DESIGN CONSIDERATIONS FOR LONGITUDINAL FORCES IN RAILWAY BRIDGES

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SUMMARY

This paper describes the longitudinal response of railway bridges when subjected to forces generated by the starting or braking of railway vehicles. By comparing experimental results and analytical results with various codes of practice from around the world it has been shown there are significant discrepancies between the codes and conclusions drawn from the analytical and experimental results. Further, the code currently used in Australia ("Australian and New Zealand Rail Conferences - Railway Bridge Design Manual. 1974"), appears to significantly underestimate the longitudinal force in certain commonly occurring circumstances.

KEYWORDS

Railway, Bridges, Longitudinal, Force, Codes

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The Public Transport Corporation (Victoria) provided time release for Mr C.F. Duffield to attend The University of Melbourne. This provision is gratefully acknowledged.
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INTRODUCTION

The use of more powerful locomotives capable of pulling heavier, longer trains has increased the longitudinal forces applied to a rail in service. Advanced braking systems are associated with these heavier locomotives and these maximise the adhesion between locomotive and rail. To cope with these increased loads trackwork requires better maintenance and alignment. Also, rails are now continuously welded and this has been shown to distribute live loads away from the point of loading and reduce dynamic effects. Detailed analytical and experimental studies to investigate both the magnitude of applied longitudinal forces and their distribution have been conducted in Europe and America. However, little correlation between this work exists, and the final conclusions of these studies vary substantially.

Design of railway bridges in Australia is currently carried out in accordance with the Australian and New Zealand Railway Bridge Design Manual (ANZRC, 1974). Both ANZRC and the American Railway Engineering Association's manual for Railway Engineering (AREA, 1990) permit high reductions in the design value of longitudinal forces to account for their distribution through the rail. This reduction far exceeds the reductions allowed in the European codes such as BS 5400, (refer Fig. 1) and the Office for Research and Experiments of the International Union of Railways (ORE).

Fig. 1 Code recommendations for longitudinal force, continuous rail

This paper presents a brief review of current analytical techniques used to predict longitudinal forces on railway bridges; some typical experimental results and some analytical predictions of longitudinal forces.

CODE APPROACHES

CURRENT CODE METHODS

ANZRC (1974), is based on AREA guideline's where longitudinal force (LF) transmitted at rail level is taken to be:

$$LF = 0.15 \times \text{vertical live load}$$
In cases where the rail is continuous over and beyond the bridge the longitudinal force is reduced to:

\[
LF = 0.15 \times (\text{live load}) \times \frac{\text{bridge length}}{365}
\]  

(1)

with the proviso that bridge length/365 does not exceed 0.8. The standard design live load is taken to be the M250 Cooper's load [refer ANZRC (1974)].

The Canadian design code (S29-1978) incorporates Equation 1 except that a heavier Cooper M357 live-load (E80) is recommended.

The British code (BS5400) uses a stepped limit bound approach, checking for both loadings of vehicles currently running or projected to run in Europe and for rapid transit vehicle systems. For bridges supporting ballasted track the design values for longitudinal force are reduced such that up to one third of the load is assumed to be transmitted beyond the bridge provided there are no rail discontinuities within 18 m of either end of the bridge.

ORE (1979) determined various values for the friction coefficient at the bridge bearing level where adhesion values of 0.3 for braking and 0.4 for acceleration (ORE 1971) were adopted. (Adhesion is defined as the ratio of applied horizontal to applied vertical loading).

Comparisons have been made (Duffield, 1989), between ANZRC (1974), BS5400 and ORE recommendations for bridges with lengths ranging up to 100 m, (refer Fig 1).

BACKGROUND TO EMPIRICAL CODE FORMULAE

The longitudinal force provisions of the major codes of practice are based on empirical formulae obtained from field trials (refer Arya et al 1982). Typically the longitudinal force formulae take the form of the Coulomb force-friction concept, ie \( LF = \mu N \), where \( \mu \) is the friction coefficient (adhesion) and \( N \) is the vertical live load.

