, placement, and operation of this devices. This implies that ESSs must be placed in the most effective nodes of the power system, depending on the intended application, to minimize the required ESS installation cost. Accordingly, in the third chapter of this thesis, a new method of placement and operation of ESS devices in transmission power systems is proposed, with a VS improvement viewpoint. The method identifies the best ESS locations in the power system, using a combination of CPF and MA methods. The operation of ESSs is also carried out such that a desired level of VSM is always guaranteed for the system.

The need for reducing the planning and operation costs of power systems, and, at the same time, the increased uncertainty and intermittency in today’s renewable-rich power systems, have given rise to various probabilistic and stochastic optimization approaches for power system’s planning and operation. Probabilistic optimal allocation of ESSs in power systems has recently received growing attention for a variety of purposes, from minimizing the system’s operational cost to grid-integration of renewables, etc. Notwithstanding the relatively extensive studies carried out in this area, to-date no optimization-based research has been reported in the open literature in respect of optimal ESS allocation to ensure a desired level of VSM for uncertain, renewables-intensive power systems. Considering small VSM in many of the today’s heavily loaded, renewable-rich power systems, ensuring a secure distance from voltage instability points, while keeping within the normal operational constraints, can be of great merit for power system operators. Hence, in the fourth chapter of this thesis, a new stochastic optimization approach for ESS allocation in wind-rich power systems is proposed. A required VSM for the system is assured through inclusion of risk-based VS constraints in the problem. The inherent stochasticity of wind is accounted for using two different methods: (i) probability density function (PDF) of wind farms’ output power, and (ii) optimally generated wind power scenarios, which are incorporated into probabilistic optimization frameworks.

From distribution power systems’ perspective, the problem of voltage instability, also referred to as load instability [10], has been addressed in several research studies [8, 10, 12–14]. Two common mechanisms of a potential long-term voltage instability in distribution systems, initiated by either large or small disturbances, can be found in reference [13]. While the small disturbance voltage instability is associated with the large load build-ups, the large disturbance is a consequence
of the interaction between voltage dependant loads and OLTCs, initiated, for example, by loss of one distribution line in a parallel distribution corridor. The ubiquity of low-inertia IM loads, such as air conditioning, pumps, etc., can also play an important role in the voltage instability problem of today’s distribution power systems, since IMs may stall under low-voltage incidents and deteriorate the VS condition of the system. There have been real-life incidents of voltage instability in distribution power systems. A case in point is the voltage instability incident in a Brazilian distribution system, which spread to the transmission system and caused a major blackout [15]. Hence, ensuring a secure distance from voltage collapse point is of great importance for distribution power systems as well. A secure level of VSM, in fact, ensures that the distribution system can be safely loaded up to a certain extent above its current loading level, from any operating condition, without violating any of the operational constraints or experiencing any potential VS issue. This problem is addressed in the second part of the fourth chapter. It is worth mentioning that violation of an operational constraint (due to unexpected changes in load or renewables output power), depending on the severity and duration of the violation, may activate the protective devices and lead to curtailment of the power supply for a part or the entire distribution system.

Another important, emerging concept in modern power systems is the so-called active network management (ANM) schemes, which includes smart, coordinated operation of the power system apparatus to meet the operational and security requirements of the grid. Considering the prohibitive costs of upgrading power networks, the use of ANM tools is an effective alternative measure to improve the VS of heavily-loaded power systems. Therefore, in the fourth chapter of this thesis, the use of some common ANM tools, such as reactive power capabilities of both ESSs and wind farms and the action of on-load tap changers (OLTCs), is incorporated in the VS improvement problem. In doing so, the ANM tools are utilized in coordination with the other power system components to meet the operational requirements of the grid and, hence, reduce the required ESS size to achieve a desired VSM for the system.

Short term (few seconds) voltage stability (ST-VS), is another increasing industry concern, which may occur in the aftermath of large disturbances such as faults, element outages (such as generators, transformers, and/or transmission lines), or large increases in the load. Many of the instability incidences, which caused considerable loss of load, have been associated to the ST-VS [16–18]. ST-VS has been shown to be tightly linked with the behaviour of induction motors
Introduction

(IMs) during and after disturbances [16–21]. This necessitates dynamic modelling of IMs for a proper ST-VS analysis of power systems. Increasing use of low-inertia IMs in air conditioning, pumps, etc., along with the growing number of voltage-insensitive, power-electronic controlled loads (drives, inverters, etc.), have made the problem of ST-VS even more challenging [16]. Besides, fixed speed induction generator (FSIG)-based wind turbines (WTs), which are broadly being used in many power systems, leave similar adverse effects to that of IMs on power system’s ST-VS condition, through drawing high amounts of reactive power during large disturbances. The capability of ESS devices to supply dynamic voltage support (DVS) to the system is a potential effective means for ST-VS improvement. To the best of the author’s knowledge, systematic control of ESS devices to provide DVS for ST-VS improvement has not been addressed in the literature yet. Accordingly, in the fifth chapter of this thesis, first, a detailed analysis of the impacts of IMs, FSIGs, and over-excitation limiter (OEL) of synchronous generators (SGs) on ST-VS of power systems is presented. Besides, dynamic control of ESS power injection, during and after a fault, to improve ST-VS of power systems, is investigated. The effects of ESSs’ fault ride through (FRT) and time-overload (TOL) capabilities, as well as their operating power factor (PF) on ST-VS are also evaluated.

1.2 Literature Review

To be able to quantitatively ascertain the VS condition of power systems, measures such as L-index [22], improved L-index [23], MA [9], VS phase and gain margin [24], etc. are introduced in the literature. Among all such methods, the ones which try to evaluate the power system VS through identifying the SNB point (SNBP) have received much attention. In the early references [25] and [26], modified Newton-Raphson power flow (NR-PF) methods are presented. These methods are able to converge to points close to SNBP, where the Jacobian matrix (JM) of load flow equations becomes ill-conditioned and close to singularity. In reference [27] quasi steady state load flow trajectories of power systems are obtained, through a non-iterative (Klopfenstein’s) method, by defining variables which are slowly varying with time in a prescribed scheme. The advantage of the proposed method is that it does not exhibit the convergence drawbacks of the conventional NR-PF method around the SNBP, at which the JM is not invertible. However, many
of the early approaches of SNBP identification do not trace the lower part of the P-V curve. This is particularly important to identify and analyse a potential LIB point (LIBP) [28, 29], as will be explained in Chapter 2. A homotopy continuation approach was later proposed in reference [30], which used a tangent predictor and perpendicular corrector, and traced the upper and lower parts of the P-V curve separately.

The CPF method (denoted by “original CPF” in this thesis), which identifies the SNBP through plotting the complete P-V curves, was first proposed in reference [3]. The method introduces an extra variable and equation into the NR-PF algorithm, employs a tangent predictor and vertical/horizontal corrector, and uses system loading, denoted by $\lambda$, as the continuation parameter. Around the SNBP, the continuation parameter is changed to either the voltage magnitude or phase angle with the largest tangent, to avoid Jacobian singularity and get around the SNBP. Since then, modified versions of CPF have been introduced which mainly aim to improve the method’s convergence and speed of SNBP identification. Despite their useful characteristics and applications, all these methods suffer from high complexity, as they retain all of the main steps of the original CPF method, i.e. parametrization of PF equations, prediction and correction steps, and local parametrization around the SNBP, albeit with some useful modifications. For example, different continuation parameters, rather than system loading, such as branch admittance [31–33], active/reactive power loss [31, 34], power at the slack generator [31], and geometric parameters [35], have been used in the literature to carry out CPF. Also, alternative predictor/correctors (secant predictor [31, 36], high order predictor [37], or perpendicular corrector [38]) and various step-size control techniques (based on number of iterations the corrector needs to converge [31, 38], norm of the tangent vector [34, 36], etc. [37]) have been adopted in different references. Fast-decoupled power-flow (FDPF) is also incorporated into the CPF algorithm in references [36] and [39], to further increase the method’s speed. A particular drawback of FDPF-based CPF methods is that the basic decoupling assumption (P-V and Q-$\delta$ are assumed decoupled) is not valid around the SNBP, due to the large deviations in voltage phase angles at high loading levels. Thus, the number of iterations required by FDPF algorithm to converge to a point around the SNBP increases, and this can significantly slow down the convergence of the method around the SNBP. This explains why some FDPF-based CPF algorithms, such as that reported in reference [39], switch back to standard NR-PF method around the SNBP. A new geometric parameterization technique for CPF
calculation is proposed in reference [35]. It introduces a line equation to the power flow formulation, which passes through an operating point in the plane of active power loss and loading condition. In reference [37] an asymptotic numerical method is used for CPF calculations which uses a high-order predictor, instead of first-order tangent predictor. Apart from high complexity of all the above-reviewed methods, none of them have addressed the identification of LIBP.

Identifying LIBP of power systems has also been addressed in a number of references, which either are based on complex predictor/corrector-based approaches [40–42], or do not address the SNBP identification [28,40,42]. In [40] and [41], predictor/corrector-based methods are proposed to find reactive power limit points (referred to as $Q_C$-limit points) of power system on the P-V curve, including LIBP. In reference [42], an arc-length corrector is used in the CPF method to identify LIBP. The method uses the tangent vector before the bifurcation point to distinguish between SNBP and LIBP. In fact, to the best of our knowledge, there are very few references, such as [2] and [42], which have addressed both SNBP and LIBP of power systems by using a single approach. However, the optimization approach of reference [2], cannot guarantee that the obtained solution would satisfy the optimality conditions, such that the solution can be associated to SNBP or LIBP with certainty. Also, the method of reference [42] can lead to wrong identification of SNBP and LIBP (especially in odd-shaped P-V curves), if the step size is not chosen carefully.

In summary, recent literature review on the existing CPF approaches shows that nearly all the methods proposed to identify SNBP or LIBP, introduced after the original CPF of reference [3], to various degrees, are based on the same steps of this algorithm, and thus suffer from the common drawbacks of high complexity and computational burden. In addition, almost all of the reviewed references address either SNBP or LIBP identification, rather than both of them, using the same approach.

On the other hand, grid integration of ESS devices has recently been investigated in the literature for various applications such as power quality assurance, load levelling, grid integration of intermittent renewables, energy cost reduction, etc. [43–49]. In reference [44], ESS’s application to smooth the fluctuation of wind farms’ output power is studied. Optimal allocation of ESS devices is also studied in references [43] and [45] to reduce the need for wind power curtailment. In reference [45], optimal sizing of ESS devices in pre-determined locations of the system, to achieve a desired level of wind power curtailment, is studied, in order to keep within bus voltage
1.2 Literature Review

and thermal limits. However, the probabilistic nature of wind is not considered in reference [45]. Reference [43] investigates optimal ESS allocation to eliminate the need for wind energy spilling. The wind speed is modelled through a generated time series data, using auto regression moving average (ARMA) technique. The charging and discharging periods of ESS devices, however, are pre-specified in reference [43] and are not determined by the model.

Recently, stochastic and robust optimization techniques have received increasing appreciation in tackling planning problems including uncertainties. In references [46–49], optimal ESS allocation is investigated to minimize the system’s cost (operational, investment, etc), using stochastic [47, 48], robust [49], and heuristic [46] optimization approaches. Optimal ESS allocation is investigated in reference [46] to minimize the total social cost, using a probabilistic optimal power flow. Reference [48] has addressed ESS allocation problem to minimize the system’s expected operation cost, by using a scenario tree to capture wind’s stochasticity. A robust optimization approach has also been proposed in [49] for ESS allocation with the aim of minimizing system’s total cost. In references [47–49], nevertheless, binary variables are used to determine the ESS locations. This is known to aggravate the problem’s scalability, since mixed integer problems are generally more difficult to solve [45]. Moreover, in all references [46–49] DC load flow is used, and system voltages and reactive power capabilities (RPCs) of ESSs and DGs are overlooked. From a VS viewpoint, ignoring voltage and reactive power constraints can lead to highly inaccurate evaluation of the system’s VS condition. Hence, the use of AC power flow constraints is imperative for proper VS consideration in such problems. Recently, convex optimal power flow has been used in [50] for ESS allocation to minimize the system’s cost and power loss. However, the ESSs’ sizes and locations are not co-optimized, and thus the approach may lead to sub-optimal allocation. The use of ESS devices for steady-state VS improvement, however, has only recently been investigated in references [51] and [52]. In reference [51] the effect of ESS on the minimum eigenvalue of the Jacobian matrix (as a criterion for system’s VS condition) is studied, while ESS is sized from an economic viewpoint, with no VS considerations. Also, sensitivity of the system voltages to load fluctuations is considered as a measure of VS evaluation in reference [52]. A required minimum VSM is not ensured in any of these references.

The VS of distribution power systems, on the other hand, has been investigated in several recent research studies, for several operational and planning purposes, such as optimal DG place-
ment [8, 10, 12, 14], optimum network reconfiguration [53], and optimal capacitors placement [54]. In references [10] and [12], DG placement is respectively carried out to maximize the expected, weighted voltage profile and minimize the power loss. While these methods can improve the system’s VS, a desired VSM cannot be guaranteed. Besides, both references [8] and [12] use simulation (rather than optimization)-based approaches. Also, the system uncertainties are not considered in any of the references [8, 12, 14, 53, 54]. Considering the small VSM in heavily loaded transmission systems, adopting a proper approach to ensure a minimum desired VSM for the system, taking into account the system’s uncertainties, is essential.

ST-VS of power systems has also been investigated in several research papers, using transient P-V curves [19, 21, 55], taking the load dynamics into consideration [16–19, 21], and by investigating the effect of remedial dynamic compensation [16, 19, 56]. Dynamic reactive power injection after a disturbance, known as dynamic voltage support (DVS), using static compensator (statcom) has traditionally been used in many references [19, 57–60], for ST-VS improvement. Lyapunov-based adaptive controller [57], feedback linearization [58], different state-feedback approaches [59], and PI adaptive controller [60] are among the utilized methods for statcom control in order to improve the ST-VS of the power system. In references [19] and [61], ST-VS have been investigated through analysis of IM’s dynamics and transient P-V curves, and dynamic compensation by photovoltaic (PV) systems has been proposed to improve ST-VS. However, a simplified first-order IM has been used which can lead to optimistic ST-VS evaluation [16]. Also, the effect of OEL of the SGs is neglected. In reference [18], ST-VS has been investigated through eigenvalue tracking approach, and IM disconnection (rather than dynamic compensation) has been proposed to counteract short-term voltage instability (ST-VI). A coordinated voltage control scheme has been proposed in reference [56] to ensure adequate dynamic reactive power reserve, provided by inverter-based DGs, during the transient events, while the OELs’ action is overlooked. Reference [62] has shown that cloud coverage in small, photovoltaic-rich distribution systems can act like a large disturbance and cause ST-VI, where ST-VI is characterized by voltage oscillation due to the repetitive connection/disconnection of the IMs’ start-up circuit, rather than by the excessive voltage drops. Despite their advantageous contributions, the proposed dynamic compensation approaches of the reviewed references have been limited to reactive power injection, except for reference [61]. Besides, the OELs’ action, which has determining impact on ST-VS, has been
1.3 Thesis Outline

Chapter 2: Improved Continuous Power Flow and Modal Analysis Approaches

In the first part of this chapter, a new, efficient method of drawing the complete P-V curves of a power system is proposed. The algorithm is based on conventional NR-PF, and thus, avoids the complexities of the existing CPF methods. Three new, simple techniques are proposed to (i) approach the SNBP, (ii) eliminate the Jacobian matrix singularity at the SNBP, and (iii) transit to the lower part of the P-V curve. The generation of the required parameters of the algorithm, namely perturbation and transition coefficients, are automated, to make the method suitable for on-line applications. Besides, a simple additional step is incorporated to identify LIBP of the power flow equations. The method is based on the eigenvalue analysis of the reduced Jacobian matrix at the reactive power limit points. The method is tested on several, including a large-scale, power systems and its efficiency, in terms of speed and accuracy, is compared with the original CPF method and some of its improved versions.

In the second part of this chapter, two improved MA methods, applicable to radial distribution systems, are proposed. The proposed MA methods are used in Chapter 4 to determine the candidate buses for ESS allocation in a distribution power system. Unlike the original MA approach,
the proposed methods do not ignore active power variation and allow taking into account any combination of active and reactive power variations. The proposed MA methods improve the accuracy of the original MA, in identifying the best buses to apply active or reactive power compensation, with the aim of improving the distribution system’s VSM. The effectiveness of the proposed methods is verified through VSM analysis and comparison with the results of original MA and L-index approaches.

Chapter 3: Placement and Operation of ESS to Improve Voltage Stability Margin of Power Systems

The first part of this chapter proposes a method for placement of grid-scale ESS devices in a transmission power system. It is shown that MA can be effectively used for clustering power system buses from a VS viewpoint. Then, considering the identified clusters, a combination of MA and CPF is proposed to optimally increase the system’s VSM up to a desired level. The effectiveness of active and reactive power compensation in improving the system’s VSM are evaluated, and the optimal power factor (PF) for the operation of ESSs is calculated, accordingly. It is shown that the proposed method also improves the voltage profile at the weak areas of power system.

In the second part of this chapter, the wind power uncertainty is taken into account in the ESS placement problem. The PDF of the wind farm’s output power, obtained by carrying out Monte-Carlo simulation, is used for this purpose. The operation of the ESS devices is also incorporated into the problem, with the aim of ensuring a desired VSM, regardless of the load and wind variations. Based on the effectiveness of the installed ESSs on the system’s VSM, power sharing between the installed ESSs is carried out. The required power injection of ESSs, at the optimal PF, is also calculated such that the required VSM is ensured at all load-wind combinations.

Chapter 4: Risk-Based Stochastic Allocation of ESS to Improve Voltage Stability of Power Systems

In the first part of this chapter, the problem of ESS allocation is formulated as a probabilistic optimization problem, through which optimal placement, sizing, and operation of ESS devices are carried out. The objective of the optimization problem is to minimize the required power rating of the ESSs to be installed such that a desired level of VSM is always ensured, at all wind-load
combinations. VS constraints are incorporated into the optimal power flow (OPF) formulation to ensure a desired VSM. The uncertainties of wind power generation and system loading are taken into account, by computing the joint PDF of wind and load, and using it in a probabilistic optimization framework. Also, ANM tools, such as reactive power capabilities of both ESS devices and wind farms, as well as a 10% extra rating of their associated inverters, are used as additional flexibilities to reduce the required ESS size. Different case studies show the impact of various ANM tools, as well as the desired VSM level, on the required ESS size.

Since optimal dimensioning of ESSs includes computation of both power and energy ratings of these devices, in the second part of this chapter, a new stochastic optimization problem is formulated to minimize the power and energy ratings of ESS, required to ensure a desired VSM for a distribution power system. The reactive power loss and reactive power import from the upstream network are also minimized through a multi-objective optimization problem. Wind uncertainty is accounted for through optimally generated wind power scenarios, using historical wind data, and a stochastic optimization framework. Risk-based VS constraints are used to limit the risk of violating the system’s constraints, at the desired maximum loading level, to a pre-set risk level. The proposed MA approach in Chapter 2, is used to identify the candidate buses for ESS installation. Besides, the action of OLTCs is modelled by using a new, simple method and incorporated into the problem formulation to further reduce the required ESS size. The effects of ANM tools, desired VSM, and acceptable risk level on the required ESS size are evaluated through different case studies.

Chapter 5: Short-Term Voltage Stability Improvement Using Dynamic Control of Storage Devices

In the first part of this chapter, dynamic simulation is carried out to demonstrate the effectiveness of the ESS devices to dynamically ensure a desired level of VSM for power systems. The required active and reactive power injections by the ESS are computed, from a steady-state viewpoint, by considering the worst-case N-1 contingency criterion. The effectiveness of the proposed compensation scheme is evaluated through dynamic simulations. In order to do so, ESSs are systematically controlled to dynamically track the computed active and reactive power references. The effect of voltage-dependent loads is also evaluated through case studies.
In the second part of this chapter, ST-VS of power systems is analysed. The detrimental effects of IMs, FSIG-based WTs, as well as the OELs of SGs on ST-VS are evaluated in detail, using several case studies and illustrative examples. The control of ESS devices is carried out to provide DVS to the system during and after large disturbances, to improve ST-VS. To implement ESS’s DVS capability, the required power injection by the ESS is dynamically determined, based on local voltage measurement, and then tracked, after occurrence of a disturbance. FRT and TOL capabilities of ESS’s inverter, are also used to increase the ESS’s effect on ST-VS improvement. Besides, the effect ESS’s PF, when providing DVS to the system, on ST-VS is studied and the optimal PF of ESS, from a ST-VS viewpoint, is determined.

Chapter 6: Conclusion and Future Research

In this chapter the thesis is concluded and future research directions are also proposed.
Chapter 2

Improved Continuous Power Flow and Modal Analysis Approaches

In this chapter, first, a new continuous power flow method is proposed to plot complete P-V curves of power system. The method is based on standard NR-PF algorithm, and, as a result, it relaxes all the complexities associated with the existing CPF methods. Simple, new techniques are used to approach the SNBP, eliminate the Jacobian singularity at the SNBP, transit to the lower part of the P-V curve, and identify the LIBP of power system. The proposed algorithm offers low execution time, ease of implementation, and automated applicability. In the second part of this chapter, two modified MA approaches, applicable to distribution power systems, are proposed. Unlike the original MA, which neglects the effect of active power variation, the proposed MA methods allow taking into account both active and reactive power variations. The effectiveness of the proposed methods are validated through VSM analysis and comparison with the results of original MA and L-index.

2.1 Introduction

The focal aim of constructing P-V curves is to ascertain how far an operating point is from the VS limit points, corresponding to either SNBP or LIBP. A main difficulty which arises when plotting the P-V curves, using standard NR-PF algorithm, is that the Jacobian matrix of load flow equation becomes ill-conditioned, i.e. near singular, in the close vicinity of the SNBP. Hence, NR-PF is unable to calculate the SNBP, since the inverse of the Jacobian matrix can not be computed at this point. CPF method is conventionally used to avoid the Jacobian singularity around the SNBP and plot the complete P-V curves of power system. However, the complexity and computation burden of existing CPFs are significant drawbacks that require attention.

On the other hand, with the recent global trend for integration of DGs and ESSs into the distribution systems, the placement of these devices is becoming more important. Improving the
VSM of the distribution system has been considered as a criterion for the mentioned problem. Accordingly, MA has been used to identify the best buses of the system to apply compensation at, from a VS viewpoint. The original MA, nevertheless, suffers from an important drawback of ignoring the active power variation. This may lead to sub-optimal placement of DGs and ESSs, when increasing the VSM of a distribution system is aimed.

In this chapter, first, a simple method of drawing complete P-V curves, using standard Newton-Raphson power-flow (NR-PF), is presented. Three novel, simple techniques are incorporated into the NR-PF algorithm to enable drawing complete P-V curves. The method is able to compute/identify both SNBP and LIBP of the power system, unlike almost all of the existing methods reported in the literature, which can only identify either SNBP or LIBP through the same algorithm. Four simple techniques are incorporated into the standard NR-PF method to enable drawing complete P-V curves and accurate identification of both SNBP and LIBP. Ease of implementation and comprehension and much reduced execution time are the noteworthy advantages of the proposed method. In the second part of this chapter, two improved MA approaches are presented which eliminate the drawback of the original MA, i.e. overlooking the active power variation. The proposed MA approaches yield more accurate placement of active and reactive power compensations, with the aim of improving the distribution system’s VSM. In general, the proposed CPF and MA methods in this chapter offer the following contributions:

1. The proposed CPF method is highly efficient, i.e. fast and accurate, compared to the existing CPF methods. This is particularly important considering the need for more frequent VS analysis in renewable-rich, highly uncertain power systems.

2. Both SNBP and LIBP can be identified, with any prescribed accuracy, unlike almost all of the existing CPF methods in the literature, which can only identify either SNBP or LIBP using the same approach.

3. The proposed CPF method is fully automated and does not need any tuning or adjustment by the user. The few required parameters are automatically tuned within the algorithm. As a result, the algorithm is suitable for application to any system with any operating condition.

4. The proposed CPF approach is fully based on the standard, widely available, NR-PF algorithm. As a result, it is easy to implement and comprehend, as it avoids the aforementioned
2.2 Efficient Continuous Power Flow to Identify SNBP and LIBP

Complexities of the existing CPF-based methods.

5. The proposed MA approaches do not ignore active power variation, unlike original MA, and allow taking into account any combination of active and reactive power variations.

6. The proposed MA methods improve the accuracy of the original MA, in identifying the best buses to apply active or reactive compensation at, verified by VSM analysis.

In the rest of this chapter, the proposed CPF and MA approaches are elucidated and the simulation results associated with each method are presented. In Section 2.2, the proposed CPF method is explained in detail. Section 2.2.1 explains the fundamental definitions required for presenting the proposed method. In Section 2.2.2 the proposed method for identification of SNBP and tracing complete P-V curves is described. The additional simple technique for identification of LIBP is explained in Section 2.2.3. Simulation results of this part are presented in Section 2.3. The results corresponding to SNBP identification and complete P-V curve drawing are presented in Sections 2.3.1 and 2.3.2 for the cases with and without considering reactive power limits (Q-limits) of generators, respectively. The results of LIBP identification are also presented in Section 2.3.3.

Section 2.4 is dedicated to the proposed new MA methods. The original MA and the proposed, improved MA approaches are, respectively, described in Sections 2.4.1 and 2.4.2, followed by the method for validation of the proposed MA approaches, presented in Section 2.4.3. Section 2.5 presents the simulation results of the proposed MA methods. The results for MA$^P$ and MA$^{PQ}$ are, respectively, presented in Section 2.5.1 and 2.5.2. An overall comparison of all the original and proposed MA approaches are also drawn in Section 2.5.3. Finally, in Section 2.6, the chapter is summarized and concluded.

2.2 Efficient Continuous Power Flow to Identify SNBP and LIBP

The proposed continuous power flow method to plot complete P-V curves and identify SNB and LIB points of power system is described in this section.
2.2.1 Fundamental Definitions

- Maximum Loading Point (MLP)

Identification of the MLP (corresponding to either SNBP or LIBP) of a power system, is conventionally done through plotting the P-V curves of the system. The SNBP, corresponding to the nose point of the P-V curves, is a point where the power flow Jacobian matrix has a simple zero eigenvalue, with non-zero eigenvectors [2]. For systems with no LISB occurring, the MLP corresponds with SNBP. On the other hand, LIB is mostly due to the limited reactive power capability of the generators (denoted by $Q_G$-limits) [40]. Two common types of LIBP are limit-induced dynamic bifurcation point (LIDBP) and limit-induced static bifurcation point (LISBP). In LIDBP, the operating point is on the upper, stable portion of both the unlimited and limited P-V curves, as seen in Fig. 2.1-(a). In this case, the system remains voltage stable, and an equilibrium point continues to exist when $Q_G$-limits are reached, but the voltage stability margin (VSM) suddenly decreases [28]. The LISBP, on the other hand, is located on the upper (stable) part of the unlimited P-V curve, and on the lower (unstable) portion of the limited P-V curve, as seen in Fig. 2.1-(b). This means that once the $Q_G$-limit is encountered, the system becomes voltage unstable immediately [2, 29].

In power systems with a LISB occurring before the SNBP, the MLP is the same as the LISBP, and thus, any load increase beyond LISBP would result in voltage instability. However, the power flow Jacobian matrix does not become singular at LISBP [68]. Thus, all methods that are based
2.2 Efficient Continuous Power Flow to Identify SNBP and LIBP

on the Jacobian singularity cannot detect the LISBP. Alternatively, the direct distance to the generators’ $Q_G$-limits should be used. This requires monitoring the $Q_G$ reserves of the generators and proper identification of the critical generator whose $Q_G$-limit leads to occurrence of LISB [40, 68]. As the maximum loadability of any power system is determined by either SNBP or LISBP (whichever is reached first), it is important to precisely identify both types of bifurcation points.

- Voltage Stability Phase Margin

Using standard NR-PF, rather than CPF, to identify MLP necessitates having a reliable criterion to indicate the proximity to the SNBP at each operating point. Although the minimum eigenvalue of the Jacobian matrix, which approaches zero as the SNBP is approached, seems an appropriate indicator of proximity to SNBP, the computational burden of the eigenvalue computation at each loading level, especially for large-size power systems, will significantly slow down the algorithm. This issue is alleviated in this thesis as follows:

It is generally known that by approaching the SNBP, the rate of change of system voltages with respect to the change in the system loading increases at all of the system’s buses, especially at the weakest bus. This rate of change is defined in reference [24] by the term “voltage stability phase margin (PM)”. At any loading condition, PM is computed from equation (2.1):

$$PM^k = \left(90 - \Phi^k\right) \times \text{sign} \left(\Delta \lambda^k\right) \quad (2.1)$$

where $\Phi^k$ is the curve angle at the $k^{th}$ loading level, and $\Delta \lambda^k$ is the change in the system’s loading
between loading levels \( k-1 \) and \( k \), which is referred to as the step-size. Fig. 2.2 shows a P-V curve illustrating the definitions of \( \Phi^k \) and \( \Delta \lambda^k \). The \( \Delta \lambda^k \) is positive for the upper part of the P-V curve, and vice versa. \( \Delta V^k \) is the magnitude of the maximum voltage change among all buses between loading levels \( k-1 \) and \( k \). As seen, the PM is large and positive at light loading levels, and approaches zero at the SNBP. Then, it becomes negative on the lower part of the curve, which indicates voltage instability. The advantages of using the PM to identify the proximity to the SNBP are: (i) Calculating the PM does not require much computational effort, as PM can also be inferred from the Jacobian matrix elements [24], (ii) Around the SNBP, PM always approaches zero for any system of any size and complexity, and thus, it is a generic and accurate index for VS. In the case of highly compensated power systems, where voltage collapse occurs at voltages near the nominal, the curve angle remains relatively unchanged up to a very close vicinity of the SNBP, and then rapidly increases to \( 90^\circ \). This implies that the generality of the PM definition remains unaffected for these systems too.

2.2.2 Calculation of SNBP and complete P-V curves

This section, presents the proposed method to identify SNBP and drawing complete P-V curves.

A. Step-size Adjustment (SSA)

A key to ensuring robustness, speed, and accuracy of CPF algorithms is to adopt an efficient step-size adjustment (SSA) method. In fact, by using standard NR-PF calculations, rather than the CPF, the operating points in a very close proximity of the SNBP can be computed, without encountering numerical difficulties, only if an efficient SSA is applied [32]. An efficient SSA should allow for the SNBP to be calculated with any prescribed accuracy. Also, it should be adjusted such that the algorithm is executed as fast as possible. Ideally, most of the execution time should be spent on identifying the SNBP. Two common SSA approaches, used in the literature, are:

- **SSA1**: This approach is based on the number of iterations the corrector step needs to converge [31, 38], where the SSA is done according to: \( \Delta \lambda^k = \Delta \lambda^{\text{max}} / N_{\text{corrector}}^{k-1} \).

- **SSA2**: This approach is based on the Euclidean norm of the tangent vector [34, 36], where the step-size is adjusted according to: \( \Delta \lambda^k = \Delta \lambda^{\text{max}} / \| \tau \|_2^k \).
2.2 Efficient Continuous Power Flow to Identify SNBP and LIBP

\[ \Delta \lambda^k = e^{-\tan(\Phi^k)} \Delta \lambda_{max} + \Delta \lambda_{min} \]  

(2.2)

Fig. 2.3-(a) illustrates the diagrams of \( \tan(\Phi^k) \) and step-size versus loading level, obtained for

\[ \Delta \lambda^\text{max} \] and \( \Delta \lambda^k \) are the maximum step-size and the step-size at \( k^{th} \) loading level, respectively. \( N^k_{\text{corrector}} \) and \( \| \tau \|^2_k \) are respectively the number of iterations required by the corrector step to converge, and the second-order Euclidean norm of the tangent vector, at the \( k-1^{th} \) and \( k^{th} \) loading levels. In the original CPF of reference [3], and its improved versions, the step-size is defined as \( \Delta \lambda \) at all points except around the SNBP, in which the step-size is defined as the change in the selected continuation parameter. In order to keep the notation consistent, the symbol \( \Delta \lambda^k \) is used here to indicate the step-size regardless of the parametrized variable or method used. In this thesis, the step-size, \( \Delta \lambda^k \), is dynamically controlled based on the curve angle at any loading level (\( \Phi^k \)). The method exploits the physical behavior of the system around the SNBP, and thus renders the algorithm generic and accurate. The SSA is captured by equation (2.2) and is explained simply as follows: As the loading condition approaches SNBP, the slope of the P-V curve, \( \tan(\Phi^k) \), approaches infinity. Thus, based on equation (2.2), at light loadings the step-size is set very close to \( \Delta \lambda^\text{max} \), but near SNBP it is automatically adjusted to \( \Delta \lambda^\text{min} \). As seen from equation (2.2), an advantages of the proposed SSA is that SNBP can be computed with a prescribed accuracy of \( \Delta \lambda^\text{min} \).
IEEE 30-bus power system [69]. For this system, the active powers of all generators are set to 10 MW, and the loading level of all buses is increased in proportion to their initial values. The values of $\Delta \lambda_{max} = 0.01$ (which is system dependent [31, 36]) and $\Delta \lambda_{min} = 1 \times 10^{-5}$ (which yields accuracy of the order of $10^{-5}$ in the SNBP identification) have been used. As seen from Fig. 2.3-(a), the step-size initially decreases with a gentle slope, and then sharply decreases to $\Delta \lambda_{min}$ around the SNBP. This considerably increases the speed of the algorithm. Note that using the small value of $\Delta \lambda_{min}$ is justified because: (i) it is used around the SNBP only, (ii) it enables the method to calculate the SNBP with high accuracy, and (iii) it allows for computing the operating points in a very close proximity to the SNBP, without encountering “any” numerical difficulties. Experiments carried out on different size power systems have shown that the proposed method is robust for the level of accuracy $\Delta \lambda_{min} \leq 10^{-3}$.

