

ON THE WELFARE EFFECTS OF CREDIT ARRANGEMENTS*

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This paper studies the welfare effects of credit arrangements and how these effects depend on the trading mechanism and inflation. In a competitive market, credit arrangements can be welfare reducing, because high consumption by credit-users drives up the price level, reducing consumption by money-users who are subject to a binding liquidity constraint. By adopting an optimal trading mechanism, however, these welfare implications can be overturned. Both price discrimination and non-linear pricing are essential features of an optimal mechanism.

Running head: Welfare and Credit

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1 Introduction

Recent policy debates on regulating the retail payments system are motivated by concerns about the efficiency and welfare implications of different payment instruments and their pricing schemes. Conducting policy analysis on these issues from first principles requires a general equilibrium model in which the fundamental roles of different payment arrangements are explicitly captured. Thanks to recent developments in monetary theory², it is now widely recognized that, in an environment with imperfect information and limited commitment, money is essential as a means of payment.³ Moreover, the allocation in a monetary economy is typically inefficient when some agents are money constrained (for example, due to sub-optimal monetary policy or liquidity shocks). As a result, some forms of credit arrangement may help improve efficiency by relaxing agents' liquidity constraints. What remain less well understood are the welfare effects of different credit arrangements and their interaction with monetary policy and the trading mechanism. This paper is an attempt to use modern, micro-founded monetary theory to address these issues.

Specifically, this paper investigates the following questions: Does availability of credit always improve social welfare in a competitive environment? If not, what is the source of inefficiency? What sorts of trading/pricing mechanisms are needed to mitigate this inefficiency? Do technologies of production, trading, and enforcement matter for these questions?

Let us briefly describe the model and give the basic intuition behind our findings. Owing to information frictions in the goods market, buyers need to acquire non-interest-bearing money in order to trade for consumption. Money-users, therefore, bear a cost of holding liquidity which is particularly onerous when inflation is high. Credit-users, however, can economize on the use of cash and (at least partially) avoid this inflation tax by acquiring credit from a bank which channels funds from unconstrained to constrained agents. There-

²Williamson and Wright (2010) and Lagos et al. (2017) describe the most recent development in this literature.

³See, for example, Kocherlakota (1998).

fore, inflation has differential effects on money-users and credit-users, and may generate a redistributive effect across these two types, reducing money-users' consumption more than credit-users' consumption. Overall, a deviation from the Friedman rule is sub-optimal, and inflation is welfare reducing in a competitive environment.

Since the cost of using money is increasing in the inflation rate, one may expect that allowing more buyers to use credit can always enhance welfare because a buyer, by gaining access to credit, can now avoid the inflation tax and enjoy a higher level of consumption. We call this the “composition effect” because an economy composing of more credit-users, *ceteris paribus*, has a higher level of welfare. But this is only a partial equilibrium argument. There is an additional general equilibrium “price effect”: an increase in consumption of new credit-users will drive up the market price and reduce money-users' consumption. At first glance, this pecuniary externality should not lead to any welfare loss according to standard arguments. One needs to notice, however, that the first welfare theorem can fail when there are distortions in the economy, and pecuniary externalities can have welfare consequences (Greenwald and Stiglitz, 1986).⁴ In the current environment, the presence of binding liquidity constraints implies that more people using credit can tighten money-users' liquidity constraints and hence lower the aggregate welfare whenever the price effect dominates the composition effect. As a result, with competitive pricing, the introduction of credit services can be welfare reducing. These are the main findings of the benchmark model discussed in Section 3.

Since the negative effects of inflation and credit hinge on the assumption of competitive pricing, it is natural to ask whether such inefficiencies can be mitigated by the optimal design of the trading mechanism and pricing protocol. Instead of focusing on one specific trading mechanism, we employ the mechanism design approach to solve for the set of optimal allocations subject to technological and incentive feasibility constraints. The welfare implications of inflation and credit arrangements are significantly different under the optimal

⁴Hart (1975) also points out that adding an additional asset to an environment with incomplete financial markets can actually decrease welfare.

trading mechanism. First, deviation from the Friedman rule is not necessarily sub-optimal. By appropriately splitting the trade surpluses of different parties, the first-best allocation can be supported for low inflation. Second, under the optimal trading mechanism, the price effect can now be internalized and thus the provision of credit by banks becomes welfare improving. We also show that non-linear pricing and price discrimination are essential features of an optimal trading mechanism, and that the optimal allocation can be supported in a competitive market with an appropriately designed tax-subsidy scheme. These results will be derived in Section 4. In Section 5, we calibrate the model to US data and illustrate that increasing access to credit can have a negative welfare impact for parameter values commonly used in the literature.

Our model follows the money and banking model developed by Berentsen et al. (2007), which in turn is based on Rocheteau and Wright (2005). Building on this framework, a growing literature examines the role of credit in a monetary economy.⁵ Some quantitative analyses have been conducted and the findings are consistent with our model's implications. For example, Rojas Breu (2013) uses a comparable model calibrated to the US economy and credit card usage, and finds that the increase in access to credit from 1990 to 2003 has had a slightly negative impact on welfare. Another closely related work is by Monnet and Roberds (2008), who show that the optimal pricing policy in a credit system internalizes externalities generated by the participation in the credit system. Interestingly, they find that credit is always welfare improving and the no-price-discrimination is necessary for the existence of a credit equilibrium. The main difference between their paper and our paper is

⁵Recent work on related ideas includes Sanches and Williamson (2010), Chiu and Meh (2011), Dong (2011), Sanches (2011), Gu et al. (2013), Gu et al. (2016) and Lotz and Zhang (2016). There are also a lot of papers on money and credit using earlier versions of monetary search models. For example, Corbae and Ritter (2004) study long-term partnerships in an indivisible goods search model of Kiyotaki and Wright (1993) by allowing matched pairs of agents to stay together, as long as it is mutually beneficial. They show that the introduction of money can affect credit partnerships by weakening incentives for partners with money to produce, leading to welfare loss.

that some agents in Monnet and Roberds (2008) are always liquidity constrained: without credit, they cannot consume even at the Friedman rule. Thus, the welfare gains from an increase in consumption via access to credit outweigh any other costs. If price discrimination is allowed, given that credit-users faces higher prices, people will have less incentive to use credit, leading to a less sustainable credit arrangement. Nevertheless, the negative externality we highlight in this paper should exist in their model if the above assumption is relaxed.

The welfare implication of our paper is also related to that of Berentsen et al. (2014) as both papers examine the potential negative effects of financial activities on welfare due to pecuniary externalities. However, the two papers present different channels through which pecuniary externalities could reduce welfare. In our paper, allowing more agents to use credit increases the demand for goods, driving up the marginal cost and the price of goods and thus tightens other agents' liquidity constraints. This channel is not modelled in Berentsen et al. (2014) because trades in their goods market are bilateral, and the marginal cost of production is constant. In contrast, in their environment, allowing more agents to go to the bond market reduces their *ex-ante* demand for money and thus decreases its equilibrium value, tightening other agents' liquidity constraints. This channel relies on the assumption that bond market entry is random, and that this entry shock is realized after the asset portfolio is made. This effect will disappear in an environment where access to the financial market is known in advance, as in our paper. We argue that this is a more appropriate assumption when we study the usage of credit for daily consumption needs. Therefore, the two papers are complementary in the sense that they focus on different channels that can give rise to non-trivial pecuniary externalities. Finally, by focusing on pecuniary externalities in normal times, our work is also very different from those macro literature studying fire sales during a financial crisis.

The rest of the paper proceeds as follows. Section 2 describes the basic model. Section 3 derives the welfare effects of monetary policy and credit arrangements in a competitive environment. Section 4 revisits the questions under the optimal trading arrangements applying the mechanism design approach. Section 5 conducts a quantitative exercise in a calibrated

economy. Section 6 discusses several extensions and policy implications and Section 7 concludes. Formal proofs of lemmas and propositions can be found in the Appendix.

2 Benchmark Model

The basic economic environment is similar to Berentsen et al. (2007). Time is discrete and runs forever. Each period is divided into two subperiods, day and night. Agents meet at a Walrasian market in both subperiods. There is a continuum of infinitely-lived agents who differ along three dimensions. First, they permanently belong to one of two groups in the day market, called *buyers* and *sellers*. We normalize the measure of each group to 1. Second, all buyers experience preference shocks during the day: with probability π , a buyer wants to consume, while with complementary probability, the buyer does not want to consume. These shocks are i.i.d. across agents and time. Third, only a fraction α of buyers have access to a banking sector.⁶ All sellers have access to banking.

In the night market all agents produce and consume, but in the day market a buyer can only consume and a seller can only produce. Goods are perishable. Moreover, all goods trades are anonymous during the day, and all histories of goods trading are private information. As a result, sellers require immediate payment and a medium of exchange is needed to facilitate trades. There exists fiat money which is perfectly divisible. The supply M grows at a constant gross rate γ . New money is injected ($\gamma > 1$) or withdrawn ($\gamma < 1$) via lump sum transfers $\tau M = (\gamma - 1) M$ to all agents at the beginning of night. We restrict attention to policies where $\gamma \geq \beta$, where $\beta \in (0, 1)$ is the discount factor.

For a seller who produces q units of output during the day, consumes x units of output and produces y units of output at night, the instantaneous utility is $-c(q) + v(x) - y$. We assume that $v'(x) > 0$, $v''(x) < 0$ for all x , and there exists $x^* > 0$ such that $v'(x^*) = 1$. The cost of production satisfies $c(0) = 0$, $c'(q) > 0$, $c''(q) \geq 0$.

⁶We assume this type is permanent. Having i.i.d. types, however, will not affect the main results of the paper.

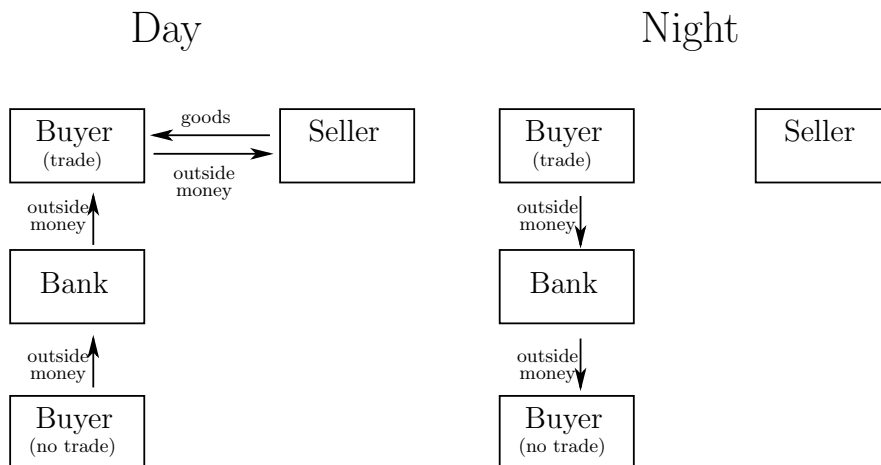


Figure 1: Flow of Funds

Similarly, the instantaneous utility of a buyer is $\varepsilon u(q) + v(x) - y$, where q is the quantity consumed during the day and $\varepsilon \in \{0, 1\}$ is the i.i.d. preference shock, with $\Pr(\varepsilon = 1) = \pi$ and $\Pr(\varepsilon = 0) = 1 - \pi$. The utility function $u(q)$ satisfies $u(0) = 0$, $u'(0) = +\infty$, $u'(q) > 0$, and $u''(q) < 0$.

In the banking sector, there are competitive banks that can make credit arrangements, as in Berentsen et al. (2007). Banks possess a record keeping technology that can keep track of financial histories, but not trading histories in the goods market. Since record keeping is only available for financial transactions, trade credit between buyers and sellers is not feasible. Instead, banks can make nominal loans and take deposits. These financial services are available only at the beginning of the day after the preference shock is realized and before goods trading. Finally, without loss of generality, we assume that all financial contracts are one-period contracts and thus loans and deposits are not rolled over across periods. Banks can commit to repay their depositors. Banks can also perfectly enforce loan repayment by the borrowers in the benchmark model. We consider limited enforcement of loan repayment as an extension in the Online Appendix. The flow of funds is described in Figure 1.

