CONTRIBUTION OF DISPERSIVE STRESS TO SKIN FRICTION DRAG IN TURBULENT FLOW OVER RIBLETS

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ABSTRACT

We carry out direct numerical simulations (DNSs) of minimal open-channel flow over riblets, which are streamwise-aligned grooves that modify the near-wall flow for drag reduction. Several riblet sizes and cross-sectional geometries are simulated, namely symmetric triangular, asymmetric triangular, blade and trapezoidal. With this unprecedented breadth and detail afforded by the DNS data, we are able to obtain more general insights into the flow physics of riblets. A generalization of the Fukagata–Iwamoto–Kasagi (FIK) identity is used to isolate the different contributions to skin friction drag changes. We show that, in the nonlinear regime of large riblet size, the dispersive contribution is comparable or larger than the turbulent one, representing an important mechanism to the breakdown of drag reduction.

INTRODUCTION

A large proportion of the energy required in transportation systems and pipe systems is used to overcome fluid-dynamic drag. In particular, skin friction constitutes 50% of the total drag on aircraft. Hence reduction in skin friction can bring a substantial energy saving. Riblets are streamwise-aligned micro-grooves that modify skin friction drag by shifting turbulence away from the wall, relative to the flat wall, an idea which can be described by protrusion heights (Luchini et al., 1991). Despite the additional insight provided by previous experimental and numerical studies, two fundamental aspects of the flow physics of riblets need further investigation: (1) the contribution of the streamwise flow, an idea which can be described by protrusion heights; and (2) the viscous-scaled square root of the groove area $\ell_g^+$, experimental data of triangles $\alpha = 60^\circ$; triangles $\alpha = 30^\circ$; blades $s/t = 5$; asymmetric triangles $\alpha = 63.4^\circ$; experimental data of triangles $\alpha = 60^\circ$ from Bechert et al. (1997); DNS data for blades $s/t = 4$ from García-Mayoral & Jiménez (2011).

Reynolds number. Figure 1 shows $\Delta U^+$ as a function of the viscous-scaled square root of the groove area $\ell_g^+ \equiv \ell_g/\delta_t$, where $\ell_g = \sqrt{\mathcal{A}_g}$. Symbols represent different flow cases: $\Delta_1$, triangles $\alpha = 30^\circ$; $\Delta_2$, triangles $\alpha = 60^\circ$; $\Delta_3$, triangles $\alpha = 90^\circ$; $\Delta_4$, trapezoids $\alpha = 30^\circ$; blades $s/t = 5$; $\Delta_6$, asymmetric triangles $\alpha = 63.4^\circ$; $\n$, experimental data of triangles $\alpha = 60^\circ$ from Bechert et al. (1997); $\bullet$, DNS data for blades $s/t = 4$ from García-Mayoral & Jiménez (2011).
Riblets are not yet fully understood: i) the effect of the groove shape and ii) the breakdown of drag reduction. On the first point most parametric studies involving the effect of the geometry have been carried out through experiments (Bechert et al., 1997), which do not provide access to the three-dimensional flow field, whereas DNS studies of riblets are limited to triangular and blade geometries (Goldstein & Tuan, 1998; García-Mayoral & Jiménez, 2011). On the second point, different mechanisms have been proposed for the breakdown in drag reduction, but often these are based on observations from distinct riblet geometries. For example Goldstein & Tuan (1998) studied the turbulent secondary flows, or turbulent dispersion using DNS of triangular riblets and attributed it to the breakdown of the linear regime, arguing that the loss of performance occurring for large riblets ($\ell^+ \geq 20$) could be tackled by disrupting these secondary velocities. Another mechanism has been proposed by García-Mayoral & Jiménez (2011), who carried out DNS of blade riblets and observed that large grooves trigger the onset of spanwise-coherent vortical structures, which are visible in the 2D pre-multiplied velocity spectrum right above the riblet crest, and are similar to Kelvin–Helmholtz rollers which represent an additional contribution to the Reynolds stress. Therefore, what is needed is a systematic study across a broad parameter space to assess the validity and generality of the various proposed mechanisms. To this end, we carry out DNS of minimal open-channel flow over many riblet sizes and geometries, namely triangular, trapezoidal and blade riblets, to understand if a common mechanism leading to the breakdown of drag reduction exists.

