Long-wave infrared magnetic mirror based on Mie resonators on conductive substrate

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Abstract: Metal films are often used in optoelectronic devices as mirrors and/or electrical contacts. In many such devices, however, the π-phase shift of the electric field that occurs upon reflection from a perfect electric conductor (for which a metal mirror is a reasonable approximation) is undesirable. This is because it results in the total electric field being zero at the mirror surface, which is unfavorable if one wishes for example to enhance absorption by a material placed there. This has motivated the development of structures that reflect light with zero phase shift, as these lead to the electric field having an anti-node (rather than node) at the surface. These structures have been denoted by a variety of terms, including magnetic mirrors, magnetic conductors, and high impedance surfaces. In this work, we experimentally demonstrate a long-wave infrared device that we term a magnetic mirror. It comprises an array of amorphous silicon cuboids on a gold film. Our measurements demonstrate a phase shift of zero and a high reflectance (of ~90%) at a wavelength of 8.4 µm. We present the results of a multipole analysis that provides insight into the physical mechanism. Lastly, we investigate the use of our structure in a photodetector application by performing simulations of the optical absorption by monolayer graphene placed on the cuboids.

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1. Introduction

Structures that reflect light with zero phase shift of the electric field have drawn much attention in recent years. These are often referred to as magnetic mirrors, but also as magnetic conductors and high impedance surfaces. Early work included radio frequency designs pursued for the goal of suppressing propagating surface waves and for the realization of low profile antennas [1,2]. More recent investigations [3–15] have explored other structures and other applications, and have included experimental studies in the visible, infrared, and terahertz portions of the electromagnetic spectrum. These previous works have generally fallen into two categories. In the first, metals are employed [4–9]. By introducing corrugations into the metal, it becomes possible for the incident light to be coupled into surface plasmon polariton modes, and for these to be subsequently coupled back out to free-space propagating modes. By choosing the geometric parameters of the corrugations appropriately, the reflected beam can be made to have the desired phase shift (of zero) at the target wavelength. In the second, high index dielectric nanoparticles supporting resonances (akin to Mie resonances of spheres) are used [10–15]. The reflected wave can be then thought of as the envelope comprising the sum of the scattered waves generated by these resonances. Judicious choice of nanoparticle geometry and material permits these resonances to be tuned, allowing control over the phase of the scattered light and thereby achieving the desired functionality. This approach benefits from recent work on nano-optical devices based on nanoparticles with high refractive indices [16–23]. The theoretical foundations for this approach have a long history, e.g. with Lewin showing in 1946 that non-unity permeability could be achieved with an array of non-magnetic, sub-wavelength dielectric resonators [24]. Phase modulators operating in reflection mode based on nano-optics have also been demonstrated.
These [25,26] and related previous works can achieve the desired zero phase shift, but often reflect light poorly as they operate near the perfect absorption regime.

For conventional metal mirrors, electromagnetic boundary conditions force a $\pi$-phase shift of the electric field upon reflection (in the ideal case that the metal can be considered as a perfect electric conductor). For the magnetic mirror, boundary conditions lead to a $\pi$-phase shift of the magnetic field, rather than electric field. As a result, when a magnetic mirror is illuminated by a plane wave, the total electric field has an anti-node at the surface. Because of this unusual property, magnetic mirrors can be much more effective than conventional metal mirrors as reflectors in optoelectronic devices for which strong light-matter interaction (e.g. absorption or emission) near the mirror surface is desirable. These structures have been thus pursued for applications that include solar cells [5], enhanced photoluminescence [7], surface-enhanced Raman scattering [8] and tunable photodetectors [9]. In previous works [3–15], it has generally been the case that either the metal or the dielectric material is patterned to realize the magnetic mirror. However, for many optoelectronic devices, it would be advantageous to realize a multi-functional magnetic mirror from hybrid structures containing both metals and dielectrics, with the metal acting as both an electrical conductor and a broadband optical reflector and the dielectric structures providing flexible tunability of the reflected wave-front, e.g. control over phase, amplitude, polarization, and radiation pattern. This approach would also be timely, given the recent trend toward optoelectronic devices that employ ultra-thin active regions, e.g. comprising two dimensional materials. These have included photodetectors [27–34] for the mid-wave and long-wave infrared (MWIR and LWIR). The use of very thin materials is partly motivated by the principle, well-established for MWIR and LWIR detectors [35], that reducing the volume of the active material in turn reduces the noise associated with thermal generation of charge carriers. Such materials, however, face the challenge of inherently weak optical absorption, which is exacerbated by the fact that it is generally desirable for them to be placed close to an electrical conductor (e.g. metal) for electrostatic gating. This motivates the development of an LWIR magnetic mirror that incorporates both high index semiconductor resonators and a metal film. This is the topic of this paper.

In this work, we harness the high reflectance of a conductive substrate (gold film) and the versatile scattering properties of high index Mie resonators (a-Si, i.e. amorphous silicon, cuboids) to achieve a thin LWIR magnetic mirror with high reflectance. A modified multipole analysis method is employed to provide insights into the resonances supported by the resonator when it is brought close to a perfect electric conductor (PEC) substrate. We experimentally demonstrate a LWIR magnetic mirror comprising an array of thin a-Si cuboids ($\sim1.1 \mu m$ thick) on gold substrate that shows zero phase shift and high reflectance of $\sim90\%$ at a wavelength of $8.4 \mu m$. We further show by simulation that the optical absorption in monolayer graphene placed on our structure is more than three orders of magnitude greater than the absorption that would result were the graphene instead placed on a gold mirror.

The organization of our paper is as follows. In the next section (Section 2), we describe the design of our LWIR magnetic mirror. The magnetic mirror functionality is based on the a-Si cuboids supporting magnetic dipole resonances. Excitation of these result in a boundary condition in which the electric field does not change sign upon reflection. In Section 2 we thus elucidate the physics of the scattering by our dielectric resonator (a-Si cuboid) when it is placed above a PEC substrate. We then present magnetic mirror designs that span the wavelength range $\sim6.6$ to $12 \mu m$. Results are shown as Figs. 1–3. In Section 3, we describe the experimental realization of our magnetic mirror, including its fabrication and optical characterization. We measure the reflection spectrum of the fabricated device using a homebuilt infrared microscope interfaced to a Fourier transform infrared (FTIR) spectrometer. We measure the phase of the reflected beam using a homebuilt Michelson interferometer, with a quantum cascade laser as the light source. In Section 3, we also explore the potential application of using the magnetic
mirror to enhance the light absorption in monolayer graphene via simulations. Results are shown as Figs. 4–6. The conclusions of this study are provided as Section 4. We provide additional details on the multipole expansion method, sample fabrication, and optical characterization in the Appendix. These details may be of interest to specialists working in the field. Results are shown as Figs. 7–10.

