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A PROBABILISTIC MODEL FOR TIME TO COVER CRACKING DUE TO CORROSION

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Running Head: Time to Cover Cracking

ABSTRACT

The time to cover cracking is widely used as a service life indicator in the assessment of deterioration of corrosion-affected reinforced concrete structures. This paper presents a general probabilistic procedure for prediction of time to cover cracking. Within this procedure, the Response Surface Method (RSM) is employed to calibrate a new model for calculating of radial displacement required for cover cracking based on the results obtained from the Finite Element (FE) analysis. By taking advantage of the Central Limit Theorem (CLT), simple but accurate probabilistic models for prediction of time to cover cracking that only rely on the knowledge of first and second moments of basic random variables are derived. Rigorous simulation analysis has proved the accuracy of these models. It is shown in the paper that using this probabilistic procedure, factors affecting randomness of time to cover cracking can be easily identified. It is also shown that the time to cover cracking is highly variable with the concrete

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cover, tensile strength of concrete, corrosion current density and the model error as the most influential factors on its randomness.

KEYWORDS

Concrete; Corrosion; Time to Cover Cracking; Probabilistic; Service life.

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INTRODUCTION

Corrosion of reinforcing steel in concrete is a worldwide problem costing managing agencies considerable expense and effort in repair and remediation (Tilly 2007; Whitmore and Ball 2004). The corrosion products, which occupy much greater space than the original iron exert pressure on the steel-concrete interface leading to cracking of concrete cover. It appears that appearance of corrosion-induced surface cracks is an aesthetics issue affecting the normal operation of a structure and does not significantly influence the strength of the structure (Val et al. 2009). Therefore, it is widely accepted that the corrosion-induced cover cracking can be identified as a serviceability limit state (Li 2004; Vu et al. 2005), and appearance of corrosion-induced crack is commonly considered as an indicator for service life prediction of concrete structures (Chen et al. 2018; El Maaddawy and Soudki 2007; Li 2004; Thoft-Christensen 2001; Val et al. 2009). Violation of this limit state warrants minor repair actions that may include local removal of damaged cover and patching. On the other hand, reduction of reinforcement area and steel-concrete bond due to corrosion leads to reduction of structural strength, and it can be classified as an ultimate limit state (Li and Melchers 2005). Recently, Bezuidenhout and van Zijl (2019) proposed that both the serviceability and the ultimate limit states can be expressed in terms of loss of reinforcement cross-sectional area. They proposed that service life is expressed as the time before the reinforcement diameter reaches 15% loss of cross-sectional area.

Prediction of time to cover cracking (following corrosion initiation) is of great interest for asset managers, and as such, in this study, it has been as an indicator for service life prediction. In many proposed models the time to cover cracking is related to variables such as mechanical properties of concrete, concrete cover and environmental factors represented by corrosion rate (Bazant 1979; Bhargava et al. 2006; Chernin et al. 2010; El Maaddawy and Soudki 2007; Li et al. 2006; Pantazopoulou and Papoulia 2001). There are also numerical models in which more complex features such as non-uniform and time-dependent creep analysis have been considered (Jang and Oh 2010; Thybo et al. 2017). Most of the available models for time to cover cracking are presented in deterministic frameworks. However, high variability of factors affecting the corrosion-induced cover cracking indicates that prediction for corrosion-induced cracking should be based on a probabilistic procedure in which variability of all the influential variables is considered. Papakonstantinou and Shinozuka (2013) simulated the corrosion process in a probabilistic procedure in which the random field theory was used to find the extent of corrosion-damaged area. Lu et al. (2017) incorporated the analytical model originally proposed by El Maaddawy and Soudki (2007) within a probabilistic procedure based on the Monte Carlo simulation. Shao et al. (2018) developed a probabilistic approach for evaluating the lifetime of reinforced concrete pipes considering corrosion initiation and crack initiation processes using the Monte Carlo simulation.

All the above-mentioned probabilistic models on corrosion-induced concrete cover cracking are based on simulation. In all these models, uncertainty associated with the model error has not been considered. Furthermore, evaluation of thickness of porous ring at the steel-concrete interface in these studies is based on rudimentary assumptions. This paper aims at addressing these shortcomings by developing a comprehensive probabilistic procedure for prediction of time to cover cracking. Based on the Response Surface Methodology (RSM) and with the aid of the Central Limit Theorem (CLT) and considering variability in all the basic random variables, analytical probabilistic models are developed, in which only the knowledge of first two moments of basic random variables is required.

It is assumed that corrosion rust grows uniformly around steel bar. Nonetheless, the developed RSM can be extended to include non-uniform distribution of corrosion rust. This is outside the scope of this paper where the focus is on development of a probabilistic model for time to cover cracking.

METHODOLOGY FOR PROBABILISTIC TIME TO COVER CRACKING

As Figure 1 shows, difference in volume of steel and rust leads to the internal pressure, p_r, at the steel-concrete interface. If the thickness ring representing the steel loss is δ_s, the thickness of ring representing the corrosion rust will be $(\alpha_v)\times(\delta_s)$, where α_v is the characteristic relative volume ratio of the corrosion product to that of parent iron. In Figure 1, δ_0 represents thickness of a porous ring, which is formed at the steel-concrete interface, and will be discussed in the subsequent sections.

Figure 1–Rust propagation

The pressure exerted by expansion of corrosion products at the steel-concrete interface would eventually lead to cracking of concrete cover and spalling. The various stages of crack growth are schematically depicted in Figure 2. The corrosion-induced damage is defined as any stress or crack developed in concrete surrounding the steel reinforcement. The initiation time is the time for the diffusion of aggressive substances through the concrete cover to the steel-concrete interface leading to activation of the corrosion process.

Figure 2–Stages of corrosion-induced damage

Following corrosion initiation, the volume expansion of corrosion products begins. However, not all of the corrosion products contribute immediately to the expansive pressure on the concrete (Liu and Weyers 1998). Some are considered to fill the voids and pores around the reinforcing bar (see Figure

1). Once this zone is filled, volume expansion of corrosion products exerts a gradually increasing pressure on concrete surrounding the steel rebar, which eventually reaches the tensile capacity of concrete leading to concrete cover cracking. The first crack will appear at the steel-concrete interface. Then, it propagates until it reaches concrete surface as is marked in Figure 2. The whole period from the end of corrosion initiation to appearance of surface crack is considered as the second stage in the corrosion process.

