An Optical Study of Sound Generation in Lean Premixed Combustion

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Abstract

This thesis presents an experimental and theoretical study of sound generation in lean, premixed, propane-air flames. Sound generation by axisymmetric, forced, laminar flames is investigated first. The sound generated by the formation and consumption of separated pockets is determined. The second part of the thesis quantifies the capability of Flame Chemiluminescence Tomography (FCT) to directly study the flame dynamics responsible for sound generation. Suitable techniques are not yet available to acquire time-resolved, 3D measurements of entire turbulent flames. FCT is therefore investigated on non-axisymmetric, forced, laminar flames.

Chemiluminescence imaging and microphone measurements are used to relate the acoustic pressure emitted by forced, laminar flames to the flame geometry at different phases of the forcing cycle. The formation and consumption of separated pockets is shown to be a significant source of sound. A model of sound generation by conical and spherical flame elements is presented, where the flame is treated as an infinitely-thin zone of heat generation. It is shown that increases in the flame displacement speed with increasing curvature is responsible for a significant increase in the amplitude of the radiated sound. The merging of the preheat and reaction zones of oppositely propagating flames that occurs in the terminal moments of both the formation and consumption of separated reactant pockets has previously been identified as a potentially significant source of combustion noise. However, in forced, laminar, lean, propane-air flames the sound produced by this merging is found to be small relative to the sound generated by negative flame stretch that occurs before opposite flames are within two thermal flame thicknesses of each other.

To assess the capabilities of FCT, a single camera is used to acquire 36 equally-spaced, coplanar views of forced, premixed, non-axisymmetric, laminar flames at 100 phases of their forcing cycle. A Multiplicative Algebraic Reconstruction Technique (MART) algorithm is then used to reconstruct the time resolved chemiluminescence field using different numbers of equally-spaced views. Algorithms for measuring the flame surface area, the mean curvature, the normal component of the flame propagation velocity and the chemiluminescent flame thickness are demonstrated and the sensitivity of these measurements to the number of views used in the reconstruction is assessed. Less than 10 views may be sufficient to measure flame surface area, flame curvature and flame
surface speed. However, at least 18 views appear to be required in order to obtain useful measurements of flame thickness.
Declaration

This is to certify that:

(i) the thesis comprises only my original work towards the PhD,

(ii) due acknowledgement has been made in the text to all other material used,

(iii) the thesis is less than 100,000 words in length, exclusive of tables, maps, bibliographies and appendices.

Samuel Wiseman
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List of symbols

Roman Letters
\( R \) Specific ideal gas constant
\( S(x,t) \) Distributed source function
LHV Fuel lower heating value
\( n \) Flame surface normal
\( q \) Conductive heat flux
\( A \) Area
\( B_j \) Basis function coefficient
\( b_j \) Basis function
\( C \) Contrast
\( c \) Speed of sound
\( c_p \) Specific heat capacity at constant pressure
\( c_v \) Specific heat capacity at constant volume
\( CTF \) Contrast Transfer Function
\( d_{mic} \) Distance from microphone to position of unforced flame tip.
\( E \) Density ratio of unburned to burned gas
\( G \) Green’s function
\( h \) Mass specific enthalpy
\( I \) Normalised light emission intensity
\( I' \) Instantaneous light emission intensity
\( I_{rms} \) RMS current
\( k \) Proportionality constant
\( k_m \) Mean curvature
\( L_d \) Markstein length for flame displacement speed
\( L_q \) Marksteins length for flame heat release speed
\( Le \) Lewis number
\( P \) Projection measurement
\( p \) Pressure
\( Q \) Heat release rate per unit volume
List of symbols

\( R \)  
Radius or radial distance

\( r \)  
Radius or radial distance

\( r_f \)  
Flame radius

\( s(\lambda) \)  
Optical wavelength dependent sensitivity function

\( s_c \)  
Flame consumption speed

\( s_d \)  
Flame displacement speed

\( S_{L,0} \)  
Planar, unstrained laminar flame speed

\( s_q \)  
Flame heat release speed

\( St \)  
Strouhal number

\( T \)  
Temperature

\( t \)  
Time

\( T_{ij} \)  
Lighthill’s stress tensor

\( u \)  
Unburned gas velocity

\( V \)  
Volume

\( v \)  
Fluid particle velocity.

\( V_f \)  
Flame propagation velocity (relative to laboratory reference frame)

\( w_i \)  
Weighting function for projection \( i \)

\( W_{i,j} \)  
Weighting for combination of basis function \( j \) and projection \( i \)

\( x \)  
Cartesian coordinates

\( y \)  
Cartesian co-ordinates

\( Y_i \)  
Mass fraction of species \( i \)

**Greek Letters**

\( \delta \)  
Flame thickness

\( \delta_{ij} \)  
Kronecker delta

\( \delta_{th} \)  
Thermal flame thickness

\( \dot{\omega}_i \)  
Mass consumption rate of species \( i \) per unit volume

\( \gamma \)  
Ratio of specific heats

\( \kappa \)  
Flame stretch

\( \lambda \)  
Acoustic wavelength

\( \mu \)  
Relaxation factor

\( \omega \)  
Angular frequency

\( \phi \)  
Equivalence ratio

\( \rho \)  
Density

\( \rho_e \)  
Excess density

\( \sigma \)  
Signal standard deviation

\( \tau \)  
Acoustic retarded time

\( \tau_{ij} \)  
Viscous stress tensor

\( \theta \)  
Angle
List of symbols

**Superscripts**

- \( ^{′} \) Fluctuating component

**Subscripts**

- \( 0 \) Constant reference value
- \( b \) Burned products
- \( F \) Fuel
- \( u \) Unburned reactants

**Accents**

- \( \bar{\cdot} \) Mean value
- \( \dot{\cdot} \) Time rate of change
- \( \hat{\cdot} \) Tomographically reconstructed field
Chapter 1

Introduction

1.1 Background and Motivation

Interest in the mechanisms of sound generation by premixed combustion is driven by two objectives: to reduce noise emissions from combustion devices, and to avoid thermoacoustic instabilities. Combustion noise is a concern in aeroengine design (Dowling and Mahmoudi, 2015; Mahan and Karchmer, 1991). This is largely a result of the progress made in reducing other noise sources and combustion noise is now estimated to be of similar magnitude to jet noise, fan noise and other aerodynamic noise.

Thermoacoustic combustion instability is an unstable coupling between the flame motion and the combustor's acoustics. It disrupts the development of combustion systems in industrial processing, rockets, ramjets, afterburners and land-based gas turbines (Zinn and Lieuwen, 2000). Large-amplitude pressure pulsations can result. Thermoacoustic instability is increasingly becoming a problem for gas turbines designed for electricity generation as lean, premixed combustion is adopted to meet increasingly strict emission standards.

Lean Premixed Gas Turbines and Thermoacoustic Instability

Land-based gas turbines are likely to account for a significant proportion of electricity generation around the world over the next few decades. The ‘new policies scenario’ reported in the World Energy Outlook (WEO) 2016 takes into account the current policies of governments and, to some extent, declared policy intentions (International Energy Agency, 2016). Under this scenario gas consumption is projected to increase on average by 1.5% per year over the period to 2040. Gas turbines are capable of responding to fluctuations in generation from intermittent renewable sources, specifically photovoltaic
solar panels and wind turbines, and therefore are projected to play a significant supporting role in the decarbonization of electricity generation.

Reducing the pollutant emissions from gas turbines is therefore important. Nitrogen Oxide (NO) and Nitrogen Dioxide (NO₂), together known as NOₓ, are of particular concern. NOₓ contributes to the formation of photochemical smog and to acid rain (Lefebvre and Ballal, 2010). NO is formed during the combustion of hydrocarbons with air by three main mechanisms: the thermal or Zeldovich mechanism, the N₂O mechanism, and the prompt mechanism (Bowman, 2000). The amount of thermal NO produced is a function of the maximum temperature and the residence time of the gases at that temperature. One widely adopted strategy to keep the temperatures low and thereby minimise thermal NO in gas turbines is lean premixed combustion (Brewster et al., 1999; Correa, 1993). This does not affect the gas turbine’s thermodynamic efficiency as the combustor exit temperature is currently limited by material considerations and air dilution downstream of the combustion zone is typically used to regulate the burned gas temperature.

Gas turbines operating in lean premixed mode are particularly susceptible to thermoacoustic instabilities (Huang and Yang, 2009; Jones, 2011). A number of reasons for this susceptibility have been identified including a reduction in acoustic damping due to a reduction in the cooling air supplied along the combustor liner and the relatively short flame length relative to the wavelengths of resonant modes of the combustor (Huang and Yang, 2009; O’Connor et al., 2015). Numerous types of thermoacoustic instabilities have been observed. Resonant acoustic modes of the system can be longitudinal, transverse, radial, or azimuthal (Poinsot, 2017), and coupling with the flame can be through induced changes in flame surface area, consumption speeds, equivalence ratio, and mixture composition (O’Connor et al., 2015). Thermoacoustic instabilities often involve interactions between various aspects of a combustion system. Though current methods of predicting thermoacoustic instabilities in gas turbine combustors do not directly use models of sound production from unconfined flames, sound production is understood to be a key element in certain types of thermoacoustic instability (Candel et al., 2004). An increased understanding of sound production in unconfined flames may provide insight into some aspects of thermoacoustic instabilities that remain poorly understood. A summary of the current gaps in the understanding of thermoacoustic instabilities is given by Poinsot (2017).

Poinsot (2017) also discusses the related problems of thermoacoustic instability in rocket engines. One major difference between real engines, both gas turbines and rocket engines, and the simplified combustors more commonly studied in research laboratories, is the extremely high frequency of flame-flame interactions (Poinsot, 2017). Rocket chambers can be even more complex than gas turbine combustors since they sometimes feature hundreds of injectors. Transverse thermoacoustic instabilities in liquid rockets led to the first large-scale effort to understand and control thermoacoustic instabilities
1.1 Background and Motivation

Many thermoacoustic instability mechanisms have been identified and may involve the fuel injection system, the atomisation and vaporisation of the fuel, or just the flame dynamics. On the other hand, instabilities due to convective/entropy waves or vortex shedding are extremely uncommon in rocket engines. Comprehensive reviews of thermoacoustic instabilities in rocket engines are given by Culick (2006) and O’Connor et al. (2015).

Sound Generation in Lean Premixed Combustion

The mechanisms of combustion generated noise are commonly categorised as either direct or indirect (Candel et al., 2009; Strahle, 1978). Direct sound originates from the unsteady motion of the flame. Indirect noise, sometimes referred to as entropy noise, originates downstream of the flame and is generated by flow inhomogeneities.

Only direct sound is considered in this work. As will be detailed in Chapter 2, direct sound from open, turbulent, premixed flames has been characterised experimentally and is understood to be proportional to the rate of change of the heat release rate in the combustion region. Fluctuations in the heat release rate depend on the response of flames to turbulent velocity fluctuations. This has been consistently identified as the most challenging aspect of predicting and understanding premixed, combustion noise.

Many combustion systems operate in the wrinkled flame regime, including gas turbines used in power generation (Swaminathan and Bray, 2011, p15). In this regime, the structure of the reaction zone of the flame is not significantly perturbed by the turbulence and is substantially unchanged from the internal structure of a steady laminar flame (Veynante and Vervisch, 2002). Distortions of the flame are commonly described in terms of flame stretch. Flame stretch is the fractional rate of change of the area of an infinitesimal flame surface element and can occur due to flow velocity gradients or the propagation of curved flames. Commonly observed processes featuring high flame stretch include the separation of pockets of unburned reactants from the bulk of the reactants, cusp recovery, and the consumption of separated pockets. Many of these flame processes have been extensively studied in the premixed flame literature but there has been relatively little consideration of the sound they generate. Combustion noise theory predicts that, if a wrinkled flame is acoustically compact, flame processes will generate similar acoustic waveforms whether they occur in a turbulent or laminar flow field.

Flame annihilation has nonetheless recently received particular interest regarding its potential role in sound production. Flame annihilation is the mutual interaction and destruction of flames propagating in opposite directions and occurs in the terminal moments of tunnel narrowing and pocket consumption, and at the initiation of tunnel formation. Significant flame acceleration has been observed during annihilation, beginning when
the diffusive flame zones first overlap, and it has been demonstrated numerically and theoretically that high-amplitude, radiating, pressure fluctuations can be generated (Talei et al., 2012b). However, it is not clear how significant the sound generated by flame annihilation is compared to other sound generation processes in premixed flames.

1.2 A Note on Terminology

There is no standard terminology for the flame processes leading up to, during, and following the separation of pockets of unburned reactants. The terminology used in this thesis is outlined here. The instant at which a pocket of reactants separates from the bulk of the reactants, termed the separation reference time, marks the end of tunnel narrowing and the start of both cusp recovery and pocket consumption. The separation reference time is defined as the earliest instant when flames on opposite sides of the tunnel have merged so that there is a single peak in the heat release rate profile along a path across the tunnel.

Two cusps are formed by the separation event. In forced, laminar flames featuring separated pockets, the lower cusp is connected to the anchored flame, and the upper cusp forms part of the separated pocket. The first stages of pocket consumption therefore include cusp recovery.

The term flame annihilation is used to refer to the extinction process that occurs as oppositely propagating flames interact and overlap. The flame annihilation process is defined as beginning when the flames are separated by two thermal flame thicknesses. The location of the flame is taken to be the position of peak heat release rate. The annihilation reference time is the instant at which the heat release rate profiles of the two flames have merged so that there is a single peak in the profile. Pocket separation is an approximately axisymmetric flame annihilation event. Pocket consumption features an approximately spherically symmetric flame annihilation event in the terminal moments, termed the pocket burn-out event.

1.3 Research Aims

This work therefore aims to further the understanding of sound generation by premixed combustion. Studying these processes in turbulent flames is difficult due to the complexity and chaotic nature of turbulence. However, some flame processes can be produced in a highly repeatable and controllable manner in forced, laminar flames. Furthermore, sound generation and thermoacoustic instability in laminar, premixed flames has direct practical application for laminar, premixed burners used in heating, chemical diagnostics, and the production of some engineering materials (Kuo, 2005). In this work sound generation
by tunnel narrowing, pocket consumption, and cusp recovery is investigated in forced, laminar, premixed, conical, lean, propane-air flames. Using forced, laminar flames has the advantage that geometrical parameters of the flame can be readily manipulated by controlling the forcing frequency, forcing amplitude and unburned mixture flow rate. Combustion noise theory predicts that these flame dynamics will generate similar acoustic waveforms whether they occur within a laminar or turbulent flow field, and so the results of this work are relevant to understanding the sound generation mechanisms in turbulent, lean, premixed flames. However, many other sources of combustion noise, including other sources of direct combustion noise, are not investigated in this work. The specific aims of the project are as follows:

1. To determine the contributions of tunnel narrowing, pocket consumption, and cusp recovery processes to the acoustic waveforms emitted by forced, laminar, lean, premixed, propane-air flames.

In Chapter 4, tunnel narrowing, pocket consumption, and cusp recovery are investigated using flame chemiluminescence imaging and measurement of the emitted sound. Flame images are related to the measured acoustic pressure waveform by estimating the acoustic propagation delay to the microphone. Pocket consumption and cusp recovery are typically both initiated by the separation of a reactant pocket and therefore occur simultaneously. To determine the acoustic contribution of each process, forcing amplitudes were swept from low amplitudes that do not produce separated pockets to high amplitudes that produce separated pockets several millimetres in diameter. The effect of the forcing Strouhal number and equivalence ratio is also investigated. In Chapter 5, visible chemiluminescence from the region of the flame where the pocket is formed and consumed is used to predict the emitted sound and the predictions are compared to the measured sound.

2. To determine the effect of flame acceleration on the sound generated by tunnel narrowing and pocket consumption processes in lean, premixed flames.

The displacement speed and consumption speed of premixed flames is modified by flame stretch. Acoustic modelling presented in Chapter 6 is used to investigate the effect of flame acceleration on the sound generated by tunnel narrowing and pocket consumption. The modelling enables quantification of the contribution of different parts of the flame to the emitted sound. The flame is modelled as an infinitely thin heat-release region for this work. The modelling limitations are evaluated by comparing the measured and predicted sound in propane-air flames at multiple equivalence ratios. The dependence of the flame displacement speed on the mean curvature is determined empirically using
Chemiluminescence images of tunnel narrowing.

3. To determine the number of views required to obtain certain flame measurements when applying Flame Chemiluminescence Tomography to laboratory-scale, premixed jet flames.

Flame Chemiluminescence Tomography (FCT) is one of the most promising, time-resolved, 3D diagnostics for directly investigating sound generation in turbulent, premixed flames. Time-resolved Flame Chemiluminescence Tomography of a turbulent flame requires multiple high-speed cameras. This project instead utilises a relatively simple experimental setup comprising a single intensified camera and a non-axisymmetric, forced, laminar, premixed flame. This setup allows an arbitrary number of views to be acquired and thus enables tomographic reconstructions with greater spatial and temporal resolution than has previously been achieved. Measurements of flame surface area, flame curvature, the flame-normal component of the propagation velocity, and chemiluminescent flame thickness are obtained from tomographically reconstructed emission intensity fields. A convergence analysis is undertaken to determine the sensitivity of the measurements to the number of views used in the reconstruction.
Chapter 2

Literature Review

2.1 Premixed Combustion

Combustion is an exothermic chemical reaction involving a fuel and a oxidiser, normally air. In premixed combustion the fuel and oxidiser are fully mixed prior to the reaction. This is in contrast to non-premixed combustion where the fuel and oxidiser are initially separate and mix at the reaction region. Premixed combustion can occur homogeneously through the reactant mixture or can occur in a thin reaction zone, a flame, that propagates towards the reactant mixture. Turns (2000) defines a flame as a self-sustaining propagation of a localized combustion zone at subsonic velocities.

2.1.1 Laminar Flames

Flames propagate in a self-sustaining manner by the diffusion of thermal energy and radical chemical species from the reaction zone into the unburned mixture upstream. Flame propagation will continue until the unburned mixture is consumed or the flame is quenched. The structure of a laminar flame is shown in Figure 2.1. The premixed flame comprises a preheat zone and a reaction zone. Intermediate species are mostly confined to the reaction zone. For many species, diffusive and reactive processes dominate in the reaction zone. In the preheat zone, reactive processes are negligible but there are large gradients in temperature and species concentration. For each chemical species, the sum of the mass flux due to diffusion and advection remain constant in this region. The thickness of premixed, hydrocarbon-air flames is typically on the order of 0.1-1 mm (Veynante and Vervisch, 2002). Temperature ratios between the unburned gases at standard conditions and burned gases are typically 5-7. The laminar, planar, unstrained flame speed ($S_{L,0}$) is typically between 0.1-1 m/s and depends on the physical properties of the fuel and oxidiser, the chemical kinetics, and the temperature of the unburned gases (Turns, 2000).
Laminar Flame Speeds

Three flame speeds are routinely referred to in the literature: the flame displacement speed ($s_d$), the flame propagation velocity ($V_f$), and the flame consumption speed ($s_c$) (Driscoll, 2008; Poinsot et al., 1992). The flame propagation velocity is occasionally referred to elsewhere as the absolute flame velocity (e.g. Poinsot and Veynante, 2005). This section provides a review of the theoretical and phenomenological relations between flame speeds and flame geometry, with a focus on highly-curved, inwardly-propagating flames. Some of these relations are used in Chapter 6 to model sound generation.

In flame models that treat the flame as a surface of discontinuity that separates the unburned reactants and burned products, the flame displacement speed ($s_d$) is defined as the speed of the flame relative to the unburned gas in front of the flame. Difficulties with this definition arise when undertaking a theoretical, experimental or numerical analysis that partially or fully resolves the internal flame structure. The gas density decreases through the thin flame region, from the cool unburned reactants to the hot burned products, and the gas velocity therefore increases. The flame displacement speed can be rigorously defined as the speed of the flame relative to the gas at the flame position (Giannakopoulos et al., 2015b). The flame position is typically defined by an isosurface of a scalar property such as temperature or a species mass fraction. The surface representing the flame position is referred to as the flame surface or flame front. However, under this definition, the flame displacement speed changes significantly depending on the chosen definition of the flame surface. This has been discussed in detail elsewhere (Lipatnikov, 1996; Tien and Matalon, 1991). To overcome this problem, a density-weighted flame displacement speed is sometimes used (e.g. Giannakopoulos et al., 2015b), defined as,

$$
\tilde{s}_d = \rho s_d / \rho_u.
$$

(2.1)
The density-weighted flame displacement speed is less sensitive to the definition of the flame surface and is typically close to the intuitive definition of the flame displacement speed used in infinitely-thin flame models.

Experimentalists almost exclusively use the first definition of the flame displacement speed, i.e., the flame speed relative to the cool, unburned gases ahead of the flame. This is often out of necessity because the measurement resolution is typically not sufficient to accurately determine the gas velocity within the flame. A practical difficulty for experimentalists is the appropriate choice of a surface or contour to identify the flame position. Experimental measurement of the flame position is obtained by many different methods including chemiluminescence imaging, fluorescence imaging, imaging of mie scattering, shadowgraph imaging, and schlieren imaging. These imaging methods often produce very different signal profiles across the flame. For some of these methods, the signal profile has a peak near the reaction zone and decreases on either side. In these cases, the flame surface is typically defined as the set of points where the signal is at a peak when moving across the flame from unburned reactants to burned products. This definition avoids an arbitrary choice of isolevel and enables more meaningful comparisons of multiple experimental data sets because the signal peak is generally close to the peak reaction zone.

The flame propagation velocity \( V_f \) is the flame velocity relative to a fixed laboratory reference frame. It is equal to the vector sum of the gas velocity \( \mathbf{v} \) and the flame displacement velocity \( \mathbf{s}_d \mathbf{n} \), where \( \mathbf{n} \) is a unit vector, normal to the flame surface. The flame consumption speed \( \mathbf{s}_c \) is defined as the speed at which a reactant, often the fuel, is consumed and is defined as

\[
\mathbf{s}_c = \int_{n^-}^{n^+} \rho_f d\mathbf{n}/\rho_u Y_{F,u}.
\]

Unlike the flame displacement speed, the flame propagation velocity and the flame consumption speed are relatively insensitive to the chosen definition of the flame surface.

Flame speeds are modified by flame curvature and the strain rate in the tangent plane of the flame surface. The precise nature of this dependence remains a subject of research (Giannakopoulos et al., 2015a). An early phenomenological model of the relationship between flame curvature and the flame displacement speed was proposed by Markstein (1964) and is hereafter referred to as the Markstein relation. Markstein proposed that the flame speed is linearly proportional to the mean curvature \( k_m \) of the flame,

\[
s = S_{L,0}(1 - L_1 k_m).
\]

The proportionality constant \( L_1 \) is commonly called the Markstein length. Curvature is defined as negative when the flame curves towards the unburned reactants. The mean
curvature $k_m$ is the sum of the principle curvatures. Markstein (1964) did not distinguish between the displacement speed and consumption speed in this relation. This is typical of the earlier studies that referred ambiguously to the ‘burning velocity’ (Tien and Matalon, 1991).

It was later proposed that the flame displacement speed is linearly related to flame stretch ($\kappa$):

$$s_d = S_{L,0} - L_2 \kappa. \quad (2.4)$$

This hypothesis was based on several theoretical studies (Clavin and Williams, 1982; Frankel and Sivashinsky, 1983; Matalon and Matkowsky, 1982; Pelce and Clavin, 1982) and is here referred to as the ‘theory of flame stretch’. Flame stretch is defined using the concept of a Lagrangian flame surface element, abbreviated to ‘flame element’. A flame element is part of a flame surface bounded by a closed curve. The boundaries of the flame element move at the flame propagation velocity. Flame stretch ($\kappa$) is defined as the fractional rate of change of the surface area of an infinitesimal flame element (Poinsot and Veynante, 2005, p59). Flame stretch is a combination of the rate of strain in the tangent plane of the flame and the stretching of the surface due to the combination of its curvature and normal propagation. If the flame surface is defined as a level set of a scalar field $G(x,t)$, where $\nabla G$ is in the direction of the burned products, flame stretch can be expressed as

$$\kappa = (-n n : \nabla \mathbf{v} + \nabla \cdot \mathbf{v}) + s_d \nabla \cdot \mathbf{n}, \quad (2.5)$$

where $\mathbf{v}$ is the gas velocity and $\mathbf{n} = -\nabla G / |\nabla G|$ is the unit normal vector pointing towards the unburned reactants (Candel and Poinsot, 1990). The double dot product $nn : \nabla \mathbf{v}$ can be written in index notation as $n_i n_j \partial v_i / \partial x_j$. It is important to recognise that $\mathbf{n}$ is defined everywhere, not just at the flame surface. However, $\kappa$ has physical significance only at the flame surface.

$\nabla \cdot \mathbf{n}$ is equal to the mean curvature $k_m$ of the surface defined by the level set. If the strain rate terms (in brackets in equation 2.5) are negligible, Equations 2.4 and 2.5 yield:

$$s_d = \frac{S_{L,0}}{1 + L_2 k_m}. \quad (2.6)$$

The proportionality constant in Equation 2.4 ($L_2$) continues to be referred to as the Markstein length, though in general different values of the Markstein length will be measured when fitting experimental data to the theory of flame stretch (Equation 2.6) than to the Markstein relation (Equation 2.3) (Durox et al., 2001). Equation 2.6 predicts that the flame displacement speed becomes infinite when the flame curvature is equal to the reciprocal of the Markstein length. This has been interpreted in previous studies as the maximum possible curvature a steady flame can have (Echekki and Mungal, 1991, p460). The unburned
reactants contained by inwardly propagating spherical flame are presumed to react almost simultaneously and instantaneously when the flame curvature reaches this value.

There is a large body of experimental and numerical research aimed towards measuring Markstein lengths. Flame configurations that have been used to estimate Markstein lengths include outwardly propagating spherical flames (Giannakopoulos et al., 2015b; Lipatnikov et al., 2015), inwardly propagating spherical flames (Baillot et al., 2002; Giannakopoulos et al., 2015a), inwardly propagating cylindrical flames (Durox et al., 2001) and strained counterflow or stagnation flames (Wu and Law, 1985). However, there is a large amount of scatter in the results (Baillot et al., 2002; Sinibaldi et al., 2003). One factor contributing to the scatter is the choice of the flame surface (Tien and Matalon, 1991). This has been shown to strongly influence measurements of the flame curvature and the strain rate and hence the computed Markstein length (Giannakopoulos et al., 2015b). Further complicating the experimental data is the range of optical methods that have been used and the various post-processing and contour extraction methods performed on image data with low signal-to-noise ratios. Imaging methods that have been used include chemiluminescence imaging (Baillot and Bourehla, 1997), Schlieren imaging (Durox et al., 2001), and shadowgraph imaging Ibarreta and Driscoll (2000). Baillot et al. (2002) compared Markstein lengths determined from these three imaging methods and obtained results that differed by more than a factor of 2.

It has been widely argued that the scatter in reported measurements of the Markstein length is also partly due to the fundamental limitations of the theory of flame stretch. The theory of flame stretch is based on an asymptotic analysis that employs a number of simplifying assumptions including unity Lewis number and weak flame stretch, and the analysis does not consider the diffusion of radicals and intermediate species. As has been noted by Echekki and Chen (1999), extensions of the theory to high stretch are purely empirical. A number of experimental and numerical studies have concluded that the theory of flame stretch is unable to adequately describe the data. For example, several experimental studies on inwardly propagating flames have reported much larger Markstein lengths than those obtained by studies of outwardly propagating flames (Durox et al., 2001; Ibarreta and Driscoll, 2000). A numerical study by Ibarreta et al. (2002) produced the same result. Sinibaldi et al. (2003) measured stretch over a flame as it interacted with a vortex and found large variations in the Markstein lengths measured at different locations on the flame. Markstein lengths computed from regions of the flame undergoing negative stretch were generally larger than those from regions undergoing positive stretch, sometimes by a factor exceeding 5.

It has been suggested that a more general expression for $s_d$ would require two terms, one for a linear dependence on the strain rate, and one for a linear dependence on curvature, each term requiring a different Markstein length (Bradley et al., 1996; Clavin and Graña-
Otero, 2011; Mikolaitis, 1984; Poinsot et al., 1992). This hypothesis has support from experimental studies of inwardly propagating flames with high negative curvature but negligible strain rate that have found that the phenomenological Markstein relation fits the experimental data better than theory of flame stretch (Baillot et al., 2002; Durox et al., 2001; Ibarreta and Driscoll, 2000). It has alternatively been argued that a relation for \((s_d)\) should include a total stretch term and a curvature term (Giannakopoulos et al., 2015a).

In contrast to the extensive experimental work on flame displacement speeds, there have been no experimental studies measuring the Markstein length for flame consumption speeds. This is because the consumption speed cannot be measured with current diagnostics (Sinibaldi et al., 2003, p330). However, the available evidence suggests that the consumption speed can differ greatly from the displacement speed (Poinsot and Veynante, 2005). For example, in numerical simulations of steady 2D methane-air bunsen flames by Poinsot et al. (1992) the consumption speed at the flame tip was found to be unchanged from that of the unstretched, planar flame, though the displacement speed was almost an order of magnitude larger.

### 2.1.2 Turbulent Flames

Turbulence can be naturally occurring or may be intentionally promoted by introducing abrupt obstructions in the flow such as turbulence grids. Turbulent flames are generally categorised by how the turbulence affects the flame structure. Many turbulent flame regime diagrams have been proposed (e.g. Borghi, 1988; Peters, 2000; Pope, 1987; Renard et al., 2000; Veynante and Vervisch, 2002) and are typically constructed using the ratio of a characteristic turbulence velocity to the laminar flame speed on one axis, and the ratio of the flame thickness to a characteristic length scale of the turbulence on the other. The Karlovitz number \(Ka\) is useful in describing the transitions between flame regimes and represents the ratio of the reaction time scale \(\tau_c\) to the time scale of the smallest turbulence scales \(\tau_\kappa\) (i.e. the Kolmogorov scale).

Veynante and Vervisch (2002) describes the following turbulent regimes:

- **Thin wrinkled flame or flamelet regime \((Ka < 1)\)** - The effect of turbulence is limited to distortion of the flame surface. The structure of the flame remains close to a laminar flame structure. When the turbulent fluctuations \(u'\) exceed the laminar flame propagation speed \((u'/S_L > 1)\) turbulent motions are able to roll up the flame surface causing flame front interactions that lead to the creation of disconnected burning pockets. In this case the flame regime is often called *wrinkled flame with pockets* or *corrugated flames*. 

2.2 Flame Chemiluminescence

Chemiluminescence is the emission of light due to the radiative relaxation of molecules formed directly in an excited state by chemical reactions. Almost all of the visible and near ultraviolet light emitted by lean hydrocarbon-air flames is due to chemiluminescence (Gaydon, 1974). Flame chemiluminescence has been widely used as a marker for heat release rate in studies of sound generation by combustion (Candel et al., 2004; Hurle et al., 1968; Schuller et al., 2002). In addition, flame chemiluminescence is an important marker for the location of the reaction zone and is the field that is reconstructed by FCT. This section provides an overview of the current understanding of flame chemiluminescence in the visible and near-ultraviolet regions.

The visible and ultraviolet light emitted by lean, premixed, hydrocarbon flames is largely produced by CH*, OH*, C_2*, and CO_2*, where * represents an electronically excited state (Kathrotia et al., 2012; Lee and Santavicca, 2005). Emission from CH*, OH*, and C_2* are confined to relatively narrow emission bands, whereas CO_2* emission occurs over most of the visible spectrum from 300 nm to 600 nm (Docquier et al., 2002; Ikeda et al., 2002; Kopp et al., 2012; Lee and Santavicca, 2005), and is often referred to as the CO_2* continuum (García-Armingol et al., 2013; Hardalupas and Orain, 2004; Samaniego et al., 1995; Zhou et al., 2017). Almost 80% of CH* emission is from a spectral band at 431 nm (Nau et al., 2012; Smith et al., 2002), produced by the transition A^2Δ−X^2Π (Kojima
et al., 2005) and commonly referred to as CH(A). The rest of the CH* emission is from a band at 387 nm, referred to as CH(B), produced by the transition B²Σ⁺ → X²Π (Smith et al., 2002). The main OH* band is around 308 nm A²Σ⁺ → X²Π (Kojima et al., 2005). C₂* is responsible for the swan bands located mostly between 470 nm and 550 nm (Smith et al., 2002) but is not significant for hydrocarbon flames below φ = 0.9 (García-Armingol et al., 2013; Lee and Santavicca, 2005).

For lean, hydrocarbon-air flames with equivalence ratios less than 0.9, chemiluminescence in the visible range is therefore dominated by CH* and CO₂*. Camera images of a flame capture a combination of the emission from CH* and CO₂*, even when bandpass filters are used (Guiberti et al., 2017; Lauer and Sattelmayer, 2009). Published flame spectra provide a means to estimate the ratio of CH*/CO₂* emission over any wavelength band. In the wavelength band around the peak CH(A) emission, CO₂* emission is often assumed to be negligible (Smith et al., 2002). However, based on spectra published by García-Armingol et al. (2013) for an atmospheric methane-air flame at φ = 0.90, even with a very narrow bandpass filter (+/-1 nm) centred at the peak CH(A) emission, CO₂* emission accounts for approximately 40% of the signal and CO₂* emission would likely dominate when using a +/-5 nm filter. Guiberti et al. (2017) reported that for a methane-air flame at φ = 0.85, CO₂* accounts for approximately 80% of the signal in the band 416-449 nm. Hardalupas and Orain (2004) found in methane-air flames that CO₂* accounts for only 20-30% of the signal when using a +/-5 nm filter. However they used cassegrain optics to probe a small volume with a 100 µm diameter and obtained their measurements at the position of maximum CH* emission, which does not coincide with peak CO₂* emission. For lean flames, the ratio of CH* to CO₂* emission (I_{CH^*}/I_{CO₂^*}) decreases with equivalence ratio (García-Armingol et al., 2013; Hardalupas and Orain, 2004; Tripathi et al., 2012). Methane-air flame spectra reported by Tripathi et al. (2012) show greater I_{CH^*}/I_{CO₂^*} ratios than García-Armingol et al. (2013) but for a φ = 0.78 flame CO₂* is still found to account for about 50% of the signal between 420-440 nm.

For propane-air flames there is much less spectral data available. The primary source of CH* is the reaction C₂H + O → CH* + CO (Najm et al., 1998) and there may be significant differences in the propane flame and methane flame spectra. The available data suggests the I_{CH^*}/I_{CO₂^*} ratio is greater for propane flames. Ikeda et al. (2000) measured the spectrum from a turbulent propane-air flames at φ = 0.9 using Cassegrain optics. The light collected by the Cassegrain optics is mostly from a small local probe volume. However, due to movement of the flame surface in and out of the probe volume over the 100 ms exposure, the measured spectrum is likely to be similar to that for the entire flame. The reported CH(A) peak is much higher relative to CO₂* emission than the spectra reported by others for methane. Samaniego et al. (1995) present results that show a much greater
$I_{CH^*}/I_{CO_2^*}$ ratio for a lean propane-air flame ($\phi = 0.52$) compared to a lean methane-air flame ($\phi = 0.55$).

As has been outlined in the previous two paragraphs, there are significant discrepancies between reported chemiluminescence spectra for hydrocarbon flames. This could be due to several factors. Flame spectra are known to be dependent on equivalence ratio (García-Armingol et al., 2013; Hardalupas and Orain, 2004; Tripathi et al., 2012), temperature (García-Armingol et al., 2014; Gaydon, 1974; Nori and Seitzman, 2009), and pressure (Ikeda et al., 2002; Kojima et al., 2000). There is also a large variance in the reported ratios of heights of spectra peaks due to different resolutions of optical sensors (Wang et al., 2018). A higher resolution monochromator gives taller and narrower peaks than a low resolution spectrometer (García-Armingol et al., 2013). There is less discrepancy in the reported ratios of different integrated areas of 5 nm width or greater under the spectra curves, which are more relevant to flame imaging. Nevertheless, it is presently difficult to confidently estimate the source of the majority of chemiluminescent emission over any wavelength band using the literature.

$CH^*$, $OH^*$, and $CO_2^*$ are formed predominantly in the reaction zone and have all been recommended as markers of the location of the reaction zone (De Leo et al., 2007; Lee and Santavicca, 2005). At atmospheric pressure and for $\phi = 0.8 - 1.2$ the FWHM thickness of $OH^*$ and $CH^*$ profiles across the flame front is approximately 100 $\mu$m and the peaks are separated by approximately 50 $\mu$m with the $CH^*$ peak located closer to the burned gas (Kojima et al., 2005). Numerical results for planar flames have shown $OH^*$ and $CH^*$ peaks to be within 100 $\mu$m of the peak heat release rate (Hardalupas et al., 2010), and some simulations suggest this distance is only 10-20 $\mu$m (Kathrotia et al., 2012; Zhang et al., 2017). Less data has been published regarding the $CO_2^*$ profile. Numerical results from Samaniego et al. (Samaniego et al., 1995) found that for a very lean methane-air flame ($\phi = 0.55$), the $CO_2^*$ emission profile is approximately 400 $\mu$m wide (FWHM), significantly wider than the $CH^*$ or $OH^*$ profiles, with the peak located approximately 200 $\mu$m downstream of the peak heat release rate and a long tail in the burned gas direction.

$CH^*$, $OH^*$, and $CO_2^*$ chemiluminescence are commonly used as markers for the global heat release rate from a flame (Lee and Santavicca, 2005). This has been justified by experiments showing that the chemiluminescence from an entire turbulent flame, hereafter referred to as the global chemiluminescence, is proportional to the mass flow rate of reactants at constant equivalence ratio. Hurle et al. (1968) observed that, for a fixed equivalence ratio and relatively low Reynolds numbers (Re < 13000), $C_2^*$ and $CH^*$ global chemiluminescence is proportional to the fuel consumption rate in ethylene-air flames. Similar results for $C_2^*$ chemiluminescence were reported by Price et al. (1969). Lee and Santavicca (2005) pointed out that these measurements would have included a significant contribution from $CO_2^*$. Lee and Santavicca (2005) also reported a proportional
relationship between global CO$_2^*$ chemiluminescence and the global heat release rate for a lean, premixed combustor operating on natural gas between Re=9000 and Re=18000. All of these studies found that the proportionality constant depends on the equivalence ratio. Two studies from the 1950s reported a deviation from the proportional relationship at constant equivalence ratio. Clark (1958) reported a change in the gradient of the graph of both CO$_2^*$ and CH$^*$ chemiluminescence versus the mass flow rate at approximately Re = 7000 in a stoichiometric ethylene-air jet flame. John and Summerfield (1957) reported that CO$_2^*$, CH$^*$, and C$_2^*$ chemiluminescence normalised by the mass flow rate all decrease with increasing Reynolds number in propane-air flames.

Because of the observed linear relationship between global chemiluminescence and the global heat release rate, it is sometimes assumed that the local heat release rate is proportional to the local chemiluminescence (e.g. Palies et al., 2010). If this is not the case, any local departures must average out (Lee and Santavicca, 2005). Because the normalised chemiluminescence profiles and heat release rate profiles are not identical, the correlation clearly does not hold at spatial scales less than the flame thickness (Zhang et al., 2017). Local quantities in this case therefore refer to the path integral of profiles across the flame, or equivalently, the per flame surface area quantities. Najm et al. (1998); Samaniego et al. (1995) have concluded based largely on numerical results that local chemiluminescence is not proportional to the local heat release rate in places where the local flame structure is altered by curvature or high strain rate.

Overall, for lean, premixed, flames, there is little evidence that chemiluminescence from narrow emission bands in the near ultraviolet and visible spectrum is better correlated with either the global or local heat release rate than broadband chemiluminescence across the visible and ultraviolet bands. Li et al. (2015) reported no obvious qualitative differences between images of forced, laminar, premixed, methane-air flames acquired with CH$^*$ filters (430±15 nm), with OH$^*$ filters (310±15 nm), or without any filters. Furthermore, they found that fluctuations of the path integrated OH$^*$, CH$^*$, and the entire visible emission band, all capture the same dynamics and concluded that heat release rate fluctuations can be deduced without the use of optical filters for φ < 1.2.

