Experimental investigation of velocity and vorticity in turbulent wall flows

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A thesis submitted in total fulfilment of the requirements of the degree of Doctor of Philosophy

April 2019
Declaration of Authorship

This is to certify that:

- this thesis comprises only my original work except where indicated,
- due acknowledgement has been made in the text to all other material used,
- this thesis is fewer than 100,000 words in length, exclusive of tables, bibliographies and appendices.

Spencer J. Zimmerman

April 2019
Abstract

This thesis details the results of a research effort both to acquire sufficiently-resolved velocity and vorticity vector time-series in turbulent wall-bounded flows, as well as to use the acquired data to juxtapose two canonical wall-bounded flows: the zero-pressure-gradient boundary layer and the fully-developed pipe. Towards these ends, a novel configuration of measurement sensors has been designed, evaluated, and deployed in three of the largest canonical wall-flow facilities in existence: the Flow Physics Facility (FPF) at the University of New Hampshire, the High Reynolds Number Boundary Layer Wind Tunnel (HRNBLWT) at the University of Melbourne, and the Centre for International Cooperation in Long Pipe Experiments (CICLoPE) at the University of Bologna (Università di Bologna). The datasets presented herein are the first to contain simultaneously-acquired velocity and vorticity statistics in both pipe and boundary layer flows under matched conditions.

The capacity of the measurement probe to resolve the velocity and vorticity vectors under idealised conditions is evaluated via ‘synthetic experiments’, whereby the response of the probe to a simulated turbulent flow is modeled, and the resulting aggregate statistics compared to those of the known input. The synthetic experimental results are then compared to statistics obtained from physical experiments, and show close agreement for most quantities despite differences in Reynolds number. Disagreement between the physical and synthetic experimental results in several quantities is used to diagnose a limitation of the present sensor system. An awareness of the measurement capabilities (and limitations) afforded by the comparison between the synthetic and physical experiments pervades the ensuing analysis and discussion of additional physical experimental results.

Normalised statistical moments (up to the kurtosis) of velocity and vorticity are presented for both pipe and boundary layer cases. The two flows are shown to exhibit virtually no differences from one another wallward of the wake region aside from the transverse velocity component variances. The boundary layer
wake is characterised by higher turbulence enstrophy and turbulence kinetic energy (TKE) than the pipe, despite containing both turbulent and non-turbulent states. Although there is no ‘free-stream’ in the fully-developed pipe, a significant time-fraction of the flow can be described as ‘quasi-irrotational’ near the centre-line. This results in a departure of the velocity and vorticity kurtosis (and, when not identically zero, skewness) from Gaussian behaviour in the outer region of the pipe, as is known to occur in the boundary layer (owing to turbulent/non-turbulent intermittency).

Spectral properties of the velocity and vorticity signals acquired in both flows are examined, both for their own content as well as to compare the two flows. Despite very little difference in the observed streamwise turbulence kinetic energy, the contributions to the total differ considerably by scale between the two flows. The pipe is observed to contain more streamwise TKE than the boundary layer in scales longer than 10 times the outer length scale $\delta$, while the opposite is true for scales shorter than $10\delta$. This ‘crossover’ scale also applies to the spectra of the transverse velocity components and Reynolds shear stress. The measured enstrophy spectrum is shown to be approximately invariant under Kolmogorov normalisation for wall-distances greater than about 200 viscous-lengths. Finally, local isotropy and axisymmetry in the velocity components are evaluated. Local isotropy is observed to be satisfied over a range of scales for which the characteristic timescale of inertial energy transfer is expected to be small relative to the timescale of the mean strain rate. The relationship between the transverse velocities and the streamwise velocity, however, does not appear to approach isotropy along the pipe centreline, suggesting that the bounding wall also plays a role in imposing a sense of direction on the turbulence. Indeed, the aforementioned timescale criterion is shown to identify a cutoff scale that increases proportionally with wall-distance, confounding a conclusive statement regarding the primary source of anisotropy away from the centreline.
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## Nomenclature

### Acronyms

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<tr>
<td>AEH</td>
<td>Attached Eddy Hypothesis</td>
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<tr>
<td>CICLoPE</td>
<td>Center for International Cooperation in Long Pipe Experiments</td>
</tr>
<tr>
<td>DNS</td>
<td>Direct Numerical Simulation</td>
</tr>
<tr>
<td>FPF</td>
<td>Flow Physics Facility</td>
</tr>
<tr>
<td>HRNBLWT</td>
<td>High Reynolds Number Boundary Layer Wind Tunnel</td>
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<tr>
<td>LDV</td>
<td>Laser Doppler Velocimetry</td>
</tr>
<tr>
<td>LSM</td>
<td>Large-Scale-Motion</td>
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<tr>
<td>MEMS</td>
<td>Micro-Electro-Mechanical System</td>
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<td>MTV</td>
<td>Molecular Tagging Velocimetry</td>
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<tr>
<td>MUCTA</td>
<td>Melbourne University Constant Temperature Anemometer</td>
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<td>MVP</td>
<td>Mean Velocity Profile</td>
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<td>MWT</td>
<td>Melbourne Wind Tunnel</td>
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<td>NSTAP</td>
<td>Nano-Scale Thermal Anemometry Probe</td>
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<td>PIV</td>
<td>Particle Image Velocimetry</td>
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<td>PTV</td>
<td>Particle Tracking Velocimetry</td>
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<tr>
<td>RSS</td>
<td>Reynolds Shear Stress</td>
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<tr>
<td>TBL</td>
<td>Turbulent Boundary Layer</td>
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<tr>
<td>TKE</td>
<td>Turbulence Kinetic Energy</td>
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<tr>
<td>TNTI</td>
<td>Turbulent/Non-turbulent Interface</td>
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<tr>
<td>VLSM</td>
<td>Very-Large-Scale-Motion</td>
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<td>ZPG</td>
<td>Zero Pressure Gradient</td>
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Notation

\( \cdot \)_i \quad \text{Denotes a vector component pointing in the } x_i \text{ unit direction}

\( \overline{\cdot} \) \quad \text{Denotes a time-averaged quantity}

\( \cdot \)_{\text{rms}} \quad \text{Denotes the root-mean-square of a quantity}

\( \cdot \)\(^+\) \quad \text{Denotes nondimensionalisation by the viscous scales } u_\tau \text{ and } l_\nu

\( \cdot \)_{N,T,B} \quad \text{Denotes a vector component in the direction (N)ormal, (T)angential, or (B)i-normal to a hotwire element}

\( \cdot \)^m \quad \text{Denotes a (m)easured quantity in Chapter 4}

\( \cdot \)\( \hat{\cdot} \) \quad \text{Denotes Fourier coefficients of } (\cdot)

\( \cdot \)^* \quad \text{Denotes the complex conjugate of } (\cdot)

\( \cdot \)_{B,P} \quad \text{Denotes a quantity computed from (B)oundary layer or (P)ipe data, not to be confused with } (\cdot)_{N,T,B} \text{ coordinates above}

Greek

\( \alpha \) \quad \text{Effective cooling angle of a hotwire element}

\( \beta \) \quad \text{Magnitude of hotwire effective angle relative to mean flow } (\beta = 45^\circ \text{ unless otherwise specified})

\( \Delta x_i \) \quad \text{Probe sub-array separation (or, for } \Delta x_1, \text{ equivalent separation) in the } x_i\text{-direction}

\( \delta \) \quad \text{Domain thickness—} \delta_{99} \text{ and/or } R, \text{ where appropriate}

\( \delta_{99} \) \quad \text{Boundary layer thickness based on } 99\% \text{ of } U_\infty \ (U_1(\delta_{99}) = 0.99 \cdot U_\infty)

\( \delta_{ij} \) \quad \text{Kronecker delta}

\( \epsilon_{ijk} \) \quad \text{Alternating delta tensor}

\( \varepsilon \) \quad \text{Mean dissipation rate of turbulence kinetic energy}

\( \varepsilon_h \) \quad \text{Pseudo-dissipation rate, or homogenous dissipation rate, of turbulence kinetic energy}

\( \varepsilon_{iso} \) \quad \text{Isotropic approximation estimation of mean dissipation rate (4.24)}

\( \varepsilon_{hyb} \) \quad \text{Hybrid homogeneous/inhomogeneous approximation of mean dissipation rate}

\( \eta \) \quad \text{Kolmogorov length scale } (\eta \equiv (\nu^3/\varepsilon)^{1/4})

\( \kappa \) \quad \text{Wavenumber vector}

\( \kappa_i \) \quad \text{Wavenumber vector component in the } x_i \text{ unit direction}
\( \kappa \) \hspace{1cm} \text{Wavenumber vector magnitude} \left( \kappa \equiv (\kappa_1^2 + \kappa_2^2 + \kappa_3^2)^{1/2} \right)

\( \kappa_{1,SI}(n) \) \hspace{1cm} \text{Streamwise wavenumber at which mean strain rate timescale } \tau_S \text{ is } n \text{ times faster than the inertial timescale } \tau_I

\( \Lambda_{u,i} \) \hspace{1cm} \text{Streamwise wavelength corresponding to maximum magnitude of} \ k_1 E_{ij} \text{ (function of wall-distance)}

\( \Lambda_\omega \) \hspace{1cm} \text{Streamwise wavelength corresponding to maximum magnitude of} \ k_1 W \text{ (function of wall-distance)}

\( \lambda_T \) \hspace{1cm} \text{Anisotropic Taylor microscale} \left( \lambda_T \equiv (10 \nu \kappa^2 / \varepsilon)^{1/2} \right)

\( \lambda_{TI} \) \hspace{1cm} \text{Isotropic Taylor microscale} \left( \lambda_{TI} \equiv \left( \frac{u_1^2}{\partial u_1 / \partial x_1^2} \right)^{1/2} \right)

\( \lambda_i \) \hspace{1cm} \text{Wavelength in the } x_i \text{ unit direction}

\( \lambda_t \) \hspace{1cm} \text{Temporal wavelength} \lambda_t \equiv 2\pi / \omega

\( \lambda_{1,SI}(n) \) \hspace{1cm} \text{Streamwise wavelength at which mean strain rate timescale } \tau_S \text{ is } n \text{ times faster than the inertial timescale } \tau_I

\( \mu \) \hspace{1cm} \text{Dynamic viscosity}

\( \nu \) \hspace{1cm} \text{Kinematic viscosity} \left( \nu \equiv \mu / \rho \right)

\( \rho \) \hspace{1cm} \text{Fluid density}

\( \varrho \) \hspace{1cm} \text{Correlation coefficient}

\( \tau_I \) \hspace{1cm} \text{Inertial timescale}

\( \tau_S \) \hspace{1cm} \text{Mean strain rate timescale} \left( \tau_S \equiv \left( \frac{1}{2} \partial U_1 / \partial x_2 \right)^{-1} \right)

\( \tau_t \) \hspace{1cm} \text{Total shear stress} \left( \tau_t \equiv \tau_w - \rho \overline{u_1 u_2} \right)

\( \tau_w \) \hspace{1cm} \text{Wall shear stress} \left( \mu \partial U_1 / \partial x_2 \bigg|_{x_2=0} \right)

\( \Phi_{ij}(\kappa) \) \hspace{1cm} \text{Velocity spectrum tensor}

\( \phi_{u_i,j}(\kappa_1) \) \hspace{1cm} \text{One-dimensional (streamwise) spectrum of the velocity derivative} \ \partial u_i / \partial x_j

\( \omega \) \hspace{1cm} \text{Frequency in radians per second}

\( \tilde{\omega}_i, \Omega_i, \omega_i \) \hspace{1cm} \text{Instantaneous total, mean, and fluctuating vorticity components}

\textbf{Roman}

\( A_a, A_b \) \hspace{1cm} \text{Line-averaging factors for orthogonal slanted wires } a \text{ and } b, \text{ both inclined } 45^\circ \text{ to the mean flow (4.17)}

\( C \) \hspace{1cm} \text{Constant describing colour scale of filled-contour plots}
$C_K$ Kolmogorov constant (4.9)

$C_{o_{u,j}u_{k,l}}(\kappa_1)$ One-dimensional (streamwise) cospectrum of the velocity derivatives $\partial u_i/\partial x_j$ and $\partial u_k/\partial x_l$

$d$ Vector corresponding to separation between elements of a $x$-array

$d_2, d_3$ Wire-separation vectors for present probe geometry ($d_2 = [0 \ l_{wp}/2 \ 0]^T$, $d_3 = [0 \ 0 \ l_{wp}/2]^T$)

$E(\kappa)$ Isotropic energy spectrum function

$E_{ij}(\kappa_1)$ One-dimensional (streamwise) energy spectrum/cospectrum of $(u_i,u_j)$

$E_{11}^t(\omega)$ Temporal power-spectral-density of the streamwise velocity

$E_{11}^m(\kappa_1)$ Model for streamwise energy spectrum of $u_1$ ‘measured’ by a single-wire (4.14)

$e$ Hotwire element output (voltage)

$h$ Binormal cooling coefficient

$h_j$ Distance between probe centroid and sensor element $j$ for Taylor series expansions (3.6) and (3.7)

$K_{ii}$ Anisotropy ratio for $i = 2$ or $i = 3$, repeated index does not imply summation

$k$ Tangential cooling coefficient

$\overline{k}$ Mean turbulence kinetic energy ($\overline{k} \equiv \frac{1}{2}u_iu_i$)

$l_\nu$ Viscous length ($\nu/u_\tau$)

$l_w$ Hotwire length

$l_{wp}$ Hotwire length projected onto the $x_2-x_3$ plane ($l_w \approx \sqrt{2}l_{wp}$)

$l_{wa}, l_{wb}$ Hotwire length vectors for orthogonal slanted wires $a$ and $b$, both inclined $45^\circ$ to the mean flow

$R$ Pipe radius

$Re_\tau$ Friction Reynolds number used interchangeably with $\delta^+$ ($Re_\tau \equiv \delta u_\tau/\nu$)

$R_{\lambda T}$ Anisotropic Taylor microscale Reynolds number $R_{\lambda T} \equiv \lambda_T \sqrt{\frac{2k}{\nu}}$

$R_{\lambda T I}$ Isotropic Taylor microscale Reynolds number $R_{\lambda T I} \equiv \lambda_{TI} \sqrt{\frac{u_1^2}{\nu}}$

$s_1, s_2, s_3$ Array-separation vectors for present probe geometry ($s_1 = [\Delta x_1 \ 0 \ 0]^T$, $s_2 = [0 \ \Delta x_2 \ 0]^T$, $s_3 = [0 \ 0 \ \Delta x_3]^T$)
\( s_{ij} \) Fluctuating rate-of-strain tensor \( (s_{ij} \equiv \frac{1}{2} (\partial u_i/\partial x_j + \partial u_j/\partial x_i)) \)

\( U_\infty \) Boundary layer free-stream velocity

\( U_C \) Pipe centreline velocity

\( U_c \) Convection velocity

\( U_o \) Outer velocity scale \( U_\infty \) or \( U_C \) where appropriate

\( u_\eta \) Kolmogorov velocity scale \( (u_\eta \equiv (\nu \varepsilon)^{1/4}) \)

\( u_\tau \) Friction velocity \( (\sqrt{\tau_w/\rho}) \)

\( \tilde{u}_i, U_i, u_i \) Instantaneous total, mean, and fluctuating velocity components

\( \overline{u_i^2} \) TKE of \( n \)-th velocity component contributed by scales exhibiting isotropy

\( W(\kappa_1) \) One-dimensional (streamwise) enstrophy-spectrum

\( (x_1, x_2, x_3) \) Streamwise, wall-normal, and spanwise coordinates

\( x_{2i} \) Inner bound of logarithmic/inertial layer (taken herein as \( x_{2i} = 2.6\sqrt{\delta^+} \))

\( x_{2o} \) Outer bound of logarithmic/inertial layer (taken herein as \( x_{2o} = 0.15\delta^+ \))
Chapter 1

Introduction

Turbulence is a fundamental facet of nature. As such, nearly any scientific or engineering pursuit on the human scale (or larger) will involve a turbulent flow in some form. Although turbulent flows represent solutions to a known (in all likelihood) system of governing partial differential equations, prediction of their characteristics based on those equations alone is a notoriously difficult task. Improving the capacity to observe various features of turbulence through experimentation is, therefore, a worthwhile pursuit.

Beyond the problem of unbounded turbulence, the imposition of boundary conditions can complicate matters, but also present opportunities. On a fundamental level, examining the response of a turbulent flow field to various boundary conditions presents an opportunity to elucidate the nature of turbulence in general. On an applied level, boundary conditions can be chosen to be reflective of circumstances encountered in practice.

This thesis presents the results of an effort both to improve upon flow-sensing capacity, as well as to apply that capacity to investigate the properties of two turbulent flows of considerable practical significance—each with one shared and two differing sets of boundary conditions.

1.1 Motivation

Of all turbulent flows encountered in practice, wall-bounded turbulent flows are a particularly consequential subset. Examples include flow of the atmosphere over the surface of the Earth, of liquids and gases through pipes and ducts, of water past the hull of a ship, and of air over the fuselage and wings of an aircraft. Representation of all of these cases by a small collection of so-called canonical flows (and the subsequent investigation of those canonical flows) allows for the discovery of broadly applicable insight. Furthermore, comparison of the canonical cases to one another holds the potential to reveal the influence of the features that distinguish them.
Fully-developed pipe flow and the zero-pressure-gradient (ZPG) boundary layer are two such canonical flows. Both feature impermeable, rigid walls that impose a Dirichlet boundary condition (the velocity of the wall must match the velocity of the fluid at their interface) that terminates the problem domain at one end of one coordinate. The second set of boundary conditions for the same, ‘wall-normal’, coordinate, differ: the ZPG boundary layer is bounded from above by an infinite reservoir of uni-directional, irrotational flow, the conditions of which are matched only in the limit as distance from the wall approaches infinity; the pipe condition of axisymmetry along the centreline limits the maximum distance of any point from the bounding wall, and in so doing defines a domain that is ‘fully turbulent’. The boundary conditions of the coordinates parallel to the wall also differ: the boundary layer evolves with downstream distance and exists in a domain with a span that is (theoretically) infinite; pipe flow does not evolve with downstream distance, and is periodic through its ‘span’ (i.e. its azimuth). Furthermore, the maximum wavelength through the pipe azimuth becomes vanishingly small as the centreline is approached. Direct comparison of pipe and boundary layer flow therefore allows the following questions to be addressed:

- To what extent is the similarity (between the two flows) imposed by the rigid impermeable wall broken by the entrainment of irrotational fluid in the boundary layer?
- To what extent is the similarity (between the two flows) imposed by the rigid impermeable wall broken by the ‘crowding’ effect of the vanishing maximum wavelength through the azimuth as the centreline of the pipe is approached?

To shed light upon the answers to either question is to yield insight into the nature of both flows individually, and thus doing so is one of the primary motivations of the present research.

The present research effort is certainly not the first to draw comparisons between pipe and boundary layer flow. Previous efforts towards this end can be classified primarily into two groups: experimental studies based on measurements of the streamwise velocity component (e.g. Schubauer [1954], Monty et al. [2009]), and direct numerical simulations (DNS) at relatively low Reynolds number (e.g. Jimenez & Hoyas [2008], Jiménez et al. [2010]). Even when the scope is limited to just one of the two flows, there is a distinct paucity of three-component velocity data and (to the author’s knowledge) no previously published three-component vorticity data at Reynolds numbers substantially higher than those presently achievable through DNS. This scarcity of data is not for lack of motivation—such quantities contain details of the flow fields that are not detectable through the streamwise velocity signal alone—but rather is related to the relative difficulty of conducting the requisite experiments. Thus, further motivation for the present research is provided by the potential for new insight afforded by generating (and subsequently analysing) heretofore unavailable datasets. Furthermore, it is the hope of the author that the lessons learned through the development and evaluation of a novel configuration of measurement sensors may help pave the way for future development and deployment of sensing technology with similar capacity.
1.2 Aims and outline

The specific aims of this thesis can be stated as follows:

- detail a configuration of measurement sensors and associated data reduction scheme that allow for the estimation of both the velocity and vorticity vector;
- characterise the performance of the measurement system based on an idealised model;
- identify and address non-ideal performance through comparison of the model output to actual physical experiments;
- elucidate the influence of boundary conditions on statistical measures of the velocity and vorticity distributions in pipes and boundary layers;
- juxtapose the spectral distribution of turbulence kinetic energy and enstrophy in pipes and boundary layers;
- identify scaling behaviours and symmetries in the velocity and vorticity spectra of wall-bounded flows.

Before describing the sensor system, further motivation for the measurement of the targeted quantities—along with a summary of relevant theoretical and experimental work—is provided in Chapter 2. Chapter 3 details the experimental pipe and boundary layer flow facilities as well as the design of the measurement sensor and associated data reduction scheme. In Chapter 4, the response of the probe to a set of DNS flow fields is modeled and the capacity of the system to resolve the velocity and vorticity vectors evaluated. The predictions of this ‘synthetic experiment’ are then demonstrated to compare favorably (for most statistics) with a set of physical experiments at matched spatial resolution. Having established the capacity of the probe to yield trustworthy results, statistical measures of the distribution of velocity and vorticity components in pipes and boundary layers are presented and compared in Chapter 5. The outer regions of pipe and boundary layer flows are also characterised and compared on the basis of their content of non-turbulent or quasi-non-turbulent flow ‘patches’. Finally, Chapter 6 contains a discussion of the spectral properties of velocity in vorticity in pipes and boundary layers. The spectral content of the velocity components is compared in the context of differences observed in Chapter 5. The capacity of various length and velocity scales to describe the enstrophy spectrum across the range of measured wall-distances and Reynolds numbers is evaluated, as is the approximation of local isotropy/axisymmetry in the velocity component spectra.

This thesis presents (amongst new material) results and text from several publications/manuscripts and presentations contributed by the author throughout their candidature. The following lists summarise these contributions. Item (ii) in the publications and manuscripts list comprises the majority of Chapter 5, and excerpts from items (i) and (ii) of the same list are included in Chapter 2.
Publications and manuscripts

(i) Zimmerman, S., Morrill-Winter, C., & Klewicki, J. 2017 Design and implementation of a hot-wire probe for simultaneous velocity and vorticity vector measurements in boundary layers. *Experiments in Fluids* 58, (10), 148


Conference proceedings and presentations


Chapter 2

Review of relevant literature†

This thesis is comprised of two threads: the design and implementation of an experimental technique to measure heretofore relatively inaccessible fluid dynamical quantities, and the analysis of the results in the context of existing physical understanding and models of turbulence/wall-bounded turbulence. The former not only provides the means for the latter, but also represents a standalone effort to facilitate future development and/or evaluation of models through the lens of terms in the Navier-Stokes equations (and/or its variants) whose behaviour, until relatively recently, could only be theorised. Accordingly, the goals of this chapter are to provide (i) a brief discussion of the equations that motivate experimental measurement of the quantities targeted by the present investigation, (ii) an overview of relevant theoretical models for such quantities, and (iii) a summary of existing experimental data. These summaries are by no means exhaustive, but rather are intended to give the prospective reader a background sufficient to contextualise the results of the ensuing chapters. Further discussion of relevant literature, as it relates to more specific topics covered herein, may be found in the ensuing chapters.

2.1 Relevant transport equations

The Navier-Stokes equations, which are a statement of Newton’s second law (i.e. \( \vec{F} = m\vec{a} \)) for a fluid element, are the established mathematical model for the vast majority of fluid dynamics research. Although these equations are formulated to describe the time-rate-of-change of linear momentum, they may be used to develop expressions that describe angular velocity, kinetic energy, or a number of other quantities potentially of interest to the fluid dynamicist. By convention, the equations for zero-pressure gradient (ZPG) boundary layers are typically expressed in Cartesian coordinates, while those for pipe flow are expressed in cylindrical coordinates. Unless otherwise specified, the equations derived in this section are expressed in Cartesian coordinates.

Stated for an incompressible fluid with negligible body forcing, the Navier-Stokes equations may be expressed using Einsteinian index notation as:

\[
\rho \frac{D\tilde{u}_i}{Dt} = \rho \frac{\partial \tilde{u}_i}{\partial t} + \rho \tilde{u}_j \frac{\partial \tilde{u}_i}{\partial x_j} = -\frac{\partial \tilde{p}}{\partial x_i} + \mu \frac{\partial^2 \tilde{u}_i}{\partial x_j \partial x_j},
\]

\[
\frac{\partial \tilde{u}_j}{\partial x_j} = 0,
\]

(2.1)

(2.2)

where \( \rho \) is the fluid density, \( \mu \) is the dynamic viscosity, \( \tilde{\cdot} \) indicates an instantaneous total quantity, \( D/Dt \) is the material derivative, and \( \tilde{u}_i \) is the fluid velocity in the \( x_i \)-direction. Throughout this document, \( i = 1 \) corresponds to the direction of the mean flow, or the streamwise direction, \( i = 2 \) corresponds to the wall-normal direction, and \( i = 3 \) corresponds to the “spanwise” or “azimuthal” direction, where appropriate. The system represented by (2.1) is closed by including (2.2): a statement of conservation of mass for an incompressible fluid.

As turbulent flow is characterised by chaotic fluctuations in both the pressure and velocity fields, the instantaneous total quantities are typically split into their mean (capital) and fluctuating (lower-case) components in what is known as a Reynolds decomposition [Reynolds, 1895]:

\[
\rho \frac{D(U_i + u_i)}{Dt} = -\frac{\partial (P + p)}{\partial x_i} + \mu \frac{\partial^2 (U_i + u_i)}{\partial x_j \partial x_j},
\]

(2.3)

The time-average of (2.3) represents the balance of forces that dictate the evolution of the mean linear momentum in an Eulerian reference frame:

\[
\frac{\partial U_i}{\partial t} + U_j \frac{\partial U_i}{\partial x_j} + u_j \frac{\partial u_i}{\partial x_j} = \frac{1}{\rho} \left(-\frac{\partial P}{\partial x_i} + \mu \frac{\partial^2 U_i}{\partial x_j \partial x_j} \right),
\]

(2.4)

where \( \langle \cdot \rangle \) is used to indicate a temporal mean when capital-letter notation is inappropriate. Typically, (2.2) is invoked in (2.4), leading to the following representation:

\[
\frac{\partial U_i}{\partial t} + U_j \frac{\partial U_i}{\partial x_j} = \frac{1}{\rho} \frac{\partial P}{\partial x_i} - \frac{\partial u_i u_j}{\partial x_j} + \nu \frac{\partial^2 U_i}{\partial x_j \partial x_j},
\]

(2.5)
where $\nu$ is the kinematic viscosity $\nu \equiv \mu/\rho$. The covariances of the fluctuations are referred to as the Reynolds stresses, and the divergence of $\overline{u_i u_j}$ represents the net mean effects of the turbulent fluctuations on the mean momentum. The balance equations of mean linear momentum given by (2.5) may be simplified for the flows considered herein by invoking statistical two-dimensionality such that:

$$U_3 = \frac{\partial \overline{\cdot}}{\partial x_3} = \overline{u_1 u_3} = \overline{u_2 u_3} = 0. \tag{2.6}$$

Further, statistical stationarity is invoked, such that:

$$\frac{\partial \overline{\cdot}}{\partial t} = 0. \tag{2.7}$$

Fully developed pipe flow is statistically homogeneous in the streamwise direction so that $\partial \overline{\cdot}/\partial x_1 = 0$ (except for the mean pressure gradient $\partial P/\partial x_1$ which drives the flow), and the mean flow must be parallel to the walls such that $U_2 = 0$. By construction, the ZPG boundary layer features $\partial P/\partial x_1 = 0$, but is not strictly parallel to the wall or fully-developed. Still, the following “boundary layer approximations” apply (e.g. see Schlichting [1979]):

$$U_1 \gg U_2, \tag{2.8}$$

$$\frac{\partial \overline{\cdot}}{\partial x_2} = \frac{\partial \overline{\cdot}}{\partial x_1}, \tag{2.9}$$

$$\overline{U_1^2} \gg \overline{u_1^2}, \tag{2.10}$$

$$\overline{u_2^2} \gg \overline{U_2^2}. \tag{2.11}$$

Thus, the mean streamwise momentum transport equation for the boundary layer can be reduced to (e.g. see Pope [2000]):

$$U_1 \frac{\partial U_1}{\partial x_1} + U_2 \frac{\partial U_1}{\partial x_2} = -\frac{\partial \overline{u_1 u_2}}{\partial x_2} + \nu \frac{\partial^2 U_1}{\partial x_2^2}. \tag{2.12}$$

In the analogous equation for pipe flow, all of the terms to the left of the equals sign in (2.12) are identically zero, while the mean pressure gradient term $\partial P/\partial x_1$ is non-zero.

The central role played by the Reynolds stress in the transport of mean momentum is one reason to examine its own transport equation. The general equation for the transport of Reynolds stress can be derived from the transport of fluctuating velocity (i.e., the momentum balance for fluctuating velocity), which is simply (2.3) minus (2.5) (e.g. see Pope [2000]):

$$\frac{Du_i}{Dt} = -u_j \frac{\partial U_i}{\partial x_j} - \frac{1}{\rho} \frac{\partial P}{\partial x_i} + \overline{u_i u_j} \frac{\partial \overline{u_j}}{\partial x_i} + \nu \frac{\partial^2 u_i}{\partial x_j \partial x_j}. \tag{2.13}$$
The mean of the Reynolds stress transport equation can be obtained from (2.13) as follows:

\[
\frac{Du_iu_j}{Dt} = u_i \frac{Du_j}{Dt} + u_j \frac{Du_i}{Dt}.
\]  

(2.14)

Expanding and re-collecting terms in (2.14) eventually leads to the transport equation for each of the 9 terms in the mean Reynolds stress tensor (e.g., see Pope [2000]):

\[
\begin{align*}
\frac{\partial u_i u_j}{\partial t} + U_k \frac{\partial u_i u_j}{\partial x_k} = & - \frac{\partial u_i u_j u_k}{\partial x_k} - \left( \frac{u_i u_k}{\partial x^k} + \frac{u_j u_k}{\partial x^k} \right) \\
& - \frac{1}{\rho} \left( \frac{\partial p u_i}{\partial x_j} + \frac{\partial p u_j}{\partial x_i} \right) - \frac{p}{\rho} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) + \nu \frac{\partial^2 u_i u_j}{\partial x_k \partial x_k} - 2 \nu \frac{\partial u_i \partial u_j}{\partial x_k \partial x_k}. \\
\end{align*}
\]  

(2.15)

From left to right, the terms in (2.15) represent:

I: Eulerian time-rate-of-change of the Reynolds stress \(u_iu_j\)

II: turbulence transport/diffusion

III: turbulence production

IV: pressure transport

V: pressure-strain redistribution

VI: viscous diffusion

VII: viscous dissipation

Statistical two-dimensionality reduces the number of unique non-zero Reynolds stresses to four: the three Reynolds normal stresses \(\overline{u_1^2}, \overline{u_2^2}, \text{ and } \overline{u_3^2}\), and the Reynolds shear stress \(\overline{u_1 u_2}\). The transport of these non-zero Reynolds stresses may be further simplified by invoking (2.6)–(2.11). The end result is the following
(in Cartesian coordinates) for the transport of $\overline{u_1^2}$, $\overline{u_2^2}$, $\overline{u_3^2}$, and $\overline{u_1u_2}$ respectively:

$$0 = - \frac{\partial \overline{u_1^2}}{\partial x_2} - 2 \overline{u_1u_2} \frac{\partial U_1}{\partial x_2} - 0 - 2 \overline{p} \frac{\partial u_1}{\partial x_1} + \nu \frac{\partial^2 \overline{u_1^2}}{\partial x_3^2} - 2 \rho \frac{\partial u_1}{\partial x_2} \left( \frac{\partial U_2}{\partial x_1} + \frac{\partial U_3}{\partial x_3} \right)$$

$$0 = - \frac{\partial \overline{u_2^2}}{\partial x_2} - 0 - 2 \overline{p} \frac{\partial u_2}{\partial x_2} + \nu \frac{\partial^2 \overline{u_2^2}}{\partial x_3^2} - 2 \rho \frac{\partial u_2}{\partial x_2} \frac{\partial u_3}{\partial x_3}$$

$$0 = - \frac{\partial \overline{u_3^2}}{\partial x_2} - 0 - 0 - 2 \overline{p} \frac{\partial u_3}{\partial x_3} + \nu \frac{\partial^2 \overline{u_3^2}}{\partial x_3^2} - 2 \rho \frac{\partial u_2}{\partial x_2} \frac{\partial u_3}{\partial x_3}$$

$$0 = - \frac{\partial \overline{u_1u_2}}{\partial x_2} - \overline{u_2^2} \frac{\partial U_1}{\partial x_2} - \overline{u_1u_3} \frac{\partial U_1}{\partial x_3} - 2 \overline{p} \left( \frac{\partial u_1}{\partial x_2} + \frac{\partial u_2}{\partial x_1} \right) + \nu \frac{\partial^2 \overline{u_1u_2}}{\partial x_3^2} - 2 \rho \frac{\partial u_1}{\partial x_2} \frac{\partial u_2}{\partial x_3} \frac{\partial u_3}{\partial x_1}$$

Evaluating Equations (2.16)–(2.19) are referenced throughout this document, and thus warrant additional consideration. The production term (III) is zero for (2.17) and (2.18), but not (2.16) or (2.19). Term I is zero for all four cases to within the boundary layer approximations; terms II, IV, and VI ultimately describe a spatial redistribution of existing Reynolds stress; and term VII is strictly a sink of Reynolds stress. In the absence of direct production, the only way for the $u_2$ or $u_3$ components to maintain finite contributions to the turbulence kinetic energy (TKE) is if term V, the pressure-strain redistribution, acts as source. Evidently, conservation of mass (continuity) in the neighborhood of pressure fluctuations results in $\overline{p} \partial u_i/\partial x_1 > 0$ as well as $\overline{p} \partial u_2/\partial x_2 < 0$ and $\overline{p} \partial u_3/\partial x_3 < 0$ for turbulent flows in which there is no direct production of $u_2^2$ or $u_3^2$. From the perspective of the normal stresses alone, all production comes from term III in (2.16), and $\overline{u_1^2}$ and $\overline{u_2^2}$ ‘feed’ on the resulting $\overline{u_1^2}$ via the pressure-strain redistribution term. The wrinkle in this line of thought, however, is that the production of $\overline{u_1^2}$ depends on $\overline{u_1u_2}$, the production of which is shown in (2.19) to depend on $\overline{u_2^2}$. Although one cannot identify the ‘origin’ of the Reynolds stresses from (2.16)–(2.19), the mechanisms of intercomponent exchange are still clear.

One particularly instructive means by which to understand the interaction between the mean flow and the turbulence is to examine the kinetic energy of each. The mean turbulence kinetic energy (TKE), or $\overline{k}$ is defined in terms of the Reynolds normal stresses as follows:

$$\overline{k} \equiv \frac{1}{2} \overline{u_i u_i}$$

The TKE is (one-half) the trace of the Reynolds stress tensor, or equivalently one-half the sum of the eigenvalues of the Reynolds stress tensor (i.e. it is independent of the chosen coordinate system). Although the mean TKE can be estimated by combining multiple non-simultaneous measurements (e.g. $(\tilde{u}_1, \tilde{u}_2)$ and $(\tilde{u}_1, \tilde{u}_3)$, the results in (2.19) to depend on $\overline{u_2^2}$. Although one cannot identify the ‘origin’ of the Reynolds stresses from (2.16)–(2.19), the mechanisms of intercomponent exchange are still clear.

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×-wire measurements), simultaneous time-resolved measurements of all three components that allow estimation of the instantaneous TKE $k$ are uncommon. The measurement technique described in this thesis is capable of resolving all three components of the velocity vector about a common point, and thus is capable of producing time-resolved estimates of $k$.

In accord with (2.20), the TKE transport equation is one-half the sum of (2.16), (2.17), and (2.18):

$$0 = -\frac{1}{2} \frac{\partial u_i u_j}{\partial x_k} - \frac{1}{2} \frac{\partial \rho u_j}{\partial x_k} - \frac{1}{2} \frac{\partial^2 u_i u_j}{\partial x_k^2} - \nu \frac{\partial u_i \partial u_i}{\partial x_k \partial x_k}.$$  

(2.21)

Note that the production term also appears in the mean kinetic energy transport equation (i.e. $K = \frac{1}{2} U_i U_i$), but with the opposite sign. Thus, the production is the mechanism by which kinetic energy is exchanged between the mean flow and the turbulence. In the flows considered herein, the production term is exclusively a source of Reynolds stress/TKE.

