

## CHAPTER ONE

### *The Role of Mathematical Tasks in Different Cultures*

#### THE CENTRALITY OF TASKS IN MATHEMATICS CLASSROOM INSTRUCTION

Mathematics classroom instruction is generally organised around and delivered through students' activities on mathematical tasks (Doyle, 1988). International comparative studies on mathematics classroom instruction tend to report analyses of such students' activities engaged in tasks that occupy various amounts of lesson time in classrooms. Notably, in all of the seven countries that participated in the TIMSS 1999 Video Study, eighth-grade mathematics was most commonly taught by spending at least 80% of lesson time in mathematics classrooms working on mathematical tasks (Hiebert et al., 2003).

Classroom activities are coherent actions shaped by the instructional context, in general, and, in particular, by what is taught through the use of tasks (Stodolsky, 1988). Individual teachers arrange instruction very differently, depending on what they are teaching, and students respond to instruction very differently, depending on the structure and demands shaped by tasks enacted in the classroom. The tasks that teachers assign can determine how students come to understand what is taught. In other words, tasks serve as a context for students' thinking, during and after instruction. Doyle argues the point that

tasks influence learners by directing their attention to particular aspects of content and by specifying ways of processing information. (Doyle, 1983, p.161)

Mathematics tasks are important vehicles for classroom instruction that aims to enhance students' learning. To achieve quality mathematics instruction, then, the role of mathematical tasks to stimulate students' cognitive processes is crucial (Hiebert & Wearne, 1993).

In summary, the centrality of tasks in mathematics classroom is evident from theoretical perspectives as well as in empirical results from international comparative studies. The role of mathematical tasks provides a key to any attempt to understand teaching and learning in research on classroom practices in mathematics.

MATHEMATICAL TASKS AND THE LEARNER'S PERSPECTIVE STUDY

This book is the third in a series arising from the international collaborative project called The Learner's Perspective Study (LPS). The LPS documented sequences of at least ten lessons, using three video cameras, supplemented by the reconstructive accounts of classroom participants obtained in post-lesson video-stimulated interviews, and by test and questionnaire data, and copies of student written material (Clarke, 1998, 2001, 2003). In each classroom, formal data generation was preceded by a one-week familiarisation period in which the research team undertook preliminary classroom videotaping and post-lesson interviewing until such time as the teacher and students were accustomed to the classroom presence of the researchers and familiar with the research process. In each participating country, the focus of data generation was the classrooms of three teachers, identified by the local mathematics education community as competent, and situated in demographically different school communities within the one major city. For each school system (country), this design generated a data set of at least 30 'well-taught' lessons (three sequences of at least ten lessons), involving 120 video records, 60 student interviews, 12 teacher interviews, plus researcher field notes, test and questionnaire data, and scanned student written material. Well-taught, in the context of this study, meant that the teachers in each country were recruited according to local criteria for competence: visibility as presenters at conferences for other teachers, leadership roles in professional organisations, and, acclamation by colleagues and students. It is not surprising, therefore, that the classroom of a competent teacher in Uppsala might look a little different from the classroom of a competent teacher in Shanghai or San Diego. The local construction and enactment of competence was one of the most appealing aspects of this study. Greater detail on data generation procedures is provided in the appendix to this book. Signature elements of the LPS Research Design are (i) the commitment to studying 'competent' teachers as these are locally defined; (ii) the recording of a sequence of at least ten lessons constituting a mathematics topic; and (iii) the use of classroom videos in video-stimulated reconstructive interviews with teacher and students as soon as possible after the recorded lesson. The teacher and student interviews offer insight into both the teacher's and the students' participation in (and reconstruction of) particular lesson events and the significance and meaning that the students associated with their actions and those of the teacher and their classmates.

