The lore of Mathematics and Law
[with apologies to my master Master Wheeler]

M M Park
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ABSTRACT: the reliance upon mathematics in the law is considered by a jaundiced observer who believes that there exists too much reliance and that this reliance is “pseudo science” in an attempt to bolster a litigant’s position in a lawsuit. The observer’s view is that lawyers are ill-equipped to utilise mathematics.

KEYWORDS: mathematics — geometric progression — law — legal profession — history — legal facetiae

Quite apart from anything else, the elementary mathematics of judges are prone to error, except in the case of Lord Denning who was a wrangler, and his maths are too good for anyone else to understand.


[1] Fable has it that an ancient shah was outsmarted by his grand vizier who invented the game of chess. In gratitude for the new pastime the shah desired to reward his vizier with a gold piece for every one of the board’s 64 squares. The vizier declined and suggested an alternative reward acceptable to him. He suggested that the Shah give to him a single grain of wheat for the first square, doubling it to two grains for the second square, doubling it to four grains for the third square, doubling it to eight grains for the fourth square, and so on, doubling the grains for the next square until all 64 squares of the board had been accounted for. Quickly comparing the relative value of the gold he had offered with the wheat requested by the vizier, the Shah accepted the vizier’s counter-offer.

[2] The Shah ordered a bag of wheat to be brought into the room and bade his servants to carry out the vizier’s instructions. After the servants had placed the requisite number of grains on the tenth square, they found it difficult to continue on the eleventh. Undaunted, the Shah directed them to place the grains in a small pile adjacent to the board. The small pile soon became large. To the Shah’s amazement the bag of wheat was emptied before the sixteenth square of the board had been accounted for. He called for another bag of wheat. And another. The large pile soon became huge. He called for more bags of wheat. He finally conceded defeat. All of the wheat in India, indeed all the wheat in the then world was not enough to satisfy the bargain struck between the Shah and his vizier. The man who invented chess was smarter than the average vizier or even the average Shah.
[3] To keep the bargain the Shah required $2^{64} - 1$ or 18,446,744,073,709,551,615 grains of wheat or (approximately) 180 billion tonnes. So that the reader can appreciate the enormity (and impossibility) of the Shah’s obligation to his vizier we advise that the present day world harvest is 415 million tonnes per annum (at least it was prior to the United States’ “export enhancement programme”). Consequently, the Shah was indebted to his vizier to the tune of about 435 years of present day annual world production of wheat (Poundstone, *Labyrinth of Reason*, 1988, page 149; Peterson, *Islands of Truth*, 1990, page 196).

[4] An alternative illustration from Devlin (*Mathematics: the New Golden Age*, 1988, page 1), is that using coins, each two millimetres thick (the approximate thickness of the Australian twenty cent piece). The total number of coins stacked on top of each other “will stretch out beyond the Moon (a mere 150 million kilometres) and will in fact reach almost to the nearest star, *Proxima Centauri*, some four light years (or 40,000,000,000,000 kilometres) from Earth”.

> When I was a judge at first instance, sitting alone, I could and did do justice. But when I went to the Court of Appeal of three, I found that the chances of doing justice were two to one against!

[5] The ancient Shah was not the only one to be caught out by a geometric series — Megarry’s *Second Miscellany-at-Law* (1973, page 283) refers to the following from Charles Butler’s autobiography: Butler was instructed to prepare a partnership deed between ten landowners who were engaged in a mining venture. His instructions were to include a provision for one or more of the partners to be advanced money by any other or others of the syndicate, the moneys so advanced constituting a charge upon the share or respective shares of the borrower or respective borrowers. At that time any mortgage for an indefinite sum was subject to stamp duty of £25 and Butler advised his clients against the proposed provision on the basis that the stamp duty would amount to £90,720,000. Butler miscalculated, but his advice was to save his clients £25 duty on each of the possible 57,002 possible mortgages or a total of £1,425,050, a tidy enough sum in the early nineteenth century.

[6] Unfortunately for Butler’s credibility, doubt is cast on the story by Willock’s claim, in his volume of legal reminiscences, to advising his clients upon an identical
problem — identical even to the extent that Willock made the same miscalculation as did Butler: Willock, *Legal Facetiae* (1888) pages 352-3.