Additional insight may be gleaned by examining the flow through the lens of angular velocity instead of linear momentum, or vorticity instead of velocity. The vorticity $\tilde{\omega}_i$ is defined as the curl of the velocity vector, or in index notation:

$$\tilde{\omega}_i = \varepsilon_{ijk} \frac{\partial \tilde{u}_k}{\partial x_j},$$  

(2.22)

where $\varepsilon_{ijk}$ is the alternating tensor. Terms in the linear momentum equation may themselves be represented in terms of the vorticity. Of particular interest is the vorticity form of the divergence of the Reynolds stress term:

$$\frac{\partial u_i u_j}{\partial x_j} = \frac{1}{2} \frac{\partial u_k u_k}{\partial x_i} - \varepsilon_{ijk} u_j \omega_k.$$  

(2.23)

The Reynolds stress divergence term in (2.5) for the streamwise ($i = 1$) mean momentum becomes:

$$\frac{\partial u_i u_j}{\partial x_j} = \frac{\partial u_3 u_2}{\partial x_2} - \frac{\partial u_2 u_3}{\partial x_2} + \frac{\partial K}{\partial x_1},$$  

(2.24)

or, after invoking (2.6) and (2.8)–(2.11):

$$\frac{\partial u_1 u_2}{\partial x_2} = \frac{u_3 \omega_2}{\partial x_2} - \frac{u_2 \omega_3}{\partial x_2}.$$  

(2.25)

Equation (2.25) reveals that the contributions to the mean streamwise momentum balance in a boundary layer can be alternatively represented as the difference between two velocity-vorticity correlations. Studying the behaviour of the two constituent terms allows for a more detailed analysis of the motions which ultimately lead to the mean Reynolds stress gradient. Equation (2.25) is thus one of the chief motivations for designing and implementing a sensor capable of estimating both the velocity and vorticity vectors simultaneously. As will be shown in Chapter 4, however, satisfactory measurement of both the velocity and vorticity vectors does
not guarantee satisfactory measurement of their cross-product—even very slight
cross-talk between the two measured vectors can (and does) result in significant
events in several mean correlation values.

The transport equation for vorticity may be used instead of the transport equation
for velocity (i.e. linear momentum) to fully describe a flow field. The vorticity
transport equation is obtained by taking the curl of (2.1):

$$ \frac{D\tilde{\omega}_i}{Dt} = \tilde{\omega}_j \frac{\partial \tilde{u}_i}{\partial x_j} + \nu \frac{\partial^2 \tilde{\omega}_i}{\partial x_j \partial x_j}. \quad (2.26) $$

From left to right, the terms in (2.26) represent: the Eulerian time-rate-of-change
of vorticity; vorticity stretching and reorientation; and vorticity diffusion through
viscous effects. Of particular note is the disappearance of the pressure in (2.26),
which simplifies matters in a sense. As such, (2.26) paints a fairly simple picture
of a ZPG boundary layer (for example). The simplicity of (2.26) makes vorticity
an attractive field variable by which to describe a turbulent flow, and thus further
motivates its measurement experimentally.

## 2.2 Expected behaviour of relevant quantities

### 2.2.1 Mean velocity

#### 2.2.1.1 Background theory

The behaviour of the mean velocity profile (MVP) of turbulent wall-bounded flows
has been the subject of nearly a century of research. This research includes mea-
measurements with better spatial resolution and at higher Reynolds numbers than any
presented herein (e.g. see Zagarola & Smits [1998] (pipe) and Samie et al. [2018]
(boundary layer)). As such, expanding upon the existing body of research into the
MVP is not one of the goals of this thesis. Still, a brief discussion of the general
features of the MVP is included, since the properties of other mean statistical pro-
files will frequently be discussed in the forthcoming chapters in the context of the
local behaviour of the MVP.

Prandtl [1925] proposed that a boundary layer MVP is subject to two distinct
scaling laws corresponding to the ‘inner’ and ‘outer’ portions of the flow —i.e.,
in the neighborhood of each of the two flow boundaries \( x_2 = 0 \) and \( x_2 = \delta \),
respectively\(^1\). The ‘inner’ region MVP, it was proposed, scales with \( \rho, \mu, x_2, \) and
the wall shear stress \( \tau_w \equiv \mu \partial U_1 / \partial x_2 |_{x_2=0} \), while the ‘outer’ region MVP scales
with \( \rho, x_2, \tau_w, \) and the domain thickness \( \delta \). From these parameters, one can
define a velocity scale \( u_\tau \equiv \sqrt{\tau_w / \rho} \) and two length scales \( l_\nu \equiv \nu / u_\tau \) and \( \delta \). The
ratio of these two lengths is indicative of the range of energetic scales, and thus

\(^1\)Throughout this thesis, \( \delta \) refers to both the pipe radius \( R \) and the boundary layer thickness
\( \delta_99 \) as defined by \( U_1(\delta_99) \equiv 0.99 \cdot U_\infty \), where appropriate.
constitutes a Reynolds number \( \delta^+ \equiv \delta u_\tau / \nu \equiv Re_\tau \). Note that throughout this thesis, normalisation of a quantity by the friction velocity \( u_\tau \) and/or the viscous length \( l_\nu \) (i.e. by the viscous scales) is indicated by a superscript ‘+’.

In the ensuing decades, the original formulation of Prandtl [1925] was expanded into the classical four-layer structure: the linear sublayer, the buffer layer, the log-layer, and the wake layer. According to the classical four-layer structure, the flow in the linear sublayer and buffer layer (together referred to as the classical viscous layer) depends exclusively on the viscous scales, and is thus independent of the outer boundary condition. While modern evidence suggests that this is not strictly the case (e.g. see Hoyas & Jiménez [2006], Mathis et al. [2009]), available direct numerical simulations (DNS) of the Navier-Stokes equations suggest that pipe, channel, and boundary layer MVPs in the classical viscous layer (i.e. \( x_2^+ \lesssim 30 \)) are virtually identical under viscous normalisation up to at least \( Re_\tau \approx 2000 \).

A number of justifications have been proposed for the existence of a log-layer—i.e. a subdomain where \( U_1 \sim \log(x_2) \)—including heuristics (e.g. see Von Kármán [1930], Izakson [1937], Millikan [1939], Townsend [1976]) as well as arguments grounded in first-principles (e.g. see Oberlack [2001], Fife et al. [2005], Wei et al. [2005], Morril-Winter et al. [2017]). Justifications have also been proposed for a log-resembling (in an empirical sense) power-law layer (e.g. see Barenblatt [1993], George & Castillo [1997]) between the classical viscous layer and the wake. The exact formulation for the MVP—and the universality of the fitting coefficients between pipe, channel, and boundary layer flows in this region—are neither evaluated herein, nor are they significant to the conclusions drawn. For the purposes of this thesis, it is sufficient to note that log-like dependence of the MVP is associated with the appropriateness of distance-from-the-wall scaling. The location of the log-layer (also sometimes referred to as the inertial sublayer) has also been the subject of considerable debate. To within the certainty of available experimental data, the outer limit of the log-layer \( x_2o \)—i.e. where the MVP begins to depart from logarithmic-like dependency in the outer region—is \( x_2o \approx 0.15\delta \) for both the pipe [McKeon et al., 2004] and boundary layer [Nagib et al., 2007] cases regardless of Reynolds number. While the inner limit of the log-layer \( x_2i \) based on the classical four-layer structure is \( x_2i^+ \approx 30 \), modern studies based on analysis of the equations of motion (e.g. see Fife et al. [2005], Wei et al. [2005], Klewicki et al. [2009]) and empirical results (e.g. see Long & Chen [1981], Afzal [1982], Marusic et al. [2013], Yamamoto & Tsuji [2018]) suggest that \( x_2i \) is instead associated with the geometric mean of the inner and outer length scales. Based on the support for such dependency in the equations of motion and in available experimental and numerical data, the inner limit of the log-layer/inertial sublayer is taken as \( x_2i^+ \approx 2.6\sqrt{\delta^+} \) herein [Wei et al., 2005]. This choice is of little consequence to the discussions presented in this thesis, and as such is intended only to serve as a rough demarcation.

The remainder of the flow domain (i.e. above the log-layer) is referred to as the wake-layer, after Coles [1956] pointed out the similarity in mean velocity distribution between a wake flow half-profile and a boundary layer at the point of separation (i.e., a boundary layer without an ‘inner’ layer). Coles [1956] proposed
that the velocity profile of a boundary layer is composed of a linear combination of the log-law with a universal law of the wake function \( w(x_2/\delta) \), where \( w \geq 0 \).

In the same decade, Schubauer [1954] showed that deviation of the ZPG boundary layer from the log-law could be attributed to its ‘irregular’ outer limit, where positions above \( x_2/\delta \approx 0.4 \) would intermittently switch between turbulent and non-turbulent states. The non-turbulent regions, Schubauer observed, had approximately constant velocity equal to that of the free-stream, while the turbulent regions had a streamwise velocity that decreased with increasing ‘depth’ below the instantaneous height of the turbulence. Despite this observation, Schubauer was hesitant to conclude that the deviation from the log-law was entirely owing to the non-zero time-fraction where \( \tilde{u}_1 = U_\infty \), since pipe and channel flows (which lack a free-stream) were also observed to deviate from the log-law. More recently, however, Kwon et al. [2014] demonstrated that internal flows feature a quasi-non-turbulent core, that they termed the quiescent core, having properties similar to those of the boundary layer free-stream. Krug et al. [2017] then showed that the wake-layer MVPs of both internal (i.e. pipe/channel) and external (i.e. boundary layer) flows may be recovered from the time-fraction of quiescent/non-turbulent flow (for internal/external cases, respectively) at each wall-distance, and the assumption that turbulent patches obey the log-law while quiescent/non-turbulent patches obey \( \tilde{u}_1 = U_o \). Thus, our most recent understanding is that the wake portion of both the boundary layer and pipe can be described as a continuation of the log-layer (or log-like layer) with patches of non-turbulent/quasi-non-turbulent flow interspersed to varying degrees.

### 2.2.1.2 Experimental access

Measurement of the MVP is relatively straightforward and can be accomplished (for example) via various optical techniques (particle image velocimetry (PIV), molecular tagging velocimetry (MTV), etc.), single-element thermal anemometry (constant temperature anemometry (CTA), also referred to herein as hotwire anemometry, or constant current anemometry (CCA)), or determination of the mean dynamic pressure. The mean velocity profile may also be determined through direct numerical simulation of the Navier-Stokes equations, although the maximum \( Re_\tau \) achievable though this approach is currently significantly lower than the highest \( Re_\tau \) experiments. A selection of MVPs from pipe and boundary layer DNS and experimental studies are shown in figure 2.1 (a) in law-of-the-wall form (i.e., viscous scaling). As described in the previous section, the pipe and boundary layer profiles are virtually identical in the classical viscous layer, contain an apparent log-layer, and differ markedly from one another only in the wake. The difference in wake structure is illustrated more clearly in figure 2.1 (b), where the same profiles are shown in velocity defect form (i.e. in outer scaling and relative to the maximum velocity).
Figure 2.1: Mean velocity profiles from selected pipe and boundary layer studies in (a): law-of-the-wall form, and (b): velocity defect form. All red curves indicate pipe data, while all blue curves indicate ZPG boundary layer data. Darker shades indicate higher $Re_{\tau}$. Solid curves without symbols represent DNS, while curves with symbols represent experiments. 

- $Re_{\tau} \approx 1000$–$2000$ pipe DNS from Chin et al. [2014];
- $Re_{\tau} \approx 1300$–$2000$ boundary layer DNS from Sillero et al. [2013];
- $Re_{\tau} \approx 800$–$55000$ pipe experiments from Zagarola & Smits [1998];
- $Re_{\tau} \approx 5000$–$16100$ boundary layer experiments from Samie et al. [2018];
- $(0.41)^{-1} \log (x_2^+) + 5.0$.
2.2.2 Velocity fluctuations

2.2.2.1 Background theory

Much like the mean streamwise velocity, the streamwise Reynolds normal stress profile—or, likewise, the streamwise velocity variance profile $u_1^2$—features an ‘inner’ and an ‘outer’ region owing to the locally relevant scaling parameters. Examples of these features are given in figure 2.2 (a). Although the inner region is dominated by processes that scale with $u_\tau$ and $l_\nu$, some dependence is observed on Reynolds number (and thus on the magnitude of the outer length scale) [Wei & Willmarth [1989], Klewicki & Falco [1990], Metzger et al. [2001], Hutchins et al. [2009]]. The following specific fit for the peak magnitude of $u_1^2$ in the inner region was proposed by Lee & Moser [2015], based on existing data supplemented by their channel-flow DNS up to $Re_\tau \approx 5200$:

$$u_{1\text{max}}^2 = 3.66 + 0.656 \log (Re_\tau). \quad (2.27)$$

This fit was later corroborated by Samie et al. [2018] (i.e. by the profiles shown in figure 2.2). Mathis et al. [2009] suggest that this Reynolds number dependence is associated with amplitude modulation by large outer-region motions on the smaller near-wall motions. Indeed, Samie et al. [2018] show that contributions to $u_1^2$ from $\delta$-scale motions in the vicinity of the inner peak increase with $Re_\tau$, while contributions from the shorter-wavelength portion of the spectrum remain virtually fixed. Beyond the inner peak, $u_1^2$ decays to zero in the boundary layer and to a finite value in the pipe that depends on $Re_\tau$ (but that is on the order of one-tenth the magnitude of the inner-peak, for the $Re_\tau$ range of available data). This decay rate slows with increasing $Re_\tau$, corresponding to the growth of an outer peak/plateau.

One particular theory which has had a notable level of success in predicting behaviours of the Reynolds stresses is the attached eddy hypothesis (AEH) of Townsend [1976] (and in particular the extensions to the AEH proposed by Perry & Chong [1982] and Perry et al. [1986]). The AEH holds that the motions responsible for the bulk of the production and dissipation in a subdomain of a high-$Re_\tau$ wall-bounded flow scale with distance from the wall, and are, in that sense, attached to the wall. Perry & Chong [1982] specified the shape and population statistics of such eddies based in part on available experimental observations by Head & Bandyopadhyay [1981] and others, and were able to recover a log-layer in the MVP. Perry et al. [1986] then showed that the proposed structure also results in the following profiles for $u_1^2$, $u_2^2$, and $u_3^2$, respectively:

$$u_1^2 = B_1 - A_1 \log \left( \frac{x_2}{\delta} \right) \quad (2.28)$$

$$u_2^2 = B_3 \quad (2.29)$$

$$u_3^2 = B_2 - A_2 \log \left( \frac{x_2}{\delta} \right) \quad (2.30)$$

$$u_{1\text{max}}^2 = 3.66 + 0.656 \log (Re_\tau). \quad (2.31)$$
Figure 2.2: Streamwise velocity variance profiles from selected pipe and boundary layer studies by (a): viscous distance from the wall, and (b): wall-distance relative to $\delta$. All red curves indicate pipe data, while all blue curves indicate ZPG boundary layer data. Darker shades indicate higher $Re_\tau$. Solid curves without symbols represent DNS, while curves with symbols represent experiments. ---: $Re_\tau \approx 2000$ pipe DNS from Chin et al. [2014]; ---: $Re_\tau \approx 2000$ boundary layer DNS from Sillero et al. [2013]; ---: $Re_\tau \approx 5400$, $\approx 10500$, and $\approx 20300$ pipe experiments from Hultmark et al. [2013]; ---: $Re_\tau \approx 4900$, $\approx 8000$, $\approx 12100$, and $\approx 16100$ boundary layer experiments from Samie et al. [2018]; ---: equation (2.27); ---: equation (2.28) with $B_1 = 1.95 + \log(1.25)$, $A_1 = 1.26$ fit suggested by Samie et al. [2018], with additional additive constant factor to account for a differing definition of $\delta$. 
Returning for a moment to figure 2.2 (b), the region of the $u_1^2$ profiles beyond the outer peak appear to satisfy a logarithmic decay rate to a degree that improves with increasing $Re_{\tau}$. This is also shown to be the case by Marusic et al. [2013]. The reason for this dependence on $Re_{\tau}$ is apparent from the spectral scaling arguments which give rise to (2.28)–(2.30). Although exact adherence to the predictions of Perry et al. [1986] is not evaluated herein, these spectral scaling laws are briefly summarised below for the purposes of establishing a reference for the velocity component spectra shown in chapter 6.

One of the core tenets of the AEH is that the motions associated with the production of turbulence—in a thin subdomain where production and dissipation dominate the TKE transport equation (i.e. (2.21))—have a wall-normal extent from their ‘centre’ that is proportional to the distance of that centre to the wall. This distance-from-the-wall scaling has been shown by Fife et al. [2005] and others to be an analytic property of a similarity solution (i.e., not necessarily all solutions) to the mean momentum balance equation (2.12). Perry & Chong [1982] proposed that the A-shaped inclined ‘vortex’ filaments observed by Head & Bandopadhyay [1981] in flow-visualisation experiments be used as the model ‘eddy’. The velocity field corresponding to such a pattern has features that form the basis for the spectral scaling arguments presented in Perry et al. [1986]. Essentially, the chosen ‘eddy’ induces considerable $u_2$ only locally, but induces considerable $u_1$ and $u_3$ in both the near- and the far-field. The ‘eddies’ that contribute most heavily to $u_2^2$ at a particular wall-distance $x'_2$ are therefore expected to have a height in proportion to $x'_2$, while those contributing significantly to $u_1^2$ and $u_3^2$ have heights ranging from $x'_2$ to $\delta$. The population density of eddies of each size—proposed to follow geometric scaling such that there are twice as many eddies with height $H$ as with height $2H$—then leads to a spectrum of $u_1$ and $u_3$ for which the range of energetic scales is a function of the scale separation between $\delta$ and $x'_2$, and a spectrum of $u_2$ for which the range of energetic scales is fixed (but whose position is anchored by $x'_2$). Since the variance is equal to the integral of the spectrum, $u_1^2$ and $u_2^2$ therefore depend on the scale separation between $x_2$ and $\delta$ (i.e. equations (2.28) and (2.30)), while $u_3^2$ does not (i.e. equation (2.29)).

The AEH is based in part on the assumption that the motions contributing to the Reynolds shear stress $\rho u_1 u_2$ (RSS) scale with distance from the wall. A further assumption is that the RSS is approximately constant and equal to the wall shear stress $\tau_w$ across the subdomain over which it is proposed to apply. This assumption, along with other basic properties of the RSS in the pipe can also be inferred from the mean momentum equation. Integrating the equivalent to (2.12) for the pipe in cylindrical coordinates with respect to the wall-normal coordinate ($x_2$ in this case is distance from the wall), and plugging in boundary conditions results in the following (e.g. see Laufer [1954]):

$$-\frac{u_1 u_2^*}{\partial x_2^*} + \frac{\partial U^*}{\partial x_2^*} = \left(1 - \frac{x_2}{\delta}\right).$$

(2.32)

According to (2.32), the total shear stress $\tau_t \equiv (\rho u_1 \partial U_1/\partial x_2 - \rho u_1 u_2)$ varies linearly throughout the entire flow domain, decreasing from $\tau_t = \rho u_2^2 = \tau_w$ at the wall to
\[ \tau_1 = 0 \] at the centreline. Note that one consequence of viscous-scaling of the MVP is that the region where the mean velocity gradient is large shrinks as a fraction of \( \delta \) (i.e. since large \( \delta^+ \) and \( \delta \gg \nu/u_\tau \) are equivalent statements) (e.g. see Townsend [1976]). In other words, the region over which the mean velocity gradient is large becomes increasingly confined to the near-wall, leaving the RSS as the dominant term in \( \tau \) in the outer region. The RSS must then approach zero linearly with wall-distance in the pipe at sufficiently high \( Re_\tau \) [Tennekes & Lumley, 1972]. The RSS profile in the boundary layer case, on the other hand, does not necessarily approach zero linearly, since the mean advection terms in (2.12) are not constant with wall-distance (unlike the mean pressure in the pipe case) (e.g. see Schlichting [1979]). Finally, consider a subdomain in which, due to the aforementioned Reynolds number trend, both \( \partial U^+ / \partial x^+ \ll 1 \) and \( x^+ \ll \delta \) apply. In such a subdomain, the RSS profile according to (2.32) is approximately \( -\tilde{u}_1 \tilde{u}_2^+ \approx 1 \); this classical argument (e.g. see Tennekes & Lumley [1972]) based on (2.32) is the justification for the aforementioned assumption of the existence of a finite subdomain over which the RSS is constant and equal to \( \tau_w \).

Velocity fluctuation skewness and kurtosis are also presented in this thesis. Further discussion of higher order moments is provided in chapter 5.

### 2.2.2.2 Experimental access

Experimental determination of velocity variance is typically accomplished via hotwire anemometry. The streamwise component \( \bar{u}_1^2 \) can be measured by a single hotwire element mounted normal to the mean flow direction (hereafter referred to as a single-wire). Some additional commentary on single-wires is warranted here to contextualise some of the difficulties associated with multi-element hotwire anemometry.

The response \( \hat{e} \) of a single-wire can be described, for example, through a combination of the expressions suggested by King [1914] and Jorgensen [1971a], respectively:

\[
\begin{align*}
\hat{e}^2 &= A\hat{u}_{eff}^2 + B \\
\hat{u}_{eff}^2 &= \hat{u}_N^2 + \hat{u}_T^2 + \hat{u}_B^2,
\end{align*}
\]

where \( \hat{u}_N, \hat{u}_T, \) and \( \hat{u}_B \) are the components of velocity normal, tangential, and binormal to the sensing element, respectively; \( k \approx 0.2 \) and \( h \approx 1.05 \) [Bruun, 1995]; and \( A \) and \( B \) are constants determined via calibration. For a single-wire mounted parallel to the bounding wall (as is typical), \( \hat{u}_N = \hat{u}_1, \hat{u}_T = \hat{u}_3, \) and \( \hat{u}_B = \hat{u}_2 \). Since \( \hat{u}_1 \gg \hat{u}_2 \) and \( \hat{u}_1 \gg \hat{u}_3 \) in most locations within a wall-bounded flow [Pope, 2000], \( \hat{u}_T \) and \( \hat{u}_B \) are typically (safely) ignored [Bruun, 1995]. Thus, determination of streamwise velocity from the signal acquired from a single-wire is akin to solving a fully-determined one-equation/one-unknown system. This system is typically very well-conditioned based on the slope of the input-output calibration curve relative to the noise level, and so is capable of producing highly accurate estimates of \( \hat{u}_1 \). Two important barriers to accurate determination of \( \hat{u}_1 \) are calibration drift (typically
due to change in ambient temperature) and spatial resolution. Temperature drift can be compensated-for to an extent by conducting pre- and post-calibrations to establish reference points at two different temperatures. The finite length of the sensor effectively results in an output that is proportional to the line-average of the instantaneous velocity across the sensor rather than the instantaneous velocity at a single point [Bruun, 1995]. Thus, decreasing the sensor length (while avoiding issues associated with doing so, e.g. see Ligrani & Bradshaw [1987]) results in an output that, to an increasingly accurate degree, reflects the instantaneous velocity at a single point. Ligrani & Bradshaw [1987] and Hutchins et al. [2009] show that measurement error of $\overline{u_1^2}$ (by a single-wire) due to finite sensor length is small for sensors shorter than about $20l_w$. The limitation on sensor length is also related to a limitation on sensor diameter. Ligrani & Bradshaw [1987] suggest that the length of a hotwire element must be at least $200 \times$ its diameter to avoid issues related to heat loss through conduction to the supporting prongs. In practice, a standard hotwire element meeting this criterion is typically in the range of $0.5$–$1$mm in length ($2.5$–$5\,\mu$m in diameter). Elements of smaller diameter ($d \approx 0.6\,\mu$m) are available, but since the strength of the element is proportional to its cross-sectional area, these elements are extremely fragile. Bailey et al. [2010] (see also Vallikivi et al. [2011]) approached this problem via micro-electro-mechanical systems (MEMS) technology, resulting in the Nano-Scale Thermal Anemometry Probe (NSTAP), with a sensor length of just $60\,\mu$m and a (rectangular) $0.1 \times 2\,\mu$m cross-section (reduced to $0.1 \times 1\,\mu$m by Vallikivi & Smits [2014]). At present, measurements of $\overline{u_1^2}$ at high $Re_\tau$ collected via well-behaved NSTAPs (see Samie et al. [2018]) represent the gold-standard. As such (and as with the MVP), expanding upon the body of knowledge surrounding the detailed behaviour of $\overline{u_1^2}$ is not one of the goals of this thesis. Still, $\overline{u_1^2}$ and other statistics based on $\tilde{u}_1$ are discussed herein, and therefore NSTAP measurements of $\overline{u_1^2}$ in pipe and boundary layer flows are shown in figure 2.2 to provide context for the discussions of the ensuing chapters.

While experimental measurements of $\tilde{u}_1$ at high Reynolds number are common in both pipes and boundary layers, measurements of $\tilde{u}_2$ and $\tilde{u}_3$ are relatively scarce owing to the disproportionate effort required to accurately capture transverse velocities relative to the streamwise velocity. Two-component measurements (i.e. of either $(\tilde{u}_1, \tilde{u}_2)$ or $(\tilde{u}_1, \tilde{u}_3)$ simultaneously) may be acquired via a standard orthogonal two-element hotwire ‘$\times$-array’ (e.g. see Bruun [1995]), a standard $\times$-array with an additional element(s) normal to the mean flow direction (e.g. see Kawall et al. [1983], Morrill-Winter et al. [2015]), or any one of several optical techniques (planar PIV, laser doppler velocimetry (LDV) etc.). Planar PIV represents one potential alternative to two-component hotwire anemometry, although this technique faces its own hurdles associated with dynamic range, spatial and temporal resolution, and computational cost (for data reduction). Statistics determined via LDV hold the capacity to be highly accurate, but the measured signal is acquired with an non-uniform sampling rate, which leads to temporal resolution issues.

Determination of the streamwise and transverse velocity from the two independently acquired signals of a $\times$-array is akin to solving a fully-determined two-equation/two-unknown system. Unlike single-wires, $\tilde{u}_N$ and $\tilde{u}_T$ are functions of
both the streamwise and (targeted) transverse velocity components. This coupling, along with the property that $\tilde{u}_1$ is typically much larger than the transverse components, can exacerbate issues such as temperature drift or inaccurate determination of calibration coefficients, leading to proportionally large errors in the reported transverse velocity components (e.g., see Zimmerman et al. [2017]). Adding an additional single-wire to a standard $\times$-array as did Kawall et al. [1983] mitigates these issues to a degree, as evidenced by the quality of transverse velocity component data reported by Morrill-Winter et al. [2015]. Just as with single-wires, the elements in a multi-sensor probe respond to the line average of the instantaneous velocity along each sensor element. Unlike single-wires, however, there is an additional issue related to the separation of the sensing elements (e.g. see Vukoslavčević & Wallace [1981], Park & Wallace [1993]). Instantaneous velocity gradients across the sensing volume result in contamination of the transverse velocity signal, or gradient-aliasing, adding noise to the variance that can offset attenuation from spatial-filtering [Zimmerman et al., 2017]. This effect is clearly visible in all near-wall hotwire transverse velocity variance data presented herein.

Simultaneous three-component velocity measurements are even more scarce than two-component measurements. Measurement of the full velocity vector may be accomplished via LDV (e.g. see Anthony & Willmarth [1992], Fontaine & Deutsch [1995]), stereoscopic PIV (e.g. see Prasad [2000], Van Doorne & Westerweel [2007]), tomographic PIV (e.g. see Elsinga et al. [2006], Schröder et al. [2008]), or three (plus) component hotwire sensors (e.g. see Bruun [1995]). While accurate determination of 3D velocity statistics is possible via the PIV techniques at low $Re_\tau$ (e.g. see Hutchins et al. [2005]), dynamic range and spatial/temporal resolution issues make higher-$Re_\tau$ measurements significantly more challenging. As with two-component LDV, three-component LDV faces challenges associated with temporal resolution (especially at high-$Re_\tau$) owing to a non-uniform sampling rate. Three- and four-element hotwire arrays form the building blocks of most existing measurement probes capable of estimating the full vorticity vector [Wallace & Vukoslavčević, 2010]. Triple-wire probes targeting the full velocity vector (e.g. see Jorgensen [1971]) face the same problems relative to $\times$-arrays as $\times$-arrays face relative to single-wires: i.e. more degrees of freedom and coupling of velocity components/coupling of errors. Four-wire probes (e.g. see Dobbeling et al. [1990]) seek to mitigate these issues by over-determining the corresponding system of equations (four equations for three unknowns). Additionally, multi-sensor probes become increasingly hampered by gradient-aliasing (as described above) as additional wires are added, since additional wires typically require a larger overall sensing volume.

Figure 2.3 shows $\overline{u_2^2}$, $\overline{u_3^2}$, and $\overline{u_1u_2}$ statistics from several selected studies. The data shown are divided into three subgroups: (i) available high-$Re_\tau$ experimental data for boundary layer flow [Baidya [2015], Morrill-Winter et al. [2015]] and pipe flow [Orlu et al. [2017], Willert et al. [2017]]; (ii) available DNS for boundary layer [Sillero et al., 2013] and pipe [Chin et al., 2014] flow; and (iii) hotwire data for boundary layer flow collected via a sensor that is also capable of estimating the full vorticity vector [Balint et al. [1991], Honkan & Andreopoulos [1997]].
The results in figure 2.3 reveal additional motivation for the present measurement effort. Further high-$Re_\tau$ measurements of all velocity variances and covariances (save for $u_1^2$) in the outer region of the pipe are necessary given the disagreement in magnitude between the results of Örlü et al. [2017] and a visual extrapolation of trends at either lower $Re_\tau$ or in the inner region. This is particularly true for the spanwise variance, as various DNS profiles and the measurements of Örlü et al. [2017] are the only data known to the author for moderate to large $Re_\tau$. The sensors in group (iii) leave substantial room for improvement in estimation of the velocity vector relative to those in group (i). Furthermore, simultaneous measurements of the velocity and vorticity vectors are presently limited to fairly low $Re_\tau$. As such, one of the goals of the research effort documented herein was to design, construct, and deploy a measurement probe capable of capturing the full velocity vector with a degree of accuracy in line with group (i), while simultaneously acquiring the vorticity vector as in group (iii). It will be shown in Chapter 4 that this goal has largely been accomplished. Direct comparison between the present measurement technique and those in group (iii), for both velocity and vorticity statistics, is provided in Zimmerman et al. [2017].

2.2.3 Velocity gradients

2.2.3.1 Background theory

Arguments pertaining to the behaviour of terms in the mean momentum balance equation (2.12) as discussed in the previous section may be extended into predictions for the behaviour of TKE dissipation (which is composed of velocity gradient terms). A classical approximation (e.g. see Townsend [1976]) posits that wall-distance $x_2$ and friction velocity $u_\tau$ are the appropriate length and velocity scales within some subdomain of a turbulent wall-bounded flow. Fife et al. [2005], Morrill-Winter et al. [2017a], and others have shown that distance-from-the-wall scaling is an analytical property of one similarity solution to the mean momentum balance equation (2.12), lending credence to the classical heuristic. Regardless, acceptance of the appropriateness distance-from-the-wall scaling leads to the following expressions from dimensional analysis:

$$\frac{u_1 u_2}{\bar{u}} \sim u_\tau^2, \quad (2.35)$$

$$\frac{\partial U_1}{\partial x_2} \sim \frac{u_\tau}{x_2}, \quad (2.36)$$

$$\varepsilon \sim \frac{u_3^3}{x_2}, \quad (2.37)$$

where $\varepsilon$ is the mean dissipation rate of TKE. Note that this argument implies that the TKE production $-u_1 u_2 \partial U_1 / \partial x_2$ is proportional to the TKE dissipation. Note further that equation (2.36) integrates into a logarithmic velocity profile. A rough approximation of the bounds of the subdomain over which (2.35)–(2.37) apply is therefore given by the bounds between which the MVP is observed to be well-approximated by a log-law (i.e. $2.6\sqrt{\delta^+} \lesssim x_2^+ \lesssim 0.15\delta^+$).
Figure 2.3: (a)–(c): Wall normal velocity variance, spanwise velocity variance, and streamwise/wall-normal velocity covariance versus viscous wall-distance for selected pipe and boundary layer profiles. (d): Same data as (c), plotted against wall-distance as a fraction of $\delta$. All red curves indicate pipe data, while all blue curves indicate ZPG boundary layer data. Darker shades indicate higher $Re_\tau$. Data group (i)—high-$Re_\tau$ experimental data: $Re_\tau \approx 7000$, and $\approx 15000$ pipe hotwire experiments from Örlü et al. [2017]; $Re_\tau \approx 5200$ and $\approx 11000$ pipe PIV experiments from Willert et al. [2017]; $Re_\tau \approx 2000$, $\approx 4000$, and $\approx 8000$ boundary layer hotwire experiments from Baidya [2015]; $Re_\tau \approx 2000$, $\approx 4000$, and $\approx 8000$ boundary layer hotwire experiments from Morrill-Winter et al. [2015]. Data group (ii)—pipe and boundary layer DNS: $Re_\tau \approx 2000$ pipe DNS from Chin et al. [2014]; $Re_\tau \approx 2000$ boundary layer DNS from Sillero et al. [2013]. Data group (iii)—measurements from sensors capable of estimating the full vorticity vector: $Re_\tau \approx 1100$ boundary layer hotwire experiment from Balint et al. [1991]; $Re_\tau \approx 1200$ boundary layer hotwire experiment from Honkan & Andreopoulos [1997]. Black dashed line in (d) corresponds to $1 - x_2/\delta$. All cited experimental data (with the exception of those of Morrill-Winter et al. [2015]) have been manually digitised from plots in the original cited work to the best of the author’s ability, but may deviate slightly from actual results.
The mean TKE dissipation is related to the fluctuating velocity gradient tensor as follows:

$$\overline{\varepsilon} = 2\nu s_{ij} s_{ij} = \nu \frac{\partial u_i}{\partial x_j} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right),$$  \hspace{1cm} (2.38)

where \( s_{ij} \) is the fluctuating rate-of-strain tensor \( s_{ij} \equiv \frac{1}{2}(\partial u_i/\partial x_j + \partial u_j/\partial x_i) \).

Equation (2.38) differs from the dissipation term in the TKE transport equation (2.21) due to a difference in the way the viscous terms are expanded and re-collected. The dissipation term in equation (2.21) is termed the pseudo-dissipation, and is roughly equal to \( \overline{\varepsilon} \) in most flows (and exactly equal to \( \overline{\varepsilon} \) in homogeneous flows) [Pope, 2000]. The pseudo-dissipation (or homogeneous dissipation) \( \overline{\varepsilon}_h \) may also be expressed in terms of the vorticity as (e.g. see George & Hussein [1991]):

$$\overline{\varepsilon}_h = \nu \omega_i \omega_i.$$  \hspace{1cm} (2.39)

Out of necessity, most experimental estimates of the dissipation rate rely on simplifications based on assumed symmetries at the dissipative (small) scales. Through an assumption of complete symmetry about a rotation of any axis/axes, or isotropy, all terms in (2.38) may be written in terms of \((\partial u_1/\partial x_1)^2\) [Pope, 2000]. As discussed in the previous section, single-wire hotwires are a common and relatively straightforward means by which time-resolved measurements of \( \dot{u}_1 \) may be acquired. These time-resolved measurements may be used to estimate \( \dot{u}_1/\partial x_1 \) (and thus also its variance) according to Taylor’s frozen turbulence hypothesis [Taylor, 1938]. That is, if the distortion rate of dissipative motions is slow (on average) relative to the time it takes for them to convect past the measurement point, the spatial gradient along the presumed path of the motion may be estimated from the measured temporal signal as follows:

$$\frac{\partial (\cdot)}{\partial x_1} \approx -\frac{1}{U_c} \frac{\partial (\cdot)}{\partial t}. \hspace{1cm} (2.40)$$

In an isotropic flow, the variance of \( \partial u_1/\partial x_1 \) is related to the other terms in (2.38) as follows (e.g. see Monin & Yaglom [1975]):

$$\left( \frac{\partial u_1}{\partial x_1} \right)^2 = \left( \frac{\partial u_2}{\partial x_2} \right)^2 = \left( \frac{\partial u_3}{\partial x_3} \right)^2$$  \hspace{1cm} (2.41)

$$\left( \frac{\partial u_1}{\partial x_1} \right)^2 = \frac{1}{2} \left( \frac{\partial u_3}{\partial x_2} \right)^2 = \frac{1}{2} \left( \frac{\partial u_2}{\partial x_3} \right)^2 = \frac{1}{2} \left( \frac{\partial u_1}{\partial x_3} \right)^2$$  \hspace{1cm} (2.42)

$$\left( \frac{\partial u_1}{\partial x_1} \right)^2 = -2 \frac{\partial u_3}{\partial x_2} \frac{\partial u_2}{\partial x_3} = -2 \frac{\partial u_1}{\partial x_3} \frac{\partial u_3}{\partial x_1} = -2 \frac{\partial u_2}{\partial x_1} \frac{\partial u_1}{\partial x_2}$$  \hspace{1cm} (2.43)

While pipe and boundary layer flow cannot be globally isotropic due to the inhomogeneity induced by the wall boundary condition, local isotropy—that is, isotropy
The full velocity vector allows (among myriad other possibilities) for investigation of the properties of the instantaneous pseudo-dissipation rate $\tilde{\varepsilon}$, as well as an evaluation of the commonly-used isotropic approximation of dissipation based on $(\partial u_1/\partial x_1)^2$ alone.

2.2.3.2 Experimental access

As discussed above, measurements of $\partial u_1/\partial x_1$ are readily obtained from single-wire experiments. Likewise, $\partial u_2/\partial x_1$ and $\partial u_3/\partial x_1$ may be obtained via multi-sensor hotwire measurements of $\tilde{u}_2$ and $\tilde{u}_3$, respectively. Acquisition of transverse gradients (i.e. in the $x_2$ or $x_3$ direction) via hotwire anemometry requires adjacent sub-arrays, each capable of measuring the velocity component to be differentiated (e.g. see Vukoslavčević et al. [1991], Antonia et al. [1993], and the review of Wallace & Vukoslavčević [2010]). Spanwise and/or wall-normal gradients of the streamwise velocity may be obtained via spatially separated single-wires; acquisition of these gradients for the transverse velocity components requires spatially separated sub-arrays each containing two or more elements [Wallace & Vukoslavčević, 2010]. The focus of this section is on techniques which aim to measure the full vorticity vector.