In Berentsen et al. (2007), banks are subject to a cash-in-advance constraint when making loans, in the sense that all loans have to be backed by money deposit.⁷ An alternative credit

⁷Berentsen et al. (2007) provide an alternative interpretation of this restriction: banks can create trans-

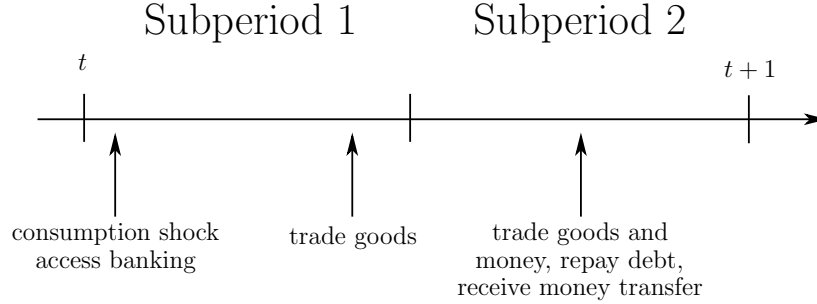


Figure 2: Timeline of Events

arrangement is where banks are not subject to the cash-in-advance constraint so that loans (or credit) can be used as a payment instrument. This extension is examined in the Online Appendix and discussed in Section 6.

The timing in our model is as follows (see Figure 2). At the beginning of each period, buyers observe their preference shocks and the banking sector opens where agents with access to it can borrow loans or make deposits. Then, the banking sector closes and agents trade goods in the day market. Agents receive lump-sum transfers τM , consume and produce as well as settle financial claims at night.

2.1 Night Market Problem

Let b denote a buyer who has access to banking, n denote a buyer who does not have access to banking, and s denote a seller. Let $W^j(m, \ell, d)$ be the value function of a type $j \in \{b, n, s\}$ agent who enters the night market holding m units of money, ℓ loans and d deposits. Denote an agent j 's value function of carrying m units of money into the day market by $V^j(m)$. We normalize the price of the consumption good in the night market to 1 and denote the value of 1 unit of money in units of this numeraire good by ϕ . The value function of agent j in the

ferrable IOU's but these IOU's have to be fully backed by outside money. That is, there is a 100% reserve requirement.

night market is

$$(1) \quad W^j(m, \ell, d) = \max_{x, y, m_+} [v(x) - y + \beta V^j(m_+)]$$

$$(2) \quad \text{st. } x + \phi m_+ = y + \phi(m + \tau M) + \phi(1 + r^d)d - \phi(1 + r)\ell,$$

where r is the nominal loan rate and r^d is the nominal deposit rate. Type n agents cannot use banks and thus have $\ell = d = 0$. Substituting (2) into (1), the problem simplifies to

$$W^j(m, \ell, d) = \phi [m + \tau M - (1 + r)\ell + (1 + r^d)d] \\ + \max_{x, m_+} [v(x) - x - \phi m_+ + \beta V^j(m_+)].$$

The first order conditions are $v'(x) = 1$ and

$$(3) \quad \beta \frac{dV^j(m_+)}{dm_+} \leq \phi, \text{ “=” if } m_+ > 0.$$

It follows that the optimal choice of (x, m_+) is independent of (m, ℓ, d) for all agents. This is a natural result from assuming quasi-linear utility in the night market, as first formalized by Lagos and Wright (2005). The envelope conditions imply that $\partial W^j(m, \ell, d) / \partial m = \phi$, $\partial W^j(m, \ell, d) / \partial \ell = -\phi(1 + r)$, and $\partial W^j(m, \ell, d) / \partial d = \phi(1 + r^d)$. The value function $W^j(m, \ell, d)$ is linear in (m, ℓ, d) .

2.2 Day Market Problem

In the day market, the value of a seller who holds m^s units of money at the beginning of the day market is given by

$$V^s(m^s) = \max_{q^s, \ell, d} [-c(q^s) + W^s(m^s + \ell - d + pq^s, \ell, d)] \text{ st. } d \leq m^s,$$

where p is the competitive price of goods during the day. Let λ_d^s be the Lagrangian multiplier, the first order conditions are $\phi r = 0$, $\phi r^d = \lambda_d^s$, and

$$(4) \quad c'(q^s) = \phi p,$$

It is immediate that a seller will not borrow unless $r = 0$, and will deposit all the money holding (i.e. $d = m^s$) whenever $r^d > 0$. The envelope condition thus gives $dV^s(m)/dm = \phi(1 + r^d)$. From (3), a seller's demand for money satisfies

$$(5) \quad r^d \leq \frac{\gamma - \beta}{\beta}, \text{ " = " if } m^s > 0.$$

That is, sellers may hold 0 or any positive amount of money depending on the deposit rate. If the deposit rate is strictly less than $\gamma/\beta - 1$, then sellers strictly prefer holding zero money balances.

All buyers experience i.i.d. preference shocks at the beginning of the day. Consider first the buyers who have access to banking. One can show that those who want to consume will never deposit money in the bank and those who do not want to consume will never take out loans. Hence, the value of a buyer holding m^b units of money in the day market is

$$(6) \quad V^b(m^b) = \max_{q^b, \ell, d} \pi [u(q^b) + W^b(m^b + \ell - pq^b, \ell, 0)] \\ + (1 - \pi) W^b(m^b - d, 0, d),$$

$$(7) \quad \text{st.} \quad pq^b \leq m^b + \ell,$$

$$(8) \quad d \leq m^b.$$

Let λ_q and λ_d denote the Lagrangian multipliers for (7) and (8), respectively. As $W^b(m, \ell, d)$

is linear in (m, ℓ, d) , we can derive the first order conditions as

$$(9) \quad \pi u' (q^b) = (\pi\phi + \lambda_q) p,$$

$$(10) \quad \pi\phi r = \lambda_q,$$

$$(11) \quad (1 - \pi) \phi r^d = \lambda_d.$$

If r is positive, (10) implies that the liquidity constraint (7) must be binding in a monetary equilibrium (i.e. $\phi > 0$). Similarly, if r^d is positive, the deposit constraint (8) must be binding. Combining (4), (9), and (10), we obtain

$$(12) \quad \frac{u' (q^b)}{c' (q^s)} = 1 + \frac{\lambda_q}{\pi\phi} = 1 + r.$$

The envelope condition of $V^b (m^b)$ gives

$$(13) \quad \frac{dV^b (m^b)}{dm^b} = \phi \left[\pi \frac{u' (q^b)}{c' (q^s)} + (1 - \pi) (1 + r^d) \right].$$

Plugging (13) into (3), we can show that a type b buyer's demand for money satisfies

$$(14) \quad \pi \left[\frac{u' (q^b)}{c' (q^s)} - 1 \right] \leq \frac{\gamma - \beta}{\beta} - (1 - \pi) r^d, \text{ " = " if } m_+^b > 0.$$

For those buyers who cannot use banks, the value of holding m^n at the beginning of the day market is

$$(15) \quad V^n (m^n) = \max_{q^n} \pi [u (q^n) + W^n (m^n - pq^n)] + (1 - \pi) W^n (m^n) \text{ st. } pq^n \leq m^n.$$

One can show that the constraint $pq^n = m^n$ must be binding. The first order condition

implies a type n buyer's money demand satisfies

$$(16) \quad \pi \left[\frac{u'(q^n)}{c'(q^s)} - 1 \right] = \frac{\gamma - \beta}{\beta}.$$

Comparing (14) and (16), we can see that $q^n < q^b$ for any $\gamma > \beta$ and $r^d > 0$. As long as the deposit rate is positive, type b buyers enjoy a higher q^b because they can take out loans to expand their consumption. Finally, the goods market clearing condition is

$$(17) \quad q^s = \pi [\alpha q^b + (1 - \alpha) q^n].$$

2.3 Banking Problem

In the benchmark economy which we call the outside-money loan economy, the sizes of the loans are constrained by the amount of deposits that a bank has, so banks can only lend out *outside-money loans*. In the day market, only (outside) money is accepted as a means of payment. Therefore, in this economy, banks take deposits and make loans to channel money balances across agents, but they do not provide any payment services. Competitive banks take as given the market rates r^d and r and choose the amount of money deposits D and money loans L to solve

$$(18) \quad \max_{L,D} (rL - r^d D)$$

subject to $L \leq D$. It is straightforward that, in equilibrium,

$$(19) \quad \left\{ \begin{array}{l} r = r^d, \\ D \geq L, \\ r^d(D - L) = 0. \end{array} \right.$$

That is, the deposit rate and the loan rate are equal. The interest rates are positive, unless there is an excess supply of deposits.

To complete the description of the banking sector, the loan market clearing condition is

$$(20) \quad D - L = m^s + \alpha m^b - \pi \alpha p q^b.$$

The amount of deposits net of loans $D - L$ equals to the supply of deposits $m^s + \alpha m^b$ net of the demand for loans $\pi \alpha p q^b$. Since banks can only make outside-money loans, they are subject to a cash-in-advance constraint, implying that $D - L \geq 0$.

2.4 Equilibrium

Having solved for the optimal decisions by buyers, sellers and banks, we combine these decisions to define a steady state equilibrium. For the outside-money loan economy, using (5), (12), (14), (16), (19) and (20), we can characterize a steady state equilibrium as a list of (r, q^b, q^n, q^s) satisfying (17) and

$$(21) \quad r = i,$$

$$(22) \quad \frac{u'(q^n)}{c'(q^s)} = 1 + \frac{i}{\pi},$$

$$(23) \quad \frac{u'(q^b)}{c'(q^s)} = 1 + i.$$

Here $i = \gamma/\beta - 1$ is the nominal interest rate for a loan between two consecutive night markets.

In equilibrium, sellers and type b buyers are just indifferent between bringing money to the banking sector or not (as long as some agents bring money to deposit). There is an indeterminacy with respect to their money holdings. However, their individual money holdings are payoff-equivalent and thus irrelevant for real allocations and welfare. Note that, q^b and q^n are not independent unless $c''(q^s) = 0$. In the special case when $\alpha = 0$, no one

has access to banking in the economy. This is equivalent to a pure monetary economy. The steady state equilibrium is a list of (q^n, q^s) that satisfies $q^s = \pi q^n$ and (22).

The aggregate welfare is defined as

$$(24) \quad \mathcal{W} = 2v(x) - 2x + \pi\alpha u(q^b) + \pi(1 - \alpha)u(q^n) - c(q^s),$$

where x is determined independently by $v'(x) = 1$. Given this definition, the first-best allocation can be found by maximizing (24) subject to (17). The solution of (q^b, q^n, q^s) satisfies

$$u'(q^b) = u'(q^n) = c'(q^s) \text{ and } q^s = \pi q^b = \pi q^n.$$

We define q^* as the solution to the first-best (q^b, q^n) so that $q^s = \pi q^b = \pi q^n = \pi q^*$.

Before we examine the central question of how the availability of credit affects the social welfare, it would be useful to understand the effects of inflation on equilibrium allocations and the social welfare. Since the derivation is standard and shared by previous monetary search models, we will only summarize the basic intuition here and leave the formal argument in the appendix. In this economy, inflation has two different effects depending on the types of buyers. First, higher inflation raises the liquidity costs for both types of buyers, inducing them to reduce consumption and thus aggregate output q^s drops. Second, when q^s falls, the marginal cost of production goes down too, which partially offsets the inflation cost. The net effect is the sum of the two. Proposition A.1 in the appendix shows that inflation always reduces a money-user's consumption, while it may increase or decrease a credit-user's consumption.⁸ In general, inflation has a smaller effect on credit-users than on money-users because using credit allows agents to partially avoid the inflation tax. Given that inflation reduces (q^n, q^s) and possibly q^b , it is not surprising that inflation reduces welfare. One can also show that the Friedman rule is the optimal monetary policy.

⁸The proof of Proposition A.1 in the appendix shows that one can impose additional assumptions to ensure that $dq^b/di < 0$.

3 Access to Credit and Welfare in Competitive Equilibrium

Given that inflation is welfare reducing, and that inflation typically is less costly for credit-users than for money-users, it is natural to expect that increasing access to credit is welfare improving. The following lemma and proposition establish that this is not true in general.

Lemma 1 *Effects of credit on allocation: $dq^n/d\alpha \leq 0$, $dq^b/d\alpha \leq 0$ and $dq^s/d\alpha > 0$, with strict inequalities if and only if $c'' > 0$.*

Lemma 1 shows that as α rises, q^s increases, but both q^n and q^b decrease. This is because a rise in α has the following two effects on the economy. First, an increase in α has a direct effect on the composition of the population. Since $q^b > q^n$, an increase in α implies that q^s tends to go up. We can label it as the *composition effect*. The second effect is a general equilibrium *price effect*: a higher q^s drives up $c'(q^s)$ whenever $c'' > 0$. It follows that the price level in the day market becomes higher and hence both types of buyers tend to consume less, leading to lower q^n and q^b . Obviously, if $c'' = 0$, the price effect disappears and (q^n, q^b) are not affected by α while q^s goes up due to the composition effect. Away from the Friedman's rule ($i > 0$), we show in Proposition 1 that allowing more buyers to use credit can decrease welfare.