Early code values for \( \mu \) were taken as 0.15 corresponding to values associated with steel to steel friction. AREA later made provision for starting forces to be associated with \( \mu \) of 0.25 and with \( \mu \) of 0.15 for braking. Only half this loading was applied if continuous rails were in place. In the current ANZRC provisions the distribution of longitudinal force on continuous rails was revised on the basis of field tests such as AREA (1955, 1961, 1964, 1966, 1967). Neither the ANZRC nor BS5400 account for the effects of dynamic loads. Moreover the ANZRC does not provide guidance with respect to the distribution of the longitudinal force for:

1. varied restraint conditions of the track beyond the bridge, eg partial fixity due to points or crossings,
2. the condition of the track. (This directly affects the longitudinal stiffness of the track),
3. the material properties of the bridge,
4. the stiffness of the bridge including bearings and foundations.

EXPERIMENTAL TESTING

To further understand the basis of the code formulae and the reasons for the differences between codes a comparison of the test results as presented by AREA (1966) and (1967) along with those of ORE (1969-1985) have been studied. These same experimental results have been used as the basis for comparison with analytical results later in this paper.
Results from these testing programs for bridges having a continuous rail (ie rail joints welded or fishplated), or non-continuous rail joints (ie joints broken by a moveable span, sliding rail expansion joints or other devices), are summarised in Tables I and II respectively. The ratio of longitudinal bearing reaction to applied vertical load is designated $\mu_B$. The percentage ratio of the averaged experimental bearing reactions to the applied longitudinal load are designated $X\%$.

Duffield (1989) provides comprehensive details of these structures, the testing programs and interpretation of the results.

**TABLE I**

**FIELD TEST RESULTS OF BRIDGES WITH CONTINUOUS RAILS**

<table>
<thead>
<tr>
<th>Bridge No</th>
<th>Type</th>
<th>Ballast</th>
<th>Bearing</th>
<th>Span</th>
<th>Max $\mu$</th>
<th>Max $\mu_B$</th>
<th>$X%$ Av $\frac{\mu_B}{\mu}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Steel</td>
<td>No</td>
<td>Steel</td>
<td>16.2</td>
<td>0.366</td>
<td>0.104</td>
<td>27.7</td>
</tr>
<tr>
<td>2</td>
<td>Steel</td>
<td>No</td>
<td>Steel</td>
<td>30.0</td>
<td>0.237</td>
<td>0.110</td>
<td>43.8</td>
</tr>
<tr>
<td>3</td>
<td>Steel</td>
<td>No</td>
<td>Steel</td>
<td>53.9</td>
<td>0.264</td>
<td>0.118</td>
<td>47.7</td>
</tr>
<tr>
<td>4</td>
<td>Steel*</td>
<td>Yes</td>
<td>Steel</td>
<td>30.0</td>
<td>0.350</td>
<td>0.118</td>
<td>54.4</td>
</tr>
<tr>
<td>5</td>
<td>Prestressed concrete</td>
<td>Yes</td>
<td>Neoprene</td>
<td>21.5</td>
<td>0.192</td>
<td>0.143</td>
<td>81.2</td>
</tr>
<tr>
<td>6</td>
<td>Prestressed concrete</td>
<td>Yes</td>
<td>Neotopf</td>
<td>15+15</td>
<td>0.387</td>
<td>0.113</td>
<td>57.4</td>
</tr>
<tr>
<td>7</td>
<td>Concrete</td>
<td>Yes</td>
<td>Tetron</td>
<td>34.5</td>
<td>0.290</td>
<td>0.188</td>
<td>64.9</td>
</tr>
<tr>
<td>8</td>
<td>Concrete</td>
<td>Yes</td>
<td>Elastomeric</td>
<td>201.1</td>
<td>0.102</td>
<td>0.05</td>
<td>49.0</td>
</tr>
</tbody>
</table>