Also, to demonstrate the robustness of the proposed SSA method, the $Q_G$-limits for the same IEEE 30-bus system, are taken into account. Fig. 2.3-(b) shows the step-size and P-V curves for bus 30, the weakest bus, with and without considering $Q_G$-limits. As seen, by encountering each $Q_G$-limit the voltage starts to drop more sharply due to the lack of reactive power support at the generator buses. However, the dynamic SSA of equation (2.2) adapts to the new operating condition and decreases the step size automatically.

B. Identifying the Weakest Bus and Perturbing the Jacobian

At the SNBP the minimum eigenvalue of the Jacobian matrix becomes zero, the Jacobian matrix is no longer invertible, and NR-PF method fails. In this thesis, a simple method is proposed to eliminate the Jacobian singularity, which is based on a slight perturbation of only one element in the Jacobian matrix, as detailed next. A matrix becomes singular (loses its full rank condition) when at least two of its rows or columns are linearly dependent. This is reflected by a rank deficiency of the Jacobian matrix. Using this fact, it is possible to avoid singularity of the Jacobian matrix by eliminating the linear dependency of its rows and columns. This can be easily done by perturbing only one of the elements of the Jacobian matrix by a small percentage, $\alpha \%$. Algebraically, this increases the angle of the perturbed vector with respect to the vector(s) it is linearly-dependent on. The Jacobian element to be perturbed corresponds to the voltage and reactive power of the weakest bus of the system, from VS perspective. This is so because eliminating the singularity condition
can be achieved by the smallest required perturbation of the mentioned element than by perturbing any other element of the Jacobian. The physical reason is the high V-Q sensitivity at the weakest bus compared to the other buses. Thus, a small perturbation in that element is all that is needed at SNBP to avoid singularity. As a result, NR-PF returns a solution very close to that obtained by CPF of [3]. The advantages are removal of the need to introduce a new variable and equation into the power flow, and to use local parametrisation around the SNBP. The effect of the proposed slight perturbation on the accuracy of the power flow results is negligible, as will be shown in the simulation section. Identifying the weakest bus of the system from VS viewpoint needs to be done just once at the SNBP, and is carried out using MA [9] in this thesis.

To illustrate the effect of the applied perturbation on the accuracy of SNBP identification, P-V curves of the same IEEE 30-bus system are drawn by using the proposed method and the CPF method of [3]. At the SNBP, identified by encountering numerical difficulty in NR-PF method, the value of $\alpha=1.02$ returned by the algorithm of Fig. 2.4 (see Section 2.2.2-D), was used to perturb the V-Q element of bus 30 (the weakest bus) to eliminate the Jacobian’s singularity. The resulted P-V curves and a magnified inset around the SNBP, seen in Fig. 2.5-(a), show that the two P-V curves exactly match, and the effect of the perturbation at the SNBP is negligible.

C. Transition to the Lower Part of the P-V Curve

Although the lower, unstable, part of the P-V curve is not practically functional, it is especially important for LISBP analysis and identification, where the lower part of the limited system’s trajectory is involved. In the original CPF [3], the transition from the upper part of the P-V curve to the lower part is carried out by changing the continuation parameter around the SNBP from the system loading, to either the voltage magnitude or phase angle with the highest tangent.

After reaching the SNBP, the system loading is gradually reduced in order to complete the lower part of the P-V curve. By using the standard NR-PF, however, and without adopting a suitable technique, the trajectory of P-V curves will trace back the upper part of the curve. To avoid this from happening, it is necessary to find at least one operating point after the SNBP on the lower part of the P-V curve. A simple technique to find this point is to exploit the symmetry of the P-V curves in close vicinity of the SNBP, to orient the trajectory towards the lower part. Note that, although the P-V curves are not symmetric in general, there always exists a neighbourhood...
Set $\lambda=0$ and Start

Find the operating point using NR-PF

Numerical difficulty? 

Yes

singularity elimination

Identify the weakest bus

$\alpha=1.00$

Perturb the V-Q element of the weakest bus in Jacobian (Multiply it by $\alpha$)

Is the Jacobian full-rank ?

Yes

No

Find the operating point at SNBP using NR-PF

End

Draw the P-V curves

Yes

No

$\lambda < 0$ ?

Calculate $\Delta \lambda$ using (2) and find the next loading level

Use the current point as initial guess for the next loading level

Run one NR-PF iteration, $N=N+1$

Use eq. (3) and generate the initial guess

$\eta = 1$

Transition to the lower part

$\Delta V_{weak} < 0$

converge

$N \geq 10$

No

Yes

$\alpha = \alpha + \Delta \alpha$

No

Yes

Yes

Yes

No

No

No

No

No

No

$\eta = 1$

Figure 2.4: The flow chart of the proposed method

Figure 2.5: (a) P-V curves around the SNBP for CPF and proposed method, (b) Symmetry of P-V curves in close neighbourhood of SNBP
around the SNBP in which the curves are, or are very close to being, symmetric. Fig. 2.5-(b) shows the magnification of some of the P-V curves for IEEE 300-bus test system around the SNBP. The degree of symmetry, seen in the figure, can be exploited to generate a suitable initial guess for NR-PF to converge to the first operating point on the lower part of the curve.

In this thesis, the first estimate of an operating point on the lower part of the P-V curve is generated using equations (2.3):

\[
\begin{align*}
V_{j}^{k+1(\text{guess})} &= V_{j}^{k} - \eta |V_{j}^{k-1} - V_{j}^{k}| \quad \forall j \in PQ \\
\delta_{j}^{k+1(\text{guess})} &= \delta_{j}^{k} - \eta |\delta_{j}^{k-1} - \delta_{j}^{k}| \quad \forall j \in PQ, PV \\
\lambda^{k+1} &= \lambda^{k-1}
\end{align*}
\]

where \(V_{j}^{k}\) and \(\delta_{j}^{k}\) are the voltage magnitude and phase angle of bus \(j\) at SNBP (i.e. loading level \(k\)). \(PQ\) stands for all load buses, and \(PV\) stands for all generator buses, except the slack bus. \(\lambda^{k+1} \pm 1\) is the loading level at one iteration before and one iteration after SNBP, and \(\eta\) is a coefficient, called in this thesis “transition constant” (see Section 2.2.2-D).

In equations (2.3) the symmetry of P-V curve in close vicinity of SNBP is exploited, to generate the initial guess for the first operation point on the lower part of the P-V curve. For example, for \(\eta = 1\) the differences between the voltages and phase angles of SNBP and the previous operating point (on the upper part of the P-V curve), denoted by \(|V_{j}^{k-1} - V_{j}^{k}|\) and \(|\delta_{j}^{k-1} - \delta_{j}^{k}|\), are deducted from the SNBP coordinate, to create the initial guess for NR-PF on the lower part of the curve. Besides, the loading level of the generated guess is the same as the point just before SNBP, i.e. \(\lambda^{k+1} = \lambda^{k-1}\). In case the generated guess did not converge to a point on the lower part of the P-V curve, the value of \(\eta\) is increased, as shown in the flowchart of Fig. 2.4, to slightly move the location of the initial guess. In fact, by changing \(\eta\) we move the initial guess downward, which increases the likelihood for initial guess to fall within the RoA of the first operating point on the lower part of the P-V curve. Once the first point on the lower part is identified, it is used as an initial estimate for the next loading condition. This process is repeated until the lower part of the P-V curve is completely traced.
D. Finding Proper Values of $\alpha$ and $\eta$

In order for the proposed method to be applicable in real-time VS analysis of power systems, the two required parameters, $\alpha$ and $\eta$, are found in this method, automatically and without the need to the user’s intervention or adjustment.

The minimum perturbation coefficient, $\alpha$, required to eliminate the Jacobian singularity is automatically determined, in few iterations, by following the simple algorithm of Fig. 2.4. The algorithm checks the invertibility of the Jacobian matrix through its rank after each perturbation (when a matrix loses full-rank property, computing its inverse is numerically impossible). As computing the rank of the Jacobian is only required for a few iterations, until a proper $\alpha$ is found, the required time for finding $\alpha$ is negligible compared to the execution time of the whole process, as will be shown in Table 2.3.

The value of the transition coefficient, $\eta$, is also automatically determined using the algorithm of Fig. 2.4. A proper value of $\eta$ is obtained when the generated estimate by equation (2.3) lies inside the region of attraction (RoA) of the existing operating point on the lower part of the P-V curve. As reported in [39] and [70], when the starting point of the NR method lies inside the RoA of an existing solution, the point is usually converged to within 10 iterations. Also, to make sure that the generated estimate by equation (2.3) converges to a point on the “lower” part of the P-V curve, the change in the voltage magnitude of the weakest bus between the SNBP and the generated point, denoted by $\Delta V_{\text{weak}}$ in Fig. 2.4, is checked. If $\Delta V_{\text{weak}}$ is negative, i.e. the voltage has decreased, the generated point lies on the lower part of the P-V curve, and transition is successful. Otherwise, the transition constant, $\eta$, is automatically re-adjusted and a new guess is generated by equation (2.3).

2.2.3 Identification of LISB Point

In this section, a simple approach to identify LISBP, using standard NR-PF, is proposed, which can be easily incorporated into the method presented in Section 2.2.2 and incurs small additional time (see Table 2.7). The eigenvalue analysis of the reduced Jacobian matrix, $J_R$ (see Section 2.4), is used to identify the LISBP. It is known that at the voltage stable operating points of power system, all the eigenvalues of the $J_R$ are positive. This implies positive $dV/dQ$ sensitivity at all load buses,
indicating voltage stability [9]. On the other hand, at least one of the eigenvalues of $J_R$ has negative real part on the lower part of the P-V curve, which means the system is voltage unstable [9, 71]. The conclusion is that at the SNBP, (at least) one of the $J_R$’s positive eigenvalues (called critical eigenvalue) crosses the imaginary axis into the left-half complex-plane and becomes negative. At LIDBP points, the rate by which the critical eigenvalue approaches zero discontinuously increases, although its sign does not change. Fig. 2.6-(a) shows these facts for IEEE 30-bus test system. With respect to LISBP, the case is different. As seen in Fig. 2.1-(b), at LISBP the stable portion of the unlimited curve intersects with the unstable part of the limited curve. It is important to notice that LISBP is typically well-conditioned and does not show any convergence issues [68, 71]. Thus, it is computable by standard NR-PF method. Also, LISBP, similar to LIDBP, can be computed by using both limited and unlimited system equations using NR-PF method, because it is the intersection point between the two systems’ trajectories. However, the important difference between LISBP and LIDBP is the sign of the critical eigenvalue(s) of $J_R$. At LIDBP, all the eigenvalues of $J_R$ are positive for both power flow solutions (obtained from limited and unlimited system’s equations), because both limited and unlimited systems are voltage stable at the LIDBP. However at the LISBP, at least one eigenvalue of $J_R$, associated with the limited system’s solution, has negative real part, because the LISBP is located on the unstable part of the limited system’s trajectory. Thus, at least one of the load buses has negative $dV/dQ$ sensitivity. This fact can be effectively used to identify the LISBP. In addition, the generator whose $Q_G$-limit has been reached at the LISBP is the critical generator.

Note that the proposed SSA method of (2.2) is designed to identify the “SNBP” with $\Delta \lambda_{\text{min}}$ accuracy. Therefore, in order to identify the LISBP with $\Delta \lambda_{\text{min}}$ precision too, the following simple steps are carried out:

- Once a $Q_G$-limit is reached, change the corresponding bus from PV to PQ. Then, compute the operating point (using the limited system’s equations), and eigenvalues of $J_R$. Notice that as $J_R$ is much smaller than the main Jacobian, the required time for calculating its eigenvalues is very small, as will be shown in Table 2.7.

- If at least one of the eigenvalues of $J_R$ is negative, the corresponding generator is the critical generator.
For illustration purpose, the same IEEE 30-bus system was modified, in order to create a case in which LISBP occurs. To do so, the loads are assigned a power factor of 0.9 leading. Fig. 2.7-(a) shows magnified P-V curves for bus 4 (the weakest bus of the modified system), for the cases where two different groups of generators have reached their $Q_G$-limits. As seen in the figure, the LISB occurs when generator 5 reaches its $Q_G$-limit, after generators 2 and 8 have already reached their $Q_G$-limits, i.e. when the system’s trajectory changes from the green curve to the red one. At this point, the real part of the critical eigenvalue of $J_R$ becomes negative, as seen in Fig. 2.6-(b). Then, the abovementioned steps to identify LISBP with $\Delta \lambda^{\text{min}}$ precision are carried out, as seen...
2.3 Simulation Results of the Proposed Continuous Power Flow

In this section, the effectiveness of the proposed method is further illustrated by using different-size test systems (IEEE 118-bus, IEEE 300-bus [69], and 1354-bus [72] test systems). Also, to demonstrate the advantages of the proposed method in terms of computational time and accuracy, its performance is compared to those of three existing methods: the original CPF [3], CPF with SSA1 [31, 38] and CPF with SSA2 [34, 36] (see Section 2.2.2-A). The tests are based on the following SSA schemes:

- Original CPF with a fixed step-size of $\Delta \lambda = 0.001$.
- Proposed method with $\Delta \lambda_{\max} = 0.01$ and $\Delta \lambda_{\min} = 10^{-5}$.
- Original CPF with SSA1 ($\Delta \lambda_k = \Delta \lambda_{\max} / \lambda_{\text{corrector}}^{k-1}$), named as CPF$_{\text{iter}}$, with $\Delta \lambda_{\max} = 0.01$.
- Original CPF with SSA2, ($\Delta \lambda_k = \Delta \lambda_{\max} / \| \tau \|^k_2$), named as CPF$_{\text{norm}}$, with $\Delta \lambda_{\max} = 0.01$.

Figure 2.7: (a) LISBP identification with $\Delta \lambda_{\min}$ precision, (b) Complete P-V curve including LISBP for modified IEEE 30-bus test system

in Fig. 2.7-(a). After LISBP, the loading direction is reversed, and the lower part of the limited curve (red) is automatically traced. The complete P-V curve is obtained passing through LISBP, as seen in Fig. 2.7-(b), with no convergence issue occurring.
Table 2.1: Weakest buses at the SNBP using Modal Analysis

<table>
<thead>
<tr>
<th>System</th>
<th>IEEE 118-bus</th>
<th>IEEE 300-bus</th>
<th>1354-bus</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case</td>
<td>Case1</td>
<td>Case1</td>
<td>Case1</td>
</tr>
<tr>
<td>Bus</td>
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<td>33</td>
<td>282</td>
</tr>
<tr>
<td>BPF</td>
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<td>0.0887</td>
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</tbody>
</table>

Note that the above-mentioned step sizes are selected such that meaningful comparisons of the methods are drawn. For instance, the chosen step-sizes for the first two methods (proposed method and CPF) yield the same accuracy in identifying the SNBP, which makes the comparison fair. Also, the same initial step-size of $\Delta \lambda_{\text{max}} = 0.01$ is used in all of the methods that use SSA techniques (i.e. last three methods). Computations are carried out using MATLAB on a Core-i7 CPU, 3.4 GHz computer. The convergence threshold of NR-PF is set to $10^{-5}$.

Three case studies have been investigated. In the first two case studies, the performance of the proposed method in identifying SNBP and in drawing complete P-V curves is examined and compared with above-mentioned CPF methods. In the third case study, the capability of the proposed method in identifying LISBP is tested on IEEE 118-bus test system.

2.3.1 Case 1 - Ignoring the $Q_G$-Limits of Generators

In this case study, the performances of the four methods mentioned above to identify SNBP and to draw complete P-V curves are tested and compared. The procedure explained in Section 2.2.2 has been applied. Uniform load increase at all buses is considered, and the increase is supplied uniformly by all generators. The P-V curves of sample buses, drawn successfully by the proposed method for all test systems, are seen in Fig. 2.8-(a). The obtained curves by three other methods are identical to those shown in Fig 2.8-(a). Table 2.1 shows the weakest bus of each test system, identified by MA [9] at the SNBP. The corresponding bus participation factors (BPFs) are also presented. The identified weakest buses, obtained at SNBP, are used in the perturbation step in order to eliminate the singularity of the Jacobian matrix.

Table 2.2 compares the loading level, $\lambda^{k^*}$, and voltage, $v^{k^*}$, of the weakest bus at the SNBP, along with the execution time for the four tested methods. In addition, the table shows the values of the perturbation coefficient, $\alpha$, transition constant, $\eta$, and $N^{cr}$ (the number of iterations NR-PF takes to converge to the first point on the lower part). It is seen from Table 2.2 that there is
Table 2.2: SNBP coordinate and other parameters of Case I

<table>
<thead>
<tr>
<th>System</th>
<th>Method</th>
<th>$\lambda^{k^{cr}}$</th>
<th>$\nu^{k^{cr}}$</th>
<th>$\alpha$</th>
<th>$\eta$</th>
<th>$N_{cr}$</th>
<th>time (s)</th>
<th>$EC$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>118-bus</td>
<td>Proposed</td>
<td>3.3152</td>
<td>0.7621</td>
<td>1.02</td>
<td>4</td>
<td>3</td>
<td>105</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td>CPF [3]</td>
<td>3.3152</td>
<td>0.7623</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>710</td>
<td>0.21</td>
</tr>
<tr>
<td></td>
<td>CPF$_{iter}$</td>
<td>3.3150</td>
<td>0.7581</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>210</td>
<td>0.66</td>
</tr>
<tr>
<td></td>
<td>CPF$_{norm}$</td>
<td>3.3152</td>
<td>0.7619</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>350</td>
<td>0.37</td>
</tr>
<tr>
<td>300-bus</td>
<td>Proposed</td>
<td>1.4303</td>
<td>0.6034</td>
<td>1.08</td>
<td>3</td>
<td>6</td>
<td>330</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td>CPF [3]</td>
<td>1.4303</td>
<td>0.6037</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>2540</td>
<td>0.14</td>
</tr>
<tr>
<td></td>
<td>CPF$_{iter}$</td>
<td>1.4297</td>
<td>0.6035</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>400</td>
<td>0.62</td>
</tr>
<tr>
<td></td>
<td>CPF$_{norm}$</td>
<td>1.4303</td>
<td>0.6034</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>1670</td>
<td>0.22</td>
</tr>
<tr>
<td>1354-bus</td>
<td>Proposed</td>
<td>1.4925</td>
<td>0.6925</td>
<td>1.06</td>
<td>2</td>
<td>6</td>
<td>2880</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td>CPF [3]</td>
<td>1.4925</td>
<td>0.6924</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>22220</td>
<td>0.12</td>
</tr>
<tr>
<td></td>
<td>CPF$_{iter}$</td>
<td>1.4919</td>
<td>0.6903</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>3440</td>
<td>0.71</td>
</tr>
<tr>
<td></td>
<td>CPF$_{norm}$</td>
<td>1.4925</td>
<td>0.6925</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>15860</td>
<td>0.18</td>
</tr>
</tbody>
</table>

high congruity between the proposed method and CPF-based approaches. However, the proposed method has the minimum execution time in all cases compared to the three other methods. As seen in Table 2.2, using the two SSA methods has considerably decreased the simulation time of CPF$_{iter}$ and CPF$_{norm}$ compared to that of original CPF. However, the accuracy of identifying the SNBP has adversely been affected. This is because a less number of points are computed around the SNBP. In order to compare the overall efficiency of all the tested methods, an index named efficiency coefficient ($EC$) is defined in this thesis as $EC = \frac{\log_{10}(\Delta \lambda^{k^{cr}})}{\text{time}}$. The index combines execution time and accuracy, represented by $|\Delta \lambda|$ around the SNBP and denoted by $\Delta \lambda^{k^{cr}}$. A larger value of $EC$ indicates higher efficiency, in terms of less execution time and higher accuracy. Table 2.2 shows the values of $EC$ for all tested methods. The $EC$ values are normalised with respect to those of the proposed method, assumed to be 1 p.u. Table 2.2 clearly shows that the proposed method is significantly more efficient than the other three ones.

Remark: The values of $\Delta \alpha = 0.02$ and $\Delta \eta = 0.5$ (see Fig. 2.4) have been used in the above simulations, even though smaller or larger values can also be used. However, too small values would slow down the process of finding proper values for $\alpha$ and $\eta$ (using the algorithm of Fig 2.4). Large value of $\Delta \alpha$ may cause larger perturbation in the element of the Jacobian matrix, which affects the accuracy of the SNBP identification. Also, too large value of $\Delta \eta$ may place the generated guess out of the RoA of the existing point on the lower part of the curve. In fact, the success of the proposed method will not be impacted as long as $\Delta \alpha$ and $\Delta \eta$ are chosen small.
2.3.2 Case 2 - Considering the $Q_G$-Limits of Generators

This case is similar to case study 1, but with the $Q_G$-limits of the generators taken into account. The aim of this case study is to demonstrate the capability of the proposed method to deal with changes in the operating condition as a result of increasing the loading level. The same four methods, with the same parameters, as in case 1 have been tested. The $Q_G$-limits are set to twice their original values for all of the test systems. The same procedure as explained in Section 2.2.2 was followed. However, once a generator reaches its $Q_G$-limit, the associated bus is changed from
2.3 Simulation Results of the Proposed Continuous Power Flow

Table 2.3: Required time for finding the proper value of $\alpha$

<table>
<thead>
<tr>
<th>Test System</th>
<th>118-bus</th>
<th>300-bus</th>
<th>1354-bus</th>
</tr>
</thead>
<tbody>
<tr>
<td>Required time to check the rank of the Jacobian matrix (s)</td>
<td>0.02</td>
<td>0.064</td>
<td>4.194</td>
</tr>
<tr>
<td>Number of required search iterations</td>
<td>1</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>Required time to find the proper $\alpha$ (s)</td>
<td>0.020</td>
<td>0.256</td>
<td>12.582</td>
</tr>
<tr>
<td>Percentage of the whole method’s time</td>
<td>0.02%</td>
<td>0.08%</td>
<td>0.44%</td>
</tr>
</tbody>
</table>

Table 2.4: Required time to find the proper value of $\eta$

<table>
<thead>
<tr>
<th>Test System</th>
<th>118-bus</th>
<th>300-bus</th>
<th>1354-bus</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of required search iterations</td>
<td>7</td>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td>Required time to find the proper $\eta$ (s)</td>
<td>1.12</td>
<td>2.40</td>
<td>66.14</td>
</tr>
<tr>
<td>Percentage of the whole method’s time</td>
<td>1.06%</td>
<td>0.72%</td>
<td>2.30%</td>
</tr>
</tbody>
</table>

$PV$ to $PQ$, resulting in a decrease in the voltage magnitude of the respective bus, and consequently reaching either LIDB or LISB points.

Table 2.5 compares the results of case 2. Here again, the results show clearly that the efficiency of the proposed method is significantly higher than the other CPF methods. The P-V curves for this case, obtained by the proposed method, are shown in Fig. 2.8-(b), which show that the SNBPs have shifted to the left, compared to those of case 1. In this case, all the $Q_G$-limit points correspond to LIDB. This is so because after each $Q_G$-limit point the system remains voltage stable; despite the fact that the slopes of the curves suddenly increase.

2.3.3 Case 3 - Identifying LISBP

The proposed LISBP identification method was tested on IEEE 30-bus system in Section 2.2.3, and the results were presented in Fig. 2.7. In this section, the method is tested on IEEE 118-bus test system. The procedure and parameters of constructing P-V curves are the same as in the previous case studies. However, the additional steps outlined in Section 2.2.3 have been incorporated to identify the LISBP with $\Delta \lambda^{\text{min}}$ accuracy. Again, to create a LISBP in the system, the power factors of the loads have all been changed to 0.95 leading. Fig. 2.9-(a) shows the P-V curve for bus 9, weakest bus of the modified system, for the cases where two different groups of generators have been fixed at their $Q_G$-limits (the red and green curves). As seen, LISB occurs when generator 10 meets its $Q_G$-limit, indicating that it is the critical generator. Fig. 2.9-(b) shows the P-V curves for
Table 2.5: SNBP coordinate and other parameters of Case 2

<table>
<thead>
<tr>
<th>System</th>
<th>Method</th>
<th>SNBP</th>
<th>α</th>
<th>η</th>
<th>N_{cr}</th>
<th>time (s)</th>
<th>EC</th>
</tr>
</thead>
<tbody>
<tr>
<td>118-bus</td>
<td>Proposed</td>
<td>2.6025 0.7217</td>
<td>1.02</td>
<td>4</td>
<td></td>
<td>115</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td>CPF [3]</td>
<td>2.6025 0.7217</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>460</td>
<td>0.26</td>
</tr>
<tr>
<td></td>
<td>CPF_{iter}</td>
<td>2.6023 0.7214</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>130</td>
<td>0.74</td>
</tr>
<tr>
<td></td>
<td>CPF_{norm}</td>
<td>2.6025 0.7218</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>290</td>
<td>0.40</td>
</tr>
<tr>
<td>300-bus</td>
<td>Proposed</td>
<td>1.2143 0.8093</td>
<td>1.04</td>
<td>4</td>
<td>7</td>
<td>170</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td>CPF [3]</td>
<td>1.2143 0.8092</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>1200</td>
<td>0.17</td>
</tr>
<tr>
<td></td>
<td>CPF_{iter}</td>
<td>1.2138 0.8087</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>250</td>
<td>0.58</td>
</tr>
<tr>
<td></td>
<td>CPF_{norm}</td>
<td>1.2143 0.8092</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>740</td>
<td>0.26</td>
</tr>
<tr>
<td>1354-bus</td>
<td>Proposed</td>
<td>1.2123 0.7945</td>
<td>1.10</td>
<td>2</td>
<td>5</td>
<td>1720</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td>CPF [3]</td>
<td>1.2123 0.7944</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>10550</td>
<td>0.18</td>
</tr>
<tr>
<td></td>
<td>CPF_{iter}</td>
<td>1.2103 0.7968</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>1990</td>
<td>0.77</td>
</tr>
<tr>
<td></td>
<td>CPF_{norm}</td>
<td>1.2123 0.7944</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>8510</td>
<td>0.29</td>
</tr>
</tbody>
</table>

Figure 2.9: (a) P-V curve including LISBP, and (b) Sample P-V curves of IEEE 118-bus system

The proposed LISBP identification method is easily applicable to every power systems of any size and description, for the simple fact that the eigenvalue analysis of the $f_R$ is a generic rule regardless of the system size and configuration. Furthermore, incorporating LISBP identification approach into the proposed method incurs negligible additional execution time. For example, in case 2, in which $Q_G$-limits are considered (although no LISBP occurred), the only additional step
Table 2.6: LISBP coordinate and execution time for both tested systems

<table>
<thead>
<tr>
<th>Test System</th>
<th>$\lambda_{LISB}$</th>
<th>$v_{LISB}$</th>
<th>time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>30-bus</td>
<td>3.8971</td>
<td>0.8727</td>
<td>34</td>
</tr>
<tr>
<td>118-bus</td>
<td>2.9910</td>
<td>0.8467</td>
<td>135</td>
</tr>
</tbody>
</table>

Table 2.7: The required additional time for LISBP identification

<table>
<thead>
<tr>
<th>Test System</th>
<th>118-bus</th>
<th>300-bus</th>
<th>1354-bus</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of generators reaching their $Q_G$-limits before SNBP</td>
<td>31</td>
<td>12</td>
<td>78</td>
</tr>
<tr>
<td>Required time to compute ( e_{ \text{ig}(J_R)(s) } ) (s)</td>
<td>0.003</td>
<td>0.035</td>
<td>0.53</td>
</tr>
<tr>
<td>Total added time to identify LISBP (s)</td>
<td>0.093</td>
<td>0.42</td>
<td>41.34</td>
</tr>
<tr>
<td>Percentage of the whole method’s time</td>
<td>0.08%</td>
<td>0.25%</td>
<td>2.40%</td>
</tr>
</tbody>
</table>

required is to check the eigenvalues of \( J_R \) when a $Q_G$-limit is reached, and the corresponding bus type is changed to $PQ$. Table 2.7 shows the required time to check the eigenvalues of \( J_R \) for all of the tested systems. It also shows the total additional time, as a percentage of the whole execution time of the method. As shown, the added execution time is less than 2.5 percent for all the tested systems. This is to be weighed against the added advantage of identifying the LISBP.

### 2.4 Modified Modal Analysis for Distribution Power Systems

MA is a commonly-used method which evaluates the VS of power system through eigenvalues and eigenvectors of the reduced Jacobian matrix, \( J_R \) [9]. It identifies weak areas (modes) in a power system, from a VS viewpoint. Besides, through computing bus participation factors (BPFs) associated with each weak mode, the best buses to apply remedial actions for stabilizing weak modes are identified [9]. As a result, MA is a useful means for identifying the best places to install compensation, when power system’s VS improvement is aimed. However, MA suffers from a significant disadvantage of ignoring active power variations, denoted by $\Delta P$, which makes it less reliable for optimal placement of active power compensation. For example, MA results are used in reference [8] for optimal placement of DGs in a radial distribution system, with the aim of maximizing the system’s VSM, i.e. the distance of the operating point from the SNBP. The DGs are assumed to operate at unity power factor, with no reactive power capability. It means that the MA results, in which the effect of $\Delta P$ is neglected, have been used for optimal placement of active
power compensation, which clearly does not result in the optimal solution. In this section, after a brief description of the original MA, two modified MA methods are proposed to alleviate the mentioned issue.

2.4.1 Original Modal Analysis [9]

This section briefly describes the original MA method, which is essential for presenting the proposed methods. The linearized form of power flow equations of a power system can be written in the following form:

\[
\begin{bmatrix}
\Delta P \\
\Delta Q
\end{bmatrix} =
\begin{bmatrix}
J_{P\delta} & J_{PV} \\
J_{Q\delta} & J_{QV}
\end{bmatrix}
\begin{bmatrix}
\Delta \delta \\
\Delta V
\end{bmatrix}
\tag{2.4}
\]

where \(\delta\) and \(V\) are the vectors of voltage phase angles and voltage magnitudes, respectively. \(\Delta P\) and \(\Delta Q\) are respectively the vectors of active and reactive power variations, and \(J\) represents the power flow equations’ Jacobian matrix. As mentioned, the original MA considers the effect of reactive power variation only by setting \(\Delta P = 0\), which leads to:

\[
\Delta Q = J_R \Delta V \Rightarrow \Delta V = J_R^{-1} \Delta Q 
\tag{2.5}
\]

\[
J_R = [J_{QV} - J_{Q\delta} J_{P\delta}^{-1} J_{PV}]
\tag{2.6}
\]

The matrix \(J_R\) is called the reduced Jacobian matrix, whose eigenvalue decomposition yields:

\[
J_R = \xi \Lambda \eta \Rightarrow J_R^{-1} = \xi \Lambda^{-1} \eta
\tag{2.7}
\]

where \(\xi, \eta,\) and \(\Lambda\) are right and left eigenvector matrices, and diagonal eigenvalue matrix of \(J_R\), respectively. Combining equations (2.5) and (2.7) yields:

\[
\Delta V = \xi \Lambda^{-1} \eta \ \Delta Q \Rightarrow \Delta V = \sum_i \frac{\xi_i \eta_i}{\lambda_i} \Delta Q
\tag{2.8}
\]

where \(\xi_i\) is the \(i^{th}\) column right eigenvector and \(\eta_i\) is the \(i^{th}\) row left eigenvector of \(J_R\). The \(i^{th}\) mode of the \(Q - V\) response is defined by the eigenvalue \(\lambda_i\) and the corresponding right and left
2.4 Modified Modal Analysis for Distribution Power Systems

eigenvectors $\xi_i$ and $\eta_i$. Inasmuch as $\xi_i^{-1} = \eta_i$, equation (2.8) can be rewritten as:

$$ \eta \Delta V = \Lambda^{-1} \eta \Delta Q \Rightarrow v = \Lambda^{-1} q$$

(2.9)

where $v = \eta \Delta V$ and $q = \eta \Delta Q$ are the vectors of modal voltage and reactive power variations, respectively. Since $\Lambda^{-1}$ is diagonal, one can write $v_i = (1/\lambda_i) q_i$ for the $i^{th}$ mode. A positive $\lambda_i$ means that the $i^{th}$ modal voltage and reactive power variations are along the same direction, indicating that the system is voltage stable, vice versa. Also, the smaller the magnitude of a positive $\lambda_i$, the closer the $i^{th}$ modal voltage to instability. In equation (2.8), assuming all elements of $\Delta Q$ are zero except for the $k^{th}$ element, the $V - Q$ sensitivity at bus $k$ would be:

$$ \frac{\partial V_k}{\partial Q_k} = \sum_i \frac{\xi_{ki} \eta_{ik}}{\lambda_i} = \sum_i \frac{P_{ki}}{\lambda_i}$$

(2.10)

where $P_{ki} = \xi_{ki} \eta_{ik}$ is the bus participation factor (BPF) of bus $k$ in mode $i$, and determines the contribution of $\lambda_i$ to the $V - Q$ sensitivity at bus $k$. The size of a BPF in a given mode indicates the effectiveness of remedial actions applied at that bus in stabilizing the mode.