Tomographic reconstruction of the chemiluminescent emission field of a flame is significantly more difficult if self-absorption is non-negligible (Worth and Dawson, 2013). This is because the degree of self-absorption is a function of the flame geometry, which is not known prior to reconstruction. Fortunately, flames are effectively transparent over most of the visible and ultraviolet region (Gaydon, 1974). OH$^*$ imaging is a notable exception for which self-absorption may be significant. Despite this, OH$^*$ imaging is a reasonably attractive option. Published methane flame spectra show $I_{OH^*}/I_{CO_2^*}$ in the peak OH$^*$ emission band to be far greater than $I_{CH^*}/I_{CO_2^*}$ (Guiberti et al., 2017; Tripathi et al., 2012).
Apart from a low intensity long tail on the burned gas side, the OH* emission profile is thought to be very similar to that of CH*.

2.3 Premixed Combustion Noise

An overview of the relevant acoustic fundamentals is provided in Section 2.3.1. The classical monopole source is described in detail because of its central importance in earlier theories of direct sound production in premixed flames. Distributed acoustic sources and solutions of the inhomogeneous wave equation in terms of Green’s functions are then described due to their application in more recent theories of combustion noise based on acoustic analogies.

A largely chronological account of the development of combustion noise theories is given in Section 2.3.2, starting from the monopole source theories of Thomas and Williams (1966) and Hurle et al. (1968), to the acoustic analogy formulations of Strahle (1971) and Dowling (1992). A summary of the important empirical observations regarding the sound emitted by open, turbulent, premixed flames is provided in Section 2.3.3. Experimental studies directly comparing the acoustic pressure waveform in the time domain to measurements of the turbulent flame’s chemiluminescent emission are also reviewed. A review of the literature on the sound generated by unsteady, laminar, premixed flames is given in Section 2.3.4.

2.3.1 Overview of the Fundamentals

The Equations of Linear Acoustics

The wave equation is derived from the mass and momentum conservation equations:

\[
\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_i} (\rho v_i) = 0 \tag{2.7}
\]

\[
\frac{\partial (\rho v_i)}{\partial t} + \frac{\partial}{\partial x_j} (\rho v_i v_j + p \delta_{i,j} - \tau_{ij}) = 0 \tag{2.8}
\]

where \( \tau_{ij} \) is the viscous stress tensor and \( \delta_{ij} \) is the Kronecker delta. The index notation and Einstein’s summation convention, used in Equations 2.7 and 2.8, are common in aeroacoustics (e.g. Dowling, 1992; Lighthill, 1952). Repetition of an index in a single term denotes summation over the values 1-3, e.g. \( x_i x_i = x_1^2 + x_2^2 + x_3^2 \). Gibbs vector notation will also be used where convenient. Many of the equations presented here may be found in both forms in Rienstra (2016). The equations of linear acoustics are derived by representing
the dependent variables with ambient and fluctuating components, e.g. $\rho = \rho_0 + \rho'$, $p = p_0 + p'$, $v_i = v_{i,0} + v_i'$. Typical sounds involve extremely small fluctuations. Human hearing is most sensitive to tones with frequencies on the order of 1 kHz. An 80 dB, 1 kHz tone is generally considered loud, equivalent to the volume of a person shouting at you from a distance of several metres away. For this tone the peak velocity fluctuation is approximately 0.7 mm/s, the peak particle displacement is approximately 100 nm, and the fluctuating components of pressure and density as a fraction of the mean components are

$$\frac{p'_{pk}}{p_0} \approx 2.8 \times 10^{-6} \quad \text{and} \quad \frac{\rho'_{pk}}{\rho_0} \approx 1.4 \times 10^{-6}.$$ 

By neglecting second order terms in the fluctuating components and imposing a homogeneous, quiescent medium, the mass and momentum equations simplify to

$$\frac{\partial \rho'}{\partial t} + \rho_0 \frac{\partial v_i}{\partial x_i} = 0,$$

(2.9)

$$\rho_0 \frac{\partial v_i}{\partial t} + \frac{\partial p'}{\partial x_i} = 0.$$

(2.10)

The wave equation results from taking the time derivative of Equation 2.9 and the divergence of Equation 2.10 and subtracting one from the other. It is assumed that for acoustic disturbances pressure can be expressed as a function solely of density. $c$ is defined as $c^2 = \frac{dp}{d\rho}$ evaluated at ambient pressure and density. This allows for the linear approximation $p' \approx c^2 \rho'$. Substituting for $\rho'$ yields the wave equation, with $p'$ as the only dependent variable

$$\frac{1}{c^2} \frac{\partial^2 p'}{\partial t^2} - \nabla^2 p' = 0.$$ 

(2.11)

$p'$ is here referred to as the acoustic pressure. Solutions of the wave equation show $c$ to represent the propagation speed of the disturbances. Acoustic disturbances are approximately isentropic processes, and for perfect gases $c^2 = \gamma R T$ where $\gamma$ is the adiabatic index and $R$ is the specific ideal gas constant.

**Classical Sources**

In classical acoustics the generation of sound is treated as a boundary value problem (Pierce, 1989; Rienstra, 2016). One of the simplest solutions to the wave equation is the planar wave, $p' = f(t - n \cdot x/c)$, where $n$ is a unit vector in the direction of propagation. This solution satisfies a pressure or velocity boundary condition that is uniform over a plane of infinite extent. The velocity fluctuations are in phase with the pressure fluctuations and are related by $v' = n p' / (\rho c)$. 
Another widely known solution is obtained by considering the solution for a uniform velocity distribution over a compact spherical surface of radius $a$. A radially pulsating sphere is accurately modelled by this boundary condition if the radial displacement amplitude is much less than the characteristic wavelength of the radiated sound ($\lambda$). The pressure and velocity fields that result are

\[
p' = \frac{\rho_0 c f}{r} \quad v'_r = \frac{f}{r} + \frac{cF}{r^2}
\]

where $r$ is the distance from the centre of the spherical surface, $F$ is a function of $t - r/c$ and $f$ is its derivative (Pierce (1989)). Unlike for a planar wave, the amplitude of $p'$ is inversely proportional to $r$. It is important to note that $p'$ and $v'$ are only in phase for $r \gg \lambda$. At $r = a$, the velocity boundary condition imposes

\[
\frac{1}{a} \frac{dF(t-a/c)}{dt} + \frac{c}{a^2} F(t-a/c) = v_s(t)
\]

(2.12)

where $v_s(t)$ is the velocity boundary condition. By the integration factor method, $F$ is found to be

\[
F(t-r/c) = a \int_{-\infty}^{t-\frac{r}{c}} e^{-\frac{\xi}{a}} (t-\frac{\xi}{c} - \tau) v_s(\tau) d\tau
\]

For an acoustically compact source ($a \ll \lambda$), the second term in Equation 2.12 dominates and integration yields

\[
p'(r,t) \approx \frac{\rho_0}{4\pi r} \left[ \frac{d}{dt} \left( \pi a^2 v_s \right) \right]_{t-(r-a)/c}
\]

(2.13)

where the subscript of the square brackets gives the value that the expression inside the square brackets is to be evaluated at. It is consistent with the acoustically compact approximation to ignore the difference between $t - (r-a)/c$ and $t - r/c$. If the displacement amplitudes are much smaller than the radiated acoustic wavelengths, $4\pi a^2 v_s$ can be interpreted as the rate of change in the volume of a pulsating sphere and is referred to as the Source Strength Function. This leads to the result that the radiated acoustic pressure is proportional to the second time derivative of the volume of an acoustically compact object. The solution may be alternatively expressed as

\[
p'(r,t) = \frac{\rho_0}{4\pi r} \left[ \frac{d}{dt} \left( \int_S v_n dS \right) \right]_{t- r/c}
\]

(2.14)

where the surface integral is the volume flux through a fixed spherical surface $S$ of radius $a$. Expressed in this way, an assumption of small displacement amplitudes is not
required. Though derived here for a spherically symmetric velocity distributions over $S$, Equation 2.14 is often a good approximation of the sound radiated from non-axisymmetric compact sources and for closed surfaces $S$ of arbitrary geometry that enclose all acoustic sources (Pierce, 1989). Exceptions occur when symmetries in the source processes lead to $\int_S v_i n_i dS \ll a^2 v_{s,\text{max}}$, where $v_{s,\text{max}}$ is the maximum velocity on $S$. An example is the velocity distribution that results from the translational vibration of a rigid object of constant volume.

Multipole expansion can be used to express the solution of an acoustically compact collection of monopoles as a combination of a monopole, dipole, quadrupole, etc. The radiation patterns for collections of point sources have analogues in the solutions for continuous source distributions. A monopole point source is an idealised, spherically symmetric, acoustic source constructed by taking the limit as the dimensions of a spherically symmetric source are reduced and the amplitude of the velocity boundary condition is simultaneously increased to maintain a finite source-strength function. Close to point sources, the fluctuations become very large and, if realised physically, would require consideration of the nonlinear terms in the mass and momentum equations. However, collections of point sources are useful models for many real sources and the solutions are often accurate in the region where the equations of linear acoustics are valid. The solution for a point dipole is constructed from the superposition of the solution for two monopole point sources that are $180^\circ$ out of phase with each other, in the limit as the distance between the sources goes to zero. Point quadrupole solutions are similarly derived by the superposition of two dipole solutions. Unlike monopoles, dipoles and quadrupoles have acoustic near fields where the $p'$ amplitude is inversely proportional to $r$. Terms in the solution that do not decay with $1/r$ are sometimes referred to as standing wave patterns.

**Distributed Sources**

The inhomogeneous wave equation is fundamental in the field of aeroacoustics,

$$\frac{1}{c_0^2} \frac{\partial^2 p'}{\partial t^2} - \nabla^2 p' = \mathcal{S}(x,t). \quad (2.15)$$

$\mathcal{S}(x,t)$ here denotes a Distributed Source Function. Solutions of the inhomogeneous wave equation, as with any linear equation, can be expressed in terms of a Green’s function. The Green’s function is the solution of an inhomogeneous linear equation when the source function is the Dirac delta function.

The free-space Green’s function for the 3D wave equation is known as the Retarded Potential, given by

$$G(x,t; y, \tau) = \frac{\delta(|x - y| - c_0(t - \tau))}{4\pi|x - y|} \quad (2.16)$$
The Green’s function may be interpreted as the response at position \( x \) and time \( t \) to point forcing applied at position \( y \) and at time \( \tau \). The solution of the inhomogeneous wave equation is the integral over the source variables \((y, \tau)\) of the product of the Green’s function and the source function \( \mathcal{S}(y, \tau) \). The acoustic pressure is therefore given by

\[
p'(x, t) = \int_{V(y)} \int_{-\infty}^{+\infty} G(x, t; y, \tau) \mathcal{S}(y, \tau) d\tau dy = \int_{V(y)} \int_{-\infty}^{+\infty} \frac{\mathcal{S}(y, \tau) \delta(|x - y| - c_0(t - \tau))}{4\pi|x - y|} d\tau dy \number{(2.17)}
\]

The acoustic pressure at a receiver position \( x \) is proportional to the volume integral of the source function evaluated at the retarded time, \( \tau = t - |x - y|/c_0 \), which varies across the source region.

The acoustic analogy approach involves rearranging the equations of mass and momentum conservation to form an inhomogeneous wave equation. The source terms are commonly classified as monopole, dipole, or quadrupole distributions. A source function that can be expressed as the divergence of some vector function \( \mathcal{S}(x, t) = \nabla \cdot f \) is called a dipole distribution. If \( f \) goes to zero outside the source region \( V_s \), then the divergence theorem implies that the integral of the source function over \( V_s \) is zero. Sources from neighbouring subregions within \( V_s \) are in destructive interference, though sound may still be emitted due to differences in the retarded times across the source region. If the wavelengths of the radiated acoustic disturbances are large compared to the typical distances between phase coherent subregions, the radiated sound is far weaker than that from a source distribution of similar magnitude that is everywhere in-phase. A source distribution that does not meet the precise definition of a multipole distribution is often referred to as a monopole distribution or a simple source.

### 2.3.2 Source Models for Premixed Combustion Noise

The earliest theories of sound generation in turbulent premixed flames were proposed in the 1960s. Theories of combustion driven acoustic resonances were proposed far earlier, but these theories had little relevance to the irregular and unrepellent pressure fluctuations generated by open turbulent flames. The combustion region was described as a distribution of burning elements that displace the surrounding gases in an unsteady manner as the unburned gases react and expand (Smith and Kilham, 1963). This process was hypothesized to be acoustically equivalent to a distribution of simple monopoles of varying strengths and frequencies. Lighthill (1952) revolutionized the study of aerodynamic noise...
with his acoustic analogy, derived from the fundamental equations of fluid mechanics. Lighthill’s formulation predated the experimental studies of turbulent flame noise conducted in the 1960s but it was not until the early 1970s that Strahle (1971) applied and extended Lighthill’s approach to combusting flows. Several other reformulations have been developed since in an effort to either provide greater clarity regarding the physical interpretation of the derived source terms (Dowling, 1992), or to derive source terms more amenable to approximation (Hassan, 1974).

Classical Source Models

Smith and Kilham (1963) proposed that the sound radiated by an open turbulent flame could be considered to arise from a statistical distribution of monopole sources, each radiating sound according to Equation 2.18. The origin of each monopole was proposed to be a ‘burning element’ that converts reactants to less dense products at an unsteady rate, hence behaving as an unsteady volume source. Thomas and Williams (1966) provided some experimental validation of this model by measuring the sound from a spherically expanding flame, produced by centrally igniting reactants initially contained by a soap bubble. This flame was proposed to behave as a single acoustic monopole, radiating sound according to

\[
p'(R,t) = \frac{\rho_0}{4\pi R} \left[ \frac{d}{dt} \left( \frac{dV}{dt} \right) \right]_{t-R/c}
\]

(2.18)

where \( R \) is the distance from a representative or average flame location. In the canonical example of an acoustically-compact, radially oscillating sphere, \( dV/dt \) is the rate of change of volume of the sphere. Here, \( V \) can be taken to be the volume of the bubble enclosing the unburned mixture. \( V \) should not be mistaken for the volume enclosed by the flame. Because the combustion occurs at approximately constant pressure, \( dV/dt \) may also be substituted by \( \int_S \mathbf{v} \cdot \mathbf{n} dS \) where \( S \) is any acoustically compact surface enclosing the flame. Importantly, Thomas and Williams (1966) recognised that \( dV/dt \) is proportional to the product of the burning velocity and the flame surface area.

Hurle et al. (1968) extended this model to turbulent flames. The rate of volume generation was reformulated as proportional to the volumetric consumption rate of the unburned reactant mixture (\( \dot{V} \)). Consistent with the model proposed by Smith and Kilham (1963), the turbulent flame was considered to be acoustically equivalent to a collection of monopole sources of varying strengths and frequencies. Equation 2.18 was reformulated for a single burning element, denoted here by subscript \( i \), to yield

\[
p'_i(R,t) = \frac{\rho_0}{4\pi R} \left[ \frac{d}{dt} \left( (E-1)\dot{V}_i \right) \right]_{t-R/c}
\]

(2.19)
where $E$ is the ratio of the specific volumes of the burned and unburned gas. For acoustically compact flames, the far-field sound was proposed to be approximately equal to

$$p'(R,t) = \sum_n p'_n(R,t) = \frac{\rho_0}{4\pi R} (E - 1) \left[ \frac{d\dot{V}}{dt} \right]_{t=R/c}$$

(2.20)

where $\dot{V} = \sum \dot{V}_i$. The low Mach number mean flow field is predicted to not contribute to the radiated sound, as would be expected. It was noted that this simple summation applies only to flames with dimensions less than a quarter wavelength of the emitted sound. For shorter wavelengths, differences in phase due to the different distances between sources and the receiver must be accounted for.

**Lighthill’s Acoustic Analogy**

Lighthill (1952) developed the first general theory of aerodynamic sound and his acoustic analogy provided the basis for much of the future work on sound from combusting and non-combusting jet flows. Lighthill’s expression, given in Equation 2.21, is derived by combining the exact equations of continuity and momentum.

$$\frac{\partial^2 \rho}{\partial t^2} - c_0^2 \nabla^2 \rho = \frac{\partial^2 T_{ij}}{\partial x_i \partial x_j},$$

(2.21)

where $T_{ij}$, now known as Lighthill’s stress tensor, is

$$T_{ij} = \rho v_i v_j + \delta_{ij} (p - c_0^2 \rho) - \tau_{ij}.$$  

(2.22)

On the left hand side of Equation 2.21 are the familiar terms of the classical acoustic wave equation with density as the primary acoustic variable. Lighthill (1952) showed that the remaining terms can be treated as an external force field acting on a uniform acoustic medium at rest and showed that this approach to determining the sound field outside the region of fluctuating flow is an exactly valid one. Therefore, if the right-hand side is known, from experiment or simulation, the sound field in the quiescent medium around the source region can be calculated.

An order of magnitude argument was used to show that the first term in Equation 2.22, which represents a quadrupole distribution, is dominant in a non-combusting jet flow and it was concluded that the sound intensity of a non-combusting jet flow should increase roughly proportional to $U^8$ (Lighthill, 1952). Non-reacting flows, combusting flows do not follow this relation (Kilham and Kirmani, 1979; Smith and Kilham, 1963), suggesting a different sound generation mechanism may dominate for combusting flows. Lighthill (1954) later applied his acoustic analogy to a detailed analysis of the sound produced by turbulence in non-combusting flows.
In Lighthill’s acoustic analogy, all effects on sound propagation due to non-acoustic variations in the medium are represented by the source terms. Such effects include convection by the flow, refraction, and variations in the speed of sound, and are together called flow-acoustic interaction (Lilley, 1991). This was considered by Lighthill to be an advantage of the approach, given that such effects ‘would be difficult to handle’. However, this approach disregards the conventional distinction between processes that generate sound, and effects on the propagation of sound due to changes in ambient properties. The limitations of the acoustic analogy approach and its usefulness in identifying sound generation mechanisms have been discussed in detail elsewhere (e.g. Doak, 1972; Lilley, 2003; Peake, 2004; Phillips, 1960; Spalart, 2007; Tam, 2002).

Aeroacoustic Theories Applied to Combustion

Strahle (1971) developed a theory of combustion noise that closely followed Lighthill’s approach. Strahle’s analysis was focussed on direct combustion noise, with origins in the combustion region, and does not consider noise generated downstream. Strahle (1971) argued that in light of the success of Lighthill’s formulation in matching experimental results for turbulent jet noise, similar success may be hoped for in turbulent combustion noise.

The viscous contribution to Lighthill’s stress tensor, \( \tau_{ij} \), was omitted on the basis of the order of magnitude arguments made by Lighthill (1952). The remaining terms were denoted as:

\[
F_1 = (\rho v_i v_j)_{x_i x_j}, \quad F_2 = p_{x_i x_j}, \quad \text{and} \quad F_3 = \rho \frac{c_0^2}{c_s^2} x_i x_j
\]

where the subscripts denote partial derivatives. In his analysis of non-combusting turbulent jets, Lighthill (1952) neglected the terms \( F_2 \) and \( F_3 \) on the basis that the departure of \( p - p_0 \) from \( c_0^2 (\rho - \rho_0) \) is a very small effect. This is clearly not the case in combusting jet flows where the temperature is highly non-uniform. Strahle (1971) argued that since the available experimental data on noise from combusting flows showed that the sound intensity did not scale with \( U^8 \) (Smith and Kilham (1963), Hurle et al. (1968)), the first term \( F_1 \), considered the dominant term by Lighthill (1952) in non-combusting flows, must be dominated by one or both of the other terms in combusting flows. Based on an order of magnitude argument, Strahle (1971) asserted that the ratio of the amplitude of the source terms \( F_2 \) and \( F_3 \) is \( O[S_L^2/c_0^2] \) which is approximately \( 10^{-6} \). The monopole source term \( F_3 \) was therefore considered the only significant source term in Strahle’s analysis. By assuming the wavelengths of the emitted sound are sufficiently large compared to the macroscopic dimensions of the combustion region, Strahle (1971) neglected the effect of variations in the retarded time of sound originating from the combustion region. The
solution to Equation 2.21 then becomes:

$$\rho'(x,t) = -\frac{1}{4\pi c_0^2 R} \frac{\partial^2}{\partial t^2} \int_V \rho \left( y, t - \frac{R}{c_0} \right) dy. \quad (2.23)$$

where $R$ is the distance from $x$ to a representative location in source region. In order to explain the results of Hurle et al. (1968) and Price et al. (1969), Strahle (1972) used a simplified, single-step energy equation to express Equation 2.23 in terms of the integral over the combustion region of the time derivative of the fuel consumption rate resulting in the following expression:

$$\rho'(x,t) = \left( \frac{\gamma - 1}{\gamma} \right) \frac{LHV \rho_b}{4\pi c_0^2 R \rho_0} \int_V \left[ \frac{\partial \dot{\omega}_F}{\partial t} \right]_{t-R/c_0} d(y) \quad (2.24)$$

where LHV is the lower heating value of the fuel and $\dot{\omega}_F$ is the mass consumption rate of the fuel per unit volume. In the intermediate steps of his analysis, Strahle (1972) showed that the acoustic density fluctuation in the far field is proportional to the volume flux through a surface enclosing the flame region.

$$\rho'(x,t) = \rho_b \frac{4}{\pi Rc_0^2} \frac{d}{dt} \left( \int_S v_i n_i dS \right)_{t-R/c_0} \quad (2.25)$$

The steps involve bringing one of the partial time derivatives in Equation 2.23 inside the volume integral, substituting for $\partial \rho / \partial t$ using the continuity equation, and applying the divergence theorem. In the far field $p' = c_0^2 \rho'$ and the above equation is therefore equivalent to Equation 2.14 if the ambient fluid were burned gas. Dowling (1992) has since pointed out that in the derivation of 2.24 the control surface should have been located outside the boundary between the plume of burned gas and the ambient air and that this would have resulted in $\rho_b$ being replaced by $\rho_0$. This mistake was apparently also realised by Strahle. A revised derivation for $p'$ by Strahle and Shivashankara (1975), starting from Equation 2.14, correctly used the density of the ambient air,

$$p'(x,t) = \left( \frac{\gamma - 1}{\gamma} \right) \frac{LHV \rho_0}{4\pi R \rho_0} \int_V \left[ \frac{\partial \dot{\omega}_F}{\partial t} \right]_{t-R/c_0} d(y). \quad (2.26)$$

The possible directional effect due to the finite size of the combustion region compared to the wavelength of the emitted sound was investigated and Strahle (1972) concluded that other effects such as convection of the sound sources, refraction due to temperature and velocity differences, or scattering due to the burner must be responsible for the measured weak directionality of combustion noise observed by Smith and Kilham (1963).
Hassan (1974) used a reformulation of Lighthill’s acoustic analogy by Doak (1972) and the conservation of energy and species equations to reformulate the source terms in a form useful for deriving scaling laws for the sound power and peak frequency. However, the resulting source terms are difficult to interpret in terms of the underlying processes responsible for combustion-generated sound. Dowling (1992) also used Doak’s formulation of Lighthill’s analogy as a basis for a theory of combustion-generated noise:

\[ \frac{1}{c_0^2} \frac{\partial^2 p}{\partial t^2} - \nabla^2 p = \frac{\partial^2}{\partial x_i \partial x_j} (\rho v_i v_j - \tau_{ij}) - \frac{\partial^2 \rho_e}{\partial t^2}, \]  

(2.27)

where \( \rho_e \) is the ‘excess density’:

\[ \rho_e = \rho - \rho_0 - (p - p_0)/c_0^2. \]  

(2.28)

Dowling (1992) used the energy equation and species conservation equation to develop an expression for the time derivative of the excess density:

\[ \frac{\partial \rho_e}{\partial t} = \frac{\alpha \rho_0}{\rho c_p} \left( \sum_{n=1}^{N} \frac{\partial h}{\partial Y_n} \left|_{\rho, p, Y_m} \right. \frac{DY_n}{Dt} - \frac{\partial q_i}{\partial x_i} + \tau_{ij} \frac{\partial v_i}{\partial x_j} \right) \]

\[ - \frac{1}{c_0^2} \left( 1 - \frac{\rho_0 c_0^2}{\rho c^2} \right) \left( \frac{DP}{Dt} - \frac{p - p_0}{\rho} \frac{D\rho}{Dt} \right) \]

\[ - \frac{\partial}{\partial x_i} (v_i \rho_e), \]  

(2.29)

where the subscripts after the vertical bar indicate the quantities that remain constant when taking the partial derivative and \( \alpha \) is the volumetric expansion coefficient. For ideal gases \( \alpha = 1/T \). This expression for \( \partial \rho_e / \partial t \) was substituted into Equation 2.27 to obtain the inhomogeneous wave equation:

\[ \frac{1}{c_0^2} \frac{\partial^2 p}{\partial t^2} - \nabla^2 p = \frac{\partial}{\partial t} \left( \frac{\alpha \rho_0}{\rho c_p} \left( - \sum_{n=1}^{N} \frac{\partial h}{\partial Y_n} \left|_{\rho, p, Y_m} \right. \frac{DY_n}{Dt} - \frac{\partial q_i}{\partial x_i} + \tau_{ij} \frac{\partial v_i}{\partial x_j} \right) \right) \]

\[ + \frac{\partial^2}{\partial x_i \partial x_j} (\rho v_i v_j - \tau_{ij}) \]

\[ + \frac{1}{c_0^2} \frac{\partial}{\partial t} \left( 1 - \frac{\rho_0 c_0^2}{\rho c^2} \right) \left( \frac{DP}{Dt} - \frac{p - p_0}{\rho} \frac{D\rho}{Dt} \right) \]

\[ + \frac{\partial^2}{\partial x_i \partial t} (v_i \rho_e). \]  

(2.30)

Dowling’s reformulation may be solved using the free-space Green’s function given in Equation 2.16. The terms on the top line on the right-hand side represent a monopole.
source distribution associated with irreversible processes. The terms on the second line are quadrupole source terms containing the fluctuating Reynold’s stresses, determined to be the dominant source term in Lighthill’s analysis of non-combusting flows. The terms on the third line are appreciable if there are regions of unsteady flow with different mean density and sound speed from the ambient fluid. The last term is a dipole source representing the acceleration of density inhomogeneities through mean flow gradients and contributes to indirect noise.

Dowling (1992) argued that the first of the three terms in the curly braces on the top line is dominant in low mach number, unconfined, combusting flows and that this term also dominates all other source terms in Equation 2.30. Considering only this term, assuming a compact source region, and neglecting the effects of diffusion such that $\dot{\omega}_n = \rho D Y_n / Dt$, the far field ($R \gg a$) pressure is approximated by

$$p'(x,t) = -\frac{1}{4\pi R} \frac{\partial}{\partial t} \int_{V(y)} \left[ \frac{\alpha \rho_0}{\rho c_p} \sum_{n=1}^{N} \frac{\partial h}{\partial Y_n} \right]_{\rho, p, Y_m} \dot{\omega}_n \left[ Q \right]_{\tau = t - R/c_0} dy,$$

(2.31)

where $\dot{\omega}_n$ is the rate of production of species $n$ with dimensions of mass per unit time per unit volume. It was shown that for the combustion of hydrocarbons with air

$$- \sum_{n=1}^{N} \frac{\partial h}{\partial Y_n} \mid_{\rho, p, Y_m} \dot{\omega}_n \approx Q,$$

(2.32)

where $Q$ is the rate of heat release per unit volume due to changes in the species concentration and is defined as

$$Q = - \sum_{n=1}^{N} \frac{\partial h}{\partial Y_n} \mid_{T, p, Y_m} \dot{\omega}_n.$$

(2.33)

Assuming ideal gas behaviour with $\gamma$ independent of temperature, and that the pressure is everywhere close to ambient pressure, Equation 2.31 becomes

$$p'(x,t) = \frac{\gamma - 1}{4\pi R c_0^2} \frac{\partial}{\partial t} \int_{V(y)} \left[ Q \right]_{\tau = t - R/c_0} dy$$

(2.34)

Using Equation 2.34 and by applying appropriate assumptions, Dowling (1992) was able to rederive the relations by Hurle et al. (1968), Strahle (1971), and Hassan (1974) and was able to reconcile the differences between them.
2.3.3 Experimental Studies of Open Turbulent Premixed Flame Noise

There is a large empirical database in the literature regarding the overall sound power, directivity, and spectral content of noise produced by open, turbulent flames (e.g. Kumar (1975); Rajaram and Lieuwen (2003); Roberts and Leventhal (1973); Shivashankara (1973); Smith and Kilham (1963)). In this section an overview is given of the key experimental findings. Studies that have compared the radiated acoustic waveform with predicted waveforms based on indirect measurements of the flame heat release rate are also described.

Sound Power, Directivity, and Spectral Content

Smith and Kilham (1963) were among the first to systematically measure the influence of geometric and flow parameters on noise from an open, premixed, turbulent flame. The turbulence was fully developed in all experiments. Through regression analysis, the total sound power \( P \) of the fully turbulent flame was found to be related to the burner diameter \( D \), the flow velocity \( U \), and the laminar flame speed \( S_L \) by the expression

\[
P \propto U^2 D^2 S_L^2.
\]

The sound field was found to be almost omnidirectional and the preferred direction of radiation, measured from the burner axis with 0 degrees in the flow direction, decreased with increasing flow velocity, and was typically between 40° and 80°. The spectrum of the turbulent combustion noise was found to have a universal shape when the frequency was scaled by the frequency of maximum spectral power density \( f_{\text{peak}} \), and the spectral power density is scaled by its maximum value. The reported universal spectrum was broad with a well defined maximum, a steep low-frequency fall-off of approximately 8.5 dB per octave, and a more gradual fall-off on the high frequency side of 2.8 dB per octave. The peak frequency was found to be a linear function of flow velocity and burning velocity and inversely proportional to the burner diameter, and it was proposed that the peak frequency may be expressed by a constant Strouhal number in terms of these parameters.

Shivashankara (1973) extended the experimental study done by Smith and Kilham (1963) to include larger burner diameters and a wider flow velocity range. The fuel mass fraction \( F \) was considered an independent variable. Data regression resulted in the scaling

\[
P \propto F^{-0.4} S_L^{1.83} U_{\text{ave}}^{2.67} D^{2.78}.
\]

Kilham and Kirmani (1979) used turbulence grids to investigate the influence of turbulence on the sound power. By increasing the turbulence intensity the sound power was more than doubled in some cases. The integral length scale was found to be independent of the turbulence grid used and its effect on the emitted sound was not able to be established.

Rajaram et al. (2006) measured the noise emission spectrum for a wide range of turbulent flames using natural gas, acetylene and propane. It was also shown that \( f_{\text{peak}} \) scales with \( U_{\text{ave}}/L_{f,\text{spread}} \), where \( U_{\text{ave}} \) is the average burner flow velocity and \( L_{f,\text{spread}} \)
2.3 Premixed Combustion Noise

is a measure of the extent along the flame of the region where most of the heat release fluctuation occurs. The universal shape of the scaled spectrum presented by Smith and Kilham (1963) was confirmed. Four representative spectrums from the work of Rajaram and Lieuwen (2009) are shown in Figure 2.2. The spectral power density can generally be parameterized by a low frequency dependence of $f^\beta$, a peak frequency $f_{\text{peak}}$, and a high frequency dependence $f^{-\alpha}$ (Rajaram and Lieuwen, 2009). Rajaram and Lieuwen (2009) estimated $\beta = 2 - 2.4$ and $\alpha = 2.1 - 3.2$ when $f_{\text{peak}} > 600\text{Hz}$. This corresponds to a low frequency roll-off of 6-7.2 dB per octave and a high frequency roll-off of 6.3-9.6 dB per octave, far greater than reported by Smith and Kilham (1963). Abugov and Obrezkov (1978) proposed a similar value $\alpha = 2.5$ in the region 2 – 10 kHz for the apparent power law dependence of the spectral power density with frequency.

**Direct Comparison of Measured Acoustic Waveforms with Flame Chemiluminescence**

The work done by Thomas and Williams (1966) on laminar flame noise was extended to turbulent flames by Hurle et al. (1968). Hurle et al. (1968) proposed that the instantaneous volumetric consumption rate of combustion mixture in a turbulent flame is proportional to the emission intensity of $\text{C}_2$ and CH radicals. $\text{C}_2$ and CH radicals are known to be confined almost exclusively to the reaction zone and, due to their low concentrations, show very little self-absorption. The emission intensity of $\text{C}_2$ and CH radicals was measured by a photo-multiplier tube (PMT) equipped with a narrow band spectrum filter to isolate the desired wavelength of light. The PMT current ($I$) was therefore assumed proportional to
the total volumetric consumption rate and Equation 2.20 was restated as

$$p'(R,t) = \frac{\rho_0}{4\pi R} (E - 1) k \left[ \frac{dT}{dt} \right]_{t-R/c_0} \quad (2.35)$$

The mean $C_2$ intensity was found to be proportional to the volumetric flow rate for a fixed equivalence ratio, and independent of burner diameter and turbulence levels. Similar results were also obtained for CH emissions. The $C_2$ emission intensity and the sound pressure were measured simultaneously and the waveform of the differentiated PMT current was compared to the microphone signal. Both signals were passed through a 50-1000 Hz bandpass filter to minimise distortion due to electrical noise. The agreement of the resulting waveforms is shown in Figure 2.3. It was found that as the bandpass filter was widened the correlation between the calculated and the measured pressure waveforms decreased. In addition to the restrictions caused by electrical noise, the simple relationship

<table>
<thead>
<tr>
<th>equiv. ratio</th>
<th>flow rate (mL/s)</th>
<th>burner diam. (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) 1.3</td>
<td>350</td>
<td>0.44</td>
</tr>
<tr>
<td>(b) 1.1</td>
<td>450</td>
<td>0.96</td>
</tr>
<tr>
<td>(c) 0.9</td>
<td>350</td>
<td>0.44</td>
</tr>
</tbody>
</table>

Fig. 2.3 Comparisons of calculated (—) and measured (- -) pressure-time waveforms obtained for ethylene-air mixtures of equivalence ratios (a) 1.3, (b) 1.1, and (c) 0.9) (Hurle et al. (1968)).
in Equation 2.35 was derived assuming the wavelength of sound is much greater than the size of the flame, thereby limiting the scope of this study to frequencies below 4 kHz.

The same optical technique was used by Price et al. (1969) to study the effect of turbulence intensity on the noise generated by open, turbulent, premixed flames. This work is essentially an extension of the investigations by Hurle et al. (1968) which considered only fully developed natural pipe-flow turbulence. Price et al. (1969) used four perforated plates to generate different turbulence intensities, measured using a constant temperature hot-wire anemometer. It was confirmed that the mean $C_2$ emission intensity from a turbulent flame is almost independent of turbulence intensity and linearly proportional to the total flow rate of the combustion mixture. However, the statistical characteristics of fluctuations in the $C_2$ emission intensity, and hence the fluctuations in volumetric combustion rate, were found to be strongly dependent on the turbulence intensity. Consistent with the simple theory derived in Hurle et al. (1968), the rms sound pressure in the far field was found to be proportional to the rms time derivative of the $C_2$ emission intensity, as demonstrated by the results in Figure 2.4. The same theory was extended to turbulent diffusion, and liquid spray combustion with reasonably good agreement (Price et al. (1969)).

**Fig. 2.4** Correlation between measured root-mean-square sound pressure and $dI/dt_{r.m.s.}$, a measure of the rms time derivative of $C_2$ emission intensity, in the frequency band 100 Hz to 5 kHz for premixed flames (Price et al. (1969)).
Zhang et al. (2017) conducted a similar study on a turbulent methane-air flame and using the global OH* chemiluminescence as a quantitative marker for the global heat release rate. They reported reasonable agreement in the spectra of the measured sound and the sound predicted using the time derivative of the chemiluminescence signal at frequencies below approximately 500 Hz. However, they did not directly compare the waveforms.

2.3.4 Sound Generation in Laminar Premixed Flames

Due to the complexity of noise generation in turbulent flames and the limitations of current experimental techniques, specific noise generation mechanisms are difficult to investigate in turbulent flames. For flames in the wrinkled flame regime, the flame structure is close to that of a laminar flame. Investigations into sound generation by laminar flame processes are therefore likely to provide insight into the sound radiating processes occurring within turbulent flames. The studies presented in this section were motivated by this line of reasoning.

Experimental Studies of Sound Generation by Laminar Premixed Flames

Thomas and Williams (1966) studied the comparatively simple case of a spherically propagating flame, produced by centrally igniting premixed, homogeneous, air-fuel mixtures contained within soap bubbles. A Schlieren optical system was used to measure the propagation of the flame front. A simple expression was developed to calculate the sound waveform from the combustion event based on Equation 2.18 and by assuming the displacement speed equals the consumption speed. Good quantitative agreement was achieved and a sample of the results are shown in Figure 2.5. The key features of the waveform are the steady pressure increase as the expanding flame front, moving at an almost constant speed $S_{L,0}$, consumes the combustion mixture, followed by the sharp rarefaction as the flame extinguishes at the boundary of the bubble. The sound pressure in the far field depends on the rate of change of the rate of volume displacement of the surrounding air. The sound intensity was therefore found to be strongly dependent on the flame speed and the geometry of the combustion. The magnitude of the rarefaction spike due to deceleration and extinction of the flame was 3-4 times higher than the peak positive pressure and was the predominant sound to the ear, due to the higher frequency content. Eccentrically ignited bubbles were far quieter and exhibited dipole behaviour; the rarefaction due to deceleration and extinction of the flame at the bubble boundary partly balancing the compression due to the expanding flame surface propagating through the bubble. This demonstrated that the radiated sound depends strongly on the geometry of the combustion system.
Fig. 2.5 Pressure waveform obtained for three mixtures of C$_2$H$_4$, O$_2$, and N$_2$, with burning velocities of (a) 55 cm/s, (b) 102 cm/s, and (c) 147 cm/s. Measured pressure waveform (+); Waveform calculated from record of flame movement (−). Bubble radius in each case is 2.5 cm (Thomas and Williams (1966)).

Thomas and Williams (1966) also recognised that inwardly propagating flames terminating in flame annihilation was almost certainly a more common occurrence in turbulent flames than outwardly propagating flames and derived the following expression for such events:

$$ p = \frac{\rho_0}{R} (E - 1) \left( 2r_f \left( \frac{dr_f}{dt} \right)^2 + r_f^2 \frac{d^2 r_f}{dt^2} \right) $$

where $r_f$ is the flame radius. The author believes a minus sign is missing from this expression. The omission resulted from a sign error in the expression for $dV/dt$. The correct expression is

$$ dV/dt = -4\pi r_f^2 (dr_f/dt)(E - 1) $$

where $V$ is the volume within a surface enclosing the burned gases. Assuming a constant displacement speed ($dr_f/dt = -S_L$, $d^2 r_f/dt^2 = 0$), Equation 2.36 predicts that a rarefaction is generated throughout the consumption of a reactant pocket and that the rarefaction magnitude decreases at a steady rate. Thomas and Williams (1966) noted that there may be additional pressure fluctuations in the terminal moments of the event, due to changes in the flame speeds, but reasoned that, because the event is so small, Equation 2.36 implies the magnitude of sound generated is likely to be small. However, at the time, there was no convenient method to confirm this prediction experimentally.

Such a method was devised by Kidin et al. (1984). A speaker was used to produce a sinusoidally varying inlet velocity to a conical methane-air laminar flame. Similar experimental arrangements had been used much earlier in studies of the flame response to sinusoidally varying inlet velocities (Blackshear, 1953) but had not previously been
applied to studying combustion noise. The velocity perturbation at the base of the flame causes a deformation of the flame surface that is convected along the flame, resulting in a wrinkled flame surface. At high enough forcing amplitudes flame deformation results in the collision of flame surfaces propagating in opposite directions, called flame annihilation (Talei et al., 2011b). Two types of flame annihilation events were identified in the study by Kidin et al. (1984). Separation events occur when a pocket of reactants is separated from the rest of the unburned mixture. The process leading up to the separation event is here called the tunnel narrowing process. The pocket of unburned mixture is subsequently consumed, terminating in a second flame annihilation event, termed pocket burn-out. Other terminologies have been adopted elsewhere. Separation events have been called ‘pinch-off events’ (Talei et al., 2011b, 2012b), ‘pocket detachment’ (Kidin et al., 1984), ‘channel closure’ (Chen et al., 1999), and ‘tunnel closure’ (Dunstan et al., 2013; Griffiths et al., 2015). Pocket burn-out events have been called ‘pocket collapse’ (Kidin et al. (1984)). Both annihilation events were described by Kidin et al. (1984) as ‘small thermal explosions’ occurring due to sufficient elevation of the temperature of the combustion mixture when the flame neck or pocket reached a ‘critical radius’.