METHODOLOGY

We solve the incompressible Navier–Stokes equations with a uniform and constant kinematic driving pressure gradient $\Pi > 0$. The equations are discretized using an unstructured finite volume solver CTI Cliff (Ham et al., 2006), and the computational domain is a minimal open channel (Chung et al., 2015; MacDonald et al., 2017) with dimensions $L_x \times L_y \times \delta$. The minimal channel allows us to extract $\Delta U^+$ at an affordable computational cost while resolving the flow–riblet interaction below the critical height $z_c \approx 0.4L_y$. The minimal channel flow results match the ones of the full channel, as long as $z_c$ is above the roughness sublayer, which is verified for all cases presented here. Simulations are set up to have the same volume and the open-channel height $\delta$, defined as the distance from the top flat surface to the mean height of the riblets. The mean wall-shear stress $\tau_w$ is thus fixed for the same pressure gradient, $\tau_w = \rho\delta\delta$$''$ and the friction Reynolds number is also fixed, $Re_\tau = \delta/\delta$ = 395.

The streamwise, spanwise and wall-normal directions are denoted by $x, y$ and $z$, respectively, and the velocity components in the corresponding directions are $u, v$ and $w$. Ensemble averages (averages in time, streamwise direction and riblets period) are indicated by the overline symbol, $\overline{\theta}(y, z)$ with $y \in [0, s]$, whereas plane averages (averages in streamwise, spanwise direction and in time) are indicated as $\langle \theta \rangle(z)$. Turbulent fluctuation are defined with respect to ensemble averages, $\theta^' = \theta - \overline{\theta}$. Variables normalized with respect to wall units ($\delta_s, u_\tau$) are denoted with a superscript $\ast$. No-slip boundary conditions are imposed at the bottom riblet wall, whereas a free-slip impermeable boundary condition is imposed at the top boundary and periodicity is imposed in the streamwise and spanwise direction. We consider four riblet geometries, both in the drag decreasing and increasing regime, and compare them to a flat wall case. Different riblet geometries are indicated as T,$''$, AT,$''$, BL,$''$, L,$''$, and T,$''$, for symmetric triangle, asymmetric triangle, blade and trapezoid, respectively, where $s$$''$ = $s/\delta$, the viscous-scaled riblet spacing (table 1).

CONTRIBUTIONS TO THE CHANGE IN DRAG

Riblets operating in the nonlinear regime show the presence of non-zero mean cross-stream velocities $\overrightarrow{\tau}$, $\overrightarrow{\pi}$ which carry additional stress. In order to understand the relation between dispersive velocities and drag change we report $\overrightarrow{\tau}$ and $\overrightarrow{\pi}$ in the cross-stream plane in figures 2 and 3, respectively. We note that the intensity of $\overrightarrow{\tau}$ and $\overrightarrow{\pi}$ increases with the riblet size, suggesting a correlation between dispersive velocities and increasing drag ($\checkmark$ indicates drag reducing cases, $\times$ drag increasing cases). The mean wall-normal velocity component $\overrightarrow{\pi}$ (figure 2) is spatially organized into three lobes, two positive ones towards the riblet crests and a negative one at the center of the groove. On the other hand, the mean spanwise velocity component $\overrightarrow{\tau}$ (figure 3) is constituted by four lobes with alternating sign, which together with $\overrightarrow{\pi}$ form two counter rotating vortices lying in the riblet groove, as shown from the stream function $\psi = \psi/\nu$, figure 4. Note that large asymmetric riblets show a small, but non-zero, net $\overrightarrow{\tau} \approx 0.1$, but this does not seem to affect the drag curve (figure 1). In order to quantify the contribution of the dispersive velocities to the breakdown of drag reduction we consider the streamwise mean momentum equation in divergence form, 

$$-\Pi + \nabla \cdot \tau_{\parallel} + \nabla \cdot \tau_{\perp} = \nu \nabla^2 \pi,$$

where $\tau_{\parallel} = (\pi, \pi, \pi)$ and $\tau_{\perp} = (u\overrightarrow{\tau}, u\overrightarrow{\tau}, w\overrightarrow{\tau})$ are the disperse and turbulent contributions to the mean momentum balance equation. In order to quantify the relative contribution of the terms in (1) to $\Delta U^+$ we use the approach of Modesti et al. (2018) who generalized the Fukagata–Iwamoto–Kasagi (FIK) identity (Fukagata et al., 2002) to arbitrary complex geometries. Indeed, the mean momentum balance equation (1) can be interpreted as a Poisson equation for the mean velocity, in which the terms at the left hand side are the source terms from DNS, representing the laminar, dispersive and turbulent contributions to the mean velocity.