To the author’s knowledge, existing studies of turbulent boundary layers that present spatially or temporally resolved measurements of the full vorticity vector include the efforts of only a small number of research groups, as represented by the studies of Balint et al. [1987, 1991], Lemonis [1995, 1997], Honkan & Andreopoulos [1997], Ong & Wallace [1998], Ganapathisubramani et al. [2006], and Schneiders et al. [2017]. These existing studies will henceforth be abbreviated as B87, B91, L97, HA97, OW98, G06, and S17 respectively. Boundary layer measurements were also presented in Tsinober et al. [1992]. This study, however, contained results from only two positions in the wake region of the flow, as the primary focus was on grid turbulence. The data presented in all of these studies, with the exception of G06 and S17, were acquired via multi-element hotwire probes. The data presented in G06 were acquired via dual-plane stereo-PIV, and thus include more spatial information than the hot-wire studies, but are not time-resolved and so lack the temporal information afforded by the present data. The practical difficulty associated with deploying the dual-plane stereo-PIV also limited the number of wall-normal positions presented in G06 to just two. More recently, volumetric measurements of the velocity gradient tensor in a boundary layer at $Re_\tau \approx 780$ were obtained via tomographic particle tracking velocimetry (PTV) by Schneiders et al. [2017]. The authors of this study note, however, that the computational cost of processing such a dataset (even at the low-$Re_\tau$ presented and for short time-resolved bursts) is substantial. Since the range of timescales associated with turbulent motions only increases with $Re_\tau$, it is presently not feasible to execute such a technique at the Reynolds numbers presented herein. Other optical
methods, such as single plane stereoscopic-PIV [Van Doorne & Westerweel, 2007], three-plane PIV [Zeff et al., 2003], and tomographic PIV [Schröder et al., 2008] have been used to measure the velocity gradient tensor in various flows, but none of these have been used to produce vorticity statistics in a turbulent boundary layer. As will become apparent by the data comparisons herein, previous studies that have presented vorticity statistics in turbulent boundary layers were performed at low Reynolds numbers, and are characterized by a relatively sparse measurement density per profile. The current effort employs a sensor design with a physical measurement volume and spatial resolution similar to previous studies, but is able to achieve Reynolds numbers up to a decade larger owing to the physical scale of the experimental facilities. Furthermore, owing to the physical durability of the present sensor, much more detailed profile information is afforded by the larger number of points per profile in the present versus these previous studies.

B87, B91, and OW98 all used a version of the same 9-wire sensor. Several flaws related to the version of this probe used in B87 were pointed out in B91 and corrected in the later two studies. B91 and OW98 acquired measurements in boundary layers with friction Reynolds numbers of $\delta^+ \approx 1100$ and $\delta^+ \approx 545$, respectively. The sensor sub-array separation (the distance over which gradients were estimated) was approximately $11l_\nu$ in both the spanwise and wall-normal directions in B91, compared to 7 in OW98. Such separation distances, while smaller than those achieved in the present work, are more readily achievable at low-$Re_\tau$ (as $l_\nu$ is inversely proportional to $Re_\tau$). Streamwise gradients for OW98, as well as for all other previous hot-wire studies, were estimated via Taylor’s hypothesis.

The measurements presented in HA97 were acquired in a turbulent boundary layer with a friction Reynolds number of $\delta^+ \approx 1200$. The sub-array separation of the probe used in this study was approximately $21l_\nu$ in the wall-normal direction, and $18l_\nu$ in the spanwise direction.

L97 improved upon the technique of L95 with slight modifications to the 20-wire probe originally used in their earlier study. The second of these two studies presented data acquired in a turbulent boundary layer with friction Reynolds number of $\delta^+ \approx 2300$. This sensor had a total sub-array separation of approximately $21l_\nu$ in both the spanwise and wall-normal directions, and featured 5 sub-arrays capable of measuring the full velocity vector, whereas the probes used by B91 and HA97 featured 3 sub-arrays. The use of two additional sub-arrays allowed for a second-order estimation of the velocity field, as well as the symmetric estimation of all velocity gradients about a common location. While the probes and data reduction methods employed by B91 and HA97 also produce both the velocity and vorticity about a common location, the gradient estimates about that location are based on measurement points that are not symmetric. The advantages of a symmetric design are noted in Vukoslavčević & Wallace [2013], and are related to the reduction in error associated with central differencing as opposed to forward or backward differencing.

Root-mean-square (RMS) profiles of each fluctuating vorticity component are shown for selected studies in figure 2.4. To the author’s knowledge, there are at
present no previously published statistics from instantaneous measurements of the vorticity vector in a pipe flow. While the existent experiments appear to capture the general trend of the vorticity RMS profiles, it is clear that there is room for improvement in the way of reducing scatter and increasing both profile density and the range of Reynolds numbers captured (to say nothing of contributing measurements for pipe flow). As such, these items are included in the goals of the present research effort. It will be shown in the ensuing chapters that these goals have, to a large extent, been met.
2.3 Direct comparison of pipe and boundary layer flow

Both physical experiments and numerical simulations have been conducted towards clarifying the onset and causes of discrepancies between internal (pipe/channel) and external (boundary layer) flows. Experimental results, however, are primarily limited to those pertaining to the streamwise component of velocity—largely owing to the relative difficulty of measuring the other two components. Monty et al. [2009] compared streamwise velocity spectra and the first four statistical moments of the streamwise velocity collected in pipe, channel, and boundary layer flows at \(Re_\tau \approx 3000\). The authors found that the statistical structure of the streamwise velocity fluctuations was virtually the same in all three flows from the wall to at least 0.5\(\delta\). Despite this statistical invariance, the authors also found that eddies with streamwise wavelength \(\lambda_1 \gtrsim 10\delta\) contribute more to the streamwise variance in the log-layer for internal (pipe/channel) flows than they do for external (boundary layer) flows. That the streamwise statistical invariance is apparently maintained despite the difference in spatial organization motivates an investigation into the behaviours of other flow variables such as the cross-stream velocities and the vorticity.

While experimentally determined profile statistics of the wall-normal and spanwise/azimuthal components of velocity are available independently for both pipes and boundary layers, no single experimental study has presented data for both flows acquired with the same probe and data-reduction scheme under matched probe resolution and Reynolds number conditions. Consequently, it is difficult to differentiate between flow-dependent features and experimental scatter based on a collection of existing experimental results alone. This is illustrated in Jimenez & Hoyas [2008], where a selection of existing experimental data from both internal and external flows is presented alongside the results of a channel flow DNS dataset. One way to approach the issue of experimental scatter is to compare DNS results of internal and external flows directly, as in Jiménez et al. [2010] and Chin et al. [2014]. Such comparisons, however, have thus far been limited to friction Reynolds numbers of \(Re_\tau \approx 1000\) or less. Since it is doubtful that wall-flows of \(Re_\tau \lesssim 1000\) contain a well-developed inertial layer [Morrill-Winter et al., 2017b], it remains to be seen whether features observed in the transverse velocity variance profiles persist at higher \(Re_\tau\). Furthermore, to the author’s knowledge, third and fourth order statistics of the transverse velocity components have not yet been reported in a comparative study of internal and external flows (nor have any vorticity statistics).
Chapter 3

Experimental Procedures

3.1 Facilities

The data presented in this thesis were collected at three of the largest wall-bounded flow facilities in the world: the Flow Physics Facility (FPF) at the University of New Hampshire; the High Reynolds Number Boundary Layer Wind Tunnel (HRN-BLWT), or the Melbourne Wind Tunnel (MWT) for short, at the University of Melbourne; and the Center for International Cooperation in Long Pipe Experiments (CICLoPE) at the University of Bologna.

The FPF, first characterized in Vincenti et al. [2013], is an open-circuit zero-pressure-gradient (ZPG) turbulent boundary layer (TBL) wind tunnel in which the boundary layer grows continuously over a streamwise development length of 72m, ultimately achieving boundary layer heights of up to 75cm. The spatial development of the boundary layer over this long fetch permits the outer flow scale to be set to any value up to the maximum by establishing a fixed measurement station at the corresponding streamwise location. Owing to the high maximum boundary layer height, the FPF allows access to the highest $Re_\tau$ values at a given viscous length scale (i.e. measurement resolution) of the three facilities. Because the air is pulled from the surrounding atmosphere, however, the flow temperature cannot be controlled. This introduces, to a greater degree than the other facilities, an effect on the measurement sensors that requires compensation. Additionally, the FPF does not have a contraction section upstream of the turbulence trip point, which may result in slight deviations from canonical behaviour in the generated boundary layer wake.

The MWT is an indoor open-circuit ZPG TBL wind tunnel with a development length of 27 m, over which the boundary layer achieves a maximum height of approximately 35 cm. A detailed characterization of the MWT can be found in Kulandaivelu [2012]. Although the MWT cannot produce $Re_\tau$ values as high as those produced by the FPF for a given viscous length scale, its lack of exposure to the atmosphere (i.e. stable temperature), and its extensive turbulence management section upstream of the tripping point make it a suitable testbed in which to
evaluate the measurement technique and search for evidence of facility-related artifacts in the ZPG boundary layer results.

The CICLoPE houses a closed-loop system that generates a fully developed turbulent pipe flow in a 90 cm diameter test section over a development length of 110.9 m (i.e. a length-to-diameter ratio of 123.2). The loop includes a heat exchanger which keeps the flow temperature constant to within $\pm 0.2^\circ$C, even for measurement durations in excess of 9 hours. Further detail on the design of the CICLoPE is available in Talamelli et al. [2009].

All three facilities are ideal for high-fidelity measurements of high Reynolds number flows, as their physical size allows for the generation of a wide range of energy-containing scales without the smallest of those being unresolvable via conventional measurement techniques. The FPF and CICLoPE are also particularly well-suited for direct flow comparisons with one another, as the operational flow speeds and physical dimensions make it possible to simultaneously match both inner and outer flow scales at considerable Reynolds numbers.

### 3.2 Measurement Probe

As discussed in chapter 1, only a few studies have presented time-resolved data of all three components of vorticity in wall-bounded flows. In these cases, however, the probes employed actually measured the entire velocity gradient tensor rather than specifically the vorticity vector. Measurement of the full velocity gradient tensor (i.e. both the normal and shear strain rates) necessitates a probe design with at least nine wires, as with Balint et al. [1987 and 1991], Ong & Wallace [1998], and Honkan & Andreopoulos [1997]. Some probes, such as those of Lemonis [1995 and 1997], have used as many as twenty wires. Velocity gradient tensor probes, for practical reasons, also require sub-arrays capable of resolving all three velocity components each. If, however, the focus is only on the velocity and vorticity vectors (as in the present study), a probe may be constructed with as few as eight wires. This eight-wire style of probe, a variant of which was first developed by Antonia et al. [1998], need not be composed of sub-arrays that each measure all three components of velocity. Reducing the number of velocity components measured by each sub-array from three to two facilitates a more rugged probe design, while significantly reducing the calibration time and processing complexity. It is for these reasons that the eight-wire approach of Antonia et al. [1998] was chosen as the starting point for the design of the present probe.

The present probe geometry is depicted in figure 3.1. As with the probes employed in Antonia et al. [1998] and Zhou et al. [2003], this probe consists of eight sensing elements, four of which are parallel to the $x_1-x_2$ plane and four to the $x_1-x_3$ plane. Unlike these previous probes, the present design positions the wall-parallel sensing elements closer together to prioritise resolution of $x_2$ gradients (over the $x_3$ gradients). The merits of this choice will be discussed in chapter 4.
The present sensing elements are lengths of 5µm-diameter tungsten wire with a thin (<5% of wire by mass) gold or platinum coating to improve solderability. The length of these elements projected into the $x_2$-$x_3$ plane is $l_{wp} = 0.8$ mm, and their overall length is approximately $l_w = \sqrt{2}l_{wp}$. This constitutes an overall length-to-diameter ratio in excess of 225, which satisfies the criterion of Ligrani & Bradshaw [1987] to mitigate the effects of conductive heat loss. In lieu of Wollaston wire ‘stubs’ to further mitigate end conduction effects, the 5µm sensing elements are soldered directly to vacuum arc remelted stainless steel ‘prongs’ that are sharpened from a starting diameter of approximately 250 µm down to a tip diameter of approximately 50µm. The use of ‘stubless’ tungsten wire significantly reduces the measurement volume (improving spatial resolution under otherwise matched conditions) and increases the tensile strength of the sensors relative to a probe that employs equivalent-diameter platinum or platinum-alloy Wollaston wire.

Due to the poor wetting of solder on stainless steel, the prongs must be plated before the sensors can be affixed. The solderability of the stainless steel prong tips is enhanced by an initial treatment with Wood’s nickel strike solution at $500 - 800$A/m² for 3-4 minutes (to establish a well-adhered base coating of nickel), followed by a treatment with an acid-style copper plating bath also at $500 - 800$A/m² for 3-4 minutes. Note that the nickel layer is required for two reasons: acid-copper plating baths produce very poor adhesion when applied directly to stainless steel; and repeated stripping of the plating/solder by nitric acid will result in passivation of the stainless steel—a condition that, left untreated by the hydrochloric acid included in Wood’s nickel strike solution, would prevent additional plating.

For illustrative purposes, it is useful to describe the present probe as being composed of four individual $x$-wire sub-arrays. The probe schematic shown in figure 3.1 is consistent with this description and, together with (3.1)–(3.3), demonstrates...
one way in which both the velocity and vorticity vectors may be obtained about the centroid of the measurement volume. Note that subscripts with two letters are used in (3.1)–(3.3) to indicate the mean of the indicated velocity component between the two sub-arrays associated with the letter subscripts.

\[
\omega_1 = \frac{\partial u_3}{\partial x_2} - \frac{\partial u_2}{\partial x_3} \approx \frac{u_{3_A} - u_{3_B}}{\Delta x_2} - \frac{u_{2_a} - u_{2_d}}{\Delta x_3} \tag{3.1}
\]

\[
\omega_2 = \frac{\partial u_1}{\partial x_3} - \frac{\partial u_3}{\partial x_1} \approx \frac{u_{1_a} - u_{1_b}}{\Delta x_3} - \frac{u_{3_ab}(t + \Delta t) - u_{3_ab}(t - \Delta t)}{-2U_c \Delta t} \tag{3.2}
\]

\[
\omega_3 = \frac{\partial u_2}{\partial x_1} - \frac{\partial u_1}{\partial x_2} \approx \frac{u_{2_a}(t + \Delta t) - u_{2_a}(t - \Delta t)}{-2U_c \Delta t} - \frac{u_{1_a} - u_{1_b}}{\Delta x_2}. \tag{3.3}
\]

In contrast to some other multi-element hot-wire probes deployed in wall-bounded flows (e.g. see the review of Wallace & Vukošavljević [2010]), the individual sub-array centroids of the present probe are symmetric about the overall measurement volume centroid. The advantage of this symmetry is that all gradient estimates (and thus vorticity component estimates) can be obtained via central finite differences about a single common point rather than by less accurate forward/backward difference approximations.

### 3.3 Acquisition

Each sensor is operated by one of eight identical custom-built Melbourne University Constant Temperature Anemometer (MUCTA) channels. The output of each channel is routed through an Alligator Technologies USBPGF-S1 programmable analog low-pass filter. The cutoff frequency \( f_c \) is set for each measurement such that \( f_c^+ > 1 \). The signals are acquired along with ambient temperature, free-stream/pipe-centreline dynamic pressure, and ambient pressure (in the MWT and CICLoPE) with a Data Translation DT9836 12-channel 15-bit signed simultaneous A/D board. For the experiments conducted at the FPF, ambient pressure is recorded manually at regular time intervals.

### 3.4 Calibration

Data collected from a two-step \textit{in situ} calibration procedure are combined to characterize the response of each sensor to a range of flow angles and speeds expected to be encountered in the profile scans. In the first procedure, the sensors are traversed to a position where they will encounter quasi-uniform flow \((x_2 > \delta_99 \text{ in the boundary layer and } x_2 = 0.93 R \text{ in the pipe})\). The sensors are then exposed to between 9 and 11 flow speeds ranging from roughly 1 m/s at the low end to 1.25\( U_o \) at the high end, where \( U_o \) is the velocity in either the free-stream or at the pipe
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The flow speeds in both cases are measured by pitot-static tubes in the free-stream or pipe centreline. A third order polynomial is then fitted to the median sensor response versus median flow speed data points. The median is used rather than the mean to remove the influence of non-uniformity in the calibrating flow, (e.g. non-zero turbulence intensity in the pipe centreline). This procedure is performed before and after every profile scan, providing two reference points for temperature-based interpolation of a single response curve for each profile measurement \( x_2 \) position. A key advantage of this procedure is that the flow incidence is known to be 0° relative to the position of the probe during the actual profile scan.

The second step of the calibration procedure utilizes an in-house built articulating jet first described in Morrill-Winter & Klewicki [2013] to generate uniform flow at both yaw and pitch angles across the same range of flow speeds as the quasi-uniform tunnel calibrations described above. Data are collected at thirteen pitch and thirteen yaw angles at each speed. The range of these angles starts at ±30° for speeds up to 4 m/s, and is gradually reduced to ±15° in yaw and ±12.5° in pitch for speeds 8–14 m/s. The reduction in calibration angles is done to avoid prong vibrations sustained under prolonged exposure to uniform cross-flow. Tangential cooling coefficients \( k \) [Jorgensen, 1971a], and effective cooling angles \( \alpha \) [Bradshaw, 1971], are determined for each sensor at each speed and used in the following expression to describe the sensor response:

\[
\begin{align*}
    u_e^2 &= \left( \frac{u_1 \sin \alpha - u_{X1} \cos \alpha}{u_N} \right)^2 + k^2 \left( \frac{u_1 \cos \alpha + u_{X1} \sin \alpha}{u_T} \right)^2 + h^2 \left( \frac{u_{X2}}{u_T} \right)^2.
\end{align*}
\]  

(3.4)

where \( u_e \) is the 'effective' cooling velocity and \( u_{X1}/u_{X2} \) are either \( u_2/u_3 \) or \( u_3/u_2 \) (depending on the orientation of the sensor), and the subscripts \( N \) and \( T \) refer to the directions normal and tangential to the sensor, respectively. The goal of (3.4) is to collapse all of the calibration data onto a single curve that can be assigned a one-to-one functional relationship to the sensor output as in (3.5):

\[
    u_{e_j} = f(e_j) = a_j e_j^3 + b_j e_j^2 + c_j e_j + d_j.
\]  

(3.5)

A cubic polynomial relating the output \( e \) of sensor \( j \) (1–8) to the effective cooling velocity has been found to have sufficient degrees of freedom to match the calibration data.

The original expression suggested by Jorgensen [1971a] includes a term to account for the sensor cooling associated with the velocity component that is bi-normal to the sensor. For the present probe, the bi-normal component is simply \( u_2 \) for the wall-parallel sensors and \( u_3 \) for those in the \( x_1-x_2 \) plane. It was found that inclusion of this term had very little effect on the velocity and vorticity statistics. An asymmetry in the response to bi-normal cooling, however, does have an effect on velocity-vorticity products and high-order vorticity statistics. This will be discussed in detail in Chapter 4. The effects of this asymmetry aside, the insensitivity of most statistics to the consideration of bi-normal cooling is not surprising.
based on its role in the governing equations. Since both sensing elements in a x-array are approximately equally sensitive to bi-normal cooling, bi-normal velocity should increase the signal from both sensors by the same amount. An increase in the output from both sensors should influence only the interpreted $u_1$ component rather than the cross-stream components, as the former is essentially dependent on the sum of the two sensor outputs, and the latter on the difference between the two sensor outputs. It therefore stands to reason that bi-normal cooling has very little effect on the reduced data, as the magnitudes of the cross-stream components $u_2$ and $u_3$ are typically much smaller than $u_1$ in the present flow and measurement domain, and are thus unlikely to produce enough bi-normal cooling to dramatically affect the eventual $u_1$ output.

While alternative models of wire response have been proposed (e.g. see Dobbeling et al. [1990]), the fundamental tasks of the model (when the input data are free from noise) are to fit the calibration data, and reduce the error associated with interpolation between calibration points. When the density of calibration points is sufficiently high, all that is required of a functional form is that it produces a smooth fit of the data with minimal squared-error. The use of (3.4) with speed-dependent cooling coefficients collapses the jet calibration data onto a single curve with errors typically less than 1% for speeds above 2 m/s.

Although the jet calibration is theoretically sufficient to describe the entire probe response, there are two practical difficulties associated with the jet calibration that necessitate the tunnel calibration (i.e. the first step). First, it was shown in Zimmerman et al. [2017] that even very minor misalignment of the jet relative to the mean flow could result in substantial errors in reported Reynolds shear stress components. Second, the fan that generates the calibrating jet flow can heat the air by several degrees Celsius, resulting in a shifted speed response curve. Both of these issues are circumvented by forcing the jet calibration surface to fall upon (at 0° nominal flow incidence) the tunnel calibration curve.

### 3.5 Data Reduction

Raw sensor output is reduced to velocity and shear gradients about the centroid of the measurement volume by solving two closed four-equation four-unknown systems. These systems, obtained from (3.4), and given in (3.6) and (3.7), are first-order Taylor series expansions of two-dimensional velocity in the direction normal to the velocity plane for each set of four coplanar sensing elements. Equation (3.6) is thus appropriate for the four wires oriented in sub-arrays $a$ and $b$, and (3.7) is appropriate for those in $c$ and $d$. Note that the index $j$ indicates an individual sensing element (1–8), and $h_j$ indicates the separation between sensor ‘$j$’ and the centroid in the direction of the Taylor series expansion. For example, if elements
1 and 2 are in sub-array $a$, then $h_1 = \frac{3}{4} l_{w_p}$ and $h_2 = \frac{1}{4} l_{w_p}$ (see figure 3.1).

$$u_{e_j}^2 = \left[ \left( u_1 + h_j \frac{\partial u_1}{\partial x_2} \right) \sin(\alpha_j) - \left( u_3 + h_j \frac{\partial u_3}{\partial x_2} \right) \cos(\alpha_j) \right]^2 + \ldots$$

$$k_j^2 \left[ \left( u_1 + h_j \frac{\partial u_1}{\partial x_2} \right) \cos(\alpha_j) + \left( u_3 + h_j \frac{\partial u_3}{\partial x_2} \right) \sin(\alpha_j) \right]^2$$

(3.6)

$$u_{e_j}^2 = \left[ \left( u_1 + h_j \frac{\partial u_1}{\partial x_3} \right) \sin(\alpha_j) - \left( u_2 + h_j \frac{\partial u_2}{\partial x_3} \right) \cos(\alpha_j) \right]^2 + \ldots$$

$$k_j^2 \left[ \left( u_1 + h_j \frac{\partial u_1}{\partial x_3} \right) \cos(\alpha_j) + \left( u_2 + h_j \frac{\partial u_2}{\partial x_3} \right) \sin(\alpha_j) \right]^2$$

(3.7)

The systems are solved via a hybrid Powell method (with the Jacobian supplied analytically) from the open-source OPTI toolbox for MATLAB. The effective cooling velocity is known directly from the sensor output based on the fit shown in (3.5). Initial guesses for the velocity and gradient components are obtained via a dense lookup table generated according to the calibration fits to the functional forms given in (3.4) and (3.5). These initial guesses are also used to obtain values for $k_j$ and $\alpha_j$. The tangential cooling coefficient $k_j$ and effective cooling angle $\alpha_j$ are weak functions of $u_e$, and thus for simplicity are not updated by the iterative solution procedure once initial values are determined from the $\times$-array method. The output of this solution method is the three velocity components and the four cross-stream shear gradients $\frac{\partial u_1}{\partial x_2}$, $\frac{\partial u_3}{\partial x_2}$, $\frac{\partial u_1}{\partial x_3}$, and $\frac{\partial u_2}{\partial x_3}$.

The remaining shear strain rates not directly obtained by the solution method (i.e. those taken in the streamwise direction) are estimated via Taylor’s frozen turbulence hypothesis, using the local mean velocity as the convection velocity. Following the recommendation of Klewicki & Falco [1990], the streamwise gradients are subjected to a moving average filter to attenuate ‘spatial’ wavelengths shorter than the sensor length. The six measured gradients are then used to compute all three instantaneous vorticity components.

### 3.5.1 Resolving ambiguity

The systems given by (3.6) and (3.7) each contain four polynomials in four variables, each of which are degree two. By Bézout’s theorem, it is possible for the system to have up to $2^4 = 16$ unique roots. This section describes the approach used to avoid reporting roots that satisfy the systems given by (3.6) and (3.7) but that do not describe the incident flow field.

All roots can be obtained by first computing the reduced Gröbner basis and then solving the resulting ‘upper triangular’ polynomial system (i.e. one that can be solved by finding the roots of successive univariate polynomials). While this
Figure 3.2: Streamwise velocity roots that (when paired with corresponding $u_3$, $\partial u_1 / \partial x_2$, and $\partial u_3 / \partial x_2$ roots) satisfy the system of equations given by (3.6) over time. Complex roots are coloured blue, and real-valued roots are coloured black. Real components are projected onto the real-time plane, and imaginary components are projected onto the imaginary-time plane. Sixteen $u_1$ roots exist at each time step, and these roots form continuous chains over time.

The high computational cost of computing a Gröbner basis for each time step necessitates a more efficient approach to root-finding. The hybrid Powell minimisation method with an analytically supplied Jacobian is orders of magnitude faster, but returns only one root for a given initial guess. The found root is typically the one (of sixteen possibilities) closest to the initial guess. Although only 8 roots can possibly satisfy $u_1 > 0$, it is possible over a short time for the initial $\times$-array guess
to be closer in proximity to an incorrect root chain than it is to the correct root chain. In this case, the output of the hybrid Powell method may ‘jump’ from one root chain to another. It is assumed that the ‘correct’ root chain (i.e. the one most representative of the incident flow field) is the one that is closest (on average) to the $\times$-array solution. Thus, a process is employed to detect and re-solve regions where the output of the hybrid Powell method has ‘jumped’ from the correct to an incorrect chain.

The re-solving process is illustrated in figure 3.3. Figure 3.3(a) depicts an initial output of $u_1$ from the hybrid Powell method that contains a ‘jump’ between root chains. Discontinuities are identifiable by consecutive positive/negative spikes in the discrete second order time gradient $(u_1)_{tt}$ as shown in figure 3.3(b). These points are then paired based on their proximity to one another to define ‘patches’ that do not belong to the main root-chain. All data points within an identified ‘patch’ (with an additional buffer on both sides) are replaced by a line connecting points before and after the region in question as in figure 3.3(c). The hybrid Powell method is then re-applied to the points in these regions, using the linearly interpolated points as the initial guesses instead of the $\times$-array outputs. The percentage of points flagged as being within a discontinuous patch before and after this process is shown in figure 3.4. Re-solving with updated initial guesses greatly reduces the number of flagged points. Note that for all locations the percentage of points flagged before the re-solving process is much less than 50%, and thus the ‘correct’ root chain that is not in doubt. The dependence on wall height is related to the strength of the gradients and the high flow angles near the wall.

Where discontinuities still exist after the first attempt at re-solving, one final attempt is made to find the continuation of the accepted root chain. Initial guesses for the root are selected as the root from the previous time step. This process is reserved for the final pass as it cannot be parallelized for efficient computation. The output of this ‘paving’ method is accepted if and only if it rejoins the accepted root at the tail end of the discontinuous patch. The time-fraction of points that are still flagged after these processes is minuscule, especially away from the wall.

### 3.6 Measurement Parameters

A number of boundary layer profile scans have been conducted across a range of Reynolds numbers and spatial resolutions for both ZPG boundary layer and pipe flow. These measurements are summarized in Table 3.1 from lowest-to-highest $Re_\tau$. The parameter range and symbol convention are clarified by figure 3.5. Note that each pipe flow measurement is paired with a ZPG boundary layer measurement at approximately matched $Re_\tau$ and $l^+$.  

The friction velocity $u_\tau$ in the ZPG boundary layer case is determined by the composite fit of Chauhan et al. [2009]. An analogue of the standard $\delta_{99}$ is used due to the slight non-zero curvature of the mean velocity profile in the FPF free-stream. Since the inner-normalized streamwise velocity variance profile follows a
Experimental Procedures

Figure 3.3: Illustration of process for identifying discontinuities in reduced data and re-solving equations with updated initial guesses.

single $Re$-independent curve in the vicinity of $\delta_{99}$ (at least over the $Re$-range considered herein [Marusic et al., 2015]), $\delta_{99}$ is identified in the present ZPG boundary layer cases as the position where the inner-normalized streamwise variance equals 0.257—its value at the position corresponding to $U = 0.99U_o$ based on the DNS results of Sillero et al. [2013]. Throughout the rest of this text, $\delta_{99}$ will refer to this analogue definition. The friction velocity in the pipe is obtained from direct measurements of the pressure drop using 18 ports located along the entire working section. Wall-position in both measurements is first determined with a microscope to within $\pm 0.1\text{mm}$ and subsequently tracked via an optical encoder on the traversing apparatus.
**Figure 3.4:** Fraction of time series flagged for re-solving before and after re-solve process. Outputs of both systems (3.6) and (3.7) shown versus wall-height for one particular measurement.

<table>
<thead>
<tr>
<th>Facility</th>
<th>( u_\tau [\frac{m}{\tau}] )</th>
<th>( t^+ )</th>
<th>( \delta [m] )</th>
<th>( Re_\tau )</th>
<th>( n_{x_2} )</th>
<th>( tU_{o}/\delta [-] )</th>
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</thead>
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<tr>
<td>MWT</td>
<td>0.18</td>
<td>12</td>
<td>0.30</td>
<td>3200</td>
<td>25</td>
<td>10000</td>
</tr>
<tr>
<td>FPF</td>
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<td>14</td>
<td>0.27</td>
<td>3300</td>
<td>25</td>
<td>4100</td>
</tr>
<tr>
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<td>0.30</td>
<td>4800</td>
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<td>15000</td>
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<tr>
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<td>13</td>
<td>0.45</td>
<td>5200</td>
<td>25</td>
<td>5100 (25300)</td>
</tr>
<tr>
<td>FPF</td>
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<td>12</td>
<td>0.52</td>
<td>5600</td>
<td>25</td>
<td>3400</td>
</tr>
<tr>
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<td>0.31</td>
<td>6300</td>
<td>25</td>
<td>14000</td>
</tr>
<tr>
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<td>19</td>
<td>0.45</td>
<td>7700</td>
<td>25</td>
<td>5000 (24800)</td>
</tr>
<tr>
<td>FPF</td>
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<td>7800</td>
<td>25</td>
<td>4300</td>
</tr>
<tr>
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<td>0.76</td>
<td>9100</td>
<td>25</td>
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</tr>
<tr>
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<td>5200 (25900)</td>
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<td>0.75</td>
<td>16700</td>
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<td>3900</td>
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</table>

**Table 3.1:** Summary of experimental data. Outer scale \( \delta \) defined as pipe radius or the analogue to \( \delta_{99} \) (defined in §3.6), where applicable. Number of unique wall-normal locations per experiment indicated by \( n_{x_2} \). Large-eddy turnover times \( (tU_{o}/\delta) \) shown in parentheses represent measurement sample times for selected \( x_2 \) locations near the start, middle, and end of the log layer, as well as the centremost location (i.e. \( 2.6\sqrt{\delta^+}, 0.15\delta^+, (2.6\sqrt{\delta^+} \times 0.15\delta^+)^{1/2}, \) and \( 0.93\delta^+ \)).
Figure 3.5: Summary of experimental data $Re_\tau$, spatial resolution, and symbol convention. This convention applies to the entirety of this text unless otherwise noted. Measurements are divided into tiers based on spatial resolution, and assigned a symbol that corresponds to that tier. Downward-pointing triangles indicate low $l^+$, circles indicate mid-range $l^+$, and upward-pointing triangles indicate high $l^+$ (relative to the other measurements). Shades of red indicate pipe flow data, shades of blue indicate ZPG boundary layer data, and darker shades correspond to higher $Re_\tau$. 
In this chapter, we will characterize the effects of geometry, finite spatial resolution, and data reduction scheme on the measured quantities. A collection of high resolution DNS flow fields is used to obtain predicted outputs for statistical moments of the velocity vector components and the off-diagonal components of the strain-rate tensor. These predictions identify artifacts of the measurement technique in a number of statistical profiles that might otherwise have been mistaken as physical effects. Actual experimental results are shown to agree well with the predicted outputs, which suggests that the various measurement biases are well-characterized by the factors considered in the synthetic experiment.
4.1 Motivation

As discussed in chapter 2, experimental vorticity and cross-stream velocity data in wall-bounded flows are relatively scarce, especially at high Reynolds number. This is in no small part due to the difficulty of building and operating measurement systems capable of capturing these quantities. The lack of existing benchmark data and the difficulty of the measurement necessitate a characterization of the inherent limitations of the technique, as well as a means to evaluate the actual performance relative to these expectations. The highly resolved DNS flow volumes of Sillero et al. [2013] provide a testing platform in which both of these objectives may be achieved. Agreement between the actual experimental results and those predicted by a model applied to a DNS of the flow of interest constitutes evidence that the probe performance is well-characterized by the model, and thus that it is appropriate to interpret the experimental results through the lens of the model.

4.2 Synthetic experiment

The present ‘synthetic’ experiment is similar in design to the one presented in Vukoslavčević et al. [2009], but with several noteworthy extensions. While the authors of that study sought to characterize the effects of probe geometry alone on the measurement of the velocity gradient tensor, the goal of the present effort is to model as closely as possible the behaviour of an actual probe with physical dimensions (relative to flow scales) equal to those achieved in practice. The present model extends the work of Vukoslavčević et al. [2009] to include the effects of finite sensor length and non-ideal sensor response to tangential and bi-normal flow.

Given a probe geometry and a response equation for each sensor, a predicted probe output can be obtained for any velocity field input. Sourcing the velocity field inputs from a DNS flow volume allows estimation of the effects of the measurement technique on measured quantities. This process is illustrated below for the simple case of a \(x\times\)wire array oriented such that each sensor is parallel to the \(x_1\times x_2\) plane.

First, the DNS volume is interpolated onto a grid comprised of 5 points along each individual sensor. The probe velocity output is presumed to apply to the centroid of the array, as is the standard assumption for a \(x\times\)wire array. The average of each velocity component is then computed along each sensor. These spatially averaged velocities are then plugged into Jorgensen’s expression, shown below as (4.1), to obtain ‘effective’ cooling velocities for sensor \(a\) of the \(x\times\)array (and similarly for sensor \(b\)):

\[
\tilde{u}_{ca}^2 = (\langle \tilde{u}_{1a} \rangle s_a - \langle \tilde{u}_{2a} \rangle c_a)^2 + k_a^2 (\langle \tilde{u}_{2a} \rangle c_a + \langle \tilde{u}_{2a} \rangle s_a)^2 + h_a^2 \langle \tilde{u}_{3a} \rangle^2, \tag{4.1}
\]

where \(\langle \cdot \rangle\) notation here denotes the spatial average along each individual sensor and \(s_a \equiv \sin \alpha_a\) and \(c_a \equiv \cos \alpha_a\). The effective cooling velocities are then used to
determine the output ‘voltages’ of the sensors using a typical functional relationship, which in this case is assumed to be King’s law:
\[
\tilde{e}_a = \left( A_a \tilde{u}^{1/2} + B_a \right)^{1/2},
\]
(4.2)
\[
\tilde{e}_b = \left( A_b \tilde{u}^{1/2} + B_b \right)^{1/2},
\]
(4.3)
For the purposes of this model, all sensors are assigned tangential cooling coefficients of \( k = 0.2 \), bi-normal cooling coefficients of \( h = 1.05 \), King’s law coefficients of \( A = 1 \) and \( B = 0 \), and effective cooling angles of \( \alpha_a = 45^\circ \) and \( \alpha_b = 135^\circ \) (\( \alpha \) is defined herein as the angle of the sensor relative to the \( x_1 \) coordinate in the sensor plane). It is assumed that these coefficients are exactly recovered from the calibration procedure.

Once ‘voltage’ outputs are generated for each sensor, velocity outputs can be obtained in the same manner as they would in an actual experiment. As described in sections 3.4 and 3.5, \( \times \) array outputs are obtained from a lookup-table approach. The lookup table can be made arbitrarily dense, as it is generated from the functional forms given in (4.1) and (3.5) with coefficients determined by the calibration data. While the sensors are assumed to respond to bi-normal flow (\( h = 1.05 \)), the lookup tables do not include information about the bi-normal component. Thus, the consequences of neglecting this term in the data reduction process are reflected in the model output. For the full eight-sensor probe, the \( \times \)-array output velocities are then used as initial guesses to solve the first order Taylor-series-expanded equations given in (3.6).

The outputs of the \( \times \)-array method and the gradient-array method may be directly compared to the known DNS velocities and gradients at the locations where each quantity is assumed to be resolved, allowing a direct evaluation of the efficacy of each method. Figure 4.1 shows the deviation from unity of the model-predicted correlation coefficient \( \varrho \) between the known DNS signal (again, from Sillero et al. [2013] at \( Re_\tau \approx 2000 \)) and the probe outputs of each measured quantity for three inner-normalised wire lengths that reside within tiers 1, 2, and 3 from figure 3.5 (\( l^+ \approx 14 \approx 20, \approx 25 \)) and thus match the experimental data presented herein. Zero deviation from unity indicates that the probe perfectly recovers the input signal, while a deviation of 1 indicates that the probe output is completely independent of the input signal.