* Partially continuous

**TABLE II**

**FIELD TEST RESULTS NON-CONTINUOUS JOINTS**

<table>
<thead>
<tr>
<th>Bridge No</th>
<th>Type</th>
<th>Ballast</th>
<th>Bearing</th>
<th>Span</th>
<th>Max $\mu$</th>
<th>Max $\mu_B$</th>
<th>$X%$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Steel</td>
<td>No</td>
<td>Steel</td>
<td>16.2</td>
<td>0.438</td>
<td>0.258</td>
<td>59.7</td>
</tr>
<tr>
<td>2</td>
<td>Steel</td>
<td>No</td>
<td>Steel</td>
<td>30.0</td>
<td>0.246</td>
<td>0.175</td>
<td>70.0</td>
</tr>
<tr>
<td>3</td>
<td>Steel</td>
<td>No</td>
<td>Steel</td>
<td>53.9</td>
<td>0.264</td>
<td>0.256</td>
<td>97.2</td>
</tr>
<tr>
<td>4</td>
<td>Steel*</td>
<td>Yes</td>
<td>Steel</td>
<td>30.0</td>
<td>0.350</td>
<td>0.195</td>
<td>54.4</td>
</tr>
<tr>
<td>5</td>
<td>Prestressed concrete</td>
<td>Yes</td>
<td>Neoprene</td>
<td>21.5</td>
<td>0.161</td>
<td>0.084</td>
<td>61.9</td>
</tr>
<tr>
<td>6</td>
<td>Prestressed concrete</td>
<td>Yes</td>
<td>Neotopf</td>
<td>15+15</td>
<td>0.132</td>
<td>0.132</td>
<td>100.</td>
</tr>
<tr>
<td>7</td>
<td>Concrete</td>
<td>Yes</td>
<td>Tetron</td>
<td>34.5</td>
<td>0.214</td>
<td>0.193</td>
<td>90.3</td>
</tr>
<tr>
<td>8</td>
<td>Prestressed Concrete</td>
<td>Yes</td>
<td>Elastomeric</td>
<td>201.1</td>
<td>0.094</td>
<td>0.058</td>
<td>61.7</td>
</tr>
</tbody>
</table>

* Partially continuous

Testing of bridge number eight was reported in AREA (1967) and testing of all other bridges in Table I was reported in ORE (1969-85). Note that for bridge number five, in Table I, testing was conducted for two differing bearing type.
ANALYTICAL TECHNIQUES

The problem of estimating the longitudinal forces exerted on railway structures was first considered by Willis (1849) and Stokes (1867). Extensive linear elastic studies have subsequently been undertaken to assist in the prediction of longitudinal forces (e.g. Inglis (1951), Siekmeier (1965), Eisses (1975), Fryba (1975), Chu and Lee (1980) and Arya and Agrawal (1982)). In particular, the works of Siekmeier and Fryba have provided well recognised techniques. Much of Fryba's work was conducted under the auspices of an ORE committee set up to investigate braking and acceleration forces on bridges (1966 to 1985).

EXISTING ELASTIC THEORETICAL METHODS

Siekmeier's linear elastic model

Siekmeier (1965) used linear elastic analytical techniques and concluded that track resistance (W), is a function of frictional force (R), and the product of displacement (u) and the elastic restoring force (k).

\[ W = R + ku \]  

(2)

The solution assumes the rail to be a bar of infinite length.

If discrete bar lengths are chosen and these bars are fixed to a bridge deck, the longitudinal force transmitted to the bridge, based on Siekmeier's analysis (Duffield, 1989) can be determined from the spring reaction forces.

Fryba's Quasi-static distribution

Fryba (1974) assumed that longitudinal stresses in bridge beams and rails act separately and independently of corresponding bending stresses. Both the beam and rail were modelled as bars with the connection between them taken as an elastic layer. Fryba's model is presented in Figure 2. A continuous rail is modelled by predetermined discrete lengths of track on either side of the bridge. The longitudinal displacements \( u_j(x) \) of the bar are assumed proportional to the product of the load and the track stiffness coefficient \( k_j \), i.e. \( u_j(x) = k_j \times \text{load} \).