This paper focuses on probabilistic evaluation of the time to cover cracking represented by the length of second stage of corrosion process as shown in Figure 2. The duration of this stage is difficult to quantify due to its dependence on many random factors and assessment criteria. Assuming uniform corrosion around the reinforcement and constant corrosion current density, by using Faraday's law, the governing equation for calculating the thickness of the generated rust, δ, in micrometres can be derived as follows (Chernin et al. 2010),

$$\delta = 11.6\left(\alpha_v - 1\right) i_{corr} t \tag{1}$$

where t is time measured in years, i_{corr} is the corrosion current density in μA/cm^2. There exists a critical time, t_{cr}, at which the crack appears at concrete cover surface. Using Equation (1), this critical time can be related to the thickness of critical rust ring. The first part of rust ring thickness should fill the porous ring with a thickness of δ_0, while the second part is required to crack the concrete cover and is denoted as δ_c in this paper. Therefore, the time to cover cracking can be formulated as follows,

$$t_{cr} = \frac{\delta_0 + \delta_c}{11.6\left(\alpha_v - 1\right) i_{corr}} \tag{2}$$

From a probabilistic point of view, there are uncertainties involved in prediction of t_{cr} using Equation (2). These uncertainties can be collectively represented by a model error, which should be calibrated using field or experimental data. Here, two types of model errors are proposed. The first type of model error, shown in Equation (3a) as ξ, is a model error commonly used in the probabilistic studies. It is the ratio of the actual, i.e., test value to that predicted by the model.

$$t_{cr} = \frac{\delta_0 + \delta_c}{11.6(\alpha_v - 1) i_{corr}} \xi \tag{3a}$$

$$t_{cr} = \frac{\delta_0 + \delta_c + \delta_\varepsilon}{11.6(\alpha_v - 1) i_{corr}} \tag{3b}$$

The second type of model error proposed in this study, denoted as δ_ε in Equation (3b), is based on addition of an extra thickness to $\delta_c + \delta_0$ term in Equation (2). The logic behind this form of model error is the fact that part of corrosion products diffuses inside concrete and the formed cracks (Chernin et al. 2010; Lu et al. 2011). Chen et al. (2019) have shown that the thickness of porous ring, δ_0, can be related to properties of concrete and as such it should not be treated as part of the error. Equation (3b) shows that any uncertainty associated with ingress of corrosion products inside concrete cracks can be treated as a part of the model error.

To derive a probabilistic model for the time to cover cracking, the following steps are followed:

(i) Derivation of a new model for δ_c using the Response Surface Method (RSM) and a rigorous Finite Element (FE) analysis;

(ii) Derivation of a new empirical model that relates thickness of porous ring, δ_0, to concrete properties (water-cement ratio) using experimental results recently published by the authors;

(iii) Calibration of model error using an experimental database;

(iv) Derivation of analytical probabilistic models for time to cover cracking and comparing them with results of Monte Carlo simulation;

(v) Derivation of a probabilistic model for prediction of service life based on the criterion of cover cracking

DISPLACEMENT REQUIRED TO CRACK CONCRETE, δ_c

For the sake of derivation of analytical and numerical solutions for time to cover cracking, it is common to model concrete with embedded reinforcing steel bar as a thick-wall cylinder (Bazant 1979; Chernin et al. 2010; Pantazopoulou and Papoulia 2001). This is schematically shown in Figure 3(a), where D is the diameter of rebar and C is the clear concrete cover to rebar. δ denotes the radial displacement on the concrete-rust interface and p_r is the equivalent pressure resulted from this displacement. Furthermore, it is generally assumed that concrete is a homogenous material and cracking of concrete cover is only caused by stresses resulting from corrosion-induced pressure. The corrosion rate is also assumed as a known variable and growth of corrosion products is uniform over the perimeter of steel reinforcement.

Methods for Evaluation of δ_c

Assuming elastic behaviour, the analytical solution of a thick-wall cylinder under uniform internal pressure derived from plane-stress isotropic linear elasticity (Timoshenko 1970) has been used to evaluate the internal pressure exerted by the corroding bar on the surrounding concrete (Bazant 1979).

Using the elastic solution, a relationship between the applied pressure and the internal radial displacement, δ_c, can be established as follows,

$$\delta_c = \frac{Cf_t}{E_{ef}}\left(\frac{b^2+a^2}{b^2-a^2}+v_c\right) \tag{4}$$

where $a = D/2$ and $b = D/2 + C$. v_c is the Poisson's ratio and E_{ef} is the effective modulus of elasticity. Using a creep model, the effective modulus of elasticity can be calculated.

Figure 3–Failure criteria for surface cracking

In a more sophisticate failure criterion, depicted in Figure 3(b), the actual nonlinear behaviour of concrete can be considered. Derivation of an analytical solution is not an easy task for this criterion and usually an iterative numerical solution is required. The failure criterion is defined as the state at which the crack front [see Figure 3(b)] reaches the concrete surface (Bhargava et al. 2006; Pantazopoulou and Papoulia 2001). On the other hand, Chernin et al. (2010) suggested that the nonlinear relationship between radial displacement and pressure, δ_r and p_r, be established. The critical δ_c corresponds to the maximum pressure. Chernin et al. (2010) proposed an analytical model for establishing p_r-δ_r relationship. However, their analytical solution is dependent on a model for modulus of elasticity as a function of radial coordinate, which is derived from a FE analysis. Furthermore, their analytical solution requires application of hypergeometric functions that should be numerically evaluated.

In this paper, the criterion proposed by Chernin et al. (2010) will be used. However, the results of simple split model are used as the basis for development of a new model, which accounts for the nonlinear behaviour of concrete material. Consider the model shown in Equation (4), which can be rewritten as follows,

$$\delta_c = \frac{Cf_t}{E_{ef}}\left(\frac{D^2}{2C\left(C+D\right)}+1+v_c\right) = \left[\frac{1}{\left(\frac{C}{D}\right)\left(\frac{C}{D}+1\right)}+2\left(1+v_c\right)\right]\left(\frac{f_t}{E_{ef}}\right)\left(\frac{C}{D}\right)D \tag{5}$$

Usually the minimum cover required by design codes is larger than the rebar diameter (ACI 318 2014). Noting that $v_c = 0.20$, for large C/D ratios, the term $1/[C/D\times(1 + C/D)]$ can be neglected when it is compared with the term $2(1 + v_c)$ in Equation (5). Also, the omission of this term can be compensated by applying an exponent to the remaining C/D term. On the other hand, as Figure 4(a) shows, considering the post-cracking softening behaviour of concrete, the hoop stress at the critical state at which $p_{r,cr}$ is calculated exhibits a nonlinear variation with a maximum equal to f_t. This stress distribution can be replaced by an equivalent stress block λf_t, where $\lambda < 1.0$. Furthermore, due to cracking, the stiffness is decreasing gradually leading to higher radial displacement values as is shown in Figure 4(b). Considering all these effects, the following general function for calculation of the critical radial displacement can be proposed.

$$\delta_c = \left[\alpha\left(\frac{C}{D}\right)^{\beta}\left(\frac{f_t}{E_{ef}}\right)^{\gamma}\right]\left[2\left(1+v_c\right)\right]D \tag{6}$$

where α, β and γ are calibration coefficients. This general function is an extension of the simplified linear form, in which nonlinearity is considered through two exponents for the dimensionless quantities representing the geometric properties, C/D, and the material properties, f_t/E_{ef}. In this paper, using the Response Surface Method (RSM) and with the aid of a FE analysis, the model parameters α, β and γ are calibrated.

Figure 4–Nonlinear failure criterion

The general model shown in Equation (6) physically predicts consistent values for extreme values of the independent variables. The condition is that as C or $f_t \rightarrow 0$, $\delta_c \rightarrow 0.0$. Moreover, as C or $f_t \rightarrow \infty$, theoretically $\delta_c \rightarrow \infty$.

Finite Element Model

For the FE analysis, ANSYS (2018) program is employed. The numerical analysis follows the assumption of plain strain. As Figure 5 shows, due to symmetry, only a quarter of the cylinder is modelled. The horizontal and vertical edges of the quarter cylinder are restrained against movements normal to these edges. Displacement-controlled analysis is used for the incremental nonlinear analysis where the inner circle of the cylinder is pressed incrementally at each time step. PLANE182 element in ANSYS is used to model concrete in a two-dimensional model. The element is defined by four nodes having two translational degrees of freedom at each node. The element has plasticity and large strain capabilities. A mesh sensitivity analysis showed that maximum mesh size of near 1.0 mm is adequate for reasonably accurate results.