2. Design of LWIR magnetic mirror

It is well known that the tangential electric field on the surface of a perfect electric conductor (PEC) is zero \[36\]. There is thus a π-phase shift of the electric field upon reflection and an induced electric current on the PEC surface \[36\]. An analogous situation occurs for perfect magnetic conductors (PMCs). Regular materials do not function as PMCs, unlike the situation for PECs for which metals are a reasonable approximation. Nonetheless, PMCs are a useful conceptual tool for electromagnetics problems. The tangential magnetic field is zero on the surface of a PMC \[36\]. This leads to a π-phase shift of the magnetic field upon reflection and an induced magnetic current on the PMC surface \[36\]. The phase shift of the electric field upon reflection is zero. It has been previously shown that an array of dielectric resonators supporting magnetic dipole resonances could provide this functionality (e.g. \[12\]). One may think of the magnetic dipoles as serving the role of the magnetic current in the PMC case. In \[12\] and similar studies, the dielectric resonators were on substrates with relatively low refractive indices. As discussed, in this work our dielectric resonators are on conductive substrates. This modifies the multipole resonances of the particles \[37–44\]. In this section, we describe the design of our LWIR magnetic mirror. To elucidate the physics on how the multipolar resonances are modified by the conductive substrate, we begin this section by studying the scattering properties of our resonator as a function of its distance from a PEC. We next consider in greater detail the situation in which the resonator is on the PEC and contrast its scattering properties with the case of a resonator that is twice as thick and in a homogeneous medium. This section concludes with the presentation of designs in which the conductive substrate is gold, i.e. a realistic metal rather than a PEC.

We begin by studying the scattering properties of an a-Si cuboid (thickness \(D = 1 \mu m\), side length \(L = 3 \mu m\)) separated from a PEC surface by different distances. Figure 1(a) shows the four configurations we study where the a-Si cuboid is at distances \(H = 0, \lambda/12, \lambda/6, \lambda/2\) (\(\lambda = 10.5 \mu m\)) above the PEC surface. The origins of coordinate system, denoted as O1, O2, O3, O4, are chosen to be the points at which the perpendicular bisector of the cuboid crosses the PEC surface. Illumination is from a normally-incident plane wave linearly polarized along the \(x\) axis. In our calculation, the background field is taken as the standing wave distribution produced by the PEC upon plane wave illumination, i.e. without the a-Si cuboid. The total field is that occurring with both the PEC and the a-Si cuboid under plane wave illumination. The scattered field is the difference between the total field and the background field. Due to the presence of the PEC surface, conventional multipole analysis method - which deals with scatterers in homogeneous environments - cannot be directly applied. To apply multipole analysis in our case, we replace the PEC surface with image multipole sources based on the method of images. The corresponding image multipole coefficients can be found and related to the original multipole coefficients (calculated for a-Si cuboid by conventional method) using the PEC boundary condition.

The scattering of the a-Si cuboid can thus be decomposed into a new set of multipole scatterings with modified multipole coefficients (see Appendix A1 for derivation) that are given as follows

\[
\alpha_{E}^{\text{mod}}(l, m) = [1 - (-1)^{l+m}]\alpha_{E}(l, m)
\]

\[
\alpha_{M}^{\text{mod}}(l, m) = [1 + (-1)^{l+m}]\alpha_{M}(l, m)
\]

where, \(\alpha_{E,M}^{\text{mod}}(l, m)\) are the modified electric and magnetic multipole coefficients with order \(l\) and degree \(m\). \(\alpha_{E,M}(l, m)\) are the conventional multipole coefficients \[45\]. We note that we previously

\(1\)

\(2\)
Fig. 1. Multipole analysis of scattering from a-Si cuboid ($D = 1 \, \mu m$, $L = 3 \, \mu m$) above PEC surface. (a) Schematic of scattering configurations with a-Si cuboid above PEC surface at different height $H = 0$, $\lambda/12$, $\lambda/6$, $\lambda/2$ ($\lambda = 10.5 \, \mu m$). $O_1$, $O_2$, $O_3$, $O_4$ are coordinate origins of the four configurations. (b-e) Total scattering cross section $\sigma_{sca}$ (black curve) calculated by integrating Poynting vector over the surface of cuboid and partial scattering cross sections of electric dipole (ED, green curve) and magnetic dipole (MD, blue curve) components, calculated using the modified multipole expansion method for the four configurations of panel a. In panel b, a factor of 3 is applied to MD contribution for better visualization. (f-i) Normalized electric field distribution on cross section ($8 \, \mu m \times 8 \, \mu m$) through a-Si cuboid, calculated at wavelength corresponding to those of the longer-wavelength peak in MD scattering spectra of panels (b-e).

developed a related multipole expansion method for dielectric resonators on PECs and employed it in the visible wavelength range for structural color applications [43]. Details on the differences between the two methods are given in Appendix A1.

The calculated total scattering cross sections $\sigma_{sca}$ and multipolar scattering cross section components are shown as Figs. 1(b)–1(e). For clarity, only electric dipole (ED) and magnetic dipole (MD) are shown here, with the results for higher order multipole contributions provided in the Appendix (see Fig. 8). It can be seen that for all the cases, ED scattering contributions are negligible as the original electric dipole and its image cancel each other. Large MD scattering contributions can be seen and become more dominant as the cuboid is brought closer to the
PEC surface. When the cuboid is on the PEC surface \((H = 0 \mu m)\), the MD contribution accounts for almost the entirety of the scattering of the cuboid at the wavelength of the spectral peak \((\lambda = 10.5 \mu m, \text{Fig. 1(e)})\), which is important for the design of magnetic mirror. Figures 1(f)–1(i) show the electric field profile at wavelengths corresponding to those of the longer-wavelength peak in the MD scattering spectra of Figs. 1(b)–1(e). The characteristic field pattern of an MD resonance is visible and becomes stronger as the cuboid is brought closer to the PEC, which is consistent with the scattering cross section calculation.