![Diagram of Quasi-static model](image)

**FIG. 2 QUASI STATIC MODEL (AFTER FRYBA 1974)**

Analyses were conducted (Duffield 1989) to compare Siekmeier's and Fryba's methods for bridges varying in length up to 150 m. An input live load of M250 Coopers configuration (ANZRC 1974) was adopted with the analysed length of rail each side of the bridge being taken as 20 m. The live load was stepped across the model in 0.1 m increments to determine the peak value responses and the results are presented in Figure 3.
It should be noted that these results are based on a constant value of EA (5.0 E7 kN) and this value corresponds to the properties of bridge number two (30 m steel bridge, ORE, 1971). The results presented in Figure 3 clearly indicate a large divergence in the predicted results for bearing force reactions for bridges with spans greater than about 20 m in length.

![Graph showing predicted bearing force reactions based on Siekmeier (1965) and Fryba (1975)](image)

**FIG. 3 PREDICTED BEARING FORCE REACTIONS BASED ON SIEKMEIER (1965) AND FRYBA (1975)**

**ANALYTICAL MODELS**

Modelling of bridges where the rail is discontinuous (or free) at the end of the bridge has been undertaken using a simple beam (Case 1) and for a continuous rail a detailed finite element model has been considered (Case 2). These two cases and the input loading are considered in the following sections.

**VEHICLE MODEL**

Train axle loadings have been modelled as either a disc moving across a beam or as a series of moving forces. In the former case, the inertia effects of the loading are included. As the purpose of the model is to study longitudinal forces from vehicle braking and acceleration, adhesion effects are included.

The vertical input load \( P(t) \) for track fixed to timber sleepers has been be taken as:

\[
P(t) = \sin \omega t,
\]

where \( \omega \) = wavelength of input, \( t \) = time

For the vertical load non-dynamic inputs \( P(t) = 1.0 \) were used and the horizontal force \( (R(t)) \) is limited in magnitude by Coulomb's law of friction.

\[
|R(t)| < f(t) P(t)
\]

where \( f(t) \) is usually a function of the speed of the load. If the wheel is sliding (ie \( R(t) \geq f(t) P(t) \)):

\[
R(t) = \pm \bar{u}_0(t) f(t) P(t)
\]
where $f(t) = \text{coefficient of friction}$

Another major factor influencing the horizontal force is the relationship between the braking system of the vehicle, its mass, the wheel diameter and the horizontal velocity of the loading.

**BEAM MODEL, Case 1**

This model is based on the Bernoulli-Euler beam (refer Figure 4) with viscous damping and the solution uses the modal analysis method, where the equations of motion are formed by direct equilibration of all forces acting. A Fourier sine finite integral transformation was used to assist in the simplification of the equations and these were subsequently solved using numerical integration techniques (Duffield 1989).

![FIG 4 BERNONLI-EULER BEAM](image)

**BEAM MODEL Case 2**

To assess the longitudinal dynamic response of bridges, having continuous rail laid on ballast, bridge number two which is a 30 m single span steel structure has been investigated. A finite element model was developed and a general purpose finite element package was used for the analysis. The loads applied corresponded to measured loads from the testing programs, (eg ORE 1973) and were applied to simulate moving forces decelerating at 3 m/s².

The geometric details of the model are shown in Figure 5. The bridge and rail are modelled as two dimensional beams in a plane and the increment length of these beams has been taken as 1.0 m which involves some 366 degrees of freedom. The rail has been modelled for 30 m each side of the bridge and is connected to the ground by linear springs acting in both the vertical and horizontal directions. Similar springs have been used to connect the rail to the bridge structure. The bridge beams are supported by a pin support and roller bearing.

![FIG 5FINITE ELEMENT MODEL](image)
The model was created with dashpots at every spring location. Initially these were set to zero, and hence the springs representing the track were undamped. The spring stiffness of the track was taken to be 10 kN/m both vertically and horizontally over the full length of the rail. This spring stiffness corresponds with track in good condition (Chu 1980).

RESULTS

The experimental results for the bridges having non-continuous rail joints at each end of the bridge have been used for comparison with the simply supported beam model (i.e. Beam model case 1, refer Figures 6 and 7). Corresponding code results for bridges carrying this type of jointed track are also included.