The classroom use of mathematical tasks has been addressed in previous publications from the Learner's Perspective Study (LPS). For example, the first book in the LPS series (Clarke, Keitel, & Shimizu, 2006) included a chapter on "Setting a Task" (Keitel, 2006) and another on "The Role of the Textbook and Homework" (Kaur, Low, & Seah, 2006), and the second LPS book (Clarke, Emanuelsson, Jablonka, & Mok, 2006) included a chapter on "Learning Tasks" (Mok & Kaur, 2006). It is difficult to imagine any substantial investigation of the mathematics classroom that did not address the tasks that characterise such

settings. This book, the third in the LPS series, is devoted entirely to research into the role of mathematical tasks in the classrooms of different countries.

### THE NATURE OF MATHEMATICAL TASKS IN CLASSROOMS

A mathematical task has been defined as a set of problems or a single complex problem that focuses students' attention on a particular mathematical idea (Stein, Grover, & Henningsen, 1996). The construct 'task' also includes the intellectual and physical products that are expected of students, such as the operations that students are to use to obtain the desired results, and the resources that are available for students to produce the products (Doyle, 1983). In his elaboration of the construct 'task,' Doyle (1988) included the following components: The product(s), such as numbers in blanks on a worksheet; the operation(s) required to produce the product; the resources drawn upon in completing the task, such as notes from textbook information; and, the significance or 'weight' of a task in the accountability system of a class.

In their critique of 'minimal guidance' instruction, Kirschner, Sweller and Clark (2006) make the insightful observation that

it may be an error to assume that the pedagogic content of the learning experience is [should be] identical to the methods and processes (i.e., the epistemology) of the discipline being studied. (p. 84)

In particular, their assertion that "The practice of a profession is not the same as learning to practice the profession" (p. 83) highlights a critical issue in the design of instruction in mathematics. How is classroom mathematical activity related to the activity of the mathematician? While we may classify the tasks of the mathematics classroom in a variety of ways, we should not confuse those tasks with the tasks of the mathematician: they are fundamentally different in purpose.

Mathematical tasks employed in educational settings have been variously categorised under designations such as 'authentic,' 'rich' and 'complex.' The classification 'authentic' has particularly emotive overtones – suggesting that some mathematical tasks might be classified as 'inauthentic.' The most common usage of the term 'authentic' in this regard seems to refer to an assumed correspondence between the nature of the task and other mathematical activities that might be undertaken outside the classroom for purposes other than the learning of mathematics. The value attached to 'authentic mathematical tasks' seems to appeal to a theory of learning that measures mathematical understanding by the capacity to employ mathematical knowledge obtained in the classroom in non-classroom ('real-world') settings and which constructs the process of mathematical learning as 'legitimate peripheral participation' (Lave & Wenger, 1991) in the mathematical activities of a community larger than a mathematics class.

To illustrate another classificatory scheme: Williams and Clarke (1997) developed a framework that identified the following forms of task complexity: linguistic, contextual, numerical, conceptual, intellectual, and representational. In her subsequent research, Williams (2000, 2002) identified a phenomenon she

called ‘discovered complexity.’ This research identified situations in which students discovered complexities in the course of attempting a mathematical task posed by the teacher. These discovered complexities in combination with other instructional and learning conditions would stimulate the creation by students of their own mathematical tasks. These student-created tasks provide the focus of Chapter 9.

An academic task can be examined through a wide variety of attributes. When a teacher selects a task for her/his teaching, she/he may think of such task attributes as: context, complexity, degree of openness, form and representation. In the mathematics education research tradition, considerable research has been devoted to task attributes which affect students’ mathematical problem solving (Goldin & McClintock, 1984). Each of the attributes just listed has its own structure and variations – as has just been illustrated in the case of complexity.

As these earlier studies pointed out, a systematic analysis of task attributes has direct implications for teaching and learning in mathematics classroom, and particularly for teaching via problem solving. As discussed below, it is especially important that attention be given to the analysis of the cognitive demands enacted by tasks presented in the classroom and to the situated nature of the task as it is enacted by teacher and students in the classroom.