The advantages of placing the cuboid directly onto a PEC surface can be further seen by comparing its multipolar scattering behavior to that of a cuboid in air. Figures 2(a) and 2(b) show the calculated total scattering cross section and multipolar contributions (up to octupolar order) for two cases: an a-Si cuboid \((D = 2 \mu m, L = 3 \mu m)\) in air (Fig. 2(a)) and an a-Si cuboid (half as thick \(D = 1 \mu m, L = 3 \mu m\)) on a PEC surface (Fig. 2(b)). For both cases, the multipole calculation of the total scattering cross section matches perfectly with that calculated using the Poynting method, in which we integrate the Poynting vector over a closed surface surrounding

![Fig. 2. Multipole analysis of scattering from (a) an a-Si cuboid \((D = 2 \mu m, L = 3 \mu m)\) in air and (b) an a-Si cuboid \((D = 1 \mu m, L = 3 \mu m)\) on PEC surface. (c,d) Normalized electric field magnitude profiles on cross section \((6 \mu m \times 6 \mu m)\) in (c), \(6 \mu m \times 3 \mu m\) in (d)) through the cuboid center in XZ plane are shown at \(\lambda = 10.5 \mu m\) for scenarios in (a,b) respectively.](image-url)
the cuboid. This verifies the validity of our modified multipole analysis method. Note that for the case of the cuboid in air, the coordinate origin is chosen to be the cuboid center. Some interesting facts can be seen from the results. It can be seen that, due to their distinct mirror symmetry properties, the interaction of different multipoles with their corresponding images (resulting from the PEC) leads to either cancellation or enhancement of the overall scattering. More specifically, for the case of the cuboid on PEC, the scattering strengths of the ED, magnetic quadrupole (MQ) and electric octupole (EO) are strongly diminished, while the scattering strengths of the MD, electric quadrupole (EQ) and magnetic octupole (MO) are retained and approximately doubled. Of particular interest is the retention and enhancement of the MD when the cuboid is on the PEC. This can be understood conceptually by the schematic illustration of Fig. 2(d), i.e. the PEC can be thought of as supporting image electric fields that mirror the electric fields of the cuboid on the PEC, resulting in the formation of a circulating electric field pattern, i.e. an MD configuration. These resemble the MD that occurs when the cuboid is in air. These observations are consistent with our previous study [43] at shorter wavelengths that was motivated by structural color applications. Figures 2(c)–2(d) show the electric field profiles at MD resonance wavelength for the two cases corresponding to Figs. 2(a) and 2(b), respectively. The suppression/enhancement of the electric dipole/magnetic dipole contribution and reduction (50%) of structure thickness for the cuboid on PEC are beneficial for the realization of the magnetic mirror functionality.

The magnetic mirror structure we propose thus consists of a square array of a-Si cuboids on an Au film (100 nm thick) on a silicon wafer (Fig. 3(a)). We choose gold film to play the role of the PEC because of its high conductivity in the LWIR. We begin by simulating the reflectance and phase shift using the finite difference time domain method implemented in a commercial software package (FDTD Solutions, Lumerical). Figures 3(b) and 3(c) show the simulated reflectance and phase shift spectra for structures consisting of cuboids with $D = 1 \mu m$ and $L = 1 \mu m - 4 \mu m$. The period of the (square) array is taken to be $2L$. For the phase calculation, the reference plane

![Fig. 3.](image-url)
is taken to be the top surface of the cuboids. It can be seen that as we increase the cuboid side length, the reflection dip caused by the MD resonance (green colored diagonal Fig. 3(b)) as well as the wavelength at which the reflection phase shift is zero (see Fig. 3(c)) red shift. Multipole resonances of order higher than the MD can also be seen at shorter wavelengths. The wavelength at which the phase shift is zero and its corresponding reflectance are plotted as a function of cuboid side length as Fig. 3(d). It can be seen that magnetic mirrors (i.e. zero phase shift) with high reflectance (e.g. 96% at $\lambda = 6.6 \mu m$) can be achieved over a range of wavelengths that span the LWIR by properly choosing the structure parameters. Interestingly, the reflectance in (Fig. 3(d)) increases as the cuboid side length decreases, which is due to the reduction of MD strength as a result of the mismatch between the cuboid dimension and the excitation wavelength.

3. Experimental demonstration of LWIR magnetic mirror

To demonstrate the magnetic mirror experimentally, we fabricate a series of samples with cuboid thickness $D = 1.1 \mu m$, and side lengths $L = \{1.4 \mu m, 1.5 \mu m, 1.6 \mu m, 1.7 \mu m, 1.85 \mu m, 2 \mu m\}$. Fabrication starts with deposition of Au (100 nm thick) and a-Si (1.1 \mu m thick) films onto a silicon wafer by electron beam evaporation and plasma-enhanced chemical vapor deposition, respectively. An Al (30 nm thick) etch mask is then added by electron beam lithography, evaporation and lift off. The a-Si film is then etched down to the Au surface by inductively coupled reactive ion etching. Wet etching is then used to remove the Al mask. Figure 4(a) shows the scanning electron microscope (SEM) images of a completed magnetic mirror sample. A well-defined cuboid array is fabricated uniformly over a large area. We measure reflection spectra using a homebuilt microscope attached to a Fourier transform infrared (FTIR) spectrometer (Fig. 4(b)). For comparison, we simulate the reflection spectra of structures with the same cuboid thickness and side length as that fabricated. The reflectance in both the simulation and measurement are obtained by using the reflectance from a gold film as the reference. It can be seen that the simulated reflection spectra (Fig. 4(c)) match well with the measured spectra in terms of the reflection dip position. The dips of the measured spectra are slightly deeper than those of simulations, however. This might be caused by fabrication imperfections. The small dips that appear at shorter wavelengths in the measurement might be a consequence of the illumination in our set-up not being purely at normal incidence [46].

