Decision-making under Pressure: A Study of Tennis Professionals

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Abstract

This thesis examines the effect of pressure on a sample of highly-trained individuals in a simple strategic setting. By modelling the strategic interaction between servers and receivers in professional tennis matches, the main contribution of this research is to present an environment where pressure can be introduced in a practical and reasonable manner. Using this environment, I am able to investigate the effect that pressure has on players’ behaviour and its consequences for their payoffs. Thus, the analysis in this thesis is two-fold. The first component of the analysis examines the effect that pressure has on the decision-making ability of players. The second component examines the effect that pressure has on their point outcomes.

The results indicate that pressure does have a marked effect on the service decisions of many players in the sample. I find that a significant number of servers have a particular strategy that they choose to play more often under pressure than they do otherwise. I also investigate the effect of pressure on the level of correlation between past and present choices and find similar results. The implication of this is that servers’ behaviour changes when they are faced with high pressure situations. Since players are also found to behave in accordance with theoretical predictions in the absence of pressure, this leads to the possibility that players could be performing sub-optimally in such situations. Indeed, the results confirm that pressure has a generally negative effect on servers’ chances of winning the point. Interestingly, there is less evidence for a link between the two effects. The correlation between the players whose choices change under pressure and those players whose outcomes are affected is small. This leads me to believe that there is a possibility that either player could benefit from exploiting the sub-optimal choices of the other in high pressure scenarios.

These results suggest that the theoretical models that have been previously used to describe the serve-return interaction may be insufficient in the presence of pressure. Consequently, researchers may be led to incorrect predictions about behaviour in high-stakes environments by excluding pressure from their models. Therefore, I also provide some theoretical basis to rationalise the observed results by proposing several approaches to incorporate pressure into existing models.
Declarations

This is to certify that:

1. The thesis comprises only my original work towards the Doctor of Philosophy (Business and Economics);

2. Due acknowledgement has been made in the text to all other material used; and

3. The thesis is fewer than 100,000 words in length, exclusive of tables, figures, bibliographies, and appendices.

Signed

[Signature]

Simeon Press
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Chapter 1

Introduction

When attempting to understand the actions of individuals who are forced to compete or cooperate for scarce resources, economic models and game theory are often turned to for answers. Typically, these models present us with relatively simple predictions that can be used as a basis for modelling and assessing behaviour in situations when resources are in short supply. However, laboratory experiments explicitly designed to study these theories have failed to observe behaviour consistent with their predictions (see e.g., Ochs, 1995; Rapoport & Boebel, 1992). This failure brings into question some of the assumptions about optimal decision-making that underpin many economical models. To better replicate the environment that economic agents face, many researchers have turned towards field experiments to analyse individual behaviour (see e.g., Chiappori, Levitt, & Groseclose, 2002; Romer, 2006; Walker & Wooders, 2001). In this case greater conformity of individuals’ behaviour to theoretical predictions has been observed, but the results are still mixed. Thus, whether people will make optimal choices in strategic situations is still very much an open question.

Several arguments have been advanced as to why we do not typically observe behaviour that conforms to our theoretical predictions. The simplest explanation is that it can be extremely complex to know what optimal behaviour is in many scenarios. Since humans are not perfect calculators, it is not surprising that we are

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1For an ongoing discussion regarding the validity of laboratory experiments vs. field experiments refer to Levitt & List (2007a,b) and rebuttal by Camerer (2015).
not always capable of determining the ideal choice(s) that we should make in competitive situations. This is particularly evident in laboratory experiments where the subjects often have little experience with the scenarios presented to them (Kovash & Levitt, 2009). As people gain more experience with a given task, it is reasonable to assume that they get better at making optimal choices. Despite this, behaviour that falls short of the theoretical optimal can still be observed even amongst highly experienced individuals and teams (see e.g., Kovash & Levitt, 2009; Romer, 2006). This can be costly to those whose livelihood depends on their ability to successfully compete against others over a single prize.

A compounding factor for sub-optimal decision-making is the role played by stress and pressure on our cognitive function. It is well known that stress has psychological and physiological effects on the human brain (see e.g., Lupien, McEwen, Gunnar, & Heim, 2009; Wolf, 2009). Even if humans could perform the complex calculations required to derive their optimal behaviour under ideal conditions, the addition of pressure into this environment is likely to make this task significantly more difficult. The idea that pressure can impair our ability to make clear and rational decisions has been extensively researched in psychology, resulting in several long-standing models of pressure (see e.g., Baumeister, 1984; Wine, 1971). Despite this, economic models rarely factor pressure into consideration. The exclusion of this influence from our models is likely to result in misleading predictions about human behaviour. This issue is of substantial importance since many scenarios we face in personal or professional life involve considerable levels of pressure or stress. This thesis aims to incorporate pressure into behavioural models with the intention of bridging the gap between this well-known psychological phenomenon and our economic models.

To investigate these issues, I undertake a field study from professional tennis. Typically, field experiments have the benefit of being able to examine individuals in their natural environment and, therefore, provide researchers with a level of confidence that observed behaviour is indicative of general behaviour in that environment.
However, due to the interaction between numerous environmental factors outside the control of the experimenter, it can sometimes be difficult to identify the causes of the observed behaviour in field experiments. In an attempt to mitigate this issue, a number of researchers have turned to professional sports to address a variety of economic issues, such as profit-maximisation (Romer, 2006), risk-taking (Goldman & Rao, 2014b), loss aversion (Goldman & Rao, 2014a; Pope & Schweitzer, 2011), momentum (Cohen-Zada, Krum, & Shtudiner, 2017; Gauriot & Page, 2014; Livingston, 2012), decision-making (Chiappori et al., 2002; Walker & Wooders, 2001), gender differences (Cohen-Zada, Krum, & Shtudiner, 2017; Krum, Rosenboim, & Shapir, 2016; Cohen-Zada, Krum, Rosenboim, & Shapir, 2017) and pressure (Cohen-Zada, Krum, Rosenboim, & Shapir, 2017; Dohmen, 2008; Goldman & Rao, 2012, 2013).

Professional sports can be viewed as a favourable environment for examining many aspects of human behaviour for several reasons. First, the rules are clearly defined so the environment is highly structured. Second, the objective function can be clearly defined as the motivations of the individuals are transparent (in that, barring exceptional circumstances, they all want to win). Finally, individual behaviour is easily observable in the vast majority of circumstances. These factors imply that it is possible to extract results with more precision from studies using professional sports than in many other field studies. In addition, professional athletes typically devote a considerable portion of their lives to the pursuit of their sport so they should fully understand the environment they face. Therefore, it is likely that such experienced athletes exhibit the pinnacle of performance in their environment. Consequently, any analysis of behaviour in this environment should yield the full extent of human potential.

Moreover, using tennis to investigate strategic behaviour has numerous advantages over other professional sports. Studies using penalty kicks in soccer (see e.g., Chiappori et al., 2002) encounter the problem that penalty kicks rarely happen more than once or twice per match. Therefore, the datasets in these studies typically con-
tain few observations resulting in statistical tests with low power to reject the null hypothesis. This is even more pronounced when the researcher wishes to examine the effect of pressure, a scenario that would only arise in a small fraction of observations. On the other end of the spectrum, studies in American football (see e.g., Romer, 2006) and baseball (see e.g., Kovash & Levitt, 2009) typically have many observations per match, but tend to involve interactions with multiple players and strategies. Consequently, the theoretical models needed to describe behaviour in these environments are considerably more complex. In contrast, tennis provides an environment where a relatively simple game is repeated multiple times.

Using data from professional tennis matches, I examine the extent to which highly-trained individuals make their service choices optimally according to game theoretical models. Furthermore, I provide an extensive analysis of the effect of pressure on the decision-making of these professionals. I also examine the effect that pressure has on the outcome of the point. Lastly, a cross-analysis of the impact of these effects is provided to determine whether there is a link between the effect that pressure has on decision-making and the effect that it has on the outcome at the stages of the match where it arises.

A key feature of my research is that I present a scenario where it is possible to quantify pressure, something that is typically a subjective measure. Using the variability in the importance of a point on the outcome of a match, I provide a method to quantify the level of pressure that players experience in a tennis match. This allows me to identify the effect that pressure has on players’ decision-making and the prevalence of this effect in a sample of experienced individuals.

The results suggest that pressure has a significant effect on the choices of tennis professionals. I investigate three possible channels through which pressure can affect players’ decision-making and there are a significant number of players in the sample who are affected by pressure for each channel. Pressure also appears to have an effect on the outcome of the point. There is some evidence that the players’ who behave differently under pressure are worse off, however this link is not decisive.
This dissertation is organised as follows: Chapter 2 summarises the relevant literature. Chapter 3 provides the theoretical background for the model of decision-making in tennis. Chapter 4 outlines the data used in this analysis. Chapter 5 presents an examination of decision-making without pressure. This provides a baseline for the main analysis that incorporates pressure as a key influencing factor on behaviour. Chapter 6 develops and discusses a method to quantify pressure. Chapter 7 presents an empirical analysis of the effect of pressure on players. This forms the main analysis of this dissertation and is conducted in three sections. Section 7.1 provides an extensive analysis of the effect of pressure on players’ decision-making. Section 7.2 examines the effect of pressure on players’ outcomes. Section 7.3 attempts to link the effect that pressure has on players’ decision-making to the effect that it has on their outcomes. Chapter 8 attempts to reconcile some of the observed results with the theoretical model of decision-making in tennis. Finally, Chapter 9 concludes with a discussion of the key results and some suggestions for the direction of future research.
Chapter 2

Literature Review

There is an extensive literature investigating the ability of individuals to make strategic decisions that are consistent with optimal behaviour as predicted by game theory models. Early experiments examining the extent of individuals’ conformity to the minimax theorem, one of the most basic predictions of these models, presented overwhelmingly negative results (Brayer, 1964; Fox, 1972; Lieberman, 1960, 1962; Messick, 1967; Rapoport, Guyer, & Gordon, 1976). In a later and more innovative experiment, O’Neill (1987) found that people can play close to minimax under some circumstances, somewhat refuting the earlier results. However, this study was subsequently contested by Brown & Rosenthal (1990), who found high levels of serial correlation in individuals’ actions as evidence against minimax play. More recent experimental studies support the general consensus that the minimax hypothesis does not hold by rejecting it in a substantial number of laboratory experiments (Mookherjee & Sopher, 1994; Ochs, 1995; Rapoport & Boebel, 1992; Rosenthal, Shachat, & Walker, 2003; Shachat, 2002). Thus, the results from the laboratory strongly suggest that people do not make strategic choices optimally.

While the inability of individuals to make optimal choices in the laboratory may be inconsequential, there are scenarios where it may have more significant

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2 O’Neill (1991) defended his original work stating that his interest was to examine the question “How close is minimax to the truth?” rather than “Is minimax exactly correct?” (p. 503).

3 To the contrary, McCabe, Mukherji, & Runkle (2000) is one of the few studies that does not reject the hypothesis of equilibrium play in a three-person matching pennies game. Curiously, Okano (2013) found evidence of equilibrium play in teams that was not present at the individual level.
consequences. For example, poor decision-making may be costly to a professional athlete whose livelihood can revolve around making the right choice at the right time. Despite the negative results from the laboratory, there is some evidence that athletes choose their strategies optimally. Tennis players have been shown to behave close to equilibrium predictions in choosing their service direction (Gauriot, Page, & Wooders, 2016; Hsu, Huang, & Tang, 2007; Walker & Wooders, 2001), as have kickers and goalkeepers in soccer penalty kicks (Chiappori et al., 2002; Coloma, 2007; Dohmen & Sonnabend, 2018; Palacios-Huerta, 2003). However, studies from American football (Kovash & Levitt, 2009; Romer, 2006) and baseball (Kovash & Levitt, 2009) found evidence rejecting equilibrium behaviour.

There have been numerous explanations as to why there may be differing results between laboratory experiments and field studies. Kovash & Levitt (2009) argued that it is not possible to motivate subjects in a laboratory in such a way that mimics the situations they may face in the real world. Furthermore, they argued that these subjects may not fully understand the circumstances of the experiment in which they are participating. Consequently, the subjects may not take actions that they would otherwise take outside of the laboratory. These views are emphasised in the study of decision-making by Palacios-Huerta & Volij (2008). They found that professional soccer players made choices that were close to the theoretical equilibrium in a laboratory experiment designed to mimic a soccer penalty shoot-out. Their findings suggest that familiarity and experience are important factors influencing behaviour, and could explain some of the discrepancies between most laboratory and field experiments.

Contradictory results from within the various field studies in professional sports (e.g., optimal behaviour in tennis and soccer vs. sub-optimal behaviour in American football and baseball) have been harder to explain. Consequently, it remains an open question as to whether sports ‘experts’ do play optimally. The first contribution of

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4However, a further analysis of the results undertaken by Wooders (2010) found some deviations away from the theoretical predictions. Also a similar experiment by Levitt, List, & Reiley (2010) found little evidence supporting the equilibrium play in a different set of professionals. These results place some bounds on the initial results of Palacios-Huerta & Volij (2008).
this research will be to provide a further analysis of minimax behaviour to shed more light on this debate. These findings can then also be used to support or challenge the experimental results that suggest individuals do not make choices that are consistent with minimax behaviour.

This research is related to previous studies of decision-making in tennis beginning with Walker & Wooders (2001), which use data from professional tennis matches to study the strategic choices of servers. They found that the server’s point-winning probabilities for each action were roughly equal – an indicator of minimax play. Subsequent work by Hsu et al. (2007) and Gauriot et al. (2016) further confirmed these results. However, Walker & Wooders (2001) noted that there were high levels of negative serial correlation in the directions that players chose to serve, hinting at some irregularities in players’ decision-making on a point-by-point basis. In addition, these studies do not consider the impact of pressure and the effect that this may have on players’ ability to choose their service strategies optimally. This research builds on these studies by incorporating individual behavioural responses to pressure in an attempt to better model behaviour on a point-by-point level.

The psychology literature detailing the negative effect of pressure is vast and dominated by two prevailing theories. One of these is the self-focus hypothesis of pressure (Baumeister, 1984), which hypothesises that pressure causes experienced agents to focus too much on performing tasks that have become natural. This ‘over-thinking’ disrupts the usual performance mechanisms resulting in hindered performance. A second theory is the attentional distraction hypothesis (Wine, 1971), which hypothesises that an individual will devote less attention to the task at hand and focus more on external factors (such as crowd noise or possible rewards) in situations of high pressure, and consequently under-perform in these tasks.

These two theories of pressure have been examined extensively in a variety of circumstances. For example, the self-focus hypothesis has been used to explain detrimental performance in many situations that require high-level motor skills,

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5Hsu et al. (2007) found less evidence of serial correlation in players’ serving choices.
such as golf putting (Beilock & Carr, 2001; Beilock, Carr, MacMahon, & Starkes, 2002), soccer (Beilock et al., 2002; Jackson, Ashford, & Norsworthy, 2006), hockey (Jackson et al., 2006) and rally car driving (Wilson, Chattington, Marple-Horvat, & Smith, 2007). Alternatively, the attentional distraction hypothesis has seen some experimental verification in tasks that require high working memory, such as complex decision-making tasks (L. W. Morris & Liebert, 1969; Sarason, 1972; Wine, 1971).

Despite researchers being aware of the possibility that pressure can influence decision-making, examination of these effects can be challenging from an economic perspective. The primary reason for this is that it can be financially difficult or ethically problematic to induce a pressure environment in laboratory subjects. As a consequence, the most common form of pressure that economists investigate is the impact of time pressure. Time pressure is known to negatively affect the ability to process information in complex tasks in the psychological literature (Gilliland & Schmitt, 1993; Weenig & Maarleveld, 2002). In economic experiments this type of pressure is generally achieved by restricting the amount of time subjects have to make their choices. For example, when bargaining in the ultimatum game, Sutter, Kocher, & Strauß (2003) found that rejection rates were significantly higher under a tight time constraint. Furthermore, Kocher & Sutter (2006) found that convergence to equilibrium and agents’ payoffs in the beauty-contest game are greater under low time pressure than under high time pressure. Both studies highlight the importance of incorporating pressure as a key component of decision-making models.

The importance and difficulty of incorporating pressure into economic models highlights the second contribution of this research, which is to study an environment (professional tennis) where pressure can be introduced in a tractable manner. The pressure in this environment is not due to time constraints, but rather factors that include financial payoff and ranking points, both of which directly translate into a player’s livelihood. Arguably this kind of pressure is more important for these individuals and so this study is likely to represent a more accurate assessment of behaviour under extreme pressure than previous research.
The most relevant studies in this area were conducted by González-Díaz, Gossner, & Rogers (2012) and Cohen-Zada, Krumer, Rosenboim, & Shapir (2017). González-Díaz et al. (2012) examined the performance of several tennis players to determine whether their success was related to their performance at important points in a tennis match. Not surprisingly they found that there was a significant correlation between players’ rankings and their ability to perform well (i.e., be less negatively affected) at important moments. More generally, Cohen-Zada, Krumer, Rosenboim, & Shapir (2017) found that men ‘choke’ under competitive pressure and consistently perform worse at important points. Interestingly, they found no such behaviour in women. However, these studies focused solely on the outcome of the point/match, without consideration of the choices that players took to get there. In this study I expand upon these results by examining the role that decision-making plays in determining players’ success at important points.

Since there is considerable evidence that increasing the size of the audience or possible financial gain has negative consequences on performance (Baumeister & Showers, 1986; Lewis & Linder, 1997), it is not surprising that there have been other studies investigating pressure in sports where these factors are particularly prominent. In addition to the studies of the self-focus hypothesis mentioned above, Dohmen (2008) found that soccer players successfully made significantly fewer penalty kicks at their home ground, especially when the score was close. Goldman & Rao (2012, 2013) also found evidence that pressure inspires greater effort in home teams in the National Basketball Association (NBA) but that players perform significantly worse in situations involving concentration, such as free throws.6

These are examples of scenarios where pressure affects the execution of particular actions: the golf putt, the soccer kick or the basketball shot. However, the literature examining the effect of pressure on the initial choice of the player (e.g., the direction of the kick) is limited. A feature of the work of Romer (2006) is that he analysed play calls in American Football on the 4th down, the most important play of the team’s

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6Both these are example of the home-court disadvantage. See, for example, Wallace, Baumeister, & Vohs (2005).
possession. The key result of his analysis was that teams tended to be sup-optimal in their choice of play calls. Specifically, he found that teams chose to kick the ball too frequently when instead they would be considerably better off by running the ball more often on the final play. These findings suggest that pressure not only affects the execution of particular actions of professional athletes, but also their ability to make optimal decisions.

Therefore, the final contribution of this research is to provide an in-depth analysis of the effect of pressure on the decision-making of highly-trained individuals. This represents an attempt to extend the literature of decision-making in economics, while also providing an empirical analysis of a well-known psychological phenomenon.
Chapter 3

A Model of Tennis as a Game

Tennis (officially known as “lawn tennis”) was first played sometime between 1859 and 1865. Since then it has become one of the most popular sports in the world. The British Journal of Sports Medicine estimates that 75 million people played the sport in 2007. Tennis can be played between two (singles) or four (doubles) and has a professional tour (the ATP tour for men and the WTA tour for women) that includes events from various tiers ranging from the prestigious Grand Slam events to Masters, Challenger and Futures events. These tournaments are held all over the world and have audiences of hundreds of thousands of people annually. Tennis also became a full medal sport at the 1988 Olympics. The focus of this research will be men’s singles matches from Grand Slam and Masters events.

I begin with a brief outline of the game of tennis. Every point in tennis is started by a server who attempts to serve the ball into a designated area and a receiver attempts to return this serve. From here onwards, both players attempt to hit the ball within the boundaries of the court and the point is won when one player cannot successfully do so. To win a game a player must be the first to win four points with a margin of two. To win a set a player must be the first to win six games with a margin of two or, where applicable, win the set tie-break. Lastly, the

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7 More recent estimates from 2012 suggest that 1.2 billion people play or watch tennis regularly, putting it as the fourth most popular sport in the world behind soccer, cricket and hockey.
8 Tie-breaks were introduced in 1971 and are played in the majority of sets when the game score reaches 6–6 to prevent exceptionally long sets. Tie-breaks require a player to win seven points with a margin of two points. Tie-breaks are played in all sets except the final set at Wimbledon, the Australian Open and the French Open.
winner of the match is determined in either a best-of-three or best-of-five set format. Players alternate service after every game, and after the first point and every two points thereafter in a tie-break.

Finally, the score can be expressed as a vector of the point, game and set scores for both the Server and Receiver. Let $p$ denote the point score, $g$ denote the game score and $s$ denote the set score, the total current score, $\theta$, is given by:

$$\theta = (p_s, p_r, g_s, g_r, s_s, s_r)$$

where the subscripts $s$ and $r$ denote the Server’s and Receiver’s scores, respectively.

### 3.1 Modelling a Point in Tennis

One important aspect of a tennis match is the serve-return interaction between the two players. As approximately half of all points are decided in the first three shots – the serve, return and subsequent shot – this interaction is vital for determining the outcome of the majority of points in tennis. The aim of this section is to develop a tractable way to model this interaction as a repeated constant-sum game that allows extensions to include situations of varying pressure. This will provide the framework for examining players’ decision-making while under the influence of pressure.

Constant-sum games are typically characterised by some form of competition over a resource. In these games agents simultaneously choose actions to maximise their chance of obtaining the resource. This description can also be applied to the interaction between a server and a receiver in tennis. The agents of this game are the two players and the resource they are competing for is the outcome of the point (which affects the outcome of the match and the resulting money and ranking points). The two players choose their shots to maximise the likelihood of winning the point. Moreover, if one player can predict their opponent’s actions then they can alter their own actions to counter them and increase their chance of winning. Therefore, one would expect players to keep their opponent guessing about the shot that they will choose to play. This type of behaviour is characteristic of constant-
sum games, since any gain by one player is offset by an equivalent loss to the other, and is crucial for examining whether players behave optimally in these scenarios.

Explicitly, I model the serve-return interaction in tennis as a $2 \times 2$ constant-sum game between two risk-neutral players: the Server and the Receiver. The Server chooses which side of the service court to serve to, and the Receiver chooses which side they anticipate the Server will serve to and ‘overplay’ this side. In tennis these two sides are known as Wide ($W$) and Tee ($T$). Since every point has one winner and one loser, it is a constant-sum game where the winner gets a payoff of 1 and the loser gets a payoff of 0. Hence, every point can be represented by a game matrix where the entries in each cell represent the expected payoff to the Server, given the actions/choices of each player. Denoting the probabilities that the Server wins the point as $\pi_{SR}$, where $S$ is the Server’s choice, $\{W, T\}$, and $R$ is the Receiver’s choice, $\{W, T\}$, the payoffs (to the Server) in the game matrix are precisely the service point-winning probabilities, $\pi_{SR}$ (the Receiver’s payoff is $1 - \pi_{SR}$). Figure 3.1 illustrates how tennis can be modelled as $2 \times 2$ normal-form game and presents the corresponding game matrix:

<table>
<thead>
<tr>
<th></th>
<th>Receiver</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$W$</td>
</tr>
<tr>
<td>Server</td>
<td>$W$</td>
</tr>
<tr>
<td></td>
<td>$T$</td>
</tr>
</tbody>
</table>

**Figure 3.1:** Tennis as a $2 \times 2$ normal-form game

---

9I do not attempt to model the entire point, only the first serve-return interaction. One may argue that the likelihood of winning the point could depend on other factors, such as the second serve or the subsequent shots in the points. However, for risk-neutral players, these factors will be incorporated into the game matrix payoffs. Furthermore, the aim of the majority of professional players is to win the point using their first serve; hence, their service actions should still be chosen to maximise their chance of winning the point. This model is extended in Chapter 8, where I include players’ risk preferences in an attempt to rationalise some of the observed behaviour.

10Overplaying a side is the tennis analogue of how a goalkeeper must guess the side of a penalty kick in soccer and react accordingly. I model the game in this manner to be consistent with the original work by Walker & Wooders (2001). For further details refer to their seminal paper.

11More than two choices may exist for the Server (such as serving directly at the Receiver’s body) but have been excluded for simplicity. Some examination of these choices will be carried out in a later analysis.
3.1.1 Point-Games

If there was no variation in the winning probabilities for each point over the course of the match, then Figure 3.1 could be used to model the serve-return interaction for every point in the match. However, points in tennis begin by one player serving from one of two possible sides, or ‘courts,’ and alternate after each point. These courts are known as the ‘Deuce’ and ‘AD’ courts. Furthermore, due to the differences in players’ skills and preferences, it is highly likely that the probability of winning a point on service will vary between these distinct courts even for the same player. Additionally, due to differences in players’ abilities, there will almost certainly be different winning probabilities for each server for a given court. Therefore, there are at least four distinct game matrices in every tennis match. Each specific game matrix corresponds to one of the two players serving to one of the two courts. This leads to the following definition:

Definition. A point-game is the game matrix characterised by all the points in a match that are served by a particular player to a particular court. Explicitly, these are specified as:

- **Point-Game 1.** Player 1 serving to the Deuce court
- **Point-Game 2.** Player 1 serving to the AD court
- **Point-Game 3.** Player 2 serving to the Deuce court
- **Point-Game 4.** Player 2 serving to the AD court

I assume that this represents the full variation in the winning probabilities in the each match (until I introduce pressure). Therefore, rather than the one game matrix as shown in Figure 3.1, there are actually four different game matrices for each tennis match.

---

12This assertion is investigated initially in Chapter 4 and in greater depth in Section 7.1 and the results strongly suggest that this assumption is valid for this sample.
3.1.2 Characterising the Equilibrium

The simplest theory for describing how agents should act in constant-sum games, such as the one illustrated by Figure 3.1, is the minimax theorem proposed by von Neumann (1928). This theorem states that agents should attempt to minimise the loss they could receive and has some intuitive features. For instance, in many games where a single equilibrium is predicted, players should randomise their actions so that their opponent has limited ability to exploit their behaviour. For example, in soccer penalty kicks, the kicker should randomise between choosing to kick left or right so that the goalkeeper cannot predict where the kick will go. This concept was generalised by Nash (1950) and is the basic framework for understanding behaviour in many strategic games.

This interaction and the one presented in Figure 3.1 belong to a class of games colloquially known as the ‘matching pennies’ game. In these games one player has the incentive to choose a different action to their opponent, whereas the other benefits from the actions being the same. In the case of tennis, it is the Server who wants to serve the ball away from the Receiver, while the Receiver wants to correctly anticipate the direction of the serve. From this description, it is reasonable to believe that the Server will win more points when they serve to the side that the Receiver is not anticipating compared to the side that they anticipate. Furthermore, it is also reasonable to believe that the Server will win relatively fewer points if the Receiver overplays the side that the Server chooses to serve compared to the side that they do not serve to. These assertions are equivalent to:

**Assumption.** The entries in the game-matrix satisfy the following for each point-game:

- $\pi_{WW} < \pi_{TW}$ and $\pi_{TT} < \pi_{WT}$
- $\pi_{WW} < \pi_{WT}$ and $\pi_{TT} < \pi_{TW}$

The following theorem is a direct result of this assumption:

**Theorem** (Nash). Every point has a unique equilibrium in strictly mixed strategies.
Since this is essentially Nash’s Theorem (Nash, 1950) for the particular game shown in Figure 3.1 and characterised by the above assumptions – which rule out the possibility of pure strategies in equilibrium – I will not provide a proof. Denoting the $p$ as the probability that the Server serves $W$ and $q$ is the probability that the Receiver anticipates a $W$ serve, then the unique equilibrium solution to the specific example in Figure 3.1 is:

$$p^* = 0.5, \quad q^* = 0.625$$

with expected payoffs of

$$E(S) = 0.675, \quad E(R) = 0.325$$

Thus, the equilibrium of this game is for the Server to serve $W$ with probability 0.5 and the Receiver to anticipate $W$ with probability 0.625. Furthermore, the Server will win the point with probability 0.675.

The issue with a one-point model of tennis is that it is impossible to determine whether any observed behaviour is consistent with the equilibrium. This is because the equilibrium solution is to choose each action with a non-zero probability. Thus, to make assertions about players’ behaviour, it is necessary to study matches as a whole.

### 3.2 Modelling Tennis Matches as a Whole

The difficulty with generalising the above point-level model to the match-level is that tennis consists of an ex-ante unknown number of identical stage-games (points) where the payoff is not the sum of stage-game payoffs, but depends only on the final outcome. In this ‘super-game’ the (expected) payoff for each point to the Server given the choices of the Server and the Receiver is:

$$\Pi_{SR} = \pi_{SR}\left(\Pr(W^*|W(\theta)) - \Pr(W^*|L(\theta))\right)U = \pi_{SR}\Delta\Pi(\theta)U$$
where $\pi_{SR}$ is the probability that the Server wins the point for the specific choices of two players, $\Delta \Pi(\theta)$ is the difference between the probability that the Server wins the match given they win the point, $\Pr[W^*|W(\theta)]$, and given they lose it, $\Pr[W^*|L(\theta)]$, at score $\theta$, and $U$ is the utility from winning the match. Defining $I(\theta) := \Delta \Pi(\theta)$ and normalising $U$ to 1 gives:

$$\Pi_{SR} = \pi_{SR} I(\theta)$$

(3.1)

where $I(\theta)$ is also known as the ‘importance of a point’ on the outcome of the match. This concept will be critical to the empirical analysis in this thesis and will be discussed in great detail in Chapter 6.

As a result of this structure, the optimal strategy for the match ‘super-game’ may differ from the optimal strategy for playing a single point. However, for tennis, it is possible to show that this is not the case and that the optimal strategy is to play each point optimally. To show this I first assume the following:

**Assumption** (I.I.D.). In each point-game in a tennis match:

1. The probability of winning a point on serve is independent of all other points.
2. The probability of winning a point on serve is constant.

This assumption implies that the winner of each point is an independent draw from a Bernoulli distribution with parameter $\pi$, the probability that the Server wins the point.\(^{13}\) Nash’s theorem has the following implication for the probability of winning a service point in this scenario:

**Corollary.** Players have a non-certain probability of winning a point on their service in equilibrium, $\pi^*$. That is,

$$0 < \pi^* < 1$$

Using this, I can obtain the main result needed to generalise the model to the match-level:

\(^{13}\)For a detailed discussion on this assumption refer to Appendix B.
Proposition 1. For each point in tennis:

1. Players strictly prefer winning to losing.

2. The equilibrium strategies are homogeneous of degree zero in the payoffs.

(Proof in Appendix A.) The first part of this result states that $I(\theta) > 0$. Thus, the expected payoff of the ‘super-game’ from equation (3.1) is a non-zero constant multiplied by the point-winning probabilities. While it may be complex to calculate the exact value of this constant, is it not important for the determination of the equilibrium strategy due to second part of the result. This part states that the equilibrium strategies of the ‘super-game’ with payoffs given by equation (3.1) are the same as those for the individual point model in Figure 3.1. This provides the basis for the key lemma needed to describe behaviour at the match-level:

Lemma 1. In tennis the optimal long-run strategy for each player is to play the one-point stage-game mixed-strategy equilibrium at each point.

This result is generalised by Walker, Wooders, & Amir (2011) who showed that, for a category of games known as binary Markov games (of which tennis belongs), the optimal strategy is to play each stage game as though it is the only one.\footnote{This provides a basis for the well-known tennis adage, “Play each point as it comes.”}

The first implication of Lemma 1 is that, while it is impossible to infer whether players are choosing their actions optimally for each individual point, it is possible to determine whether their behaviour over a series of points is optimal. This can be deduced by comparing the proportion of times each action is observed to the equilibrium probabilities. For example, consider a match consisting of 100 service points and characterised by the game matrix given in Figure 3.1. If both players were playing their equilibrium strategies, then one would expect to observe the Server choosing to serve $W$ and $T$ each roughly 50 times. Similarly, one would expect to observe the Receiver anticipating the Server to serve $W$ roughly 63 times and $T$ roughly 37 times.
The second implication of Lemma 1 is that players’ service choices at each point should be independent of all past choices so that there is no discernible pattern in either player’s behaviour. In the $2 \times 2$ case, the decision variable is a binary variable (it is either to serve $W$ or $T$) and so I can simplify this further using the following proposition:

**Proposition 2.** If $X$ and $Y$ are uncorrelated Bernoulli random variables, then they are also independent.

(Proof in Appendix A.) Letting $X$ be the current choice and $Y$ the previous choice, this result states that the two choices are independent if there is no serial correlation between them. In other words, dependence and correlation are equivalent in the $2 \times 2$ case. Consequently, players’ service choices should be serially uncorrelated in equilibrium.

Lemma 1 implies that there are two conditions required for optimal behaviour. These are formalised in the following corollary:

**Corollary.** In equilibrium, each player chooses their actions over the course of the match such that:

1. The probability of choosing a specific action is consistent with the minimax equilibrium probability of choosing that action.

2. The choices of actions are serially uncorrelated.

Determining the minimax probabilities requires the true winning probabilities from the payoff matrix (the $\pi_{SR}$’s), which are not known here. In theory, these values could be estimated from the observed winning probabilities for each action of the Server and Receiver. However, this cannot be done in tennis since it is hard to observe which side the Receiver’s anticipates. The elements that can be observed are the Server’s actions and which player wins the point. From this it is possible to determine the winning probabilities for each action of the Server, $\pi_S$. Using the indifference property of the equilibrium, the observed winning probabilities should
be the same for all choices of service direction if players are behaving optimally. This provides an alternative way to test for optimal behaviour, despite not having information on the Receiver’s choices, by reformulating the above conditions into the following key lemma:

**Lemma 2.** In equilibrium the Server chooses their service direction such that:

1. The probability of winning the point is equal across their service choices.

2. The choices of actions are serially uncorrelated.

Note that the equilibrium conditions concern the players’ choices but, in reality, it is not possible to observe their choices, only their actions. However, I assume that players at this level are capable of implementing their decisions (close to) perfectly from years of training, so:

**Assumption.** The observed choice is always the action chosen.

The equilibrium conditions presented in Lemma 2 provide the basis for my analysis of the ability of tennis players to make optimal decisions. In Chapter 5, I use these equilibrium conditions to examine whether players’ behaviour is consistent with theoretical predictions. This will provide a general assessment of whether tennis players behave optimally. In Chapter 7, I use the behaviour observed from this analysis to examine the impact that pressure may have on players’ decision-making. This is complemented with an analysis of the impact of pressure on players’ outcomes to determine whether the effect that pressure has on decision-making impacts players’ outcomes as well.

The empirical analysis in this dissertation will be predominantly from the Server’s perspective. This is because, as mentioned, the Receiver’s decisions are substantially harder to observe in practice and so the dataset does not contain information on the Receiver’s choices. Regardless, I argue that the Server’s choice is more important for an analysis of decision-making under pressure than the Receiver. This is because the Server has full control over their action, whereas the Receiver chooses
a direction to anticipate but also has a small amount of time to react to the serve. This implies that the Receiver’s action is comprised of a decision component and a reactionary component. Consequently, if there is an effect of pressure on decision-making, it should be more prominent in the observed actions of the Server than the Receiver. There will be some discussion of the Receiver’s behaviour when I attempt to rationalise the key findings in Chapter 8.
Chapter 4

Data

The data used for this research come from the publicly available website Tennis Abstract.\textsuperscript{15} This is an open source platform for tennis enthusiasts dedicated to the detailed charting of professional tennis matches. At the time of writing their Match Charting Project has 3,303 matches in their database from the men’s Association of Tennis Professionals (ATP) and the Women’s Tennis Association (WTA) tours. The matches in this database are being updated on a daily basis. These matches range from Challenger to Grand Slam level, covering all the top tiers in professional tennis. For each match, a point-by-point description of play is provided along with the score and the player serving for each point. Using these descriptions, it is possible to extract the variables required for this analysis.\textsuperscript{16} The following is an example taken from the 2012 Australian Open Final between Novak Djoković and Rafael Nadal, with Nadal serving at a score (30 – 15, 3 – 2, 0 – 0):

1st serve down the T, fault (net). 2nd serve down the T; forehand return down the middle (shallow); forehand inside-out; forehand down the middle; forehand cross-court; backhand crosscourt; forehand down the middle; forehand crosscourt; backhand slice down the middle; forehand drop shot crosscourt; backhand slice down the line; backhand down the line, winner. (12-shot rally)

\textsuperscript{15}The data are available from the following URL: http://www.tennisabstract.com/.

\textsuperscript{16}Refer to Appendix C for a detailed description of the data and the construction of the key variables used in this analysis.
In this study the data are used from 941 tennis matches from the professional ATP (men’s) tennis tour spanning the years 2008–2017. These matches comprise a total of 148,144 individual points across 199 different players. Furthermore, the matches are only from Masters or Grand Slam tier tournaments to ensure that each player has a strong incentive to behave optimally.\textsuperscript{17} Lastly, the vast majority of the players have achieved a career highest ranking inside the top 100; therefore, the sample should be representative of the top professional players.

Although it is possible to examine professional women’s tennis, this analysis is confined to the men’s game as the serve in the men’s game is generally considered more potent than in the women’s game.\textsuperscript{18} Consequently, while not exclusively true, it is more likely that male players will actively try to win the point based on their serve. This implies that the decision of where to serve plays a more crucial role for males than females. This restriction is an attempt to ensure that the individuals in the sample will endeavour to choose their strategies in the server-returner game optimally. This behaviour is essential for any analysis of decision-making.\textsuperscript{19}

Table 4.1 below summarises the properties of the matches in the sample. In particular, I provide the breakdown of each match in the sample by court surface, format, stadium and the tournament tier. This is presented separately for the number of points, matches and point-games (of which there are four per match).

\textsuperscript{17}These matches offer the highest prize money and ranking points therefore it is reasonable to assume that each player has a powerful incentive to win the match. Top players will often use lower tier tournaments when they are recovering from injury as practice and hence, may not always try as hard as they would in a match in a top tier tournament.

\textsuperscript{18}The average men’s service speed at Grand Slams in 2009 was 184.1km/h and 150.4km/h for 1\textsuperscript{st} and 2\textsuperscript{nd} serves, respectively; whereas it was only 158.5km/h and 133.4km/h for women (Cross, 2014). Furthermore, the percentage of points won for men on serve at Grand Slams in 2010 ranges from 62\% to 67\% depending on the surface, compared to 55\% to 59\% for women (Klaassen & Magnus, 2014, p.29).

\textsuperscript{19}It is worth noting that Gauriot et al. (2016) found behavioural differences between genders in decision-making in tennis. Specifically, they found that, while male tennis players appear to conform to some of the theoretical predictions of minimax, the same is not true for female players.
Table 4.1: Summary of Match Characteristics

<table>
<thead>
<tr>
<th>Surface</th>
<th>Points</th>
<th>Matches</th>
<th>Point-games</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clay</td>
<td>37,062</td>
<td>243</td>
<td>972</td>
</tr>
<tr>
<td>Grass</td>
<td>18,060</td>
<td>102</td>
<td>408</td>
</tr>
<tr>
<td>Hard</td>
<td>93,022</td>
<td>596</td>
<td>2,384</td>
</tr>
<tr>
<td>Best-of-3 Sets</td>
<td>109,781</td>
<td>775</td>
<td>3,100</td>
</tr>
<tr>
<td>5 Sets</td>
<td>38,363</td>
<td>166</td>
<td>664</td>
</tr>
<tr>
<td>Stadium</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Outdoor</td>
<td>125,144</td>
<td>778</td>
<td>3,112</td>
</tr>
<tr>
<td>Indoor</td>
<td>23,000</td>
<td>163</td>
<td>652</td>
</tr>
<tr>
<td>Tier</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Grand Slam</td>
<td>37,930</td>
<td>164</td>
<td>656</td>
</tr>
<tr>
<td>Olympics</td>
<td>3,336</td>
<td>21</td>
<td>84</td>
</tr>
<tr>
<td>Masters 1000</td>
<td>42,543</td>
<td>307</td>
<td>1,228</td>
</tr>
<tr>
<td>Masters 500</td>
<td>21,919</td>
<td>157</td>
<td>628</td>
</tr>
<tr>
<td>Masters 250</td>
<td>42,416</td>
<td>292</td>
<td>1,168</td>
</tr>
<tr>
<td>Total</td>
<td>148,144</td>
<td>941</td>
<td>3,764</td>
</tr>
</tbody>
</table>

Table 4.2 and Figure 4.1 also present the distribution of the number of points per match for all the matches in the sample. Specifically, Table 4.2 provides the summary statistics, while Figures 4.1a and 4.1b plot the (kernel-smoothed) density and the empirical cumulative distribution function of the number of points for each match. This information is presented separately for the best-of-three and best-of-five set matches, as well as the entire sample.

Table 4.2: Summary of Points per Match

<table>
<thead>
<tr>
<th></th>
<th>Obs.</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Min.</th>
<th>Max.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Best-of-3</td>
<td>775</td>
<td>141.65</td>
<td>38.84</td>
<td>70</td>
<td>336</td>
</tr>
<tr>
<td>Best-of-5</td>
<td>166</td>
<td>231.10</td>
<td>70.04</td>
<td>116</td>
<td>436</td>
</tr>
<tr>
<td>Total</td>
<td>941</td>
<td>157.43</td>
<td>57.16</td>
<td>70</td>
<td>436</td>
</tr>
</tbody>
</table>
Based on Table 4.2 and Figure 4.1 there is substantial variation in the number of points in each match. This stems from the scoring format of tennis, which allows matches to continue as long as necessary until a result is obtained. As expected, the best-of-three set matches tend to have fewer points than the best-of-five set matches. Furthermore, the distributions are positively skewed. Therefore, despite the number of points in the majority of matches being close to the sample average, there are a few matches with substantially more points. These longer matches will likely involve ‘pressure points,’ which I will use to identify the effect of pressure on player’s decision-making in the subsequent analysis.

The key component of this analysis is conducted at the server-level; thus, I also examine the data at the server-level. Table 4.3 presents the summary statistics for the number of points for each server in the sample.