Proposition 1 *Negative welfare effects of credit (sufficient conditions): $dW/d\alpha < 0$ if $c'' > 0$, $c' > 0$, and π is small.*

According to Lemma 1, when $c'' = 0$, an increase in α has no price effect. As trades involving credit-users generate larger trade surpluses, a higher fraction of credit-users always raises social welfare as the composition effect dominates. In the case where $c'' > 0$, the price effect associated with a higher α can reduce the welfare because of lower q^n and q^b . In the presence of two opposing effects of α on welfare, Proposition 1 provides a sufficient condition under which the price effect dominates.

It seems counterintuitive that an increase in α can reduce the social welfare. After all, the use of credit is completely voluntary and can only expand a credit-user's feasibility set.

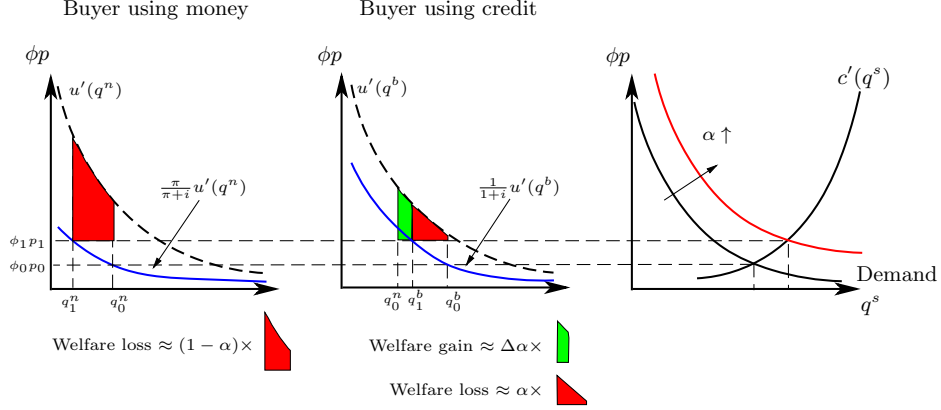


Figure 3: Welfare Effects of Increasing α

Moreover, while the rest of the economy is affected by an increase in α through the price effect, it is only a pecuniary externality. The standard argument suggests that pecuniary externalities by themselves are not a source of inefficiency in a competitive equilibrium. One needs to notice, however, that pecuniary externalities can have welfare consequences when there are distortions in the economy. In the current setting, the first welfare theorem fails in the presence of binding liquidity constraints. A rise in α leads to a higher market price which then tightens other agents' liquidity constraints, and potentially creates inefficiencies. To understand the intuition, first note that the change in welfare as a result of a rise in α can be decomposed as follows,

$$\begin{aligned}
 (25) \quad dW &= \underbrace{\pi \{ [u(q^b) - q^b \phi p] - [u(q^n) - q^n \phi p] \}}_{\text{increase in credit-user}} d\alpha \\
 &\quad + \underbrace{\pi \alpha \left(1 - \frac{1}{1+i}\right) u'(q^b) dq^b}_{\text{decrease in } q^b} \\
 &\quad + \underbrace{\pi (1-\alpha) \left(1 - \frac{\pi}{\pi+i}\right) u'(q^n) \cdot dq^n}_{\text{decrease in } q^n}
 \end{aligned}$$

We illustrate this decomposition in Figure 3. The left diagram plots the individual demand curve (blue curve) for a buyer using money, derived from (22), as well as the marginal

utility $u'(q^n)$ (dashed line). The wedge between the two curves captures the inefficiency due to the presence of the liquidity constraint. The inefficiency wedge is determined by two parameters: i and π . The higher the inflation rate and the higher the trading friction, the more costly it is for a money-user to carry liquidity to finance the trade. Similarly, the middle diagram shows the demand curve for a buyer using credit, derived from (23) and his marginal utility. The right diagram plots the aggregate demand and the aggregate supply which correspond to (4).

When α increases, the aggregate demand curve (red curve) in the right diagram shifts out, driving up the market price. There are three effects on welfare. First, an increase in α implies that there are more credit-users. Since credit-users can consume more than money-users, there is a welfare gain from the composition effect reflected by the green area in the middle diagram. This also corresponds to the first term in (25). Second, the higher price level reduces money-users' consumption from q_0^n to q_1^n and reduces credit-user consumption from q_0^b to q_1^b since it tightens their liquidity constraints. The welfare loss from the price effect is depicted by the red areas in the left and middle diagrams. These correspond to the second and third terms in (25). Whenever the sum of the red areas is larger than the green area, an increase in α leads to lower aggregate welfare. This is more likely when π is small. Not surprisingly, high trading frictions and convex production costs are necessary for generating the negative welfare effect of credit.

Proposition 2 *Negative welfare effects of credit (necessary conditions): $d\mathcal{W}/d\alpha < 0$ only if $c'' > 0$ and π is sufficiently smaller than 1.*

Under the assumptions made above, we can summarize the results in this section. In a competitive equilibrium, inflation lowers aggregate consumption and welfare, and has a bigger impact on money-users than on credit-users. An increase in access to credit reduces consumption of both money-users and credit-users, and lowers welfare when π is small.

4 Welfare under Optimal Trading Mechanism

The previous section considers the benchmark case in which the day market is competitive, and illustrates how credit arrangements can lead to inefficient outcomes, partly due to a general equilibrium price effect. It is then natural to ask whether such inefficiencies can be mitigated by adopting an optimal pricing arrangement. To address this question, we follow the mechanism design approach developed by Hu et al. (2009) and Rocheteau (2012) to solve for the efficient allocation and find out the optimal pricing mechanism. We then characterize properties of the optimal allocation and pricing mechanism, derive effects of credit on welfare, and finally discuss the implementation of the optimal allocation by a tax-subsidy scheme.

The mechanism design problem can be described as follows. First, suppose that the buyer/seller status, the realization of the consumption shock and whether an agent has access to banking or not are public information.⁹ During the day, agents play a two-stage game specified by a mechanism. In the first stage of the game, everyone announces his real money balance. A mechanism maps the announced real money holdings of a type $j \in \{b, n, s\}$ agent to a proposed allocation $(q, z) \in R_+ \times Z$, where q is the quantity consumed by a type $j \in \{b, n\}$ agent or the quantity produced by a type s agent, and z is a transfer of real money balances from the agent.¹⁰ In the second stage of the game, everyone says “yes” or “no” simultaneously. If everyone says “yes”, each agent receives (q, z) according to the rule specified by the mechanism. If anyone says “no”, then the mechanism assigns $(0, 0)$ to all agents.

Following similar arguments as in Rocheteau (2012), the allocation (q^j, z^j) for $j \in$

⁹We also explore the case in which whether an agent has access to credit is private information. Section 6 summarizes the implications of this additional informational constraint and we provide formal derivations in the Online Appendix.

¹⁰Note that, unlike Rocheteau (2012), there are two types of buyers b and n with different payment technologies.

$\{b, n, s\}$ should not depend on a seller's money balance for truthful announcement of the seller's money balance. To ensure that the allocation satisfies a buyer's truthful announcement of his money balance, the mechanism will propose (q^j, z^j) for $j \in \{b, n\}$ if the announced money balance is no less than z^j , and will propose $(0, 0)$ otherwise. Under this mechanism, a buyer carrying less than z^j has no (strict) incentive to misreport because any report below z^j gives him zero payoff, while any report above z^j is infeasible. Similarly, a buyer carrying more than z^j has no incentive to misreport because reporting below z^j generates zero payoff while all reports above z^j are payoff equivalent. So a type j buyer will carry z^j to the first stage of the game.

Rocheteau (2012) considers decentralized, pairwise trading and implements allocations that are in the pairwise core. Here, we consider centralized, multilateral trading and the mechanism implements allocation that is immune to individual deviation (Nash implementable). In this regard, the mechanism is more powerful than a competitive market because it can rule out "side trades". If side trades were allowed, the allocation prescribed by the mechanism may not be immune to group deviation. Therefore, the set of optimal allocation that can be supported by this mechanism (with side trades ruled out) is at least as big as the set under a mechanism with side trades allowed.^{11,12}

¹¹The assumption of no side trades may be strong for certain trading environments. We view our finding as a useful benchmark because it provides an upper bound on the achievable level of welfare. If an allocation is not implementable by the mechanism under the assumption of no side trades, then it will also be not implementable when side trades are allowed. For completeness, we also examine how a planner can use a simple tax and subsidy scheme to support the optimal allocation in competitive markets in which side trades are allowed. Interestingly, the scheme resembles real-world arrangements in credit systems such as merchant fees and surcharging.

¹²In addition, the mechanism designer can potentially make use of the payment technology to enforce non-linear pricing and cross-subsidization, without the need to prohibit side trades. For example, Chiu and Wong (2015) consider the roles of money and e-money in a related trading environment and show how an e-money technology can help implement the optimal allocation when it can impose restrictions on payments within the

The timing of events is the following. At the beginning of the day, agents can choose to deposit money or take out loans taking the market interest rates of deposits and loans as given. Afterwards, they play the game specified by the mechanism. Trading takes place according to the allocation generated by agents' actions. Activities in the night market remain the same as before.

The optimal mechanism maximizes the aggregate welfare subject to the relevant technological constraints and incentive constraints by choosing (q^j, z^j) for $j \in \{b, n, s\}$. The technological constraints are the feasibility constraints with respect to goods and money holdings. The incentive constraints include the participation constraints (PCs) that ensure all agents participate in the mechanism, and the truth-telling constraints that no one wants to misreport his money holdings.¹³ The PC for a buyer who does not have access to credit is

$$-\gamma z^n + \beta z^n + \beta \pi [u(q^n) - z^n] \geq 0,$$

which guarantees that the buyer does not gain by unilaterally deviating from the proposed allocation. The loan market clearing condition implies that $\pi \phi \ell = (1 - \pi) \phi k = (1 - \pi) \phi \hat{m}^b$. The amount of money available for a buyer with access to credit is thus $z^b = \phi \hat{m}^b + \phi \ell = \phi \hat{m}^b / \pi$. The PC for a buyer who has access to credit is

$$-\pi \gamma z^b + \beta \pi z^b + \beta \pi [u(q^b) - z^b] \geq 0.$$

For a seller, the PC is

$$-c(q^s) + z^s \geq 0.$$

e-money system. We expect that similar results will apply to our setting if the mechanism can restrict transfers of credit payments. We will leave this extension for future study.

¹³Actually, the truth-telling constraints coincide with the participation constraints in this model since the mechanism itself ensures that agents have no incentives to over or under report their money balances.

As we do not restrict to pairwise trading, the market clearing conditions imply (17) and $z^s = \pi\alpha z^b + \pi(1 - \alpha)z^n$. The PCs can be rearranged as

$$(26) \quad -iz^n + \pi [u(q^n) - z^n] \geq 0,$$

$$(27) \quad -iz^b + [u(q^b) - z^b] \geq 0,$$

$$(28) \quad -c(q^s) + z^s \geq 0.$$

Having specified the trading mechanism and the PCs, we focus on the optimal mechanism that maximizes (24) such that (26) to (28) are satisfied.

Notice that the trading mechanism is more flexible than a competitive market. First, the mechanism is not restricted to linear pricing. Second, the mechanism has an option to specify (q, z) contingent on the agent's type and on the (self-reported) money holding. These flexibilities allow the mechanism to achieve better allocations than a competitive market. We will show that these features are essential in the sense that, without them, the welfare is strictly lower.

4.1 Properties of the Optimal Allocation

In this subsection, we derive some properties of the optimal allocation and compare the allocation with the competitive equilibrium allocation. The following proposition characterizes how the allocation and welfare vary with inflation.

Proposition 3 *Effects of inflation on welfare: there exists a unique $i_1 > 0$ such that the first-best allocation can be implemented if and only if $i \leq i_1$. Moreover, $dW/di = 0$ for $i \leq i_1$, and $dW/di < 0$ for $i > i_1$.*

As shown in the Appendix, i_1 denotes the interest rate that solves

$$(29) \quad c(\pi q^*) = \left[\frac{\pi\alpha}{1 + i_1} + \frac{\pi^2(1 - \alpha)}{\pi + i_1} \right] u(q^*).$$

Under the optimal trading mechanism, when $i \leq i_1$, both money-users and credit-users consume the first-best quantity q^* , and thus inflation does not affect welfare. This finding is in sharp contrast with Proposition A.1 in the appendix in which inflation is always welfare reducing in a competitive market.

