Supporting Information for
“Generalized method of images and reflective color
generation from ultra-thin multipole resonators”

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Overview
In the first section of this Supporting Information document, we derive the expressions for the effective multipole coefficients that are presented in the main manuscript. These constitute the generalized method of images, an intuitive way for understanding the scattering of light by an individual nanoparticle on a conductive substrate. We demonstrate the application of this approach to understanding several physical phenomena that occur for scattering from nanoparticles on conductive substrates, including substrate-induced magnetic resonances. We evaluate the applicability of this method to real metals with finite conductivity, i.e. which are not perfect electrical conductors (PECs). These metals consist of gold, silver and aluminum.

In the second section, we consider the case for which the nanoparticles are not formed in regular arrays, but rather with random positions in ensembles. We study through experiments what effect this randomness has upon the color the ensembles exhibit. We conclude that color can be understood as arising chiefly from the properties of single disks, rather than being an outcome of the disks being formed in ordered arrays.

Derivation of the effective multipole coefficients

In solving scattering problems with the multipole expansion, the scattered waves are treated as comprising radiation from currents induced in the scatterer. When the object is small, this radiation can be approximated by dipole radiation. As the object’s size parameter increases, however, higher-order radiation sources need to be considered. This is done by decomposing the electric field scattered from the object in a basis comprising vectorial spherical harmonics about a chosen origin. This is often called the Mie or multipole expansion. For arbitrarily-shaped particles, the electric field in the object induced by the incident light is treated as a volume current source that in turn generates the scattered radiation, meaning that the expansion can be performed through numerical integration. However, the application of multipole expansions to analyze scattering from an object on a surface is still limited, as the interaction between the object and the substrate is complex, for example sometimes leading to an effect that has been termed magnetolectric coupling. Hence, it is of interest to the nano-optics community to explore an accurate way to describe multipole scattering from an object at an interface.

In this section, starting from the Dirichlet boundary condition, we derive an orthogonal form (i.e. free of coupling among the poles) of the multipole description of scattering from an object on a perfect electrical conductor (PEC) substrate. We then generalize our method to practical substrates, i.e. those with finite conductivity. We test our formulation with gold, silver, and aluminum.
**Effective multipole coefficient for scattering by nanoparticle on a perfect electrical conductor.** We first consider the case of a silicon nanorod sitting on a PEC. This case is relatively simple, facilitating physical intuition, but is also of practical importance as many metals are reasonably approximated as PECs, especially at long wavelengths.\textsuperscript{17} We choose to consider silicon nanorods because the high refractive index of this material at visible and near-infrared wavelengths means that, in addition to the fundamental electric dipole mode, higher order multipole resonances such as the magnetic dipole occur.\textsuperscript{18}

The electric dipole and the magnetic dipole are the two modes of interest here.\textsuperscript{19} They occur at similar wavelengths because they are the two lowest order modes of the multipole expansion. It can be difficult to separate these modes.\textsuperscript{18} However, this can be achieved when the resonant particle is sitting on a PEC substrate, as we consider below. Normally-incident plane-wave illumination of a particle on a substrate induces dipoles oriented in the plane of the substrate. In the method of images, the fields generated by a dipole above a PEC are written as the sum of the fields generated by the original dipole and those generated by a virtual (image) dipole source.\textsuperscript{20} The image dipole is located at the same distance below the PEC surface as the original dipole is above it. With this in mind, we consider the electromagnetic simulations of Figure S1(a). The instantaneous electric field distributions are calculated for a silicon sphere in air (radius 100 nm, Figure S1(a-I)), a silicon rod in air (radius 70 nm, height 190 nm, Figure S1(a-II)) and a silicon rod above a PEC substrate (radius 70 nm, height 95 nm, Figure S1(a-III)). Inspection of the field patterns of Figure S1(a-I, II) indicates that they are very similar to those of magnetic dipole resonances, as illustrated schematically in the upper half of the Figure. We now consider the rod above the PEC. Inspection of Figure S1(a-III) reveals that the field distribution resembles that of the upper half of the rod in air (Figure S1 (a-II)). This is consistent with the formation of image electric and magnetic dipoles that are antiparallel and parallel to the original electric and magnetic dipoles, respectively. This leads to the cancellation of the electric dipole and the formation of a pure magnetic dipole.