To determine the phase shift of our magnetic mirror samples, we employ an interferometric method based on a homebuilt infrared Michelson interferometer system that employs a quantum cascade laser (QCL) operating at $\lambda = 8.4 \mu m$. The two interfering beams are from a gold mirror (whose position is scanned) and from the sample under test. Using this setup, we measure interferograms from the magnetic mirror samples and from a gold reference sample. These interferograms consist of the photodetector signals measured as a function of position of the scanning gold mirror. The relative phase of the reflection between a magnetic mirror sample and the gold reference sample is retrieved by comparing the interferograms. Two of the measured interferograms are shown in the inset of Fig. 5(a). The top interferogram (blue dots) is from a magnetic mirror sample (with $D = 1.1 \mu m$ and $L = 1.7 \mu m$). The bottom interferogram (green dots) is from the gold reference sample. We fit the measured interferograms to sinusoidal functions, from which we extract the relative phase of reflection from different samples. Note that the height difference (of $1.1 \mu m$) between the gold reference sample and the magnetic mirror samples is taken into account in the phase determination. This height difference arises from the fact that the gold reference sample comprises a region of the magnetic mirror chip in which there are no a-Si cuboids (i.e. beam reflects from gold film). The results for our phase shift measurements are shown as Fig. 5(a). It should be noted that these results take the phase shift of reflection from the gold reference sample to be $-\pi$. The measured phase shifts of the fabricated samples vary from $-0.12\pi$ to $0.69\pi$ as the cuboid side length increase from $1.4 \mu m$ to $2 \mu m$, which is in reasonable agreement with the simulation. It can be seen that the desired zero phase shift,
Fig. 4. (a) SEM images (tilted view) of fabricated magnetic mirror sample containing cuboids with side length $L = 1.7 \, \mu m$ and thickness $D = 1.1 \, \mu m$. Scale bars: 50 $\mu m$, 20 $\mu m$ and 2 $\mu m$ respectively. (b,c) Measured and simulated reflection spectra of magnetic mirrors containing cuboids with cuboid dimensions $D = 1.1 \, \mu m$ and $L = \{1.4 \, \mu m, 1.5 \, \mu m, 1.6 \, \mu m, 1.7 \, \mu m, 1.85 \, \mu m, 2 \, \mu m\}$. i.e. magnetic mirror functionality, is realized for the sample with cuboid side length $L = 1.7 \, \mu m$. Figure 5(b) shows simulated and measured reflectance at the operating wavelength (8.4 $\mu m$). The measurements (of Fig. 5(b)) are extracted from the spectra of Fig. 4(c). It can be seen that the measured reflectance for the sample with the magnetic mirror behavior ($L = 1.7 \, \mu m$) is as high as 90%. Possible reasons for the small optical losses include absorption in a-Si cuboids and the ohmic loss in the metal caused by the near field coupling between cuboids and the gold substrate on magnetic dipole resonance.

To investigate a potential application of our magnetic mirror, we study its use to increase the optical absorption in monolayer graphene placed directly on it via simulations. For comparison, we consider two other cases, in which the graphene monolayer is situated in free space and in which it is placed on a gold mirror. The results are shown as Fig. 6(a) and consist of the absorption spectra simulated for monolayer graphene placed on the magnetic mirror designs we consider in this paper (a-Si cuboids on gold). The cuboids have $D = 1 \, \mu m$, and $L = 1–2.2 \, \mu m$ in steps of 0.2 $\mu m$ (solid curves). It can be seen that the absorption in graphene is strongly enhanced at wavelengths corresponding to zero phase shift wavelengths of the magnetic mirrors (Fig. 6(b)). Further details on the simulation method are provided in Appendix A3.2. The peak absorption of each device is around four times greater than that of the graphene suspended in air (grey dashed curve in Fig. 6(a)). The peak absorption is around three orders of magnitude larger than the case when graphene is on a gold mirror (grey dotted curve in Fig. 6(a)). This is a direct result of the enhancement of the electric field intensity (roughly four times of incident field intensity) at the
surface of the magnetic mirror when it reflects light with zero phase shift. Likewise, due to the electric field intensity being very small on the gold surface, absorption in graphene is strongly suppressed when it is placed on a gold mirror. In a photodetector application, the use of our magnetic mirror would allow the graphene to be electrostatically gated (by applying a voltage to the metal) while allowing optical absorption to be substantial. We furthermore note that while in each device of Fig. 6, cuboids have the same design, this does not have to be the case. One could for example have a device in which the cuboid side length is varied to produce a phase shift distribution akin to that of a concave mirror, i.e. so that the reflected wave is focused to a small spot in the device center, at which a photodetector could be located. In this way, magnetic mirrors could perform the function normally performed by microlenses in optically-immersed photodetector devices [35].

4. Conclusions

In summary, in this work we describe a LWIR magnetic mirror that consists of an array of a-Si cuboids on a gold substrate. A modified multipole analysis method is employed to reveal the
physical mechanism of the magnetic mirror functionality of the structure, showing efficient excitation of the magnetic dipole resonance of the cuboid on the PEC surface. The image effect (due to the presence of conductive substrate) allows us to use resonators that are half the thickness of dielectric resonators in a homogeneous environment. We fabricate magnetic mirror samples and demonstrate the magnetic mirror functionality at wavelength $\lambda = 8.4 \mu m$ from one of the samples (cuboid side length $L = 1.7 \mu m$, thickness $D = 1.1 \mu m$) by measuring the phase shift using a homebuilt Michelson interferometer. The magnetic mirror shows a measured phase shift of zero and reflectance of 90% at $\lambda = 8.4 \mu m$. The proposed magnetic mirror is also shown to offer significantly stronger light absorption for monolayer graphene placed on it than were the graphene to be suspended in air or placed on a gold surface. Our findings provide important information for the development of thin and low-loss magnetic mirrors, especially in the long wave infrared (LWIR) spectral range.