When $i > i_1$, the allocation (q^b, q^n, q^s) is characterized by

$$(30) \quad \frac{u'(q^b) - c'(q^s)}{u'(q^n) - c'(q^s)} = \frac{c'(q^s) - \frac{1}{1+i}u'(q^b)}{c'(q^s) - \frac{\pi}{i+\pi}u'(q^n)},$$

$$(31) \quad \frac{\pi\alpha}{1+i}u(q^b) + \frac{\pi^2(1-\alpha)}{i+\pi}u(q^n) = c(q^s),$$

and (17). The allocation features $\pi u'(q^n)/(\pi + i) < c'(q^s) < u'(q^n)$ and $u'(q^b)/(1 + i) < c'(q^s) < u'(q^b)$, while in a competitive market, equilibrium conditions imply that $\pi u'(q^n)/(\pi + i) = c'(q^s)$ and $u'(q^b)/(1 + i) = c'(q^s)$.

4.2 Properties of Optimal Mechanism

To understand the difference between a competitive market and an optimal trading mechanism, we should notice that a competitive market does not distinguish between the two different types of buyers and that type b buyers do not internalize the effect of their consumption on type n buyers. Therefore, pecuniary externalities can hurt the economy when the liquidity constraint is binding. The optimal mechanism instead can distinguish between the two types of buyers and assign proper allocations and payment schemes to ensure that both types participate in the mechanism. Compared with the market equilibrium, the mechanism can redistribute from type b buyers to type n buyers to mitigate any externality generated by the price effect. This redistribution can be done via an appropriate price discrimination. Denote $p^b \equiv z^b/q^b$ and $p^n \equiv z^n/q^n$ as the prices paid by type b and type n respectively. We highlight the price discrimination in the following proposition.

Proposition 4 *Price discrimination. There exists $\underline{i} < i_1$ and $\bar{i} > i_1$ such that the relationship between p^b and p^n is ambiguous for $i \leq \underline{i}$, and $p^b > p^n$ for $i \in (\underline{i}, \bar{i})$. Moreover, if the utility*

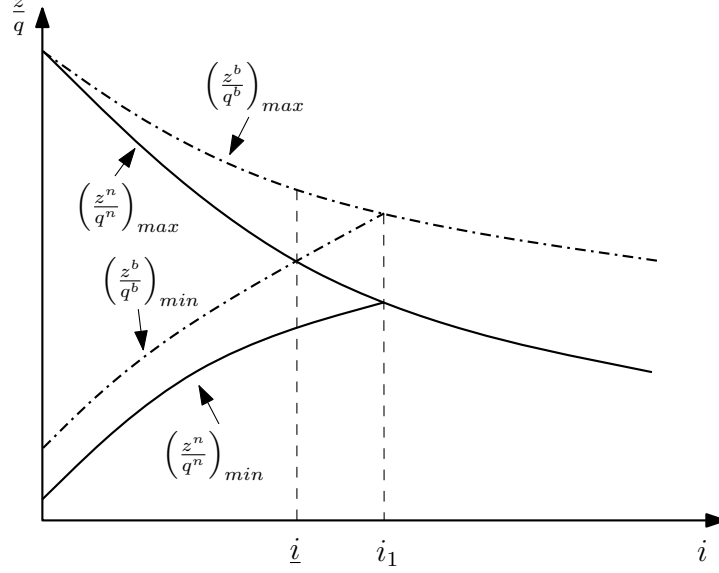


Figure 4: Optimal pricing mechanism

function is isoelastic (i.e., $u'(q)q/u(q)$ is constant), then $p^b > p^n$ for all $i > \bar{i}$.

Figure 4 illustrates how the trading mechanism supports the (constrained) optimal allocation by adjusting prices. The dash-dot curve represents the optimal price range for credit-users while the solid curve represents the price range for money-users. When i is lower than i_1 , incentive constraints are not binding. There are multiple (z^b, z^n) pairs consistent with the first-best allocation $q^b = q^n = q^*$. Therefore, there is a range of prices that allows the trade surplus to be optimally split among the three parties (i.e. money-users, credit-users and sellers) to satisfy all the incentive constraints given the first-best allocation. That is why the relationship between p^b and p^n is ambiguous for $i \leq \bar{i}$. As i increases, the liquidity cost for buyers goes up and it becomes harder to support the first-best allocation. As a result, the upper bounds for z^b and z^n drop while the lower bounds for z^b and z^n go up, until getting to the threshold i_1 beyond which the first-best allocation is not implementable and the optimal prices are uniquely determined. Because credit-users can access credit to partially offset the effect of inflation, their participation constraints are less binding than money-users, the maximum prices for credit-users are higher than those for money-users. Given this, for any $i \in (\bar{i}, i_1)$, in order to implement the (constrained) optimal allocation, it is necessary to charge credit-users a higher price and cross subsidize money-users.

We can further characterize the optimal prices when $i > \bar{i}$ when the utility function exhibits constant elasticity. In that case, $p^b > p^n$ for all $i > \bar{i}$. The general idea is that credit-users generate a negative price externality on money-users. In order to internalize this externality, the pricing mechanism needs to price-discriminate between different types by charging credit-users a higher price as indicated in the graph. Note that this type of welfare-improving price discrimination is infeasible in a centralized, competitive market because it will induce side-trades to exploit arbitrage opportunities. These arbitrage activities, however, are prohibited under the current trading mechanism.

The optimal allocation achieved by the trading mechanism requires both price discrimination and non-linear pricing. If we restrict the mechanism from using price discrimination, we find that the mechanism can achieve the first-best allocation only when $i \leq \hat{i}_1$, where \hat{i}_1 is expressed as

$$\hat{i}_1 = \pi \left(\frac{\pi u(q^*)}{c(\pi q^*)} - 1 \right) = \underline{i}.$$

It implies that the optimal trading mechanism can achieve higher welfare for $i \in (\hat{i}_1, i_1)$. That is, when $i \in (\hat{i}_1, i_1)$, the optimal mechanism can still implement the first-best allocation, but the mechanism without price discrimination cannot. Without price discrimination, credit-users do not internalize the externality generated by the price effect.

The optimal mechanism also cross-subsidizes money-users by extracting trade surpluses from sellers and credit-users. Without non-linear pricing, this cross-subsidization cannot be achieved. Suppose that the mechanism can use only linear pricing on both buyers and sellers. We find that the linear-pricing mechanism cannot implement the first-best allocation for any $i > 0$. If the mechanism uses linear pricing only on buyers, then the first-best allocation can be achieved if and only if $i \leq \tilde{i}_1$, where $\tilde{i}_1 < i_1$ is the interest rate that satisfies

$$c(\pi q^*) = \pi \left(\frac{\alpha}{1 + \tilde{i}_1} + \frac{1 - \alpha}{1 + \frac{\tilde{i}_1}{\pi}} \right) u'(q^*) q^*.$$

Proposition 5 *Essential features of the optimal trading mechanism. (1) Price discrimina-*

tion: without price discrimination, the trading mechanism achieves lower welfare than the optimal trading mechanism for all $i \in (\hat{i}_1, i_1)$. (2) Non-linear pricing: without non-linear pricing, the trading mechanism achieves lower welfare than the optimal trading mechanism for all $i \in (0, i_1)$. If non-linear pricing is allowed only on sellers but not on buyers, the trading mechanism achieves lower welfare than the optimal trading mechanism for all $i \in (\tilde{i}_1, i_1)$.

In the appendix, we explore more details of these two features and show that in some cases, one is more important than the other for supporting the optimal allocation. For example, if the utility is close to linear, then price discrimination is of first-order importance.

4.3 Access to Credit and Welfare

We proceed to examine the effect of access to credit on welfare under the optimal trading mechanism and summarize the result in the following proposition.

Proposition 6 *Effects of access to credit on welfare: $d\mathcal{W}/d\alpha > 0$ for any $i > i_1$, and $di_1/d\alpha > 0$.*

Note that for $i < i_1$, the first-best allocation can always be implemented. When $i > i_1$, in contrast to Proposition 1, price discrimination and non-linear pricing make the usage of credit always welfare improving. Moreover, an increase in access to credit makes it easier to support the first-best allocation by raising the threshold inflation rate i_1 .

4.4 Implementation in Competitive Markets

The above analysis is based on the assumption that the mechanism can rule out side trades. For completeness, we discuss whether a planner can use a simple tax and subsidy scheme to support the optimal allocation in competitive markets in which side trades are allowed. Suppose that the planner can create two markets. One is for credit-users and the other is for

money-users.¹⁴ Sellers are free to enter either market and have the choice of avoiding taxes by staying inactive. Sellers who actively trade in the markets need to pay a lump-sum tax η_s , which is used to subsidize proportionally to buyers who actively trade in the markets. Let τ_b and τ_n denote the proportional subsidies. In the Appendix, we show that the following tax-subsidy scheme can implement the first-best allocation when $i \leq \tilde{i}_1$, $\tau_b = i$, $\tau_n = i/\pi$, and

$$\eta_s = \pi u'(q^*) q^* \left[\frac{\alpha i}{1+i} + \frac{(1-\alpha) \frac{i}{\pi}}{1 + \frac{i}{\pi}} \right].$$

Finally, we summarize the results in this section. Under the optimal trading mechanism, moderate inflation does not reduce aggregate consumption or welfare. Optimal trading mechanism typically involves non-linear pricing and price discrimination between money-users and credit-users. Both features are essential for credit to be welfare improving. An increase in access to credit makes it easier to support the first-best allocation. Furthermore, an increase in access to credit does not affect welfare when inflation is not too high and can always improve welfare when inflation is high. The optimal allocation can be supported in a competitive market with an appropriately designed tax-subsidy scheme.

5 Numerical Exercise

To further understand the implications from our model, we present quantitative results on the welfare effects of credit and inflation. Following Lagos and Wright (2005), we assume

¹⁴Our finding is robust even when credit-users are allowed to mimic money users and to enter the market for money-users (but not vice versa because by definition, money-users cannot use credit). In the appendix, we show that they do not have an incentive to do so because in equilibrium credit-users earn the same payoff as money-users by self-selecting to the corresponding market. Hence, it does not matter whether the planner can distinguish the types of buyers or not.

$v(x) = B \log x$ for utility from consumption in the night market. In the day market, the utility function is $u(q) = \log(q + \epsilon) - \log(\epsilon)$, with $\epsilon \simeq 0$. The cost function in the day market is

$$c(q) = \frac{(q + \epsilon)^{1+\chi} - \epsilon^{1+\chi}}{1 + \chi}.$$

Let $q^b(i)$, $q^n(i)$ and $q^s(i)$ denote the solution to (17), (22) and (23). The aggregate money stock is calculated as $M(i) = p[\alpha\pi q^b(i) + (1 - \alpha)\pi q^n(i)]$. Nominal expenditure in each period includes $2x/\phi$ in the night market (by all agents) and $pq^s(i)$ in the day market (by buyers). Consequently, the ratio of nominal money stock to nominal expenditure (or nominal output) as a function of the interest rate is given by

$$\tilde{m}(i) = \frac{\alpha\pi q^b(i) + (1 - \alpha)\pi q^n(i)}{\frac{2x}{c'[q^s(i)]} + q^s(i)},$$

where we have replaced ϕ by $c'[q^s(i)]/p$. The variable $\tilde{m}(i)$ can be interpreted as money demand, or the inverse of velocity of money.

The model is then parameterized in order to fit the aggregate money demand in the model $\tilde{m}(i)$, to the US money demand in the data M/GDP . The length of a period is defined to be a year. The discount factor is set to $\beta = 0.973$, implying an annual real interest rate of 2.8 percent. We then follow Aruoba et al. (2011) to set $\alpha = 0.15$ so that in 15 percent of trades in the day market, credit is available.¹⁵ The values of the remaining parameters, B , π and χ , are chosen such that the money demand function matches the US historical data of the average M1-GDP ratio.¹⁶ We find that $B = 0.8983$, $\pi = 0.0528$ and $\chi = 2.2525$. Figure 5 shows the data and the model fit.

¹⁵Aruoba et al. (2011) justify the choice of the value of α : “First, Klee (2008) finds that shoppers use credit cards in 12 percent of total transactions in the supermarket scanner data... Second, using earlier consumer survey data, Cooley and Hansen (1991) come up with a similar measure of around 16 percent.”

¹⁶The three parameter values are chosen to minimize the mean square error, defined by the difference between model predicted money demand and historical data.