In general, the velocity components are recovered more effectively than the gradients over this range of probe sizes. The fidelity of all measured signals improves when the probe size is reduced relative to the viscous length scale. The gradient-array processing improves the recovery of the velocity signals and their streamwise gradients relative to the \( \times \)-array, and has virtually no effect on the cross-stream gradients. The most substantial improvement associated with the gradient-array method is seen in the \( u_3 \) signal. It was noted in Zimmerman et al. [2017] that this result was consistent with expectations, since the \( u_3 \) signal is subject to contamination from \( \partial u_1 / \partial x_2 \), which is the only gradient with a non-zero mean (to within the boundary layer approximations) and also generally has a high RMS relative to
Figure 4.1: Estimated deviation from unity of correlation coefficient $\rho$ between measured and actual quantities. Smaller plotted values indicate superior measurement quality. Blue curves indicate gradient-array processing; black curves indicate $x$-array processing. Dashed, dash-dotted, and dotted lines indicate wire lengths of $l^+ \approx 14$, $\approx 20$, and $\approx 25$, respectively. Line key also given in table 4.1.
other velocity gradients. As the improvement is substantial throughout the entire flow domain (where the mean gradient is nearly zero and there is greater parity in the gradient RMS values), however, it is likely that another factor is in play. It was suggested in Zimmerman et al. [2017] that the success of the gradient-array method also depends on the separation ratio—the distance between sub-arrays divided by the distance between sensors in each sub-array. The greater improvement in the $u_3$ signal than the $u_2$ signal from the gradient-array method, even in the outer region of the flow, supports this notion, since the $u_3$ sub-arrays are closer together than the $u_2$ sub-arrays.

The quality of each gradient measurement depends at least on the effective distance over which the velocity difference is computed and the portion of the spectrum of each gradient that resides in scales smaller than that effective distance. If the flow were truly isotropic, the spectra of all six shear gradients would be identical, and so the signal fidelity would depend only on the separation distance. Based on the physical dimensions of the probe, this distance is $l_{wp}$ in the $x_2$ direction and $2.5 \cdot l_{wp}$ in the $x_3$ direction (see figure 3.1). The effective separation in the $x_1$ direction requires a more detailed explanation. In a real physical experiment, the sample frequency is set such that all flow scales could theoretically be captured (if not for the spatial averaging associated with the probe). Taylor’s frozen turbulence hypothesis is then used to convert the temporal signal to a spatial signal. Thus, the ‘spatial’ separation over which $x_1$ gradients may be taken is $U_1/f_s$, where $U_1$ is the local mean velocity and $f_s$ is the sample rate. Following the recommendation of Klewicki & Falco [1990], however, this signal is filtered (via moving-average) to remove wavelengths affected by measurement noise—i.e. those smaller than the sensor length. Thus, the ‘effective’ separation in the $x_1$ direction is approximately $\sqrt{2} \cdot l_{wp}$, or the total sensor length. This is replicated in the synthetic experiment. As the DNS is not time-resolved, Taylor’s frozen turbulence hypothesis is not invoked. Instead, the fields are interrogated at closely spaced streamwise locations (smaller than the sensor length). Then, just as with the actual experimental output, the modeled probe velocity outputs are filtered such that wavelengths smaller than the sensor length are largely removed.

Were the flow truly isotropic, the expectation based on these separations would be that the $x_2$ gradients would be the best resolved, followed by the $x_1$ gradients, followed by the $x_3$ gradients. In reality, the anisotropy in the boundary layer results in higher signal fidelity of the measured $x_1$ gradients than of the measured $x_2$ and $x_3$ gradients. The fidelity of $\partial u_1/\partial x_3$ at the highest resolution is approximately matched by the fidelity of $\partial u_1/\partial x_2$ at the second highest resolution, and the fidelity of $\partial u_2/\partial x_3$ at the highest resolution is approximately matched by the fidelity of $\partial u_3/\partial x_2$ at the second highest resolution. Thus, spanwise gradients measured over a viscous separation of $\Delta x_3 = 25\nu/\nu_t$ in a boundary layer are captured with approximately the same average fidelity as wall-normal gradients measured over a viscous separation of $\Delta x_2 = 13\nu/\nu_t$. This validates the probe design choice of prioritizing resolution of the $x_2$ gradients over the $x_3$ gradients.

Figures 4.1(b) and (d) also show the difference in the signal fidelity of $u_2$ and $\partial u_2/\partial x_1$ between the probe-centered signals and the sub-array-centered signals.
The ‘measured’ sub-array-centered signals (thinner lines) more closely resemble the DNS signals at the centroid of the sub-array than do the probe-centered signals. This is expected, since the process of obtaining the probe-centered signal is tantamount to applying a spatial filter in the spanwise direction that attenuates wavelengths less than $2.5l_w$ (see figure 3.1). While the individual sub-arrays are therefore able to more faithfully reproduce the statistical profiles of $u_2$ and its streamwise gradient, it is the probe-centered signal which more faithfully reproduces quantities such as $\omega_3 = \partial u_2/\partial x_1 - \partial u_1/\partial x_2$, where collocation with the other components of the signal is of value.

4.3 Probe effects: expected and actual

While the correlation coefficients shown in figure 4.1 are helpful to compare measurement quality of various quantities relative to others and across a range of probe dimensions, there is no obvious way to interpret their particular magnitudes as either ‘sufficient’ or ‘insufficient’. Instead, the aggregate effects of the probe dimensions and data reduction scheme on one-point statistics can be computed. These modeled statistical profiles not only contextualise the effects of the measurement technique on some of the quantities of interest, they also serve as a benchmark against which experimental measurements can be compared. In this section, model-predicted statistical profiles of the measured velocity and gradient components will be compared to experimental results at matched resolution. Mismatch between experimental results and model outputs reflects the influence of one or more of the following factors: wall-position uncertainty, $u_\tau$ uncertainty, Reynolds number trends (model and three experiments are at different $Re_\tau$), calibration error, aerodynamic blockage by the probe, thermal cross-talk between sensors, violation of Taylor’s frozen turbulence hypothesis (for streamwise gradients), conductive heat transfer to the bounding wall, frequency response, sensor drift, and electrical noise. The data presented in this chapter are summarised in Table 4.1.

4.3.1 Velocity statistics

Figure 4.2 shows the mean, variance, skewness, and kurtosis profiles of the streamwise velocity. As suggested by figure 4.1(a), linearising the velocity field across the probe (gradient-array method) has only a slight effect on the $u_1$ statistics. The model predicts attenuation of the variance and an increase in the skewness coefficient, both of which are observed in the experimental results. The near-wall variance is known to increase slightly with $Re_\tau$, particularly owing to increased contributions from large scales (e.g. see Metzger et al. [2001] and Hutchins & Marusic [2007]), which are less susceptible to spatial attenuation. This may be the cause of the lack of downward trend in the experimental near-wall $u_1$ variance.
Table 4.1: Summary of experiments and DNS comparisons discussed in this chapter. Boundary layer DNS from the dataset of Sillero et al. [2013]. Black lines and symbols are associated with the standard $\times$-array data reduction scheme. Shades of blue are associated with the gradient-array data reduction scheme. Symbols represent physical experimental spatial resolution: downward triangles (▽) indicate lowest $l^+$, circles (○) indicate mid-range $l^+$, and upward triangles (△) indicate highest $l^+$. Synthetic experimental ‘effective’ spatial resolution is indicated by line style.
Figure 4.3 shows the mean, variance, skewness, and kurtosis profiles of the wall-normal velocity. The accuracy of the mean $u_2$ measurement is subject to the agreement between the jet calibration $0^\circ$ pitch angle curve and the free-stream calibration as described in section 3.4. The present data reduction process includes a step where the jet calibration is forced into agreement with the free-stream calibration, and so the accuracy of the present mean $u_2$ measurements reflects the efficacy of this method. The errors in interpreted mean flow angle are very small (i.e. $U_1/U_2 \gg 1$) in all cases. The experimental variance and skewness profiles exhibit the expected near-wall trends for both data reduction schemes. The best-resolved $\times$-array kurtosis profile (in figure 4.3(d)) exhibits the expected behaviour in the near-wall region, but the other two $\times$-array profiles move in the opposite direction of the expected trend. In contrast, the gradient-array scheme produces profiles that exhibit the trend predicted by the synthetic experiment. That the two lower-resolution $\times$-array kurtosis profiles appear erroneously high near the wall may be associated with inaccurate extrapolation to high flow-angles. This would not affect the synthetic experimental $\times$-array outputs, because the probe response is assumed to be perfectly described by Jorgensen’s expression at all incident angles. If, for example, a large $\partial u_1/\partial x_3$ gradient produced an output differential in the two adjacent sensors in sub-array (c) that was greater than any produced by the jet calibration, the flow angle interpreted by the $\times$-array reduction scheme would be based on extrapolation. Removal of this gradient effect could therefore reduce the incidence rate of spurious $u_2$ values by reducing the number of data points that are interpreted as being outside the calibration domain. While this explanation is consistent with the observed profiles, further investigation of this phenomenon is warranted.

Figure 4.4 shows the mean, variance, skewness, and kurtosis profiles of the spanwise velocity. As suggested by figure 4.1(c), the gradient-array data reduction method affects the $u_3$ statistics more significantly than it does either the $u_1$ or $u_2$ statistics. The mean of the $\partial \tilde{u}_1/\partial x_2$ gradient is responsible for the predicted and observed near-wall bias in the odd moments of $u_3$ for the $\times$-array reduction method. Allowing and accounting for linear variation in the velocity across the sub-array domain with the gradient-array method completely eliminates this bias in the $u_3$ mean and skewness profiles. The sensitivity of a standard $\times$-array to the $x_2$ gradients (regardless of their mean value) manifests as the ‘peelup’ in the variance. This effect is also mitigated with the gradient-array approach. The predicted increase in the near-wall kurtosis due to gradient-aliasing suggests that the gradient-influenced values tend to be of larger magnitude than the actual $u_3$ fluctuations. As with the other statistical moments of $u_3$, accounting for linear variation in the velocity profile across the measurement domain largely removes this bias.

Figure 4.5 shows the mean, variance, skewness, and kurtosis profiles of the non-zero Reynolds shear stress (i.e. $u_1u_2$). The second, third, and fourth-order moment profiles exhibit good agreement with the model predictions, although the same near-wall kurtosis trend that was observed in the $u_2$ fluctuations is observed here in the $\times$-array profiles (and is also removed by the the gradient-array approach). The mean Reynolds shear stress is a particularly challenging quantity to measure
Figure 4.3: Wall-normal velocity statistics from DNS, synthetic experiment, and physical experiment. Symbols and lines as in Table 4.1

Figure 4.4: Spanwise velocity statistics from DNS, synthetic experiment, and physical experiment. Symbols and lines as in Table 4.1
Figure 4.5: Fluctuating Reynolds shear stress statistics from DNS, synthetic experiment, and physical experiment. Symbols and lines as in Table 4.1

accurately. It was shown in Zimmerman et al. [2017] that even slight calibration jet misalignment could lead to large errors in the mean value. Additionally, the highly super-Gaussian kurtosis of the $u_1u_2$ fluctuations suggest that the mean is composed of a delicate balance of instantaneous values that often far exceed the standard deviation. Thus, a calibration error that would be inconsequential to the $u_1$ and/or $u_2$ statistics individually could potentially shift their covariance substantially. This can be avoided by including an additional sensor normal to the mean flow as in Morrill-Winter et al. [2015]. This essentially turns the problem from a two equation, two unknown system (two sensors to determine both $u_1$ and $u_2$) into an overdetermined regression problem. The flow-normal sensor is used to determine $u_1$ independently, and this value is used along with the responses of the two slanted wires to determine the best-fit value of $u_2$. This was not included in the design of the present sensor, as the resulting increase in overall probe size would adversely affect the resolution of the vorticity vector.

4.3.2 Shear strain rate statistics

Profiles of the first four statistical moments of the $\partial \tilde{u}_2/\partial x_1$ gradient are shown in figure 4.6. The mean value is extremely close to zero, indicating the lack of temporal sensor drift throughout each record. That is, if one of the sensors responsible
for detecting \( u_2 \) drifted during a record, \( U_2 \) at the end of the record would presumably differ from \( U_2 \) at the start of the record, resulting in a non-zero \( \partial U_2 / \partial x_1 \) after Taylor’s frozen turbulence hypothesis is applied. The RMS profiles shown in figure 4.6(b) match the model predictions very closely. This suggests that Taylor’s frozen turbulence hypothesis is, at least in a mean sense, appropriate for differentiating \( u_2 \) in the streamwise direction of a turbulent boundary layer. A similar near-wall sign change to the one that was predicted and observed in the \( u_2 \) skewness profile is predicted and observed in the \( \partial u_2 / \partial x_1 \) skewness profile. The predicted spatial resolution trend in the kurtosis profiles is also observed. The gradient-array approach and the \( x \)-array approach produce essentially the same profiles with the exception of the near-wall kurtosis, in which a magnitude difference is predicted by the synthetic experiment and observed in practice.

Statistical profiles of the other streamwise shear gradient, \( \partial u_3 / \partial x_1 \), are shown in figure 4.7. As with \( \partial U_2 / \partial x_1 \) profiles shown in figure 4.6(a), the nearly identically zero mean \( \partial U_3 / \partial x_1 \) profiles suggest the lack of temporal sensor drift during each record. The agreement between the model and the experiment for the RMS profiles shown in figure 4.7(b) also suggest that Taylor’s frozen turbulence hypothesis is, at least in a mean sense, appropriate for differentiating \( u_3 \) in the streamwise direction of a turbulent boundary layer. As with the \( u_3 \) variance, the gradient method largely eliminates the ‘peelup’ observed in the \( \partial u_3 / \partial x_1 \) RMS and skewness profiles. The model-predicted spatial resolution trends in the kurtosis are also observed.
Figure 4.7: Statistics of $\partial u_3/\partial x_1$ from DNS, synthetic experiment, and physical experiment. Symbols and lines as in Table 4.1

Figure 4.8 shows profiles of the first four statistical moments of $\partial u_1/\partial x_2$. The model predicts attenuation of the RMS and decrease in the skewness and kurtosis profiles with worsening spatial resolution for both data reduction schemes. The trends in the RMS and skewness are observed, albeit the highest-resolution RMS is slightly lower than the model prediction. The predicted trend in the kurtosis is observed near the wall, but the opposite of the prediction is observed in the log-layer. This is related to low magnitude noise associated with potential flow around the front of the probe, which will be discussed in more detail in section 4.3.3.

The model-predicted trends in $\partial \tilde{u}_3/\partial x_2$ are shown in figure 4.9, and are closely matched by the experimental results. The mean value is reflective of the second gradient in the mean streamwise velocity $\partial^2 U_1/\partial x_2^2$. Since the measured mean $U_3$ value shown in figure 4.4 is proportional to $\partial U_1/\partial x_2$ (owing to gradient aliasing), the difference in measured mean $U_3$ (i.e. $\partial U_3/\partial x_2$) is proportional to the difference in $\partial U_1/\partial x_2$ between the two measurement locations (i.e. proportional to $\partial^2 U_1/\partial x_2^2$). The expected mitigation of this effect from the gradient-array method is observed in the experimental results. The potential flow effect noted above and discussed further in section 4.3.3 is not expected to induce $u_3$, and so the $\partial u_3/\partial x_2$ kurtosis does not exhibit the same flattening in the log and wake layers as is observed with $\partial u_1/\partial x_2$.

Figure 4.10 shows the mean, standard deviation, skewness, and kurtosis profiles
Figure 4.8: Statistics of $\partial u_1/\partial x_2$ from DNS, synthetic experiment, and physical experiment. Symbols and lines as in Table 4.1.

Figure 4.9: Statistics of $\partial u_3/\partial x_2$ from DNS, synthetic experiment, and physical experiment. Symbols and lines as in Table 4.1.
Figure 4.10: Statistics of $\partial u_1/\partial x_3$ from DNS, synthetic experiment, and physical experiment. Symbols and lines as in Table 4.1

of the spanwise gradient of the streamwise velocity, $\partial \tilde{u}_1/\partial x_3$. The experimental mean value shows a slight negative bias near the wall, likely caused by a slight calibration imbalance in the low-speed regime between sub-arrays (c) and (d). The experimental RMS and skewness profiles closely match the model-predictions. The potential flow effect noted above and observed in 4.8(d) (to be described in detail in section 4.3.3) is expected to generate low-magnitude noise in the spatial gradients of the streamwise velocity. Consistent with this phenomenon, the flattening of the kurtosis observed first in the $\partial u_1/\partial x_2$ profiles in figure 4.8(d) is also observed in the $\partial u_1/\partial x_3$ profiles in figure 4.10(d).

The experimental and model-predicted statistical profiles of $\partial u_2/\partial x_3$, shown in figure 4.11, exhibit very close agreement beyond the very near-wall. In agreement with the model output, none of the $\partial u_2/\partial x_3$ profiles are sensitive to the inclusion of gradient information in the data reduction scheme.

4.3.3 Velocity-vorticity products

Products of the velocity and vorticity components are of interest for several reasons. The cross-product of the velocity and vorticity vectors, also known as the Lamb vector, appears in a decomposition of the convective term of the Navier-Stokes
equation:
\[ \tilde{u}_j \frac{\partial \tilde{u}_i}{\partial x_j} = \frac{1}{2} \frac{\partial \tilde{u}_k \tilde{u}_k}{\partial x_i} - \epsilon_{ijk} \tilde{u}_j \tilde{\omega}_k. \]  
(4.4)

Additionally, the dot product of the velocity and vorticity vectors, also known as the helicity, is of consequence in topological analysis of turbulence. Indeed, the potential value of accurate measurements of these products was one of the motivating factors for the present research effort. The capacity of a physical sensor to accurately report the mean values of these quantities is discussed in this section, and suggestions are made to aid in future efforts toward this end.

Figure 4.12 shows the model-predicted and experimental mean profiles of all nine velocity-vorticity products. All of the profiles show reasonably good agreement with the model predictions, with the exception of \( u_2 \omega_3 \) and \( u_3 \omega_2 \), shown in figures 4.12(f) and (h), respectively. Both profiles exhibit reasonable agreement with the model near the wall, but then reach non-zero plateau values that are not predicted by the model. We believe this is related to aerodynamic blockage of the incident flow by the measurement probe itself. This effect is also visible in \( u_1 \omega_3 \) due to the non-zero correlation between \( u_1 \) and \( u_2 \), although the magnitude of the ‘plateau’ region is smaller than the plateaus in both \( u_2 \omega_3 \) and \( u_3 \omega_2 \).

Consider a two-dimensional model of the flow around the probe as shown in figure 4.13, where the incident flow is not aligned with the \( x_1 \) direction. The curvature of the streamlines around the front of the probe (as well as the underlying...
Figure 4.12: Mean velocity-vorticity correlations from DNS, synthetic experiment, and physical experiment. Symbols and lines as in Table 4.1

colour map of $U_1$) indicate acceleration of the flow at the leeward side of the leading edge and deceleration of the flow at the windward side of the leading edge. In the notation of figure 4.13, $u_1(a) < u_1(b)$ when $u_2 < 0$ and $u_1(a) > u_1(b)$ when $u_2 > 0$. Such conditions would result in $u_2 \partial u_1 / \partial x_2 > 0$. The same argument can be made for a 90° rotation of the diagrams in figure 4.13 about the $x_1$ axis, which would result in $u_3 \partial u_1 / \partial x_3 > 0$. The mean velocity-vorticity products of interest can be simplified by the boundary layer approximations according to the following:

$$\bar{u}_2 \omega_3 = u_2 \left( \frac{\partial u_2}{\partial x_1} - \frac{\partial u_3}{\partial x_2} \right) = \frac{1}{2} \frac{\partial u_2^2}{\partial x_1} - \frac{u_2}{2} \frac{\partial u_1}{\partial x_2} \approx -u_2 \frac{\partial u_1}{\partial x_2}, \quad (4.5)$$

$$\bar{u}_3 \omega_2 = u_3 \left( \frac{\partial u_3}{\partial x_3} - \frac{\partial u_3}{\partial x_1} \right) = u_3 \frac{\partial u_1}{\partial x_3} - \frac{1}{2} \frac{\partial u_3^2}{\partial x_1} \approx u_3 \frac{\partial u_1}{\partial x_3}. \quad (4.6)$$

The positive correlations between the cross-stream velocities and the induced gradients implied by figure 4.13 should therefore be expected to produce $\bar{u}_2 \omega_3 < 0$ and $\bar{u}_3 \omega_2 > 0$, both of which are observed in the experimental data.

This hypothesis was evaluated against the jet calibration data, which is expected to produce the same effect. Figure 4.14 shows a scatter plot of the measured $\partial U_1 / \partial x_3$ when the probe is subjected to a known $U_3$ input from the calibration jet. Despite the uniform incident flow, the probe detects a difference in $U_1$ between
sub-arrays (c) and (d) (see figure 3.1) that is approximately proportional to $U_3$. This is consistent with the potential flow model—which predicts dependence of the $u_1$ differential on $|V_o|$ and the flow incidence angle $\psi$—since $u_3 = |V_o| \sin \psi$.

The aerodynamic blockage effect can be mitigated by reducing its magnitude relative to the contributions from the turbulence and/or including compensation in the data-reduction scheme. The former approach is demonstrated inadvertently by increasing the wind-tunnel speed to achieve higher $Re_\tau$. To illustrate this point, it is useful to rewrite the rightmost term in equation 4.6 in terms of the actual measured quantities:

$$u_3 \frac{\partial u_1}{\partial x_3} \approx u_3 \frac{u_{1d} - u_{1c}}{\Delta x_3},$$

where the subscripts $c$ and $d$ refer to the velocities measured at sub-arrays (c) and (d), respectively (see figure 3.1). Absent any nonlinear effects associated with the turbulence, the mean correlation value can be modeled by substituting the relationship implied by figure 4.14 (i.e. $C_1(u_{1d} - u_{1c}) \simeq u_3$) into equation 4.7:

$$u_3 \frac{u_{1d} - u_{1c}}{\Delta x_3} \simeq C_1 \frac{\overline{u_3^2}}{\Delta x_3}.$$  

Since and $\overline{u_3^2}$ approximately scales with $u_3^2$ over the $Re_\tau$ range covered by the three measurements discussed in this chapter, and $\Delta x_3$ is fixed in physical units, the mean correlation value owing to the aerodynamic blockage effect should scale with $u_3^2$. Viscous normalization of a velocity-vorticity product requires division by $u_3^2/\nu$, which is the product of a velocity scale, a velocity scale, and (the inverse of) a length scale $(u_\tau \cdot u_\tau \cdot u_\tau/\nu)$. Thus, the viscous-normalized magnitude of

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**Figure 4.13:** Streamlines around a simplified measurement probe based on a potential flow calculation.
Figure 4.14: Differential streamwise velocity of jet calibration interpreted by data reduction scheme versus known $u_3$ input velocity.

The aerodynamic blockage effect on the affected velocity-vorticity correlations is expected to itself scale with $\nu/u_\tau$. This is indeed observed in the ‘plateau’ regions of figures 4.12(c), (f), and (h), as the magnitude of these regions approaches zero with increasing $Re_\tau$, or, equivalently (in this case), decreasing $\nu/u_\tau$. While decreasing the viscous length scale improves the signal-to-noise ratio for the velocity-vorticity products, it has the counterproductive side-effect of worsening spatial attenuation of the velocity components and their gradients.

The second approach, to account for the aerodynamic blockage effect in the data reduction scheme, is more easily pursued. This effort is ongoing, and so only a preliminary attempt will be presented here. The effect is quantified as a voltage output deviation from the $u_2 = u_3 = 0$ case across a range of speeds, and is assumed to depend only on the flow speed and the bi-normal velocity. For example, the voltage deviation $\Delta e$ in a sensor in sub-array (a) (in the $x_1$-$x_3$ plane) due to a bi-normal velocity $u_2$ is computed as $\Delta e(|\vec{u}|, u_2) = e(|\vec{u}|, u_2) - e(|\vec{u}|, 0)$.

As described in section 3.5, the $x$-array outputs are computed as a first step to obtain estimates of the velocity. Next, $\Delta e$ values are computed for each sensor based on the initial estimates of flow speed and bi-normal velocity. These voltage deviations are then subtracted from the measured sensor voltages, and the data are reprocessed as normal. This model is likely an oversimplification of the effects of aerodynamic blockage, but serves to illustrate the potential for a compensatory method to correct the target statistics. Those statistics that were discussed as having been affected by aerodynamic blockage are re-plotted in figure 4.15 alongside the results of the preliminary attempt at compensation. The gradient kurtosis profiles are relatively unchanged near the wall, but shift closer to the model predictions after the compensation for aerodynamic blockage is included in the data reduction scheme. This is consistent with the notion that the downward shift in the kurtosis is related to a decrease in the signal to noise ratio, where the aerodynamic blockage ‘noise’ eventually competes with the gradient signal as distance
from the wall increases. The ‘plateau’ regions in $u_2\omega_3$ and $u_3\omega_2$ are very sensitive to the compensation step. While the magnitude of the correction leaves room for improvement, the response of the profiles to the compensation step suggests that the underlying issue is manageable with a targeted calibration and data-reduction scheme.

![Figure 4.15](image)

**Figure 4.15:** Various statistical profiles before compensation for aerodynamic blockage (filled symbols) and after (open symbols).

### 4.3.4 Spectra

Given the ‘true’ velocity spectrum tensor $\Phi_{ij}$, one can compute model one-dimensional spectra of all velocity components and their gradients. This process is described for the velocity outputs of single normal hot-wires and $\times$-arrays by Wyngaard [1968], for the gradient outputs of two separated single normal hot-wires by Wyngaard [1969], and for the gradient outputs of two separated $\times$-arrays by Zhu & Antonia [1995]. To follow these procedures, it must be assumed that the sensing elements respond only to the normal velocity component (i.e. $k = h = 0$ in equation 3.4). Furthermore, since the full velocity spectrum tensor is required for the calculations, it is typically necessary to assume local isotropy. The functional forms of the modeled ‘measured’ spectra are given below for reference.

The model isotropic spectrum $E(\kappa)$ of Pope [2000] is given as:

$$E(\kappa) = C \varepsilon^{2/3} \kappa^{-5/3} f_L(\kappa L) f_\eta(\kappa \eta) ,$$  \hspace{1cm} (4.9)
where $C = 1.5$ is a constant, $L \equiv k^{1/2}/\varepsilon$ is a large-eddy lengthscale, and the functions $f_L$ and $f_\eta$ describe the shape of the spectrum in the energy-containing and dissipative ranges, respectively:

$$f_L (\kappa L) = \left( \frac{\kappa L}{[(\kappa L)^2 + c_L]^{1/2}} \right)^{5/3 + p_0},$$  \hspace{1em} (4.10)

$$f_\eta (\kappa \eta) = \exp \left( -\beta \left[ (\kappa \eta)^4 + c_\eta^4 \right]^{-1/4} - c_\eta \right),$$  \hspace{1em} (4.11)

and $p_0 = 2$, $c_L = 6.78$, $\beta = 5.2$, and $c_\eta = 0.4$. Note that the actual shape of the spectrum used herein is of relatively little consequence, since the predicted ‘measured’ spectrum divided by the ‘true’ spectrum highlights only the proportional attenuation. Equation (4.9) may be used to obtain the velocity spectrum tensor as follows:

$$\Phi_{ij} (\kappa) = \frac{E(\kappa)}{4\pi \kappa^4} \left( \kappa^2 \delta_{ij} - \kappa_i \kappa_j \right).$$  \hspace{1em} (4.12)

The energy spectrum/cospectrum of $u_i$ and $u_j$ computed from a hotwire-obtained time-series is an approximation of the one-dimensional (streamwise) spectrum $E_{ij}$, which is related to the velocity spectrum tensor as follows:

$$E_{ij}(\kappa_1) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \Phi_{ij} (\kappa) \, d\kappa_2 \, d\kappa_3.$$  \hspace{1em} (4.13)

Assuming (as Wyngaard [1968]) that the ‘measured’ velocity is the line-average of the actual velocity field along the length of the wire, the ‘measured’ streamwise energy spectrum of a wall-parallel single-wire $E_{11m} (\kappa_1)$ may be computed as:

$$E_{11m} (\kappa_1) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \sin^2 \left( \frac{\kappa_3 l_w/2}{\kappa_3 l_w/2} \right) - \Phi_{11} (\kappa) \, d\kappa_2 \, d\kappa_3,$$  \hspace{1em} (4.14)

For the remainder of this chapter, superscript $m$ will indicate a ‘measured’ spectrum (as opposed to a ‘true’ spectrum). The analysis leading to (4.14) is extended in the same 1968 paper by Wyngaard to obtain a model of $E_{11m}$ and $E_{22m}$ (or $E_{33m}$) for a standard $\times$-wire array. Equation (4.1) is first linearised by assuming that $k = h = 0$ (i.e. by considering only the normal component of velocity). Following the same process that led to (4.14) leads to the following expressions for $E_{11m}$ and
\[ E_{22}^m (\text{or } E_{33}^m) = \int \int \frac{1}{4} \left[ (A_a^2 + 2A_a A_b \cos(\kappa \cdot d) + A_b^2) \Phi_{11} + 
\right. \n\]
\[ \left. 2 \cot \beta (A_b^2 - A_a^2) \Phi_{12} + \cot^2 \beta \left( A_a^2 - 2A_a A_b \cos(\kappa \cdot d) + A_b^2 \right) \Phi_{22} \right] d\kappa_2 d\kappa_3, \quad (4.15) \n\]
\[ E_{11}^m (\kappa_1) = \int \int \frac{1}{4} \left[ (A_a^2 + 2A_a A_b \cos(\kappa \cdot d) + A_b^2) \Phi_{22} + \n\right. \n\]
\[ \left. 2 \tan \beta (A_b^2 - A_a^2) \Phi_{12} + \tan^2 \beta \left( A_a^2 - 2A_a A_b \cos(\kappa \cdot d) + A_b^2 \right) \Phi_{11} \right] d\kappa_2 d\kappa_3, \quad (4.16) \n\]

where \( \beta \) is the magnitude of the effective cooling angle (\( |\alpha_a| = |\alpha_b| = 45^\circ \) is assumed), \( d \) is the vector describing the separation between the two elements of the \( x \)-array, and \( A_a \) and \( A_b \) are the coefficients associated with line-averaging along wires \( a \) and \( b \) of a \( x \)-array, or for example:

\[ A_a = \frac{\sin(\kappa \cdot l_{wa}/2)}{(\kappa \cdot l_{wa}/2)}. \quad (4.17) \n\]

Zhu & Antonia [1995] extend this result to estimate the streamwise velocity gradient spectra ‘measured’ by two adjacent \( x \)-wire arrays. Assuming parallel wires have equal line-averaging coefficients \( A_a \) and \( A_b \), the ‘measured’ streamwise spectra of (for example) \( \partial u_1/\partial x_2 \), \( \partial u_2/\partial x_3 \), and \( \partial u_2/\partial x_1 \) (i.e. \( \phi_{u1,2}^m(\kappa_1) \), \( \phi_{u2,3}^m(\kappa_1) \) and \( \phi_{u2,1}^m(\kappa_1) \)) may be computed as follows:

\[ \phi_{u1,2}^m(\kappa_1) = \frac{1}{\Delta x_2^2} \int \int_{-\infty}^{\infty} \sin^2 \left( \frac{\kappa \cdot s_2}{2} \right) \left[ 2 \cot \beta (A_b^2 - A_a^2) \Phi_{13} + \left( A_b^2 + A_a^2 + 2A_a A_b \cos(\kappa \cdot d) \right) \Phi_{11} + \cot^2 \beta \left( A_b^2 + A_a^2 - 2A_a A_b \cos(\kappa \cdot d) \right) \Phi_{33} \right] d\kappa_2 d\kappa_3, \quad (4.18) \n\]
\[ \phi_{m_{u,2,3}}(\kappa_1) = \frac{1}{\Delta x_3} \int_{-\infty}^{\infty} \sin^2 \left( \frac{\kappa \cdot s_3}{2} \right) \left[ 2 \tan \beta (A_b^2 - A_a^2) \Phi_{12} + \left( A_b^2 + A_a^2 + 2A_a A_b \cos(\kappa \cdot d) \right) \Phi_{22} + \tan^2 \beta (A_b^2 + A_a^2 - 2A_a A_b \cos(\kappa \cdot d)) \Phi_{11} \right] d\kappa_2 d\kappa_3, \]  
\[ (4.19) \]

\[ \phi_{m_{u,2,1}}(\kappa_1) = \frac{1}{\Delta x_1} \int_{-\infty}^{\infty} \cos^2 \left( \frac{\kappa \cdot s_3}{2} \right) \sin^2 \left( \frac{\kappa \cdot s_1}{2} \right) \left[ 2 \tan \beta (A_b^2 - A_a^2) \Phi_{12} + \left( A_b^2 + A_a^2 + 2A_a A_b \cos(\kappa \cdot d) \right) \Phi_{22} + \tan^2 \beta (A_b^2 + A_a^2 - 2A_a A_b \cos(\kappa \cdot d)) \Phi_{11} \right] d\kappa_2 d\kappa_3, \]  
\[ (4.20) \]

where \( \Delta x_2 \) and \( \Delta x_3 \) are the sub-array separations as given in figure 3.1 (here \( \Delta x_2 = l_{wp} \) and \( \Delta x_3 = 2.5l_{wp} \)); \( \Delta x_1 \) is the equivalent streamwise separation (i.e. \( \Delta x_1 = l_w \) owing to the moving average filter described above); \( s_1 = [\Delta x_1 \ 0 \ 0]^T \), \( s_2 = [0 \ \Delta x_2 \ 0]^T \) and \( s_3 = [0 \ 0 \ \Delta x_3]^T \) correspond to the separation (or equivalent separation) of \( \times \)-arrays; and \( d_2 = [0 \ l_{wp}/2 \ 0]^T \), and \( d_3 = [0 \ 0 \ l_{wp}/2]^T \) correspond to the separation between \( \times \)-array elements (cf. figure 3.1).

Alternatively, the relationship between the ‘true’ and ‘measured’ one-dimensional spectra can be deduced from the ‘brute-force’ simulation described in section 4.2. Although the ZPG boundary layer is not strictly homogeneous in the streamwise direction, it is assumed to be locally homogeneous over a streamwise slice of thickness \( \Delta x_1 \approx 1\delta_{99} \). While many of the measured quantities contain spectral density at wavelengths far in excess of \( \lambda_1 \approx \delta_{99} \), the ratios of measured to ‘true’ spectra approach constancy as \( \kappa_1 \rightarrow 2\pi/\delta_{99} \).

Spectra can be computed along lines in the \( x_1 \) direction for both the unfiltered DNS and the simulated probe output. The ratios of these spectra are shown in figures 4.16 (for velocity components) and 4.17 (for velocity gradients). Unlike the relations derived by Wyngaard [1968, 1969] and Zhu & Antonia [1995], these ratios do not require an assumption of isotropy to evaluate, and reflect the effects of sensing element sensitivity to tangential/bi-normal cooling.

In general, close agreement is observed between the spectral ratios predicted by the Wyngaard [1969] model applied to the isotropic spectrum of Pope [2000] and those computed from the synthetic experiment. The exceptions to this agreement for the velocity components are for the streamwise velocity at wavenumbers \( \kappa_1 l_w >\)
Figure 4.16: Simulated ratio between ‘measured’ and ‘true’ velocity spectra. Black curves represent spectral ratios computed from wall-positions within the log-layer of the DNS volumes of Sillero et al. [2013]. Dark and light grey curves represent the spectral ratios for wall-positions below and above the log-layer, respectively. Blue curves are computed from the model proposed by Wyngaard [1968] using the isotropic spectrum of Pope [2000] with the wire length set to \( l_w = 4\eta \) (to approximately match actual experimental conditions). DNS ratios are truncated for each wall-position where \( \kappa_1\eta(x_2) = 1 \).

1 and for all three components below the log-layer. The near-wall disagreement is reflective of the effects of the strong \( \partial u_1 / \partial x_2 \) and \( \partial u_1 / \partial x_3 \) RMS in this region. As discussed in Chapter 3, these gradients are interpreted by standard \( x \)-wire pairs as contributions to the transverse velocity. This effect is accounted-for in the Wyngaard [1968] model, but the input isotropic spectrum does not reflect the relative strength of the near-wall gradients and thus the model output under-predicts the aliasing in this region. The discrepancy in the \( u_1 \) ratios at high wavenumbers may result from the fact that the bi-normal term \( h \) is neglected in the Wyngaard [1968] model. In theory, small scale energy from the bi-normal velocity component could inflate the measured \( u_1 \) spectrum at large wavenumbers and increase the ratio of the measured to the ‘true’ spectrum without significantly increasing the total measured \( u_1 \) energy.
Figure 4.17: Simulated ratio between ‘measured’ and ‘true’ velocity gradient spectra. Black curves represent spectral ratios computed from wall-positions within the log-layer of the DNS volumes of Sillero et al. [2013]. Dark and light grey curves represent the spectral ratios for wall-positions below and above the log-layer, respectively. Blue curves are computed from the model proposed by Zhu & Antonia [1995] using the isotropic spectrum of Pope [2000], with the wire length set to $l_w = 4\eta$ (to approximately match actual experimental conditions). DNS ratios are truncated for each wall-position where $\kappa_1 \eta(x_2) = 1$. 