The flame surface was imaged using a stroboscopic shadowgraph technique. A microphone, positioned 160 mm from the burner port, measured two positive pressure pulses purportedly corresponding to the separation and pocket burn-out events. A typical waveform is shown in Figure 2.6. A theory was developed to predict the amplitude of the

Fig. 2.6 Time history of the acoustic pressure as measured on a digital oscilloscope. The first peak coincides with the separation event and the second with the pocket burn-out (Kidin et al. (1984)).

sound pulses based on Equation 2.18 and by estimating the characteristic timescale of the
chemical reaction. The theoretical prediction was reported to be in reasonable agreement with the experimental results. This study was later extended to investigate the effect of parameters such as the equivalence ratio, activation energy and flame speed on the far field peak pressure produced by pocket burn-out (Kidin et al. (1988)). The theory was found to under-predict the peak pressure and it was speculated that the consumption of the pocket may be more violent than predicted by the theory. Based on these results, Kidin et al. (1984) proposed that the terminal explosion of small pockets of unburned mixture may be a significant source of sound in turbulent premixed flames. It was also suggested that chemiluminescent emission may provide further insight into the sound production mechanism.

There have since been several similar studies of the noise produced by acoustically excited laminar flames in various configurations (Schuller et al. (2002), Candel et al. (2004), Candel et al. (2009)). The consistent conclusion of these studies is that flame surface destruction as a result of flame interactions with cold walls, flame-flame interactions and flame-vortex interactions is a strong source of sound and it has been proposed that the same mechanisms are significant in turbulent continuous combustors (Candel et al. (2004)).

In these studies a microphone and photo-multiplier tube (PMT) were used to simultaneously measure the sound radiation and the chemiluminescent light emission from forced, unconfined, laminar, premixed flames. Due to space constraints and to minimise the impact of acoustic reverberation on the results the microphone was not placed in the far field and was instead positioned 24-30 cm from the burner axis. Phase locked images of the flames were also taken using an intensified CCD camera to identify flame dynamics that may provide insight into sound generation mechanisms and to measure flame surface area fluctuations. The PMT, equipped with a narrow band filter, was used to measure the global chemiluminescent CH or OH emission intensity. The chemiluminescent emission was assumed to be proportional to the heat release rate. The PMT signal was thereby used to predict the shape of the sound waveform at the microphone using Equation 2.35.

Reasonable correlation was achieved for most flame configurations. For forced M-flame configurations, the largest amplitude acoustic pressure fluctuations were negative, and the peaks were found to coincide with the separation, referred to as a pinch off, of toroidal volumes of unburned mixture near the tips of the M-flames (Candel et al., 2009, 2004). It was concluded that the flame noise was primarily due to the fast rate of destruction of flame surface area at this phase of the cycle. Lower amplitude positive fluctuations in the acoustic pressure were generated by the generation of flame surface area, primarily at the base of the flame. For forced V-flame configurations, flame elements interact near the top of the flame resulting in a large rate of flame surface area destruction (Durox et al., 2005). This rapid flame surface destruction has also been correlated with a large negative peak in the acoustic pressure (Candel et al., 2009). It has also been observed that for
V-flames forced at relatively low amplitudes, the proportional relationship between the time derivative of the flame surface area and the acoustic pressure does not hold. This is thought to be because heat release fluctuations will also be generated by variations in the local consumption rate, as well as variations in the flame surface area. The variation in the local consumption rate is believed to be due to changes in the equivalence ratio from mixing between the jet and the surrounding air (Candel et al., 2009; Durox et al., 2005). Forced, jet flames that interact with a cooled plate have also been studied (Candel et al., 2004; Schuller et al., 2002). For this configuration also, the largest acoustic pressure fluctuation is negative and coincides with the fast destruction of flame surface area as the flame interacts and is extinguished by the cold plate.

Conical flame configurations featuring pocket separation and burn-out events were included in the studies of Candel et al. (2009) and Schuller et al. (2002). The study of Candel et al. (2009) included a methane-air flame with $\phi = 1.11$, an average bulk flow velocity of $1.7 \text{ ms}^{-1}$, and an rms velocity fluctuation of $0.8 \text{ ms}^{-1}$ at an excitation frequency of 50 Hz. This corresponds to very strong forcing as shown by the flame images in Figure 2.7. The flame neck annihilated in the separation event extends almost over the entire length of the flame. A small separated pocket is likely to have been formed near the tip of the flame. Figure 2.8 shows the measured and derived acoustic pressure for this flame. The flame surface areas shown in Figure 2.8 were calculated from the camera images using an Abel deconvolution (Candel et al., 1998). The flame surface area was found to correlate well with the OH emission intensity, supporting the assumption that the heat release rate per unit surface area is relatively constant. For this strongly forced conical flame, the separation and burn-out events were not correlated with high amplitude sound. The peak acoustic pressure was positive and coincided with the production of flame surface area at the base of the flame.

The study of Schuller et al. (2002) included a forced, conical, methane-air, $\phi = 0.95$ flame more similar to that described by Kidin et al. (1984). The excitation frequency was 101 Hz and the mean flow velocity is 1.2 m/s. A separation event is clearly observed near the tip of the flame, as shown by the Abel-deconvolved image in Figure 2.9. The acoustic pressure waveform was measured at a distance 250 mm from the burner axis.
2.3 Premixed Combustion Noise

Fig. 2.8 Time traces for the forced conical flame shown in Figure 2.7. The forcing is at 50Hz with $u'/\bar{u}=0.47$: OH light intensity $I$, flame surface area $A$, acoustic pressure $p'$, and predicted acoustic pressure $\alpha dI/dt$. (Candel et al. (2009)).

and was compared to a predicted waveform derived from the CH chemiluminescence. Both waveforms are shown in Figure 2.10. Interestingly, there is no evidence of the short duration compression pulses observed by Kidin et al. (1984). The sound generated by the annihilation events may be dominated by the sound associated with the flame stretch occurring in the circumferential cusps. A cooled plate was fixed several millimeters above the tip of the flame in this study and this is also expected to have affected the measured sound.

Fig. 2.9 An Abel-deconvolved image a conical flame acoustically forced at a frequency of 101 Hz. A separation event is clearly identifiable (Schuller et al. (2002)).

Fig. 2.10 Measured ($\tilde{p}_{ex}$) and derived ($\tilde{p}_{pl}$) acoustic pressure waveforms for the forced flame shown on the left (Schuller et al. (2002)).
Numerical Simulations of Sound Generation by Laminar Premixed Flames

Talei et al. (2009) conducted a 2D direct numerical simulation (DNS) study of an acoustically excited laminar, premixed, conical, jet flame. The simulations used single step chemistry and both the flame dynamics and the radiated sound waves were resolved. The acoustic excitation was modelled by a sinusoidally varying inflow velocity. The configuration is equivalent to the acoustically excited flames studied experimentally by Kidin et al. (1984) and Candel et al. (2009). Separation events and pocket burn-out events were identified as strong sources of sound with the separation event the stronger of the two. The flame position and the sound produced by the events can be simultaneously visualized by plotting the dilatation field, as shown in Figure 2.11. In the far field, the dilatation field is proportional to the time derivative of the acoustic pressure:

$$\frac{\partial v_i}{\partial x_i} = -\frac{1}{\rho_0 c_0^2} \frac{\partial p'}{\partial t}$$

(2.37)

This result is derived from the linearised mass conservation equation (Equation 2.10) and using $p' \approx c_0^2 \rho'$. 

Fig. 2.11 Instantaneous dilatation field at four instants during a forcing cycle, $St=0.024$, clockwise (Talei et al., 2009).

Simpler 1D DNS studies of planar, axisymmetric, and spherically symmetric flame annihilation events were conducted to provide insight into the flame annihilation process (Talei et al., 2010, 2011a,b, 2012b). It was suggested that many flame annihilation events observed in unsteady flames can be approximated by one of these three configurations (Talei et al., 2010). The DNS results for all three 1D annihilation configurations were
compared to the pressure field calculated from the source terms in Dowling’s rearrangement of Lighthill’s acoustic analogy (Talei et al., 2011b). Considering only the heat release rate source term in Dowling’s equation, the pressure field was calculated using the reaction rate distribution obtained from the DNS. The pressure field calculated in this manner agreed well with the pressure field obtained directly from the DNS results, supporting the assumption that the source term associated with heat release rate is the only significant source term for these 1D annihilation events.

It was found that the flame thickness and the change in flame propagation velocity leading up to flame annihilation had a significant effect on sound generation (Talei et al., 2011b). The flame propagation velocity, defined as the rate of change of the position of maximum reaction rate, was observed to increase leading up to the annihilation event for all three geometrical configurations, as shown in Figure 2.12 (Talei et al., 2011a). For unity Lewis number the sound produced was found to be well predicted using the flame propagation velocity obtained from the DNS if the consumption speed \( \dot{\omega}_d \frac{d\zeta}{\rho u S_L} \) was assumed constant (Talei et al., 2011a). The assumption of constant consumption speed was found to not be valid for non-unity Lewis numbers (Talei et al., 2011a), as is also shown in Figure 2.12. Sound production was found to increase with Lewis number due to the increased consumption speed just prior to annihilation (Talei et al., 2011a). This set of DNS studies led to the derivation of scaling laws for unity Lewis number flame annihilation events, relating the far field pressure fluctuation amplitude to several parameters.

Talei et al. (2012a) also conducted a 2D DNS study to investigate the contribution of different source terms in Dowling’s reformulation of Lighthill’s acoustic analogy to the overall sound produced by a laminar, premixed, conical jet flame. The source term
associated with the temporal variation in the heat release rate was not the only significant source term and sound associated with a different term was found to be comparable for many cases (Talei et al., 2012a). This term has been interpreted as representing sound generation due to changes in the momentum of density inhomogeneities in the flow. Talei et al. (2012c) also investigated the ability of the flamelet model to predict the far field $p'_{\text{rms}}$. In the flamelet model a constant consumption speed is assumed and the heat release rate is therefore proportional to the flame surface area (Clavin and Siggia, 1991). Reasonable agreement was achieved only for unity Lewis numbers and low to intermediate Strouhal numbers.

2.4 Flame Chemiluminescence Tomography

Flame Chemiluminescence Tomography (FCT) is arguably the most promising, time-resolved, 3D diagnostic for directly investigating sound generation in turbulent, premixed flames. FCT can determine the reaction zone position with an uncertainty less than the flame thickness. Furthermore, flame chemiluminescence is a reasonable marker for the flame heat release rate and has been widely used in previous studies of combustion noise.

This section is organised as follows. Alternative methods for spatially and temporally resolving turbulent flames are discussed in Section 2.4.1. An overview of the fundamentals of Computed Tomography is included in Section 2.4.2. A categorisation of previous implementations of FCT is given in Section 2.4.3. Section 2.4.4 describes the practical considerations in acquiring projection measurements, This is followed by a review of the reconstruction methods used in FCT (Section 2.4.5), the achieved resolution in previous implementations (Section 2.4.6), and the flame measurements that have been acquired using FCT (Section 2.4.7).

2.4.1 Alternative Time-resolved 3D Flame Diagnostics

There has recently been significant progress in the development of measurement methods capable of obtaining pseudo-instantaneous and time-resolved, 3D flame measurements over an entire combustion region. Two main approaches have been used: laser scanning methods, and tomographic methods.

Planar laser diagnostics are well developed but scanning methods for time resolved measurements over a volume of interest are relatively recent (Cho et al., 2014; Olofsson et al., 2006; Weinkauf et al., 2015; Wellander et al., 2014). Scanning introduces new challenges and conventional difficulties, such as the non-uniform spatial profile of the laser sheet and laser attenuation, are more difficult to compensate for (Wellander et al., 2014). Moving mirrors are used to control the position of the laser sheet and accurate
control of the laser sheet position is a non-trivial task (Weinkauff et al., 2015). High repetition lasers rarely exceed 50 kHz and in choosing the scanning frequency a tradeoff must be made between spatial and temporal resolution. Temporal and spatial interpolation is required to obtain a 3D representation of the flame and the low signal-to-noise ratio from high repetition rate lasers typically necessitates substantial filtering of the images. Further development and technological progress is required before the technique is able to be applied affordably to highly turbulent flames with resolution on the order of the flame thickness. Currently temporal resolution on the order of 1 ms and spatial resolution on the order of 1 mm is achievable on relatively small flames using state of the art equipment. A more complete summary of previous studies involving laser scanning techniques is provided by Weinkauff et al. (2015).

Tomographic methods involve reconstructing a three-dimensional scalar field from integral measurements. The use of tomography in combustion has been limited primarily by the difficulties and cost associated with obtaining measurements of the required number of projections. As reconstruction algorithms are optimized for combustion applications, and optical equipment continues to decrease in price, tomographic methods are anticipated to have greater applicability in both combustion research and for on-line diagnostics of industrial combustors (Yong et al., 2012). Most high resolution flame tomography has used one of two approaches: light scattered by oil droplets or solid particles in the unburned premixture, and using the light emitted from the flame itself. The latter is known as Flame Chemiluminescence Tomography (FCT), or alternatively as Computed Tomography of Chemiluminescence (CTC).

Upton et al. (2011) demonstrated tomographic reconstruction of oil droplet evaporation on turbulent premixed flames using 12 flame images. Tomographic particle image velocimetry (TPIV) is a related method that uses lower seeding densities to allow reconstruction of the scattering from individual droplets (Ebi and Clemens, 2016a,b) or solid particles (Osborne et al., 2017). The resolution of the flame front position is limited by the sparse seeding density, typically to approximately 1 mm.

In comparison, FCT involves imaging the natural light emission from the flame, and can in principle provide a more accurate determination of the location of the reaction zone. FCT has recently been demonstrated on several turbulent, premixed, rim-stabilised jet flames and on turbulent, premixed, opposed-jet flames. Chemiluminescence imaging is non-intrusive and passive and the high-speed cameras required for the acquisition of projections are relatively common in combustion research laboratories.

Two additional approaches to flame tomography have recently been demonstrated. Tomographic reconstruction of laser induced fluorescence (LIF), termed ‘volumetric LIF’ was demonstrated in Ma et al. (2017a,b) and Li et al. (2018). Only five views were used in Ma et al. (2017a,b), while 8 views were used in Li et al. (2018). This approach
has the advantage that the entire flame need not be reconstructed, but in many ways is similar to FCT, including in the reconstruction algorithms and the methods used to obtain measurements from the reconstructed fields. The disadvantages of time-resolved volumetric LIF are that it requires a high repetition rate laser, and obtaining a sufficient signal-to-noise ratio is challenging. Background-oriented schlieren (BOS) tomography has also been recently demonstrated on a bunsen burner flame (Grauer et al., 2018). BOS tomography reconstructs the refractive index field from images of a patterned background, viewed through the flame. An advantage is that images with short exposure times and high signal to noise ratio can be obtained without expensive equipment. The refractive index field features a high gradient between the unburned and burned gases but the refractive index remains high in the burned gases, unlike flame chemiluminescence. Methods used in BOS tomography are therefore usually not applicable to FCT.

2.4.2 Fundamentals of Computed Tomography

Computed or computerized tomography is the reconstruction of an unknown scalar function $I$ from a finite number of projections. It has wide applicability in the fields of science, engineering and medicine (Herman, 2009), for example in CT scans. In the terminology of applied tomography, a projection ($P_i$) is a spatially weighted integral of a scalar function for which the spatial weighting function ($w_i$) can be estimated. The term ‘view’ is often used to mean a set of projections measured by sensors arranged in a structured manner on a plane at a defined orientation to the object. For example in an x-ray image, the intensity value at one pixel is a single projection, and the total image is a single view.

The general problem of calculating $I$ from its projections was solved by Radon in 1917 (Madych, 2004). This analytical expression, known as the inverse Radon transform, is not directly useful in practical applications as it requires the complete set of projections, i.e. continuous in both projection angle and transverse co-ordinate (Upton et al., 2011). Reconstruction methods using a finite number of projections can be categorised as either transform methods or series expansion methods (Herman, 2009). Transform methods are based on calculating an estimate of $I$ using a discretized form of the Radon inversion formula. Series expansion methods involve approximating $I$ as a linear combination of $J$ basis functions ($b_j$) over the reconstruction domain,

$$\hat{I} = \sum_{j=1}^{J} B_j b_j,$$  \hspace{1cm} (2.38)
where $B_j$ is the coefficient of basis function $b_j$ and $\hat{I}$ is an approximation of the scalar function $I$. A projection of $\hat{I}$ may be expressed as

$$\hat{P}_i = \sum_j W_{i,j} B_j \tag{2.39}$$

where $W_{i,j}$ is a weighting representing the contribution of the $j^{th}$ basis function to the $i^{th}$ projection, given by

$$W_{i,j} = \int_V b_j(x,y,z)w_i(x,y,z) dV \tag{2.40}$$

The problem becomes one of solving for the basis function coefficients $B_j$ so that the projections of $\hat{I}$ match the measured projections (Herman, 2009), i.e. to solve the system

$$\sum_j W_{i,j} B_j = P_i \tag{2.41}$$

Series expansion methods are commonly preferred for applications where a relatively small number of views are available, as is commonly the case in flame tomography (Verhoeven, 1993).

Projections for optical flame tomography are acquired using scientific cameras, or in some cases, commodity cameras. Each pixel is a projection measurement $P_i$ whose value represents a wavelength weighted integral of the emission intensity field over some volume and from some time $t_0$ through the exposure time $\Delta t$.

$$P_i(t_0) = \int_V \int_{t_0}^{t_0+\Delta t} w_i(x,y,z)I'(x,y,z,\lambda,t) dtdV d\lambda \tag{2.42}$$

where $s$ is the wavelength dependent sensitivity function of the imaging system, $I'$ is the instantaneous emission intensity field, and $w_i$ is a function describing the spatial weighting associated with pixel $P_i$ and is determined largely by the optics. Refraction and absorption can also be accounted for in $w_i$. In general, $w_i$ is therefore a function of wavelength and time. However, in practice $w_i$ is almost always assumed independent of both time and of wavelength over the range that significantly contributes to the projection measurements. This yields

$$P_i(t_0) = \int_V w_i(x,y,z)I(x,y,z,t_0,\Delta t) dV \tag{2.43}$$

where

$$I(x,y,z,t_0,\Delta t) = \int_{t_0}^{t_0+\Delta t} s(\lambda)I'(x,y,z,t,\lambda) d\lambda dt \tag{2.44}$$
is the field to be reconstructed.

2.4.3 A Categorisation of Implementations of Flame Chemiluminescence Tomography

FCT is here categorised as either steady or unsteady, depending on whether the target flame is steady or unsteady. Steady FCT of non-axisymmetric flames have been used to demonstrate the capabilities of FCT, and to develop the methods (e.g. Cai et al., 2013; Denisova et al., 2013; Floyd and Kempf, 2011; Goyal et al., 2014). Steady FCT has two main advantages over unsteady FCT: (i) an arbitrary number of views can be acquired using a single camera, and (ii) the exposure time can be arbitrarily long, allowing optical filters to be used without compromising the Signal-to-Noise Ratio (SNR).

In principle, results obtained from steady FCT are applicable to reconstructions of the instantaneous emission field of turbulent flames in the wrinkled flame regime. This is because in both steady, laminar flames and wrinkled, turbulent flames, chemiluminescence is restricted to thin, connected regions. Nevertheless, the closer the steady flame topology is to a wrinkled, turbulent flame, the greater confidence one can have that a similar level of accuracy would be achieved in reconstructions of wrinkled, turbulent flames.

FCT of unsteady, turbulent flames can be further categorised as instantaneous FCT (e.g. Floyd et al., 2011; Ishino and Ohiwa, 2005; Ma et al., 2016b; Mohri et al., 2017), time-averaged FCT (e.g. Moeck et al., 2013; Samarasinghe et al., 2016), and phase-averaged FCT (e.g. Samarasinghe et al., 2013; Worth and Dawson, 2013). Time averaged and phase-averaged FCT reconstructs the emission field of the turbulent flame brush, as opposed to an ‘instantaneous’ emission field. These applications therefore share the advantages described for steady FCT. However, the distribution of the chemiluminescent emission intensity is considerably different. Flame brushes are typically an order of magnitude thicker than the instantaneous flame, gradients in the emission field are smaller, and the emission field is considerably less wrinkled. Results from these studies therefore have limited relevance to instantaneous FCT. For the following sections, the focus is on steady FCT and instantaneous, turbulent FCT of lean, premixed, non-axisymmetric flames.

A comprehensive review of tomography applied to axisymmetric flames is also not included. The reconstruction methods used for, and results obtained from, axisymmetric flames are generally not relevant to non-axisymmetric flames because only one view is required to resolve the 2D (in cylindrical co-ordinates) emission field.
2.4 Flame Chemiluminescence Tomography

2.4.4 The Acquisition of Projection Measurements

Obtaining projection measurements with a high signal-to-noise ratio is challenging due to the low intensity of light typically emitted by premixed flames. Short exposure times, usually less than 1 ms, are required when imaging turbulent flames to limit motion blur. The combination of a low intensity source and short exposure time requires the use of highly sensitive cameras. The expense of suitable cameras motivated Ishino and Ohiwa (2005) to build a custom 40 lens camera that used high-speed film, with the images digitized after the film was developed. Floyd and Kempf (2011) has recently showed that relatively cheap low speed cameras, costing less than US$1000 each, are now capable of producing acceptable images of turbulent flames with exposure times less than 250 $\mu$s. High resolution cameras are widely available and the spatial resolution of tomographic reconstruction techniques is typically limited by the number of views rather than the image resolution (Floyd et al., 2011). Due to the still considerable cost of suitable cameras, mirror assemblies are often used to project two or more images onto one camera/detector (e.g. Floyd et al., 2011; Gilabert et al., 2007; Hertz and Faris, 1988; Upton et al., 2011).

View calibration or view registration is sometimes necessary to estimate the volume weighting functions ($w_i$) associated with each projection measurement with sufficient accuracy. This is non-trivial for camera images due to perception distortion and optical aberrations. Self-absorption and refraction due to the large temperature gradients in the flame region make the task considerably more difficult (Soloff et al., 1997) and these effects are generally neglected. Refraction in particular depends on the flame geometry and therefore would require iteratively reconstructing the flame and estimating the weighting function. This is computationally prohibitively expensive. Perception distortion and image blur is often ignored due to the large distance between the object and the lens compared to the object dimensions and the use of sufficiently large depth of fields (Hertz and Faris, 1988; Ishino and Ohiwa, 2005). This allows for a relatively simple view registration procedure, requiring only the camera magnification and orientation relative to the reconstruction volume, often accomplished using a calibration object (Floyd et al., 2011; Upton et al., 2011). If projections are acquired in the same plane (i.e. coplanar views) and the flame images are treated as orthographic projections, horizontal slices of the reconstruction volume can be individually reconstructed (Floyd and Kempf, 2011; Ishino and Ohiwa, 2005; Upton et al., 2011). This approach has advantages in terms of algorithm simplicity and memory requirements. However, coplanar views are generally not considered optimal for maximising the tomographic reconstruction accuracy (Cai et al., 2013). It is, nonetheless, one of the simpler arrangements for the acquisition of images, and in many applications it is the only practical arrangement.
In steady, time-averaged, and phase-averaged FCT, optical bandpass filters are often used to restrict the imaged wavelengths to bands where OH* or CH* emission is greatest. CH* and OH* bandpass filters typically have a center wavelength in the respective ranges 430-440 nm (Cai et al., 2013; Denisova et al., 2013; Floyd and Kempf, 2011; Samarasinghe et al., 2013) and 300-310 nm (Denisova et al., 2013; Worth and Dawson, 2013). The filter full-width at half-maximum is typically 10-25 nm. Despite the use of bandpass filters, CO$_2$* emission unavoidably accounts for some of the light forming the image in these studies (Lee and Santavicca, 2005). The chemiluminescent source proportions in the filter band depend not just on the filter characteristics, but also on flame parameters such as fuel composition, equivalence ratio, pressure, and unburned gas temperature (Docquier et al., 2002; García-Armingol et al., 2013; Hardalupas and Orain, 2004; Ikeda et al., 2002; Tripathi et al., 2012). Optical filters decrease the signal-to-noise ratio in the images, and in the cited experiments either long exposures greater than 10 ms were used or multiple images were averaged. Neither of these options are possible when attempting to tomographically resolve the flame surface of turbulent flames due to limitations placed on exposure times due to flame movement. In the few reported instantaneous, turbulent FCT implementations, broadband chemiluminescence across the visible spectrum was imaged (Floyd et al., 2011; Ishino and Ohiwa, 2005; Ma et al., 2016b; Mohri et al., 2017).

### 2.4.5 Reconstruction Algorithms

Series expansion methods involve representing the field to be reconstructed with a set of basis functions and solving a system of equations for the basis function coefficients (Herman, 2009; Verhoeven, 1993). The system is typically underdetermined and often inconsistent. An inconsistent system of equations can arise from error in the projection measurements, error in the calculated weightings, or can be inherent in the choice of basis functions. Many methods to find an optimal solution have been proposed. The additive and multiplicative variants of the algebraic reconstruction techniques (ART/MART) are the most widely used in previous CTC implementations (Floyd et al., 2011; Hertz and Faris, 1988; Mohri et al., 2017; Wang et al., 2015; Worth and Dawson, 2013). The ART/MART family of algorithms are popular for a number of reasons. They can accurately reconstruct a field from a small number of views, are robust in the presence of noise, can incorporate a-priori information such as maximum and minimum values, place no restrictions on the geometry of the projections and can be equally applied to 2D or 3D reconstructions with little modification (Floyd and Kempf, 2011).

Other reconstruction methods involve formulating the problem with additional explicit optimization criteria, often to incorporate some prior knowledge about the character of the field. Anikin et al. (2012) reconstructed the OH* emission of a turbulent bluff body...
flame using a regularization parameter based on the Euclidean norm. Ishino and Ohiwa (2005) used the Maximum Likelihood Expectation Method (MLEM) to reconstruct the broadband chemiluminescence of a turbulent jet flame. Cai et al. (2013) reported improved results over ART by using a total variation regularization method in reconstructing synthetic data representing a modified McKenna burner. Denisova (2004) demonstrated an improvement using the maximum entropy (MENT) method over ART in the reconstruction of two synthetic emission fields, but did not consider examples representative of flame chemiluminescence. Denisova et al. (2013) later reported reconstructions of an axisymmetric, conical flame using MENT and showed the MENT method could be improved by local smoothing of the projections. Further study was said to be required for applying this method to non-axisymmetric flame tomography. Overall, there is some evidence that improved reconstructions can be obtained using reconstruction algorithms other than ART. However the reported improvements are typically small, are dependent on the criteria used to assess the reconstruction error, and are demonstrated on a small range of emission fields.

2.4.6 Resolution

Resolution is a description of the size of the minimum separation between two closely related forms that allows the two forms to be discriminated. However, different studies often use different methods of estimating the resolution of reconstructed emission fields. Floyd et al. (2011) investigated reconstruction resolution using synthetic data representing a turbulent opposed jet (TOJ) flame. A low pass filter was applied to the original and reconstructed fields. The correlation coefficient between the filtered fields was computed for different cut-off frequencies in the low pass filter. The resolution was determined to be equal to the cutoff wavelength at which the correlation coefficient exceeded some threshold. Their results were supported by qualitative analysis of reconstructions of a real TOJ flame. For similar flames, they concluded that 20 views are sufficient to reliably resolve wavelengths of 0.035 object diameters. Cai et al. (2013) reconstructed the CH* emission from a customized McKenna burner using 8 non-coplanar views. Resolution on the order of 1.25 mm was reported based on measurements from the reconstructed emission field of the average dimensions of controlled extinction regions although the reported standard deviation in the measurement had a similar magnitude. Anikin et al. (2012) reconstructed the OH* emission from turbulent non-premixed flames with 10 views and reports a spatial resolution equal to the resolution of the optical measuring equipment, approximately 1 mm. This was justified by the observation that adding noise to the measured projections does not significantly affect the reconstructed field.

Resolution in FCT is typically limited by the number of views used. There is currently no standardised methodology for determining the resolution of reconstructed emission
fields. Furthermore, it is far from straightforward to extrapolate from the reconstruction resolution to the uncertainty in specific flame measurements derived from FCT. It is also generally agreed that resolution is dependent on the field to be reconstructed, that features with similar spatial wavelengths may be reconstructed well in some part of a reconstruction and poorly in others, and that there is no simple and reliable method of predicting whether a feature will be resolved.

2.4.7 Measurements Acquired Using FCT

For unsteady, wrinkled, premixed flames, there are only two instances in the literature where 3D flame measurements at points on an identified “flame surface” have been made using FCT.

Ishino et al. (2009) report measuring the flame propagation velocity on a small part of a turbulent flame surface. The measurement was obtained using two instantaneous chemiluminescence fields from a weakly turbulent propane-air flame using 20 views for each reconstruction, with the 1.29 ms exposures for each reconstruction occurring in succession. Details of the velocity measurement method were not provided.

Ma et al. (2016b) reconstructed a weakly turbulent slot flame using six coplanar cameras and reported measurements of the 3D flame curvature. The curvature measurement was obtained by first computing an isosurface of the chemiluminescence field on the unburned gas side. However, from cross-sections of the reconstructions (Ma et al., 2016b, Fig. 4), it is clear that the identified “flame location” is not coincident with the location of peak chemiluminescent emission, and it is equally evident that the thin emission region was not resolved. The stated claim that all the chemiluminescence away from the identified “flame location” is due to species in the combustion products is not consistent with other works on the visible chemiluminescence emission from stoichiometric methane-air flames (e.g. Li et al., 2015), particularly considering the reconstructed cross-sections show emission intensity far exceeding that at the extracted isosurface, at distances of more than 20 mm away from the isosurface. Furthermore, combusting separated pockets are observed 100 mm above the flame base in the projections and reconstructed cross-sections, but are not seen in the extracted isosurfaces on which the curvature is measured.

FCT-based measurements have also been reported in several other combustion studies. Ma et al. (2015, 2016a) reconstructed a combustion region in a Mach-2 combustor using 8 views. Flame surface area and volume were computed using an isosurface and voxel thresholding respectively. Cross-sections of the combustion zone showed a distributed, connected emission region and thin emission zones characteristic of wrinkled flames either were not present or were not resolved. FCT has also been applied to the reconstruction of time-averaged turbulent flame brushes (Geraedts et al., 2016; Moek et al., 2013;
Samarasinghe et al., 2013, 2016; Worth and Dawson, 2013) and reported measurements include the turbulent flame surface area. However, chemiluminescence fields from turbulent flame brushes typically have very different characteristics to ‘instantaneous’ flame surfaces.

2.5 Summary

This chapter reviewed the literature on sound generation in unconfined, premixed flames. Fundamental aspects of premixed combustion, flame chemiluminescence, and direct combustion noise were reviewed first, leading to a discussion of the most recent work on sound generation by unsteady, laminar, premixed flames.

Direct sound from open, premixed flames is approximately proportional to the time derivative of the heat release rate in the flame region. This has been shown theoretically using the acoustic analogy framework, and experimentally using the correlation between flame chemiluminescence and the heat release rate. The sound radiated by open, turbulent, premixed jet flames has also been characterised. However, these observations provide limited and indirect evidence of the processes responsible for the generation of the radiated sound. Because of the complexity of turbulent flames, features of the radiated acoustic waveform have not been directly related to observed flame dynamics, such as are involved in the separation of pockets.

There have been several experimental and numerical studies of sound production in well-controlled, laminar flames featuring flame annihilation events. The experimental studies of Kidin et al. (1984) reported compression waves generated in the terminal moments of tunnel narrowing and pocket consumption processes. No other significant acoustic features were observed. Candel et al. (2004) reported the generation of significant rarefaction waves as a result of the rapid flame surface destruction occurring during flame annihilation and cusp recovery but did not report the compression waves described by Kidin et al. (1984). It is not clear whether differences in the flame configurations used in these two studies can explain these seemingly contradictory observations. Furthermore, in both studies sound generation by flame annihilation events was not temporally resolved from sound generation by the subsequent cusp recovery.

In the existing literature on sound generation by laminar, premixed flames, it has not been established whether flame annihilation is a significant source of sound. It is also unclear whether the evolution of highly curved flames before and after flame annihilation events contribute to the emitted sound. Given the known relationship between the far-field acoustic pressure and the time derivative of the total heat release rate from the combustion, changes in the flame displacement speed and consumption speed in response to curvature are also likely to affect the generated sound. However, except during flame annihilation,
the importance of accurately modelling flame acceleration in order to predict the emitted sound has not yet been explored.

This chapter then reviewed the literature on time-resolved, 3D flame diagnostics, with a focus on the imaging method used in this thesis, Flame Chemiluminescence Tomography (FCT). Prior implementations of FCT have demonstrated that the method is capable of resolving the instantaneous flame surface using approximately 10-20 views. Resolution depends on a number of factors including the optical resolution, the signal-to-noise ratio in projection measurements, the reconstruction grid resolution and size, and uncertainty in the computed weightings. However, the limiting factor is often the number of views. This directly relates to the degree to which the problem is underdetermined. Uncertainties in projection measurements, weightings, or inherent in the choice of basis functions often result in the formulated system of equations being inconsistent. A number of reconstruction algorithms have been developed to reliably compute approximate solutions, the most commonly implemented being the Algebraic Reconstruction Techniques (ART).

The number of views is also the critical parameter determining the cost of FCT implementations. The low emission intensity of lean, premixed flames and the magnitude of the flame propagation velocity in turbulent flames necessitates the use of multiple, sensitive cameras. For time-resolved measurements high-speed cameras must be used, significantly increasing the cost. There are no convincing reports in the literature of FCT measurements relating to premixed flame theory such as flame speeds, curvature, thickness, or flame surface area. Furthermore, few algorithms to obtain such measurements from reconstructed emission fields have been proposed and the number of views necessary to measure the flame quantities of interest has not yet been quantified.
Chapter 3

Experimental Methods

3.1 Introduction

This chapter includes descriptions of the forced, laminar, premixed flame rig, the imaging set-up, and sound measurements. The image processing methods, including tomographic reconstruction methods, are also described. The optical resolution is measured and uncertainties in the projection measurements used in tomographic reconstructions are quantified.

The methods used for analysing sound measurements are also described. These include the identification of distinctive features and the computation of mean and representative pressure traces. The decay in the amplitude of the acoustic pressure waveform with distance is measured to determine at what distances the $1/R$ acoustic monopole decay is observed and to identify acoustic reflections. Lastly, high speed Schlieren imaging is used to investigate the effects of positioning an acoustically-lined stabilisation stack downstream of the flame to reduce buoyancy-induced flame flickering.

3.2 Forced Laminar Premixed Flame Rig

All the results presented in this thesis were obtained on the same combustion rig. The rig was originally designed and built by Karimi et al. (2009), and was recommissioned for this work. New flame holders were designed for this work and drawings are included in Appendix B. A schematic of the burner plenum is shown in Figure 3.1.

All experiments were conducted using propane. Propane was chosen because, in contrast to methane, the Lewis number of propane-air flames varies with equivalence ratio. The Lewis number has been shown in previous studies to be important to the sound generated by flame annihilation (Talei et al., 2012b). The propane and air streams are mixed approximately three metres upstream of the plenum to ensure a well mixed mixture.
at the burner port. The burner plenum contains multiple flow conditioning sections. The mixture passes first through a 60 mm honeycomb flow straightener and subsequently through two layers of fine steel mesh, installed before and after a contraction that reduces the diameter from 224 mm to 50 mm. The velocity profile at the flame holder port is almost uniform, as is detailed by Karimi et al. (2009).

Flow rates of dry, filtered, compressed air and propane (99.95% purity) were controlled using MKS thermal flow meters, models 1559A and M100B respectively. The full scale ranges of the flow meters were 80 SLM and 6 SLM respectively and the uncertainty stated by the manufacturer is ±0.5% of the full scale range.

Two flame holders were used in this work: an axisymmetric flame holder with a circular port, and a non-axisymmetric flame holder with a square port. Both are shown in Figure 3.1 Flames were stabilised on the flame holder lip. The lip has a vertical inner surface and is tapered to a knife edge at a 60° angle. The circular port has a diameter of 25 mm. The square port has sides that are 22 mm long and has rounded corners of 2 mm radius. The flame holders can be rotated about the vertical axis. This feature is used in the work on Flame Chemiluminescence Tomography in Chapter 7 and allows any number of coplanar images of the flame to be obtained without moving the camera. A vernier angle scale is used to measure the viewing angle $\beta$ to an accuracy of ±0.1 degrees (see drawing in Appendix B).

An RCF 8 inch speaker driver (model no. L8S800) is mounted at the bottom of the burner plenum and is used to produce sinusoidal velocity modulations in the unburned gases at the burner port. The speaker is driven by a B&K Type 2706 amplifier and the sinusoidal signal is generated by Direct Digital Synthesis on an Arduino Due microcontroller. A phase locked square wave is also generated and is used by a second microcontroller to control the image timing. The operation of the second microcontroller is described in

![Fig. 3.1 Left: Schematic of burner plenum and flow conditioning. Centre: Circular port flame holder. Right: Square port flame holder.](image-url)
Section 3.3.2. The Arduino Due has a clock rate of 84 MHz. The analog output is updated 1000 times per cycle of the sine wave. As an example, for a 50 Hz sine wave, the output is updated every 1680 clock cycles. Because the output can only be updated on an integer number of clock cycles, only a discrete number of frequencies can be generated. For example, a 49 Hz sine wave cannot be generated exactly. The closest approximation is 49.0082 Hz which results from updating the output every 1714 clock cycles. The resolution is more than sufficient for the purposes of this work. The period of the sine wave was measured using a Tektronix TDS 2012 Digital Storage Oscilloscope for five frequencies from 25 Hz to 125 Hz and fluctuated by less than 5 µs over 100 cycles.

3.2.1 Flame Confinement Configurations

Unconfined, premixed flames are susceptible to a buoyancy driven instability known as flame flickering (Durox et al., 1990; Kostiuk and Cheng, 1995). Flame flickering occurs due to the formation of toroidal vortices in the shear layer between the burned gas plume and the ambient air. This affects the repeatability of the forced flame cycles and induces air movements around the flame. The associated non-radiating pressure fluctuations are significant up to 20 cm from the flame. For the results presented in Chapters 4 and 6, flame flickering was largely eliminated by installing a 540 mm long, 150 mm diameter cylindrical stack downstream of the flame. The bottom of the stack is mounted 55 mm above the burner port. The stack is lined with a 25 mm thick ceramic-fibre blanket, giving an internal diameter of 100 mm. The ceramic-fibre lining attenuates acoustic reflections. Acoustic reflections due to the stabilisation stack were found to be negligible. Further details of the effects of the stabilisation stack are included in Section 3.6.4.

For the work on Flame Chemiluminescence Tomography presented in Chapter 7, the flame sound was not measured and the flame could therefore be confined. A 230 mm long, 50 mm diameter, fused silica tube was placed around the flame holder as can be seen in Figure 3.1. The tube protected the flame from air disturbances and prevented flame flickering while still allowing optical access.

3.3 Chemiluminescence Imaging

3.3.1 Optical Arrangement

Chemiluminescence images were captured with a LaVision Flowmaster 3S camera system with a LaVision high-speed IRO image intensifier. A Sigma 150 mm f/2.8 lens with a Kenko MC UV low pass filter was used for most of the imaging work. A Nikon 60 mm f/2.8 lens was used to acquire images of the entire flame presented in Chapters 4 and 6.
The lens aperture was fully open for all imaging. The camera sensor has a 12 bit dynamic range, a full-frame resolution of $1280 \times 1024$, and a maximum frame rate of 8 frames per second. The camera was positioned so that the image plane was parallel to the flame holder’s axis of rotation.

The imaged emission field, here denoted $I$, is the integral of the instantaneous, visible, chemiluminescent, emission field over the camera exposure, denoted $I'$. For the forced flames presented here, the flame velocity relative to the laboratory is rarely expected to exceed $3 \text{ ms}^{-1}$. For the images used in Chapters 4 and 6, exposure times are up to 20 $\mu$s. The flame may therefore move by up to $60 \mu\text{m}$, or 2 pixels, during the exposure. In Chapter 7 a 100 $\mu$s exposure time is used and the flame may move up to 0.3 mm, or 4 pixels, over the image exposure. However, at most flame locations the flame movement will be much less.