The method of image dipoles for the PEC originates from the uniqueness theorem and the solution to the scalar potential for Dirichlet boundary conditions:

\[
\Phi(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \int_S \rho(\vec{r'}) G_D(\vec{r},\vec{r'}) d\vec{r'} - \frac{1}{4\pi} \int_S \Phi(\vec{r'}) \frac{\partial G_D(\vec{r},\vec{r'})}{\partial n'} ds',
\]

(1)

The first term on the right-hand side is the potential due to the charges above the boundary, where \(\rho(\vec{r'})\) is the charge density at a position \(\vec{r'}\) above the PEC and \(G_D(\vec{r},\vec{r'})\) is a Green’s function. The second term is an integration of surface potential at the boundary. This means that the contribution to the potential from the other side of the boundary is completely specified by the potential at the boundary, regardless of the details of the charge distribution on the other side. This confirms that the method of images is rigorous. The problem then becomes that of finding the image charge distribution on the other side of the boundary that will produce the same \(\Phi(\vec{r'})\) on the surface \(s\) of the original problem.
We note that \( \Phi(\vec{r}') \) is a constant on the PEC surface. We are then able to perform the multipole expansion to find the strength of each pole and, using the boundary condition, the strengths of the corresponding image poles. We choose the origin of the expansion to be on the PEC surface, so the source and image poles will be co-located. The overlapped origins facilitate a definition of what we call the **effective multipole coefficients**, which is a set of coefficients for point multipole sources with the image multipoles taken into account.

We start by analyzing the magnetic field generated by individual magnetic multipole sources. For an arbitrary multipole source, the electric field due to magnetic multipole is expressed as \( \vec{E}^m(l, m) = Z g(l) \vec{L} (Y_{l,m}) \), where \( Z \) is the impedance of the medium, \( g(l) \) is the radial component of the solution of order \( l \), \( Y_{l,m} \) is the spherical harmonic of order \( l \) and degree \( m \), and \( \vec{L} \) is the orbital angular-momentum operator, defined as \( \vec{L} = \frac{1}{i}(\vec{r} \times \nabla) \).

The spherical harmonics \( Y_{l,m} \) is

\[
Y_{l,m} = \frac{\sqrt{2l+1} \begin{pmatrix} \frac{l+m}{2} \\ \frac{l-m}{2} \end{pmatrix}}{4\pi (l+m)!} P^m_l(\cos \theta) e^{im\phi}
\]

, where \( P^m_l(\cos \theta) \) is associated Legendre polynomial.

We can expand the term on the right-hand side of the equation to write the electric field in Cartesian coordinates explicitly,

\[
\vec{E}^m(l, m) = Z g(l) \begin{cases}
0.5 \begin{pmatrix} \sqrt{(l+m)(l-m+1)} Y_{l,m-1} + \sqrt{(l-m)(l+m+1)} Y_{l,m+1} \\ \sqrt{l+m} \sqrt{l-m} \end{pmatrix} \hat{x} \\
0.5i \begin{pmatrix} \sqrt{(l+m)(l-m+1)} Y_{l,m-1} - \sqrt{(l-m)(l+m+1)} Y_{l,m+1} \\ \sqrt{l+m} \sqrt{l-m} \end{pmatrix} \hat{y} \\
- i m Y_{l,m} \hat{z}
\end{cases}
\]

, where the spherical harmonic terms \( (Y_{l,m-1} \text{ and } Y_{l,m+1}) \) are the only terms with angular dependence (i.e. have \( \theta \) and \( \phi \) as independent variables). The Dirichlet boundary condition requires that the \( y \) and \( z \) components of the electric field vanish at \( \phi = \pi/2 \) and \( 3\pi/2 \), for the choice of coordinates in which the PEC lies in the \( y-z \) plane and the \( x \) axis is normal to its surface. With this condition, we find that the image pair term of the magnetic multipole term \( (l, m) \) is the term \( (l, -m) \).