Task attributes need to be considered in relation both to the teacher and the learners in a mathematics classroom as well as in relation to broader contextual influences such as the curriculum, social expectations and so on. Also, considerations of the ternary relations such as Teacher-Task-Learner or Learner-Task-Mathematics are needed to explore fundamental aspects in mathematics teaching and learning (Christiansen & Walther, 1986). Study of the nature of such relations will reflect the choice of theoretical framework.

Although attention to the nature of mathematical tasks is important, attention to the classroom processes associated with mathematical tasks is equally needed. Such student activities as making conjectures, abstracting mathematical properties, explaining their reasoning, validating their assertions, and discussing and questioning their own thinking and the thinking of others do not fit well with the tasks and task use employed in many ‘traditional classrooms’ (Lampert, 1990). Yet such activities are the explicit goal of many curricular initiatives (for example, NCTM, 1989) and it is mathematical tasks that provide the pretext and the catalyst for such activities.

Any consideration of the nature of mathematical tasks in classrooms must attend to elements such as: task complexity, social participants and the nature of participation, socio-cultural context, and, most importantly, the purpose for which the task has been introduced into the mathematics classroom. Some of the chapters of this book explore such purposes: mathematical tasks as catalysts for student talk (Chapter 3 by Bergem and Klette); the function of mathematical tasks in connecting mathematical ideas (Chapter 5 by Shimizu and Chapter 6 by Novotná and Hošpesová); and the use of mathematical tasks “to scaffold student entry to idiosyncratic exploration” (Chapter 9 by Williams). In Chapter 2, Kaur distinguishes tasks according to their use for learning, review, practice or

assessment, and then employs an analytical framework synthesised from other studies to carry out a more fine-grained analysis of task use in three Singapore classrooms.

It is the intention of this book to contribute to the extensive and diverse literature on mathematical tasks in two ways: by examining the classroom use of mathematical tasks from a variety of perspectives; and, by making comparison possible between forms of classroom use of mathematical tasks in different classrooms around the world. Neither approach is intended to be comprehensive, but it is hoped that the chapters that follow, both individually and in combination, will address issues of interest to the mathematics education community internationally. The remainder of this introductory chapter sets out some of the theoretical and contextual considerations that should be taken into account in any reading of this book.

#### THEORETICAL ALTERNATIVES IN CONSIDERING THE CLASSROOM USE OF MATHEMATICAL TASKS

There are many different theories currently being employed in mathematics education. Activity Theory, for example, is an obvious contender in considering how the classroom use of mathematical tasks might be situated theoretically. Recent developments in the conceptualisation of Activity Theory (eg Engeström, 2001) have increased the breadth of phenomena and contexts able to be addressed using Activity Theory. In particular, mathematical tasks can be situated naturally within the tools available for use in pedagogic activity systems. Our purpose here is not to catalogue all the different theories that might be employed in researching the classroom use of mathematical tasks, but to examine briefly a selection of relevant theories that informed the analytical work of the LPS community represented in the chapters of this book. In this context, Activity Theory serves as a useful example of an eminently eligible theory, used in other LPS publications for consideration of classroom discourse, but which was not explicitly employed in any of the chapters that follow. This example provides an opportunity to re-emphasise the commitment of the LPS community to an inclusive selectivity, by which the theory guiding each analysis is chosen pragmatically for its consistency with the purposes of that analysis.

Gellert (2008) usefully contrasts ‘interactionist’ and ‘structuralist’ perspectives on mathematics classroom practice. In the consideration of the classroom use of mathematical tasks, the interactionist perspective offers insight into the negotiative processes that interact with individuals’ use in classroom settings for the socially-mediated constitution of learning. Chapters 4 (Gallos Cronberg) and 7 (Mok) report interactionist analyses that emphasise student engagement and mediated learning respectively.