**Table 4.3: Summary of Points per Server**

<table>
<thead>
<tr>
<th>Obs.</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Min.</th>
<th>Max.</th>
</tr>
</thead>
<tbody>
<tr>
<td>199</td>
<td>744.44</td>
<td>1542.58</td>
<td>41</td>
<td>12272</td>
</tr>
</tbody>
</table>

The number of points for each server in the sample is also highly skewed and so there is a significant proportion of points that stem from only a few players. This may be problematic for an aggregate analysis since any results may be driven by the behaviour of these players. However, the majority of the analysis is conducted
at the individual level and hence this should not significantly affect the final results. Regardless, I drop players with a small number of observations to ensure that this issue is appropriately addressed. This is discussed in more detail in the estimation procedure in Chapter 7.

4.1 Some Trends in the Data

In this section I present a discussion of some patterns in the data to address the validity of some of the assumptions raised in Chapter 3, such as whether there is a difference between the Deuce and AD courts. I also provide motivation for a deeper analysis into the effect of pressure by using break-points as a crude measure of pressure.

4.1.1 Difference between Deuce and AD courts

The model of tennis as a constant-sum game in Chapter 3 makes the assumption that there are two independent courts in each tennis match (the Deuce and AD courts), where players’ strategies and winning probabilities are likely to be different. This assumption can be examined in the data. If there is no difference between the two courts then one would expect the proportion of $W$ and $T$ serves to be the same for both courts. To investigate this, Table 4.4 provides a breakdown of the number of $W$ and $T$ serves for each court.

<table>
<thead>
<tr>
<th>Court</th>
<th>Deuce</th>
<th>AD</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wide</td>
<td>35,093 (47.2%)</td>
<td>37,030 (54.9%)</td>
<td>72,123 (50.9%)</td>
</tr>
<tr>
<td>Tee</td>
<td>39,206 (52.8%)</td>
<td>30,372 (45.1%)</td>
<td>69,578 (49.1%)</td>
</tr>
<tr>
<td>Total</td>
<td>74,299</td>
<td>67,402</td>
<td>141,701</td>
</tr>
</tbody>
</table>

TABLE 4.4: Service Direction by Court
The numbers in Table 4.4 highlight that there is a clear difference between the frequency with which players choose to serve $W$ on the Deuce court (47% of the time) compared to serving $W$ on the AD court (almost 55% of the time).\textsuperscript{20} Note that the figures in Table 4.4 represent the aggregate statistics for the entire sample.

I also investigate whether individual players treat these two courts differently. Figure 4.2 illustrates this by plotting the (kernel-smoothed) density of the mean proportion of serves to a particular direction for each court across all servers in the sample.

![Density of Proportion of Serves Wide](image)

Figure 4.2: Distribution of Mean Proportion of Wide Serves

The distribution of the mean proportion of $W$ serves for the Deuce court in Figure 4.2 appears to be significantly different to that of the AD court. It is centred below 0.5 for the Deuce court, suggesting that the majority of players serve $T$ more than 50% of the time on this court. On the other hand, the distribution for the AD court is centred above 0.5, suggesting that the majority of players serve $W$ more than 50% of the time on this court. These findings are consistent with the aggregate results.

\textsuperscript{20}As a side note, these results conform to a common belief that one should serve more to a player’s backhand – generally considered a weaker side. Since the majority of professional players are right-handed, their backhand side corresponds to a $T$ serve on the Deuce court and $W$ serve on the AD court.
Lastly, I examine this observation on an individual level by testing the hypothesis that the probability of serving $W$ on the Deuce court is equal to the probability of serving $W$ on the AD court for each player. I find that for 87 out of a total of 199 servers (44%) the null hypothesis is rejected at the 5% level of significance. This number is substantially higher than one would expect to see for this sample. Specifically, for a sample of 199 servers, approximately 10 would be expected to show a significant difference by chance. Therefore, the assumption from Chapter 3, that there are two separate courts involving separate behaviour, appears valid.

As a result of players’ distinct behaviour on the separate courts, an analysis of player behaviour should be separated by court and examined individually. Another benefit of examining each court independently is that the analysis is robust to the fact that the majority of pressure points occur on the AD court. This may bias the results if the observations for the two courts were pooled together. In general, there are numerous factors that may affect points between the Deuce and AD courts; however, it appears that there is no interaction between the Server, Receiver and the court, and so the analysis should be robust to these factors.

### 4.1.2 The Impact of Pressure

The motivation for this dissertation arises from the idea that pressure affects players’ behaviour in some (possibly negative) way. To provide preliminary support for this idea I examine the outcomes for players at break-points compared to other points. Break-points are the points in a game where the Server will lose their service game if they lose the point. These points are traditionally seen as crucial points for the Server since losing a service game typically has a large detrimental effect on that player’s chance of winning the set (and hence the match). For this reason break-points are commonly quoted by players and coaches as “pressure points.” Therefore, a comparison between break-points and other points can be used as an initial check to see whether pressure has an effect on players.

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21 The significance of these points can be seen mathematically in Chapter 6 where I quantify the importance of each point in a game.
Table 4.5 examines players’ winning probabilities at break-points vs. other points as motivation for an analysis into the effect of pressure on tennis professionals.

**Table 4.5: Service-Winning Probabilities Break-Points vs. Other Points**

<table>
<thead>
<tr>
<th></th>
<th>Winning Probability</th>
<th>Total Observations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Break-points</td>
<td>61.1%</td>
<td>12,186</td>
</tr>
<tr>
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</table>

The figures from Table 4.5 show that players win significantly fewer points on break-points than other points. On average, players win approximately 64% of points on their serve compared to only 61% of points at break-points. The difference in means is significant at the 1% level ($p$-value = $1.17 \times 10^{-12}$). This suggests that pressure may have a significant (negative) impact on the outcome of the point for the Server. However, care must be taken when interpreting this result as there is likely to be some selection bias in these figures. Overall, weaker players typically have lower service-winning probabilities and they are also the players most likely to be those who face break points; consequently, the winning-probabilities at break points will be biased downward. This motivates and highlights the need for a more in-depth analysis into the effect of pressure on players’ performance. Sections 7.1 to 7.3 of Chapter 7 scrutinise this observation further by presenting a detailed analysis of the effect of pressure on players and their outcomes. Specifically, they are predominately devoted to investigating the effect that pressure has on players’ decision-making and whether this is the cause of their below average performance at ‘pressure points.’

Before I attend to the primary aim of this study which is to understand decision-making under pressure, I first need to establish how individuals behave without pressure. This is the focus of the next chapter and will be used as a baseline to compare how pressure affects players when I introduce it into this environment. This comparison will allow me to examine the key question of the dissertation: whether pressure affects the likelihood of players choosing their strategies optimally and the impact this has on their outcomes.
Chapter 5

Baseline Analysis

In Chapter 3 I presented a number of equilibrium predictions that can be derived from the model of the serve-return interaction that players should adhere to in order to maximise their chances of winning a tennis match. This chapter is devoted to examining whether these theoretical predictions are observed in practice. This involves investigating the two main predictions of minimax theory formalised in Lemma 2. The first is that players equalise their winning probabilities across their service choices. The second is that the service choices are serially uncorrelated. If players do exhibit this behaviour, then it suggests that they are playing the game optimally. However, if these predictions are not observed then it is possible that some facet of their behaviour could be exploited by their opponent. The behaviour observed in this chapter will be used a baseline for the analysis in later chapters when pressure is introduced.

In addition to serving as a benchmark for examining how pressure affects players’ decision-making, this analysis also supplements the previous literature that examines decision-making in tennis; originally by Walker & Wooders (2001) and followed by Hsu et al. (2007) and Gauriot et al. (2016).

To examine the equality of winning probabilities I utilise two approaches. The first is a direct test for the equality of the point-winning probabilities between each service direction choice. This method allows a direct comparison with the analysis by Walker & Wooders (2001) and subsequent work by Hsu et al. (2007). However,
this method only has sufficient power in longer matches, therefore the analysis must be restricted to the longest matches in the sample to draw meaningful conclusions. The second method uses a randomised version of the Fisher exact test. This test provides a comparison with the analysis by Gauriot et al. (2016). This method has the advantage of providing a test that is robust even when using matches with fewer points and so it can be applied to the whole sample of matches. The results I observe suggest that, in the vast majority of cases, players choose their service direction to equalise the probability of winning the point across all choices. These results are consistent with the first prediction of minimax theory and also with the findings of previous research.

To investigate the extent of serial correlation in players’ service choices, I test whether there is a relationship between the current service choices and past service choices. The results suggest that, on average, there is negative serial correlation in a player’s choice of service direction. This finding is at odds with the second prediction of minimax theory, but is consistent with the majority of previous experimental studies and previous research of decision-making in tennis, which have concluded that players generally switch directions too frequently.

5.1 Prediction 1: Equal Winning Probabilities

The first prediction of minimax theory is that players should equalise their winning probabilities across their possible choice of service direction. As mentioned above, there are two ways to test this prediction. The first method is a direct test for the equality of the Server’s point-winning probabilities between each of their choice of service direction. The second method uses a randomised version of the Fisher exact test.
5.1.1 Direct Test

The first test involves directly examining point-games to determine whether players equalise the probability of winning the point between each service direction choice. Since this test only has sufficient power to reject the null hypothesis in long matches, I restrict the sample to the 25 longest matches. Every match consists of four distinct point-games, so the observations are separated by player and court and analysed individually. Since there are 25 matches each consisting of four distinct point-games, this results in a total of 100 point-games. For each of these point-games I test whether the observed probability of winning the point, \( \pi \), is the same across the Server’s action set: Wide (W) or Tee (T). To examine this I test the following hypothesis:

**Hypothesis 1.** For each point-game, \( i = 1, \ldots, 100 \)

\[
H_0 : \quad \pi^i_W = \pi^i_T \\
H_1 : \quad \pi^i_W \neq \pi^i_T
\]

Under the null hypothesis, the Pearson test statistic for the equality of the service-winning probabilities is asymptotically distributed as chi-squared with one degree of freedom.

Table 5.1 below provides the results of the test of equality with two service choices (W and T). From these results it can be seen that the null hypothesis is rejected at the 1% level of significance in only four point-games. These are: (1) Federer serving on the Deuce court to del Potro at the 2012 Olympics semi-final, (2) Federer serving on the Deuce court to Nadal at the 2009 Australian Open final, (3) Federer serving on the AD court to Čilić at the 2016 Wimbledon quarter-final and (4) Dimitrov serving on the Deuce court to Nadal at the 2017 Australian Open semi-final. Furthermore, the null hypothesis is rejected at the 5% level of significance in only one additional point-game and at the 10% level of significance in a further three.

\[\text{For a more detailed description of the data used for the minimax analysis refer to Appendix D.1.}\]
In total, the results from Table 5.1 show that the null hypothesis is rejected in only four point-games at the 1% level of significance, five point-games at the 5% level of significance and eight point-games at the 10% level of significance. If instead these results were generated on a purely random basis, then roughly one, five, and ten rejections would be expected at these levels of significance, respectively. This evidence suggests that players do tend to adhere to the theoretical predictions of minimax behaviour in the majority of situations.

This conclusion is supported by the joint test of the equality of winning probabilities across all point-games. In this case the Pearson test statistic is the sum off all the test statistics for each point-game and is distributed as chi-squared with 100 degrees of freedom. The test-statistic is 103.533 and the associated $p$-value for this test is 0.384. Clearly the null hypothesis cannot be rejected in this case. Once again, I conclude that the probabilities of winning the point are equal across the Server’s choices in each point-game and players tend to adhere to the theoretical predictions of minimax behaviour.

I also examine a three-choice specification (Wide, Body and Tee) as a robustness check.\textsuperscript{23} Examining the results from using this specification, three, five and nine rejections are observed at the 1%, 5% and 10% levels of significance, respectively. The results are similar to the two-choice specification and in neither case can I conclude that deviations from the theoretical predictions are anything but random occurrences.

Following Walker & Wooders (2001), I provide further evidence by examining the distribution of the $p$-values obtained from the above analysis. If the data are consistent with the minimax hypothesis, then the $p$-values should be distributed $U[0,1]$. This provides a method to compare the observed data against the theoretical expectation.\textsuperscript{24} Figure 5.1 plots the empirical $p$-values (in blue) against the uniform distribution (in black).

\textsuperscript{23}Refer to Appendix D.2 for the details.
\textsuperscript{24}This method of examination is discussed in greater depth in Section 7.1.
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<td>0.748</td>
</tr>
<tr>
<td>P1</td>
<td>Ad</td>
<td>32</td>
<td>41</td>
<td>73</td>
<td>0.44</td>
<td>0.56</td>
<td>0.589</td>
</tr>
<tr>
<td>P2</td>
<td>Deuce</td>
<td>30</td>
<td>51</td>
<td>81</td>
<td>0.37</td>
<td>0.63</td>
<td>1.905</td>
</tr>
<tr>
<td>P2</td>
<td>Ad</td>
<td>48</td>
<td>26</td>
<td>74</td>
<td>0.65</td>
<td>0.35</td>
<td>0.186</td>
</tr>
<tr>
<td>P1</td>
<td>Deuce</td>
<td>45</td>
<td>42</td>
<td>87</td>
<td>0.52</td>
<td>0.48</td>
<td>1.313</td>
</tr>
<tr>
<td>P1</td>
<td>Ad</td>
<td>32</td>
<td>41</td>
<td>73</td>
<td>0.44</td>
<td>0.56</td>
<td>0.589</td>
</tr>
<tr>
<td>P2</td>
<td>Deuce</td>
<td>30</td>
<td>51</td>
<td>81</td>
<td>0.37</td>
<td>0.63</td>
<td>1.905</td>
</tr>
<tr>
<td>P2</td>
<td>Ad</td>
<td>48</td>
<td>26</td>
<td>74</td>
<td>0.65</td>
<td>0.35</td>
<td>0.186</td>
</tr>
</tbody>
</table>

**Indicates rejection at the 1-percent level of significance.**

***Indicates rejection at the 5-percent level of significance.**

*Indicates rejection at the 10-percent level of significance.
Observation of Figure 5.1 shows that the difference between the empirical and uniform distributions is small. Indeed, using the Kolmogorov-Smirnov (KS) test for the equality of two continuous and one-dimensional probability distributions, it is possible to test whether they are statistically different. The KS statistic is:

\[ K = \sqrt{n} \sup_{x \in [0,1]} |\hat{F}(x) - F(x)| \]

where \( n = 100 \) is the number of point-games, \( \hat{F}(x) \) is the empirical distribution of the \( p \)-values and \( F(x) = x \) is the cumulative distribution function of the uniform distribution. From this the KS test statistic becomes:

\[ K = 10 \sup_{x \in [0,1]} |\hat{F}(x) - x| \]

The KS test statistic for this analysis is 0.784, with associated \( p \)-value equal to 0.901. The evidence overwhelmingly suggests that the empirical distribution is equivalent to the uniform distribution.

A variety of tests indicate that it is not possible to reject the Hypothesis 1. Consequently, I conclude that players equate their service winning probabilities across their choice of service direction in accordance with minimax predictions.
5.1.2 Randomised Fisher Exact Test

Following the work of Gauriot et al. (2016), I employ a more powerful test based on the Fisher exact test to determine whether players choose their strategies optimally and can be used on the entire sample. Furthermore, they noted that this test provides a more detailed and powerful test of Hypothesis 1 when the number of points in each match is small relative to the overall number of matches.

Let \( f(n_{WS}; n_S, n_W, n_T) \) denote the probability that, under the null hypothesis, the Server wins \( n_{WS} \) serves \( W \), conditional on winning \( n_S \) serves in total, after delivering \( n_W \) and \( n_T \) serves \( W \) and \( T \) respectively, that is, \( f(n_{WS}; n_S, n_W, n_T) \equiv \Pr(n_{WS}|n_S, n_W, n_T) \). This conditional probability is given as:

\[
f(n_{WS}; n_S, n_W, n_T) \equiv \frac{\Pr(n_{WS}, n_S, n_W, n_T)}{\Pr(n_s, n_W, n_T)} = \frac{B(n_{WS}; n_W, \pi)B(n_{TS}; n_T, \pi)}{B(n_S; n_W + n_T, \pi)}
\]

where \( n_{TS} = n_S - n_{WS} \) is the number of winning \( T \) serves, and \( B(n_j; n_j, \pi) \) is the binomial probability of winning \( n_j \) of \( n_j \) serves in direction \( j \in \{W, T\} \) when the winning probability is \( \pi \). The equality follows from the independence of the binomial processes for \( W \) and \( T \) serves. By direct calculation it follows that:

\[
f(n_{WS}; n_S, n_W, n_T) = \frac{n_W}{n_{WS}} \pi^{n_{WS}}(1 - \pi)^{n_W - n_{WS}} \left( \frac{n_T}{n_{TS}} \right)^{n_{TS}}(1 - \pi)^{n_T - n_{TS}}
\]

where the density is defined over the domain \( n_{WS} \in \{\max(n_S - n_T, 0), \ldots, \min(n_s, n_W)\} \).\(^{25}\)

This conditional probability is exact for finite samples and does not depend on \( \pi \). The associated CDF, \( F(n_{WS}; n_S, n_W, n_T) \), is given by:

\[
F(n_{WS}; n_S, n_W, n_T) = \sum_{k=\max(n_S-n_T,0)}^{n_{WS}} f(k; n_S, n_W, n_T)
\]

The standard application of the Fisher exact test would reject the null hypothesis at the 5% significance level when \( F(n_{WS}; n_S, n_W, n_T) \leq 0.025 \) or \( F(n_{WS}; n_S, n_W, n_T) \geq 0.975 \). However, since the density \( f \) is discrete, this test will typically not have a

\(^{25}\)The number of successful \( W \) serves cannot be negative or less than the total winning serves minus the number of \( T \) serves. Similarly, it cannot be greater than the total number of winning serves or the total number of \( W \) serves.
size of exactly 5%. To correct for this Gauriot et al. (2016) employed a randomised version of the above test. For each point-game, i, define \( t^i \) as a random draw from the distribution \( U[0, F(n_{WS}; n_S, n_W, n_T)] \) when \( n_{WS} \) takes its minimum value and from the distribution \( U[F(n_{WS} - 1; n_S, n_W, n_T), F(n_{WS}; n_S, n_W, n_T)] \) otherwise. Under this definition, \( t^i \) is a random variable and the following proposition arises:

**Proposition 3.** \( t^i \) is distributed \( U[0, 1] \) under the null hypothesis, \( H_0 : \pi^i_W = \pi^i_T \).

(Proof in Appendix A.) Using this result to correct for the discontinuity in the density \( f \), it is possible to construct a test that has a size of exactly 0.05 by rejecting the null hypothesis when \( t^i \leq 0.025 \) or \( t^i \geq 0.975 \).

Table 5.2 presents the percentage of point-games for which the equality of winning probabilities is rejected at the 1%, 5% and 10% levels of significance using the randomised Fisher exact test. Since \( t^i \) is a random test statistic, the table shows the mean and standard deviation for the number of rejections for 10,000 trials.

<table>
<thead>
<tr>
<th>Significance Level</th>
<th>1%</th>
<th>5%</th>
<th>10%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean (%)</td>
<td>1.07</td>
<td>5.16</td>
<td>10.06</td>
</tr>
<tr>
<td>Std. Dev. (%)</td>
<td>0.11</td>
<td>0.21</td>
<td>0.26</td>
</tr>
</tbody>
</table>

To understand these results, consider the 5% column. The results state that the hypothesis of equalised winning probabilities can only be rejected in 5.16% of point-games (with standard deviation of 0.21%). The expected percentage of rejections under the null hypothesis is 5%. This difference is statistically insignificant. Therefore, this provides further evidence supporting the hypothesis that players equalise their winning probabilities across their service choice as suggested by minimax theory. The percentage of rejections is also consistent with what would be expected from random fluctuations in the data for the other significance levels examined in Table 5.2. Once again, this supports the theoretical predictions of minimax behaviour.
5.2 Prediction 2: No Serial Correlation

The assertion that players choose their strategies according to minimax predictions has two implications. First, players’ service strategies should be chosen to maximise the probability of winning the point. This property was investigated and confirmed in the previous discussion. Second, they should not be predictable in their strategic choices so that there are no exploitable patterns in their choices. This is equivalent to saying that the choice of service direction on a point should be independent of previous service choices. The following discussion examines this property.

Past research is inconclusive as to whether tennis players’ service choices are serially independent.\(^{26}\) This property is a theoretical prediction of optimal behaviour; thus it is important to properly investigate it for the players in this sample. This forms the second component of establishing players’ general behaviour that will later be used to compare against the behaviour observed under pressure.

If serial correlation is present, then players’ service choices for each point-game will be dependent on their choice at previous points. To capture the possibility of this dynamic behaviour, I specify a player’s service choices as an AR(1) process:

\[
d_{scm,t} = \beta_{sc0}^c + \rho_{sc}^c d_{scm,t-1} + \lambda_{scm}^c + \varepsilon_{scm,t}
\]

where \(d_{scm,t}\) is the direction of the serve at point \(t\) for Server \(s\) in match \(m\) on court \(c\). This depends on some average behaviour, \(\beta_{sc0}^c\), and the direction of the serve for the previous point, \(d_{scm,t-1}\), through the correlation coefficient, \(\rho_{sc}^c\). I also include a set of match fixed-effects, \(\lambda_{scm}^c\). Note that this model corresponds to estimating one regression per player per court.

Previous research uses non-parametric tests such as a runs test to test for the presence of serial correlation. However, a more detailed analysis can be performed by running regression (1) for each player-court and studying the coefficient estimates \(\hat{\rho}\). These coefficients provide an estimate of the serial correlation for each point-

\(^{26}\)Walker & Wooders (2001) have found evidence that, on average, tennis players switch their direction too often. However, Hsu et al. (2007) re-examined this property and have found less evidence of serial correlation in players service decisions.
game in the sample. Examining the coefficients in this manner has the advantage of providing information on the strength and distribution of the serial correlation in the sample. Also, since there are some missing observations, the run test is invalid here. Figure 5.2 plots the (kernel-smoothed) density of the coefficient estimates, $\hat{\rho}$, for every player-court in the sample.

![Kernel-Smoothed PDF](image)

**Figure 5.2: Density of the Correlation Coefficients Estimates, $\hat{\rho}$**

Figure 5.2 indicates that there is some heterogeneity with respect to players’ serial correlation. The majority of serial correlation observed lies in the range $-0.4 < \rho < 0.4$, which implies that the choices are not strongly serially dependent. However, the distribution appears to be centred below zero, which suggests that players may be slightly biased towards negative serial correlation on average. This observation points to the necessity of more rigorously testing to determine whether players exhibit serial correlation in their service direction. To do this, I formally test the hypothesis:

**Hypothesis 2.** *In the sample of 399 player-court observations*

- $H_0$: The sample exhibits no serial correlation
- $H_1$: The sample exhibits some serial correlation
To test Hypothesis 2, I use the following:

**Proposition 4.** Under the null hypothesis, the number of significant coefficients from $n$ independent regressions follows a binomial distribution, $B(n, p)$, where $p$ is the level of significance level chosen for the regression coefficients.

(Proof in Appendix A.) This result provides a method to test Hypothesis 2 by restating it as the following equivalent hypothesis:

**Hypothesis 2a.** The total number of observed significant correlation coefficients follows a binomial distribution, $B(n, p)$, where

$$
H_0 : \quad p = \alpha \\
H_1 : \quad p > \alpha
$$

with $n = 399$ and $\alpha$ is the significance level.

To test this hypothesis, I estimate regression (1) and count the number of point-games that exhibit some serial correlation. I compare this to the number of point-games that the binomial model theorises should arise from random fluctuations in the data. If the number/proportion of instances where serial correlation is observed is significantly greater than what is expected under the null hypothesis then it suggests that some form of serial correlation is present.

Table 5.3 gives the observed and expected number (first panel), and proportion (second panel) of significant estimates of first-order correlation at a significance level given by the column. The first row presents the results for either type of correlation (the correlation coefficients that are significantly different from zero). The second and third rows present the number and proportion of negative (the correlation coefficients that are significantly less than zero), and positive (the correlation coefficients that are significantly greater than zero) instances, respectively. The breakdown into positive and negative effects is presented to determine if the effect is asymmetric. I test the proportion of instances where serial correlation is observed against the null hypothesis in Hypothesis 2a in the second panel.
Table 5.3: Proportion of Observed Rejections

<table>
<thead>
<tr>
<th></th>
<th>Significance Level</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1%</td>
</tr>
<tr>
<td>Number</td>
<td></td>
</tr>
<tr>
<td>Either ($\rho \neq 0$)</td>
<td>89</td>
</tr>
<tr>
<td>Negative ($\rho &lt; 0$)</td>
<td>87</td>
</tr>
<tr>
<td>Positive ($\rho &gt; 0$)</td>
<td>18</td>
</tr>
<tr>
<td>Expected</td>
<td>3.98</td>
</tr>
<tr>
<td>Proportion</td>
<td></td>
</tr>
<tr>
<td>Either ($\rho \neq 0$)</td>
<td>0.224*</td>
</tr>
<tr>
<td>Negative ($\rho &lt; 0$)</td>
<td>0.219***</td>
</tr>
<tr>
<td>Positive ($\rho &gt; 0$)</td>
<td>0.045***</td>
</tr>
<tr>
<td>Expected</td>
<td>0.010</td>
</tr>
</tbody>
</table>

*** Indicates rejection at the 1-percent level of significance.
** Indicates rejection at the 5-percent level of significance.
* Indicates rejection at the 10-percent level of significance.

For example, there are 128 out of a total of 399 player-court observations where either form of serial correlation is observed at the 5% level of significance. Assuming these player-courts are independent and the likelihood of observing a significant coefficient is the same for each player-court, the number of observed coefficients follows a binomial distribution with $n = 399$ and $p_0 = 0.05$ under the null hypothesis. This implies that a significant serial correlation coefficient should be observed in approximately 20 player-courts. Using a one-tailed binomial test, the difference in proportions ($p_0 = 0.050$ vs. $\hat{p} = 0.322$) is highly significant. Therefore, the null hypothesis is rejected, and I conclude that there is a significant number of instances where some form of serial correlation is present. Similar results hold for the number of $\rho$ estimates that are significantly different from zero at the 1% and 10% level of significance.

Past research of decision-making in tennis suggests that any serial correlation in players’ service choices is likely to be negative (see e.g., Walker & Wooders, 2001). To investigate the validity of this observation, the next row of Table 5.3 examines the instances of correlation coefficients that are significantly less than zero. In this case, I observe 135 of 399 player-courts where negative serial correlation is present.
Baseline Analysis

at the 5% level. Once again, the difference in expected and observed proportions 
\( p_0 = 0.050 \) vs. \( \hat{p} = 0.339 \) is highly significant. Therefore, I reject the null hypothesis and conclude that there is a significant number of instances where negative serial correlation is present. Similar results also hold for the number of \( \rho \) estimates that are significantly less than zero at the 1% and 10% level. This finding is consistent with those of Walker & Wooders (2001) for professional tennis players, as well as the behaviour observed in several laboratory experiments (see e.g., Wagenaar, 1972).

For the sake of completeness, I also examine whether there is any evidence of positive serial correlation in the sample. However, in this case, the observed number of significant coefficients is only significantly greater than the expected number at the 1% level of significance. At both the 5% and 10% level of significance the number of \( \rho \) estimates is almost exactly what would be expected from random fluctuations in a sample this size. This is in complete contrast to the results for negative serial correlation above. Therefore, I cannot reject the null hypothesis and suggest that there is evidence that positive serial correlation is not prevalent in the sample.

5.3 Discussion

In this chapter I have attempted to replicate the results of Walker & Wooders (2001) using more than twice as many matches from a more recent dataset. I have successfully replicated their results suggesting that, in general, professionals choose their strategies according to one prediction of game theory. This prediction states that the probabilities of winning the point should be equivalent for each choice of serve. This implies that neither player has an incentive to change their strategies, which is a key consequence of an equilibrium strategy. These results were robust to the specification of the Server’s choice of actions. Specifically, the conclusions were the same regardless of whether the Server’s action set was \{Wide, Tee\} or \{Wide, Body, Tee\}. In both cases it is not possible to reject the null hypothesis that the winning probabilities are equal.
Furthermore, I employed a more powerful statistical test developed by Gauriot et al. (2016) to examine the same hypothesis. The increased power of this test has enabled me to perform the analysis on a larger sample of matches. Once again, the findings are consistent with the previous results. The evidence strongly suggests that players choose their service direction in accordance with this theoretical prediction.

The results were not as convincing for the prediction of minimax theory that posits that players should choose their service direction independently of their previous choices. The evidence from this analysis suggests that negative serial correlation is present in players’ service choices. This finding is at odds with those of Hsu et al. (2007), but consistent with the original work by Walker & Wooders (2001) and the subsequent work by Gauriot et al. (2016). The implication of these results is that, while tennis players appear to choose their service direction to equate the probability of winning the point across serves, they still have elements of predictability in their choices. Specifically, they tend to switch directions too frequently.

The fact that players do not adhere to the full set of equilibrium predictions when choosing their service strategies leads to the possibility that they do not make their decisions optimally. Specifically, any predictable patterns in players’ service choices could be exploited by their opponent, resulting in lower point-winning probabilities. However, individuals have been shown to be poor at recognising and generating random sequences (Wagenaar, 1972). Therefore, the failure of this second prediction of minimax may not be of serious consequence if the player’s opponent is unable to recognise and exploit any sub-optimal behaviour. For this reason, I argue that the first prediction is more critical in assessing players’ decision-making. A failure in this regard would imply that players are actively making choices that decrease their expected payoffs, which is clearly detrimental to their livelihoods. This is not observed in this analysis. Thus, despite their inability to make choices that are serially independent, tennis professionals still appear to be more adept at strategic decision-making than laboratory subjects who do not appear to conform to any predictions of minimax theory.
The analysis presented here provides a replication of previous literature designed to examine decision-making in tennis professionals. Moreover, it is intended to provide a baseline for when I study the effects of pressure in the same set of players. As noted, the results suggest that players do generally play according to one of the key predictions of von Neumann’s minimax theorem. Using this as a baseline, the next phase of the analysis is to incorporate pressure into this environment to determine whether this affects players’ ability to make optimal strategic choices. Thus, the following chapter is devoted to introducing pressure into this environment.
Chapter 6

Introducing Pressure

The baseline analysis in Chapter 5 suggests that players’ choice of service direction is consistent with some of the predictions of minimax theory. However, this analysis does not consider the impact of pressure on players’ decisions. If pressure has a detrimental effect on complex decision-making tasks, then it is not unreasonable to assume that it may also affect players’ ability to choose their service direction optimally. If true, then the conclusions from the previous analysis may no longer hold when pressure is present. This chapter outlines a method to quantify pressure in tennis that will provide the foundation for the subsequent analysis of the effect of pressure on decision-making and player outcomes.

Despite being a subjective phenomenon, there is some consensus amongst researchers that there are certain scenarios where people may be most susceptible to the effects of pressure. These include situations where individuals are forced to make decisions under tight time constraints (Gilliland & Schmitt, 1993; Weenig & Maarleveld, 2002), or when exposed to uncertain future events with large payoffs (Markman, Klein, & Suhr, 2012). Time constraints are not likely to be a factor in tennis since the amount of time allotted for each decision is constant and determined by the rules of the game. However, players are constantly exposed to some degree of uncertainty in the outcome of the match and a successful outcome often carries a substantial financial gain; thus, this factor is likely to have an impact in tennis.

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27 According to the laws of tennis, players are allowed 20 seconds between the conclusion of a point and the player beginning their service action for the next point (within the same game).
The remainder of this chapter formalises the idea of the ‘uncertainty in the outcome of a match’ by properly introducing the concept of a point’s importance. This concept allows me to quantify the uncertainty to provide an apt measure for the level of pressure faced by tennis players. Following that, I present some conditions that allow me to model tennis as a time-homogeneous and separable Markov process. Finally, I discuss how I specify players’ beliefs regarding their service point-winning probabilities. In combination, this allows me to calculate an explicit value for the level of pressure felt by players. As a parting note, I conclude with discussion of the importance of some points for two matches in the sample.

### 6.1 Importance of a Point

The discussion above suggests that the best approach to measure the amount of pressure at any given point in a match is to consider how much that point affects the uncertainty that each player feels about the outcome of the match. I illustrate this by considering two separate points. The first has a negligible impact on the outcome of the match and the second completely determines the winner. Since the first point has no bearing on the outcome, it has no effect on the uncertainty of the result. Therefore, both players should not care whether they win or lose the point and so are unlikely to feel any pressure. Alternatively, the second point has a substantial bearing on the outcome and so there is a large effect on the uncertainty of the result (since the result is certain at the conclusion of the point). Consequently, players should care greatly whether they win or lose the point and so are likely to feel a considerable amount of pressure.

What this highlights is that players are most likely to care about how their probability of winning the match changes from point to point. This is captured by the concept of the *importance of a point* proposed by C. Morris (1977) and used extensively in tennis research.\(^{28}\) This concept is defined as:

\[^{28}\text{For further examples using this method see Klaassen & Magnus (2001), Abramitzky, Einav, Kolkowitz, & Mill (2012) and González-Díaz et al. (2012).}\]
Definition. The importance of a point, $I$, is the change in the probability of winning the match based on the outcome of the point. Formally:

$$I(\theta) := \Pr[W^*|W(\theta)] - \Pr[W^*|L(\theta)]$$  \hspace{1cm} (6.1)

where $\theta$ is the current score, $W^*$ is the event that the player wins the match, and $W(\theta)$ and $L(\theta)$ are the events that they win and lose the current point, respectively.

Note that this is precisely the same quantity that transforms the point payoffs to the match payoffs in equation (3.1) from Chapter 3. The first term is the probability that a player wins the match given that they win the point and the second term is the probability that they win the match given that they lose the point. The difference is the extent to which the probability of winning the match hinges on this point, precisely what I theorised is a sensible method to measure pressure above.

Using the same approach to define total importance of a point in (6.1), it is possible to define the importance of a point in a game, the importance of a game in a set and the importance of a set in a match:

Definition. The importance of

- a point in a game, $PiG$, is the change in the probability of winning the game based on the outcome of the point:

$$PiG(\theta) := \Pr[W_G|W_P(\theta)] - \Pr[W_G|L_P(\theta)]$$  \hspace{1cm} (6.2)

- a game in a set, $GiS$, is the change in the probability of winning the set based on the outcome of the game:

$$GiS(\theta) := \Pr[W_S|W_G(\theta)] - \Pr[W_S|L_G(\theta)]$$  \hspace{1cm} (6.3)

- a set in a match, $SiM$, is the change in the probability of winning the match based on the outcome of the set:

$$SiM(\theta) := \Pr[W^*|W_S(\theta)] - \Pr[W^*|L_S(\theta)]$$  \hspace{1cm} (6.4)

where the subscripts $P$, $G$ and $S$ refer to the point, game and set, respectively.
These definitions are interpreted in the same way as the measure for the total importance of a point in a match. For example, the importance of a point in a game ($PiG$) is the degree to which the outcome of the game depends on the outcome of the current point. Similar interpretations hold for the importance of a game in a set ($GiS$) and the importance of set in a match ($SiM$).

6.2 Calculating the Importance

Calculating the importance of a point from (6.1) is quite involved. To make the calculations more tractable I utilise the i.i.d. assumption from Chapter 3. This provides a number of results that can be used to find a closed-form solution for the importance of a point. First, the independence of points implies that points in one game will have no bearing on the outcome of another point in another game, which allows (6.1) to be simplified using the following result:

**Proposition 5.** The importance of a point is the product of the importance of each point to a game, each game to a set, and each set to a match. That is,

$$I(\theta) = PiG(\theta) \cdot GiS(\theta) \cdot SiM(\theta)$$

(Proof in Appendix A.) This states that the total importance can be factorised into the importance of a point in a game, a game in a set and a set in a match. Furthermore, since $\theta = (p_s, p_r, g_s, g_r, s_s, s_r)$, and the point, game and set scores are independent of each other, this can be re-formulated as:

**Corollary.** The importance of a point is:

$$I(\theta) = PiG(p_s, p_r) \cdot GiS(g_s, g_r) \cdot SiM(s_s, s_r)$$ (6.5)

Expression (6.5) states that the importance can be factorised and that each factor depends only on the relevant component of the score. For example, the importance of a point in any game depends only on the point scores, but not the game or set scores. The total importance of a point in the match clearly depends on the total score vector, but only through the product of these independent factors.
A further implication of the independence assumption is the following:

**Proposition 6.** Let $\Omega_\theta$ be the set of all possible (countably infinite) score vectors, then the score of a tennis match, $\theta_t \in \Omega_\theta$, is a Markov process.

(Proof in Appendix A.) This proposition provides the first step for calculating an explicit value for the importance of a point. Again, as a consequence of the separability of the score vector $\theta$, I refine Proposition 6 to the following:

**Corollary.** Let $\Omega_P, \Omega_G, \Omega_S$ be the set of all possible (countably infinite) point, game and set scores respectively, then the point score, $p_t \in \Omega_P$, game score, $g_t \in \Omega_G$, and set score, $s_t \in \Omega_S$, are Markov processes.

This allows me to solve the individual point, game and set importances specified by expressions (6.2), (6.3) and (6.4), by providing a structure to the evolution of the possible future score trajectories.

Second, the identically distributed assumption further simplifies this calculation by assuming that players’ outcomes are not state dependent.\(^{29}\) In combination with the above results, this allows me to present the key lemma of this section that is used to determine the importance of a point. This is given by:

**Lemma 3.** The point score, $p_t \in \Omega_P$, game score, $g_t \in \Omega_G$, and set score, $s_t \in \Omega_S$, are time-homogeneous Markov processes.

Importantly, Lemma 3 is utilised to derive a closed-form recursive solution for the importance of a point in a game, a game in a set and a set in a match. Using these expressions in conjunction with (6.5), I am able to calculate the value of the importance of a point. The subsequent discussion outlines the solution procedure.

---

\(^{29}\)This assumption is stronger than I require to calculate the importance of a point. In practice, the probabilities do not need to be constant throughout the entire match, simply that they are known deterministically at every future state. However, making this assumption vastly simplifies the calculations since the future winning probabilities are not path dependent. I relax this assumption somewhat when I discuss the empirical procedure to approximate players’ beliefs of the importance of a point.
6.2.1 The Importance of a Point in a Game

From equation (6.5), the first factor to calculate is the importance of a point in a game (PiG). The first step to do this is to determine the probability that the Server wins the game for each possible point score within the game. Lemma 3 allows me to calculate these by starting at the end of the game and working backwards to find the probabilities at each remaining point. At a score of 3–3 (equivalent to “Deuce”) a player will win the game if they are the first to win two points more than their opponent. Denoting $f_{j,2}^i$ as the probability that Player $i$ will gain an advantage of two points first given they have currently won $j \in \{-1, 0, 1\}$ points more than their opponent, the following system of equations arises:

\[
\begin{align*}
    f_{1,2}^i &= \pi_i + (1 - \pi_i)f_{0,2}^i \\
    f_{0,2}^i &= \pi_i f_{1,2}^i + (1 - \pi_i)f_{-1,2}^i \\
    f_{-1,2}^i &= \pi_i f_{0,2}^i
\end{align*}
\]

Solving this system of equations for the 3–3 point, $f_{0,2}^i$, gives:

\[
f_{0,2}^i = \frac{\pi_i^2}{1 - 2\pi_i(1 - \pi_i)}
\]

The remaining points can be solved using the following recursive relationship:

\[
\Pr[W_G|(p_s, p_r)] = \pi_i \Pr[W_G|(p_s + 1, p_r)] + (1 - \pi_i)\Pr[W_G|(p_s, p_r + 1)]
\]

along with the boundary conditions:

\[
\Pr[W_G|(4, p_r)] = 1 \quad \text{and} \quad \Pr[W_G|(p_s, 4)] = 0
\]

From the solution to this system of equations the importance can then be calculated from (6.2). Table 6.1 presents the Point in Game (PiG) winning probabilities and importances with $\pi = 0.6$, which roughly corresponds to men’s average service-winning probability.\(^{30}\)

\(^{30}\)Note, it is only necessary to specify the probabilities and importances up to 3–3 since the winning probabilities are equivalent at every Deuce point (and every AD point). For example, the 3–4 and 4–5 points are equivalent to 2–3, the 4–4 and 5–5 points are equivalent 3–3, and so on.
### Table 6.1: Point in Game (PiG) for an Average Server ($\pi = 0.6$)

Table 6.1a shows the probability of the Server winning the game given the point scores of the Server, $p_s$, and the Receiver, $p_r$. Table 6.1b shows the importance of a point (in the game) given a score ($p_s, p_r$). The importances are exactly the $(p_s, p_r + 1)$ entry subtracted from the $(p_s + 1, p_r)$ entry in Table 6.1a, by the definition of importance from (6.2). For example, at the start of the game when the score is 0–0, the Server has a 74% chance of winning the game and the importance of this point is a 27% change in the probability of winning the game. When the score is 2–3 (equivalent to 30–40) the probability of winning changes to 42% and the importance is now a 69% change in winning probability. Figure 6.1 provides a visual representation of the importance of a point in a game.

![Graph of PiG](image)

**Figure 6.1:** Plot of the Point Importance in Game, $\pi = 0.6$
A quick examination of Table 6.1 and Figure 6.1 shows that the most important point in a game is the 2–3 point, known as 30–40 or ‘AD Out’ in the traditional tennis scoring system. These are also known as ‘break points,’ which are points where the Receiver has a chance to break (win) the Server’s service game. Importantly, this observation is consistent with evidence from professional tennis matches where coaches and commentators commonly quote break points as the most important points for the Server. These points are important to the Server because their point-winning probabilities are typically greater than 0.5 and so they are generally expected to win their service games. Clearly, this will not happen if they lose a break point and so their chance of winning the game critically depends on the outcome of the break point. The other important points occur when the Receiver is ahead in a game and when the score is close.

Table 6.2 provides another example of the probabilities and importance of each point in a game, but for a stronger server with a probability of winning a point on serve equal to 0.7. Once again Tables 6.2a and 6.2b show the probabilities of winning and the importance, respectively, for a given score.