Appendix

A1. Multipole analysis of scatterer above a perfect electric conductor (PEC) substrate

Multipole analysis is a well-established method and has proven successful for many scattering problems [40–42,44,45,47,48]. Nonetheless, most conventional multipole analysis methods have the constraint of requiring the scatterer to be situated in a homogeneous environment. In case of the scatterer being situated in an inhomogeneous environment, such as being above a perfect electric conductor (PEC) substrate, conventional methods cannot be directly applied. For this reason, various approaches have been developed to study light scattering by scatterers in the presence of a substrate. Early efforts include studies of simple geometries such as a circular cylinder parallel to a reflecting flat surface illuminated by a plane wave [38,39]. In these works, a cylindrical-wave approach is employed to decompose the fields. This takes into account the fields reflected from the surface. But multipole contributions to the scattering are not investigated in this approach, and the method imposes restrictions on the geometry and orientation of the scatterer, i.e. an infinite cylinder parallel to the reflective surface. Multipole analysis of light scattering by arbitrary-shaped nanoparticles located near or on a plane surface is studied up to the orders of magnetic quadrupole and electric octupole based on the decomposed discrete dipole approximation method, where the induced polarization is decomposed into multipole moments around a point at the center of mass of the scatterer [40]. The multipole decomposition of the scattered field in the far-field zone is then obtained using the far-field approximation of Green’s tensors for reflection from the surface. A method termed the generalized point-dipole approximation was devised to study substrate-induced resonant magnetoelectric effects for a dielectric sphere on a substrate [41]. In this method, the dielectric particle is replaced by a pair of point electric and magnetic dipoles located at the center of the particle with the substrate-modified polarizabilities that contain cross-coupling terms between electric and magnetic dipoles. The applicability of the method is limited for particles with certain geometries such as sphere or cylinder since the effective polarizabilities are expressed through Mie’s dipole scattering coefficients. More recently, an analytical model based on a modified Mie theory is used to study the fields generated by plane-wave illumination of a dielectric cylinder above a reflective mirror [42]. The method neglects the near-field interaction between the cylinder and the mirror, thus requires the cylinder to be sufficiently away from the mirror. It also imposes restrictions on the shape of the scatterer as it is based on Mie’s theory. Here we present a set of modified multipole coefficients that describe the scattering from a scatterer above a PEC substrate. These modified multipole coefficients are found by introducing a set of image multipole coefficients based on the method of images. The method we present is applicable for arbitrarily-shaped particles above a PEC surface with any direction of the incident wave. While we have employed it to study the
where work was shown, but with some differences. In that work, we studied the case of a scatterer on the PEC substrate (situated in the YZ plane). In this work, the presented method is employed to study the case of a scatterer above (with separation) the PEC substrate (situated in the XY plane). In addition, in this work, we derive the result in a different way. We hope that the derivation we present in this paper will provide additional physical insight. The conventional multipole analysis method states that for a monochromatic plane wave with electric field amplitude $E_0$, angular frequency $\omega$ and wavevector $\mathbf{k}$, incident on a particle in an otherwise homogeneous lossless dielectric medium, the scattered electric field can be written in spherical coordinates in the form of multipole expansion [45]:

$$E_s(r) = E_0 \sum_{l=1}^{\infty} \sum_{m=-l}^{l} \frac{i}{k} \alpha_E(l, m) \nabla \times [h_1^{(1)}(kr) \mathbf{X}_{lm}(\theta, \varphi)] + \alpha_M(l, m) h_1^{(1)}(kr) \mathbf{X}_{lm}(\theta, \varphi)$$  (3)

where $h_1^{(1)}(kr)$ is the spherical Hankel function of the first kind, $\mathbf{X}_{lm}(\theta, \varphi)$ is the normalized vector spherical harmonics, $\alpha_E(l, m)$ and $\alpha_M(l, m)$ are electric and magnetic multipole coefficients of order $l$ and degree $m$ respectively. The vector functions in the multipole expansion form a complete basis for representing the electromagnetic field outside an arbitrary localized source. Using the orthogonality of the basis, the multipole coefficients can be calculated as:

$$\alpha_E(l, m) = \frac{(-i)^l k^2 \eta O_{lm}}{E_0 \pi (2l + 1)^{1/2}} \int e^{-i m \phi} \left[ \frac{\varphi_0(kr)}{k} P_l^m(\cos \theta) \mathbf{J}_{sca}(\mathbf{r}) \right] d^3r$$  (4a)

$$\alpha_M(l, m) = \frac{(-i)^{l+1} k^2 \eta O_{lm}}{E_0 \pi (2l + 1)^{1/2}} \int e^{-i m \phi} j_l(kr) \left[ i P_l^m(\cos \theta) \mathbf{J}_{sca}(\mathbf{r}) + \tau_{lm}(\theta) \nabla \times \mathbf{J}_{sca}(\mathbf{r}) \right] d^3r$$  (4b)

$$\mathbf{J}_{sca}(\mathbf{r}) = -i \omega |\epsilon| \mathbf{E}(\mathbf{r})$$  (4c)

where $\mathbf{E}(\mathbf{r})$ is the total electric field, $\epsilon_h$ is the permittivity of the host medium, $\varphi_0(kr) = kr j_l(kr)$ are the Ricatti-Bessel functions, $P_l^m(\cos \theta)$ are the associated Legendre polynomials, and $O_{lm}$, $\tau_{lm}$ and $\pi_{lm}$ are defined as follows:

$$O_{lm} = \frac{1}{[l(l + 1)]^{1/2}} \left[ \frac{2l + 1}{4 \pi} \frac{(l - m)!}{(l + m)!} \right]^{1/2}$$  (5a)

$$\tau_{lm}(\theta) = \frac{d}{d \theta} P_l^m(\cos \theta)$$  (5b)

$$\pi_{lm}(\theta) = \frac{m}{\sin(\theta)} P_l^m(\cos \theta)$$  (5c)

The scattering cross section of the particle can thus be decomposed into multipole contributions by $\sigma_{sca} = \pi \sum_{l=1}^{\infty} \sum_{m=-l}^{l} (2l + 1)(|\alpha_E(l, m)|^2 + |\alpha_M(l, m)|^2)$.