We now move forward to find the image sources for the electric multipoles. Instead of finding the electric field directly for electric multipoles, it is simpler to evaluate the scalar product between the unit position vector \( \vec{r} \) and the electric field \( \vec{E} \) generated by each multipole source. The Dirichlet condition implies that \( \vec{r} \cdot \vec{E} \) needs to be zero when \( \vec{r} \) is parallel to the PEC boundary, i.e. when \( \phi = \pi/2 \) or \( 3\pi/2 \) (Figure S2). Furthermore, the electric multipole field due to each pole satisfies the following equation
\[ \vec{r} \cdot \vec{E}^e(l,m) = -Z \frac{l(l+1)}{k} f(l) Y(l,m), \]

where the radial dependence is included in the term \( f(l) \). Similar to magnetic multipoles, we find that the image pair term of the multipole with the order \((l, m)\) is the term \((l, -m)\), but of opposite sign.

We name a new set of multipole coefficients, termed the **effective multipole coefficients**, by taking the image multipoles into consideration. We express these in terms of the original multipole coefficients using the following formulae,

\[
\begin{align*}
a^e_{\text{eff}}(l,m) &= -a^e_{\text{eff}}(l,-m) = a^e(l,m) - a^e(l,-m) \quad (2) \\
a^m_{\text{eff}}(l,m) &= a^m_{\text{eff}}(l,-m) = a^m(l,m) + a^m(l,-m) \quad (3)
\end{align*}
\]

As an example, the Cartesian dipole moments can then be expressed with the multipole coefficients obtained as follows,\(^\text{21}\)

\[
\begin{align*}
p_x &= C_1 \left( a^e_{\text{eff}}(1,-1) - a^e_{\text{eff}}(1,1) \right) = 2C_1 \left[a^e(1,-1) - a^e(1,1) \right] \\
p_y &= -iC_1 \left( a^e_{\text{eff}}(1,-1) + a^e_{\text{eff}}(1,1) \right) = 0 \\
p_z &= \sqrt{2}a^e_{\text{eff}}(1,0) = 0 \\
m_x &= cC_1 \left( a^m_{\text{eff}}(1,-1) - a^m_{\text{eff}}(1,1) \right) = 0 \\
m_y &= cC_1 \left( a^m_{\text{eff}}(1,-1) + a^m_{\text{eff}}(1,1) \right) = -icC_1 \left[a^m(1,1) + a^m(1,-1) \right] \\
m_z &= \sqrt{2}cC_1 a^m_{\text{eff}}(1,0) = 2\sqrt{2}cC_1 a^m(1,0)
\end{align*}
\]

, where \( C_i = \frac{-3\pi E_i}{ik^3} \) and \( E_i \) is the incident field strength. These results agree with the method of images for dipoles.\(^\text{20-21}\)

In Figure S1(b), we plot the contributions to the scattering cross-section from ED and MD components for the nanorod in air (full length, i.e. height is 190 nm) and for the nanorod sitting on the PEC surface (half-length, i.e. height is 95 nm). The contributions of different poles are calculated as follows for the nanorod in air:\(^\text{6}\)

\[ C^{e,m}(l) = \frac{\pi}{k} \sum_{m=-l}^l \left( 2l+1 \right) | a^{e,m}(l,m) |^2 \]

For the case of the nanorod on the PEC, Eqn (3) from the main text is used with the effective multipole coefficients. The total cross-section is the incoherent summation of all orders of multipoles, as the basis of the effective multipole expansion is the same as the orthogonal basis of the direct multipole expansion.

For comparison, we calculate the scattering cross-section \( (C_{\text{Scat}}) \) by integrating the Poynting vector of the scattered field at the surface of the nanorod,
\[
C_{\text{Scat}} = \frac{\hat{S}_s \cdot \hat{n} da}{I} = \frac{\int \text{real}(\vec{E}_s \times \vec{H}_s^*) \cdot \hat{n} da}{\frac{1}{2} \sqrt{\epsilon_r \epsilon_i |E_i|^2}}
\]