<table>
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<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
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<td>0.59</td>
<td>0.29</td>
</tr>
<tr>
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<td>0.88</td>
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<td>0.41</td>
</tr>
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<td>3</td>
<td>1.00</td>
<td>0.99</td>
<td>0.95</td>
<td>0.84</td>
</tr>
</tbody>
</table>

<table>
<thead>
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<th></th>
<th>0</th>
<th>1</th>
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<th>3</th>
</tr>
</thead>
<tbody>
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<td>0.29</td>
<td>0.43</td>
<td>0.41</td>
</tr>
<tr>
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<td>0.10</td>
<td>0.23</td>
<td>0.43</td>
<td>0.59</td>
</tr>
<tr>
<td>2</td>
<td>0.05</td>
<td>0.14</td>
<td>0.36</td>
<td>0.84</td>
</tr>
<tr>
<td>3</td>
<td>0.01</td>
<td>0.05</td>
<td>0.16</td>
<td>0.36</td>
</tr>
</tbody>
</table>

Table 6.2: Point in Game (PiG) for a Strong Server ($\pi = 0.7$)

The same patterns emerge in this case, except they are even more pronounced. The most important points are those where the Server is down a break point. A strong server has a greater probability of winning a point on their serve and therefore a greater probability of winning a service game. Therefore, if they are faced with the possibility of a break point and a chance to lose their serve, it becomes even more
critical that they win the point. This is illustrated in Figure 6.2, which provides a
visual representation of the importance of a point in a game. The general structure
of the surface tracing out the importance of the point in the game is similar to the
previous example.

![Graph of PiG](image)

**Figure 6.2:** Plot of the Point Importance in Game, $\pi = 0.7$

### 6.2.2 The Importance of a Point in a Tie-Break

Since 1971, a tie-break is played in the majority of sets when game score reaches
6–6 to determine the winner of the set. This is a special ‘game,’ and it is possible to
determine the importance of each point in a tie-break by a similar method that was
used to calculate the importance of a point in a game.\(^{31}\) Figure 6.3 illustrates the
importance of each point in a tie-break played between an average server ($\pi_1 = 0.6$)
and a strong server ($\pi_2 = 0.7$) with the average server serving first.

---

\(^{31}\)The full derivation of the importance of a point in a tie-break is provided in Section E.1.1 of Appendix E.1.
Figure 6.3: Plot of the Point Importance in Tie-Break, $\pi_1 = 0.6$, $\pi_2 = 0.7$

Figure 6.3 shows that the most important points are those near the end of the tie-break where the score is close, or when the weaker server is marginally ahead. These are also intuitive because the points that determine the winner of the tie-break (and hence the set) are likely to substantially influence the game, especially when the weaker player has a chance to take an unexpected lead.

6.2.3 The Importance of a Game in a Set

I calculate the importance of a game in a set using a similar procedure to the importance of a point in a game/tie-break.\textsuperscript{32} To illustrate this, Figure 6.4 shows the importance of each game in a (tie-break) set between an average server ($\pi_1 = 0.6$) and a strong server ($\pi_2 = 0.7$) where the average server serves first.

\textsuperscript{32}The full derivation of the importance of a game in a set is provided in Section E.1.2 of Appendix E.1.
Figure 6.4: Plot of the Game Importance in Set, $\pi_1 = 0.6$, $\pi_2 = 0.7$

Figure 6.4 shows that the important games are near the end of the set when the weaker server is marginally leading. The valley between the peaks corresponds to the games where the scores are tied, or the stronger server is slightly ahead. There is less importance here since the stronger player is likely to win the set regardless of the outcome of the game. The peaks correspond to the games where the weaker server has a chance to take an unexpected lead. These games have high importance because if the weaker server wins the game then they have a good chance to win the set, whereas if they lose the game they will likely lose the set. The same intuition applies if the stronger server serves first.

6.2.4 The Importance of a Set in a Match

I calculate the importance of a set in a match again using a similar method to the previous sections. The importance of a set in a match is plotted in Figure 6.5 for both a best-of-three and a best-of-five set match.

---

33The full derivation of the importance of a set in a match is provided in Section E.1.3 of Appendix E.1.
These are relatively simple, but the main observation to note is that importance is highest in the final set and when the weaker server is leading by a set.

6.2.5 Putting it Together

The total importance of a point in a match is the product of the importance of a point in game, game in set and set in match. By way of illustration, Figure 6.6 plots of the importance of a point in a set (PiS) for a tie-break set.

Figure 6.5: Importance of Set in Match, $\pi_1 = 0.6$, $\pi_2 = 0.7$

Figure 6.6: Plot of the Point Importance in Set, $\pi_1 = 0.6$, $\pi_2 = 0.7$
This plot does not illustrate the total importance of a point in a match (which is unnecessarily complicated to display), however, it does provide some interesting insights as to the dynamics of the importance of points in a match. First, as the set progresses and the total points won by the Server and Receiver increases, the importance of each point tends to increase. This is particularly prominent when the scores are close, that is, where players are within one service break of each other. On the other hand, at the end of a one-sided set, the importance of points is low. Second, importance is not constant throughout the course of the match. It peaks and troughs as the match progresses, which is a result of the unique scoring mechanism used in tennis. Both these observations should be intuitive to anyone with some experience on a tennis court.

6.3 Service-Winning Probabilities

Up to now I have assumed that the service-winning probabilities, $\pi_i$, are given. From the recursive calculations in the previous section, it is clear that these probabilities are required to calculate the importance of a point in a game, set and match. Thus, a more complete specification for the importance of a point should be stated as:

$$I(\theta; \pi_1, \pi_2) := \Pr[W^*|W(\theta), \pi_1, \pi_2] - \Pr[W^*|L(\theta), \pi_1, \pi_2]$$

where $\pi_1$ and $\pi_2$ are the probabilities that Player 1 and Player 2 win a point on their serve respectively. However, exactly how each player experiences the importance will depend on their beliefs of these probabilities, not the actual probabilities. To capture this idea, I introduce a subjective importance measure that is based on a player’s belief of the winning-probabilities (instead of the true probabilities). Specifically, Player $i$’s subjective importance, $\tilde{I}_i$, is defined as:

$$\tilde{I}_i(\theta; \tilde{\pi}_1^i, \tilde{\pi}_2^i) := \Pr[W^*|W(\theta), \tilde{\pi}_1^i, \tilde{\pi}_2^i] - \Pr[W^*|L(\theta), \tilde{\pi}_1^i, \tilde{\pi}_2^i]$$

where $\tilde{\pi}_1^i$ and $\tilde{\pi}_2^i$ are Player $i$’s beliefs about the probabilities that Player 1 and Player 2 will win a point on their serve, respectively.
Each player will form their beliefs based on numerous factors, utilising both public and private information about themselves and their opponents. Since it is impossible to observe in practice, I ignore any effect of players’ private information. However, the public information is available to both players. Moreover, in the absence of any data to the contrary, I assume that this information is utilised in a similar manner by each player. This leads to the following:

**Assumption.** *Players share the same beliefs about themselves and their opponents.*

The following corollary is a direct result of this assumption:

**Corollary.** *Players’ subjective importance of a point is given by:*

\[
\tilde{I} (\theta; \tilde{\pi}_1, \tilde{\pi}_2) = \Pr[W^*|W(\theta), \tilde{\pi}_1, \tilde{\pi}_2] - \Pr[W^*|L(\theta), \tilde{\pi}_1, \tilde{\pi}_2]
\]  

(6.7)

where \( \tilde{\pi}_1 \) and \( \tilde{\pi}_2 \) are the probabilities that both players believe Player 1 and Player 2 will win a point on their serve, respectively.

The expression above is identical to equation (6.6) except that the beliefs of players’ service-winning probabilities are independent of the particular player making the beliefs. Importantly, I use this expression as the basis for approximating the level of pressure that a player experiences on the tennis court and determining the effect it has on their decision-making.

### 6.3.1 Specifying Winning Probabilities

The difficulty calculating the importance from (6.7) is that the beliefs are subjective by definition and so to use the importance as a proxy for pressure it is necessary to approximate these beliefs accurately. In this discussion I present a method to specify the point-winning probabilities from the data that I argue best represents how players will form their beliefs over the course of a match. This allows me to develop a measure for pressure that I believe most aptly describes what an average professional tennis player would experience. Before doing so, I first discuss the mechanisms through which the players’ beliefs are likely to be formed.
It seems reasonable that players’ beliefs about the service-winning probabilities will depend on a permanent component and a match-specific component. The permanent component accounts for the average ability of the Server, who should be expected to have a better chance of winning the current point if they win more service points on average. The match-specific component accounts for the form of the Server in the current match, which adjusts for when players are having an exceptionally strong or weak serving performance in a specific match.

To estimate players’ beliefs of the probability that the Server will win the point, I follow González-Díaz et al. (2012) and present a measure that incorporates both a player’s potential to win a point on serve and any match-specific variations.\(^{34}\) This method uses players’ career point-winning averages as a starting point and allows players to update their beliefs as the match progresses. Specifically, players initially assume that they win their service points with their career average service-winning probabilities. These initial probabilities, \(\pi^0_i\), are adjusted at the beginning of each set with the winning probabilities from all the previous sets in the match, \(\pi^{s-1}_i\). To keep this transparent, the updating procedure will use the simple average of all the probabilities up to set \(s\).\(^{35}\) Therefore the current winning probability in set \(s\) is estimated as:

\[
\pi_i = \frac{1}{s} \sum_{j=1}^{s} \pi^{j-1}_i
\]

This allows players to form their beliefs regarding the point-winning probability based on inherent serving ability, given by \(\pi^0_i\), while also incorporating any match-specific characteristics that may influence their probabilities through \(\pi^{s-1}_i\), \(\forall s \geq 2\). Players’ initial beliefs correspond to the historical averages and, by responding to information from previous sets, approach the realised winning probabilities as the match progresses. Moreover, the returning ability of the Receiver is indirectly incorporated through the updating process. For example, if the Receiver returns

\(^{34}\)Refer to Appendix E.2 for a discussion of some alternative methods for specifying the winning probabilities.

\(^{35}\)Rather than a simple average, different weights could also be used. These are investigated and have little effect on the value of the importance. Therefore, I use the simple average for simplicity.
well, they will lower the Server’s probability of winning the point. This, in turn, will directly influence how the players perceive the probabilities in the next set. Note that this method also allows the assumption that the probabilities remain constant throughout the course of the match to be relaxed.

### 6.4 Empirical Properties of Importance

Now that I have a method to explicitly calculate the importance of a point, I can examine some of its empirical properties. Table 6.3 below presents the summary statistics for the importance of the points in this sample using the previous method for specifying the service-winning probabilities.

<table>
<thead>
<tr>
<th>Importance</th>
<th>Obs.</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Min.</th>
<th>Median</th>
<th>Max.</th>
</tr>
</thead>
<tbody>
<tr>
<td>141,144</td>
<td>0.0369</td>
<td>0.0456</td>
<td>0.0000</td>
<td>0.0237</td>
<td>0.6165</td>
<td></td>
</tr>
</tbody>
</table>

Table 6.3 shows that the mean importance across all points is 0.0369. This implies that a player who wins an ‘average’ point can expect to increase their likelihood of winning the match by approximately 3.7%. The minimum importance is approximately zero, which shows that there are some points that have essentially no impact on the outcome of the match. Alternatively, the maximum importance observed in this sample is 0.6165. This implies that the player who wins this point will change their probability of winning that match by almost 62%. From these figures it is clear to see that the importance is highly positively skewed. Consequently, there will be many points that are insignificant in determining the outcome of the match and a small number of points that play a big role in determining the winner. The latter set of points is those that will correspond to ‘high pressure’ scenarios and are the ones that I wish to focus on in this analysis.

To illustrate these properties, Figure 6.7 plots the (kernel-smoothed) probability density and the cumulative density for the importance of all points in this sample.
Figure 6.7: Densities of the Importance

Figure 6.7 confirms the figures in Table 6.3 that the distribution is positively skewed. In particular, the points at the upper end of the distribution will be used to identify the points where pressure is present. This allows for an analysis of the effect of pressure and is discussed further when I present the estimation methodology in Section 7.1.

6.5 Examples of Important Points

As a final note I provide a comprehensive discussion of a selection of the important points in two matches to examine how it compares with the level of pressure that may be felt in actual tennis matches. For this discussion I use one long match and one average length match to see how the importance evolves in each of these cases. In each match I highlight and discuss some of the interesting features and how they relate to the pressure that may be perceived by the players.

The first match is the 2012 Australian Open Final between Novak Djoković and Rafael Nadal, won by Djoković 5–7, 6–4, 6–2, 6–7⁵, 7–5. As the longest Grand Slam final in history at 5 hours and 53 minutes, a match this tight is bound to have many situations that would be perceived as high pressure. Figure 6.8 shows precisely this by plotting the importance of each point for the entire match.
Two interesting spikes in the importance come around the 160th point. These are marked as (1) and correspond to Nadal serving at 4–5 and leading one set to love. During this game Nadal faced a score of 15–30 followed by a break point, which are the most important points for a server. Since he was also serving to stay in the set, these moments should be vitally important to his chances of winning and so it is reasonable to assume that this scenario should also involve a high degree of pressure. Referring to Figure 6.8, there is a spike in the importance at this time. Nadal did end up having his serve broken in this game and lost the set as a result.

The next spike is marked at (2) and occurs around the 290th point. At this point Djoković is leading two sets to one and 5–3 in the tie-break – only two points away from winning the match. As Nadal proceeds to win the next few points, the importance builds to over 10% at the set point that was won by Nadal forcing the match into a deciding set. It is reasonable to believe that this situation would induce a significant amount of pressure, which is again illustrated in Figure 6.8.

Finally, there should be many pressure points in the fifth set since it is also the deciding set. To start this set Nadal broke serve early to sway the match in his favour. However, Djoković broke back to level and again to win the match. Marked as (3) and (4), each break-point faced has a substantial impact on the outcome and so again one would expect each point to carry a lot of pressure.
I will now discuss the importance of some points in an average game (by total points played) as a comparison. This match is the 2015 Wimbledon Semi-final between Roger Federer and Andy Murray, won by Federer 7–5, 7–5, 6–4. Although the match was won in straight sets, each set was tight and would likely contain some pressure points. The importance of the points is plotted in Figure 6.9.

![Figure 6.9: Importances for the 2015 Wimbledon Men’s Semi Final](image)

There is a small spike almost immediately after the match begins marked as (1). Indeed, Federer faces a break-point in the first game of the match. Having a serve broken right at the start of a match can put a lot of pressure on a player to recover as they find themselves at an immediate disadvantage in the match.

The second spike, marked as (2), occurs around the 60th point and refers to the stage of the match where Murray is serving at 5–6 in the first set. He finds himself at 15–40, and then 30–40, before getting broken and losing the set. Both these points are set points, which can have a major influence on the outcome of the match. Therefore, again one would expect this to be a high-pressure situation.

Finally, there are numerous spikes marked as (3) between the 130th and the 150th points. At these points Murray faces a break point while serving to stay in the second set. Had he lost these points (as was the case with the final spike) then he would lose the set and find himself down two sets to zero. This is a hard position from which to recover and so one would also expect a large amount of pressure here.
These matches demonstrate that the importance measure maps closely to what one would reasonably expect to be the high-pressure situations in a tennis match. Points that are traditionally seen as pressure points, such as break-points and set points, are classified as important points using this approach. Moreover, the importance method also identifies some additional scenarios, such as points near the end of close sets and matches, where it seems reasonable that pressure would arise. In light of this, I maintain that the importance method is a good representation of the dynamics of pressure in a tennis match.

Having now defined an appropriate way to capture the amount of pressure a player experiences, I can now examine the effect that pressure has on the decision-making ability of tennis players. The remainder of this dissertation is focused on investigating this issue.
Chapter 7

Analysis of the Effect of Pressure

There is substantial evidence from psychology that pressure can impair individuals’ ability to perform high-level motor skills (see e.g., Baumeister, 1984) or complex decision-making tasks (see e.g., Wine, 1971). This is known to be the case even in those with substantial experience in high-pressure environments (see e.g., Dohmen, 2008). As a consequence, it is crucial to consider the effects of pressure on the subjects in this analysis to gain a more complete understanding of the optimality of their decision-making and performance. Thus, the aim of this chapter is to provide an analysis of the effect of pressure on a sample of highly-trained professionals. This addresses the main research question of this dissertation: Does pressure have a significant effect on the behaviour and outcomes of professional athletes in high-stakes contests?

In the previous chapter I develop a method for measuring the level of pressure in a tennis match. This is used as the basis for the empirical analysis of the effect of pressure on professional players that is conducted in this chapter. The analysis is divided into three sections. The first section examines the effect of pressure on players’ choice of service direction. The second section examines the effect of pressure on the probability that the Server wins the point. The third section examines whether there is a link between the effect pressure may have on service choice and point-winning probabilities. This allows for an in-depth analysis of the effect of decision-making and outcome to address the research question above.
7.1 The Effect of Pressure on Decision-making

The baseline analysis in Chapter 5 suggests that players equalise their service-winning probability across service direction. This observation is consistent with predictions from theoretical models of strategic decision-making. However, evidence from the psychological literature suggests that pressure may impair decision-making. If that is correct, then the presence of pressure may cause instances when players no longer choose their strategies optimally. This consideration is not addressed in Chapter 5 or previous studies. Thus, the conclusions from these studies should aptly describe behaviour over a match, but may not necessarily describe it in instances where a high level of pressure has the potential to substantially influence behaviour. A detailed understanding of decision-making therefore calls for an examination of whether and how behaviour changes in response to variations in pressure. The aim of this section is to develop the methodology required to estimate the effect of pressure on players’ decision-making.

To provide the foundation for the estimation methodology in this section, I present the key hypothesis of decision-making under pressure. This is based on psychological evidence that suggests that pressure can negatively influence cognitive function:

**Key Hypothesis 1.** The decision-making of players will be affected in high pressure situations, and consequently, their service strategy will deviate from their average (or long-run) behaviour.

The empirical analysis of this hypothesis will be to test for the presence of short-run shifts from players’ long-run behaviour as a function of pressure. I expand upon this in the following discussion where I investigate how players can deviate from their average behaviour as a basis for developing several models that can be used to estimate the prevalence of the effect of pressure in a sample of professional tennis players.
7.1.1 Deviations in Behaviour

Lemma 2 in Chapter 3 presents two conditions that should be observed in tennis players’ behaviour if they are making their service choices optimally. The first condition is that they should choose their service direction to equalise their winning probabilities across all choices. The second condition is that they should keep their choices unpredictable (uncorrelated) over time. This gives two natural approaches to investigate how behaviour can “deviate” under pressure and the possible effects these may have on decision-making. First, pressure could affect the ability of players to choose their service direction according to the minimax equilibrium probabilities. Second, pressure could affect the ability of players to keep their choices unpredictable over time. The presence of either type of effect would imply that pressure has an effect on decision-making. This leads to the key definition:

**Definition.** A player has a **behavioural deviation** if the presence of pressure has any significant effect on their behaviour.

**Deviation in Service-Mixing Probabilities**

Suppose a player has a behavioural deviation due to the impact of pressure on their ability to equalise their winning probabilities across service choices. In this case pressure causes a change in the player’s service-mixing probabilities. This may lead the player to gravitate to one service direction more under pressure than they do on average. For example, suppose that a player serves W with probability \( p \) on average but with probability \( p' \) under pressure. If \( p' \neq p \) then pressure has caused this player to deviate from their usual strategy. Specifically, if \( p' \) is greater (less) than \( p \) then pressure causes this player to serve W more (less) frequently under pressure than usual.\(^{36}\) This leads to the following definition:

**Definition.** A player has a **go-to deviation** if the presence of pressure causes them to choose one action significantly more often than they do on average.

\(^{36}\)One explanation for behaviour of this nature stems from the common belief among professionals and commentators that some players have a ‘go-to’ serve that they will choose in times of need.
**Deviation in the Correlation of Service Direction Choices**

Suppose a player has a behavioural deviation due to the impact of pressure on their ability to keep their service choices independent of each other. In this case pressure causes a change in the correlation between the direction of past and present serves. This will change the rate at which they switch from one service direction to the other. For example, suppose that a player has a level of serial correlation $\rho$ on average but level $\rho'$ under pressure. If $\rho' \neq \rho$ then pressure has caused this player to deviate from their usual behaviour. Specifically, if $\rho'$ is less (greater) than $\rho$ then pressure causes this player to switch directions more (less) frequently under pressure than usual. This leads to the following definition:

**Definition.** A player has a correlation deviation if the presence of pressure causes them to vary the level of correlation between this choice and any previous choices.

The most obvious way to examine any deviations in the predictability of players’ choices is to study the effect of pressure on the (one-period) serial correlation in their choices over time. This leads to the following definition:

**Definition.** A player has a serial correlation deviation if the presence of pressure causes them to vary the level of correlation between the current choice and the previous choice.

There is also another way to think of pressure affecting players. If players are able to correctly identify points played under pressure, then they may base their current choices on what they did on the previous pressure point rather than on what they did on the previous point. For example, a player’s decision of where to serve at a break point may depend on where they decided to serve to on the previous break point. In this case pressure affects the correlation between this point and the last point played under pressure. This scenario leads to the following definition:

---

37One explanation for behaviour of this nature stems from the notion that individuals believe sequences with excessive switching appear more random than truly random sequences (see e.g., Wagenaar, 1972). If true, then players may switch too frequently when stressed in an attempt to be unpredictable.
**Definition.** A player has a **pressure correlation deviation** if the presence of pressure causes them to vary the level of correlation between the current choice and the previous choice made under pressure.

To illustrate the difference between the two types of correlation deviations, suppose that a specific player-court sample consists of ten points where the second, fifth and ninth points are pressure points. An analysis of serial correlation deviations would examine whether the choice in the first point affects the second, the fourth affects the fifth and the eighth affects the ninth. Alternatively, an analysis of pressure correlation deviations would examine whether the choice in the second point affects the fifth and the fifth affects the ninth. How exactly the effects of pressure will manifest themselves is unknown and may vary from player to player, so I will allow for both possibilities in the estimation.

### 7.1.2 Estimation

The simplest specification for estimating the effect of pressure on players’ decision-making is to assume that it only affects them through the probability that they choose a particular service direction. To examine this behaviour, I propose the following linear probability model (LPM):

\[
d_{smc,t} = \beta^{sc}_p P_t + \beta^{sc}_l d_{smc,t-1} + \Gamma^{sc} X_t + \lambda^{sc}_m + \varepsilon_{smc,t}
\]

where \(d_{smc,t}\) is the choice of service direction (W or T) at point \(t\) for Server \(s\) in match \(m\) on court \(c = \{\text{Deuce, AD}\}\). This is dependent on a variable for the presence of pressure, \(P_t\), the construction of which I explicitly detail below. I also include a lagged dependent variable, \(d_{smc,t-1}\), to control for general serial correlation in players’ decisions over time. I include this because the analysis in Chapter 5 suggests that there is some underlying serial correlation in the service choices of players. This is separate from any investigations of serial correlation deviations and is simply included to better model players’ choices over time. Furthermore, I assume only

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\(^{38}\)Appendix F.1 examines the probit model as a robustness analysis.
The Effect of Pressure

one-point correlations (within each point-game) and that there is no direct effect of greater lags. I include match-fixed effects, $\lambda_m^{sc}$ and a set of controls, $X_t$, to account for other factors that may also influence the service choices of players.\footnote{Explicitly, the variable $X_t$ contains court surface dummies and controls for fatigue by accounting for the number of sets points played up to point $t$. See Appendix C for details.} Finally, the errors are clustered at the match level.

The coefficient of interest, $\beta_{sp}^{sc}$, captures the extent to which pressure influences the chance that the Server opts for a particular direction when faced with pressure. A value significantly different from zero suggests that pressure has a significant effect on the service choices of Server $s$ on court $c$. Note, that the analysis is performed separately for each player and each court. This specification investigates deviations in service-mixing probabilities (i.e., go-to deviations), but ignores other possible deviations. Regression (1) will be referred to as the Base regression hereafter.

The Base specification presented above can only examine whether pressure affects players’ service-mixing probabilities. However, as discussed previously, this is just one way that pressure can affect behaviour. Pressure could also affect the correlation in the players’ service choices. Therefore, the following specifications build upon the Base model to allow for the possibility of such effects.

The first extension that I examine is the effect of pressure on the baseline level of serial correlation in players’ choices by estimating the following regression:

$$d_{smc,t} = \beta_{sp}^{sc} P_t + \beta_{ls}^{sc} P_t \times d_{smc,t-1} + \beta_{sp}^{sc} S^{sc} P_t \times d_{smc,t-1} + \Gamma^{sc} X_t + \lambda_m^{sc} + \varepsilon_{smc,t}$$  \hspace{1cm} (2)

where the variables are defined similarly to the previous specification, but includes the interaction term $P_t \times d_{smc,t-1}$. In this specification pressure can affect behaviour through two dimensions. The first is the effect on the likelihood of choosing a service direction given by the pressure coefficient $\beta_{sp}^{sc}$. The second is the effect on the degree of correlation between consecutive serves given by the interaction coefficient $\beta_{ls}^{sc}$. This specification investigates deviations in service-mixing probabilities (i.e., go-to deviations) and serial correlation, but ignores pressure correlation. Regression (2) will be referred to as the Serial Correlation (SC) regression hereafter.
The second extension to the Base model that I investigate is the correlation between consecutive pressure points. To examine this effect, I estimate the following regression:

\[ d_{smc,t} = \beta_{sc}^{P} P_t + \beta_{sc}^{I} d_{smc,t-1} + \beta_{sc}^{P} P_t \times d_{smc,t}^{LP} + \Gamma^{sc} X_t + \lambda_{m}^{sc} + \varepsilon_{smc,t} \]  

where all the variables are defined similarly to the previous specifications, but includes an interaction term \( P_t \times d_{smc,t}^{LP} \), where \( d_{smc,t}^{LP} \) is the service direction on the last pressure point. This specification also allows pressure to influence behaviour through two dimensions. The first is still the effect on the likelihood of choosing a direction given by the pressure coefficient \( \beta_{sc}^{P} \). The second is the relationship between the service direction chosen on the previous pressure point and the current one given by the interaction coefficient \( \beta_{sc}^{I} \). This specification investigates deviations in service-mixing probabilities (i.e., go-to deviations) and pressure correlation, but ignores serial correlation. Regression (3) will be referred to as the Pressure Correlation (PC) regression hereafter.

Finally, I allow players to exhibit both serial and pressure correlations deviations under pressure. Therefore, the final specification includes both serial correlation and pressure correlation interaction terms:

\[ d_{smc,t} = \beta_{sc}^{P} P_t + \beta_{sc}^{I} d_{smc,t-1} + \beta_{sc}^{P} P_t \times d_{smc,t-1} + \beta_{sc}^{I} P_t \times d_{smc,t}^{LP} + \Gamma^{sc} X_t + \lambda_{m}^{sc} + \varepsilon_{smc,t} \]  

where all variables are defined similarly to previous specifications. The coefficient \( \beta_{sc}^{P} \) is the effect of pressure on the likelihood of choosing a particular service direction, holding all else constant. The interpretation of the interaction coefficients is a little more complex in this specification. The coefficient \( \beta_{sc}^{I} \) is the contribution of the previous service choice on the current service choice. The coefficient \( \beta_{sc}^{P} \) is the contribution of the previous pressure point on the current service choice. Due to the interactions between the variables, the total effect of pressure is a complex combination of these three partial effects. This will be discussed in detail when I present the estimation results. Regression (4) will be referred to as the Multiple Interaction (MI) regression hereafter.
Lastly, to ensure that I have a sufficiently large sample to identify players’ career-level performance, I drop any player who has less than 220 serves on the Deuce court or 200 on the AD court.\textsuperscript{40} This leads to a total of 162 player-court regressions. I re-perform the randomised Fisher exact test on this restricted sample. Again, I cannot reject the hypothesis that players choose their service to equalise winning probabilities at the 1%, 5% or 10% level of significance. Thus, I conclude that the results of the baseline analysis in Chapter 5 remain valid for the restricted sample.

\textbf{The Pressure Variable}

The next step is to explicitly construct the pressure variable, $P_t$, to capture the pressure that players experience, while allowing for an examination of all possible behavioural deviations. I initially argued in Chapter 6 that the importance of a point provides an apt description of the evolution of pressure.\textsuperscript{41} However, defining pressure as a continuous variable cannot capture pressure correlation as there is no precise definition of what constitutes the last pressure point (i.e., the $d^{LP}$ variable cannot be defined). Additionally, the majority of studies analysing decision-making under pressure treat pressure as a variable that is either present or absent.\textsuperscript{42} Lastly, professionals talk about “pressure points” as specific instances in tennis. For these reasons I specify $P_t$ as a dummy variable to indicate whether pressure is present, while still basing it on the importance via:

\[
P_t = \begin{cases} 
0 & \text{if } \text{Imp}_t < I^* \\
1 & \text{if } \text{Imp}_t \geq I^* 
\end{cases}
\]

where $\text{Imp}_t$ is the importance of point $t$ and $I^*$ is some cut-off value. The aim of this thesis is to examine the impact of rare instances of extreme pressure on players’ behaviour and so I specify a cut-off such that the pressure points are only a small number of extremely important points. Specifically, I opt to classify points with

\textsuperscript{40}The average importance was approximately 10% lower on the Deuce compared to the AD court, so I use a cut-off that is 10% higher on the Deuce court to counteract this.

\textsuperscript{41}Appendix F.1 provides an analysis using pressure as a continuous variable.

\textsuperscript{42}For example, studies of decision-making under time pressure, such as those by Sutter et al. (2003); Kocher & Sutter (2006), have examined only two time pressure treatments: high and low.
an importance above the 95\textsuperscript{th} percentile (a cut-off value $I^* = 0.120$ or 12\%) as pressure points.\textsuperscript{43} If players experience pressure, then it seems reasonable that a point with such a high importance would correspond to the “pressure points” that they describe. Therefore, I specify the pressure variable as:

$$P_t = \begin{cases} 
0 & \text{if } Imp_t < 0.1203430 \\
1 & \text{if } Imp_t \geq 0.1203430 
\end{cases}$$

Lastly, I drop 14 additional player-courts observations due to a lack of pressure points faced at this cut-off, giving a total of 148 player-court regressions.

**Identifying the Deviations**

I estimate each response to pressure through a distinct variable in each of the previous model specifications. However, in certain cases, these deviations may be observationally equivalent. For example, if a player chooses to serve to one side every time they are faced with pressure then they clearly have a go-to deviation. In addition, their choices at pressure points will be perfectly positively correlated and so they also have a pressure correlation deviation. Another example occurs when pressure points arise consecutively. In this case the previous pressure point is also the previous point. Thus, if pressure affects the serial correlation between points then it will also affect the pressure correlation between pressure points. From a behavioural perspective, it would be impossible to determine exactly through which channel pressure is influencing behaviour in these scenarios. From an econometric perspective, there will be a high degree of multicollinearity between the variables. This discussion outlines these hypothetical scenarios and their implications.

To formally see how the first issue arises, consider the phi coefficient of association between two binary variables defined by:

$$\phi := \frac{p_{11}p_{00} - p_{10}p_{01}}{\sqrt{p_{1}p_{0}p_{0}p_{1}}} \quad (7.1)$$

where $p_{ij}$ is the probability that the first variable takes the value of $i = \{0, 1\}$

\textsuperscript{43}Appendix F.1 provides an analysis using the 90\textsuperscript{th} percentile as the cut-off for pressure points.
and the second variable takes the value of \( j = \{0, 1\} \). For the above problem, consider the association between the current serve, \( d_t \), and the previous serve, \( d_{t-1} \), specifically. This is analogous to the Pearson correlation coefficient for binary data (and is equivalent in the \( 2 \times 2 \) case). Furthermore, in the limit as \( n \to \infty \), the number of 01 combinations is the same as the 10 combinations since this is the association between a variable and its lag, so \( p_{10} = p_{01} \equiv p_c, p_{11} = p \) and \( p_0 = p_0 = 1 - p \). Using \( \{W, T\} \) notation instead of \( \{0, 1\} \) notation, this simplifies equation (7.1) to:

\[
\phi = \frac{p_{WW}p_{TT} - p_c^2}{p(1 - p)}
\]

where \( p \) is the probability of serving \( W \), and \( p_c \) is the probability of serving in the opposite direction to the previous serve. This leads to the following proposition:

**Proposition 7.** The phi coefficient of the association between the direction variable and its lag, \( \phi \), is bounded by

\[
-\frac{\min\{p, 1 - p\}}{\max\{p, 1 - p\}} \leq \phi \leq 1
\]

(Proof in Appendix A.) For example, considering only the pressure points, the degree of correlation between consecutive serves under pressure is bounded by a function of the probability of serving a particular direction under pressure. In the extreme case where \( p = 0 \) or \( p = 1 \) (under pressure) then \( 0 \leq \phi \leq 1 \), and so it will only be possible to observe positive pressure correlation in this instance. Alternatively, if \( p = 0.5 \) then \( -1 \leq \phi \leq 1 \), and so there is no restriction on the pressure correlation that can be observed.

I do not believe that the example outlined above poses a significant issue in practice. For this scenario to be problematic, a player must serve close to 100% to one particular side under pressure. In reality, players do not tend to be this one-dimensional even under pressure. Therefore, the probabilities of serving to a particular direction under pressure should not dramatically restrict the value of the pressure correlation coefficient.

The second issue may be problematic in some instances. As I have described, when the pressure points arise consecutively, it may be difficult to separately identify
The Effect of Pressure

the serial correlation and pressure correlation effects since the interaction variables $P_t \times d_{t-1}$ and $P_t \times d_{t-P}^P$ will commonly take the same value. This results in a high degree of multicollinearity in the estimation, which could affect the magnitude and sign of the coefficient estimates and increase their estimated variance. However, this is only problematic in the Multiple Interaction specification where I attempt to estimate both effects simultaneously. Thus, care must be taken when interpreting the serial and pressure correlation effects in the MI specification.

As a consequence of the scoring in tennis, pressure generally increases as the match progresses and so there may be some consecutive pressure points. However, most of the important points are break/game points after which pressure subsides substantially (refer to discussion in Chapter 6). Therefore, there are likely to be several instances where the previous pressure point is not also the previous point. In these cases it is possible to identify both of the interaction coefficients.

Model Selection

The discussion of the econometric procedure to determine the effect of pressure on decision-making presents four different models. The first model allows for pressure to cause players to have a tendency towards one particular action but to have no other impact. The second and third models allow for an extra response to pressure, namely an effect on the baseline level of serial correlation, or an effect I have called pressure correlation. Finally, the fourth model allows for all three possible responses to pressure. The discussion in this section describes the process used to determine the preferred specification for examining the overall effect of pressure.

To determine which of the models best match the data, I employ a model selection mechanism based on two common information criteria. By using these methods I endeavour to choose the model specification that best satisfies the data whilst penalising the addition of extra variables, thus avoiding the temptation to over-fit a model. The two criteria that I use are the Akaike Information Criterion (AIC) and the Bayesian Information Criterion (BIC). To determine the optimal specification,
I perform each player-court regression for each model and count the number of times a particular specification is chosen as the preferred model (i.e., has the lowest AIC/BIC). Table 7.1 provides the number of times each model is selected as the optimal model.

**Table 7.1: Optimal Model According to the AIC and BIC**

<table>
<thead>
<tr>
<th>Model</th>
<th>AIC</th>
<th>BIC</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) Base</td>
<td>0</td>
<td>12</td>
</tr>
<tr>
<td>(2) SC</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>(3) PC</td>
<td>86</td>
<td>95</td>
</tr>
<tr>
<td>(4) MI</td>
<td>62</td>
<td>41</td>
</tr>
<tr>
<td>Total</td>
<td>148</td>
<td>148</td>
</tr>
</tbody>
</table>

The evidence from Table 7.1 favours model (3) as the specification that best fits the data. This result suggests that the model should allow players to be affected by pressure in two dimensions. The first as a consequence of a propensity to gravitate towards one particular service direction under pressure and the second through a tendency to make choices that depend on the previous choice made when faced with pressure. The second most preferred model is (4), which allows players to be affected in three dimensions. Where there are discrepancies in the results between models (1) to (4), I will focus predominately on these model specifications.

### 7.1.3 Main Results

The analysis is performed at the player-court level, one for each player on each court. The coefficient estimates of interest for each of the four model specifications and each player-court are the $\beta^{sc}_j$ coefficients, where the subscript $j = \{p, i_S, i_P\}$ corresponds to the three different types of possible channels that pressure can affect (service-mixing probabilities, serial correlation and pressure correlation). These coefficients are used to determine the prevalence of the effect of pressure on decision-making through the different possible responses. This discussion presents the results for the partial and total effect of pressure on each possible channel.


**Pressure Coefficients – Partial Effects**

The coefficient estimates from each regression can be interpreted as the partial effect of pressure on a player for a specific channel while holding all other channels constant. For example, the coefficient $\beta_{sc}^p$ corresponds to the direct effect of pressure on the player’s choice of service direction while holding the level of serial and pressure correlation constant. In practice, there could be interactions between the different channels meaning that it is impossible to observe this effect in isolation (see later discussion of total effects). Nevertheless, this provides a starting point to examine the effect of pressure on decision-making.

Figure 7.1 plots the (kernel-smoothed) distribution of the coefficients governing the effect of pressure on players’ service-mixing probabilities, serial correlation and pressure correlation. Figure 7.1a plots the estimates of the effect of pressure on the service-mixing coefficients, $\beta_{sc}^p$. Figure 7.1b plots the estimates of the effect of pressure on the serial correlation coefficients, $\beta_{si}^{sc}$. Figure 7.1c plots the estimates of the effect of pressure on the pressure correlation coefficients, $\beta_{ip}^{sc}$. These are plotted for all the relevant models.

\[\text{Figure 7.1} \quad \text{(A) Mixing Probability} \quad \text{(B) Serial Correlation}\]

---

\[\text{44} \text{The service-mixing probabilities coefficients can be examined by all models (1) – (4). The serial correlation coefficients can only be examined by model (2) and model (4). Finally, the pressure correlation coefficients can only be examined by model (3) and model (4).}\]
Figure 7.1a plots the density of the pressure coefficients. The models are roughly centred on zero indicating the behaviour of the ‘average’ player may not be affected by pressure. However, there is substantial dispersion in the estimates away from zero suggesting the prevalence of a significant effect of pressure for a number of players.\(^{45}\) This provides preliminary evidence that an effect of pressure may be present in this sample. A rigorous analysis undertaken in the subsequent section confirms this observation.

Figures 7.1b and 7.1c plot the distributions of the effect of pressure on serial correlation and pressure correlation respectively. Interestingly, in both these cases the distribution is centred below zero. This indicates that pressure may cause the ‘average’ player to switch service directions more frequently (even after controlling for any negative serial correlation that may be prevalent in the absence of pressure). Although the distributions in model (4) have somewhat more variation (possibly driven by some multicollinearity between the \(d_{t-1}\) and \(d_{t}^{LP}\) variables), a similar same pattern is also observed.

\(^{45}\)Due to measurement error, the variance of the estimates may be biased upwards and so the true dispersion is likely to be smaller than is presented here. To adjust for this C. Morris (1983) has suggested that the estimates be shrunk via an Empirical Bayes shrinkage technique. This has been commonly used in studies that perform further estimation based on the coefficients (see e.g., Branch, Hanushek, & Rivkin, 2012; Helal & Coelli, 2016; Jacob & Lefgren, 2005). However, I do not require the estimates to be used in subsequent regressions. Furthermore, the shrinkage technique does not affect the standard errors (and \(p\)-values) of the coefficient estimates. Thus, my analysis would remain unaffected if I were to shrink the estimates.
To address these observations formally I investigate the first component of Key Hypothesis 1, which states that pressure should have a significant and prevalent effect on the choice of service direction made by servers. To determine whether this effect is persistent in the sample, I examine the distribution of the $\beta_{sc}^k$ coefficients from each of the player-court regressions. The $\beta_{sc}^k$ coefficients from these regressions determine the extent to which Server $s$ is affected by pressure. For the player-court regressions where the $\beta_{sc}^k = 0$ hypothesis is rejected, I conclude that pressure has resulted in this player deviating from their long-run strategy (the precise deviation depends on which of the $\beta_{sc}^k$ coefficients is significantly different from zero). Therefore, an examination of the distribution of the coefficients and the prevalence of significant coefficients can be used to determine whether there is a noticeable effect of pressure on the population as a whole.

This analysis is analogous to the work of Walker & Wooders (2001) and involves determining whether the observed results are due to an effect of pressure or simply a product of random fluctuations in the data. If the observed effects occur more frequently in the sample than one would theoretically expect if there was no effect, then it suggests that pressure has a significant effect on the population as a whole.

To provide the foundation for the main analysis in this section, I formalise the above discussion into the following hypothesis. This will provide the basis for developing two separate tests to examine the pervasiveness of the effect of pressure on players’ decision-making.

**Hypothesis 3.** For the entire sample of players:

- $H_0$: The deviations are generated by random fluctuations in the data.
- $H_1$: The deviations are not generated by random fluctuations in the data.