Therefore, the associated velocity fields $\pi_\parallel, \pi_\perp$ and $\pi_\parallel$, induced by dispersion, turbulence and pressure gradient, respectively, can be obtained as solutions of three separate Poisson problems,

$$\nabla \cdot \tau_{\parallel} = \nu \nabla^2 \pi_\parallel, \quad \nabla \cdot \tau_{\perp} = \nu \nabla^2 \pi_\perp, \quad -\Pi = \nu \nabla^2 \pi_\parallel,$$

where homogeneous Dirichlet and zero gradient boundary conditions are used at the wall and at the top boundary, respectively and periodicity is used in the spanwise direction. Note that the first two of (2) satisfy Green’s identity and by construction $\pi_\parallel$ and $\pi_\perp$ do not contribute to the mean wall-shear stress. The total mean velocity field can be recovered by summing the three contributions,

$$\pi = \pi_\parallel + \pi_\perp + \pi_\parallel.$$
Figure 2. Mean wall-normal velocity component $\overline{w}$ in the cross-stream plane. $z_v$ indicates the riblet valley. The green tick (✓) indicates drag decreasing cases, the red cross (X) drag increasing cases. Positive values (red) indicate flow going upward.

Figure 3. Mean spanwise velocity component $\overline{v}$ in the cross-stream plane. $z_v$ indicates the riblet valley. The green tick (✓) indicates drag decreasing cases, the red cross (X) drag increasing cases. Positive values (red) indicate flow going rightward.

Figure 4. Cross-flow stream function $\Psi^+$, $\tau = -\partial \Psi/\partial z$, $\overline{\tau} = -\partial \Psi/\partial y$. $z_v$ indicates the riblet valley and $s$ the riblet spacing. The green tick (✓) indicates drag decreasing cases, the red cross (X) drag increasing cases.
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Drag) from (8); turbulence \( \Delta U^+ \), dispersion \( \Delta U_D^+ \), slip \( \Delta U_s^+ \), for (a) triangles \( \alpha = 30^\circ \), (b) triangles \( \alpha = 60^\circ \), (c) triangles \( \alpha = 90^\circ \), (d) asymmetric triangles \( \alpha = 63.4^\circ \), (e) trapezoids \( \alpha = 30^\circ \) and (f) blades \( s/t = 5 \).

Table 1. DNS cases of minimal open-channel flow. \( s^+ = s/\delta_s \) and \( \ell_g^+ = \ell_g/\delta_s \), the viscous-scaled riblet spacing and square root of the groove area, \( \ell_g = \sqrt{A_g} \). \( \Delta c^+ \) is the viscous-scaled mesh spacing in the streamwise direction and \( \Delta y^+_{\min} - \Delta y^+_{\max} \) and \( \Delta z^+_{\min} - \Delta z^+_{\max} \) are the minimum-to-maximum range of mesh spacings in the spanwise and wall-normal directions, respectively. \( L_g^+ \) and \( L_s^+ \) are the viscous-scaled dimensions of the computational domain in the streamwise and spanwise direction, respectively. \( L_g^+ = 2054 \) for trapezoidal riblets and \( L_s^+ = 1027 \) for all other cases, whereas \( L_s^+ \approx 250 \) for all cases. \( T \) is the time averaging interval. \( R = k/s^+ \) is the riblet equivalent aspect ratio and \( s^* \) the groove spacing, \( s^* = s \) for triangles and trapezoid \( s^* = s - t \) for blades.
due to the linearity of the Laplacian operator. Further averaging equation (3) in the spanwise direction we obtain,

\[ \langle u \rangle = \langle u_f \rangle + \langle u_T \rangle + \langle u_D \rangle. \tag{4} \]