, where \( \hat{S}_s \) is the time-averaged Poynting vector of the scattered fields on the surface of the particle, \( \hat{n} \) is the surface normal, and \( da \) is the differential surface area. The scattered electric field \( \vec{E}_s \) is defined as the difference between total electric field \( \vec{E}_t \) and the incident electric field \( \vec{E}_i \) (i.e. \( \vec{E}_s = \vec{E}_t - \vec{E}_i \)). The scattered magnetic field \( \vec{H}_s \) is defined in the same way (i.e. \( \vec{H}_s = \vec{H}_t - \vec{H}_i \)). The incident electric and magnetic fields are those that would occur in the absence of the particle. \( I \) is the intensity of the incident light. It can be seen from Figure S1(b) that there is good agreement between the scattering cross section found by direct integration, and that found by considering the contributions of the ED and MD. This confirms the accuracy of our approach. The multipole coefficients are obtained by the integration of the scattering current in the particle, which is defined as

\[
\vec{J}_s = -i \omega \epsilon_0 (\epsilon_r - \epsilon_i) \vec{E}_s,
\]

where \( \epsilon_r \) and \( \epsilon_i \) are the relative permittivities of the particle and of the medium in which it is embedded, respectively. The detailed formulation is described in details in the references\textsuperscript{6-8}. The coefficients are obtained through the following equations:

\[
a^e(l,m) = \frac{k^2}{E_0 i \sqrt{l(l+1)}} \int \left\{ Y_{lm}^* \left( \nabla \cdot \vec{J}_t \right) \frac{d}{dr} \left[ r f_l (kr) \right] + ik (\vec{r} \cdot \vec{J}_t) f_l (kr) \right\} dr
\]

\[
a^o(l,m) = \frac{k^2}{E_0 i \sqrt{l(l+1)}} \int \left\{ Y_{lm}^* \nabla \cdot (\vec{r} \times \vec{J}_t) f_l (kr) \right\} dr
\]

, where \( k \) is the modulus of the wavevector, \( r \) is the radial distance from the origin of integration. The vector \( \vec{J}_s \) may be obtained by many methods and we use COMSOL FEM solver here. To calculate the scattering cross-section, the multipole coefficients are normalized by the electric field strength of the incident wave \( E_0 \).

To test the generality of our method, we reduce the symmetry by considering the case where the incident plane wave is tilted to an angle of 60 degrees from the normal. We also rotate the cylindrical nanorod about a randomly chosen axis (Figure S2(a)). The sum of the effective multipole coefficients (dipoles and quadrupoles only) still matches perfectly with the scattering cross section \( C_{\text{Scat}} \) found by direction integration of the scattered field (Figure S2(b)). On the other hand, the total scattering cross-section calculated from the original multipole expansion deviates significantly from \( C_{\text{Scat}} \), shown in Figure S2(c). This deviation can be explained as the quenching of radiation sources by the PEC boundary, which is not taken into account by the original multipole expansion.
Extension to conductive surfaces with the loss. We next consider the case of the substrate having finite conductivity (i.e. not a PEC). We must therefore modify the virtual radiation sources to take into account losses in the substrate. We postulate the following generalized formulae,

\[
\alpha_{\text{eff}}^e(l,m) = a^e(l,m) - \frac{\varepsilon_s - \varepsilon_1}{\varepsilon_s + \varepsilon_1} a^e(l,-m) \tag{4}
\]

\[
\alpha_{\text{eff}}^m(l,m) = a^m(l,m) + \frac{\varepsilon_s - \varepsilon_1}{\varepsilon_s + \varepsilon_1} a^m(l,-m) \tag{5}
\]

where \(\varepsilon_s\) is the relative permittivity of the substrate and \(\varepsilon_1\) is the relative permittivity of the medium in which the scattering particle is situated. It can be seen that these equations are an extension of the image dipole approximation for a planar interface (see e.g. Ref [22]). These equations are consistent with Equations (2) and (3). That is, as \(\varepsilon_s\) approaches \(-\infty\) in the limit in Equations (4) and (5), i.e. for a PEC substrate, we recover Equations (2) and (3). Furthermore, the original result for multipole expansion in a homogeneous environment is obtained when we set \(\varepsilon_s = \varepsilon_1\), as the second terms on the right-hand sides of Equations (4) and (5) are then equal to zero. We conclude that this approach behaves well asymptotically. Note that in the latter case (i.e. setting \(\varepsilon_s = \varepsilon_1\)), the origin of the multipole expansion is at the edge of the scattering object, which is not favorable as it is not at the center of symmetry. We denote \(\frac{\varepsilon_s - \varepsilon_1}{\varepsilon_s + \varepsilon_1}\) as \(\Gamma_i\), termed the image strength.