The first method to test this hypothesis is identical to the test of Hypothesis 2a in Chapter 5. Assuming that the likelihood of observing each significant coefficient is an independent and identically distributed occurrence, the number of significant coefficients follows a binomial distribution $B(n, p)$. The number of trials, $n$, is
equal to the number of player-court regressions and the probability of success, \( p \), is equal to the significance level under the null hypothesis (i.e., \( p = 0.05 \)). Using this observation, I can test Hypothesis 3 by reformulating it as the following equivalent hypothesis:

**Hypothesis 3a.** The total number of observed significant coefficients follows a binomial distribution, \( B(n, p) \), with \( n \) equal to the number of player-court regressions and

\[
\begin{align*}
H_0 : & \quad p = 0.05 \\
H_1 : & \quad p > 0.05
\end{align*}
\]

Table 7.2 below presents the results. Each panel represents one of the three possible channels that pressure can affect. The first panel corresponds to the effect of pressure on players’ service-mixing coefficients. The second and third panels correspond to the effect of pressure on serial correlation and pressure correlation, respectively. In each panel I provide the number of observed significant coefficients and the expected number under the null hypothesis. The expected number is equal to the total number of regressions multiplied by the probability of observing a significant effect under the null hypothesis. In addition, the observed proportion of significant coefficients, \( \hat{p} \), is also provided. The proportion of observed significant coefficients is tested against the null hypothesis that \( p = 0.05 \) using a one-tailed binomial test. If the observed proportion of significant coefficients is significantly greater than the expected proportion then this indicates that the number of significant coefficients in the sample is not random.
Table 7.2: Number of Observed Significant Coefficients

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>β_p</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>N</td>
<td>148</td>
<td>148</td>
<td>148</td>
<td>148</td>
</tr>
<tr>
<td>Obs</td>
<td>21</td>
<td>24</td>
<td>37</td>
<td>35</td>
</tr>
<tr>
<td>Exp</td>
<td>7.40</td>
<td>7.40</td>
<td>7.40</td>
<td>7.40</td>
</tr>
<tr>
<td>p</td>
<td>0.142***</td>
<td>0.162***</td>
<td>0.250***</td>
<td>0.237***</td>
</tr>
</tbody>
</table>

| β_is        |     |     |     |     |
| N           | 146 | 135 |
| Obs         | 34  | 47  |
| Exp         | 7.30| 6.75|
| p           | 0.233***| 0.348***|

*** Indicates rejection at the 1-percent level of significance.
** Indicates rejection at the 5-percent level of significance.
* Indicates rejection at the 10-percent level of significance.

Consider the first column in the top panel of Table 7.2. This corresponds to the analysis of the significant coefficients on the pressure variable (i.e., the service-mixing probabilities) in the Base model. Table 7.2 highlights that 21 of the possible 148 cases show signs of a deviation towards a particular action under pressure. The observed proportion of significant coefficients is 0.142, equivalent to 14% of the total sample. However, if the results were random then we would expect to see only approximately 5% of the population exhibiting this effect. Using the one-tailed binomial test, the observed proportion is significantly greater than the expected proportion and so the null hypothesis is rejected. The remaining results show a similar pattern. The model specifications (2) – (4) display even larger percentages of significant coefficients. For these models a significant coefficient is observed in 16%, 25% and 24% of times, respectively. Once again, all these values are significantly greater than 5%, which is the expected value under the null hypothesis. Therefore, there appears to be strong evidence that these deviations are prevalent in this sample.
Similar conclusions can be drawn for the serial correlation and pressure correlation coefficients. For the serial correlation coefficients, I find that pressure has a significant effect in 14% of cases in model (2) and 27% of cases in model (4). These are both significantly greater than the expected level if the observed results were a product of random fluctuations in the data. For the pressure correlation coefficients, I find that pressure has a significant effect in 23% of cases in model (3) and 35% of cases in model (4). Hence, I conclude that there is a significant number of players who are affected by pressure in the level of correlation between consecutive serves (i.e., serial correlation) and a significant number of players who are affected by pressure in the level of correlation between consecutive serves under pressure (i.e., pressure correlation).

The analysis to this point has focused on players with an extreme (statistically significant) effect. To supplement this, I also present a method that allows the behaviour of the full set of players to be considered. This method utilises information on the $p$-values of the coefficient estimates from all regressions and so does not just consider the significant ones. To formalise this test, the following is proposition is required:

**Proposition 8.** The $p$-values of a continuous distribution are distributed $U[0, 1]$ under the null hypothesis.

(Proof in Appendix A.) Therefore, I can investigate the likelihood that pressure has some effect in the sample by comparing the distribution of $p$-values from the player-court regressions to the uniform distribution. Consequently, I also examine Hypothesis 3 by testing the equivalent hypothesis:

**Hypothesis 3b.** Let $P$ be the distribution of the observed $p$-values obtained from testing the significance of each of the $\beta^{sc}_j$ coefficients then

$$H_0 : P \sim U[0, 1]$$

$$H_1 : P \not\sim U[0, 1]$$
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To illustrate, Figure 7.2 plots the empirical distribution of \( p \)-values for the coefficients of interest for each type of pressure response for the relevant models. Figure 7.2a plots \( p \)-values for the service-mixing coefficients, \( \beta_{sc} \). Figure 7.2b plots the \( p \)-values for the serial correlation coefficients, \( \beta_{sc} \). Finally, Figure 7.2c plots the \( p \)-values for the pressure correlation coefficients, \( \beta_{sp} \). The cumulative distribution function (CDF) of the uniform distribution is included in black for comparison.

![Empirical CDF](image)

(A) Mixing Probability  
(B) Serial Correlation  
(C) Pressure Correlation

**Figure 7.2:** Empirical CDF of the \( p \)-values vs. Uniform distribution

In each of these plots the empirical distribution lies above the uniform distribution. The implication is that the \( p \)-values tend to be lower than would be predicted if the results were obtained through randomness. Hence, the significant \( \beta_{sc} \) coefficients appear to occur more frequently than one would expect if there were no effect.
These conclusions are based on visual inspection, so a formal test of Hypothesis 3b is required. This requires testing whether there is a significant difference in the distance between the empirical and theoretical distributions in Figure 7.2. Since the function of the empirical distribution of \( p \)-values, \( \hat{F}(x) \), defines a function over the domain \( x \in [0, 1] \) in the space of Lebesgue integrable functions, I can use \( \ell_p \)-norms for the distance metrics. These are the most common distance metrics between two functions \( f(x) \) and \( g(x) \) in the space of \( L^p \) functions, and are defined as:

\[
||f(x) - g(x)||_p := \left( \int |f(x) - g(x)|^p \, dx \right)^{\frac{1}{p}} \tag{7.2}
\]

The first metric I use to examine the distance between the distributions is the \( \ell_\infty \)-metric. This metric reduces expression (7.2) to:

\[
||f(x) - g(x)||_\infty = \sup_{x \in [0, 1]} |f(x) - g(x)|
\]

This is essentially the Kolmogorov-Smirnov (KS) test statistic:

\[
K = \sqrt{n} \sup_{x \in [0, 1]} |\hat{F}(x) - F(x)|
\]

The critical values of the test statistic are known, so I use the KS-test to determine if there is a significant difference between the empirical distribution, \( \hat{F}(x) \), and the uniform distribution, \( F(x) = x \). Thus, the KS test provides one method to test Hypothesis 3b. However, the KS test has some potential problems. First, it requires a large number of observations to provide sufficient power to adequately reject the null hypothesis. Second, it is poor at picking up differences in the tails of the distributions. This is because the \( \ell_\infty \)-metric measures the maximum distance between functions, which is unlikely to be close to the end points \( x = 0 \) or \( x = 1 \) as \( \hat{F}(0) \equiv F(0) \equiv 0 \) and \( \hat{F}(1) \equiv F(1) \equiv 1 \), by definition. Thus, this metric may not pick up a significant difference if the difference is predominately around the end points. Since the binomial test suggested that there are a substantial number of coefficients with \( p \)-values < 0.05, a significant difference should be expected between the distributions in the interval \( 0 < x < 0.05 \). This interval contains the end point \( x = 0 \) so this is precisely the problematic scenario that I have just discussed.
Therefore, I also test Hypothesis 3b using the $\ell_1$-metric, simplifying (7.2) to:

$$||f(x) - g(x)||_1 = \int |f(x) - g(x)| \, dx$$

This metric calculates the area between the two distributions and resolves some of the potential issues surrounding the KS-test. Specifically, it uses all the observations to calculate the distance between the distributions rather than just the maximum distance. Therefore, it can provide a more accurate assessment of the difference between the distributions, particularly when the differences are most pronounced near the end points. However, the distribution of the area is not known and so I resort to simulations to find the critical values. To do this I simulate a sequence of $p$-values under the null hypothesis and calculate the area between the simulated sample and the theoretical value using the trapezoidal method. This simulation is repeated 10,000 times to find the distribution of the area under the null hypothesis. From this it is possible to derive the appropriate critical values for the test statistic.

Table 7.3 presents the results of the test of Hypothesis 3b for the two methods discussed above. The results are provided for each coefficient and for each model. The first row of each panel presents the KS-test statistic and the second row presents the area difference between the two distributions.

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_p$</td>
<td>KS</td>
<td>Base</td>
<td>SC</td>
<td>PC</td>
</tr>
<tr>
<td>KS</td>
<td>0.1526***</td>
<td>0.1581***</td>
<td>0.2362***</td>
<td>0.2493***</td>
</tr>
<tr>
<td>Area</td>
<td>0.0812***</td>
<td>0.0876***</td>
<td>0.1224***</td>
<td>0.1283***</td>
</tr>
<tr>
<td>$\beta_{iS}$</td>
<td>KS</td>
<td>0.1096*</td>
<td></td>
<td>0.2351***</td>
</tr>
<tr>
<td>Area</td>
<td>0.0600**</td>
<td></td>
<td></td>
<td>0.1379***</td>
</tr>
<tr>
<td>$\beta_{iP}$</td>
<td>KS</td>
<td></td>
<td>0.2005***</td>
<td>0.2994***</td>
</tr>
<tr>
<td>Area</td>
<td></td>
<td>0.1167***</td>
<td>0.1534***</td>
<td></td>
</tr>
</tbody>
</table>

*** Indicates rejection at the 1-percent level of significance.
** Indicates rejection at the 5-percent level of significance.
* Indicates rejection at the 10-percent level of significance.
The results of the KS-test and the Area-test suggest that the distribution of the $p$-values from the service-mixing probability coefficients, $\beta_{sc}^{p}$, is significantly different from the uniform distribution under all models. The results for the $p$-values from the serial correlation and pressure correlation coefficients are similar. Therefore, under all models and for all possible responses, I reject the null hypothesis that the observed effects are the product of random fluctuations in the data and conclude that some effect of pressure is present in the sample. This is consistent with the conclusion from the binomial test presented in Table 7.2.

**Behavioural Deviations – Total Effects**

The discussion to this point focused on the regression coefficients, which represent the partial effect of pressure on different facets of players' service choices (i.e., the reaction through one channel, while holding all others constant). However, there are interactions between the different types of deviations that imply it is unlikely that one type of deviation will not affect another. For example, if a particular player has a go-to deviation then the correlation in their service direction between pressure points may be affected as well (see discussion on identifying deviations). Thus, it will rarely be the case that behaviour will change under pressure through one type of response while the others are held constant. This implies that the partial effects (i.e., the regression coefficients) are not necessarily equivalent to the behavioural deviations (i.e., go-to, serial correlation and pressure correlation deviations).

To understand the effect of pressure on the behavioural deviations, I require the total effect that pressure has on players' service direction, serial correlation and pressure correlation. Mathematically these correspond to:

\[
\begin{align*}
\text{Go-to:} & \quad \frac{\Delta E[d_t]}{\Delta P_t} \\
\text{Serial Correlation:} & \quad \frac{\Delta}{\Delta P_t} \mathbb{E} \left[ \frac{\Delta E[d_{t+1}|P_t]}{\Delta d_{t-1}} \right] \\
\text{Pressure Correlation:} & \quad \frac{\Delta}{\Delta P_t} \mathbb{E} \left[ \frac{\Delta E[d_{t+1}|P_t]}{\Delta d_{t-1}^{\text{LP}}} \right]
\end{align*}
\]

(7.3)
Importantly, these quantities do not require the others to be held fixed and so represent the actual behavioural responses of players to pressure. It can be seen that in the more complex specifications the interactions between the variables mean that the partial effects no longer represent the total effect and the presence of one does not necessarily imply the presence of the other.

Equation (7.3) can be used to find the expressions for the behavioural deviations as functions of the regression coefficient estimates. The exact function for the total effect of pressure varies for each of the four model specifications. These functions are given in Table 7.4 below.

**Table 7.4: Functional form for each type of Deviation**

<table>
<thead>
<tr>
<th>Model</th>
<th>Go-to</th>
<th>SC</th>
<th>PC</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) Base</td>
<td>( \beta_p )</td>
<td></td>
<td>( \beta_i S )</td>
</tr>
<tr>
<td>(2) SC</td>
<td>( \frac{\beta_p(1-\beta_l)+\beta_i S \Gamma E[X_t]}{1-\beta_l-q\beta_i S} )</td>
<td>( \beta_i S )</td>
<td></td>
</tr>
<tr>
<td>(3) PC</td>
<td>( \frac{\beta_p(1-\beta_l)+\beta_i P \Gamma E[X_t]}{1-\beta_l-\beta_i P+(1-q)\beta_i P} )</td>
<td>( \beta_i P )</td>
<td></td>
</tr>
<tr>
<td>(4) MI</td>
<td>( \frac{\beta_p(1-\beta_l)+[\beta_i S + \beta_i P + 2(1-q)\beta_i S \beta_i P \Gamma E[X_t]}{1-\beta_l-q\beta_i S-\beta_i P+(1-q)\beta_i P} \beta_i S + q\beta_i P )</td>
<td>( q\beta_i S + \beta_i P )</td>
<td></td>
</tr>
</tbody>
</table>

However, to perform hypothesis tests on these values I require the distribution of the quantities in Table 7.4. To derive this distribution, I use the delta method approximation. This method provides an approximate distribution of a function of estimators, \( h(\beta) \), given by:

\[
\sqrt{n}(h(B) - h(\beta)) \xrightarrow{d} N\left(0, \nabla h(\beta)^T \cdot \Sigma \cdot \nabla h(\beta)\right)
\]

where \( n \) is the number of observations and \( \Sigma \) is a (symmetric positive semi-definite) covariance matrix. The function \( h(\cdot) \) is defined to be the total effect of pressure for a particular deviation given in Table 7.4 above.

---

\(^{46}\)A full derivation for each method is given in Appendix F.2. The returner fixed-effects are excluded here for clarity, but can be thought of as a subset of the control variables, \( X_t \).
With these expressions I can analyse the presence of the behavioural deviations in the sample in a similar manner to the significant coefficients. I test for the presence of these deviations employing exactly the same hypothesis tests as in the previous section on the approximate distribution for each deviation from the multivariate delta method.

Table 7.5 presents the results of the test for the presence of any significant behavioural deviations given by Hypothesis 3a. The $h(\beta)$ functions given in Table 7.4 are used to determine the functional form for each type of behavioural deviation. The delta method given by (7.4) provides an approximate distribution to examine the hypothesis. This table is interpreted in the same manner as Table 7.2 above. The first panel corresponds to the results for the go-to deviations. The second panel corresponds to the results for the serial correlation deviations. Finally, the third panel corresponds to the pressure correlation deviations.

<table>
<thead>
<tr>
<th>Deviation</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Go-to</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$N$</td>
<td>148</td>
<td>148</td>
<td>148</td>
<td>148</td>
</tr>
<tr>
<td>Obs</td>
<td>25</td>
<td>31</td>
<td>42</td>
<td>43</td>
</tr>
<tr>
<td>Exp</td>
<td>7.40</td>
<td>7.40</td>
<td>7.40</td>
<td>7.40</td>
</tr>
<tr>
<td>$\hat{p}$</td>
<td>0.169***</td>
<td>0.210***</td>
<td>0.284***</td>
<td>0.291***</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Serial Correlation</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$N$</td>
<td>148</td>
<td>148</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Obs</td>
<td>27</td>
<td>47</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Exp</td>
<td>7.40</td>
<td>7.40</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\hat{p}$</td>
<td>0.182***</td>
<td></td>
<td>0.318***</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pressure Correlation</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$N$</td>
<td>146</td>
<td>148</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Obs</td>
<td>39</td>
<td>53</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Exp</td>
<td>7.30</td>
<td>7.40</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\hat{p}$</td>
<td>0.267***</td>
<td></td>
<td>0.358***</td>
<td></td>
</tr>
</tbody>
</table>

* Indicates rejection at the 10-percent level of significance.
** Indicates rejection at the 5-percent level of significance.
*** Indicates rejection at the 1-percent level of significance.
For example, go-to deviations are observed in approximately 17% of cases in model (1), 21% in model (2), 28% in model (3) and 29% in model (4). In all these cases the number is significantly greater than 5%, which is the expected number under the null hypothesis. Serial correlation deviations are observed in 18% of the sample in model (2) and 32% in model (4), which are both significantly greater than 5%. Finally, pressure correlation deviations are observed in 27% of the sample in model (3) and 36% in model (4), again significantly greater than 5%. The results for the total effects are not dissimilar to the partial effects presented in Table 7.2. However, since these figures represent the total behavioural change, I can conclude that a substantial number of players change their behaviour under pressure through a go-to deviation, a serial correlation deviation or a pressure correlation deviation.

I also examine the presence of behavioural deviations by testing Hypothesis 3b. Table 7.6 below presents the results. This is interpreted similarly to Table 7.3, except that I test for a significant difference between the uniform distribution and empirical distribution of \(p\)-values for the behavioural deviations from the delta method.

<table>
<thead>
<tr>
<th>Deviation</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Go-to</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>KS</td>
<td>0.1804***</td>
<td>0.2254***</td>
<td>0.2503***</td>
<td>0.2955***</td>
</tr>
<tr>
<td>Area</td>
<td>0.0971***</td>
<td>0.1144***</td>
<td>0.1532***</td>
<td>0.1746***</td>
</tr>
<tr>
<td>Serial Correlation</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>KS</td>
<td></td>
<td>0.1391***</td>
<td></td>
<td>0.2706***</td>
</tr>
<tr>
<td>Area</td>
<td></td>
<td>0.0731***</td>
<td></td>
<td>0.1465***</td>
</tr>
<tr>
<td>Pressure Correlation</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>KS</td>
<td></td>
<td></td>
<td>0.2322***</td>
<td>0.3225***</td>
</tr>
<tr>
<td>Area</td>
<td></td>
<td></td>
<td>0.1301***</td>
<td>0.1744***</td>
</tr>
</tbody>
</table>

*** Indicates rejection at the 1-percent level of significance.
** Indicates rejection at the 5-percent level of significance.
* Indicates rejection at the 10-percent level of significance.

These results are similar to the test of Hypothesis 3a, suggesting that there are go-to, serial correlation and pressure correlation behavioural deviations present. The instances of these effects are significantly more prevalent than random chance for all of the models considered under the both KS-test and the Area-test.
In summary, evidence from Tables 7.5 and 7.6 suggests that there are go-to, serial correlation and pressure correlation behavioural deviations present in the sample. This leads me to conclude that pressure has a substantial effect on decision-making through one of three possible avenues. The first is by causing players to gravitate towards one particular action more often than expected. The second is by causing players to change the rate that they switch service directions. The third is by causing players to make decisions based on choices made at previous pressure points.

7.1.4 Additional Results

The key results from this analysis suggest that pressure affects the average service behaviour of a significant number of players. This result holds regardless of whether the partial effects (the regression coefficients) or the total effects (the behavioural deviations) are considered. In addition, this methodology can be used to examine further questions regarding decision-making. Questions such as whether the two courts are independent as initially assumed, whether the effect on the correlation of choices can undo any baseline correlation, and whether the top ranked players are immune to these effects, can also be investigated using this framework. The focus of this further discussion will be to examine these questions to better understand the mechanisms through which pressure influences decision-making.

Independence of Court

In the previous analysis I assume that the player-court regressions are independent to test whether pressure has a significant effect on player’s decision-making. It seems natural that the behaviour of each player under pressure will be independent of other players. However, it is less certain that each player’s behaviour under pressure will be independent across courts. Therefore, the first question that I investigate is whether there is any correlation between a player’s response to pressure on the Deuce court and their response on the AD court. To answer this, I examine the correlation between the pressure estimates for the same player across the two courts.
However, due to estimation error, the correlation between the observed estimates is biased downward relative to the true correlation, as shown by the following result:

**Proposition 9.** Let $\hat{\alpha} = \alpha + \epsilon_\alpha$ and $\hat{\beta} = \beta + \epsilon_\beta$ be the estimates of the true effects $\alpha$ and $\beta$ with estimation errors $\epsilon_\alpha$ and $\epsilon_\beta$, respectively. Then the true correlation between the effects, $\rho(\alpha, \beta)$, is:

$$
\rho(\alpha, \beta) = \rho(\hat{\alpha}, \hat{\beta}) \sqrt{\frac{\text{Var}[\hat{\alpha}]\text{Var}[\hat{\beta}]}{\text{Var}[\alpha]\text{Var}[\beta]}} > \rho(\hat{\alpha}, \hat{\beta})
$$

where $\rho(\hat{\alpha}, \hat{\beta})$ is the correlation between the estimated effects.

(Proof in Appendix A.) Furthermore, this result provides the adjustment required to calculate the true correlation coefficient. The true variance is unknown so I estimate it by subtracting the mean error variance from the variance of the observed estimates; that is, $\text{Var}[\alpha] = \text{Var}[\hat{\alpha}] - \text{Var}[\epsilon_\alpha]$ and $\text{Var}[\beta] = \text{Var}[\hat{\beta}] - \text{Var}[\epsilon_\beta]$.

Table 7.7 presents the estimates of the true correlation coefficient between the absolute value of the Deuce and AD estimates. I use the absolute value since I am only interested in the correlation of the strength of the effects between the two courts; the direction of the deviation is irrelevant. The first row corresponds to the correlation coefficients between the two courts of the effect pressure on service-mixing probability, the second row corresponds to the effect on serial correlation and the third row corresponds to pressure correlation. The standard errors are calculated using a bootstrap.

<table>
<thead>
<tr>
<th></th>
<th>Partial Effect</th>
<th>Total Effect</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
</tr>
<tr>
<td>$\hat{\rho}_j$</td>
<td>Base</td>
<td>SC</td>
<td>PC</td>
</tr>
<tr>
<td>Mix</td>
<td>0.195</td>
<td>-0.243</td>
<td>0.048</td>
</tr>
<tr>
<td>SC</td>
<td>0.922*</td>
<td>0.251</td>
<td></td>
</tr>
<tr>
<td>PC</td>
<td>0.667**</td>
<td>0.074</td>
<td></td>
</tr>
</tbody>
</table>

*** Indicates rejection at the 1-percent level of significance.  
** Indicates rejection at the 5-percent level of significance.  
* Indicates rejection at the 10-percent level of significance.
For example, consider model (4) in Table 7.7. The correlation coefficient between the pressure estimates of the service-mixing probabilities on the Deuce and AD courts is 0.167 for the partial effect and 0.086 for the total effect. The correlation coefficient between the serial correlation estimates for the two courts is 0.251 for the partial effect and 0.307 for the total effect. Finally, the correlation coefficient between the pressure correlation estimates for the two courts is 0.074 for the partial effect and 0.052 for the total effect. All these effects are positive but insignificant, implying that there is no definite dependence between the estimates for the same player across the two courts for any possible response to pressure.

Considering the results overall, the correlations are generally positive but small. The direction of this effect is to be expected since it would be surprising to observe players who are more likely to change their behaviour on one court but less likely to change on the other. However, as the magnitude of the correlation is small, the link between the effects across the two courts for the same player is weak overall. There are a couple of outliers, in particular the serial correlation effects in model (2) and the pressure correlation effects in model (3), that both display large correlations. However, on closer inspection these results are substantially driven by one extreme outlier in the coefficient estimates that substantially inflates the adjusted correlation coefficient. Therefore, I conclude that the response that players have to pressure appears to be independent across courts.

**Sign of the Correlation**

The next question relates to the level of correlation in players’ service strategies under pressure. The main results suggest that pressure may have an effect on the correlation between service directions from point to point in some specifications. However, the analysis did not consider the sign of the effect, which has implications for the way pressure changes behaviour. If the sign of the effect is negative (positive) then the player tends to switch more (less) frequently under pressure than average. This is an important factor for understanding players’ behaviour under pressure.
To investigate this further, Table 7.8 provides the breakdown of the number of $\beta_{is}^{sc}$ and $\beta_{ip}^{sc}$ coefficients into positive and negative effects. Moreover, the lag coefficients, $\beta_{l}^{sc}$, are included for comparison. These coefficients represent the players’ baseline level of serial correlation and so they can also provide a check to the results from baseline analysis from Chapter 5, which suggest that players have some underlying negative serial correlation in their service decisions. I also provide a normalised test statistic that is equal to the difference between the proportion of positive and negative coefficients; that is, the test statistic is equal to $\frac{n_{pos}-n_{neg}}{n_{tot}}$, where $n_{pos}, n_{neg}$ and $n_{tot}$ are the number of positive, negative and total instances of a particular type of correlation coefficient. The intuition of the test statistic is that a value of 1 or –1 implies that the coefficients are all positive or all negative, respectively, whereas a value of 0 implies the number of positive and negative coefficients is equal. The standard errors are constructed using a bootstrap.

### Table 7.8: Sign of Correlations Estimates

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Base</td>
<td>SC</td>
<td>PC</td>
<td>MI</td>
</tr>
<tr>
<td>$\beta_l$</td>
<td>Total</td>
<td>148</td>
<td>148</td>
<td>148</td>
</tr>
<tr>
<td></td>
<td>Positive</td>
<td>43</td>
<td>45</td>
<td>45</td>
</tr>
<tr>
<td></td>
<td>Negative</td>
<td>105</td>
<td>103</td>
<td>103</td>
</tr>
<tr>
<td>Test stat.</td>
<td>$-0.419^{***}$</td>
<td>$-0.392^{***}$</td>
<td>$-0.392^{***}$</td>
<td>$-0.392^{***}$</td>
</tr>
<tr>
<td>$\beta_{is}$</td>
<td>Total</td>
<td>148</td>
<td></td>
<td>148</td>
</tr>
<tr>
<td></td>
<td>Positive</td>
<td>61</td>
<td></td>
<td>70</td>
</tr>
<tr>
<td></td>
<td>Negative</td>
<td>87</td>
<td></td>
<td>78</td>
</tr>
<tr>
<td>Test stat.</td>
<td>$-0.176^{*}$</td>
<td></td>
<td>$-0.054$</td>
<td></td>
</tr>
<tr>
<td>$\beta_{ip}$</td>
<td>Total</td>
<td>146</td>
<td>135</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Positive</td>
<td>57</td>
<td>51</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Negative</td>
<td>89</td>
<td>84</td>
<td></td>
</tr>
<tr>
<td>Test stat.</td>
<td>$-0.219^{**}$</td>
<td>$-0.244^{***}$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*** Indicates rejection at the 1-percent level of significance.

** Indicates rejection at the 5-percent level of significance.

* Indicates rejection at the 10-percent level of significance.
Considering model (2) in Table 7.8, the number of positive serial correlation coefficients under pressure, $\beta_{is}$, is 61, whereas the number of negative coefficients is 87. The difference in the proportion of positive and negative coefficients is $-0.176$, which is insignificant at the 5% level of significance. Therefore, I cannot reject the hypothesis that the number of positive and negative effects is equal. Observation of the lagged coefficients, $\beta_{il}$, provides a different picture. In this case the number of players who have some general positive serial correlation in their service choices is 45, whereas the number that have some general negative serial correlation is 103. The difference in proportions in this case is $-0.392$, which is significant at the 1% level. This suggests that players have a tendency to switch their service direction too frequently in general (i.e., without pressure).

Overall, the second panel shows that the directional effect of pressure on the serial correlation in service choices is ambiguous. Under pressure some players tend to switch directions more than average and some less. The number of players who switch more is not significantly different from the number of players who switch less. On the other hand, the third panel shows evidence that there are more negative instances of pressure correlation than positive instances. In both models the number of players who experience positive pressure correlation is significantly different to the number of players who experience negative pressure correlation, implying that players have a general tendency to alternate service direction on pressure points.

Lastly, the lagged coefficients (the first panel) are consistently more negative than positive. This result is consistent with the analysis from Chapter 5, and the conclusions from Walker & Wooders (2001) and Gauriot et al. (2016), which suggest that there is some underlying negative serial correlation in players’ service choices.

The ambiguity in the sign of the serial correlation coefficients, $\beta_{is}$, raises an interesting possibility. Since the previous results suggest that players switch serves sub-optimally in general, the addition of pressure could lead to one of two scenarios depending on the relative direction of the effect to the baseline. One scenario is that pressure could alleviate the baseline level of serial correlation, which could actually
move players closer to true randomisation (i.e., zero correlation). This would be the case if the additional component of serial correlation under pressure, \( \beta_{iS} \), was the opposite sign to the baseline level, \( \beta_l \), causing the total correlation under pressure, \( \beta_l + \beta_{iS} \), to move closer to zero. In this case pressure may have a positive influence on individuals’ behaviour. Alternatively, the additional effect of pressure could amplify the already non-random choices and cause players to be even more predictable in their choices. This would be the case if the additional component of serial correlation under pressure, \( \beta_{iS} \), was the same sign as the baseline level, \( \beta_l \), causing the total correlation under pressure, \( \beta_l + \beta_{iS} \), to move farther away from zero. If this is the case, then pressure will likely have a negative influence on behaviour.

To explore this possibility further, I investigate the correlation coefficient between the correlation estimates under pressure and the baseline level of serial correlation. If the two coefficients have a positive correlation, then pressure will tend to amplify any serial correlation initially present. However, if they have a negative correlation then the effect of pressure could cancel out the inherent baseline serial correlation. Table 7.9 presents the correlation coefficient between the baseline coefficients, \( \beta_{lsc} \), and correlation coefficients under pressure, \( \beta_{iS}^{sc} \) and \( \beta_{lp}^{sc} \).\(^{47}\) I use Proposition 9 to adjust the correlation coefficients from the estimated coefficients to calculate the true correlation. The standard errors are obtained by a bootstrap.

**Table 7.9: Correlation between Baseline and Correlation Estimates**

<table>
<thead>
<tr>
<th></th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline vs.</td>
<td>Serial Correlation</td>
<td>Pressure Correlation</td>
<td></td>
</tr>
<tr>
<td></td>
<td>−0.492***</td>
<td>−0.298***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>−0.043</td>
<td>0.123</td>
<td></td>
</tr>
</tbody>
</table>

*** Indicates rejection at the 1-percent level of significance.
** Indicates rejection at the 5-percent level of significance.
* Indicates rejection at the 10-percent level of significance.

\(^{47}\)The Pressure Correlation vs. Baseline comparison is not as direct as the Serial Correlation vs. Baseline since the baseline corresponds to the level of serial correlation (not pressure correlation) without pressure. Nevertheless, it is included to provide a contrast to the effect of pressure on the level of serial correlation in players’ service strategies.
Interestingly, there is evidence that the effect of pressure may counteract the inherent over-switching in players’ service decisions. The correlation between the baseline level of serial correlation and the additional effect of pressure is $-0.492$ in model (2) and $-0.298$ in model (4), both of which are significant at the 1% level. The implication of this is that pressure could ‘undo’ some of the general serial correlation in players’ service choices. Consequently, pressure may actually cause players to behave closer to their theoretical optimal behaviour (which is to keep their service choices independent of previous choices). On the other hand, there is no evidence that there is any positive or negative impact of pressure correlation on players’ baseline level of correlation. In both model (3) and model (4) the correlation coefficient between the pressure correlation estimates and the baseline level of serial correlation is small and insignificant.

**The Effect of Rank**

The final question that I examine is whether top-ranked players are also susceptible to the effects of pressure. While it is likely that a substantial component of a player’s success could be due to superior shot execution, it could also be the case that their success could be attributed to superior decision-making skills. Consequently, one would expect those at the top echelon of tennis to be less likely to exhibit potentially sub-optimal deviations under pressure than those who are not as highly ranked.

To study whether players prone to making systematic deviations under pressure are unable to make it to the top of the rankings, I look for evidence of a relationship between the estimates and career highest ranking. The rationale behind this is that players who make it to the top of the rankings should be less likely to display behaviour that is exploitable and potentially detrimental to their performance.\footnote{Even if this is the case, there is still a high likelihood that pressure has a negative influence in other aspects. For example, there is still strong evidence from the previous discussion that pressure has a negative impact on players’ service-mixing probability. Hence, I do not claim that pressure has a universally positive impact in all facets of behaviour.}

\footnote{Career highest rank can be biased towards older players who have had more time to obtain a higher rank and against young players who have yet to reach their full potential. Nevertheless, one should not expect to observe a pressure effect in players who achieve a high ranking if it has a significant detrimental effect on performance; thus this comparison is still highly informative.}
I investigate this for all players with a ranking in the top 100 by examining the correlation between the absolute value of the coefficients for the effect of pressure on players’ decision-making and their career highest ranking. Again, I use the absolute value because I am only interested in the strength of the deviation. I separate by court to ensure that the results are not influenced by having two observations for the same player (who will clearly have the same rank). Lastly, I use Proposition 9 to adjust for the bias in the correlation stemming from any estimation error in the decision-making estimates. Table 7.10 presents the correlation coefficient between (the absolute value of) the decision-making estimates and career highest rank separated by court.

**Table 7.10: Correlation between Rank and Decision-making Estimates**

<table>
<thead>
<tr>
<th></th>
<th>Partial Effect</th>
<th>Total Effect</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>$\hat{\rho}_j$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Deuce</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mix</td>
<td>$-0.030$</td>
<td>$-0.151$</td>
</tr>
<tr>
<td>SC</td>
<td>$-0.019$</td>
<td>$0.084$</td>
</tr>
<tr>
<td>PC</td>
<td>$0.092$</td>
<td>$0.107$</td>
</tr>
<tr>
<td>AD</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mix</td>
<td>$0.458^{**}$</td>
<td>$0.810^{***}$</td>
</tr>
<tr>
<td>SC</td>
<td>$0.808^{***}$</td>
<td>$0.254$</td>
</tr>
<tr>
<td>PC</td>
<td>$0.655^{***}$</td>
<td>$0.272^{*}$</td>
</tr>
</tbody>
</table>

---

*** Indicates rejection at the 1-percent level of significance.
** Indicates rejection at the 5-percent level of significance.
* Indicates rejection at the 10-percent level of significance.

Table 7.10 presents some interesting results. On the Deuce court there is no significant correlation between any of the possible behavioural deviations and players’ rank in any of the models. This is in striking contrast to the results for the AD court, where there appears to be a significant positive correlation between players’ ranking and the strength of their behavioural deviations in the majority of cases. This implies that players who are more likely to have behavioural deviations under pressure are also more likely have a worse (i.e., higher in number) ranking.
The implication of this result is that players who are not as successful appear to be more susceptible to the effect of pressure on their behaviour (at least on the AD court). I hypothesise that there could be two reasons for this observation. The first is that decision-making is an important factor in the success of a player and those who have behavioural deviations under pressure are less likely to win matches. However, I cannot claim that this is a causal relationship because I am not considering the player’s outcome (i.e., their probability of winning these points). Instead it could simply be that players who are ranked lower may have less experience dealing with pressure. If so, then they may have been explicitly coached to employ certain strategies at these moments as a way of coping with the pressure scenario. Since the majority of the well-known pressure points, such as break-points, are typically on the AD court, if players are coached specifically for these points then this could also explain why there is a significant correlation on the AD court that is not present on the Deuce court.

Regardless, the existence of a relationship between success (in terms of ranking) and decision-making provides strong motivation to consider the effect of pressure on players’ outcome. This is the focus of the following section.

7.2 The Effect of Pressure on Outcomes

The analysis so far has shown that pressure affects players’ decision-making on their direction of serve. However, ultimately what matters to players is not whether their decision-making was optimal from a theoretical point of view, but whether they win or lose. Thus, in this section I supplement the analysis on decision-making with an analysis of the effect of pressure on outcomes; that is, whether pressure affects the likelihood of a server winning the point. I first examine whether servers are more/less likely to win points with greater importance using aggregate data. I then allow for heterogeneity between individuals and examine whether the presence of pressure affects outcomes at the player-level. The player-level effects provide the
next step in determining whether there is a link between the effect of pressure on individual’s decision-making and its effect on their outcomes. This last question will be examined in Section 7.3, which combines the decision-making results (Section 7.1) with the outcome results (this section).

To provide the foundation for the subsequent analysis of the effect of pressure on players’ outcomes, I present the key hypothesis for this section. This arises primarily from the psychological literature that suggests that pressure is detrimental to mental and physical performance.

**Key Hypothesis 2.** *Servers will win fewer points in high pressure situations.*

Key Hypothesis 2 forms the basis for the subsequent estimation methodology.

### 7.2.1 Pressure vs. Outcome

There is evidence in the literature that pressure has an adverse effect on players’ point-winning probabilities. In particular, researchers have found that breakpoints – a crude measure of pressure – can have a negative impact on outcome (O’Malley, 2008; Magnus & Klaassen, 2008).\(^5\) To investigate whether there is a relationship between pressure and outcome, I estimate the following regression:

\[
\pi_t = \alpha_{Imp_t} + \Gamma Z_t + \mu_t
\]  

(5)

where \(\pi_t\) is the (binary) outcome of the point \(t\), and the independent variable of interest is the importance of point \(t\), \(Imp_t\), constructed in Chapter 6. I use the full specification of importance (rather than the pressure dummy, \(P_t\)) in this instance to provide a more comprehensive examination of the effect of increasing pressure on players’ outcomes. A set of controls, \(Z_t\), accounts for factors such as fatigue, quality difference, server fixed-effects and match conditions that may influence winning probabilities. Table 7.11 presents the results of regression (5) for the entire sample of players.

\(^5\)Specifically, O’Malley (2008) found that servers win 55% of breakpoints, whereas they win 67% of points on average.
Table 7.11: The Effect of Pressure on Point-winning Probabilities

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Imp</td>
<td>-0.0061</td>
<td>0.0032</td>
<td>0.0097</td>
<td>-0.0300</td>
<td>-0.0780***</td>
<td>-0.0733***</td>
</tr>
<tr>
<td></td>
<td>(0.0309)</td>
<td>(0.0311)</td>
<td>(0.0310)</td>
<td>(0.0298)</td>
<td>(0.0305)</td>
<td>(0.0306)</td>
</tr>
<tr>
<td>Set</td>
<td>-0.0037***</td>
<td>-0.0037**</td>
<td>-0.0043***</td>
<td>-0.0034**</td>
<td>-0.0034**</td>
<td>-0.0034**</td>
</tr>
<tr>
<td></td>
<td>(0.0015)</td>
<td>(0.0015)</td>
<td>(0.0015)</td>
<td>(0.0016)</td>
<td>(0.0016)</td>
<td>(0.0016)</td>
</tr>
<tr>
<td>Qual Diff</td>
<td>0.0739***</td>
<td>0.0736***</td>
<td>0.0739***</td>
<td>-0.0016</td>
<td>-0.0016</td>
<td>-0.0016</td>
</tr>
<tr>
<td></td>
<td>(0.0045)</td>
<td>(0.0054)</td>
<td>(0.0046)</td>
<td>(0.0090)</td>
<td>(0.0090)</td>
<td>(0.0090)</td>
</tr>
</tbody>
</table>

Notes: Number of observations is 148,144. Dependent variable is a binary indicator for point won. Independent variable of interest is Imp. Specification (1) includes only additional court surface controls. Specification (2) includes the number of sets played up to point \( t \). Specification (3) controls for quality differences between the Server and the Receiver by using the difference in the expected tournament losing round of both players. Specification (4) includes server-fixed effects, (5) includes match-fixed effects and (6) includes both. Standard errors are clustered at the match level and given in parentheses. ***\( p < 0.01 \), **\( p < 0.05 \), *\( p < 0.1 \).

The results displayed in Table 7.11 highlight some intuitive features. First, the effect of the length of the match (in sets) up until point \( t \) – the Set variable – is significant and negative for all specifications. This is likely due to the fact that the potency of the serve, and the corresponding advantage it brings, becomes less pronounced as players become more fatigued.\(^{51}\) Second, the quality difference between the Server and Receiver is also significant in all specifications (except for Specification (6) when it is absorbed by the combination of server- and match-fixed effects). This is not surprising since, as the skill of the Server becomes greater (lesser) compared to the Receiver, one would expect them to be more (less) likely to win the point.

In the more extensive models (specifications (4)–(6)) the importance coefficient is negative. In particular, the effect is negative and significant when match-fixed effects are included. This suggests that pressure, as measured by importance, tends to decrease the likelihood that the Server will win the point. The simpler specifi-

\(^{51}\) There may also be some element of learning by the Receiver as the match progress and as they get better at reading, or anticipating, the direction of the serve.
cations paint a slightly different picture. In these specifications any negative effect of pressure disappears when variables for fatigue and the quality difference between the players are included. However, there is likely to be substantial heterogeneity in match conditions (e.g., variable weather conditions) that may impact on service point-winning probabilities. Therefore, I argue that the specifications that control for this heterogeneity provide better estimates of the factors influencing point-winning probabilities. Consequently, I conclude that pressure has a significantly negative effect on the likelihood of the Server winning the point.

Some 'back-of-the-envelope' calculations can put these figures into context. Considering specification (6), the change in the point-winning probability for a one unit increase in the importance is $-0.073$. Recall that the $95^{th}$ percentile for importance is 0.120, therefore the change in probability that a server will win a point at this level of importance is $-0.009$. This is equivalent to approximately a 1% decrease in the probability of winning the point. By definition of the importance of a point, this translates to decrease in the probability of winning the match by approximately 0.1%.