### 3.3.2 Image Timing

An Arduino MEGA 2560 was used as a pulse delay generator to control the image timing. The camera has a maximum frame rate of 8 frames per second and the forced flames under investigation are forced at between 25 Hz and 125 Hz. Therefore, only one image can be acquired every few flame cycles. The Arduino MEGA 2560 functions as follows. The falling edge of the square wave generated by the Arduino Due (see Section 3.2) is connected to an external interrupt on the MEGA 2560. An interrupt service routine counts the number of flame cycles. After a specified number of flame cycles the external interrupt is redirected to a second interrupt service routine. On the next falling edge the second interrupt service routine starts a timer. After a specified delay, digital triggers are sent to both the camera and the image intensifier. A new delay is set, i.e. the Compare Match Register is updated, for the next time the timer is run. The external interrupt is then reattached to the first interrupt service routine and the cycle is repeated.

The splitting of the signal generator and pulse delay generator tasks between two microcontrollers enables high precision on both tasks. Delays were measured using a Tektronix TDS 2012 Digital Storage Oscilloscope and were found to be repeatable to within 5 $\mu$s. The MEGA 2560 microcontroller is idle for a large majority of the time and additional functionality is therefore able to be included without affecting the delay generator precision. Implemented functionality included interactive selection of the image timing using a potentiometer connected to an analogue input. In practice, a part of the flame cycle of interest such as the pocket separation or pocket burn-out event was found interactively. A programmed imaging sequence then acquired images with incremented delays from a specified time before this event to a specified time after the event.
The factor limiting the temporal resolution was generally the flame cycle repeatability. For flames confined by the fused silica quartz tube, as used in Chapter 7, the phases of flame events such as the separation event and pocket burn-out event were observed to vary relative to the phase of the forcing signal by up to 50 µs. For unconfined flames stabilised by the downstream stack, as used in Chapters 4 and 6, the separation event and pocket burn-out event were observed to vary relative to the phase of the forcing signal by up to 0.3 ms. This was highly dependent on flame parameters such as the equivalence ratio and the mixture flow rate and for some cases the variation in the phase of the separation events and burn-out events was less than 20 µs.

3.3.3 Uniformity Correction

For all flame images, a background image is subtracted and the image is corrected for the spatial non-uniformity of the imaging system. Background images were acquired for each combination of camera exposure time, image intensifier gain and gate width that was used. Ten background images were acquired with the flame extinguished. A mean background image was computed from these 10 images.

After background subtraction, each pixel of a flame image is multiplied by a correction factor to correct for the non-uniform spatial response of the imaging system. Uniformity correction factors were determined from images of a uniform target. The target comprised a backlit ground-glass diffuser. The lens was focussed 5 cm in front of the diffuser. The light source was an LED array (Visual Instrumentation Corp., Model 900405R). Sets of correction factors were obtained at all image intensifier gain settings used in this work. At each gain setting, images of the uniform target were acquired with six different gate widths, to obtain average pixel counts over the image of approximately 100, 200, 400, 800, 1600, and 3200. The maximum count of the 12 bit sensor is 4095.

A background image was subtracted from each of the nominally uniform images. The average pixel count was then computed and a multiplicative correction factor was determined for each pixel by dividing the average pixel count by the individual pixel count. At each gain setting, a set of six multiplicative factors were therefore generated for each pixel (one from each of the six nominally uniform images) and are stored along with the six corresponding pixel counts. The multiplicative correction factors were found to be relatively insensitive to pixel count but did show some dependency on the gain setting, becoming more uniform, i.e. closer to 1, with increasing gain. Correction factors were individually applied to each pixel in a flame image. For a pixel in a flame image, the pixel count does not generally match any of the six pixel counts for which correction factors
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have been determined. Linear interpolation is therefore used to determine the correction factor.

3.3.4 Image Resolution

The resolving power of the imaging system is quantified using a 1951 USAF resolution target. The target pattern consists of bars, also referred to as lines, of different widths. The width of the bars varies from 0.25 line pairs/mm to 228 line pairs/mm. For each bar width, there is a set of three horizontal bars and a set of three vertical bars. The width of the space between the bars is equal to the width of the bars and the length of each bar is five times its width. The 1951 USAF resolution target is used to determine the Contrast Transfer Function (CTF). The contrast ($C$) is defined

$$C = \frac{I_{\text{max}} - I_{\text{min}}}{I_{\text{max}} + I_{\text{min}}}$$

where $I$ is either the object radiosity or the image irradiance depending on whether one is referring to the contrast of light leaving an object’s surface or the contrast of an image. The CTF is defined by $\text{CTF} = C_{\text{image}}/C_{\text{object}}$. Intensity profiles were taken along a centreline through the image of each group of 3 bars to determine the image contrast. The resolving power reported here is twice the width of the bars in the smallest set with $\text{CTF} > 0.05$. The resolving power is dependent on the exposure time, image intensifier gain, and magnification. The imaging parameters and the resolving power for imaging arrangements using the Sigma 150 mm lens are summarised in Table 3.1. Figure 3.2 shows

<table>
<thead>
<tr>
<th>Thesis Sections</th>
<th>Object length imaged on a single pixel (µm)</th>
<th>Resolving Power (µm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chapter 6</td>
<td>74</td>
<td>210</td>
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<tr>
<td>Sections 4.3, 5.4-5.6</td>
<td>30</td>
<td>76</td>
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<tr>
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<td>48</td>
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</table>

the CTF versus wavelength.

3.4 Abel Deconvolution

For an axisymmetric field $f(r)$, the Abel transform $P(y)$ is the set of path integrals along straight, parallel lines through the domain of $f(r)$,

$$P(y) = 2 \int_{y}^{R} f(r) \frac{r}{\sqrt{r^2 - y^2}} dr. \quad (3.1)$$
3.4 Abel Deconvolution

The inverse Abel transform is

\[ f(r) = -\frac{1}{\pi} \int_{y}^{R} \frac{dP(y)}{dy} \frac{dy}{\sqrt{y^2 - r^2}}. \]  

(3.2)

In practice, projection data consists of a finite set of noisy measurements. Due to the derivative in Equation 3.2, the inverse Abel transform is highly sensitive to measurement noise. A number of methods of reconstructing the radial intensity field have therefore been developed (Pretzler et al., 1992) and are known generally as Abel deconvolution methods.

Abel deconvolution is applied in Section 6.7 to investigate the chemiluminescent emission field of axisymmetric flame necks during tunnel narrowing. Each pixel of a flame image represents a spatial integral of the emission intensity field, termed a projection. For imaging arrangements with negligible perspective distortion, images are well approximated by integrals along straight, parallel paths through the flame region. An Abel deconvolution code distributed on the MathWorks website was used in this work (Killer, 2017). This code is an implementation of the method described by Pretzler (1991). This method approximates the function \( f(r) \) as a sum of cosine functions with unknown amplitudes, i.e.

\[ f(r) = \sum A_n f_n(r), \]

where

\[ f_0(r) = 1, \quad f_n(r) = 1 - (-1)^n \cos \left( n\frac{\pi}{R} r \right). \]  

(3.3)

The Abel deconvolution code was tested on synthetic data designed to resemble the emission fields under investigation. The synthetic radial emission profile is shown in Figure 3.3 along with the computed Abel transform. An Abel transform with added gaussian noise is also shown. The Signal-to-Noise Ratio (SNR) for the ‘noisy’ Abel transform is 10 dB. Figure 3.3 also shows the reconstructed emission fields from both the noisy

![Fig. 3.2 The Contrast Transfer Function as determined using a 1951 USAF resolution target for the imaging arrangements used in: (a) Chapter 6, (b) Sections 4.3 and 5.4-5.6, (c) Section 5.7.](image)
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Fig. 3.3 Left: Synthetic radial emission profile (blue line) and the Abel transform with and without gaussian noise (black lines). Centre: Deconvolved radial emission profiles computed from the noiseless Abel transform. Deconvolved profiles are shown using 1 to 40 cosine terms. Right: Deconvolved radial emission profiles computed from the Abel transform with added gaussian noise.

Fig. 3.4 The convergence of peak position measurements (left) and width measurements (right) obtained from reconstructed radial profiles computed from the noisy (triangles) and noise-free (circles) Abel transform. True values are shown by the dashed lines.

and the noise-free Abel transform when using different numbers of terms in the cosine expansion. Noise is seen to be amplified at positions where the emission intensity should be low but the synthetic profile is still relatively well reconstructed. Figure 3.4 shows the measured width (full-width at half-maximum) and the peak position versus the number of cosine terms used. The peak position converges to within one discretisation increment (0.1 distance units) of the actual peak position with only 9 terms and is relatively unaffected by noise. The width converges to within 20% of the actual width with approximately 15 terms but noise results in an error of approximately 10% in the converged result. On the basis of these results, 30 cosine terms are used for the measurements presented in Section 6.7.

3.5 Non-axisymmetric Tomography

In Chapter 7, non-axisymmetric tomographic methods are applied to two forced flames stabilised by the square-port flame holder described in Section 3.2. Coplanar flame
images are obtained by rotating the flame holder. The imaging system remains fixed in position with the optical axis perpendicular to the rotation axis of the flame holder. In all experiments, the flame was allowed to stabilise for at least 3 hours before imaging was commenced.

### 3.5.1 Tomographic Reconstruction Method

The Multiplicative Algebraic Reconstruction Technique (MART) is implemented in this study. It was chosen on the basis of its ease of implementation, flexibility and widespread use (e.g., Floyd et al., 2011; Hertz and Faris, 1988; Worth and Dawson, 2013). MART is a series expansion method, as described in Section 2.4.2. In this implementation, the reconstruction volume is divided into cubic volume elements, known as voxels. Voxels are basis functions that have a value of one inside the volume element, and zero outside. The voxel coefficient $B_j$ represents the average value of the scalar field over the voxel. The MART algorithm starts with an initial guess of the voxel coefficients, typically $B_j^0$ uniform, and a correction is sequentially applied to each voxel coefficient. One iteration of the MART algorithm is given below (Elsinga et al., 2007). The superscript $k$ is used to denote the iteration.

\[
B_j^{k+1} = B_j^k + \mu W_{i,j} \frac{P_i - \sum_{l \in N_i} W_{i,l} B_l^k}{\sum_{l \in N_i} W_{i,l} B_l^k} \quad (3.4)
\]

where $\mu$ is a scalar relaxation parameter which for MART must be $\leq 1$. The applied correction is based on the difference between the measured projection intensity and the calculated projection intensity using the current estimate of the voxel coefficients. Voxel coefficients were constrained to positive values.

Flame images are treated as parallel projections in this study, i.e., perspective distortion is assumed to be negligible. This is a common assumption (e.g., Floyd et al., 2011; Ishino and Ohiwa, 2005; Upton et al., 2011; Worth and Dawson, 2013) and justification for this assumption is provided in Section 3.5.2. Each weighting $W_{i,j}$ was determined by computing the intersection volume between a voxel and the volume created by projecting a square representing the sensor pixel along a straight line through the reconstruction volume. A common simplification has been implemented where voxels are treated as cylinders with
an equal volume (Atkinson and Soria, 2009). The weightings are then independent of the angle of intersection and are only dependent on the perpendicular distance between a line projected from the centre of the square representing the sensor pixel and the centre of the voxel.

Because the acquired views are coplanar, horizontal cross-sections of the flame can be independently reconstructed. The algorithm used here is essentially a 2D image reconstruction algorithm, using the same row of camera pixels from each view to reconstruct the intensity field of a cross-section. The weighting array is the same for each cross-section and so need only be calculated for one cross-section. The 3D emission intensity field is obtained by stacking the reconstructed slices. For each cross-section, five iterations of the MART algorithm were used to reconstruct the emission intensity field on a $400 \times 400$ grid, with a grid spacing of $74 \, \mu m$. The grid spacing was chosen to be equal to the image sensor pixel length ($6.7 \, \mu m$) divided by the image magnification (0.0904), so that the individual cross-sections form a 3D cartesian grid when stacked. A relaxation factor of 0.3 was chosen based on extensive testing.

Uncertainty in limited-view tomography is predominantly determined by the number of views. With a small number of views, the system will be underdetermined. However, when a sufficiently large number of views are used, the problem becomes overconstrained, and uncertainty in the reconstructed field is instead determined by uncertainty in the input projection measurements and in the estimated weighting functions. In the following section, these sources of uncertainty are discussed and quantified.

### 3.5.2 Uncertainty in the Projection Measurements

Projection measurements are spatially-weighted and wavelength-weighted volume integrals of the chemiluminescent emission field (see Equation 2.42). Uncertainty in the inputs to the tomographic reconstruction algorithm arises from two sources: (i) uncertainty in the measured flame images, (ii) uncertainty in the estimated weightings.

#### Flame Images

Uncertainty in the flame images results from image noise, imperfect repeatability of the flame cycle, and motion blur. Uncertainty in the image timing relative to the forcing signal has already been discussed in Section 3.3.2 and is on the order of $10 \, \mu s$. At each phase and for each view, three images were acquired. This was done for redundancy, to check repeatability, and to reduce image noise. Figure 3.5 shows three images acquired from the same view and at the same forcing cycle phase for both Cases studied. The images were taken 1785 cycles (or 36 seconds) apart.
The Signal-to-Noise Ratio (SNR) was measured using the definition used by Floyd et al. (2011),

$$\text{SNR} = \frac{\bar{I}^2}{\sigma_e^2}$$

where $\bar{I}^2$ is a representative mean signal power and $\sigma_e^2$ is the signal variance. To measure the SNR, sample areas of the images were used where the imaged flame surfaces are perpendicular to the imaging axis and the true image intensity is therefore nearly uniform. For Case 1, the SNRs in the three images in Figure 3.5 were 30.2, 31.3, and 36.0. For Case 2, the SNRs in the three images were 115.0, 115.9, and 107.5. These SNRs are conservative as the signal increases in areas where the flame surface is at a lesser angle to the imaging axis.

The images input to the reconstruction algorithm were composite images, constructed using the median intensity of each pixel in three images taken at the same phase. The composite images have an increased SNR when measured in the manner just described, but imperfect repeatability of the flame cycle may limit the resolution. The images in Figure 3.5 demonstrate high repeatability. For both cases, the flame edge was found to move by no more than five pixels at any position. For a moving flame, the spatial resolution is also dependent on the motion of the flame through the camera exposure time. For the two Cases studied, the flame velocity relative to the laboratory does not exceed $3 \text{ ms}^{-1}$ anywhere on the flame surface. The 100 $\mu$s exposure time used for the results presented here limits the movement of a flame surface element travelling at $3 \text{ ms}^{-1}$ to 0.3 mm or 4 pixels.
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Fig. 3.6 RGB composite images comprising 3 monochrome images of a calibration target at different positions to measure perspective distortion. (a) The blue channel is of the target at the focal plane, the red channel is the target 5 mm closer to the camera, the green channel is the target 5 mm further away. (b) A similar composite image with the target shifted 15 mm towards and away from the camera for the red and green channels respectively.

Weightings

To compute the weightings, flame images are treated as parallel projections. Image blur and perspective are the two main sources of error when the parallel projection approximation is used.

Image blur at the near and far edges of the object field is related to the depth of field and can be controlled by the lens aperture. The optical resolving power is a measure of image blur. The resolving power was quantified using the USAF 1951 resolution target, as described in Section 3.3.4. The resolving power for objects at the focal plane was measured to be 0.21 mm. However, the resolving power is reduced away from the focal plane. Flame zones are a maximum of 15.6 mm from the focal plane, occurring when the flame holder is rotated so that the optical axis is aligned with the burner diagonal. The measured optical resolution using the USAF 1951 resolution target was 0.57 mm and 0.45 mm respectively when the target was positioned 15 mm behind, and 15 mm in front of, the focal plane.

Perspective is the change in magnification with distance from the image plane. For a fixed magnification, perspective is governed by the focal length of the lens and does not depend on the lens aperture. To quantify the uncertainty due to perspective, a calibration target with a pattern of crosses was traversed along the imaging axis towards and away from the camera. Figure 3.6a shows a composite RGB image of the target where each colour channel is a monochrome image of the calibration target at a different position. The blue channel is an image of the target at the focal plane; the red channel is the target 5 mm closer to the camera, and the green channel is 5 mm further away. Figure 3.6b shows similar images with the target 15 mm closer to, and 15 mm further from, the camera as
the red and green channels respectively. The change in position of the imaged crosses depends on the distance from the optical axis. Images of the flame occupied the part of the sensor where the perspective distortion 15 mm from the focal plane was less than five pixel lengths, equivalent to a 0.37 mm shift of a point at the focal plane.

The uncertainties described in this section are expected to result in a slightly thickened chemiluminescence region in the reconstructions. This is likely to have little effect on measurements of curvature and velocity, which, as will be described in Chapter 7, are computed using the layer of peak emission, but an increase in the measured thickness of approximately 3 pixel lengths, or 0.2 mm, is expected. However, the aims of this study, which are to estimate the number of views required for certain measurements to converge, and to assess the relative sensitivity of the measured quantities to the number of views, are unlikely to be significantly affected.

### 3.6 Sound Measurements

A free-field, 1/2" microphone (B&K Type 4891 with B&K Type 2671 preamplifier) was used to acquire all sound measurements reported in this work. The microphone and preamplifier combination has an open-circuit sensitivity of 50.5 mV/Pa and the 0° incidence flat frequency-response range is 2.7 Hz to >20 kHz (±3 dB). Due to the low level of the acoustic signals measured in this study, a B&K Type 1704 signal conditioning unit was used to provide 40 dB amplification prior to acquisition using a National Instruments 6040E data acquisition card, sampling at 100 kS/s with 12 bit resolution.

Reported microphone distances ($d_{mic}$) are measured from the tip of the unforced flame. For all sound measurements the microphone distance was between 70 mm and 200 mm. The microphone was directed at the unforced flame tip and was angled slightly upwards at 5° from horizontal, as indicated in Figure 3.7. This was done to increase the distance of the acoustic wave propagating to the microphone from the stabilisation stack.
3.6.1 Acoustic Event Identification

Approximately six cycles of the pressure waveform generated by a forced flame are shown in Figure 3.8. The equivalence ratio is $\phi=0.75$ and the flame is forced at 46 Hz. This forced flame case is denoted Case A3. Further details can be found in Table 4.1. Case A3 is used throughout Sections 3.6.1-3.6.3 to demonstrate the data processing methods used in Chapters 4 and 6. Several distinctive features are observed in the pressure waveform. These include a rapid drop in the acoustic pressure just prior to the minimum acoustic pressure and a smaller rarefaction spike approximately 2 ms later. Event identification algorithms are used to reliably locate identifiable points in these features. An acoustic data point that meets a specific set of criteria is referred to as an ‘event’. Event identification enables quantitative analysis of changes in features in response to changes in the forced flame parameters. Additionally, it enables the selection of representative traces, and the calculation of mean traces, as will be described in Section 3.6.2.

Figure 3.9a shows part of a trace with four events indicated. Event A is a point that:

i) has the maximum negative gradient in the set of points within 80 $\mu$s of a point that has a voltage drop exceeding 0.2 V over the preceding 200 $\mu$s.
The criteria used to identify the other three events are listed below:

- **B** The nearest local minimum following \( t_A \).
- **C** The minimum in the interval from 40 \( \mu \)s after \( t_A \) to 120 \( \mu \)s after \( t_A \).
- **D** A local minimum occurring between 1.5 ms and 2.8 ms after \( t_A \), that has a voltage decrease greater than 0.03 V over the preceding 40 \( \mu \)s, and a voltage increase exceeding 0.04 V over the following 40 \( \mu \)s.

In this example, events **B**, **C**, and **D** include \( t_A \) in their criteria. Not all pressure traces include comparable features and event identification criteria vary depending on the forced flame parameters and the features of interest.

### 3.6.2 Characteristics and Representative Trace Selection

Using identified events, measurements of each feature can be made, as well relationships between features. These measurement are here referred to as ‘characteristics’. Characteristics used in this work include the time or voltage separation between events, and the voltage change over an interval around an event. Five example characteristics are shown in Figure 3.9b and their definitions are given below:

\[
\begin{align*}
\text{c}_1 & : & V(t_B) - V(t_B - 50\mu s) \\
\text{c}_2 & : & t_C - t_A \\
\text{c}_3 & : & t_D - t_A \\
\text{c}_4 & : & V(t_D) - V(t_C) \\
\text{c}_5 & : & V(t_D + 30\mu s) - V(t_D)
\end{align*}
\]

**Variability in Measured Pressure Traces**

Figure 3.10 shows pressure traces from 10 consecutive cycles, aligned by event A and with the mean cycle pressure subtracted. The variability is due to extraneous noise and a small amount of variability in the flame cycle, due mostly to hydrodynamic communication from chaotic turbulent flow downstream of the flame. PDFs of the characteristics shown in Figure 3.9 were calculated using over 750 consecutive traces and are shown in Figure 3.11.
Fig. 3.11 PDFs of the characteristics demonstrated in Figure 3.9b.

**Mean Traces**

Mean traces are calculated by aligning multiple traces by one of the identified events, and finding the mean voltage at every time point in the cycle. Figures 3.12a and 3.12b show mean traces calculated from traces aligned by event A and event D respectively. A representative trace is also shown. The short timescale features in the 0.2 ms interval

Fig. 3.12 Mean traces (gray lines), calculated by averaging traces aligned by: (a) event A; (b) event D. A representative trace is overlaid (black lines).
following event A are well represented in the mean trace calculated from traces aligned by event A (Figure 3.12a). However, the rarefaction pulse at event D is reduced significantly in amplitude relative to its magnitude in the representative trace. This is due to the variability in its position relative to event A, as shown in Figure 3.11 by the distribution of characteristic $c_3$. This variability is a result of small changes in the size and geometry of the pocket when formed. However, when the mean trace is calculated from traces aligned by event D (Figure 3.12b), the rarefaction spike at event D in the mean trace is almost indistinguishable from the rarefaction spike observed in individual traces. In this case, the short time-scale features around event A are not well captured, for the same reason.

**Representative Trace Selection**

Representative traces are selected by finding the trace that returns the smallest value of the cost function:

$$D_i = \sum_k R_k \left( \frac{c_{k,i} - \bar{c}_k}{\bar{c}_k} \right)^r_k + R_m \delta_{m,i}.$$  \hfill (3.6)

The first term on the right hand side is a measure of the distance of trace characteristics $c_k$ of trace $i$ from the median value from all measured traces, denoted by the tilde overbar. $R$ and $r$ are constants that control the weighting of each characteristic. The second term $\delta_{m,i}$ is the rms distance of trace $i$ from a mean trace:

$$\delta_{m,i} = \sqrt{\frac{1}{\Delta t_2 + \Delta t_1} \int^{t_{0,i} + \Delta t_2}_{t_{0,i} - \Delta t_1} (p_i - \bar{p})^2 dt}.$$  \hfill (3.7)

Inclusion of the term $\delta_{m,i}$ avoids selecting a pressure trace that has a significant feature caused by extraneous noise.

**3.6.3 Waveform Variation with Distance**

The spherical spreading law predicts that the amplitude of pressure waveforms decreases with $1/d$. Reflection and diffraction cause deviations from the spherical spreading law and can distort the waveform. For measurements acquired in the near field the acoustic velocity and pressure are generally not in phase. The measured pressure can therefore deviate from the ‘free-field’ pressure due to reflections off the microphone that do not resemble the plane wave reflections that the microphone is calibrated for. Figure 3.13 shows the pressure traces measured at four distances from the flame tip, from 70 mm to 200 mm. All the signals have been scaled by multiplying by the distance from the flame tip. By the spherical spreading law the resulting scaled signals can be considered the predicted waveforms at a distance of 1 m if the propagation were unimpeded. Figure 3.13a shows representative individual traces while Figure 3.13b shows the mean traces using 200 consecutive cycles.
There is very little change in the waveforms with distance from the flame and the spherical spreading law appears to hold. Individual traces are more greatly affected by extraneous noise when measured further from the flame and the increased extraneous noise can be seen in the individual traces in Figure 3.13a. A small decrease in the low frequency contribution to the signal is observed with greater distance. For forcing frequencies on the order of 50 Hz, 200 mm is not a sufficient distance to be considered the far field of the speaker driver. This is particularly important because the speaker driver cannot be considered a monopolar source. The rear of the speaker driver is not contained and the near field is not expected to follow the spherical spreading scaling law. This is demonstrated in Figure 3.14 which shows the speaker tone measured at different distances after the flame has been extinguished. The waveforms have been scaled to give the predicted sound at 1 m using the spherical spreading law. The amplitudes differ significantly, showing that the direct sound produced by the speaker driver decreases faster than $1/d$ in the near field. However, the low-frequency speaker tone does not significantly affect the analysis presented in this work. Cases where the near field of the speaker driver is suspected to influence the reported observations are discussed in Chapters 4 and 5.

### 3.6.4 Acoustic Effect of the Stabilisation Stack

As has been described in Section 3.2.1, an acoustically-lined stack was installed downstream of the flame tip to prevent flame flickering. It is demonstrated here that there are no significant acoustic reflections at the microphone position due to the presence of the stabilisation stack. Sound measurements were obtained with and without the stabilisation stack installed for 24 forced flame cases. Results from one of the investigated cases are presented here. The presented case is a $\phi = 0.75$ flame forced at 46 Hz. The forcing amplitude is reduced relative to the case presented in Sections 3.6.1-3.6.3. A smaller pocket is formed by the separation event and the burn-out event therefore occurs closer to the separation event. This case is denoted Case A2. Further details of the experimental conditions can be found in Table 4.1.

Figure 3.15 shows acoustic waveforms measured at a distance of 100 mm, over 0.5 seconds, both with and without the stabilisation stack. The narrow, downward-extending lines correspond with separation events. A notable difference is a low frequency fluctuation at approximately 6 Hz, observed when the stabilisation stack is not present. This is consistent with the frequencies typically observed for flame flickering. Furthermore, the 6 Hz fluctuation is also present when the flame is not being forced, as shown by the grey, dashed line in Figure 3.15a. The amplitude and frequency of the vortex-induced fluctuation is unaffected by the presence of the forced modulations in the unburned gases. This pressure fluctuation does not radiate as sound and is associated with the formation of
3.6 Sound Measurements

Fig. 3.13 Pressure waveforms measured at four distances from the flame tip. Traces are scaled to the predicted pressure at 1 m distance assuming spherical spreading ($p' = f(r - ct)/r$). $d_{mic} = 70$ mm (solid line); $d_{mic} = 100$ mm (dash-dotted line); $d_{mic} = 150$ mm (dashed line); $d_{mic} = 200$ mm (dotted line). (a) Representative traces; (b) Mean traces, each calculated from over 500 cycles, aligned by event A.

Fig. 3.14 Pressure waveforms produced by the speaker driver in the absence of the flame, measured at four distances from the flame tip. Traces are scaled to the predicted pressure at 1 m distance assuming spherical spreading ($p' = f(r - ct)/r$). $d_{mic} = 70$ mm (solid line); $d_{mic} = 100$ mm (dash-dotted line); $d_{mic} = 150$ mm (dashed line); $d_{mic} = 200$ mm (dotted line).
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Fig. 3.15 Pressure waveforms measured at $d_{mic}=100$ mm when the flame is being forced (black lines) and without forcing (grey dashed lines). The indicated sections of the waveform are magnified in Figure 3.16.

The toroidal vortices in the shear layer. The stabilisation stack removes flame flickering almost entirely (see Figure 3.15b). The elimination of flame flickering is also evident from visual observation of the flame.

Figures 3.16 shows pressure waveforms associated with five consecutive separation events from the waveforms shown in Figure 3.15, with and without the stabilisation stack. Without the stabilisation stack, flame flickering results in significant variations in the size of the separated pockets and in the geometrical parameters of the flame neck during tunnel narrowing. This accounts for most of the variability between the waveforms shown in Figure 3.16a. The rest of the variability is due to extraneous noise from the background acoustic environment and from flow noise. Extraneous noise and flow noise also accounts for most of the small variability between waveforms measured when the stabilisation stack
Fig. 3.16 Magnified pressure waveforms from the labelled sections in Figure 3.15.

(a) Without stabilisation stack.

(b) With stabilisation stack.
Fig. 3.17 Complete cycles of the mean pressure waveforms measured at $d_{mic}=100$ mm.

is present, as shown in Figure 3.16b. A sample of the extraneous noise is included as the grey dashed line in Figure 3.15b.

Figure 3.17 shows mean pressure traces measured with and without the stabilisation stack, computed by aligning over 500 individual traces by the position of the maximum negative gradient. It should be noted that the rarefaction pulse associated with the burn-out event is not resolved in either mean trace due to cycle-to-cycle variability. A difference in the peak amplitude of the rarefaction pulse is the most notable difference between the mean traces. The observed differences can be explained by two factors. One is the greater smoothing of the waveform when the stabilisation stack is not present due to greater cycle-to-cycle variability. The other factor is a lengthening of the flame, and more specifically a lengthening of the flame neck undergoing tunnel narrowing, caused by changes in the mean flow field when the stabilisation stack is installed. This was confirmed by simultaneous high-speed schlieren imaging and sound measurement.

Schlieren images of five consecutive separation events are shown in Figure 3.18 alongside the pressure waveforms measured over a 3.5 ms interval around the separation events, for set-ups with and without the stabilisation stack. A Phantom v1610 camera was used and images were acquired at 50,000 fps with a 9 $\mu$s exposure time. The microphone signal was acquired simultaneously, also at 50 kHz. The light source was a Xenon short-arc lamp (Olympus CLV-U20) equipped with a liquid light guide. 6" spherical mirrors with a
3.6 Sound Measurements

Fig. 3.18 Schlieren images from consecutive separation events alongside pressure waveforms acquired over the 3.5 ms interval around the image exposure time. Pressure measurements were acquired at $d_{mic}=100$ mm.
60° focal length were arranged in a z-type Schlieren arrangement. The objective lens was a 750 mm achromatic doublet. A vertical knife edge was used at the Schlieren cut-off.

Noise emitted by the light source affected the sound measurements. Nevertheless, the acoustic signatures of the separation and burn-out events are generally observed. Without the stabilisation stack there is significant cycle-to-cycle variation in the geometrical parameters of pocket separation events. Separation events with shorter flame necks are observed to generate acoustic rarefactions with smaller amplitudes than separation events with longer flame necks. When the stabilisation stack is installed, the flame neck during the separation event is typically longer. This explains the greater amplitude in the mean pressure trace. Importantly, there is no indication of acoustic reflections from the stabilisation tube. Over 20 other flame cases were similarly investigated. Equivalence ratios from $\phi=0.70$ to $\phi=0.90$ and a range of forcing frequencies and amplitudes were included. Acoustic reflections from the stabilisation stack were not observed in any of the cases.

3.7 Summary

The design of a forced, laminar, premixed flame rig has been described. Two interchangeable flame holders are used. An axisymmetric flame holder is used for investigating the sound generated by forced, laminar, premixed flames. A non-axisymmetric square-port flame holder is used for investigating the capabilities of flame chemiluminescence tomography.

The tomographic reconstruction methods have been described. The resolving power of the imaging arrangements was measured using a 1951 USAF resolution target. The resolving power was 210 $\mu$m at the focal plane for images used in the non-axisymmetric tomographic reconstructions. The resolving power at points coincident with the furthest and closest positions of the flame from the camera are 0.57 mm and 0.45 mm. The perspective distortion was measured to have a maximum magnitude of 0.37 mm, though it was significantly less for most positions through the flame region. The combination of these effects, plus the movement of the flame during the image exposure and imperfect repeatability of the flame cycle, is expected to produce a thickened chemiluminescence region in tomographic reconstructions.

The arrangement for sound measurements has also been described. The monopole nature of the forced flame as a source of sound was confirmed. The shape of the acoustic pressure waveform generated by the flame was unchanged with distance ($R$) from 70 mm to 200 mm and the amplitude decayed with $1/R$. The effect of including an acoustically-lined stabilisation stack on the sound generated by a forced flame was investigated. Though some differences were observed in the mean acoustic waveform emitted with and without the
3.7 Summary

stabilisation stack, high speed Schlieren imaging and simultaneous sound measurements showed that the differences are attributable to increased repeatability of the flame cycle due to reduced flame flickering when the stack is present, and to a lengthening effect on the flame. No acoustic reflections were observed.
Chapter 4

Measurements of the Sound Generated by Unsteady Laminar Flames

4.1 Introduction

This chapter presents an experimental study of the sound generated leading up to, during, and after the separation of reactant pockets. These processes include tunnel narrowing, cusp recovery, and pocket consumption. To study these processes, chemiluminescence images and sound measurements were acquired from experiments on forced, laminar, premixed, propane-air flames with different forcing amplitudes, frequencies, and equivalence ratios. The results are organised in two sections. In Section 4.3, image sequences and sound measurements for complete forcing cycles are presented. The effects of forcing amplitude, frequency, and equivalence ratio on the flame cycle are demonstrated. The significance of the flame dynamics that occur near the tip of the flame to the emitted pressure waveform is confirmed by demonstrating changes in the pressure waveform in response to small differences in the geometric parameters of the pocket separation process.

In Section 4.4, chemiluminescence images are related to the part of the pressure waveform generated during the image exposure time to identify the combustion processes responsible for the significant acoustic features. A temporal resolution of 20 µs is achieved. The contribution of tunnel narrowing, cusp recovery, and pocket consumption processes to the measured acoustic waveforms is determined by comparing the acoustic waveforms emitted by flame cycles with separated pockets of different sizes. From this data it is determined whether the flame processes contribute to an acoustic rarefaction or compression, the duration of the generated acoustic disturbance, and the shape of the acoustic waveform. The acoustic signatures of flame annihilation events are also identified.
Measurements of the Sound Generated by Unsteady Laminar Flames

Table 4.1 Experiment Conditions

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<th>Case</th>
<th>( \phi )</th>
<th>( \omega ) (s(^{-1}))</th>
<th>St</th>
<th>( \bar{u} ) (ms(^{-1}))</th>
<th>( I_{rms} ) (mA)</th>
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4.2 Experiment Conditions

The experimental rig has been described in Section 3.2. For the results presented in this chapter, the circular-port flame holder was used. Different flame annihilation geometries were produced by changing the forcing Strouhal number (St) and the forcing amplitude. Forcing amplitude was controlled by the speaker current \( I_{rms} \). Flame length was kept constant at 40 mm in this study and the Strouhal number was controlled by adjusting the forcing frequency. Two Strouhal numbers, St=13 and St=20, were used. The Strouhal number is defined as \( St = \omega R / S_L \cos \theta_0 \) (Karimi et al., 2009; Schuller et al., 2003). Here, \( \omega \) is the angular forcing frequency, \( R \) is the burner radius, \( S_L \) is the laminar flame speed, and \( \theta_0 \) is the flame half apex angle for the unforced flame. This Strouhal number has been used extensively in studies of the transfer function between heat release rate fluctuations and upstream velocity fluctuations in axisymmetric, forced flames (Ducruix et al., 2000; Karimi et al., 2009; Lieuwen, 2003, 2005, 2012; Schuller et al., 2003). Equivalence ratios in the range \( \phi = 0.73 \) to \( \phi = 0.82 \) were used.

Three sets of experiments were conducted. The experiment conditions are summarised in Table 4.1. For Set A, the forcing amplitude was varied while Strouhal number and equivalence ratio were kept constant. The three Cases in Set A include a forced flame with no separated pocket, a small separated pocket, and a large separated pocket. Set B comprises flames with forcing amplitudes between the first and second Case in Set A. Set B demonstrates the sensitivity of the emitted sound to small changes in the flame tip behaviour. For Set C, the Strouhal number was kept constant as the equivalence ratio was
4.3 Sound Measurement and Imaging of Complete Cycles

Methods

The microphone was located 100 mm from the unforced flame tip position. The data acquisition system is described in Section 3.6. Sound measurements were acquired with the camera turned off, so as to avoid sound contamination from the camera fan. The microphone signal was acquired using a National Instruments NI-6040E 12 bit data acquisition card, sampling at 100 kHz.

Images of the entire flame, through a complete cycle, were acquired using the optical arrangement and image timing unit described in Sections 3.3.1 and 3.3.2. The 60 mm f/2.8D lens was used with the aperture fully open. Images were taken at equally-spaced intervals through the forcing signal cycle. A gate width of 200 µs was used.

Results and Discussion

Set A

Figure 4.1 shows chemiluminescence images from the forced flames in Set A. Figure 4.2 shows the corresponding pressure fluctuations measured 100 mm from the flame tip. Representative traces and mean traces are shown. The methods for the representative trace selection and mean trace computation are described in Section 3.6.2. For the computation of the mean trace for Cases A2 and A3, pressure traces were aligned by the point of maximum $\partial p'/\partial t$ in the rapid drop in acoustic pressure that will be shown in Section 4.4 to be associated with pocket separation. For the Case A1 mean trace, pressure traces were aligned by the pressure minimum. The time given on the x-axis of Figure 4.2 is relative to these identified events.

At low forcing amplitudes (Case A1), the flame height and the curvature of the flame tip varies considerably through the cycle, though there is no separation event. The pressure rarefaction for Case A1 is generated by the flame tip behaviour around the moment when the flame tip curvature is highest. At comparatively high forcing amplitudes (Case A3), pocket separation and pocket burn-out events are observed. The pressure waveform shows short time-scale features, on the order of 10 µs. It will be shown in Section 4.4 that the generation of the rapid drop in acoustic pressure coincides with the separation event, while varied. The forcing amplitude was adjusted for each case in Set C to produce separated reactant pockets of similar size.
Fig. 4.1 Equally spaced images through the 46 Hz forcing cycles for the flames in experiment Set A. Velocity forcing ($u'/\overline{u}$) increases from A1 to A3. Image intensifier gate width is 200 µs.

the generation of the smaller rarefaction spike at approximately $t = 2$ ms coincides with the pocket burn-out event. These features are labelled on the acoustic waveform for Case A3 in Figure 4.2. An approximately sinusoidal component at the speaker forcing frequency also becomes significant at higher forcing amplitudes. At intermediate forcing amplitudes (Case A2), the reactant pocket is smaller and the pocket burn-out event therefore occurs much sooner after the separation event. However, the gradient of the waveform associated with the tunnel narrowing process ($t = -2$ ms to $t = 0.5$ ms) is almost identical to that of the more highly forced flame (Case A3). The acoustic waveforms generated by the separation events occurring in Cases A2 and A3 are also very similar. The pressure change over the 50 µs interval prior to the local minimum that follows the rapid drop is -49.73 mPa and -49.57 mPa respectively for Cases A2 and A3 (Figures 4.1b and 4.1c). These values are mean values obtained from measurements on over 500 pressure traces for both forcing cycles.

Set B

To investigate the significance of flame annihilation, the lowest forcing amplitude that unambiguously produces a separated reactant pocket was identified as Case B3. The
forcing amplitude was varied in small increments about this point and sound measurements and chemiluminescence images obtained. Chemiluminescence images of the flame cycles are shown in Figure 4.3. The forcing amplitude increases through Case B1 to B5. The image sequences of the flame cycles look almost identical, though small differences in the flame shape are observed near the flame tip at 240° phase. The measured pressure fluctuations are shown in Figure 4.4. In all cases the largest magnitude pressure fluctuations occurred in a small part of the cycle close to where fast tip propagation was observed. Significant changes in the generated sound in response to small increases in the forcing amplitude show that sound generation is dominated by processes occurring near the flame tip. The measured pressure waveform for Case B3 features a sharp rarefaction pulse. A further increase in the forcing amplitude decreases the peak amplitude of this pulse (Case B4). Developments in the waveform in response to further increases in the forcing amplitude include:

(i) An increase in the rarefaction magnitude following the separation event.
Fig. 4.3 Equally spaced images through the 46 Hz forcing cycles for the flames in experiment Set B. Velocity forcing ($u'/\bar{u}$) increases from B1 to B5. Image intensifier gate width is 200 $\mu s$. 
Fig. 4.4 Representative (black line) and mean (grey line) pressure waveforms for the flames in experiment Set B, measured 100 mm from the tip of the unforced flame tip position.
(ii) A second local minimum corresponding with the pocket burn-out event. This becomes distinguishable when the pocket burn-out occurs more than approximately 100 µs after the separation event.

Further increases in the forcing amplitude produce Cases A2 and A3.

No separation event is observed in Case B2. However, an event resembling cylindrical flame annihilation occurs at the very tip of the flame. At lower forcing amplitudes (Case B1), there is no point in the cycle where parallel flame surfaces annihilate. However, very high curvature is produced at the flame tip and the flame length decreases by several millimeters over a short duration on the order of 1 ms. The cusp propagation is associated with significant sound production, as demonstrated by the corresponding waveform for Case B1. By comparing Cases B1 and B2, the occurrence of flame annihilation is seen to be accompanied by an additional rarefaction spike of duration on the order of 200 µs. It will be shown in the subsequent section that much of this sound is generated during the cusp recovery that follows the separation event.