In Chapter 3, Bergem and Klette suggest that greater emphasis be given to mathematical communication in research and theory. They argue that

cognitive psychology, social constructivism, distributed cognition, semiotics and socio-cultural theory all draw our attention to the essential role that

reflective discourse and discursive practices have for fostering mathematical understanding (p. 36).

For Bergem and Klette, therefore, mathematical conversations and the discourses such conversations might embody warrant close research scrutiny. The situatedness of any such conversations is seen as critical to their realisation in consequent student learning.

The structuralist perspective potentially offers very different insights into the deployment and function of mathematical tasks in classrooms. Focusing attention on differentiated participation, a structuralist analysis aspires to explain such differentiation in terms of hierarchies and power relationships. In the case of mathematical tasks, these hierarchies reflect the enactment of an entrenched social order and the privileging of particular forms of knowledge. Within the structuralist perspective, particular pedagogies can be seen as embodying systems of social and academic privilege (Bernstein, 1996) and in the mathematics classroom it is primarily through the performance of mathematical tasks that these pedagogies are enacted. The chapter by Kaur (Chapter 2) reports an analysis that could be considered structuralist. The analysis differentiates usefully between forms of knowledge and the key resources that structure instruction in the three Singapore classrooms studied.

The chapter by Mesiti and Clarke (Chapter 10) argues that mathematical tasks should only be considered 'as performed,' since the same mathematics problem can provide a vehicle for the realisation of very different social and educational purposes. This performative emphasis is echoed in Chapter 5 by Shimizu, where it is asserted that the functions of mathematical tasks posed in classroom settings need to be considered within the contexts in which they are undertaken.

The *Theory of Didactical Situations in Mathematics* (TDSM) (Brousseau, 1997) is a coherent, well-elaborated instructional theory, capable of supporting the explicit advocacy of particular practices. Within TDSM, Brousseau carefully distinguishes the work of the mathematician, the work of the student and the work of the teacher. Particular attention is given to 'The notion of problem.'

A student isn't really doing mathematics unless she is asking herself questions and solving problems. (Brousseau, 1997, p. 79)

Similar care is given within TDSM to critical considerations such as 'epistemological obstacles' and 'didactic problems.' Novotná and Hošpesová make use of TDSM to theorise about the teacher's scaffolding of linkages between different mathematical concepts and procedures, using specific examples from two mathematics classrooms in the Czech Republic to illustrate the key points.

By way of comparison, Variation Theory is a similarly coherent theory that provides clear criteria for instructional advocacy. Variation Theory privileges constructs such as 'the object of learning' and 'dimensions of variation' (Marton & Tsui, 2004). In Chapter 7, Mok draws attention to one Shanghai teacher's deliberate partial variation of the content and constraints between problems and questions.

The choice of the theoretical lens focuses analytical attention on some aspects of the role of mathematical tasks and ignores others. This is inevitable. It is a strength of the combination of analyses reported in this book that they are not restricted to the application of a single theory. Instead, the tasks of the mathematics classroom are examined from a variety of theoretical perspectives. In several cases, the same tasks occur in different chapters, to be re-examined from the perspective of tasks tasks occur in different chapters, to be examined from the perspective of different theories. The reader is encouraged to compare the treatment of the same task in the different analyses and to reflect on which analysis connects most usefully with the reader's concerns, interests and purposes.

Another entry point for consideration of the theories relevant to the classroom use of mathematics tasks are the three related issues of Abstraction, Context and Transfer. In some discussions, abstract mathematics seems to be treated as simply decontextualised mathematics. Clarke and Helme have argued that there is no such thing as decontextualised mathematics (Clarke & Helme, 1998), since all mathematical activity is undertaken in a context of some sort. If abstraction in mathematics is to have any legitimacy or relevance, then it must reside in some form of generalisability of the mathematical matter under consideration, in the sense that the principle, concept or procedure can be thought of as transcending any particular context or instance. But, to argue that an exercise in Euclidean geometry or in pure number is an abstract task is to deny the social situatedness that has become accepted even from the most cognitivist of perspectives (Lave & Wenger, 1991).