While at first glance these figures may seem inconsequential, a server in a tight match may face 15 or more points at this level of importance or greater. Therefore, they could increase their chances of winning the match by approximately 1.6% if they were not adversely affected by pressure. In 2018, the difference between the prize money for a first- and second-round loser at the Australian Open was AUD30,000. In expectation this could cost that player close to AUD500 and, more importantly, the chance to go further into the tournament. The consequences are even more significant for those who make it to the final. The difference between prize money for the winner and the runner-up is AUD2million. Therefore, the cost of this effect to a server in this scenario is AUD36,000.
7.2.2 Player-level Effects

The previous results indicate that players’ service-winning probabilities are negatively influenced by pressure in aggregate. However, it is more likely that the influence of pressure will be different for each player. In light of this, I also examine the impact at an individual player level. This involves investigating the effect that pressure has on the service-winning probabilities for each player-court in the sample. To do this I estimate the following specification:

\[ \pi_{smc,t} = \alpha_{sc}^p P_t + \Gamma_{sc} Z_t + \lambda_{sc}^m + \mu_{smc,t} \]  

(6)

where \( \pi_{smc,t} \) is the probability of Server \( s \) winning the point \( t \) in match \( m \) on court \( c \) = \{Deuce, AD\}. The key independent variable is the indicator for presence of pressure, \( P_t \), defined in the same manner as in Section 7.1. Match-fixed effects, \( \lambda_{sc}^m \), and other controls, \( Z_t \), are also included to account for differences in player ability, other factors that may also influence the probability of the Server winning the point.\(^52\) The coefficient of interest, \( \alpha_{sc}^p \), captures the extent to which pressure influences the likelihood of Server \( s \) winning the point on court \( c \) when faced with pressure. A value significantly different from zero suggests that pressure has an effect on the outcome for this player-court.

This specification differs from the aggregate analysis in two ways. First, I use the pressure dummy variable, \( P_t \), as the explanatory variable instead of the \( Imp_t \) variable. I do this to allow for a direct comparison between the players who perform worse under pressure and those whose decision-making is affected by pressure. This is the final component of the analysis and the focus of Section 7.3 below. Second, the analysis is performed separately for each player and each court so that it represents the effect for each individual player rather than the aggregate sample.

Figure 7.3 below plots the (kernel-smoothed) distribution of the estimates from regression (6) above for all the player-court observations.

\(^52\)Robustness results are presented in Appendix F.1.
\(^53\)Again, the variable \( Z_t \) contains court surface dummies and controls for fatigue by accounting for the number of sets points played up to point \( t \).
Figure 7.3: Distribution of Pressure Estimates – Outcome

Figure 7.3 shows that the distribution of estimates is centred close to zero, which implies that the outcomes for the majority of the players are not significantly affected by pressure. However, there is substantial dispersion in the estimates suggesting that an effect of pressure may still manifest itself more frequently than expected as a result of random fluctuations in the data. Furthermore, careful observation shows that the distribution peaks slightly below zero and has more mass in the negative coefficients. This suggests that pressure has a more negative effect in general. This observation is consistent with the aggregate results. These observations are tested in the subsequent discussions.

Prevalence of Pressure Effects

The analysis of the effect of pressure on outcomes is similar to the analysis of its effect on decision-making from Section 7.1. Regression (6) is performed at the player-court level and the $\alpha^{sc}_p$ coefficients from each regression are extracted. For the instances where a player has a pressure coefficient significantly different to zero, pressure is said to have a significant effect on the outcome for that player. I provide the foundation for the analysis with the following hypothesis, which can be used to determine whether any effect is prevalent in the sample:
Hypothesis 4. For the entire sample of players:

\[ H_0 : \text{The effects are generated by random fluctuations in the data.} \]

\[ H_1 : \text{The effects are not generated by random fluctuations in the data.} \]

As for Hypothesis 3 (refer to Section 7.1), there are two methods I use to test Hypothesis 4. The first utilises the fact that total number of significant effects has a Binomial distribution, \( B(n, p) \), with \( n \) equal to the number of regressions and \( p = 0.05 \). The second method utilises the result that the \( p \)-values of the coefficients from each regression should be distributed \( U[0, 1] \) under the null hypothesis.

Table 7.12 presents the results of the two tests of Hypothesis 4. The first column displays the proportion of the sample that exhibits a significant pressure effect, \( \hat{p} \). The remaining columns display the KS- and Area-test statistics for the difference between the empirical and theoretical (i.e., uniform) distributions of the \( p \)-values.

<table>
<thead>
<tr>
<th></th>
<th>Binomial Test</th>
<th>( U[0, 1] ) Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \hat{p} )</td>
<td>0.1081***</td>
<td>0.1919***</td>
</tr>
<tr>
<td>KS</td>
<td></td>
<td>0.1001***</td>
</tr>
<tr>
<td>Area</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*** Indicates rejection at the 1-percent level of significance.
** Indicates rejection at the 5-percent level of significance.
* Indicates rejection at the 10-percent level of significance.

The results of Table 7.12 are interpreted in the same manner as Tables 7.2 and 7.3 from Section 7.1. First, the proportion of significant effects in the sample, \( \hat{p} \), is 0.1081. This is significantly different from the expected proportion under the null hypothesis (i.e., \( p = 0.05 \)) at the 1% level of significance. Second, the KS-test statistic is 0.1919, which is significant at the 1% level. Finally, the Area-test statistic is 0.1001, which is again significant at the 1% level. Overall, this evidence strongly favours the conclusion that pressure influences the likelihood of winning the point on serve for a significant number of players. This is confirmed by Figure 7.4, which illustrates the prevalence of these effects by plotting the empirical distribution of the \( p \)-values against the uniform distribution.
Visual inspection of Figure 7.4 supports the conclusion of the above tests. If there was no pervasive effect of pressure, then any observed effects at the player level should be due to random chance. This would result in an empirical distribution of \( p \)-values that is approximately uniform in the \([0, 1]\) interval. However, the observed empirical distribution clearly lies above the uniform CDF. This implies that the significant effects (i.e., the \( \alpha_{sc} \neq 0 \) coefficients) are more common than would be expected as a result of randomness. In summary, the results presented in this discussion suggest that pressure influences the service-winning probability for a significant number of players in the sample.

**Positive vs. Negative Effects**

Individual-level analysis reported thus far has found that pressure has a significant effect on the outcomes of service points. I now go on to consider the sign of that effect; that is, whether pressure has a positive or negative impact on the likelihood of the Server winning the point. If pressure has more positive effects than negative effects then it is evidence of the presence of individual players with above average, or ‘clutch’ performance under pressure. Alternatively, if there are more negative effects than positive ones then it is evidence that pressure has a prevalent detrimental
The Effect of Pressure

Effect on performance at the player-level. This will allow for a comparison with the findings from the aggregate analysis, where it was concluded that pressure has an overall negative impact.

To investigate this question, I examine the sign of the effect that pressure has on players’ service-winning probabilities from the player-court regressions using model (6). If the sign on the $\alpha_{sc}^p$ coefficient is positive (negative) then the presence of pressure increases (decreases) the likelihood of Server $s$ winning the point on court $c$. Table 7.13 presents the number of instances where a positive or negative effect of pressure is observed at a significance level given by the column. I also provide the same normalised test statistic equal to the difference between the proportion of positive and negative coefficients as before. The standard errors are constructed using a bootstrap.

<table>
<thead>
<tr>
<th>Significance level</th>
<th>1%</th>
<th>5%</th>
<th>10%</th>
<th>All</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total</td>
<td>3</td>
<td>16</td>
<td>25</td>
<td>148</td>
</tr>
<tr>
<td>Positive</td>
<td>1</td>
<td>4</td>
<td>7</td>
<td>75</td>
</tr>
<tr>
<td>Negative</td>
<td>2</td>
<td>12</td>
<td>18</td>
<td>73</td>
</tr>
<tr>
<td>Test-stat.</td>
<td>$-0.33$</td>
<td>$-0.50^*$</td>
<td>$-0.44^*$</td>
<td>$0.01$</td>
</tr>
</tbody>
</table>

*** Indicates rejection at the 1-percent level of significance.
** Indicates rejection at the 5-percent level of significance.
* Indicates rejection at the 10-percent level of significance.

For example, for the players who are significantly affected by pressure at the 5% level, there are 12 players who perform significantly worse under pressure; whereas there are four players who actually perform significantly better under pressure. The difference in the proportion of positive and negative effects is $-0.50$, which is significant at the 10% level. Although there is some evidence that there are more negative effects than positive ones, I cannot reject the hypothesis that the number of positive and negative effects is the same at the 5% level of significance. In general, there appears to be more negative effects than positive effects at high significance levels. However, it is not possible to conclude this definitively.
These results are not as conclusive as the aggregate results above. When the players are pooled together pressure appears to have a negative effect on the likelihood of winning a point. However, if the question is examined on a player-by-player case then this effect is no longer significant. In this case, the number of instances where pressure has a negative effect on the outcome of the point is not significantly different to those where it has a positive effect.

Overall, pressure has a negative impact on players’ outcomes in aggregate. This effect cannot be conclusively supported at the individual level where the number of players who have an extreme negative response to pressure is not significantly larger than those who have an extreme positive response. The discrepancy in these findings is likely driven by two factors. The first is that the test for a sample this size is likely to have insufficient power to detect a significant difference at the individual level. The second is that the aggregate results could stem from an asymmetry in the size of the individual positive and negative effects, and the fact that the number of observations is not constant for each player.

**The Effect of Rank**

Lastly, I wish to determine whether the effect of pressure on players’ outcomes is different for the top ranked players compared to the lower ranked players. Therefore, I examine the correlation coefficient between the players’ outcome estimates and their career highest ranking for players with a ranking in the top 100. Again, I separate by court to ensure that the results are not influenced by having two observations for the same player and use Proposition 9 to adjust for the bias in the correlation stemming from any estimation error in the outcome estimates.

The results suggest that the correlation between the players’ outcome estimates and their (career highest) rank is $-0.276$ for the Deuce court and $0.030$ for the AD court. Although the correlation on the AD court is essentially zero, the sign of the effect on the Deuce court is consistent with the expectation that the top ranked players are less negatively impacted by pressure. This observation is also consistent
with the research by González-Díaz et al. (2012) that there is a positive relationship between a player’s success and their ability to perform better on critical points. However, even in this case the correlation is not statistically significant.

Interestingly, the correlation between the outcome and rank is larger (although insignificant) on the Deuce court than on the AD court; however, the correlation between the decision-making estimates is more pronounced on the AD court than the Deuce court (see Section 7.1). This is a curious result and may indicate that the link between decision-making and outcome is not as strong as expected.

This observation provides some further motivation to examine the relationship between decision making and outcome. In particular, I aim to determine whether players’ negative (or positive) outcomes under pressure are correlated with the change in choices they make under pressure. This brings together the current analysis of the effect of pressure on players’ outcomes and that of players’ decision-making in Section 7.1, and is the focus of the next section.

### 7.3 Decision-making and Outcome

The final component of this thesis is to tie the two key findings together. The first key result suggests that there is a prevalent effect of pressure on individuals’ decision-making. This was the main finding of Section 7.1. The second is that there is a prevalent effect of pressure on point outcomes. This was the main finding of Section 7.2. To determine whether there is any link between the impact of pressure on decision-making and the likelihood of winning a point, it is imperative to examine the relationship between the two effects. If a player makes decisions that are potentially sub-optimal under pressure and they perform differently under pressure, then there is evidence of a relationship between decision-making and outcome. However, if the set of players who make potentially sub-optimal decisions under pressure is independent of those who have different winning probabilities, then there may be other factors that explain the difference in performance with and without pressure.
The analysis in this section attempts to understand the reasons why players tend to perform differently under pressure. I will first put forth the key hypothesis that forms the foundation of the subsequent analysis. This hypothesis arises as a direct consequence of the results of the previous two sections. First, Section 7.1 provided evidence that a significant number of players deviate from their long-run strategy under pressure, and so their performance should be affected. Second, Section 7.2 established that pressure does affect the performance of a significant number of players. When taken in conjunction, I arrive at the following hypothesis:

**Key Hypothesis 3.** The effect that pressure has on a player’s outcome is (somewhat) driven by the effect that it has on their decision-making.

The subsequent analysis examines Key Hypothesis 3 to determine whether there is a link between a player’s decision-making under pressure and their service-winning probabilities under pressure. Note that pressure is known to affect the execution of players’ shots, which may affect their outcomes. Thus, I do not claim it is solely driven by the effect on decision-making, just that there should be some effect.

### 7.3.1 Results

To investigate Key Hypothesis 3 I examine the correlation between the decision-making estimates from Section 7.1 and the outcome estimates from Section 7.2. If there is any negative correlation between the likelihood of changing behaviour under pressure and the probability of winning the point, then this would provide evidence in support of Key Hypothesis 3. To investigate this possibility, Table 7.14 presents the correlation coefficients between the outcome estimates and the absolute value of the partial and total effects from the decision-making analysis in Section 7.1. I use the absolute value of the decision-making estimates because I am only interested the correlation between the outcome estimates and the strength of any deviation, and so the sign of the decision-making estimates is not important. Again, I use Proposition 9 to adjust the correlation coefficients from the estimated coefficients to calculate the true correlation and the standard errors are obtained by a bootstrap.
Table 7.14: Correlation between Outcome and Decision-making Estimates

<table>
<thead>
<tr>
<th></th>
<th>Partial Effect</th>
<th>Total Effect</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1) (2) (3) (4)</td>
<td>(1) (2) (3) (4)</td>
</tr>
<tr>
<td>$\hat{\rho}_j$ Base</td>
<td>SC</td>
<td>PC</td>
</tr>
<tr>
<td>Mix</td>
<td>$-0.103$</td>
<td>$-0.202$</td>
</tr>
<tr>
<td>SC</td>
<td>$-0.230$</td>
<td>$-0.075$</td>
</tr>
<tr>
<td>PC</td>
<td></td>
<td>$-0.266$</td>
</tr>
</tbody>
</table>

*** Indicates rejection at the 1-percent level of significance.  
** Indicates rejection at the 5-percent level of significance.  
* Indicates rejection at the 10-percent level of significance.

Observation of the figures in Table 7.14 shows that the sign of the correlation coefficients between the outcome estimates and the decision-making estimates is negative for possible deviations in all models. This implies that the probability of a player winning a pressure point tends to decrease as the likelihood of them deviating from their baseline service strategy increases. This observation does not come as a surprise and is in agreement with Key Hypothesis 3, which posits that worse outcomes are linked to changes in behaviour under pressure.

However, despite being negative, the magnitudes of the correlations in Table 7.14 are relatively small for all possible deviations in all models. For example, in the Pressure Correlation model, the correlation coefficient between the outcome estimates and the service-mixing estimates is $\rho_p = -0.323$ for the partial effect, and $\rho_p = -0.214$ for the total effect. Additionally, the correlation coefficient between the outcome estimates and the pressure correlation estimates is $\rho_{ip} = -0.266$ for the partial effect, and $\rho_{ip} = -0.181$ for the total effect. In all these cases the correlation coefficients between (the absolute values of) the decision-making estimates and the outcome estimates are not significantly different from zero. The results from the remaining models and responses are similar. Therefore, although there appears to be some negative correlation between the strength of the effect on players’ decision-making and their point outcomes, I cannot definitely conclude that this relationship is present.
The strength of the correlations that I observe between decisions and outcome is a little surprising. Given Key Hypothesis 3, I would expect to see a stronger and more significant correlation between a player’s deviations and their outcomes. Instead, the observed correlations imply only a weak link between the two under pressure. This observation requires further investigation. Therefore, in the next chapter I present a discussion of the implications of the key observations from this thesis and suggest a few ways to understand these results.
Chapter 8

Discussion

In this thesis I develop a novel measure of pressure that allows me to address three fundamental aspects of behaviour under pressure. The first aspect that I investigate is the players’ decision problem. In the absence of pressure, the baseline analysis in Chapter 5 suggests that players’ service choices are consistent with many facets of optimal behaviour. In Chapter 7 I expand upon the baseline analysis conducted in Chapter 5 by introducing pressure to examine its effects on professional tennis players. Specifically, in Section 7.1 I introduce the possibility of pressure affecting players’ decision-making to determine whether their behaviour changes under pressure. The second aspect I investigate is the effect of pressure on players’ outcomes. This is studied in Section 7.2, where I examine the impact of pressure on the probability of a player winning a point on their serve. The final issue that I investigate is whether these two effects are related. Thus, in Section 7.3, I combine the findings from Sections 7.1 and 7.2 to examine whether there is any link between the players who behave differently under pressure and their chances of winning. This discussion provides additional ideas to understand and rationalise the key observations from each of these sections. I first summarise the key results as a reminder of the main findings of this analysis.
8.1 Results Summary

The main result from Chapter 5 is that, without considering pressure, tennis players’ behaviour conforms to one of the key predictions of minimax theory. Namely, players appear to choose their service-mixing strategy to equalise the probability of winning the point across all choices of service direction. Players do not fare as well conforming to the second prediction of minimax theory, which states that their service direction should be serially uncorrelated. Specifically, I observe a significant number of instances of negative serial correlation in players’ service direction.

The main result from Section 7.1 is that pressure influences the decision-making of a significant number of players. I investigate three possible behavioural responses that players could exhibit under pressure and the results are the same for each response. The first response is through a propensity to choose to serve to a particular direction more frequently than they do on average. The second response is through a propensity to change the rate at which they switch between different service directions. Finally, the third response is through a propensity to make service decisions that are influenced by their choice of service direction from the previous pressure point. I find that the instances of each of these three responses are significantly greater than one would expect to observe from chance. The presence of these responses suggests that pressure has a significant effect on players’ decision-making.

The main result from Section 7.2 is that pressure has a significant effect on players’ ability to win their serve. In aggregate, this effect is negative and so increasing the level of pressure decreases the likelihood that the Server wins the point. At the individual level, pressure also has a significant effect on the service-winning probabilities for a significant number of players.

The main result from Section 7.3 is that there appears to be a negative correlation between players’ behavioural deviations and their outcomes; however, these correlations are not statistically significant. This holds for all of the possible deviations considered. Therefore, I cannot definitively conclude that there is a correlation between any of the behavioural deviations and players’ outcomes in the sample.
For clarity, I summarise the key results as the following:\textsuperscript{54}

\textbf{R-0.} Service-mixing probabilities are optimal without pressure (Chapter 5).

\textbf{R-1.} Behavioural deviations are observed under pressure (Section 7.1).

\textbf{R-2.} Changes in outcomes are observed under pressure (Section 7.2).

\textbf{R-3.} Deviations in decision-making $\not\Rightarrow$ change in outcome (Section 7.3).

\textbf{R-4.} No deviations in decision-making $\not\Rightarrow$ no change in outcome (Section 7.3).

As a robustness check, I also investigate three different models and/or variable specifications.\textsuperscript{55} First, I examine a probit model instead of the linear probability model. Second, I examine a model using the importance of a point from Chapter 6 as a continuous variable for pressure. Finally, I examine a model using a different cut-off for classifying ‘pressure points.’ The results were robust to all alternative specifications and the conclusions remain the same as those presented above.

\section*{8.2 Implications}

Assuming the Receiver behaves optimally, and pressure does not affect the payoffs in the game matrix, these results have several implications regarding how pressure affects players in this environment, particularly with respect to players’ service-mixing probabilities.\textsuperscript{56} Specifically, Result R-0 from the baseline analysis suggests that players’ service choices are optimal in the absence of pressure. Since the number of pressure points is small relative to the total number of points, this implies the Server’s optimal behaviour under pressure is also given by their baseline behaviour. If true, then the observed Results R-1 to R-4 are somewhat unexpected.

\textsuperscript{54}Result R-4 is equivalent to: change in outcome $\not\Rightarrow$ deviations in decision-making. However, I present it as above because this description suggests that a change in outcome drives a deviation from the Server. This is nonsensical since the service choice always comes before the outcome.

\textsuperscript{55}Refer to Appendix F.1 for details.

\textsuperscript{56}I maintain that, if pressure has an effect, then it will have a substantially more prominent impact on the Server than the Receiver (refer to discussion in Chapter 3). Consequently, it is not unreasonable to assume the Receiver continues to anticipate optimally under pressure. Nevertheless, I provide some analysis of the Receiver’s behaviour in the subsequent discussion. In that discussion I also relax the assumption that pressure does not affect the payoffs in the game matrix.
Firstly, if behaving optimally, there should be no reason for the Server to change their behaviour or experience different outcomes in scenarios of heightened pressure. Therefore, neither Result R-1 nor Result R-2 should be observed. Furthermore, the theoretical model in Chapter 3 put forth a model of tennis as a game with a unique (mixed-strategy) equilibrium and so any deviation away from the equilibrium should make a player worse off by definition. Thus, Result R-3 is puzzling in this framework. Finally, each player’s outcome is a function of their choices and so it should be tied to the probability that each action is chosen. Thus, Result R-4 is also unexpected.

These results have two implications. The first is that players who behave ‘sub-optimally’ are not necessarily worse off than those who behave optimally. The second is that those who are worse off are not necessarily so because of their decision-making.

The fact that these results are observed leads to one of two possibilities. The first is that pressure can confound players’ ability to determine their optimal strategies so they could be observed making mistakes and behaving irrationally. For example, incorrect conjectures about an opponent’s behaviour, or correct conjectures about an opponent’s irrational behaviour, could explain the observed results. However, irrational behaviour can be used to explain the vast majority of non-optimal observations. More interestingly, the second possibility is that more extensive models are needed – ones that incorporate pressure – to better understand human behaviour. To do this, the following discussion presents a few extensions to the theoretical model in Chapter 3 in an attempt to rationalise the main observations of this thesis.

8.3 Rationalising the Results

The remainder of this chapter presents a few explanations as to why Results R-1 to R-4 might be observed by attempting to rationalise some of the key observations of this thesis. The discussion will focus on the effect of pressure on players’ service-mixing probabilities (i.e., the go-to deviations). To provide a theoretical basis for the subsequent discussion, I first present the game representing the serve-return
interaction in its most general form:

\[
\begin{array}{c|cc}
& W & T \\
\hline
W & a & b \\
T & c & d \\
\end{array}
\]

Let \( p \) be the probability that the Server serves \( W \) and \( q \) be the probability that the Receiver anticipates a serve \( W \). Then the Server’s point-winning probability is:

\[
\pi(p, q) = d + p(b - d) + q(c - d) + pq(a - b - c + d) \tag{8.1}
\]

Solving each player’s expected utility maximisation problem gives their best response correspondences, which is to choose \( W \) with probability:

\[
BR_S(q) = \begin{cases} 
0 & \text{if } q > \frac{d-b}{a-b-c+d} \\
[0,1] & \text{if } q = \frac{d-b}{a-b-c+d} \\
1 & \text{if } q < \frac{d-b}{a-b-c+d} 
\end{cases}
\]

and

\[
BR_R(p) = \begin{cases} 
0 & \text{if } p < \frac{d-c}{a-b-c+d} \\
[0,1] & \text{if } p = \frac{d-c}{a-b-c+d} \\
1 & \text{if } p > \frac{d-c}{a-b-c+d} 
\end{cases}
\tag{8.2}
\]

with the equilibrium probabilities for the Server and Receiver:

\[
(p^*, q^*) = \left( \frac{d-c}{a-b-c+d}, \frac{d-b}{a-b-c+d} \right) \tag{8.3}
\]

Finally the expected point-winning probability to the Server (i.e., the Server’s expected payoff) in equilibrium is:

\[
\pi^* \equiv \pi(p^*, q^*) = \frac{ad - bc}{a-b-c+d} \tag{8.4}
\]

Using this example, I examine some theoretical reasons for the observed results. In particular, I use it to explain how deviations may arise under pressure, R-1 and R-2, why the effect pressure has on decision-making may not translate to an effect on players’ outcomes, R-3, and why a change in outcomes might not stem from any deviations in decision-making, R-4.
8.3.1 Different game under Pressure

- **Can explain:** Why there could be deviations in decision-making (R-1), why there could be changes in point-winning probability (R-2), why deviations in decision-making probability \( \not \Rightarrow \) change in outcome (R-3) and why no deviations in decision-making \( \not \Rightarrow \) no change in outcome (R-4)

- **Cannot explain:** None applicable

Since it is known that pressure can impact an individual’s ability to perform high-level motor tasks, the simplest explanation for the observations in this study is that pressure can have an effect on the success probabilities (payoffs) of the players’ actions. For example, if the Server had a tendency to miss a particular serve more under pressure, then they will likely have a lower chance of winning the point with that action. This will lead to different payoffs in the game matrix with and without pressure. In this case it is possible to observe deviations under pressure and changes in players’ outcomes. Moreover, it is also possible to derive instances when the presence of one does not necessarily imply the presence of the other. To illustrate all these possibilities formally, suppose that pressure generated the following change in the game matrix as shown below:

```
<table>
<thead>
<tr>
<th></th>
<th>Receiver</th>
<th>Receiver</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>W</td>
<td>T</td>
</tr>
<tr>
<td>Server</td>
<td>W</td>
<td>a</td>
</tr>
<tr>
<td></td>
<td>T</td>
<td>c</td>
</tr>
</tbody>
</table>
```

First, to understand how it is possible to observe deviations in the service mixing probability under pressure, suppose that the Server has a tendency to miss their serve \( W \) considerably more under pressure than usual, while their \( T \) serve remains unaffected. This will change the point-winning probability (i.e., the game matrix
payoffs) for the \( W \) action, but leave the \( T \) action unchanged. That is:

\[
d' < a, \quad b' < b, \quad c' = c, \quad d' = d
\]

From (8.3), the change in the equilibrium probability of serving \( W \), \( \Delta p^* \), is:

\[
\Delta p^* \equiv p'^* - p^* = \frac{d' - c'}{a' - b' - c' + d'} - \frac{d - c}{a - b - c + d} = \frac{(d - c)[(a - a') + (b - b')]}{(a - b - c + d)(a' - b' - c' + d')} < 0
\]

In this scenario different service-mixing probabilities would be observed manifesting in the data as a deviation under pressure (R-1). However, this result will only arise under certain conditions in practice. To illustrate this, note that:

\[
p^*(ta, tb, tc, td) = \frac{td - tc}{ta - tb - tc + td} = \frac{a - b - c + d}{a' - b' - c' + d'} = p^*(a, b, c, d)
\]

and similarly for \( q^* \), and so the equilibrium mixing probabilities are homogeneous of degree zero in \( a, b, c, d \). Therefore, if pressure decreases the Server’s winning probability by, say, 10% for all service choices then a change in equilibrium behaviour should not be observed. Hence, pressure must have an uneven effect on each of the Server’s and/or Receiver’s actions for it to affect observed equilibrium behaviour.

Second, to illustrate how it is possible to observe a change in winning probability under this framework, assume that pressure causes the Server to miss all their serves more often. Assuming pressure affected each action equally, then the decrease in probability of getting their first serve in would lead to a decrease in the Server’s winning probabilities in the game matrix by some factor \( t \) with \( 0 < t < 1 \). That is:

\[
a' = ta, \quad b' = tb, \quad c' = tc, \quad d' = td
\]

From (8.4), this implies:

\[
\pi^*(ta, tb, tc, td) = \frac{(ta)(td) - (tb)(tc)}{ta - tb - tc + td} = \frac{ad - bc}{a - b - c + d} = t\pi^*(a, b, c, d)
\]

So the equilibrium point winning probability is homogeneous of degree one in \( a, b, c, d \). Therefore, the change in the Server’s equilibrium point-winning probability is

\[
\Delta \pi^* \equiv \pi'^* - \pi^* = \pi^*(t - 1) = \frac{ad - bc}{a - b - c + d} (t - 1) < 0
\]
This shows that it is possible to derive a scenario where the equilibrium point-winning probability can change under pressure (R-2). Since this is precisely a proportional change in the payoffs in the game matrix, the equilibrium service-mixing probabilities will remain unchanged with and without pressure (see above). This provides a scenario where it is possible to observe changes in outcome that are not driven by deviations in service-mixing probabilities (R-4).

Finally, it is theoretically possible to observe a deviation in decision-making that does not translate to a change in players’ outcomes in this framework with the following transformation in game matrix payoffs under pressure:

\[ a' = d, \quad b' = c, \quad c' = b, \quad d' = a \]

Therefore, the change in the Server’s equilibrium service mixing-probability is given by:

\[ \Delta p^* = p'^* - p^* = \frac{d' - c'}{a' - b' - c' + d'} - \frac{d - c}{a - b - c + d} = \frac{a - b + c - d}{a - b - c + d} = 0 \]

if \((a - b) \neq (d - c)\). However, the change in equilibrium point-winning probability is:

\[ \Delta \pi^* = \pi'^* - \pi^* = \frac{a'd' - b'c'}{a' - b' - c' + d'} - \frac{ad - bc}{a - b - c + d} = \frac{(da - ad) - (cb - bc)}{a - b - c + d} = 0 \]

This example illustrates that it is possible to derive a scenario where deviations under pressure with regard to decision-making have no effect on the players’ outcomes (R-3).

Importantly, these three cases are all examples where the change in behaviour is due to an optimal response to pressure. In other words, given that pressure changes the payoffs in the game matrix, then these responses are all driven by behaviour that is theoretically optimal. Hence, the key findings in this study could be the result of equilibrium (optimal) behaviour. An obvious question is whether any of these three scenarios are realistic for professional tennis players.

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57 It is hard to contrive a scenario where this transformation would actually occur in a tennis match. Nevertheless, it is presented to illustrate that it is theoretically possible to observe Result R-3.
The first scenario described above could arise if pressure affects the payoffs of one or both serve/s unevenly. There is evidence that suggests that pressure affects highly-trained individuals’ ability to execute complex motor tasks (Beilock & Carr, 2001; Beilock et al., 2002; Jackson et al., 2006; Wilson et al., 2007). Based on this evidence it is likely that pressure could also affect a tennis player’s serve. Moreover, due to the different techniques that are required to hit the various serves, it is reasonable to assume that this effect could impact one service direction more than the other. If this is the case, then this could explain why I observe behavioural deviations under pressure in professional tennis players.

The second scenario described above could arise if pressure has a general overall effect on all serves. There is also some empirical evidence to support this hypothesis. In particular, there is evidence that players take less risk on their serve on important points by opting for a safer serve that is easier to return (Magnus & Klaassen, 1997; Klaassen & Magnus, 2014). If this is true then the outcome (payoff) of these points will be lower for each action since the Server loses some of the advantage the serve has on winning the point.

The third scenario is a little contrived. Consequently, it is hard to justify this as a realistic reason for the observation of these results; nevertheless, it is a theoretical possibility.

8.3.2 Deviations are hard to Observe/Not worth Exploiting

- **Can Explain**: Why deviations in decision-making \(\neq\) change in outcome (R-3)

- **Cannot explain**: Why there are deviations in decision-making (R-1) and changes in outcome (R-2) and why no deviations \(\neq\) no change in outcome (R-4)

A more realistic explanation as to why an effect can be observed in players’ decision-making without any change in winning-probability is that any deviations

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58Paserman (2007) has provided further supporting evidence for this hypothesis finding that critical points last longer due to less risks being taken by players in the rally.
in the Server’s strategy may be difficult for the Receiver to observe. For example, in a typical match of 200 points there may be 20 important points. If the Server increases their likelihood of serving a particular direction by 20% under pressure, then this equates to only an extra four serves to their preferred direction over the course of the match. This subtle difference in behaviour may be hard for the Receiver to recognise in just one match.

To see how this situation could result in a change in the probabilities that are not reflected by an equivalent change in the outcome, consider the following scenario. Under standard conditions (i.e., without pressure) both players choose their actions optimally. However, the Server changes their probability of serving $W$ by some small amount $\delta$ under pressure. Furthermore, this deviation is not observed by the Receiver, so their behaviour remains the same regardless of the level of pressure. The change that would be observed in the Server’s service direction is:

$$\Delta p \equiv p_{\text{pres}} - p_{\text{standard}} = (p^* + \delta) - p^* = \delta \neq 0$$

In contrast the change in winning probability would be:

$$\Delta \pi \equiv \pi_{\text{pres}} - \pi_{\text{standard}} = \pi(p^* + \delta, q^*) - \pi(p^*, q^*)$$

$$\approx \pi(p^*, q^*) + \delta \frac{\partial \pi(p, q^*)}{\partial p} \bigg|_{p=p^*} - \pi(p^*, q^*)$$

$$= \delta \frac{\partial \pi(p, q^*)}{\partial p} \bigg|_{p=p^*}$$

where the second line follows from a first order Taylor expansion in $p$ around the point $(p^*, q^*)$. From (8.1), this gives:

$$\frac{\partial \pi(p, q)}{\partial p} = (b - d) + q(a - b - c + d)$$

(8.5)

which, when evaluated at the equilibrium $(p^*, q^*)$, gives:

$$\frac{\partial \pi(p, q^*)}{\partial p} \bigg|_{p=p^*} = (b - d) + \frac{(d - b)(a - b - c + d)}{a - b - c + d} = 0$$

which is precisely the indifference property of the equilibrium solution of these
games. Therefore, this gives:

\[ \Delta \pi \approx 0 \]

This line of reasoning demonstrates how it is possible to observe a change in the Server’s strategy that may not be observed in the point-winning probabilities (R-3). While this scenario may arise within a single match, where a deviation in the Server’s behaviour of 20% may equate to a noticeable difference in only a few points, I do not believe this argument is valid in general. Assuming players get the chance to observe their opponents in several matches – as is typically the case – they are likely to know their opponent’s deviations from the extensive coaching analysis that occurs in professional tennis.

However, even if players do know the strategies and deviations of their opponents, it still may be the case that there is little incentive for players to exploit these deviations. In general, the expected payoff function is flat around the equilibrium. This implies that there may be little to be gained from perfectly predicting their opponent’s deviations.\(^{59}\) Hence, the Receiver may be reluctant to change from a strategy that they know is optimal in the majority of situations. This is one of the main criticisms of the indifference property that is the foundation of Nash equilibrium.

To see this issue formally, consider a scenario where players again choose their strategies optimally in the absence of pressure. However, there is a small change \(\varepsilon\) in the Server’s probability of choosing to serve \(W\) under pressure that is observable and can be acted upon by the Receiver if they so choose. Hence

\[ \Delta p \equiv p_{\text{pres}} - p_{\text{standard}} = (p^* + \varepsilon) - p^* = \varepsilon \neq 0 \]

If \(\varepsilon > 0\) then, from (8.2), the Receiver’s best response to the Server’s choice is to anticipate \(W\) with probability 1, so the change in the winning probability between

\(^{59}\)This implication has been extensively examined in other economic environments (see e.g., Akerlof & Yellen, 1985).
exploiting the deviation and not is:

\[ \Delta \pi \equiv \pi_{\text{exploit}} - \pi_{\text{not}} = \pi(p^* + \varepsilon, 1) - \pi(p^* + \varepsilon, q^*) \]

\[ \approx \pi(p^*, 1) + \varepsilon \left. \frac{\partial \pi(p, 1)}{\partial p} \right|_{p=p^*} - \pi(p^*, q^*) - \varepsilon \left. \frac{\partial \pi(p, q^*)}{\partial p} \right|_{p=p^*} \]

\[ = \pi(p^*, q^*) + \varepsilon \left. \frac{\partial \pi(p, 1)}{\partial p} \right|_{p=p^*} - \pi(p^*, q^*) \]

\[ = \varepsilon \left. \frac{\partial \pi(p, 1)}{\partial p} \right|_{p=p^*} \]

where the second line follows from a first order Taylor expansion in \( p \) around the points \((p^*, 1)\) and \((p^*, q^*)\), and the third line follows from the indifference property about the equilibrium, that is \( \forall q \in [0, 1], \pi(p^*, q) = \pi(p^*, q^*) \), and that \( \left. \frac{\partial \pi(p, q^*)}{\partial p} \right|_{p=p^*} = 0 \) from before. From (8.5) it must be that:

\[ \left. \frac{\partial \pi(p, 1)}{\partial p} \right|_{p=p^*} = (b - d) + (a - b - c + d) = a - c \]

Therefore,

\[ \Delta \pi \approx \varepsilon(a - c) \]

Similarly, if \( \varepsilon < 0 \) then the Receiver’s best response is to anticipate \( W \) with probability 0, the change in the winning probability between exploiting the deviation and not is:

\[ \Delta \pi \approx \varepsilon \left. \frac{\partial \pi(p, 0)}{\partial p} \right|_{p=p^*} = \varepsilon(b - d) \]

Thus, if \( \varepsilon > 0 \) and the Receiver’s choice for \( W \) serves has a small impact on the outcome (i.e., \( a - c \approx 0 \)), then there will only be a small observed difference in the point-winning probability between exploiting this deviation and not. The same conclusion is arrived at if \( \varepsilon < 0 \) and the Receiver’s choice on \( T \) serves has a small impact on the outcome. In either case, there may be little incentive for the Receiver to exploit the Server’s deviations and so it is possible to observe a deviation in decision-making that does not necessarily translate to noticeable change in the outcome of the point (R-3).
8.3.3 Variable Risk Characteristics

- **Can Explain**: Why there could be deviations in decision-making (R-1) and why there could be changes in point-winning probability (R-2)

- **Cannot explain**: Why deviations in decision-making probability $\neq$ change in outcome (R-3) and why no deviations in decision-making $\neq$ no change in outcome (R-4)

As mentioned earlier, there is evidence that players tend to take fewer risks when faced with pressure. If true, this effect could also extend to their service decisions. The implication of this is that players’ risk preferences may be able to rationalise some of the behaviour observed in this thesis. For example, given two service choices, the Server may prefer to choose a ‘safe’ outcome over a ‘risky’ one under pressure. This type of behaviour could result in a deviation towards a particular strategy in the data. The theoretical model presented to this point is not able to rationalise this possibility. Therefore, I expand the model to illustrate how players’ risk preferences could explain some of the observed behaviour.

The original model assumes that a tennis point is played as a one-shot game where the Server has only one service opportunity. In reality, if the Server misses their first serve then they have another opportunity to serve and begin the point. To account for this, I extend the model of the serve-return interaction. Explicitly, if the Server makes their first serve, then they play the usual game; however, if they miss their first serve, then they hit a second serve and win the point with some fixed probability $\pi_2$. Furthermore, assume that the Server has a probability of making their first serve given by $r_S$ that depends on the action chosen, $\{W, T\}$. This example can be illustrated by the following game matrix, where the cell entries

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60The optimal strategy for this second serve is to hit it safer than the first serve (Klaassen & Magnus, 2009). This typically results in a substantially slower second serve relative to the first serve. Indeed, the average men’s service speed at Grand Slams in 2009 was 184.1km/h for 1st serves compared to 150.4km/h for 2nd (Cross, 2014). Consequently, there is little need for the Receiver to attempt to anticipate the second serve; thus, it does not make sense to also model the second serve-return interaction as a simultaneous move game.
give the expected payoff for the first serve choices of both players:

\[
\begin{array}{c|cc}
\text{Server} & W & T \\
\hline
W & r_W \pi_{WW} + (1 - r_W)\pi_2 & r_W \pi_{WT} + (1 - r_W)\pi_2 \\
T & r_T \pi_{TW} + (1 - r_T)\pi_2 & r_T \pi_{TT} + (1 - r_T)\pi_2 \\
\end{array}
\]

For example, consider the case where the Server chooses to serve \(W\) on their first serve and the Receiver also anticipates a \(W\) first serve. This corresponds to the first cell in the game matrix above. The probability that the Server wins the point is \(\pi_{WW}\) if the first serve is made and \(\pi_2\) if it is missed. Since the probability of making a first serve is \(r_W\), then the expected probability that the Server wins the point for these choices is \(r_W \pi_{WW} + (1 - r_W)\pi_2\). The same logic applies for the entries in the cells corresponding to the other possible choices of the Server and the Receiver.

Note that, for this to be consistent with what is observed in professional tennis matches, the entries in this game matrix must still satisfy the assumptions outlined in Chapter 3. Furthermore, it is also natural to assume that players have a lower chance of winning the point on their second serve than their first serve, regardless of the choices of the Server and Receiver.\(^{61}\) These statements are equivalent to the following:

**Assumption.** The entries in the game matrix satisfy the following conditions:

- \(\pi_{WW} < \pi_{WT} < \pi_{TW}\)
- \(r_W(\pi_{WW} - \pi_2) < r_T(\pi_{TW} - \pi_2)\) and \(r_T(\pi_{TT} - \pi_2) < r_W(\pi_{WT} - \pi_2)\)
- \(\pi_2 < \pi_{WW}, \pi_{WT}, \pi_{TW}, \pi_{TT}\)

The first two points above ensure that this game has a unique mixed-strategy equilibrium given by Nash’s Theorem. The third additional point implies that a

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\(^{61}\)This assumption is backed by data from professional tennis matches. The average service winning probability of male tennis players on their second serve is 51.4% compared to 73.3% their first serve (Klaassen & Magnus, 2014, p.74).
first serve is preferred to a second serve for all possible choices of the two players. Furthermore, if the probability of making the first serve is different for each choice, then the following result arises:

**Proposition 10.** Given the Server makes their first serve, then the serve that has a higher chance of going in has a lower (expected) payoff in equilibrium.

(Proof in Appendix A.) The implication of this result is that, if the probability of making a first serve varies according to the choice of direction, then the Server’s choice of action is equivalent to a choice between two different lotteries with the same expected value. This change has no effect on risk-neutral players as the payoff of the second serve will be linearly incorporated into that of the first serve.\(^{62}\) Since risk-neutral players aim to maximise the probability that they win the point and both lotteries have the same expected value, neither is preferred. However, if players are not risk-neutral, then their risk characteristics may dictate their choices.

The following simple example can help to illustrate how risk preferences may influence the choices of the players. Assume that the Server has one ‘risky’ serve and one ‘safe’ serve. Letting \(W\) be the risky serve with \(r_W = 0.5\) and \(T\) be the safe serve with \(r_T = 1\), the game matrix above simplifies to:

\[
\begin{array}{c|cc}
\text{Receiver} & W & T \\
\hline
\text{Server} & & \\
W & 0.5\pi_{WW} + 0.5\pi_2 & 0.5\pi_{WT} + 0.5\pi_2 \\
T & \pi_{TW} & \pi_{TT} \\
\end{array}
\]

As mentioned, the Server in this model can be thought of as choosing between two lotteries – one with a risky strategy \((W)\) and one with a safe one \((T)\). The risky strategy has a better (expected) outcome than the safe option when the first serve is made. The trade-off for choosing the risky serve is that it has a significantly lower chance of being made in the first place.