Consider the configuration of a particle with arbitrary size and shape in a homogenous medium above a PEC substrate that extends infinitely in the XY plane (Fig. 7(a)). The $z = 0$ plane is assumed to be the surface of the PEC substrate. A plane wave linearly polarized along the $x$ direction ($\mathbf{E}_{in} = E_0 \exp(-ikz\mathbf{X})$) is normally incident on the PEC substrate and is scattered by the particle. Here, a time dependence of $\exp(-i\omega t)$ is assumed. This scattering problem is fully specified by the electromagnetic field above the PEC (which satisfies the time harmonic form of Maxwell’s equations) and the boundary condition at $z = 0$. These are given as follows:
\[
\begin{align*}
\nabla \times \mathbf{H} + i \omega (\varepsilon + i \frac{\sigma}{\varepsilon} \omega) \mathbf{E} = 0 \\
\nabla \times \mathbf{E} - i \omega \mu \mathbf{H} = 0 \\
\n\nabla \cdot (\varepsilon \mathbf{E}) = 0 \\
\n\nabla \cdot (\mu \mathbf{H}) = 0 \\
\end{align*}
\] (6)

We first prove that this scattering problem is equivalent to the scattering problem described in Fig. 7(b), for which the PEC substrate is replaced by an image particle that mirrors the original particle with respect to \( z = 0 \), and an image source comprising a plane wave described by \( \mathbf{E}_{ib} = -E_0 \exp(ikz)\hat{x} \). The image source can be thought of as the wave reflected by the PEC, i.e. \( \mathbf{E}_{ib} = \mathbf{E}_r \).

![Fig. 7. Schematic illustration of two equivalent scattering problems. (a) Particle is above PEC \((z = 0\) plane) with \(x\)-polarized plane wave excitation. (b) Equivalent scattering problem, in which PEC is replaced by an image particle and image plane wave.](image)

To prove this claim, we have to verify that in the new configuration, the field in the upper half space \((z \geq 0)\) also satisfies Eqs. (6) and (7). By the principle of linear superposition, the scattered fields caused by the incident waves \( \mathbf{E}_{ia} \), and \( \mathbf{E}_{ib} \) can be considered separately. Suppose the solution of the scattering problem with incident wave \( \mathbf{E}_{ia} \) is:

\[
\begin{align*}
\mathbf{E}_a(x, y, z) &= E_x(x, y, z)\hat{x} + E_y(x, y, z)\hat{y} + E_z(x, y, z)\hat{z} \\
\mathbf{H}_a(x, y, z) &= H_x(x, y, z)\hat{x} + H_y(x, y, z)\hat{y} + H_z(x, y, z)\hat{z}
\end{align*}
\] (8)

By definition, this solution must satisfy Maxwell’s Eq. (6). For the scattering problem with incident wave \( \mathbf{E}_{ib} \), due to mirror symmetry of the problem, its solution can be related to the
solution in Eq. (8) as follows:

\[
\begin{align*}
E_b(x, y, z) &= -E_x(x, y, -z)\hat{x} - E_y(x, y, -z)\hat{y} + E_z(x, y, -z)\hat{z} \\
H_b(x, y, z) &= H_x(x, y, -z)\hat{x} + H_y(x, y, -z)\hat{y} - H_z(x, y, -z)\hat{z}
\end{align*}
\]  

(9)

Since one can substitute the expressions for \(E_b\) and \(H_b\) (Eq. (9)) into Eq. (6) and quickly find that they also satisfy Maxwell’s equations. Furthermore, \(E_{ib}\) and \(E_{ia}\) are also related to each other by Eqs. (8)–(9) because by definition \(E_{ib} = E_r\). The total electric and magnetic field for the scattering problem in Fig. 7(b) are thus linear superpositions of the fields described in Eqs. (8)–(9)

\[
\begin{align*}
E_{total} &= E_{ia} + E_b \\
H_{total} &= H_{ia} + H_b
\end{align*}
\]  

(10)

By substituting Eqs. (8) and (9) into Eq. (10), we can easily prove that the total field satisfy both Maxwell’s Eq. (6) and boundary condition for Eq. (7) at \(z = 0\). According to the uniqueness theorem, the solution described by Eq. (10) must be the same as the solution solved for configuration in Fig. 7(a). Thus, we have proven the two scattering problems illustrated in Figs. 7(a) and 7(b) are equivalent.

Since the new configuration (Fig. 7(b)) only consists particles in a homogeneous environment, we can apply conventional multipole analysis method directly. In this case, the multipole coefficients can be calculated from the electric fields in both the original particle and the image particle, with the incident field being the superposition of two counter propagating plane waves \(E_{ia}\) and \(E_{ib}\). Furthermore, since the fields above \(z = 0\) is the same for Fig. 7(a) and Fig. 7(b), the multipole coefficients calculated from fields in the original particle is also the same for the two configurations. We next find the relation between the multipole coefficients calculated from fields in the original particle and fields in the image particle, namely the relation between the original multipole coefficients and the image multipole coefficients.

Let us consider the equivalent configuration in Fig. 7(b). From the mirror symmetry relation in Eqs. (8)–(9), it can be understood that the total electric field described in Eq. (10) also satisfies mirror symmetry with respect to \(z = 0\),

\[
\begin{align*}
E_r(x, y, z) &= -E_r(x, y, -z) \\
E_\theta(x, y, z) &= -E_\theta(x, y, -z) \\
E_\varphi(x, y, z) &= E_\varphi(x, y, -z)
\end{align*}
\]  

(11)

The spherical components of the total electric field can be related to their Cartesian components as,

\[
\begin{align*}
E_r &= \sin \theta \cos \varphi E_x + \sin \theta \sin \varphi E_y + \cos \theta E_z \\
E_\theta &= \cos \theta \cos \varphi E_x + \cos \theta \sin \varphi E_y - \sin \theta E_z \\
E_\varphi &= -\sin \varphi E_x + \cos \varphi E_y
\end{align*}
\]  

(12)

With Eqs. (11)–(12), we can obtain the following relation in spherical coordinate,

\[
\begin{align*}
E_r(r, \theta, \varphi) &= -E_r(r, \pi - \theta, \varphi) \\
E_\theta(r, \theta, \varphi) &= E_\theta(r, \pi - \theta, \varphi) \\
E_\varphi(r, \theta, \varphi) &= -E_\varphi(r, \pi - \theta, \varphi)
\end{align*}
\]  