In relation to mathematical tasks, Clarke and Helme distinguished the social context in which the task is undertaken from any 'figurative context' that might be an element of the way the task is posed. In this sense, the task:

Siu Ming's family intends to travel to Beijing by train during the national holiday, so they have booked three adult tickets and one student ticket, totalling \$560. After hearing this, Siu Ming's classmate Siu Wong would like to go to Beijing with them. As a result they buy three adult tickets and two student tickets for a total of \$640. Can you calculate the cost of each adult and student ticket? (Shanghai School 3, Lesson 7, Train task)

has a figurative context that integrates elements such as the family's need to travel by train and the familiar difference in cost between an adult and a student ticket. The social context, however, could take a wide variety of forms, including: an exploratory instructional activity undertaken in small collaborative groups; the focus of a whole class discussion, orchestrated by the teacher to draw out existing student understandings; or, an assessment task to be undertaken individually. In each case, the manner in which the task will be performed is likely to be quite different, even though we can conceive of the same student as participant in each setting. The significance of the context in which mathematical tasks are undertaken has been persuasively demonstrated by Nunes, Schliemann, Carraher and their co-workers through the well-known series of studies contrasting the performance of

mathematically equivalent tasks in school and everyday settings (Nunes, Schliemann, & Carraher, 1993).

If we take ‘transfer’ not as a description of a particular cognitive process, but as a metaphor for a skill developed in one context being used in a different context, then it is reasonable to ask, “Under what conditions (and through the instructional use of what tasks) will the likelihood of transfer be maximised?” A cognitivist might direct attention to the selective variation of task attributes with the intention of successively focusing student attention on salient aspects of the mathematical concept or procedure to be learned. Variation Theory (Marton & Tsui, 2004) identifies learning with an increasing capacity to discern relevant attributes in the object of learning. From such a perspective, particular tasks and particular sequences of tasks can be critiqued as more or less conducive to directing student attention appropriately and thereby to the optimal promotion of the discernment that is identified with learning.

Distributed Cognition (Hutchins, 1995) and other theories with a material semiotic character accord significance to artefacts as participating in cognition. Once representational forms are included in the broad class of artefacts, then mathematical tasks cease to be either the objects to which we apply our cognitive tools nor merely the social catalysts for their deployment. Rather, mathematical tasks become the embodiment of performed cognition, integrating, as they do, representational forms, socio-cultural imperatives and mathematical entities. In Chapter 10, Mesiti and Clarke attempt to portray mathematical tasks performatively in order to examine the role each task plays in affording or constraining agency and voice in the social settings in which the tasks are communally performed.

#### COGNITIVE DEMANDS OF DIFFERENT TASKS FOR DIFFERENT LEARNERS

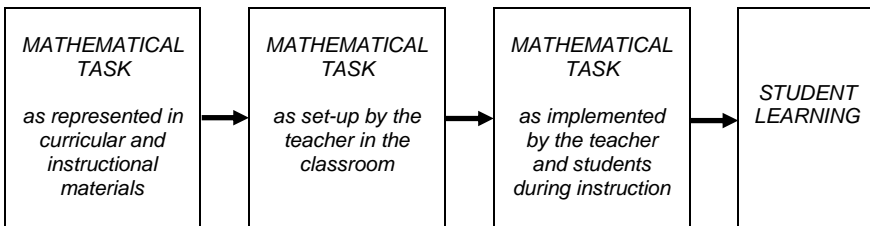
Selecting and setting appropriate tasks is key to the success of teaching mathematics (Doyle, 1988; Hiebert & Wearne, 1993; Stein & Lane, 1996; Martin, 2007). Tasks can vary not only with respect to mathematics content but also with respect to the cognitive processes involved in working on them. Worthwhile tasks are those that offer students the opportunity to extend what they know and stimulate their learning.

