**Time Resolved Measurements of Wake Characteristics from Vertical Surface-Piercing Circular Cylinders**

S. J. Keough\(^1\,2\), I. L. Kermonde\(^1\), A. Amiet\(^1\), J. Philip\(^2\), A. Ooi\(^2\), J. P. Monty\(^2\) and B. Anderson\(^1\)

\(^1\)Maritime Division
Defence Science and Technology Group, Fishermans Bend, Victoria 3207, Australia
\(^2\)Department of Mechanical Engineering
University of Melbourne, Parkville, Victoria 3010, Australia

**Abstract**

Towing tank experiments were conducted at the Australian Maritime College facility in Launceston, measuring the size and shape of the wake generated by two different diameter cylinders pulled through the free surface at a range of speeds. Measurements of the bow wave run up height have been compared with previous experimental data and have been shown to be consistent with these results. For low Froude numbers, the maximum bow wave height scales with velocity as predicted by Bernoulli’s equation for inviscid flow. As velocity increases, the deviation from the inviscid flow prediction increases accordingly and can be modelled by adding a dissipation term to Bernoulli’s equation.

Previous studies have presented measurements of the bow wave run up height, mostly measured at a single point in time. Each of the previous works present measurements of the bow wave height at various Reynolds and Froude numbers, with only two of the runs exceeding the expected threshold for generation of the vortex street. Vortex shedding produces a small variation in the drag force on the cylinder, at twice the vortex shedding frequency. This effect is likely to cause the pressure at the front of the cylinder to fluctuate at the same rate, potentially generating a cyclic variation in the bow wave height as predicted by Bernoulli’s equation.

A Kármán vortex street is expected to occur with a Reynolds number in the range \(300 < Re < 3 \times 10^5\). In this range, the Strouhal number is expected to be approximately constant at \(St = 0.2\) \(^4\). For the experiments presented here the Reynolds numbers were in the range \(4.5 \times 10^4 < Re < 3.8 \times 10^5\) (Table 1), with only two of the runs exceeding the expected threshold for generation of the vortex street. Vortex shedding produces a small variation in the drag force on the cylinder, at twice the vortex shedding frequency. This effect is likely to cause the pressure at the front of the cylinder to fluctuate at the same rate, potentially generating a cyclic variation in the bow wave height as predicted by Bernoulli’s equation.

A feature of the wake behind cylinders pulled through a fluid is the Kármán vortex street; whereby a series of vortices are shed from alternating sides of the cylinder, at a frequency \(f\) dependent on the Strouhal number \((St)\) \(^4\).\n
\[ f = \frac{St \cdot V}{d} \]

\[(2)\]

**Introduction**

A common method for experimentally determining the size and shape of wake from a cylinder moving through a free surface is to conduct a towing tank experiment. Hay \(3\) presents the most comprehensive collection of such experiments and provides a template of measurements by which the size and shape of the wake can be characterised. The bow wave height \((D_1)\) shown in Figure 1, is one such measurement and is the focus of the present study. Other investigations \([1, 2]\) have been carried out similar to this work, while focusing on relatively large diameter cylinders penetrating the free surface to greater depths, so as to eliminate end effects. Chaplin and Teigen \([1]\) describe the theoretical maximum bow wave height by rearranging Bernoulli’s equation for inviscid flow \((V^2/2 = gD_1)\) such that the non-dimensional height \((D_1/d)\) depends on the Froude number based on the diameter \((Fr_d = V^2/\sqrt{gd}\), where \(V\) is the velocity and \(d\) is the diameter of the cylinder\), suggesting that this equation is expected to hold true for a circular cylinder at high Reynolds numbers and low Froude numbers.

\[
\frac{D_1}{d} = \frac{Fr_d^2}{2}
\]

\[(1)\]

Each of the previous works present measurements of the bow wave run up height, mostly measured at a single point in time. At low Froude numbers, the flow in the bow wave is steady, so a single measurement provides an accurate description of the shape. However, as the Froude number increases, the flow in the bow wave becomes chaotic, causing the height of the bow wave to vary. As such, the \(D_1\) value can no longer be measured by a snapshot approach without introducing an error term representing the variability. Using video capture to record the flow allows investigation of these unsteady variations and calculation of accurate mean and standard deviation values for the bow wave height measurement.