Set C

Figure 4.5 shows image sequences for the forced flames in Set C. In this set the equivalence ratio varies but the Strouhal number is kept constant at St=20, 50% greater than for sets A and B. The wavelength of the transverse wave in the flame front decreases with increased Strouhal number and the pinched-off pocket is smaller than for similarly forced flames at lower Strouhal numbers, such as Case A3. The forcing amplitude was adjusted for each case so that the pocket size at the moment of separation was similar. As a consequence, the flame geometries are also similar at other phases of the forcing cycle. However, the forcing amplitudes required to produce similar pockets for each case typically resulted in some differences in amplitude of the transverse flame disturbances, as can be seen by the increased prominence of the circumferential flame cusps with increasing equivalence ratio. Unsurprisingly, an increase in chemiluminescence intensity with increasing equivalence ratio was also observed. This is illustrated by the increasing image contrast.

The corresponding pressure fluctuations at 100 mm are shown in Figure 4.6. The waveforms associated with the tunnel narrowing, cusp recovery, and pocket consumption processes are similar for the investigated range of equivalence ratio. This suggests that the magnitude of the pressure fluctuation caused by these processes has a weak dependence on this parameter. The pressure change over the 50 µs preceding the local minimum at the bottom of the rapid drop increases monotonically but slowly with $\phi$, from -34.6 mPa at $\phi = 0.73$ to -38.3 mPa at $\phi = 0.82$. The dependence on equivalence ratio will be investigated in greater detail in the next chapter. For the $\phi = 0.75$ flame (Case C2), the magnitude of the pressure rarefaction is seen to decrease with increased Strouhal number.
4.3 Sound Measurement and Imaging of Complete Cycles

Fig. 4.5 Equally spaced images through the forcing cycles for the flames in experiment Set C. The equivalence ratio increases from C1 to C5. Image intensifier gate width is 200 µs.
Fig. 4.6 Representative (black line) and mean (grey line) pressure waveforms for the flames in experiment Set C, measured 100 mm from the tip of the unforced flame tip position.
At St=20, the pressure change over the 50 µs preceding the local minimum at the bottom of the rapid drop is -36.59 mPa, 13 mPa less than for the separation event in the St=13 flame cycle (Cases A2 and A3). From these results, it is clear that the magnitude of the pressure fluctuation produced by the tunnel narrowing process and separation event has a greater dependence on the annihilation geometry than equivalence ratio in the range studied. It should also be noted that the acoustic energy associated with the measured fluctuations also depends on the duration of fluctuations, which decreases with increased flame speed.

In the pressure waveforms from Set C, an approximately sinusoidal component at the forcing frequency is apparent. The main source of this low frequency component is direct sound from the speaker driver. Variable flame surface creation at the base of the flame and development of the circumferential flame cusps are also likely to have a small contribution.

### 4.4 Temporally Correlating Flame Images and the Generation of Sound

In this section, high spatial resolution images of tunnel narrowing, cusp recovery, and pocket consumption are presented. The region where these processes occur is referred to as the annihilation region. Cases B2, A2, and A3 are studied in detail. Cases A2 and A3 feature separated pockets of different size. The separation events produce two cusps and the measured pressure trace is a superposition of the sound produced by the recovery of both cusps. In contrast, Case B2 features a single cusp and no separated pocket. The development of the single cusp will be compared to that of a lower cusp formed by the separation events in Cases A2 and A3. The recovery of the upper cusp initially contributes to consumption of the pocket. However, the pocket will be shown to terminate in a near-spherical annihilation event.

#### Methods

A Sigma 150 mm f/2.8 lens with the aperture fully open was used to image the annihilation region over the time interval containing the annihilation events. A gate width of 20 µs was used and images were taken at a resolution of 28 µm/pixel. The optical resolution is 76 µm and was determined using a USAF 1951 target, as described in Section 3.3.4.

During image acquisition, the sound produced by the flame was measured to enable the image timing to be accurately determined relative to an event in the acoustic signal. The microphone was positioned 70 mm from the unforced flame tip. This distance maximised the amplitude of the sound signal relative to extraneous noise such as the camera fan noise while maintaining a low risk of radiated heat affecting the microphone diaphragm.
TTL signal from the image intensifier, indicating when the intensifier gate was open, was simultaneously acquired with the sound measurements, both at 100 kHz. This enabled the image timing relative to an event on the acoustic signal to be determined to within $\pm 10 \, \mu s$ in post-processing.

Case A3 is used to demonstrate the method for identifying the temporal position of the flame images on the measured pressure traces. Figure 4.7(i) shows images of the annihilation region. Figure 4.7(ii) shows the temporal position of the camera exposures on the unique pressure traces that were measured while acquiring each image. Upon close inspection, there is noticeable variation in each individual pressure trace due to extraneous noise that is caused predominantly by the camera fan. Nevertheless, several distinctive features can still be reliably identified. To relate the flame images to the sound produced at the time the image is taken, the propagation delay must be taken into account.

The propagation delay for sound originating from the flame tip, and travelling 70 mm to the microphone at $340 \, \text{ms}^{-1}$, would be $205 \, \mu s$. However, sound travels at a higher speed through the plume of hotter burned gases and so a decrease in the propagation delay is to be expected. Schlieren imaging showed the radius of the plume to vary by less than 1 mm through forcing cycles, and that it is between 23 mm to 27 mm depending on the equivalence ratio and unburned gas velocity. Assuming the unburned gas to be at the adiabatic flame temperature for propane-air flames at $\phi = 0.90$ ($T_{ad,\phi=0.9} = 2220 K$ (Law, 2006)) and a plume radius of 27 mm, the minimum plausible propagation delay to the microphone would be $150 \, \mu s$, where $\gamma R$ has been assumed constant. The plume temperature will be somewhat cooler than $T_{ad}$, and the propagation delay can be confidently estimated to be between $150 \, \mu s$ and $205 \, \mu s$ at all equivalence ratios. In the following sections, a constant propagation delay of $180 \, \mu s$ is assumed. The systematic uncertainty in the propagation delay is therefore in the range $\pm 20 \mu s$. The random uncertainty in the timing of the images is $\pm 10 \mu s$.

The maximum gradient in the rapid drop follows the separation reference time (Figure 4.7(iib)) by $170 \, \mu s$, measured from the end of the camera exposure. The minimum on the second rarefaction spike follows the pocket burn-out event (Figure 4.7(iif)) by $200 \, \mu s$. The same time delays were observed for all forced flames that featured pocket separation and burn-out events with a precision of $\pm 10 \, \mu s$, irrespective of Strouhal number, forcing amplitude, and equivalence ratio.

There are several reliably identifiable events in the measured pressure traces. The temporal distance of a camera exposure from one of these events can be measured and the exposure times can therefore be plotted on a pressure trace acquired after the camera fans have been turned off, or on mean traces, measured at different distances from the flame tip. In Figure 4.7(iii), the camera exposures associated with all the images in Figure 4.7(i), adjusted for the propagation delay, are shown on a representative trace acquired at a
4.4 Temporally Correlating Flame Images and the Generation of Sound

(i) 20 µs exposures of the annihilation region at 0.5 ms intervals.

(ii) Pressure signals measured 70 mm from the flame tip during acquisition of the images in (i). The position and length of the camera exposures are indicated by the grey lines.

(iii) Representative pressure trace at a distance of 100 mm, measured with camera fans off. The position of the camera exposure times for the images in (i), relative to the maximum gradient in the rapid drop in $p'$, and corrected for the propagation delay, are indicated by the grey lines. The width of the grey lines represents the 20 µs image exposure.

Fig. 4.7 Method for temporally locating images relative to acoustic events. Results are from Case A3.
Measurements of the Sound Generated by Unsteady Laminar Flames

distance of 100 mm. In this case, the temporal distance between the start of the camera exposure and the maximum gradient in the rapid drop in \( p' \) was used to locate the image exposures.

Results and Discussion

Figures 4.8 and 4.9 show the sound generated during tunnel narrowing, cusp recovery, and pocket consumption in Cases A3 and B2 respectively. Below each image, the temporal location of the start of the image exposure, relative to the rapid drop in \( p' \), is given. Dot markers at these times are also included on the representative pressure traces. The propagation delay has been taken into account so that each image shows the flame at the moment the sound at the indicated position on the acoustic waveform is being generated.

Tunnel Narrowing

The top row of images in Figures 4.8 and 4.9 shows the annihilation region of Cases A3 and B2 over the 2 ms prior to the axisymmetric annihilation event. Due to the similarity of the flame development between the two cases over this time interval, both processes are referred to as tunnel narrowing processes, despite there being no reactant pocket separation in Case B2. However the more general term ‘axisymmetric annihilation event’ is used instead of ‘separation event’ for the terminal moment. The decreasing acoustic pressure over this interval is directly attributable to the tunnel narrowing process. In Chapter 6 it will be shown that this decrease is due to negative flame stretch throughout the annihilation region.

Figure 4.10 compares the representative pressure waveforms produced by Cases B2, A2, and A3. In Figure 4.10 the pressure traces have been vertically aligned so that \( p' = 0 \) at \( t = 2.5 \) ms. By this time, cusp recovery has effectively ended and the flame tip is approximately steady. No sound production is therefore expected from the annihilation region at this time. However, it is important to recognise that an imbalance between flame surface creation at the base of the flame and flame surface reduction in the rest of the flame generally results in a non-zero acoustic pressure at this instant. Both flame surface creation at the base of the flame and flame surface reduction in the flame outside the annihilation region is expected to vary smoothly over the cycle period of 21 ms. The absence of a significant waveform component at the forcing frequency indicates that the variations are small. The processes of interest occur in a comparatively short 3-5 ms interval. Offsets in the pressure traces in Figure 4.10 therefore represent differences in the rarefaction contributed by the tunnel narrowing processes for each case.

During tunnel narrowing a relatively constant difference is observed in the pressure waveforms. This is more clearly shown by the mean traces in Figure 4.11a, that have also
4.4 Temporally Correlating Flame Images and the Generation of Sound

(a) A section of a representative pressure trace acquired at $d_{mic} = 100$ mm. The position of the start of the exposure times for the images in (b), with the propagation delay added, are indicated by the dot markers.

Tunnel Narrowing:
-2.05 ms -1.65 ms -1.25 ms -0.85 ms -0.45 ms -0.05 ms

Separation:
-30 µs -10 µs +10 µs +30 µs +50 µs +70 µs

Cusp Recovery & Pocket Consumption:
+0.1 ms +0.32 ms +0.54 ms +0.76 ms +0.98 ms +1.20 ms

Pocket Consumption & Burnout:
+1.45 ms +1.55 ms +1.65 ms +1.75 ms +1.85 ms +1.95 ms

(b) Times beneath each image correspond to the position on the waveform in (a) generated by the flame at the start of the image exposure.

Fig. 4.8 Flame images and the generated sound for Case A3.
(a) A section of a representative pressure trace acquired at $d_{mic} = 100$ mm. The position of the start of the exposure times for the images in (b), with the propagation delay added, are indicated by the dot markers.

Tunnel Narrowing:

-2.05 ms  -1.65 ms  -1.25 ms  -0.85 ms  -0.45 ms  -0.05 ms

Annihilation:

-30 µs  -10 µs  +10 µs  +30 µs  +50 µs  +70 µs

Cusp Recovery:

+0.1 ms  +0.28 ms  +0.46 ms  +0.64 ms  +0.82 ms  +1 ms

(b) Times beneath each image correspond to the position on the waveform in (a) generated by the flame at the start of the image exposure.

Fig. 4.9 Flame images and the generated sound for Case B2.
been vertically aligned to $p' = 0$ at $t = 2.5$ ms. The rarefaction increases with increased forcing amplitude. However, the pressure gradient is similar. This is clearly demonstrated by vertically aligning the traces at $t = -2$ ms, as in Figure 4.11b. Also shown in Figure 4.10 are images of the annihilation region for each case at $t = -1.8$ ms and $t = -0.50$ ms. At both instants significant differences in the flame shape are observed but the narrowest part of the flame neck has a similar diameter, as would be expected. The similarity in the generated sound suggest a common feature is responsible for the majority of the sound production during tunnel narrowing.

**Cusp Recovery**

The axisymmetric annihilation events, shown in the second row of images in Figures 4.8 and 4.9, coincide with the rapid drops in $p'$. However by $t = 10$ µs, cusp recovery is already under way. The rest of the waveform is produced during the cusp recovery process and, for Cases A2 and A3, the consumption of the separated pocket. This conclusion is easily misunderstood and should be interpreted carefully.

In this experimental study, the flame location is taken to be the set of points at the peak of the chemiluminescent emission profile along a flame surface normal, starting in the reactants and ending in the products. Based on this definition, the images show that 10 µs after the annihilation the flame is no longer present in the region where the annihilation occurred. By our definition, the cusp recovery process has begun. However, reactions contributing to a non-zero heat release rate at the location of separation may continue to occur more than 10 µs after the position of peak heat release rate reaches the symmetry axis. The radial heat release rate distribution would have a stationary peak at the flame symmetry axis but the profile would decay in magnitude. This is supported by the observation of continued measurable chemiluminescent emission from the region where the cylindrical annihilation took place up to 200 µs after the annihilation. A similar decay is also observed in images taken after the pocket burn-out event in Case A3 as well as in the pressure waveform associated with it.

The recovery process of the lower cusp is similar for the three cases. This is demonstrated in Figure 4.12 which shows the vertical position of the flame tip and cusps over a 5 ms interval containing the annihilation events. Cusp positions were identified by the maximum gradient in the vertical chemiluminescence profile, generated by summing pixel rows. The appearance of two cusps at $t = 0$ ms follows the separation event for Cases A2 and A3. In all cases, the lower cusp initially propagates at speeds exceeding 8 ms$^{-1}$ but decelerates quickly, travelling 5-7 mm in 2 ms. 2 ms after the separation event, the flame tip is travelling at approximately 0.9 ms$^{-1}$, 0.7 ms$^{-1}$ and 0.4 ms$^{-1}$ for Cases B2, A2 and A3 respectively.
Measurements of the Sound Generated by Unsteady Laminar Flames

(a) Representative pressure traces vertically aligned at $t = 2.5$ ms.

(b) Chemiluminescence images at selected instants.

Fig. 4.10 A comparison of Cases B2, A2, and A3.
Fig. 4.11 A comparison of mean traces for Cases B2, A2, and A3. The traces have been vertically aligned at: (a) $t = +2.5$ ms, (b) $t = -2$ ms, and (c) $t = +2.5$ ms. The mean traces in (a) and (b) were computed by aligning traces at the cylindrical annihilation event. The mean traces for Cases A2 and A3 in (c) were computed by aligning traces at the burn-out event.

**Pocket Consumption and Burn-Out**

Given the similarity in the lower cusp recovery, it is likely that pocket consumption is largely responsible for the differences observed in the pressure waveforms shown in Figure 4.10 after the axisymmetric annihilation event. The similarity between Cases A2 and A3 over the 0.4 ms interval following the separation event shows sound production by the consumption of the reactant pocket over this time interval to be almost identical. This is despite the difference in size and shape of the reactant pocket, as shown in Figure 4.10. Both pockets feature a flame cusp and the similarity in the waveforms over the 0.4 ms interval following the separation event, as well as the similarity in the images of the upper flame cusp, suggests the upper cusp recovery processes are very similar in both cases. The
Fig. 4.12 Vertical position of the flame tip and the flame cusps formed by the separation event. The lower cusp position at \( t = 2 \) ms is used as the reference position for all three cases.

The similarity in the acoustic waveform also implies that the top half of the pocket, which is approximately spherical, generates the same acoustic pressure regardless of size.

As predicted by the analysis of the lower flame cusp, the pressure waveform for Case A2 after the pocket burn-out event closely follows that of Case B2. There is an almost constant pressure difference between Case B2 and Case A3 from \( t = 0.5 \) ms until the burn out event. This is further evidence that the reactant pocket consumption generates an almost constant rarefaction until the terminal moments. The flame annihilation in the terminal moments, termed the pocket burn-out event, produces a small but distinctive decrease in pressure followed by a larger increase.

### 4.5 Summary

The acoustic waveforms emitted by forced, conical, laminar, lean, premixed, propane-air flames have been measured and chemiluminescence images were used to determine the contribution of tunnel narrowing, cusp recovery, and pocket consumption processes to the emitted sound. The effects of equivalence ratio, Strouhal number and forcing amplitude were investigated. Relatively small changes in the geometry of the separation event and separated pocket were observed to produce large changes in the peak magnitude of the emitted acoustic waveform. The equivalence ratio was found to have little effect on the peak acoustic magnitude, though the investigated range was small. For equivalence ratios between 0.73 and 0.82, a high degree of similarity was observed between the emitted acoustic waveforms when the flame geometry at the separation event was kept constant.

A detailed study of a \( \phi = 0.75 \) flame, forced with a Strouhal number of 13 and at a range of forcing amplitudes, was undertaken to investigate the effect of changes in the geometrical
parameters of the flame processes on the emitted sound. This flame was chosen because 
the combination of a relatively low flame speed and a low forcing frequency enabled the 
generation of large flame disturbances without significant sound from the speaker driver.

Tunnel narrowing was observed to generate an acoustic rarefaction. The amplitude of 
the rarefaction is approximately steady 2 ms prior to the separation event. As the separation 
event is approached, the amplitude of the rarefaction increases, as does the rate of increase 
\( \frac{d|p'|}{dt} \). The amplitude of the rarefaction increased with forcing amplitude. However, in 
the 2 ms interval prior to the separation event, \( \frac{dp'}{dt} \) was similar regardless of the forcing 
amplitude. An increased forcing amplitude increases the length of the flame neck involved 
in the tunnel narrowing process and it will be shown in the subsequent chapter that this is 
the cause of the increased rarefaction magnitude.

Cusp recovery and pocket consumption are both typically initiated by a separation 
event and were also observed to generate acoustic rarefactions. The rarefaction caused 
by cusp recovery decreased in amplitude at a generally decreasing rate over the 1-2 ms 
following the separation event. Pocket consumption contributed an additional rarefaction 
that remained approximately constant until the burn-out event. Small but distinctive 
increases in the rarefaction magnitude were observed at pocket burn-out, followed by a 
rapid decrease in the magnitude of the rarefaction.
Chapter 5

The Prediction of Combustion Noise Using Visible Chemiluminescence

5.1 Introduction

In this chapter, visible chemiluminescence is used as a quantitative marker for the volumetric heat release rate. The sound generated during tunnel narrowing, cusp recovery, and pocket consumption is then predicted using the proportional relationship between the far field acoustic pressure and the time derivative of the heat release rate. The predicted sound is compared to the measured sound and instances are identified where the correlation is poor.

This chapter considers the forced flame denoted Case A3 in the previous chapter. Case A3 features a separated pocket that is approximately 5 mm in length and 2.5 mm in diameter at its widest point. Approximately 2000 images were taken in a time interval of approximately 5 ms duration that contains the tunnel narrowing, pocket consumption, and cusp recovery processes. The image timing unit was programmed to step through the interval in increments between 4 \( \mu \text{s} \) and 24 \( \mu \text{s} \) and multiple images were taken at each time step. A greater density of images was acquired closer to the annihilation events where the chemiluminescence signal is observed to change rapidly.
5.2 Calibrating Chemiluminesce as a Marker of Heat Release Rate

To determine the far field acoustic pressure from the recorded chemiluminescence images, it is assumed that the volumetric chemiluminescent emission over the camera exposure, is proportional to the volumetric heat release rate \( Q \). Given that the total number of pixel counts \( N \) is linearly proportional to the chemiluminescent emission from any point in space within the field of view, the number of pixel counts is proportional to \( Q \). If the subject dimensions are small relative to the distance to the lens, as is the case in this study, the proportionality constant will be approximately uniform throughout the subject region. Substituting for \( \int_V Q dV \), Equation 2.34 becomes:

\[
p'(x,t) = \left( \frac{\gamma - 1}{\gamma} \right) \frac{ \rho_0 k_p}{4 \pi r p_0} \left[ \frac{dN}{dt} \right],
\]

where \( k_p \) is the proportionality constant between the number of pixel counts \( N \) and the heat release rate from the imaged region \( (\int_V Q dV) \). To determine \( k_p \), two images of the steady flame were obtained with different magnifications, as shown in Figure 5.1.

For the image on the right, the same image acquisition parameters (camera position, II settings, lens, etc.) are used as for the high resolution imaging in the previous chapter, and for the imaging used later in this chapter to estimate the sound generated by the flame in the annihilation region. The image on the left captures the entire flame. The total heat...
5.2 Calibrating Chemiluminescence as a Marker of Heat Release Rate

The heat release rate from the flame is approximately:

\[ \int_{V_1} QdV = m_f LHV, \]  

where \( V_1 \) is the region containing the entire flame, \( m_f \) is the mass flow rate of propane, and \( LHV \) is the lower heating value. A section of both images is selected that corresponds to the same imaged subregion \( V_2 \). The heat release rate from this region can be estimated from the image of the entire flame as:

\[ \int_{V_2} QdV = \frac{N_{L,V2}}{N_{L,V1}} \int_{V_1} QdV \]  

where \( N_{L,V2} \) denotes the number of counts from the area of the lower magnification image of the entire flame, that captures chemiluminescent emission from the region \( V_i \). \( k_p \) is then:

\[ k_p = \frac{\int_{V_2} QdV}{N_{H,V2}}, \]  

where \( N_{H,V2} \) denotes the number of counts from the area of the higher magnification image that contains the image of the subregion \( V_2 \).

For lean propane-air flames, the chemical equation is:

\[ C_3H_8 + \frac{5}{\phi} (O_2 + 3.76N_2) \rightarrow 3CO_2 + 4H_2O + 5(\frac{1}{\phi} - 1)O_2 + \frac{18.8}{\phi} N_2 \]

where \( \phi \) is the equivalence ratio. If \( T_{ad} < 2000\text{K} \), CO\(_2\) dissociation can be ignored. The LHV is equal in magnitude to the heat of reaction.

\[ \text{LHV} = -\Delta h^\circ = -\frac{\Delta H^\circ}{M_{C_3H_8}} = -(3\Delta f H^\circ_{CO_2} + 4\Delta f H^\circ_{H_2O} - \Delta f H^\circ_{C_3H_8}) \]

\[ = 46.33 \text{ kJ/g} \]

Figure 5.2 shows \( k_p \) as a function of equivalence ratio, keeping all imaging system parameters constant. Non-monotonic behaviour is observed. Both the visible chemiluminescent emission intensity and heat release rate from a steady planar lean flame increase with increasing equivalence ratio and the non-monotonic behaviour of their ratio is not particularly surprising given the complexity of the chemical kinetics underlying chemiluminescence.
Fig. 5.2 The proportionality constant between flame heat release rate and pixel counts as a function of equivalence ratio. Parameters of the imaging system were kept constant.

Fig. 5.3 Spatially integrated chemiluminescent emission during the annihilation events, as determined from summing the pixel counts from an image. Each point corresponds to a single image, and is shifted by the propagation delay. For (a), the same area of the image sensor is used for each image. For (b), the image sensor area is shifted for each image to account for flame height variations.

5.3 Estimating the Time Derivative of the Chemiluminescent Emission

Figure 5.3a shows the chemiluminescent emission from the annihilation region of the flame during the annihilation event. Each point represents the summed pixel counts over a flame image. 1200 images were obtained in the 5 ms interval around the annihilation events. As stated in Equation 5.1, the derivative of the chemiluminescent emission is proportional to the pressure fluctuation in the far field. However, due to the scatter in the points and multiple points at some time steps, the derivative at any instant is not accurately determined by simple finite difference methods.
5.3 Estimating the Time Derivative of the Chemiluminescent Emission

The scatter is primarily due to changes in the flame geometry from cycle to cycle. By adjusting the image selection area, flame heights in the images were adjusted to follow the fitted curves shown in Figure 5.4. Height adjustments were determined using the flame tip up to 50 µs after the separation reference time, after which the lower cusp position was used.

Figure 5.3b shows the same data as in Figure 5.3a, but with height adjustments and temporal filtering applied to individual pixel rows where the summed chemiluminescence varies slowly with time. The scatter is greatly reduced. The separation reference time is noticeable as an apparent discontinuity in the gradient at $t = 0$ ms, and was shown in the previous chapter to correspond to a rapid drop in the acoustic pressure. Over the following 2 ms the chemiluminescence signal approaches a constant value.

Adaptive degree polynomial filtering (ADPF) was used to estimate the time derivative of the chemiluminescence signal at each instant. Gaussian weighting with a standard deviation of 80 µs was applied and the polynomial degree was constrained to be 3 or less. Starting with a linear fit, the degree of the polynomial is increased if the higher degree polynomial yields a significantly improved fit, as determined by applying an F-test (Barak, 1995). The test-statistic is a function of the weighted sum of squared residuals from both fits, the polynomial degree of the fitted polynomials, and the number of data points used in the fit. The test statistic is compared to a critical F-value at a probability level of 1%. That is, the critical F-value would only be exceeded by the test statistic 1% of the time if the actual function underlying the noisy data was a polynomial of lower degree. The derivative of the signal was computed directly from the coefficients of the fitted polynomial.
5.4 Results and Discussion

The derived pressure trace is shown in Figure 5.5. The measured mean trace is also shown. A mean pressure trace is shown for comparison instead of a representative trace because the derived pressure trace uses images taken from hundreds of cycles. The image timing is determined relative to the same event used to align traces for calculating the mean trace, i.e., the separation reference time. Many of the important features resolved in the mean trace are resolved in the derived trace, including the rapid drop in $p'$ at the separation reference time and the subsequent more gradual return to the mean pressure. The smaller rarefaction associated with the pocket-burn out event at $t = 2$ ms is resolved in the mean measured trace though it is smoothed compared to the rarefaction pulse observed in the individual traces. A similar feature is also observed in the derived trace.

However there are several significant differences between the predicted and measured sound. In the measured waveform, the magnitude of the rarefaction generated during tunnel narrowing increases at an increasing rate as the separation event is approached. This trend is not captured by the predicted waveform, as is also shown in Table 5.1. Furthermore, close inspection of the chemiluminescence signal shows there is a positive fluctuation in

Table 5.1 Predicted and measured changes in the acoustic pressure at a distance of 100 mm over three time intervals prior to the separation reference time.

<table>
<thead>
<tr>
<th>Time Interval</th>
<th>predicted $\Delta p'$ (mPa)</th>
<th>measured $\Delta p'$ (mPa)</th>
<th>% error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t=-1.25$ to $t=-0.75$ ms</td>
<td>-4.3</td>
<td>-7.6</td>
<td>-43%</td>
</tr>
<tr>
<td>$t=-0.75$ to $t=-0.25$ ms</td>
<td>-1.4</td>
<td>-11.3</td>
<td>-87%</td>
</tr>
<tr>
<td>$t=-0.25$ to $t=-0.10$ ms</td>
<td>+1.0</td>
<td>-8.0</td>
<td>-113%</td>
</tr>
</tbody>
</table>
5.4 Results and Discussion

The chemiluminescence signal in the 50 µs interval prior to the separation reference time, as shown in Figure 5.6a. This is caused by an increase in the chemiluminescent emission intensity in the flame during flame annihilation, as observed in Figure 4.8. The increase in emission intensity dominates the effect of decreasing flame surface area over this short interval. After the separation reference time, the derived pressure trace consistently over-predicts the measured acoustic pressure by an approximately constant value.

Figure 5.6a shows that the fitted polynomial used to estimate the time derivative of the chemiluminescence signal during the separation event is unsuitable for capturing the rapid changes occurring here. Two other options for estimating the derivative in this region were attempted. Figures 5.6b and 5.6c show the fitted polynomials that result from treating $t = 0$ as a discontinuity. In Figure 5.6b, none of the data points after $t = 0$ were used in the fit. In Figure 5.6c, none of the data points before $t = 0$ were used. Visually, both fits are reasonable. The fits shown are for determining the derivative at $t = 0$ only. For
Fig. 5.7 Mean measured pressure trace at $d_{mic} = 100$ mm (gray lines) plotted with predicted pressure traces (black lines) when: (a) including all data points in the polynomial fitting; (b) treating $t = 0$ as a discontinuity; (c) neglecting the chemiluminescence signal from $t = -0.05$ ms to $t = 0$ ms.

$t < 0$, the same fitting method may be applied using a weighted fit centred at the time of interest and excluding all data points after $t = 0$. For $t > 0$, all data points before $t = 0$ are neglected. At $t = 0$, two estimates of the derivative are obtained and the mean value is likely to provide the best estimate.

Figure 5.7b shows the derived pressure trace when $t = 0$ is treated as a discontinuity. The positive fluctuation in the chemiluminescence signal prior to the separation reference time has a highly significant effect from $t = -50 \mu s$ to $t = +50 \mu s$. Given the short duration of this event and the fall off in the microphone sensitivity at high frequencies, it may be expected that a positive pressure fluctuation of such duration would not be accurately resolved. However, the complete absence of a positive fluctuation in the measured trace suggests the chemiluminescent emission intensity is not well correlated with the heat release rate in the terminal 50 $\mu s$ of the annihilation event. This suggests it may be better to exclude the chemiluminescence data in this 50 $\mu s$ interval and to instead interpolate the signal over this interval. The fit, at $t = 0$, that results from excluding the data points in the
interval from $t = -50 \mu s$ to $t = 0$ is shown in Figure 5.6d. Figure 5.7c shows the derived pressure trace when data points in the terminal $50 \mu s$ of tunnel narrowing are excluded. The agreement between the derived and measured trace is improved. This suggests that flame acceleration in the $50 \mu s$ prior to the separation reference time, when the opposing flames’ preheat and reaction zones are merging, is not important for predicting the sound generated by this forced flame. As will be discussed in more detail in Chapter 6, this does not mean that flame acceleration is irrelevant to the sound produced by tunnel narrowing prior to the terminal $50 \mu s$, or to the sound generated by the flame during pocket consumption and cusp recovery.

Other possible sources of the discrepancies relate to the propagation of the acoustic wave, rather than to the measurement of the rate of change of heat release rate from the flame. Reflections off the stabilisation tube, the surfaces of the laboratory, or the burner plenum can affect acoustic measurements. However, tests were undertaken to detect acoustic reflections, as described Section 3.6, and no evidence of acoustic reflections were found. Equation 2.34 relates the heat release rate in an acoustically compact region to the acoustic pressure in the far field, and is the basis of Equation 5.1. The assumptions used to derive this equation are also potential sources of error. However, measurements of the acoustic waveform at four different distances, reported in Section 3.6.3, showed that the waveform shape does not change significantly with distance and that the spherical spreading law applies to the magnitude of the acoustic pressure. These measurements support the assumption of acoustic compactness and that the measurements are taken at a sufficient distance.

For the Case A3 results presented so far in this chapter, the image timings were determined based on the temporal separation from the start of the camera exposure to the position of maximum gradient on the rapid drop in the acoustic pressure associated with the separation event. This is optimal for resolving the separation event but is not optimal for resolving the terminal moments of pocket consumption. For Case A3, the temporal separation between the position of maximum gradient on the rapid drop associated with the separation event and the minimum of the small rarefaction pulse associated with the pocket burn-out event has a mean value of 1.94 ms over 1000 cycles, with a standard deviation of 0.03 ms, a minimum of 1.84 ms and a maximum of 2.02 ms. The variation is primarily due to small variations in the size of the pocket at its moment of formation. For mean traces computed by aligning traces by the separation reference time, the rarefaction pulse associated with the pocket burn-out event is smoothed out. Similarly for traces derived from images with timings determined relative to the rapid drop in $p'$ associated with separation, the pocket burn-out event is likely to be smoothed out. To better resolve the pocket burn-out event, image timings are instead determined by their temporal distance from the local minimum associated with the pocket burn-out event. Figure 5.8a shows the
Fig. 5.8 (a) The summed pixel counts from the flame pocket formed in Case A3. (b) The predicted (black line) pressure waveform and measured mean waveform (grey line) at a distance of 100 mm.

summed pixel counts from images of just the flame pocket using the pocket burn-out event as the reference feature.

Figure 5.8b compares the predicted and measured sound from the pocket consumption. The general shape of the waveform is reproduced using the time derivative of the visible chemiluminescence. However, the rarefaction prior to the pocket burn-out event is consistently overpredicted. The smaller fluctuations in the predicted waveform may be due to the scatter in the chemiluminescence signal data points, that can have a significant effect on estimates of the time derivative. From 1-0.05 ms prior to the annihilation reference time, the acoustic pressure is overpredicted by 18% on average. The overprediction is greatest 20 µs prior to the annihilation reference time, where the acoustic pressure is overpredicted by 66%.

5.5 Summary

The measured sound emitted by a forced, laminar, premixed, propane-air flame during the formation and consumption of a separated pocket was compared to the predicted sound computed using the time derivative of visible chemiluminescence from the region where the pocket is formed and consumed. Many features of the acoustic pressure waveform were reproduced. This is further evidence that tunnel narrowing, pocket consumption, and cusp recovery are responsible for the observed changes in the far-field acoustic pressure, and that the contribution from the rest of the flame is relatively insignificant.

However, the sound predicted using the time derivative of the total visible chemiluminescent emission was found to be poorly correlated with the measured sound during the separation event and the pocket burn-out event. Images showed that the local vol-
umetric chemiluminescent emission intensity increases markedly during the separation event, resulting in an increase in the total visible chemiluminescent emission of the flame over approximately 50 $\mu$s. This is an abrupt change in the otherwise negative rate of change in total visible chemiluminescent emission either side of the separation event. A corresponding increase in the emitted acoustic pressure is not observed. To the contrary, a large drop in the emitted acoustic pressure is observed during the separation event. This result suggests that visible chemiluminescence may be a poor marker of the heat release rate during flame annihilation events.

Though widely used as a marker of heat release rate, the chemiluminescence signal provides little insight into the mechanisms underlying changes in the heat release rate. The mechanism of flame stretch due to the propagation of curved flames and the significance of flame acceleration are explored in the next chapter.
Chapter 6

A New Model of Sound Production from Laminar Premixed Flames

6.1 Introduction

A model is presented for the generation of direct sound by acoustically-compact, axisymmetric, forced, laminar, premixed, jet flames. The model predicts the acoustic pressure generated by a flame at any instant using the flame geometry at that instant, and enables the identification of the flame dynamics that are responsible for rapid changes in the acoustic pressure. The model is applied to spherical pockets and to the flame neck during the tunnel narrowing process prior to pocket separation. The flame geometry is determined from chemiluminescence images. In this model, the flame is represented by a collection of flame elements that are advected by the flow and propagate into the unburned gases with a speed that is linearly related to the mean curvature. Predictions from the model are compared to measurements of the sound emitted by tunnel narrowing and pocket consumption processes. The significance of flame acceleration during these processes is investigated.

The chapter is structured as follows: A model of sound generation by flame elements is described in Section 6.2. The forced, laminar flames used to evaluate the sound generation model are described in Section 6.3. Empirical Markstein lengths are determined in Section 6.4. Markstein lengths are used in the sound generation model to estimate the flame displacement speed over the flame surface. In Sections 6.5 and 6.6, the sound generation model is applied to flame necks during tunnel narrowing and to the terminal moments of pocket consumption. The model predictions are compared to the measured sound. Lastly, in Section 6.7, the radial chemiluminescence profile is determined by Abel deconvolution. The thickness of the chemiluminescence profile is used to estimate the error introduced by treating the flame as infinitely thin. In addition, flame radii determined using the peaks in
radial chemiluminescence profiles provide a more accurate estimate of the position of the peak heat release rate than using the position of peaks in flame images.

### 6.2 A Flame Element Model of Sound Production

The model presented here is based on the relation given by Dowling (1992):

$$p'(x,t) = \left( \frac{\gamma - 1}{\gamma} \right) \frac{\rho_0}{4\pi R p_0} \int_{V(y)} \left[ \frac{\partial Q}{\partial t} \right]_\tau dy. \quad (6.1)$$

A derivation of this expression is presented in Section 2.3.2. Far field pressure fluctuations are proportional to the rate of change of the volumetric heat release rate evaluated at the retarded time and integrated over the flame volume. The partial time derivative may be replaced by a time derivative outside the spatial integral. For a compact source, for which differences in retarded time across the source region are negligible, the volume integral can be reformulated as

$$\frac{d}{dt} \int_{V(y)} [Q]_{\tau} dy = \left[ \frac{d}{dt} \int_{V(y)} Q dy \right]_{\bar{\tau}}, \quad (6.2)$$

where $\bar{\tau}$ is a representative retarded time. For open, acoustically-compact flames that emit negligible flow noise or indirect noise, the sound in the far field at some time $t$ is therefore proportional to the time derivative of the integral of the heat release rate over the flame region at a representative retarded time.

In the subsequent sections, Equation 6.2 is reformulated in terms of the flame geometry and flame speeds. Flame speed relations from premixed flame theory are then applied to predict the sound generated by the evolution of different flame geometries and to investigate the role of flame acceleration. The flame element model of sound generation is described first for spherical flames. Then, in Section 6.2.2, the model is applied to axisymmetric flames that are approximated by conical flame elements.

Further aspects of the model are described in Appendix A. Treatment of the flame base and flame tip is described in Sections A.1 and A.2 respectively. Sections A.3 and 6.2.3 demonstrate how, in a steady flame, a flame element model of sound production predicts a cancellation of the contributions of the flame tip, the flame base, and the rest of the flame, to the radiated sound, and how disturbances to the flame geometry result in the emission of sound.
6.2 A Flame Element Model of Sound Production

6.2.1 Spherical Flames

The model presented here for sound generated by inwardly propagating spherical flames is similar to the approach first used in studies of sound generation by Thomas and Williams (1966) and more recently applied by Talei et al. (2012b) to 1D spherical annihilation events featuring curvature-dependent flame speeds. However, the two key features of the acoustic waveform predicted by the model: (i) a sustained but slowly decreasing rarefaction, and (ii) a step change in pressure following the burn-out event, have not previously been identified. These predictions match the experimental observations in Section 6.6.

For spherically symmetric combustion events, Equation 6.2 is more conveniently expressed in spherical co-ordinates,

\[ \frac{d}{dt} \int_{V(y)} Qdy = \frac{d}{dt} \int_{0}^{r^+} 4\pi r^2 Qdr, \]  

(6.3)

where \( r^+ \) is greater than the largest radius where \( Q \) is not negligible. The flame position \( r_f \) is taken to be the position of the peak in the heat release rate profile across the flame. The thickness of the heat release rate profile is denoted \( \delta_Q \). For a spherical flame where \( \delta_Q \ll r_f \),

\[ \frac{d}{dt} \int_{0}^{r^+} 4\pi r^2 Qdr \approx \frac{d}{dt} \left( 4\pi r_f^2 \int_{0}^{r^+} Qdr \right). \]

(6.4)

Equation 6.4 is the time derivative of the product of the flame surface area \( A_f \) and the heat release rate per flame surface area,

\[ \frac{d}{dt} \left( 4\pi r_f^2 \int_{0}^{r^+} Qdr \right) = \frac{d}{dt} \left( A_f \int_{0}^{r^+} Qdr \right). \]

(6.5)

The flame surface area decreases during pocket consumption, suggesting a rarefaction event. However, \( \int_{0}^{r^+} Qdr \) may increase or decrease, depending on flame parameters such as the Lewis number, and increases in \( \int_{r_f}^{r^+} Qdr \) could conceivably produce compression events, though ultimately there must be a rarefaction event during flame annihilation, i.e. in the terminal moments. At this point, it is useful to define a flame ‘heat-release speed’:

\[ s_q = \frac{\int_{n}^{n^+} Qd\mathbf{n}}{\rho_a Y_{F,a} LHV} \]

(6.6)

where \( \mathbf{n} \) is the flame surface normal direction. \( s_q \) is defined in a similar manner to the familiar consumption speed \( s_c = \int_{n}^{n^+} \dot{\omega}_F d\mathbf{n}/\rho_a Y_{F,u} \). Under the condition \( \int_{n}^{n^+} Qd\mathbf{n} = \int_{n^+}^{n^+} \dot{\omega}_F d\mathbf{n} LHV \), the two speeds are identical. By defining the constant \( \beta = 2\pi \rho_a Y_{F,a} LHV \), and the displacement speed \( s_d = -dr_f/dt \), the following expression is obtained for the
rate of change of the heat release rate integrated over the flame region:

\[
\frac{d}{dt} \int_{V(y)} Q d\mathbf{y} = 2\beta \left( r_f^2 \frac{ds_q}{dt} - 2r_f s_q s_d \right).
\]  \quad (6.7)

The first term on the right hand side represents the contribution of a changing heat release speed to the time derivative of the heat release rate from a flame element, and the second term the contribution of changes in its flame surface area. The same separation is commonly used in formulations of the turbulent flame speed in terms of the stretch factor (Driscoll, 2008). Most of the early work on combustion noise neglected changes in the heat release speed and considered only changes in the flame surface area (e.g., Abguf and Obrezkov, 1978; Clavin and Siggia, 1991; Thomas and Williams, 1966). However, this separation of contributions to the sound emitted by flames has been previously proposed by Candel et al. (2009) and has been applied in studies of sound generation from flame annihilation by Talei et al. (2011b).