2.4.2 Proposed Improved MA Methods

A. Considering Active Power Variation ($MA^P$)

As seen in the Section 2.4.1, the original MA identifies the system’s weakest modes and associated buses through calculating modal $V - Q$ sensitivities, and by neglecting the effect of active power variation. However, the main function of DGs and ESSs is to supply active power to the system. Thus, the results of the original MA may not be accurate for the purpose of optimal placement of DGs or ESSs in distribution systems, from a VS viewpoint. It means that the weakest buses identified by original MA, may not be the best locations for DGs or ESSs, in terms of maximizing the system’s VSM.

Now, let us take a look at the Jacobian matrix and its sub-matrices as shown below:

$$ J = \begin{bmatrix} P_{\delta(n_b-1 \times n_b-1)} & P_{PV(n_b-1 \times n_pq)} \\ Q_{\delta(n_pq \times n_b-1)} & Q_{PV(n_pq \times n_pq)} \end{bmatrix} $$

(2.11)
where \( n_b \) and \( n_{pq} \) are the number of buses and number of \( PQ \) (load) buses, respectively. The reduced Jacobian matrix, \( J_R \), calculated by using equation (2.6), is a feasible \( n_{pq} \times n_{pq} \) matrix, owing to the compatible sizes of the Jacobian’s sub-matrices. Now let us, instead of assuming \( \Delta P = 0 \), neglect the reactive power variations (\( \Delta Q = 0 \)) and study the effect of active power variation on system’s VS. From equation (2.4) one can write:

\[
\Delta P = J'_R \Delta V
\]

(2.12)

\[
J'_R = [J_{PV} - J_{P\delta}J_{Q\delta}^{-1}J_{QV}]
\]

(2.13)

For a typical power system with more than one voltage regulated bus where \( n_b - 1 \neq n_{pq} \), matrix \( J'_R \) in equation (2.13) is not feasible, due to the incompatible sizes of the Jacobian sub-matrices. This problem, nevertheless, does not exist for many of the existing distribution systems with no voltage-regulated bus, other than the main substation bus, modelled as a slack generator. In these systems, the relation \( n_b - 1 = n_{pq} \) holds and the matrix \( J'_R \) in (2.13) is feasible. The matrix \( J'_R \) can be used for eigenvalue decomposition and BPF calculation, similar to what is done for \( J_R \) in the original MA using equations (2.7)-(2.10). In this case, the resulted BPFs reflect the effectiveness of the active power compensation in stabilizing system’s weak modes, and therefore can be used for optimal placement of active power compensation in the system, with the aim of VSM improvement. Since many of the available DGs operate at unity power factor due to the economic considerations [8], the proposed MA approach, denoted by \( MA^P \), can be used to optimally place these devices in distribution systems, from a VS viewpoint.

B. Considering Both Active and Reactive Power Variations (\( MA^{PQ} \))

Many of the recently available DG technologies have considerable reactive power capability. Doubly-Fed Induction Generator (DFIG) or all inverter-interfaced DGs can be mentioned as cases in point. The same is true for ESS devices which can generate any desired combination of active and reactive powers, constrained by their operational limits. Thus, if optimal placement of these devices is carried out with the aim of steady-state VS improvement, the effect of both active and reactive power variations must be evaluated.

Once again, consider the equation (2.4). Suppose that the operating power factor (PF) of the
2.4 Modified Modal Analysis for Distribution Power Systems

DGs or ESSs, is \( PF = \cos(\theta) \). It should be noted that even if the DGs or ESSs are not operated at constant PF, their average operating PF can be estimated and used. Therefore, the following relation holds between the active and reactive power variations:

\[
\Delta P = K \times \Delta Q
\]  

(2.14)

where \( K \) is a diagonal matrix with the diagonal elements being equal to \( \cot(\arccos(PF)) \). Expanding equation (2.4) yields:

\[
\Delta P = J_{P\delta} \Delta \delta + J_{PV} \Delta V
\]  

(2.15)

\[
\Delta Q = J_{Q\delta} \Delta \delta + J_{QV} \Delta V
\]  

(2.16)

From equation (2.15), it can be derived that:

\[
\Delta \delta = J_{P\delta}^{-1} \Delta P - J_{P\delta}^{-1} J_{PV} \Delta V
\]  

(2.17)

Substituting equation (2.17) in (2.16) yields:

\[
\Delta Q = J_{Q\delta} J_{P\delta}^{-1} \Delta P - J_{Q\delta} J_{P\delta}^{-1} J_{PV} \Delta V + J_{QV} \Delta V
\]  

(2.18)

Again, substituting equation (2.14) in (2.18) yields:

\[
(I - K J_{Q\delta} J_{P\delta}^{-1}) \Delta Q = (J_{QV} - J_{Q\delta} J_{P\delta}^{-1} J_{PV}) \Delta V
\]  

(2.19)

where \( I_{(n_1 \times n_1 - 1)} \) is the identity matrix. Now define:

\[
J''_R = (I - K J_{Q\delta} J_{P\delta}^{-1})^{-1} (J_{QV} - J_{Q\delta} J_{P\delta}^{-1} J_{PV})
\]  

(2.20)

Experiments carried out on several, different size, distribution power systems have shown that the matrix \( (I - K J_{Q\delta} J_{P\delta}^{-1}) \) is generally non-singular, and thus, invertible. Consequently, the reactive power variations can be related to the voltage variations as:

\[
\Delta Q = J''_R \Delta V
\]  

(2.21)
Notice that the active power variations are also included in the $J''$ through the matrix $K$. Substituting $K$ with zero yields the same $J_R$ of the original MA. Again, due to the equality $n_b - 1 = n_{pq}$ being held in most distribution systems, all the matrix operations are feasible and $J''$ can be simply calculated. This matrix can be used for eigenvalue decomposition and BPF calculation using equations (2.7)-(2.10). The obtained BPFs reflect the effects of both active and reactive power compensations in stabilizing weak modes of the system. This version of MA is denoted by $MA^{PQ}$.

It is worth mentioning that the only requirement for the proposed MA methods to work, is that the distribution power system should have only one voltage-regulated (i.e. PV) bus, which is the case for many of the existing distribution systems. Hence, even if the system buses contain generation units such as DGs like residential solar panels, or storage devices, as long as they do not attempt to regulate voltage at their buses, the proposed MA methods can be applied. Notice that in most cases, the residential DGs like solar panels are considered as negative loads and do not regulate the voltage, and thus their corresponding buses remain as load (i.e. PQ) buses.

### 2.4.3 Validation of the Proposed MA methods

The described MA approaches identify the weakest buses of the system, from VS viewpoint, by considering the effect of either active, reactive, or both active and reactive power variations. The obtained BPFs by each MA method indicate the effectiveness of the corresponding compensation at different buses in stabilizing the associated weak modes [9]. In the other word, applying compensation at the bus with the highest BPF, improves the steady-state VS better than compensating any other buses. The BPFs resulted from the original MA, which only considers reactive power variations, reflect the effectiveness of the reactive power compensation at different buses, and similarly for $MA^P$ and $MA^{PQ}$.

In order to evaluate the effectiveness of any type of compensation (active, reactive, or a combination of them) in improving the system’s steady-state VS, the sensitivity of the system’s VSM ($\Delta VSM$) to the applied active/reactive power injection at system buses is used in this section. Calculation of the VSM requires implementing CPF. To calculate the value of $\Delta VSM$ corresponding to any bus and any type of compensation, the proposed CPF method of Section 2.2 is implemented twice, once for the base case system, and once for the case when the corresponding compensation is applied at the bus. The difference between the values of VSM in these two cases is the corre-
2.5 Simulation results of the proposed MA Methods

The proposed MA approaches are tested on a commonly used 33-bus radial distribution system [73], with nominal voltage of 12.66 kV, shown in Fig. 2.11. The system reaches the maximum loading point (MLP), corresponding to the SNBP, at the loading level \( \lambda = 3.56 \), as shown in Fig. 2.10. Also, Fig. 2.12 shows the voltage profile of the test system at the nominal and critical loading conditions. The weak areas of the system are also identified in the figure. Table 2.8 presents the results of the original MA, carried out at the SNBP. Two weakest modes and the first five buses associated with each weak mode are presented along with the corresponding BPFs. The table also shows the weakest buses of the system identified by using L-index method. Briefly, L-index uses the re-configured admittance matrix \( Y_{bus} \) to assign a quantitative measure, varying between 0 and 1, to each load bus of the power system. The higher the value of the index, the closer the associated bus, and the whole system, to the voltage instability point [22]. As seen from the Table 2.8, the results of MA and L-index are not identical, although similar.

For further evaluation, the BPFs of the weakest mode obtained from the original MA, along with the L-index results, both for the first 20 weak buses are presented in Table 2.9. Again, there are some differences between the results of the two methods in terms of the order of the identified weak buses. Also, Fig. 2.13 shows the diagram of the (sorted) \( \Delta V_{SM} \)s, calculated for all buses as a result of 0.1 MVar reactive power compensation applied at each bus, one at a time. As seen, the
Figure 2.11: Single-line diagram of 33-bus radial distribution test system

Figure 2.12: Voltage Profile at the nominal and critical loading conditions

order of the weakest buses identified by the original MA (shown in Table 2.9), is highly congruent with the order of the most effective buses to apply reactive power compensation (shown in Fig. 2.13), in terms of $\Delta VSM$ improvement.

Table 2.8: Original Modal Analysis and L-index results

<table>
<thead>
<tr>
<th>Original Modal Analysis</th>
<th>2$^{nd}$ Weak Mode</th>
<th>L-index</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weakest Mode ($\lambda=0.0009$)</td>
<td>($\lambda=0.0194$)</td>
<td>Bus</td>
</tr>
<tr>
<td>BPF</td>
<td>BPF</td>
<td>Bus</td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>18</td>
<td>0.0877</td>
<td>33</td>
</tr>
<tr>
<td>17</td>
<td>0.0849</td>
<td>32</td>
</tr>
<tr>
<td>33</td>
<td>0.0761</td>
<td>31</td>
</tr>
<tr>
<td>32</td>
<td>0.0710</td>
<td>18</td>
</tr>
<tr>
<td>31</td>
<td>0.0664</td>
<td>17</td>
</tr>
</tbody>
</table>
2.5 Simulation results of the proposed MA Methods

Table 2.9: Original MA and L-index results for the first 20 weak buses

<table>
<thead>
<tr>
<th>Bus</th>
<th>Original MA</th>
<th>BPF</th>
<th>L-index</th>
</tr>
</thead>
<tbody>
<tr>
<td>18</td>
<td>0.087</td>
<td>0.087</td>
<td>0.998</td>
</tr>
<tr>
<td>17</td>
<td>0.085</td>
<td>0.085</td>
<td>0.98</td>
</tr>
<tr>
<td>33</td>
<td>0.076</td>
<td>0.076</td>
<td>0.97</td>
</tr>
<tr>
<td>32</td>
<td>0.071</td>
<td>0.071</td>
<td>0.96</td>
</tr>
<tr>
<td>31</td>
<td>0.066</td>
<td>0.066</td>
<td>0.95</td>
</tr>
<tr>
<td>16</td>
<td>0.066</td>
<td>0.066</td>
<td>0.94</td>
</tr>
<tr>
<td>15</td>
<td>0.064</td>
<td>0.064</td>
<td>0.93</td>
</tr>
<tr>
<td>14</td>
<td>0.069</td>
<td>0.069</td>
<td>0.92</td>
</tr>
<tr>
<td>30</td>
<td>0.054</td>
<td>0.054</td>
<td>0.91</td>
</tr>
<tr>
<td>13</td>
<td>0.059</td>
<td>0.059</td>
<td>0.9</td>
</tr>
<tr>
<td>12</td>
<td>0.064</td>
<td>0.064</td>
<td>0.83</td>
</tr>
<tr>
<td>11</td>
<td>0.068</td>
<td>0.068</td>
<td>0.82</td>
</tr>
<tr>
<td>10</td>
<td>0.073</td>
<td>0.073</td>
<td>0.8</td>
</tr>
<tr>
<td>9</td>
<td>0.060</td>
<td>0.060</td>
<td>0.77</td>
</tr>
<tr>
<td>8</td>
<td>0.054</td>
<td>0.054</td>
<td>0.75</td>
</tr>
<tr>
<td>28</td>
<td>0.036</td>
<td>0.036</td>
<td>0.7</td>
</tr>
<tr>
<td>7</td>
<td>0.041</td>
<td>0.041</td>
<td>0.67</td>
</tr>
<tr>
<td>26</td>
<td>0.051</td>
<td>0.051</td>
<td>0.65</td>
</tr>
</tbody>
</table>

Figure 2.13: Diagram of ∆VSM for 0.1 MVar reactive power compensation applied at all buses

2.5.1 Results of $MA^P$

The $MA^P$ approach, proposed in Section 2.4.2-A, was carried out on the test system and the results, i.e. the weakest buses of the weakest mode and their corresponding BPFs, are presented in Table 2.10. Also, Fig. 2.14 presents the diagram of the (sorted) $\Delta$VSMs, calculated for all system buses as a result of 0.1 MW active power compensation applied at each bus. Comparing the results of Table 2.10 and Fig. 2.14 shows that the obtained bus orders are highly congruent. It shows that the $MA^P$ method accurately identifies the most effective buses for active power compensation to improve system’s VSM.

2.5.2 Results of $MA^{PQ}$

The proposed $MA^{PQ}$ method, presented in Section 2.4.2-B, was implemented on the same system. The value of diagonal elements of matrix $K$ in equation (2.14) was taken 1, which implies investigating equal active and reactive power variations. Notice that equal active and reactive power

Table 2.10: $MA^P$ results for the first 20 weak buses

<table>
<thead>
<tr>
<th>Bus</th>
<th>BPF</th>
</tr>
</thead>
<tbody>
<tr>
<td>18</td>
<td>0.003</td>
</tr>
<tr>
<td>17</td>
<td>0.009</td>
</tr>
<tr>
<td>16</td>
<td>0.087</td>
</tr>
<tr>
<td>15</td>
<td>0.080</td>
</tr>
<tr>
<td>14</td>
<td>0.073</td>
</tr>
<tr>
<td>33</td>
<td>0.063</td>
</tr>
<tr>
<td>32</td>
<td>0.054</td>
</tr>
<tr>
<td>31</td>
<td>0.054</td>
</tr>
<tr>
<td>30</td>
<td>0.051</td>
</tr>
<tr>
<td>12</td>
<td>0.044</td>
</tr>
<tr>
<td>11</td>
<td>0.041</td>
</tr>
<tr>
<td>9</td>
<td>0.039</td>
</tr>
<tr>
<td>8</td>
<td>0.036</td>
</tr>
<tr>
<td>28</td>
<td>0.028</td>
</tr>
<tr>
<td>7</td>
<td>0.019</td>
</tr>
<tr>
<td>26</td>
<td>0.014</td>
</tr>
<tr>
<td>27</td>
<td>0.013</td>
</tr>
<tr>
<td>25</td>
<td>0.013</td>
</tr>
</tbody>
</table>
variations are used here, just to demonstrate the effectiveness of the proposed $M A^{P Q}$ method, in accounting for combination of both active and reactive powers. However, any other combinations of active and reactive power (which means different $P F$s) can be used. By using a different value of $P F$ (which, as mentioned, should be determined based on the estimated or average $P F$ of the installed DGs or ESSs), the diagonal elements of matrix $K$ (i.e. $\cot(\cos(P F))$) changes, however the generality of the proposed method is not affected.

The identified weak buses and their associated BPFs are presented in Table 2.11. An interesting observation is the complete consistency between the results of $M A^{P Q}$ method and those of $L$-index (Table 2.9), which shows that $L$-index captures the effect of both active and reactive power variations. Furthermore, Fig. 2.15 presents the diagram of the (sorted) $\Delta V S M$s, calculated for system buses as a result of 0.1 MVA ($P F=0.707$) active and reactive power compensations at each bus. Comparing the results of Table 2.11 and Fig. 2.15 shows that a high degree of consistency exists between them. It proves that the $M A^{P Q}$ method accurately identifies the most effective buses for both active and reactive power compensations, with the aim of VSM improvement.

### 2.5.3 Overall Comparison of All Methods

In this Section, the relative similarities between the results of each described MA approaches and the results obtained from $\Delta V S M$ calculations (with all types of applied compensations) are compared. In order to do so, the correlation coefficients (CCs) between all the bus arrays of Figures 2.13 to 2.15 and Tables 2.9 to 2.11 were calculated. The CC between two arrays $A$ and $B$ of length $n$ with respective elements of $A_i$ and $B_i$, which is calculated through equation (2.22), is
2.5 Simulation results of the proposed MA Methods

a measure of degree of dependency between two arrays.

\[ R_{A,B} = \frac{n \sum_i A_i B_i - \left( \sum_i A_i \right) \left( \sum_i B_i \right)}{\sqrt{\left[ n \sum_i A_i^2 - (\sum_i A_i)^2 \right] \left[ n \sum_i B_i^2 - (\sum_i B_i)^2 \right]}} \]  

(2.22)

Table 2.11: \( MA_{PQ} \) results for the first 20 weak buses

<table>
<thead>
<tr>
<th>Bus</th>
<th>BPF</th>
</tr>
</thead>
<tbody>
<tr>
<td>18</td>
<td>0.097</td>
</tr>
<tr>
<td>17</td>
<td>0.093</td>
</tr>
<tr>
<td>16</td>
<td>0.082</td>
</tr>
<tr>
<td>15</td>
<td>0.076</td>
</tr>
<tr>
<td>33</td>
<td>0.061</td>
</tr>
<tr>
<td>32</td>
<td>0.058</td>
</tr>
<tr>
<td>31</td>
<td>0.056</td>
</tr>
<tr>
<td>30</td>
<td>0.047</td>
</tr>
<tr>
<td>29</td>
<td>0.043</td>
</tr>
<tr>
<td>14</td>
<td>0.040</td>
</tr>
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<td>31</td>
<td>0.039</td>
</tr>
<tr>
<td>13</td>
<td>0.038</td>
</tr>
<tr>
<td>30</td>
<td>0.028</td>
</tr>
<tr>
<td>28</td>
<td>0.019</td>
</tr>
<tr>
<td>12</td>
<td>0.015</td>
</tr>
<tr>
<td>10</td>
<td>0.013</td>
</tr>
</tbody>
</table>

Figure 2.15: \( \Delta VSM \) values for 0.1 MVA (PF=0.7071) active and reactive power compensation

Figure 2.16: Correlation Coefficients between the results of L-index, MA approaches and \( \Delta VSM \) calculations

The results are shown in Fig. 2.16. The original MA is denoted by \( MA_Q \) in this figure. As seen, the highest correlations exist between three pairs of (\( \Delta VSM_P, MA_P \)), (\( \Delta VSM_Q, MA_Q \)), and (\( \Delta VSM_{PQ}, MA_{PQ} \)) arrays. It means that, for example, the results of \( \Delta VSM_P \) is most correlated with those of \( MA_P \). This clearly proves that there are close bonds between the results of each MA
approach and the corresponding $\Delta VSM$ analysis and validates the efficacy of the proposed MA approaches. Besides, Fig. 2.16 presents the CC between L-index results and those of different MA approaches. As seen, there is a complete correlation ($R = 1$) between the L-index results and $MA^{PQ}$ ones, as mentioned before. The advantage of $MA^{PQ}$ over L-index, however, is that $MA^{PQ}$, similar to other MA approaches, can identify weak areas (modes) of the system and determine the best buses to apply remedial action to stabilize each weak mode, based on the calculated BPF values. This information cannot be obtained by L-index, as it only identifies the weakest buses of the whole system.

### 2.6 Summary

In this chapter, first, an efficient, i.e. fast and accurate, method for identifying bifurcation points of power system, i.e. SNBP and LIBP, is proposed. High efficiency, easy implementation, and automated applicability, are some characteristics of the proposed algorithm which make it highly suitable for VS assessment of today’s renewable-rich, uncertain, power systems. The method is based on standard NR-PF calculations with the advantage of relaxing all the complexities of the established CPF methods. Three novel techniques have been used in the proposed method to identify SNBP and plot complete P-V curves. These are: (i) dynamic step-size control based on the slope of the P-V curve, (ii) perturbation of the Jacobian matrix at the SNBP to avoid singularity, and (iii) an initialization process to trace the lower part of the P-V curve. LIBP is also identified through a simple additional step of checking the eigenvalues of the $J_R$ at the $Q_C$-limit points, with negligible additional time. The advantages of the proposed method are demonstrated by the fact that it requires less execution time than the other established methods as well as its ease of implementation. The method is tested on different-size test systems under various operating conditions. Through quantitative comparisons with some established CPF-based methods, it is shown that the proposed method yields as accurate results as the other ones, but with much less execution time and higher efficiency than the others.

Two new MA approaches, suitable for distribution power systems, are also proposed in this chapter. The core idea of the new MA methods is the feasibility of considering both active and reactive variations (or either of them) in the MA calculations, due to the fact that in these systems
all the Jacobian sub-matrices are square matrices of the same size. The results of the proposed MA methods have been validated through computing the sensitivity of the system’s VSM, denoted by $\Delta VSM$, to the applied compensation at system buses. The $\Delta VSM$ analyses corresponding to different MA methods, was carried out to validate the results of each MA approach. For example to validate the results of MA$^P$, at which active power variation is considered, $\Delta VSM$ values were calculated by applying active power compensation at system buses. It was shown that the results of each proposed MA are most consistent with those of the associated $\Delta VSM$ calculations. Besides, it was shown that the L-index results are fully correlated with those of MA approach in which both active and reactive power variations are taken into consideration.
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This chapter, first, proposes a method for placement of grid-scale ESS devices in a transmission power system. MA is used for clustering power system buses, from a VS perspective. The VSM of the system is optimally increased, using a combination of MA and CPF methods. The optimal PF for the operation of ESSs is calculated as well, to achieve the maximum VSM. In the second part, the effect of wind power uncertainty in the ESS placement problem is considered, through the PDF of the wind farm’s output power. ESS operation is carried out, with the aim of obtaining a desired level of VSM, at all possible wind-load combinations. A power sharing method between the ESSs is proposed, based on the effectiveness of the ESSs on improving the system’s VSM. Simulation results are presented to verify the effectiveness of the proposed method for ensuring a desired VSM for the power system.

3.1 Introduction

Voltage stability is linked with the ability of the power system to supply both active and reactive power required by the loads. Static compensator (statcom), as one of the common flexible AC transmission system (FACTS) devices, has widely been used to provide power systems with dynamic regulation of reactive power and, thus, voltage levels. With the ongoing and recent technological advances in power electronics and storage devices, integration of large-scale energy storage systems (ESSs) into power systems is becoming economically feasible and prevalent. The main advantage of ESS over statcom is its ability to inject active power into the grid, owing to the energy storage capability of the battery. In fact, in a traditional statcom, although both angle and magnitude of the inverter output voltage can be controlled, they cannot be independently adjusted
due to the lack of active power capability [63]. In presence of battery, the voltage of the DC bus capacitor, behind the inverter, can be kept almost constant and both active and reactive powers can be adjusted independently. This can be effectively used to improve the power system’s voltage controllability and stability.

In this chapter, ESS’s application for steady-state VS improvement is investigated. MA approach, combined with the proposed CPF method of Chapter 2, are used for identifying the most effective places for ESS installation. It is shown that using the proposed ESS placement approach, the voltage profile at the weak buses of the system are also improved. The effects of active and reactive power compensations on VSM of the system are also evaluated. In the second part of this chapter, the stochastic nature of wind is accounted for in the ESS placement problem, through the PDF of wind farm’s output power. The operation of ESSs are also carried out, such that a desired VSM is always obtained for the system, regardless of the load and wind variations. According to the aforementioned descriptions, the contributions of this chapter can be summarized as follows:

1. A MA-based method for clustering power system into buses of similar VS behaviour is proposed, which provides foundation for systematic ESS placement with the aim of VS and voltage profile improvement.

2. Exploiting the obtained clusters of buses, the required numbers, locations, and optimal PFs of ESSs are identified in order to achieve any desired VSM, in an optimal manner.

3. Wind’s uncertainty is incorporated in the ESSs placement problem, with a VSM improvement viewpoint, using the PDF of the wind farm’s output power.

4. A method of power sharing between the installed ESSs, based on their effectiveness on systems VSM, is proposed. The required power injections by the ESSs are also determined such that the minimum required VSM is ensured at all load-wind combinations.

The rest of this chapter is structured as follows. In Section 3.2 the proposed method of optimal ESS placement is presented. Section 3.2.1 gives a brief overview of MA. In Sections 3.2.2, 3.2.3, and 3.2.4 power system clustering approach, ESS placement method, and optimal PF calculation are explained, respectively. Simulation results of this part are presented in Section 3.3, comprising the cases without (Section 3.3.1) and with (Section 3.3.1) considering the reactive power limits of
the generators. Section 3.4 presents the proposed approach for considering wind’s uncertainty in the ESS placement problem, in the context of highly-compensated power systems. More specifically, Section 3.4.1 elaborates on the VS issue of highly compensated power systems. The PDF of wind farm’s output power is derived in Section 3.4.2. The proposed ESS placement, power sharing between ESSs, and operation of ESSs are also explained in Sections 3.4.3, 3.4.4, and 3.4.5, respectively. In Section 3.5, the simulation results of this part are presented, comprising the results of ESS placement (Section 3.5.1), power sharing scheme between ESSs (Section 3.5.2), and power injection levels of ESSs (Section 3.5.3). Finally, the chapter is concluded in Section 3.6.

3.2 ESS Placement To Improve Power System’s VSM

In this section, first, a brief overview of the MA is presented, followed by a detailed description of the proposed method for optimal ESS placement.

3.2.1 Modal Analysis

MA [9] is a frequently used approach for steady-state VS evaluation of power systems. It evaluates the power system VS by computing the eigenvalues and eigenvectors of the reduced Jacobian matrix, $J_R$. Besides, MA divides the power system into several weak areas (modes) from VS viewpoint. Through computing bus participation factors (BPF) associated with each weak mode, the effectiveness of applied compensation at different buses are determined too. It is shown in this chapter that the buses with large BPFs in the weakest modes are the best candidate buses to apply the remedial actions required to stabilise those weak modes [9]. These advantageous properties of MA are exploited in this chapter for ESS placement, from a VS viewpoint. A more detailed description of MA is presented in Chapter 2.

3.2.2 Clustering the Power System Buses

As mentioned, MA is able to identify the weak modes (areas) of a power system. A significant property of the obtained identification is that, by taking remedial measures at any bus associated with each identified weak mode, the stability of other buses of that mode will be improved as well
This makes MA suitable for clustering the power system buses. In this sense, each weak mode of the system can be regarded as a cluster of buses with similar behaviour, from a VS point of view.

To clarify the proposed clustering approach, let us consider the IEEE 118-bus test system. Table 3.1 presents the weakest modes and their associated buses, identified by MA. Note that it suffices to consider the weakest 5 to 10 modes only to identify all critical areas of systems with thousands of buses as argued in reference [9]. Thus, in this section, the first 5 buses associated with the 5 weakest modes of the IEEE 118-bus test system are considered, as they cover all the weakest buses of the system. The changes in the system’s VSM (denoted by $\Delta V_{SM}$) due to the injection of 60 MW active and 60 MVar reactive power at the weak buses, one bus at a time and sequentially, are shown in Fig. 3.1. The value of $\Delta V_{SM}$ associated with each bus is calculated by running the CPF algorithm twice, once for the base system and once more after applying the compensation at that bus. The difference in the value of the VSM in these two cases is termed the $\Delta V_{SM}$ associated with that bus.

An important fact, found through the information given in Table 3.1 and Fig. 3.1, is that the weakest bus of each mode is not necessarily the most effective bus of that mode from $\Delta V_{SM}$ improvement viewpoint, i.e. applying compensation at that bus does not obtain the highest $\Delta V_{SM}$. Two cases in point are buses 2 and 3 (1st mode in Table 3.1), having little effect on the system’s VSM. Thus, in order to find the most effective buses in each mode to apply compensation at, it is required to calculate $\Delta V_{SM}$ for all the weak buses.

Fig. 3.2 shows the voltage profile of this system at the SNBP. It also shows the weakest buses associated with the 5 weakest modes of the system. In addition, it shows the voltage profiles when active power compensation (which is more effective than reactive power compensation as shown in Fig. 3.1) of 60 MW is applied once at buses 44 (mode 3) and once more at bus 95 (mode 5).
Table 3.1: Modal Analysis Results for the Base Case

<table>
<thead>
<tr>
<th>weak mode</th>
<th>1st</th>
<th>2nd</th>
<th>3rd</th>
<th>4th</th>
<th>5th</th>
</tr>
</thead>
<tbody>
<tr>
<td>eigenvalue</td>
<td>0.0016</td>
<td>0.2610</td>
<td>0.5663</td>
<td>0.9989</td>
<td>1.2913</td>
</tr>
<tr>
<td>5 buses with the highest BPF</td>
<td>2</td>
<td>21</td>
<td>44</td>
<td>52</td>
<td>95</td>
</tr>
<tr>
<td></td>
<td>3</td>
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<td>43</td>
<td>51</td>
<td>96</td>
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<tr>
<td></td>
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<td>38</td>
<td>52</td>
<td>44</td>
<td>94</td>
</tr>
</tbody>
</table>

As seen for both cases, by compensating any bus in a weak mode the voltages of all the other buses of that mode are improved, while the effect on the other modes is rather limited. Based on the above-mentioned facts, clustering the power system buses based on MA results provides a solid foundation for the systematic compensation of the power system for VS and voltage profile improvement.

### 3.2.3 ESS Placement

After identifying the power system clusters using MA results, the next step is to find the most effective buses in each cluster to place ESS at. Based on the observations made in the previous section, the following approach is adopted in this thesis for this purpose. First, the values of $\Delta V_{SM}$ resulted from the injection of active and reactive power, one at a time, in all the buses of the weak clusters are calculated. The calculated $\Delta V_{SM}$s are then used to identify the most effective buses in each weak cluster where ESSs are to be installed at. Therefore, in each installation round, one ESS is installed in each weak cluster. The rationale behind this approach is that compensating any weak cluster alone will not improve the stability of the other clusters appreciably, thus every cluster should be compensated at each round. This has the advantage of simultaneous enhancement of the stability and voltage profile of all the weak clusters, and therefore the entire system.

### 3.2.4 Optimising Power Factors of ESS

Although VS is generally linked to the reactive power flow in the power system, the injection of active power at the system buses considerably affects the VS condition and therefore the VSM of the system. In fact, and as is shown in the simulation section, the effect of active power injection on the system’s VSM can be considerably higher than that of reactive power, especially when the
system is highly compensated with reactive power, as is the case with the IEEE 118-bus system. As the interfaced inverter of the ESS allows injection of any desired combination of active and reactive power, the effect of ESSs on system’s VSM can be maximised if they are operated at their optimal PFs. Thus, an optimisation problem can be formulated in which the objective is to maximise the system’s VSM, and the decision variables are the PFs of all the selected ESSs. The optimisation problem can therefore be stated as follows:

\[
\max_{PF_i} \lambda
\]

\[
\text{s.t. } (\lambda + K_{Gi}) P_{Gi} - \lambda P_{Di} + (I_i \times S_{ESS} \times PF_i) = \text{real} \left( \sum_{j=1}^{Nb} V_j^* Y_{ij} \right) \quad \forall i
\]

\[
Q_{Gi} - \lambda Q_{Di} + \left( I_i \times S_{ESS} \times \sqrt{1 - PF_i^2} \right) = \text{imag} \left( \sum_{j=1}^{Nb} V_j^* Y_{ij} \right) \quad \forall i \quad (3.1)
\]

\[
Q_{Gi}^{\min} \leq Q_{Gi} \leq Q_{Gi}^{\max} \quad \forall i \in g
\]

where \( \lambda \) is the system’s loading level, \( V_i \) is the complex voltage at bus \( i \), \( Y_{ij} \) is the complex \( ij^{th} \) element of admittance matrix, \( P_{Gi} \) and \( Q_{Gi} \) are the active and reactive power generations at bus \( i \), respectively, \( P_{Di} \) and \( Q_{Di} \) are the active and reactive power consumption at bus \( i \), respectively, \( Q_{Gi}^{\max} \) is the maximum reactive power capability of the generator \( i \), \( g \) is the set of generator buses, \( I_i \) is a binary variable already determined in the previous stage of the proposed method (\( I_i = 1 \) indicates ESS placement at bus \( i \) and \( I_i = 0 \) indicates no ESS placement at bus \( i \)), \( S_{ESS} \) is the rated MVA of the ESS, and \( PF_i \) is the PF of the ESS placed at bus \( i \). The parameter \( K_{Gi} \)}
distributes the active power losses among all generators. The first two equality constraints ensure active and reactive power balance at all buses at the loading level $\lambda$. The third constraints indicate a simplified presentation of the reactive power limit of the generators. An informative discussion about how to model in such optimisation problems the behaviour of a PV bus when it changes to a PQ bus, due to the reactive power limit being reached, can be found in reference [2]. Notice that the main decision variables which should be determined to maximise $\lambda$ are $PF_i$s.