Figure 5–Schematic FE model and boundary conditions

For modelling concrete material, the Menetrey-Willam (Menetrey and Willam 1995) yield surface is employed. This model utilizes the non-associated flow rule that considers invariants of stress tensors as well as invariants of deviatory stress tensors. To define the yield surface of the Menetrey-Willam model, uniaxial tensile and compressive strength and the biaxial compressive strength are needed.

The uniaxial compression stress-strain, shown in Figure 6(b), is based on well-known Scott and Park model (Scott et al. 1982). It is assumed that the biaxial compressive strength of concrete is 1.2 times the uniaxial compressive strength. It is worth noting that the problem of corrosion-induced cracking is predominantly controlled by tension and the compressive behaviour has minimal effect on the overall results. For cracking, the concept of smeared crack is used. The tensile stress-strain model is shown in Figure 6(b). This model is similar to the bilinear softening model proposed by CEB-FIP (1990) and used by Pantazopoulou and Papoulia (2001) with the difference that the bilinear softening is replaced by a linear softening. This simplification was also adopted by other researchers, e.g., Bhargava et al. (2006).

Figure 6–Uniaxial stress-strain models for concrete

For probabilistic analysis in ANSYS program using the Monte Carlo simulation, the whole required calculation was programmed in an ANSYS Parametric Design Language (APDL) macro, which sets up the geometry, material parameters, solution controls and solves for the internal stresses and strains and finally processing and exporting the required results.

Verification of the FE Model

To validate the FE model used in this study, comparison of the numerical results with some experimental results is necessary. Chernin et al. (2010) have argued that as there are many unknown factors involved in corrosion tests, it is preferred to compare the numerical results with test data in which pressure induced by corrosion is simulated using internal hydraulic pressure. The results of tests conducted by Williamson and Clark (2000) are used for validation of the proposed FE model. Williamson and Clark investigated cracking of the cover concrete due to internal pressure representing the pressure exerted by corrosion products expansion. The test specimens were 150 mm concrete cubes, and the bars were placed in either side or corner locations. Uniform corrosion was simulated using a hydraulic jack to pressurise a soft PVC tube inserted in the hole representing the rebar. Hand pump was used to incrementally pressure the specimen to failure. Failure load was identified as the maximum load sustained by the specimen at cracking of the cover. Comparison of the results of the proposed FE model and the experimental results on cracking pressure according to C/D ratio of 0.5 and 1.0 for 8 mm rebar diameter is shown in Figure 7. Similar numerical results were obtained by Val et al. (2009).

Figure 7–Comparison of FE and experimental results from Williamson and Clark (2000)

It can be noted from these results that the proposed FE model correlates well with the experimental data. It is worth noting that in the Williamson and Clark tests, prismatic concrete specimens with holes representing the rebar location were used. Nonetheless, the 2D thick-wall cylinder analysed using the FE method could predict the cracking pressure with reasonable accuracy.

Calibration of δ_c model using RSM

The goal of RSM is to formulate an implicit closed form function relating a dependent variable, Y, to independent vector of variables, $\boldsymbol{X}$ (X_1, X_2, X_3, …, X_k), through polynomials. It builds functional relationship between input variables and the dependent output. Second-order polynomials are usually used as a response surface approximation function with the following general form,

$$Y = \beta_0 + \sum_{j=1}^{k} \beta_j X_j + \sum_{j=1}^{k} \beta_j X_j^2 + \sum_{i=1}^{k-1} \sum_{j=i+1}^{k} \beta_{ij} X_i X_j + \varepsilon \tag{7}$$

where k is the number of variables. β_i, β_i and β_{ij} are the regression coefficients and ε is the residual on the approximation error. To use the RSM, the model formulated in Equation (6) should be linearized. Using the natural logarithm, the model is transformed to a linear model as follows,

$$\ln\left[\frac{\delta_c}{\left[2\left(1+v_c\right)\right]D}\right] = \ln\left(\alpha\right) + \beta \ln\left(\frac{C}{D}\right) + \gamma \ln\left(\frac{f_t}{E_c}\right) + \varepsilon \tag{8}$$

where $\ln[C/D]$ and $\ln[f_t/E_c]$ are the independent variables and the term $\ln(\delta_c/[2(1 + v_c)D])$ is the dependent variable. For the RSM calibration, only the linear terms are needed. The basic random variables considered for the RMS are shown in Table 1. Most of the statistical models are taken from Joint Committee on Structural Safety (JCSS 2018) probabilistic model code. To cover the practical range for concrete cover and concrete properties, three different cover thicknesses as well as three characteristic compressive strength are used. According to JCSS, the 28-day compressive strength of standard cylinder, f_{c0}, is the basis for calculation of concrete compressive strength, tensile strength and modulus of elasticity as is shown in Equations 9(a-c).

$$f_c^{'} = \left(f_{c0}^{0.96} \right) Y_1 \tag{9a}$$

$$f_t = \left(0.30 f_{c0}^{2/3} \right) Y_2 \tag{9b}$$

$$E_{ef} = \frac{\left[10.5 f_{c0}^{1/3} \right] Y_3}{1 + \varphi(t_0, t)} \tag{9c}$$

where E_{ef} is measured in GPa and f'_c and f_t are in MPa. For the sake of model calibration, the creep function φ is taken as 0. The additional random variables, Y_1, Y_2 and Y_3 account for uncertainty in actual properties of concrete. Using the Monte Carlo simulation technique, a large pool of samples can be generated. Then, using the FE method, described in the previous sections, each of the simulated cases can be analysed. The result of the FE analysis is the radial displacement that corresponds to the maximum pressure at the steel-concrete interface, δ_c, which is used to calculate the dependent variable as can be seen in Equation (8).

Table 1–Statistical models for the basic random variables used in the RSM

For each of the cases shown in Table 1, 5,000 simulation runs are performed. The total combined simulated runs are 15,000 covering a wide range for each design variable. To check the validity of the function formulated for determination of δ_c, Pearson's correlation coefficient is used to find the correlation between δ_c and the main dimensionless variables C/D and f_t/E_c. The statistical analysis shows that the correlation between δ_c as the output and the C/D and f_t/E_c ratios as the inputs are 0.76 and 0.67, respectively, showing strong dependency of the dependent variable δ_c to the input variables. The scatter of the simulated results and the best-fit model resulted from the RSM is shown in Figure 8. As can be seen, the calibrated exponent for f_t/E_{ef} variable, γ, is very close to 1.0, indicating a linear relationship between the output variable and this variable.

Figure 8–Simulated results versus the best-fit response surface model

It should be noted that in the calibrated model the effective modulus of elasticity is in gigapascal, tensile strength of concrete is in megapascal, and concrete cover and rebar diameter are in millimetre. The predicted δ_c is micrometre. Table 2 summarises the performance of different forms (with 1, 2 and

3 fitting parameters) of the proposed model. Using the coefficient of determination, R^2, and the Residual Mean Square Error (RMSE), the performance of these calibrated models is assessed. Statistical performance of these models shows that all these models can produce the corresponding FE results with excellent accuracy confirming the appropriateness of the formulated function. Nonetheless, the model with two parameters is a compromise between simplicity and accuracy.