The Zhu & Antonia [1995] model based on linearised \((k = h = 0)\) sensing element response to isotropic turbulence closely match the synthetic experimental results for the streamwise and spanwise gradients in the log-layer. Below and above the log-layer, the spectral ratios (in most cases) depart significantly from the isotropic models. These departures reflect the influence of strong anisotropy due to mean shear or external intermittency. These spectral ratios represent a means to ‘correct’ the measured spectra for the purposes of determining the sensitivity of results to sensor effects.

In addition to limitations associated with finite resolution, the \(\partial u_1/\partial x_2\) and \(\partial u_1/\partial x_3\) gradients are affected by the physical presence of the probe in the flow, as discussed in section 4.3.3. As illustrated in figure 4.13, instantaneous velocity vectors having non-zero \(u_2\) or \(u_3\) components (in addition to \(u_1 > 0\)) result in a flow field around the probe which contains non-zero \(\partial u_1/\partial x_2\) and \(\partial u_1/\partial x_3\) gradients, respectively. These gradients are in turn measured by the probe, resulting in distortions to several velocity-vorticity correlations. This phenomenon also, however, results in “leakage” of the \(u_2\) and \(u_3\) spectra into the \(\partial u_1/\partial x_2\) and \(\partial u_1/\partial x_3\) spectra. The extent of this leakage is illustrated in figure 4.18, wherein the \(\partial u_1/\partial x_3\) premultiplied spectrum is shown as determined by both the \(\times\)-array processing described in Chapter 3 and the preliminary blockage-compensated method described in Chapter 4. Also shown in figure 4.18 is a \(\partial u_1/\partial x_3\) spectrum computed from the synthetic experiment at the same spatial resolution as the physical experiment. Note that premultiplication by wavenumber is applied as a visual aide to illustrate the contributions to the signal variance per wavenumber range. The spectra shown in figure 4.18 correspond to the outer edge of the log-layer of the lowest \(Re_\tau\) case from Table 4.1, a combination of parameters for which the leakage effect is particularly strong relative to the uncontaminated signal level. The synthetic experimental spectrum is scaled to account for the difference in Reynolds number between the DNS \((\delta^+_D \approx 2000)\) and the experimental case \(\delta^+_E \approx 3300\). The scaling is predicated on the validity of the following approximations near \(x_2/\delta \approx 0.2\):

\[
x^+_2 \tau \approx f \left( \frac{x_2}{\delta} \right), \tag{4.21}
\]

\[
\frac{\partial u_1}{\partial x_3} \sim \tau. \tag{4.22}
\]

Equation (4.21) is valid if and where (for example) the classical approximation \(\tau \sim x_2^{-1}\) (as discussed in Chapter 2) applies—it will also be shown in figure 4.20 below that (4.21) is approximately valid over a domain that includes \(x_2/\delta \approx 0.2\) for the Reynolds numbers considered herein.

The blockage-compensated correction scheme significantly reduces the variance of \(\partial u_1/\partial u_3\) (i.e. the area under the spectrum), and brings the spectrum of the compensated case into much closer agreement with the scaled synthetic experimental spectrum relative to the \(\times\)-array processing scheme. The remaining disagreement between the compensated experimental data and the DNS is likely related to a combination of (i) imperfect compensation for the blockage effect and/or (ii) Reynolds number differences not captured by the \(\delta^+_D/\delta^+_E\) scaling factor. In keeping with the
Figure 4.18: Effect of aerodynamic blockage compensation on $\partial u_1/\partial u_3$, $\partial u_3/\partial u_1$, and $u_3$ premultiplied spectra. Experimental data correspond to $x_2/\delta \approx 0.2$ from the $Re_\tau \approx 3300$ ZPG case. Synthetic experimental curve computed at $x_2/\delta \approx 0.2$ from the fields made available by Sillero et al. [2013]. Note that the synthetic experimental curve is scaled by the ratio of DNS to experimental Reynolds numbers (i.e. $\delta^+ / \delta_E^+$), as described in the text.

The notion that the blockage effect is one-directional ($u_3$ induces $\partial u_1/\partial x_3$, but not vice versa), the compensation processing scheme has virtually no effect on $u_3$ or its streamwise gradient. The reduction in contributions to the $\partial u_1/\partial x_3$ variance from each wavenumber also appears to be proportional to the $u_3$ energy contribution at each wavenumber, further supporting the notion that the two signals are directly linked. While the blockage-compensated processing scheme appears to remove most of the influence of $u_3$ on the $\partial u_1/\partial u_3$ gradient, the results shown in figure 4.15 suggest that this preliminary scheme may slightly over-correct the issue, and thus the results of this scheme should not be considered to be completely free of bias.

4.3.5 Dissipation estimate

One of the goals of Chapter 6 is to examine the enstrophy spectra through the lens of the dissipation-based Kolmogorov scales. Thus, it is crucial to first examine the capacity of the measurement technique to produce accurate estimates of these scales. Any trend in the fidelity of the scale estimates with wall-position could result in the introduction of spurious trends into the comparison of spectra at different positions. This section compares and evaluates several different approaches to obtaining an estimate of the dissipation rate.
The full dissipation rate (per unit mass) in an incompressible flow is given by:

\[ \varepsilon = 2\nu s_{ij}s_{ij} = \nu \frac{\partial u_i}{\partial x_j} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right), \]  

(4.23)

where \( s_{ij} \) is the fluctuating rate of strain tensor. Due to the difficulty of accurate measurement of \( s_{ij} \), however, it is typical to assume local isotropy at the dissipative scales, which yields the following simpler equation for the mean of \( \varepsilon \):

\[ \varepsilon_{iso} = 15\nu \left( \frac{\partial u_1}{\partial x_1} \right)^2. \]  

(4.24)

While the present measurement technique does not measure all components of \( s_{ij} \), it is also not restricted to the fully isotropic estimate given by (4.24). For example, the less restrictive assumption of homogeneity (at least at the dissipative scales) yields the relation \( \varepsilon_h = \nu \omega_i \omega_i \) (as discussed in Chapter 2). An estimate of the ‘true’ dissipation rate can therefore be obtained from the measured values of vorticity. Further, Antonia et al. [1998] point out that an assumption of full homogeneity is not necessary to approximate the full dissipation rate using the outputs of a similar 8-wire vorticity probe. The following is the expanded version of the right-hand side of (4.23) with underbrace/checked and × symbols included to respectively indicate terms that are or are not measured by the present measurement technique (as well as by Antonia et al. [1998]):

\[ \varepsilon = \nu \left[ 2 \left( \frac{\partial u_1}{\partial x_1} \right)^2 + \left( \frac{\partial u_2}{\partial x_2} \right)^2 + \left( \frac{\partial u_3}{\partial x_3} \right)^2 \right] \]

\[ + \left( \frac{\partial u_1}{\partial x_2} \right)^2 + \frac{\partial u_1}{\partial x_3} + \frac{\partial u_2}{\partial x_1} + \left( \frac{\partial u_3}{\partial x_3} \right)^2 + \left( \frac{\partial u_3}{\partial x_2} \right)^2 \]

\[ + 2 \left( \frac{\partial u_1}{\partial x_2} \frac{\partial u_2}{\partial x_1} + \frac{\partial u_1}{\partial x_3} \frac{\partial u_3}{\partial x_1} + \frac{\partial u_3}{\partial x_2} \frac{\partial u_3}{\partial x_2} \right) \right]. \]  

(4.25)

Antonia et al. [1998] use the following approximation based on the (squared) incompressible statement of mass conservation, with an assumption of homogeneity for one term:

\[ \frac{\partial u_2}{\partial x_2} + \left( \frac{\partial u_3}{\partial x_3} \right)^2 = \left( \frac{\partial u_1}{\partial x_1} \right)^2 - 2 \frac{\partial u_2}{\partial x_2} \frac{\partial u_3}{\partial x_3} = \left( \frac{\partial u_1}{\partial x_1} \right)^2 - 2 \frac{\partial u_2}{\partial x_2} \frac{\partial u_3}{\partial x_3}, \]  

(4.26)

to estimate of the sum of the two unmeasured gradient variance terms in (4.25). The hybrid dissipation rate \( \varepsilon_{hyb} \) is then obtained by substituting the right-most side of (4.26) for the unmeasured terms in (4.25). Figure 4.19 shows the ratio of the isotropic and hybrid dissipation estimates to the ‘true’ dissipation for three spatial
resolution cases, as measured by the synthetic experiment. The hybrid estimate is effectively identical to the ‘true’ dissipation for $x_2^+ \gtrsim 70$, and outperforms the isotropic estimate by a considerable margin throughout the entire boundary layer. Accounting for sensor effects, however, limits the over-performance of the hybrid estimate relative to the isotropic estimate to the near-wall region. Above $x_2^+ \approx 100$, the isotropic and hybrid estimates of $\varepsilon$ reported by the measurement technique are essentially equivalent. While the hybrid estimate is able to account for most of the effects of anisotropy, it is ultimately based on gradient measurements that may be less well-resolved than $\partial u_1 / \partial x_1$. These effects approximately offset in the log-layer and wake for the three spatial resolutions shown. Regardless of the estimate, it will be necessary to account for the underestimation of the dissipation both in the normalised wavenumbers and the spectral densities. The necessary correction, however, differs depending on the quantity being normalised. The Kolmogorov and (anisotropic) Taylor lengthscales $\eta$ and $\lambda_T$ depend on $\varepsilon$ as follows:

$$\eta \equiv (\nu^3 / \varepsilon)^{1/4}, \quad (4.27)$$

$$\lambda_T \equiv (10 \nu k / \varepsilon)^{1/2} \quad (4.28)$$

In effect, a 20% underestimation of $\varepsilon$ results in just a 5.7% overestimation of $\eta$ and an 11.8% overestimation of $\lambda_T$. The effects of the dissipation estimate on the Kolmogorov-normalisation of enstrophy spectra will be discussed in further detail in Chapter 6.
The dissipation estimate may be corrected in the same way as the power-spectral densities (i.e. by the ratios shown in figure 4.17), since the dissipation terms are simply integrals of spectral and co-spectral densities. The corrected dissipation is therefore not obtained by simple multiplication by a fixed correction factor, but rather depends on the measured spectral/co-spectral signal content and the predicted losses at each scale. Figure 4.20 shows the effect of applying this correction scheme to the present data. Wallward of the log-layer ($x_2^+ < 2.6\sqrt{\delta^+}$), the ‘transfer functions’ (i.e. spectral ratios) used at each wall-position are interpolated from those determined by the synthetic experiment (as shown in figure 4.17) based on distance to the wall in viscous units. The average transfer function determined from the synthetic experiment within the log-layer ($x_2^+ < 2.6\sqrt{\delta^+}$, $x_2/\delta < 0.15$), and the average within the wake ($x_2/\delta > 0.15$), are used to correct the experimental data within the log-layer and wake, respectively. An alternative to direct interpolation by wall-distance is necessary in order to account for the differences in $Re_\tau$ between the DNS and the experiments. The interpolation scheme presented here has not been optimised, but is suitable to demonstrate the general effectiveness of the method. Indeed, the spectral correction shifts the experimentally-determined hybrid mean dissipation estimates closer (both in value and in shape) to those calculated from the DNS. This is particularly true for the data wallward of the wake, where the effects of increasing $Re_\tau$ on the viscous-scaled dissipation are expected to be small.
Chapter 5

A comparison of pipe and boundary layer statistics *

Comparison of canonical turbulent flows offers insight into the role(s) played by features that are unique to one or the other. Pipe and zero pressure gradient boundary layer flows are often compared with the goal of elucidating the roles of geometry and a free boundary condition on turbulent wall-flows. Prior experimental efforts towards this end have focused primarily on the streamwise component of velocity, while direct numerical simulations are at relatively low Reynolds numbers. In contrast, this study presents experimental measurements of all three components of both velocity and vorticity from $5000 \lesssim Re_\tau \lesssim 10000$. Differences in the two transverse Reynolds normal stresses are shown to exist throughout the log-layer and wake layer at Reynolds numbers that exceed those of existing numerical data sets. The turbulence enstrophy profiles are also shown to exhibit differences spanning from the outer edge of the log-layer to the outer flow boundary. Skewness and kurtosis profiles of the velocity and vorticity components imply the existence of a ‘quiescent core’ in pipe flow, as described by Kwon et al. (J. Fluid Mech., vol. 751, 2014, pp. 228–254) for channel flow at lower $Re_\tau$, and characterise the extent of its influence in the pipe. Observed differences between statistical profiles of velocity and vorticity are then discussed in the context of a structural difference between free-stream intermittency in the boundary layer and ‘quiescent core’ intermittency in the pipe that is detectable to wall-distances as small as 5% of the layer thickness.

5.1 Experiments

5.1.1 Measurement Parameters

The relevant measurement parameters of this study are summarized in table 5.1. The friction velocity $u_\tau$ in the boundary layer case is determined by the composite fit of Chauhan et al. [2009] with the von Kármán and intercept constants chosen as $\kappa = 0.39$ and $B = 4.3$, respectively [Marusic et al., 2013]. An analogue of the standard $\delta_{99}$ is used due to the slight non-zero curvature of the mean velocity profile in the FPF boundary layer free-stream (cf. figure 5.1 in the next section). Since the inner-normalized streamwise velocity variance profile as a function of $x_2/\delta$ follows a single $Re$-independent curve in the vicinity of $\delta_{99}$, at least over the $Re$-range considered herein (e.g. see Marusic et al. [2015]), $\delta_{99}$ is identified in the present ZPG cases as the position where the inner-normalized streamwise variance equals 0.257—it’s value at the position corresponding to $U = 0.99U_o$ based on the DNS results of Sillero et al. [2013]. Throughout the rest of this text, $\delta_{99}$ will refer to this analogue definition. The friction velocity in the pipe is obtained from direct measurements of the pressure drop using 18 ports located along the entire working section. Wall-position in both measurements is first determined with a microscope to within $\pm 0.1$mm and subsequently tracked via an optical encoder on the traversing apparatus. Integration of the measured mean velocity profiles yields the average, or bulk velocity, from which (along with the measured mean pressure gradient) the friction factor $\lambda$ and Reynolds number $Re_D$ based on pipe diameter and bulk velocity are obtained. The present measured values of $\lambda$ (for each measured $Re_D$) are all within 0.75% of those based on the curve suggested by McKeon et al. [2004] at the corresponding values of $Re_D$.

Also summarized in table 5.1 are the numerical data sets used for comparison. These include the boundary layer DNS of Sillero et al. [2013], the pipe DNS of Chin et al. [2014], and a computer simulation of our probe when exposed to the six DNS flow volumes made available by Sillero et al. [2013]. This simulation, or “synthetic experiment”, seeks to predict the effects of physical scale, probe geometry, and data reduction method on each measured statistic. More detail on the synthetic experiment is available in Zimmerman et al. [2017]. Statistics from the fields of Sillero et al. [2013] that are not published online, such as velocity fluctuation kurtosis and vorticity skewness/kurtosis, are computed from the six available fields, and so may not be fully converged. Pipe DNS statistics are limited to only those which were published in Chin et al. [2014]. Pipe synthetic experimental “results” are not computed directly, but rather we normalize the pipe DNS statistics with the ratio of boundary layer simulation statistics to boundary layer DNS statistics.

5.2 Velocity statistics

This section presents profiles of the statistical moments (up to kurtosis) of the three velocity components and the Reynolds shear stress.
### Table 5.1: Summary of present experiments and DNS comparisons.

Boundary layer and pipe DNS respectively from the datasets of Sillero et al. [2013] and Chin et al. [2014]. † Pipe synthetic experiment based on boundary layer results, see text for details. Outer scale $\delta$ refers to pipe radius or the analogue to $\delta_{99}$ (defined in §5.1.1), where applicable. Measurement sample times correspond to all samples for each case, with the exception of four selected $x_2$ locations in the pipe cases for which longer samples were collected—these longer sample times (given in parentheses) correspond to the centremost location, and (near) the start, middle, and end of the log layer (i.e. $x_2^+ = 0.93\delta^+, \approx 2.6\sqrt{\delta^+}, \approx 0.15\delta^+$, and $\approx (2.6\sqrt{\delta^+} \times 0.15\delta^+)^{1/2}$).

<table>
<thead>
<tr>
<th>Method</th>
<th>$u_\tau$ [m/s]</th>
<th>$t_w^+$</th>
<th>$t_{w_p}^+$</th>
<th>$\Delta x_2^+$</th>
<th>$\Delta x_3^+$</th>
<th>$\delta$ [m]</th>
<th>$Re_\tau$</th>
<th>$t U_o/\delta$ [-]</th>
</tr>
</thead>
<tbody>
<tr>
<td>TBL Exp.</td>
<td>0.16</td>
<td>12</td>
<td>9</td>
<td>9</td>
<td>22</td>
<td>0.52</td>
<td>5600</td>
<td>4700</td>
</tr>
<tr>
<td>Pipe Exp.</td>
<td>0.18</td>
<td>13</td>
<td>9</td>
<td>9</td>
<td>23</td>
<td>0.45</td>
<td>5200</td>
<td>5100 (25300)</td>
</tr>
<tr>
<td>TBL Exp.</td>
<td>0.23</td>
<td>18</td>
<td>13</td>
<td>13</td>
<td>32</td>
<td>0.51</td>
<td>8100</td>
<td>5700</td>
</tr>
<tr>
<td>Pipe Exp.</td>
<td>0.26</td>
<td>19</td>
<td>14</td>
<td>14</td>
<td>34</td>
<td>0.45</td>
<td>7700</td>
<td>5000 (24800)</td>
</tr>
<tr>
<td>TBL Exp.</td>
<td>0.31</td>
<td>24</td>
<td>17</td>
<td>17</td>
<td>42</td>
<td>0.47</td>
<td>9900</td>
<td>6500</td>
</tr>
<tr>
<td>Pipe Exp.</td>
<td>0.34</td>
<td>25</td>
<td>18</td>
<td>18</td>
<td>44</td>
<td>0.45</td>
<td>10000</td>
<td>5200 (25900)</td>
</tr>
<tr>
<td>TBL DNS</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>2000</td>
<td>—</td>
</tr>
<tr>
<td>Pipe DNS</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>2000</td>
<td>—</td>
</tr>
<tr>
<td>Synth Exp.</td>
<td>—</td>
<td>24</td>
<td>18</td>
<td>18</td>
<td>45</td>
<td>—</td>
<td>2000</td>
<td>—</td>
</tr>
<tr>
<td>Synth Exp.†</td>
<td>—</td>
<td>24</td>
<td>18</td>
<td>18</td>
<td>45</td>
<td>—</td>
<td>2000</td>
<td>—</td>
</tr>
</tbody>
</table>

### 5.2.1 Streamwise

Figures 5.1(a) and (b) show the mean streamwise velocity $U_1$ in log-law and defect form, respectively. As noted in §5.1.1, $u_\tau$ is determined for the boundary layer by fitting the measured points to the composite profile of Chauhan et al. [2009] with a von Kármán constant $\kappa = 0.39$ and an intercept $B = 4.3$. Thus, agreement between the boundary layer cases and the boundary layer DNS of Sillero et al. [2013] in the log-layer is essentially prescribed. It is for this reason that the exact slopes of the profiles are not compared herein. Plotting the mean velocity in defect form, as in figure 5.1(b), reveals slight departures from canonical behaviour in the FPF boundary layer cases. For the purposes of this plot, the boundary layer free-stream velocity $U_o$ is chosen to force the log-law portions of each curve to lie on the expected ZPG boundary layer curve. This reveals that the two lower-$Re_\tau$ boundary layer measurements show good agreement in the wake with the DNS of Sillero et al. [2013] out to $x_2/\delta \approx 0.8$, at which point $U_1$ reaches a maximum and begins to decrease. In addition to exhibiting the same local maxima in $U_1$, the wake of the highest-$Re_\tau$ boundary layer case is also slightly weaker than that of a canonical ZPG boundary layer wake. In spite of these departures from the expected $U_1$ profile shape, the turbulence statistics shown in the figures that follow (i.e. figures 5.2–5.9) show close agreement with the boundary layer DNS as well as with the (lower-$Re_\tau$) measurements collected at the HRNBLWT (shown in figures A5.1–A5.5 in Appendix A to this chapter), even for $x_2/\delta_{99} > 1$. The present pipe measurements are virtually indistinguishable from the DNS curve of Chin et al.
Pipe versus boundary layer statistics

Figure 5.1: (a) Mean streamwise velocity in log-law form. Experimental profiles plotted as solid lines capped by symbols for clarity. Symbols at start/end of each line correspond to Table 3.1. (b) Mean streamwise velocity in defect form, with $U_0$ chosen for the boundary layer cases to force agreement (for illustrative purposes) with log-law (see text for details).

[2014] through the log-layer and wake. Likewise, the HRNBLWT $U_1$ profiles shown in figure A5.1 in Appendix A to this chapter show very close agreement with the DNS curve of Sillero et al. [2013].

Figures 5.2(a), (b), and (c), respectively show the variance, skewness coefficient, and kurtosis coefficient profiles of the fluctuating streamwise velocity $u_1$ for all present data and DNS. Apart from the expected difference in $u_2^+$ near $x_2/\delta \approx 1$, the $u_1$ variance profiles do not exhibit any systematic differences that are distinguishable with the present dataset. The similarity between the $u_2^+$ profiles of the two flows is in agreement with Monty et al. [2009], who presented streamwise velocity statistics up to the fourth order for pipe, channel, and boundary layer flows. The HRNBLWT $u_1$ variance profiles, shown in figure A5.2(a) in Appendix A to this chapter, are also very similar in shape to the pipe profiles, although they do indicate that the boundary layer features slightly higher $u_1$ variance than the pipe over the range $0.3 \lesssim x_2/\delta \lesssim 0.8$. The difference being slight, however, means that this conclusion is particularly sensitive to the choice of outer length scale for the boundary layer. Still, it will be shown below that the enstrophy and the other components of the Reynolds stress tensor are all higher in the boundary layer than the pipe over approximately the same range.

The streamwise velocity skewness coefficient profiles of the pipe and boundary layer are very similar from the wall until at least $x_2/\delta \approx 0.5$. The profiles of both flows grow more negative at a rate that is approximately logarithmic across the sub-domain where the mean velocity is logarithmic. Monty et al. [2009] also observed this similarity out to $x_2/\delta \approx 0.5$, but remarked that “it could be argued that the boundary layer skewness exhibits a slightly different trajectory for $x_2^+ \gtrsim 200$”. Indeed, the boundary layer $u_1$ exhibits (on average) slightly less negative skewness than the pipe in the vicinity of $x_2/\delta \approx 0.1$. The same is clearly true of the boundary
layer data acquired in the HRNBLWT, as shown in figure A5.3 in Appendix A to this chapter. This difference is part of a general trend: the boundary layer exhibits higher variance and probability density functions that are less dominated by extreme events for most measured quantities near the outer edge of the log-layer/wallward edge of the wake. It is surmised in §5.4 that this results (at least in part) from the difference between intermittency associated with the turbulent/non-turbulent interface (TNTI) in the boundary layer and that associated with the turbulent/quiescent-core interface in the pipe.

As with the skewness coefficient profiles, the pipe and boundary layer kurtosis profiles remain very similar moving outward from the wall until the emergence of a super-Gaussian peak in the wake of the boundary layer that far exceeds the more modest peak in the pipe (see figure 4.2(c)). The HRNBLWT measurements, shown in figure A5.4 in Appendix A to this chapter, suggest that the boundary layer kurtosis is slightly lower than that of the pipe in the range $0.3 \lesssim x_2/\delta \lesssim 0.5$. Again, this is consistent with the differences between the intermittency in the pipe and boundary layer. Both profile sets remain sub-Gaussian from the nearest-wall measured points until approximately $0.5\delta$. The pipe centreline kurtosis is higher in

\[
\frac{u_i^2}{u_r^2} = 1.95 - 1.26\log(x_2/\delta) + \log(1.15) \quad \text{for reference (note the additional log(1.15) constant accounts for differing definitions of $\delta$).}
\]

The dashed lines in (b) and (c) represent the Gaussian values of the plotted statistics.
Figure 5.3: (a, b, c) Wall-normal velocity variance, skewness, and kurtosis, respectively. Shaded region in (a) corresponds to the range of inner log-layer boundaries (i.e. $x_2^+ \approx 2.6\sqrt{\delta^+}$) for the present experiments. Dashed black lines indicate Gaussian values of the plotted statistics.

Figure 5.3 shows the variance, skewness, and kurtosis profiles of the wall-normal velocity component. Based on the DNS-based synthetic probe predictions for the experimental data in figure 5.3(a), the wall-normal velocity variance is expected to suffer noticeable attenuation much farther from the wall than the streamwise velocity variance. Thus, while a slight positive slope is observed in the pipe $u_2$ variance across the domain where the mean velocity is logarithmic, it is unlikely that this trend would be observed in the absence of spatial filtering. Still, if either the absolute or proportional attenuation of the boundary layer and pipe $u_2$ signals are

the lowest-$Re_\tau$ case than the two higher-$Re_\tau$ cases. This higher value is observed individually by all sub-arrays, and at a position where the time-record length exceeded 25000 radius turnover times. Thus, it is unlikely that this observation results from spurious probe behaviour or insufficient statistical convergence. As no two pipe measurements are collected at the same $Re_\tau$ or sensor resolution, the cause of the difference in centreline kurtosis is left unclear.

5.2.2 Wall-normal

Figure 5.3 shows the variance, skewness, and kurtosis profiles of the wall-normal velocity component. Based on the DNS-based synthetic probe predictions for the experimental data in figure 5.3(a), the wall-normal velocity variance is expected to suffer noticeable attenuation much farther from the wall than the streamwise velocity variance. Thus, while a slight positive slope is observed in the pipe $u_2$ variance across the domain where the mean velocity is logarithmic, it is unlikely that this trend would be observed in the absence of spatial filtering. Still, if either the absolute or proportional attenuation of the boundary layer and pipe $u_2$ signals are...
equal, the present data indicate that the $u_2$ variance differs between the two flows starting at least at the inner edge of the log-layer. The same conclusion is reached via inspection of the HRNBLWT data, as shown in figure A5.2 in Appendix A to this chapter. According to both the experimental and DNS data, the difference in profiles is the most pronounced in the outer region $x_2/\delta \approx 0.2$. At this location, the boundary layer case exhibits an outer peak that, according to Morrill-Winter et al. [2015], continues to grow with increasing $Re_\tau$. The present pipe data does not exhibit an outer peak or any obvious trend with $Re_\tau$, and is of considerably lower magnitude than the boundary layer cases, in agreement with the findings of Jimenez & Hoyas [2008], which were observed at lower $Re_\tau$.

As with the streamwise velocity, the pipe and boundary layer skewness and kurtosis profiles for $u_2$ (shown in figures 5.3(b) and (c)) are very similar from the near-wall to the onset of the wake region. The synthetic probe results predict that combined probe filtering/aliasing effects near the wall result in positive near-wall skewness, rather than the negative values reported by the fully-resolved DNS. This effect is indeed observed in both the pipe and boundary layer experimental results. Once this effect becomes negligible, the DNS, pipe, and boundary layer skewness profiles show very close agreement over the domain where the mean velocity is logarithmic. In all cases, the wall-normal fluctuations have an approximately constant skewness coefficient on this domain, varying only between 0.1-0.15. The pipe and boundary layer cases both exhibit a positive peak in the $u_2$ skewness in the wake region, although the magnitude of the boundary layer peak far exceeds that of the pipe. The existence of this skewness peak in the boundary layer is not altogether surprising given the non-zero skewness in the log-layer and the presumed tendency of free-stream intermittency to increase the probability density of $u_2 \approx 0$. Although there is no source of truly non-turbulent ‘free-stream’ in fully-developed pipe flow, the intermittency associated with the varying boundary of the quiescent core [Kwon et al., 2014] would also presumably increase the probability density of $u_2 \approx 0$ (as well as of $u_3 \approx 0$), and thus produce the observed outer peak in the pipe $u_2$ skewness profiles. Indeed, all of the velocity fluctuation skewness and kurtosis profiles presented herein exhibit a tendency to increase in magnitude as one moves outward from $x_2/\delta \approx 0.3$, further supporting the existence of a quiescent core in the pipe. As with the $u_1$ component, the measurements of $S_{u_2}$ collected in the HRNBLWT (shown in figure A5.3 in Appendix A to this chapter) indicate that the boundary layer skewness is slightly more Gaussian (closer to zero) near the outer edge of the log-layer/inner edge of the wake. Again, this is consistent with the differences shown in §5.4 between the intermittency associated with the boundary layer TNTI and with the quiescent core in the pipe.

In contrast to the streamwise velocity fluctuations, the kurtosis of the $u_2$ fluctuations is super-Gaussian across the entire flow domain of both the pipe and the boundary layer. The kurtosis profiles of both flows exhibit an increase in magnitude as one moves from $x_2/\delta \approx 0.3$ towards the centerline, although the magnitude increase in the boundary layer far exceeds that of the pipe. While the difference between the $u_2$ skewness profiles of the two flows is detectable as close to the wall as $x_2/\delta \approx 0.35$, the kurtosis profiles do not appear to rapidly diverge until $x_2/\delta \approx 0.55$, which is close to the point at which the $u_2$ variance profiles intersect.
Using the ‘intermittency factor’ $\gamma$ (defined as the time-fraction of non-turbulent flow) to account for the effects of external intermittency, Schubauer [1954] found close agreement in the kinetic energy between the pipe and the turbulent portion of a boundary layer in the region above $x_2/\delta \approx 0.6$. The author argued based on this agreement that the distribution of turbulent energy in a pipe and turbulent portion of a boundary layer are most likely the same in this region. The fact that the point at which the boundary layer and pipe $u_2$ variances are equal is approximately coincident with the point at which the kurtosis profiles begin to diverge is consistent with this hypothesis (at least for $x_2/\delta \gtrsim 0.6$).

5.2.3 Spanwise/azimuthal

As the odd moments of the spanwise/azimuthal velocity are identically zero in both pipes and boundary layers in theory (and to within experimental error in actuality), figure 5.4 shows only the even-moment statistics of $u_3$. Similar to $u_2$, the $u_3$ boundary layer variance profiles show a sharp outer slope change, while the slope change in the pipe profiles is less pronounced. These slope changes occur at approximately $x_2/\delta \approx 0.3$ for both the DNS and experimental results, which is approximately coincident with the outer ‘bump’ feature in the $u_3$ variance profile, and is in agreement with the findings of Jimenez & Hoyas [2008] at lower $Re_e$. Further, this location is near the lower $x_2/\delta$ limit for turbulent/non-turbulent intermittency suggested by Chauhan et al. (2014a and 2014b). Thus, the ‘knee’ in the profile and subsequent rapid decay of $u_3$ variance is caused (at least in...
part) by an increasing time-fraction of signal containing quasi-irrotational flow with near-zero spanwise velocity as one moves above $x_2/\delta \simeq 0.3$.

That the $u_3$ variance is higher in magnitude (and features a different slope) in the boundary layer than in the pipe and channel below $0.3\delta$ is apparently a separate issue. Log-lines of best fit computed from the present boundary layer and pipe $u_3$ variance data over the range $2.6\sqrt{\delta} < x_2^+ < 0.15\delta^+$ have slopes of $-0.34$ and $-0.42$, respectively. The HRNBLWT $u_3$ variance measurements, shown in figure A5.2 in Appendix A to this chapter, feature a log-layer fit of $u_3^2/u_r^2 = 1.66 - 0.26\log(x_2/\delta)$. If the difference between the pipe and boundary layer cases were predominantly related to flow geometry, one would expect the channel flow profile to closely resemble the boundary layer profile. Instead, the channel flow profile of Hoyas & Jiménez [2006] in figure 5.4 is virtually indistinguishable from the pipe flow profile of Chin et al. [2014] at the same $Re_\tau$. The $Re_\tau \approx 5200$ channel DNS of Lee & Moser [2015] is also shown in figure 5.4(a) for comparison. Although the higher $Re_\tau$ DNS features slightly higher $u_3$ variance in the outer region than the $Re_\tau \approx 2000$ pipe and channel DNS, it is still much closer to these cases than it is to the boundary layer DNS of Sillero et al. [2013]. Jiménez et al. [2010] suggested that the discrepancies between channel and boundary layer cases in both the $u_2$ and $u_3$ variance profiles are caused by higher pressure fluctuation RMS in the boundary layer, resulting in an increased redistribution of the $u_1$ energy to the $u_2$ and $u_3$ components. This explanation is also appropriate for pipe flow as noted by Chin et al. [2014], because the RMS profiles of the pressure fluctuations in the pipe and channel also closely resemble one another, at least at $Re_\tau \approx 1000$. Jiménez et al. [2010] showed for a boundary layer and channel both at $Re_\tau \approx 550$ that the difference in pressure RMS in this region is due almost exclusively to the negative fluctuations, which are generally associated with the ‘cores’ of vortices. In support of this notion, it will be shown in §5.3 and §5.4 that the boundary layer features higher mean enstrophy (at the resolved scales) and less quasi-irrotational flow in the same region.

Like the $u_2$ fluctuations, the $u_3$ fluctuations exhibit slightly super-Gaussian kurtosis throughout the entire flow domain of both pipes and boundary layers (see figure 5.4(b)). The kurtosis for all three pipe profiles in the domain where the mean velocity is logarithmic remains constant at approximately 3.35 to within the scatter of the data. In contrast, all three boundary layer kurtosis profiles trend toward the Gaussian value of 3 with increasing distance from the wall within the log layer, moving from $\simeq 3.35$ to $\simeq 3.25$. The local minimum of the boundary layer kurtosis profiles corresponds approximately to the location of the ‘knee’ in the variance profiles (see figure 5.4(a)). The difference between the pipe and boundary layer $u_3$ kurtosis is also clear when using the HRNBLWT measurements as in figure A5.4(c) in Appendix A to this chapter. This feature, again, is consistent with the difference in the time fractions of ‘fully’ turbulent flow (as opposed to quiescent or non-turbulent) near the outer edge of the log-layer that will be discussed further in §5.4.

As is the case with $u_2$ fluctuations, the point at which the boundary layer and pipe $u_3$ variance profiles intersect is approximately coincident with the point
at which the kurtosis profiles rapidly diverge, with both features occurring near
\( \frac{x_2}{\delta} \approx 0.6 \). Again, this is consistent with the hypothesis of Schubauer [1954] that
the distribution of turbulent energy between the pipe and the turbulent patches
of the boundary layer is the same over this region.

5.2.4 Reynolds shear stress

Owing in part to its strong dependence on probe alignment [Zimmerman et al.,
2017], the mean Reynolds shear stress is one of the more difficult statistics to
accurately measure. The magnitude of all present kurtosis profiles of the instantaneous $u_1u_2$ signal reveal its highly intermittent nature, and indicate that the mean value is composed of a delicate balance of instantaneous motions that often far exceed the magnitude of the mean in both directions. That said, the present mean Reynolds stress profiles in the boundary layer (see figure 5.5(a)) do exhibit the expected outer region deviation from the pipe cases in the same region as the observed boundary layer ‘knees’ in both the $u_2^2$ and $u_3^2$ profiles. This difference is shown more clearly in the HRNBLWT measurements (cf. figure A5.2(d) in Appendix A to this chapter).

The slightly higher mean Reynolds stress values in the outer region of the boundary layer in figure 5.5(a) are matched by slightly higher signal RMS boundary layer profiles compared to the pipe profiles as shown in figure 5.5(b). In agreement with the findings of Morrill-Winter [2016] and the computation from the available DNS fields, the Reynolds shear stress signal is negatively skewed across the entire flow domain with a coefficient of $S_{u_1u_2} \approx -1.6$ in the region where the mean velocity is logarithmic (see figure 4.5(c)). The skewness and kurtosis profiles for the pipe experiments closely resemble those of the boundary layer out to $x_2/\delta \approx 0.5$. As with the constituent components $u_1$ and $u_2$, the HRNBLWT measurements indicate that the $u_1u_2$ skewness and kurtosis magnitudes are slightly lower in the boundary layer than in the pipe near the outer edge of the log-layer. This is also shown by the FPF measurements (at least in a mean sense), albeit less convincingly. This slight discrepancy is likely related to the departures from the canonical ZPG wake discussed in §5.2.1.

The $(u_1u_2)'$ skewness reaches a negative peak in the wake regions of both the pipe and boundary layer. As with the skewness profiles of the constituent velocity components, the peak magnitude in the boundary layer far exceeds that of the pipe. As noted above, the instantaneous fluctuating $u_1u_2$ signal is characterized by extreme events, resulting in a kurtosis greater than 10 across the entire flow domain (see figure 5.5(d)). The pipe and boundary layer kurtosis profiles appear to match one another everywhere except in the wake region. Here, a substantial increase in the pipe profiles is outpaced by an even more substantial peak in the boundary layer profiles.