Using the above prescription, we calculate light scattering from a silicon sphere. The permittivity values of gold, silver, and aluminum used here are from Palik.\textsuperscript{22} In Figure S3(a-c), we show the contributions to the scattering cross-section from the multipole components using Equations (4) and (5) for gold, silver and aluminum, respectively. We furthermore show the sum of the scattering cross section contributions from the multipoles, up to the octopole terms. The sum of multipole scattering cross sections is in general agreement with \(C_{\text{Scat}}\) computed by direct integration of the fields on the nanoparticle surface. For each metal considered, both curves show two spectral peaks, whose locations are in good agreement. It can be seen however that for all of the metals, the multipole scattering sum is smaller than \(C_{\text{Scat}}\). The contributions of the octopole are small, and we thus believe that the fact that the higher multipoles are not included in the summation is unlikely to be the reason for the difference between the two approaches. However, we note that some of the field scattered by the particle is absorbed by the metal substrate and not radiated to the far field. This is taken into account by the effective multipole method but ignored when directly integrating the Poynting vector on the surface of the nanoparticle. Comparison between the sum of multipole contributions and the scattering cross-section calculated from direct integration of the Poynting vector on a hemisphere whose radius is one wavelength (\(C_{\text{Scat,farfield}}\)) shows better agreement.

For comparison, we also calculate the PEC case (Figure S3(d)). It can be seen that the results for the PEC are in broad agreement with those for the real metals (Figure S3(a)-
(c)), but with some differences that are most pronounced for the gold substrate. The resonances of the latter are significantly broader than the PEC case. The electric quadrupole mode is significant and peaks at $\lambda = 470 \text{ nm}$ for the PEC case, as indicated by the dashed vertical line in Figure S3(d). For the gold substrate case, the electric quadrupole mode is weaker and is red-shifted (again indicated by dashed vertical line, Figure S3(a)). The instantaneous electric field distribution for the PEC-substrate case is shown as Figure S3(e). This is calculated for illumination at the wavelength of the electric quadrupole resonance peak, as denoted by both the vertical dashed line and the letter “A” in Figure S3(d). It can be seen that the field distribution has a quadrupole nature, and is complemented by its image pole.

For the gold substrate case, there is some electric dipole component in the spectral range between 450 nm and 500 nm that is not present in the PEC case. We note that these differences between the gold and PEC cases are at spectral locations for which the gold has high loss and where the image strength has a large imaginary component (Figure S3(f)). It is further confirmed by that the broadening effect is less in silver and in aluminum than in gold. In the case of aluminum, the multipole expansion result agrees well with $C_{\text{Scat}}$. This is consistent with the fact that, for aluminum, the image strength $\Gamma_i$ is approximately unity across the visible spectrum (Figure S3(f)).

In summary, we formulate a multipole expansion method to account for the scattering of a particle on a conductive substrate. We verify the physical validity of this method by considering individual particles and a particle array. We furthermore extend our approach beyond perfect electrical conductor substrates to those with finite conductivity. In particular, we examine the predictions of our model for gold, silver and aluminum substrates at visible wavelengths. The effective multipole coefficients we obtain are consistent with the scattering cross sections predicted by integration of the Poynting vector. We find that the best agreement is obtained for the case an aluminum substrate. Our multipole expansion method is based on considering the fields internal to the particle as being a scattering current source. Provided that these internal fields are known (e.g. by numerical simulation such as FDTD or FEM), further consideration of the illumination conditions is unnecessary.