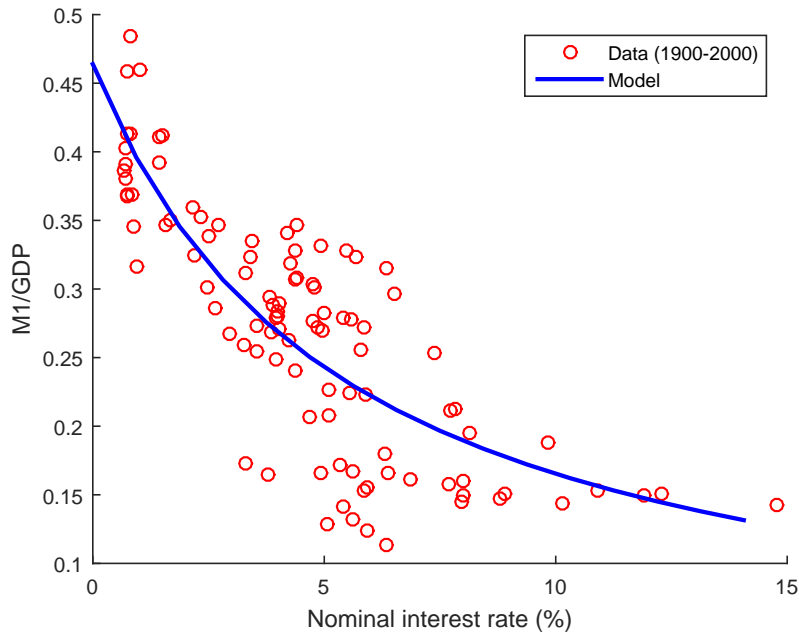


Figure 5: Model and Data

Using these parameter values, we calculate the welfare effects of changing access to credit. Figure 6 illustrates the effects of changing α on welfare under 2 percent inflation. The red dot indicates the benchmark economy where $\alpha = 0.15$. We can see that increasing α in the day market has a non-monotonic effect on welfare. In this example, when $\alpha \leq 0.26$, a marginal increase in access to credit has a negative welfare effect. When $\alpha > 0.26$, further credit expansion generates positive welfare effects. Hence, the benchmark economy with $\alpha = 0.15$ is in the range of parameter values where credit expansion is welfare reducing. This exercise also suggests that, starting from $\alpha = 0.15$, an increase in credit access will only improve welfare when credit is widely available (e.g., when α is larger than 0.38 at 2 percent inflation).

We check the robustness of the above results by varying parameter values. Table 1 reports the effects of increasing credit from $\alpha = 0$ to $\alpha = 0.15$ on steady state welfare and consumption in the day market.¹⁷ The benchmark economy is labelled as economy i and the

¹⁷Consumption and welfare in the night market are invariant to changes in i and α . Also, the percentage changes in q_n and q_b are the same.

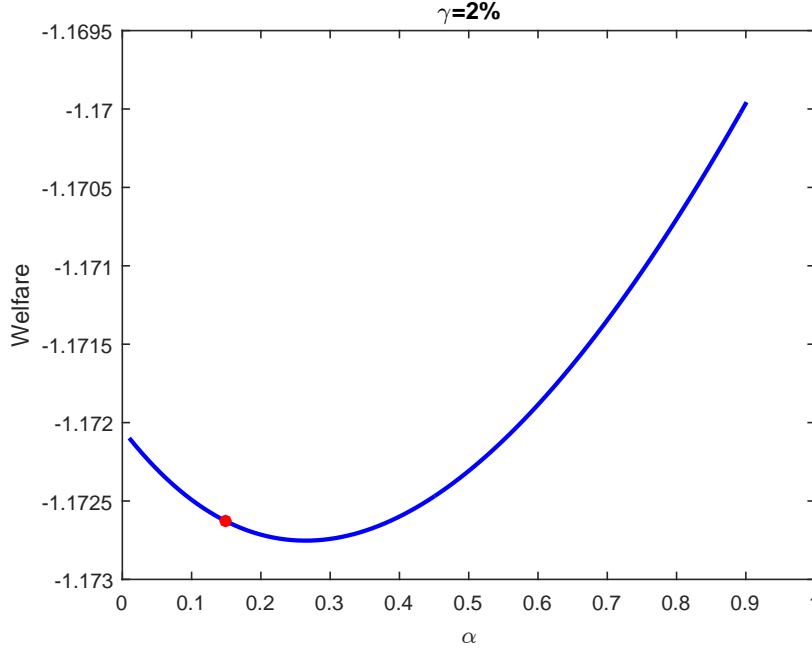


Figure 6: Welfare (average utility) and Credit

results are shown in the first row. As inflation goes up, both consumption and welfare in the day market drop. In economy ii, the value of B is lower than that in the benchmark economy. This reduces the relative size of the night market but has no impact on day consumption or welfare. In economy iii, since the probability of consumption π in the day market is smaller, type n buyers face a tighter liquidity constraint. The price effect is stronger and credit has bigger impacts on welfare and consumption. In economy iv, χ is bigger and the cost function becomes more convex. The price of day-market goods rises more when α increases. Therefore, the welfare cost of increasing access to credit is again larger. Overall, we expect that the negative welfare effect of credit is stronger when inflation is higher, when trading becomes more frictional, and when the cost function is more convex.

Table 1: Effects of Access to Credit on Consumption and Welfare

	B	π	χ	Day output share ($\gamma = 2$)	Day welfare ($\gamma = 2$)	Day consumption q_b, q_n ($\gamma = 2$)	Day welfare ($\gamma = 10$)	Day consumption q_b, q_n ($\gamma = 10$)
i.	0.90	0.05	2.25	1.67%	-0.66%	-8%	-2.96%	-17%
ii.	0.10	0.05	2.25	13.27%	-0.66%	-8%	-2.96%	-17%
iii.	0.90	0.01	2.25	0.16%	-5.61%	-31%	-14.86%	-50%
iv.	0.90	0.05	10.00	1.67%	-0.89%	-10%	-3.37%	-22%

6 Extensions

This section provides a general discussion of the main results of the paper and explores several extensions. First of all, the result on welfare reduction of credit access relies on the presence of the price effect. To have this effect, the cost of producing for one buyer has to be dependent on the cost for other buyers. For example, in our centralized trading environment, when a credit-user buys more from a seller, the marginal cost of producing for money-users will be driven up. This effect will disappear if the costs of production for different buyers are *completely* independent. One example is a decentralized market with bilateral trades where each seller uses his own labor to operate an independent production technology. However, we want to point out that the market structure alone is not sufficient for eliminating the price effect. What matters is the cost structure. This price effect will be present even in a decentralized goods market, if the sellers employ some factors of production that are traded in a centralized market.¹⁸

The main results of the paper are robust to several different ways of arranging credit. In the Online Appendix, we study two extensions of our benchmark model. In the benchmark

¹⁸Suppose meetings in the day market are bilateral, and the terms of trade are determined by take-it-or-leave-it offers from the buyers. Each buyer makes an offer (D, q) to buy q goods by paying D dollars. Suppose a seller produces output q by employing labor hours h according to a linear production function $q = F(h) = h$. Labor hours are traded in a centralized market at a (real) wage rate w . Each seller chooses how much labor to supply H to the market and how much labor to be hired h . The disutility of supplying labor is $\psi(H)$ with $\psi' > 0, \psi'' < 0$. The payoff of a seller with an offer (D, q) is thus

$$V^s(q, D) = \phi D - wq + \max_H [wH - \psi(H)].$$

Obviously, $w = \psi'(H)$ in a competitive labor market. Moreover, any offer from the buyer to the seller must satisfy $V^s(q, D) = V^s(0, 0)$. It is then straightforward to show that, the same set of equilibrium conditions can be derived, by appropriately relabeling H and $\psi(H)$ as q^s and $c(q)$.

case, banks can only issue loans in terms of government-issued outside money, and cannot issue their own inside money. We relax this assumption and allow banks to issue credit in the form of inside money which is accepted as a means of payment in goods transactions. In this modified environment, banks provide an additional payment service (i.e. imagine a private payment system without the use of outside money). Surprisingly, we show that the price effect dominates the composition effect more strongly in this case than in the benchmark model, implying that the introduction of credit as a means of payment is even more welfare reducing. In the Online Appendix, we also study an extension in which loan repayments are only imperfectly enforceable: a defaulter is punished by being excluded from access to credit in the future. In this setting, we uncover a new channel operating through the implied endogenous credit limit. For example, an increase in access to credit will reduce the value of default through the price effect, and hence relax credit-users' credit limits, potentially improving the social welfare. Finally, allowing banks to take deposits from sellers will not change any results of the paper.

In Section 4, we assume that the mechanism can distinguish buyers who have access to credit from those who don't. In the Online Appendix, we examine different cases when this assumption is relaxed. Specifically, we study two cases depending on whether the usage of credit is observable by the mechanism or not. If the usage of credit is observable, then a money-user cannot imitate a credit-user, but a credit-user can mimic a money-user by consuming less. If it is not observable, then buyers can mimic each other. We show that the solutions from these two cases coincide in the sense that the money-user's incentive constraint never binds. Most results such as Proposition 3 and Proposition 6 under full information still carry through under private information. The main difference is that the unique cutoff value of the interest rate for implementing the first-best is lower in the private information case than in the full information case. The welfare under private information is weakly dominated by that under full information. When α is small, increasing access to credit still has a positive effect on welfare regardless of the presence of informational

friction.¹⁹

7 Conclusion

This paper uses modern monetary theory to study the welfare effects of credit arrangements. There are a few useful implications for policy makers. Firstly, the general message is that the welfare effect of credit expansion is not as obvious as it appears. It depends on market structure (pricing arrangement and trading frictions), production technology (returns to scale), the banking system (issuance of inside money) and monetary policy. In addition, increasing access to credit can have a non-monotonic welfare effect. Hence, the level of financial development also matters. Finally, there is a role for intervention to enhance welfare but its effectiveness also depends on enforcement and information technologies.

Our analysis highlights that the optimal mechanism involves non-linear pricing and price discrimination. These features resemble such features as interchange fees and membership fees on credit cards.²⁰ One interesting policy implication is that, in contrast to common belief, these elements can be essential features of the optimal credit arrangement. This finding suggests that imposing a credit card fee regulation might distort the pricing arrangement that implements the optimal allocation. This implication is particularly relevant for the discussion of limiting the interchange fees on payment cards, which was implemented by the recent Durbin amendment as part of the Dodd-Frank Act.

¹⁹While the general analytical condition is not available, quantitative results suggest that \mathcal{W} is always increasing with α in the private information case. The results are available upon request.

²⁰Masters (2014) considers an environment in which product quality is unobservable and develops a theory to explain why retailers pay merchant fees to credit card issuers. In this model, retailers use credit card acceptance as a signal of product quality. The reason is that cash transactions do not provide any customer recourse in the case of mis-sold items while credit cards do. Therefore, retailers who accept credit cards send a message to potential customers that they stand behind their product.

Appendix

A Effects of Inflation

Proposition A.1 *Effects of inflation: $dq^n/di < 0$, $dq^s/di < 0$, $dq^b/di \geq 0$. Also, $dW/di < 0$.*

Proof. In this proof, we compute the comparative statics with respect to i by totally differentiating the equilibrium conditions. We will first show the effects of inflation on quantities and then the effects on welfare. Totally differentiating the system (17), (22), (23), we get

$$\Phi \begin{pmatrix} dq^n \\ dq^b \\ dq^s \end{pmatrix} = \begin{pmatrix} \frac{1}{\pi} c'(q^s) & 0 \\ c'(q^s) & 0 \\ 0 & \pi(q^b - q^n) \end{pmatrix} \begin{pmatrix} di \\ d\alpha \end{pmatrix},$$

where

$$\Phi = \begin{pmatrix} u''(q^n) & 0 & -(1 + \frac{i}{\pi})c''(q^s) \\ 0 & u''(q^b) & -(1 + i)c''(q^s) \\ -\pi(1 - \alpha) & -\pi\alpha & 1 \end{pmatrix}.$$

Thus, the determinant is given by

$$|\Phi| = u''(q^n) [u''(q^b) - \pi\alpha(1 + i)c''(q^s)] - (1 - \alpha)(\pi + i)u''(q^b)c''(q^s) > 0.$$

We can then obtain the comparative statics with respect to i as follows

$$(A.1) \quad dq^n/di = c'(q^s) \left[\frac{1}{\pi} u''(q^b) - \alpha(1 - \pi)c''(q^s) \right] / |\Phi| < 0,$$

$$(A.2) \quad dq^b/di = c'(q^s) [u''(q^n) + (1 - \pi)(1 - \alpha)c''(q^s)] / |\Phi| \leq 0,$$

$$(A.3) \quad dq^s/di = c'(q^s) [(1 - \alpha)u''(q^b) + \pi\alpha u''(q^n)] / |\Phi| < 0.$$

Note that, in general, the sign of dq^b/di is ambiguous and depends on the curvature of u and

c .²¹ This is the first part of the proposition.