\(^{62}\)This can be seen by defining \(a \equiv r_W\pi_{WW} + (1 - r_W)\pi_2, b \equiv r_W\pi_{WT} + (1 - r_W)\pi_2, c \equiv r_T\pi_{TW} + (1 - r_T)\pi_2\) and \(d \equiv r_T\pi_{TT} + (1 - r_T)\pi_2\) in the general game matrix above.
For example, let $\pi_{WW} = 0.7, \pi_{WT} = 0.9, \pi_{TW} = 0.79, \pi_{TT} = 0.69$ and $\pi_2 = 0.5$.

If players are risk-neutral, then the game matrix for the serve-return interaction (where the Receiver payoffs are included for illustration) is:

<table>
<thead>
<tr>
<th></th>
<th>W</th>
<th>T</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>W</strong></td>
<td>0.6, 0.4</td>
<td>0.7, 0.3</td>
</tr>
<tr>
<td><strong>T</strong></td>
<td>0.79, 0.21</td>
<td>0.69, 0.31</td>
</tr>
</tbody>
</table>

The equilibrium solution and the Server’s point-winning probability for this game are given by:

$$ (p^*, q^*) = (0.5, 0.05) \quad \text{and} \quad \pi^* = 0.695 $$

In this scenario, the equilibrium is a mixed-strategy equilibrium where the Server chooses to serve $W$ and $T$ each with probability 0.5. Furthermore, the Server will win the point with probability 0.695.

Now assume that the Server becomes risk-averse under pressure with utility function $u(x) = \sqrt{x}$, but the Receiver remains risk-neutral (i.e., they still only care whether they win or lose the point). The change in the Server’s utility function under pressure changes the game-matrix to:

<table>
<thead>
<tr>
<th></th>
<th>W</th>
<th>T</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>W</strong></td>
<td>0.772, 0.4</td>
<td>0.828, 0.3</td>
</tr>
<tr>
<td><strong>T</strong></td>
<td>0.889, 0.21</td>
<td>0.831, 0.31</td>
</tr>
</tbody>
</table>

The equilibrium solution and the Server’s point-winning probability for this game are now given by:

$$ (p^*, q^*) = (0, 0) \quad \text{and} \quad \pi^* = 0.69 $$
Therefore, due to the increased risk aversion of the Server, this game now has a pure-strategy equilibrium under pressure. In this new equilibrium the Server chooses the safe $T$ serve with probability 1 and wins the point with probability 0.69 – a decrease of 0.5%. This should not come as a surprise since the objective of the Server on pressure points is no longer to maximise their expected value, but also to minimise their exposure to risky outcomes. In contrast, the Receiver’s only objective is to win the point. As a consequence, the Receiver is able to exploit the Server’s increased risk aversion under pressure to lower the Server’s probability of winning the point.

While this is just a specific example, it does illustrate how it is possible for the Server’s behaviour and their outcomes to change under pressure depending on their risk characteristics. The remainder of this discussion formalises the above example to explain how it is possible to rationalise some of the key observations of this study using variable risk preferences.

In general, if the Server has a safe and a risky service strategy and their risk aversion increases under pressure, then the change in preferences will result in a change in the equilibrium properties. The following proposition provides the basis for determining the resulting equilibrium:

**Proposition 11.** In a mixed-strategy equilibrium, if the Server becomes more risk-averse, then they prefer the safe serve for the initial equilibrium choices.

(Proof in Appendix A.) If the Server becomes risk-averse under pressure, then this result implies that they will prefer the safer serve for the initial equilibrium choice of the Receiver. To keep the Server indifferent and restore the equilibrium the Receiver must anticipate the safer serve more frequently. This will lower the expected probability that the Server will win the point with the safe serve relative to the risky serve, counteracting the change in utility from their increased risk aversion. Moreover, the Receiver’s preferences have not changed and so the Server does not need to change their behaviour to keep the Receiver indifferent. The result is a new mixed-strategy equilibrium where the Server’s probability of winning the point is also unchanged.
However, if the change in the Server’s risk aversion is large enough then the Receiver must anticipate the safe serve with probability 1 to counteract the increase in utility from the Server’s increased risk aversion. At this point the Receiver is utilising a pure strategy and any additional increase in the Server’s risk aversion can no longer be counteracted by a change in the Receiver’s behaviour. Since the Receiver’s actions cannot change further, Proposition 11 states that any additional increase in risk aversion will cause the Server to prefer the safe serve over the risky serve. Therefore, if the change in risk preferences is large enough, the Server will choose the safe serve with probability 1 and, hence, also play a pure strategy. The result is a new pure-strategy equilibrium where the Server’s probability of winning the point is less than the initial equilibrium.

This intuition can be summarised in the following proposition:

**Proposition 12.** There exists some risk-averse utility function, $\tilde{u}$, such that any utility function with lower risk aversion results in a mixed-strategy equilibrium and any utility function with greater risk aversion results in a pure-strategy equilibrium. Moreover,

- In the mixed-strategy equilibrium the Server’s behaviour does not change, the Receiver anticipates the safer serve more frequently and the probability the Server wins the point is unchanged.

- In the pure-strategy equilibrium the Server chooses the safer serve with probability 1, the Receiver anticipates the safer serve with probability 1 and the probability the Server wins the point is the less than the initial equilibrium.

(Proof in Appendix A.) Importantly, this result shows that it is possible to observe deviations in the Server’s behaviour (R-1) and deviations in outcome (R-2), if the change in the Server’s risk aversion is large enough that it results in a pure-strategy equilibrium.

\[63\text{This scenario will occur when the change in the Server’s risk aversion is large enough that it violates the assumptions for a mixed-strategy equilibrium. In particular, the } r_s(u(\pi_{ss}) - u(\pi_2)) < r_r(u(\pi_{rs}) - u(\pi_2)) \text{ can be violated if the change in risk aversion is large enough.}\]
Chapter 9

Conclusion

In this thesis I examine the effect of pressure on the decision-making of a sample of highly-trained individuals – professional tennis players. First, I provide an analysis of decision-making without pressure and find that the probability of winning a point is the same regardless of the choice of service direction. Importantly, this observation is indicative of optimal strategic decision-making within a game theory model of the serve-return interaction in tennis. Despite this, I find evidence that pressure has an impact on a significant proportion of players’ service strategies implying that players may not be behaving optimally in these specific instances. I also find that a significant proportion of players are more likely to lose their service points at pressure points. There is some evidence to suggest that the players perform worse because of a change in their behaviour, however this link is not conclusive.

I have provided a comprehensive analysis of tennis professionals’ service choices that incorporates pressure into a simple model of strategic decision-making to underscore the importance of this effect in understanding human behaviour in competitive environments. From a broader perspective, this research confirms the notion held in the psychological literature that pressure can impair complex decision-making tasks. This has substantial implications for the development of economic models, where this factor is often absent or overlooked.

The implications of this research with respect to testing models of decision-making are twofold. The first is that researchers should explicitly model pressure
as a key variable that can influence behaviour. This has been done in studies that examine behaviour under time constraints. However, in cases where pressure can arise from substantial variations in the financial reward/cost of the same (repeated) decision problem, this rarely occurs. The second is that care should be taken when generalising results even when agents appear to be acting close to theoretically optimal. While aggregate behaviour may conform to equilibrium predictions of theoretical models, the presence of additional factors, such as pressure, may cause significant departures from individuals’ aggregate behaviour in specific situations.

The primary limitation of this research is that, apart from the baseline analysis, this thesis studies the behaviour of the Server and focuses predominantly on rationalising the observed behaviour from their perspective. However, the serve-return interaction is a strategic game between both the Server and the Receiver, and so the Receiver may also be responding to pressure. I argue in the body of this thesis that the Server’s side of the game is more important for examining the effect of pressure on decision-making; however, it is important to recognise that an analysis of only the Server’s response may not provide the full picture.

There is also scope for future research in this area. I find that pressure affects decision-making but not necessarily the outcomes from this decision. I propose a few explanations to rationalise this observation, but in the absence of additional data I am unable to test these hypotheses rigorously. For example, these issues could be further investigated in tennis matches with access to the Hawk-Eye technology, which tracks the exact trajectory of the ball of every single shot and is used in the majority of professional matches. I hypothesise that players’ risk characteristics may affect their service decisions. Since Hawk-Eye provides data on the exact position that the ball lands for each serve, it would be possible to examine whether players take more risk on their serve by seeing how close the ball landed to line; that is, the closer the ball to the extremity of the court, the more risky the serve. This could be used to supplement the analysis in this thesis to determine the extent to which risk plays a role in strategic decision-making under pressure. The Hawk-Eye
technology might also be able to provide data on the Receiver by tracking their movement across the court. Such information may provide a more comprehensive analysis of the strategic component since it would be possible to examine both the Server’s and Receiver’s responses to pressure.

An obvious extension of this research is to investigate this issue in the laboratory. By examining if and how behaviour responds to varying agents’ payoffs in multiple repetitions of the same game, it may be possible to replicate a scenario similar to those observed in this research. The experimenter may also be able to investigate the underlying causes for why pressure may influence behaviour and why this may (not) be exploited. These are difficult to identify in my environment as it is hard to observe one agent’s (the Receiver’s) choices. Regardless, I believe the results of this thesis are significant since I examine a sample of individuals that are highly trained and incentivised to behave as optimally as possible. Therefore, the behaviour of these agents should represent the highest level of strategic decision-making possible.

In conclusion, the results of the research suggest that pressure plays an important role in decision-making of experienced agents in competitive environments. Since the most proficient individuals are susceptible to pressure, it may play an even greater role in individuals with less experience in coping with these situations. If true, predictions of behaviour in high-stakes environments without consideration of pressure are likely to be distorted or incorrect. As a consequence, it is vital to develop economic models that incorporate pressure to better understand and rationalise human behaviour in the many scenarios that involve large payoffs.
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Glossary

Ace: A serve that lands in and is not touched by the Receiver.

AD court: The left side of the court for each player where the advantage point is played.

Advantage: The point score after one player has won the deuce point.

Backhand: A shot that is hit by swinging the back of the hand forwards. For a right-hander this is on the left side of their body and for a left-hander this is on their right side.

Break (of serve): A game won by the Receiver.

Break point: A point where the Receiver needs one more point to win the game. Also results in a break of serve if won by the Receiver.

Body/B serve: A serve directed down the middle of the service box to the body of the Receiver.

Deuce: The score of 40–40 and repeated after a lost advantage point.

Deuce court: The right side of the court for each player where the deuce point is played.

Double fault: Two service faults in a row. Results in the Server losing the point.

Doubles (match): A match involving two pairs of players.

Fault: A serve that does not land in the designated service area.
**Final set:** The third (fifth) set in a best-of-three (best-of-five) set match.

**Forehand:** A shot that is hit by swinging the front of the hand forwards. For a right-hander this is on the right side of their body and for a left-hander this is on their left side.

**Game point:** A point where the Server needs one more point to win the game.

**Grand Slam:** One of the four major tournaments each year: the Australian Open, the French Open, Wimbledon and the US Open.

**Hawk-Eye:** An electronic ball tracking system that is used for umpiring purposes.

**Match point:** A point where one player only needs one more point to win the match.

**Masters:** A tier of tennis tournaments that sit just below Grand Slams in terms of prestige and prize money.

**Overplay:** The act of anticipating and committing to a direction of the opponent’s serve before seeing where it is going.

**Rally:** A sequence of shots that constitute a point.

**(ATP/WTA) Ranking:** An ordering of (male/female) players by ranking points that determines the seeds of every tournament. The best player in the world has a rank of 1.

**(ATP/WTA) Ranking points:** A reward for winning a professional (men’s/women’s) tennis match.

**Receiver/Returner:** The player who is being served to.

**Seed:** A player who has a prearranged position in the tournament draw as a result of having a high number of ranking points.

**Service box:** A region on the court where the serve must land.
Serve/Service: A special shot that is used to start every point in tennis by the Server.

Server: The player who is serving.

Service game: With respect to one player, the complete game that they serve.

Set point: A point where one player only needs one more point to win the set.

Singles (match): A match involving two players.

Tee: A position near the centre of the court where the lines that trace out the boundaries of the service box create a ‘T’ shape.

Tee/T serve: A serve that is directed to the ‘T’ position on the court.

Tie-break/Tie-breaker: A special game played at 6–6 in most sets to decide the winner of the set.

Unseeded player: Any player that is not given a seed in the tournament.

Wide: The extreme left and extreme right edges of the court. Also, the direction of a serve that is directed to this position on the court.

Wide/W serve: A serve that is directed to the Wide position on the court.
Appendix A

Proofs of Propositions

Additional Proposition A.1

Proposition A.1. For a function $u$ in the set of $C^2$ functions, then $\forall a < b < c$, $\exists \xi_{ab} \in (a, b)$ and $\xi_{ac} \in (a, c)$ such that $u'(\xi_{ab}) = \frac{u(b) - u(a)}{b - a}$ and $u'(\xi_{ac}) = \frac{u(c) - u(a)}{c - a}$. Moreover, if $u''(\cdot) < 0$ then $\xi_{ab} < \xi_{ac}$.

Proof. The first part is a direct result of the mean value theorem (m.v.t.) applied to the function $u$ over the intervals $(a, b)$ and $(a, c)$, respectively. To prove the second part note that, by the m.v.t., $\exists \xi_{bc} \in (b, c)$ such that $u'(\xi_{bc}) = \frac{u(c) - u(b)}{c - b}$. Therefore,

$$u'(\xi_{ac}) = \frac{u(c) - u(a)}{c - a} = \frac{1}{c - a} \left[ (c - b) \frac{u(c) - u(b)}{c - b} + (b - a) \frac{u(b) - u(a)}{b - a} \right] = \frac{c - b}{c - a} u'(\xi_{bc}) + \frac{b - a}{c - a} u'(\xi_{ab})$$

where $\xi_{ab} < \xi_{bc}$. Since $u''(\cdot) < 0 \Rightarrow u'(\xi_{ab}) > u'(\xi_{bc})$, giving:

$$u'(\xi_{ac}) = \frac{c - b}{c - a} u'(\xi_{bc}) + \frac{b - a}{c - a} u'(\xi_{ab}) < \frac{c - b}{c - a} u'(\xi_{ab}) + \frac{b - a}{c - a} u'(\xi_{ab}) = u'(\xi_{ab})$$

Finally, since $u''(\cdot) < 0$, this implies $\xi_{ab} < \xi_{ac}$. \qed
Appendix: Proofs

Proof of Proposition 1

**Proposition.** For each point in tennis:

1. Players strictly prefer winning to losing.

2. The equilibrium strategies are homogeneous of degree zero in the payoffs.

**Proof of 1.** For risk-neutral players, players prefer winning the point if increases their probability of winning the match, that is $I(\theta) > 0$. By induction, I prove that players have a strictly higher probability of winning the game if they win the point. The proof can be generalised to the set- and match-level following a similar procedure.

Starting at the end of the game, the final point occurs at either a game point or a break point.

- **Game point:**

  \[
  \Pr[W_G|W(\theta_t)] = 1
  \]

  \[
  \Pr[W_G|L(\theta_t)] = \pi \Pr[W_G|W(\theta_{t+1})] + (1 - \pi) \Pr[W_G|L(\theta_{t+1})] < 1
  \]

  since $\Pr[W_G|W(\theta_{t+1})] \leq 1$ and $\Pr[W_G|L(\theta_{t+1})] < 1$. The second condition stems from the fact that $\pi < 1$ so the Server has some non-zero chance to lose the game if the Receiver won all the remaining points. Therefore, the following condition holds

  \[
  I_G(\theta_t) = \Pr[W_G|W(\theta_t)] - \Pr[W_G|L(\theta_t)] > 0
  \]

- **Break point:**

  \[
  \Pr[W_G|W(\theta_t)] = \pi \Pr[W_G|W(\theta_{t+1})] + (1 - \pi) \Pr[W_G|L(\theta_{t+1})] > 0
  \]

  \[
  \Pr[W_G|L(\theta_t)] = 0
  \]

  since $0 < \Pr[W_G|W(\theta_{t+1})]$ and $0 \leq \Pr[W_G|L(\theta_{t+1})]$. The first condition stems from the fact that $\pi > 0$ so the Server has some non-zero chance to win the
game if they won all the remaining points. Therefore, again the following condition holds

\[ I_G(\theta_t) = \Pr[W_G|W(\theta_t)] - \Pr[W_G|L(\theta_t)] > 0 \]

Assume that \( \Pr[W_G|W(\theta_{t+1})] - \Pr[W_G|L(\theta_{t+1})] > 0 \) holds at all possible scores. Then the following arises:

\[ I_G(\theta_t) = \Pr[W_G|W(\theta_t)] - \Pr[W_G|L(\theta_t)] = \pi \Pr[W_G|W(\theta_t), W(\theta_{t+1})] + (1 - \pi) \Pr[W_G|L(\theta_t), L(\theta_{t+1})] \]

where \( I_G(\theta_{t+1}^W) \) and \( I_G(\theta_{t+1}^L) \) are the differences in the probability of winning the game between winning and losing point \( t + 1 \) given the player won and lost point \( t \), respectively. The second equality follows from the law of total probability. The third follows from the fact that the points are independent so the probability of winning the game is the same regardless of whether the player wins the next point and loses the following one or they lose the next point and win the following one (as the score in both scenarios is the same); that is, \( \Pr[W_G|W(\theta_t), L(\theta_{t+1})] = \Pr[W_G|L(\theta_t), W(\theta_{t+1})] \). Finally, the last line follows from the inductive hypothesis, which holds regardless of the score. \( \square \)

**Proof of 2.** For simplicity let \( a = \pi_{WW}, b = \pi_{WT}, c = \pi_{TW}, d = \pi_{TT} \). The mixed strategy equilibrium for the Server, \( p^* \), and the Receiver \( q^* \), is given by:

\[ (p^*, q^*) = \left( \frac{d - c}{a - b - c + d}, \frac{d - b}{a - b - c + d} \right) \]

Multiplying all the payoffs by a non-zero constant \( t \), gives the Server’s equilibrium
strategy:

\[ p^*(a, b, c, d) = \frac{td - tc}{ta - tb - tc + td} = \frac{d - c}{a - b - c + d} = p^*(a, b, c, d) \]

and the Receiver’s equilibrium strategy:

\[ q^*(a, b, c, d) = \frac{td - tb}{ta - tb - tc + td} = \frac{d - b}{a - b - c + d} = q^*(a, b, c, d) \]

Thus, multiplying the payoffs by a non-zero constant does not change the equilibrium strategies of the two players. \(\square\)

**Proof of Proposition 2**

**Proposition.** If \(X\) and \(Y\) are uncorrelated Bernoulli random variables, then they are also independent.

**Proof.** Let \(X\) and \(Y\) be uncorrelated Bernoulli random variables with parameters \(p\) and \(q\), respectively. The joint distribution is given by:

\[ f_{X,Y}(x, y) = \begin{cases} 
  p_{00}, & \text{if } X = 0, Y = 0 \\
  p_{01}, & \text{if } X = 0, Y = 1 \\
  p_{10}, & \text{if } X = 1, Y = 0 \\
  p_{11}, & \text{if } X = 1, Y = 1 
\end{cases} \]

With the following expectations:

\[ E[XY] = p_{11}, \quad E[X(1 - Y)] = p_{10}, \quad E[(1 - X)Y] = p_{01}, \quad E[(1 - X)(1 - Y)] = p_{00} \]

Since \(X\) and \(Y\) are uncorrelated \(E[XY] = E[X]E[Y]\), this gives:

\[ E[XY] = pq, \quad E[X(1 - Y)] = p(1 - q), \quad E[(1 - X)Y] = (1 - p)q, \quad E[(1 - X)(1 - Y)] = (1 - p)(1 - q) \]

Equating these to the above expressions gives:

\[ p_{11} = pq, \quad p_{10} = p(1 - q), \quad p_{01} = (1 - p)q, \quad p_{00} = (1 - p)(1 - q) \]
Therefore, the joint density is:

\[ f_{X,Y}(x,y) = \begin{cases} 
  pq, & \text{if } X = 0, Y = 0 \\
  p(1 - q), & \text{if } X = 0, Y = 1 \\
  (1 - p)q, & \text{if } X = 1, Y = 0 \\
  (1 - p)(1 - q), & \text{if } X = 1, Y = 1 
\end{cases} = f_X(x)f_Y(y) \]

hence the two distributions are independent.

**Proof of Proposition 3**

**Proposition.** \( t^i \) is distributed \( U[0,1] \) under the null hypothesis, \( H_0 : \pi^i_W = \pi^i_T \).

**Proof.** Under the null hypothesis

\[ \pi^i_W = \pi^i_T \Rightarrow \frac{n_{WS}}{n_W} = \frac{n_{TS}}{n_T} \Rightarrow \frac{n_{WS}}{n_W} = \frac{n_S - n_{WS}}{n_T} \Rightarrow n_{WS} = \frac{n_{WS}n_S}{n_W + n_T} \]

Since this is conditioned on \( n_S, n_W \) and \( n_T \) this implies that \( n_{WS} \) is known. That is,

\[ f(n_{WS}; n_S, n_W, n_T) = \begin{cases} 
  1 & \text{if } n_{WS} = \frac{n_{WS}n_S}{n_W + n_T} \\
  0 & \text{otherwise} 
\end{cases} \]

Thus, \( F(n_{WS} - 1; n_S, n_W, n_T) = 0 \) and \( F(n_{WS}; n_S, n_W, n_T) = 1 \) and \( t^i \sim U[0,1] \).

**Proof of Proposition 4**

**Proposition.** Under the null hypothesis, the number of significant coefficients from \( n \) independent regressions follows a binomial distribution, \( B(n, p) \), where \( p \) is the level of significance level chosen for the regression coefficients.

**Proof.** The presence of a significant coefficient in each regression is equivalent to a success in a Bernoulli trial with probability equal to the significance level because, by definition, the significance level is the likelihood of randomly obtaining the observed coefficient under the null hypothesis. Furthermore, since the regressions are
independent, the total number of significant coefficients is the sum of independent
Bernoulli trials. This is precisely a binomial distribution, $B(n, p)$, with $n$ equal to
the number of regressions and $p$ equal to the significance level.

Proof of Proposition 5

Proposition. The importance of a point is the product of the importance of each
point to a game, each game to a set and each set to a match

$$I(\theta) = PiG(\theta) \cdot GiS(\theta) \cdot SiM(\theta)$$

Proof. Suppressing the dependence on $\theta$ and using the notation above, gives

$$I(\theta) = \Pr[W^*|WP] - \Pr[W^*|LP]$$

$$= \Pr[W^*|WP, WS] \Pr[WS|WP] + \Pr[W^*|WP, LS] \Pr[LS|WP]$$

$$- \left\{ \Pr[W^*|LP, WS] \Pr[WS|LP] + \Pr[W^*|LP, LS] \Pr[LS|LP] \right\}$$

$$= \Pr[W^*|WS]\{\Pr[WS|WP] - \Pr[WS|LP]\}$$

$$+ \Pr[W^*|LS]\{\Pr[LS|WP] - \Pr[LS|LP]\}$$

$$= \Pr[W^*|WS]\{\Pr[WS|WP] - \Pr[WS|LP]\}$$

$$- \Pr[W^*|LS]\{\Pr[WS|WP] - \Pr[WS|LP]\}$$

$$= (\Pr[W^*|WS] - \Pr[W^*|LS]) (\Pr[WS|WP] - \Pr[WS|LP])$$

$$= SiM \cdot (\Pr[WS|WP] - \Pr[WS|LP])$$

$$= SiM \cdot (\Pr[WS|WP, WG]\Pr[WG|WP] + \Pr[WS|WP, LG]\Pr[LG|WP]$$

$$- \left\{ \Pr[WS|LP, WG]\Pr[WG|LP] + \Pr[WS|LP, LG]\Pr[LG|LP] \right\})$$

$$= SiM \cdot (\Pr[WS|WG]\Pr[WG|WP] + \Pr[WS|LG]\Pr[LG|WP]$$

$$- \left\{ \Pr[WS|WG]\Pr[WG|LP] + \Pr[WS|LG]\Pr[LG|LP] \right\})$$

$$= SiM \cdot (\Pr[WS|WG]\{\Pr[WG|WP] - \Pr[WG|LP]\}$$
\[ + \Pr[W_S|L_G]|\{\Pr[L_G|W_P] - \Pr[L_G|L_P]\} \]
\[ = SiM \cdot (\Pr[W_S|W_G]|\{\Pr[W_G|W_P] - \Pr[W_G|L_P]\}) \]
\[ \quad - \Pr[W_S|L_G]|\{\Pr[W_G|W_P] - \Pr[W_G|L_P]\} \]
\[ = SiM \cdot (\Pr[W_S|W_G] - \Pr[W_S|L_G]) (\Pr[W_G|W_P] - \Pr[W_G|L_P]) \]
\[ = SiM \cdot GiS \cdot PiG \]

The first equality follows by definition, the second follows from the law of total probability, the third follows from independence of each point, and the seventh follows from the definition in Chapter 6. The remainder of the proof follows similarly. \(\square\)

**Proof of Proposition 6**

**Proposition.** Let \(\Omega_\theta\) be the set of all possible (countably infinite) score vectors, then the score of a tennis match, \(\theta_t \in \Omega_\theta\), is a Markov process.

**Proof.** Let \(\pi_{i,t}\) be the probability of Player \(i\) winning point \(t\) on their serve. There are two possible states for \(\theta_{t+1}\), corresponding to the state where the Server wins or loses the point. Define these states as \(\theta^W\) and \(\theta^L\) respectively.

\[
\Pr(\Theta_{t+1} = \theta^W|\Theta_1 = \theta_1, \ldots, \Theta_t = \theta_t) = \pi_{i,t}|\Theta_t = \theta_t = \pi_{i,t}\]
\[
= \Pr(\Theta_{t+1} = \theta^W|\Theta_t = \theta_t)
\]

where the second line follows from the independence of winning probabilities. Also,

\[
\Pr(\Theta_{t+1} = \theta^L|\Theta_1 = \theta_1, \ldots, \Theta_t = \theta_t) = 1 - \Pr(\Theta_{t+1} = \theta^W|\Theta_1 = \theta_1, \ldots, \Theta_t = \theta_t)
\]
\[
= 1 - \Pr(\Theta_{t+1} = \theta^W|\Theta_t = \theta_t)
\]
\[
= \Pr(\Theta_{t+1} = \theta^L|\Theta_t = \theta_t)
\]

Thus, the random variables \(\Theta_t\) satisfy the Markov property. Therefore, the score follows a Markov process. \(\square\)
Proof of Proposition 7

**Proposition.** The phi coefficient of the association between the direction variable and its lag, $\phi$, is bounded by

$$-\frac{\min\{p, 1-p\}}{\max\{p, 1-p\}} \leq \phi \leq 1$$

**Proof.** The association between the service direction and its lag

$$\phi = \frac{p_{WW}p_{TT} - p_c^2}{p(1-p)}$$

where $p_{ij}$ is the observed proportion of serving direction $i \in \{W, T\}$ and point $t$ and $j \in \{W, T\}$ at point $t-1$, $p_c$ is the proportion of serving to the opposite direction to the previous serve, and $p$ is the proportion of $W$ serves. Furthermore, $0 \leq p_{WW} \leq p$, $0 \leq p_{TT} \leq 1 - p$ and $0 \leq p_c \leq \min\{p, 1-p\}$. Therefore:

$$0 - \frac{\min\{p, 1-p\}}{p(1-p)} \leq \phi \leq \frac{p(1-p) - 0}{p(1-p)}$$

which simplifies to:

$$-\frac{\min\{p, 1-p\}}{\max\{p, 1-p\}} \leq \phi \leq 1$$

$\Box$

Proof of Proposition 8

**Proposition.** The $p$-values of a continuous distribution are distributed $U[0, 1]$ under the null hypothesis.

**Proof.** First let $F = F(X)$ be the random variable given by the distribution function of the random variable $X$ that follows a given continuous distribution, then for all $0 \leq f \leq 1$:

$$\Pr[F \leq f] = \Pr[F(X) \leq f]$$

$$= \Pr[F^{-1}(F(X)) \leq F^{-1}(f)]$$

$$= \Pr[X \leq F^{-1}(f)]$$
which is precisely the uniform CDF and so \( F \) is distributed \( U[0, 1] \). Next consider the random variable \( X \). The two-sided \( p \)-value for an observation \( x \) is defined as:

\[
p-value = 2 \min \{ \Pr[X \leq x|H_0], \Pr[X \geq x|H_0] \}
\]

where \( H_0 \) is the null hypothesis. Let \( P \) be the distribution of the \( p \)-values, then:

\[
\Pr[P \leq p] = \Pr[2 \min \{ \Pr[X \leq x|H_0], \Pr[X \geq x|H_0] \} \leq p] = \Pr \left[ \min \{ \Pr[X \leq x|H_0], \Pr[X \geq x|H_0] \} \leq \frac{p}{2} \right]
\]

Let \( A \) be the event that \( \Pr[X \leq x|H_0] \leq \Pr[X \geq x|H_0] \) and its compliment \( \bar{A} \) be the event \( \Pr[X \leq x|H_0] > \Pr[X \geq x|H_0] \), then by the law of total probability:

\[
\Pr[P \leq p] = \Pr[P \leq p \cap A] + \Pr[P \leq p \cap \bar{A}]
\]

\[
= \Pr \left[ \min \{ \Pr[X \leq x|H_0], \Pr[X \geq x|H_0] \} \leq \frac{p}{2} \cap A \right] + \Pr \left[ \min \{ \Pr[X \leq x|H_0], \Pr[X \geq x|H_0] \} \leq \frac{p}{2} \cap \bar{A} \right]
\]

\[
= \Pr \left[ \Pr[X \leq x|H_0] \leq \frac{p}{2} \right] + \Pr \left[ \Pr[X \geq x|H_0] \leq \frac{p}{2} \right]
\]

since \( \min \{ \Pr[X \leq x|H_0], \Pr[X \geq x|H_0] \} = \Pr[X \leq x|H_0] \) under the event \( A \), likewise \( \min \{ \Pr[X \leq x|H_0], \Pr[X \geq x|H_0] \} = \Pr[X \geq x|H_0] \) under the event \( \bar{A} \). Finally, let \( F \) be the distribution of \( X \) under the null hypothesis, then:

\[
\Pr[P \leq p] = \Pr \left[ F \leq \frac{p}{2} \right] + \Pr \left[ 1 - F \leq \frac{p}{2} \right]
\]

\[
= \Pr \left[ F \leq \frac{p}{2} \right] + \Pr \left[ F \geq 1 - \frac{p}{2} \right]
\]

\[
= \Pr \left[ F \leq \frac{p}{2} \right] + \left( 1 - \Pr \left[ F \leq 1 - \frac{p}{2} \right] \right)
\]

\[
= \frac{p}{2} + \left[ 1 - \left( 1 - \frac{p}{2} \right) \right]
\]

\[
= p
\]

when the penultimate equality follows from the first result above. Once again this is the CDF of the uniform distribution, therefore \( P \sim U[0, 1] \). \( \square \)
Proof of Proposition 9

**Proposition.** Let $\hat{\alpha} = \alpha + \epsilon_\alpha$ and $\hat{\beta} = \beta + \epsilon_\beta$ be the estimates of the true effects $\alpha$ and $\beta$ with estimation errors $\epsilon_\alpha$ and $\epsilon_\beta$, respectively. Then the true correlation between the effects, $\rho(\alpha, \beta)$, is:

$$\rho(\alpha, \beta) = \rho(\hat{\alpha}, \hat{\beta}) \sqrt{\frac{\text{Var}[\hat{\alpha}] \text{Var}[\hat{\beta}]}{\text{Var}[\alpha] \text{Var}[\beta]}} > \rho(\hat{\alpha}, \hat{\beta})$$

where $\rho(\hat{\alpha}, \hat{\beta})$ is the correlation between the estimated effects.

**Proof.** Assuming that the estimation errors are uncorrelated with the true effects and each other, then:

$$\rho(\hat{\alpha}, \hat{\beta}) = \frac{\text{Cov}[\hat{\alpha}, \hat{\beta}]}{\sqrt{\text{Var}[\hat{\alpha}] \text{Var}[\hat{\beta}]}}$$

$$= \frac{\text{Cov}[\alpha, \beta]}{\sqrt{\text{Var}[\alpha] \text{Var}[\beta]}}$$

$$= \frac{\text{Cov}[\alpha, \beta]}{\sqrt{\text{Var}[\alpha] \text{Var}[\beta]}} \sqrt{\frac{\text{Var}[\alpha] \text{Var}[\beta]}{\text{Var}[\hat{\alpha}] \text{Var}[\hat{\beta}]}}$$

$$= \rho(\alpha, \beta) \sqrt{\frac{\text{Var}[\alpha] \text{Var}[\beta]}{\text{Var}[\alpha] \text{Var}[\beta]}}$$

where the second equality comes from the assumption that $\text{Cov}[\alpha, \epsilon_\beta] = \text{Cov}[\beta, \epsilon_\alpha] = \text{Cov}[\epsilon_\alpha, \epsilon_\beta] = 0$, so that $\text{Cov}[\hat{\alpha}, \hat{\beta}] = \text{Cov}[(\alpha + \epsilon_\alpha), (\beta + \epsilon_\beta)] = \text{Cov}[\alpha, \beta]$. Rearranging gives:

$$\rho(\alpha, \beta) = \rho(\hat{\alpha}, \hat{\beta}) \sqrt{\frac{\text{Var}[\hat{\alpha}] \text{Var}[\hat{\beta}]}{\text{Var}[\alpha] \text{Var}[\beta]}}$$

Moreover,

$$\sqrt{\frac{\text{Var}[\hat{\alpha}] \text{Var}[\hat{\beta}]}{\text{Var}[\alpha] \text{Var}[\beta]}} = \sqrt{\frac{\text{Var}[\alpha + \epsilon_\alpha] \text{Var}[\beta + \epsilon_\beta]}{\text{Var}[\alpha] \text{Var}[\beta]}} > 1$$

Therefore,

$$\rho(\alpha, \beta) > \rho(\hat{\alpha}, \hat{\beta})$$

\qed
Proof of Proposition 10

**Proposition.** Given the Server makes their first serve, then the serve that has a higher chance of going in has a lower (expected) payoff in equilibrium.

**Proof.** Let \(0 < q^*_s < 1\) be the equilibrium probability that the Receiver anticipates the safe serve \(s\), then the equilibrium payoff to the Server for choosing the safe serve \(s\) is:

\[
E[s] = q^*_s [r_s \pi_{ss} + (1 - r_s) \pi_2] + (1 - q^*_s) [r_s \pi_{sr} + (1 - r_s) \pi_2]
\]

\[
= r_s [q^*_s \pi_{ss} + (1 - q^*_s) \pi_{sr} - \pi_2] + \pi_2
\]

Similarly, the equilibrium payoff to the Server for choosing the risky serve \(r\) is:

\[
E[r] = q^*_r [r_r \pi_{rs} + (1 - r_r) \pi_2] + (1 - q^*_r) [r_r \pi_{rr} + (1 - r_r) \pi_2]
\]

\[
= r_r [q^*_r \pi_{rs} + (1 - q^*_r) \pi_{rr} - \pi_2] + \pi_2
\]

Due to the indifferent property of the equilibrium these expected payoffs are equal, that is, \(E[s] = E[r]\). Therefore, in equilibrium:

\[
r_s [q^*_s \pi_{ss} + (1 - q^*_s) \pi_{sr} - \pi_2] + \pi_2 = r_r [q^*_r \pi_{rs} + (1 - q^*_r) \pi_{rr} - \pi_2] + \pi_2
\]

\[
\Rightarrow r_s [q^*_s \pi_{ss} + (1 - q^*_s) \pi_{sr} - \pi_2] = r_r [q^*_r \pi_{rs} + (1 - q^*_r) \pi_{rr} - \pi_2]
\]

\[
\Rightarrow \frac{q^*_s \pi_{ss} + (1 - q^*_s) \pi_{sr} - \pi_2}{q^*_r \pi_{rs} + (1 - q^*_r) \pi_{rr} - \pi_2} = \frac{r_r}{r_s}
\]

Since \(r_r < r_s\) then the above equilibrium condition gives:

\[
P = \frac{q^* \pi_{ss} + (1 - q^*) \pi_{sr} - \pi_2}{q^* \pi_{rs} + (1 - q^*) \pi_{rr} - \pi_2} < 1
\]

\[
\Rightarrow q^* \pi_{ss} + (1 - q^*) \pi_{sr} < q^* \pi_{rs} + (1 - q^*) \pi_{rr}
\]

\[
\Rightarrow E[\text{Safe}|1^{st} \text{ serve made}] < E[\text{Risky}|1^{st} \text{ serve made}]
\]

since \(\pi_2 < \pi_{ss}, \pi_{sr}\) and where the last line follows by definition. \(\Box\)
Proof of Proposition 11

**Proposition.** In a mixed-strategy equilibrium, if the Server becomes more risk-averse, then they prefer the safe serve for the initial equilibrium choices.

**Proof.** Suppose that the Server has an initial utility function $u(\cdot)$ that satisfies the traditional properties for a utility function, that is, it is continuous and differentiable with $u'(\cdot) > 0$. The expected utility for the two possible service choices are:

$$u_s = r_s u(\pi_{1,s}) + (1 - r_s) u(\pi_2) \quad \text{and} \quad u_r = r_r u(\pi_{1,r}) + (1 - r_r) u(\pi_2)$$

where $\pi_{1,s}$ and $\pi_{1,r}$ are the probabilities of winning the point given the first serve goes in for the safe and the risky serve, respectively. Due to the indifference property of the equilibrium, the two choices give the same utility, that is, $u_s = u_r$. Therefore:

$$\gamma \equiv r_W [u(\pi_{1,W}) - u(\pi_2)] = r_T [u(\pi_{1,T}) - u(\pi_2)] > 0$$

Now suppose the risk preferences of the Server change to the utility function $\tilde{u}(\cdot)$, which is a transformation given by $\hat{u}$ of the initial utility function $u(\cdot)$, that is, $\tilde{u}(\cdot) = \hat{u}(u(\cdot))$. Given no change in the Receiver’s behaviour, $\pi_{1,s}$ and $\pi_{1,r}$ remain the same and the difference in the utility between the two options becomes:

$$\tilde{u}_s - \tilde{u}_r = r_s [\hat{u}(u(\pi_{1,s})) - \hat{u}(u(\pi_2))] - r_r [\hat{u}(u(\pi_{1,r})) - \hat{u}(u(\pi_2))]$$

From the mean value theorem $\exists \xi_s \in (u(\pi_2), u(\pi_{1,s}))$ and $\xi_r \in (u(\pi_2), u(\pi_{1,r}))$, such that $\hat{u}'(\xi_s) = \frac{\hat{u}(u(\pi_{1,s})) - \hat{u}(u(\pi_2))}{u(\pi_{1,s}) - u(\pi_2)}$ and $\hat{u}'(\xi_r) = \frac{\hat{u}(u(\pi_{1,r})) - \hat{u}(u(\pi_2))}{u(\pi_{1,r}) - u(\pi_2)}$. The difference in the utility between the two options becomes:

$$\tilde{u}_s - \tilde{u}_r = \gamma [\hat{u}'(\xi_s) - \hat{u}'(\xi_r)]$$

Proposition 10 implies that $\pi_{1,s} < \pi_{1,r}$ and Proposition A.1 gives $\xi_s < \xi_r$. If the utility transformation is risk-averse, $\hat{u}''(\cdot) < 0$, then $\xi_s < \xi_r \Rightarrow \hat{u}'(\xi_s) > \hat{u}'(\xi_r)$ and:

$$\tilde{u}_s > \tilde{u}_r$$

and so the Server prefers the safe serve. \qed
Proof of Proposition 12

**Proposition.** There exists some risk-averse utility function, $\bar{u}$, such that any utility function with lower risk aversion results in a mixed-strategy equilibrium and any utility function with greater risk aversion results in a pure-strategy equilibrium.

Moreover,

- In the mixed-equilibrium the Server’s behaviour is unchanged, the Receiver’s anticipates the safer serve more frequently and the probability that the Server wins the point is unchanged.

- In the pure-strategy equilibrium the Server’s chooses the safer serve with probability 1, the Receiver’s the safer serve with probability 1 and the probability that the Server wins the point is the less than the initial equilibrium.

**Proof.** To prove the first part, consider the expressions:

\[
\psi_{ss} := r_s(u(\pi_{ss}) - u(\pi_2)) \quad \psi_{sr} := r_s(u(\pi_{sr}) - u(\pi_2)) \\
\psi_{rs} := r_r(u(\pi_{rs}) - u(\pi_2)) \quad \psi_{rr} := r_r(u(\pi_{rr}) - u(\pi_2))
\]

Let $\psi_{SR}$ be the specific expression of the choices for the Server, $S \in \{W,T\} = \{s,r\}$, and the Receiver, $R \in \{W,T\} = \{s,r\}$, represented above, then:

\[
\psi_{SR} \equiv r_s(u(\pi_{SR}) - u(\pi_2)) = r_s \left( \frac{u(\pi_{SR}) - u(\pi_2)}{\pi_{SR} - \pi_2} \right)(\pi_{SR} - \pi_2)
\]

By the mean value theorem (m.v.t.) $\exists \xi_{SR} \in (\pi_2, \pi_{SR})$ such that $u'(\xi_{SR}) = \frac{u(\pi_{SR}) - u(\pi_2)}{\pi_{SR} - \pi_2}$.