Table 2–Best-fit models for δ_c

THICKNESS OF THE POROUS RING, δ_0

A combination of the pores and voids and cement paste with high porosity is collectively referred to as a porous ring around the steel-concrete interface. In most of the models proposed for prediction of time to corrosion-induced cover cracking, it is assumed that the size of porous ring is similar to that of Interfacial Transition Zone (ITZ), formed between mortar and aggregate, in the order of 10 µm to 20 µm (El Maaddawy and Soudki 2007; Liu and Weyers 1998; Lu et al. 2011). However, it seems that the thickness of 10 µm to 20 µm for porous ring is not supported by experimental evidence, with the thickness dependent on concrete properties, e.g., water to cement (*w*/*c*). (Horne et al. 2007; Soylev and François 2003)

Even though Backscattered Electrons (BSE) imaging has been widely used to investigate the steel-concrete interface in concrete, few studies have quantitatively investigated the porous ring at the steel-concrete interface and there is little or no experimental data on the thickness distribution of the porous ring in the published literature. Chen et al. (2019) conducted a comprehensive experimental program to investigate the effects of water-cement ratio, aggregate size and concrete cover on the size of the porous ring systematically. More than 125 BSE images were used to investigate spatial variation of thickness of porous ring around steel rebar. Using the experimental data reported in Chen et al. (2019) study, a model relating the thickness of porous ring at the steel-concrete interface and *w*/*c* ratio and concrete cover could be developed. As only two cover thicknesses were used in Chen et al. (2019) study, here the cover thickness is not considered in model calibration, and thickness of porous ring is only related to the *w*/*c* ratio. As *w*/*c* ratio would equally affect thickness of porous ring at all points around the steel bar, the non-uniform thickness of steel-concrete interface is replaced by an equivalent thickness. This equivalent thickness includes effect of measurements at different angles around rebar, and physically represents the volume of the porous zone per unit length at the steel-concrete interface.

In Figure 9, the scatter of equivalent thickness of porous ring, in micrometre unit, with water cement ratio is shown. A power function is used to establish a relationship between the thickness of porous ring and the water cement ratio. The exponent 2.5 in the calibrated model shows high sensitivity of thickness of porous ring to w/c ratio. It is worth noting that as this model is fitted using water cement ratio data in the range of 0.40-0.50, which is the practical range for concrete, extrapolating beyond this range may not be appropriate. Nonetheless, this favourable relationship between water-cement ratio and thickness of porous ring is preserved in this model. Future availability of more experimental data would help in building more reliable models. The choice of this model does not affect generality of the proposed probabilistic procedure.

Figure 9–Calibration of model for δ_0

MODEL UNCERTAINTY

To assess the accuracy of the time to cover cracking model proposed in this paper, an experimental database is established from nine different sources covering a wide range of variables. The inputs in relation to corrosion and concrete properties as well as the recorded time to cover cracking are listed in Table 3. The value of time to cover cracking recorded in all these experiments is defined as the time from the outset of the corrosion test to the first appearance of a visible crack. In cases where information on modulus of elasticity and tensile strength of concrete were not available, basic models from JCSS (2018) are used. These models are similar to those proposed by CEB-FIP (1990) and later editions. The Poisson's ratio for all cases is taken as 0.20.

Table 3–Experimental database for calibration of model error

The water-cement ratio (*w*/*c*) in Table 3 will be used in calculation of the thickness of porous ring based on the proposed model shown in Figure 9.

It is usually assumed that for long-term loadings the creep function φ takes its maximum according to concrete standards, while for short-term loadings (few days accelerated testing) this function is zero, i.e., effective modulus of elasticity equals that of 28 days concrete (Chen et al. 2018; Chernin et al. 2010; Liu and Weyers 1998). In this paper, by using the average of the creep function over the cracking time, an equivalent method is employed to consider the time-dependent effect of creep in a linearly time varying load. If the modulus of elasticity at the initiation of corrosion-induced pressure, i.e., $t = 0$, is the code-specified concrete modulus of elasticity, E_c, the effective modulus of elasticity can be formulated as follows,

$$E_{ef} = \frac{E_c}{1 + \frac{1}{t_{cr}} \int_0^{t_{cr}} \varphi(t, t_{cr}) dt} \tag{10}$$

Here, the creep function $\varphi(t, t_{cr})$ from ACI 209.2R guideline (2008), shown in Equation (11), is adopted. In Equation (11), t_0 and ψ are the creep model parameters and t is measured in days. In the absence of more accurate experimental and field data for the creep function, the t_0 and ψ parameters are recommended as 10 days and 0.60.

$$\varphi(t, t_{cr}) = \frac{\left(t_{cr} - t\right)^{\psi}}{t_0 + \left(t_{cr} - t\right)^{\psi}} \varphi_u \tag{11}$$

According to ACI 209.2R guideline the φ_u can be taken as 2.35. For large time to cover cracking, which is the case for the real-life corrosion rate, the average creep factor approaches the asymptotic value of φ_u.

Statistical evaluation of δ_ε

In Equation (3b), the time to cover cracking model based on δ_ε model error is shown. Given the recorded information for each set of tests and the developed models for δ_0 and δ_c, the model error can be evaluated as follows,

$$\delta_\varepsilon = 11.6\left(\alpha_v - 1\right) i_{corr} t_{cr} - 165\left(w/c\right)^{2.5} - 0.92\left(\frac{C}{D}\right)^{1.24}\left(\frac{f_t}{E_{ef}}\right)\left[2(1+v_c)\right]D \tag{12}$$

The α_v parameter is taken as 3.0 and the Poisson ratio is 0.20. All other variables can be seen in Table 3. The scatter of the error δ_ε is shown in Figure 10. The extra error, which may account for factors such as ingress of corrosion products in concrete, has a mean value close to zero. Nonetheless, with a standard deviation of 26 μm, this error has wide variation.

Figure 10–Scatter of δ_ε error

The statistics of the model error is dependent on the size and reliability of the experimental database. Consistency is very important in experimental data on time to cover cracking. More comprehensive experimental program covering adequately wide range for the basic variables on time to cover cracking is needed.

Calibration of the model error, ξ

The model error ξ shows the ratio of the actual time to cover cracking to that of predictive model. This type of formulation for the model error has more application in probabilistic analyses. According to Equation (3a) and using the models developed for δ_0 and δ_c, this model error can be evaluated as follows,

$$\xi = \frac{11.6(\alpha_v - 1) i_{corr} t_{cr}}{165(w/c)^{2.5} + 0.92\left(\frac{C}{D}\right)^{1.24}\left(\frac{f_t}{E_{ef}}\right)\left[2(1+v_c)\right]D} \tag{13}$$

Histogram of this model error is shown in Figure 11. Further statistical analysis shows that the Lognormal distribution is the best-fit probability density function for this model error. The corresponding mean and coefficient of variation of the model error are also shown in Figure 11. The high coefficient of variation for the model error indicates the many unknowns involved in the process of calculating the time to cover cracking. Sources of uncertainty can be classified in the uncertainties in corrosion product properties, uncertainties in recording concrete properties such as the water-cement ratio and uncertainties related to the use of models such as Faraday's law and those models used to predict thickness of porous δ_0 band and δ_c.