A similar reasoning can be applied to the flat wall mean velocity profile,

\[ \langle u_f \rangle = \langle u_{f,f} \rangle + \langle u_{f,T} \rangle, \tag{5} \]

for which the dispersive component is identically zero. In order to study the contributions to the drag change \( DR \) we use the mean velocity shift with respect to the flat wall \( \Delta U^+ = \langle u_f \rangle - \langle \hat{u} \rangle \). \( \hat{z} \), measured at the critical height \( z^* \) in the log-layer, as \( DR \sim \Delta U^+ \). The riblet mean velocity profile is evaluated taking into account the effective virtual origin of turbulence \( \ell^*_T \) measured downward from the riblet crest. \( \ell^*_T \) is evaluated as the shift of the plane averaged turbulent stress of the riblet \( (\hat{u} \hat{w})^+ \) with respect to the turbulent stress of the flat wall. Hence, subtracting equation (4) from (5),

\[ \Delta U^+ = \Delta U_{f,f}^+ + \Delta U_{f,T}^+ + \Delta U_{f,D}^+, \tag{6} \]

where \( \Delta U_{f,f}^+ = \langle u_{f,f} \rangle - \langle \hat{u} \rangle \), \( \Delta U_{f,T}^+ = \langle u_{f,T} \rangle - \langle \hat{u} \rangle \), \( \Delta U_{f,D}^+ = \langle u_{f,D} \rangle - \langle \hat{u} \rangle \). An equation equivalent to (6) can also be obtained using a 1D FK identity (García-Mayoral & Jiménez, 2011; MacDonald et al., 2016), with the main difference that equation (4) retains the flow contributions below the crest. Moreover, it is convenient to reformulate equation (6) by introducing the slip velocity at the crest \( \Delta \tilde{u}_S \), allowing to explicitly introduce the effect of the virtual origin,

\[ \Delta U_{f,S}^+ = \Delta U_{f,f}^+ + \Delta U_{f,T}^+ + \Delta U_{f,D}^+, \tag{7} \]

where, \( \Delta U_{f,f}^+ = \langle u_{f,f} \rangle - \langle \hat{u} \rangle \), \( \Delta U_{f,T}^+ = \langle u_{f,T} \rangle - \langle \hat{u} \rangle \), \( \Delta U_{f,D}^+ = \langle u_{f,D} \rangle - \langle \hat{u} \rangle \). The laminar slip velocity \( \Delta U_{f,S}^+ \) represents the viscous (Stokes) slip contribution of Luchini et al. (1991), which should correspond to the total slip velocity \( \Delta U_{f,S}^+ \sim \Delta U_{f,f}^+ + \Delta U_{f,T}^+ + \Delta U_{f,D}^+ \), \( \Delta U_{f,S}^+ \) is the difference between the streamwise and spanwise Stokes protrusion heights (Luchini et al., 1991).

![Figure 6](image_url)