The optimisation problem stated in equation (3.1) is highly non-linear, non-convex, and of high complexity, especially when the number of buses are large. In this chapter, PSO method, as one of the most efficient heuristic optimisation approaches, is used to find the optimal PF of all ESSs. Briefly, PSO is a search-based optimisation technique, based on movement and intelligence of swarms, in which individuals orient their movements towards the personal ($P_{best}$) and overall ($G_{best}$) best solutions determined by the associated objective function of each particle [74]. Detailed description of the PSO algorithm can be found in reference [74]. The effect of PF optimisation is emphasised in Fig. 3.3, which shows a 3-D presentation of the VSM surface for different lagging PFs of two 60-MVA ESSs installed at buses 2 and 3 (the weakest buses) of the IEEE 118-bus system. The optimal PFs of 0.88 and 0.74 lagging, respectively, found for buses 2 and 3 by using the PSO algorithm, are confirmed in this figure.

The whole process explained in Subsections 3.2.2 to 3.2.4 above is called one round of ESS
allocation/placement in this thesis. After each round, if the obtained VSM is insufficient, the process is repeated for as many rounds as required until a desired VSM is achieved.

3.3 Simulation Results for the Optimal ESS Placement

As mentioned above, the IEEE 118-bus test system [69] is used in this section to show the efficacy of the proposed method. The system includes 54 generators, which 21 of them are synchronous condensers supplying reactive power only to the system, making the system highly reactive power compensated. In the following, two cases are considered.

3.3.1 Case A. Ignoring Generators Reactive Power Limits

The weak modes of the system in this case are presented in Table 3.1. The values of $\Delta VSM$ for 60 MW active and 60MVar reactive power injections at the weakest buses, once at a time, are shown in Fig. 3.1. The important fact observed in this figure is that the active power injection is much more effective than reactive power injection for improving the system’s VSM. This observation can be explained by the fact that the system is highly reactive power compensated, as stated above, and injecting more reactive power has little effect on the system’s VS. The advantage of ESS over conventional statcom, i.e. the ability to provide active power compensation, to be used to achieve higher VSMs, is clearly demonstrated in Fig. 3.1. In fact, Fig. 3.1 shows that the conventional statcom, by injecting reactive power only, would have very limited effect on the system’s VSM.

Three rounds of ESS allocation are carried out for this case. Here again, we consider the five weakest modes, each with the five weakest buses, as system clusters. The selected buses at each round and their associated modes, the optimal PFs of each selected bus, identified by PSO, and the VSMs obtained at each round, denoted by $VSM^*$, are presented in Table 3.2. It is seen that the optimised PFs by the PSO algorithm are very close to unity, due to the much higher effect of active power injection on system’s VSM.

Fig. 3.4 depicts the voltage profile of the system after each round of ESS placement at the loading level corresponding to the system’s initial SNBP ($\lambda=3.3152$). It is seen that the proposed method, along with VSM improvement, enhances the voltage profile by compensating the weak areas of the system simultaneously. Besides, it is seen that compensating any bus in a cluster
3.3 Simulation Results for the Optimal ESS Placement

Figure 3.4: Voltage profile at $\lambda=3.3152$ after each round of ESS installation (case A)

Table 3.2: Results of ESS Placement Algorithm for Case A

<table>
<thead>
<tr>
<th>Round</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bus</td>
<td>53 44 38 20 95</td>
<td>52 43 38 21 82</td>
<td>53 45 38 20 95</td>
<td></td>
</tr>
<tr>
<td>Mode</td>
<td>4 3 1-3 2 5 4 3 1-3 2 5 4 3 1-3 2 5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PF</td>
<td>1 0.98 0.99 1 0.98 1 0.98 1 0.99 1 0.99 1 0.99 1 0.98</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$V_{SM}^*$</td>
<td>3.32</td>
<td>3.0755</td>
<td>3.9654</td>
<td>4.1480</td>
</tr>
<tr>
<td>$V_{SMQ}$</td>
<td>3.32</td>
<td>3.3384</td>
<td>3.3551</td>
<td>3.3620</td>
</tr>
</tbody>
</table>

Finally, in order to compare the advantage of using ESS over that of statcom, the same placement approach is carried out for the conventional statcom. The obtained VSMs, denoted by $V_{SMQ}$, are shown in Table 3.2. Notice that in this case, the PF optimisation stage is not required as statcom is only able to inject reactive power into the system. As seen, using ESSs with optimal PFs leads to considerably higher VSMs compared to using conventional statcoms.

3.3.2 Case B. Considering Generators’ Reactive Power Limits

The reactive power shortage is the main difference of this case compared with the previous one. As a generator reaches its reactive power ($Q_G$) limit, the associated bus changes from PV to PQ and thus the generator is no longer able to regulate the voltage at its bus. This considerably decreases the system’s VSM. The MA results for this case are presented in Table 3.3. Notice that at the SNBP, 31 generators, out of the total 54, reach their $Q_G$ limits. This dramatically changes the
The initial P-V curve

The P-V curve after the 1st round

The P-V curve after the 2nd round

The P-V curve after the 3rd round

Critical point

Figure 3.5: P-V curves for bus 2 after each round (case A)

Table 3.3: Modal Analysis Results for Case B

<table>
<thead>
<tr>
<th>weak mode</th>
<th>1st</th>
<th>2nd</th>
<th>3rd</th>
<th>4th</th>
<th>5th</th>
</tr>
</thead>
<tbody>
<tr>
<td>eigenvalue</td>
<td>0.0018</td>
<td>0.1740</td>
<td>0.4778</td>
<td>0.4998</td>
<td>0.8077</td>
</tr>
</tbody>
</table>

5 buses with the highest BPF

|         | 33  | 104 | 21  | 76  | 84  |
|         | 35  | 105 | 22  | 118 | 83  |
|         | 36  | 106 | 20  | 74  | 118 |
|         | 34  | 103 | 43  | 75  | 85  |
|         | 37  | 109 | 44  | 84  | 76  |

The results of three rounds of ESS allocation for this case are shown in Table 3.4. It is seen that the optimised PFs by the PSO algorithm are much lower than those obtained in case A, reflecting higher effectiveness of the reactive power injection in this case. Also, the resulted VSM$^Q$ values, obtained by using the conventional statcom, are presented in Table 3.4. Again, the results clearly show the advantage of PF optimisation, as a result of active power capability of ESS, to maximise system’s VSM.
3.3 Simulation Results for the Optimal ESS Placement

Figure 3.6: $\Delta$VSM values for 60 MW/MVar power injection at the weak buses (case B)

Table 3.4: Results of ESS Placement Algorithm for Case B

<table>
<thead>
<tr>
<th>Round</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bus</td>
<td></td>
<td>36</td>
<td>109</td>
<td>43</td>
</tr>
<tr>
<td>Mode</td>
<td></td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>PF</td>
<td>0.91</td>
<td>0.99</td>
<td>0.94</td>
<td>0.97</td>
</tr>
<tr>
<td>VSM</td>
<td>2.60</td>
<td>2.8523</td>
<td>3.0035</td>
<td>3.1758</td>
</tr>
<tr>
<td>$VSM^Q$</td>
<td>2.60</td>
<td>2.6726</td>
<td>2.7332</td>
<td>2.7815</td>
</tr>
</tbody>
</table>

It is to be noted that after each placement round, the weak modes of the system may change. This implies that after compensating several weak clusters in each round, some other areas may become the weakest clusters of the system, albeit with much better VSM than the original one. In this case, the new weak areas after each installation round are also identified by MA and compensated in the next round. A case in point is bus 52, which is identified as a weak bus in the third allocation round. Notice, however, that after two allocation rounds, system’s VSM has improved 15% compared to that of the original system (from 2.6025 to 3.0035).

Fig. 3.7 shows the voltage profile for case B after each allocation round at the initial SNBP ($\lambda=2.6025$). Again, it is seen that the voltages of all the weak clusters are improved after each placement round due to the simultaneous compensation of the system’s weak areas. Also, it is seen that after two allocation rounds bus 52 becomes a system’s weak bus (red dash-dotted curve), although its voltage is much better than the initial weak buses (dotted black curve). The P-V curves of this case after each ESS installation round are also shown in Fig. 3.8, reflecting the considerable improvement in the system’s VSM.

An important fact found by comparing the results of MA and $\Delta$VSM calculation, for both cases, is that the weakest buses identified by MA, are exactly the most effective buses of the system for reactive power compensation. However, MA results do not identify the best buses for active power compensation. This is rooted in the fact that in MA the active power variations are ignored, and only the effect of reactive power variations is taken into account [9]. This fact has
been investigated in more detail in Chapter 2 (Section 2.4).

### 3.4 ESS Placement and Operation Considering Wind’s Uncertainty

In this section, the issues of placement and operation of ESS devices are addressed in a wind-embedded power system. It is shown that a desired level of VSM can be attained efficiently, regardless of variations of wind power generation and loading condition. The VS issue of highly compensated power systems are explained first, followed by the description of the proposed method.
3.4 ESS Placement and Operation Considering Wind’s Uncertainty

3.4.1 VS Issue in Highly Compensated Power Systems

As mentioned in Chapter 1, continuous increase in power consumption, along with economic and environmental barriers against infrastructure upgrade, has made many of the today’s power systems operate close to their VS limits. In order to extend their VSM, many of the power systems around the world are heavily compensated with reactive power. In such systems, the voltage collapse occurs at voltages very close to the nominal and this makes operating the power system quite challenging [1, 75]. In highly compensated power systems, voltage instability can occur quickly and unpredictably after small increases in the loading condition. This is despite the fact that the system voltage remains healthy in close proximity of the collapse point [1, 76]. Figure 3.9 shows the P-V curves for bus 30 of the IEEE 30-bus power system for three different levels of reactive power compensation, and thus different PFs of the loads. As seen, the more the system is reactive power compensated, the more the SNBP is shifted to the both right and up. For example, in the heavily compensated case (corresponding to the PF of the loads being 0.95 leading), the voltage collapse occurs at a voltage very close to the permissible operating range (0.94 to 1.1 p.u based on AS61000 grid code [77]). This means that returning the voltages to the permissible range will not guarantee a secure VSM for the system. As seen from Fig. 3.9, by operating the system at \( \lambda=5.8 \) (where the voltage of bus 30 is at the boundary of the permissible range) there exists only around 5% (from 5.8 to the SNBP which corresponds to \( \lambda=6.1 \)) VSM, which is too small.

The discussed VS issue for heavily compensated power systems will be more profoundly felt...
in the coming years when higher penetration of ESSs and inverter-interfaced renewables, with considerable reactive power capability, will become reality. Consequently, a careful investigation and control of system’s VSM is required in order to maintain system’s stability under different operating conditions.

3.4.2 PDF of the Output Power of Wind Farm

Wind speed variation, typically modelled by using the Weibull distribution function (WDF) [78, 79], directly effects the wind farms’ output power. The WDF with scale and shape parameters of 9 and 4, respectively, is shown in Fig. 3.10-(a). The power-wind speed (or $P_w - \nu$) characteristic of a typical wind turbine, using the same parameters as in reference [78], is also shown in Fig. 3.10-(b), where $P_r$ is the turbine’s rated power, $\nu_c$ is the cut-in speed, $\nu_r$ is the rated wind speed, and $\nu_{lim}$ is the limiting wind speed.

By combining the PDF of the wind speed and $P_w - \nu$ characteristic of the wind farm, the PDF of the wind farm’s output power is obtained in this thesis, by using Monte-Carlo simulation. In order to do so, a 30,000 number of random wind speeds corresponding to the WDF of Fig. 3.10-(a) were generated. For each generated wind speed, the output power of the wind farm was calculated by using the $P_w - \nu$ characteristic of the wind farm. Then, a 10-level histogram of all obtained output powers was constructed. By dividing the histogram by the total number of the generated data (30,000), the discretized PDF of the wind farm’s output power was obtained, as shown in the Fig. 3.11. This figure is consistent with the PDF calculated using the random-variable transformation method proposed in reference [78].
3.4 ESS Placement and Operation Considering Wind’s Uncertainty

Fig. 3.11 shows that, although the wind farm output power varies in the whole range between zero to rated power, more highly probable outputs are around zero and rated power. This fact is used in the next section to identify the best places for installing ESS.

3.4.3 VSM-Oriented ESS Placement

The goal of this section is to identify the best locations to install ESSs in order to efficiently improve the systems VSM, taking into account wind farm output power variation and its PDF. For this purpose, the buses at which the ESS is to be installed should (i) belong to one of the VS weak areas (modes) of the system (identified by MA), and (ii) have the highest impact on the systems VSM when the compensation is applied at those buses. However, as will be shown in Section 3.5 (Table 3.5), wind power variation can significantly change the location of weak areas, and consequently the most effective buses to install the compensation devices at, from VS viewpoint. Therefore, all the possible wind farm output powers should be considered in the installation procedure. Alternatively, only the most probable wind farm output powers may be considered for ESS installation. As illustrated in Fig. 3.11, the most probable wind output powers occur at zero and rated power. Accordingly, the most effective buses for VSM improvement at any wind power level are identified as follows:

1. Set the wind power \( P_w \) at a desired specified level.
2. Run CPF to identify the initial VSM (denoted by \( \text{VSM}^0 \)), at that wind level.
3. At the SNBP, perform MA, and identify the weakest modes and buses of the system.
4. Apply increments of active power (P) and reactive power (Q) compensations (say 5 MW/MVar), one at a time, at the identified weak buses. Calculate the new VSMs, and then \( \Delta VSM \) from 
\[
\Delta VSM = VSM - VSM^0.
\]

5. Identify the most effective bus for the specified level of \( P_w \) as the bus associated with the highest \( \Delta VSM \).

6. Change to the next \( P_w \) level and return to the step 2.

In this way, the most effective buses for ESS installation at each level of \( P_w \) are identified. Using the identified buses and considering the PDF of Fig. 3.11, the ESS installation buses are decided, as will be shown in Section 3.5.1.

### 3.4.4 Power Sharing Between the ESSs

As will be shown in Section 3.5.1 (Fig. 3.13), the effectiveness of power injection at different buses for VSM improvement varies with the wind power level variations. In fact, the most effective ESSs at a wind power level may not be so influential for upgrading the system’s VSM at other wind power levels. This implies that power injection by the ESSs should be carried out based on their effectiveness in upgrading system’s VSM at each wind power level. Hence, a power sharing scheme between the ESSs should be implemented. In this thesis, the sharing coefficient \( \alpha_{P_w}^k \), determining the power injection share of \( k^{th} \) ESS at wind power level \( P_w \), is calculated using equation (3.2):

\[
\alpha_{k}^{P_w} = \frac{\Delta VSM_{k}^{P_w}}{\sum_{i=1}^{N_{st}} \Delta VSM_{k}^{P_w}} \quad (3.2)
\]

where \( \Delta VSM_{k}^{P_w} \) is the amount of obtained VSM improvement as a result of injecting incremental active power by the \( k^{th} \) ESS at wind power level \( P_w \), and \( N_{st} \) is the number of installed ESSs. By using equation (3.2) at each wind power level, the ESS with the higher effect on system’s VSM injects more power into the system. Besides, applying the power sharing as per equation (3.2) prevents power injection by the ESSs with low effect on system’s VSM.

As shown in Section 3.2, to achieve the highest possible VSM, ESSs should be operated at
their optimal PFs. Hence, a similar optimization problem to equation (3.1) is also formulated to find the optimal PF of each ESS. Notice that as wind power variation alters the VS condition of the system, the optimal PF of ESSs may also vary with the wind power level variation. Thus, calculating optimal PFs should be done for every wind power level.

### 3.4.5 Operation of the ESSs

The required VSM of power systems operating under normal condition has commonly been defined to be in the range of 5 to 10% of their loading condition [80]. However, for the highly compensated power systems with embedded renewables, the high variability and unpredictability of wind and solar power generations necessitate a higher level of VSM. In this section, the required VSM is set to be at least 15% at any loading condition. Thus, at each wind power level and each loading condition, the required (reference) power injection level of each installed ESS should be determined such that the required VSM is obtained for the system. This reference value is determined through defining a variable named *injection coefficient*, $\mu_{\lambda,P_w}$, at any loading level, $\lambda$, and wind farms’ output power, $P_w$. In this way, the required reference active, $P_{\lambda,P_w}^k$, and reactive, $Q_{\lambda,P_w}^k$, powers of the $k^{th}$ ESS are computed from the following equation:

$$P_{\lambda,P_w}^k = \mu_{\lambda,P_w} \times \alpha_{P_w}^k \times S_{\text{max}}^k \times P_{F_k}^{P_w}$$

$$Q_{\lambda,P_w}^k = \mu_{\lambda,P_w} \times \alpha_{P_w}^k \times S_{\text{max}}^k \times \sqrt{1 - \left(\frac{P_{F_k}^{P_w}}{S_{\text{max}}^k}\right)^2}$$

(3.3)

where $S_{\text{max}}^k$ is the rated MVA of the ESSs. Also, $P_{F_k}^{P_w}$ and $\alpha_{P_w}^k$ are the optimal PF and power sharing coefficient of the $k^{th}$ ESS at wind power level $P_w$, respectively. The values of $\mu_{\lambda,P_w}$ is computed through off-line simulations such that, at any loading level $\lambda$ and wind output power $P_w$, the injected $P_{\lambda,P_w}^k$ and $Q_{\lambda,P_w}^k$ lead to at least 15% VSM for the system. In order to do so, for any $\lambda$ and $P_w$ levels, the value of $\mu_{\lambda,P_w}$ is changed incrementally and the resulted $P_{\lambda,P_w}^k$ and $Q_{\lambda,P_w}^k$ from equation (3.3) are injected into the system by the corresponding ESSs. Then, the system’s VSM is calculated by running CPF. This process is continued until an injection coefficient $\mu_{\lambda,P_w}$ leading to a VSM equal to $0.15 \times \lambda$ is obtained.

The obtained values of $P_{\lambda,P_w}^k$ and $Q_{\lambda,P_w}^k$ can be used as reference signals, continuously adjusted with variations in the loading and wind power conditions; and fed into the inverter controller of the
respective ESS. Through this feedback control loop, the required 15% VSM can be dynamically maintained for the system. This concept is schematically shown in Fig. 3.12.

### 3.5 Simulation Results for ESS Placement Under Wind’s Uncertainty

The proposed method of ESS placement and operation, considering wind’s uncertainty, has been tested on IEEE 30-bus power system. To fully demonstrate the advantages of the method, the following assumptions and modifications in the system have been made:

- The power factors of the loads have all been changed to 0.95 leading to make the system highly reactive power compensated. The corresponding P-V curve for this level of compensation was shown in Fig. 3.9 (with MLP corresponding to $\lambda = 6.1$).

- A wind farm with 40 MW rated power is connected to the weakest bus, identified as bus 30. The output power of the wind farm varies according to the PDF of Fig. 3.11.

- The wind farm is operated at PF=1, which is a common practice in many power systems to avoid interference with the main voltage regulations of the system [81].

- The peak loading condition corresponds to $\lambda^{\text{peak}} = 6$. The implication is that, with the 15% VSM criterion, the SNBP of the system should be extended to least 6.9 (i.e. $6 \times 1.15$) at the peak loading level.
3.5 Simulation Results for ESS Placement Under Wind’s Uncertainty

3.5.1 ESS Placement for VSM Improvement

The sensitivity of the system’s VSM to incremental active and reactive power injections of 5 MW/MVar, one at a time, at the weak buses identified by MA, for two wind power levels of 0 and 40 MW are shown in Fig. 3.13.

The following important observations are made:

- Injection of P at the identified weak buses for VSM improvement is much more effective than injecting Q.

- The most effective buses for VSM improvement at a $P_{\text{wind}}$ level may not be so effective at the other $P_{\text{wind}}$ levels. For instance, as seen in Fig. 3.13, bus 7 is the most effective bus for VSM improvement at high wind powers but is quite ineffective at low wind power levels.

The most effective buses for ESS placement at different wind power levels, found by the procedure explained in Section 3.4.3, are shown in Table 3.5, which also shows the system’s loading level at SNBP, denoted by $\lambda^{\text{max}}$, corresponding to each wind power level. Based on these results and considering the PDF of the wind farms’ output power, shown in Fig. 3.11, buses 30 and 7 are chosen for ESS placement, as they are the most effective buses for the most probable states of wind power, $P_{\text{wind}}=0$ MW and $P_{\text{wind}}=40$ MW.
Table 3.5: The Most Effective Buses for VSM Improvement at Different Wind Power Levels

<table>
<thead>
<tr>
<th>( P_w ) (MW)</th>
<th>0</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
<th>25</th>
<th>30</th>
<th>35</th>
<th>40</th>
</tr>
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<tbody>
<tr>
<td>( \lambda_{\text{max}} )</td>
<td>6.10</td>
<td>6.35</td>
<td>6.54</td>
<td>6.65</td>
<td>6.71</td>
<td>6.75</td>
<td>6.78</td>
<td>6.81</td>
<td>6.84</td>
</tr>
<tr>
<td>5 weakest buses</td>
<td>30</td>
<td>30</td>
<td>29</td>
<td>27</td>
<td>27</td>
<td>26</td>
<td>7</td>
<td>7</td>
<td>7</td>
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<td>26</td>
<td>26</td>
<td>26</td>
<td>30</td>
<td>26</td>
<td>25</td>
<td>29</td>
<td>25</td>
<td>25</td>
</tr>
<tr>
<td>Best Bus</td>
<td>30</td>
<td>30</td>
<td>30</td>
<td>30</td>
<td>30</td>
<td>26</td>
<td>7</td>
<td>7</td>
<td>7</td>
</tr>
</tbody>
</table>

Figure 3.14: \( \Delta \text{VSM} \) values for buses 30 and 7, and sharing coefficients \( \alpha_{k}^{P_w} \) at different wind power output levels

### 3.5.2 Power Sharing Scheme of the ESSs

As explained in Section 3.4.4, the sharing coefficients of ESSs are determined based on their effectiveness on improving system’s VSM, using equation (3.2). Fig. 3.14 shows the values of \( \Delta \text{VSM} \) resulted from injecting 5 MW active power at buses 30 and 7 and at different wind power levels. The sharing coefficients corresponding to these \( \Delta \text{VSMs} \) are calculated, and a quadratic curve has been fitted to the obtained data. Figure 3.14 presents the resulted sharing coefficients, \( \alpha_{k}^{P_w} \), curves for ESSs of bus 30 (red) and bus 7 (blue). It is seen that, for instance, at \( P_w = 0 \) the sharing coefficient of ESS of bus 7 is almost zero due to its small effect on system’s VSM compared to the ESS of bus 30.

The optimal PFs of the ESSs are also calculated for different wind power levels using the PSO algorithm. The results are shown in Table 3.6. As seen in the table, the optimal PFs slightly change with wind power variation. Also, it is seen that the optimal PFs are close to unity due to the much higher effect of P injection compared to that of Q injection on system’s VSM.
3.5 Simulation Results for ESS Placement Under Wind’s Uncertainty

Table 3.6: Optimal PFs of ESSs at each wind power level identified by PSO

<table>
<thead>
<tr>
<th>$P_w$ (MW)</th>
<th>0</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
<th>25</th>
<th>30</th>
<th>35</th>
<th>40</th>
</tr>
</thead>
<tbody>
<tr>
<td>$PF^{P_{w}}$</td>
<td>0.96</td>
<td>0.97</td>
<td>0.98</td>
<td>0.98</td>
<td>0.99</td>
<td>0.99</td>
<td>0.99</td>
<td>0.99</td>
<td>0.99</td>
</tr>
<tr>
<td>$PF_{I}^{P_{w}}$</td>
<td>0.92</td>
<td>0.93</td>
<td>0.94</td>
<td>0.94</td>
<td>0.94</td>
<td>0.93</td>
<td>0.93</td>
<td>0.93</td>
<td>0.93</td>
</tr>
</tbody>
</table>

Figure 3.15: Existing and desired VSM surfaces of the system

3.5.3 Power Injection Level of ESSs

As explained in Section 3.4.5, at any loading condition $\lambda$, and wind power level $P_w$, 15% VSM must be obtained to ensure the system’s safe operation. By increasing $\lambda$, the existing VSM decreases as the system gets closer to the SNBP. Also, as seen from Table 3.5, by increasing $P_w$, the VSM increases. Thus, the worst case of the system under study corresponds to the maximum loading ($\lambda = 6$) and minimum wind power ($P_w = 0$), in which the system’s VSM is minimum, as is also shown in Fig. 3.15. Figure 3.15 implies that the maximum power injection of the ESSs is required at $\lambda = 6$ and $P_w = 0$, where the VSM is minimum. Besides, it shows that, for a wide range of low $\lambda$s and high $P_w$s, the existing VSM is higher than the desired one, and, thus, no power injection by ESSs is required. Notice, however, that in cases of very large wind powers, the resulted large reverse power flows may lead to increasing the power loss in the transmission lines, and thus, decreasing the system’s VSM. This is not the case in the current test system though.

Let the rating MVA of the ESSs, $S_{\text{max}}$, in equation (3.3), be 50 MVA. For an ESSs of this
The total required ESS installation is less than 4% of the peak loading level of the system (around 1700 MW, considering the assumptions of $\lambda^{peak} = 6$), which is reasonably low.

Finally, the resulted VSMs using the obtained $P_k^{\lambda,P_w}$ and $Q_k^{\lambda,P_w}$ surfaces of Fig. 3.16 are calculated and the results are drawn in Fig. 3.17 (red surface). It is seen that the minimum 15%
3.5 Simulation Results for ESS Placement Under Wind’s Uncertainty

Figure 3.17: The initial and final obtained VSM surfaces

required VSM has been ensured for the system by using the proposed method. At the load-wind combinations where ESSs do not inject power into the system, however, the existing VSM (which is already above 15%) has not changed. In fact, those load-wind combinations (with excess of VSM) are where the ESSs should be charged. The charging scheme of ESSs should also be carried out such that the system’s VSM remains above the required 15%. This issues are addressed in Chapter 4.

The proposed ESS allocation method of this chapter is designed for the cases with random pattern of renewable generation, when PDF of wind power is taken into account to allocate ESS. Even though the load has not been considered stochastic in this chapter, all possible ranges of the loading level are accounted for, as seen in figures 3.15 to 3.17. This ensures that the allocated ESS is sufficient for the worst case scenario, from voltage stability viewpoint, which is when the loading level is maximum and wind power is zero. In this case, the storage size is determined by the required power injection by the ESS at peak loading condition and minimum wind power generation, such that the desired VSM is ensured, as explained in section 3.4.5 (see Fig. 3.16). In case of random load pattern (which can be modelled by its PDF), the required size of the ESSs is still determined by the minimum wind - maximum loading condition. However, since the optimal location of ESSs may change, depending on the PDF of wind power and loading level, the sizes of the installed ESSs may be different with those computed by the approach of this chapter. The
probabilistic nature of the load is considered in Chapter 4, where the joint PDF of load and wind power is used in the probabilistic optimal ESS allocation problem.

3.6 Summary

In this chapter, the optimal allocation of ESS for the purpose of static VS improvement is studied. In the first part of this chapter, two objectives of optimally increasing the system’s VSM, and simultaneously improving the voltage profile at the system’s weakest areas are achieved. It is shown that MA can be effectively used for clustering the system buses into areas where the compensation is most effective. In addition, it is shown that by operating ESSs at their optimal PFs, found by using PSO, the best composition of active and reactive powers can be injected into the system in order to maximise system’s VSM. The results verify that injection of active power can be much more effective than that of reactive power for upgrading system’s VSM, especially in highly reactive power compensated systems. This indicates the advantage of ESS over conventional statcom.

In the second part of this chapter, placement and operation of ESS in highly compensated power systems with embedded wind farms is studied. It is shown that wind power variation can considerably change the weak areas of the system, from a VS viewpoint. The ESS placement is carried out by considering the most probable level of the wind farm’s output power, derived from the wind’s PDF. The power injection share of each ESS is calculated at each wind power level based on their effects on improving the system’s VSM. In addition, the required power injection level of ESSs at any wind power level and loading condition are calculated to ensure 15% VSM at all operating conditions. The simulation results of this part clearly verify that the required VSM for the system can always be achieved by using the proposed method.
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Chapter 4
Risk-Based Stochastic Allocation of ESS to Improve Voltage Stability of Power Systems

In this chapter, optimal placement, sizing, and operation of ESS devices in power systems with embedded wind farms are studied, within two different probabilistic optimization frameworks. In the first part of the chapter, the minimum required “power” ratings of ESSs is computed such that a desired level of VSM is always ensured for the power system. The uncertainties of wind power generation and loads are taken into account, by considering their corresponding probability density functions (PDFs) and using a probabilistic optimization approach. Active network management (ANM) tools, such as reactive power capabilities of both ESS devices and wind farms, as well as a 10% extra rating of their associated inverters are also used as additional flexibilities to reduce the required ESS sizes. In the second part of this chapter, minimization of both “power” and “energy” ratings of ESSs, required to ensure a desired VSM for a distribution power system is carried out. The reactive power loss and reactive power import from the upstream network are also minimized through a multi-objective optimization approach. Wind uncertainty has been accounted for through ARIMA-based generated wind scenarios and using a risk-based stochastic optimization framework. Besides, the use of ANM tools, including the action of on-load tap changers (OLTCs), modelled by using a new method, are incorporated into the problem. The results show that the required ESS, to ensure a desired VSM, can be considerably curtailed by using proper risk-based constraints and ANM tools.

4.1 Introduction

STEADY increase in load consumption, and environmental and economical barriers against reinforcing power network’s infrastructure, has made many power systems operate close to their VS limits. In these situations, alternative measures have to be taken to ensure secure operation
of power system under various operating conditions. Grid integration of large-scale ESSs, which is becoming technologically and economically possible, is considered as a potential solution for the above-mentioned problem. The interfaced inverters of ESS devices can be appropriately designed to provide additional levels of flexibility for reactive power control. The current relatively high prices of ESS devices, nevertheless, necessitate the use of optimization-based approaches to achieve the minimum required ESS sizes. Emerging ANM tools, as other potential alternatives for network reinforcement, have been used to control power flow and mitigate voltage deviations and transmission congestions [45, 82]. Hence, by incorporating ANM facilities into the ESS allocation problem, potentially, smaller ESS sizes could be required. In tackling power system planning problems, accounting for the system’s uncertainties is imperative. Hence, different probabilistic, stochastic, or robust optimization approaches are used to take into account the system’s uncertainty. This chapter, presents a stochastic optimization approach for ESS placement, sizing, and operation, in order to secure a desired level of VSM for power systems. VSM is ensured by including VS constraints in the ESS allocation problem. ANM tools are also incorporated into the problem to further decrease the required ESS sizes.

Although VS is more known to be an issue for transmission power systems, there have been reported incidents of real-life voltage instability in distribution power systems too, with the potential of spreading to the associated transmission system, as mentioned in Chapter 1. Hence, in this chapter, the problem of optimal ESS allocation is investigated for both transmission and distribution power systems. In summary, this chapter offers the following contributions:

1. Using a stochastic optimization framework, the minimum required ESS (power and energy) to secure a desired VSM for transmission/distribution power systems is computed. Compared to references [8, 10, 12], which do not guarantee a secure voltage stability level, the approach of this chapter ensures a desired VSM, through inclusion of the risk-based VS constraints in the problem formulation.

2. Co-optimization of ESSs’ location, size, and operation is carried out, taking into account the wind’s uncertainty. Unlike references [45, 50, 83] which have only considered either placement or sizing of ESSs, this chapter co-optimizes ESSs’ location and size. Besides, unlike references [46–49], AC power flow constraints and RPC of ESSs and wind farms have not been ignored. Moreover, no binary variables are used to determine ESSs’ charg-
4.1 Introduction

ing/discharging periods (unlike reference [47]) and locations (unlike references [47–49]), which improves the scalability of the formulated problem.

3. A new simple method to model the transformers’ OLTC in the OPF problems is presented. Compared to existing OLTC models, which are either in rectangular coordinate [84, 85], only applicable to branch flow model (BFM) of the OPF [86], or too simplistic [45], the proposed OLTC model is expressed in polar coordinate, easily applicable to both BFM and bus injection model (BIM) of the OPF problem, and accurate.

4. The effects of ANM tools, such as OLTCs’ tap position and RPCs of both ESSs and wind farms, are investigated through case studies. While ANM tools have been used for purposes such as reducing the wind power curtailment [45] and system’s operating costs [82], they have not been utilized for VSM improvement purposes.

The proposed method is tested on IEEE 30-bus transmission system and a commonly used rural 41-bus distribution system, using two different approaches. The wind uncertainty is modelled in two different ways, i.e. PDF of wind farm’s output power and wind power scenarios. Although the proposed method of ESS integration and operation in power systems can bring about other technical and economic benefits for the system and utility, apart from securing the system’s VSM, investigating these benefits is beyond the scope of this thesis.