![Image of bow wave measurement](image)

**Figure 1:** Measured maximum height of the bow wave \((D_1)\). Here the cylinder is towed from right to left.

A feature of the wake behind cylinders pulled through a fluid is the Kármán vortex street; whereby a series of vortices are shed from alternating sides of the cylinder, at a frequency \((f)\) dependent on the Strouhal number \((St)\) \(^4\).\n
\[ f = \frac{St \cdot V}{d} \]

\[(2)\]

**Methodology**

**Towing Tank Experiments**

For these experiments, the towing tank located at the Australian Maritime College at the University of Tasmania has been used. This towing tank measures 100 m long by 3.55 m wide and was filled to a nominal water depth of 1.5 m. Cylinders of 100 mm and 216 mm diameter were mounted to the towing carriage such that the bottom of the cylinder reached 840 mm below the surface. The towing tank is capable of towing models up
to 4.6 m s\(^{-1}\) however due to the large drag force produced, the experimental runs were limited to a maximum of 3 m s\(^{-1}\) and 2 m s\(^{-1}\) for the 100 mm and 216 mm cylinders respectively. The cylinders were coloured with alternating yellow and black horizontal stripes of 10 mm increment and wooden rulers were attached to the towing carriage to allow for scaled measurements to be extracted from recorded imagery. Cameras were placed in a range of locations attached to both the towing carriage and to the viewing window on the side of the towing tank and video was recorded at 120 fps. For analysis of the bow wave height, video taken from in front of the cylinder was used (Figure 2).

Figure 2: 100 mm cylinder as viewed from the front of the towing carriage

### Video Processing

Using the yellow and black stripes on the cylinders as a calibration scale, the bow wave height at the front of the cylinder can be extracted from the video footage as a function of time by measuring the height in each frame. Doing these measurements manually is prohibitively time consuming so a method of automatically tracking the height of the bow wave was created. The software written to track the bow wave height takes each frame of the video and crops it to include only the cylinder. An image of the cylinder while stationary is used as a background and subtracted from the current frame. The result is converted to black and white such that only areas where the image has changed from the background show white. Spurious regions of white—created by changing lighting as the towing carriage moves and small movements in the camera position—are removed by dilating and filling then eroding the white regions, leaving a clear edge highlighting the top of the bow wave. A simple edge detection process is used to remove any remaining outliers and the instantaneous bow wave height is taken as the root mean square of the maximum height in each pixel column across the cropped image. Figure 3 shows the progression of the processed image using this method.

Figure 3: Progression of image processing used to isolate the bow wave for automated tracking. (a) cropped raw video; (b) background subtracted black and white; (c) dilated and filled; (d) final image used for tracking.

Figure 4 shows a small sample of one run, comparing the result of the automated bow wave height tracking with manual measurement of the same run. Comparison between the automatic and manually measured data for this run shows a strong Pearson correlation of 0.839, highlighting that the automated process is able to accurately track the variations in time. The difference between measured values is seen to be less than the experimental error expected in the manual process (±5 mm), as seen in Table 2, which shows a comparison between the calculated characteristic values.

### Table 1: Froude and Reynolds numbers for experimental runs

<table>
<thead>
<tr>
<th>Diameter (mm)</th>
<th>Velocity (m/s)</th>
<th>Fr</th>
<th>Re</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>0.5</td>
<td>0.505</td>
<td>45,656</td>
</tr>
<tr>
<td></td>
<td>1.0</td>
<td>1.01</td>
<td>91,312</td>
</tr>
<tr>
<td></td>
<td>1.5</td>
<td>1.515</td>
<td>136,968</td>
</tr>
<tr>
<td></td>
<td>2.0</td>
<td>2.02</td>
<td>182,623</td>
</tr>
<tr>
<td></td>
<td>2.5</td>
<td>2.525</td>
<td>228,279</td>
</tr>
<tr>
<td></td>
<td>3.0</td>
<td>3.03</td>
<td>273,935</td>
</tr>
<tr>
<td>216</td>
<td>0.5</td>
<td>0.349</td>
<td>95,877</td>
</tr>
<tr>
<td></td>
<td>1.0</td>
<td>0.697</td>
<td>191,755</td>
</tr>
<tr>
<td></td>
<td>1.5</td>
<td>1.046</td>
<td>287,632</td>
</tr>
<tr>
<td></td>
<td>1.75</td>
<td>1.22</td>
<td>335,570</td>
</tr>
<tr>
<td></td>
<td>2.0</td>
<td>1.394</td>
<td>383,509</td>
</tr>
</tbody>
</table>

Figure 4: Sample of measured bow wave height \(D_1\) for the 216 mm cylinder at 2 m s\(^{-1}\), comparing automated and manual measurement.