Jamali et al. (2013) have shown that the level of sophistication in predictive models has less effect in reducing the level of reliability in prediction of time to cover cracking. Chen et al. (2018) have shown that despite its simplicity the model proposed by Liu and Weyers (1998) outperforms other more sophisticated models.

Figure 11–Statistical distribution of the model error ξ

PROBABILISTIC EVALUATION OF TIME TO COVER CRACKING

For prediction of time to cover cracking, Equation (3a) is used as the basis. The randomness in time to cover cracking has three basic sources. The first source is the uncertainty in geometric properties and concrete material properties. These uncertainties are reflected in δ_c variable. The second source of randomness is the uncertainty in prediction of thickness of porous ring δ_0, which in this paper is related to water-cement ratio. The third source is related to properties of corrosion products represented in α_v variables and to the environmental corrosion rate represented by i_{corr} variable. The model error is the final source of randomness, which accounts for inability of the employed model in evaluation of the time to cover cracking. In this section, firstly, probabilistic models for assessing uncertainties in δ_c and δ_0 variables are developed. Then, using these models, probabilistic models for the time to cover cracking are proposed. These models only require the knowledge of the first two moments of each random variable.

Probabilistic model for δ_c

For evaluating δ_c, the second model shown in Table 2 is adopted. It is assumed that the corrosion is a slow processes so that the final asymptotic creep can be used, i.e., $E_{ef} = E_c/(1 + \varphi_u)$ where $\varphi_u = 2.35$. For the ease in the probabilistic analysis, the models given in JCSS (2018), shown in Equations (9a-c), are used to formulate the final form of the model for δ_c based on model 2 in Table 2 as follows,

$$\delta_c = 0.92\left(C\right)^{1.24}\left(D\right)^{-0.24}\left(\frac{1}{35}f_{c0}^{1/3}\frac{Y_2}{Y_3}\right)\left[2(1+v_c)\right]\left(1+\varphi_u\right) \tag{14}$$

As the dependent variable, δ_c is the product of independent variables, C, D, f_{c0}, Y_2, Y_3, and $1 + v_c$. If all these random variables in Equation (14) are Lognormal, the Probability Density Function (PDF) of the response δ_c will also be Lognormal. Otherwise, according to the CLT, regardless of their actual distribution, product of large number of random variables can be approximated by the Lognormal distribution (Benjamin and Cornell 1975). In this case, only the first two moments of each random variable are adequate to fully define the random variable. It should also be noted that regardless of distribution of the random variables, the following procedure for derivation of first and second moments is accurate.

For natural logarithm of a random variable X, the following relationships between first and second moment of the random variable and its logarithm can be established,

$$\mu_{\ln X} = \ln\left(\mu_X\right) - \frac{1}{2}\sigma_{\ln X}^2 \approx \ln\left(\mu_X\right) \tag{15a}$$

$$\sigma_{\ln X} = \sqrt{\ln\left(1+V_X^2\right)} \approx V_X \tag{15b}$$

For random variables with coefficient of variation less than 0.30, the approximation shown in the above equations has adequate accuracy. Following this simplification, the mean and coefficient of variation of δ_c can be calculated as follows (see model 2 in Table 2),

$$\mu_{\delta_c} = 0.92\left(\frac{\mu_C}{\mu_D}\right)^{1.24}\left(\frac{\mu_{f_t}}{\mu_{E_{ef}}}\right)\left[2(1+\mu_{v_c})\right]\mu_D \tag{16a}$$

$$V_{\delta_c} = \sqrt{1.24^2 \times V_C^2 + 0.24^2 \times V_D^2 + (1/3)^2 V_{f_{c0}}^2 + V_{Y_2}^2 + V_{Y_3}^2 + \left(\frac{\mu_{v_c}}{1+\mu_{v_c}}\right)^2 V_{v_c}^2} \tag{16b}$$

As Table 1 shows, except the Poisson's ratio, rebar diameter, *D*, and cover thickness, *C*, all other variables are Lognormally distributed. Nonetheless, it is known that for random variables with coefficient of variation less than 0.20 (which is the case in this study), Normal distribution can be approximated by Lognormal distribution with adequate accuracy (Benjamin and Cornell 1975). Therefore, δ_c approximately follows the Lognormal distribution with the mean and coefficient of variation shown in Equations (16a) and (16b), respectively.

The coefficient of variation of all the random variables is given in Table 1. Equation (16b) clearly shows that the cover thickness, tensile strength of concrete and modulus of elasticity of concrete are the governing variables that determine level of randomness in δ_c. For instance, considering case 1 in Table 1, the coefficient of variation of δ_c is 0.418, while only considering the dominant variables, the coefficient of variation is 0.417. Although simulation methods can also be used for assessment of variability of δ_c, as Equation (16b) shows, the analytical method proposed in this paper gives clearer insight into the randomness of this variable.

Probabilistic model for δ_0

As shown in Figure 9, if probabilistic models for *w*/*c* ratio is known, the probabilistic model for δ_0 can be determined. However, probabilistic models for *w*/*c* ratio are rare. Nevertheless, there is strong relationship between *w*/*c* ratio and the concrete compressive strength. Thus, by having a probabilistic model for concrete compressive strength, an indirect method can be used to establish a probabilistic model for *w*/*c* ratio. The well-known Abrams' formula gives an adequately accurate relationship between concrete compressive strength and *w*/*c* ratio (Popovics and Ujhelyi 2008) as follows,

$$w/c = 2.3256 - 0.5056 \times \ln\left(f_c^{'}\right) \tag{17}$$

Probabilistic model for compressive strength of concrete, f'_c, can be seen in Equation (9a). As a random variable, the compressive strength of concrete follows the Lognormal distribution. Therefore, logarithm of this variable follows the Normal distribution. Using the relationship between statistics of a random variable and its logarithm, statistics of water-cement ratio, *w*/*c*, can be derived as follows,

$$\mu_{w/c} = 2.3256 - 0.5056 \times \mu_{\ln\left(f_c^{'}\right)} \approx 2.3256 - 0.5056 \times \ln\left(\mu_{f_{c0}^{'}}^{0.96}\right) \tag{18a}$$

$$\sigma_{w/c} = 0.5056 \times \sigma_{\ln\left(f_c^{'}\right)} \approx 0.5056 \times V_{f_c^{'}} = 0.5056 \times \sqrt{0.96^2 \times V_{f_{c0}}^2 + V_{Y_1}^2} \tag{18b}$$

Statistics of compressive strength of standard concrete cylinder, f_{c0}, are listed in Table 1. Now, based on the model presented in Figure 9 statistics of the water-cement ratio can be used to derive expression for the first and second moment of thickness of porous ring, δ_0.

$$\mu_{\delta_0} = 165\left(\mu_{w/c}\right)^{2.5} \tag{19a}$$

$$V_{\delta_0} = 2.5 \times V_{w/c} = 2.5 \frac{\sigma_{w/c}}{\mu_{w/c}} \quad (19b)$$

By knowing the mean and coefficient of variation of basic random variables, as shown in Table 1, statistics of the thickness of porous ring can be determined. For instance, for a case where mean concrete compressive strength is 39.0 MPa as shown in Table 1, the mean and coefficient of variation of the *w/c* ratio are 0.547 and 0.156, respectively. If these values are used in Equations (19a and b), the mean and coeffect of variation for δ_0 will be 36.5 μm and 0.39, respectively. Considering that the *w/c* ratio follows the Normal distribution and its coefficient of variation is less than 0.20, the Lognormal distribution provides a close approximation to the actual Normal distribution. Adopting this approximation and using the proposed model for δ_0, it can be concluded that δ_0 also follows the Lognormal distribution.