The reason for doing this controversial work, much of it unpaid or *pro bono*, seemed obvious to the partners. “We have to take it on”, said Fortas, “because if we don’t nobody else will”. To which Thurman Arnold later added, “Isn’t it wonderful to work on something in which you really believe?” In the same spirit was Paul Porter’s motto for the firm: “When in doubt, do the right thing” (later changed by Fortas, once on the Supreme Court, to “We are never in doubt. We always do the right thing. Sometimes we have to do it 5-4.”).

Murphy, *Fortas: the rise and ruin of a Supreme Court Justice* (1988) 82.

[7] A century before Butler, a Mr Whitacre was sufficiently imprudent to agree with a Mr Thornborow that in return for 2s. 6d. down with a further £4 17s. 6d. to be paid to him in a years time upon Whitacre performing his part, Whitacre was to deliver to Thornborow two grains of rye corn on Monday 29 March and four grains on the Monday then next following and eight grains on the Monday next and sixteen grains on the next Monday and so on for one year when Thornborow would pay the outstanding £4 17s. 6d. to Whitacre: *Thornborow v Whitacre* (1705) 2 Ld Raym 1164, 92 ER 270; 6 Mod Rep 305, 87 ER 1044; 3 Salk 97, 91 ER 715.

[8] Salkeld for Whitacre sought to maintain the defendant’s demurrer on the basis that the amount of grain required to fulfil the agreement was in excess of all the rye in the world and thus the agreement was void for impossibility of performance.

[9] Holt CJ construed the contract terms so as to require Whitacre to double his delivery of grain every other Monday (or each fortnight) instead of weekly every Monday (this would amount to a total of less than four tonnes while a weekly delivery would require more than 175 million tonnes).

[10] Salkeld, noting the way the judicial wind was blowing, offered the plaintiff the return of his half crown and costs which was accepted.

[11] During argument the case of *James v Morgan* (1664) 1 Lev 111, 83 ER 323; 1 Keb 569, 83 ER 1116 was referred to. This case, before Hide CJ, was that the defendant was to pay for a gelding with a barley-corn for each nail, doubling it every nail with a total of eight nails for each of the four feet. The estimated “price” was 500 quarters of barley and the Chief Justice directed the jury to award the plaintiff damages in the sum equal to the value of the horse (£8) which they did.
[12] Given that the authoritative constitutional (but hitherto non-judicial) dictum of Sir Robert Garran:

… imagine a law student approaching the study of a law student approaching the study of the Constitution for the first time [and in particular the first fifty years of case law on section 92]. He buys his law books, opens his notebook and begins with a historical survey. … The student [understandably] closes his notebook, sells his law books, and resolves to take up some easy study, like nuclear physics or higher mathematics.

Prosper the Commonwealth (1958) 413-5

has only recently been conferred with the highest judicial imprimatur: Cole v Whitfield (1988) 165 CLR 360 at 392 per the Full Court; consider the career path chosen by the son of Harlan Fiske Stone, Associate Justice of the US Supreme Court (1925-41) and Chief Justice (1941-46). The narrator is Stanislaw Ulam:

Marshall Stone, whom I met when he came through Warsaw with [John] von Neumann and Birkhoff in 1935 on the way back from [the] Moscow Conference, had had a meteoric career at the university, although he was only thirty-one years old. Already a full professor, he was quite influential in the affairs of the department and of the university for that matter. He wrote a classic work, a comprehensive and authoritative book on Hilbert space, an infinitely dimensional generalization of the three-dimensional or n-dimensional Euclidean space, mathematically basic to modern quantum theory in physics. He was the son of Harlan Stone, Chief Justice of the Supreme Court. It is said that his father proudly said of Marshall’s mathematical achievements, “I am puzzled but happy that my son had written a book of which I understand nothing at all.”

Adventures of a Mathematician (1976) 92.

[13] Unlike King Shivam, the Shah bested by his grand vizier Sissa Ben Dahiv, the Honourable John Street, Judge of the 352nd District Court, Tarrant County, Texas, is not a man mystified by geometric progressions. The judge had the conduct of a personal injury action brought by Andrew Cavrizales against Wal-Mart Stores, Inc.