This gives:

\[
\psi_{SR} = r_s u'(\xi_{SR})(\pi_{SR} - \pi_2) \quad \text{(A.1)}
\]

Next, for a pure-strategy equilibrium to exist it must be that either

\[
\psi_{ss} > \psi_{rs} \quad \text{and} \quad \psi_{sr} > \psi_{rr} \quad \text{or} \quad \psi_{ss} < \psi_{rs} \quad \text{and} \quad \psi_{sr} < \psi_{rr}
\]

Moreover, $\psi_{ss} < \psi_{sr}$ and $\psi_{rr} < \psi_{rs}$ therefore $\psi_{ss} > \psi_{rs} \Rightarrow \psi_{sr} > \psi_{rr}$ and $\psi_{sr} < \psi_{rr} \Rightarrow \psi_{ss} < \psi_{rs}$. This simplifies the conditions for a pure-strategy equilibrium to
either
\[ \psi_{ss} > \psi_{rs} \quad \text{or} \quad \psi_{rr} > \psi_{sr} \]

Let \( \bar{u} \) be the utility function that satisfies \( \psi_{ss} = \psi_{rs} \). (A.1) implies:
\[ \frac{\bar{u}'(\xi_{ss})}{\bar{u}'(\xi_{rs})} = \frac{r_r(\pi_{rs} - \pi_2)}{r_s(\pi_{ss} - \pi_2)} \quad \text{(A.2)} \]

The assumptions of the initial risk-neutral mixed-strategy equilibrium state that \( r_s(\pi_{ss} - \pi_2) < r_r(\pi_{rs} - \pi_2) \), which gives:
\[ \frac{\bar{u}'(\xi_{ss})}{\bar{u}'(\xi_{rs})} > 1 \quad \Rightarrow \quad \bar{u}'(\xi_{ss}) > \bar{u}'(\xi_{rs}) \]

Moreover, these assumptions imply that \( \pi_{ss} < \pi_{rs} \) so that \( \xi_{ss} < \xi_{rs} \) from Proposition A.1. Therefore, this condition only holds if
\[ \bar{u}''(\cdot) < 0 \]

Therefore, the utility function \( \bar{u} \) is a risk-averse utility function. Next consider \( \Psi \equiv \frac{\psi_{ss}}{\psi_{rs}} \) for some general risk-averse utility function \( u \):
\[ \Psi = \frac{r_s u'(\xi_{ss})(\pi_{ss} - \pi_2)}{r_r u'(\xi_{rs})(\pi_{rs} - \pi_2)} \]

Using equation (A.2), this gives:
\[ \Psi = \frac{u'(\xi_{ss})}{u'(\xi_{rs})} \cdot \frac{\bar{u}'(\xi_{rs})}{\bar{u}'(\xi_{ss})} \]

Consider the pure-strategy equilibrium defined by \( \psi_{ss} > \psi_{rs} \). In this equilibrium:
\[ \Psi > 1 \quad \Rightarrow \quad \frac{u'(\xi_{ss})}{\bar{u}'(\xi_{ss})} > \frac{u'(\xi_{rs})}{\bar{u}'(\xi_{rs})} \]

Let the function \( U(x) \equiv \frac{u'(x)}{u'(\xi)} \). The above condition is equivalent to the following:
\[ U(\xi_{ss}) > U(\xi_{rs}) \quad \Leftrightarrow \quad U(\xi_{rs}) - U(\xi_{ss}) < 0 \]

By the m.v.t. \( \exists \xi_U \in (\xi_{ss}, \xi_{rs}) \) such that \( U'(\xi_U) = \frac{U(\xi_U) - U(\xi_{ss})}{\xi_{rs} - \xi_{ss}} \). Therefore, the above condition is equivalent to:
\[ U'(\xi_U)(\xi_{rs} - \xi_{ss}) < 0 \]
Appendix: Proofs

\[ U'(\xi_U) < 0 \]
\[ \iff \frac{u''(\xi_U)\bar{u}'(\xi_U) - u'(\xi_U)\bar{u}''(\xi_U)}{[\bar{u}'(\xi_U)]^2} < 0 \]
\[ \iff u''(\xi_U)\bar{u}'(\xi_U) - u'(\xi_U)\bar{u}''(\xi_U) < 0 \]
\[ \iff -\frac{\bar{u}''(\xi_U)}{\bar{u}'(\xi_U)} < -\frac{u''(\xi_U)}{u'(\xi_U)} \]
\[ \iff A_{\bar{u}} < A_u \]

where \( A_u \) and \( A_{\bar{u}} \) are the coefficients of absolute risk aversion for the utility functions \( u \) and \( \bar{u} \), respectively. Lastly, note that it is not necessary to check to see whether it is possible to obtain the pure-strategy equilibrium with \( \psi_{rr} > \psi_{sr} \) where the Server chooses the risky strategy with probability 1. This is because Proposition 11 states that, as the Server becomes more risk-averse, they prefer the safer choice. Therefore, it cannot be possible for an increase in risk aversion to produce a pure-strategy equilibrium where the risky choice is made from a risk-neutral equilibrium.

In summary, for a unique pure-strategy equilibrium to exist it must be that the utility function \( u \) has a higher coefficient of risk aversion than the utility function \( \bar{u} \).

To prove the second part, let \( u(\cdot) \) be the risk-averse utility function of the Server under pressure. Assume that this is a function of their expected winning probabilities, \( \pi_S \), which themselves are a function of the Receiver choice to anticipate the safe serve, \( q_s \), thus \( u(\cdot) \equiv u(\pi_S(q_s)) \). By the first part, there are two possibilities: either the new equilibrium is a mixed-strategy equilibrium or a pure-strategy equilibrium.

- **Mixed-strategy equilibrium**: The change in the expected payoff for the Receiver for choosing to anticipate \( R \) is given by:

\[ \Delta \tilde{\pi}_R = \Delta p \frac{\tilde{\pi}_R(p + \Delta p) - \tilde{\pi}_R(p)}{\Delta p} \]

where \( \tilde{\pi}_R = 1 - \pi_R \) is the expected probability that the Receiver wins the point if they anticipate \( R \). If \( \Delta p \neq 0 \), then by the m.v.t. \( \exists \xi_R \in (\min\{p, p + \Delta p\}, \max\{p, p + \Delta p\}) \) such that \( \tilde{\pi}'_R(\xi_R) = \frac{\tilde{\pi}_R(p + \Delta p) - \tilde{\pi}_R(p)}{\Delta p} \), therefore

\[ \Delta \tilde{\pi}_R = \Delta p \tilde{\pi}'_R(\xi_R) \]
In a mixed-strategy equilibrium, the Receiver must remain indifferent between choosing to anticipate $W$ or $T$ and so $\Delta \tilde{\pi}_W = \Delta \tilde{\pi}_T$. Therefore,

$$\Delta p (\tilde{\pi}_W'(\xi_W) - \tilde{\pi}_T'(\xi_T)) = 0$$

Since $\tilde{\pi}_W'(\cdot) < 0$ and $\tilde{\pi}_T'(\cdot) > 0$, the above condition only holds if

$$\Delta p = 0$$

which contradicts the initial hypothesis that $\Delta p \neq 0$. Therefore, $\Delta p = 0$ and so the Server’s behaviour is unchanged.

Now consider the change in the expected payoff for the Server for choosing to serve $S$:

$$\Delta u_S = u_S(\pi_S(q_s)) - \pi_S(q_s)$$

$$+ \Delta q_s \frac{u_S(\pi_S(q_s + \Delta q_s)) - u_S(\pi_S(q_s)) \pi_S(q_s + \Delta q_s) - \pi_S(q_s)}{\pi_S(q_s + \Delta q_s) - \pi_S(q_s)} \Delta q_s$$

By the m.v.t. $\exists \xi_{S_1} \in (\min\{\pi_S(q_s), \pi_S(q_s + \Delta q_s)\}, \max\{\pi_S(q_s), \pi_S(q_s + \Delta q_s)\})$ such that $u_S'(\xi_{S_1}) = \frac{u_S(\pi_S(q_s + \Delta q_s)) - u_S(\pi_S(q_s))}{\pi_S(q_s + \Delta q_s) - \pi_S(q_s)}$, furthermore $\exists \xi_{S_2} \in (\min\{q_s, q_s + \Delta q_s\}, \max\{q_s, q_s + \Delta q_s\})$ such that $\pi_S'(\xi_{S_2}) = \frac{\pi_S(q_s + \Delta q_s) - \pi_S(q_s)}{\Delta q_s}$, giving:

$$\Delta u_S = u_S(\pi_S(q_s)) - \pi_S(q_s) + \Delta q_s u_S'(\xi_{S_1}) \pi_S'(\xi_{S_2})$$

In a mixed-strategy equilibrium, the Server must remain indifferent between serving safe and risky, so $\Delta u_s = \Delta u_r$. Furthermore, the initial equilibrium condition states that $\pi_s(q_s) = \pi_r(q_s)$. Therefore,

$$\Delta q_s \left( u_r'(\xi_{r_1}) \pi_r'(\xi_{r_2}) - u_s'(\xi_{s_1}) \pi_s'(\xi_{s_2}) \right) = u_s(\pi_s(q_s)) - u_r(\pi_r(q_s))$$

Proposition 11 states that the Server prefers the safe choice as their risk-aversion increases for a fixed choice of the Receiver implying that $u_s(\pi_s(q_s)) > u_r(\pi_r(q_s))$. Therefore:

$$\Delta q_s \left( u_r'(\xi_{r_1}) \pi_r'(\xi_{r_2}) - u_s'(\xi_{s_1}) \pi_s'(\xi_{s_2}) \right) > 0$$
Since \( u_r' (\cdot), \pi_r' (\cdot), u_a' (\cdot) > 0 \) and \( \pi_a' (\cdot) < 0 \), the above condition only holds if \( \Delta q_s > 0 \)

Thus, the Receiver anticipates the safe serve more frequently as the Server’s risk aversion increases.

Finally, the change in the Server’s point-winning probability is given by:

\[
\Delta \pi = \pi (p^* + \Delta p, q^* + \Delta q) - \pi (p^*, q^*)
\]

where \((p^*, q^*)\) is the initial risk-neutral equilibrium. Since \( \Delta p = 0 \),

\[
\Delta \pi = \pi (p^*, q^* + \Delta q) - \pi (p^*, q^*)
\]

\[
= \Delta q \left[ r_T (\pi_{TW} - \pi_{TT}) + r_T (\pi_{TT} - \pi_{TW}) \right]
\]

\[
= 0
\]

using equation (8.3) and that \( p^* = \frac{r_T (\pi_{TT} - \pi_{TW})}{r_T (\pi_{TT} - \pi_{TW}) + r_T (\pi_{TT} - \pi_{TW})} \). Thus, the Server’s wins the point with the same probability as the initial equilibrium.

- **Pure-strategy equilibrium**: First note that, from the first part of the proof, the only pure-strategy equilibrium that can exist for a risk-averse utility is where the Server chooses the safe serve with probability 1. Furthermore, since \( \pi_{ss} < \pi_{sr} \), the Receiver maximises the probability that they win the point by also choosing to anticipate the safe serve with probability 1. Lastly, the change in the probability that the Server wins the point is given by:

\[
\Delta \pi = \pi (1, 1) - \pi (p^*_s, q^*_s)
\]

\[
= \frac{r_s (\pi_{ss} - \pi_{sr}) [r_s (\pi_{ss} - \pi_{2}) - r_r (\pi_{sr} - \pi_{2})]}{r_s (\pi_{ss} - \pi_{sr}) + r_r (\pi_{rr} - \pi_{rs})}
\]

\[
< 0
\]

using (8.3) and \((p^*_s, q^*_s) = \left( \frac{r_s (\pi_{sr} - \pi_{rs})}{r_s (\pi_{ss} - \pi_{sr}) + r_r (\pi_{rr} - \pi_{rs})}, \frac{r_r (\pi_{rr} - \pi_{rs}) - r_s (\pi_{sr} - \pi_{rs})}{r_s (\pi_{ss} - \pi_{sr}) + r_r (\pi_{rr} - \pi_{rs})} \right) \). Thus, the Server’s wins the point with a lower probability than the initial equilibrium.
Appendix B

The I.I.D. Assumption Revisited

Recall that a couple of assumptions were used to provide a theoretical model for a tennis match in order to make the model more tractable. Specifically:

**Assumption (I.I.D.).** *In each point-game in a tennis match:*

1. *The probability of winning a point on serve is independent of all other points.*
2. *The probability of winning a point on serve is constant.*

The first part of this assumption states that each point is independent, while the second states that each point is identically distributed (from this point forward). Essentially, this assumption states that the probability of winning each point served by Player $i$ is an *i.i.d.* draw from a Bernoulli distribution with parameter $\pi_i$. Therefore, a pertinent question is whether this assumption is actually confirmed in the data and if not, do the deviations have a substantial impact on the inferences drawn from the analysis conducted in this dissertation.

There are several ways to test whether the points on serve are identically distributed. One method would be to see whether the observed winning frequencies differ significantly across sets over the course of a match. This would be one indication of a trend occurring in a match that may suggest that these probabilities are not identically distributed across the course of a match. Another option would be to see whether the winning frequencies vary according to the importance of the point being played (which is one of the key components of the analysis in this thesis).
If there is evidence that these winning probabilities change based on the importance of a point, then again there is evidence against the assumption that points are identically distributed.

To test for independence, other properties of the data are of interest. For example, if points were not independent then winning one point may have a positive effect on the chance of winning the next point (e.g., a momentum effect) or a negative effect on the winning the next (e.g., a break-rebreak effect). If there is evidence of these effects, then the assumption of independence of each point is also suspect.

What does research suggest about the validity of the *i.i.d.* assumption? First, there is evidence that winning frequencies do vary depending on the importance of the point based on the results in this dissertation. This is consistent with past research. O’Malley (2008) found that servers win a significantly lower proportion of break-points than the average point (55% v. 67%). Furthermore, Klaassen & Magnus (2001) found that servers do have a slightly lower chance of winning a point on their serve the more important the point. This effect is smaller for the top players (judged by ATP ranking) who have a higher chance of winning the important points (i.e., a less negative effect). These results are further confirmed by González-Díaz et al. (2012), who found that there is significant heterogeneity amongst tennis players in this regard and a strong correlation between this ‘critical ability’ and ATP ranking. The suggestion put forward as to the reason for this deviation is that players play safer at these points – for example, go for less aces – and hence deviate from their optimal service strategy.

There is also some evidence against the independence of each point. Klaassen & Magnus (2001) has found that there is a significant and positive effect of winning

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64 The momentum effect (a.k.a. ‘hot hand’ or ‘winning mood’) has been extensively studied in many sports with mixed results. Early work by Gilovich, Vallone, & Tversky (1985) found no evidence of a ‘hot hand’ in basketball, although both players and fans alike believe in its existence. More recently, Livingston (2012) has found slightly mixed results of a momentum effect in professional golf but no strong conclusive evidence of its existence. On the other hand, Klaassen & Magnus (2014) found no evidence of the break-rebreak effect. In fact, if anything, the authors found a slight positive effect on the probability of winning a service game given that they had just broken in the previous game. This runs counter to the break-rebreak hypothesis. For a comprehensive review of the existence of momentum in sport refer to Bar-Eli, Avugos, & Raab (2006) or Reifman (2012).
the previous point on winning the current point. They also note that this effect is smaller for higher ranked players, suggesting that part of the reason for their success is the ability to be stable and to treat each point as it comes. More recent work by Gauriot & Page (2014) has utilised the quasi-randomness of shots landing either side of the line to estimate the causal effect of winning a point on the chance to win the next. Their fuzzy discontinuity regression analysis has found a significant momentum effect for men that was absent in women. This is further evidence against the assumption of independence of each point.

Although there is evidence that the *i.i.d.* assumption used in this analysis is not strictly true, it is unlikely to be too problematic for the analysis. While the deviations are significant in a statistical sense, they are generally small and hence should not have a large effect on the results. Indeed Klaassen & Magnus (2001) found these effects to be smaller on average and almost non-existent for the top players which make up the majority of the sample used in this thesis. Moreover, Newton & Aslam (2006) found that small deviations from the *i.i.d.* assumption produce virtually the same results as those models based only on the *i.i.d.* assumption. In light of this, it appears that this assumption provides a great deal of simplification without sacrificing too much accuracy. Lastly, Walker et al. (2011) show that it is indeed theoretically optimal to play every point as it comes, that is, play each point independently. Based on this, it appears valid to proceed under the *i.i.d.* assumption.
Appendix C

Data Construction

The majority of the data used in the analysis come from the *Tennis Abstract* website. This is an open source match charting platform that provides detailed descriptions and the corresponding score of each point in every match in their database. The descriptions include the direction (*Wide*/*Body*/*Tee*) of the 1st and, if needed, 2nd serves, as well as whether the serves were faults. They also describe the return (*Forehand*/*Backhand*) when the point is not decided by an ace, service winner or double fault, and the side and placement of the subsequent shots in the rally.

Using this information from the *Tennis Abstract* website, I create a variable for the service direction and, from the progression of the score, a variable describing whether the Server won the point. I also create a variable for the importance/pressure at each point from the score vector. Along with player and court/tournament information taken from the ATP official website, this is all the relevant information required to perform the analysis in this research.\(^6\)

Some data points from the *Tennis Abstract* website were labelled as “unknown” and these observations were omitted from the analysis. These comprise 934 observations, or approximately 0.6% of the total number of observations. Since these are likely caused by factors totally unrelated to the match, such as disruptions in the broadcast, these were deemed random omissions. Therefore, they should not have any effect on the analysis.

\(^6\)This additional data can be obtained from the following URL: [www.atpworldtour.com/en](http://www.atpworldtour.com/en)
The remainder of this discussion details the variables used in the estimation of the effect of pressure on tennis players. I include a discussion of all variables except for the pressure explanatory variable, which is discussed in detail in the body of this dissertation. I first outline the construction of the dependent variable for the decision-making analysis in Section 7.1. I then outline the construction of the dependent variable for the outcome analysis in Section 7.2. Finally, I detail the additional explanatory variables that are used in the estimation across these two sections.

C.1 Construction of Decision Variable

The dependent variable, $d_t$, in this analysis is a variable for the direction of the serve that is defined as a binary variable in the following manner:

$$d_t = \begin{cases} 
1, & \text{if the serve is } W \\
0, & \text{if the serve is } T 
\end{cases}$$

However, the data specify the service direction as three possibilities \{W, B, T\} rather than two \{W, T\}. To reduce the number of service actions from three to two required consideration of several issues. Since I did not want to throw observations away, I reassigned the $B$ serves to either $W$ or $T$ based on whether the Receiver returned the serve on their forehand or backhand. Assuming that the Receiver does not strongly favour one return over the other, this information should identify the precise half that the $B$ serves are going and should be an accurate way to reassign these serves.

This was possible for observations where the Server made their first serve. However, this assignment was impossible when the first serve was missed and so these observations had to be dropped. This should not introduce any bias into the results as long as two conditions are satisfied. First, the probability of missing a $B$ serve that I reclassify as a $W$ serve is the same as for a $B$ serve that I reclassify as a $T$ serve. Second, the probability of winning the point is the same for a $B$ serve
that I reclassify as a $W$ serve as for a serve that is initially classified as a $W$ serve, and similarity for the $T$ serves. The first assumption seems reasonable since there should be little difference between serves at the borderline between $W$ and $T$, and so the probabilities of making these serves should not be vastly different. The second assumption also seems reasonable.

By performing this transformation, the number of observations dropped from 148,144 in the three-choice specification to 141,701 in the two-choice specification. The remaining 6,443 observations were dropped. Table C.1 shows the distribution of service direction across the entire sample for both the three-choice and two-choice specifications.

<table>
<thead>
<tr>
<th></th>
<th>3-Directions</th>
<th>2-Directions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wide</td>
<td>65,965 (44.8%)</td>
<td>72,123 (50.9%)</td>
</tr>
<tr>
<td>Body</td>
<td>18,561 (12.6%)</td>
<td>–</td>
</tr>
<tr>
<td>Tee</td>
<td>62,684 (42.5%)</td>
<td>69,578 (49.1%)</td>
</tr>
<tr>
<td>Total</td>
<td>147,210</td>
<td>141,701</td>
</tr>
</tbody>
</table>

It can be seen from Table C.1 that the ratio of $W$ serves to $T$ serves remains roughly constant in both specifications. This provides further evidence that the transformation from the three-choice specification to the two-choice specification is valid. In general, it appears that the distribution of $B$ that are reclassified to $W$ and $T$ is roughly even, something that should be expected for the serves that are at the borderline between the two halves.

## C.2 Construction of Outcome Variable

The dependent variable, $\pi_t$, in the regressions in this dissertation is a binary variable indicating whether point $t$ was won by the Server:

$$
\pi_t = \begin{cases} 
1, & \text{if the Server won the point} \\
0, & \text{if the Server lost the point}
\end{cases}
$$
This variable was constructed directly from the progression of the score. For points within the same game, if the Server won the point then their point score increases by one and if the Receiver won the point then their score increases by one. For points within the same set but not the same game, the winner can be determined by the player whose game score increased by one. For points within the same match but not the same set, the winner can be determined by the player whose set score increased by one. For the final point of the match, the winner of the point is the player who has the match point.

C.3 Control Variables

The control variables in the estimations consist of any factors that may influence the choice of where to serve and the likelihood of the Server winning the point. In particular, it is important to include any variables that may be correlated with pressure so as not to bias the regression estimates. I also include match-fixed effects in all the individual-level regressions and Server- and match-fixed effects in the more extensive aggregate regressions to account for difference in players’ behaviour/outcomes against different opponents in different matches.

The first additional factor that I account for is fatigue. Hitting a serve (like every shot in tennis) requires exerting some amount of physical effort. Since different service techniques are used to serve to the different directions, it is possible that some players may find some serves less tiring than others. If this is the case then players could choose to hit those serves more as they get more tired. Furthermore, the advantage that the serve brings may diminish as players get tired and cannot hit the ball as hard, and so fatigue may also have a causal effect on point-winning probabilities. Lastly, fatigue may be correlated with the importance of a point since longer and more tiring matches tend to have more important points. I control for these potential issues by using the number of sets played up to point $t$ as a measure of fatigue.
The second factor that I account for is the court surface. Different court surfaces cause the ball to react differently when it makes contact with the surface. Amongst other things, this can result in the ball losing more speed or gaining more spin depending on the surface. Since different serves are hit with different speeds and spins, this implies that different court surfaces will reward some serves more than others. Knowing this, players are likely to adjust their service strategies based on the surface that they are playing on to maximise their chances of winning. Therefore, court service is likely to affect both the service decision and the likelihood that the Server wins the point. Moreover, slower surfaces are typically known to have longer and potentially tighter matches; thus, there may be some correlation with the pressure points. I control for these factors by including court surface dummies.

In the aggregate regressions I also include a measure for player quality. Player quality is potentially correlated with importance since players of similar quality are more likely to have tighter matches with more critical points. Furthermore, it will likely influence point-winning probabilities. Thus, a measure of quality should be incorporated to prevent bias in the estimates from regression (5). Explicitly, I use the difference in players’ individual quality. This is based on the concept of an ‘expected tournament losing round,’ the round at which the player is expected to be knocked out of the tournament given by their seeding. For example, the first seed is expected not to be knocked out (i.e., to lose after the tournament concludes), the second seed is expected to be knocked out in the final, the third and fourth seeds are expected to be knocked out in the semi-finals, etc. Using the concept of the expected losing round, the (normalised) quality for Player $i$, $Q_i$, is given by:

$$Q_i = \begin{cases} 
\left\lceil \log_2 DS \right\rceil + 1 - \log_2 \text{seed}_i & \text{if Player } i \text{ is seeded} \\
\frac{\left\lceil \log_2 DS \right\rceil + 1}{\left\lceil \log_2 DS \right\rceil + 1} & \text{if Player } i \text{ is unseeded}
\end{cases}$$

66 Magnus & Klaassen (1999) argued that absolute quality could also play a role in influencing winning probabilities (i.e., the quality sum of the two players). However, they examined only one tournament and so ranking and seeding are interchangeable. The tournaments in my sample span several years and ranking tiers so using the quality sum based on the seeds could potentially lead to biased results in my sample. This is because high seeded matches in lower tier tournament represent different absolute quality than high seeded matches in higher tier tournaments.
where $DS$ is the tournament draw size and $seed_i$ is the seed of Player $i$. $\lceil \log_2 DS \rceil$ gives the total number of rounds in a tournament of draw size $DS$. Those that are unseeded are always expected to lose in either the first or second round, hence I assume that their expected losing round is 1.5. Quality is normalised so that the top seed has quality rating of 1. This variable is only required in the aggregate analysis. The Receiver-fixed effects, in conjunction with the fact that the regressions are performed independently for each server in the sample, absorbs the relative quality between the Server and the Receiver in the player-level analysis. For illustration, Figure C.1 plots the quality of seeded players for a Grand Slam ($DS = 128$).

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure.png}
\caption{Player Quality}
\end{figure}

Figure C.1 shows that the quality of the players, $Q_i$, decreases as the seed of the player increases. This is precisely what one would expect since a player’s seed is directly related to their ranking and the players with the higher rank/seed are not as good as those with lower rank/seed. Furthermore, it can be seen that the rate of change of quality with respect to seed is also decreasing. This captures the notion that quality is often pyramid-shaped so that, for example, the quality difference between the top ranked player and the 10th ranked player is likely to be greater than for the 100th and the 110th ranked player.\textsuperscript{67}

\textsuperscript{67}This idea is discussed in Klaassen & Magnus (2001) and the method presented here is similar. The key difference is that I use the seed of Player $i$ rather than their rank because the size and quality of the tournaments varies across the sample.
Appendix D

Appendices for Chapter 5

D.1 Summary of Replication Matches

In Chapter 5 I provide a direct test of the minimax hypothesis that players equalise the probability that they win the point for every choice of service direction. To test this hypothesis, I compare the probability of winning the point for each service direction chosen by each player on each court. This analysis is similar to that of Walker & Wooders (2001) except that I examine more than twice as many matches.

For this analysis I use the 25 longest matches from the sample. I choose these matches because this test requires a large number of observations to provide enough power to appropriately reject the null hypothesis when it does not hold. As a consequence of this, it is only possible to draw meaningful conclusions regarding equilibrium play in matches with a large number of points.\textsuperscript{68} The matches used have a total of 8,834 points. There are 17 missing observations, giving a total of 8,817 observations for the 25 matches.

Table D.1 provides a summary of the 25 matches used in the minimax analysis in Chapter 5. For each match I provide the details of the players involved and the tournament in which the specific match was played. I also provide the winner of the match, the final score and the number of points played in the match.

\textsuperscript{68}Walker & Wooders (2001) noted that, by choosing the longest matches there may be some sample selection issues in the analysis. That is, these matches are long because the players play close to optimally. However, they do not provide further discussion.
Table D.1: Matches used in Direct test of Minimax

<table>
<thead>
<tr>
<th>Year</th>
<th>Tournament</th>
<th>Rnd</th>
<th>Player 1</th>
<th>Player 2</th>
<th>Winner</th>
<th>Score</th>
<th>Pts.</th>
</tr>
</thead>
<tbody>
<tr>
<td>2009</td>
<td>Wimbledon</td>
<td>F</td>
<td>A. Roddick</td>
<td>R. Federer</td>
<td>R. Federer</td>
<td>5–7, 7–6, 7–6, 3–6, 16–14</td>
<td>436</td>
</tr>
<tr>
<td>2008</td>
<td>Wimbledon</td>
<td>F</td>
<td>R. Federer</td>
<td>R. Nadal</td>
<td>R. Nadal</td>
<td>6–4, 6–4, 6–7, 6–7, 9–7</td>
<td>413</td>
</tr>
<tr>
<td>2013</td>
<td>Aus Open</td>
<td>R16</td>
<td>N. Djoković</td>
<td>S. Wawrinka</td>
<td>N. Djoković</td>
<td>1–6, 7–5, 6–4, 6–7, 12–10</td>
<td>409</td>
</tr>
<tr>
<td>2011</td>
<td>Aus Open</td>
<td>R128</td>
<td>D. Nalbandian</td>
<td>L. Hewitt</td>
<td>D. Nalbandian</td>
<td>3–6, 6–4, 3–6, 7–6, 9–7</td>
<td>386</td>
</tr>
<tr>
<td>2009</td>
<td>Aus Open</td>
<td>SF</td>
<td>F. Verdasco</td>
<td>R. Nadal</td>
<td>R. Nadal</td>
<td>6–7, 6–4, 7–6, 6–7, 6–4</td>
<td>385</td>
</tr>
<tr>
<td>2012</td>
<td>Aus Open</td>
<td>F</td>
<td>N. Djoković</td>
<td>R. Nadal</td>
<td>N. Djoković</td>
<td>5–7, 6–4, 6–2, 6–7, 9–7</td>
<td>369</td>
</tr>
<tr>
<td>2016</td>
<td>Aus Open</td>
<td>R16</td>
<td>N. Djoković</td>
<td>G. Simon</td>
<td>N. Djoković</td>
<td>6–3, 6–7, 6–4, 4–6, 6–3</td>
<td>369</td>
</tr>
<tr>
<td>2012</td>
<td>Olympics</td>
<td>SF</td>
<td>J.M. del Potro</td>
<td>R. Federer</td>
<td>R. Federer</td>
<td>3–6, 7–6, 19–17</td>
<td>352</td>
</tr>
<tr>
<td>2014</td>
<td>Wimbledon</td>
<td>F</td>
<td>N. Djoković</td>
<td>R. Federer</td>
<td>N. Djoković</td>
<td>6–7, 6–4, 7–6, 5–7, 6–4</td>
<td>366</td>
</tr>
<tr>
<td>2017</td>
<td>Aus Open</td>
<td>SF</td>
<td>R. Nadal</td>
<td>G. Dimitrov</td>
<td>R. Nadal</td>
<td>6–3, 5–7, 7–6, 6–4, 6–4</td>
<td>365</td>
</tr>
<tr>
<td>2016</td>
<td>Aus Open</td>
<td>R128</td>
<td>R. Nadal</td>
<td>F. Verdasco</td>
<td>F. Verdasco</td>
<td>7–6, 4–6, 3–6, 7–6</td>
<td>362</td>
</tr>
<tr>
<td>2009</td>
<td>US Open</td>
<td>F</td>
<td>R. Federer</td>
<td>J.M. del Potro</td>
<td>J.M. del Potro</td>
<td>3–6, 7–6, 4–6, 7–6, 6–2</td>
<td>352</td>
</tr>
<tr>
<td>2013</td>
<td>Aus Open</td>
<td>F</td>
<td>R. Federer</td>
<td>R. Nadal</td>
<td>R. Nadal</td>
<td>7–5, 3–6, 7–6, 3–6, 6–2</td>
<td>347</td>
</tr>
<tr>
<td>2012</td>
<td>Aus Open</td>
<td>SF</td>
<td>N. Djoković</td>
<td>A. Murray</td>
<td>N. Djoković</td>
<td>6–3, 3–6, 6–7, 6–4, 6–1, 7–5</td>
<td>345</td>
</tr>
<tr>
<td>2013</td>
<td>French Open</td>
<td>SF</td>
<td>N. Djoković</td>
<td>R. Nadal</td>
<td>R. Nadal</td>
<td>6–4, 3–6, 6–1, 6–7, 9–7</td>
<td>335</td>
</tr>
<tr>
<td>2016</td>
<td>Wimbledon</td>
<td>QF</td>
<td>M. Čilić</td>
<td>R. Federer</td>
<td>R. Federer</td>
<td>6–7, 4–6, 6–3, 7–6, 6–3</td>
<td>332</td>
</tr>
<tr>
<td>2013</td>
<td>US Open</td>
<td>SF</td>
<td>N. Djoković</td>
<td>S. Wawrinka</td>
<td>N. Djoković</td>
<td>2–6, 7–6, 3–6, 6–3, 6–4, 3–6, 6–4</td>
<td>331</td>
</tr>
<tr>
<td>2013</td>
<td>Aus Open</td>
<td>SF</td>
<td>R. Federer</td>
<td>A. Murray</td>
<td>A. Murray</td>
<td>6–4, 6–7, 6–3, 6–7, 6–2</td>
<td>328</td>
</tr>
<tr>
<td>2016</td>
<td>Wimbledon</td>
<td>SF</td>
<td>M. Raonić</td>
<td>R. Federer</td>
<td>M. Raonić</td>
<td>6–3, 6–7, 4–6, 7–5, 6–3</td>
<td>327</td>
</tr>
<tr>
<td>2017</td>
<td>Aus Open</td>
<td>R64</td>
<td>N. Kyrgios</td>
<td>A. Seppi</td>
<td>A. Seppi</td>
<td>6–1, 7–6, 6–4, 6–2, 10–8</td>
<td>321</td>
</tr>
<tr>
<td>2016</td>
<td>Aus Open</td>
<td>R16</td>
<td>S. Wawrinka</td>
<td>M. Raonić</td>
<td>M. Raonić</td>
<td>6–4, 6–3, 5–7, 4–6, 6–3</td>
<td>320</td>
</tr>
<tr>
<td>2015</td>
<td>US Open</td>
<td>R64</td>
<td>B. Tomic</td>
<td>L. Hewitt</td>
<td>B. Tomic</td>
<td>6–3, 6–2, 3–6, 5–7, 7–5</td>
<td>319</td>
</tr>
<tr>
<td>2015</td>
<td>US Open</td>
<td>R16</td>
<td>A. Murray</td>
<td>K. Anderson</td>
<td>K. Anderson</td>
<td>7–6, 6–3, 6–7, 7–6, 6–3</td>
<td>319</td>
</tr>
<tr>
<td>2016</td>
<td>Aus Open</td>
<td>SF</td>
<td>A. Murray</td>
<td>M. Raonić</td>
<td>A. Murray</td>
<td>4–6, 7–5, 6–7, 6–4, 6–2</td>
<td>318</td>
</tr>
<tr>
<td>2014</td>
<td>Aus Open</td>
<td>QF</td>
<td>N. Djoković</td>
<td>S. Wawrinka</td>
<td>S. Wawrinka</td>
<td>2–6, 6–4, 6–2, 3–6, 9–7</td>
<td>314</td>
</tr>
</tbody>
</table>
D.2 Baseline Robustness Check

This section tests the robustness of the analysis conducted in Chapter 5. Rather than the two-choice model \((\text{Wide/Tee})\), this robustness check consists of a three-choice model. For the subsequent analysis the following three services choices are considered: \text{Wide} \((W)\), \text{Body} \((B)\), or \text{Tee} \((T)\). Therefore, the following hypothesis is to be tested:

**Hypothesis 5.** For each point game, \(i = 1, \ldots, 100\)

\[
H_0 : \; \pi^i_W = \pi^i_B = \pi^i_T \\
H_1 : \; \text{At least one equality fails}
\]

Since each \(\pi^i\) has to be estimated, the Pearson test statistic for each point-game is asymptotically distributed as chi-squared with two degrees of freedom under the null hypothesis. Table D.2 shows the results for the 100 point-games. In only three point-games the equality of winning probabilities is rejected at the 1% level of significance. These are: (1) Dimitrov serving to Nadal on the Deuce court at the 2017 Australian Open semi-final, (2) Federer serving on the AD court to Čilić at the 2017 Wimbledon quarter-final and (3) Djoković serving on the Deuce court to Wawrinka at the 2013 US Open semi-final. In two additional point-games the null hypothesis is rejected at the 5% level, and a further four at the 10% level. If these results were generated randomly, one, five and ten rejections would be expected at these significance levels (compared to three, five and nine from this data). Therefore, it appears that players adhere to the predictions of minimax behaviour in the majority of games. This conclusion is supported by the joint test of equality across all point-games (but allowing the probabilities to vary across point-games). The Pearson test statistic is the sum of the statistics for each point-game and is distributed as chi-squared with 200 degrees of freedom. The test statistic is 210.649 and the \(p\)-value is 0.289. Clearly the null hypothesis cannot be rejected in this case and I conclude that the probabilities of winning the point are equal across the Server’s choices in each point-game.
In the majority of these point-games there are two common serves (Wide and T) and Body serves are much less common. In many cases the number of Body serves is less than five – usually an order of magnitude less than the other two choices. When there is a discrepancy such as this, the Pearson statistic can be inaccurate since it assumes that discrete probabilities can be approximated by a chi-squared distribution. Therefore, the associated $p$-values could be more severe (i.e., lower probabilities of random occurrence). This might cause the rejection of the null hypothesis in cases when it should not be rejected. In particular, the results could be driven by the Body serves which could be inaccurately estimated due to their small sample size. Therefore, these results are only presented to provide a check to the main results given in Chapter 5. Regardless they still confirm the conclusion of the analysis in the body of the dissertation that I cannot reject the belief that players are choosing their strategies optimally.
### TABLE D.2: Equality of Winning Probabilities (Three Service Choices)

<table>
<thead>
<tr>
<th>Server</th>
<th>Court</th>
<th>Serves</th>
<th>Mixture</th>
<th>Points Won</th>
<th>Win Rates</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1</td>
<td>Deuce</td>
<td>10</td>
<td>8</td>
<td>10</td>
<td>13</td>
</tr>
<tr>
<td>P1</td>
<td>Ad</td>
<td>31</td>
<td>14</td>
<td>67</td>
<td>112</td>
</tr>
<tr>
<td>P2</td>
<td>Deuce</td>
<td>4</td>
<td>4</td>
<td>56</td>
<td>10</td>
</tr>
<tr>
<td>P2</td>
<td>Ad</td>
<td>45</td>
<td>3</td>
<td>43</td>
<td>91</td>
</tr>
<tr>
<td>P1</td>
<td>Deuce</td>
<td>49</td>
<td>6</td>
<td>45</td>
<td>100</td>
</tr>
<tr>
<td>P1</td>
<td>Ad</td>
<td>55</td>
<td>2</td>
<td>38</td>
<td>95</td>
</tr>
<tr>
<td>P2</td>
<td>Deuce</td>
<td>25</td>
<td>24</td>
<td>67</td>
<td>115</td>
</tr>
<tr>
<td>P2</td>
<td>Ad</td>
<td>46</td>
<td>24</td>
<td>28</td>
<td>103</td>
</tr>
<tr>
<td>P1</td>
<td>Deuce</td>
<td>48</td>
<td>24</td>
<td>67</td>
<td>101</td>
</tr>
<tr>
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### Notes
- P Stat: Probability of equality of winning probabilities.
- p-value: Significance level for the test.
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**Indicates rejection at the 1-percent level of significance.**

*Indicates rejection at the 5-percent level of significance.

*Indicates rejection at the 10-percent level of significance.
Appendix E

Appendices for Chapter 6

E.1 Further Derivations of the Importance

The discussion in this appendix provides the additional calculations required to quantify the importance of a point from Chapter 6. These calculations use a similar procedure to the importance of a point in a game outlined in the body of this dissertation, but are a little more involved since they involve the service-winning probabilities of both players. In this discussion I provide the full derivation of the importance of a point in a tie-break, the importance of a game in a set and the importance of a set in a match. Along with the importance of a point in a game, these are all the necessary calculations required to determine the total importance of a point.

E.1.1 The Importance of a Point in a Tie-Break

As mentioned, there is one complication that arises in tie-breaks due to players alternating service after every two points. Therefore, both the current Server’s and Receiver’s point-winning probabilities are required. Denoting $f_{j,2}^i$ as the probability that Player $i$ will win the tie-break, given that they are serving the current point and $g_{j,2}^i$ as the probability that Player $i$ will win the tie-break given their opponent is serving the current point, the initial set of equations that must be solved at 6–6
and 7–7 are:

\[ f_{1,2}^i = \pi_i + (1 - \pi_i)f_{0,2}^i, \quad g_{1,2}^i = (1 - \pi_i) + \pi_i g_{0,2}^i \]
\[ f_{0,2}^i = \pi_i g_{1,2}^i + (1 - \pi_i)g_{-1,2}^i, \quad g_{0,2}^i = (1 - \pi_i)f_{-1,2}^i + \pi_i f_{1,2}^i \]
\[ f_{-1,2}^i = \pi_i f_{0,2}^i, \quad g_{-1,2}^i = (1 - \pi_i)g_{0,2}^i \]

where \( \pi_i \) and \( \pi_{-i} \) are the probabilities that Player \( i \) and their opponent wins a point on their serve, respectively. Solving for the 6–6 and 7–7 points

\[ f_{0,2}^i = \frac{\pi_i (1 - \pi_{-i})}{1 - \left[ \pi_i \pi_{-i} + (1 - \pi_i)(1 - \pi_{-i}) \right]} = g_{0,2}^i \]

The remaining points can be solved by a similar recursive relationship as the one described in the analysis in Section 6.2.1, with the following boundary conditions:

\[ \mathbb{1}_{\{i \text{ serving}\}} \text{Pr}[W_{TB}|(7, p_r)] = 1, \quad \left(1 - \mathbb{1}_{\{i \text{ serving}\}}\right) \text{Pr}[W_{TB}|(7, p_r)] = 0 \]
\[ \mathbb{1}_{\{i \text{ serving}\}} \text{Pr}[W_{TB}|(p_s, 7)] = 0, \quad \left(1 - \mathbb{1}_{\{i \text{ serving}\}}\right) \text{Pr}[W_{TB}|(p_s, 7)] = 1 \]

where \( \mathbb{1}_{\{i \text{ serving}\}} \) is an indicator variable that takes a value of 1 if player \( i \) is serving and 0 if not. Care must be taken since the server alternates every two points, and so it is necessary to keep track of who is serving and receiving. Alternatively, it is sufficient to specify who serves the first point (since that determines the server at any given score). Once again, due to the repeating nature of a tie-break, it is only necessary to specify up to a score of 7–7. Scores above this can be mapped back to one of the following scores: 5–5, 5–6, 6–5, 6–6, 6–7, 7–6.

### E.1.2 The Importance of a Game in a Set

The parameters required to calculate the importance of a game in a set are the game, \( \pi_i^G \), and tie-break winning probabilities, \( \pi_i^{TB} \). These parameters come from the 0–0 entries in the probability of winning matrix for a game or a tie-break. Considering a tie-break set, the initial condition at 6–6 is simply:

\[ \text{Pr}[W_S|(6, 6)] = \mathbb{1}_{\{i \text{ serving}\}} \pi_i^{TB} + \left(1 - \mathbb{1}_{\{i \text{ serving}\}}\right) \left(1 - \pi_{-i}^{TB}\right) \]
Then it is possible to solve via the recursive relation:

$$\Pr[W_S|(g_s, g_r)] = \mathbb{1}_{\{i \text{ serving}\}}\left(\pi_i^G \Pr[W_S|(g_r, g_s + 1)] + (1 - \pi_i^G) \Pr[W_S|(g_r + 1, g_s)]\right)$$

$$+ \left(1 - \mathbb{1}_{\{i \text{ serving}\}}\right)\left(\pi_i^G \Pr[W_S|(g_s, g_r + 1)] + (1 - \pi_i^G) \Pr[W_S|(g_s + 1, g_r)]\right)$$

Along with the boundary conditions

$$\mathbb{1}_{\{i \text{ serving}\}} \Pr[W_G|(6, g_r)] = 1, \quad \left(1 - \mathbb{1}_{\{i \text{ serving}\}}\right) \Pr[W_G|(6, g_r)] = 0$$

$$\mathbb{1}_{\{i \text{ serving}\}} \Pr[W_G|(g_s, 6)] = 0, \quad \left(1 - \mathbb{1}_{\{i \text{ serving}\}}\right) \Pr[W_G|(g_s, 6)] = 1$$

A similar process for the importance of a point in a tie-break is used to determine the values in sets without a tie-break, and so is omitted here.