The acoustic pressure in the far field is approximately given by

\[
p'(R,t) = \frac{1}{R} \left( \frac{\gamma - 1}{\gamma} \right) \frac{\rho_0 \rho_u Y_{F,u} LHV}{p_0} \left[ r_f^2 \frac{ds_q}{dt} - 2r_f s_q s_d \right].
\]  \quad (6.8)

This expression agrees with those previously given in the literature. Using \( Y_{F,u} LHV = c_p(T_b - T_u) \), \( \mathcal{R} = c_p - c_v \), and \( p_0 = \rho_0 \mathcal{R} T_0 \), the expression in front of the square brackets can be reformulated as:

\[
\frac{\rho_u}{R} \left( \frac{\gamma - 1}{\gamma} \right) \frac{\rho_0 Y_{F,u} LHV}{p_0} = \frac{\rho_u}{R} \frac{1}{T_0} (T_b - T_u).
\]  \quad (6.9)

In the case that \( s_q = s_d \) and the receiver position \( R \) is located in burned gas, Equations 6.8 and 6.9 give Equation 5.43 in Talei et al. (2011b). The Heaviside functions in the expressions in Talei et al. (2011b) simply express time boundaries for the process. Equation 5.48 from Talei et al. (2011b) is obtained if it is assumed that \( s_q = s_d = S_{L,0} \). In the case that \( s_q = S_{L,0} \) but \( s_d \) is allowed to vary independently, Equation 19 from Talei et al. (2012b) is obtained. Equation 7 from Thomas and Williams (1966) is also obtained from Equations 6.8 and 6.9 if it is assumed that \( s_q = s_d = S_{L,0} \). In that case the ambient air at position \( R \) should be assumed at the same temperature as the unburned mixture and the approximation \( T_b / T_u = \rho_b / \rho_u \) is made. A minus sign was omitted from their expression but this does not affect their conclusion that no sound is produced in the terminal moments of the burn-out event.

To predict sound generation using Equation 6.8, expressions for \( s_q \) and \( s_d \) as are required. This analysis follows that of Talei et al. (2012b) and uses the Markstein relation
Table 6.1 Combinations of flame speed relationships used for predicting sound production by inwardly propagating spherical flames. $\beta = 2\pi \rho_Y YF_{\infty}LHV$. 

<table>
<thead>
<tr>
<th>Case</th>
<th>$s_q/S_{L,0}$</th>
<th>$s_d/S_{L,0}$</th>
<th>$\frac{d}{dt} \int_{V(y)} Qdy$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>$-4\beta S_{L,0}^2 r_f$</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>$1 + \frac{2L_d}{r_f}$</td>
<td>$-4\beta S_{L,0}^2 (r_f + 2L_d)$</td>
</tr>
<tr>
<td>3</td>
<td>$1 + \frac{2L_q}{r_f}$</td>
<td>$1 + \frac{2L_d}{r_f}$</td>
<td>$-4\beta S_{L,0}^2 (r_f + L_q + 2L_d + \frac{2L_q L_d}{r_f})$</td>
</tr>
</tbody>
</table>

(Markstein, 1964),

$$s = S_{L,0}(1 - L k_m). \quad (6.10)$$

The flame speeds (both displacement and heat-release) are assumed linearly related to the mean curvature $k_m$, though the Markstein length ($L$) may be different for the two flame speeds (Poinsot and Veynante, 2005, p70). Subscripts $d$ and $q$ will be used to refer to the displacement speed and heat-release speed Markstein lengths respectively. Here we use the convention that the mean curvature is equal to the sum of the principal curvatures and that curvature is negative for flames curved towards the unburned reactants.

As is discussed in the literature review in Section 2.1.1, prior experimental studies on inwardly-propagating, propane-air flames with high curvature and negligible strain rate conclude that the Markstein relation fits the experimental data for flame displacement speed better than the theory of flame stretch (Baillot et al., 2002; Durox et al., 2001; Ibarreta and Driscoll, 2000), particularly at the highest observed curvatures. The strain rate in conical, laminar, premixed flames forced at similar frequencies and amplitudes has been previously shown to be negligible (Durox et al., 2001).

The heat release speed is closely related to the consumption speed. However, the relationship between the flame consumption speed and high flame curvature is not well understood in lean, propane-air flames. For unity Lewis number mixtures, there is evidence to suggest the flame consumption speed remains relatively constant even at high negative stretch (Poinsot et al., 1992). Studies based on asymptotic theories suggest that, at least for flames with low curvature, the flame consumption speed will increase with increasing negative curvature if the Lewis number is positive (Clavin, 1985), as is the case for lean, propane air flames (Giannakopoulos et al., 2015b). An increasing consumption speed at high negative flame curvature for mixtures with $Le > 1$ has also been demonstrated in numerical simulations (Talei et al., 2012b).

Three cases are now considered to investigate the effect of flame acceleration on the predicted sound. The three cases are summarised in Table 6.1. In Case 1, $s_q$ and $s_d$ are
Table 6.2 Parameters for simulations of sound generation by spherical flames and conical flame elements.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>LHV</td>
<td>kJ/g</td>
<td>46.33</td>
</tr>
<tr>
<td>$\rho_u$</td>
<td>kg/m³</td>
<td>1.2</td>
</tr>
<tr>
<td>$\rho_0$</td>
<td>kg/m³</td>
<td>1.2</td>
</tr>
<tr>
<td>$p_0$</td>
<td>N/m²</td>
<td>101325</td>
</tr>
<tr>
<td>$r_{f,0}$</td>
<td>m</td>
<td>1.5x10⁻³</td>
</tr>
<tr>
<td>$S_{L,0}$</td>
<td>m/s</td>
<td>0.31</td>
</tr>
<tr>
<td>$L_d$</td>
<td>m</td>
<td>6x10⁻⁴</td>
</tr>
<tr>
<td>$L_q$</td>
<td>m</td>
<td>3x10⁻⁴</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>-</td>
<td>1.4</td>
</tr>
<tr>
<td>$Y_F$</td>
<td>-</td>
<td>0.0489</td>
</tr>
</tbody>
</table>

assumed constant at the unstretched laminar flame speed. In Case 2, $s_d$ is assumed to follow Markstein’s relation and $s_q$ is assumed to remain constant. In Case 3, both $s_d$ and $s_q$ are assumed to follow Markstein’s relation. Expressions for the time derivative of the heat release rate integrated over the flame region (derived from Equation 6.7) are given in Table 6.1 as a function of the flame radius for the three cases.

Pressure fluctuations have been predicted 100 mm from an inwardly-propagating, spherical, propane-air flame with $\phi = 0.80$. The simulation parameters are listed in Table 6.2. The value for $L_d$ is an average of the measured values, reported later in Table 6.5, for a propane-air flame at an equivalence ratio of 0.8. There are no reliable methods to measure or estimate the value of $L_q$ in propane-air flames. Here, a non-zero value of $L_q = 0.3$ mm is somewhat arbitrarily chosen to compare to the case where the heat release speed is assumed to remain constant. However, it will be shown in Section 6.6 that the assumption of a constant heat release speed yields reasonable agreement between measured and predicted changes in the acoustic pressure during pocket consumption. Furthermore, it will be shown in Section 6.2.2 that the value of $L_q$ does not significantly affect the sound generated by the conical flame elements involved in tunnel narrowing.

Figure 6.1 shows the predicted acoustic pressure waveform over the consumption of the reactant pocket. The predicted $p'$ spatial distribution at three time instants after the annihilation event are shown in Figure 6.2. The simulations treat the acoustic source function as a point source located in ambient air. The effect of an interface with burned gases is therefore not shown. With constant $s_q$ and $s_d$ (Case 1), a steadily decreasing rarefaction is generated until the burn-out event. No sound is generated in the terminal moments. If $s_d$ follows Markstein’s relation (Case 2) and $L_d$ is positive, the displacement speed increases as the
6.2 A Flame Element Model of Sound Production

![Image of two graphs showing acoustic pressure waveforms and distributions.](image_url)

**Fig. 6.1** Simulations of the acoustic pressure waveform 100 mm from the centre of the spherical flame. Case 1 (dash-dotted line), Case 2a (dashed line), Case 2b (grey solid line) and Case 3 (black solid line). Cases 2a and 2b are almost coincident. The solid circles indicate the retarded time when the flame radius is equal to \( \delta_{th} \). (a) Acoustic waveform through complete event with \( \tau = 0 \) at \( r_f = 1.5 \text{ mm} \). (b) Magnified plot of the waveforms with \( \tau = 0 \) at the annihilation event.

![Image of three graphs showing acoustic pressure distributions.](image_url)

**Fig. 6.2** Simulations of the acoustic pressure distribution with distance from the point of annihilation at 0.1 ms, 0.4 ms, and 0.9 ms after a spherical annihilation event. The solid circles indicate the location where the flame radius at the retarded time is equal to \( \delta_{th} \). The wave travels from left to right.
pocket is consumed. The generated rarefaction is greater in magnitude than the constant \( s_d \) case, though shorter in duration given the same initial pocket radius. Notably, the limit of the time derivative of the volume integrated heat release rate is a non-zero negative constant as the radius goes to zero. A step increase to the ambient pressure is therefore predicted immediately following the burn-out event. The magnitude of the positive step is proportional to \( L_d \) and to \( S_q^2 L_{d0} \). In reality, given the finite flame thickness, a return to ambient pressure would not be instantaneous, but would occur over a reaction time scale on the order of the reaction zone thickness \( \mathcal{O}(100 \, \mu m) \) divided by the displacement speed \( \mathcal{O}(1 \, \text{ms}^{-1}) \). Time scales on the order of 100 \( \mu s \) are therefore expected for propane-air flames. Case 2a is computed using Equation 6.8. Case 2b is computed numerically and assumes a Gaussian \( Q \) profile. Case 2b is included to demonstrate the error that results from treating the flame as infinitely thin. This is discussed further near the end of this section.

For Case 3, where both \( s_q \) and \( s_d \) follow Markstein’s relation, the rarefaction magnitude is greater still and the rarefaction magnitude starts increasing near the burn-out event, tending to infinity. Increases in the rarefaction magnitude occur only after the flame radius becomes smaller than \( r_f = \sqrt{\Sigma L_q L_d} \). The Markstein lengths are typically on the order of the thermal flame thickness \( (\delta_{th}) \).

Two approximations were used in the formulation of the model and both become increasingly inaccurate as the flame radius \( r_f \) approaches zero. The first approximation is stated in Equation 6.4. The error is small when \( r_f \gg \delta_Q \) but may become large when the reaction zones merge. This is the only assumption in Equation 6.7, which expresses the time derivative of the total heat release rate in terms of the flame radius, the flame displacement speed, and the flame heat-release speed. The error introduced by this assumption was estimated by considering a flame with a Gaussian \( Q \) profile with a thickness (FWHM) of \( \delta_Q=150 \, \mu m \). This corresponds to the measured thickness of the chemiluminescent emission distribution, as will be reported in Section 6.7. Table 6.2 lists the other parameters used. Figure 6.3a compares \( \int_0^+ 4\pi r^2 Qdr \) with the approximation \( 4\pi r_f^2 \int_0^+ Qdr \) for every flame position from \( r_f = 0 \, \text{mm} \) to \( r_f = 0.5 \, \text{mm} \). The error is almost constant, even for \( r_f < \delta_Q \). Because the acoustic pressure is proportional to the time derivative of the total heat release rate, the error in the sound predicted using the infinitely thin flame approximation is negligible. This is further demonstrated in Figure 6.1. Case 2b shows the sound generated by a flame with a constant gaussian \( Q \) profile with \( \delta_Q = 150 \, \mu m \) and \( s_d \) following the Markstein relation. The predicted sound is almost indistinguishable from Case 2a, computed using Equation 6.8, prior to the terminal 10 \( \mu s \).

The second source of error arises from the relations used for \( s_d \) and \( s_q \). The Markstein relation for the flame displacement speed is examined in Section 6.4 and is found to be inaccurate when \( r_f < \delta th \), where \( \delta th \) is the thermal flame thickness. Points where the
6.2 A Flame Element Model of Sound Production

Fig. 6.3 The volume integrated heat release rate from a flame with a gaussian heat-release rate profile with $\delta_0 = 150 \, \mu m$ (solid lines), compared against estimates computed using $A_f \int_0^{r_f} Q dr$ (dashed lines), for every flame position from $r_f = 0$ mm to $r_f = 0.5$ mm. (a) A spherical flame; (b) A cylindrical flame with a length of 1 mm.

Flame radius reaches $\delta_{th}$ are indicated on the waveforms in Figure 6.1. The thermal flame thickness was estimated based on kinetic modelling. The AramcoMech 2.0 mechanism (Li et al., 2017) was used in a Chemkin premixed laminar flame-speed model, using an unburned mixture at $25^\circ C$ and 100 kPa. The calculated planar flame thicknesses were 0.49 mm, 0.44 mm and 0.42 mm for $\phi=0.75$, $\phi=0.80$, and $\phi=0.85$ flames respectively, based on the definition $\delta_{th} = (T_b - T_u)/|dT/dx|_{max}$, where $\delta_{th}$ is the flame thickness, $T_u$ and $T_b$ are the unburned and burned gas temperatures, and $dT/dx$ is the temperature gradient.

### 6.2.2 Conical Flame Elements

In this section a similar analysis to that applied to predicting sound generation by spherical flames will be applied to conical flame elements. Axisymmetric flames will be approximated by a collection of conical flame elements. The boundaries of a flame element propagate into the unburned mixture in the flame normal direction with a speed $s_d$ and are simultaneously advected by the gas velocity $u$.

The volume occupied by a flame can be divided into $n$ moving, non-overlapping subvolumes, each containing the region associated with a flame element. The integral of $Q$ over the flame volume can be expressed as the sum of the integral of $Q$ over each subvolume,

\[
\int_{V(y)} Q dy = \sum_{i=0}^{n} \int_{V_i(y)} Q dy
\]  

(6.11)
Figure 6.4 shows a flame neck during tunnel narrowing. The flame neck region is divided into $n$ axisymmetric flame elements. Each element is approximately conical, with the exception of the flame tip. In the limit as the number of axisymmetric flame elements is increased, and the height of each element is decreased, the flame elements become identical to conical elements. The same approach can also be applied to an entire forced laminar flame if it is acoustically compact, as illustrated in Figure 6.5. A single conical flame element is depicted in Figure 6.6.

Fig. 6.4 A flame neck during tunnel narrowing, divided into $n$ vertically stacked subvolumes.

Fig. 6.5 A forced, laminar, premixed flame region divided into $n$ vertically stacked subvolumes.

Fig. 6.6 Approximation of a flame element by a conical flame element.
6.2 A Flame Element Model of Sound Production

The time derivative of the heat release rate for a conical flame element is approximately equal to the time derivative of the product of the flame surface area and the heat release rate per flame surface area,

\[
\frac{d}{dt} \left( \int_{V(y)} Q \, dy \right) \approx \frac{d}{dt} \left( A_{f,i} \int_{n^-}^{n^+} Q \, dn \right). \tag{6.12}
\]

This was also the case for the spherical flame (see Equation 6.5). The surface area of a cone segment is \(2\pi lr_c\), where \(l\) is the slant length, and \(r_c\) is the distance from the symmetry axis to the flame position at the vertical centre of the flame element. The heat release rate from subvolume \(V_i\), which contains the \(i\)th flame element, is

\[
\int_{V_i(y)} Q \, dy \approx 2\pi l_i r_{c,i} \left[ \int_{n^-}^{n^+} Q \, dn \right] \bar{z}_i. \tag{6.13}
\]

Similarly to spherical flames, the approximation in Equations 6.12 and 6.13 become less accurate as \(r_c\) decreases, i.e., as the flame approaches the symmetry axis. Using the definition of \(s_q\) (Equation 6.6) gives

\[
\frac{d}{dt} \int_{V(y)} Q \, dy = \sum_{i=0}^{n} \beta \frac{d}{dt} \left( r_{c,i} s_q, i \right). \tag{6.14}
\]

where \(\beta = 2\pi \rho_0 Y_F \text{LHV}\).

Expanding the right hand side of Equation 6.14 for flame element \(i\) and dropping the subscript \(i\) gives

\[
\beta \left( l_{s,q} \frac{dr_c}{dt} + r_c \frac{ds_q}{dt} + r_c s_q \frac{dl}{dt} \right). \tag{6.15}
\]

For the conical flame elements to which equation 6.15 will be applied in this work, the last term is negligible compared to the other two terms. Contributions to this term arise from flame curvature along the axial direction, aerodynamic strain, or differences in the displacement speed over the height of the flame segment. All three contributions are negligible during the tunnel narrowing processes that will be the focus of this chapter. This term has therefore been dropped from subsequent expressions. Using \(\frac{dr_c}{dt} = -s_q \cos \theta\) and \(l_{c}/l = \cos \theta\), the contribution of each conical flame element to an acoustic pressure fluctuation in the far field is approximated by:

\[
p'_i(R,t) = \left( \frac{\gamma - 1}{\gamma} \right) \frac{\rho_0}{4\pi R p_0} \beta \left[ -l_{c,i} s_d, i s_q, i + r_{c,i} \frac{ds_q, i}{dt} \right] \bar{z}. \tag{6.16}
\]

Sound production is therefore dependent on the height and orientation (or slant length) of the flame element, its average radius, the flame displacement speed, and the flame
Table 6.3 Combinations of flame speed relationships used in predictions of sound production from conical/cylindrical flame elements. \( \beta = 2\pi \rho_0 Y_f LHV \). \( l_z \) is the vertical height of a conical/cylindrical flame element.

<table>
<thead>
<tr>
<th>Case</th>
<th>( s_q/S_{L,0} )</th>
<th>( s_d/S_{L,0} )</th>
<th>( \frac{d}{dt} \int_{V(y)} Q d\bar{y} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>(-\beta l_z S_{L,0}^2)</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>1 + \cos \theta \frac{L_d}{r_c}</td>
<td>(-\beta l_z S_{L,0}^2 (1 + \cos \theta \frac{L_d}{r_c}))</td>
</tr>
<tr>
<td>3</td>
<td>1 + \cos \theta \frac{L_d}{r_c}</td>
<td>1 + \cos \theta \frac{L_d}{r_c}</td>
<td>(-\beta l_z S_{L,0}^2 (1 + \cos \theta \frac{L_d}{r_c}) - \beta l S_{L,0} L_q \sin \theta \frac{d\theta}{dt})</td>
</tr>
</tbody>
</table>

heat-release speed. \( s_q \) and \( s_d \) are assumed to follow Markstein’s relation (Equation 6.10). For a point on a conical surface the mean curvature is related to the radius \( r \) and the cone apex half angle \( \theta \) by

\[
k_m = \frac{-\cos \theta}{r}. \quad (6.17)
\]

As was done for a spherical flame, three cases will be used to examine the effect of flame acceleration. The three cases are summarised in Table 6.3. In Case 3, both \( s_d \) and \( s_q \) follow Markstein’s relation. The time derivative of the heat release rate integrated over the \( i \)th subvolume, derived from Equation 6.14 for Case 3, is

\[
\frac{d}{dt} \int_{V_i(y)} Q d\bar{y} = \left\{ \begin{array}{l}
-\beta l_z S_{L,0}^2 (1 + \cos \theta \frac{L_d}{r_c}) \\
-\beta l S_{L,0} L_q \sin \theta \frac{d\theta}{dt}
\end{array} \right. \quad \text{Term I}
\]

where \( \theta \) is in radians. Case 2 and Case 1 are obtained by setting one or both of the Markstein lengths to zero. Term II is generally negligible for the flame elements that the model will be applied to in this work. This is demonstrated by an order of magnitude analysis. Conservative estimates give \( S_{L,0} = O(1 \text{ m/s}) \), \( L_q = O(10^{-3} \text{ m}) \), and \( d\theta/dt = O(10 \text{ rad/s}) \). This yields Term I/Term II = \( O(10^2) \). Sound generation by conical flame elements is therefore only very weakly dependent on \( L_q \). This is a particularly useful result given the difficulties in measuring or estimating \( L_q \). Neglecting Term II yields

\[
p'_i(R,t) = -\left( \frac{\gamma-1}{\gamma} \right) \frac{\rho_0}{4\pi R p_0} \beta S_{L,0}^2 \left[ l_z,i (1 + \cos \theta \frac{L_d}{r_c,i}) \right]. \quad (6.19)
\]

The dependence on flame angle (\( \theta \)) is usually weak, though it can be significant during separation events when \( L_d/r_c \) approaches or exceeds unity. This is shown in Figure 6.7. For \( L_d/r_c \ll 1 \), the sound contribution of conical flame elements with the same vertical height is approximately equal, regardless of the flame angle (\( \theta \)). A low-order model of
sound generation by weakly forced laminar flames could therefore consider flame height as the only variable. When the flame height is lower than the steady flame height a positive pressure fluctuation is predicted and vice-versa. However, rapid changes in the geometry of the flame tip can dominate this effect over short periods, as will be shown.

For cylindrical flames, \( \cos \theta = 1 \) and \( d\theta/dt = 0 \), and there is no difference in sound production for Cases 2 and 3. This is demonstrated by earlier substitution of the Markstein relation for \( s_q \) into Equation 6.14:

\[
\frac{d}{dt} \left( r_c l S_{L,0} (1 + L_q/r_c) \right) = S_{L,0} \frac{d}{dt} (l r_c). \tag{6.20}
\]

An increased heat-release speed increases the volume-integrated heat release rate \( Q \), but does not affect \( dQ/dt \) if the heat-release speed is linearly related to the curvature. However, in the terminal moments, the heat release rate must go to zero, and a greater heat-release speed is therefore predicted to produce a larger terminal rarefaction pulse. The amplitude and duration of the terminal pulse depends on the details of the reactions as reaction zones merge and cannot be predicted by an infinitely-thin flame model.

Figure 6.8 shows the predicted acoustic pressure waveform 100 mm from a cylindrical flame element of 1 mm length, as it propagates inwards, consuming reactants. The simulation parameters are the same as for the spherical flame simulation and are given in Table 6.2. The predicted \( p' \) spatial distribution at three time instants after the annihilation event are shown in Figure 6.9.

For Case 1, where \( s_d \) and \( s_q \) are assumed constant, a constant rarefaction is sustained until the burn-out event. The magnitude of the rarefaction is proportional to \( S_{L,0}^2 \) and
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Fig. 6.8 Simulations of the acoustic pressure waveform 100 mm from a cylindrical flame element. Case 1 (dash-dotted line), Case 2a (dashed line), Case 2b (grey solid line) and Case 3 (black solid line). The waveforms for Case 2a and Case 3 are identical. Case 2b is also almost coincident. The solid circles indicate the retarded time when the flame radius is equal to $\delta_{th}$. (a) Acoustic waveform through complete event with $\tau = 0$ at $r_c = 1.5$ mm. (b) Magnified plot of the waveforms with $\tau = 0$ at the annihilation event.

Fig. 6.9 Simulations of the acoustic pressure distribution with distance from the point of annihilation at 0.1 ms, 0.4 ms, and 0.9 ms after a cylindrical annihilation event. The solid circles indicate the location where the flame radius at the retarded time is equal to $\delta_{th}$. The wave travels from left to right.
to the height of the cylindrical flame element. A step increase in pressure is predicted immediately following the annihilation event.

For Case 2 and Case 3, where \( s_d \) follows Markstein’s relation, the generated rarefaction is greater in magnitude than the constant \( s_d \) case, though shorter in duration given the same initial cylinder radius. This was also true for spherical flames. However, for cylindrical flame elements, the rarefaction monotonically increases prior to the separation event and tends to infinity at the terminal moment. This is the case regardless of \( L_q \). This has significant implications for the relative significance of different parts of a laminar jet flame to sound generation. These implications are further discussed in Section A.3. Discussions of the significance of flame surface creation at the flame base and the sound generated by the flame tip are also included in Appendix A.

The same considerations apply to this analysis of conical flame elements as were discussed in regards to the spherical flames, namely that the flame has a finite thickness and that Markstein relation does not accurately model the flame speeds when \( r_f < \delta h \).

Figure 6.3b compares \( \int_0^{r_f} 2\pi r Q dr \) with the approximation \( 2\pi r_f \int_0^{r_f} Q dr \) for a cylindrical flame element with a Gaussian profile with \( \delta Q = 150 \) µm. The flame element has a length of 1 mm. The error is almost zero for \( r_f > \delta Q \). This is further demonstrated in Figure 6.8. Case 2b shows the sound generated by a flame with a constant Gaussian Q profile and \( s_d \) following the Markstein relation. The predicted sound is almost indistinguishable from Case 2a, computed using Equation 6.8, prior to the terminal 100 µs.

### 6.2.3 Application of the Model to Forced Flames

Predictions from the model outlined in the previous sections will be compared to measurements of the sound emission from forced, laminar, premixed, propane-air flames. Given the predicted significance of conical flame elements close to the central axis, disturbances to the flame tip are predicted to correspond with the largest amplitude sound. This has already been confirmed by observations detailed in the previous chapter. Tunnel narrowing and pocket consumption processes will be considered in detail.

During the the formation, separation, and consumption of a separated pocket, the flame geometry near the flame tip changes significantly over a time interval of several milliseconds. In weakly forced flames, the contribution of flame elements away from the flame tip is predicted to remain relatively constant over this time interval. The behaviour of flame elements near the flame tip is therefore likely to dominate sound production and become increasingly dominant as the separation event is approached. Errors in Equation 6.19 are small for flame elements temporally and spatially close to the point of separation, provided \( r_f \gg \delta Q \). Changes in the radiated acoustic pressure during the tunnel narrowing process will be predicted from changes in the geometry of the flame neck, identified
Table 6.4 Experiment Conditions

<table>
<thead>
<tr>
<th>$\phi$</th>
<th>Case</th>
<th>$\omega$ (s$^{-1}$)</th>
<th>St</th>
<th>$\bar{u}$ (ms$^{-1}$)</th>
<th>$I_{rms}$ (mA)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.75</td>
<td>A2</td>
<td>290</td>
<td>13</td>
<td>0.97</td>
<td>2.4</td>
</tr>
<tr>
<td></td>
<td>A3</td>
<td>290</td>
<td>13</td>
<td>0.97</td>
<td>5.8</td>
</tr>
<tr>
<td>0.80</td>
<td>D1</td>
<td>346</td>
<td>13</td>
<td>1.14</td>
<td>2.9</td>
</tr>
<tr>
<td></td>
<td>D2</td>
<td>346</td>
<td>13</td>
<td>1.14</td>
<td>8.8</td>
</tr>
<tr>
<td>0.85</td>
<td>E1</td>
<td>383</td>
<td>13</td>
<td>1.27</td>
<td>3.7</td>
</tr>
<tr>
<td></td>
<td>E2</td>
<td>383</td>
<td>13</td>
<td>1.27</td>
<td>8.3</td>
</tr>
</tbody>
</table>

from flame images. Predictions will be compared to measured changes in the acoustic pressure. This analysis demonstrates how the rarefaction associated with tunnel narrowing is generated and identifies the cause of the similarity between the waveforms generated by tunnel narrowing processes in flames with different amplitude forcing.

Based on the analysis of spherical flames in Section 6.2.1, the consumption of separated reactant pockets generates a sustained rarefaction. A positive change in the acoustic pressure is predicted to immediately follows the burn-out event. In the previous chapter the cusp recovery that immediately follows a separation event was observed to be a significant source of sound. Relatively high forcing amplitudes are therefore necessary to produce pockets of sufficient size so that the burn-out event occurs after sound production from cusp recovery has become negligible.

### 6.3 Experiment Conditions

Forced propane-air flames at three equivalence ratios are included in this study: $\phi = 0.75$, $\phi = 0.80$, and $\phi = 0.85$. Two flame cycles, differing only by the forcing amplitude, are used at each equivalence ratio. Cases A2 and A3 from the previous chapter are used as the $\phi = 0.75$ cases. Additional cases are used at $\phi=0.80$ (Cases D1 and D2) and $\phi=0.85$ (Cases E1 and E2). The cases are summarised in Table 6.4. Images of the flame cycles are shown in Figures 6.10, 6.12, and 6.14. Representative pressure traces and mean traces acquired at a distance of 100 mm are given in Figures 6.11, 6.13, and 6.15. Also shown by the dashed lines are measurements of the speaker sound, acquired after the flame had been extinguished.

For each equivalence ratio the case with the lower forcing amplitude is used to study the tunnel narrowing process (Cases A2, D1, and E1). This is to minimize the influence of direct sound from the speaker and to minimise distortions in the rest of the flame. Higher forcing amplitudes (Cases A3, D2, and E2) are used to test the predictions of
6.3 Experiment Conditions

Fig. 6.10 Equally spaced images through a forcing cycle for $\phi = 0.75$ flames. Image intensifier gate width is 200 $\mu$s.

Fig. 6.11 Representative (black line) and mean (gray line) pressure waveforms for $\phi = 0.75$ flames, measured 100 mm from the unforced flame tip position. Dashed lines show the waveform generated by the speaker in the absence of a flame.
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Fig. 6.12 Equally spaced images through a forcing cycle for $\phi = 0.80$ flames. Image intensifier gate width is 200 $\mu$s.

Fig. 6.13 Representative (black line) and mean (gray line) pressure waveforms for $\phi = 0.80$ flames, measured 100 mm from the unforced flame tip position. Dashed lines show the waveform generated by the speaker in the absence of a flame.
6.3 Experiment Conditions

Fig. 6.14 Equally spaced images through a forcing cycle for $\phi = 0.85$ flames. Image intensifier gate width is 200 $\mu$s.

Fig. 6.15 Representative (black line) and mean (gray line) pressure waveforms for $\phi = 0.85$ flames, measured 100 mm from the unforced flame tip position. Dashed lines show the waveform generated by the speaker in the absence of a flame.
Table 6.5 Measured Markstein lengths and unstretched, laminar flame speeds.

<table>
<thead>
<tr>
<th>$\phi$</th>
<th>Case</th>
<th>$S_L$ (m/s)</th>
<th>$L_d$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.75</td>
<td>A2</td>
<td>0.27</td>
<td>1.11</td>
</tr>
<tr>
<td></td>
<td>A3</td>
<td></td>
<td>1.00</td>
</tr>
<tr>
<td>0.80</td>
<td>D1</td>
<td>0.31</td>
<td>0.62</td>
</tr>
<tr>
<td></td>
<td>D2</td>
<td></td>
<td>0.57</td>
</tr>
<tr>
<td>0.85</td>
<td>E1</td>
<td>0.35</td>
<td>0.67</td>
</tr>
<tr>
<td></td>
<td>E2</td>
<td></td>
<td>0.61</td>
</tr>
</tbody>
</table>

a sustained rarefaction from pocket consumption and a positive change in the acoustic pressure following the burn-out event. As seen from Figures 6.11, 6.13, and 6.15 the near-field of the speaker is likely to affect the sound measurements for these three cases. Sound associated with variable flame surface creation at the base of the flame and the development of circumferential cusps may also influence the results at the higher forcing amplitudes. However, the predicted step-like increase in pressure immediately following the burn-out event should still be observable, given it is expected to occur over a time interval on the order of 100 $\mu$s.

### 6.4 Empirical Displacement Speed Relations

The derived expressions for sound production by conical and spherical flame elements contain the unstretched, laminar, flame speed ($S_{L,0}$) and the displacement speed Markstein length ($L_d$). Unstretched, laminar, flame speeds were estimated using the cone angle method on steady flames. The measured values are reported in Table 6.5. $L_d$ was measured by least squares fitting of Equation 6.10 to measurements of the radius of the narrowest part of the flame neck ($r_{min}$) during tunnel narrowing. The mean curvature was approximated as $1/r_{min}$ and the small positive flame curvature on the vertical normal plane was neglected. I.e., the narrowest part of the flame neck was assumed to propagate like a cylindrical flame. The image timing relative to the separation reference time ($t_{sep}$) was determined using the distinctive acoustic signature of the separation event, as described in Section 4.4. To obtain flame neck radius measurements from the noisy flame images, images were filtered by convolution with a uniform, rectangular kernel with a height of 50 pixels and a width of 1 pixel. Figure 6.16 shows a filtered and unfiltered image. The difference in the positions of peaks in the projected and radial emission profiles is investigated in Section 6.7 and is found to be approximately 60 $\mu$m. An additional 60 $\mu$m was added to the measured flame radius ($r_{min}$). Flame radius measurements less than the numerically estimated thermal flame thickness ($\delta_{th}$) were excluded.
The displacement speed Markstein lengths were measured for all six cases. Figure 6.17 shows the flame radius measurements versus time and the fitted curves. The results are presented in Table 6.5. The two measurements of the Markstein length at each equivalence ratio differ by a maximum of 11%. At each equivalence ratio, the measured Markstein length is larger for the less strongly forced flame. This is likely to be due to the increase in positive curvature on the vertical normal plane.

The measured Markstein lengths are consistent with those reported in the literature for propane-air flames, though there is large scatter due to the range of imaging methods that have been used. It should be noted that numerous studies have shown that Markstein lengths for inwardly propagating propane-air flames are significantly larger than those for outwardly propagating flames and results from these different flame configurations should therefore not be compared (Durox et al., 2001; Ibarreta and Driscoll, 2000; Sinibaldi et al., 2003). See Section 2.1.1 for further discussion. Durox et al. (2001) measured the Markstein length by fitting the Markstein relation to measurements of the flame radius versus time, obtained from a tunnel narrowing process. Their method is similar to that used here, though the flame radius is determined from Schlieren images. At $\phi = 0.80$ they reported $L_d = 0.34$ mm using $S_{L,0} = 0.32$ m/s and at $\phi = 0.90$ they reported $L_d = 0.30$ mm using $S_{L,0} = 0.36$ m/s. The Markstein lengths measured in this study exceed those reported by Durox et al. (2001) by almost a factor of 2. However, this discrepancy is reasonable in light of the results reported by Baillot et al. (2002) who found similar differences between Markstein lengths determined using Schlieren imaging and chemiluminescence imaging.
Fig. 6.17 Measurements of the minimum flame neck radius versus time relative to the separation reference time (dot markers) and the fitted Markstein relation (solid lines).
Baillot et al. (2002) also measured Markstein lengths by fitting the Markstein relation to measurements of the flame radius versus time, but obtained the measurements from near-spherical pockets instead of from the tunnel narrowing process. They conducted their experiments and analysis using both chemiluminescence imaging and Schlieren imaging. At $\phi = 0.80$ they reported $L_d = 0.75$ mm based on chemiluminescence imaging. $L_d$ obtained from Schlieren imaging was only 0.38 mm, approximately half the value obtained using chemiluminescence images. A planar, unstretched flame speed ($S_{L,0}$) of 0.30 m/s was used. The Markstein length obtained by Baillot et al. (2002) using chemiluminescence imaging at $\phi = 0.80$ is approximately 30% larger than the result measured in this study. This may be explained by differences in how the flame radius was measured from the chemiluminescence images. Baillot et al. (2002) used a contour at the ‘outside boundary of the luminous zone’. This is likely to give larger radii than the contour of peak brightness and therefore a slightly higher Markstein length is expected. It should also be noted that multiple pairs of $S_{L,0}$ and $L_d$ fit the data well. The measured Markstein length is therefore sensitive to the assumed unstretched, planar flame speed. This has been investigated in detail by Lipatnikov et al. (2015).

Baillot et al. (2002) also reported results at $\phi = 0.90$. Differences in $L_d$ determined from chemiluminescence and Schlieren imaging were again observed. A planar, unstretched flame speed ($S_{L,0}$) of 0.365 m/s was used. They obtained $L_d = 0.66$ mm using chemiluminescence images and $L_d = 0.46$ mm using Schlieren images. A value of $L_d = 0.16$ mm was also reported when using shadowgraph images, approximately 25% of the value obtained using chemiluminescence images. Interestingly, even the direction of the trend of $L_d$ versus $\phi$ is dependent on the imaging method used.

The Markstein lengths reported in this study show an increase of approximately 79% in the Markstein length from $\phi = 0.80$ to $\phi = 0.75$. To the authors knowledge, Ibarreta and Driscoll (2000) report the only empirical Markstein length measurements for inwardly-propagating propane-air flames leaner than $\phi = 0.80$. Ibarreta and Driscoll (2000) measured the Markstein length for a $\phi = 0.65$ flame by fitting the Markstein relation to measurements of flame displacement speed versus curvature, obtained from shadowgraph images of inwardly propagating pockets. They reported $L_d = 1.3$ mm using a planar, unstretched, flame speed of $S_{L,0} = 0.19$ m/s. The increase in the Markstein length from $\phi = 0.80$ and $\phi = 0.75$ flame observed in this study is therefore reasonable, particularly given the reported differences in Markstein lengths obtained from shadowgraph imaging and chemiluminescence imaging (Baillot et al., 2002).

To accurately predict sound generation using the flame geometry, an accurate relation between the displacement speed and flame curvature is required. As shown by the plots in Figure 6.17 that the Markstein relation is an accurate model, up until the preheat zones of oppositely propagating flames begin to merge. The Markstein relation, using the measured
combinations of $L_d$ and $S_{L,0}$, should therefore accurately predict sound generation until the preheat zones merge.

### 6.5 Tunnel Narrowing

Equation 6.19 predicts the sound generated by a conical flame element given its geometrical parameters: the radius at the vertical centre ($r_c$), the vertical height ($l_z$), and the flame angle ($\theta$). In this section, the geometry of flame necks during tunnel narrowing is determined from flame images. Equation 6.19 is then used to predict the sound generated by the evolution of the flame neck.

During the formation, separation, and consumption of separated pockets, changes in acoustic pressure are generated by rapid changes in the geometry of the flame in the annihilation region, while the contribution of the rest of the flame to the acoustic pressure is predicted to remain relatively constant. Changes in the acoustic pressure over short time intervals during tunnel narrowing can therefore be estimated from the geometry of the flame neck at the beginning and end of the time intervals, and compared to measured changes in the acoustic pressure.

Images of the flame neck were acquired at a resolution of 30 $\mu$m/pixel. The image timing relative to the separation reference time ($t_{sep}$) was obtained using the method described in Section 4.4. Flame positions were manually identified by locations of peak intensity on the images. Figure 6.18 shows images of the flame neck for Case A2 at four instants. The manually identified flame positions are shown in red.

Conical flame elements were constructed from each pixel row using the measured flame radius and flame angle, and with a vertical length equal to the image resolution (i.e. 30 $\mu$m). In Section 6.7, it is shown that the radial position of peak emission is approximately 60 $\mu$m greater than that determined from flame images. 60 $\mu$m is therefore added to the flame radii ($r_c$) identified from images. Equation 6.19 is used to predict the sound contribution from each flame element at a distance of 100 mm. Flame neck geometries from Case A2 and plots of the predicted sound generated per vertical flame length ($p'_i/l_z,i$) are shown in Figure 6.19. Integration over the vertical axis gives the net acoustic pressure generated at that moment by flame elements comprising the flame neck. The area between two curves therefore gives the change in the generated acoustic pressure due to changes in the flame neck. The plots clearly show the generation of an increasing rarefaction during tunnel narrowing, particularly from flame elements in the vicinity of the narrowest part of the flame neck.

Figure 6.20 shows the measured acoustic pressure for Case A2 during tunnel narrowing and the predicted acoustic pressure based on a summation of the contributions of the conical flame elements. Predicted waveforms using both the measured $L_d$ and $L_d = 0$
6.5 Tunnel Narrowing

Fig. 6.18 Chemiluminescence images of the annihilation region of Case A2 during tunnel narrowing. Red lines show the manually determined flame positions used in predictions of changes in the acoustic pressure.