(13)
Let us denote the original multipole coefficients as $\alpha_E(l,m)$, $\alpha_M(l,m)$, and the image multipole coefficients as $\alpha'_E(l,m)$, $\alpha'_M(l,m)$. These can be calculated in the same way as Eqs. (4a)–(4c),

$$\alpha'^{(s)}_E(l,m) = \frac{(-i)^l l^2 \eta O_{lm}}{E_0 \pi (2l+1)^{1/2}} \int e^{-i m \phi} \left[ \varphi_i(kr) + \varphi'_i(kr) P_l^m(\cos \theta) \hat{r} \cdot J_{\text{scat}}(r) + \varphi''_i(kr) [\tau_{lm}(\theta) \hat{\theta} \cdot J_{\text{scat}}(r) - i \pi_{lm}(\theta) \hat{\varphi} \cdot J_{\text{scat}}(r)] \right] d^3 r$$

(14a)

$$\alpha'^{(s)}_M(l,m) = \frac{(-i)^l l^2 \eta O_{lm}}{E_0 \pi (2l+1)^{1/2}} \int e^{-i m \phi} j_i(kr) [i \pi_{lm}(\theta) \hat{\theta} \cdot J_{\text{scat}}(r) + \tau_{lm}(\theta) \hat{\varphi} \cdot J_{\text{scat}}(r)] d^3 r$$

(14b)

Note that in Eqs. (14a)–(14b), the integration for the original multipole coefficients is over the volume of the original particle, while the integration for the image multipole coefficients is over the volume of the image particle. By substituting Eq. (14c) into Eqs. (14a)–(14b), we have

$$\alpha'^{(s)}_E(l,m) = \frac{(-i)^l \omega k^2 \eta O_{lm}}{E_0 \pi (2l+1)^{1/2}} \int e^{-i m \phi} [\psi_i(r) - \psi_h] \left[ \varphi_i(kr) + \varphi'_i(kr) P_l^m(\cos \theta) E_0(r) + \varphi''_i(kr) [\tau_{lm}(\theta) E_0(r) - i \pi_{lm}(\theta) E_\varphi(r)] \right] d^3 r$$

(15a)

$$\alpha'^{(s)}_M(l,m) = \frac{(-i)^l \omega k^2 \eta O_{lm}}{E_0 \pi (2l+1)^{1/2}} \int e^{-i m \phi} j_i(kr) [\psi_i(r) - \psi_h] [i \pi_{lm}(\theta) E_0(r) + \tau_{lm}(\theta) E_\varphi(r)] d^3 r$$

(15b)

Since the integration spaces for the original and image multipole coefficients possess a one-to-one correspondence, each integration point $P(r, \theta, \varphi)$ in the original particle corresponds to an image integration point $P'(r, \pi - \theta, \varphi)$ in the image particle. With Eq. (13) and the following symmetry properties for $P_l^m(\cos \theta)$, $\tau_{lm}$, and $\pi_{lm}$

$$P_l^m(\cos(\pi - \theta)) = (-1)^{l+m} P_l^m(\cos \theta)$$

$$\tau_{lm}(\pi - \theta) = -(-1)^{l+m} \tau_{lm}(\theta)$$

$$\pi_{lm}(\pi - \theta) = -(-1)^{l+m} \pi_{lm}(\theta)$$

(16)

It can be easily shown that the image multipole coefficients and original multipole coefficients satisfy the following relation

$$\alpha'^E(l,m) = (-1)^{l+m} \alpha_E(l,m)$$

$$\alpha'_M(l,m) = (-1)^{l+m} \alpha_M(l,m)$$

(17)

Thus, we can define a new set of multipole coefficients as follows:

$$\alpha^{\text{mod}}_E(l,m) = [1 - (-1)^{l+m}] \alpha_E(l,m)$$

$$\alpha^{\text{mod}}_M(l,m) = [1 + (-1)^{l+m}] \alpha_M(l,m)$$

(18)

with $\alpha_E(l,m)$ and $\alpha_M(l,m)$ being the original multipole coefficients that can be calculated from the field distribution in the original particle in Fig. 7(a).

The scattering cross section is then given:

$$\sigma_{\text{scat}} = \frac{\pi}{2k^2} \sum_{l=1}^{\infty} \sum_{m=-l}^{l} (2l+1) |\alpha^{\text{mod}}_E(l,m)|^2 + |\alpha^{\text{mod}}_M(l,m)|^2$$

(19)

where a factor of 2 is added in the denominator to account for the fact that only the scattering into the upper half space is relevant.
Fig. 8. Multipole expansion of scattering cross section for an a-Si cuboid ($D = 1 \mu m$, $L = 3 \mu m$) above PEC calculated using modified multipole coefficients at different heights (a) $H = 0 \text{nm}$, (b) $H = 875 \text{nm}$, (c) $H = 1750 \text{nm}$, (d) $H = 5250 \text{nm}$, and calculated using conventional method under the equivalent configuration as described in Fig. 7(b) for (e) $H = 0 \text{nm}$, (f) $H = 875 \text{nm}$, (g) $H = 1750 \text{nm}$, (h) $H = 5250 \text{nm}$. Total scattering cross section denoted “Poynting method” is calculated by integrating the Poynting vector over the surface of the cuboid and dividing the result by the illumination intensity. The curve labeled “sum of poles” is the summation of scattering cross sections from all multipoles considered (i.e. up to octupolar).
A2. Multipole expansion of scattering cross section for an amorphous silicon (a-Si) cuboid above PEC at different heights

Using the modified multipole coefficients derived in A1, we study the scattering properties of an a-Si cuboid with thickness $D = 1 \, \mu m$ and side length $L = 3 \, \mu m$ that is placed at different distances ($H$) from a PEC. This is also considered in Fig. 1 of the main manuscript, but only the dipole contributions are plotted there. Figures 8(a)–8(d) show the multipole contributions to the scattering cross section when the cuboid is at distances of $H = \{0 \, nm, 875 \, nm, 1750 \, nm, 5250 \, nm\}$ above the PEC. The total scattering cross section is also calculated by the integral of the Poynting vector associated with scattered field over the surface of the cuboid, divided by the illumination intensity (“Poynting method”). Note that we only calculate the multipolar scattering cross section up to octupolar order. It can be seen that, for all cases considered, the contributions to the scattering cross section from the ED, MQ, and EO are strongly suppressed. On the other hand, scatterings from the MD, EQ and MO together account for nearly the entire scattering cross section. For the case $H = 5250 \, nm$, the sum of the multipolar scattering cross sections (up to octupolar order, orange solid line of Fig. 8(d)) is less than the total scattering cross section (blue dashed line, Fig. 8(d)). This is because higher order multipoles (i.e. beyond octupolar) are non-negligible for $H = 5250 \, nm$. The contribution to the scattering cross section from the MD becomes larger as the cuboid is brought closer to the PEC. Indeed, when the cuboid is on the PEC, the scattering cross section for wavelengths longer than $9 \, \mu m$ is dominated by the MD. In Figs. 8(e)–8(h), we plot the result of performing the multipole expansion by applying the conventional method to two cuboids (original and image) illuminated with counter-propagating plane waves, i.e. the equivalent configuration described in Fig. 7 We can see the results agree with those calculated using our modified multipole coefficients (Figs. 8(a)–8(d)).