Probabilistic model for t_{cr}

As the final step of the probabilistic analysis, a probabilistic model for the time to cover cracking is derived. The analytical model for prediction of the time to cover cracking is shown in Equation (3a). As a random variable, time to cover cracking, t_{cr}, is a product of four random variables: ($\delta_c + \delta_0$), ξ, (α_v - 1) and i_{corr}. Mean and standard deviation of δ_c and δ_0 were derived in the previous subsections. Probabilistic model for the model error, ξ, was also derived in the previous section (see Figure 11). For variables α_v and i_{corr}, probabilistic models from Papakonstantinou and Shinozuka (2013) are adopted. According to these models, α_v follows the general Beta distribution with a mean of 3.01 and a coefficient of variation of 0.27. The corrosion current density, i_{corr}, follows the Lognormal distribution with coefficient of variation of 0.34 and the mean value varies with corrosion rate.

For random variable ($\delta_c + \delta_0$), using the probabilistic model developed for δ_c and δ_0 mean and standard deviation of this linear distribution can be determined. It is assumed that these two random variables are not correlated. However, as both variables are related to concrete compressive strength (for δ_0, through the water-cement ratio), correlation exists between these variables. Given that higher concrete compressive strength leads to higher δ_c and lower δ_0 (due to lower water-cement ratio) these variables are negatively correlated. Therefore, neglecting the correlation leads to more conservative results.

Following the same procedure as the one for δ_c, the mean and coefficient of variation of t_{cr} can be determined. However, as the coefficients of variation of random variables in Equation (3a) are high, the approximations in Equation (15a) and (15b) cannot be used. The mean and coefficient of variation of the time to cover cracking as a function of the four basic random variables described above can be calculated as follows,

$$\mu_{t_{cr}} = \frac{\mu_{\delta_0} + \mu_{\delta_c}}{11.6\left(\mu_{\alpha_v} - 1\right)\mu_{i_{corr}}}\left[1 + V_{i_{corr}}^2\right]\left[1 + \left(\frac{\mu_{\alpha_v}}{\mu_{\alpha_v} - 1}\right)^2 V_{\alpha_v}^2\right] \quad (20a)$$

$$V_{t_{cr}} = \sqrt{\left(1 + \frac{(\mu_{\delta_c} V_{\delta_c})^2 + (\mu_{\delta_0} V_{\delta_0})^2}{(\mu_{\delta_c} + \mu_{\delta_0})^2}\right)\left(1 + V_{\xi}^2\right)\left(1 + V_{i_{corr}}^2\right)\left(1 + (\frac{\mu_{\alpha_v}}{\mu_{\alpha_v} - 1})^2 V_{\alpha_v}^2\right) - 1} \quad (20b)$$

Having the mean and coefficient of variation of the basic random variables, and using the general solution given in Equations (20a and b) it is possible to calculate the mean and coefficient of variation of the time to cover cracking. Regardless of actual distribution of the random variables involved, this general solution is accurate. However, as some of these random variables do not follow the Lognormal distribution, the probability density of function of the time to cover cracking can only be approximated by the Lognormal distribution. Nonetheless, according to the CLT, distribution of a random variable as a product of multiple random variables approaches the Lognormal distribution. In the next subsection, accuracy of this approximation is compared with the accurate results from direct simulation.

Verification of the proposed probabilistic models

Simulation of time to cover cracking is carried out based on Equation (3a). The model for prediction of δ_c is the second model in Table 2. Also, the model for prediction of thickness of porous ring δ_0 is shown in Figure 9, and the Abrams' formula, shown in Equation (17), is used to relate *w*/*c* ratio to concrete compressive strength. The probabilistic models for corrosion current density, i_{corr}, and the volume ratio of the corrosion product to that of parent metal, α_v, are taken from Papakonstantinou and Shinozuka (2013) as is discussed before. The corrosion current density of 1.0 μA/cm^2 is used in the probabilistic analysis. Probabilistic models for all other random variables are shown in Table 1. The Monte Carlo simulation with 1,000,000 runs is used to find statistics and the empirical PDF of the response random variables.

In Table 4, a comparison between the mean and coefficient of variation of the response random variables resulted from the proposed analytical models and the simulation are shown. Probabilistic models for cases 1 to 3 are listed in Table 1. There is an excellent agreement between the results from the proposed models and those obtained from direct simulation. The maximum error in predicting the time to cover cracking is only 6.2%. Considering the simplicity of the proposed analytical models, this level of error is negligible.

Table 4–Simulation versus the proposed analytical models

The statistical analysis using simulation shows that the correlation between δ_c and δ_0 variables is only around -0.10, which is negligible. A comparison between the Cumulative Distribution Functions (CDF) resulted from the simulation and the proposed models based on the Lognormal distribution is shown in Figure 12 for cases 1 and 3. As can be seen, there is very good agreement between the results of the simulation and the approximate Lognormal distribution, especially in the lower tail region, which is the area of interest in the service life prediction of the time to cover cracking.

Figure 12–Simulation versus the proposed approximate Lognormal distribution

APPLICATION TO SERVICE LIFE PREDICTION

The final aim of probabilistic assessment of time to cover cracking is to predict the service life of corrosion-affected reinforced concrete structures based on the criterion of concrete cover cracking. Using the CDF for time to cover cracking, the service life of corrosion-affected concrete structure can be calculated. Using the lower part of the cumulative distribution function, and by assuming an acceptable exceedance probability, *p*, the service life, t_{service}, can be calculated, as follows,

$$\Pr(t_{cr} < t_{\text{service}}) = p \tag{21}$$

This means that the probability of having a time to cover cracking less than the defined service life is *p*. The *p* value depends upon policies set by asset owners. It was shown in this paper that the time to cover cracking, t_{cr}, can be reasonably modelled by the Lognormal distribution with the mean and coefficient of variation shown in Equations (20a and b). Considering the Lognormal distribution for

t_{cr}, the relationship between the service life, $t_{service}$, and the mean time to cover cracking can be obtained as follows,

$$t_{\text{service}} = \mu_{t_{cr}} \exp\left[\Phi^{-1}(p) V_{t_{cr}}\right] \quad (22)$$

where Φ is the CDF of the standard Normal distribution. This equation is important as it relates statistics of the time to cover cracking and the acceptable probability, p, to the probabilistically-defined service life, $t_{service}$. All the information required to calculate the service life are formulated in simple analytical expression with no need to numerical analysis and probabilistic simulation. As an application to service life prediction, consider the second case listed in Table 1. Four different corrosion rates represented by four corrosion current densities of 0.25, 0.50, 0.75 and 1.00 $\mu A/cm^2$ are used. In Figure 13, the resulted CDFs and the service life based on an acceptable probability of failure of 0.10 are shown.

Figure 13–Service life prediction for different corrosion rates

The results of the probabilistic procedure, as is shown in Figure 13, can be used by asset manager as a simple but reasonably accurate tool for prediction of service life of corrosion-affected concrete structure.