By the same token, the results for a scatterer on a PMC surface can be easily obtained. We present the result here for completeness of this discussion:

$$a_{\text{eff,PMC}}^e(l,m) = a^e(l,m) + a^e(l,-m)$$  \hspace{1cm} (6)
$$a_{\text{eff,PMC}}^m(l,m) = a^m(l,m) - a^m(l,-m)$$  \hspace{1cm} (7)

**Design, Fabrication, and Evaluation of Randomized Arrays**

The nano-disks we present in the main manuscript are in square arrays whose periods are 500 nm and smaller. Such arrays are relatively straightforward to fabricate and to simulate. However, this leads to the question of whether lattice modes are important in determining the color that an ensemble of nano-disks exhibit. To address this question, we here present studies of randomized arrays and compare them with their periodic counterparts. We conclude that the contribution of lattice modes to the color of the arrays is not substantial.
The fabrication process for the randomized arrays is the same as that used for the ordered arrays, as presented in the main manuscript. The pattern used in the electron beam lithography step is produced as follows. We begin by generating square arrays of nano-disks with five different filling fractions, ranging from 0.05 to 0.25 in steps of 0.05. Each disk in the each array is then shifted along the two principal axes of the array by a random step size that is uniformly distributed over the range \(-p/2\) to \(p/2\), where \(p\) is the pitch of the original square array.

Atomic force microscope (AFM) images of ordered (square lattice) and randomized nano-disk arrays are shown as Figure S4(a) and S4(b), respectively. The Fourier transforms (FTs) of these AFM images are shown as Figure S4(c) and S4(d). It can be seen that the FT pattern of the ordered array exhibits a dot array, while the pattern from the randomized array only possesses broad ring patterns. This implies that these arrays have short range order, which is consistent with the algorithm used to generate them. Note that anisotropy in the FTs of randomized arrays is due to the artefacts that arise from the fact that the tip-sample convolution that occurs in the AFM imaging process is dependent on the scan direction.\(^{23}\)

In Figure S4(e) to S4(g), we show AFM images of arrays of three different densities and their FT patterns. Due to the fact that, for the high density arrays, the disks are shifted by smaller distances, the array whose filling fraction is 0.25 is more ordered. Nevertheless, the FT pattern shows no dot arrays.

In Figure S5, we show the reflectance spectra measured for the randomized arrays of disks with different filling fractions, together with images obtained with a bright-field optical microscope. We also plot the spectra and images for the ordered arrays under the same lighting conditions and microscope settings. Five different sets of arrays are shown. The nano-disks vary in radius from 75 nm to 150 nm, in increments of 15 nm. It can be seen that the positions of the reflectance dips are consistent between arrays of nano-disks of the same radius at different filling fractions. It can be seen from the optical microscope images that the color saturation of the arrays decreases as the filling fraction decreases. It can also be seen that the resonances red-shift as the filling fraction decreases, a consequence of the weaker near-field coupling between nano-disks. From the spectra and visual appearance of the arrays under the microscope, we can conclude that the lattice resonances as shown in the main text do not play a significant role in determining the color. Because the color is not a lattice resonance effect, the color saturation of the arrays can be varied by changes to the filling fraction.

**Understanding the selection and re-ordering of multipoles that occurs with the introduction of the PEC using the generalized method of images**