Fig. 6.19 Case A2 results: (a) Flame edges at four times instants. (b) Predicted $p'$ per flame element height at each instant.
Fig. 6.20 The acoustic pressure during pocket pinch for Case A2, measured at $d_{mic}=100$ mm (solid line). The predicted changes in the acoustic pressure using Equation 6.19 and $L_d = 1.11$ mm (circles). The predicted changes in the acoustic pressure using Equation 6.19 and $L_d = 0$ mm (triangles). The pressure waveforms are vertically aligned to $p' = 0$ at $t = -1.75$ ms. Changes in the acoustic pressure over the three marked time intervals are given in Table 6.6.

are shown. The points corresponding to the images shown in Figures 6.18 and 6.19 are indicated by filled markers. As is evident from Figure 6.20 almost no change in the acoustic pressure is predicted during tunnel narrowing if the displacement speed remains constant. The reasons for this have been discussed in Section 6.2.2. In the remainder of this section, predictions of the acoustic pressure therefore use the measured $L_d$.

Three time intervals, bounded by the instants shown in Figure 6.18 and indicated on Figure 6.20, are now considered. The first time interval has a 1 ms duration starting 1.75 ms prior to $t_{sep}$. The second time interval has a 0.5 ms duration starting 0.75 ms prior to $t_{sep}$. The third time interval has a 0.1 ms duration starting 0.25 ms prior to $t_{sep}$. Table 6.6 compares predictions of the change in the acoustic pressure over each time interval to the measured changes for Case A2. The magnitude of the percentage error between the predicted and measured pressure changes is 20% in the first time interval, 28% in the second time interval, and 50% in the third time interval. The magnitude of the change in the acoustic pressure is underpredicted in the first time interval but overpredicted in the second and third time interval.
Table 6.6 Predicted and measured changes in the acoustic pressure for Case A2 at a distance of 100 mm over three time intervals.

<table>
<thead>
<tr>
<th>Time Interval</th>
<th>predicted $\Delta p'$ (mPa)</th>
<th>measured $\Delta p'$ (mPa)</th>
<th>% error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t=-1.75$ to $t=-0.75$ ms</td>
<td>-12.7</td>
<td>-15.9</td>
<td>-20%</td>
</tr>
<tr>
<td>$t=-0.75$ to $t=-0.25$ ms</td>
<td>-14.0</td>
<td>-10.9</td>
<td>+28%</td>
</tr>
<tr>
<td>$t=-0.25$ to $t=-0.10$ ms</td>
<td>-11.7</td>
<td>-7.8</td>
<td>+50%</td>
</tr>
</tbody>
</table>

Table 6.7 Predicted and measured changes in the acoustic pressure for Cases D1 and E1 at a distance of 100 mm over three time intervals.

<table>
<thead>
<tr>
<th>Case</th>
<th>$\phi$</th>
<th>Time Interval</th>
<th>predicted $\Delta p'$ (mPa)</th>
<th>measured $\Delta p'$ (mPa)</th>
<th>% error</th>
</tr>
</thead>
<tbody>
<tr>
<td>D1</td>
<td>0.80</td>
<td>$t=-1.75$ to $t=-0.75$ ms</td>
<td>-11.2</td>
<td>-16.9</td>
<td>-34%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$t=-0.75$ to $t=-0.25$ ms</td>
<td>-17.4</td>
<td>-16.5</td>
<td>+5%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$t=-0.25$ to $t=-0.10$ ms</td>
<td>-8.6</td>
<td>-10.4</td>
<td>-17%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$t=-1.75$ to $t=-0.75$ ms</td>
<td>-17.4</td>
<td>-15.6</td>
<td>+12%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$t=-0.75$ to $t=-0.25$ ms</td>
<td>-18.9</td>
<td>-20.3</td>
<td>-7%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$t=-0.25$ to $t=-0.10$ ms</td>
<td>-17.8</td>
<td>-13.7</td>
<td>+30%</td>
</tr>
</tbody>
</table>

The underprediction in the first time interval is due to changes in the flame outside the region considered. During this interval, the rarefaction generated per flame element height at the narrowest part of the flame neck reaches only approximately double that near the base of the flame. Over the second time interval, because of the larger amplitude sound generated per flame element height associated with the smaller flame radii, the change in the acoustic pressure over this time interval is dominated by changes in the flame neck. It will be shown in the following section that a majority of the overprediction in the second time interval is due to the contribution of the near-hemispherical flame tip. The overprediction in the third time interval is due to the flame neck radius being on the order of $\delta_{th}$ at the narrowest point. Error from approximating the flame as infinitely-thin becomes significant and the Markstein relation does not accurately model the flame acceleration when $r_c < \delta_{th}$.

The same method was applied to Cases D1 and E1, with equivalence ratios of $\phi = 0.80$ and $\phi = 0.85$ respectively. Figure 6.21 shows the measured acoustic waveform during tunnel narrowing for both cases. Table 6.7 compares the predicted and measured changes in the acoustic pressure. The prediction error is between -34% and +30%. There are two sources of sound, not originating from the annihilation region, that contribute to the measured sound for Cases D1 and E1 to a greater degree than for Case A2: (i)
A New Model of Sound Production from Laminar Premixed Flames

\[ \phi = 0.80 \]

\[ \phi = 0.85 \]

Fig. 6.21 The acoustic pressure during pocket pinch for Cases D1 and E1, measured at \( d_{mic} = 100 \text{ mm} \). The pressure waveforms are vertically aligned to \( p' = 0 \) at \( t = -1.75 \text{ ms} \). Changes in the acoustic pressure are indicated over the three time intervals given in Table 6.7.

direct sound from the speaker, and (ii) sound from the evolution of circumferential cusps. The circumferential cusps feature greater flame curvature in the axial direction at higher equivalence ratios.

Lastly, Case A3 is considered. Case A3 is at the same equivalence ratio as Case A2 \( (\phi = 0.75) \) but is forced at a higher amplitude and features a larger separated pocket. Figure 6.22 shows the measured acoustic pressure over the tunnel narrowing process. Table 6.8

\[ \phi = 0.75 \]

Fig. 6.22 The acoustic pressure during pocket pinch for Case A3, measured at \( d_{mic} = 100 \text{ mm} \). The pressure waveforms are vertically aligned to \( p' = 0 \) at \( t = -1.75 \text{ ms} \). Changes in the acoustic pressure are indicated over the three time intervals given in Table 6.8.
Table 6.8 Predicted and measured changes in the acoustic pressure for Case A3 at a distance of 100 mm over three time intervals.

<table>
<thead>
<tr>
<th>Time Interval</th>
<th>predicted $\Delta p'$ (mPa)</th>
<th>measured $\Delta p'$ (mPa)</th>
<th>% error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t=-1.75$ to $t=-0.75$ ms</td>
<td>-11.7</td>
<td>-12.4</td>
<td>-6%</td>
</tr>
<tr>
<td>$t=-0.75$ to $t=-0.25$ ms</td>
<td>-14.3</td>
<td>-11.3</td>
<td>+27%</td>
</tr>
<tr>
<td>$t=-0.25$ to $t=-0.10$ ms</td>
<td>-11.4</td>
<td>-8.0</td>
<td>+43%</td>
</tr>
</tbody>
</table>

compares the predicted and measured changes in the acoustic pressure. All predicted and measured changes in acoustic pressure are within 20% of those for Case A2. The reason for this similarity is apparent from plots of the sound generated per vertical flame length, shown in Figure 6.24. Comparison of Figures 6.19 and 6.24 shows that at all instants through the tunnel narrowing process, the generated acoustic pressure per vertical length at the narrowest part of the flame neck is approximately the same for Cases A2 and A3. This is because the minimum flame radius is almost identical for the two cases at an equal time relative to the separation reference time.

**Separation Events**

During the terminal moments of tunnel narrowing there is an axisymmetric flame annihilation event called the separation event. Flame annihilation refers to the terminal moments of flame processes that involve the consumption of reactants between oppositely propagating flames. Flame annihilation events are defined in this work as beginning when oppositely propagating flames are separated by a distance of $2\delta_{th}$. At this point, the flame propagation behaviour is modified by its proximity to another flame.

It can be seen in the measured pressure waveforms that there is a rapid drop in the acoustic pressure during the pocket separation event. The majority of the drop is generated within 50 $\mu$s of the separation reference time. This observation is consistent with the prediction obtained from the infinitely-thin flame model, i.e., that the magnitude of the rarefaction will increase as the flame approaches the symmetry axis. Following the separation reference time, i.e. during cusp recovery, the relatively high magnitude rarefaction is sustained for approximately 100 $\mu$s, and then decreases relatively slowly over several hundred microseconds. The majority of the sound is therefore more accurately attributed to the cusp recovery process, rather than the tunnel narrowing process.

Part of the rarefaction during cusp recovery can be explained by flame elements in the near-spherical flame tip and by the conical flame elements that have a distance from the symmetry axis that is greater than the thermal flame thickness. However, the flame development near and at the cusps appears to also be a significant source of sound.
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Fig. 6.23 Chemiluminescence images of the annihilation region of flames during tunnel narrowing for Case A3. Red lines show the manually determined flame positions used in predictions of changes in the acoustic pressure.

Fig. 6.24 Case A3 results: (a) Flame edges at four times instants. (b) Predicted $p'$ per flame element height at each instant.
6.6 Pocket Consumption and Pocket Burn-out Events

Fig. 6.25 Representative pressure waveforms (gray, solid lines) measured during pocket consumption at a distance of 100 mm. Pressure waveforms predicted assuming $s_d$ follows Markstein’s relation (dashed line) and assuming constant $s_d$ (dash-dotted line) are also shown. $s_q$ is assumed constant for both cases.

Very high flame curvature is observed at the cusps and cusp recovery could therefore be considered a continuation of the flame annihilation event. The sustaining of a relatively high amplitude rarefaction during cusp recovery is therefore consistent with the presented model, though accurate prediction of the sound generated during cusp recovery requires a model that is able to account for the development of the flame cusps.

6.6 Pocket Consumption and Pocket Burn-out Events

Figure 6.25 shows the predicted and measured pressure waveforms during pocket consumption and the burn-out event for three equivalence ratios. The heat-release speed has been assumed constant for the computation of the predicted waveforms. Agreement is best for the $\phi = 0.75$ case (Case A3). A slowly decreasing rarefaction magnitude prior to the burn-out event is observed, consistent with predictions. From 1-0.05 ms prior to the annihilation reference time, the average error in the predicted acoustic pressure is 11% and
the maximum error is 31%. The increase in the acoustic pressure from when the flame radius is predicted to be equal to \( \delta_{th} \), to 0.5 ms after the annihilation reference time, is overpredicted by 19%.

Errors in the predicted acoustic waveform prior to the burn-out event may be due to the non-sphericity of the pocket and changes in the heat-release speed. The small increase in the rarefaction magnitude during the burn-out event is not predicted by the infinitely-thin flame model and the rarefaction magnitude at the annihilation reference time is therefore underpredicted by 28%. However the details of the flame annihilation event do not appear to be significant to the generated sound. This is because the duration of the increase is only approximately 50 \( \mu \)s and the magnitude of the additional pulse is small in comparison to the sustained rarefaction due to the prior pocket consumption. The measured waveforms also show that the increase in acoustic pressure at the burn-out event does not occur instantaneously, but occurs over approximately 0.5 ms. This is because the heat release profile has a finite thickness and a long tail towards the burned gas. Figure 6.25 also shows that almost no sound generation is predicted if a constant displacement speed is assumed. The acoustic pressure 1 ms prior to the burn-out event is underpredicted by greater than 90% when assuming a constant displacement speed and the error increases further as the burn-out event is approached.

For the \( \phi = 0.85 \) case (Case E2), the burn-out event occurs within 1 ms of the separation event and the measured sound is likely to be affected by the cusp recovery. In addition, the circumferential cusps are more prominent in the \( \phi = 0.80 \) and \( \phi = 0.85 \) cases and are likely to contribute to the emitted sound. This is supported by the large changes in the acoustic pressure that are observed more than 0.5 ms after the burn-out events. Direct sound from the speaker driver is also more significant for these two cases. Nonetheless, for all three equivalence ratios the predicted rapid increase in the acoustic pressure following the burn-out event is clearly observed.

**The Contribution of the Near-Hemispherical Flame Tip During Tunnel Narrowing**

In contrast to the increasing rarefaction generated by conical flame elements, the magnitude of the rarefaction produced by inwardly propagating spherical flames decreases leading up to the annihilation event. During tunnel narrowing, the flame tip is approximately hemispherical. The flame tip is therefore predicted to contribute to a positive change in the acoustic pressure over any time interval during tunnel narrowing. The derived waveforms for pocket consumption can be used to estimate this contribution over the time interval from 0.75 ms prior to \( t_{sep} \) to 0.25 ms prior to \( t_{sep} \). For Case A2, the burn-out event occurs 0.47 ms after \( t_{sep} \). The predicted change in the rarefaction generated by a spherical reactant pocket from 1.22 ms prior to the burn-out event to 0.72 ms prior to the burn-out event is
4.4 mPa. The additional +2.2 mPa for Case A2 reduces the error in the predicted pressure change from +28% to +8%. For Case A3, the burn-out event occurs 1.95 ms after the separation. The predicted change in the rarefaction generated by a spherical reactant pocket from 2.70 ms prior to the burn-out event to 2.20 ms prior to the burn-out event is 3.24 mPa. The additional +1.6 mPa for Case A3 reduces the error in the predicted pressure change from +27% to +12%.

### 6.7 Radial Emission Intensity Profiles

Measurements of the flame position during tunnel narrowing were used in this chapter to measure the flame displacement speed and to enable the comparison of measurements and predictions of the emitted sound. The position of peak chemiluminescent emission intensity across the flame is used in this work as a marker for the flame position. Chemiluminescence images consist of path integral measurements of emission intensity and the flame radius measured by intensity peaks in images does not in general coincide with the radius obtained from peaks in the deconvolved emission profile. In this Section, radial emission profiles are therefore obtained by Abel deconvolution of flame images. Measurements of the thickness of the chemiluminescent emission zone are also obtained. The results have implications for the limitations of infinitely-thin flame models in predicting sound generation and are also relevant to the subsequent chapter on the capabilities of non-axisymmetric flame tomography using visible chemiluminescence.

To determine the radial emission profile the Abel deconvolution algorithm described in Section 3.4 is applied to images of the tunnel narrowing process. Abel deconvolution is sensitive to image noise. Average projection profiles of the narrowest part of the flame neck were therefore created using up to 50 individual images. For this purpose, images with a pixel resolution of 18 µm/pixel were obtained in the 0.3 ms interval prior to \( t_{sep} \). The position of the narrowest part of the flame neck and the position of the symmetry axis was determined in individual images using the method demonstrated in Figure 6.16. Individual images taken at the same time (+/- 10 µs) relative to \( t_{sep} \) were then averaged to create reduced noise images. Figure 6.26 shows averaged images through the terminal 210 µs of tunnel narrowing for three equivalence ratios (\( \phi = 0.75 \), \( \phi = 0.80 \), and \( \phi = 0.85 \)). The timing indicated at the bottom of Figure 6.26 is the time at the start of the image intensifier gate relative to \( t_{sep} \).

Figure 6.27 shows the identified position of the narrowest part of the flame neck on one of the averaged images. The projection profile taken from the image at this vertical position and the deconvolved profile are also shown. The Abel deconvolution shows the majority of the visible light to be emitted from a narrow region at the edges of the flame neck, with a Full Width at Half-Maximum (FWHM) of approximately 150 µm. The fluctuations in
Fig. 6.26 Average images of the flame neck from forced flame cases A3, D2, and E1, at 40 μs intervals during tunnel narrowing and the separation event.
6.7 Radial Emission Intensity Profiles

Fig. 6.27 Left: Image of a flame neck from a $\phi = 0.75$ flame, 120 $\mu$s prior to $t_{sep}$. Right: Emission intensity projection (solid line) from the position indicated on the image by the dashed line, and the deconvolved emission profile (dashed line).

the emission intensity near the origin are artefacts caused by the increasing sensitivity of Abel deconvolution algorithms to noise in the projection data near the symmetry axis.

Deconvolved radial emission profiles were obtained at 10 $\mu$s intervals leading up to $t_{sep}$. The distance between the profile peaks was measured using both the projected and deconvolved profiles. Figure 6.28 shows the results. The flame radius measured from the deconvolved profile is approximately 30-90 $\mu$m greater than that measured using the projection profile and this does not change significantly during tunnel narrowing.

At $t = -10$ $\mu$s, the distance between the peaks of the deconvolved profile is less than 0.2 mm for all three cases. At this point, the emission distributions have merged to a significant degree. At this time the projection profile has a single peak located at the symmetry axis for the $\phi = 0.75$ and $\phi = 0.80$ cases. The flame radius using the projection profile is therefore zero. However, deconvolution still resolves two peaks. 10 $\mu$s later, the peak of the deconvolved profile is also at the symmetry origin.

The FWHM of the deconvolved profile is also shown in Figure 6.28. At each instant, the mean FWHM from the two distributions obtained from each half of a flame image is plotted. There is no clear increase or decrease in flame thickness through the separation events. At $t = -20$ $\mu$s for $\phi = 0.75$, and $t = -30$ $\mu$s for $\phi = 0.80$ and $\phi = 0.85$, the distributions have merged to the point where the emission intensity at the symmetry axis is greater than half the peak value, and the FWHM is no longer defined.
Fig. 6.28 Measurements of chemiluminescent emission profiles during tunnel narrowing and the separation event. Top row: The flame radius determined by the projected emission profile (solid circles) and deconvolved profiles (open circles). Middle row: Full-width at half-maximum measurements from deconvolved emission profiles. Bottom row: The average distance between peaks in the projected and deconvolved emission profiles.
6.8 Implications for Sound Generation in Turbulent Flames

As has been demonstrated in this chapter, the contribution of spherical and conical flame elements to the time derivative of the volume-integrated heat release rate can be accurately approximated using the time derivative of the product of flame surface area and the flame heat-release speed. A more general formulation for the contribution per flame surface area can be expressed in terms of flame stretch ($\kappa$) and the flame heat-release speed:

$$\frac{1}{A} \frac{d(Asq)}{dt} = sq\kappa + \frac{dsq}{dt}$$ \hspace{1cm} (6.21)

It has been shown that this approach predicts the sound from forced, laminar, flames when the mean curvature is less than approximately $1/\delta_Q$. The same approach may therefore yield reasonable results when applied to acoustically compact, turbulent, premixed flames. Negative (rarefaction) contributions from parts of the flame surface under high negative curvature have been demonstrated to change rapidly. Positive contributions due to flame surface creation at the flame base and positive flame stretch are known to change more slowly and it is plausible that the net positive contribution may remain relatively constant. Similarly it is plausible that the net contribution from flame surface elements undergoing low negative flame stretch will also remain relatively constant. If this is the case, sound emission from turbulent flames may be modelled with acceptable accuracy by considering only the flame surface undergoing high negative flame stretch. The predicted waveform from just these flame elements will be almost always negative. However the mean may be shifted to the ambient pressure to account for the almost constant net positive contribution of the rest of the flame.

Flame annihilation events feature critical points in scalar fields such as temperature, species mass fraction or progress variables where the values of these variables are intermediate between the burned gas and unburned gas values. These events are therefore readily identified in numerical simulations. Tunnel narrowing and pocket consumption processes both terminate in annihilation events. Flame annihilation events may therefore be used to identify regions of the flame where the sound generating processes are occurring. Significant sound production has also been observed immediately following separation events. It was demonstrated in the previous chapter that axisymmetric cusps can be generated without an annihilation event. However, the experimental conditions required to produce such an event were highly contrived and are expected to be rare in turbulent flames.

One type of flame annihilation event, known as tunnel formation, was not observed in this study. Tunnel formation refers to the formation of a tunnel of burned gas that usually
occurs when low-curvature flames collide. It is an approximately planar-symmetric flame annihilation. An interesting feature of tunnel formation is that for flames with positive Lewis numbers a short duration compression wave is expected to precede a rarefaction. Cusps are also produced by tunnel formation processes. Tunnel formation typically generates an expanding tunnel of burned gases surrounded by outwardly propagating wedge-shaped flame cusps. Unlike the pointed flame cusps produced by separation events, these cusps feature high curvature only at the cusp and at a distance of several flame thicknesses away from the cusps the flame may undergo very low stretch. Given the infinitely-thin flame assumption used in the proposed model, and the extremely high curvature at flame cusps, the contribution of flame cusps in this case is likely to require separate consideration.

In the subsequent chapter, the equipment requirements to tomographically reconstruct the visible chemiluminescence field with sufficient spatial resolution to resolve tunnel narrowing, cusp recovery, and pocket consumption processes are investigated. Measurement algorithms are also developed to measure the quantities relevant to sound generation.

### 6.9 Summary

In previous studies, the rate of change in flame chemiluminescence has been correlated with the generated sound. Flame images have also been used to identify the flame geometries associated with the generation of large magnitude acoustic pressure fluctuations. However, these observations provide little insight into how flames with certain geometries generate sound, and give no indication of the significance of flame acceleration.

In this chapter, a novel model has been presented for the generation of direct sound by acoustically-compact, axisymmetric, forced, laminar, premixed flames. The model involves estimating the time derivative of the heat release rate from flame elements. The integral of the contribution of all flame elements yields the radiated sound. The model enables identification of the flame elements that are responsible for most of the generated sound and also clarifies the relative significance of flame acceleration and flame geometry. The model was used to predict sound emission during tunnel narrowing and pocket consumption processes and the predictions were compared with measured waveforms obtained from axisymmetric, forced, laminar, premixed, propane-air flames.

In lean, propane-air flames, the model predicts that acoustically-compact, inwardly-burning, conical flame elements generate a sustained rarefaction at large flame radii. The rarefaction increases in magnitude as the flame curvature increases, and the flame element annihilates at the symmetry axis with a positive step in the acoustic pressure. Flame necks were modelled as a collection of conical flame elements. Predicted and measured changes in the acoustic pressure were compared over several time intervals prior to the
6.9 Summary

Changes in the rarefaction magnitude during tunnel narrowing were consistently predicted to within 35% of measured values when the flame radius was greater than $\delta_{th}$. Accounting for flame acceleration associated with flame curvature was found to be important for accurate prediction of the emitted sound. In fact, assuming a constant flame displacement speed yields the prediction that the acoustic pressure remains constant during tunnel narrowing, in contradiction with the observed increasing rarefaction.

The consumption of a spherical reactant pocket is predicted to generate a sustained acoustic rarefaction that decreases in magnitude as the pocket is consumed. A distinctive positive change in the acoustic pressure is predicted to occur immediately following the burn-out event. Predicted acoustic waveforms were compared to measurements of the acoustic pressure during the consumption of pockets formed in forced, laminar flames. It was challenging to distinguish the sound generated by pocket consumption from the sound generated by the cusp recovery that immediately follows pocket separation. This was best achieved at the lowest equivalence ratio included in the study, $\phi = 0.75$. For this case, the predicted rarefaction prior to the pocket burn-out event, and the relatively fast decrease in the rarefaction magnitude following the burn-out event, were both observed. From 1 ms to 0.05 ms prior to the pocket burn-out event the error in the predicted acoustic pressure did not exceed 31%. Flame acceleration was again found to be important for accurately predicting the sound. In models assuming a constant flame displacement speed, the acoustic pressure during the final 1 ms of pocket consumption is underpredicted by more than 90% and the positive change in the acoustic pressure observed after the burn out event is not captured.

The main limitation in the proposed model is its questionable applicability to flame elements with mean curvature exceeding $1/\delta_{th}$. This is due to two approximations used in the model: an infinitely-thin heat release region, and Markstein’s relation for flame speeds. Very high flame curvature occurs during flame annihilation events, such as separation events and pocket burn-out events. During the pocket burn-out event, experimental observations show an approximately 50% increase in the rarefaction magnitude in the terminal 100 $\mu$s. This is not captured by the model. However, due to the short duration of this additional pulse, it is considered to be relatively insignificant. A considerably larger magnitude rarefaction is generated by the pocket separation event. The largest gradient in the acoustic pressure is generated at the moment the peak of the heat-release rate profile reaches the symmetry axis, termed the separation reference time. Despite this, flame annihilation prior to the separation reference time is not considered a significant source of sound, due to its short duration. However, the large magnitude rarefaction is sustained for approximately 100 $\mu$s after the separation reference time and gradually decreases in magnitude over the following 1-2 ms. The generation of this rarefaction is attributed to cusp recovery. The maximum mean curvature at the flame cusps exceed $1/\delta_{th}$ and further development of the
model would therefore be required to predict the sound generated by flame elements at the flame cusp.
Chapter 7

Measurements from Flame Chemiluminescence Tomography of Forced Laminar Premixed Propane Flames


7.1 Introduction

In this chapter, algorithms for measuring the flame surface area, flame surface curvature, the normal component of the flame propagation velocity (surface speed), and the flame thickness are proposed and demonstrated on the time-resolved chemiluminescence field of two non-axisymmetric, forced, laminar, premixed, propane-air flames at different equivalence ratios: $\phi = 0.71$ and $\phi = 0.84$. Flame Chemiluminescence Tomography (FCT) is used to determine the 3D chemiluminescence fields. This work includes the first reported measurements of flame thickness and surface speed over the entire surface of an instantaneous, unsteady, non-axisymmetric, premixed flame. Measurements are obtained from flame chemiluminescence fields reconstructed using between 3 and 36 views and the relative sensitivity of the measurements to the number of views is assessed.

For unsteady, wrinkled, premixed flames, there are only two previous instances in the literature where 3D flame measurements at points on an identified “flame surface” have been made using FCT. Ishino et al. (2009) report measuring the flame propagation
velocity on a small part of a turbulent flame surface reconstructed using 20 equally-spaced, coplanar views, but do not report their methods, and Ma et al. (2016b) recently reported 3D curvature measurements on a turbulent slot flame calculated on an isosurface of the CH* chemiluminescence field, reconstructed using 6 coplanar views.

The results presented in this chapter: (i) show the relative sensitivity of the measured quantities to the number of views used in the reconstruction, and (ii) provide an estimate of how many views are likely to be required to obtain statistical measurements of these quantities in flames of similar character, that is, weakly-turbulent, lean, premixed flames of similar dimensions. Such measurements could be used to identify the dominant sound-producing turbulent flame processes and are likely to have many other applications in premixed combustion. To reconstruct the “instantaneous” flame surface of a turbulent flame, as opposed to the turbulent flame brush, all views must be acquired simultaneously. Unless mirror arrangements are used to acquire multiple views on a single camera, the number of cameras required equals the number of views required. The affordability and sensitivity of high-speed cameras continues to increase and the results of this work clarify the equipment requirements for future studies of turbulent flames.

7.2 Test Cases and Flame Imaging

Two lean, premixed flames of different stoichiometry are used as test cases. These are referred to as Case 1 and Case 2 (Table 7.1), and were chosen given their strong flame dynamics, including annihilation events that have been investigated as sources of sound in earlier chapters. The forcing frequency was 50 Hz.

The flame rig, tomographic reconstruction method, and flame imaging are described in Section 3.5. Images from 36 coplanar views, equally-spaced through 180 degrees, were taken of both flames. One hundred phases of each flame cycle were imaged, which is equivalent to imaging at 5,000 Hz. At each phase and for each view, three images were

Table 7.1 Key experimental parameters for Cases 1 and 2. The mean bulk velocity is estimated based on the measured mass flow rates and the component gas densities at 25°C and 1 atm. Reynolds number is based on the length of the square port edges.

<table>
<thead>
<tr>
<th></th>
<th>Case 1</th>
<th>Case 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>φ</td>
<td>0.71</td>
<td>0.84</td>
</tr>
<tr>
<td>Air flow rate</td>
<td>0.53</td>
<td>0.94</td>
</tr>
<tr>
<td>(gs⁻¹)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Propane flow rate</td>
<td>0.024</td>
<td>0.051</td>
</tr>
<tr>
<td>(gs⁻¹)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Unburned gas mean bulk velocity</td>
<td>0.87</td>
<td>1.6</td>
</tr>
<tr>
<td>(ms⁻¹)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Reynolds number</td>
<td>1100</td>
<td>1900</td>
</tr>
</tbody>
</table>
acquired. For 36 views, this amounts to 10,800 images acquired for each of the flame cycles.

7.3 Post-Processing Algorithms

7.3.1 Flame Surface Area

MATLAB’s isosurface function was applied to the reconstructed emission fields. The algorithm produces a set of triangular faces, each defined by their vertices \((v_1, v_2, v_3)\). The surface area of face \(p\) is then

\[
A_p = \frac{|(v_2 - v_1) \times (v_3 - v_1)|}{2}.
\]  

(7.1)

Two isosurfaces are created either side of the peak emission layer and the flame surface area is therefore \(A_f = \frac{1}{2} \sum_p A_p\). Three dimensional mean filtering was applied to the reconstructed fields prior to computing the isosurfaces. A sensitivity analysis was undertaken to determine appropriate values for the filter kernel size and the isosurface value. A filter kernel size of 11 voxels was used and the isosurface value is 15% of the maximum emission intensity \(I_{\text{max}}\) at a chosen instant in the cycle. A wide range of isosurface values, from less than 7% to greater than 20% of \(I_{\text{max}}\), were found to result in computed areas within 1%, though a greater amount of filtering is required when using smaller isosurface values.

7.3.2 Local Surface Fitting

Measuring curvature, surface speed, and thickness requires identifying a surface to represent the flame location. This study defines the flame surface as the set of points that are at the peak of the emission profile along a flame surface normal. This is chosen instead of the more common isosurface method to avoid processing issues, such as sensitivity to thresholds levels in regions of high curvature, and failure of surface identification near flame holes.

The algorithm comprises two key steps:

1. identifying points near the flame surface;
2. locally fitting polynomial surfaces to the points.

The flame curvature and surface normal direction can then be determined from the coefficients of the fitted polynomial surface.
Identifying points near the flame surface

Voxels identified as near the flame surface are referred to as ‘flagged’ voxels. The algorithm for identifying voxels near the flame surface is described on a 2D cross-section to simplify the description. The emission intensity and gradient magnitude on a 2D cross-section are shown in Figure 7.1. As in the 3D field, the emission intensity in this 2D cross-section is characterised by thin continuous areas of high intensity with low gradients in the direction tangential to the flame surface. High intensity regions are observed in regions of high curvature.

The proximity of a pixel to the flame surface was determined using a two part algorithm. The first part (Algorithm A) flags a given pixel if the difference in direction of the gradient between any of the four opposing pairs of pixels is greater than 90 degrees (Figure 7.1c). Gradient vectors typically point towards the pixels that are flagged by Algorithm A. The exception is in regions of high intensity, such as near the flame tip (Figure 7.1d). Here the gradient vectors can point away from the flagged pixels and towards the flagged pixels in the high intensity region. Algorithm A then fails to flag pixels near these regions, as shown in the dashed box in Figure 7.1c.

The second part (Algorithm B) is then applied (Figure 7.2). This algorithm only considers pixels neighbouring those that have already been flagged. If the gradient vector at these pixels is directed away from previously flagged pixels then these pixels are also flagged. Algorithm B loops until no further pixels are flagged, proceeding from the ‘typical’ flamelet regions that were identified by Algorithm A to the crest of the more highly curved flame regions, following a path with the highest emission intensity.

This two part algorithm is generalised for the 3D data in this study by using the neighbouring 26 voxels (13 opposing pairs) for Algorithm A. Algorithm B was unchanged.

Locally Fitting Polynomial Surfaces to the Identified Points

The linear regression method described by Flynn and Jain (1989) has been used to locally fit polynomial surfaces of the form:

\[ h(x',y') = a_{30}x'^3 + a_{21}x'^2y' + a_{12}xy'^2 + a_{03}y'^3 + a_{20}x'^2 + a_{11}xy' + a_{02}y'^2 + a_{10}x' + a_{01}y' + a_{00} \]  

(7.2)

Each local surface fit is centred on one of the flagged voxels. The centre points of other flagged voxels within a specified radius, called the inclusion sphere, are included in the surface fit. Prior to the surface fit, the coordinate system is transformed to put the centre of the inclusion sphere at the origin and rotated such that the support plane \((x',y')\) is perpendicular to the third principal component direction (Jolliffe, 2002). This plane is parallel to the plane for which the sum of squared perpendicular distances between the
Fig. 7.1 Example of results from Case 2 reconstructed using 36 views; a) vertical cross-section of normalized $\hat{I}$, b) magnitude of the normalized gradient, c) pixels flagged by algorithm A with the box indicating a region where the contiguous flame surface was unsuccessfully identified and d) the gradient magnitude and direction.

Fig. 7.2 Pixels identified by Algorithms A and B inside the region indicated in Figure 7.1c.
points and the plane is a minimum. Rotation of the coordinate system in this manner increases the probability of a good surface fit.

A root mean square error (RMSE) criteria of 1.4 voxel lengths and a maximum residual (MR) criteria of 4 voxel lengths were used as goodness of fit criteria. An inclusion radius of 20 voxel lengths was used in the first surface fit and the inclusion radius was decreased by a factor of $2^{1/4}$ until the goodness of fit criteria were met or until an inclusion radius of 10 voxel lengths was reached. This factor was used for reducing both the computational effort and the sensitivity of the surface fit to the scatter of the points around the flame surface. Once a surface meets the goodness of fit criteria, the inclusion volume centre point, originally at $(x', y', z') = (0, 0, 0)$ is projected onto the fitted surface to obtain the flame surface point at $(x', y', z') = (0, 0, h(x', y'))$.

### 7.3.3 Calculating the Curvature

The principal curvatures $(k_1, k_2)$ at a point on a surface $\sigma(x', y')$ are the maximum and minimum normal curvatures. The definition of the mean curvature that has been used in previous studies measuring 3D curvature is the mean of the principal curvatures (e.g. Ma et al., 2016b) and that definition is adopted in this chapter. In other contexts the mean curvature is often defined as the sum of the principal curvatures, as was done in Chapters 4 and 6. The principle curvatures of a surface are given by the roots of the equation:

$$
\begin{vmatrix}
L - kE & M - kF \\
M - kF & N - kG
\end{vmatrix} = 0 \quad (7.3)
$$

where $E, F, G$ and $L, M, N$ are the coefficients of the first and second fundamental forms respectively (Pressley, 2001). For a surface $\sigma(x', y') = (x', y', h(x', y'))$ these coefficients are:

$$
E = 1 + h_x^2 \quad (7.4) \quad L = \frac{h_{x'y'}^2 + h_{yy}^2}{\sqrt{1 + h_{xx}^2 + h_{yy}^2}} \quad (7.5)
$$

$$
F = h_x h_y \quad (7.6) \quad M = \frac{h_{x'y'}^2}{\sqrt{1 + h_{xx}^2 + h_{yy}^2}} \quad (7.7)
$$

$$
G = 1 + h_y^2 \quad (7.8) \quad N = \frac{h_{x'y'}^2}{\sqrt{1 + h_{xx}^2 + h_{yy}^2}} \quad (7.9)
$$

### 7.3.4 Calculating the Normal Component of the Flame Propagation Velocity

The velocity of the flame surface in its normal direction at some point $x_0$ is estimated from the distance between intersections of that normal with the flame surfaces that are 200 microseconds either side of the current instant. See Figure 7.3 for a 2D example. The
7.3 Post-Processing Algorithms

flame surface normal at $x_0$ is $(h_x', h_y', -1)$. Starting from $x_0$ in the earlier (or later) instant, voxels intercepted by the surface normal are stepped through in both directions. When a flagged voxel is encountered the intersection point of the normal line and the surface tangent plane at the flame surface point associated with the intercepted voxel is calculated. If a flagged voxel is not encountered when moving along the flame surface normal, the speed is not calculated at that point.

The flame speed ($s_u$) and flame propagation velocity ($V_f$) are related by:

$$V_f = v + s_u n$$

(7.10)

where $v$ is the unburned gas velocity. The surface speed measured in this paper is an estimate of $V_f \cdot n$ (Law, 2006; Poinsot and Veynante, 2005).

7.3.5 Calculating the Flame Thickness

The Full Width at Half Maximum (FWHM) of the emission profile in the flame surface normal direction is calculated and reported as the flame thickness. Trilinear interpolation was used to determine the emission intensity in steps of 19 µm, equal to one quarter of the voxel length. As can be seen from Figure 7.4, filtering of the reconstructed emission field significantly affects the measured profile maximum and FWHM. Thickness measurements are therefore derived from the unfiltered emission field.

7.3.6 Determining the Unburned and Burned Gas Sides

Flame velocity and curvature measurements are only useful if the side of the unburned gas is known. A segmentation method was applied to the entire reconstruction volume to assign voxels to the flame region, unburned gas region, or the burned gas region. The flame region was identified first by thresholding the emission intensity field. For the two cases studied here, there is only one connected region of burned gas. Voxels in the top corners of
the reconstruction volume are known to be located in the burned gas region. All voxels that are not part of the flame region or separated from the voxels in the top corners by the flame region are assigned to the burned gas region. All remaining unassigned voxels are assigned to the unburned gas region. The unburned gas side of the flame was then determined by analysing the assigned regions intercepted by the flame surface normal in both directions.

7.4 Results and Discussion

The effect of reducing the number of views used in FCT is first demonstrated qualitatively using a single cross-section from each case. The effect of reducing the number of views on measurements of flame surface area is then examined. In Section 7.4.3, the coincidence of the locally fitted polynomial surfaces with the layer of peak chemiluminescent emission is measured and is found to be acceptably small. Measurements of the mean curvature, the normal component of the flame propagation velocity, and the flame thickness are then presented and the sensitivity of the measurements to the number of views is quantified.

7.4.1 Reconstructed Emission Field

The emission field reconstructed using $x$ views is referred to as EF$_x$. A comparison between camera images and computed views from EF$_{36}$ and EF$_{3}$ is given in Figure 7.5. The observed high degree of similarity between computed and measured images is a necessary but not sufficient condition for accurate reconstructions. Sample cross-sections from EF$_{36}$ are shown for both cases in Figure 7.6. Thin, connected, high intensity areas
7.4 Results and Discussion

Fig. 7.5 A comparison of camera images (left images) and computed images from EF\textsubscript{36} (centre images), and EF\textsubscript{3} (right images).

Fig. 7.6 Sample unfiltered cross-sections of the emission field reconstructed using 36 views (EF\textsubscript{36}). The z values refer to the height above the burner lip. Dashed boxes indicate slices in Figure 7.7.

Fig. 7.7 A sample cross-section of the emission field indicated in Figure 7.6 and reconstructed using different numbers of equally-spaced views.

are observed in the cross-sections, characteristic of forced, laminar, premixed flames (Law and Sung, 2000).

The accuracy of the approximate solution computed using the MART algorithm is understood to generally increase as the number of views is increased (Frieder and Herman, 1971; Herman, 2009). Figure 7.7 demonstrates the effects of reducing the number of views using the sample cross-sections indicated in Figure 7.6 for each of the cases. Tomographic artefacts progressively appear, starting with breaks in the flame surface and followed by the appearance of lines perpendicular to the projection planes. The flame surface begins
Measurements from Flame Chemiluminescence Tomography of Forced Laminar Premixed Propane Flames

to lose coherence with fewer than nine views. A similar trend was observed for other cross-sections and at other instants in the forcing cycle, with Figure 7.8 showing that 3D isosurfaces of these two flames also feature these artefacts as the number of views is reduced.

7.4.2 Flame Surface Area Measurements

Figure 7.9 shows the flame surface area fluctuations ($A' / \bar{A} = (A - \bar{A}) / \bar{A}$) calculated using 36 views for 100 equally-spaced instants through the flame cycle for both cases. The flame surface area fluctuations are less than 2% for Case 1, despite large changes in the flame geometry. For Case 2, area fluctuations reach 6%. Also shown are the fluctuations obtained using different numbers of views at eight instants through the cycle. The cycle is well represented with as few as nine views for both cases.

The difference in the area fluctuation between the two cases is related not only to the amplitude of the velocity forcing but also the Strouhal number ($St = \omega R / S_L \cos \theta_0$). The flame speed for Case 2 is 39% higher than for Case 1. The Strouhal number is 28% lower as a consequence and the wavelength of the transverse wave along the flame surface is larger. The impact of these variations has been studied extensively in other works and so is not considered further (Ducruix et al., 2000; Karimi et al., 2009; Lieuwen, 2003, 2005, 2012; Schuller et al., 2003).

Table 7.2 shows $\bar{A}$ calculated from eight instants, using different numbers of views.