The correlation coefficient $\rho_{u_1u_2}$ is shown in figure 5.5(e). While the present experimental results are of lower magnitude than the synthetic experiment and DNS, a slight decrease in magnitude is expected with increasing $Re_{\tau}$. This decrease in magnitude is expected since (at least) the $u_1$ fluctuations are known to increase in strength with increasing $Re_{\tau}$ (albeit slowly) in the region where the mean Reynolds shear stress remains close to $-u_2^2$. The correlation coefficient in the pipe remains fairly constant over the majority of the flow domain (in logarithmic space) before turning sharply toward zero, passing through $-0.3$ at $x_2/\delta \approx 0.7$. In contrast, the boundary layer correlation coefficient profiles slope gently away from zero for the majority of the flow domain before turning sharply toward zero, passing through $-0.3$ at $x_2/\delta_{99} \approx 0.8-1$. 
5.3 Vorticity

This section presents statistics of all three components of vorticity, as well as the mean enstrophy $\frac{1}{2} \omega_i \omega_i$. Where the velocity statistics elucidate the distribution of motions contributing to the kinetic energy of the turbulence, the vorticity statistics describe (to a close approximation) the distribution of contributions to the dissipation of turbulence kinetic energy. Furthermore, the distribution of the spanwise/azimuthal vorticity describes the motions which underlie the slope of the mean velocity profile (i.e. since $\Omega_3 \approx -\partial U_1 / \partial x_2$).

Figures 5.6 through 5.8 show statistics of the streamwise, wall-normal, and spanwise/azimuthal components of vorticity, respectively. Each component exhibits similar features, including an outer ‘bump’ in the boundary layer RMS (at a location that is coincident with ‘bumps’ in the Reynolds stresses described above) and universal super-Gaussian kurtosis.

The fraction of boundary layer vorticity RMS that is resolved by the present measurement technique may be predicted via the “synthetic” experiment described briefly in §5.1.1 and in detail in Zimmerman et al. [2017]. The effects of physical scale, probe geometry, and data reduction method are reproduced in a DNS volume by using the known velocity fields and model calibration functions to generate synthetic sensor “voltages”, which are then reduced to velocities and velocity gradients according to the process outlined in §3.5. The ratio of ‘measured’ to ‘true’ vorticity predicted by the synthetic experiment is reported for the three resolution cases in figure B5.1 in Appendix B to this chapter. The effects of spatial resolution on the vorticity RMS and kurtosis values, as predicted by the synthetic experiment, are given herein by the dashed light-blue lines in each plot. This synthetic case corresponds to the least-resolved physical experimental cases (see Table 5.1), and so all the experimental data is expected to approximately lie between the DNS computations and the synthetic experimental curve in the absence of effects not captured by the synthetic experimental model.

When both spatial resolution and $Re_\tau$ are matched, the streamwise vorticity RMS profiles of the pipe and boundary layer closely resemble one another. Zimmerman et al. [2017] argued that the variability among vorticity RMS profiles (of all three components) for unmatched cases is primarily a function of spatial resolution, and should not be confused for a Reynolds number trend. This argument was based on the observed agreement between two physical experimental ZPG cases with matched resolution but disparate Reynolds numbers, as well as agreement between physical and synthetic experimental results across a range of spatial resolutions.

The outer boundary condition for all three vorticity components differs between the pipe and boundary layer cases in that the pipe RMS profiles do not go to zero. Thus, the change in concavity observed in the boundary layer DNS profiles (see figures 5.6(a), 5.7(a), and 5.8(a)) in the wake region is not expected to exist, at least to the same degree, in the pipe profiles. Although this result is somewhat obfuscated by non-zero free stream RMS in the boundary layer cases, it is still...
visible in the insets of figures 5.6(a), 5.7(a), and 5.8(a). The pipe profiles are shown in the insets as lines without symbols for clarity.

With the exception of the highest $Re_\tau$ boundary layer case, both the pipe and boundary layer kurtosis profiles increase across the flow domain. As the streamwise component of vorticity is composed exclusively of cross-stream gradients of cross-stream velocity components, any imbalance in the measurement of these sensitive velocity components between two sub-arrays may result in signal contamination. Even a slight increase in the denominator of the kurtosis coefficient due to contamination may be the cause of not only the flattening of the highest $Re_\tau$ boundary layer kurtosis profile, but also the lack of outer peaks in the two higher-$Re_\tau$ boundary layer kurtosis profiles.
Pipe versus boundary layer statistics

The boundary layer $\omega_2$ RMS profiles shown in figure 5.7(a) clearly exhibit the expected change in concavity in the wake region while the pipe profiles do not. The $\omega_2$ kurtosis profiles are very similar to the $\omega_1$ kurtosis profiles for both the pipe and boundary layer cases, except that the predicted outer peak in the boundary layer profiles is present in the experimental results. The $\omega_2$ kurtosis of the boundary layer cases is also less than that of the pipe in the outer region, a feature which was also observed in at least the $u_2$ and $u_3$ component kurtosis profiles over approximately the same wall-normal domain. Again, this is related to the differing properties of intermittency associated with the TNTI in the boundary layer and the quiescent core in the pipe.

As with the two zero-mean vorticity components (i.e. $\omega_1$ and $\omega_2$), the pipe and boundary layer $\omega_3$ vorticity RMS profiles closely resemble each other with the exception of the change of concavity observed in the boundary layer wake. The spanwise/azimuthal vorticity fluctuations for both the pipe and boundary layer cases are skewed with the same sign as the mean across the entire flow domain (see figure 5.8(b)). This is reflective of the existence of spatially concentrated regions of strong $\partial U_1/\partial x_2$ shear (e.g. as observed by Meinhart & Adrian [1995]). The pipe skewness profiles in particular appear to follow a steady logarithmic curve toward
Pipe versus boundary layer statistics

**Figure 5.9:** (a) Inner-normalized turbulence enstrophy profiles plotted as solid lines capped by symbols for clarity. Symbols at start/end of each line correspond to Table 3.1. (b) Ratio of turbulence enstrophy between boundary layers and pipes of approximately matched Re, and spatial resolution. Symbol shape corresponds to spatial resolution in accordance with the trend established in Table 5.1.

zero, but with a slope that is too shallow to intersect zero at the centreline. An abrupt turn toward zero skewness is observed in all three pipe profiles between the two centremost points at 0.74δ and 0.93δ that hints at the path taken by the curve to satisfy the symmetry condition of $S_{\omega_3} = 0$ at the centreline. Continuing the trend observed in a number of third and fourth-order statistics of other quantities, the boundary layer $\omega_3$ skewness is of smaller magnitude than that of the pipe near the outer edge of the log-layer.

The $\omega_3$ kurtosis profiles, shown in figure 5.8(c), exhibit qualitatively different behavior than the two zero-mean vorticity component kurtosis profiles. Where the $\omega_1$ and $\omega_2$ signals become increasingly dominated by large fluctuations as one moves away from the wall (kurtosis increasing gradually from 5 to 10), the $\omega_3$ kurtosis profiles exhibit no such monotonic increase. The slight trends in kurtosis profiles observed for the different Reτ cases are likely related to changes in spatial resolution. With the exception of the highest-Reτ boundary layer case, the pipe and boundary layer kurtosis profiles track each other closely.

Figure 5.9(a) shows the mean turbulence enstrophy, $\frac{1}{2} \overline{\omega_i^2}$, on a logarithmic scale. Since the enstrophy is related to the turbulence dissipation rate $\tau$ by $\tau \approx \nu \overline{\omega_i^2}$, the classical $-1$ power-law slope (based on equality of production and dissipation, e.g. see Townsend [1976]) is also included in figure 5.9(a) for reference. Figure 5.9(b) shows the ratio of the enstrophy profiles of the boundary layer and pipe cases with matched Reτ and spatial resolution. The outer peaks observed in the enstrophy ratio profiles coincide with the change-of-concavity discussed above in the context of figures 5.6 through 5.8 as well as the outer ‘bumps’ in the boundary layer $u_2$ and $u_3$ variance profiles relative to those of the pipe. Figure 5.9 is replotted as figure A5.5 in Appendix A to this chapter using the HRNBLWT measurements in place of the FPF measurements. The same peak in the enstrophy
ratio is also clearly visible in figure A5.5(b), although the region where the ratio departs from unity is more clearly discernible as $0.1 \lesssim x_2/\delta \lesssim 0.7$.

As noted above in §5.2.3, the cross-stream velocity variances and enstrophy are linked through the pressure RMS. Increased levels of mean enstrophy are associated with stronger negative pressure fluctuations (which were indeed observed by Jiménez et al. [2010]), while increased pressure RMS is linked to increased redistribution of $u_1$ energy to $u_2$ and $u_3$ through the pressure-strain redistribution term in the Reynolds stress transport equations (e.g. see Tennekes & Lumley [1972]). The differences in mean enstrophy are also indicative of increased levels of viscous dissipation of turbulence in the boundary layer wake relative to the pipe wake, in agreement with El Khoury et al. [2013] wherein channel, pipe, and ZPG boundary layer DNS results are compared at $Re_\tau \approx 1000$. The following section shows that the difference in enstrophy levels observed in the pipe and boundary layer are related to the differing time-fractions of highly-turbulent flow. These time-fractions are themselves a product of the properties of the TNTI in the boundary layer and the quiescent core boundary in the pipe.

### 5.4 Intermittency

Two overarching features of the RMS, skewness, and kurtosis profiles are shown in §5.2 and §5.3 to consistently differentiate between pipe and boundary layer flow: outer magnitude peaks in the boundary layer skewness and kurtosis cases that emerge at $x_2/\delta \approx 0.5$; and higher RMS/lower skewness and kurtosis magnitude of boundary layer quantities over a domain roughly spanning $0.1 \lesssim x_2/\delta \lesssim 0.5$. The emergence of an outer peak in the kurtosis profiles of boundary layer statistics has long been understood to be related to turbulent/non-turbulent intermittency in the boundary layer. Early studies of this phenomenon even used the departure of the $u_1$ kurtosis from the Gaussian value as a measure of intermittency [Klebanoff, 1955]. Some profile features that are typically understood to be a consequence of intermittency, however, are also observed (albeit to a lesser degree) in the present pipe cases. Indeed, all of the third and fourth order velocity statistics presented herein for the pipe trend away from Gaussian values in the outer flow region. Recently, Kwon et al. [2014] have shown that channel flows contain in their wake a region of nearly uniform momentum, or a ‘quiescent core’, which has characteristics similar to those of the boundary layer free-stream. The role of these quasi-‘non-turbulent’ patches is thus of interest. The aim of this section is to identify differences in the structure and prevalence of flow having characteristics reminiscent of the boundary layer free-stream—namely irrotationality and unidirectionality.

As the vorticity signal is not fully resolved by the present measurement technique (see Appendix B for details), it is not possible to identify portions of the signal that are strictly irrotational. Instead, a range of thresholds for instantaneous enstrophy, $\tilde{\omega} \omega_i$, are used to identify ‘irrotational’ or ‘quasi-irrotational’ flow. The conclusions drawn from these data are then shown to be independent of
the threshold level over the range employed. As shown in figure 5.9, the enstrophy roughly decreases as \( x_2^- \) from \( x_2^- > 30 \) to \( x_2^-/\delta \approx 0.5 \). In effect, the mean turbulence enstrophy at, say, \( x_2^-/\delta = 0.1 \) for \( Re_\tau \approx 10000 \) is therefore roughly half that at the same position for \( Re_\tau \approx 5000 \). Since the present intermittency analysis is primarily focused on the outer region of the flow, it is therefore logical to define a threshold relative to the enstrophy at some point in the outer region for each \( Re_\tau \) case rather than, for example, one fixed in viscous units. For the purposes of the present analysis, the threshold enstrophy is expressed relative to the mean enstrophy at \( x_2^* \), the location where the pipe and boundary layer enstrophy values are equal (\( x_2^*/\delta \approx 0.7 \) according to figure 5.9(b)). The threshold level for each measurement may then be represented in terms of the coefficient \( \Psi \), defined as

\[
\Psi = C_{\omega}\left(\frac{1}{2}\omega_i(x_2^*)\omega_i(x_2^*)\right)^{-1}
\]  

(5.1)

where \( C_{\omega} \) is the actual threshold value in \( s^{-2} \).

Although the velocity measurements are less susceptible to attenuation in the outer region than the vorticity measurements, a range of thresholds will also be used to determine ‘unidirectionality’ to ensure independence of the conclusions from the chosen threshold level. The criterion for ‘unidirectionality’ is defined herein as the total velocity flow angle relative to the \( x_1 \) direction, which is calculated according to (5.2) below:

\[
\theta = \tan^{-1}\left(\frac{\tilde{u}_1}{(\tilde{u}_1^2 + \tilde{u}_2^2 + \tilde{u}_3^2)^{1/2}}\right).
\]  

(5.2)

Figures 5.10(a) and (b) respectively show examples of the flow-angle \( \theta \) and the enstrophy time series that are used to evaluate irrotationality and unidirectionality. As both quantities are expected to be close to zero within a patch of ‘non-turbulent’ flow, it is expected that the two time series exhibit some degree of correspondence. Indeed, the correlation coefficient between the two is positive everywhere for both flows (\( \approx 0.1 \)), and forms a peak in the ZPG wake (\( \approx 0.35 \)). The time-fractions of signal that lie above the unidirectional and irrotational thresholds (shown in red) are denoted as \( \gamma_u \) and \( \gamma_i \), respectively. These time-fractions are computed for a range of thresholds at each wall-normal position, and are shown (as \( 1 - \gamma_u \) and \( 1 - \gamma_i \)) in figure 5.11. Note that darker shades in figure 5.11 correspond to lower thresholds.

Figure 5.11 reveals several fundamental differences in the organization of pipe and boundary layer flows. From just interior to the outer boundary of the log-layer to the middle of the wake (i.e. \( x_2^-/\delta \approx 0.05-0.5 \)), the boundary layer can be characterized as more ‘well-stirred’ than the pipe, as a smaller time-fraction of boundary layer flow falls below each threshold for both the unidirectionality and irrotationality criteria. Magnitude differences between the two \( Re_\tau \) cases, particularly for the lowest thresholds, are most likely influenced by spatial resolution, and thus conclusions drawn from figure 5.11 should be limited to those based on the
relative magnitudes of the pipe and boundary layer, and the dependence of these relative magnitudes on wall-distance.

The location of the discrepancy between ‘non-turbulent’ time fractions in the two flows roughly corresponds to the outer ‘bump’ in the boundary layer enstrophy relative to pipe enstrophy, the largest differences in Reynolds stresses between the two flows, and the lower magnitudes of boundary layer skewness and kurtosis profiles for a number of quantities as discussed throughout §5.2 and §5.3. Beginning at $x_2/\delta \approx 0.5$, the boundary layer sees a rapid rise in the portion of flow that can be characterized as ‘non-turbulent’. The location of this abrupt change is consistent with the onset of peaks in numerous third and fourth order boundary layer statistics, and thus supports the notion that these phenomena are related to the onset of ‘external’ intermittency. At $x_2/\delta \approx 0.55$, the pipe and boundary layer have simultaneously equal fractions of both quasi-irrotational and quasi-unidirectional flow. Beyond this crossover point, however, the boundary layer fully transitions to the free-stream while the pipe maintains a finite level of enstrophy and turbulence intensity.

The statistics first shown in figure 5.9 are plotted again in figure 5.12 for the $Re_\tau \approx 5400$ case, along with a family of curves showing the effect of removing ‘non-turbulent’ regions. Figure 5.12(b) reveals that the outer ‘bump’ in the ratio of boundary layer enstrophy to pipe enstrophy is diminished when quasi-irrotational flow is removed from consideration. When the threshold is taken as 90% of the value at $x_2^*$ (i.e. $\Psi = 0.9$), the ratio of boundary layer to pipe enstrophy is essentially flat. Furthermore, the enstrophy ratio is unchanged by the removal of
quasi-irrotational signal at $x_2/\delta \approx 0.55$, which corresponds to the intersections of $1 - \gamma_i$ for the two flows as shown in figure 5.11. Thus, the observed discrepancy in enstrophy profile shape in the wake region is due to the difference in the time fraction of each flow in which instantaneous enstrophy is very low. The highly super-Gaussian kurtosis of all three vorticity components suggests that the enstrophy signal is composed of ‘bursts’ having magnitudes that far exceed the signal RMS. That the ratio shown in figure 5.12(b) becomes flat and close to unity by removing ‘non-turbulent’ regions suggests that the mean enstrophy of these ‘bursts’ is the same in pipe and boundary layer flows of the same $Re_\tau$ at any wall distance.
Figure 5.12: Effect of removing ‘non-turbulent’ patches on quantities plotted in figure 5.9. (a) Mean enstrophy of ‘turbulent’ patches in pipe and boundary layer flow at $Re_\tau \approx 5400$. (b) Ratio of ‘turbulent’ patch mean enstrophy between boundary layer and pipe at $Re_\tau \approx 5400$.

5.5 Conclusions

A multi-sensor hotwire probe capable of measuring both the velocity and vorticity vectors has been deployed in a set of three turbulent pipe flows and three zero-pressure-gradient boundary layers with nominally matched inner and outer scales. The present results represent the first physical measurements of kinetic energy and enstrophy in pipe and ZPG boundary layer flows with matched $Re_\tau$ and spatial resolution conditions, as well as the highest $Re_\tau$ simultaneous measurements of these quantities in pipe flow. Basic statistical results of these measurements are presented and highlight differences between the two flows and identify the subdomain over which they occur. A number of the observed differences in the present study match the observations of several lower-$Re_\tau$ DNS and experimental studies, including those of Jimenez & Hoyas [2008], El Khoury et al. [2013], and Monty et al. [2009].

Differences are observed in the $u_2$ and $u_3$ variance profiles from at least $x_2^+ \approx 2.6\sqrt{\delta^+}$, with the maximum difference occurring at $x_2/\delta \approx 0.3$. The location of the maximum difference in $u_2$ and $u_3$ variance profiles is also characterised by smaller boundary layer skewness and kurtosis magnitudes (of both velocity and vorticity components), as well as higher boundary layer turbulence enstrophy (or to a close approximation, turbulence dissipation) relative to the pipe. It is then shown that the same region features a higher time fraction of pipe flow that can be characterized quasi-irrotational and quasi-unidirectional. Accounting for this difference largely eliminates the discrepancy in enstrophy between the two flows.
Aside from slight deviations near $x_2/\delta \approx 0.3$ as noted above, third and fourth order statistics of both velocity and vorticity components for the pipe cases closely match those of the ZPG boundary layer from the near-wall until the emergence of intermittency-related outer magnitude peaks in the boundary layer profiles. With the exception of the outer peak in the boundary layer $u_2$ skewness, which emerges at $x_2/\delta \approx 0.35$, these peaks generally emerge near $x_2/\delta \approx 0.5–0.6$. The pipe and boundary layer velocity variances also intersect at approximately $x_2/\delta \approx 0.6$, beyond which the boundary layer variances decay to zero while those in the pipe do not. The agreement in position between the point at which the velocity variances intersect and the point at which the higher order statistics rapidly increase in magnitude supports the hypothesis of Schubauer [1954]—that the distribution of turbulent energy in a pipe is the same as in the ‘turbulent’ patches of a boundary layer in the region above $x_2/\delta \approx 0.6$. Despite the absence of purely irrotational potential flow at the outer boundary, velocity fluctuations in the pipe also trend away from Gaussian behaviour in the wake. This is consistent with the existence of a ‘quiescent core’ in a pipe flow (Kwon et al. [2014] detected this feature in a channel flow) and the associated intermittency between high- and low-level turbulent regions.

Cross-stream velocity component fluctuations exhibit super-Gaussian kurtosis throughout the entire flow domain for both pipe and boundary layer flows, while streamwise velocity fluctuations remain sub-Gaussian until the wake. The kurtosis coefficients of the vorticity fluctuations of all three components are super-Gaussian across the flow domain, with the kurtosis of the zero-mean components ($\omega_1$ and $\omega_2$) tending to increase with distance from the wall. The spanwise vorticity skewness is of the same sign as the mean vorticity across the flow domain, and trends toward zero at an approximately logarithmic rate in the region where the mean velocity is logarithmic.

**Appendix A: HRNBLWT measurements**

Owing to the slight departures from the expected canonical ZPG wake behaviour of the Flow Physics Facility (FPF) measurements (cf. figure 5.1), additional measurements collected at the High Reynolds Number Boundary Layer Wind Tunnel (HRNBLWT)—and adhering more strictly to the canonical wake shape—are included in this Appendix. These measurements are not included in the main text because they correspond to lower-$Re_\tau$ cases (about $2/3$ the magnitude of those collected at the FPF and in the pipe flow, for matched spatial resolution), and to avoid overcrowding the figures. The parameters of these measurements are given in Table 5.2. The parameters of the pipe measurements are also reproduced in Table 5.2 (from Table 5.1) for reference.

Figure A5.1 contains the same plots of mean velocity in log-law and deficit form as shown in figure 5.1, but with the HRNBLWT measurements in place of the FPF measurements. As can be seen from figure A5.1(b) in particular, the
Pipe versus boundary layer statistics

Facility | $u_\tau$ [m/s] | $t_\tau^+$ | $t_w^+$ | $\Delta x_2^+$ | $\Delta x_3^+$ | $\delta$ [m] | $Re_\tau$ | $tU_o/\delta$ [-]
--- | --- | --- | --- | --- | --- | --- | --- | ---
HRNBLWT (BL) | 0.18 | 12 | 9 | 9 | 22 | 0.30 | 3300 | 9900
CICLoPE (Pipe) | 0.18 | 13 | 9 | 9 | 23 | 0.45 | 5200 | 5100 (25300)
HRNBLWT (BL) | 0.25 | 18 | 12 | 13 | 31 | 0.30 | 4800 | 15000
CICLoPE (Pipe) | 0.26 | 19 | 14 | 14 | 34 | 0.45 | 7700 | 5000 (24800)
HRNBLWT (BL) | 0.33 | 23 | 17 | 17 | 41 | 0.31 | 6300 | 14000
CICLoPE (Pipe) | 0.34 | 25 | 18 | 18 | 44 | 0.45 | 10000 | 5200 (25900)

Table 5.2: Summary of experiments presented in Appendix A. Measurement sample times correspond to all samples for each case, with the exception of four selected $x_2$ locations in the pipe cases for which longer samples were collected—these longer sample times (given in parentheses) correspond to the centremost location, and (near) the start, middle, and end of the log layer (i.e., $x_2^+ = 0.93\delta^+, \approx 2.6\sqrt{\delta^+}, \approx 0.15\delta^+, and \approx (2.6\sqrt{\delta^+} \times 0.15\delta^+)^{1/2}$).

HRNBLWT data closely match the DNS of Sillero et al. [2013] through the log-layer and wake. Thus, the wake-region features seen in the HRNBLWT data in the following figures are not expected to feature any artifacts associated with departures from the canonical wake shape.

Figure A5.2 shows the profiles of Reynolds stress of the pipe and boundary layer cases as measured in the CICLoPE and the HRNBLWT, respectively, along with selected results from existing studies. The deviations in the $u_2$ and $u_3$ variance profiles are the same as those identified using the FPF data in §5.2. The present $\overline{u_2^2}$ and $\overline{u_3^2}$ profiles exhibit very close agreement with those of Baidya [2015], which were obtained via a ×-wire hot-wire array. The present $\overline{u_2^2}$ profiles agree in trend with those of Morrill-Winter et al. [2015], which were obtained with a specialised 4-wire hotwire probe, though with a slight difference in magnitude. Such differences highlight the advantage of using the same measurement technique under matched
Figure A5.2: (a)–(c) Streamwise, wall-normal, and spanwise Reynolds normal stress, and (d) Reynolds shear stress profiles as measured in the HRNBLWT and the CICLoPE. Shaded regions indicate range of inner log-layer boundaries for present experimental cases. ‘—’ $Re_\tau \approx 5200$ channel DNS from Lee & Moser [2015]; ‘—’ $Re_\tau \approx 10500$ pipe data from Hultmark et al. [2013]; ‘—’ $Re_\tau \approx 8000$ boundary layer data from Samie et al. [2018]; ‘+’ $Re_\tau \approx 8000$ boundary layer (HRNBLWT) data from Baidya [2015]; ‘*’ $Re_\tau \approx 7900$ boundary layer (HRNBLWT) data from Morrill-Winter et al. [2015]. Log-line in (a) corresponds to $\overline{u_1^2}/\overline{u_2^2} = 1.95 - 1.26 \log{(x_2/\delta_9)} + \log(1.15)$ from Marusic et al. [2013], where the additional $\log(1.15)$ constant accounts for a difference in definition of $\delta$. Log lines in (c) correspond to $\overline{u_3^2}/\overline{u_2^2} = 1.66 - 0.26 \log{(x_2/\delta_9)}$ for the boundary layer and $\overline{u_3^2}/\overline{u_2^2} = 1.11 - 0.42 \log{(x_2/\delta)}$ for the pipe. See Tables 5.1 and 5.2 for remaining symbols/lines.
Figure A5.3: (a)–(b) Skewness coefficient profiles for the fluctuating streamwise and wall-normal velocity signals, respectively, and (c) the fluctuating Reynolds shear stress signal. Symbols given in Table 5.2.

conditions to compare pipe and boundary layer flow. Differences between the two flows in the wake portion of the Reynolds shear stress profiles are more clearly visible in figure A5.2(d) than in figure 5.5(a). Although no clear difference in the $u_1$ variance profile shapes could be identified from the comparison of the FPF and CICLoPE data, it could be argued that the HRNBLWT $u_1$ variance systematically exceeds that of the pipe over the region $0.3 \lesssim x_2/\delta \lesssim 0.8$ when $\delta_{99}$ is used as the outer scale. This difference in $u_1$ variance in the outer region is highlighted in the inset to figure A5.2(a), which shows the two highest-$Re_\tau$ cases along with highly resolved single-element hotwire measurements for pipe flow at $Re_\tau \approx 10500$ from Hultmark et al. [2013] and for boundary layer flow at $Re_\tau \approx 8000$ from Samie et al. [2018]. When $\delta$ for the boundary layer is chosen as $\delta_{99}$, this feature is reminiscent of the outer 'bumps' in the $u_2$ and $u_3$ variance profiles. While a slight change in the definition of $\delta$ for the boundary layer case could remove the difference between the two $u_1$ profiles in the outer region, it could not remove the observed differences in the $u_2$ or $u_3$ variance profiles.

Figure A5.3 shows the skewness profiles of $u_1$, $u_2$, and $u_1u_2$ as measured in the HRNBLWT and CICLoPE. These profiles clearly exhibit a trend of slightly lower boundary layer skewness magnitude from near the outer edge of the log-layer out to $x_2/\delta \approx 0.5$. These features are consistent with the differences in intermittency
factor $\gamma$ discussed in §5.4. Quasi non-turbulent flow increases the probability density of fluctuating quantities close to 0, which in turn increases the magnitudes of skewness and kurtosis factors (if the fully-turbulent portions of the flow remain relatively unchanged). Indeed, it was shown in §5.4 that the time-fraction of quasi non-turbulent flow is higher in the pipe than the boundary layer from near the outer edge of the log-layer out to $x_2/\delta \approx 0.5$.

As with the skewness profiles shown in figure A5.3, the kurtosis profiles shown in figure A5.4 exhibit lower boundary layer profile magnitude near the outer edge of the log-layer out to $x_2/\delta \approx 0.5$. This feature is also clearly visible in the FPF $u_3$ kurtosis shown in figure 5.4(b), and to a lesser degree in the $u_2$ kurtosis shown in figure 5.3(c). Again, it is proposed that this feature is related to the difference in intermittency associated with the TNTI in the boundary layer and the quiescent core in the pipe.

Figure A5.5 contains the same plots of mean turbulence enstrophy and enstrophy ratio as figure 5.9, but with the HRNBLWT data plotted in place of the FPF data. The same outer ‘bump’ feature in the boundary layer enstrophy relative to the pipe enstrophy is visible in figure A5.5(b), but its onset (at $x_2/\delta \approx 0.1$) is clearer and more consistent. Note that the ratio shown in figure A5.5(b) is multiplied by the ratio of Reynolds numbers $\delta_{BL}^+ / \delta_P^+$ to account for the difference in enstrophy.

Figure A5.4: (a)–(c) Kurtosis coefficient profiles respectively for the fluctuating streamwise, wall-normal, and spanwise velocity signals, and (d) the fluctuating Reynolds shear stress signal. Symbols given in Table 5.2.
Figure A5.5: (a) Inner-normalized turbulence enstrophy profiles plotted as sold lines capped by symbols for clarity. Symbols at start/end of each line correspond to Table 5.2. (b) Ratio of turbulence enstrophy between boundary layers and pipes of approximately matched spatial resolution. Note the shorthand $\omega^2 = \omega_i^2$ is used in (b). Each ratio is multiplied by $\delta_{BL}^+ / \delta_P^+$ to account for the apparent Reynolds number trend when ratios are computed based on $x_2/\delta$ coordinates (i.e. that associated with $\overline{\omega_i^2} \sim x_2^{-1}$ dependence).

at a fixed $x_2/\delta$ location owing to the dependence of said enstrophy on (to an approximation) $x_2^{-1}$.

Appendix B: Vorticity resolution

The synthetic experimentally predicted effects of probe size on vorticity resolution are summarised in figure B5.1. Figure B5.1(a) shows the length of an individual wire relative to the local Kolmogorov length scale $\eta$ for the three spatial resolution cases corresponding to the present experimental data. Figure B5.1(b) shows the synthetic experimental predicted ratio of ‘measured’ (subscript $m$) to ‘true’ (subscript $m'$) vorticity RMS magnitude $|\omega|$, defined as

$$|\omega| = (\omega_1^2 + \omega_2^2 + \omega_3^2)^{1/2},$$

(5.3)
a function of $l_w/\eta$. The light and dark shaded regions in both figures B5.1(a) and (b) correspond to the inner and outer boundaries of the log layer for the range of experimental data presented herein. Recall from figure 3.1 and Table 5.1 that the sub-array separation distances are related to the wire length as $\Delta x_2 = l_w/\sqrt{2}$ and $\Delta x_2 = 2.5l_w/\sqrt{2}$. Figure B5.1(b) indicates that about 85% of the vorticity is resolved at the inner edge of the log-layer for the highest resolution cases, and about 73% for the lowest resolution (highest-$Re_\tau$). These percentages increase to about 90% and 85% respectively for the highest and lowest resolution cases at the outer edge of the log-layer. The focus of most of the conclusions surrounding vorticity in the present paper is on the region near the outer edge of the log-layer out to the middle of the wake, where the vorticity resolution will improve
Pipe versus boundary layer statistics

Exp. $x_2^+ = 25$

Exp. $x_2^+ = 19$

Exp. $x_2^+ = 14$

Figure B5.1: (a) Synthetic experiment predicted magnitude of reference length $l_w$ relative to Kolmogorov length scale $\eta$. Light and dark shaded regions respectively indicate the range of inner and outer boundaries of the log-layer ($x_2^i$ and $x_2^o$) for the physical experimental datasets presented herein. (b) Synthetic experiment ‘measured’ (subscript $m$) versus ‘true’ (subscript $t$) vorticity RMS magnitude captured by the present probe geometry as a function of reference length $l_w$ relative to $\eta$. Light and dark shaded regions respectively indicate range of probe resolutions at the start and end of the log-layer for the physical experimental cases.

Given that the synthetic experiment corresponds to a lower Reynolds number ($Re_\tau \approx 2000$) than the present measurements, an additional comment on the effects of $Re_\tau$ is warranted. As can be seen from figure B5.1(b), resolution of the vorticity signal is (to a close approximation) dependent on the local value of the Kolmogorov length scale $\eta$ relative to the size of the probe (e.g. see also Zhu & Antonia [1995]). To the degree that the dissipation rate scales with $u_\tau$ and $l_w$ from the wall through the log-layer (which is quite well based on available DNS and $\partial u_1/\partial x_1$-based experimental estimates), $\eta$ is (at least for the present purposes) independent of $Re_\tau$ in the near-wall region and where the log-layers of two disparate $Re_\tau$ flows overlap. Thus, the synthetic experimental predictions of vorticity resolution should apply even to the higher-$Re_\tau$ experiments out to a viscous wall distance that corresponds to $x_2/\delta_{DNS} \approx 0.15$, i.e. $x_2^+ \approx 300$. Above this position, the $\eta \sim x_2^{1/4}$ trend can be used to predict the values of $l_w/\eta$ at higher Reynolds numbers. The resolved fraction of vorticity magnitude may then be estimated according to figure B5.1(b).

For reference, figure B5.2 shows the present CICLoPE and HRNBLWT vorticity RMS profiles alongside existing published vorticity data from the multi-sensor hotwire studies of Balint et al. [1991], Honkan & Andreopoulos [1997], and Lemonis [1997], and the dual-plane PIV study of Ganapathisubramani et al. [2006] (see also figure 2.4).
Pipe versus boundary layer statistics

Figure B5.2: (a)–(c): Streamwise, wall-normal, and spanwise vorticity RMS versus viscous wall-distance for selected pipe and boundary layer profiles. Filled symbols as in table 5.2. —: $Re_\tau \approx 1000$ pipe DNS from El Khoury et al. [2013]; —: $Re_\tau \approx 1300$ boundary layer DNS from Sillero et al. [2013]; —: $Re_\tau \approx 1100$ boundary layer hotwire experiment from Balint et al. [1991]; —: $Re_\tau \approx 1200$ boundary layer hotwire experiment from Honkan & Andreopoulos [1997]; —: $Re_\tau \approx 2300$ boundary layer hotwire experiment from Lemonis [1997]; ◊: $Re_\tau \approx 1200$ boundary layer dual-plane stereo-PIV from Ganapathisubramani et al. [2006]. All cited experimental data have been manually digitised from plots in the original cited work to the best of the author’s ability, but may deviate slightly from actual results.
Chapter 6

Spectral characteristics of velocity and vorticity

In this chapter, we examine spectral characteristics of velocity and vorticity in pipe and ZPG boundary layer flows, and identify differences between the two cases. In the previous chapter, differences between pipes and boundary layers were identified in the kinetic energy of the transverse velocity components and the enstrophy. The wavenumber ranges contributing to the differences observed in the previous chapter are examined via direct comparison of spectra. In agreement with Monty et al. [2009], it is found that pipe flow receives higher contributions than the boundary layer to the TKE from wavelengths $\lambda_1/\delta > 10$, while the boundary layer receives higher contributions than the pipe to the TKE from wavelengths $\lambda_1/\delta < 10$.

Spectral criteria for local isotropy and local axisymmetry in the velocity components are also examined. It is found that the energy content in scales shorter in streamwise wavelength than half their distance to the wall is well-approximated by both local isotropy and local axisymmetry.

The Kolmogorov length scale is shown to describe the enstrophy spectrum more consistently than the Taylor microscale throughout the flow domains of both the pipe and boundary layer. Under normalisation by the Kolmogorov scales, the enstrophy spectrum is found to be invariant to within experimental uncertainty for wall-distances above $x_2^+ \approx 200$. 
Spectral Characteristics

6.1 Introduction

Differences in the turbulence enstrophy and the TKE of the transverse velocity components in pipe and boundary layer flows were identified in Chapter 5. In this chapter, we examine the spectral content of the velocity and vorticity components to determine the scales that contribute to the observed differences. The following notation will be used throughout for the one-dimensional velocity component spectra/cospectra $E_{ij}$, velocity gradient spectra $\phi_{ui,j}$, velocity gradient cospectra $Co_{ui,juk,l}$, and enstrophy spectra $W$:

\[
E_{ij}(k_1) = \int_{-\infty}^{\infty} E(k) \left( \delta_{ij} - \frac{k_i k_j}{k^2} \right) dk_2 dk_3 \quad (6.1)
\]

\[
\phi_{ui,j}(k_1) = \int_{-\infty}^{\infty} k_j^2 E(k) \left( 1 - \frac{k_i^2}{k^2} \right) dk_2 dk_3 \quad (6.2)
\]

\[
Co_{ui,juk,l}(k_1) = \int_{-\infty}^{\infty} k_j k_l E(k) \left( \delta_{ik} - \frac{k_i k_k}{k^2} \right) dk_2 dk_3 \quad (i, j) \neq (k, l) \quad (6.3)
\]

\[
W(k_1) = \phi_{u3,2} + \phi_{u2,3} + \phi_{u1,3} + \phi_{u3,1} + \phi_{u2,1} + \phi_{u1,2} - 2Co_{u3,2u2,3} - 2Co_{u3,1u1,3} - 2Co_{u2,1u1,2} \quad (6.4)
\]

The quantities in (6.1)–(6.4) are computed from the measured time series at each wall-distance $x_2$ from the squared magnitudes of the Fourier transforms of the constituent signals. For example, $E_{ij}$ is computed at wall-distance $x_2$ according to the following expressions:

\[
E_{ij}(x_2, k_1) = \frac{U_c}{\pi f_s N} \hat{u}_i(x_2)\hat{u}^*_j(x_2), \quad (6.5)
\]

where $\hat{\cdot}$ indicates the (one-sided) Fourier coefficients computed via discrete Fourier transform, $(\cdot)^*$ denotes a complex conjugate, $N$ is the number of points from which the discrete Fourier transform is computed, $U_c$ is the convection velocity, and $f_s$ is the sample frequency in Hertz. The convection velocity is chosen at each wall-distance as the local mean streamwise velocity. Equation (6.5) yields the specific energy per unit wavenumber; the analogous temporal spectrum with units of specific energy per unit frequency (in radians per second) is computed as the following (note the superscript $t$ will refer to the temporal spectrum throughout this chapter):

\[
E_{it}(x_2, \omega) = \frac{1}{\pi f_s N} \hat{u}_i(x_2)\hat{u}^*_j(x_2). \quad (6.6)
\]

The values of $E_{ij}(x_2, k_1)$, $W(x_2, k_1)$ etc. are typically plotted against logarithmically scaled wavenumber/wavelength to clearly illustrate the entire range of contributing scales (which span several decades). Stretching the coordinates in this
Table 6.1: Summary of experiments featured in this chapter. Outer scale \( \delta \) defined as pipe radius or the analogue to \( \delta_{99} \) (defined in §5.1.1), where applicable. Outer-scale (i.e. \( \delta \)) turnover times shown in parentheses represent measurement sample times for selected \( x_2^+ \) locations near the start, middle, and end of the log layer, as well as the centremost location (i.e. \( 2\sqrt{\delta^+}, 0.15\delta^+, (2\sqrt{\delta^+} \times 0.15\delta^+)^{1/2} \), and \( 0.93\delta^+ \)).

way, however, leads to a misleading representation of the scales that contribute to the signal variance (i.e. the integral of the spectrum through all wavenumbers). This skewing may be visually compensated-for through ‘premultiplication’ by the wavenumber, e.g. by plotting \( \kappa_1 E_{ij} \) in place of \( E_{ij} \). Plots depicting the premultiplied spectrum of any quantity against both wall-distance and wavenumber (or wavelength) simultaneously are referred to here as ‘spectrograms’ or ‘cospectrograms’, where appropriate.