The rest of this chapter is organized as follows: In Section 4.2, the proposed probabilistic ESS allocation for transmission power systems is presented. Section 4.2.1 describes the method of computing the joint PDF of wind power and loading level, Section 4.2.2 explains the approach of determining the candidate buses for ESS installation, and Section 4.2.3 presents the optimization problem formulation. The simulation results of this part are presented in Section 4.3. Section 4.4, presents the risk-based stochastic allocation of ESS to ensure VSM in a distribution power system. The wind scenario generation approach (Section 4.4.1), the proposed OLTC modelling approach (Section 4.4.2), and the problem formulation (Section 4.4.3) are described. The investigated case study and numerical results are presented in Sections 4.5.1 and 4.5.2, respectively. Finally, the chapter is summarized and concluded in Section 4.6.
4.2 Probabilistic ESS Allocation to Ensure VSM

4.2.1 PDFs of the Wind Power and Loading Level

In this section, the wind’s uncertainty is modelled using its PDF, in the same way as explained in Section 3.4.2 and shown again in Fig. 4.1-(a). The load variation, on the other hand, is assumed to follow the IEEE-RTS load profile [43]. This provides the hourly loading level variation for each typical day of a season as a fraction of the system’s annual peak load. Using the loading level data of the four seasons, a 10-level histogram, and then the discretized PDF of the loading level was obtained, similar to what was done for the wind power, as is shown in Fig. 4.1-(b). Having the PDFs of the wind power and loading level, the effects of their corresponding uncertainties can be combined by computing their joint PDF, as shown in Fig. 4.2. In this way, a total number of 100 wind-load combinations with corresponding probabilities, adding up to one, is generated, to be incorporated into the probabilistic optimization framework. Although load and wind generation are not statistically independent, as there exists a minor correlation between them [87, 88], this correlation has been ignored in this section for simplicity.

4.2.2 Candidate Buses for ESS Placement

In this chapter, the candidate buses for installing ESS devices are determined from a VS viewpoint, using MA. The identified weak buses are the ones with the highest voltage sensitivity due to the bus power variations [9]. It is shown in Chapter 3 that these buses are also the best ones for
4.2 Probabilistic ESS Allocation to Ensure VSM

applying remedial measures for the purpose of improving the system’s VSM. Thus, based on the MA results, in this section, the 5 weakest buses associated with the first two weak modes of the test system (IEEE 30-bus) are used as candidate buses for ESS installation. As load and wind variations may change the weak modes of the system, as seen in Chapter 3, the candidate buses are selected based on MA results at all wind-load levels, as will be shown in Section 4.3.

4.2.3 Problem Formulation

This section presents the formulation of the proposed ESS allocation method to achieve a desired VSM for transmission power systems.

Remark 1: In the following, $X_{i\omega\lambda}$ is used to indicate the value of variable $X$ at bus $i \in I$ (set of all buses), wind power level $\omega \in \Omega$ (set of all wind power levels), and loading level $\lambda \in \Lambda$ (set of all loading levels).

- Objective Function

The objective function of the problem consists of two terms. The first term is the total required power rating of the ESS devices to be installed, and the second term is the expected required active power injection by the ESSs over all possible wind-load combinations (both terms to be
minimized), as stated in equation (4.1):

$$\text{Min} \quad \sum_{i \in C} P_{\text{ess},i}^{r} + \sum_{\omega \in \Omega} \sum_{\lambda \in \Lambda} \pi_{\omega \lambda} \left( \sum_{i \in C} P_{\text{ess},i}^{r} \right)$$

(4.1)

where $P_{\text{ess},i}^{r}$ is the rated active power of the ESS installed at bus $i$, and $P_{\text{ess},i}^{r}$ is the active power injection by the ESSs. $\theta_{\text{ess},i}^{\omega \lambda}$ and $\theta_{\text{w},i}^{\omega \lambda}$ are, respectively, the PF angles of the ESSs and wind farms, which are the angles between the active and apparent powers injected by these devices. $\pi_{\omega \lambda}$ is the probability of simultaneous occurrence of wind power level $\omega$ and loading level $\lambda$ (obtained using the joint PDF of Fig. 4.2). $C \subset I$ is the set of candidate buses for ESS placement, identified by MA, as explained in Section 4.2.2.

**Remark 2:** It is to be noted that minimizing the required energy capability of the ESSs is carried out in this section through inclusion of the ESSs’ expected active power injection in the objective function. Thus, the ESSs’ energy capacity does not explicitly appear in the problem formulation. To calculate the ESSs’ energy requirement, either a time series data for wind speed or a scenario based approach should be used, which is different from the approach of this section. The ESS’s energy rating calculation is carried out in Section 4.4.

**- System Constraints**

Active and reactive power balances are the main equality constraints of the problem and formulated as expressed in equations (4.2) and (4.3), respectively. These equations ensure that the net active/reactive power generation at each bus (i.e. power generation minus power demand, denoted in the left hand sides of equations (4.2) and (4.3)) should be equal to the amount of active/reactive power injected into the transmission grid. The latter is ensured using the Kirchhoff’s circuit laws, as per the right hand sides of equations (4.2) and (4.3). These equality constraints must hold under all load-wind combinations.

$$P_{\text{g},i}^{\omega \lambda} - P_{\text{d},i}^{\lambda} + P_{\text{ess},i}^{\omega \lambda} + P_{\text{w},i}^{\omega \lambda} = |V_{i,\omega \lambda}|$$

$$\sum_{j \in I} |V_{j,\omega \lambda}| \left( G_{ij} \cos \delta_{ij,\omega \lambda} + B_{ij} \sin \delta_{ij,\omega \lambda} \right) \quad \forall i, \omega, \lambda \quad (4.2)$$
\[ Q_{i\omega\lambda}^g - Q_{i\lambda}^d + P_{i\omega\lambda}^{\text{ess}} \tan \theta_{i\omega\lambda}^{\text{ess}} + P_{i\omega\lambda}^{w} \tan \theta_{i\omega\lambda}^{w} = |V_{i\omega\lambda}| \]
\[ \sum_{j \in I} |V_{j\omega\lambda}| (G_{ij} \sin \delta_{ij\omega\lambda} - B_{ij} \cos \delta_{ij\omega\lambda}) \forall i, \omega, \lambda \tag{4.3} \]

where \( V_{i\omega\lambda} \) and \( \delta_{i\omega\lambda} \) are, respectively, the bus voltage magnitudes and phase angles, and \( \delta_{ij\omega\lambda} = \delta_{i\omega\lambda} - \delta_{j\omega\lambda} \). \( B_{ij} \) and \( G_{ij} \) are real and imaginary parts of the \( ij \)th element of the admittance matrix. \( P_{i\omega\lambda}^{g} \) and \( Q_{i\omega\lambda}^{g} \) are, respectively, the active and reactive power generations. In this section, the values of \( P_{i\omega\lambda}^{g} \) are obtained from off-line OPF calculations carried out for each wind-load combination. \( P_{i\lambda}^{d} \) and \( Q_{i\lambda}^{d} \) are active and reactive power demands, and \( P_{i\omega}^{w} \) denotes the active power generation of the wind farms.

**Remark 3:** Inasmuch as the VS is a problem of peak-load periods, and ESS devices are typically discharged over these periods, a power injection convention (like generators) is used in this section for ESSs. In fact, the goal of this section is to find the minimum required power injection by ESSs during peak-load periods to achieve a desired VSM. The ESSs’ charging scheme which relates to the off-peak periods is addressed in Section 4.4.

Other operational constraints, including bus voltage, reactive power generation, and thermal constraints are formulated as shown in equations (4.4)-(4.6), respectively:

\[ 0.94 \leq |V_{i\omega\lambda}| \leq 1.1 \forall i, \omega, \lambda \tag{4.4} \]
\[ Q_{i}^{\text{gmin}} \leq Q_{i\omega\lambda}^{g} \leq Q_{i}^{\text{gmax}} \forall i, \omega, \lambda \tag{4.5} \]
\[ (G_{ij}^2 + B_{ij}^2) \left( |V_{i\omega\lambda}|^2 + |V_{j\omega\lambda}|^2 - 2 |V_{i\omega\lambda}| |V_{j\omega\lambda}| \cos \delta_{ij\omega\lambda} \right) \leq (I_{ij}^{\text{max}})^2 \forall i, \omega, \lambda \tag{4.6} \]

where \( Q_{i}^{\text{gmin}} \), \( Q_{i}^{\text{gmax}} \), and \( I_{ij}^{\text{max}} \) are, respectively, the minimum and maximum reactive power capabilities of the generator at bus \( i \), and thermal limit of the line between buses \( i \) and \( j \).

Equation (4.4) ensures that the stationary voltages at each bus of the power system are within the acceptable operational range, which is considered to be between 0.94 p.u. and 1.1 p.u., as common values in Australian power grid. Equation (4.5) guarantees that no generator violates its reactive power capability limitations, i.e. \( Q_{i}^{\text{gmin}} \) and \( Q_{i}^{\text{gmax}} \), depending on the generator’s stator thermal limit and over/under excitation limits. Besides, equation (4.6) constrains the apparent power flow through the transmission lines within their corresponding thermal limits, which is essential to avoid premature ageing of the lines.
- Voltage Stability Constraints

It is known that simple constraints on bus voltages alone cannot guarantee VS of the power system. In fact, voltage instability can occur even when all the bus voltages are within the normal range (e.g. in highly reactive power compensated systems, as shown in Chapter 3). In this chapter, the VS constraints introduced in reference [89] are used to ensure a required VSM for the system. These are similar constraints to (4.2) to (4.6), but all at the desired maximum (or critical) loading point (MLP) denoted by $\lambda^*$. The value of $\lambda^*$ is considered to be $\alpha^* \times \lambda$ in this section, for every loading level $\lambda$. For example, $\alpha^* = 1.2$ ensures 20% VSM at all loading levels. The MLP can be associated with either bus voltage, thermal, or VS (either saddle node or limit induced bifurcation) limits [89]. The active and reactive power balance constraints for the critical loading condition ($\lambda^*$) are formulated as shown in equations (4.7) and (4.8). These constraints, in fact, guarantee the existence of an operating point at the loading level $\lambda^* = \alpha^* \times \lambda$.

\[
\left(\alpha^* + K^g_{\omega\lambda}\right) P^g_{i\omega\lambda} - \alpha^* P^d_{i\lambda} + P^{ess}_{i\omega\lambda} + P^{w}_{i\omega} = \left| V^\lambda_{i\omega\lambda} \right| \sum_{j \in \mathcal{I}} \left| V^\lambda_{j\omega\lambda} \right| \left( G_{ij} \cos \delta_{ij\omega\lambda} + B_{ij} \sin \delta_{ij\omega\lambda} \right) \quad \forall i, \omega, \lambda (4.7)
\]

\[
Q^g_{i\omega\lambda} - \alpha^* Q^d_{i\lambda} + P^{ess}_{i\omega\lambda} \tan \theta^{ess}_{i\omega\lambda} + P^{w}_{i\omega} \tan \theta^{w}_{i\omega} = \left| V^\lambda_{i\omega\lambda} \right| \sum_{j \in \mathcal{I}} \left| V^\lambda_{j\omega\lambda} \right| \left( G_{ij} \sin \delta_{ij\omega\lambda} - B_{ij} \cos \delta_{ij\omega\lambda} \right) \quad \forall i, \omega, \lambda (4.8)
\]

Note that all the variables with superscript $\lambda^*$ represent the system at the critical loading condition. Also, $K^g_{\omega\lambda}$ is a scalar variable which distributes active power loss at the critical loading condition among all generators in proportion to their generation values, as discussed in reference [89]. Similar constraints to (4.4)-(4.6) are also applied for the critical loading condition, $\lambda^*$, which are not included here for brevity. These constraints, along with constraints (4.7)-(4.8), guarantee that at any loading level $\lambda$, the system can be safely loaded up to the loading level $\lambda^* = \alpha^* \times \lambda$, without violating any of the operational constraints or experiencing voltage collapse.
- ESS and Wind Farm Constraints

As mentioned, the RPCs of ESSs and wind farms are utilised in this chapter to achieve smaller ESS sizes. In order to do so, the PF angles of ESSs and wind farms are considered as decision variables. The sizes (rated power) of the selected ESS devices at any bus \( i \) are modelled by using the variable \( P_{i,\text{ess}}^r \), which constrains the active power injection of the ESS at bus \( i \), at all wind power and loading levels, as expressed in equation (4.9). Also, it is assumed that the apparent power rating of the ESSs’ inverters is 10% higher than the battery’s rated active power, as expressed in equation (4.10). This Extra Inverter Capability (EIC) provides the ESS with up to 46\% (\( 0.46 \approx \sqrt{1.1^2 - 1^2} \)) extra capability to inject reactive power, when full active power capability of the battery is used. Constraint (4.11) ensures that no ESS installation at non-candidate buses occurs, and constraint (4.12) limits the PF of the ESSs between \( P_{\text{min}}^{\text{ess}} \) leading and lagging.

\[
0 \leq P_{i,\omega\lambda}^{\text{ess}} \leq P_{i,\text{ess}}^r \quad \forall \omega, \lambda, i \in C \tag{4.9}
\]
\[
(P_{i,\omega\lambda}^{\text{ess}})^2 \left( 1 + \tan^2 \theta_{i,\omega\lambda}^{\text{ess}} \right) \leq (1.1 P_{i,\text{ess}}^r)^2 \quad \forall \omega, \lambda, i \in C \tag{4.10}
\]
\[
P_{i,\text{ess}}^r = 0 \quad \forall i \not\in C \tag{4.11}
\]
\[
-\text{acos}(P_{\text{min}}^{\text{ess}}) \leq \theta_{i,\omega\lambda}^{\text{ess}} \leq \text{acos}(P_{\text{min}}^{\text{ess}}) \quad \forall \omega, \lambda, i \in C \tag{4.12}
\]

Finally, constraints (4.13) and (4.14) limit the apparent power and PF of the wind farms. Wind farms are assumed to be non-dispatchable, i.e. all available power generation by them is injected into the system. Similar to equations (4.10) and (4.12), it is assumed that the apparent power rating of the wind farms’ inverters is 10\% higher than the wind farm’s rated active power, as per equation (4.13), to provide additional capability for reactive power injection. Also, the PF of the wind farms is limited between \( P_{\text{min}}^{\text{ess}} \) leading and lagging, as per constraint (4.14). This ensures that the inverter’s thermal capability is mainly used for active power (rather than reactive power) injection, considering the acceptable PF limits.

\[
(P_{i,\omega}^{\text{w}})^2 \left( 1 + \tan^2 \theta_{i,\omega}^{\text{w}} \right) \leq (1.1 P_{i,\text{w}}^r)^2 \quad \forall \omega, \lambda, i \in \mathcal{W} \tag{4.13}
\]
\[
-\text{acos}(P_{\text{min}}^{\text{w}}) \leq \theta_{i,\omega}^{\text{w}} \leq \text{acos}(P_{\text{min}}^{\text{w}}) \quad \forall \omega, \lambda, i \in \mathcal{W} \tag{4.14}
\]

where \( P_{i,\text{w}}^r \) denotes the wind farms’ rated active power, and \( \mathcal{W} \subset \mathcal{I} \) is the set of all wind farm
Table 4.1: MA results (weak buses) at different wind-load levels

<table>
<thead>
<tr>
<th>weak buses</th>
<th>wind power level</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0 %</td>
</tr>
<tr>
<td>34 %</td>
<td>25, 24, 19, 20</td>
</tr>
<tr>
<td>67 %</td>
<td>25, 24, 19, 20</td>
</tr>
<tr>
<td>100 %</td>
<td>25, 19, 20, 18</td>
</tr>
</tbody>
</table>
\[ C \subset I \]

buses.

4.3 Simulation Results of the Probabilistic ESS Allocation

The proposed ESS allocation method is tested on modified IEEE 30-bus power system. Two wind farms with rated powers of 20 MW and 10 MW are respectively connected at buses 30 and 19, which are found to be the weakest buses of the first two weak modes (identified by MA). For simplicity, it is assumed that both wind farms are exposed to the same wind regime, and their output powers vary according to the PDF of Fig. 4.1-(a). The loading level of the system is assumed to be 60% higher than the system’s nominal loading condition, to creates a challenging case for system’s VS. The thermal limits of the lines are taken to be 50% higher than their original values to be able to accommodate the mentioned loading level.

The weakest buses of the system, belonging to the first two weak modes identified by MA, for different wind power and load levels are presented in Table 4.1. As seen, both load and wind power variations can change the weakest buses and their order of weakness. The resulted set of candidate buses for ESS installation (C), i.e. the union of all weak buses at all wind power and load levels, is also shown in Table 4.1.

To determine the maximum loading level the system can accommodate without ESS, the system’s maximum loadability (\( \lambda^{max} \)) at different wind power and loading levels were computed. In order to find the value of \( \lambda^{max} \) at each wind power and loading level, loads and generations are incrementally scaled, in proportion to their nominal values, as expressed in (4.15). A unity PF is
4.3 Simulation Results of the Probabilistic ESS Allocation

Figure 4.3: Existing maximum loadability ($\lambda^{\text{max}}$) without ESS, versus the desired loadability ($\lambda^* = 1.1 \times \lambda$)

considered for the wind farms at this stage.

$$P_{i\omega \lambda}^{\text{g,}\lambda} = \alpha P_{i\omega \lambda}^{\text{g,}\lambda^0}, \quad P_{i\lambda}^{\text{d,}\lambda} = \alpha P_{i\lambda}^{\text{d,}\lambda^0}, \quad Q_{i\lambda}^{\text{d,}\lambda} = \alpha Q_{i\lambda}^{\text{d,}\lambda^0}$$

(4.15)

The smallest value of $\alpha$ at which any of the system’s constraints (bus voltage, thermal, voltage stability) are violated, determines the maximum loadability ($\lambda^{\text{max}} = \alpha^{\text{max}} \times \lambda^0$) corresponding to that wind power and loading level. The obtained values of $\alpha^{\text{max}}$ are shown in Fig. 4.3, using colour bars (with bold lines). It is observed that by increasing the wind power level, the system’s loadability increases, due to the decrease in the line power flows, and thus in the voltage drops. Fig. 4.3 also shows the surface of the desired loadability level (white bars), for $\alpha^* = 1.1$, indicating 10% desired VSM. As seen, for a wide range of high wind powers and low loading levels the system’s existing $\lambda^{\text{max}}$ exceeds the desired $\lambda^*$. However, at high loading and low wind power levels, the system’s existing VSM is below the desired one (The colour bars fall below the white bars in Fig. 4.3). This indicates that the installed ESS devices should discharge power into the system at these wind-load combinations, such that the desired VSM is achieved for the system. The obtained results of ESS allocation problem (case1) will confirm these initial analysis.

The optimization problem was formulated in GAMS software and solved using SNOPT (NLP) solver. To evaluate the effects of different factors being considered in the problem, the following
cases are studied:

**Case 1:** In this case, ESS installation with unity PF is considered to achieve a 10% VSM. Neither the ESSs nor the wind farms have RPC and 10% EIC. The results of this case and all the subsequent cases, are presented in Table 4.2. The desired VSM in each case is denoted by $VSM^*$ in the table. It should be noted that as the R/X ratio in transmission systems is typically low (0.38 in average for this system), the effect of reactive power compensation on upgrading the bus voltages is considerably higher than that of active power. Thus, as seen in Table 4.2, in this case a large amount of 49.6 MW ESS is installed at buses 19, 24, 26, and 30. The expected total power injection by the installed ESSs is, however, much smaller (4.7 MW), since full power injection is not required by the ESSs at all wind-load combinations, to achieve the desired $VSM^*$.

Figure 4.4 shows the active power injection by the ESS installed at bus 30, for example. It is seen that at all wind-load combinations at which the system’s existing loadability is lower than the desired $VSM^*$ (10%), as is shown in Fig. 4.3, the ESS at bus 30 has injected power into the system, which verifies the previous analysis in this regard. Also, it is seen that the amount of power injection increases as the loading level increases and wind level decreases, to achieve the same desired $VSM^*$.

**Case 2:** In this case, the RPC of wind farms (only), with $PF_{min}^w = 0.95$ as expressed in equation (4.14), is taken into consideration. As seen in Table 4.2, compared to case 1, RPC of wind farms is not effective in decreasing the required ESS installation. The reason is that the ESSs’ maximum power injections (which determine their sizes) occurs at zero wind power level
Table 4.2: ESS allocation results for different cases

<table>
<thead>
<tr>
<th>case</th>
<th>VSM*</th>
<th>Additional Flexibilities</th>
<th>Installed ESS size per bus (MW)</th>
<th>Expected $\sum P_{ess}^C$ (MW)</th>
<th>Total Installed ESS (MW)</th>
<th>Objective function</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10%</td>
<td>-</td>
<td>4.7 - 18.4 - 17.7</td>
<td>4.7</td>
<td>49.6</td>
<td>54.3</td>
</tr>
<tr>
<td>2</td>
<td>10%</td>
<td>RPC</td>
<td>3.4</td>
<td>49.6</td>
<td>50.6</td>
<td>53.1</td>
</tr>
<tr>
<td>3</td>
<td>10%</td>
<td>RPC, RPCs</td>
<td>1.5</td>
<td>34.5</td>
<td>37.3</td>
<td>26.8</td>
</tr>
<tr>
<td>4</td>
<td>10%</td>
<td>RPC, RPCs, RPCs, EIC</td>
<td>0.3</td>
<td>24.9</td>
<td>26.8</td>
<td>41.6</td>
</tr>
<tr>
<td>5</td>
<td>20%</td>
<td>RPC, RPCs, RPCs, EIC</td>
<td>6.3</td>
<td>21.9</td>
<td>8.9</td>
<td>26.8</td>
</tr>
<tr>
<td>6</td>
<td>20%</td>
<td>RPC, RPCs, RPCs, EIC</td>
<td>6.3</td>
<td>14.7</td>
<td>6.3</td>
<td>8.9</td>
</tr>
</tbody>
</table>

(see Fig. 4.4), where wind farms’ reactive power injection is zero too (due to the minimum PF constraint of equation (4.14)). Thus, the RPC of wind farms cannot reduce the required ESS sizes. However, the expected total power injection by ESSs has significantly reduced (from 4.7 to 3.4 MW) compared to case1. The reason is that due to the reactive power injection by wind farms at different wind levels, less active power injection is required by the ESSs to provide the desired VSM*. This results in smaller required energy capacity of ESSs.

**Case3:** In this case, the RPC of ESSs, with $P_{ess}^{min} = 0.95$ as expressed in equation (4.12), is added to the problem. As seen in Table 4.2, RPC of ESSs has significantly decreased the required ESS installation, compared to the previous cases (from 49.6 to 34.5 MW). The expected total power injection by the ESSs has also decreased to 2.8 MW. The reason, apart from the RPC of ESSs, is the optimal placement of the ESSs at the weak buses, where reactive power compensation is most effective. Therefore, the RPC of ESSs directly improves the voltage levels of the weak buses, which leads to smaller required ESS sizes.

**Case4:** In this case, all ESSs and wind farms are allowed to have RPC (with $P_{ess}^{min} = P_{w}^{min} = 0.95$) and 10% EIC. It is seen from Table 4.2 that the least ESS installations and expected power injections by ESSs, compared to all previous cases, are obtained in this case. The reason, apart from the RPC of ESSs and wind farms, lies in the 46% reactive power capabilities of ESSs and wind farms when their full active power ratings are utilized, which would not exist in the absence of 10% EIC. It is worth mentioning that by increasing the apparent power rating of inverters by only 10%, the required ESS installation in this case has been decreased by 28% (from 34.5 to 24.9 MW), compared to case3.

**Case5 and case6:** Cases 5 and 6 are similar to cases 1 and 4, respectively, with the difference that the desired VSM* is set to 20% in these cases. As seen from Table 4.2, the required ESS
installation and expected total power injection by ESSs has noticeably increased compared to their corresponding cases. However, the increase in the ESSs’ expected power injections is higher than that of ESS installation capacity. For example, in case5 the total ESS installation has increased by 58% (from 49.6 to 78.5 MW) compared to case1, due to the higher required power injections by ESSs to achieve 20% VSM. It is while the expected total power injections by ESSs has increased by 89% (from 4.7 to 8.9 MW). The reason for this higher increase (compared to that of ESS installation) is that in this case there are more wind-load combinations at which the system lacks sufficient VSM and thus, the ESSs need to inject power into the system. This fact is shown in Fig. 4.5 for case5. Thus, it can be concluded that by increasing the desired VSM\(^*\), the required energy capacity of ESSs increases more significantly compared to their active power ratings.

### 4.4 Risk-Based Stochastic ESS Allocation

This section presents a new stochastic formulation for optimal ESS allocation in a distribution power system to ensure any desired VSM. Both required power and energy ratings of ESSs are computed. To be able to accurately formulate the ESS allocation problem, thorough comprehension of the ESS installation purpose is essential. Generally, available wind energy is not synchronized with load profile. Wind is normally more abundant during off-peak night periods and more scarce during on-peak day periods [43]. Wind also undergoes different seasonal variations. For instance, it blows the least during the summer days, when the load is usually at its highest level. In winter, however, the wind speed is generally high, especially during the night time, while the
load is relatively low. From the steady-state VS viewpoint, summer days are considered the worst cases, in which high demand and low wind push the system towards its VS limit. Accordingly, in this section, the ESSs are operated as per the following scheme:

- ESSs are charged during the high-wind, low-load (usually off-peak night) periods, when the system’s VSM is higher than the desired level. As a side benefit, the large reverse power flows during high-wind, low-load periods is mitigated and reactive power loss/import is reduced. The ESSs may be charged by either the excess of the wind energy, which would be curtailed in the absence of ESSs to keep within the system limits [43, 45], or by the power utility, through the substation bus. Notice that ESSs act as loads when being recharged, and thus, degrade the system’s VSM. Hence, the ESSs’ charging powers should be adjusted such that the system’s VSM remains above the desired level.

- ESSs are discharged during the low-wind, high-load periods, when the system’s VSM is below the desired level. The required discharging power and energy depend on the desired VSM. The stored energy into the ESSs during the charging periods should be sufficient to accommodate the required amount of discharged energy over high-load periods. As a side benefit, the lost/imported reactive powers are reduced. Notice that the charging/discharging periods of ESSs are decided by the model, and are not imposed to the problem.

4.4.1 Wind Power Uncertainty Modelling Through Scenarios

Wind power uncertainty can be modelled through scenarios, evolving in time, in which the wind forecasting error is accounted for. The number of scenarios is a matter of compromise between computational burden and accuracy in modelling the wind’s uncertainty. In this thesis, each scenario spans 24 hours. Scenarios are assigned corresponding probabilities, adding up to one. Auto Regressive Integrated Moving Average (ARIMA)-based scenario generation [90], and fast-forward scenario reduction [91] approaches are used here to generate an optimal set of scenarios. The main steps of the scenario generation approach used in this section are as follows:

1. Wind speed, denoted by stochastic process $Y$, normally follows Weibull distribution. Hence, to be able to use the modelling capability of ARIMA, which is based on Gaussian (Normal) distribution assumption, define a new Gaussian stochastic process through the trans-
formation $Z = \Phi^{-1}[F_Y(Y)]$, where $\Phi$ and $F_Y$ are the cumulative distribution functions of standard normal random variable, $N(0, \sigma)$, and the wind speed stochastic process $Y$, respectively [90].

2. Using the transformed historical wind data, generate an appropriate ARIMA $(p, d, q)$ model as described in equation (4.16):

$$y_t = \sum_{j=1}^{p} \phi_j y_{t-j} + \epsilon_t - \sum_{j=1}^{q} \theta_j \epsilon_{t-j}$$

where $\phi_j$ and $\theta_j$ are auto-regressive and moving-average parameters, respectively, and $\epsilon_t$ is the forecast error at time $t$, also referred to as white noise.

3. Generate a large number of scenarios, say 1000, over the 24 hours using the obtained ARIMA parameters and repeated generation of white noises $\epsilon_t \sim N(0, \sigma)$ [90] to incorporate the wind speed forecasting error in the problem.

4. Apply fast-forward scenario reduction approach of reference [91] to optimally find a subset of selected scenarios $\Omega_s$, with the desired cardinality and assigned probability to each scenario, such that the Kantorovich distance between the original and reduced sets of scenarios is minimized.

5. To obtain the scenarios characterizing the original stochastic process of wind speed, $Y$, apply the inverse normal transformation, $Y = F_Y^{-1}[\Phi(Z)]$, on the set of the obtained reduced set of scenarios.

6. Use the power-wind speed (or $P_w - \nu$) characteristic of the wind turbine as described in equation (4.17) [78] to generate the scenarios for the output power of the wind farms.

$$P_w = \begin{cases} 
0 & \nu \leq \nu_c \text{ or } \nu \geq \nu_{\text{lim}} \\
\frac{\nu - \nu_c}{\nu_r - \nu_c} P_r & \nu_c \leq \nu \leq \nu_r \\
P_r & \nu_r \leq \nu \leq \nu_{\text{lim}} 
\end{cases}$$

In equation (4.17), $P_r$ is the turbine’s rated power, $\nu_c$ is the cut-in speed, $\nu_r$ is the rated wind speed, and $\nu_{\text{lim}}$ is the limiting wind speed.
4.4 Risk-Based Stochastic ESS Allocation

4.4.2 Modelling the OLTCs’ Action

A new approach is proposed here to model the transformer’s OLTC in the OPF formulation. The method is suitable for both BFM and BIM of OPF problem, and is based on modelling the secondary side of the OLTC as a generator bus with controlled voltage magnitude and phase angle, as illustrated in Fig. 4.6. The line $ij$ with an OLTC at the end, can be divided into two sections $im$ and $mj$, in which the section $mj$ includes an ideal OLTC, and the impedances of the transformer and branch are included in section $im$ [84, 86]. The bus $m$ is called OLTC auxiliary bus in this section. As seen, the system is split into two parts from the two ends of the OLTC. These two parts should be linked such that the same load flow results as the original system are obtained. This is achieved through adding a fictitious load and generator at buses $m$ and $j$, respectively, as seen in Fig. 4.6. The active and reactive loads at bus $m$ are set to be equal to those generated by the generator at bus $j$, which is treated like a slack generator (its power generations is not pre-specified and is computed by the power balance law at bus $j$). On the other hand, the voltage magnitude of the generator at bus $j$ must be equal to that of bus $m$ divided by the tap ratio, $\tau$. The voltage phase angles of both sides of the OLTC are also set to be the same. In the optimization problem, the OLTCs’ tap positions are considered to be continuous, to avoid using integer variables. Hence, a post-optimization adjustment needs to be made, as in references [45] [92], to round the tap positions to the nearest feasible discrete value. Since OLTCs can provide up to 33 tap/voltage steps, each voltage step being as small as 0.0065 p.u., the effect of this adjustment on the bus voltages or branch flows, and thus on the feasibility of the obtained solution, is virtually negligible. The accuracy of the proposed OLTC model has been verified through comparison with Matpower power flow tool, and is demonstrated through simulation studies.

Figure 4.6: The Proposed Method of OLTC Modelling
4.4.3 Stochastic Optimization Problem for ESS Allocation

The stochastic ESS allocation problem involves two sets of decision variables. The first, scenario-independent, set is decided on before knowing the actual realizations of the wind speed, and is called planning or here-and-now decision set (locations and sizes of ESSs). The second, scenario-dependent, set includes variables which are decided on when the actual wind powers are gradually realized, and is named operational or wait-and-see decision set (ESSs’ power, energy, and power factor (PF), wind farm’s PF, and OLTCs’ tap positions) [90]. The realizations of the wind power are modelled through the generated scenarios as discussed in Section 4.4.1.

**Remark:** Let \( X_{i,s} \) indicate the value of the variable \( X \) at bus \( i \in I \) (set of all buses), scenario \( s \in S \) (set of all wind scenarios), time \( t \in T \) (set of all hours in a day), and day \( d \in D \) (set of all considered days, i.e. one day per season).

- **Objective Function**

The objective function (OF) of the problem, to be minimized, consists of four terms, as per equation (4.18):

\[
\min_{S_i^r, E_i^r, P_{Ch}^{istd}, P_{Dis}^{istd}, \theta_{ESS}^{istd}, \theta_w^{istd}, \tau_{oltc}^{istd}} \left[ C_S \sum_{i \in C} S_i^r + C_E \sum_{i \in E} E_i^r + 90 \times C_Q \sum_{s \in S} \pi_s \sum_{t \in T, d \in D} Q_{loss}^{s, std} + 90 \times C_Q \sum_{s \in S} \pi_s \sum_{t \in T, d \in D} Q_{loss}^{s, std} \right]
\]

(4.18)

The first two terms indicate the total costs of power and energy of ESSs. \( S_i^r \) and \( E_i^r \) are, respectively, the rated apparent power and rated energy capacity of the ESS installed at bus \( i \). \( C_S \) and \( C_E \) are, respectively, the annualized normalized costs of apparent power and energy ratings of the ESSs, and are equal to \( C_E = 1 \) and \( C_S = 0.67 \) [45] (assuming Lithium-Ion battery with a lifetime of 10 years [47]). \( \pi_s \) is the associated probability of scenario \( s \).

\( C \) is the set of candidate buses for ESS placement. The candidate buses are determined from a VS viewpoint, using the proposed MA\(^{PQ}\) approach of Chapter 2, with the parameter \( K \) set to be 0.707. It is shown in Chapter 2 that the identified buses by MA\(^{PQ}\) are the most effective buses to apply active/reactive compensation, in order to improve the distribution system’s VSM. The
4.4 Risk-Based Stochastic ESS Allocation

The weakest buses of the first two weak modes of the test system, identified by using MA\(^{PQ}\) at all hours, are used as candidate buses for ESS installation.