In these analyses the ratio of the applied horizontal to applied vertical loading (adhesion, \( \mu \)) of 0.3 was applied to both the 30 m and 16 m long bridges and the computed longitudinal forces were found to be greater than those measured (Figures 6 and 7). The measured non-dimensional force at the bearing (\( \mu_B \)) of the 16 m bridge was 0.258 corresponding to a maximum applied input loading of \( \mu_{max} = 0.438 \). The results for bridge number two correspond to an applied \( \mu_{max} \) of 0.246, whilst in the partially non-continuous rail jointed bridge number four \( \mu_{max} = 0.35 \). For bridge number six (30 m length) \( \mu_{max} \) equalled 0.132. In bridges two and five the rail to rail joint at the end of the bridges was partially restrained hence the simply supported structure model does not truly represent these bridges. However, the results do confirm that the use of \( \mu = 0.3 \) for braking situations is quite reasonable, even though the tested bridges were specially prepared with sand to produce maximum adhesion (i.e. an upper bound condition). The computed results generally ranged up to 0.34 with isolated higher results, indicating they are of reasonable magnitude.

**FIG 6 AXIAL FORCE, NON-CONTINUOUS RAIL JOINTS, 30 M SPAN BRIDGE**

When compared with the code results the value assumed for adhesion becomes critical. In braking the ORE use \( \mu = 0.3 \), BS5400 use \( \mu = 0.25 \) and ANZRC use \( \mu = 0.15 \). The varying initial adhesion values are reflected in the correlation with the computed values.

From the limited test results, shown as \( \mu_B \) on Figures 6 and 7, it is evident that the ORE recommendations are conservative for the 30 m and 16 m long bridges tested. Also, the ANZRC recommendation appears non-conservative. The recommendations of BS 5400 reflect the test results quite well.

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Analytical results for bridge number 2 are presented in Table III. The results are for a vertical live load of 2064 kN, with adhesion of 0.3 travelling along a corrugated rail having a corrugation wavelength (λ) of 48 mm.

These analytical results compare well with the experimental results (Figures 8 to 11). This indicates that the analysis of a detailed finite element model such as shown in Figure 5 provides a satisfactory method for predicting the longitudinal forces transmitted.

Figures 8 to 11 compare the results using the methods proposed by Siekmeier, Fryba and the various codes with test results.

### TABLE III

**HORIZONTAL REACTION FOR CORRUGATED LOADING, λ = 0.048 MM, TOTAL VERTICAL LOAD OF 2064 KN**

<table>
<thead>
<tr>
<th>Track damping (kN/s/m)</th>
<th>Response on load stopping (kN)</th>
<th>μB%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nil</td>
<td>-294.5</td>
<td>14.3</td>
</tr>
<tr>
<td>50.0</td>
<td>-316.6</td>
<td>15.3</td>
</tr>
<tr>
<td>75.0</td>
<td>-320.6</td>
<td>15.5</td>
</tr>
</tbody>
</table>

Figure 10 indicates the analytic technique presented by Siekmeier compares reasonably well with the test results over a limited range, being best for spans of the order of 35 m. The results from Fryba are most consistent where the load is totally on the structure and does not continue to the approach embankments. The code results all appear to be reasonable for bridges of the order of 100 m length however, the small number of tests conducted mean that conclusions should be substantiated by further testing.

![Figure 8](image1.png) **CONTINUOUS RAIL FRYBA VS EXPERIMENTS**

![Figure 9](image2.png) **CONTINUOUS RAIL, LOAD ONLY OVER BRIDGE, FRYBA VS EXPERIMENTS**
SUMMARY AND CONCLUSIONS

In summary the computed results compare well with the experimental results when the effect of partial fixity of the rail is taken into account. Further the ORE and BS 5400 recommendations match the results well except for the very short span bridges.

In the range 30 to 70 m the predictions of BS5400 and ORE seem reasonable except that they are some 25% low for structures of approximately 50 m span. The prediction of the ANZRC is shown to be unsatisfactory in all cases other than for long length structures.

REFERENCES


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