CONCLUSION

In this paper, a probabilistic study on the time to cover cracking in corrosion-affected concrete structures has been conducted. First, with the aid of the RSM a new model for calculation of radial displacement required for surface cracking of concrete based on a rigorous FE is derived. Secondly, using recently published experimental data, a new model relating the thickness of porous ring to water-cement ratio is proposed. These two models are combined with the well-known Faraday's law to predict the time to cover cracking. A large experimental database is used to calibrate the model error associated with the proposed model. By taking advantage of the multiplicative form of functions required for evaluation of time to cover cracking, simple yet accurate analytical models are derived. To use these models only the first and second moments of the basic random variables are needed. These probabilistic models easily show the relative contribution of variable on the overall uncertainty level of time to cover cracking. The proposed models are in very good agreement with the accurate simulation results. It has been shown that the radial displacement required to crack concrete and the thickness of porous ring exhibit similar level of uncertainty. It has also been shown that the concrete cover, tensile strength of concrete, thickness of porous ring, model error and the corrosion rate are the dominant variables controlling the variability of time to cover cracking. The proposed probabilistic procedure is general and availability of more robust models especially for thickness of porous ring and the model error which both rely on experimental data would boost the applicability of this procedure.

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REFERENCES

ACI 209.2R (2008). "Guide for modeling and calculating shrinkage and creep in hardened concrete." *ACI Report*, 209.

ACI 318 (2014). "Building Code Requirements for Structural Concrete and Commentary." American Concrete Institute, Farmington Hills, MI, USA.

Alonso, C., Andrade, C., Rodriguez, J., and Diez, J. M. (1998). "Factors controlling cracking of concrete affected by reinforcement corrosion." *Materials and structures*, 31(7), 435-441.

Andrade, C., Alonso, C., and Molina, F. (1993). "Cover cracking as a function of bar corrosion: Part I-Experimental test." *Materials and structures*, 26(8), 453-464.

ANSYS. 2018. ANSYS Reference Manual, Swanson Analysis Systems, Houston, PA.

Baji, H. (2014). "The effect of uncertainty in material properties and model error on the reliability of strength and ductility of reinforced concrete members." PhD, The University of Queensland, Brisbane, Australia.

Bazant, Z. P. (1979). "Physical model for steel corrosion in concrete sea structures--application." *Journal of the Structural Division*, 105(ST6), 1155-1166.

Benjamin, J., and Cornell, C. (1975). *Probability, statistics and decision for civil engineers*, McGraw-Hill, New York, United States.

Bezuidenhout, S. R., and van Zijl, G. P. J. S. C. (2019). "Corrosion propagation in cracked reinforced concrete, toward determining residual service life." *Structural Concrete*, 1-11.

Bhargava, K., Ghosh, A. K., Mori, Y., and Ramanujam, S. (2006). "Analytical model for time to cover cracking in RC structures due to rebar corrosion." *Nuclear Engineering and Design*, 236(11), 1123-1139.

Bhargava, K., Ghosh, A. K., Mori, Y., and Ramanujam, S. (2006). "Model for cover cracking due to rebar corrosion in RC structures." *Engineering Structures*, 28(8), 1093-1109.

Cabrera, J., and Ghoddoussi, P. "The effect of reinforcement corrosion on the strength of the steel/concrete bond." *Proc., International conference on bond in concrete*, CEB Riga, Latvia, 10.

CEB-FIP (1990). "CEB-FIP model code 1990." Lausanne, Switzerland, 214.

Chen, F., Baji, H., and Li, C. Q. (2018). "A comparative study on factors affecting time to cover cracking as a service life indicator." *Construction and Building Materials*, 163(2018), 681-694.

Chen, F., Li, C. Q., Baji, H., and Ma, B. (2019). "Effect of design parameters on microstructure of steel-concrete interface in reinforced concrete." *Cement and Concrete Research*, 119, 1-10.

Chernin, L., Val, D. V., and Volokh, K. Y. (2010). "Analytical modelling of concrete cover cracking caused by corrosion of reinforcement." *Materials and Structures*, 43(4), 543-556.

El Maaddawy, T., and Soudki, K. (2007). "A model for prediction of time from corrosion initiation to corrosion cracking." *Cement and Concrete Composites*, 29(3), 168-175.

El Maaddawy, T. A., and Soudki, K. A. (2003). "Effectiveness of impressed current technique to simulate corrosion of steel reinforcement in concrete." *Journal of materials in civil engineering*, 15(1), 41-47.

Horne, A. T., Richardson, I. G., and Brydson, R. M. D. (2007). "Quantitative analysis of the microstructure of interfaces in steel reinforced concrete." *Cement and Concrete Research*, 37(12), 1613-1623.

Jamali, A., Angst, U., Adey, B., and Elsener, B. (2013). "Modeling of corrosion-induced concrete cover cracking: A critical analysis." *Construction and Building Materials*, 42, 225-237.

Jang, B. S., and Oh, B. H. (2010). "Effects of non-uniform corrosion on the cracking and service life of reinforced concrete structures." *Cement and Concrete Research*, 40(9), 1441-1450.

JCSS (2018). "Probabilistic model code." *The Joint Committee on Structural Safety*, Technical University of Denmark.

Li, C. Q. (2004). "Reliability based service life prediction of corrosion affected concrete structures." *ASCE Journal of Structural Engineering*, 130(10), 1570-1577.

Li, C. Q., and Melchers, R. E. (2005). "Time-dependent risk assessment of structural deterioration caused by reinforcement corrosion." *ACI Structural Journal*, 102(5), 754-762.

Li, C. Q., Melchers, R. E., and Zheng, J.-J. (2006). "Analytical model for corrosion-induced crack width in reinforced concrete structures." *ACI Structural Journal*, 103(4), 479-487.

Liu, Y., and Weyers, R. E. (1998). "Modeling the time-to-corrosion cracking in chloride contaminated reinforced concrete structures." *ACI Materials Journal*, 95(6), 675-681.

Lu, C., Jin, W., and Liu, R. (2011). "Reinforcement corrosion-induced cover cracking and its time prediction for reinforced concrete structures." *Corrosion Science*, 53(4), 1337-1347.

Lu, C., Yuan, S., and Liu, R. (2017). "Experimental and probabilistic analysis of time to corrosion-induced cover cracking for marine reinforced concrete structures." *Corrosion Engineering, Science and Technology*, 52(2), 124-133.

Mangat, P. S., and Elgarf, M. S. (1999). "Bond characteristics of corrding reinforcement in concrete beams." *Materials and Structures*, 32(2), 89-97.

Menetrey, P., and Willam, K. J. (1995). "Triaxial failure criterion for concrete and its generalization." *ACI Stuctural Journal*, 92(3), 311-318.

Pantazopoulou, S., and Papoulia, K. (2001). "Modeling cover-cracking due to reinforcement corrosion in RC structures." *Journal of Engineering Mechanics*, 127(4), 342-351.

Papakonstantinou, K. G., and Shinozuka, M. (2013). "Probabilistic model for steel corrosion in reinforced concrete structures of large dimensions considering crack effects." *Engineering Structures*, 57, 306-326.

Popovics, S., and Ujhelyi, J. (2008). "Contribution to the concrete strength versus water-cement ratio relationship." *Journal of Materials in Civil Engineering*, 20(7), 459-463.