The third term of the OF is the expected total reactive power loss (denoted by \(Q_{\text{loss}}\)). Reducing \(Q_{\text{loss}}\) is known to improve the system’s VS, which is linked with the system’s ability to transfer reactive power to the loads [1]. The fourth term of the OF minimizes the expected imported reactive power from the substation bus (denoted by \(Q_{\text{ss}}\)). Reducing \(Q_{\text{ss}}\) contributes to the VS improvement of the upstream transmission grid, by reducing the reactive power transfer through long distances in the transmission network [93]. The last two terms of the OF are weighted by the term \(90 \times C_Q\), where 90 is the number of days in a season, and \(C_Q\) is the average reactive power price for a typical distribution systems, i.e. 10 $/MVAr-h [94], normalized with the same ratio as \(C_E\) and \(C_S\) are normalized by.

Even though active power loss (\(P_{\text{loss}}\)) can also be added to the OF, only \(Q_{\text{loss}}\) has been included here, since (i) it is more effective on VS, and (ii) reducing \(Q_{\text{loss}}\) directly leads to a reduction in \(P_{\text{loss}}\) too. Although considering the ESS’s energy arbitrage with the grid is beyond the scope of this chapter, the proposed method of ESS operation leads to the profitable practice of charging over low-load periods, and discharging over high-load periods, as shown in the simulation results.

- System Constraints

Active and reactive power balances are the problem’s main equality constraints, as per equations (4.19) and (4.20). These equations ensure that the net active/reactive power generation at each bus (i.e. power generation minus power demand, denoted in the left hand sides of equations (4.19) and (4.20)) should be equal to the amount of active/reactive power injected into the distribution grid. The latter is ensured using the Kirchhoff’s circuit laws, as per the right hand sides of equations (4.19) and (4.20). These equality constraints must hold at any time and under all wind scenarios.

\[
p_p^{\text{grid}} + p_w^{\text{grid}} + p_{\text{Dis}}^{\text{grid}} - p_{\text{Ch}}^{\text{grid}} - p_{\text{oltc}}^{\text{grid}} = |V_{\text{grid}}| \sum_{j \in I} |V_{j\text{std}}| (G_{ij} \cos \delta_{ij\text{std}} + B_{ij} \sin \delta_{ij\text{std}}) \quad \forall i, s, t, d \tag{4.19}
\]

\[
Q_q^{\text{grid}} + p_w^{\text{grid}} \tan \theta_{\text{grid}}^{\text{grid}} + (p_{\text{Dis}}^{\text{grid}} - p_{\text{Ch}}^{\text{grid}}) \tan \theta_{\text{ESS}}^{\text{grid}} - Q_{\text{grid}}^{D} - Q_{\text{grid}}^{\text{oltc}} = |V_{\text{grid}}| \sum_{j \in I} |V_{j\text{std}}| (G_{ij} \sin \delta_{ij\text{std}} - B_{ij} \cos \delta_{ij\text{std}}) \quad \forall i, s, t, d \tag{4.20}
\]
where $|V_{\text{istd}}|$ and $\delta_{\text{istd}}$ are the voltage magnitudes and phase angles, respectively, and $\delta_{ij,\text{std}} = \delta_{\text{istd}} - \delta_{j,\text{std}}$. $B_{ij}$ and $G_{ij}$ are real and imaginary parts of the $ij^{\text{th}}$ element of the admittance matrix. $P_{\text{istd}}^g$ and $Q_{\text{istd}}^g$ are the active and reactive power generations, and correspond to two groups of variables: (i) the imported powers from the substation bus, $ss$, which is considered as the slack bus ($\delta_{ss,\text{std}} = 0$), (ii) the fictitious power generations at the buses after OLTC (denoted by $j$ in Fig. 4.6), as explained in Section 4.4.2. Similarly, $P_{\text{oltc}}^g$ and $Q_{\text{oltc}}^g$ are the fictitious active and reactive power loads at the OLTC auxiliary buses (bus $m$ in Fig. 4.6). $P_{\text{Ditd}}^D$ and $Q_{\text{Ditd}}^D$ are the active and reactive power demands. The same load profile is considered for all wind scenarios of a typical day of a season (see Section 4.5.1). Also, $P_{\text{wistd}}^\text{w}$ is the wind farm’s active power generation, obtained from the scenario generation scheme of Section 4.4.1. $P_{\text{Chitd}}^\text{Ch}$ and $P_{\text{Disitd}}^\text{Dis}$ are, respectively, the charging and discharging active powers of ESS devices, and $\theta_{\text{ESSitd}}^\text{ESS}$ and $\theta_{\text{wistd}}^\text{w}$ are the PF angles of the ESSs and wind farms.

Other operational constraints, including bus voltage and thermal constraints, are as shown in equations (4.21) and (4.22), respectively. Equation (4.21) ensures that the stationary voltages at each bus of the power system are within the acceptable operational range, i.e. between 0.94 p.u. and 1.1 p.u. Also, equation (4.22) constrains the apparent power flow through distribution lines within their corresponding thermal limits, being essential to avoid premature ageing of the lines.

$$
0.94 \leq |V_{\text{istd}}| \leq 1.1 \quad \forall i \notin T^\text{oltc}_1, s, t, d \quad (4.21)
$$

$$
\left( G^2_{ij} + B^2_{ij} \right) \left( |V_{\text{istd}}|^2 + |V_{j,\text{std}}|^2 - 2 |V_{\text{istd}}| |V_{j,\text{std}}| \cos \delta_{ij,\text{std}} \right) = I^2_{ij,\text{std}} \leq \left( I_{ij}^\text{max} \right)^2 \quad \forall i, j, s, t, d \quad (4.22)
$$

where $I_{ij,\text{std}}$ and $I_{ij}^\text{max}$ are, the current flow and thermal limit of the branch $ij$. The limits on the substation bus’s power transfer capability, ($p_{\text{ss,istd}}^\text{min}$ and $p_{\text{ss,istd}}^\text{max}$), and the slack bus constraints are as shown in (4.23) and (4.24), respectively:

$$
p_{\text{ss,istd}}^\text{min} \leq p_{\text{ss,istd}}^g \leq p_{\text{ss,istd}}^\text{max} \quad \forall s, t, d \quad (4.23)
$$

$$
\delta_{ss,\text{std}} = 0, \quad |V_{ss,\text{std}}| = V_{ss} \quad \forall s, t, d \quad (4.24)
$$
4.4 Risk-Based Stochastic ESS Allocation

- OLTC Constraints

As mentioned in Section 4.4.2, for each OLTC, an auxiliary bus is added to the system at the end of the associated transmission line and just before the OLTC. Let us denote the set of these auxiliary buses with \( I_{\text{oltc}} \subset \mathcal{I} \). Also, denote the set of buses after OLTCs with \( I_{\text{oltc}}^2 \subset \mathcal{I} \). Accordingly, as explained in Section 4.4.2, the following constraints describe the OLTCs’ action:

\[
\begin{align*}
\forall i \in I_{\text{oltc}}^1, s, t, d: \\
\left| V_{u_i \text{ std}} \right| &= \left| V_{\text{istd}} \right| / \tau_{\text{oltc}} \\
\delta_{u_i \text{ std}} &= \delta_{\text{istd}} \\
\tau_{\text{oltc}}^\text{min} &\leq \tau_{\text{oltc}} \leq \tau_{\text{oltc}}^\text{max}
\end{align*}
\]

where \( u_i \) maps the bus \( i \in I_{\text{oltc}}^1 \) (the bus before OLTC) to its associated bus \( u_i \in I_{\text{oltc}}^2 \) (the bus after OLTC). \( \tau_{\text{oltc}} \) is the tap ratio of the OLTC before bus \( i \), and \( \tau_{\text{oltc}}^\text{min} \) and \( \tau_{\text{oltc}}^\text{max} \) are the minimum and maximum limits of OLTCs’ tap position. For all the buses without OLTC the following constraints apply:

\[
\begin{align*}
\forall i \notin I_{\text{oltc}}^1, s, t, d: \\
P_{\text{oltc}} = 0, & Q_{\text{oltc}} = 0 \\
P_{\text{oltc}}^\text{std} = 0, & Q_{\text{oltc}}^\text{std} = 0 \\
\tau_{\text{oltc}}^\text{std} = 1
\end{align*}
\]

- Risk-Based Voltage Stability Constraints

As mentioned in Section 4.2.3, simple constraints on bus voltages alone cannot guarantee VS of the distribution system, since voltage instability can occur even when all the bus voltages are within the normal range, particularly when the system is highly compensated with reactive power [1]. Hence, ensuring a secure level of VSM is of great value for distribution system operators. A minimum VSM guarantees that the distribution system can be safely loaded up to a certain level, without violating any of the system constraints or experiencing (potential) voltage instability. In this section, a desired VSM is ensured through risk-based VSM constraints, which consist of two main sets of constraints as follows:
• As a main goal, the existence of the operating point at the desired maximum (or critical) loading level, denoted by $\lambda_{td}^*$, is ensured, through deterministic equality constraints. The value of $\lambda_{td}^*$ is considered to be $\alpha^* \times \lambda_{td}^0$, where $\lambda_{td}^0$ defines the base loading level at hour $t$ of day $d$. For instance, $\alpha^* = 1.2$ indicates 20% VSM. Accordingly, the VSM deterministic constraints are obtained by replacing $P_{itd}^D$ and $Q_{itd}^D$ in equations (4.19) and (4.20), with $\alpha^* P_{itd}^D$ and $\alpha^* Q_{itd}^D$, respectively. These constraints are not re-written for brevity.

• While ensuring the existence of a stable operating point at the desired $\lambda_{td}^*$ is crucial, it is also important to account for the operational (bus voltage and thermal) constraints at $\lambda_{td}^*$, to an acceptable extent. The operational constraints at $\lambda_{td}^*$ are fulfilled here through risk-based probabilistic constraints [95]. These constraints ensure that the operational limits at $\lambda_{td}^*$ are not exceeded more than a pre-determined risk level, $\mu$. The VSM probabilistic constraints are presented in equations (4.31) and (4.32) for bus voltage and thermal limits, respectively:

$$\sum_{s \in S} \pi_s \Delta V^\lambda_{istd} \leq \mu \quad \forall i, t, d \tag{4.31}$$

$$\sum_{s \in S} \pi_s \Delta I^\lambda_{ij,std} \leq \mu \quad \forall i, t, d \tag{4.32}$$

where $\Delta V^\lambda_{istd}$ and $\Delta I^\lambda_{ij,std}$ are the voltage violation and percentage thermal violation at $\lambda_{td}^*$, as per equations (4.33) and (4.34), respectively:

$$\Delta V^\lambda_{istd} = \max \left( V^\lambda_{istd} - 1.1 , \ 0 \right) + \max \left( 0.94 - V^\lambda_{istd} , \ 0 \right) \quad \forall i, s, t, d \tag{4.33}$$

$$\Delta I^\lambda_{ij,std} = \max \left( \left( I^\lambda_{ij,std} - I_{ij}^{max} \right) / I_{ij}^{max} , \ 0 \right) \quad \forall i, s, t, d \tag{4.34}$$

The variables with superscript $\lambda^*$ represent the operating point of $\lambda_{td}^*$. Similar (deterministic) constraints to equations (4.23)-(4.30) are also applied at $\lambda_{td}^*$, which are not included here for brevity.

The above-mentioned VSM constraints guarantee that at any given time, the system not only is able to safely supply the existing loading level, $\lambda_{td}^0$, but can also securely accommodate any sudden increase in the loading level up to $\lambda_{td}^* = \alpha^* \times \lambda_{td}^0$, without experiencing voltage collapse (if is an issue) or exceeding the operational constraints more than the pre-defined risk level, $\mu$. 
4.4 Risk-Based Stochastic ESS Allocation

- ESS Devices Constraints

As mentioned, the RPC of ESS devices are modelled in this chapter as additional flexibilities to achieve smaller ESS sizes. The active and apparent power injections/absorptions of the ESSs are constrained by the corresponding rated apparent power of the ESS at each bus $i$, $S'_i$, as expressed in equations (4.35) and (4.36), respectively. These constraints ensure that the maximum thermal capability of the inverter is not violated. Constraint (4.37) ensures no ESS installation at non-candidate buses, since only candidate buses are to be selected for ESS installation. Also, constraint (4.38) limits the PF of the ESSs between $PF^{ESS}_{\min}$ leading and lagging. As mentioned before, the aim of this constraint is to allocate most of the inverters’ thermal capability to active, rather than reactive, power transfer, depending on the acceptable PF limits.

\[
0 \leq P^\text{Dis}_{tstd} + P^\text{Ch}_{tstd} \leq S'_i \quad \forall i \in \mathcal{C}, s, t, d \tag{4.35}
\]

\[
\left(P^\text{Dis}_{tstd} + P^\text{Ch}_{tstd}\right)^2 \left(1 + \tan^2 \theta^\text{ESS}_{tstd}\right) \leq S'^2_i \quad \forall i \in \mathcal{C}, s, t, d \tag{4.36}
\]

\[
S'_i = 0 \quad \forall i \notin \mathcal{C} \tag{4.37}
\]

\[-\acos\left(PF^\text{ESS}_{\min}\right) \leq \theta^\text{ESS}_{tstd} \leq \acos\left(PF^\text{ESS}_{\min}\right) \quad \forall i \in \mathcal{C}, s, t, d \tag{4.38}
\]

In order to prevent simultaneous charging and discharging of ESSs, which leads to overstatement of ESSs’ power conversion loss [45], constraint (4.39) is added to the problem formulation:

\[
P^{\text{Dis}}_{tstd} \times P^{\text{Ch}}_{tstd} = 0 \quad \forall i \in \mathcal{C}, s, t, d \tag{4.39}
\]

Notice that the charging and discharging periods of ESSs are not imposed to the problem, and are decided by the model. Besides, no binary variable is required to determine ESS locations, which improves the method’s scalability [45].

In order to incorporate computation of the required energy capacity of the installed ESS devices, the energy conversion losses of the ESSs need to be taken into account. This is done through the inter-temporal constraint of (4.40), in which $E^\text{ESS}_{tstd}$ is the stored energy in the ESS devices. $\Delta t$ is the time step duration, which is considered to be 1 hour in this chapter, and $\eta_{ch}$ and $\eta_{dis}$ are, respectively, the charging and discharging efficiencies of ESS devices. Equation (4.41) constrains the stored energy level of ESSs by their corresponding maximum depth of discharge, $DoD^m$, and
rated energy capacity, $E^r_i$.

$$E^{ESS}_{istd} = E^{ESS}_{istd, t-1} + P^{Ch}_{istd} \times \eta_{ch} \times \Delta t - P^{Dis}_{istd, \Delta t} \times \eta_{dis} \times \Delta t \quad \forall i \in C, s, t, d \quad (4.40)$$

$$(1 - DoD^w)E^{R,ESS}_{i, \Delta t} \leq E^{ESS}_{istd} \leq E^r_i \quad \forall i \in C, s, t, d \quad (4.41)$$

The initial energy level of ESSs, $E^{ESS}_{istd, 0, \Delta t}$, is considered to be at the minimum (worst case) level, as stated in equation (4.42):

$$E^{ESS}_{istd, 0, \Delta t} = (1 - DoD^w) \times E^r_i \quad \forall i \in C, s, d \quad (4.42)$$

### - Wind Farms’ Constraints

The wind farms in this section are considered to be non-dispatchable, i.e. all available wind power generation is either injected into the system or stored into the ESSs (or both). This assumption, i.e. eliminating the need for wind power curtailment, has been the main goal of references [43] and [45]. In addition, inverter-interfaced wind turbines (types 3 and 4), which offer controllable RPC, are considered. The RPCs of wind turbines are limited by the wind turbines’ rated active power, $P^{r,w}_i$, and minimum permissible PF, $PF_{min}^w$, as per equations (4.43) and (4.44), where $W \subset I$ is the set of all wind farm buses. Equation (4.43) ensures that apparent power flow through the wind farms’ inverters does not violate their corresponding thermal limits. Besides, equation (4.44) is applied to utilize the inverters’ thermal capability mainly for active, rather than reactive, power injection.

$$\left( P^{aw}_i \right)^2 (1 + tan^2 \theta^{aw}_{istd}) \leq \left( P^{r,aw}_i \right)^2 \quad \forall i \in W, s, t, d \quad (4.43)$$

$$-acos \left( PF_{min}^w \right) \leq \theta^{aw}_{istd} \leq acos \left( PF_{min}^w \right) \quad \forall i \in W, s, t, d \quad (4.44)$$
4.5 Simulation Results of the Risk-Based Stochastic ESS Allocation

4.5.1 Case Study

The proposed ESS allocation approach has been tested on a typical 41-bus, 27.6 KV, rural distribution system, with the peak demand of 16.18 MVA [10], shown in Fig. 4.7. Three wind farms are connected at buses 19, 28, and 40, with respective power ratings of 8.5 MW, 4 MW, and 10.3 MW [10]. The system has 4 OLTCs installed before buses 8, 16, 21, and 41. The voltage magnitude at the substation bus is 1.025 p.u.

The $P_{w} - \nu$ curve parameters of the wind farms, as expressed in equation (4.17), are $\nu_c = 4 \text{ (m/s)}$, $\nu_r = 14 \text{ (m/s)}$, and $\nu_{lim} = 24 \text{ (m/s)}$ [43]. Five-year historical data of wind [96, 97] for the system under study is used to generate an ARIMA model for each season, using the forecast package of R software. For example, for the summer day with the minimum average wind speed, which is August 2nd, the ARIMA(2,0,2) is obtained, as shown in equation (4.45):

$$y_t = 1.528 y_{t-1} - 0.558 y_{t-2} + \epsilon_t - 0.249 \epsilon_{t-1} + 0.124 \epsilon_{t-2} \quad (4.45)$$

Since the daily load profile in a season is not likely to vary too much [48], the mentioned day has the minimum average VSM, and thus VSM, throughout the summer, and is considered as a challenging case for the proposed method. The scenario generation approach of Section 4.4.1, with the ARIMA parameters as in equation (4.45), and a standard deviation of the forecasting
error term, \( N(0, \sigma) \), of \( \sigma = 0.2 \), has been used to generate a final (reduced) set of 6 scenarios for the mentioned summer day, as shown in Fig. 4.8. The same procedure has been used to generate 6 wind scenarios for the most critical day of each season.

The hourly load variation is assumed to follow the IEEE-RTS load profile [43], which provides the hourly load variation for a typical day of each season as a fraction of the system’s annual peak load, as seen in Fig. 4.9 (red curve) for a typical summer day. The peak loading level of the system is taken 45% higher than the original peak load, to create a challenging case for the system’s VS. The thermal limits of the lines have also been increased by 25% to be able to accommodate this loading level.

### 4.5.2 Numerical Results

**- System’s Preliminary Evaluation**

The weakest buses of the test system, belonging to the first two weak modes, identified by the proposed MA\(PQ\) of Chapter 2, for the summer day and for the 1st corresponding wind power scenario (which is an average scenario, as seen in Fig. 4.8), were calculated at all day hours. The resulted set of candidate buses for ESS installation, i.e. the union of all weak buses at all hours, is found to be as (4.46):

\[
C \subset \mathcal{I} = \{27, 28, 29, 30, 31, 35, 36, 37, 38, 39, 40, 41\} \tag{4.46}
\]

As mentioned in Chapter 3, the required VSM for the power systems operating under normal
operating condition is reported to be between 5-10% of their nominal loading level [98]. It means that the system should be able to be safely loaded up to 5-10% above its base loading level at any time without encountering any of the operational or stability limits. Hence, a 10% VSM criteria is used here as the base value; however, simulation results for 15% VSM are also provided (see Table 4.3).

To find out the maximum loading level the system can accommodate without ESS, the system’s maximum loadability, $\lambda_{td}^{\text{max}}$, at each hour of the summer day and for the 6th corresponding wind power scenario (which is more critical than the others), is computed and shown in Fig. 4.9. At this stage, the wind farms are operated at unity PF. In order to find the value of $\lambda_{td}^{\text{max}}$ at each hour $t$, loads are incrementally scaled in proportion to their initial values, as shown in equation (4.47), and the load increase is supplied by the substation bus.

\[
P^{D,\lambda}_{itd} = \alpha \times P^{D,\lambda}_0, \quad Q^{D,\lambda}_{itd} = \alpha \times Q^{D,\lambda}_0
\]  

(4.47)

The smallest value of $\alpha$ at which any of the system’s constraints (bus voltage, thermal, voltage stability) are violated, determines the maximum loadability corresponding to that hour of the considered day ($\lambda_{td}^{\text{max}} = \lambda_{td}^{\text{max}} \times \lambda^{0}_{td}$). Fig. 4.9 (blue curve) shows that during the first few hours of the summer day, when load is low and wind is high, the system’s $\lambda_{td}^{\text{max}}$ is higher than the required 10% VSM. However, between the hours 9 and 23, in which load is at the highest levels and wind power is low, the system’s VSM is lower than the required 10%. For example, at $t = 15\text{hr}$ the system’s loadability is only 0.79 %. This means that the system is unable to securely accommodate even its base loading level. The implication is that for the 6th wind scenario of the typical summer day, ESSs must be charged before $t = 9\text{hr}$ and discharged between hours $t = 9\text{hr}$ and $t = 23\text{hr}$. The simulation results presented in the following confirm this preliminary evaluation. The system’s total expected reactive power import, $Q_{\text{ess}}$, and $Q_{\text{loss}}$ (as in equation (4.18)) are initially 572.4 MVar and 100.4 MVar, respectively (over all hours and considered days).

### - Optimal ESS Allocation Results and Discussion

The proposed stochastic ESS allocation approach was formulated in GAMS software and solved by using CONOPT solver, on a Core-i7 CPU, 3.4 GHz computer. The round trip efficiency of the
Figure 4.9: Normalized load profile and wind power, and system’s existing VSM for the 6th scenario of the considered summer day, before ESS installation

Figure 4.10: Simulation results of case 1 for the 6th scenario of the summer day

ESSs is taken to be 85%, which implies $\eta_{ch} = \eta_{dis} = 0.92$. The value of $DoD^m$ is set to 0.8. Four days, representing the four seasons, and 6 wind power scenarios for each day are considered in the problem formulation.

To evaluate the effects of different considered factors, the following case studies are carried out, with the results presented in Table 4.3. The desired VSM (denoted by VSM*) and risk level ($\mu$) are also shown for each case. In the table, for brevity, the ANM tools are denoted by the following numbers: 1 for RPC of wind farms, 2 for RPC of ESSs, and 3 for OLTCs’ action.

**Case 1:** In this case, the ESS installation is carried out without applying ANM tools (i.e. $\theta_{ESS_{istd}} = \theta_{w_{istd}} = 0$, $\tau_{oltc_{istd}} = 1$). A 10% desired VSM and zero risk level, $\mu$, are considered. Notice that although the $R/X$ ratio of the system is relatively high (around 0.6), the effects of reactive...
### Table 4.3: ESS Placement Results for Different Case Studies

<table>
<thead>
<tr>
<th>Case</th>
<th>VSM</th>
<th>$\mu$</th>
<th>ANM tools</th>
<th>Installed ESS size per bus (MVA/MWh)</th>
<th>Total ESS (MVA/MWh)</th>
<th>Total Cost</th>
<th>Total $Q_{ESS}$</th>
<th>Total $Q_{ss}$</th>
<th>Exec. time</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10%</td>
<td>0</td>
<td>-</td>
<td>0.54/2.63, 0.23/0.79</td>
<td>1.43/6.72</td>
<td>6.92/38.9</td>
<td>13.9/73.0</td>
<td>82.3</td>
<td>53.5</td>
</tr>
<tr>
<td>2</td>
<td>10%</td>
<td>0</td>
<td>1</td>
<td>0.14/0.64</td>
<td>-</td>
<td></td>
<td></td>
<td>73.8</td>
<td>51.1</td>
</tr>
<tr>
<td>3</td>
<td>10%</td>
<td>0</td>
<td>1,2</td>
<td>0.56/2.6, 0.39/1.59</td>
<td>1.05/5.75</td>
<td>6.59/42.3</td>
<td>11.1/66.5</td>
<td>365.1</td>
<td>11</td>
</tr>
<tr>
<td>4</td>
<td>10%</td>
<td>0</td>
<td>1,2,3</td>
<td>0.39/1.59</td>
<td>-</td>
<td></td>
<td></td>
<td>279.0</td>
<td>13</td>
</tr>
<tr>
<td>5</td>
<td>10%</td>
<td>0.01</td>
<td>1,2,3</td>
<td>0.87/3.53</td>
<td>-</td>
<td></td>
<td></td>
<td>289.1</td>
<td>14</td>
</tr>
<tr>
<td>6</td>
<td>10%</td>
<td>0.02</td>
<td>1,2,3</td>
<td>0.56/2.6, 0.57/2.34</td>
<td>-</td>
<td></td>
<td></td>
<td>289.1</td>
<td>14</td>
</tr>
<tr>
<td>7</td>
<td>10%</td>
<td>0.02</td>
<td>1,2,3</td>
<td>-</td>
<td>-</td>
<td></td>
<td></td>
<td>289.1</td>
<td>14</td>
</tr>
<tr>
<td>8</td>
<td>15%</td>
<td>0</td>
<td>1,2,3</td>
<td>1.39/7.61</td>
<td>-</td>
<td></td>
<td></td>
<td>289.1</td>
<td>14</td>
</tr>
<tr>
<td>9</td>
<td>15%</td>
<td>0.02</td>
<td>-</td>
<td>1.76/9.24</td>
<td>0.29/1.70</td>
<td></td>
<td></td>
<td>289.1</td>
<td>14</td>
</tr>
</tbody>
</table>
power injection on bus voltages and reactive power losses is still larger than those of active power. Thus, in case 1 with no RPC of ESSs and wind farms, the total ESS capacity to be installed is a large 13.9 MVA. Figure 4.10-(a) shows the charging/discharging powers and energy levels of four larger ESSs installed in this case. The results relate to the 6th scenario of the summer day. Comparing these results with those shown in Fig. 4.9 confirms the outcome of Section 4.5.2 in respect of charging and discharging periods of the installed ESSs. The voltage of bus 41 (the weakest bus) and the current flow through line 1–2 (the mostly loaded line) are also shown in Fig. 4.10-(b), both at $\lambda_{td}^* = 1.1 \times \lambda_{td}^0$. As seen, the voltage of bus 41 (red curve) is at the minimum level (0.94 p.u.) at almost all of the day hours, indicating that in this case the bus voltage limits prevail in determining the system’s VSM. It is worth reminding that ESSs act like loads during the charging periods and cause a decrease in the system’s VSM. However, the amounts of ESSs’ charging powers are adjusted such that the system’s VSM is always kept above the desired 10% level. Figure 4.10-(a) also shows that the power ratings of ESSs are determined during the charging period. It is so because in this case the charging period is shorter than the discharging period, as the system is heavily loaded, with insufficient VSM most of the time. Thus, the installed ESSs need to be charged more quickly to store sufficient energy to discharge into the system during the high-load periods.

Figure 4.11 shows more results for this case. Figs. 4.11-(a) and 4.11-(b), respectively, show that both $Q_{ss}$ and $Q_{loss}$ are decreased by using ESS, during both charging and discharging periods (see Fig. 4.11-(c)). During the charging periods, high wind and low load cause large reverse power flows in the system, which lead to large amounts of $Q_{loss}$ (Fig. 4.11-(b)) and $Q_{ss}$. Charging of ESSs with high power magnitude (Fig. 4.11-(c)) greatly mitigates the reverse power flow and reduces both $Q_{loss}$ and $Q_{ss}$, by around 66% and 30% (in average), respectively. During the discharging hours, ESSs reduce $Q_{loss}$ and $Q_{ss}$ by decreasing the active power flow in the system (Notice that in this case ESSs have no RPC.). The decreasing effect is, however, less noticeable, compared to the charging periods, due to the smaller discharging power magnitudes (Fig. 4.11-(c)). Figure 4.11-(c) also shows that ESSs take one charge/discharge cycle each day, without being imposed to the model.

**Case 2:** Wind farms’ RPC, with $PF_{w_{min}}^{eq} = 0.95$ as stated in equation (4.44), is added in this case, compared to case 1. Table 4.3 shows that the ESS installation cost decreases by 10% from...
4.5 Simulation Results of the Risk-Based Stochastic ESS Allocation

82.3 to 73.8 unit cost), in comparison with case 1. Also, $Q_{ss}$ has considerably been decreased (30%) compared to case 1, owing to the distributed reactive power supply by the wind farms. Figure 4.12-(a) shows $Q_{ss}$ variations for case 2. As seen, $Q_{ss}$ decreases more considerably during the early hours of each day, when wind generation is higher. However, when the voltage at the wind farm buses (like $V_{40}$ in Fig. 4.12-(b)) reach the maximum limit of 1.1 p.u., the reactive power supply of the wind farms decrease (to avoid over-voltage) and thus, the $Q_{ss}$ increases. Table 4.3 also shows that $Q_{loss}$ has only slightly been decreased compared to case 1. The reason is that the decrease in $Q_{loss}$ due to the RPC of wind farms is almost cancelled as a result of smaller ESS sizes (which means larger power flows).

Case 3: In this case, the RPC of both wind farms and ESSs, with $PF_{w, min} = PF_{ESS, min} = 0.95$ are taken into account ($V_{SM}^{*}=1.1$, $\mu=0$). As seen in Table 4.3, much less ESS ratings (40% smaller than case 2), and in fewer places, have been installed. The reason, in addition to the RPC of ESSs, is the optimum placement of ESSs in the weakest buses of the system (31, 40 and 41), in which the reactive power compensation is the most effective. The value of $Q_{ss}$ has also been decreased by 24% compared to case 2. However, smaller ESS sizes, as well as less distributed ESS placement in this case, has caused $Q_{loss}$ to increase by 15%, compared to case 2 (This also implies that the $Q_{loss}$
term is dominated by the other terms of OF). Figure 4.13 shows the system’s VSM (represented by $\alpha_{sd}^{\text{max}}$) before and after utilising wind farms’ RPC and ESSs, for case 3. It also shows ESSs’ total charging/discharging powers. As seen, although the system’s VSM decreases when ESSs are charged, the minimum desired 10% VSM is achieved at all times (bold, black curve). In addition, the figure demonstrates how wind farms’ RPC can improve the system’s VSM (blue curve). This improvement, however, only exists when the wind farms generate active power ($t \leq 11hr$ and $t \geq 21hr$), which is due to the minimum PF constraint of equation (4.44).

**Case 4:** In this case, OLTCs’ tap changing capability with $0.95 \leq \tau_{\text{oltc}}^{\text{std}} \leq 1.05$ is added, in comparison with case 3. Table 4.3 shows that in this case the required ESS cost is 18% reduced, compared to case 3. This is so because OLTCs’ tap changing actions (Figs. 4.14-(a) and (b)) contribute in satisfying system’s constraints at both loading levels $\lambda_{sd}^*$ and $\lambda_{sd}^0$. Table 4.3 shows, however, that both $Q_{\text{loss}}$ and $Q_{ss}$ have been increased (i.e. dominated by the ESS cost terms), by
4.5 Simulation Results of the Risk-Based Stochastic ESS Allocation

7% and 4%, respectively, compared to case 3. This is due to the smaller ESS ratings (i.e. less RPC), located in fewer places, and thus, larger line flows. Figure 4.14-(a) shows that between the hours of 10 and 22 the voltage of bus 40 (and thus, 41) is boosted due to the action of OLTC at bus 16. During these hours, the system’s VSM is determined by the increased current through line 1-2 (green curve in Fig. 4.14-(a)), as a result of smaller ESSs. Figure 4.14-(a) also shows that the bus voltages are boosted by OLTCs, as much as the OLTCs’ capability ($\tau_{\text{oltc}}^\text{min} = 0.95$) and maximum voltage limit (1.1 p.u.) allow, to decrease the line currents, and thus $Q_{\text{loss}}$ and $Q_{ss}$. For example, OLTC at bus 16 is at its lower limit (blue curve in Fig. 4.14-(a)) at all times, except during the high-wind periods (in which over voltage could happen), when the OLTC’s tap increases to keep voltage of bus 40 within the limits (Fig. 4.14-(a) and (b)).

**Cases 5 to 7:** Cases 5 and 6 are similar to case 4, with the difference that risk levels of 1% ($\mu = 0.01$) and 2% ($\mu = 0.02$), respectively, are considered in the VSM constraints of equations (4.31) and (4.32). Thus, the operational limits at $\lambda_{td}^\ast$ are allowed to be exceeded, as long as the expected violation at each hour is not larger than the risk level, as stated in equations (4.31) and (4.32). Table 4.3 shows that the required ESS cost is reduced by 10% and 20% for cases 5 and 6, respectively, compared to case 4 (at the expense of slight increases in $Q_{\text{loss}}$ and $Q_{ss}$). The higher the acceptable risk level, the smaller the ESS sizes required to be installed. Figure 4.15-(a) shows the current flow of the branch 1-2 in all scenarios, for case 6. The expected violation of thermal
limit for this line at $\lambda^*_td$ is also shown in Fig. 4.15-(b), which is seen to be bounded to the pre-set 2% risk level.

Notice that because of reactive power terms in the OF, the ESSs are distributed in three locations, which leads to smaller power losses. To emphasize this fact, case 7 is carried out, which is the same as case 6, but with terms $Q_{\text{loss}}$ and $Q_{ss}$ removed from the OF (Only ESSs’ cost is minimized.). Table 4.3 shows that this case yields smaller ESS (and much larger $Q_{\text{loss}}$ and $Q_{ss}$), located centrally at bus 41.