For the second part, even though the quantities may not be monotone, we now show that the welfare is strictly decreasing with inflation. Again, totally differentiating welfare in (24) with respect to i , we can obtain

$$\begin{aligned}
\frac{d\mathcal{W}}{di} &= \pi\alpha u'(q^b) \frac{dq^b}{di} + \pi(1-\alpha) u'(q^n) \frac{dq^n}{di} - c'(q^s) \frac{dq^s}{di} \\
&= \pi\alpha \frac{dq^b}{di} [u'(q^b) - c'(q^s)] + \pi(1-\alpha) \frac{dq^n}{di} [u'(q^n) - c'(q^s)] \\
\text{(A.4)} \quad &= \pi\alpha i c'(q^s) \frac{dq^b}{di} + (1-\alpha) i c'(q^s) \frac{dq^n}{di},
\end{aligned}$$

where the last equality is derived by using (22) and (23). Finally, replacing dq^b/di and dq^n/di in (A.4) by equations (A.1) and (A.2), we get

$$\begin{aligned}
\frac{d\mathcal{W}}{di} &= \frac{i [c'(q^s)]^2}{|\Phi|} \left[\pi\alpha u''(q^n) + (1-\alpha) \frac{1}{\pi} u''(q^b) - \alpha(1-\alpha)(1-\pi)^2 c''(q^s) \right] \\
&< 0.
\end{aligned}$$

■

B Proof of Lemma 1

Proof. In this proof, we totally differentiate the system (17), (22), (23) with respect to α . Following the steps in the proof of Proposition A1, we obtain the following comparative statics with respect to α

$$\text{(B.1)} \quad dq^n/d\alpha = (\pi + i)(q^b - q^n)u''(q^b) c''(q^s) / |\Phi| \leq 0,$$

$$\text{(B.2)} \quad dq^b/d\alpha = \pi(1+i)(q^b - q^n)u''(q^n) c''(q^s) / |\Phi| \leq 0,$$

$$\text{(B.3)} \quad dq^s/d\alpha = \pi(q^b - q^n)u''(q^n) u''(q^b) / |\Phi| > 0.$$

²¹However, if $\pi \rightarrow 1$ or $\alpha \rightarrow 1$, then $dq^b/di < 0$.

Since $|\Phi| > 0$ (see the proof of Proposition A.1), $dq^n/d\alpha$ and $dq^b/d\alpha$ are strictly negative whenever $c''(q^s) > 0$. ■

C Proof of Proposition 1

Proof. In this proof, we obtain a sufficient condition for $d\mathcal{W}/d\alpha < 0$. To show that, we first totally differentiate welfare with respect to α . Then, we use the comparative statics obtained in the proof of Lemma 1 to simplify the derivative. Finally, we establish that the derivative is negative when π is small.

Following the steps in the proof of Lemma 1, we totally differentiate the welfare in (24) with respect to α to obtain

$$\begin{aligned}
\frac{d\mathcal{W}}{d\alpha} &= \pi [u(q^b) - u(q^n)] + \pi\alpha u'(q^b) \frac{dq^b}{d\alpha} + \pi(1-\alpha) u'(q^n) \frac{dq^n}{d\alpha} - c'(q^s) \frac{dq^s}{d\alpha} \\
&= \pi [u(q^b) - u(q^n)] + \pi\alpha u'(q^b) \frac{dq^b}{d\alpha} + \pi(1-\alpha) u'(q^n) \frac{dq^n}{d\alpha} \\
&\quad - \pi\alpha c'(q^s) \frac{dq^b}{d\alpha} - \pi(1-\alpha) c'(q^s) \frac{dq^n}{d\alpha} - \pi c'(q^s) (q^b - q^n) \\
&= \pi \{ [u(q^b) - q^b c'(q^s)] - [u(q^n) - q^n c'(q^s)] \} \\
\text{(C.1)} \quad &+ \pi\alpha i c'(q^s) \frac{dq^b}{d\alpha} + (1-\alpha) i c'(q^s) \frac{dq^n}{d\alpha}.
\end{aligned}$$

We then substitute $dq^b/d\alpha$ and $dq^n/d\alpha$ into (C.1) using (B.1) and (B.2) from the proof of Lemma 1 to yield

$$\begin{aligned}
\text{(C.2)} \quad \frac{d\mathcal{W}}{d\alpha} &= \pi \{ [u(q^b) - q^b c'(q^s)] - [u(q^n) - q^n c'(q^s)] \} \\
&+ \frac{i(q^b - q^n) c'(q^s) c''(q^s)}{|\Phi|} \{ \alpha \pi^2 (1+i) u''(q^n) + (1-\alpha) (\pi+i) u''(q^b) \}
\end{aligned}$$

Notice that (17), (22) and (23) imply that q^n, q^b, q^s are continuous functions w.r.t. π . Therefore, the RHS of (C.2) is also continuous w.r.t. π .

We now examine the limiting case as π converges to zero. As $\pi \rightarrow 0$, we have $q^n \rightarrow 0$,

$q^s \rightarrow 0$. Also, $q^b \rightarrow \bar{q}^b$, which satisfies $u'(\bar{q}^b) = (1+i)c'(0)$. To derive the marginal effect of credit in the limit, we first note that, as $\pi \rightarrow 0$,

$$\begin{aligned} |\Phi| &\rightarrow |\bar{\Phi}| \equiv u''(0)u''(\bar{q}^b) - (1-\alpha)(1+i)u''(\bar{q}^b)c''(0) > 0, \\ dq^n/d\alpha &\rightarrow (1+i)\bar{q}^b u''(\bar{q}^b)c''(0)/|\bar{\Phi}| < 0, \\ dq^b/d\alpha &\rightarrow (1+i)\bar{q}^b u''(0)c''(0)/|\bar{\Phi}| < 0. \end{aligned}$$

Therefore, the marginal effect on welfare converges to

$$\lim_{\pi \rightarrow 0} \frac{d\mathcal{W}}{d\alpha} = (1-\alpha)ic'(0)(1+i)\bar{q}^b u''(\bar{q}^b)c''(0)/|\bar{\Phi}| < 0.$$

■

D Proof of Proposition 2

Proof. In this proof, we show that $d\mathcal{W}/d\alpha < 0$ only if (i) $c'' > 0$ and (ii) π is sufficiently smaller than 1. For the first part, notice from the proof of Lemma 1 that, if $c'' = 0$, then $dq^b/d\alpha = 0$ and $dq^n/d\alpha = 0$. This implies that $d\mathcal{W}/d\alpha > 0$ from equation (C.1). Hence, $c'' > 0$ is necessary for $d\mathcal{W}/d\alpha < 0$.

For the second part, we first rewrite the marginal effect of credit on welfare in (C.2) as a product of two terms:

$$\begin{aligned} \frac{d\mathcal{W}}{d\alpha} &= \\ &(q^b - q^n) \times \\ &\left\{ \pi \left[\frac{u(q^b) - u(q^n)}{q^b - q^n} - c'(q^s) \right] + \frac{ic'(q^s)c''(q^s)}{|\Phi|} \left\{ \alpha\pi^2(1+i)u''(q^n) + (1-\alpha)(\pi+i)u''(q^b) \right\} \right\} \end{aligned}$$

For any $\pi < 1$, the first term, $q^b - q^n$, is strictly positive. As $\pi \rightarrow 1$, the second term

converges to

$$\begin{aligned}
& u'(q^s) - c'(q^s) + \frac{ic'(q^s)c''(q^s)}{u''(q^s) - (1+i)c''(q^s)}(1+i) \\
&= ic'(q^s) \left[1 + \frac{c''(q^s)}{u''(q^s) - (1+i)c''(q^s)}(1+i) \right] \\
&= ic'(q^s) \frac{u''(q^s)}{u''(q^s) - (1+i)c''(q^s)} > 0.
\end{aligned}$$

Therefore, $\frac{dW}{d\alpha} > 0$ for π sufficiently close to one. Thus, π sufficiently less than one is necessary for credit to have a negative effect on welfare. ■

E Proof of Proposition 3

Proof. In this proof, we solve the optimization problem faced by the mechanism designer. We first show that there is a threshold level of interest rate i_1 such that the first-best allocation can be implemented iff $i \leq i_1$ (i.e. $dW/di = 0$). We then show that $dW/di < 0$ for $i > i_1$.

First of all, the Lagrangian of the mechanism designer's problem is given by

$$\begin{aligned}
\mathcal{L} = & \max_{q^n, q^b, z^n, z^b} \pi\alpha u(q^b) + \pi(1-\alpha)u(q^n) - c[\pi\alpha q^b + \pi(1-\alpha)q^n] \\
& + \lambda_1 \{-iz^n + \pi[u(q^n) - z^n]\} \\
& + \lambda_2 \{-iz^b + [u(q^b) - z^b]\} \\
& + \lambda_3 \{ \pi\alpha z^b + \pi(1-\alpha)z^n - c[\pi\alpha q^b + \pi(1-\alpha)q^n] \}.
\end{aligned}$$

where $\lambda_1, \lambda_2, \lambda_3$ are the multipliers associated with the constraints (26), (27) and (28). The

FOCs with respect to the four choice variables are

$$\begin{aligned}
\pi(1 - \alpha + \lambda_1)u'(q^n) &= (1 + \lambda_3)\pi(1 - \alpha)c'(q^s), \\
(\alpha + \lambda_2)u'(q^b) &= (1 + \lambda_3)\alpha c'(q^s), \\
\lambda_1(i + \pi) &= \lambda_3\pi(1 - \alpha), \\
\lambda_2(1 + i) &= \lambda_3\alpha.
\end{aligned}$$

Notice that there are two cases: if $\lambda_3 = 0$, then $\lambda_1 = \lambda_2 = 0$ and if $\lambda_3 > 0$, then $\lambda_1, \lambda_2 > 0$. We analyze below these two cases.

In the first case, $\lambda_1 = \lambda_2 = \lambda_3 = 0$, it is straightforward to show that $q^n = q^b = q^*$. The necessary and sufficient condition to implement this allocation is

$$(E.1) \quad c(\pi q^*) \leq \pi \left[\frac{\alpha}{1+i} + \frac{\pi(1-\alpha)}{i+\pi} \right] u(q^*).$$

Notice that at the Friedman rule ($i = 0$), the condition is satisfied because $c(\pi q^*) \leq \pi c(q^*) \leq \pi u(q^*)$. Also, note that the LHS of (E.1) does not depend on i , while the RHS of (E.1) is decreasing in i . Therefore, there exists a unique i_1 such that $i \leq i_1$, the allocation $q^n = q^b = q^*$ and $q^s = \pi q^*$ can be implemented.

In the second case, $\lambda_1, \lambda_2, \lambda_3 > 0$, all PCs are binding. Substituting z^n and z^b out using (26), (27) and (28) with equalities, we then obtain the allocation (q^n, q^b, q^s) which are characterized by (17), (30) and (31). Therefore the threshold interest rate is i_1 that solves (E.1) with equality.

Next, we derive the effect of inflation on the welfare. For any $i \leq i_1$, the first-best allocation is always implemented. Hence, $d\mathcal{W}/di = 0$ for $i \leq i_1$. Above the threshold i_1 , consider any arbitrary interest rates \hat{i}, \tilde{i} such that $\hat{i} > \tilde{i} > i_1$. Suppose $(\hat{q}^n, \hat{q}^b, \hat{z}^n, \hat{z}^b)$ and $(\tilde{q}^n, \tilde{q}^b, \tilde{z}^n, \tilde{z}^b)$ are respectively the optimal allocation for the given interest rates \hat{i} and \tilde{i} .

Since the participation constraints are binding, we must have

$$\begin{aligned} u(\hat{q}^n) &= \left(1 + \frac{\hat{i}}{\pi}\right) \hat{z}^n > \left(1 + \frac{\tilde{i}}{\pi}\right) \tilde{z}^n, \\ u(\hat{q}^b) &= (1 + \hat{i}) \hat{z}^b > (1 + \tilde{i}) \tilde{z}^b. \end{aligned}$$

Hence, equation (28) is trivially satisfied. Therefore, the allocation $(\hat{q}^n, \hat{q}^b, \hat{z}^n, \hat{z}^b)$ is also feasible under \tilde{i} . It immediately implies that

$$\begin{aligned} \mathcal{W}(\hat{i}) &= \pi\alpha u(\hat{q}^b) + \pi(1 - \alpha) u(\hat{q}^n) - c(\pi\alpha\hat{q}^b + \pi(1 - \alpha)\hat{q}^n) \\ &< \pi\alpha u(\tilde{q}^b) + \pi(1 - \alpha) u(\tilde{q}^n) - c(\pi\alpha\tilde{q}^b + \pi(1 - \alpha)\tilde{q}^n) \\ &= \mathcal{W}(\tilde{i}). \end{aligned}$$

The inequality comes from the uniqueness of the solution. Notice that when $i > i_1$, the solution is unique. Given that $(\tilde{q}^n, \tilde{q}^b, \tilde{z}^n, \tilde{z}^b)$ is not chosen when $i = \hat{i}$, it is associated with a lower level of welfare. We obtain the last equality because $(\tilde{q}^n, \tilde{q}^b, \tilde{z}^n, \tilde{z}^b)$ is optimum under \tilde{i} . Thus, $d\mathcal{W}/di < 0$ for any $i > i_1$. ■

F Proof of Proposition 4

Proof. In this proof, we derive the relationship between p^b and p^n . We will show that (i) for $i < \underline{i}$: p^n can be higher or lower than p^b ; (ii) for $i \in (\underline{i}, i_1)$: $p^b > p^n$; for $i \in (i_1, \bar{i})$: $p^b > p^n$; (iv) if the utility is isoelastic, then $p^b > p^n$ for all $i > \underline{i}$.