### E.1.3 The Importance of a Set in a Match

The parameter required to calculate the importance of a set in a match is the set-winning probability, $\pi^S$, which comes from the 0–0 entry in the set-winning probability matrix from the previous sections. The exact matrix has four possibilities depending on who served the set first and whether the set has a tie-break. However, the following result simplifies this substantially:

**Proposition 13.** The probability of Player $i$ winning the set is the same (at the beginning of the set) whether they serve first or second.

**Proof.** (Sketch) For Player $i$ to win a (non-tiebreak) set they must break their opponent’s service at least once. Since points are independent, the probability of breaking the opponent’s service is the same regardless of whether they serve the odd games (i.e., they served first) or the even games (i.e., they served second). In either case the player must win at least one of these games to win the set, but when exactly they do is irrelevant. This holds for any set which is decided by one break only; however, a similar idea holds for sets with multiple breaks of service. For full proof refer to Newton & Keller (2005).
This is useful since it is no longer required to know who serves first in the set to determine the importance of the set in the match. Furthermore, let $f$ denote the format of the match, which is either best-of-three ($f = 3$) or best-of-five ($f = 5$). The winner of the match is the first to reach $w = (f + 1)/2$ sets (i.e., $w = 2$ for best-of-three set matches and $w = 3$ for best-of-five set matches). Therefore, the starting condition at the set score $(w - 1, w - 1)$

$$
\Pr[W|(w-1, w-1)] = \mathbb{1}\{\text{tie-break}\} \pi^S_{i, TB} + \left(1 - \mathbb{1}\{\text{tie-break}\}\right) \pi^S_{i, NTB}
$$

where $\mathbb{1}\{\text{tie-break}\}$ is an indicator variable that takes a value of 1 if the set is determined by a tie-break and 0 if not. Then it is possible to solve via the recursion relation:

$$
\Pr[W|(s_i, s_{-i})] = \mathbb{1}\{\text{tie-break}\} \cdot \left(\pi^S_{i, TB} \Pr[W|(s_i + 1, s_{-i})] + (1 - \pi^S_{i, TB}) \Pr[W|(s_i, s_{-i} + 1)]\right) + \left(1 - \mathbb{1}\{\text{tie-break}\}\right) \cdot \left(\pi^S_{i, NTB} \Pr[W|(s_i + 1, s_{-i})] + (1 - \pi^S_{i, NTB}) \Pr[W|(s_i, s_{-i} + 1)]\right)
$$

Along with the boundary conditions

$$
\Pr[W|(w, s_{-i})] = 1 \quad \text{and} \quad \Pr[W|(s_i, w)] = 0
$$
E.2 Robustness of the Importance Measure

In this discussion I outline two alternative methods for estimating the beliefs of the service-winning probabilities. I discuss the strengths and weakness of each alternative method against the one used in the analysis in Chapter 7. A brief discussion of the alternative methods is important because different specifications may change the classification of the ‘high pressure’ points in the estimation.

E.2.1 Alternative Methods of the Importance

The first method I investigate uses the averages from the current match to determine the probability that the Server will win the point. The second uses the Server’s total average winning probability from all matches in the dataset to determine this probability. The specification I use in the body of this thesis is essentially a combination of these two, allowing each player to update their belief of the service-winning probabilities as the match progresses.

Alternative Method 1 – Match-Specific Probabilities

The first alternative method calculates the probability of a player winning a point on their service in a match as a simple average of all the points they serve in the match. The advantage of this method is that is should best represent the state of the match. Moreover, the calculations for the match do not depend on what other matches are included in the sample. However, there are disadvantages which are potentially problematic.

The first issue is that this measure does not incorporate the inherent ability of the player serving and all that matters is the points in the current match. However, it seems reasonable that a player’s experience of pressure would depend on the perceived strength of their opponent. For example, this may not distinguish a top player having an average serving match from a weak player having a strong serving match. Whereas, it is likely that players will feel more pressure playing a top
player than playing a lower ranked player, regardless of how they are serving on this particular day. This method will not factor this into account since it only accounts for the averages from the current match.

A potentially more important issue is that this method uses points that have yet to be played to calculate the importance of the current point. This is because the winning probabilities are calculated using the averages taken across the entire match ex post. For example, the importance of a point in the first game uses the average of all points in the match, which contain points in the second game that have yet to be played, and so on. This implies that players at the early stages of a match are using information that is currently unavailable to them to form their beliefs. Obviously, this is an undesirable feature. Moreover, it can bias the importance measure, especially early in the match when the majority of this future information would still be unknown.

To see this last point explicitly, consider two different matches between the same two players where the score is a break point (e.g., 30–40) in the first game of the first set. The first match turns out to be one-sided, while the second turns out to be tight. By definition of one-sided, one player will have a higher average service-winning probability in the first match compared to the other player. Whereas, in the second match, the same player will have a similar average service-winning probability to the other. As a result, this specification will attribute a lower importance to this point in the first match than in the second.\textsuperscript{69} However, this is the same point of the match between the same players; thus, the players’ beliefs should be the same in both scenarios. This is clearly not the case and so the importance may be biased, especially near the beginning of the match, in undesirable and unpredictable ways.

\textbf{Alternative Method 2 – Player-Specific Probabilities}

The second method attempts to correct for the inconsistencies of the first by using each player’s average service-winning probabilities from the sample as $\pi_1$ and $\pi_2$

\textsuperscript{69}This is a consequence of the mathematical formulation of the importance. See Chapter 6 for a discussion.
for the point importance. The advantage of this is that it captures each player’s inherent serving ability (which does not vary on a match-to-match basis). This should best represent the importance of each point based on players’ capabilities. However, there are also some significant disadvantages to this method.

The main disadvantage of this method is that it ignores match-specific conditions. This will not account for one player having a unusually good/bad serving performance and may result in some bias in the importance at specific points. For example, consider a match consisting of two players who usually have similar service-winning probabilities. However, one is having a strong serving performance and the other is having a weak serving performance in this particular match. This method would assign similar winning probabilities to both players in this match (because their career averages are similar), whereas the winning probabilities that the players experience are substantially different in reality. As a consequence, the importance will be overstated in this scenario.\(^7\)

This method also suffers a similar problem to the last where players form beliefs based on information that is currently unknown to them. Specifically, matches from later in the sample are used to calculate the winning probabilities for the early matches. However, I deem this to be less of a problem since players have usually played their opponents several times over the course of their career. Consequently, they should have a good idea of their opponent’s inherent serving ability. This is unlike the previous method where it is hard to form accurate assessments of player’s match-level variations until after the match concludes because conditions can vary substantially from day to day. Therefore, this method should map more closely to what the players believe about themselves and their opponents than the previous method.

Another potential issue of this method relates to the selection of matches in the sample. Specifically, each player’s ability is directly calculated from only the observations in the sample. Clearly this may not adequately capture a player’s

\(^7\) Again this is a consequence of the formulation of the importance measure (see Chapter 6 for more details).
ability if there are only a few matches in the sample on which to base the analysis. I circumvent this issue by dropping players who do not have a sufficient number of observations to adequately assess their serving ability. Assuming the sample contains enough observations for each player then I do not believe this will pose a serious problem.

**E.2.2 Comparison of Methods using Tennis Matches**

Next, I examine how the specific values of the point importance vary with the exact specification. In particular, I compare and contrast the preferred specification from the body of the dissertation against the two alternative specifications. To do this, I calculate the importance of each point in four different matches from my sample using the three methods for specifying the service-winning probabilities. This allows for a comparison of how these methods identify pressure situations in each match.

The sample of matches is chosen to contain two long matches and two average matches by the total number of points played.\(^{71}\) The intuition is that longer matches are also generally tighter and so should have more points with higher importance/pressure. This can be contrasted with an average match, which is likely to have fewer points of high importance. The details of the matches used in this comparison are given in Table E.1:

<table>
<thead>
<tr>
<th>Year</th>
<th>Tournament</th>
<th>Rnd</th>
<th>Player 1</th>
<th>Player 2</th>
<th>Winner</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>2015</td>
<td>Wimbledon</td>
<td>SF</td>
<td>Federer</td>
<td>Murray</td>
<td>Federer</td>
<td>7–5, 7–5, 6–4</td>
</tr>
<tr>
<td>2015</td>
<td>Miami</td>
<td>F</td>
<td>Djoković</td>
<td>Murray</td>
<td>Djoković</td>
<td>7–6(^1), 4–6, 6–0</td>
</tr>
<tr>
<td>2012</td>
<td>Aus Open</td>
<td>F</td>
<td>Djoković</td>
<td>Nadal</td>
<td>Djoković</td>
<td>5–7, 6–4, 6–2, 6–7(^3), 7–5</td>
</tr>
<tr>
<td>2009</td>
<td>Aus Open</td>
<td>SF</td>
<td>Verdasco</td>
<td>Nadal</td>
<td>Nadal</td>
<td>6–7(^4), 6–4, 7–6(^2), 6–7(^1), 6–4</td>
</tr>
</tbody>
</table>

The first two rows in Table E.1 correspond to the average matches while the second two rows correspond to the long matches in the sample.

\(^{71}\)The ‘average’ matches were determined by randomly choosing two matches with total number of points approximately equal to the mean of the entire sample of matches.
**Preferred Method**

Figure E.1 illustrates the trajectory of the importance of each point in the four different matches using the preferred method to calculate the service-winning probabilities. To interpret the plots, the $x$-axis denotes the duration (in points) of the match, while the $y$-axis denotes the importance of the point. For example, when the importance is 0.2 this means that the difference between winning and losing this point affects the probability of winning the entire match by 0.2 (20%).

![Figure E.1: Plot of 4 Matches and their Importances (Preferred Method)](image)

The first point to note is that, even in the average matches, there are some points that have significant importance. Although it would be reasonable to expect that the longer matches have more important points, the average match should still have some points scattered throughout the match that have a non-trivial effect on the outcome of the match. This can be observed in Figure E.1a. Both matches have a few obvious spikes in the importance over the course of the match. Observing Figure E.1b, it can also be seen that the longer matches tend to have points with high importance, particularly near the end of the match. Since long matches are generally close, the points near the end of the match are going to have a significant influence on determining the winner and, hence, are likely to have high importance. Therefore, both these plots align closely with how pressure can evolve over the course of a tennis match.
This specification appears to identify trends in the evolution of importance that are similar to the other two specifications (see below). The slight differences between the preferred method and the alternatives arise from correcting the biases in the alternative methods and accounting for any match-specific variation that may be lacking. The result is a method for calculating pressure that captures the persistent trends as well as satisfying a number of criteria that appear reasonable.

**Alternative Method 1**

Figure E.2 plots the importance of the four matches using the Alternative Method 1 to calculate the service-winning probabilities:

![Figure E.2: Plot of 4 Matches and their Importances (Alternative Method 1)](image)

There are many similarities here compared to the preferred method. In general, the peaks and troughs of the importance measure qualitatively match those from the preferred method. However, as mentioned previously, this method has some potential flaws that may bias the values. Specifically, it could downplay the importance of some points in one-sided matches. This would be most noticeable at the start of the match when few points have been played. Comparing this method to the others, this appears to be the case. Most noticeable in Figure E.2a, the absolute values for the importance are generally lower using Alternative Method 1 compared to the other two specifications.
**Alternative Method 2**

Figure E.3 plots the importance of the four matches using the Alternative Method 2 to calculate the service-winning probabilities:

![Figure E.3: Plot of 4 Matches and their Importances (Alternative Method 2)](image)

Similar trends are observed using this method. There is a fair spread of important points in both the long and average matches, with the long matches tending to have more points of high importance. Moreover, this specification avoids the issues of the previous one and should not bias the importance of points in the same fashion. However, it does not take into account any match specific factors, which may also bias the results slightly.

**Discussion**

I have presented three methods for determining the probability that a player wins a point on their serve. Observing Figures E.1–E.3 there is not much difference in the qualitative results from each method. The general shape and trends are similar, and so each of these methods will rank the importance of points similarly within each match. That is, if one point is considered more important than another within a match by using one method, then it will (almost certainly) be assigned a higher importance by another. However, this may not necessarily true across matches. That is, one method may specify a point in one match as more important than
another point in a completely different match, whereas an alternative method may say the opposite.

To obtain a better understanding of how the precise value of the importance of a point varies across matches, I examine the distribution of pressure points for each of the different specifications. Table E.2 below presents the distribution of the importance in ten percentile increments.

Table E.2: Importance of Point at each Percentile

<table>
<thead>
<tr>
<th>Percentile</th>
<th>Preferred Method</th>
<th>Alternative Method 1</th>
<th>Alternative Method 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>10.</td>
<td>0.0016</td>
<td>0.0001</td>
<td>0.0051</td>
</tr>
<tr>
<td>20.</td>
<td>0.0053</td>
<td>0.0012</td>
<td>0.0109</td>
</tr>
<tr>
<td>30.</td>
<td>0.0103</td>
<td>0.0037</td>
<td>0.0171</td>
</tr>
<tr>
<td>40.</td>
<td>0.0164</td>
<td>0.0078</td>
<td>0.0244</td>
</tr>
<tr>
<td>50.</td>
<td>0.0237</td>
<td>0.0137</td>
<td>0.0316</td>
</tr>
<tr>
<td>60.</td>
<td>0.0319</td>
<td>0.0214</td>
<td>0.0396</td>
</tr>
<tr>
<td>70.</td>
<td>0.0419</td>
<td>0.0317</td>
<td>0.0507</td>
</tr>
<tr>
<td>80.</td>
<td>0.0575</td>
<td>0.0473</td>
<td>0.0668</td>
</tr>
<tr>
<td>90.</td>
<td>0.0856</td>
<td>0.0766</td>
<td>0.0974</td>
</tr>
</tbody>
</table>

For example, the preferred method attributes a value of 8.56% of a point at the 90% percentile of importance, while Alternative Method 1 and Alternative Method 2 attribute a value of 7.66% and 9.74% respectively. As discussed previously, there is the possibility for Alternative Method 1 and Alternative Method 2 to underestimate and overestimate the importance of some points, respectively. While not proving this directly, Table E.2 is consistent with this conjecture. As such, the preferred method appears to strike the balance between these two alternative methods. To illustrate, Figures E.4a and E.4b plot the (kernel-smoothed) density and the empirical cumulative distribution of the point importances for each of the three methods.
Figure E.4 confirms the assertion that the preferred method provides a middle-ground between the other two specifications. Both the probability density and the cumulative density functions show that distribution of the preferred method lies between the two alternative methods. This provides visual evidence that this method does a reasonable job at correcting any possible underestimation in Alternative Method 1 and overestimation in Alternative Method 2.

Finally, I can also examine the correlation between the different methods by measuring the extent of the overlap of points that each classify as high pressure. The next discussion addresses this issue.

E.2.3 Overlap of Methods

To further investigate the effect that the different specifications have on the importance of points in a given match, I examine how each ranks the top 20% points in each match by importance. By studying the overlap between this set of points for each of the different methods, I can obtain the extent of the agreement between the various specifications. This provides a robustness check, since high levels of overlap imply that the level of pressure predicted by each method does not vary substantially, at least within each match. As mentioned previously, it is natural that each method should have some variations across matches. Therefore, this discussion is restricted to studying the within-match variation.
To inspect the within-match variation between the specifications, I examine the mean percentage of the overlap between the preferred method and the alternative methods for the points in the top 20% of importance for each match. Moreover, I examine the overlap for the preferred method and the alternatives by excluding matches with lower average point importances. Excluding these matches should remove some bias resulting from the one-sided matches. This arises from the fact that these matches tend to be comprised of points with low importances and so the overlap will likely be lower due to the greater noise in the small importance values. Specifically, the restricted sample consists only the matches with their match average point importance in the top 50% across all matches.

Table E.3 presents the mean overlap of the points in the top 20\textsuperscript{th} percentile between the preferred method and the two alternative methods for both the full sample and the restricted sample. The standard deviations are presented in parentheses.

<table>
<thead>
<tr>
<th>Sample</th>
<th>Alt. Method 1</th>
<th>Alt. Method 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full</td>
<td>73.2%</td>
<td>75.0%</td>
</tr>
<tr>
<td></td>
<td>(20.1%)</td>
<td>(21.2%)</td>
</tr>
<tr>
<td>Restricted</td>
<td>81.2%</td>
<td>84.9%</td>
</tr>
<tr>
<td></td>
<td>(14.1%)</td>
<td>(11.7%)</td>
</tr>
</tbody>
</table>

Table E.3 shows that, on average, approximately three quarters of the points classified in the top 20\% of importance by the preferred method are also classified similarly by the alternative methods and further increases to over 80\% when examining the restricted sample. The high levels of overlap suggest that the importance measure is relatively robust to the exact specification of the method chosen to calculate the probabilities.

Given the discussion in this appendix, I believe the preferred method offers a fair representation of how pressure may manifest in a tennis match and so it is used to calculate the measure for pressure in the body of the thesis.
Appendix F

Appendices for Chapter 7

F.1 Robustness of Results

The body of this thesis presents an analysis of the effect of pressure by estimating a linear probability model (LPM) for four specifications for decision-making and one outcome specification. In this discussion I examine some alternative models to provide a robustness check to the key results in Chapter 7. The results in question are whether pressure has a prevalent effect on players’ decision-making and their outcomes, and whether these two effects are linked.

I address these questions using three approaches. The first uses the probit model instead of the LPM, which can have some potential issues for estimating binary outcomes. The second uses the 90th percentile (equating to an importance value of 0.0855) as the cut-off for defining pressure points. Finally, the third uses the importance from Chapter 6 as the (continuous) pressure variable.

First, the LPM can produce predicted probabilities outside the unit interval. This is of little concern here because the probabilities are not the focus of the research. Moreover, most players serve W approximately half of the time (β0 ≈ 0.5) and so the predicted probabilities rarely fall outside of this interval. Second, the errors terms are heteroscedastic and not normally distributed, so the standard errors are not exactly correct. Heteroscedastic errors can be controlled for by using heteroscedasticity-consistent (HC) robust standard errors (Eicker, 1967; Huber, 1967; White, 1980). Non-normality can be problematic for statistical inference; however it can be insignificant in practice (Hellevik, 2009), particularly in large samples (Chatla & Shmueli, 2017). On the other hand, due to the ‘incidental parameters problem’ first examined by Neyman & Scott (1948), care must be taken when examining nonlinear models with fixed-effects where the estimators can be biased and inconsistent (Greene, 2004). Therefore, while solving some issues, the use of nonlinear models also gives rise to others. This has led some to argue that there is much to say for the simplicity and transparency of using the LPM despite its shortcomings (see e.g., Angrist & Pischke, 2012).
Appendix: The Effect of Pressure

The first question to investigate is whether pressure affects players’ decision-making. Thus, I re-estimate the decision-making regressions from Section 7.1 and use the estimates to test Hypothesis 3. Table F.1 shows the results of the test of the alternative Hypotheses 3a and 3b, and is interpreted in the same manner as Tables 7.2 and 7.3. The results strongly suggest that there is evidence that pressure affects player’s service decisions for all alternative models. In all instances the effects of pressure appear more frequently in the sample than would occur randomly. This is consistent with the results in the body of the dissertation.

The second question to investigate is whether pressure affects player’s chances of winning a service point. Thus, I re-estimate the outcome regression from Section 7.2 to test Hypothesis 4. Table F.2 shows the results of the test of this hypothesis and is interpreted in the same manner as Table 7.12. The results of the probit model are the same as the results in the body of the thesis that pressure affects the outcomes of a significant number of players. The results are a little less convincing for the $p_{90}$ model, but this should not be too surprising. Extreme pressure is known to affect individuals and so if the definition of a pressure point becomes less extreme then this may dilute some of the observed effect that pressure will have on players’ outcomes. Finally, knowing that the KS-test has the potential to underestimate the significance of the effect in the tails of the distribution, the results from the continuous pressure specification also suggest that pressure has an effect on a substantial number of players in the sample.

The final question to study is whether there is a link between the effect of pressure on decision-making and outcomes. Thus, I re-examine the correlation between the decision-making and the outcome estimates. Table F.3 presents the correlation coefficients for the decision and outcome variables and is interpreted in the same manner as Table 7.14. The observed correlations are typically small (insignificant) and negative. This is consistent with the results from the body of the thesis that there is no definitive link between the effect that pressure has on players’ service choices and the effect that it has on their chances of winning the point.
### Table F.1: Analysis of Decision-making Coefficients (Robust)

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{p}$</td>
<td>0.1565***</td>
<td>0.1973***</td>
<td>0.2361***</td>
<td>0.2569***</td>
</tr>
<tr>
<td>$\beta_p$</td>
<td>0.1808***</td>
<td>0.1733***</td>
<td>0.2109***</td>
<td>0.2482***</td>
</tr>
<tr>
<td>Area</td>
<td>0.0970***</td>
<td>0.0966***</td>
<td>0.1177***</td>
<td>0.1269***</td>
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<table>
<thead>
<tr>
<th>Probit</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{p}$</td>
<td>0.1667***</td>
<td>0.1667***</td>
<td>0.3750***</td>
<td>0.3750***</td>
</tr>
<tr>
<td>$\beta_{i_S}$</td>
<td>0.1313**</td>
<td>0.1313**</td>
<td>0.3362***</td>
<td>0.3362***</td>
</tr>
<tr>
<td>Area</td>
<td>0.0640***</td>
<td>0.0640***</td>
<td>0.1477***</td>
<td>0.1477***</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$p_{90}$</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{p}$</td>
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<td>0.1313***</td>
<td>0.1250***</td>
<td>0.1250***</td>
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<tr>
<td>$\beta_p$</td>
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<td>0.1153**</td>
<td>0.1462***</td>
<td>0.1464***</td>
</tr>
<tr>
<td>Area</td>
<td>0.0513**</td>
<td>0.0513**</td>
<td>0.0825***</td>
<td>0.0820***</td>
</tr>
</tbody>
</table>

<table>
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</thead>
<tbody>
<tr>
<td>$\hat{p}$</td>
<td>0.2099***</td>
<td>0.2099***</td>
<td>0.1543***</td>
<td>0.1543***</td>
</tr>
<tr>
<td>$\beta_p$</td>
<td>0.1922***</td>
<td>0.1922***</td>
<td>0.1955***</td>
<td>0.1955***</td>
</tr>
<tr>
<td>Area</td>
<td>0.1132***</td>
<td>0.1132***</td>
<td>0.1263***</td>
<td>0.1263***</td>
</tr>
</tbody>
</table>

*** Indicates rejection at the 1-percent level of significance.
** Indicates rejection at the 5-percent level of significance.
* Indicates rejection at the 10-percent level of significance.
Table F.2: Prevalence of Pressure Effects on Outcome (Robust)

<table>
<thead>
<tr>
<th></th>
<th>Binomial Test</th>
<th>$U[0, 1]$ Test</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\hat{p}$</td>
<td>KS</td>
</tr>
<tr>
<td>Probit</td>
<td>0.1419***</td>
<td>0.2050***</td>
</tr>
<tr>
<td>$p_{90}$</td>
<td>0.0938**</td>
<td>0.0847</td>
</tr>
<tr>
<td>Continuous</td>
<td>0.1111***</td>
<td>0.0955*</td>
</tr>
</tbody>
</table>

*** Indicates rejection at the 1-percent level of significance.
** Indicates rejection at the 5-percent level of significance.
* Indicates rejection at the 10-percent level of significance.

Table F.3: Correlation between Outcome and Decision-making Estimates (Robust)

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\hat{\rho}_j$</td>
<td>Base</td>
<td>SC</td>
<td>PC</td>
</tr>
<tr>
<td>Probit</td>
<td>Mix</td>
<td>-0.109</td>
<td>0.015</td>
<td>-0.078</td>
</tr>
<tr>
<td></td>
<td>SC</td>
<td>0.036</td>
<td>0.040</td>
<td></td>
</tr>
<tr>
<td></td>
<td>PC</td>
<td></td>
<td>-0.042</td>
<td>-0.137</td>
</tr>
<tr>
<td>$p_{90}$</td>
<td>Mix</td>
<td>0.070</td>
<td>0.053</td>
<td>-0.045</td>
</tr>
<tr>
<td></td>
<td>SC</td>
<td>0.000</td>
<td></td>
<td>-0.684*</td>
</tr>
<tr>
<td></td>
<td>PC</td>
<td></td>
<td>-0.443</td>
<td>-0.592*</td>
</tr>
<tr>
<td>Continuous</td>
<td>Mix</td>
<td>-0.225</td>
<td>-0.099</td>
<td></td>
</tr>
<tr>
<td></td>
<td>SC</td>
<td></td>
<td>-0.328</td>
<td></td>
</tr>
</tbody>
</table>

*** Indicates rejection at the 1-percent level of significance.
** Indicates rejection at the 5-percent level of significance.
* Indicates rejection at the 10-percent level of significance.
F.2 Delta Method

This discussion outlines the key derivations required to calculate the total effect of the variables of interest. This enables direct calculation of the prevalence of the behavioural deviations discussed in Section 7.1. These include go-to deviations, serial correlation deviations and pressure correlation deviations. In the simplest specifications these are equivalent to the coefficients in the regression models. However, in the more complex specifications, where there are multiple interactions between the terms, these coefficients only give the partial effect of these variables. As such, the delta method provides a means to examine the total effects as a function of the partial effects.

F.2.1 Go-to Deviations

First, I outline the calculations required to determine the total effect of pressure on players’ service decisions, which is equivalent to an expression for the go-to deviations. This is only required for specifications (2) – (4), since the total effect is the same as the partial effect for the Base specification. For clarity, I drop the server-court superscripts and ignore the fixed effects (which can be thought of as part of the controls $X_t$).

**Specification: Serial Correlation**

Consider the Serial Correlation regression for each server-court given by:

$$d_t = \beta_p P_t + \beta_1 d_{t-1} + \beta_{is} P_t \times d_{t-1} + \Gamma X_t + \varepsilon_t$$  \hspace{1cm} (F.1)

The effect of pressure on service direction is given by:

$$\frac{\Delta E[d_t]}{\Delta P_t} = \beta_p + \beta_{is} E[d_{t-1}]$$  \hspace{1cm} (F.2)

Using equation (F.1), it is possible to find an expression for $E[d_t]$, and hence $E[d_{t-1}]$:

$$E[d_t] = \beta_p E[P_t] + \beta_1 E[d_{t-1}] + \beta_{is} E[P_t \times d_{t-1}] + \Gamma E[X_t]$$
Appendix: The Effect of Pressure

Assuming that pressure points occur i.i.d. with probability $q$, this simplifies to:

$$E[d_t] = q\beta_p + \beta_l E[d_{t-1}] + q\beta_{ls} E[d_{t-1}] + \Gamma E[X_t]$$

This process must be stationary, hence $E[d_t] = E[d_{t-1}] = \mu$ and this gives:

$$\mu = q\beta_p + \beta_l \mu + q\beta_{ls} \mu + \Gamma E[X_t]$$

Solving gives:

$$\mu = \frac{q\beta_p + \Gamma E[X_t]}{1 - \beta_l - q\beta_{ls}}$$

Using this it is possible to obtain an expression for (F.2), the total effect of pressure on direction:

$$\frac{\Delta E[d_t]}{\Delta P_t} = \beta_p + \beta_{ls} \left\{ \frac{q\beta_p + \Gamma E[X_t]}{1 - \beta_l - q\beta_{ls}} \right\}$$

$$= \frac{\beta_p (1 - \beta_l) + \beta_{ls} \Gamma E[X_t]}{1 - \beta_l - q\beta_{ls}}$$

This expression gives the function $h(\beta)$ for pressure in the Serial Correlation model.

**Specification: Pressure Correlation**

Consider the Pressure Correlation regression for each server-court given by:

$$d_t = \beta_p P_t + \beta_l d_{t-1} + \beta_{ls} P_t \times d_{tP} + \Gamma X_t + \epsilon_t \quad \text{(F.3)}$$

The effect of pressure on service direction is given by:

$$\frac{\Delta E[d_t]}{\Delta P_t} = \beta_p + \beta_{lp} E[d_{tP}] \quad \text{(F.4)}$$

Using equation (F.3) it is possible to find an expression for $E[d_{tP}]$:

$$E[d_t | P_t = 1] = \beta_p + \beta_l E[d_{t-1}] + \beta_{lp} E[d_{tP}] + \Gamma E[X_t]$$

Since this process is stationary, $E[d_t | P_t = 1] = E[d_{tP}] = \mu_p$ and:

$$\mu_p = \beta_p + \beta_l E[d_{t-1}] + \beta_{lp} \mu_p + \Gamma E[X_t] + \sum_{i=1}^{R} \lambda_i \Pr[r = i]$$
Solving gives:

\[ \mu_p = \frac{\beta_p + \beta_t E[d_{t-1}] + \Gamma E[X_t]}{1 - \beta_{i_p}} \]

Again, using (F.3) to find an expression for \( E[d_t] \):

\[ E[d_t] = \beta_p E[P_t] + \beta_t E[d_{t-1}] + \beta_{i_p} E[P_t \times d_{tL}^P] + \Gamma E[X_t] \]

Assuming that pressure points occur i.i.d. with probability \( q \), then this equation simplifies to:

\[ E[d_t] = q\beta_p + \beta_t E[d_{t-1}] + q\beta_{i_p} E[d_{tL}^P] + \Gamma E[X_t] \]

This process is also stationary, hence \( E[d_t] = E[d_{t-1}] = \mu \). Using this, along with \( E[d_{tL}^P] = \mu_p \), gives:

\[ \mu = q\beta_p + \beta_t \mu + q\beta_{i_p} \mu_p + \Gamma E[X_t] \]

Solving gives:

\[ \mu = \frac{q\beta_p + q\beta_{i_p} \mu_p + \Gamma E[X_t]}{1 - \beta_t} \]

Substituting this into the equation for \( \mu_p \), gives:

\[ \mu_p = \frac{\beta_p + \beta_t \left( \frac{q\beta_p + q\beta_{i_p} \mu_p + \Gamma E[X_t]}{1 - \beta_t} \right) + \Gamma E[X_t]}{1 - \beta_{i_p}} \]

Again, solving to give:

\[ \mu_p = \frac{[1 - (1 - q)\beta_i] \beta_p + \Gamma E[X_t]}{1 - \beta_t - \beta_{i_p} + (1 - q)\beta_t \beta_{i_p}} \]

Therefore, this gives an expression for (F.4), the total effect of pressure on direction:

\[ \frac{\Delta E[d_t]}{\Delta P_t} = \beta_p + \beta_{i_p} \left\{ \frac{[1 - (1 - q)\beta_i] \beta_p + \Gamma E[X_t]}{1 - \beta_t - \beta_{i_p} + (1 - q)\beta_t \beta_{i_p}} \right\} \]

\[ = \frac{\beta_p (1 - \beta_t) + \beta_{i_p} \Gamma E[X_t]}{1 - \beta_t - \beta_{i_p} + (1 - q)\beta_t \beta_{i_p}} \]

This expression gives the function \( h(\beta) \) for pressure in the Pressure Correlation model.
**Specification: Multiple Interaction**

Consider the Multiple Interaction regression for each server-court given by:

\[ d_t = \beta_p P_t + \beta_l d_{t-1} + \beta_i s P_t \times d_{t-1} + \beta_i P_t \times d_{t-1}^P + \Gamma X_t + \varepsilon_t \]  

(F.5)

The effect of pressure on direction (i.e., the strength of the go-to deviation) is given by

\[ \frac{\Delta E[d_t]}{\Delta P_t} = \beta_p + \beta_i s E[d_{t-1}] + \beta_i P E[d_{t-1}^P] \]  

(F.6)

Using equation (F.5), it is possible to find an expression for \( E[d_t^P] \):

\[ E[d_t | P_t = 1] = \beta_p + \beta_l E[d_{t-1}] + \beta_i s E[d_{t-1}] + \beta_i P E[d_{t-1}^P] + \Gamma E[X_t] \]

This process is stationary, hence \( E[d_t | P_t = 1] = E[d_t^P] = \mu_p \) and:

\[ \mu_p = \beta_p + (\beta_l + \beta_i s) E[d_{t-1}] + \beta_i P \mu_p + \Gamma E[X_t] \]

Solving gives:

\[ \mu_p = \frac{\beta_p + (\beta_l + \beta_i s) E[d_{t-1}] + \Gamma E[X_t]}{1 - \beta_i P} \]

Again, using (F.5) to find an expression for \( E[d_{t-1}] \):

\[ E[d_t] = \beta_p E[P_t] + \beta_l E[d_{t-1}] + \beta_i s E[P_t \times d_{t-1}] + \beta_i P E[P_t \times d_{t-1}^P] + \Gamma E[X_t] \]

Assuming that pressure points occur i.i.d. with probability \( q \), then this equation simplifies to:

\[ E[d_t] = q \beta_p + (\beta_l + q \beta_i s) E[d_{t-1}] + q \beta_i P E[d_{t-1}^P] + \Gamma E[X_t] \]

This process is also stationary, hence \( E[d_t] = E[d_{t-1}] = \mu \). Using this, along with \( E[d_{t-1}^P] = \mu_p \), gives:

\[ \mu = q \beta_p + (\beta_l + q \beta_i s) \mu + q \beta_i P \mu_p + \Gamma E[X_t] \]

Solving gives:

\[ \mu = \frac{q \beta_p + q \beta_i P \mu_p + \Gamma E[X_t]}{1 - \beta_l - q \beta_i s} \]
Substituting this into the equation for \( \mu_p \), gives:

\[
\mu_p = \frac{\beta_p + (\beta_l + \beta_{is})}{1 - \beta_{ip}} \left( \frac{q\beta_p + q\beta_{ip}\mu_p + \Gamma E[X_t]}{1 - \beta_l - q\beta_{ip}} \right) + \Gamma E[X_t]
\]

Again, solving to give:

\[
\mu_p = \frac{[1 - (1 - q)\beta_l]\beta_p + [1 + (1 - q)\beta_{is}]\Gamma E[X_t]}{1 - \beta_l - q\beta_{is} - \beta_{ip} + (1 - q)\beta_l\beta_{ip}}
\]

Substituting this back into the equation for \( \mu \), gives:

\[
\mu = \frac{q\beta_p + q\beta_{ip} \left( \frac{[1-(1-q)\beta_l]\beta_p + [1 + (1 - q)\beta_{is}]\Gamma E[X_t]}{1 - \beta_l - q\beta_{is} - \beta_{ip} + (1 - q)\beta_l\beta_{ip}} \right)}{1 - \beta_l - q\beta_{is}} + \Gamma E[X_t]
\]

Finally, solving to get:

\[
\mu = \frac{q\beta_p + [1 + (1 - q)\beta_{ip}]\Gamma E[X_t]}{1 - \beta_l - q\beta_{is} - \beta_{ip} + (1 - q)\beta_l\beta_{ip}}
\]

Therefore, this gives an expression for (F.6), the total effect of pressure on direction:

\[
\frac{\Delta E[d_t]}{\Delta P_t} = \beta_p + \beta_{is} \left\{ \frac{q\beta_p + [1 + (1 - q)\beta_{ip}]\Gamma E[X_t]}{1 - \beta_l - q\beta_{is} - \beta_{ip} + (1 - q)\beta_l\beta_{ip}} \right\} + \beta_{ip} \left\{ \frac{[1 - (1 - q)\beta_l]\beta_p + [1 + (1 - q)\beta_{is}]\Gamma E[X_t]}{1 - \beta_l - q\beta_{is} - \beta_{ip} + (1 - q)\beta_l\beta_{ip}} \right\}
\]

\[
= \frac{\beta_p(1 - \beta_l) + [\beta_{is} + \beta_{ip} - 2(1 - q)\beta_{is}\beta_{ip}]\Gamma E[X_t]}{1 - \beta_l - q\beta_{is} - \beta_{ip} + (1 - q)\beta_l\beta_{ip}}
\]

This expression gives the function \( h(\beta) \) for pressure in the Multiple Interaction model.

### F.2.2 Serial Correlation Deviations

Next, I outline the calculations required to determine the total effect of pressure on the serial correlation in players’ service choices, which is equivalent to an expression for the serial correlation deviations. This is only required for specification (4), since the total effect is the same as the partial effect for the Serial Correlation specification.

For clarity, I drop the server-court superscripts and ignore the fixed effects (which can be thought of as part of the controls \( X_t \)).
Appendix: The Effect of Pressure

**Specification: Multiple Interaction**

Consider the Multiple Interaction regression for each server-court given by:

\[ d_t = \beta_p P_t + \beta_t d_{t-1} + \beta_{is} P_t \times d_{t-1} + \beta_{ip} P_t \times d_{LP}^t + \Gamma X_t + \varepsilon_t \]

The effect of pressure on serial correlation is given by the total difference:

\[
\frac{\Delta}{\Delta P_t} \mathbb{E}\left[ \frac{\Delta \mathbb{E}[d_t|P_t]}{\Delta d_{t-1}} \right]
\]

For clarity I will switch to the derivative operator, \( d \), for the correlation term instead of the difference operator, \( \Delta \). This is the continuous variable analogue to the difference operator and is simply used to make the following steps clearer. With this notation, the effect of pressure on serial correlation is:

\[
\frac{\Delta}{\Delta P_t} \mathbb{E}\left[ \frac{d \mathbb{E}[d_t|P_t]}{d d_{t-1}} \right] = \frac{\Delta}{\Delta P_t} \mathbb{E}\left[ \frac{\partial \mathbb{E}[d_t|P_t]}{\partial d_{t-1}} + \frac{\partial \mathbb{E}[d_t|P_t]}{\partial d_{LP}^t} \frac{\partial d_{LP}^t}{\partial d_{t-1}} \right]
\]

where the equality comes from the definition of a total derivative. In essence, this is the change in the total derivative of the expectation of the current serve (given that it is under pressure) on the previous serve. Furthermore, this gives the following expressions:

\[
\frac{\partial \mathbb{E}[d_t|P_t]}{\partial d_{t-1}} = \beta_t + \beta_{is} P_t, \quad \frac{\partial \mathbb{E}[d_t|P_t]}{\partial d_{LP}^t} = \beta_{ip} P_t, \quad \frac{\partial d_{LP}^t}{\partial d_{t-1}} = P_{t-1}
\]

Substituting these back into (F.7), gives the expression for the total effect of pressure on serial correlation:

\[
\frac{\Delta}{\Delta P_t} \mathbb{E}\left[ \frac{d \mathbb{E}[d_t|P_t]}{d d_{t-1}} \right] = \frac{\Delta}{\Delta P_t} \mathbb{E}\left[ \frac{d \mathbb{E}[d_t|P_t]}{d d_{t-1}} \right] = \beta_{is} + q \beta_{ip}
\]

This expression gives the function \( h(\beta) \) for serial correlation in the Multiple Interaction model.
F.2.3 Pressure Correlation Deviations

Finally, I outline the calculations required to determine the total effect of pressure on the correlation in players’ service choices under pressure, which is equivalent to an expression for the pressure correlation deviations. This is only required for specification (4), since the total effect is the same as the partial effect for the Pressure Correlation specification. For clarity, I drop the server-court superscripts and ignore the fixed effects (which can be thought of as part of the controls $X_t$).

**Specification: Multiple Interaction**

Consider the Multiple Interaction regression for each server-court given by:

$$d_t = \beta_p P_t + \beta_t d_{t-1} + \beta_{is} P_t \times d_{t-1} + \beta_{ir} P_t \times d_{LP} + \Gamma X_t + \varepsilon_t$$

The pressure correlation effect is given by the total difference:

$$\Delta \Delta P_t E\left[ \frac{\Delta E[d_t|P_t]}{\Delta d_{LP}} \right]$$

For the sake of clarity, I use the derivative operator for the correlation term instead of the difference operator, $\Delta$. Doing so the effect on pressure correlation is:

$$\frac{\Delta}{\Delta P_t} E\left[ \frac{\Delta E[d_t|P_t]}{\Delta d_{LP}} \right] = \frac{\Delta}{\Delta P_t} E\left[ \frac{\partial E[d_t|P_t]}{\partial d_{LP}} + \frac{\partial E[d_t|P_t]}{\partial d_{t-1}} \frac{\partial d_{t-1}}{\partial d_{LP}} \right]$$

(F.8)

where the equality comes from the definition of a total derivative. This is the change in the total derivative of the expectation of the current serve (under pressure) on the previous pressure serve. Moreover, this gives the following expressions:

$$\frac{\partial E[d_t|P_t]}{\partial d_{LP}} = \beta_{ir} P_t, \quad \frac{\partial E[d_t|P_t]}{\partial d_{t-1}} = \beta_t + \beta_{is} P_t, \quad \frac{\partial d_{t-1}}{\partial d_{LP}} = P_{t-1}$$

Substituting these into (F.8), gives the total effect on pressure correlation:

$$\frac{\Delta}{\Delta P_t} E\left[ \frac{\Delta E[d_t|P_t]}{\Delta d_{t-1}} \right] = \frac{\Delta}{\Delta P_t} E\left[ \beta_{ir} P_t + (\beta_t + \beta_{is} P_t) P_{t-1} \right]$$

$$= q \beta_{is} + \beta_{ir}$$

as the function $h(\beta)$ for pressure correlation in the Multiple Interaction model.
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