Cases 8 and 9: Case 8 is similar to case 4, but with 15% desired VSM ($\mu=0$). As seen from Table 4.3, compared to case 4, more ESSs (55%), and in more distributed places, have been installed (Notice that installing large ESSs in few places leads to over/under voltage during discharging/charging periods.). Also, compared to case 4, $Q_{\text{loss}}$ and $Q_{ss}$ have been decreased by 16% and 13%, respectively, due to the larger ESSs (with RPC) installed in this case. Case 9 is carried out to compare the effects of using risk-based VSM constraints and ANM tools on ESSs’ size, $Q_{\text{loss}}$, and $Q_{ss}$. Compared to case 8, the value of $\mu$ is set to 0.02, however no ANM tools have been considered. Table 4.3 shows that although the ESS cost is almost similar to that of case 8 (and $Q_{\text{loss}}$ is only 5% higher), a remarkable increases in $Q_{ss}$ (110%), compared to case 8, is resulted. Therefore, even though risk-based approach can lead to less ESS installation, large reductions in $Q_{\text{loss}}$ and $Q_{ss}$ is not possible without using RPCs of wind farms and ESSs.

It is worth mentioning that in solving case studies 1 to 7, the results of each case were used as a feasible initial point for the next case study, using the initialization capability of GAMS.
Consequently, as seen from Table 4.3, the execution times of cases 2 to 7 were noticeably reduced.

**Summary of findings:** In summary, the case studies clearly demonstrate that the proposed optimal allocation and operation of ESSs is able to guarantee a desired level of VSM for distribution systems, regardless of wind and load variations. Besides, it is shown that $Q_{ss}$ and $Q_{loss}$ can also be significantly reduced, by using the proposed method. It has been demonstrated that by proper regulation of RPCs of wind farms and ESSs, as well as tap positions of OLTCs, the required ESS capacity to achieve the same VSM* can be noticeably (up to 55%) reduced. Significant additional reduction in the required ESSs capacity (around 20%) can also be achieved through a risk-based approach, where a slight relaxation is allowed in voltage and thermal limits at the critical loading level, $\lambda^*$.

### 4.6 Summary

This chapter proposes two probabilistic optimization methods to allocate and operate ESSs in transmission/distribution power systems, with the aim of obtaining a required VSM. The stochasticity of wind power is modelled through two methods using: (i) the PDF of wind farm’s output power and (ii) optimally generated wind power scenarios. It is demonstrated that a desired VSM* can be secured, under all possible load and wind conditions, using proper allocation and operation of ESSs, accompanied by coordinated adjustment of ANM tools. ESSs are shown to go through one charging/discharging cycle each day by using the proposed method. The results show that with only active power capabilities of ESSs and wind farms, large ESS sizes are required to secure a desired VSM*. However, the required ESS size can be remarkably reduced by optimally regulating the RPCs of wind farms and ESSs and tap positions of OLTCs, to achieve the same VSM*. The RPC of ESSs is shown to be more effective in reducing the ESSs’ size, due to the optimal placement of ESSs. More importantly, it is shown that by slight, risk-based, relaxation of voltage and thermal constraints at the critical loading condition, the required ESS can be further reduced. Both the imported (from the upstream transmission system) and lost (in the distribution system) reactive powers are also significantly decreased by the proposed method, as a result of local supply of reactive power by ESSs and wind farms and reduced line flows. Also, it is observed that inclusion of reactive power terms, i.e. $Q_{ss}$ and $Q_{loss}$, in the objective function leads to more
distributed placement of ESSs, which leads to less power flow through the lines.
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Chapter 5
Short-Term Voltage Stability Improvement Using Dynamic Control of Storage Devices

In this chapter, first, the use of ESS devices to dynamically attain a prescribed level of VS for power systems is investigated. The objectives are to (i) optimally compute and (ii) dynamically control the required active and reactive power compensation by using ESSs such that a desired VSM is always ensured, taking into account single-contingency criterion. The performance of the proposed method is evaluated through case studies, considering both constant-power and voltage-dependent loads. In the second part of this chapter, the effects of induction motors (IMs), fixed-speed induction generator (FSIG)-based wind turbines (WTs), and over-excitation limiter (OEL) of synchronous generators (SGs) on short-term voltage stability (ST-VS) are analysed. Then, systematic control of ESS to provide dynamic voltage support (DVS) to power systems, during and after large disturbances, is investigated, as a countermeasure against short term voltage instability (ST-VI). To implement ESS’s DVS capability, the required power injection by the ESS is dynamically determined, and then tracked, by a suitably designed control system, after occurrence of a disturbance. The effects of implementing fault ride through (FRT) and time-overload (TOL) capabilities of ESS on ST-VS are also analysed. Optimal power factor (PF) adjustment for power injection by ESS, from a ST-VS viewpoint, is discussed as well. It is shown that the capability of ESS in providing DVS to the system can be effectively used against detrimental effects of IMs, FSIGs, and OELs on ST-VS. The proposed method is tested on IEEE 30-bus test system.

5.1 Introduction

Short-term (few seconds) voltage stability (ST-VS), has been a serious concern for power systems over the recent decades. It has been identified to be the root cause of several major
blackouts throughout the world [16–18]. A main factor contributing in power system ST-VI is known to be the large reactive power demand of, widely used, IM loads during and after large disturbances. Also, despite the growing ubiquity of the inverter-interfaced WTs (types 3 and 4), FSIG-based WTs are still broadly in use in many power systems. FSIGs show similar behaviour to that of IMs, in terms of large reactive power demand requirement over the disturbances which cause large voltage dips (called low-voltage disturbances in this thesis). In the absence of immediate proper corrective measures, IMs may stall and FSIGs may over-accelerate during the faults. In those cases, IMs and FSIGs keep drawing high amounts of reactive power from the grid. This suppresses the system voltages and prevents complete voltage recovery even after the fault clearance. As a result, protective devices may become activated and a cascading failure of elements may be triggered, which can lead to a partial or global voltage collapse. Hence, it is paramount to limit the adverse affects of the low-voltage disturbances, either by fast-enough clearance of the fault or through boosting the voltages during and after the fault, using proper, fast response corrective measures. As a result, stalling of IMs and over-acceleration of FSIGs can be avoided, and, eventually, power system ST-VS can be maintained.

The capability of ESSs in supplying dynamic active and reactive powers to the grid can be used for providing DVS to the system during the large disturbances. FRT and TOL capabilities of many of the today’s inverters, can significantly increase the DVS capability of ESSs, to be used for ST-VS improvement. In this chapter, first, systematic, decoupled control of active and reactive power injection by ESS is carried out. The designed controllability of ESS, is then used for dynamically preserving a desired level of VSM for the power systems. The required amount of power injection by ESS at all load-wind combinations is determined through steady-state VS studies, considering the worst-case N-1 contingency criterion. Dynamic simulations are carried out to show the effectiveness of the proposed ESS control method in maintaining system’s VS after single contingency events. The effect of voltage-dependent loads is also evaluated through case studies. In the second part of the chapter, first, detailed investigation of the adverse impacts of IMs, FSIGs, and OEL of SGs on ST-VS of power systems is carried out. Then, dynamic control of ESS’s power injection, based on local voltage measurement during and after a fault, is utilised to improve ST-VS, taking into account the FRT and TOL capabilities of ESSs as well as their operating PFs. In summary, the main contributions of this chapter are as follows:
1. Systematic control of ESS is carried out, to dynamically control active and reactive power injection at the PCC of ESS. Two different approaches are used to systematically design the controller coefficients: (i) decoupled current control in d-q coordinates, and (ii) state-feedback approach.

2. A new method for dynamic procurement of a desired level of VSM for power systems, using the obtained controllability of ESS devices, is presented. Single-contingency criterion and voltage-dependant behaviour of the loads are taken into account.

3. The effects of IMs, FSIG-based WTs, and OEL of SGs on ST-VS of power systems are analysed in detail, through proper modelling of these components and using several illustrative examples and case studies.

4. DVS capability of ESSs are proposed as a countermeasure against ST-VI. The effects of FRT and TOL capabilities of ESS’s inverter, as well as the optimal PF for operation of ESS during the fault, are also investigated through case studies and analysis of the dynamic behaviour of the power system components.

The rest of this chapter is organized as follows. In Section 5.2, the proposed method of dynamic VSM procurement is described. The used approach for decoupled control of ESS (Section 5.2.1), the adopted VSM criterion (Section 5.2.2), the effect of voltage dependent loads (Section 5.2.3), and generation of reference active and reactive powers for ESS (Section 5.2.4) are discussed in detail. Section 5.3 presents the simulation results of this part, including the load modelling (Section 5.3.1), and the results for voltage-dependant loads (Section 5.3.2) and constant-power loads (Section 5.3.3). In Section 5.4 the ST-VS analysis and improvement is investigated. The state-feedback approach to control injected active and reactive powers of ESS is presented in Section 5.4.1. Then, the effects of IMs (Section 5.4.2) and FSIG-based WTs (Section 5.4.3) on ST-VS are presented using illustrative examples. The used model of OEL of SGs and the proposed DVS scheme by ESSs are also presented in Section 5.4.4 and 5.4.5, respectively. Section 5.5 presents the simulation results related to the ST-VS analysis and improvement. The utilized test system is described in Section 5.5.1. Then the detailed effects of FSIGs (Section 5.5.2), OELs (Section 5.5.3), DVS capability of ESS (Section 5.5.4), and operating PF of ESS (Section 5.5.5) on ST-VS
of the power system are presented and discussed. Finally, the chapter is concluded and summarised in Section 5.6.

5.2 Dynamic VSM Procurement Using ESS Devices

5.2.1 Control of ESS Device

The schematic diagram of ESS, along with its three-phase voltage source converter (VSC) and control system components is shown in Fig. 5.1. The elements $R_f$ and $L_f$ represent the interfaced filter of the ESS. By regulating the magnitude and phase angle at the inverter terminal of ESS, i.e. $v_{c}\angle\delta_c$, reactive and active powers, respectively, can be independently controlled at the point of common coupling (PCC), denoted by the voltage $v_{g}\angle\delta_g$ in Fig. 5.1.

Equation (5.1) describes the KVL equation for the ESS model of Fig. 5.1, in time domain [99].

$$v_c(t) - v_g(t) = R_f i_f(t) + L_f \frac{di_f(t)}{dt}$$

(5.1)

Decomposing this equation into d-q components [99], and separating the equations in each axis, yield:

$$v_{cd} - v_{gd} = R_f i_{fd} - \omega L_f i_{fq} + L_f \frac{di_{fd}}{dt}$$

$$v_{cq} - v_{gq} = R_f i_{fq} + \omega L_f i_{fd} + L_f \frac{di_{fq}}{dt}$$

(5.2)

The reference frame coordinate is chosen such that the d-axis is always in line with the instantaneous voltage vector at the PCC, i.e. $v_g(t)$, and the q-axis is in quadrature with that. This is done by the phase-locked loop (PLL), as shown in Fig. 5.1. As seen in equation (5.2), the equations of d and q components are coupled through the terms $-\omega L_f i_{fq}$ and $\omega L_f i_{fd}$. The ESS’s internal voltages, i.e. $v_{cd}$ and $v_{cq}$, are determined by modulation indexes in d and q coordinates, i.e. $m_d$ and $m_q$, respectively, and the capacitor’s DC voltage $v_{DC}$, as stated in equations (5.3):

$$v_{cd} = m_d \times v_{DC}$$

$$v_{cq} = m_q \times v_{DC}$$

(5.3)
Notice that the ESS is assumed to keep the voltage $v_{DC}$ almost constant [63]. The generated signals $m_d$ and $m_q$ are converted to the three phase signal $m_{abc}$ and then fed into the pulse width modulation (PWM) generator, as shown in Fig. 5.1, which generates the required pulses to control the ESS's converter.

The active and reactive powers at the PCC, in d-q coordinate, are as per equations (5.4).

$$P^{out} = \frac{3}{2} \left( v_{gd} i_{fd} + v_{gq} i_{fq} \right)$$

$$Q^{out} = \frac{3}{2} \left( v_{gq} i_{fd} - v_{gd} i_{fq} \right)$$

(5.4)

Considering the instantaneous voltage vector at the PCC as the reference for d-axis, the value of $v_{gq}$ is always zero. Thus, the equations of the reference currents $i^*_d$ and $i^*_q$, for given reference powers of $P^*$ and $Q^*$, respectively, are simplified as per equation (5.5).

$$i^*_d = \frac{2}{3} \frac{P^*}{v_{gd}} = \frac{2}{3} \frac{P^*}{|v_g|}$$

$$i^*_q = -\frac{2}{3} \frac{Q^*}{v_{gd}} = -\frac{2}{3} \frac{Q^*}{|v_g|}$$

(5.5)

As seen in equation (5.2), the KVL equations of the ESS in $d$ and $q$ axes are coupled. Decoupling these equations, and at the same time tracking the desired $i^*_d$ and $i^*_q$, using a PI controller, can be done by constructing the inverter terminal voltages as per equation (5.6) [99].

$$v_{cd} = \left( k_{P1} + k_{I1} \int \right) \left( i^*_d - i_{fd} \right) + v_{gd} - \omega L_f i_{fq}$$
\[ v_{cq} = \left( k_{P1} + k_{I1} \int \right) (i_{fq}^* - i_{fq}) + v_{gq} + \omega L_f i_{fd} \]  

(5.6)

Substituting equation (5.6) in equation (5.2) gives:

\[
L_f \frac{di_{fd}}{dt} + R_f i_{fd} = \left( k_{P1} + k_{I1} \int \right) (i_{fq}^* - i_{fq})
\]

\[
L_f \frac{di_{fq}}{dt} + R_f i_{fq} = \left( k_{P2} + k_{I2} \int \right) (i_{fq}^* - i_{fq})
\]

(5.7)

As seen, the equations in the \( d \) and \( q \) axes are now decoupled. The block diagram for the dynamic equation of \( d \) axis is shown in Fig. 5.2. The same dynamic system exists for the \( q \) axis too. In order to calculate the PI controller coefficients, the simple method proposed in reference [99] is applied as follows: The controller’s zero at \( s = -k_{I1}/k_{P1} \) is designed such that it cancels the slow pole of the ESS’s plant at \( s = -R_f/L_f \). By doing so, the closed loop transfer function of the system, \( G_i(s) \), becomes:

\[
G_i(s) = \frac{i_{fd}}{i_{f}^{*d}} = \frac{1}{(L_f/k_{P1})s + 1} = \frac{1}{\tau_i s + 1}
\]

(5.8)

where \( \tau_i \) is the time constant of the closed-loop system. Hence, for any desired value of \( \tau_i \), the PI controller coefficients can be obtained as stated in equation (5.9).

\[
k_{P1} = \frac{L_f}{\tau_i}
\]

\[
k_{I1} = \frac{R_f}{\tau_i}
\]

(5.9)

The described control of ESS, corresponding to the controller box in Fig. 5.1, is depicted in Fig. 5.3.
5.2 Dynamic VSM Procurement Using ESS Devices

5.2.2 Case Study and VSM Criterion

IEEE 30-bus test power system has been used as the case study. Single line diagram of the test system is shown in Fig. 5.4. Two wind farms with active power ratings of 20 MW and 10 MW are respectively installed at buses 19 and 26, which are among the weakest buses of the system, as identified in Chapter 3. Both wind farms are assumed to be exposed to the same wind regime and operate at unity power factor. We consider the case where the loading level of the system is increased by 60%, compared to the original loading, to create a challenging case for the VS of the system.

The conventional VSM criterion for secure operation of power systems has been defined by Western Electricity Coordinating Council (WECC) to be around 10% for normal operating condition and 5% for $N-1$ contingency condition [98]. In modern power systems with high integration of renewables, however, larger VSMs should be assured, in order to account for the inherent uncertainty of renewable energy resources. In this section, the desired VSM is defined to be 15% and 10%, respectively, for normal and single-contingency operating conditions. Fig. 5.5 presents the power-voltage (P-V) curves of the IEEE 30-bus test system for the normal condition and the most severe contingency conditions, i.e. the outage of line 27-28. As seen, the system’s loadability is considerably reduced after the occurrence of the contingency.

5.2.3 The Effect of Voltage-Dependent Loads

It is seen from Fig. 5.5 that although at the nominal loading level ($\lambda = 1$) the system’s VSM under the normal operating condition is more than the required 15% (black arrow), the VSM for
the worst contingency condition (line 27-28 outage) is negative (red arrow). It means that, for the case where the loads are assumed to be constant power, the outage of the line 27-18 makes the system voltage unstable immediately, since there is no post-contingency operating point at loading level $\lambda = 1$. The voltage-dependent behaviour of the loads, however, mitigates this issue to some extent. It is known that the power consumption of many kinds of loads varies with the voltage. This occurs in different patterns (exponential, polynomial, etc.) depending on the load’s characteristics. In this cases, the operating point of the system is the intersection of the system’s P-V curves and the load’s P-V characteristics [1]. As shown in Fig. 5.5, with the voltage dependent loads, the system converges to a new operating point on the post-contingency P-V curves after occurrence of the contingency. This new operating point, however, may or may not exist depending on the load’s characteristics and severity of the contingency. In fact, among all types of the static loads

Figure 5.4: The decoupled control of ESS
5.2 Dynamic VSM Procurement Using ESS Devices

5.2.4 Generation of $P^*$ and $Q^*$ From VS Viewpoint

This section deals with the generation of the proper reference signals, $P^*$ and $Q^*$, for the ESS’s control system, to be able to ensure a desired VSM for the power system under all operational circumstances. By applying appropriate active and reactive compensations, the system’s P-V curves can be stretched such that a required VSM is achieved. Fig. 5.5 (green curve) shows the effect of applying active power compensation at bus 30 on the VSM of the post contingency system. As seen, by applying the compensation, not only the post-contingency VSM can be guaranteed (even for constant-power loads), but also the voltage of the post-contingency operating point is improved.

Fig. 5.6 shows the existing VSM for the worst contingency condition (outage of line 27-28), without any compensation applied, and for different wind-load combinations. As seen, at high loadings and low wind levels the existing VSM (colour bars) are smaller than the desired 10% VSM (white bars). This indicates that at those load-wind combinations appropriate compensation
Figure 5.6: The existing VSM for the most severe contingency and different load and wind levels is required to provide the system with the required VSM.

In order to achieve the desired VSM with the minimum required size of ESS, the ESS should be placed at the most effective bus, i.e. the bus at which the active/reactive compensation leads to the highest improvement in the system’s VSM. It is shown in Chapter 3 that bus 30, which is the weakest bus of the system from a VS viewpoint, is the most effective bus in terms of VSM improvement. Hence, bus 30 has been selected for ESS installation. Also, it is shown in Chapter 3 that active and reactive power compensations have different effects on the system’s VSM, and there exists an optimum PF at which the injected power by ESS leads to the maximum VSM improvement. Figure 5.7 illustrates the effect of PF of ESS at bus 30 on the system’s VSM. It is seen that at PF = 0.85 lagging, maximum VSM improvement is obtained. As load and wind variation’s effects on the optimum PF are small, the ESS is operated with PF=0.85 at all load-wind combinations throughout this section.

In order to compute the required power references, $P^*$ and $Q^*$, to be fed to the ESS’s control system such that the required 15% and 10% VSMs are achieved, respectively, for the normal and worst contingency conditions, a similar approach to that of Chapter 3 is used. At each load-wind combination, the active/reactive power injection of ESS, with the optimum PF, is incrementally increased. For each level of ESS’s power injection, the CPF method is used to compute the system’s VSM for both normal and worst contingency conditions. The smallest power injection by the ESS which can obtain both required VSM criteria, as mentioned above, is the proper power reference
5.2 Dynamic VSM Procurement Using ESS Devices

Figure 5.7: The effect of PF of the ESS on system’s VSM

Figure 5.8: The required apparent power injection by the ESS to ensure desired VSM criteria for the system

at that load-wind combination. The same process is repeated for all other wind-load combinations. Figure 5.8 shows the obtained apparent power references for different load-wind combinations. From the ESS’s power factor, the active and reactive power references can be obtained. These powers are used as the input reference signals to the ESS’s control system, in dynamic simulations, to dynamically ensure the system’s required VSM.
5.3 Simulation Results of Dynamic VSM Procurement

The proposed ESS control approach has been tested on the modified IEEE 30-bus test system (see Section 5.2.2). The transmission system has been modelled by the algebraic load flow equations. A 6th order dynamic model of the synchronous generators along with their first-order automatic voltage regulator (AVR) models have also been incorporated. The ESS’s filter parameters are considered to be $R_f = 2\Omega$ and $L_f = 0.2H$. Considering the time constant for the current control loop of ESS, $\tau_i$, equal to 0.01s, the PI controller coefficients are computed as $k_{P1} = 20$ and $k_{I1} = 200$, using equation (5.9).

The applied load and wind profiles are presented in Table 5.1. As seen, after $t = 4$ s the most critical wind-load combination, from a VS viewpoint, i.e. maximum load and zero wind, is applied to the system. Also, at time $t = 4.5$ s the most severe contingency, i.e. the outage of line 27-28, occurs. With the constant-power loads, this scenario (after t=4.5s) forms the most critical condition for the system, from a VS viewpoint, and as shown in Fig. 5.5, can lead to immediate voltage instability.

5.3.1 Modelling of Loads

Static loads are modelled in this chapter as equivalent current sources, calculated by equation (5.10) as a function of the loads’ powers, $P_l$ and $Q_l$, and the terminal voltage, $V_l \angle \theta_l$.

$$I_l(P_l, Q_l, V_l \angle \theta_l) = (P_l - jQ_l) / (V_l \angle \theta_l) \quad (5.10)$$

Also, the loads’ characteristics have been modelled by equation (5.11), where $P_{l0}$ and $Q_{l0}$ denote the load’s power consumption at the nominal voltage $V_0 = 1$ p.u.

$$P_l = \alpha P_{l0} \times (V_l / V_0)^\gamma, \quad Q_l = \alpha Q_{l0} \times (V_l / V_0)^\gamma \quad (5.11)$$

The parameter $\gamma$ determines the voltage-dependent behaviour of the loads. Voltage-dependant loads are modelled as quadratic loads with $\gamma = 2$, as shown in Fig. 5.5 (blue curve). Also, constant-power loads are modelled by setting $\gamma = 0$ in equation (5.11). The loading condition of the system is changed through the constant $\alpha$, with $\alpha = 1$ denoting the peak loading level. For
Table 5.1: Applied load and wind profiles and required ESS injections

<table>
<thead>
<tr>
<th>time (sec.)</th>
<th>2</th>
<th>2.5</th>
<th>3</th>
<th>3.5</th>
<th>4</th>
<th>4.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>loading level (% of the peak)</td>
<td>80</td>
<td>85</td>
<td>90</td>
<td>95</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>wind power (% of the rated power)</td>
<td>75</td>
<td>100</td>
<td>50</td>
<td>25</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>outage event (line 27-28)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>power injected by ESS (MVA)</td>
<td>0</td>
<td>0</td>
<td>0.3</td>
<td>2.7</td>
<td>5.5</td>
<td>5.5</td>
</tr>
</tbody>
</table>

Figure 5.9: Simulation results for voltage-dependent load, without ESS

simplicity, the same method has been used in this section to control the output power of the wind farms (with PF = 1, $\gamma = 0$, and $\alpha$ denoting the percentage of the wind farm’s rating power).

5.3.2 Results for Voltage-Dependant Loads

Fig. 5.9 presents the simulation results for the case where no ESS has been used. As seen, the bus voltages gradually decrease (Fig. 5.9-(a)) by increasing the loading level of the system (Fig. 5.9-(b)). Besides, it is seen that with the outage of line 27-28, at time $t=4.5s$, the bus voltages sharply drop to around 0.75 p.u. Notice that since the loads are voltage-dependent, the outage of line 27-28 does not lead to an immediate voltage instability, as explained in Section 5.2.3, although it results in an excessive voltage drop. In this case, the system converges to the new operating point on the post-contingency P-V curve, as seen in Fig. 5.5. Fig. 5.9-(b) also shows that the loads’ power consumption significantly decreases after the contingency, due to the resulted voltage drop.

In the next stage, according to the applied loading level and wind power variations (Table 5.1)
the corresponding required apparent power injections of ESS at bus 30 (see Fig. 5.8) are applied, to assure the required 10% VSM for the worst-case contingency. The applied apparent powers of ESS are shown in the last row of Table 5.1. Having these values, and considering the ESS’s optimal power factor (see Fig. 5.7), the corresponding $P^*$ and $Q^*$ signals are computed and fed into the ESS’s control system. Hence, by tracking the reference signals, using the proposed control approach, ESS dynamically injects the required active and reactive power into the system, such that the required VSM is always ensured. The simulation results for this case are presented in Fig. 5.10. Fig. 5.10-(a) shows that the active and reactive power injections of ESS accurately track the corresponding reference signals. The voltage variation of buses 30 and 26 are also seen in Fig. 5.10-(b). It is seen that at the presence of the ESS with the proposed control scheme, not only a better voltage regulation at the normal operating condition is achieved, but also the post-contingency voltages are significantly improved compared to the case with no ESS. Thus, the adverse impacts of the excessive voltage drop after the contingency are highly mitigated. As seen in Fig. 5.10-(c), the loads’ power consumption also undergo smaller drops after the contingency, at the presence of ESS’s compensation.

5.3.3 Results for Constant-Power Loads

In this section, the more challenging static load type, i.e. constant-power loads, is investigated. Notice that investigating dynamic loads, such as induction motors, which can be even more challenging for VS is beyond the scope of this section (It will be studied in Section 5.4). Fig. 5.11 compares the load’s power and voltage variations for constant-power and voltage-dependent loads, at normal operating conditions. As seen, with constant-power loads not only larger voltage drops occur at higher loading levels (Fig. 5.11-(a)), but also the loads consume larger amounts of power compared to voltage dependent loads (Fig. 5.11-(b)).

Fig. 5.12 presents the system’s response to the worst case contingency for this case. As seen, the system experiences an excessive voltage drop followed by an immediate voltage instability after the contingency occurrence. The reason, as explained in Section 5.2.3, is non-existence of a post-contingency operating point for constant-power loads. In practice, the exceeding voltage drops can trigger the protective devices and initiate a cascading failure, leading to a partial or global voltage collapse.
The simulation results for this case and with ESS are shown in Fig. 5.13. As seen, by using the proposed ESS control, the post-contingency voltage instability issue has been resolved and the system converges to an operating point on the post-contingency P-V curve. Notice that although the post-contingency voltage is still low (around 0.7 p.u.) in this case, the system operator has the time and opportunity to apply the required corrective measures, such as using compensation devices or load curtailment, to return the voltages to the normal range [66]. This is possible owing to the prevention of immediate voltage instability by the ESS.

Fig. 5.14 shows the system’s response for different fault (i.e. contingency) clearing times (FCTs). As seen, while with ESS the system never becomes voltage unstable, the FCT needs to be very small (less than 0.04s) without ESS, to ensure VS. Finally, it is worth mentioning that in large, interconnected power systems there may be no critical single contingency, causing an immediate voltage instability, like the case shown for IEEE 30-bus test system. However, the idea proposed in this section can be applied to larger power systems, by considering higher order (N-2, N-3, etc.) contingency criteria. Hence, the existence of a post contingency operating point can be
Short-Term Voltage Stability Improvement Using Dynamic Control of Storage Devices

Figure 5.11: (a) voltage and (b) load variations, for different types of load

Figure 5.12: Simulation results for constant-power load, without ESS

guaranteed even after occurrence of the worst-case multiple contingency condition.

5.4 ST-VS Improvement Using DVS Capability of ESSs

In this section, ST-VS of power systems is investigated. The impact of different power system components, affecting the ST-VS of the system, is elaborated on and some countermeasures against ST-VI are also proposed. First, a simple, new method of ESS control using state-feedback approach is presented.

5.4.1 ESS Control Using State-Feedback Approach

Consider the schematic diagram of ESS, shown in Fig. 5.1, along with the equations (5.1) to (5.5). Equation (5.2) can be rewritten, after some manipulations, in the state-space form, as presented in
Figure 5.13: Simulation results for constant-power load, with ESS

Figure 5.14: The effect of FCT on system’s VS, with and without ESS

equation (5.12):

\[
\begin{align*}
\dot{x} &= Ax + Bu \\
y &= Cx
\end{align*}
\]

where \(x\) and \(y\) are, respectively, vectors of state and output variables, both comprise \(i_{fd}\) and \(i_{fq}\) (i.e. \(C = I_{2 \times 2}\)), and \(u\) is the input vector, defined to be the difference between ESS’s internal voltage and the PCC’s voltage in \(d\) and \(q\) axes. \(A, B,\) and \(C\) are the state-space, constant, matrices.
In this section, state feedback approach is used to control the output of the ESS to track the desired active and reactive powers, i.e. $P^*$ and $Q^*$, as per equation (5.5) and Fig. 5.15. Thus, the control objective is to design the PI controller coefficients, $K_I$ and $K_P$, such that the reference current signals, $i_{f_d}^*$ and $i_{f_q}^*$, and consequently $P^*$ and $Q^*$ as per equation (5.5), are effectively tracked. Notice that ESS is a zero type system, i.e. $\det(sI - A) = 0$ has no root at the origin.

Hence, to be able to track a step input, a PI controller is required to increase the system type to at least one. The need for an integral controller gives rise to new state variables, denoted by matrix $z$ in Fig. 5.15. Let’s define the error signals, $e_d$ and $e_q$, as the difference between the ESS’s reference and actual currents in $d$ and $q$ axes, respectively. Then, the new state variables, $z_d$ and $z_q$, can be defined as per equation (5.14):

$$
\begin{align*}
    z_d &= \int e_d dt = \int (i_{f_d}^* - i_{f_d}) dt \\
    z_q &= \int e_q dt = \int (i_{f_q}^* - i_{f_q}) dt \\

e_d &= i_{f_d}^* - i_{f_d} \quad \Rightarrow \quad z_d = e_d = i_{f_d}^* - i_{f_d} \\
e_q &= i_{f_q}^* - i_{f_q} \quad \Rightarrow \quad z_q = e_q = i_{f_q}^* - i_{f_q}
\end{align*}
\tag{5.14}
$$

This can be written in a matrix form as stated in equation (5.15):

$$
\begin{bmatrix}
    \dot{z}_d \\
    \dot{z}_q
\end{bmatrix} =
[-C]
\begin{bmatrix}
    i_{f_d} \\
    i_{f_q}
\end{bmatrix} +
\begin{bmatrix}
    i_{f_d}^* \\
    i_{f_q}^*
\end{bmatrix} \Rightarrow \dot{z} = -Cx + r
\tag{5.15}
$$
where $r$ is the matrix of the reference currents. Adding equation (5.15) to the state-space equations of ESS, as per equation (5.12), yields:

\[
\begin{bmatrix}
\dot{x} \\
\dot{z}
\end{bmatrix} =
\begin{bmatrix}
A & 0 \\
-C & 0
\end{bmatrix}
\begin{bmatrix}
x \\
z
\end{bmatrix} +
\begin{bmatrix}
B \\
0
\end{bmatrix} u +
\begin{bmatrix}
0 \\
I_{(2\times2)}
\end{bmatrix} r
\]  
(5.16)

which can be rewritten in compact form as:

\[
\dot{\tilde{X}} = \tilde{A}\tilde{X} + \tilde{B}u +
\begin{bmatrix}
0 \\
I_{(2\times2)}
\end{bmatrix} r
\]  
(5.17)

From Fig. (5.15) the control signal can be expressed as:

\[
u = -K_P x + K_I z \Rightarrow u = -
\begin{bmatrix}
K_P & -K_I
\end{bmatrix}
\begin{bmatrix}
x \\
z
\end{bmatrix} = -\tilde{K}\tilde{X}
\]  
(5.18)

where $\tilde{K}$ is a $2\times4$ gain matrix, the first $2\times2$ elements are associated with $K_P$ coefficients and the rest are associated with $K_I$ coefficients. Substituting equation (5.18) into equation (5.17) gives:

\[
\dot{\tilde{X}} = \tilde{A}\tilde{X} - \tilde{B}\tilde{K}\tilde{X} +
\begin{bmatrix}
0 \\
I
\end{bmatrix} r = [\tilde{A} - \tilde{B}\tilde{K}] \tilde{X} +
\begin{bmatrix}
0 \\
I
\end{bmatrix} r
\]  
(5.19)

The output equation can also be written as:

\[
y = [C \ 0]
\begin{bmatrix}
x \\
z
\end{bmatrix} = \tilde{C}\tilde{X}
\]  
(5.20)

The last two equations show a state-space representation of the closed loop system. Using these equations, the elements of matrix $\tilde{K}$ can be properly determined using linear quadratic regulator (LQR), pole placement, or other controller design approaches, such that a desired tracking performance is achieved. The proposed ESS control approach, allows dynamic control of active and reactive power injections by the ESS during contingency events, in order to improve the ST-VS of the power system, as will be shown in the simulation section.
5.4.2 Induction Motor Model and Effect on ST-VS

- The effect of IM on ST-VS

ST-VS is tightly linked with the load’s dynamic behaviour [16–19, 21], and static load models cannot reproduce real life ST-VI events [17]. Hence, appropriate modelling of IMs, which form a significant portion of the loads in today’s power systems (up to 50% of the summer peak load [16]), is particularly essential for analysis of ST-VS and delayed voltage recovery after the faults. Fig. 5.16 shows basic characteristic curves of a typical IM. As seen, at large rotor slips the electromagnetic torque is significantly reduced, exacerbating the rotor’s deceleration, and IM draws a high amount of current. The power factor (PF) of the motor also quickly drops at high slips, indicating large reactive power demand by IM during the deceleration phase.