Finally, there are several instances in this chapter in which reference is made to the wavelengths that correspond to the peaks in the premultiplied spectra of interest at each wall-distance. Thus, we define \( \Lambda_{u_{ij}}(x_2) \) in (6.7) as a function of wall-distance that describes the streamwise wavelength \( \lambda_1 \) corresponding to the peak magnitude of \( \kappa_1 E_{ij} \) at each wall-distance. The analogous function for the enstrophy spectrum is \( \Lambda_\omega(x_2) \), defined in (6.8).

\[
\kappa_1 E_{ij}(x_2 = x_2', \lambda_1 = \Lambda_{u_{ij}}(x_2')) = \max (|\kappa_1 E_{ij}(x_2 = x_2', \lambda_1)|)
\]

\[
\kappa_1 W(x_2 = x_2', \lambda_1 = \Lambda_\omega(x_2')) = \max (\kappa_1 W(x_2 = x_2', \lambda_1))
\]

The experimental datasets shown in this chapter are the same as those shown in the previous chapter. The relevant experimental entries from Table 3.1 are repeated here in Table 6.1 for reference.

6.2 Velocity

This section is split into two parts: one compares and contrasts the contributions to the Reynolds stresses in the pipe and ZPG boundary layer, and the other examines the satisfaction of symmetry conditions—namely local axisymmetry and local isotropy—in the Reynolds normal stresses.
6.2.1 Pipe versus boundary layer

Figure 6.1 contains spectrograms of $u_1^2$, $u_2^2$, and $u_3^2$ and cospectrograms of $u_1 u_2$ for both the pipe and boundary layer cases at $Re_\tau \approx 10000$. The main purpose of figure 6.1 is to expose the range of scales that contribute most to the Reynolds stresses; the differences in these contributions in pipe and boundary layer flows are examined more directly in figure 6.3. Note that the contour levels differ for each of the three velocity components, as indicated by the values of $C$. In all cases, the lowest contour is at $0.05 \cdot C$.

The streamwise energy spectrogram (at sufficiently high $Re_\tau$) is expected to be dominated by two distinct regions: an ‘inner’ region whose position is fixed (both in wall-distance and wavelength) in viscous units, and an ‘outer’ region centered in the log-layer at a streamwise wavelength that is $O(\delta)$ [Hutchins & Marusic, 2007]. Although the centre of the inner peak is expected to reside at a wall-distance not probed by the present experiments, its outer portion is visible in figures 6.1 (a) and (b). The centre of the outer peak is more difficult to pinpoint, as it lies along a relatively flat ‘ridge’ spanning the log-layer and wake regions, and is thus sensitive to incomplete convergence. Still, the shape of the outer ridge(s) in $\kappa_1 E_{11}$ at each wall-distance reveal the different organisation of streamwise TKE in pipe and boundary layer flow. The values of $\Lambda_{u_{11}}$ (defined above in (6.7)) in the outer region of the boundary layer show very little dependence on wall-distance. As shown in figures 6.1 (a) via the white dash-dotted line $\cdots$, the crest of the ridge in the outer region of the boundary layer resides at $\lambda_1/\delta \approx 3$. A similar ridge is also observed in the pipe case at $\lambda_1/\delta \approx 3$. A number of authors (e.g. see Kim & Adrian [1999]) have identified this ridge in both pipe and boundary layer flow and associated it with the large-scale turbulent ‘bulges’ first observed by Kovasznay et al. [1970]. These turbulent ‘bulges’ are now generally referred to as Large-Scale-Motions, or LSMs. Kim & Adrian [1999] also identified a second local peak in the outer region of the $u_1$ spectra of pipe flow at wavelengths an order of magnitude larger than those associated with LSMs. Based on the long wavelength of this second peak, the ‘eddies’ responsible for its existence have been termed “Very-Large-Scale-Motions”, or VLSMs. The power-law (6.9) suggested by Monty et al. [2009] to describe the wavelengths of the VLSM ridge (based on an empirical fit) is included in figure 6.1 for reference ( $\cdots$ ).

$$\frac{\lambda_1}{\delta} = 23 \left(\frac{x_2}{\delta}\right)^{3/7}. \tag{6.9}$$

Although there is no evidence of a VLSM ridge in the boundary layer spectrogram, Hutchins & Marusic [2007] found that similarly long-wavelength ‘superstructures’ occur in boundary layers as well, but meander in $x_3$ and so appear shorter when measured by single-point probes such as hot-wires. Despite the mounting number of studies to report a second peak in $\kappa_1 E_{11}$ in internal flows, it is worth noting that Del Álamo & Jiménez [2009] used time-resolved DNS to show that the use of Taylor’s frozen turbulence hypothesis with wavelength-independent convection velocity significantly inflates the magnitude of $\kappa_1 E_{11}$ the VLSM wavelength range. Still, the magnitude difference in $\kappa_1 E_{11}$ between the pipe and boundary layer
Figure 6.1: Viscous-scaled premultiplied velocity spectrograms for pipe and boundary layer flow at \( Re_\tau \approx 10000 \). Note that the contour levels for each component are dependent on \( C \).

- \( \lambda_1/\delta = 3 \)
- \( \lambda_1/\delta = 23 \left( \frac{x_2^+}{\delta} \right)^{3/7} \)
- \( \lambda_1 = 2x_2 \)

Symbols represent \( \Lambda_{uij} \) as defined by (6.7). Vertical white lines indicate bounds of the log-layer as defined by \( x_2^+ = 2.6\sqrt{\delta^+} \) and \( x_{2o} = 0.15\delta^+ \).
cases in the VLSM range reflects a significant difference at least in the temporal spectra of internal and external flows. Figures 6.1 (a) and (b) are re-plotted as temporal spectrograms in figures 6.2 (a) and (b). Note the following notation in figure 6.2: \( \omega \) denotes frequency in radians per second, \( \lambda_t \equiv 2\pi/\omega \) is the temporal wavelength, and \( E_{11} \) is the power-spectral density in units of velocity-squared per frequency (in radians per second). In effect, \( \omega u^{-2} E_{11}^l (\omega) \) has the same magnitude as \( \kappa_1 u^{-2} E_{11} (\kappa_1) \) and is used only to map the function to frequency space (as opposed to wavenumber space). Although the temporal wavelengths associated with VLSMs (in the pipe case) also appear to grow in the log-layer, it is not clear that this growth continues through the wake. Instead, as shown in figure 6.2 (c), the temporal wavelength of the VLSM ridge appears more-or-less fixed at \( \lambda_t U_o/\delta \approx 20 \) (twenty large-eddy turnover times). Furthermore, the contours at temporal wavelengths greater than 20 large-eddy turnover times are essentially horizontal in the log-layer of both flows. Since Taylor’s frozen turbulence hypothesis is at best flawed at such wavelengths, it is proposed that the VLSM mode convects (on average) with a single velocity scale (i.e. not with the local mean velocity).

Returning to figure 6.1, the spectrograms of the wall-normal component are shown in plots (c) and (d). Note that the lines \( \cdots \cdots \) and \( \cdots \cdots \) in figures (c)-(f) are included for reference, and are the same as those in (a) and (b). The values of \( \Lambda_{u22} \) (defined above in (6.7)) are also plotted using the symbols given in Table 6.1. The red/black dashed line \( \cdots \cdots \) through \( \lambda_1 = 2x_2 \) closely approximates the values of \( \Lambda_{u22} \) in the log-layer of both flows. Isocontours of \( \kappa_1 E_{22} \) at wavelengths greater than \( \Lambda_{u22} \) also scale with distance from the wall in the log-layer for both flows, while those below \( \Lambda_{u22} \) do not. The proportionality between \( \Lambda_{u22} \) (and larger scales) and wall-distance in both flows is consistent with the notion that the impermeable wall has a ‘blocking’ effect on \( u_2 \)-containing motions. As discussed in Chapter 2, there is no direct ‘production’ term for \( u_2 \). Instead, \( u_2 \) energy must ‘feed’ off of \( u_1^2 \) through the pressure-strain redistribution term. Despite the ample ‘source’ energy (i.e. \( E_{11} \)) that could support higher values of \( E_{22} \) in large near-wall scales via pressure-strain redistribution (this will be discussed in further detail in section 6.2.2), the actual magnitude of \( E_{22} \) is very small in this domain and exhibits clear distance-from-the-wall scaling. Thus, it is surmised that scales longer in streamwise extent than twice their distance to the wall (i.e. \( \lambda_1 \gtrsim 2x_2 \)) receive energy through the pressure-strain correlation with decreasing efficiency to a degree that varies monotonically with their length. The proportionality of scales larger than \( \Lambda_{u22} \) is shown more clearly in the appendix to this chapter, where figures 6.1 (c) and (d) are replotted against \( \lambda_1/x_2 \) rather than \( \lambda_1/\delta \) and \( \lambda^+_1 \). With the contours above \( \Lambda_{u22} \) increasing in wavelength as \( \sim x_2 \), and those below \( \Lambda_{u22} \) increasing in wavelength slower than \( \sim x_2 \), the result is a widening of the range of energetic (at least in terms of \( u_2 \)) scales for both the pipe and the boundary layer. The magnitude of \( \kappa_1 E_{22} \) at \( \Lambda_{u22} \) decreases more slowly in the log-layer/lower wake-layer of the boundary layer than in the pipe (cf. figure B6.1 in the appendix to this chapter). This feature combined with the widening range of contributing scales explains the differing behaviour of the \( u_2^2 \) profiles of the pipe and boundary layer cases observed in Chapter 5. The boundary layer \( \overline{u_2^2} \) increases in the log-layer owing to a widening range of contributing scales combined with
Figure 6.2: Viscous-scaled premultiplied temporal streamwise velocity spectrograms for pipe and boundary layer flow at $Re_\tau \approx 10000$. Red curves correspond to the pipe case; blue curves correspond to the boundary layer. $\lambda U_\infty /\delta = 20$. Symbols represent $\Lambda_{n11}$ as defined by (6.7). Vertical white lines in (a) and (b) demarcate the boundaries of the log-layer. Curves in (c) represent wall-distances $x_2/\delta \geq 0.15$; their exact locations match the symbols in (a) and (b).
a more-or-less constant peak magnitude. The pipe $\overline{u'^2}$ remains relatively flat in the log-layer because the widening range of contributing scales is offset by a slight decrease in peak magnitude with increasing wall-distance.

The $u_3$ component spectrograms are plotted in figures 6.1 (e) and (f) along with the values of $\Lambda_{u_{33}}$ and the LSM/VLSM guide lines for reference. As with the $u_1$ component, the $u_3$ spectrograms exhibit a distinct ‘jump’ in $\Lambda_{u_{33}}$ associated with the shift from the inner to outer regimes. Also like the $u_1$ component (and in stark contrast to the $u_2$ component), $\overline{u'^2}$ contains appreciable contributions from wavelengths $O(\delta)$ and greater as close to the wall as the present measurements afford. The values of $\Lambda_{u_{33}}$ are nominally constant in the wake regions of both the boundary layer and pipe cases, with a value $\Lambda_{u_{33}} \approx \delta$. This relative constancy suggests that the ridge in the $u_3$ spectrogram is associated with LSMs that are believed to contribute to the $\lambda_1/\delta \approx 3$ ridge in the $u_1$ spectrograms. Although the values of $\Lambda_{u_{33}}$ are smaller than those of $\Lambda_{u_{11}}$, it is reasonable to expect that those LSMs which contain an appreciable $u_3$-component meander in the $x_3$ direction, resulting in a shorter interpreted streamwise wavelength. The differing trends in $\overline{u'^2}$ compared to $\overline{u'^2}$ in the log-layer ($\overline{u'^2}$ decreases, while $\overline{u'^2}$ increases (boundary layer) or stays flat (pipe), can be explained through the differing trends in the ranges of contributing scales. Where isocontours of $\kappa_1 E_{22}$ scaled with $x_3^2$ for wavelengths $\lambda_1 > \Lambda_{u_{22}}$, those of $\kappa_1 E_{33}$ for wavelengths $\lambda_1 > \Lambda_{u_{22}}$ are horizontal, scaling instead with $\delta$. Meanwhile, isocontours of $\kappa_1 E_{22}$ and $\kappa_1 E_{33}$ at wavelengths less than $\Lambda_{u_{22}}$ and $\Lambda_{u_{33}}$, respectively, are inclined with slopes less than $x_3^2$. Thus, the range of scales contributing to $\overline{u'^2}$ increases with wall-distance (in the log-layer), while the range contributing to $\overline{u'^2}$ decreases. The distinct outer peak in the boundary layer case at the outer edge of the log-layer slows the decay rate of $\overline{u'^2}$, leading to the ‘bump’ in the variance profile discussed in Chapter 5. The profiles of $\overline{u'^2}$ in the pipe decay more rapidly with wall-distance, as they feature an outer plateau in $\kappa_1 E_{33}$ rather than a distinct peak.

The premultiplied cospectrograms of the Reynolds shear stress (RSS), $\kappa_1 E_{12}$, are shown for both the pipe and boundary layer cases in figures 6.1 (g) and (h). These cospectrograms show the distribution of scales that contribute to the mean RSS as a function of wall-distance. Note that the contour colours range from 0 (white) to −0.3 (blue), since $E_{12} < 0$ everywhere to within experimental scatter in both flows. Despite contributing a significant fraction of $\overline{u'^2}$, scales that lie below the $\lambda_1 = 2x_2$ line contribute very little to the mean RSS. The isocontours of $\kappa_1 E_{12}$ in much of the log-layer are inclined with a slope of $\sim x_2$ to within the scatter of the data (see also figure A6.1 in the appendix to this chapter). This scaling is consistent with one of the key assumptions of the attached-eddy hypothesis [Townsend, 1976]—that the motions expected to contribute the most to the Reynolds shear stress are geometrically self-similar with distance from the wall (i.e. scale with $x_2$). Wall-distance scaling (in the log-layer) of $\kappa_1 E_{12}$ has also been directly observed by Morrill-Winter et al. [2017b] and Baidya et al. [2017]. As the log-layer transitions into the wake ($x_2/\delta \approx 0.15$), an outer peak/ridge in $\kappa_1 E_{12}$ emerges (for both flows) centered at $\lambda_1/\delta \approx 3$—coincident with the LSM mode observed in $\kappa_1 E_{11}$. As the LSMs are believed to result from the concatenation of
the RSS-carrying, geometrically self-similar eddies described by Townsend [1976] [Adrian et al., 2000], it stands to reason that this mode should contribute significantly to the RSS. An outer ridge in \( \kappa_1 E_{12} \) is also visible in the VLSM range in the pipe case. Contributions from the VLSM range have been previously reported by Guala et al. [2006], who found that wavelengths larger than \( \lambda_1 / \delta = 3 \) contribute between 50% and 60% of the mean RSS in pipe flow for \( Re_\tau \approx 4000–8000 \). This ridge, however, appears to scale more closely with \( x_1^2 \) than with the \( x_1^{2/7} \) line (i.e. \( \cdots \)) proposed by Monty et al. [2009] to describe the wavelengths of the VLSM ridge (for \( \kappa_1 E_{11} \)) in pipe and channel flow. Given that the RSS appears in the production term for \( u_1^2 \), it is perhaps more instructive to deduce the scaling behaviour of the VLSM mode from \( \kappa_1 E_{12} \) than from \( \kappa_1 E_{11} \). Kim & Adrian [1999] and others have suggested that the VLSM mode is formed from a concatenation of LSMs, which themselves form from a concatenation of geometrically self-similar eddies of the type described by Townsend [1976]. Distance-from-the-wall scaling is therefore plausible for both LSMs and VLSMs, although LSMs appear to be more appropriately scaled by \( \delta \). Also noteworthy is the lack of contribution to \( u_1^2 \) from scales \( \lambda_1 > \delta \) near the wall. Mathis et al. [2009] and other have shown that large scale motions in the outer region have been shown to exert an influence on the motions near the wall, resulting in appreciable contributions to \( u_1^2 \) from scales \( \lambda_1 > \delta \) near the wall. This is evident in the upper-left corner of figures 6.1 (a) and (b). The lack of contributions to \( u_2^2 \) from this region, however, prevents such structures near the wall from carrying appreciable RSS.

Figure 6.3 shows the result of subtracting the premultiplied energy spectra and cospectra of the three pipe cases from those of the corresponding boundary layer cases. In other words, the rightmost column of subfigures in figure 6.3 (i.e. (c), (f), (i), and (l)) is constructed by subtracting the right column of subfigures from figure 6.1 from the left column. As with figure 6.1, the contour levels differ between the \( i = j = 1, 2, 3 \), and \( i = 1, j = 2 \) cases. The total ranges of the contours for each quantity in figure 6.3 are exactly half of those used in figure 6.1. Both ends of the colour map used in figure 6.3 correspond to a deviation equal to 25% of the maximum contour for the corresponding quantity in figure 6.1. Note that, although the highest \( Re_\tau \) cases exhibit differences in magnitude from the lower two \( Re_\tau \) cases, for the \( u_1 \) component (figure 6.3 (c)) this is believed to be the result of slight sensor drift in one of the eight sensing elements in the high \( Re_\tau \) boundary layer case and not a true Reynolds number trend. The effect of this drift on the statistics shown in Chapter 5 is insignificant to the conclusions drawn therein.

The difference between the boundary layer and pipe streamwise component is shown in figures 6.3 (a), (b), and (c). In agreement with the findings of Monty et al. [2009], the contributions to \( u_1^2 \) are greater in the boundary layer cases for \( \lambda_1 / \delta < 10 \), and greater in the pipe cases for \( \lambda_1 / \delta > 10 \). This 10\( \delta \) limit is constant to within the experimental scatter for all three \( Re_\tau \) cases across nearly the entire flow domain, until \( x_2 / \delta \gtrsim 0.6 \) where the free-stream boundary condition brings the boundary layer TKE to zero. The apparent constancy of the \( \lambda_1 / \delta \approx 10 \) zero-crossing is somewhat surprising given the diverging trends of the LSM and VLSM wavelengths with wall-distance. Since the rate at which these two peaks
Figure 6.3: Difference between boundary layer (subscript $B$) and pipe (subscript $P$) premultiplied velocity component spectrograms and cospectrograms for three $Re_\tau$ cases. Note that the contour levels for each component are dependent on $C$. $\lambda_1/\delta = 10$. Lines and filled symbols are as in Table 6.1, and represent the values of $\Lambda_{uij}$ for the both the pipe and boundary layer cases used to construct the figure.
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The difference between pipe and boundary layer transverse velocity component energy density is shown in figures 6.3 (d)–(i). Also shown for reference are the values of $\Lambda_{u_{ij}}$ for both the pipe and boundary layer cases using the symbols from Table 6.1. For the most part, the boundary layer receives more contributions to $u_2^2$ and $u_3^2$ from all scales that actually contribute appreciably to these quantities (cf. figure 6.1). The peak differences in $\kappa_1 E_{22}$ and $\kappa_1 E_{33}$ between the boundary layer and pipe cases occur near the outer edge of the log-layer ($x_2/\delta \approx 0.3$) and across wavelengths near their respective $\Lambda_{u_{ij}}$ values. This is consistent with the maximum difference in transverse component TKE observed in Chapter 5. As discussed in Chapter 2, there is no direct production of $u_2^2$ or $u_3^2$. Instead, the transverse components receive energy from the $u_1$ component (for which production is non-zero) through the pressure-strain redistribution terms. If it can be assumed that the streamwise wavelength of $u_2^2$- and $u_3^2$-containing eddies is proportional to that of the $u_1$ ‘eddy’ from which it received kinetic energy, then it follows that $E_{22}$ and $E_{33}$ are greater in the boundary layer cases than in the pipe, since the energy ‘source’ ($E_{11}$) is greater at all wavelengths that contribute appreciably to $u_2^2$ and $u_3^2$. The inverse of this holds true for wavelengths in the range $\lambda_1/\delta > 10$ for the $u_3$ component, i.e. $E_{33}$ is greater in the pipe than in the boundary layer for wavelengths $\lambda_1/\delta > 10$. As discussed above, however, wavelengths in this range contribute very little to the overall value of $u_3^2$ (cf. figure 6.1).

Figures 6.3 (j)–(l) show the difference between the pipe and boundary layer contributions to $u_1 u_2$. The red/black dashed-line boxes in these figures span the LSM and VLSM range in the outer region of the flow. The common thread within the enclosed area is a greater contribution to $u_1 u_2$ from the LSM range in the boundary layer cases, and a greater contribution from the VLSM range in the pipe cases. The zero crossing also appears to fall at $\lambda_1/\delta \approx 10$. Of note is that the greater contributions to $u_1 u_2$ in the boundary layer cases are relatively confined to the outer region of the flow at wavelengths $\lambda_1 > \delta$. Given that the mean shear is slightly higher in the boundary layer wake than the pipe wake, larger values of $\kappa_1 E_{12}$ are indicative of higher local turbulence production. The greater local production in the boundary layer cases is connected to the larger values of $\kappa_1 E_{11}$ at smaller wavelengths and wall-distances through turbulence transport and inertial energy transfer. The larger values of $E_{11}$ subsequently support greater $E_{22}$ and $E_{33}$ through pressure-strain redistribution.

6.2.2 Symmetry

Local isotropy and local axisymmetry are conditions which, where valid, allow for useful simplifications to transport equations. While global isotropy is only mathematically guaranteed in flows of little practical significance (e.g. turbulence in an unbounded box), global axisymmetry is guaranteed in a number of practically significant flows, including along the centreline of a pipe. The degree to which other
locations in the pipe (or any locations in the boundary layer case) exhibit axisymmetry, and across which scales (if any), is therefore of interest. Also of interest is the overlap between scales/wall-positions at which local isotropy is satisfied and those at which local axisymmetry is satisfied.

Global isotropy requires equality of the Reynolds normal stresses (i.e. $\overline{u_1^2} = \overline{u_2^2} = \overline{u_3^2}$), while global axisymmetry about the $x_1$ coordinate requires equality only between the two transverse components (i.e. $\overline{u_2^2} = \overline{u_3^2}$). Even when these conditions are violated, a range of scales may exhibit axisymmetry/isotropy locally. Satisfaction of local isotropy can be evaluated via the following relations between the one-dimensional energy spectra of each component [Pope, 2000]:

$$E_{22}(\kappa_1) = E_{33}(\kappa_1) = \frac{1}{2} \left( E_{11}(\kappa_1) - \kappa_1 \frac{dE_{11}(\kappa_1)}{d\kappa_1} \right),$$

(6.10)

while local axisymmetry can be evaluated by:

$$E_{22}(\kappa_1) = E_{33}(\kappa_1).$$

(6.11)

Saddoughi & Veeravalli [1994] tested each of these equalities at various positions in a ZPG boundary layer that developed over roughness to $Re_\tau \approx 35,000$ and 175,000. The authors found that each of these equalities were indeed satisfied over a wavenumber range that depends on the local mean shear. It is worth noting, however, that the presence of an impermeable wall is also capable of imposing a sense of direction on the turbulence by redirecting the wallward motion of eddies for which $\lambda_2 \approx x_2$ into the spanwise and streamwise directions, and by limiting the maximum possible $\lambda_2$ but not $\lambda_1$ or $\lambda_3$. In the special case where the distance to an impermeable wall is fixed with respect to a rotation about the $x_1$-direction (as in the centreline of the pipe), the impermeable wall may disrupt the balance between longitudinal and transverse TKE, but will affect the two transverse components equally. Although mean shear affects all scales, Corrsin [1958] reasoned that local isotropy (and thus also local axisymmetry) remains plausible for scales that lose their sense of direction via inertial breakdown more rapidly than a sense of direction can be imposed via the mean strain rate. The time scale of the mean strain rate is simply $\tau_S \equiv \left( \frac{1}{2} \frac{\partial U_1}{\partial x_2} \right)^{-1}$. Estimates of the time scale of inertial and viscous energy transfer have been derived by Onsager [1949]. The derivation provided by Corrsin [1958] is briefly summarised here.

A time scale for inertial energy transfer $\tau_l$ at a given wavenumber may be constructed through dimensional analysis and an assumption of geometric progression in the inertial cascade of energy at any given length scale (i.e. that $\Delta \kappa \approx \kappa$, or the change in a length scale through inertial energy transfer is proportional to the length scale itself). The time scale of inertial energy transfer is reasoned to be the kinetic energy in a particular ‘stage’ of the cascade divided by the energy transfer rate. With the assumption of a geometric progression in the inertial cascade, the kinetic energy per ‘stage’ is $\kappa E$ (i.e. energy per wavenumber $E \left[ m^3/s^2 \right]$ times the range of wavenumbers $\Delta \kappa \approx \kappa$ included in the ‘stage’). The energy transfer rate at a particular stage is reasoned to be the cube of the characteristic velocity of the
stage divided by the characteristic length of the stage, or \((\sqrt{\kappa E^{3}}/\kappa^{-1})\). Thus, the time scale for inertial energy transfer at wavenumber \(\kappa\) can be expressed as:

\[
\tau_{I}(\kappa) = (\kappa^{3}E(\kappa))^{-1/2}.
\]  
(6.12)

In a region where the energy spectrum can be approximated as \(E(\kappa) \approx 1.5\pi^{2/3}\kappa^{-5/3}\), for example, the inertial time scale is \(\tau_{I} \approx 0.82\pi^{-1/3}\kappa^{-2/3}\). This expression can be used to solve for the wavenumber \(\kappa_{SI}\) at which \(\tau_{I} = \tau_{S}\):

\[
\kappa_{SI} \approx \left(0.41\pi^{-1/3}\frac{\partial U_{1}}{\partial x_{2}}\right)^{3/2}.
\]  
(6.13)

The wavenumber at which \(\tau_{I}\) is approximately \(n\) times faster than \(\tau_{S}\) (i.e. \(n\tau_{I} \approx \tau_{S}\)) is then:

\[
\kappa_{SI}(n) \approx \left(0.41n\pi^{-1/3}\frac{\partial U_{1}}{\partial x_{2}}\right)^{3/2}.
\]  
(6.14)

The three-dimensional wavenumber \(\kappa_{SI}\) can be related to a one-dimensional analogue \(\kappa_{1SI} = \frac{3}{5}\kappa_{SI}\) for comparison to the present experiments [Corrsin, 1958]. Sadowdoughi & Veeravalli [1994] found that the local isotropy and axisymmetry were satisfied for the equalities in (6.10) for \(\kappa_{1SI}^{3/2}(\partial U_{1}/\partial x_{2})^{-3/2} \geq 3\), or equivalently \(n \gtrsim 10\). Values for \(\lambda_{1SI} = 2\pi/\kappa_{1SI}\) non-dimensionalised by \(\delta\) are shown in figure 6.4 for \(n = 10\) for the present measurements. In addition to the experimental data, classical scaling arguments can be used to approximate the dependence of (6.13)
on wall-distance. That is, the distance-from-the-wall scaling discussed in Chapter 2 yields $\partial U_1/\partial x_2 \sim x_2^{-1}$ and $\tau \sim x_2^{-1}$ in the log-layer. Plugging these relations into 6.13 yields $\kappa_{SI} \propto x_2^{-1}$, or equivalently $\lambda_{1_{SI}} \propto x_2$. The data in figure 6.4 indicate that $\lambda_{1_{SI}} \approx x_2/2$ in the log-layer for $n = 10$. The available DNS at lower $Re_\tau$ offers insight into the behaviour of $\lambda_{1_{SI}}$ in the wake region. The present measurements match the pipe and boundary layer DNS results in the region bounded by $0 < x_2/\delta < 0.7$, but above this location the mean shear becomes extremely small and therefore difficult to measure. The pipe DNS profile of El Khoury et al. [2013] shown in figure 6.4 indicates that $\lambda_{1_{SI}}(n = 10)$ drops below the $\lambda_1 = x_2/2$ line near $x_2/\delta \approx 0.4$, before crossing from below to above at $x_2/\delta \approx 0.8$ on its asymptotic approach to $\infty$ at the pipe centreline (due to the disappearance of the mean shear). In practice, the rapidly increasing values of $\lambda_{1_{SI}}(n = 10)$ should be clearly distinguishable from the $\lambda_1 = x_2/2$ guideline by $x_2/\delta \approx 0.9$, where the DNS results indicate that the two will deviate from one another by approximately a factor of 2. Since the effects of the shear are felt by the turbulence regardless of measurement capability, this asymptote should be visible in our spectral evaluation of the isotropic condition if the shear is indeed the leading order cause of anisotropy for such scales in this region.

It is worth noting that figure 6.4 and $\kappa_{1_{SI}}$ only describe the time scales of inertial energy transfer. Wavenumbers for which viscous dissipation is the dominant or co-dominant mechanism of energy transfer, however, have time scales that are shorter than those for which inertial effects are dominant [Corrsin, 1958]. Smaller wavenumbers for which the $-5/3$ power law is not an appropriate approximation are presumed to be in the range that is affected by the boundary conditions for the present flows, and thus the shear is not expected to be the primary cause of anisotropy for such scales.

The Taylor microscale Reynolds numbers of the present cases are shown in figure 6.5 to allow the reader to compare the ensuing observations with existing literature dealing with the subject of local isotropy in various other (non-wall-bounded) turbulent flows. Note that figure 6.5 shows both the anisotropic Taylor microscale Reynolds number $R_{\lambda_T} \equiv \lambda_T \left(\frac{2}{3}k\right)^{1/2}/\nu$ (open symbols) and the more frequently-encountered (at least in experimental studies) isotropic Taylor microscale Reynolds number $R_{\lambda_T} \equiv \lambda_{TI} \left(u_1^2\right)^{1/2}/\nu$, where $\lambda_{TI} \equiv \left(u_1^2/(\partial u_1/\partial x_1)^2\right)^{1/2}$. The pipe and boundary layer values of $R_{\lambda_T}$ (and of $R_{\lambda_{TI}}$) are effectively matched (considering the generally slow variation of turbulence properties with Reynolds number) at all wall-distances for matched $Re_\tau$ cases. In all cases (and at virtually, if not all wall-distances), $R_{\lambda_T}$ is sufficiently large for plausible Kolmogorov similarity [Kolmogorov [1941], Antonia et al. [2014]] and a clearly discernible sub-range between the energetic and dissipative scales.

Figure 6.6 shows the ratio of $E_{22}$ to $E_{33}$ at all wavelengths and wall-distances for the pipe and boundary layer high-$Re_\tau$ cases. Guidelines are also plotted corresponding to $\lambda_1 = x_2/2$ ( — ), $\lambda_1/\delta = 10$ ( — — — — ), and $x_2/\delta = 0.1$ ( — — — — — ) that divide the areas into four regions labeled with Roman numerals I–IV. Region I is
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**Figure 6.5:** Isotropic (filled symbols) and anisotropic (open symbols) Taylor microscale Reynolds numbers of all present pipe and boundary layer cases.

**Figure 6.6:** Spectral ratio of $E_{22}$ to $E_{33}$ versus outer-scaled wavelength and wall-distance for pipe and boundary layer high-$Re_\tau$ cases. 
- $\lambda_1 = \frac{1}{2} x_2$, 
- $\lambda_1/\delta = 10$, 
- $x_2/\delta = 0.1$. 
bounded above by the ‘crossover point’ wavelength ($\lambda_1/\delta = 10$, where the difference between pipe and boundary layer energy spectral density has a zero-crossing) and below by those wavelengths for which the effects of shear are expected to be minimal ($\lambda_1 \approx \frac{1}{2} x_2$). The isocontours of $E_{22}/E_{33}$ within region I vary linearly with wall-distance. To understand this, consider a particular streamwise wavelength $\lambda_1 = \lambda'_1$ within region I observed at a particular wall-distance $x_2 = x'_2$. The ratio of $u_2$-component energy density to $u_3$-component energy density has the following similarity property:

$$\frac{E_{22}(x'_2, \lambda'_1)}{E_{33}(x'_2, \lambda'_1)} = \frac{E_{22}(nx'_2, n\lambda'_1)}{E_{33}(nx'_2, n\lambda'_1)},$$

so long as the coordinates $(nx'_2, n\lambda'_1)$ also lie within region I. Such a property is consistent with distance from the wall scaling in multiple components. In the language of the attached eddy hypothesis of Townsend [1976], the velocity fields induced by two geometrically self-similar eddies with matched aspect ratio $\lambda_1/\lambda_2$ are also self-similar by construction; this property would also lead to the scaling exhibited in region I.

As discussed above, the distortion effects of the mean shear are expected to be insignificant on scales $\lambda_1 \lesssim \frac{1}{2} x_2$. Indeed, the values of $E_{22}/E_{33}$ are more-or-less constant and close to unity within region II (which is comprised exclusively of such scales).

Region III consists of wavelengths longer than those believed to be associated with LSMs. As discussed in section 6.2, this range contributes very little to either $u_2^2$ or $u_3^2$. The contours of $E_{22}/E_{33}$ in region III are more-or-less vertical, indicating that the content of $u_2$ versus $u_3$ energy for such large scales is a function only of wall-distance. There are two regions in the boundary layer case in which $E_{22} > E_{33}$. One such region corresponds to the outer flow boundary, where $u_3^2$ decays to zero faster than $u_2^2$. The highest excursions of turbulence in the ZPG boundary layer must be associated with positive $u_2$ in order to reach such heights; this is also evident in the large positive peak in the $u_3$ skewness shown in figure 4.3(a). More interesting dynamically is region IV, in which $E_{22} > E_{33}$ for both the boundary layer and pipe cases. Although only the high-$Re$ cases are shown here, $E_{22} > E_{33}$ within region IV in all of the available data. While the contributions to $u_2^2$ are very small in region IV, it was shown in figures 6.1 (g) and (h) that this portion of the spectrum is “stress-active”, or that it contributes appreciably to $\overline{u_1 u_2}$. Thus, while the transverse-component TKE contributed by wavelengths within region IV is essentially 0, some level of $E_{22}$ is maintained and is associated with significant contributions to the Reynolds shear stress.

1The actual ratio does not equal unity in this region due to an apparent slight sensitivity bias in the measurements. This bias is identified via the ratio of $u_2^2$ to $u_3^2$ extrapolated to the centreline of the pipe (for all three $Re$ cases). The difference from unity of this ratio indicates that the present measurement probe and/or data reduction scheme result in a $\sim 5\%$ greater sensitivity to $u_3$ than to $u_2$. This sensitivity is approximately constant across all scales based on the ratio of $E_{22}$ to $E_{33}$ at the centre-most measurement location in the pipe.
Figure 6.7: Spectrograms of $K_{ii}$ (as defined by (6.16)) for $i = 2$ and $i = 3$ pipe and boundary layer middle-$Re_\tau$ cases. Note that the repeated index does not imply summation. $\lambda_1 = \frac{1}{2} x_2$, $\lambda_1/\delta = 3$, $x_2/\delta = 0.1$.

Figure 6.7 shows the ‘anisotropy ratio’ $K_{ii}$, as defined by 6.16, for both $i = 2$ and $i = 3$.

$$K_{ii} = \frac{2E_{ii}}{E_{11} - \kappa_1 \frac{dE_{11}}{dx_1}}. \quad (6.16)$$

Note that the repeated index in $K_{ii}$ does not imply summation. Regions I–IV are bounded by the same guidelines as in figure 6.6. The grey shaded regions indicate where differentiation of the $E_{11}$ spectrum is untenable due to noise amplification. The $K_{22} = 1$ and $K_{33} = 1$ conditions for local isotropy are approximately satisfied in all cases in region II, where $E_{22}/E_{33} = 1$ is also satisfied. The departure from isotropic behaviour in region I highlights clear differences between $E_{22}$ and $E_{33}$. Region I in the $i = 2$ cases features isocontours that show 1 : 1 proportionality between $\lambda_1$ and $x_2$. This can be interpreted in a similar manner as was region I in figure 6.6. For a particular $\lambda_1 = \lambda'_1$ and $x_2 = x'_2$ within the domain corresponding
Figure 6.8: Anisotropy ratio $K_{22}$ for the $u_2$ component versus wavelength scaled by wall-distance for both pipe (red curves) and boundary layer (blue curves) high-$Re_\tau$ cases. Curves are truncated at long wavelengths at $\lambda_1/\delta = 10$ (upper bound of region II) and at short wavelengths at $\lambda_1/\eta = 2\pi$ (noise floor for this quantity).