The plaintiff’s lawyer wished to interrogate Sam Walton who, until his death in early 1992, was the founder, major shareholder, Chairman of the Board, and past President of the defendant company. He was also reputed to be the wealthiest person in the US and one of the wealthiest in the world.

[14] Originally Judge Street had ordered that Sam Walton present himself for deposition at the District Court, Fort worth. The defendant company opposed this direction and, on appeal, the Texas Supreme Court ordered a modification to Judge

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* The reverse career path (from higher mathematics to law) was chosen by the eminent US constitutional lawyer Laurence Tribe: Park, “Grendel’s champion wages battle with Judge Beowulf”, (Autumn 1995) 92 Vic BN 47.
Street’s order such that the deposition be taken in the county of Walton’s residence at Bentonville, Arkansas.

[15] Consequently, Judge Street modified his order to provide that Walton’s deposition be taken at the Wal-Mart headquarters in Bentonville — this location being agreed to by all parties to the suit although not necessarily by Walton himself who was not a party. Sam wasn’t too happy about this and he sought and obtained a protective order from the District Court in Bentonville directing that the deposition be taken at the same time as that ordered by Judge Street but at the local courthouse in Bentonville rather than at the Wal-Mart headquarters. Presumably ol’ Sam sought the change in venue to allow for the presence of a judge to rule on any legal questions arising from the taking of Sam’s deposition upon which he might seek a judicial ruling.

[16] On the day appointed for taking the deposition Wallace Craig, counsel for Cavrizesales, attended at the Wal-Mart head office and declined to attend at the Bentonville courthouse to take Sam’s deposition although it was only about one mile from the head office.

[17] Thereafter the plaintiff sought and obtained from Judge Street an order that was a real zinger: the judge struck out Wal-Mart’s pleadings, granted a default judgment regarding liability against Wal-Mart and ordered Wal-Mart to produce Sam for deposition in Judge Street’s Fort Worth courtroom and imposed sanctions on Wal-Mart for failing to so produce Sam. The sanctions would have taken even the breath of Sissa Ben Dahiv away.

[18] The sanctions imposed by Judge Street penalized Wal-Mart for failing to produce Sam for deposition as directed to the tune of $10,000 for the first day of non-compliance, $20,000 for the second day of non-compliance and doubling thereafter for each day until the eighth day of failure to comply was to cost the defendant company $1,280,000. Thereafter a penalty of $1,000,000 per day was to be imposed for each subsequent day of failing to comply. While $1m per day may sound a lot to us mere mortals; Sam’s fortune, estimated at $9 billion would have permitted him to hold out for nearly 25 years before he was forced to seek bankruptcy protection. But then again, the sanctions were imposed against Wal-Mart and not
against Sam who was not, after all, a party to the suit. Let us not forget the authoritative (albeit non-judicial) pronouncement of Nelson Bunker Hunt (and he is one who should know):

A billion dollars ain’t what it used to be.

[19] Of course Wal-Mart appealed: 761 SW 2d 587, Court of Appeals of Texas (Fort Worth, December 12, 1988). Although the Texas Rules of Civil Procedure provide for sanctions in the event of discovery abuse there seems little point in seeking to compel Walton’s deposition given that Judge Street had granted a default judgment to the plaintiff. Wal-Mart’s appeal before Joe Spurlock II, Hill and Lattimore JJ failed although the concurrence of Lattimore J was tinged with reluctant misgiving:

… The drivers of this unfortunate series of events appears to be the desire of skilled and strong-willed lawyers and [a] judge to prevail regardless of the imbalance between procedure and results. This does little to improve the standing of the civil justice system with the public

I concur only because I must follow the law as it now stands. 

Wal-Mart Stores, Inc. v Street, 761 SW 2d 587 at 591-2 (1988)

[20] Well might Australian lawyers (even those who participated in the epic Holmes a Court-BHP and Bond battles) envy Wallace Craig’s windfall provided by Judge Street’s appreciation of the geometric series.

Each equation … in the book [A Brief History of Time] would have halved the sales.

British theoretical physicist and author, Stephen Hawking.

[21] Whereas some taxation statutes incorporate mathematical formulae or equations the late Sir Richard Eggleston was gentleman enough to place them in appendices. Some law reviews are sufficiently bold (and perhaps imprudent) to include them in their published journal articles.