For $i < i_1$, the mechanism can implement the first-best allocation that $q^b = q^n = q^*$ and $q^s = \pi q^*$. However, there is a set of (z^b, z^n) to achieve the first-best as long as they satisfy

the participation constraints

$$(F.1) \quad u(q^*) \geq (1+i)z^b,$$

$$(F.2) \quad \frac{\pi}{\pi+i}u(q^*) \geq z^n,$$

$$(F.3) \quad \pi\alpha z^b + \pi(1-\alpha)z^n \geq c(\pi q^*).$$

From (F.1) and (F.2), the maximum payments for different buyers are

$$(F.4) \quad z_{max}^b = \frac{u(q^*)}{1+i},$$

$$(F.5) \quad z_{max}^n = \frac{\pi u(q^*)}{\pi+i},$$

which are decreasing in i . Given z_{max}^b , constraint (F.3) gives us

$$z^n \geq \frac{c(\pi q^*)}{\pi(1-\alpha)} - \frac{\alpha z^b}{1-\alpha} \geq \frac{c(\pi q^*)}{\pi(1-\alpha)} - \frac{\alpha z_{max}^b}{1-\alpha}.$$

Therefore, the minimum payment for a money-user is

$$z_{min}^n = \frac{c(\pi q^*)}{\pi(1-\alpha)} - \frac{\alpha u(q^*)}{(1-\alpha)(1+i)},$$

which is increasing in i . Similarly, given z_{max}^n , constraint (F.3) yields the minimum payment for a credit-user as

$$z_{min}^b = \frac{c(\pi q^*)}{\pi\alpha} - \frac{(1-\alpha)\pi u(q^*)}{\alpha(\pi+i)},$$

which is also increasing in i . Given the properties of maximum and minimum payments with i , it is easy to see that $z_{max}^b = z_{min}^b$ and $z_{max}^n = z_{min}^n$ when $i = i_1$. The set of optimal payments is then characterized by

$$\{(z^b, z^n) : z_{min}^b \leq z^b \leq z_{max}^b, z_{min}^n \leq z^n \leq z_{max}^n\}$$

which is depicted in Figure 4.

Notice that for any $\pi < 1$, $z_{max}^b > z_{max}^n$ for any $i < i_1$. Hence if $z_{min}^b > z_{max}^n$, the set of optimal payments must have the property that $z^b > z^n$ so that the implied prices satisfy $p^b = z^b/q^* > z^n/q^* = p^n$. This happens when $i > \underline{i}$ where \underline{i} is the solution for $z_{min}^b = z_{max}^n$ which is

$$\underline{i} = \pi \left(\frac{\pi u(q^*)}{c(\pi q^*)} - 1 \right).$$

Since i_1 solves

$$c(\pi q^*) = \left[\frac{\pi \alpha}{1+i} + \frac{\pi^2(1-\alpha)}{\pi+i} \right] u(q^*),$$

it is straightforward to verify that $\underline{i} < i_1$. This proves the results (i) and (ii).

We now prove result (iii). For $i \geq i_1$, the allocation is no longer the first-best. The participation constraints in (F.1), (F.2) and (F.3) must be binding. Hence the payments are unique (see Figure 4). In this case, $z^b = u(q^b)/(1+i)$ and $z^n = \pi u(q^n)/(\pi+i)$ and (q^b, q^n, q^s) satisfy equations (17), (30) and (31). Because $p^b(i_1) > p^n(i_1)$, p^b and p^n are continuous in i as $(q^b, q^n, q^s, z^b, z^n)$ are continuous in i , we can conclude that there is a $\bar{i} > i_1$ such that $p^b > p^n$. Combining with the previous case, it must be true that $p^b > p^n$ for any $i \in (\underline{i}, \bar{i})$.

Before we prove result (iv), we first establish that $q^b > q^n$ for any $\alpha, \pi \in (0, 1)$ and $i_1 < i < +\infty$. We prove by contradiction. Suppose it is not true, then $u'(q^b) \geq u'(q^n)$ for all $i > i_1$. From equilibrium condition (30), we have $u'(q^b) - c'(q^s) \geq u'(q^n) - c'(q^s)$ which implies

$$c'(q^s) - \frac{1}{1+i}u'(q^b) \geq c'(q^s) - \frac{\pi}{\pi+i}u'(q^n),$$

or equivalently

$$\frac{\pi}{\pi+i}u'(q^n) \geq \frac{1}{1+i}u'(q^b).$$

Because $\pi/(\pi+i) < 1/(1+i)$ for any $\pi < 1$, it must be true that $u'(q^n) > u'(q^b)$ which leads to a contradiction.

Finally, we prove result (iv). Recall that $p^b = \frac{u(q^b)}{(1+i)q^b}$ and $p^n = \frac{\pi u(q^n)}{(\pi+i)q^n}$ for $i > \underline{i}_1$, if u has a constant elasticity, then $p^b = \frac{u'(q^b)}{\rho(1+i)}$ and $p^n = \frac{u'(q^n)\pi}{\rho(\pi+i)}$ where $\rho \equiv u'(q)q/u(q)$. Hence, $p^b > p^n$ if and only if $u'(q^b)/(1+i) > \pi u'(q^n)/(\pi+i)$. We prove by contradiction. Suppose the last inequality is not true, then

$$c'(q^s) - \frac{u'(q^b)}{1+i} \geq c'(q^s) - \frac{\pi u'(q^n)}{\pi+i}.$$

From the equilibrium condition (30), we must have $u'(q^b) - c'(q^s) \geq u'(q^n) - c'(q^s)$ which contradicts to $u'(q^n) > u'(q^b)$. Therefore, $p^b > p^n$ for $i > \underline{i}$ by combining with previous results. ■

G Proof of Proposition 5

Proposition 5 consists of two parts relating to price discrimination and non-linear pricing. We will prove these results by a series of lemmas.

G.1 No Price Discrimination

Suppose the mechanism is restricted to non-contingent pricing, i.e., can only propose (q, z) to both types of buyers. We will show that under this mechanism, the allocation is dominated by the allocation (q^n, q^b, z^n, z^b) under the original mechanism.

Lemma G.1 *If the mechanism is not allowed to price discriminate, there exists $\hat{i}_1 = \underline{i} < i_1$, such that for all $i \leq \hat{i}_1$, $q = q^*$, and for $i > \hat{i}_1$, $q < q^*$. Moreover, for any $i > \hat{i}_1$, $q < q^n \leq q^b$.*

Proof. We will show that (i) for $i < \hat{i}_1$, $q = q^n = q^b = q^*$; (ii) for $i \in (\hat{i}_1, i_1)$, $q < q^n = q^b = q^*$; (iii) for $i > i_1$, $q < q^n \leq q^b < q^*$.

Under uniform pricing, the participation constraints from (26) to (28) can be written as

$$(G.1) \quad -iz + \pi (u(q) - z) \geq 0,$$

$$(G.2) \quad -iz + (u(q) - z) \geq 0,$$

$$(G.3) \quad -c(\pi q) + \pi z \geq 0.$$

The Lagrangian of the mechanism designer becomes

$$\begin{aligned} \max_{q,z} & \pi u(q) - c(\pi q) + \lambda_1 [-iz + \pi (u(q) - z)] \\ & + \lambda_2 [-iz + (u(q) - z)] + \lambda_3 [-c(\pi q) + \pi z]. \end{aligned}$$

The first order conditions imply that

$$\begin{aligned} (1 + \lambda_1 + \lambda_2) u'(q) &= (1 + \lambda_3) c'(\pi q), \\ \pi \lambda_3 &= (i + \pi) \lambda_1 + \pi (i + 1) \lambda_2. \end{aligned}$$

Observe that if (G.2) binds, then (G.1) will never hold. Therefore, (G.2) never binds. We then have two cases to consider: if $\lambda_3 = 0$, then $\lambda_1 = \lambda_2 = 0$; and if $\lambda_3 > 0$, $\lambda_1 > 0$. In the first case, all the constraints are not binding, the first-best allocation can be implemented, while in the second case, it cannot. The necessary and sufficient condition to achieve the first-best is

$$\frac{c(\pi q^*)}{\pi u(q^*)} \leq \frac{\pi}{i + \pi}.$$

Since the right hand side is decreasing in i , there must exist a cutoff \hat{i}_1 , such that for all $i \leq \hat{i}_1$, $q = q^*$. The cutoff is given by

$$\hat{i}_1 = \pi \left(\frac{\pi u(q^*)}{c(\pi q^*)} - 1 \right) = \underline{i}.$$

From the proof of Proposition 4, we know that $\underline{i} < i_1$. Hence, for any $i \in (\hat{i}_1, i_1]$, $q < q^n = q^b = q^*$. The above proves result (i) and (ii).

We now show result (iii). If $i > i_1$, we have already established that $q^n < q^b < q^*$. From equation (31), we obtain

$$(G.4) \quad \left(\frac{\pi\alpha}{1+i} + \frac{\pi(1-\alpha)}{\frac{i}{\pi} + 1} \right) u(q^n) = c(q^s) - \frac{\pi\alpha}{1+i} (u(q^b) - u(q^n)) > c(\pi q_1^n).$$

The last inequality in (G.4) is derived as follows. For any $q^n < q^b$, by Cauchy's mean value theorem, there exists $\hat{q} \in (q^n, q^b)$ such that

$$(G.5) \quad \frac{c(q^s) - c(\pi q_1^n)}{\pi\alpha c'(\pi\alpha\hat{q} + \pi(1-\alpha)q^n)} = \frac{\frac{1}{1+i} [u(q^b) - u(q^n)]}{\frac{1}{1+i} u'(\hat{q})}.$$

From (22) and (23), we know that $c'(q^s) > \frac{1}{1+i} u'(q^b)$ for any q^b and q^s . Then condition (G.5) yields the inequality in (G.4). Under the uniform pricing mechanism, when $i > i_1$, the constraint (G.1) binds and leads to

$$(G.6) \quad \frac{\pi}{\frac{i}{\pi} + 1} u(q) = c(\pi q).$$

If we compare (G.4) and (G.6) for given $q = q^n$, we see that i in (G.4) has to be higher than that in (G.6). This implies that $q^n > q$ for any given i , since q^n and q are decreasing in i . ■

G.2 No Non-linear Pricing

In this section, we consider two cases: (i) no non-linear pricing at all; (ii) no non-linear pricing on buyers. We first consider case (i). Suppose the mechanism is restricted to use only linear pricing. That is, the mechanism can only propose prices $\{p^b, p^n, p^s\}$ to corresponding agents instead of directly assigning allocations. The agents receive the prices specified by the mechanism and then choose the optimal quantities. The individual choice of q is given

by

$$q^b = \arg \max_q -i\phi m^b + \pi [u(q^b) - \phi p^b q^b], \text{ st. } p^b q^b \leq m^b + \ell.$$

$$q^n = \arg \max_q -i\phi m^n + \pi [u(q^n) - \phi p^n q^n], \text{ st. } p^n q^n \leq m^n.$$

$$q^s = \arg \max_q \phi p^s q^s - c(q^s).$$

The resource constraint on loans implies that $\pi \ell = (1 - \pi) m^b$. The first order conditions yield

$$(G.7) \quad \phi p^b = \frac{u'(q^b)}{1+i},$$

$$(G.8) \quad \phi p^n = \frac{\pi u'(q^n)}{\pi+i},$$

$$(G.9) \quad \phi p^s = c'(q^s).$$

Then the optimal mechanism chooses the prices $\{p^b, p^n, p^s\}$ to maximize the social welfare taking each agent's choice of q as given:

$$(G.10) \quad \max_{p^b, p^n, p^s} \pi \alpha u(q^b(p^b)) + \pi(1-\alpha) u(q^n(p^n)) - c(q^s(p^s))$$

st. $q^s \geq \pi \alpha q^b + \pi(1-\alpha) q^n,$

$$(G.11) \quad p^s q^s \leq \pi \alpha p^b q^b + \pi(1-\alpha) p^n q^n.$$

Constraint (G.10) is the resource constraint on consumption goods, and (G.11) is the resource constraint on money payments. They basically state that the total demand cannot exceed the total supply available. Of course, the maximization problem is also subject to the participation constraints, but it is straightforward to verify that they are not binding.