To region I, the following similarity relationship applies (in a mean sense):

$$K_{22}(x'_2, \lambda'_1) = K_{22}(nx'_2, n\lambda'_1),$$

so long as the coordinates $(n\lambda'_1, nx'_2)$ are also within region I. Furthermore, the isocontours of $K_{22}$ are more-or-less equally spaced in region I of both flows, implying that the anisotropy ratio grows as a power law within region I. This is illustrated clearly in figure 6.8, which shows the anisotropy ratio associated with the $u_2$ component against streamwise wavelength non-dimensionalised by $x_2$ for both the pipe and boundary layer high-$Re_\tau$ cases. The curves shown represent all measured wall-positions for the pipe, and all for the boundary layer that satisfy $x_2/\delta \leq 0.9$ (those above this position suffer from low signal-to-noise ratios). All curves except those very close to the wall depart from the isotropic condition $K_{22} = 1$ at $\lambda_1/x_2 \approx 1$. It is unclear whether the near-wall drop in $K_{22}$ for small scales is physical, or an artifact of the measurement technique. For wavelengths $\lambda_1/x_2 > 1$, the anisotropy ratio $K_{22}$ decays as $\lambda_1^{-2/3}$. The physical significance of this particular relationship is presently unclear. The components of $K_{22}$ (i.e. $2E_{22}$, $E_{11}$, and $\kappa_1 \frac{\partial E_{11}}{\partial x_1}$) are shown individually in figure C6.1 in the appendix to this Chapter.

In contrast to the isocontours of $K_{22}$, those of $K_{33}$ are inclined as $x_2$ only near the boundary between regions I and II. In agreement with the findings of Saddoughi & Veeravalli [1994], $K_{33}$ remains close to unity up to slightly higher wavelengths than does $K_{22}$. The isocontours of $K_{33}$ flatten with increasing wavelength, becoming
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approximately horizontal at the boundary with region III. The values of \( E_{33} \) are depressed by an approximately constant (but finite) amount relative to an isotropic relationship to \( E_{11} \) in region III (\( K_{33} \approx 0.25 \)), while \( K_{22} \) essentially goes to 0 entirely. Moving from region III to region IV, \( E_{33} \) becomes even more attenuated relative to an isotropic relationship to \( E_{11} \), while the opposite is true for \( E_{22} \). This is consistent with the description of region IV in figure 6.6 and the maintenance of some finite level of \( E_{22} \) associated with very-large-scale contributions to the Reynolds shear stress.

The behaviour of \( K_{22} \) and \( K_{33} \) close to the centreline of the pipe warrants additional consideration. While axisymmetry must be satisfied across all scales at the centreline of the pipe, the same is not necessarily true of isotropy. Indeed, neither \( K_{22} \) nor \( K_{33} \) seem to approach global isotropy near the centreline for any of the present pipe cases. Instead, the range of scales satisfying local isotropy seems well-approximated by the \( \lambda_1 = x_2/2 \) guideline. If this trend extends to \( x_2/\delta = 1 \) (recall that the present measurements stop at \( x_2/\delta = 0.94 \) due to equipment limitations), then the distance to the impermeable wall would remain a successful indicator of departure from local isotropy, while the criterion based on mean shear would not. Available pipe DNS results from Chin et al. [2014] and El Khoury et al. [2013] indicate that \( u_2^2 = u_3^2 \approx \frac{2}{3} u_1^2 \) at the pipe centreline, and thus some portion of the spectrum must remain in violation of \( K_{22} = 1 \) and \( K_{33} = 1 \) at the centreline. Although it is tempting to extend this result and conclude that the impermeable wall is responsible everywhere for the departure from local isotropy/axisymmetry, it is worth noting that the isocontours of \( K_{22} \) and \( K_{33} \) both become slightly more horizontal with increasing wall distance for \( x_2/\delta \gtrsim 0.3 \). This is consistent with the flattening of \( \lambda_{1st} \) observed in the DNS curves shown in figure 6.4. Thus, both non-zero mean shear and the presence of an impermeable wall are plausible culprits for the observed departures from local isotropy/axisymmetry.

6.3 Vorticity

Spectrograms of the turbulence enstrophy \( W \) non-dimensionalised by wall scales are shown for the middle-\( Re_\tau \) pipe and boundary layer cases in figure 6.9. The wavelengths in figure 6.9 are non-dimensionalised by \( \delta \) to facilitate comparison with the plots in section 6.2. The colourmap and contour lines are spaced logarithmically (rather than linearly) to more clearly show the shapes of the spectrograms across the full flow domain. Figure 6.9 also shows the values for \( \Lambda_\omega \)—i.e. the wavelengths corresponding to the peak value of \( \kappa_1 W \) at each wall-height as defined by (6.8). The middle-\( Re_\tau \) datasets are highlighted in this section because they represent the cases for which the aerodynamic blockage compensation scheme was the most effective\(^2\). Still, the effects of aerodynamic blockage can be seen in figure 6.9.

\(^2\) Determination of the effectiveness of the compensation scheme is based on the observed correlation coefficients of the terms known to be coupled through the blockage effect. That is, the correlation coefficients of \( u_2 \) with \( \partial u_1/\partial x_2 \) and \( u_3 \) with \( \partial u_1/\partial x_3 \) (which are expected to be close to 0 in the wake based on available DNS evidence) are brought closest to 0 (of the three \( Re_\tau \) cases) in the wake when the blockage compensation scheme is applied.
Figure 6.9: Inner-scaled turbulence enstrophy spectrogram for boundary layer and pipe flow at nominally matched Reynolds number. Note that the contours are logarithmically spaced for clarity; each solid line contour differs from the next by a factor of 2. White lines and symbols indicate $\Lambda_{\omega}$ as defined by (6.8). Symbol shapes and fill colours as in Table 6.1.

(a), highlighted by the dotted-line box in the long-wavelength, outer-region portion of the spectrogram. This region represents the worst-case for the aerodynamic blockage effect, as it corresponds to the location of maximum $\kappa_1 E_{22}$ and $\kappa_1 E_{33}$ and very low turbulence enstrophy. Indeed, the values of $\kappa_1 W$ shrink dramatically in this region (relative to their initial values) for all present pipe and boundary layer cases when the aerodynamic blockage compensation scheme is applied. Aside from the region believed to have been influenced by the aerodynamic blockage effect, the pipe and boundary layer enstrophy spectrograms appear to be very similar to one another under inner normalisation. Alternative choices for non-dimensionalising variables are explored below, starting with the length used to scale the wavelengths at each wall-distance.

Values of $\Lambda_{\omega}$ are shown for all present pipe and boundary layer cases in figure 6.10. Also shown are the values of $\Lambda_{\omega/10}$, defined by (6.18) as the wavelength at which $\kappa_1 W$ is equal to 10% of its peak value (on the small-wavelength end of the spectrum):

$$\kappa_1 W(x_2 = x'_2, \lambda_1 = \Lambda_{\omega/10}(x'_2)) = \frac{1}{10} \max(\kappa_1 W(x_2 = x'_2, \lambda_1)) \quad (6.18)$$

The values of $\Lambda_{\omega}$ and $\Lambda_{\omega/10}$ are shown for the present experimental data and the DNS data of Sillero et al. [2013] in subfigures (a), (b), and (c) as respectively
Figure 6.10: Wavelengths corresponding to maximum- and 10%-of-maximum of enstrophy spectra normalised by viscous, Kolmogorov and Taylor length scales for both pipe and boundary layer cases. Symbols and colors as in Table 6.1. Solid symbols represent $\Lambda_\omega$ as defined by (6.8), open symbols represent $\Lambda_\omega/10$ as defined by (6.18). ---: Results computed from $Re_\tau \approx 2000$ DNS fields of Sillero et al. [2013], - - - : $\lambda_1/\eta = 45$, : $\lambda_1/\lambda_T \propto x_2^{1/4}$

non-dimensionalised by $\nu/u_\tau$, $\eta$, and $\lambda_T$. Outward of $x_2^+ \approx 100$, $\Lambda_\omega$ is offset from $\Lambda_\omega/10$ by a constant factor. Thus, a length scale suitable to describe the position of one is also suitable for the other. As shown in figure 6.10(a), $\Lambda_\omega$ initially decreases with wall-distance before beginning to increase at $x_2^+ \approx 100 - 200$. The initially longer wavelengths are reflective of the wall-imposed organisation of the turbulence that tends to result in streamwise-elongated motions (e.g. see the “near-wall cycle” model put forth by Waleffe [1997] and others). This streamwise-elongated organisation is lost as the vorticity field three-dimensionalises, and the $\lambda_1$-wavelengths of the vorticity become more directly reflective of the fine scales of the dissipation. At $x_2^+ \approx 100 - 200$, the length scales associated with the vorticity increase as $\tau$ drops (as a consequence of decreasing steepness of the fluctuating gradients that contribute to it). The apparent near-wall trend with Reynolds
number in the present data is more likely a spatial resolution trend, as the inability to fully represent the small scales is expected to move the observed peak to larger wavelengths. Indeed, compensating for spatial resolution via the spectral transfer function scheme described in 4 largely removes this trend (see figure D6.1 in the appendix to this chapter). The rate at which $\Lambda_\omega$ increases with wall-distance is most closely described by the rate at which the Kolmogorov length scale $\eta$ increases with wall-distance. The peak in $\kappa_1W$ is observed to occur at $\lambda_1/\eta \approx 45$ for the experimental data and $\lambda_1/\eta \approx 40$ for the DNS data for all wall-distances outward of $x_2^+ \approx 200$. Jiménez [2012] also shows that the peak in $\kappa_1W$ resides near $\lambda_1/\eta \approx 40$ for a channel-flow DNS at $Re_\tau \approx 2000$. Note that correction for spatial resolution via the spectral transfer function scheme (see figure D6.1) also moves $\Lambda_\omega$ close to 40 ($\kappa_1W$ is increased at small wavelengths more than at large wavelengths, thereby moving the peak to smaller wavelengths). Interestingly, the magnitude of $\Lambda_\omega$ is $O(\lambda_T)$ for the present $Re_\tau$ range. Non-dimensionalisation by the Taylor length scale reveals a $\Lambda_\omega/\lambda_T \propto x^{-1/4}$ downward trend through the log-layer followed by a ‘peel-up’ in the wake. If this trend extends to arbitrarily high Reynolds number, $\Lambda_\omega/\lambda_T$ can become arbitrarily small toward the outer edge of the log-layer.

Not only does the Kolmogorov length scale consistently describe the location of the peak (and that of 10% of the peak) in $\kappa_1W$ (for wall-distances $x_2^+ \gtrsim 200$), scaling $W$ by the Kolmogorov scales reveals that the enstrophy spectrum depends only on kinematic viscosity and dissipation rate to within the experimental uncertainty (again, for wall-distances $x_2^+ \gtrsim 200$). The spectrograms plotted in figure 6.9 are replotted in figure 6.11 against Kolmogorov-scaled wavelengths and with $W$ scaled by $u_\eta$ and $\eta$, where $u_\eta$ is the Kolmogorov velocity scale. These scales are determined here using the hybrid anisotropic dissipation estimate from (4.25) and (4.26). Simplification of the non-dimensionalising coefficients makes evident the rationalisation for scaling $W$ with Kolmogorov scales:

$$ (\kappa_1 \eta) \frac{\eta}{u_\eta^2} W = (\kappa_1 (\nu^{3/4} \varepsilon^{-1/4})) \frac{\nu^{3/4} \varepsilon^{-1/4}}{(\nu^{1/4} \varepsilon^{-1/4})^2} W = \kappa_1 \frac{\nu}{\varepsilon} W. \quad (6.19) $$

Although wall-bounded flows are not homogeneous, sufficiently far from the wall the following can be assumed:

$$ \varepsilon \approx \nu \omega_1 |\omega_1|. \quad (6.20) $$

Thus, to the degree that (6.20) holds:

$$ 2 \frac{\nu}{\varepsilon} \int_0^\infty W d\lambda_1 \approx 1, \quad (6.21) $$

and the area obtained by integrating through $\lambda_1$ in figure 6.11 is the effectively the same for all wall-distances (for $x_2^+ \gtrsim 200$). Although the scaling all but forces the area represented at each wall-distance to remain constant, and the peak locations have been shown to align under $\eta$-scaling, the shape(s) of the spectra are not influenced by any scaling. Thus, it is not by construction that the contours
shown in figure 6.11 are essentially horizontal for $x_2^+ \gtrsim 200$. Rather, the horizontal contours indicate that the distribution of enstrophy in wavenumber space is self-similar with respect to the Kolmogorov length scale at all wavelengths. The apparent (slight) departure from this self-similar shape in the outer region of the experimental data is (at least in large part) caused by the influence of the aerodynamic blockage effect. The increased values of $\kappa_1 W$ at large wavelengths in the outer region of the flow result in a higher value of $\overline{\tau}$ without increasing any values of $\kappa_1 W$ at smaller wavelengths. Thus, the relatively constant area represented at each wall-distance shifts weight from the actual peak to the blockage-influenced region at larger wavelengths. This effect is less noticeable in the pipe case because the actual (non-blockage-induced) enstrophy does not go to zero at $x_2/\delta = 1$. Furthermore, it seems unlikely that the smallest (unresolved) scales would cause the deviation in the outer region, since resolution of scales $O(10\eta)$ should only improve as wall-distance increases and $\eta$ becomes larger.

An alternative explanation for the horizontal contours in figure 6.11 is that only the density of resolved enstrophy scales with the Kolmogorov length scale. In theory, the unresolved scales could deviate from Kolmogorov scaling and yet the measured contours would still appear horizontal. Indeed, attenuation effects lead to higher measured values of $\eta$, which when used as a scaling parameter then balances the attenuation-induced shift in $W$ to larger scales. Although this effect does obfuscate the effects of spatial resolution to an extent, the self-similarity does not seem to be merely an artifact, as the Kolmogorov-scaled spectrogram computed from the highly-resolved DNS fields of Sillero et al. [2013] (see figure E6.1 in the appendix to this chapter) also features horizontal contours above $x_2^+ \gtrsim 200$ for both the unfiltered and synthetic experimental cases.

If the enstrophy spectrum indeed scales with the dissipation-based Kolmogorov scales for both the pipe and boundary layer cases for $x_2^+ \gtrsim 200$, then the ratio $W_B(\kappa_1 \eta)/W_P(\kappa_1 \eta)$ is constant and equal to $\overline{\tau}_B/\overline{\tau}_P$ at all wavelengths. Thus, the difference in outer-region enstrophy shown in figure 5.9, Chapter 5 is composed of contributions that are proportional to the value of $\kappa_1 W(\kappa_1 \eta)$ at all wavelengths, and is not localised to one particular range of wavelengths.

### 6.4 Conclusions

In this chapter, the spectral and cospectral content of the velocity components of pipe and boundary layer flow were examined and compared. In agreement with Monty et al. [2009], it was found that $\lambda_1/\delta \approx 10$ is the approximate ‘crossover’ point for $\kappa_1 E_{11}$ in the two flows; wavelengths $\lambda_1/\delta > 10$ contribute more heavily to $u_1^2$ in the pipe, and wavelengths $\lambda_1/\delta < 10$ contribute more heavily to $u_1^2$ in the boundary layer. The shift in relative contribution strength to the other components of Reynolds stress is also observed to occur at $\lambda_1/\delta \approx 10$. It is surmised that this shift, at least in part, results from a difference in the production associated with two prominent modes in the outer region of both flows. Contributions to the Reynolds shear stress (and thus the production of streamwise TKE) from
the domain bounded by $0.1 \lesssim x_2/\delta \lesssim 1$ and $1 \lesssim \lambda_1/\delta \lesssim 10$ are greater in the boundary layer, while those from the region bounded by $0.1 \lesssim x_2/\delta \lesssim 1$ and $10 \lesssim \lambda_1/\delta \lesssim 100$ are greater in the pipe. The former domain has been identified by a number of authors as reflective of the influence of so-called LSMs, while the latter domain is reflective of the influence of VLSMs [Kim & Adrian, 1999]. Using this terminology, VLSMs are associated with a region of greater relative production in the pipe, while LSMs are associated with a region of greater relative production in the ZPG boundary layer. The higher production from the LSM mode in the case is surmised to support (via inertial energy exchange and turbulence transport) higher values of $E_{11}$ than those observed in the pipe at other wall-distances and scales not associated with the LSM mode (but still below the $\lambda_1/\delta \approx 10$ ‘crossover’ point). Although $\overline{u_1^2}$ is not responsible for the ‘production’ of $\overline{u_2^2}$ or $\overline{u_3^2}$, it is the source term from which the two transverse components receive energy through the action of pressure-strain redistribution (e.g. equations (2.16)–(2.18)). Thus, in a similar fashion as the relationship between $\overline{u_1u_2}$ and $\overline{u_1^2}$, it is surmised that the higher levels of $E_{11}$ in the boundary layer case for $\lambda_1/\delta < 10$ support the observed higher levels of $E_{22}$ and $E_{33}$. The inverse is also true in that the pipe receives greater contributions than the boundary layer to $\overline{u_1u_2}$, $\overline{u_1^2}$, and $\overline{u_3^2}$ from wavelengths $\lambda_1/\delta > 10$.

The satisfaction of local axisymmetry and local isotropy in the energetic motions was also examined throughout the domains of both flows. Saddoughi & Veeravalli
[1994] proposed that the range of scales for which the velocity component spectra satisfy the conditions of isotropy and axisymmetry is limited to those for which the inertial time scale is approximately 10 times smaller than that of the mean shear (or, equivalently, \( \kappa_1(\pi/(\partial U_1/\partial x_2)^3)^{1/2} \approx 3 \)). Although this criterion closely describes the boundary of locally isotropic/axisymmetric behaviour in the present data, it is also very closely approximated by \( \lambda_1 = \frac{1}{2}x_2 \). The turbulence at the centreline of the pipe must become axisymmetric at all scales (and is observed to do so to within the capacity of the present experiments), but does not become isotropic at all scales despite the absence of mean shear. Indeed, the anisotropy ratios \( K_{22} \) and \( K_{33} \) depart from unity in the pipe at \( \lambda_1 \approx \frac{1}{2}x_2 \) at the centremost measurement location (i.e. \( x_2/\delta = 0.94 \)). The scales that exhibit isotropy and axisymmetry in the relationships between the energy spectra contribute to a significant fraction of the overall \( \overline{u_2^2} \), but fairly little to the overall \( \overline{u_1^2} \) or \( \overline{u_3^2} \) and not at all to the overall \( \overline{u_1 u_2} \) (by construction). The contributions to \( \overline{u_1^2} \), \( \overline{u_2^2} \), and \( \overline{u_3^2} \) are of course equal to one another across these scales by construction, but \( \overline{u_1^2} \) and \( \overline{u_3^2} \) are larger than \( \overline{u_2^2} \) across the majority of the flow domain. Interested readers may find the fraction of the overall component-wise TKE contributed by isotropic/axisymmetric scales in figure F6.1 in the appendix to this chapter.

Finally, viscous wall-scales, the Taylor microscale, and the Kolmogorov scales were evaluated for their capacity to determine the spectrum of fluctuating enstrophy. Outward of \( x_2^+ \approx 200 \), the peak in the premultiplied enstrophy spectrum is found to consistently reside at \( \lambda_1/\eta \approx 45 \). Attempting to correct for spatial resolution issues (as in figure D6.1 in the appendix to this chapter) moves the peak location to \( \lambda_1/\eta \approx 40 \) in agreement with the value quoted by Jiménez [2012] for channel flow at \( Re_\tau \approx 2000 \). For each of the present datasets, the peak in the premultiplied enstrophy spectrum is found to reside near \( \lambda_1/\lambda_T \approx 1.5 \). The exact location relative to \( \lambda_T \), however, moves to ever-decreasing values with wall-distance. If the observed trend continues to arbitrarily high Reynolds numbers, the peak in the premultiplied enstrophy spectrum will change relative to \( \lambda_T \) by one full decade (i.e. \( \lambda_1 \approx 0.15\lambda_T \)) at \( x_2/\delta \approx 0.3 \) for \( Re_\tau \approx 3 \times 10^7 \)—more than ten times higher than those typical of the atmospheric surface layer [Priyadarshana et al., 2007]. Furthermore, scaling the enstrophy density with the Kolmogorov length and velocity scales (or, equivalently, by the approximate area under the enstrophy spectrum at each wall-distance) reveals a self-similar distribution and magnitude of enstrophy with respect to the mean dissipation rate. Outward of \( x_2^+ \approx 200 \), the enstrophy spectrum \( W(\lambda_1/\eta) \) is approximately invariant when scaled by the local mean dissipation rate.
Appendix A

Figures A6.1 (a), (b), (c), and (d) show the same data as figures 6.1 (c), (d), (g) and (h), respectively, but with the streamwise wavelength $\lambda_1$ non-dimensionalised by wall-distance instead of by $\delta$. Note the occurrence of horizontal isocontours in the log-layer of all cases above the line indicating $\lambda_1/x_2 = 2$. The contours of $\kappa_1 E_{12}$ in the pipe case remain more-or-less horizontal in the domain where the bimodal ridges are observed in $\kappa_1 E_{11}$ (i.e. $x_2/\delta \approx 0.1$, $\lambda_1/x_2 \approx 50$).

![Figure A6.1: $E_{22}$ spectrograms and $E_{12}$ cospectrograms for pipe and boundary layer flow with wavelength scaled by wall-distance. Symbols are as in Table 6.1, and represent $\Lambda_{u_{22}}$ and $\Lambda_{u_{12}}$.](image-url)

$\lambda_1/x_2 \approx 2$. 

**Figure A6.1:** $E_{22}$ spectrograms and $E_{12}$ cospectrograms for pipe and boundary layer flow with wavelength scaled by wall-distance. Symbols are as in Table 6.1, and represent $\Lambda_{u_{22}}$ and $\Lambda_{u_{12}}$. 

$\lambda_1/x_2 = 2$. 

Appendix B

Figure B6.1 shows the peak magnitude of $\kappa_1^+E_{22}^+$ as a function of wall-distance for all present pipe and boundary layer cases. The domain bounded by $2.6\sqrt{\delta^+} < x_2^+ < 0.25\delta^+$ for each case is highlighted by the darker-shaded symbols. The pipe peaks decreases steadily across the highlighted domain, while the boundary layer flattens towards its outer edge. This is reflected in the $\overline{u_2^2}$ profiles shown in Chapter 5, where the boundary layer cases feature and outer ‘bump’ relative to the pipe cases.

Figure B6.1: Peak values of $\kappa_1^+E_{22}^+$ versus wall-distance for pipe and boundary layer cases. Darker shades correspond to profile points within the log-layer/lower wake-layer ($2.6\sqrt{\delta^+} < x_2^+ < 0.25\delta^+$).

Appendix C

Figure C6.1 shows the components of $K_{22}$ from the centre of the log-layer to illustrate the origin of the isotropic and $K_{22} \propto \lambda_1^{-2/3}$ behaviour observed in figure 6.8. $K_{22}$ achieves approximate unity (i.e. satisfies isotropy) at wavelengths shorter than $x_2/2$ (those to the left of the dashed line in C6.1) owing to a near-balance between $2E_{22}$ and $-\kappa_1\frac{\partial E_{22}}{\partial x_1}$, supplemented (particularly as $\lambda_1/x_2 \approx 0.5$ is approached) by $E_{11}$. The numerator of $K_{22}$ (i.e. $2E_{22}$) is approximately proportional to $\lambda_1^{1/3}$ ($\kappa_1E_{22} \propto \lambda_1^{-2/3}$), while the denominator (i.e. $E_{11} - \kappa_1\frac{\partial E_{22}}{\partial x_1}$) is approximately proportional to $\lambda_1$ (approximately constant when premultiplied by $\kappa_1$). Note that the apparent $E_{11} \propto \lambda_1$ region is an approximation intended to illustrate the origin of the (also approximate) $K_{22} \propto \lambda_1^{-2/3}$ behaviour, and should not be taken as evidence of the existence of the $E_{11} \propto \kappa_4^{-1}$ region predicted by the Perry et al. [1986] extension to the attached eddy hypothesis.
Appendix D

Figure D6.1 shows the values of $\Lambda_\omega$ and $\Lambda_\omega/10$ non-dimensionalised by the viscous, Kolmogorov, and Taylor length scales after applying the spectral correction for spatial resolution described in Chapter 4. Note that the Kolmogorov and Taylor lengths scales are also compensated for spatial resolution with the same scheme. The compensation scheme reduces the spread between the three $Re_\tau$ cases near the wall, indicating that this spread is indeed a function of resolution. Beyond the near-wall, the compensation moves $\Lambda_\omega$ closer to the DNS results (at least for the viscous and $\eta$ normalisations), but $\Lambda_\omega/10$ farther away. Evidently, the present correction scheme fails to completely capture the sensor behaviour at the smallest scales. The compensation factor at such wavelengths is of course much higher than near the peak, and consequently the results much more sensitive to potential errors.

Appendix E

Figure E6.1 shows the Kolmogorov-scaled enstrophy spectrogram computed from the DNS fields made available by the authors of Sillero et al. [2013]. The unfiltered DNS results are shown in (a), and the synthetic experimental results for the three present spatial resolution tiers in (b)–(d). The contours in the unfiltered DNS case are approximately horizontal beyond $x_2^+ \approx 100$, and are relatively unchanged by worsening spatial resolution. The contour levels and location relative to $\eta$ are also
Insensitive to worsening spatial resolution due to the self-compensatory effect of non-dimensionalisation by measured dissipation-based scales. Still, however, note the slight migration of $\Lambda_\omega$ from below-to-above the $\lambda_1/\eta = 45$ line (red) with decreasing resolution.

Appendix F

Figures F6.1 (a)–(c) show the TKE associated with the isotropic/axisymmetric range of scales for each velocity component at each wall-distance. Figure F6.1 (d)
Spectral Characteristics

shows the ratio of the isotropic TKE in each component to the total TKE in that component. For the purposes of this plot, the energy in the range of isotropic scales is computed as follows:

\[
\overline{u_{i,iso}^2}(x_2) = \int_{4\pi/x_2}^{\infty} E_{ii} d\kappa_1,
\]

(6.22)

(the repeated index does not imply summation) while \(\overline{u_{i,tot}^2}(x_2)\) is used to refer to the total TKE in the \(i\)'th component. Note that the line \(\lambda_1 = x_2/2\) is chosen as the upper wavelength bound (i.e. \(\kappa_1 = 4\pi/x_2\)). The fractions of each TKE component contributed by scales which satisfy the isotropic relations given by 6.10 are approximately the same for all \(Re_\tau\) cases and between the pipe and boundary layer (cf. figure F6.1 (d). The isotropic TKE for \(i = 2\) and \(i = 3\) is equivalent by

**Figure E6.1:** Dissipation-scaled turbulence enstrophy spectrograms computed from boundary layer DNS fields of Sillero et al. [2013]. (a): unfiltered DNS, (b)–(d): synthetic experimental results for three tiers of spatial resolution. White lines indicate \(\Lambda_\omega\) as defined by (6.8). --

\[\lambda_1/\eta = 45.\]
construction, but the \(i = 1\) isotropic TKE ranges from \(0.3\bar{u}_{2iso}^2(x_2) - 0.75\bar{u}_{2iso}^2(x_2)\) as the range of isotropic wavenumbers increases. The isotropic fraction of the \(u_1\) TKE is relatively insignificant throughout most of the flow domain, but peaks at approximately 20% of \(u_1^2\) at \(x_2/\delta \approx 1\). With \(\bar{u}_{2iso}^2(x_2)\) being equal to \(\bar{u}_{3iso}^2(x_2)\) but \(\bar{u}_2 \approx \bar{u}_3\) essentially everywhere, the isotropic fraction of \(u_2\) TKE is larger than the isotropic fraction of \(u_3\) TKE. Both of the transverse fractions reach a peak of about 40% of their component-wise TKE at \(x_2/\delta \approx 1\), as indicated by the black dashed line in figure F6.1 (d).

**Figure F6.1:** Isotropic/axisymmetric contribution to the TKE of each velocity component. Solid symbols represent \(\bar{u}_i^{+iso}\), open symbols represent \(\bar{u}_i^{+tot}\). Colors and symbol shapes as in Table 6.1. Black dashed line in (d) at \(\frac{\bar{u}_i^{+iso}}{\bar{u}_i^{+tot}} = 0.4\), the approximate value for \(i = 2\) and \(i = 3\) in the centreline of the pipe.
Chapter 7

Conclusions

7.1 Summary of principal results

This thesis has presented the results a two-stage research effort to (i) design and validate a sensor system capable of estimating the velocity and vorticity vectors about a common point with predictable accuracy, and (ii) juxtapose fully-developed turbulent pipe flow and zero-pressure-gradient boundary layer flow using data collected via the same sensor system. Although experimental measurements of (simultaneous) velocity and vorticity have been collected in turbulent boundary layers prior to this thesis, the Reynolds number range \( Re_\tau \approx 3300 \)–\( \approx 10000 \) and data density (25 wall-normal locations per profile) are unique to the present study. Furthermore, to the author’s knowledge, the present study is the first (excluding Chapter 5 of this thesis, which is comprised of a manuscript that is in revision following review as of writing) to present simultaneous time-resolved measurements of the full velocity and vorticity vectors in a fully-developed turbulent pipe flow. Even considering quantities (e.g. \( u_2 \) and \( u_3 \) statistics) for which experimental data are available for both boundary layer and pipe cases, the data presented herein are unique in that they were acquired via the same sensor system (and data reduction scheme), under matched Reynolds number and spatial resolution conditions. The aims of this study stated in Chapter 1 have been addressed in Chapters 3–6, and the key findings will be summarised here.

7.1.1 Sensor system

The 8-wire hotwire probe described in Chapter 3 is capable of estimating the full velocity and vorticity vectors with predictable accuracy based on its size relative to the viscous scales of the flow, but introduces significant errors into several velocity-vorticity correlations owing to its physical presence in the flow. The errors associated with the aerodynamic blockage of the probe may be addressed with the data reduction scheme, but their complete removal requires additional treatment.
Inclusion of velocity gradient information in the data reduction scheme yields significant improvements in the accuracy of \( \tilde{u}_3 \) for the present probe geometry (relative to a standard \( x \)-array reduction scheme). The improvements to \( u_2 \) by including velocity gradient information are, however, perhaps outweighed by additional computational cost. The differing level of success for \( u_2 \) and \( u_3 \) is owing to the distance over which the gradients are estimated relative to the individual sensor separation (the geometric parameter that controls susceptibility to gradient aliasing).

The ratio of ‘measured’ to ‘true’ velocity and velocity gradient spectra in the log-layer of a turbulent boundary layer is largely predictable from an assumption of isotropy and probe insensitivity to tangential and bi-normal cooling. The models derived by Wyngaard [1968] and Zhu & Antonia [1995] evaluated by assuming isotropy are less successful, however, in the presence of strong anisotropy due to high near-wall mean shear and external intermittency.

### 7.1.2 Pipe versus boundary layer flow

Pipe and boundary layer normalised statistical moments (up to kurtosis) of velocity and vorticity—with the exception of \( u_2^2 \) and \( u_3^2 \)—are indistinguishable (by the present measurement technique) over the domain spanning from the nearest-wall measurement locations \( (x_+^2 = \mathcal{O}(10)) \) up to \( x_2/\delta \approx 0.3 \). This suggests that these quantities are largely unaffected by the outer boundary condition below \( x_2/\delta \approx 0.3 \).

The turbulence kinetic energy, production, and dissipation are higher in the boundary layer than in the pipe near the outer edge of the log-layer/inner edge of the wake-layer. These differences are surmised to be (at least in part) reflective of the difference between the turbulent/non-turbulent interface (in the boundary layer) and the turbulent/quiescent-core interface (in the pipe). The boundary layer interface is characterised by higher mean vorticity (cf. Chauhan et al. [2014b], Kwon et al. [2014]), which is connected to strong negative pressure fluctuations [Jiménez et al., 2010], which then support increased inter-component exchange of \( u_1^2 \), \( u_2^2 \), and \( u_3^2 \) through pressure-strain redistribution.

Although the total \( u_1^2 \) in pipes and boundary layers is the same (to within experimental uncertainty), the pipe contains more \( u_1^2 \) than the boundary layer in streamwise wavelengths greater than \( 10\delta \), while the opposite is true for streamwise wavelengths shorter than \( 10\delta \). The \( 10\delta \) ‘crossover’ wavelength also appears in the spectra of \( u_2 \) and \( u_3 \), but since wavelengths greater than \( 10\delta \) contain very little \( u_2^2 \) and \( u_3^2 \), the net result is higher \( u_2^2 \) and \( u_3^2 \) in the boundary layer.

### 7.1.3 Properties of wall-flows

To within experimental uncertainty, the enstrophy spectrum is invariant with respect to the dissipative Kolmogorov scales above \( x_2^+ \approx 200 \). The wavelength of
the peak in the premultiplied enstrophy spectrum is $O(\lambda T)$ for the Reynolds number range considered herein, but the trend implied by the present data suggests that the wavelength at which the peak is observed will continue to shrink without bound (albeit extremely slowly) relative to the Taylor scale as Reynolds number is increased.

The velocity components in both pipes and boundary layers satisfy the conditions of local isotropy (to within experimental uncertainty) at streamwise wavelengths shorter than $x_2/2$ regardless of wall-distance (across the measured domain). The degree of departure from isotropy observed in the wall-normal velocity component spectrum depends on the ratio of streamwise wavelength to wall-distance for wavelengths bounded below by $x_2/2$ and above by $10\delta$. That is, the ratio of the observed energy density in the $u_2$ spectrum to that predicted by isotropy given either the $u_1$ or $u_3$ spectra is constant along lines of constant $\lambda_1/x_2$ (in the wavelength domain bounded below by $x_2/2$ and above by $10\delta$).

7.2 Suggestions for future work

7.2.1 Sensor improvements

As discussed in Chapter 1, one of the original motivations of the present research effort was to collect time-resolved measurements of the Lamb vector (i.e. $\varepsilon_{ijk}u_j\omega_k$) to investigate its role in Reynolds stress gradient generation. This goal was not pursued herein, however, as the measured streamwise component (i.e. $u_2\omega_3 - u_3\omega_2$) was found in Chapter 4 to contain significant errors associated with the physical presence of the probe in the flow. As these errors are not predictable via the synthetic experiment, the exact degree of contamination is difficult to discern. While the issue can be addressed to an extent via the data reduction scheme, it can also be mitigated through modifications to the probe design. One potential approach is to decrease the ‘solidity ratio’ of the support prongs, either by reducing the diameter of the prongs, increasing the spacing between the prongs, or some combination of both. These approaches are not, however, without drawbacks. Decreasing the prong diameter increases resistance per unit length, leading to more heat generation and subsequently increased sensitivity to cross-flow. Increasing the spacing between the prongs reduces spatial resolution. The author has observed that the prong heat generation issue is capable (for other attempted probe geometries) of inducing a directional sensitivity bias to the probe as a whole, with leeward sensors responding less intensely than windward sensors due to the thermal wakes of the windward prongs. Prong thermal wakes, however, may also be mitigated by material choice; tungsten is far less resistive than stainless steel (the present material), for example. Preliminary results collected by the author suggest that the use of tungsten prong material, while more practically challenging, does indeed mitigate the thermal prong effect.

As with nearly any turbulence measurement technique, improvements to spatial resolution are desired. In the course of their candidature, the author constructed
a version of the present 8-sensor probe featuring 2.5µm-diameter sensors instead of the 5µm-diameter sensors used herein—all dimensions of the 2.5µm-capable probe were therefore 1/2 those of the 5µm version. This construction, however, necessitates very close proximity of the supporting prongs, and consequently very small-diameter prong-tips. These features result in unacceptably high levels of thermal crosstalk as described above.

The recommended path forward for improved spatial resolution and decreased influence of aerodynamic blockage is therefore to (i) replace the stainless steel support prongs with thinner-diameter tungsten prongs, (ii) arrange the tungsten prongs so as to support 45°-inclined, 2.5µm-diameter sensor elements, (iii) determine the minimum sub-array separation such that the effects of aerodynamic blockage on the components of the Lamb vector are insignificant.

7.2.2 Further analysis

The analysis of the experimental data presented herein is by no means exhaustive. While the list of potential additional analysis can be made arbitrarily long, several areas strike the author as particularly promising. Ong & Wallace [1998] explore features of the correlation between $\omega_1$ and $\omega_2$. This correlation is reflective of the tendency of vortex lines in wall-bounded flows to form “horseshoe” or “hairpin” shapes [Theodorsen [1952, 1954], Head & Bandyopadhyay [1981]]. Indeed, preliminary results indicate that the present probe detects a positive correlation between $\omega_1$ and $\omega_2$ with a correlation coefficient approximately matching that predicted by the synthetic experiment. Time-records of $\omega_1\omega_2$ may be probed for information regarding, for example, the length scales associated with contributions to $\omega_1\omega_2$, the tendency (or lack thereof) of intense $\omega_1\omega_2$ to occur in bursts, and the orientation of the vorticity vector in space associated with contributions to $\omega_1\omega_2$.

Although not discussed in this thesis, the pipe and boundary layer data presented herein were collected simultaneously with wall-mounted hot-film sensors. The boundary layer data were acquired simultaneously with one hot-film sensor located on the bounding wall directly below the probe, while the pipe data were acquired simultaneously with 51 wall-mounted hot-film sensors covering the entire circumference of the pipe. This permits analysis pertaining to the degree of coherence between velocity and/or vorticity and the instantaneous wall-shear stress as a function of wall-distance.
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