### Table 2: Calculated values comparing manual measurement with automated bow wave height tracking for the run shown in Figure 4

<table>
<thead>
<tr>
<th></th>
<th>Manual</th>
<th>Automatic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean (D_1) (cm)</td>
<td>19.85</td>
<td>19.58</td>
</tr>
<tr>
<td>Standard Deviation (D_1) (cm)</td>
<td>0.55</td>
<td>0.52</td>
</tr>
<tr>
<td>Pearson Correlation Coefficient</td>
<td>0.839</td>
<td></td>
</tr>
</tbody>
</table>

### Frequency Analysis

Frequency analysis was performed by calculating an estimate of the power spectral density (PSD) using the Welch method [8]. A Savitsky-Golay [6] filter was applied to the data to remove noise observed to be present at smaller and relatively larger scales. Note that the high frequency lobes in the PSD are a consequence of this filter, which was tolerable since we were mostly interested in relatively lower frequencies. Subsequently, the spectra were generated by implementing a Nuttall windowing function (minimum 4-term Blackman-Harris window [5]) on 5 second segments of signal with 50% overlap and averaging the discrete Fourier transform of each segment. In the cases where multiple runs had been completed with the same cylinder at the same speed, the PSD’s for each run were then averaged to
further reduce noise.

Results

At very low Froude numbers ($\text{Fr}_d \leq 0.5$) the flow in the bow wave is steady, with very little variation in the run up height over time. As the Froude number increases, unsteady chaotic motion appears and increases in size until the bow wave begins to break, at which point the flow is dominated by the unsteady chaotic motion; Figure 5 highlights this transition. The breaking wave introduces a cyclic mechanism in which water is pushed forward and crashes in front of the cylinder before recirculating back into the bow wave and contributing to the unsteady variation in the run up height.

![Figure 5: As $\text{Fr}_d$ increases, the flow transitions from steady flow at $\text{Fr}_d \leq 0.5$ (a), through the appearance of some unsteady chaotic motion $0.5 \leq \text{Fr}_d \leq 1.5$ (b), to unsteady breaking waves $\text{Fr}_d \geq 1.5$ (c).](image)

Figure 6 shows the measured height ($D_1$) for the 100 mm cylinder across a range of velocities from 0.5 m/s to 3 m/s ($\text{Fr}_d$ from 0.505 to 3.03). It can be seen that as the velocity increases, both the mean height and the amplitude of transient variations increase.

![Figure 6: Time resolved $D_1$ measurement for the 100 mm cylinder at a range of speeds from 0.5 m/s to 3 m/s.](image)

For the purpose of this experiment, we write Bernoulli’s equation as:

$$D_1 = \frac{V^2}{2g} \tag{3}$$

This equation highlights the important point that the bow wave height is, in theory, independent of the diameter of the cylinder. Figure 7 shows the theoretical bow wave height plotted against our experimental results for both the 100 mm and 216 mm cylinders, as well as for data taken from previous experiments [1, 2], showing an identical near linear relationship between the mean $D_1$ and $V^2$ for each experimental dataset.

![Figure 7: Mean $D_1$ values calculated from the present experiments, compared with results from previous works [1, 2]. The solid line shows the expected value, estimated using Bernoulli’s equation for inviscid flow. The dashed line shows Bournoulli’s equation including a dissipation term.](image)

As the mean $D_1$ increases with velocity, the transient variation in $D_1$ increases approximately linearly as shown in Figure 8, which compares the standard deviation of each measurement to the mean. This linear variation is likely a consequence of the fact that a small variation in velocity (or more precisely $V^2$) due to any experimental uncertainty or other unknown physical phenomenon, is directly translated to a variation in the height as per Bernoulli’s equation.

![Figure 8: Standard deviation of $D_1$ against the mean](image)

The power spectral density was calculated for runs at $\text{Fr}_d > 1.5$. For Froude numbers below $\text{Fr}_d = 1.5$, the flow in the bow wave is predominantly steady, with variation in the height generally less than the experimental error in the measurements. The results of the frequency analysis are shown as a stack plot in Figure 9. Each result exhibits a broad frequency peak in the region of 1-10 Hz however no clear single frequency peaks are observed, suggesting that any frequency effect expected due to
vortex shedding is likely to be too small to be easily distin-
guished from other effects in the signal.