As discussed in the main manuscript, when a scatterer is placed on a PEC surface, the multipoles can experience different effects. Some multipoles can be eliminated, while others are unaffected. In addition, a resonance that occurs in the response of a certain multipole in the isolated scatterer case can appear in the response of a different multipole
when the scatterer in placed on a PEC surface. Here we provide some physical insight into this process by considering a few examples of multipolar excitations (a quadrupole and an octupole) and breaking them down into electric current sources. \(^{24}\) We begin by considering a quadrupole comprising two antiparallel time-harmonic electric current sources (red arrows of Figure 6-S(a)). We assume that these currents are located infinitesimally close to the surface of the PEC. This is consistent with our method, as it integrates the current sources about an origin that is on the PEC. It can be seen that the combination of the currents and their images will not radiate, as each current source cancels with its image current. This implies that these quadrupole mode will be eliminated with the presence of the PEC. We next consider an octupole (Figure 6-S(b)). It can be seen that some of the current sources orient in the plane of PEC but some of the sources orient out of the plane. The in-plane current sources cancel with their images, while the out-of-plane sources interfere constructively with their images. The resulting radiation pattern is then as if it is from a quadrupole polarized out of the plane.
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Figure S1. **Concept and numerical results of dipolar mode of a particle scattering on PEC.** (a) Schematic illustrations (upper panels) and field distributions (lower panels) of magnetic resonances induced by normal incidence illumination (propagation direction: -x) of (I) Si sphere (100 nm radius, at wavelength of $\lambda = 772 \text{ nm}$) in air, (II) Si nanorod (70 nm radius, 190 nm height, at wavelength of $\lambda = 590 \text{ nm}$) in air, (III) Si nanorod (70 nm radius, 95 nm height, at wavelength of $\lambda = 590 \text{ nm}$) on PEC. Illumination is polarized in z direction. Field distributions are found by finite element modelling. Arrows and colors show direction and magnitude of instantaneous electric field, respectively. (b) Partial scattering cross sections for ED and MD components, calculated for the configurations (II) and (III) of panel a. Total scattering cross section is also plotted.
Figure S2. **Comparison between effectively multipole expansions and the original multipole expansion of light scattering by an arbitrary particle on PEC by tilted incident light.** (a). Schematic illustration of coordinate system chosen in this work. PEC boundary is $yz$ plane, depicted as brown-yellow surface. Illumination is at 60 degrees from the normal, with plane of incidence being $xy$ plane. Grey object at origin of coordinate system is part of an arbitrarily-oriented nanorod. Orientation of the nanorod is as follows. Nanorod axis is originally along $x$-direction, but it is then rotated by 27 degrees about an axis that is defined by a vector that includes the origin and the point with coordinates $(0.76, 0.4, 0.1)$. (b). Partial scattering cross sections for the effective multipole terms and their summation. Total scattering cross section found by Poynting
vector integration is also shown. (c) Partial scattering cross sections found for the multipole terms by the original (traditional) method and their summation. Total scattering cross section (via Poynting vector integration) is also shown.

Figure S3. Numerical results of effective multipole expansions on different conductive substrates. (a-d) Contributions to scattering cross-section from different multipoles (up to octopoles, \( l = 3 \)) for silicon nanosphere (radius is 65 nm) on (a) gold, (b) silver, (c) aluminum, and (d) PEC surfaces. Red solid curve (“total”): summation of multipole contributions to scattering cross-section. Thick light blue solid curve (“\( C_{\text{scat}} \)”: scattering cross section found by integration of Poynting vector over nanosphere surface. Orange solid curve (“\( C_{\text{scat, Farfield}} \)”: scattering cross section found by direct integration of Poynting vector over hemisphere with radius equal to one wavelength. (e) Electric field distribution for nanosphere on PEC, calculated for normal incidence illumination at wavelength of quadrupole peak (denoted “A”) of panel d. (f) Real and imaginary parts of image strength \( \Gamma_I \) as a function of wavelength for the three different metals.
Figure S4. AFM images of periodic and randomized silicon particle arrays. (a). Atomic force microscope (AFM) image of ordered array of silicon nano-disks on aluminum substrate. Nano-disks have radii of 150 nm and are on a square array of pitch 500 nm, so the filling fraction is FF~0.2827. (b). AFM image of randomized array (FF=0.2). Nano-disks have radii of 150 nm. (c). Fourier transform (FT) of array of panel a. (d). FT of array of panel b. (e)-(g). AFM images (left) and FT patterns (right) of randomized arrays with three different filling fractions (0.05, 0.15, 0.25).
Figure S5 **Spectra and optical images of arrays with different densities.** In each of the sub-figures from (a) to (e), we plot the reflectance spectra of arrays of randomly distributed nano-disks with five different filling fractions (FF ranges from 0.05 to 0.25, in steps of 0.05) together with the reflectance spectrum from an ordered array with F~0.283. The radii of the disks increase from (a) to (e), from 75 nm to 150 nm, in steps of 15 nm. In the insert of each sub-figure, a bright-field optical microscope image of the arrays is shown. The array FFs increases from top (0.05) to bottom (0.283).
Figure S6 **Schematics of multipole mode selection and transformation.** (a) Illustration of a quadrupole configuration that is supported in free space, but is eliminated with the inclusion of the PEC. (b) Illustration of a current configuration that is octupolar in free space, but strongly modified with the inclusion of the PEC. It can be seen that the in-plane currents are cancelled by their images, while the out-of-plane components interfere constructively with their images. This transforms the radiation pattern into being that of a quadrupole. The grayish blocks on the left panels denote the semi-infinite PEC. The semi-transparent arrows on the right panels represent the image currents.
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