Scott, B., Park, R., and Priestley, M. (1982). "Stress-strain behavior of concrete confined by overlapping hoops at low and high strain rates." *ACI Journal Proceedings*, 79(1), 13-27.

Shao, W., Shi, D., and Tang, P. (2018). "Probabilistic Lifetime Assessment of RC Pipe Piles Subjected to Chloride Environments." *Journal of Materials in Civil Engineering*, 30(11), 04018297.

Soylev, T. A., and François, R. (2003). "Quality of steel–concrete interface and corrosion of reinforcing steel." *Cement and Concrete Research*, 33(9), 1407-1415.

Thoft-Christensen, P. (2001). "What happens with reinforced concrete structures when the reinforcement corrodes." Department of Building Technology and Structural Engineering.

Thybo, A. E. A., Michel, A., and Stang, H. (2017). "Smeared crack modelling approach for corrosion-induced concrete damage." *Materials and Structures*, 50(2), 146.

Tilly, G. (2007). "The durability of repaired concrete structures." *IABSE Symposium Report*, International Association for Bridge and Structural Engineering, 1-8.

Timoshenko, S. (1970). *Theory of Elasticity*, McGraw-Hill, New York.

Val, D. V., Chernin, L., and Stewart, M. G. (2009). "Experimental and numerical investigation of corrosion-induced cover cracking in reinforced concrete structures." *Journal of Structural Engineering*, 135(4), 376-385.

Vu, K., Stewart, M. G., and Mullard, J. (2005). "Corrosion-induced cracking: experimental data and predictive models." *ACI Structural Journal*, 102(5), 719-726.

Whitmore, D. W., and Ball, J. C. (2004). "Corrosion management." *ACI Concrete International*, 26(12), 82-85.

Williamson, S., and Clark, L. (2000). "Pressure required to cause cover cracking of concrete due to reinforcement corrosion." *Magazine of Concrete Research*, 52(6), 455-467.

List of Tables

Table 1–Statistical models for the basic random variables used in the RSM

Variable	Case	μ	COV	Distribution	Reference
C (mm)	1	25	0.20	Normal	JCSS (2018)
	2	50	0.20		
	3	75	0.14		
D (mm)	1	10	0.10	Normal	JCSS (2018)
	2	16	0.10		
	3	20	0.10		
f_{c0} (MPa)	1	39.0	0.165	Lognormal	JCSS (2018)
	2	47.3	0.124		
	3	53.8	0.096		
Y_1	1-3	1.0	0.06	Lognormal	JCSS (2018)
Y_2	1-3	1.0	0.30	Lognormal	JCSS (2018)
Y_3	1-3	1.0	0.15	Lognormal	JCSS (2018)
v_c	1-3	0.20	0.12	Normal	Baji (2014)

Table 2–Best-fit models for δ_c

Model	Expression	R^2	RMSE
1	$\delta_c = 0.83\left(\frac{C}{D}\right)^{1.24}\left(\frac{f_t}{E_{ef}}\right)^{0.96}\left[2(1+v_c)\right]D$	0.9955	0.01314
2	$\delta_c = 0.92\left(\frac{C}{D}\right)^{1.24}\left(\frac{f_t}{E_{ef}}\right)\left[2(1+v_c)\right]D$	0.9944	0.01468
3	$\delta_c = \left(\frac{C}{D}\right)^{1.18}\left(\frac{f_t}{E_{ef}}\right)\left[2(1+v_c)\right]D$	0.9931	0.01628

Table 3–Experimental database for calibration of model error

D mm	*C* mm	*w/c* -	f'_c MPa	E_c GPa	f_t MPa	i_{corr} μA/cm^2	t_{cr} hr	Reference
16.0	20.0	0.54	15.52	17.73	3.70	100.00	112.00	Lu et al. (2017)
16.0	20.0	0.54	15.52	17.73	3.70	150.00	75.00	
16.0	30.0	0.54	15.52	17.73	3.70	100.00	142.00	
16.0	30.0	0.54	15.52	17.73	3.70	150.00	87.00	
16.0	30.0	0.50	20.00	31.00	3.55	100.00	120.00	Andrade et al. (1993)
16.0	20.0	0.50	20.00	31.00	3.55	100.00	96.00	
16.0	20.0	0.50	20.00	31.00	3.55	10.00	552.00	
16.0	20.0	0.52	20.00	21.00	3.85	100.00	113.00	Alonso et al. (1998)
16.0	50.0	0.52	20.00	21.00	3.85	100.00	234.00	
10.0	70.0	0.52	20.00	21.00	3.85	100.00	517.00	
16.0	30.0	0.52	20.00	21.00	3.85	100.00	185.00	
16.0	70.0	0.52	20.00	21.00	3.85	100.00	385.00	
12.0	50.0	0.52	20.00	21.00	3.85	100.00	415.00	
12.0	70.0	0.52	20.00	21.00	3.85	10.00	2643.00	
16.0	30.0	0.54	15.50	24.40	3.70	100.00	147.50	Lu et al. (2011)
16.0	30.0	0.54	15.50	24.40	3.70	150.00	87.50	
16.0	20.0	0.54	15.50	24.40	3.70	100.00	112.00	
16.0	33.0	0.55	40.00	42.00	4.90	150.00	95.00	El Maaddawy and Soudki (2003)
10.0	20.0	0.50	45.00	37.00	3.80	800.00	14.40	Mangat and Elgarf (1999)
12.0	69.0	0.55	45.00	37.30	3.80	244.00	108.00	Cabrera and Ghoddoussi (1992)
16.0	25.0	0.45	52.70	39.32	4.55	100.00	223.10	Vu et al. (2005)
16.0	50.0	0.45	52.70	39.32	4.55	100.00	490.70	
16.0	25.0	0.50	20.00	28.47	3.06	100.00	134.00	
16.0	50.0	0.50	20.00	28.47	3.06	100.00	194.70	
16.0	25.0	0.50	43.00	36.74	4.16	100.00	116.00	
16.0	50.0	0.50	43.00	36.74	4.16	100.00	155.70	
16.0	25.0	0.58	42.25	36.52	3.94	100.00	136.10	
16.0	50.0	0.58	42.25	36.52	3.94	100.00	402.80	
16.0	48.0	0.43	31.50	27.00	3.30	2.41	16118.40	Liu and Weyers (1998)
16.0	70.0	0.43	31.50	27.00	3.30	1.79	31010.40	
16.0	27.0	0.45	31.50	27.00	3.30	3.75	6307.20	
12.7	52.0	0.49	31.50	27.00	3.30	1.80	20848.80	

Table 4–Simulation versus the proposed analytical models

Response	Statistics	Model	Case 1	Case 2	Case 3
δ_c (μm)	Mean	Simulation	22.822	50.946	82.482
		Proposed	23.052	51.890	84.832
	COV	Simulation	0.431	0.428	0.385
		Proposed	0.422	0.420	0.380
δ_0 (μm)	Mean	Simulation	39.490	24.359	16.755
		Proposed	36.579	22.818	15.834
	COV	Simulation	0.376	0.359	0.345
		Proposed	0.391	0.372	0.355
t_{cr} (year)	Mean	Simulation	3.523	4.263	5.611
		Proposed	3.315	4.153	5.596
	COV	Simulation	0.821	0.840	0.848
		Proposed	0.816	0.829	0.836

List of Figures