<table>
<thead>
<tr>
<th>Number of Views</th>
<th>36</th>
<th>18</th>
<th>12</th>
<th>9</th>
<th>6</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>18.9</td>
<td>18.9</td>
<td>19.0</td>
<td>19.0</td>
<td>19.2</td>
<td>18.9</td>
</tr>
<tr>
<td>Case 2</td>
<td>21.6</td>
<td>21.7</td>
<td>21.9</td>
<td>22.1</td>
<td>22.4</td>
<td>22.6</td>
</tr>
</tbody>
</table>

The computed flame surface area usually, but not always, increases as the number of views used in the reconstruction decreases. Figure 7.10 compares the flame surface area fluctuations, calculated at 100 instants through the cycle using 36 views, to fluctuations in the total chemiluminescent emission. The total chemiluminescent emission was obtained by summing the pixel counts in the flame images for each instant. There is close agreement, as has been observed previously in axisymmetric, forced, laminar flames Schuller et al. (2002).
Fig. 7.8 Selection of isosurfaces used to calculate the flame surface areas at phase angles 0°, 90°, 180°, and 270°. A semi-transparent rendering is used to show the isosurfaces on either side of the reaction layer.
7.4.3 Flame Surface Location

Flame curvature and flame speed are defined with reference to a flame surface. As defined in Section 7.3.2, the flame surface used in this work comprises the set of points located at the peak of the emission profile along the surface normal direction. To be consistent with this definition, the flame surface points identified on the locally fit surfaces should therefore be located at the peak of the profile. Figure 7.11 compares the distance from these flame surface points to the peak of the normal profiles using EF_{36}. The PDF of these measurements is given in Figure 7.12. The length of the stepping distance used in creating the profiles was 19 µm. 7 × 7 × 7 mean filtering was applied to the emission field prior to running the surface fitting algorithm. The distance to the profile peaks in both the unfiltered and filtered emission fields are shown. Using the filtered emission field, the distance to the profile peak exceeds 50 µm in fewer than 1% of the measurements. A total of 72% of the profile peaks are located on the unburned gas side of the fitted surface, suggesting an acceptably small bias. Figure 7.11a does not show any decrease in performance in the more curved regions.

Fig. 7.9 Flame surface area fluctuations measured using different numbers of views: solid line 36, O 18, □ 12, ▽ 9, △ 6, ▽ 4.

Fig. 7.10 Fluctuations in the flame surface area and total chemiluminescent emission.
7.4 Results and Discussion

Fig. 7.11 Distance along the flame surface normal from the flame surface point to the peak of the emission profile with: (a) No filtering of $I$; (b) 7x7x7 mean filtering. Positive sign indicates that the peak is located on the burned gas side of the identified surface point. The points shown are at an instant of Case 2 reconstructed using 36 views.

Fig. 7.12 PDF of the distance from flame surface points to the peak of the emission profile along the flame surface normal, with and without 7x7x7 mean filtering. Positive sign indicates that the peak is located on the burned gas side of the identified surface point. Measurements are from the reconstructed emission field of Case 2, using 36 views, at the instant shown in Figure 7.11.

The effect of filtering on the location of the profile peak is assessed by considering the distance to the profile peak in the unfiltered emission field. Due to noise in the unfiltered emission field, the standard deviation of the distance distribution increases from 19 $\mu$m for the filtered emission field to 46 $\mu$m. The PDF peak for the unfiltered emission field is located approximately 20 $\mu$m towards the unburned gas side. Figure 7.11b also does not reveal any decrease in performance in regions of high curvature. Importantly, both biases in Figure 7.12 are an order of magnitude smaller than the FWHM of the chemiluminescence profile.
7.4.4 Probability Density Functions of the Flame Curvature, Speed, and Thickness

Probability density functions (PDFs) are presented to assess convergence as the number of views used in the reconstruction varies. The overlapping coefficient (OVL) (Inman and Bradley Jr, 1989) is a measure for comparing two probability distributions,

$$\text{OVL} = \sum_i \min(P_i, Q_i) \times 100\%$$  \hspace{1cm} (7.11)$$

where \(P_i\) and \(Q_i\) are the \(i\)th element, corresponding to the \(i\)th bin, of two discrete probability distribution vectors \(P\) and \(Q\). An OVL of 100% denotes two identical PDFs.

The OVL reported for \(x\) views compares the distribution found using \(\text{EF}_x\) to the distribution found using \(\text{EF}_{16}\), which is the best available estimate of the true measurement distribution. The bin widths for these PDFs are based on estimates of the measurement resolution: \(0.01/\delta_{th}\) for the curvature measurements, \(0.1S_L\) for the speed measurements and \(0.05\delta_{th}\) for flame thickness.

The laminar flame properties used for normalization were estimated based on kinetic modelling. The AramcoMech 2.0 mechanism (Li et al., 2017) was used in a Chemkin premixed laminar flame-speed model, using an unburned mixture at 25°C and 100 kPa. The calculated planar laminar flame speeds were 0.23 ms\(^{-1}\) and 0.32 ms\(^{-1}\) for Case 1 and Case 2 respectively. The planar thermal flame thicknesses were 0.53 mm and 0.43 mm respectively, based on the definition \(\delta_{th} = (T_b - T_u)/|dT/dx|_{max}\), where \(\delta_{th}\) is the flame thickness, \(T_u\) and \(T_b\) are the unburned and burned gas temperatures, and \(dT/dx\) is the temperature gradient. The planar CH thicknesses (\(\delta_{\text{CH}}\)) were 119 \(\mu\)m and 113 \(\mu\)m respectively, based on the full width at half-maximum (FWHM) of the CH molar concentration profile.

To reduce the computational expense, flame measurements were taken at a sample of points. The sampling algorithm progresses down a randomly ordered list of the flame surface points, adding a point to the measurement sample if it is further than 5 voxel lengths from any flame surface point already selected for the sample.

Calculated OVLs are presented in Figure 7.13 for these three quantities. The observed trends and their causes are considered in the following sections.

7.4.5 Normalized Mean Curvature

Measurements of normalized mean curvature for both cases are shown in Figure 7.14 and the corresponding PDFs are given in Figure 7.15. Mean curvature measurements are normalized by the planar flame thermal thickness. From Figure 7.13, when increasing from...
6 to 18 views the OVL increases from 85% to 97% for Case 1 and from 81% to 94% for Case 2. Figure 7.14 demonstrates how these measurements are affected as the number of views varies. For Case 1, the results obtained with 36 views and 18 views are very similar, and this is reflected in the high OVL of 97%. With fewer views, measurement omissions and errors continue to increase. A similar trend is observed for Case 2.

7.4.6 Normalized Surface Speed

Measurements of the normal component of the propagation velocity are shown in Figure 7.16 and the corresponding PDFs in Figure 7.17. The mean unburned bulk gas velocities, when normalized by the laminar flame speed, are 3.8 for Case 1 and 4.8 for Case 2. Figure 7.13 shows the convergence of the speed distribution as the number of views varies. The OVL is generally higher for the speed measurement distribution than for the curvature distribution, indicating that surface speed can be measured with fewer views than curvature. This is despite higher numbers of measurement omissions for speed measurements. Omissions occur for both curvature and speed measurements when a poor surface fit is identified in the present instant. Speed measurements are also omitted when the surface normal does not intersect with the flame surface in the previous or next instants. Two factors are likely to be involved in the faster convergence for speed measurements: a weaker correlation between the occurrence of measurement omission and the surface speed; and a greater tolerance to reconstruction errors. Quantification of the relative contribution of each factor is beyond the scope of this study.

7.4.7 Flame Thickness

Measurements of the flame thickness are shown in Figures 7.18 and the corresponding PDFs in Figure 7.19. Chemiluminescent flame thickness measurements are normalized
Measurements from Flame Chemiluminescence Tomography of Forced Laminar Premixed Propane Flames

Fig. 7.14 Normalized mean curvature at a sample of flame surface points for a different number of views.

Fig. 7.15 PDFs for the normalized mean curvature measurements shown in Figure 7.14.
7.4 Results and Discussion

Fig. 7.16 Normalized $V_f \cdot n$ at a sample of flame surface points for a different number of views.

Fig. 7.17 PDFs for the normalized flame speed measurements shown in Figure 7.16.
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Fig. 7.18 Normalized flame thickness at a sample of flame surface points for a different number of views.

Fig. 7.19 PDFs for the normalized flame thickness measurements shown in Figure 7.18.
by the planar flame CH thickness. The position of the PDF peak monotonically decreases from 0.89 mm ($7.5\delta_{CH}$) with 6 views to 0.49 mm ($4.12\delta_{CH}$) with 36 views for Case 1 and from 1.07 mm ($9.5\delta_{CH}$) with 6 views to 0.49 mm ($4.3\delta_{CH}$) with 36 views for Case 2. The OVLs in Figure 7.13 are lower than for curvature or speed measurements, reaching only 76% with 18 views for Case 1 and 59% with 18 views for Case 2. Based solely on this convergence analysis, it cannot be determined whether the flame thickness distribution measured using 36 views has converged.

The FWHM of the visible emission profile, after filtering by the frequency dependent sensitivity of the image intensifier, was measured in the previous chapter to be approximately 150 $\mu$m in the equivalence ratio range from $\phi = 0.75$ to $\phi = 0.85$. The FWHM of CH profiles in planar flames, obtained from chemical kinetic simulations, are 119 $\mu$m and 113 $\mu$m for cases 1 and 2 respectively. Clearly, the flame thickness measured here using flame chemiluminescence tomography is significantly greater than the expected values, even with 36 views. However, as discussed in Section 3.5.2, imperfect repeatability of the flame cycle may increase the flame thickness in the reconstructions in this study, and the difference between the expected and measured chemiluminescent flame thicknesses may therefore not be due to an insufficient number of views. Nevertheless, flame thickness measurement is significantly more sensitive to the number of views than the other parameters studied in this work.

One reason is that the curvature and speed measurement methods fit surfaces to the emission field within an inclusion radius that is between 10-20 voxel lengths, and in doing so incorporate information from a large volume of the reconstructed emission field, and so are relatively insensitive to changes in intensity values of any single voxel. The radius of curvature is also much larger than the flame thickness over most of the flame surface, allowing for the use of large inclusion radii. Measurement of the flame thickness may also be more sensitive to errors in the projection measurements than measurements related to the flame surface topology.

As this appears to be the first demonstration of flame thickness measurement on a non-axisymmetric wrinkled flame using any form of tomography, future studies should consider such effects further. The relatively large number of coplanar views required to obtain reasonable measurements in this work suggest that accurate reconstructions of the flame thickness will be a significant practical challenge for turbulent flames.

### 7.4.8 Implications for FCT of Turbulent Premixed Flames

The potential study of turbulent premixed flames is a key motivation for developing FCT. Reconstruction resolution is known to be dependent on the field to be reconstructed (Floyd et al., 2011; Frieder and Herman, 1971), as demonstrated by the differences observed
between the two test cases. However, the cross-sections of the chemiluminescence field for the test cases are similar to those reported for weakly turbulent jet flames by Ishino and Ohiwa (2005) and Ishino et al. (2009). It is therefore reasonable to expect similar results for weakly turbulent jet flames of similar dimensions. For highly turbulent flames entering the thickened flame regime (Veynante and Vervisch, 2002), where both the preheat and reaction zones are strongly affected by the turbulence, changes in the thickness of the chemiluminescence region may occur, as well as changes in the intensity of the chemiluminescence. Local extinctions may also occur. All of these complex changes to the chemiluminescent emission may require significant modifications to the methods presented here in order to obtain useful results.

The methods presented in this chapter rely on having optical access to the flame from many directions, and the complete absence of reflections off surfaces. This may be difficult to achieve for some burner arrangements. Furthermore, it is assumed that the optical density of the burned and unburned gases, and of the reaction zone, is so low that self-absorption of the chemiluminescence is negligible. This is known to be a good assumption for lean, premixed, propane-air flames (Gaydon, 1974) but may not be a good assumption in other flames. Sooting flames present further challenges due to the high level of incandescent emission and the high optical density.

The number of unknowns scales with the volume within the convex hull of the flame (Badea and Gordon, 2004) whereas the number of equations scales with the area of the flame images. More views are therefore likely to be required to achieve the same accuracy when applying these methods to more highly wrinkled jet flames or to larger flames.

7.5 Summary

The relative sensitivity of several key flame measurements to the number of views used in the tomographic reconstruction of the visible chemiluminescence field has been assessed. A single, intensified camera was first used to acquire 36 equally-spaced, coplanar views of two forced, premixed, non-axisymmetric, laminar flames at 100 phases of their forcing cycle. A standard MART algorithm was then used to reconstruct the time-resolved chemiluminescence field using different numbers of equally-spaced views.

Algorithms for measuring the flame surface area, mean curvature, the normal component of the flame propagation velocity (surface speed) and the flame thickness were then demonstrated on the reconstructed chemiluminescence fields. The flame surface area was calculated using isosurfaces of the chemiluminescence field. Fluctuations in the flame surface area calculated using as few as nine views were within 1% of those calculated using 36 views for both flames. Mean curvature, surface speed and thickness were then measured at a sample of positions over the entire flame surface, defined as the layer of
peak chemiluminescent emission. The relative sensitivity of these measurements to the number of views was assessed by comparing the similarity of measurement distributions acquired using different numbers of views. A similarity coefficient was used to quantify the similarity between two probability density functions (PDFs). For both flames, the mean curvature and the surface speed measurements achieved over 80% similarity between the 6 view and 36 view reconstructions. In contrast, the flame thickness measurements achieved less than 80% similarity between the 18 view and 36 view reconstructions, and the similarity with the 36 view reference fell significantly as the number of views was further reduced. Therefore, whilst this study appears to be the first that measures the flame thickness over a wrinkled flame surface using FCT, a large number of views are required to obtain accurate measurements of this quantity in comparison to measurements of flame curvature and surface speed.

The results obtained from the two forced flame cases do not suggest a strong relationship between the degree of flame wrinkling and the number of views required. Similar results would therefore be expected from more wrinkled flames.
Chapter 8

Conclusions and Recommendations for Further Work

Chapters 4, 5, and 6 presented a study of sound generation by tunnel narrowing, pocket consumption and cusp recovery processes in unconfined, lean, premixed flames. Experiments were conducted on a forced, laminar flame rig. Chemiluminescence imaging was used to determine the flame geometry and relate flame dynamics to the measured sound. A model of sound generation by flame elements was developed and applied to explain the observed acoustic waveforms.

In Chapter 7 the capabilities of Flame Chemiluminescence Tomography (FCT) were investigated on two non-axisymmetric, forced, laminar, lean, premixed flames with equivalence ratios of 0.71 and 0.84. Algorithms for measuring the flame surface area, flame curvature, the normal component of the flame propagation velocity, and the chemiluminescent flame thickness were demonstrated on reconstructed emission fields. The convergence of measurement distributions was analysed as the number of views used in the tomographic reconstruction was increased.

8.1 Conclusions

The research aims from Chapter 1 are restated below, together with a summary of the main findings of this work.

1. To determine the contributions of tunnel narrowing, pocket consumption, and cusp recovery processes to the acoustic waveforms emitted by forced, laminar, lean, premixed, propane-air flames.
This aim was addressed using a combination of experiments and modelling.

Experiments on forced, laminar flames using different forcing amplitudes and frequencies showed that the relative significance of tunnel narrowing, cusp recovery and pocket consumption processes is largely dependent on the length of the flame neck involved in tunnel narrowing, and the size and shape of the separated pocket. All three of these processes contribute to the generation of an acoustic rarefaction. The largest magnitude rarefactions are typically associated with the moment of separation of the reactant pocket.

For tunnel narrowing the magnitude of the generated rarefaction was observed to increase as the separation event is approached, and the rate of increase \( (\frac{d|p'|}{dt}) \) also increased. Cusp recovery begins immediately following the moment of separation. The peak rarefaction magnitude reached in the terminal moments of tunnel narrowing is sustained in the initial 100 \( \mu s \) of cusp recovery. The rarefaction magnitude then decreases over approximately 1-2 ms, depending on the equivalence ratio and the flame geometry at the moment of separation. In forced, laminar flames, the acoustic waveform generated by pocket consumption is often superposed with the sound generated by cusp recovery. Nevertheless, by generating large pockets that persist after cusp recovery has ended, pocket consumption was observed to generate a sustained rarefaction with a slowly decreasing magnitude prior to the burn-out event. The rarefaction magnitude is similar to that generated during tunnel narrowing, 1 ms prior to the separation event. A small, additional, distinctive increase in the rarefaction magnitude over approximately 50 \( \mu s \) was observed during the pocket burn-out event, prior to a larger increase in the acoustic pressure over approximately 200 \( \mu s \).

The measured sound during tunnel narrowing, cusp recovery, and pocket consumption was compared to the sound predicted using the rate of change of total visible chemiluminescence from the region of the flame where these processes occur. There were several significant differences between the predicted and measured waveforms. The observed increase in the rarefaction magnitude during tunnel narrowing was not reproduced in the predicted waveform and a positive fluctuation in the chemiluminescence signal during the separation event did not correlate with an increase in the emitted acoustic pressure. Over the 1 ms interval prior to the burn-out event, the rarefaction magnitude generated by pocket consumption was overpredicted by 18% on average and the error exceeded 60% during the burn-out event. These results are consistent with previous studies showing chemiluminescence to be an imperfect marker for the heat release rate.

Measurements of the total heat release rate provide limited insight into how flame processes such as tunnel narrowing or pocket consumption generate specific pressure waveforms. A new model of sound generation was therefore developed where the flame is considered to consist of a set of flame elements. Each flame element is advected by the flow and propagates into the unburned reactants in the flame-normal direction with
a displacement speed proportional to the mean curvature of the flame, i.e., Markstein’s relation is applied. The unstretched, flame speed and the displacement speed Markstein length were determined empirically. Expressions were derived for the time derivative of the heat release rate from, and thereby the sound generated by, individual flame elements. The sound generated by an acoustically-compact, unconfined flame is given by the integral of the contributions of the flame elements.

Tunnel narrowing was modelled by dividing the flame neck into conical flame elements. Modelling predicted that at large tunnel radii, a conical flame element sustains a rarefaction of almost constant magnitude. The rarefaction magnitude increases at an increasing rate as the flame element approaches the symmetry axis. This is consistent with the measured pressure waveform generated during tunnel narrowing. Furthermore, quantitative predictions of changes in the acoustic pressure during tunnel narrowing were consistently within 35% of the measured changes while the flame radius exceeded the thermal flame thickness. Sound generation by a spherical pocket was also considered by assuming a constant consumption speed. Separated pockets, though not initially spherical, tend to become spherical prior to the burn-out event. Consistent with observations, the model predicted the generation of a rarefaction with a relatively slowly decreasing amplitude as the pocket is consumed. The largest pockets were formed at the lowest equivalence ratio included in the study, $\phi=0.75$. Due to the relatively large initial size of the pockets, the sound generated by pocket consumption could be observed following cusp recovery. The average error in the predicted acoustic pressure over the 1 ms interval prior to the burn-out event was 11% and the measured increase in the acoustic pressure from prior to the burn-out event to after the burn-out event was predicted to within 20%. The model predicts this increase to occur instantaneously. However, due to the finite thickness of the heat release region, the increase occurs over approximately 0.5 ms.

A limitation of the model is that the error may be large when applied to flame elements with mean curvature exceeding $1/\delta_h$. This arises from two approximations in its formulation: (i) that the flame’s heat release region is infinitely thin, and (ii) that the flame speeds follow Markstein’s relation. The peak rarefaction magnitudes during separation events and pocket burn-out events were therefore not able to be accurately predicted. However, the peak rarefaction magnitude is sustained for durations on the order of 10 $\mu$s and is therefore relatively insignificant to the emitted sound. Cusp recovery also features flame elements with mean curvature exceeding $1/\delta_h$. The error in predictions of the contribution of highly curved flame elements at the cusp to the emitted sound may be significant and the flame cusp may therefore need to be considered separately in some cases.
2. **To determine the effect of flame acceleration on the sound generated by tunnel narrowing and pocket consumption processes in lean, premixed flames.**

To construct a model of sound generation by tunnel narrowing and pocket consumption, expressions were formulated in terms of geometrical parameters of flame elements, and the flame’s displacement speed and consumption speed. The expressions demonstrate the significance of flame acceleration to the generated sound. Furthermore, the dependence of the emitted sound on many other parameters has also been made explicit. In forced, laminar, lean, premixed, propane-air flames the flame displacement speed was observed to increase linearly with the mean flame curvature for flame radii greater than the thermal flame thickness.

For tunnel narrowing, positive acceleration of the flame, i.e., an increasing displacement speed, was found to be responsible for the observed increase in the rarefaction magnitude as the separation event is approached. This is because the magnitude of the rarefaction generated by conical flame elements is linearly related to the flame displacement speed, but is otherwise independent of the flame radius. Without flame acceleration, the expressions show that the rarefaction generated during tunnel narrowing would be almost constant until the separation event. It was also shown that the generated sound would be unchanged if the consumption speed were to change linearly with the mean curvature.

For the consumption of spherical pockets, the derived expression shows that flame acceleration increases the magnitude of the generated rarefaction compared to if the displacement speed were constant. Flame acceleration is also responsible for the almost constant rarefaction magnitude prior to the burn-out event in lean propane-air flames and therefore explains the observed increase in acoustic pressure following burn-out events. The rarefaction magnitude generated by spherical flames is proportional to the product of the flame displacement speed and the flame radius ($r_f$). Therefore, if the flame displacement speed increases linearly with $1/r_f$ until the burn-out event, as observed in lean, propane-air flames, the rarefaction magnitude will be non-zero just prior to the burn-out event. In contrast, if the displacement speed were constant, the rarefaction magnitude would decrease at a constant rate, going to zero at the burn-out event, as has previously been shown by Thomas and Williams (1966).

The displacement speed was observed to increase at a greater rate than predicted by Markstein’s relation during the final stages of tunnel narrowing and pocket consumption, when oppositely propagating flames were within two thermal flame thicknesses of each other. The merging and subsequent extinction of the flames has been called flame annihilation. Distinctive acoustic signatures are generated during this time but due to the short duration of these events, the sound is found to be negligible compared to that of the sound generated by the prior tunnel narrowing and pocket consumption. However, cusp recovery
could be considered a continuation of an annihilation event and was observed to sustain a high magnitude rarefaction for considerably longer durations.

3. **To determine the number of views required to obtain certain flame measurements when applying Flame Chemiluminescence Tomography to laboratory-scale, premixed jet flames.**

The chemiluminescence fields of two forced flames at equivalence ratios of 0.71 and 0.84 were reconstructed using different numbers of equally spaced views, from 3 views to 36 views. Each flame was reconstructed at 100 phases of the forcing cycle.

Measurements of flame surface area were obtained by computing the surface area of isosurfaces. Measurements of fluctuations in the flame surface area ($A'/\bar{A}$) over a complete forcing cycle were measured from reconstructions using different numbers of views. Nine views were found to be sufficient to measure surface area fluctuations to within 1% of those measured using 36 views.

Using 6, 9, 12, 18, and 36 views, measurements of mean curvature, the normal component of the flame propagation velocity (flame surface speed), and chemiluminescent flame thickness were obtained from a sample of points over the entire reconstructed surface of both forced flames at a single instant of the forcing cycle. PDFs of measurement distributions, acquired using different numbers of views, were compared using a similarity coefficient that is 100% when the PDFs are identical. For measurements of mean curvature and flame surface speed, only six views were required to achieve 80% similarity with the PDFs obtained using 36 views. However, for measurements of flame thickness, similarity between PDFs acquired using 18 views had less than 80% similarity with PDFs acquired using 36 views. It was also found that even with 36 views, the measured flame thickness was more than 100% greater than was measured by Abel deconvolution of axisymmetric flames. This is thought to be due to uncertainty in the projection measurements. The uncertainty arises from the optical resolution of the imaging arrangement, perspective error, refraction by temperature gradients in and around the combustion region, and imperfect repeatability of the flame cycles.

The results suggest that FCT with fewer than 10 intensified cameras may have acceptable accuracy for measuring flame surface area, flame curvature, or flame speed on laboratory scale, turbulent flames. This number of views can be obtained using 5 or more high speed intensified cameras with 2 or more megapixel resolution and image doubling optics, which is already practical in some laboratories.
8.2 Recommendations for Future Research

1. Quantify the sound produced by flame-vortex interactions and tunnel formation.

Flame-vortex interactions and tunnel formation are two additional processes that were not studied in this work but that may have a significant role in combustion generated sound. Renard et al. (2000) summarised the experimental studies on the interaction of vortices with laminar flames. Vortices occur in many forms and there is little known about sound generation from flame-vortex interactions. Generating well-characterised vortices in an unconfined experimental rig suitable for the study of sound generation may require novel experimental rigs.

Tunnel formation could be studied with a similar forced flame rig to the one used in this work. An appropriate combination of flame holder geometry and flow profile would be required. There are good reasons to suspect that tunnel formation may be important in sound generation in turbulent flames. The cusps that expand from around the initialisation site may propagate over distances up to the dimensions of the combustion region and may result in the destruction of a large fraction of the total flame surface area.

2. Identify the dominant sound-generating flame processes occurring in turbulent, premixed flames.

Given recent advances in time-resolved, 3D flame diagnostics it may soon be possible to test the hypothesis that a subset of flame behaviours are responsible for most of the sound generated by turbulent, premixed flames. Flame Chemiluminescent Tomography can resolve the flame surface and measurements of flame curvature and flame surface speed can be obtained. Using this information, processes such as tunnel narrowing, cusp recovery, and pocket consumption could be identified. The predicted sound from these processes could be compared to the measured sound to determine how much of the sound is attributable to these processes.

Recent DNS of turbulent flames resolving the acoustic field suggest that it may also be possible to undertake a similar numerical study. Recent work has classified and identified certain flame topologies in numerical data sets of turbulent, premixed flames (Dunstan et al., 2013; Griffiths et al., 2015). The robust identification of flame processes is likely to be a considerably easier task using numerical data sets since the velocity field is known. However, numerical simulation of turbulent flames at realistic scales remains computationally expensive, particularly if detailed chemical mechanisms are employed.
3. Measure profiles of flame chemiluminescence by using the distinctive acoustic signatures of flame annihilation events.

In this work it was found that the distinctive acoustic signatures of separation and burn-out events could be used to determine the timing of an image relative to these events to an accuracy of \( \pm 10 \ \mu s \). This was used to collect many images of the flame neck during tunnel narrowing that were taken in the same 20 \( \mu s \) interval relative to the separation event. Because of the approximately cylindrical geometry of the flame neck, the flame edges could be accurately identified despite image noise. Despite small cycle-to-cycle variability in the spatial and temporal position of the separation event, images could be accurately aligned and then averaged to create a composite image that was comparatively noise free and that could be deconvolved to determine the radial visible emission intensity. The same process could be applied using optical bandpass filters to determine the emission profile from OH*, CH* or other excited species.

By this method, narrow-band chemiluminescent emission profiles could be readily obtained for cylindrical flames with different flame radii. This could provide valuable data about the response of the reaction zone to high curvature.

4. Quantify the contribution of highly curved flame elements to the emitted sound.

Highly curved flame elements, where the mean curvature exceeds \( 1/\delta_{th} \), are observed in the flame cusps formed by pocket separation events. Highly curved flame elements are also formed by tunnel formation, and occur in the terminal moments of tunnel narrowing and pocket consumption. Though they make up a small fraction of the total flame surface area, it is not known whether sound generated by highly curved flame elements is a significant component of the sound generated by turbulent premixed flames.

An increased understanding of sound generation by highly curved flame elements would therefore be valuable. However, predicting the heat release rate from these regions of the flame is challenging. The distribution of chemical species and the reaction rates across highly curved flames is substantially different from the planar, laminar flame structure. To accurately model sound generation by highly curved flames in numerical simulations may therefore require using detailed chemical mechanisms. A numerical study of sound generation by cusp recovery using chemical mechanisms of varying complexity would therefore be valuable, and could be directly compared to experimental results obtained on forced, laminar flames.

5. Determine the importance of accurately modelling the heat release rate distribution during flame annihilation for predicting the emitted sound.
Flame annihilation is the interaction and extinction of oppositely-propagating flames. In this thesis flame annihilation was defined as beginning when the distance between the peaks of the heat release rate distributions of two opposing flames is equal to twice the thermal thickness of the unstretched flame.

The time interval from the beginning of annihilation until the heat release rate profiles have merged to form a single peak is typically on the order of 0.1 ms in propane-air flames. However, though the reaction zones have merged, the heat release rate does not immediately become zero. Sound production therefore continues. This was apparent in the decay of the acoustic rarefaction over approximately 0.5 ms following the pocket burn-out events. A similar decay in the total visible chemiluminescence was observed over a similar time interval.

A similar process is likely to occur during the separation event after the reaction zones have merged and in the region behind the flame during cusp recovery. 1D numerical simulations of spherical annihilation events could determine the complexity of the chemical mechanism required to accurately model the sound generated by the decay of the merged flames.

6. **Measure the sound generated by tunnel narrowing, pocket consumption, and pocket burn-out when using other fuels.**

A number of experimental parameters that have been shown to be important to the sound generated by tunnel narrowing, pocket consumption, and pocket burn-out can be varied by using different fuels. These include the flame speeds, the volumetric heat release rate, and the Lewis number. Flames with different fuels are known to show different flame tip behaviours (Poinsot et al., 1992). Comparing the measured sound to the sound predicted using the methodology described in Chapter 6 may provide insight into the differences in sound production in turbulent flames using different fuels, and into the limitations of the methodology.
References


References


References


Appendix A

Extensions to the Flame Element Model of Sound Generation

A.1 The Flame Base

Flame surface creation occurs primarily at the base the flame. The contribution of this flame surface creation to sound generation can be estimated using Equation 6.14. The base has the fixed $z$ position $z_0$. The upper boundary of the flame element volume $z_1(t)$ has the $z$-velocity $u - s_d \sin \theta$. Because the initial length of the flame element at the base is zero ($l_0 = 0$), Equation 6.14 simplifies to:

$$\frac{d}{dt} \int_{y_0(y)} Q dy = \beta r_0 s_{q,0} \frac{dl_0}{dt}.$$  (A.1)

At low forcing amplitudes, the base of the flame can be assumed to be in quasi-steady state with respect to the flow of unburned gases such that $u \sin \theta = s_d$. In this case $dl_0/dt = u \cos \theta$. Substituting for $\cos \theta$ yields:

$$\beta r_0 s_{q,0} \frac{dl_0}{dt} = \beta r_0 s_{q,0} \sqrt{u^2 - s_d^2}.$$  (A.2)

Substituting the Markstein relations for $s_q$ and $s_d$, the contribution of flame surface creation at the flame base to the far field pressure fluctuation is given by:

$$p_{base}'(R,t) = \left( \frac{\gamma - 1}{\gamma} \right) \frac{\rho_0}{4\pi R^2 P_0} \beta r_0 S_{L,0} \left[ (1 + \cos \theta_0 \frac{L_d}{r_0}) \sqrt{u^2 - S_{L,0}^2 \left( 1 + \cos \theta_0 \frac{L_d}{r_0} \right)} \right]^{\frac{\gamma - 2}{2}}.$$  (A.3)
If $S_{L,0} \ll u$ the sound generated by flame surface creation at the flame base is approximately proportional to the unburned gas velocity $u$. If $r_0 \gg L_d$ and $r_0 \gg L_q$, a good approximation is obtained by assuming that $s_q$ and $s_d$ are equal to the unstretched, planar, laminar flame speed. In that case:

$$p'_{\text{base}}(R, t) \approx \left(\frac{\gamma - 1}{\gamma}\right) \frac{\rho_0}{4\pi R p_0} \beta r_0 S_{L,0} \left[\sqrt{u^2 - S_{L,0}^2}\right] \bar{\tau}, z_0 \quad \text{(A.4)}$$

### A.2 The Flame Tip

Flame elements in the top subvolume $V_n(t)$ also undergo negative flame stretch and therefore contribute to sound production. Because the flame radius near the flame tip approaches 0 and the flame angle approaches 90 degrees, the assumptions and approximations employed in the conical flame element analysis outlined in Section 6.2.2 are not well justified for flame elements in the flame tip. However, the contribution of flame elements in the $n^{th}$ volume ($V_n(t)$) to the time derivative of the volume integral of $Q$ can be reformulated using Reynold’s transport theorem in terms of $Q$ at $z_n$, and the time derivative of $Q$ integrated over a volume that moves so as to always contain the tip of the flame ($V_{\text{tip}}(t)$). The lower boundary of the tip volume is denoted $z_{\text{tip}}(t)$ and may be defined in any convenient manner. Obvious choices include a fixed distance from the flame tip or where the flame radius is equal to some constant. $z_n$ is chosen to be equal to $z_{\text{tip}}$ at the instant the model is being applied. However, in general the boundary velocities $dz_n/dt$ and $dz_{\text{tip}}/dt$ will not be the same. The time derivative of the volume integral of $Q$ over the tip volume can be expressed as:

$$\frac{d}{dt} \int_{V_{\text{tip}}} Qdy = \frac{d}{dt} \int_{V_n} Qdy + 2\pi \left(\frac{dz_n}{dt} - \frac{dz_{\text{tip}}}{dt}\right) \left[\int_{r}^{r+} Q rdr\right]_{z_n}. \quad \text{(A.5)}$$

If $z_{\text{tip}}$ is chosen to be a sufficient distance from the flame tip, the same steps used from Equation 6.12 to Equation 6.14 can be applied to obtain:

$$\frac{d}{dt} \int_{V_n} Qdy = \frac{d}{dt} \int_{V_{\text{tip}}} Qdy - \left[\frac{\beta}{\cos \theta} \left(u - s_d \sin \theta - \frac{dz_{\text{tip}}}{dt}\right) r fs_q\right]_{z_n}. \quad \text{(A.6)}$$

When the time rate of change of the volume integral of $Q$ within the tip volume ($V_{\text{tip}}$) is small, the first term on the right hand side of Equation A.6 is negligible.
A.3 A Dynamic Equilibrium Model of Sound Generation by Flame Elements

For an acoustically compact flame, the sound emitted by the flame is the sum of the contributions from all the flame elements. In this section, the model is applied to a steady flame. This facilitates discussion of sound generation by forced flames and provides partial validation the model assumptions. Steady, laminar, jet flames do not produce any sound. The sum of the contributions of the flame base, flame tip, and the conical flame elements comprising the rest of the flame should therefore sum to zero. For a steady flame, the following relation applies everywhere:

\[ u \sin \theta = s_d. \]  

(A.7)

Using \( Dr_f / Dt = -s_d \cos \theta \) and \( Dz / Dt = u - s_d \sin \theta \) for a lagrangian flame element, an ordinary differential equation for the flame radius is obtained,

\[ \frac{dr_f}{dz} = \frac{-s_d}{\sqrt{u^2 - s_d^2}}. \]  

(A.8)

Using the Markstein relation for \( s_d \) and computing the mean curvature using local conical approximations the steady flame shape can be found by numerical integration. Close to the flame tip, the flame surface is not well approximated by a conical surface and the curvature on the vertical normal plane becomes significant. However, Equation A.6 can be used to predict the sound generation from the tip of the flame.

As an example, a steady, propane-air jet flame issuing from a circular port with radius \( r_0 = 12.5 \text{ mm} \) and an unburned mixture velocity of 1.2 ms\(^{-1}\) will be used. \( S_L \) and \( L_d \) are as given in Table 6.2. Figure A.1 shows the flame shape and compares it to a flame with \( s_d = S_{L,0} \) everywhere.

Equation 6.19 can be applied to the derived flame profile to estimate the sound production. This model is applied to the flame up to where the radius equals 1 mm. \( l_{z,i} \) may be arbitrarily small and dividing Equation 6.19 by \( l_{z,i} \) gives the contribution to \( p' \) from the flame at each vertical position, per vertical length of the flame. This is shown in Figure A.2. The sum of the \( p' \) contributions from the part of the flame that has been represented by conical flame elements is -188 mPa at a distance of 100 mm. The contribution of flame surface creation at the flame base is estimated using Equation A.3 to be 211 mPa.
The contribution of the flame tip is given by Equation A.6. For a steady flame, \( \frac{dz_{\text{tip}}}{dt} = 0 \). Using Equation A.7, Equation A.6 simplifies to:

\[
\frac{d}{dt} \int_{V_{n(t)}} Qdy = -\beta \left[ r_f s_q \sqrt{u^2 - s_d^2} \right]_{z_n} \tag{A.9}
\]

This is the same expression as was derived for the flame base, but with opposite sign. This must be the case because if the tip volume were defined such that \( z_{\text{tip}} = z_0 \), the magnitude of the predicted sound from flame elements in the ‘flame tip’ must equal that of the predicted sound from flame surface creation at the flame base. The estimated contribution of the flame tip to the current case is -20 mPa. The sum of all three contributions is 3 mPa. This discrepancy arises from the assumptions in the derivation of Equation 6.19. The error is small compared to the magnitude of the positive and negative contributions and is also small relative to the measured pressure fluctuations from the forced laminar flames that are the subject of this study.

Figure A.2 demonstrates the importance of the flame tip to sound production. Sound production per flame element height almost doubles that near the base of the flame. The importance of flame elements near the flame tip becomes even more apparent when the \( p' \) contribution per flame surface area is plotted, as in Figure A.3.

On a per flame surface area basis, flame elements near the flame tip are more than an order magnitude more significant to sound production than flame elements near the base of the flame. Disturbances at the flame tip would therefore be expected to have a significant impact on the sound produced by forced, axisymmetric flames. This was observed in the Chapter 4.
Fig. A.3 Predicted $p'$ contribution per flame element area vs the $z$-position of the flame element, at a position 100 mm from a steady flame.
Appendix B

Technical Drawings of the Laminar Flame Rig

Detailed drawings of the laminar flame rig are provided in the PhD thesis of Nader Karimi, who originally designed and built the rig (Karimi et al., 2009). Drawings of the flame holders used in this thesis, and of the vernier scale used to measure the rotation of the flame holders, are provided in this Appendix. All other dimensions were unchanged from those reported in Karimi et al. (2009).
Circle rim stabiliser flame holder

**Drawing No. 1**

**Scale:** 1:1

**MATERIAL:** Stainless Steel 316

**Section A-A****

**Scale 1:1**

**DIMENSIONS:**

- **ø25**
- **ø30**
- **ø49**
- **ø55.2**
- **0.1**
- **0.25**
- **0.30**

**UNLESS OTHERWISE SPECIFIED:** DIMENSIONS ARE IN MILLIMETERS

**SURFACE FINISH:** 0.8 TOLERANCES: LINEAR 0.1 ANGULAR 0.5°

**DEBUR AND SURFACE FINISH:**

- **ø27.0**
- **ø26.1**
- **ø4.9**

**NOTE:**

- **BREAK SHARP**
- **CHECKED**
- **APPROVED**
- **DRAWN**
- **SCALE 1:1**
- **SECTION A-A**

**SIGNATURE:**

- **NAME**
- **DATE**
- **REVISION**
- **EDGES**
- **HOLDER**
- **0.5**

**NOTE:**

- **DIMENSIONS ARE IN MILLIMETERS**

**NOTE:**

- **UNLESS OTHERWISE SPECIFIED:** DIMENSIONS ARE IN MILLIMETERS

**NOTE:**

- **SURFACE FINISH:** 0.8 TOLERANCES: LINEAR 0.1 ANGULAR 0.5°

**NOTE:**

- **DEBUR AND SURFACE FINISH:**
- **ø27.0**
- **ø26.1**
- **ø4.9**

**NOTE:**

- **BREAK SHARP**
- **CHECKED**
- **APPROVED**
- **DRAWN**
- **SCALE 1:1**
- **SECTION A-A**

**NOTE:**

- **SIGNATURE:**
- **NAME**
- **DATE**
- **REVISION**
- **EDGES**
- **HOLDER**
- **0.5**
Square rim stabiliser flame holder

UNLESS OTHERWISE SPECIFIED, DIMENSIONS ARE IN MILLIMETERS

MATERIAL: Stainless Steel 316

DRAWING NO. 2

MILLIMETERS

Scale: 1:1

WEIGHT: A3

SHEET 1 OF 1

Engravings

SECTION B-B

SCALE 1 : 1

TOLERANCES:

<table>
<thead>
<tr>
<th>Linear</th>
<th>Angular</th>
</tr>
</thead>
<tbody>
<tr>
<td>±0.01</td>
<td>±0.01</td>
</tr>
</tbody>
</table>

Engravings

Scale 5 : 1

DIMENSIONS

15

40

37

45

27

22

0.49

0.55

2

10 x 0.9°

49

55

22

27

30

81°

R35

R4

Q.A
Angle Scale

Drawing No. 3

5 mm Stainless Steel 316 plate

Dimensions are in millimeters

Tolerances:
- Linear: ±0.1 mm
- Angular: ±0.5°
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Author/s:
Wiseman, Samuel

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Date:
2018

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