Figure 6. Contributions to the slip velocity \( \Delta U_{f,S}^+ \) (○) from (7): turbulence \( \Delta U_{f,f}^+ \) (△), dispersion \( \Delta U_{f,D}^+ \) (♣) and viscous (Stokes) \( \Delta U_{f,T}^+ \) (□), for (a) triangles \( \alpha = 30^\circ \), (b) triangles \( \alpha = 60^\circ \), (c) triangles \( \alpha = 90^\circ \), (d) asymmetric triangles \( \alpha = 63.4^\circ \), (e) trapezoids \( \alpha = 30^\circ \) and (f) blades \( s/t = 5 \). The Stokes prediction (——) \( \Delta U^+ = -\Delta h^+ \) is also reported, where \( \Delta h = h_{||} - h_{\perp} \) is the difference between the streamwise and spanwise Stokes protrusion heights (Luchini et al., 1991).
contribution $\Delta U_D^+ > \Delta U_D^-$. On the contrary, triangles with opening angle $\alpha = 90^\circ$, asymmetric triangles, and trapezoids (figure 5c,d,e) show a larger dispersive contribution $\Delta U_D^+ > \Delta U_D^-$. This observation suggests that the riblet geometry influences the preferential contribution of turbulence or dispersion to the breakdown. Therefore, we classify the grooves depending on their effective aspect ratio $A = k/s'$, where $s'$ is the fluid groove spacing, namely $s' = s$ for triangles and trapezoid and $s' = s - t$ for blades. Grooves with $A \leq 0.5$ (triangular $\alpha = 90^\circ$, asymmetric and trapezoidal) present a non-negligible dispersive contribution to $\Delta U^+$, whereas grooves with $A > 0.5$ (triangular $\alpha = 30^\circ-60^\circ$ and blade) present a larger turbulent contribution, see table 1. This observation suggests that disruption of the secondary flows for geometries with $A < 0.5$ might retard or suppress the breakdown of drag reduction. We recall here that even though splitting (2) is mathematically exact, the various contributions are not independent of each other (Modesti et al., 2018). Even though the splitting (2) is linear, the underlying equations are nonlinear, therefore the analysis only provides diagnostic information. We further note that the total slip contribution $\Delta U_S$ tends to saturate ($\ell_g^s \approx 30$) and eventually becomes positive (drag increasing) for $\ell_g^s > 30$ (figure 5e). This trend can be traced back to the idea that the concept of virtual origin does not strictly apply for increasing $\ell_g^s$, and the Stokes, linear, prediction of Luchini et al. (1991) fails, as the near wall turbulence is substantially altered. In order to understand the mechanism that leads to deviations from the linear trend we report the single contributions to $\Delta U_S$ in figure 6. The laminar slip contribution (C) approximately corresponds to the total slip (A) $\Delta U_S^L \approx \Delta U_S^{L_g}$, for small $\ell_g^s$, consistent with the fact that small riblets operate in the linear regime. $U_S^{L_g}$ (A) decreases linearly with $\ell_g^s$, in agreement with Stokes calculations (---------) (Luchini et al., 1991), whereas $\Delta U_S^{L_g}$ (A) increases, eventually leading $\Delta U_S^L$ to deviate from the linear behavior, consistently with the fact that the near wall cycle changes substantially for large $\ell_g^s$. The dispersive contribution to the slip $\Delta U_S^{D_g}$ (W) instead is negligible for all geometries, apart for the large trapezoidal grooves (figure 6e), for which it reaches $\Delta U_S^{D_g} \approx 1$, although remaining lower than $\Delta U_S^{L_g}$. This analysis presents a method to quantify the drag penalty due to the hypothesised effect of turbulence descending below the riblet crest (Lee & Lee, 2001). The results suggest that turbulent fluctuations play a primary role in the breakdown of the Stokes regime, and show that the turbulent flow descending below the riblet crest has a detrimental effect on the slip contribution.

**DISCUSSIONS AND CONCLUSIONS**

We carried out DNS of minimal channel flow over several riblet geometries to shed light on the physical mechanisms that lead to the breakdown of drag reduction. The use of minimal channels has been instrumental to develop a larger dataset than in previous studies, allowing us to reach more general and solid conclusions. A generalized 2D FIK analysis (Modesti et al., 2018) allows us to isolate the different contributions to $\Delta U^+$ and to gain additional insight into the flow physics of riblets. In agreement with previous studies, riblets operating in the linear regime reduce drag by shifting the virtual origin of turbulence farther from the wall, increasing the effective mean velocity at the crest, and thus the bulk flow velocity. Large grooves instead operate beyond the viscous sublayer generating a turbulent flow, which inevitably carries additional stress. We show that part of this stress can be traced back to purely turbulent fluctuations, whereas the rest is associated with dispersion, namely the (time-averaged) secondary flows filling the riblet groove. The relative contribution of turbulence and dispersion depends on the riblet geometry, and grooves with aspect ratio $A < 0.5$ show a dominant contribution of dispersion to $\Delta U^+$, whereas riblets with $A > 0.5$ show a larger contribution of turbulence. The FIK analysis suggests (although it does not guarantee) that disruption of the secondary flow for geometries with $A < 0.5$ might lead to larger drag reduction. Moreover, this analysis also allows us to isolate the different contributions to the slip velocity and shows that deviations from the linear (Stokes) regime can be attributed to the turbulent stress descending below the riblet crest.

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