Doyle (1988) argues that tasks with different cognitive demands are likely to induce different kinds of learning. Tasks that require students to solve complex problems can be considered to be cognitively demanding tasks. In contrast, cognitively undemanding tasks are those that give less opportunity for the students to engage in high-level cognitive processes. Thus, the nature and the role of tasks that entail different types of cognitive demand are key to students’ opportunities for learning. Chapter 8 by Huang and Cai compares the cognitive demands of the mathematical tasks used in the US and Chinese classrooms. In this chapter, reference is made to ‘declining cognitive demand’ and the important point is introduced that the cognitive demand of a task depends not just on the



mathematical form of the initial task, but on the manner in which the task is enacted in the classroom.

The cognitive demands of mathematical tasks can change as tasks are introduced to students and/or as tasks are enacted during instruction (Stein, Grover, & Henningsen, 1996). The progression of mathematical tasks can be modeled by identifying the phases of enactment from their original form as they appear in the pages of textbooks or other curriculum materials to the tasks that teachers actually provide to students and then to the tasks as enacted by the teacher and students in classroom lessons (see Figure 1).



*Figure 1. Progression of mathematical tasks (adapted from Stein, Grover, & Henningsen, 1996)*

The first two arrows in the diagram identify critical phases in the instructional life of tasks at which cognitive demands are susceptible to being altered. The tasks, especially as enacted, have consequences for student learning of mathematics, as is shown by the third arrow in the figure. The features of an instructional task, especially its cognitive demand, often change as a task passes through these phases (Stein, Grover, & Henningsen, 1996).

In order to track changes in cognitive demand, tasks can be classified at different levels of cognitive demand. Implementing cognitively challenging tasks in ways that maintain students' opportunities to engage in high-level cognitive processes is not a trivial endeavour (Henningsen & Stein, 1997). In the TIMSS 1999 video study, the ability to maintain the high-level demands of cognitively challenging tasks during instruction was the central feature that distinguished classroom teaching in different countries (Hiebert et al., 2003). One of the questions addressed in several of the chapters in this book is whether it is useful or even feasible to classify a task as cognitively challenging, independent of the manner in which it is performed in the classroom. Instead, the progression in task form and demand as depicted in Figure 1 resembles the distinction made by Variation Theory between the intended object of learning, the enacted object of learning, and the experienced object of learning. The TIMSS 1999 Video Study sensibly distinguished between the potential demands of a task and the extent to which those demands were realised in the actual practice of the classroom. As can be seen, there is a strong convergence of view towards seeing mathematical tasks as performatively defined rather than as static objects enshrined in text.

DISTINGUISHING BETWEEN THE USE OF MATHEMATICAL TASKS FOR  
INSTRUCTION AND ASSESSMENT

If the use of a mathematical task is intended to promote the student's learning of mathematics, that use can reasonably be designated *instructional*. If the use of a mathematical task is intended to generate information about student learning or, relatedly, about the effectiveness of instruction, then we can characterise that use as being for the purposes of assessment. Of course, the same task can serve both purposes, even at the same time, as is evident from the extensive literature on formative assessment (William & Black, 1996). It does not necessarily follow, however, that a given task (or task type) is equally effective when used for instructional purposes or assessment purposes. This distinction must be seen as indicative rather than definitive. It can be argued that all tasks constitute assessment in as much as all tasks reveal information regarding any students' attempts at the task (particularly given the contemporary prioritisation of formative assessment). Equally, even test items can be instructional, serving to convey messages about what is and what is not correct and valued mathematical performance.

The chapter by Kaur (Chapter 2) classifies tasks as learning, review, practice and assessment tasks. The distinction between the instructional and assessment uses of mathematical tasks in the classrooms studied here draws attention to the lack of alignment of the espoused curricular goals with either instructional or assessment practices in the three Singapore classrooms.