A3. Electromagnetic simulations

A3.1. Scattering cross section of an a-Si cuboid above PEC

The calculation of scattering cross section of an a-Si cuboid above a PEC and its multipole expansion is done using a commercial finite element method software package (COMSOL). A three-step simulation is carried out in COMSOL. In the first step, we perform a full field simulation of a plane wave incident on an infinitely-extended PEC substrate without the cuboid and obtain the reflected field. Periodic boundary conditions are employed at the $x$- and $y$- boundaries of the simulation domain to mimic the infinite extent of the PEC. Perfectly matched layers (PMLs) are used at the $z$- boundaries. In the second step, we use the reflected field obtained in the first step as the background field and simulate the scattering of the a-Si cuboid above the PEC substrate with PMLs at all boundaries ($x$, $y$ and $z$). The obtained scattered field is used to calculate the total scattering cross section of the cuboid by integrating the Poynting vector over the surface of the cuboid and normalizing it to the illumination intensity. In the last step, we implement the multipole analysis model in COMSOL with our modified multipole coefficients to calculate the multipolar scattering cross sections based on fields calculated in former steps. At the time of writing, a simple multipole expansion model based on the conventional method is also available at the following Internet link:


A3.2. Reflection, absorption and phase shift simulation

The simulations of amplitude and phase of the reflection coefficient in the main manuscript are carried out using the finite difference time domain method with a commercial package (FDTD Solutions, Lumerical). Periodic boundaries are applied at the $x$- and $y$- boundaries to simulate the square lattice of the cuboids. PMLs are used at the $z$- boundaries to avoid unwanted backscattering of the electromagnetic field. Illumination is from a plane wave at normal incidence.
For the phase shift calculation, the top surface of the cuboids is chosen as the reference plane. The simulations of absorption spectra for monolayer graphene placed on magnetic mirrors are performed in FDTD, where the graphene is modeled based on a surface conductivity approach. The scattering rate and chemical potential of graphene are taken as 0.0427 eV and 0.1895 eV respectively in the simulation.

**A4. Fabrication of long-wave infrared magnetic mirror**

Figure 9 shows the fabrication process for the magnetic mirror. The fabrication starts with deposition of Ti (10 nm), Au (100 nm) films by electron beam evaporation (Intlvac Nanochrome II Electron Beam and Thermal Evaporation System) and deposition of a-Si (1.1 µm) film by Plasma-enhanced chemical vapor deposition (PECVD, Oxford Plasma Lab100) on a silicon wafer. The cuboid arrays are then patterned in a PMMA layer (A4, 950 K, 200 nm thick) by electron beam lithography (Vistec EBPG5000 Plus EBL) and developed in 3:1 isopropanol–methyl isobutyl ketone (MIBK, 60 seconds). After that, Al is deposited to a thickness of 30 nm by evaporation and lift-off is performed, resulting in the etching mask. The sample is then etched by inductively coupled plasma reactive ion etching (Oxford Plasma Lab100 (ICPRIE)-General Etch) for 3 minutes and 25 seconds. The following gases are used: sulfur hexafluoride (SF6, 40 sccm) and perfluorocyclobutane (C4F8, 90 sccm). The RF and ICP powers are 30 W and 1200 W, respectively. The remaining Al mask is etched away in Al etchant for 10 minutes.

![Fig. 9. Schematic of the fabrication process for magnetic mirror structures.](image)

**A5. Interferometry measurements**

We use a home-built infrared Michelson interferometer as shown in Fig. 10(a) to measure the phase shift of light upon reflection from our magnetic mirror samples. A continuous-wave Fabry-Perot quantum cascade laser (QCL, ThorLabs) with center wavelength 8.4 µm is used as the light source. A ZnSe lens with a focal length of 100 mm is used to focus the laser beam through a ZnSe beam splitter (1 inch) onto the sample. The scanning Au mirror is mounted on a one-axis translation stage (ThorLabs XR25P-K2/M) driven by a Thorlabs PIA50 piezo inertial actuator. According to the manufacturer, this has a typical step size of 20 nm with a variation ~20%. The sample is mounted on another 3-axis motorized linear translational stage (Thorlabs LNR50S) with bidirectional repeatability of 0.3 µm and backlash less than 6 µm, which is sufficient for sample positioning. A liquid nitrogen cooled MCT detector from Infrared Associates is used to detect the light from the interferometer. A source meter unit (Keithley 2450) is used to provide bias voltage to the MCT detector and to measure the photocurrent. Control over the translation stages, source supply and current readout is achieved using a Matlab control script. Figures 10(b)–10(d) show the interferograms measured for a gold reference sample and six magnetic mirror samples.
with cuboid thickness $D = 1.1 \, \mu m$, and cuboid side length $L$ ranging from $1.4 \, \mu m$ to $2 \, \mu m$. To avoid hysteresis of the piezo inertial actuator, the scanning mirror is scanned in one direction for all measurements. A height difference of $1.1 \, \mu m$ between the gold reference sample and the magnetic mirror samples is taken into account in the phase determination.

![Fig. 10.](attachment:image)

**Fig. 10.** (a) Schematic of the infrared Michelson interferometer setup for the phase measurement. (b-d) Measured interferogram for a gold reference sample (b) and magnetic mirror samples with cuboids ($D = 1.1 \, \mu m$, $L = 1.4 \, \mu m$, $1.5 \, \mu m$, $1.6 \, \mu m$) in (c) and cuboids ($D = 1.1 \, \mu m$, $L = 1.7 \, \mu m$, $1.85 \, \mu m$, $2 \, \mu m$) in (d).

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The authors declare no conflicts of interest.

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