A common scenario of ST-VI induced by the IM’s dynamic behaviour can be explained as follows. Following a fault, the resulted voltage drop at the IM’s terminal causes the motor’s electromagnetic torque (related with the square of the voltage) to sharply decrease, which slows down the rotor and highly increases the reactive current drawn by the IM. After the fault clearance, the terminal voltage of the motor, and subsequently its electromagnetic torque, partially recovers. The rotor’s inertia prevents it from quick re-acceleration and IM keeps drawing high reactive current from the grid, preventing complete voltage recovery. The rotor cannot re-accelerate and may stall if the electromagnetic torque, after the fault clearance, is smaller than the motor’s mechanical torque. A stalled IM, drawing high amount of reactive power, keeps suppressing the grid voltages, which may lead to immediate ST-VI or trigger protective devices and initiate a cascading voltage collapse.
5.4 ST-VS Improvement Using DVS Capability of ESSs

\[ V_{ds} + jV_{qs} = (R_s + jX'_r) (i_{ds} + ji_{qs}) + e'_{dr} + je'_{qr} \]  \hspace{1cm} (5.21)
\[ \dot{s} = \frac{1}{2H_m} (T_L - T_e) \]  \hspace{1cm} (5.22)
\[ T'_{do} e'_{qr} = -e'_{qr} + (X - X'_r) i_{ds} - T'_do s \omega_s e'_{dr} \]  \hspace{1cm} (5.23)
\[ T'_{do} e'_{dr} = -e'_{dr} - (X - X'_r) i_{qs} + T'_do s \omega_s e'_{qr} \]  \hspace{1cm} (5.24)

where \( X' = X_s + X_m X_r / (X_m + X_r) \) is the transient reactance, \( X = X_s + X_m \) is the rotor’s open circuit reactance, \( T_e = e'_{qr} i_{qs} + e'_{dr} i_{ds} \) is the electromagnetic torque, \( T'_{do} = (L_m + L_r) / R_r \) is the transient rotor time constant, \( s \) is the rotor slip, pairs \( (e'_{dr}, e'_{qr}) \) and \( (i_{ds}, i_{qs}) \) are direct and quadratic axes components of transient emf and stator current, respectively, \( T_L \) is the mechanical torque, \( H_m \) is the inertia constant, \( X_r, X_s, \) and \( X_m \) are respectively rotor, stator, and magnetizing reactances, and \( R_r \) and \( R_s \) are rotor and stator resistances. The IM’s parameters are as per those used in reference [100].

- IM’s effect on ST-VS: an illustrative example

To evaluate the effect of IM on ST-VS, a simple system, shown in Fig. 5.18, has been used. The per unit values in the figure have been computed considering a base power and base voltage of 100 MVA and 400 KV, respectively. The system’s load comprises a combined lump IM and constant impedance load, connected through a transmission line to an infinite bus. IM load forms 50% of the total load. At \( t = 1 \) s a three-phase to ground fault occurs at the middle of the line. Two cases,
namely cases I and II, have been considered, in which the fault is cleared 0.09s and 0.12s after the fault, respectively. Fig. 5.19 shows the simulation results of these two cases. As seen, in case I the load bus voltage, rotor slip, and other IM signals recover to their pre-fault conditions after clearing the fault, the reason being the fault clearing time is shorter than the critical clearing time (CCT), which is 0.11s in this test study. However, it is seen that in case II the electromagnetic torque cannot reach the mechanical torque after fault clearance, since fault clearing time is longer than CCT. As seen in Fig. 5.19, in this case the voltage, rotor slip, and active/reactive power of the IM cannot recover to their pre-fault values. It is also seen that IM keeps drawing a high amount of reactive power (with very low PF) after the fault clearance, which prevents the voltage from recovering. This renders the system prone to possible cascading voltage collapse.

5.4.3 Fixed-Speed Induction Generator Model and Effect on ST-VS

- Model of WT and FSIG

Variable speed, power electronic-interfaced wind turbines (WTs), such as doubly-fed induction generator (DFIG), which can regulate reactive power or voltage at their terminals, are becoming more prevalent in modern power systems. However, FSIG-based WTs still form an important portion of the installed WTs in many power systems around the world [18, 101]. WT is characterized by the output mechanical torque, $T_m$, as stated in equation (5.25) [101][102], which acts as the input to the FSIG.

$$T_m = 0.5 \rho A R C_p V_{\omega}^2 / \lambda$$

(5.25)

where $\rho$ is the air density, $A$ is the swept area by the blades, $R$ is the radius of the WT, $C_p$ is the power coefficient, $V_\omega$ is the wind speed, and $\lambda$ is tip speed ratio, i.e. the ratio between the linear
speed at the tip of the blades to the wind speed. $C_p$ is a function of $\lambda$ and the pitch angle $\beta$ as per equation (5.26) [102]:

$$C_p = (0.44 - 0.017\beta) \sin \left( \frac{\pi}{15 - 0.3\beta} (\lambda - 3) \right) - 0.0018 (\lambda - 3) \beta$$  \hspace{1cm} (5.26)

The 3rd order IG model, reported in reference [101], which is similar to the IM’s load discussed in Section 5.4.2, is used in this section. Notice that in the IG model, the equivalent inertia constant of both WT and IG is used [102].

**- The effect of FSIG on ST-VS: an illustrative example**

While IGs provide active power to the system, they use system’s reactive power for magnetization. During the transient events, similar to the IMs, IGs draw high amounts of reactive power and may contribute to possible ST-VI. A difference between IGs and IMs is that during the faults, IGs
over-accelerate rather than stall [18], due to the decreased electromagnetic torque of the machine.

To demonstrate the effect of SFIG on ST-VS, the same system of Fig. 5.18 is used with the difference that the IM is replaced with a FSIG, with nominal active power of 1 p.u and nominal wind speed of 12 m/s. The other parameters of FSIG are the same as in Ref. [102]. Here again, two cases, named cases I and II, are considered, in which the fault is cleared at 0.11s and 0.13s after the fault occurrence, respectively. Fig. 5.20 shows the simulation results of these two cases. As seen, FSIG draws reactive power from the grid at the steady-state before the fault. Also, it is seen that in case I, the terminal voltage, rotor speed, and active/reactive powers of FSIG recover their pre-fault values after fault clearance, since in this case the fault clearing time is shorter than the CCT, which is 0.12s for this test study. However, it is seen that in case II, where fault clearing time is longer than CCT, FSIG over-accelerates and draws high reactive power from the grid (around triple the steady-state value), which greatly suppresses its terminal voltage and makes the system prone to ST-VI.

5.4.4 OEL Model and Effect on ST-VS

It is known that VS is tightly linked with the ability of power systems to supply and transfer reactive power to the loads. SGs, as the main sources of reactive power, play a determining
The reactive power capability of SG is limited by the maximum permissible field current. OEL limits the field current to its maximum permissible values at steady-state and transient conditions, denoted by \( i_{f}^{\text{lim}} \) and \( i_{f}^{\max} \), respectively [18]. While the maximum steady-state field current \( i_{f}^{\lim} \) can be exceeded for up to several seconds, the maximum transient field current, \( i_{f}^{\max} \) (typically 140% of \( i_{f}^{\lim} \)), should not be violated for more than several milliseconds. The transient OEL model reported in reference [18] is used in this thesis, as displayed in Fig. 5.21. When the field current exceeds \( i_{f}^{\max} \) and this lasts for a pre-set period of time, denoted by \( T_{d} \) in Fig. 5.21 (assumed to be 0 in this thesis), the binary variable \( Z_{OEL} \) takes the value of 1, which activates the transient OEL block. As a result, an output signal \( V_{OEL} \) is generated which is subtracted from the \( V_{\text{ref}} \) of the SG. This returns the field current within the permissible transient limit, and at the same time causes a decrease in the terminal voltage of the SG. This voltage drop propagates throughout the system and degrades the ST-VS of the grid. The effect of OEL on ST-VS of power system will be demonstrated in the simulation section.

Similar model to that of Fig. 5.21 applies for the steady-state OEL, with the difference that \( i_{f}^{\max} \) is replaced with \( i_{f}^{\lim} \) and the time delay is around 10 seconds [18]. The generated voltage signal of the steady-state OEL model is also added to the \( V_{OEL} \) signal output of the transient OEL model of Fig. 5.21. Since the time frame of interest in this section is few seconds after the fault, only the effect of transient OEL limit is shown here.

### 5.4.5 DVS Capability of ESSs

The capability of ESS devices for dynamic control of active and reactive power injections into the grid, as discussed in Section 5.4.1, renders them promising devices to provide DVS to power
Figure 5.22: (a) Time-overload capability of ESS’s inverter [103], (b) The applied function $f(\Delta V)$

systems during emergency events. DVS capability of ESSs, both during the fault and after the fault clearance, is investigated in this section as a potential countermeasure against ST-VI. The degree of DVS provided by the ESS is limited by the maximum current rating of its inverter, $I_{\text{max}}$. This can greatly limit the amount of power injection by the ESS during the fault, when considerable voltage drops occur. To alleviate this issue, the following countermeasures are proposed in this section:

1. **FRT capability of ESS:** Conventionally, the inverter-interfaced components in a power system (such as photovoltaic systems, statcoms, and ESSs) used to stop operating under exceedingly low voltages, for protection purposes [19]. However, some recent industrial inverters have the low-voltage ride through (or FRT) capabilities, which allows them to continue operation over the fault periods [103, 104]. Even though this capability is currently being used only in distribution level statcoms, by potential prevalence of this important feature of inverters in near future, more effective dynamic power injection by the ESSs can be carried out during the faults, to better improve ST-VS.

2. **TOL capability of inverters:** In addition to the FRT option, many of the existing new inverters can provide time-overload (TOL) capability as well [103]. It means that they can accommodate currents up to three times their rating currents for a limited period of time, during transient events. A typical TOL capability curve of the industrial inverters is shown in Fig. 5.22-(a). TOL capability can be used to increase the amount of power injection by the ESSs during the fault, to improve ST-VS.

The DVS capability of ESS in this section is implemented based on the following scheme: In
the normal operating condition (i.e. no-fault condition, when the voltages are within the acceptable range), ESS injects/absorbs power as per its operational scheme. The normal operation of ESS can be set based on economic, or reliability criteria, and is out of the scope of this study (transient and fault analysis). When a deep voltage sag (less than 0.9 p.u.) is detected, the DVS capability of ESS is activated. In this section, the DVS is implemented by dynamically regulating the apparent power injection of ESS as stated in equation (5.27).

\[
S_{ESS}(t) = |V_g(t)| \times |I_{max}(t)| \times f(\Delta V(t))
\]  

(5.27)

where \(V_g(t)\) is the instantaneous voltage measured at the PCC of the ESS, and \(I_{max}(t)\) is the maximum current capability of the ESS’s inverter. The TOL capability of the ESS’s inverter, as per Fig. 5.22-(a), is considered in specifying the value of \(I_{max}(t)\). \(f(\Delta V(t))\) is an exponential function of voltage drop, \(\Delta V = 1 - |V_g(t)|\), defined in this section as per Fig. 5.22-(b). It varies between 0 and 1 to dynamically adjust the percentage usage of the maximum power injection capability of ESS, according to the instantaneous voltage drop. As seen from Fig. 5.22-(b), \(f(\Delta V(t))\) converges to \(a\) (the power injection percentage of ESS at normal, pre-fault condition) when \(\Delta V\) approaches \(\Delta V_0\) (the pre-fault voltage drop at the ESS bus) and exponentially approaches 1 when \(\Delta V\) increases. Hence, an advantage of the proposed function \(f(\Delta V(t))\) is that both the voltage level and the ESS’s power injection automatically return to their pre-fault values after the voltage recovery, following a fault. It is also seen that by using \(f(\Delta V(t))\), the maximum current capability of inverter is used when the voltage drop exceeds 0.2 p.u. Having the value of \(S_{ESS}(t)\) from equation (5.27), the share of active and reactive power injections of ESS can be determined, as stated in equation (5.28):

\[
P_{ESS}(t) = S_{ESS}(t) \times PF_{ESS}
Q_{ESS}(t) = S_{ESS}(t) \times \sqrt{1 - PF_{ESS}^2}
\]  

(5.28)

where \(PF_{ESS}\) is the PF of ESS and determines the proportion of active and reactive power injections of the ESS. \(PF_{ESS}\) can be optimally identified, as explained in Section 5.5.5, to provide the most improvement in ST-VS. The obtained reference active and reactive power signals, stated in equation (5.28), are dynamically fed into the ESS’s control system to implement the proposed
5.5 Simulation Results for ST-VS Improvement Using ESS

5.5.1 Test System

Modified IEEE 30-bus test system, as shown in Fig. 5.4, is used as the case study in this section to demonstrate the effectiveness of the proposed method on ST-VS. Here again, the loading level of the system is increased by 60%, compared to the original loading, to create a more challenging case for the VS of the system. An ESS with a power rating of 25 MVA is installed at bus 30, i.e. the weakest bus of the test system, from VS viewpoint. Similar to Section 5.3, the transmission system is modelled by the algebraic load flow equations, and a 6th order dynamic model of the synchronous generators along with their first-order automatic voltage regulator (AVR) models are incorporated. AVRs are equipped with OELs as modelled in Section 5.4.4. 50% of the system load consists of IMs, and the rest of the load is modelled as static loads, 25% constant impedance and 25% constant power loads. The same static load models as used in Section 5.3.1 (equations (5.10) and (5.11)) are used here, with the parameter $\alpha$ in equation (5.11) being set to 1. Two FSIG-based WTs, as described in Section 5.4.3, with power ratings of 10 MW each, are installed at buses 19 and 26, which are among the weakest buses of the system. The FSIGs are assisted with capacitor banks to supply the reactive power required by the IG. Throughout this section, the wind farms are assumed to operate at the nominal wind speed of 12 m/s.

A symmetric three-phase to ground fault is assumed to occur at the middle of the transmission line between buses 27 and 28, the most critical line in the system from VS viewpoint, as shown in Fig 5.5. The fault is set to occur close to the load area, and relatively far from the generation centre, which is a common approach for analysis of ST-VS (also referred to as load stability). In order to evaluate the effects of FSIGs, OELs, and FRT and TOL capabilities of ESS on ST-VS, the following case studies are performed. In all cases, the fault occurs at $t = 1s$, and is cleared at $t = 1.1s$. 

DVS scheme.
5.5 Simulation Results for ST-VS Improvement Using ESS

5.5.2 The Effect of FSIG on ST-VS

To demonstrate the effect of FSIG on ST-VS two case studies have been simulated in this section. In the first case (case1), no FSIG is installed in the system, while in the second case (case2), two FSIGs, as described in Section 5.5.1, are installed at buses 19 and 26. No ESS is used at this stage. Fig. 5.23 shows the results of these two cases. Rotor slips of sample IMs are shown in Fig. 5.23-(a), for case1. As seen, three of the IMs stalled after the fault. As a result, ST-VI occurred and the system voltages cannot recover after the fault, as seen in Fig. 5.23-(b) for bus 30. In case2 (with FSIGs), ST-VS condition of the system is deteriorated, as seen in Fig. 5.23-(b), compared to the first case, characterized by lower voltages after the fault. The reason is that the FSIG at bus 26 has over-accelerated after the fault (Fig. 5.23-(c)) and, as a result, it draws high amount of reactive power from the grid, as seen in Fig. 5.23-(d), which further suppresses the system voltages.

5.5.3 The Effect of OEL on ST-VS

In this section, the effects of OELs of SGs on ST-VS are investigated. Three case studies are carried out, namely case3 to case5. In case3, the OEL is not included, i.e. there is no limit on the field current of the SGs. In case4 and case5, the field currents of all SGs are respectively limited to 80% and 60% of their maximum values, identified from case3. No ESS is used in these cases.
The results are presented in Fig. 5.24. Fig. 5.24-(a) shows that OELs’ action can greatly aggravate ST-VS, in terms of further suppressing the system voltages during and after the fault. It is seen that in case 5, where tighter limits are imposed on the field current of SGs (see Fig. 5.24-(b) for field current of G8, as an example), the post-fault voltages reduced to about 0.4 p.u. The reason can be explained by using figures 5.24-(c) to Fig. 5.24-(f). It is seen that in case 4 (Fig. 5.24-(e)) the terminal voltages of the SG’s dropped during the fault, due to the action of the OELs, which limit the field current, and thus, the reactive power output of the SGs. This caused more IMs to stall, compared to case 3. However, the SGs managed to retain their terminal voltages after the fault in this case. The situation is more critical in case 5 (Fig. 5.24-(f)), where excessive voltage drops after the fault clearing (due to the more number of stalled IMs and over-accelerated FSIGs in this case, as shown in figures 5.24-(c) and 5.24-(d)), cause the field current of some of the SGs (such as
5.5 Simulation Results for ST-VS Improvement Using ESS

G8 in Fig. 5.24-(b)) to reach their maximum limits. As a result, even after the fault clearance, the terminal voltages of the SGs cannot be maintained, as seen in Fig. 5.24-(f). Figures 5.24-(c) and 5.24-(d) show that many of the IMs stalled and both FSIGs over-accelerated due to the excessive voltage drops during the fault, which also deteriorates the voltage drop after the fault in case5. In fact, high reactive power demands of IMs and FSIGs after the fault push the SGs toward their field current limits, and this, again, exacerbates the voltage drop in the system.

5.5.4 The Effect of DVS Capability of ESSs on ST-VS

In this section, the use of the proposed ESS control approach to provide DVS to the power system, as a potential countermeasure against ST-VS, is investigated, taking into account the FRT and TOL capabilities of ESSs. As mentioned in Section 5.5.1, a 25 MVA ESS is installed at the weakest bus, i.e. bus 30. The field currents of the SGs are limited to 80% of their maximum values, as in case4 of Section 5.5.3. Several case studies are carried out as follows:

- **case6**: ESS has neither FRT nor TOL capabilities. It means that ESS starts power injection only when the fault is cleared, and its power injection is limited to its nominal current rating (which is 62.5 A).

- **case7**: ESS has FRT capability (it can inject power during the fault), however its power injection is limited to its nominal current rating.

- **case8**: ESS only has TOL capability, i.e. it starts power injection after fault clearance, and its maximum current capability is as per the TOL curve shown in Fig. 5.22-(a). The transient current capability of ESS is considered to be twice its continuous current capability throughout this section.

- **case9**: ESS has FRT and TOL capabilities, as per Fig. 5.22-(a).

In all of the cases above, the PF of the ESS during the transient events is set to be 0.7071 (equal active and reactive power injection). The effect of ESS’s power factor will be studied in Section 5.5.5. The results of the aforementioned case studies are illustrated in Fig. 5.25. Fig. 5.25-(a) shows the voltage of bus 30. Fig. 5.25-(b) presents the rotor speed of the FSIG at bus 26 (denoted by FSIG-26). Figures 5.25-(c) and 5.25-(d), respectively, show rotor slip of the IM at bus...
Figure 5.25: Effect of ESS’s DVS, FRT, and TOL capabilities

29 (denoted by IM-29) and the active power injection by the ESS during and after the fault (Note that ESS’s reactive power injection is identical to the active power injection in all these cases).

As seen, without DVS of ESS, FSIG-26 over-accelerates and IM-29 stalls. As a result, the voltages cannot recover after the fault. In case6, where the ESS is used, but without FRT or TOL capabilities, the system’s ST-VS is improved, compared to the no-DVS case, due to the less number of stalled IMs in this case. However, since there is no voltage support during the fault, and the power injection of the ESS after the fault clearing is limited to its nominal current rating, the injected power by the ESS, seen in Fig. 5.25-(d), is not sufficient to recover the voltages. In case7, in which the FRT capability of the ESS has been added, compared to case6, the ESS starts power injection immediately after the fault occurrence, up to its nominal current rating. As seen from Fig. 5.25-(a), this has helped the system voltages to recover, although with a delay of about 1.5 seconds. As seen in Fig. 5.25-(d), the power injection by the ESS reduces to the pre-fault level after the voltage recovery, as per the designed function $f(\Delta V(t))$. In case8, the ESS has no FRT capability, but it can inject up to twice its nominal current rating after the fault clearing, owing to its TOL capability. As shown, the ST-VS of the system is considerably improved, i.e. the FSIGs’ and IMs’ speed and bus voltages recovered much faster, compared to case7. Note that, as seen in Fig. 5.25-(d), the transient current capability of ESS is used only for a very short period of time
Table 5.2: CCT (sec.) for different cases and different PFs of ESS

<table>
<thead>
<tr>
<th>PF of ESS</th>
<th>No DVS</th>
<th>case6</th>
<th>case7</th>
<th>case8</th>
<th>case9</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0000</td>
<td>0.075</td>
<td>0.078</td>
<td>0.085</td>
<td>0.091</td>
<td>0.098</td>
</tr>
<tr>
<td>0.7071</td>
<td>0.075</td>
<td>0.096</td>
<td>0.105</td>
<td>0.113</td>
<td>0.131</td>
</tr>
<tr>
<td>0.3000</td>
<td>0.075</td>
<td>0.100</td>
<td>0.111</td>
<td>0.120</td>
<td>0.139</td>
</tr>
<tr>
<td>0.0000</td>
<td>0.075</td>
<td>0.098</td>
<td>0.109</td>
<td>0.117</td>
<td>0.135</td>
</tr>
</tbody>
</table>

(less than a second). Finally, it is observed from Fig. 5.25 that in case9, where both FRT and TOL capabilities of the ESS are included, the ST-VS of the system improved compared to all the previous cases, in terms of faster voltage recovery. Fig. 5.25-(d) shows that the power injection capability of ESS greatly increased during the fault by using both FRT and TOL capabilities of ESS in this case, which better boosted the voltages during the fault (Fig. 5.25-(a)).

The CCT values for all the above-mentioned cases are compared in Table 5.2 (second row, shown in bold). As seen, in no-DVS case and case6, where ST-VI occurs, the CCT is smaller than 0.1s, i.e. the fault clearing time (FCT) of the mentioned cases. Also, it is seen that the CCT increases from case6 to case9, owing to the added FRT and TOL capabilities. In addition, Fig. 5.26 illustrates the effect of different FCTs for case9. As seen, the longer the fault remains in the system, the later the system state variables (IMs’ rotor slip, FSIGs’ rotor speed, and system voltages) recover, and eventually, if the fault remains in the system beyond CCT, the system voltages collapse (the red curves in Fig. 5.26).

5.5.5 The Effect of ESS’s PF on ST-VS

In this section, the effect of ESS’s PF ($PF_{ESS}$) on ST-VS is investigated. Table 5.2 shows the CCT values for the cases similar to those of Section 5.5.4, but with different PFs of ESS. $PF_{ESS}$ determines the share of active and reactive power injections by the ESS as per equation (5.28). As seen, the $PF_{ESS}$ affects the CCT in all cases. It is seen from Table 5.2 that operating the ESS at lower PFs (i.e. more reactive power injection) after the fault, results in higher CCTs, i.e. better ST-VS for the system, compared to when higher PFs are used. Also, it is seen that when the $PF_{ESS}$ equals 0.3, the highest CCT values are achieved. The implication is that in order to make the most of the existing current capability of the ESS during and after the fault, ESS should be operated at an optimal PF; which is the PF at which the system voltages are boosted the most. The reason
lies in the dynamic behaviour of the IMs and FSIGs. As discussed in Sections 5.4.2 and 5.4.3, electromagnetic torques of IMs and FSIGs greatly decrease with the voltage drop. The deeper the voltage drop during the fault is and the longer it lasts in the system, the more likely th IMs stall and the FSIGs over-accelerate, causing the system voltages to collapse.

The voltage increase at bus 30 ($\Delta V$) due to a 25 MVA apparent power injection by the ESS at bus 30, at different PFs, has been computed and the results are presented in Fig. 5.27-(a). As seen, the largest voltage increase occurs at PF=0.3, which indicates higher effect of reactive power injection in boosting the bus voltages. This is consistent with the fact that the system has a relatively low $R/X$ ratio, i.e. 0.37 in average. Hence, operating ESS at 0.3 PF will best improve the ST-VS too. This outcome is also verified by the simulation results shown in Fig. 5.27-(b), where the effect of ESS’s PF on ST-VS of the system is evaluated. The results are for the cases similar to case7 in Section 5.5.4, but with different $P_{\text{ESS}}$. As seen, the fastest voltage recovery occurs at 0.3 PF. Also, it is seen that unity PF leads to the worst performance, which is consistent with the results of Fig. 5.27-(a).

Notice that although the operating condition of the power systems continuously changes, it is unlikely to affect the optimal $P_{\text{ESS}}$ considerably. The optimal $P_{\text{ESS}}$ depends on the system’s structure and its $R/X$ ratio, both of which do not vary frequently. However, in case of any signifi-
In this chapter, dynamic control of ESSs is used to improve the VS of power system. First, a method is proposed to dynamically provide a desired VSM for the power system at normal and single-contingency operating conditions. The reference powers for the control system of the ESSs are generated, such that a desired VSM is achieved for the system, under all load-wind combinations. It is shown that using the proposed approach the post-contingency voltage levels (for voltage-dependent loads) and VS of the system (for constant-power loads) are significantly improved. Also, it is observed that in the case of constant-power loads, a post-contingency voltage collapse can be effectively prevented by using the proposed operation and control of ESS. This allows the system operator to take necessary corrective actions to recover the system to the normal condition.

In the second part of this chapter, the ST-VS of power systems is analysed. The detrimental effects of IMs, FSIG-based WTs, as well as the OELs of SGs on ST-VS are evaluated in detail, using several case studies. It is shown that ESS can be controlled, based on local voltage measurement, to provide DVS to the system during and after the fault to maintain ST-VS. Two other countermeasures, namely FRT and TOL capabilities of ESS, are also proposed and their effects on ST-VS are evaluated. The effect of PF of the ESS’s power injection on ST-VS is studied too. It is shown
that the DVS capability of ESS, obtained through the control of ESS’s active and reactive power injection, can significantly improve the ST-VS, in terms of faster voltage recovery and increased CCT. It is also shown that this improvement greatly increases if the ESS is equipped with FRT and TOL capabilities. In addition, it is shown that operating the ESS at the optimal PF, i.e. the PF providing the largest boost in the ESS’s terminal voltage, results in the best ST-VS improvement. This was explained based on the dynamic behaviours of IMs and FSIGs.
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Chapter 6

Conclusion

6.1 Conclusions

In this thesis, different aspects of power system VS are investigated. These include steady-state VS analysis and improvement, VS-constrained stochastic allocation of ESS devices, and dynamic ST-VS evaluation and enhancement of power systems. Improved, efficient tools for evaluating VS of the power system are presented, which eliminate some of the disadvantages of the existing methods. Also, new approaches, mainly using ESS devices, are proposed to improve/ensure both steady-state and short term VS of power systems.

In the second chapter of this thesis, a new, efficient method is presented for drawing complete P-V curves and identifying both SNB and LIB points of power systems, with any prescribed accuracy. The significant advantages of the proposed method, compared to the existing CPF-based approaches, are its high efficiency, in terms of low execution time and high accuracy, ease of implementation and comprehension, and automated applicability. The simulation results carried out on several test power systems demonstrate the advantages of the proposed method, when applied to power systems of different sizes and configurations. In the second part of Chapter 2, two modified MA approaches, applicable to distribution power systems, are presented. The advantage of the propose MA method, compared to the original MA, is that they do not ignore the active power variation. This enables the proposed MA methods to more accurately identify the best buses for applying active/reactive power compensation, with the objective of increasing the VSM of distribution power system. The effectiveness of the proposed MA methods is verified through VSM analysis and comparison with the results of L-index approach.

In the third chapter of the thesis, the use of ESS devices for improving VSM of power systems
is investigated. A method for clustering power system buses, using MA results, is presented. It is shown that the proposed clustering method provides a foundation for systematic placement of ESSs, with the aim of improving VS and voltage profile of the entire system. ESS devices are, then, optimally placed and operated in the network, using a combination of CPF and MA, with the aim of securing a desired VSM for the system. It is shown that by operating the storage devices at their optimal PFs, the most improvement in system’s VSM can be achieved. In addition, a method for power sharing between ESSs, based on their effectiveness on system’s VSM at different wind power levels, is presented. The simulation results show that a desired VSM can be effectively ensured at all operating condition, i.e. all wind-load combinations, using the proposed optimal placement and operation of ESSs. This ensures a secure distance from voltage instability points, regardless of wind and load variations.

In Chapter 4, the problem of optimal ESS allocation is formulated as two different probabilistic optimization approaches. A desired VSM is ensured through inclusion of VS constraints in the optimal allocation problems, and with the use of ANM schemes. The operational (voltage and thermal) constraints of the system are also taken into account. In the first approach, presented in this chapter, the required power rating of ESSs are computed to assure a desired VSM for a transmission system. The stochastic behaviours of wind and load are modelled using their corresponding PDFs. It is shown that a desired VSM can be ensured for the system, under all load-wind combinations, through optimal placement, sizing, and operation of ESS devices. The simulation results show that while securing higher levels of VSM requires much larger ESS installation, the use of ANM tools can considerably decrease the required ESS size to achieve the same VSM. Also, it is shown that by 10% over-rating the inverters of ESSs and wind farms, used for reactive power injection, the required ESS installation can be significantly reduced. In the second approach, a new, risk-based, VS-constrained, stochastic ESS allocation framework is presented to achieve a desired VSM for a distribution power system. The new approach allows for computing both required power and energy ratings of ESSs. The reactive power import, from the upstream transmission network, and the reactive power loss, in the distribution system, are also minimized, using a multi-objective optimization approach. Wind uncertainty in accounted for using wind power scenarios, generated using ARIMA forecast method and considering the wind forecasting error. In addition, a new method for inclusion of OLTCs’ action in optimal power flow formulation
is presented. The simulation results for this approach show that the minimum required power and energy rating of ESS devices, optimally sized and placed in the system, can be computed by the proposed method, such that a desired VSM is always ensured. It is also shown that by inclusion of the OLTCs’ action, less ESS is required to secure the same level of VSM. In addition, risk-based VS constraints, which allow for small violation of the operational constraints at the maximum desired loading level, are shown to significantly reduce the required ESS installation.

Finally, ST-VS of power systems is investigated in Chapter 5. First, a method of dynamically assuring a desired VSM for power systems, using dynamic control of ESS devices, is presented. In order to do so, ESS is controlled to track any reference active and reactive power signals. The proposed method accounts for single contingency criterion and the effects of voltage-dependant and constant-power loads are evaluated as well. It is shown that the existence of a post contingency operating point can be ensured by proper operation and control of ESS devices. In the second part of this chapter, the problem of ST-VS is investigated. The study takes into account the detailed analysis of dynamic behaviour of IM loads, FSIG-based WTs, and OEL of SGs, using several illustrative examples and case studies, which provide a deeper insight into the ST-VS problem. Then, a new method of providing DVS to the system by dynamic control of ESSs is also presented, taking into account the FRT and TOL capabilities of ESS’s inverter. The simulation results verify that the proposed method improves ST-VS of the system, in terms of faster voltage recovery after the fault and larger CCT values. Also, it is shown that using the ESS’s DVS capability at the optimal PF results in the most improvement in ST-VS of the grid.

6.2 Future work

Despite the extensive research carried out on power system VS, there are still potential research directions in this area. From an analytical viewpoint, inclusion of more accurate limitations of power system components in the CPF method can be considered. The P-Q curve capability of synchronous generators, the reactive power limits of the inverter-interfaced DGs and ESSs, the action and limitation of OLTCs, etc. can be considered in this regard. This can result in a more accurate identification of SNB and LIB points, reflecting the real operation of power systems.

From a steady-state point of view, optimal ESS dimensioning to increase the hosting capaci-
ity of a distribution power system, while ensuring a desired VSM, can be an interesting, possible research direction. The stochastic nature of load and wind, as well as the statistical correlation between them, can be incorporated into the probabilistic optimization framework. More ANM schemes, including distribution system reconfiguration, can be considered too. Besides, the required cost of different storage technologies, including their degradation and installation costs, to achieve certain levels of hosting capacity and VSM, can be evaluated and compared, to find out the most economic storage technology.

VS-constrained OPF (VSC-OPF) is used to include VS considerations in the power market solutions of heavily-loaded power systems. The generation schedule resulted from VSC-OPF deviate from the cheaper generation schedule obtained from normal OPF, due to the added VS constraints. The cost-benefit analysis of using storage devices in conjunction with generators, such that the deviation (between the results of OPF and VSC-OPF) is compensated by the ESSs, is an interesting potential research direction. Besides, to achieve any desired level of VSM, the costs of the required network upgrade can be evaluated against the costs of ESS installation, to find out the more economical option.

From a dynamic viewpoint, the FRT capability of inverter-interfaced renewable energy resources, such as DFIGs and PV systems, can be used in coordination with ESS devices, for improving ST-VS of the power system. Also, preventive adjustment of OLTCs to increase the reactive power reserve of ESSs and inverter-interfaced DGs, to be used during the fault conditions against ST-VI, is a potential research direction which deserves further investigation.
Bibliography


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Title:
Voltage stability issues in power grids: analysis and solutions

Date:
2017

Persistent Link:
http://hdl.handle.net/11343/208897

File Description:
Main thesis document

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