Earlier, reference was made to the designation 'rich mathematical tasks.' One context in which this characterisation of tasks has been used is assessment. In that context, 'rich' is taken to signify a task of sufficient complexity as to admit a variety of approaches and therefore to have the capacity to reveal differences in student conceptions of relevant mathematical concepts and procedures.

Assessment tasks should be sufficiently open to invite a range of responses from students and so allow students to disclose their level of competence and understanding at a number of levels. Such rich assessment tasks:

- connect naturally with what has been taught
- address a range of outcomes in the one task
- allow all students to undertake the activity
- can be successfully undertaken using a range of methods or approaches
- encourage students to disclose their own understanding of what they have learned
- allow students to show connections they are able to make between the concepts they have learned
- are themselves worthwhile activities for children's learning
- draw the attention of teachers to important aspects of mathematical activity

- help teachers to decide whether it is appropriate to move on for most students and what specific help others may require. (Clarke, 1996, pp. 361-362)

There is no question that many of the tasks discussed in this book are ‘rich tasks’ by the criteria just listed. Whether a particular task should be categorised as an instructional task or an assessment task or as both depends on the way in which the task was utilised by the particular teacher. Certainly, it is clear in several chapters that teachers made use of the information provided by many tasks to guide their instruction.

The premise of Kaur’s chapter is an important one: however we might classify tasks (whether as serving instructional or assessment purposes), a crucial consideration is the coherence with which valued mathematical performance is nurtured, promoted, revealed, evaluated and acknowledged in the classroom and it is through our choice and use of mathematical tasks that this coherence is displayed and communicated.

#### THE FOCUS AND THE STRUCTURE OF THIS BOOK

The following chapters in this book explore and discuss the nature and roles of mathematical tasks in LPS classrooms with a focus on their impact on students’ learning. It is an essential theme of the Learner’s Perspective Study that international comparative research offers unique opportunities to interrogate established practice, existing theories and entrenched assumptions (Clarke, Emanuelsson, Jablonka, & Mok, 2006; Clarke, Keitel, & Shimizu, 2006). In this book, focusing on the nature, role and implementation of mathematical tasks, we offer the reader a variety of images of classrooms from the countries participating in the Learner’s Perspective Study.

It was of interest in the development of this book whether the enactment of mathematical tasks observed in the classrooms of one country showed consistency of form and purpose, sufficiently different from other classrooms, such as to suggest a culturally-specific character. Because of the highly selective nature of the classrooms studied in each country, no claims can be made about national typification of practice, however any regularities of practices sustained across thirty lessons demand some consideration as to the possible causes of such consistency. Whether or not such identifiable characteristics in the treatment of tasks exist as cultural traits, the Learner’s Perspective Study was predicated on a belief that international comparative studies are likely to reveal patterns of practice less evident in studies limited to a single country or community.

‘Teaching’ is not only about teaching what is conventionally called content, but also teaching what a lesson is and how to participate in it. Students learn not only what the answer is to a task but also how to approach academic tasks. The performative nature of tasks and the nature of student participation in the classroom activities catalysed by mathematics tasks are explored in many different ways in the chapters that follow.

This book includes chapters with very different approaches, such as: comparisons of the features of tasks within a lesson sequence in different cultures, the cognitive demands of different tasks for different learners, mathematical tasks as experienced from the learner's perspective, the role of tasks as vehicles for interaction among classroom participants, or the analysis of particular emphases within tasks: representations, real-world contexts, proof, problem solving, or types of reasoning and argumentation. The chapters have been organised as a progression from the consideration of mathematical tasks in the classrooms of single countries to comparative analyses across classrooms situated in several countries. While each individual chapter rewards careful reading, it is our hope that in combination the different perspectives on the classroom use of mathematical tasks will provide insights through differences in the settings in which the tasks are performed, differences in ways in which classroom participants are positioned in relation to the tasks, and profound differences in how each teacher utilises mathematical tasks, in partnership with their students, to create a distinctive form of mathematical activity.

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