[22] Consider, for example, equation A4 from Craswell, “Insecurity, Repudiation and Cure”, 19 Journal Legal Studies 399 at 433 (1990):

\[ [1 - \theta(x)][U_s + P - C] + \theta(x)[V_s - D_v + (D_v - E_v) + K] - x \]

or equation A13 from the very next article: Shavell, “Deferences and the Punishment of Attempts”, 19 Journal Legal Studies 435 at 466 (1990):

\[ \int_0^1 (1 - q)p_1^* \{qh - [(1 - q)p_1^*s_1 + qp_2^*w]\} f((1 - q)p_1^*s_1 + qp_2^*w)g(q) dq \]

[23] For a real humdinger you have to hand it to the economists like Keith N Hylton’s equation 17 (“Costly Litigation and Legal Error under Negligence”, 6(2) Journal of Law, Economics and Organization 433 at 443 (1990).
However the one that really grabbed this writer was contained in footnote 110 of “Note — the use of DNA Typing in Criminal Prosecutions: a flawless partnership of law and science?”, 34 New York Law School Review 485 at 504 (1989). The footnote begins:

“The Hardy-Weinberg principle is expressed algebraically as $P^2 + 2PQ + Q^2$ …”

and continues by asserting that when that expression equals one (or unity), there is Hardy-Weinberg equilibrium between the qualities $P$ and $Q$.

The first thing to note is that the fine print of the footnote has forced the printer to abandon superscripts and the above expression should have read: $P^2 + 2PQ + Q^2$.

As for the assertion regarding Hardy-Weinberg equilibrium, when $P + Q = 1$ we will always have equilibrium because

If $P + Q = 1$ it follows that $(P + Q)^2 = 1^2 = P^2 + 2PQ + Q^2 = 1$

that is, when $P + Q = 1$, then $P^2 + 2PQ + Q^2 = 1$

Consequently, in reliance upon Josie Jo Barr’s assertion we can confidently conclude that there is Hardy-Weinberg equilibrium between those Supreme Court justices descended from Afghani free-range yak herders and those justices not so descended. If we let $P$ represent the proportion of female legal practitioners, and $Q$ the proportion of male legal practitioners we will see that there is always Hardy-Weinberg equilibrium between male and female legal practitioners. This is so whether female practitioners make up 5% or 90% of all legal practitioners (or whether female practitioners make up 45% or 50% of all legal practitioners).

Thus, the Hardy-Weinberg principle, as expostulated by Ms Barr of the New York Law School Law Review, is nothing more than a fancy way of stating that $1 \times 1 = 1$. It is apposite to dust off and trot out the well-known (but perhaps not sufficiently well-known) dictum of Sullivan J in People v Collins, 438 P 2d 33, 68 Cal Rptr 497 (1968):

Mathematics, a veritable sorcerer in our computerized society, while assisting the trier of fact in the search for truth, must not cast a spell over him.

*One must question whether the inclusion of formulae and equations in legal journal articles is a greater sin than excessively long titles for those same legal journal articles?
… in part, that’s because an innumerate public is easily impressed by a bit of mathematics.  

[29] Lest readers conclude that this rubbish can only be found in US Law Reviews, their attention is directed to Volume 17, Part 4 of our own *Melbourne University Law Review* (the 1990 Law and Economics Symposium issue).


*There was one very deaf judge who was known throughout the Temple for his kindness. Since he was not only deaf but also absent-minded he shall be nameless here, for he fell into many errors. He once tried a commercial case which involved a mass of very complicated figures, which counsel on both sides did their best to unravel for his Lordship.*

*When he came to deliver judgment, he ended a long speech by saying, “I must come to the conclusion that the plaintiff has made out his case, and I find for him in the sum of £ 24,759 15s. 6d. If my figures are incorrect, counsel on either side will, of course, correct me.”*

*The judge cast upon counsel that benign and hopeful expression which is often assumed by deaf persons. Counsel stared at each other in amazement. Then one of them burst out, “Why, the damned old fool has added the date to these figures!”*

*There is none so alert as he that strives to hear, even if he be deaf. The judge leaned forward and said in an amiable voice, “Dear me, so I have!”*

Author/s:
Park, M. M.

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