Lemma G.2 *If $i > 0$, the linear-pricing mechanism cannot implement the first-best allocation.*

Proof. To support first-best consumption $q^n = q^b = q^*$, buyer's prices have to be

$$\begin{aligned}\phi p_n &= \frac{u'(q^*)}{1 + \frac{i}{\pi}}, \\ \phi p_b &= \frac{u'(q^*)}{1 + i}.\end{aligned}$$

To support first-best production $q_s = \pi q^*$, seller's price has to be

$$\phi p_s = c'(\pi q^*).$$

Constraint (G.11) requires that

$$c'(\pi q^*) \leq \alpha \frac{u'(q^*)}{1 + i} + (1 - \alpha) \frac{u'(q^*)}{1 + \frac{i}{\pi}}.$$

This is not possible for any $i > 0$. ■

Now we consider case (ii) in which the mechanism can use non-linear pricing on sellers only. That is, the mechanism proposes $\{p^b, p^n, (q^s, z^s)\}$. The buyers still choose q that satisfies (G.7) and (G.8). The seller's participation constraint is

$$(G.12) \quad c(q^s) \leq z^s = \pi \alpha p^b q^b + \pi (1 - \alpha) p^n q^n.$$

Therefore, the optimal mechanism chooses $\{p^b, p^n\}$ to maximize the social welfare as

$$\max_{p^b, p^n} \pi \alpha u(q^b(p^b)) + \pi (1 - \alpha) u(q^n(p^n)) - c(\pi \alpha q^b(p^b) + \pi (1 - \alpha) q^n(p^n))$$

subject to (G.12).

Lemma G.3 *If the mechanism uses linear-pricing only on sellers, then the first-best allocation can be implemented if and only if $i \leq \tilde{i}_1$, where $\tilde{i}_1 < i_1$. And if u is close to linear, then $\tilde{i}_1 > \hat{i}_1$.*

Proof. Using (G.7), (G.8) and (G.12), the necessary and sufficient condition to implement the first-best is

$$(G.13) \quad c(\pi q^*) \leq \pi \left(\frac{\alpha}{1+i} + \frac{1-\alpha}{1+\frac{i}{\pi}} \right) u'(q^*) q^*.$$

Hence the cutoff \tilde{i}_1 exists such that for any $i \leq \tilde{i}_1$, (G.13) holds. Since u is strictly concave, $u'(q^*) q^* < u(q^*)$. Therefore, comparing (G.13) with (E.1), we can see that $\tilde{i}_1 < i_1$. Moreover, if u is close to linear, $qu' \rightarrow u$, then $\tilde{i}_1 \rightarrow i_1 > \hat{i}_1$. ■

H Proof of Proposition 6

Proof. In this proof, we first examine $d\mathcal{W}/d\alpha$ for $i > i_1$ by totally differentiating the equilibrium conditions and welfare with respect to α . Then we combine the conditions to show that the derivative is always positive. Finally, we show that the threshold i_1 is also increasing in α .

For any $i > i_1$, the equilibrium allocation is given by (17), (30), and (31). Notice that the following conditions must hold for all α , π , and $i > 0$:

$$(H.1) \quad \begin{aligned} \frac{1}{1+i} u'(q^b) &< c'(q^s) < u'(q^b), \\ \frac{\pi}{i+\pi} u'(q^n) &< c'(q^s) < u'(q^n). \end{aligned}$$

Also, for any $\alpha \neq 0$ and $\pi \in (0, 1)$, $q^n < q^b$ and

$$(H.2) \quad \frac{\pi}{i+\pi} u'(q^n) < \frac{1}{1+i} u'(q^b).$$

Totally differentiating (17) and (31) and replacing $dq^s/d\alpha$ with $dq^b/d\alpha$ and $dq^n/d\alpha$, we

obtain

$$(H.3) \quad \begin{aligned} & \pi\alpha (u'(q^b) - c'(q^s)) \frac{dq^b}{d\alpha} + \pi(1-\alpha) (u'(q^n) - c'(q^s)) \frac{dq^n}{d\alpha} \\ &= \frac{u'(q^n) - c'(q^s)}{c'(q^s) - \frac{\pi}{i+\pi}u'(q^n)} \pi \left[\left(\frac{u(q^b)}{1+i} - q^b c'(q^s) \right) - \left(\frac{\pi}{i+\pi}u(q^n) - q^n c'(q^s) \right) \right]. \end{aligned}$$

Also, totally differentiating welfare with respect to α , we get

$$(H.4) \quad \begin{aligned} \frac{d\mathcal{W}}{d\alpha} &= \pi [u(q^b) - u(q^n) - c'(q^s)(q^b - q^n)] \\ &+ \pi\alpha (u'(q^b) - c'(q^s)) \frac{dq^b}{d\alpha} + \pi(1-\alpha) (u'(q^n) - c'(q^s)) \frac{dq^n}{d\alpha}. \end{aligned}$$

Replacing the last two terms in (H.4) with (H.3) generates

$$(H.5) \quad \begin{aligned} \frac{d\mathcal{W}}{d\alpha} &= \pi [u(q^b) - u(q^n) - c'(q^s)(q^b - q^n)] \\ &+ \pi \frac{u'(q^n) - c'(q^s)}{c'(q^s) - \frac{\pi}{i+\pi}u'(q^n)} \left[\frac{u(q^b)}{1+i} - \frac{\pi u(q^n)}{\pi+i} - c'(q^s)(q^b - q^n) \right]. \end{aligned}$$

If we rewrite $u(q^b) - u(q^n)$ as $u'(\hat{q})(q^b - q^n)$ where $\hat{q} \in (q^n, q^b)$, then equation (H.5) becomes

$$(H.6) \quad \begin{aligned} \frac{d\mathcal{W}}{d\alpha} &= \pi [u'(\hat{q}) - c'(q^s)] (q^b - q^n) \\ &+ \pi \frac{u'(q^b) - c'(q^s)}{c'(q^s) - \frac{1}{i+1}u'(q^b)} \left\{ \left[\frac{u'(\hat{q})}{1+i} - c'(q^s) \right] (q^b - q^n) + \left(\frac{1}{1+i} - \frac{\pi}{i+\pi} \right) u'(q^n) \right\}. \end{aligned}$$

Note that the second term on the right hand side of (H.6) is derived using equation (30).

Rearranging terms, we have

$$(H.7) \quad \begin{aligned} \frac{d\mathcal{W}}{d\alpha} &= \pi (q^b - q^n) \left[u'(\hat{q}) - c'(q^s) - \frac{c'(q^s) - \frac{u'(\hat{q})}{1+i}}{c'(q^s) - \frac{1}{i+1}u'(q^b)} (u'(q^b) - c'(q^s)) \right] \\ &+ \pi \left(\frac{1}{1+i} - \frac{\pi}{i+\pi} \right) u'(q^n) \frac{u'(q^b) - c'(q^s)}{c'(q^s) - \frac{1}{i+1}u'(q^b)}. \end{aligned}$$

From (H.1), the last term in (H.7) is positive for any $\pi \in (0, 1)$. So the sign of the RHS depends on the sign of the first term. There are two cases. First, if $c'(q^s) \leq u'(\hat{q}) / (1 + i)$, then the first term in (H.7) is positive. Second, if $c'(q^s) > u'(\hat{q}) / (1 + i)$, then

$$\frac{c'(q^s) - \frac{u'(\hat{q})}{1+i}}{c'(q^s) - \frac{1}{i+1}u'(q^b)} (u'(q^b) - c'(q^s)) < u'(q^b) - c'(q^s) < u'(\hat{q}) - c'(q^s),$$

since $u'(\hat{q}) > u'(q^b)$. Thus, the first term is still positive. We can then conclude that $d\mathcal{W}/d\alpha > 0$ for any $i > i_1$, $\pi \in (0, 1)$ and $\alpha > 0$.

For the last part of the proposition, because the RHS of (29) is increasing in α while the LHS is constant, it follows that an increase in α raises i_1 . ■

I Implementation by Tax-Subsidy Scheme in Competitive Markets

Consider tax and subsidy on sellers only. Suppose the planner can create two markets, one is for credit-users and the other is for money-users. We do not impose credit-users' participation constraint at this moment, but will verify it later. Sellers are free to enter either market. The planner can enforce a lump-sum tax η_s on all sellers and subsidize proportionally to sellers in different markets. Let τ_b and τ_n denote the rate of subsidies.

The individual choice of q is given by

$$\begin{aligned} q^b &= \arg \max_q -i\phi m^b + \pi [u(q^b) - \phi p^b q^b], \text{ st. } p^b q^b \leq m^b + \ell. \\ q^n &= \arg \max_q -i\phi m^n + \pi [u(q^n) - \phi p^n q^n], \text{ st. } p^n q^n \leq m^n. \\ q_b^s &= \arg \max_q \phi p^b (1 + \tau_b) q_b^s - c(q_b^s) - \eta_s. \\ q_n^s &= \arg \max_q \phi p^n (1 + \tau_n) q_n^s - c(q_n^s) - \eta_s. \end{aligned}$$

The sellers are free to enter the markets, therefore

$$\phi p^b (1 + \tau_b) q_b^s - c(q_b^s) = \phi p^n (1 + \tau_n) q_n^s - c(q_n^s).$$

The resource constraint on loans implies that $\pi \ell = (1 - \pi) m^b$. The first order conditions yield

$$(I.1) \quad \phi p^b = \frac{u'(q^b)}{1 + i},$$

$$(I.2) \quad \phi p^n = \frac{\pi u'(q^n)}{\pi + i},$$

$$(I.3) \quad \phi p^b (1 + \tau_b) = c'(q_b^s),$$

$$(I.4) \quad \phi p^n (1 + \tau_n) = c'(q_n^s).$$

The first-best allocation requires that $q^b = q^n = q^*$, and $q_b^s = q_n^s = \pi q^*$. This implies that from (I.1) to (I.4)

$$(I.5) \quad \tau_b = i,$$

$$(I.6) \quad \tau_n = \frac{i}{\pi},$$

$$(I.7) \quad \phi p^b = \frac{u'(q^*)}{1 + i},$$

$$(I.8) \quad \phi p^n = \frac{u'(q^*)}{1 + \frac{i}{\pi}}.$$

Hence, $p^b > p^n$ means that there is price discrimination. The lump-sum tax that is levied to finance the subsidies must satisfy

$$(I.9) \quad \begin{aligned} \eta_s &= \alpha \phi p^b \tau_b q_b^s + (1 - \alpha) \phi p^n \tau_n q_n^s \\ &= \pi u'(q^*) q^* \left[\frac{\alpha i}{1 + i} + \frac{(1 - \alpha) \frac{i}{\pi}}{1 + \frac{i}{\pi}} \right]. \end{aligned}$$

Since taxes and subsidies are voluntary, the participation constraints of sellers must be satis-

fied,

$$\begin{aligned}\phi p^b (1 + \tau_b) q_b^s - c(q_b^s) - \eta_s &\geq 0, \\ \phi p^n (1 + \tau_n) q_n^s - c(q_n^s) - \eta_s &\geq 0.\end{aligned}$$

Plugging (I.5)-(I.9) into the constraints, we find that the tax-subsidy scheme can implement the first-best if and only if

$$c(\pi q^*) \leq \pi \left(\frac{\alpha}{1+i} + \frac{1-\alpha}{1+\frac{i}{\pi}} \right) u'(q^*) q^*,$$

which is the same as inequality (G.13). Therefore, the same interest cutoff \tilde{i}_1 is obtained.

Finally, we want to verify if credit-users' participation constraints hold, i.e.,

$$(I.10) \quad -i\phi m^b + \pi [u(q^b) - \phi p^b q^b] \geq -i\phi m^n + \pi [u(q^n) - \phi p^n q^n].$$

Replacing m^b and m^n with q^b and q^n at q^* , (I.10) becomes

$$\pi u(q^*) - \pi(1+i)\phi p^b q^* \geq \pi u(q^*) - (\pi+i)\phi p^n q^*.$$

Using (I.7) and (I.8), the above inequality holds with equality.

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