![Figure 9: Stack plot of estimated power spectral density (PSD) for runs with \( F_{Rd} \geq 1.5 \). Dashed lines show the polynomial fit of the broad peak between 1-10 Hz.](image)

As the Froude number increases, the broad peak in the spec-
trum appears to shift slightly towards lower frequencies. If we
assume that for the inviscid flow prediction, the kinetic energy of
the flow is entirely conserved and translated into potential en-
ergy at the peak of the bow wave, then the time taken for wa-
ter in the breaking wave fall to back to the free surface level,
accelerated only by gravity, would be given by \( V/g \). If we
then assume, for the perfectly conservative model problem, that
the time taken to rise to the peak is also given by \( V/g \), the cy-
cle period for a crashing wave will be given by \( 2V/g \). Invert-
ing this to extract a theoretical frequency for the cyclic wave
.crashing gives \( f = 0.5g/V \), which would produce a peak shift
lower toward lower frequencies with increasing velocity. In a non-
conservative case, it may be possible to model the frequency as
\( f = C_{frig}g/V \), where \( C_{frig} \) is some constant less than 0.5, de-
pendent on the energy dissipation. By applying a simple poly-
nomial fit to the broad peak part of the spectra (shown as the
dashed lines in Figure 9), we can approximate the peak value and plot it against \( g/V \) to determine if such a relationship ex-
ists. Figure 10 shows this data along with a linearly fitted line
\( f = 0.46g/V + 0.64 \). While this appears to show that a re-
lationship of \( f \approx g/V \) could exist, the small amount of data and the noise level in the frequency analysis make it difficult to come to any firm conclusions in this regard. In order to improve the confidence in this result, more data at the same Froude num-
ers is required to reduce the noise in the signal. Further data at higher Froude numbers would also enable better investiga-
tion of the relationship between the frequency and the Froude
number.

**Conclusions**

Video analysis was used to investigate the transient nature of
the bow wave run up height measurements during towing tank ex-
periments of cylinders being pulled through the free surface. It was observed that for Froude numbers below \( F_{Rd} = 0.5 \) the flow in the bow wave was steady, with unsteady chaotic flow appearing and increasing in size up to approximately \( F_{Rd} = 1.5 \) at which point the bow wave began to break and the flow be-
came dominated by unsteady flow. A simple bow wave track-
ing algorithm was created and found to be as accurate as man-
ual tracking methods to within the experimental error, allowing
for accurate high frequency measurements of a large number of
experimental runs. The time resolved data was used to calcu-
late accurate mean bow wave height values, which were com-
pared with results from previous experiments and shown to be
consistent. The measurements confirmed that for low Froude
numbers, the bow wave height can be predicted by Bernoulli’s
equation for inviscid flow, which is independent of the size of
the cylinder. As velocity increases, the deviation from the invi-
sid flow prediction can be modelled by including a dissipation
term in Bernoulli’s equation with a constant of dissipation ap-
proximately given by \( C_{dissp} = 0.005 \). Frequency analysis of the
data showed that while effects due to vortex shedding from the
cylinder could not be detected experimentally in the height of
the bow wave, a cyclic effect from the breaking wave does oc-
cur that interacts with the height variation and is likely to affect
the frequency according to some relationship \( f \approx g/V \).

**Acknowledgements**

The authors would like to thank the staff at AMC for their as-
sistance in preparing and operating the towing tank facility.

**References**

surface-piercing circular cylinder, *Journal of Fluids and

Experimental and numerical analysis of submarine mast
surface wakes, in *Pacific International Maritime Confer-
ence*, 2015, 1–12.

length*, Princeton University, 1947.

data for rigid circular cylinders*, Technical Extension Ser-
vice, Washington State University, 1966.

[5] Nuttall, A., *Some windows with very good sidelobe behav-
ior*, *IEEE Transactions on Acoustics, Speech, and Signal

[6] Savitzky, A. and Golay, M. J. E., Smoothing and differenti-
ation of data by simplified least squares procedures., *Ana-


[8] Welch, P., The use of fast fourier transform for the estima-
tion of power spectra: A method based on time averaging
over short, modified periodograms, *IEEE Transactions on
Audio and Electroacoustics*, 15, 1967, 70–73.
Author/s: Keough, SJ; Kermonde, IL; Amiet, A; Philip, J; Ooi, A; MONTY, J; Anderson, B

Title: Time Resolved Measurements of Wake Characteristics from Vertical Surface-Piercing Circular Cylinders

Date: 2016

Citation: Keough, SJ; Kermonde, IL; Amiet, A; Philip, J; Ooi, A; MONTY, J; Anderson, B, Time Resolved Measurements of Wake Characteristics from Vertical Surface-Piercing Circular Cylinders, Proceedings of the 20th Australasian Fluid Mechanics Conference, 2016

Persistent Link: http://hdl.handle.net/11343/214570

File Description: Published version