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Dedication:

For Joan, Alison and Janette

† † † † † †

Of all the intellectual hurdles which the human mind has confronted and has overcome in the last 1500 years the one which seems to me to have been the most amazing in character and most stupendous in the scope of its consequences is the one relating to the problem of motion . . . . . .

Herbert Butterfield (1900 –1979)

If you have knowledge, let others light their candles at it.

Thomas Fuller (1608 - 1661)

† † † † † †

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Preface

This monograph and the associated computer program arose out of the author's experience in teaching and research in agricultural engineering at the University of Melbourne. This work included a course on fluid-particle mechanics within larger courses on the properties of agricultural materials and the design of machines to handle them.

The program was prompted by a certain frustration with the fact that existing methods were largely limited to a one-dimensional analysis and the use of terminal velocity as a basis for separation processes. This occurs in the context of a range of separation, distribution, and other problems that require an analysis in two dimensions.

It is true that the two-dimensional analyses in the research literature, which are often based on various approximations, are satisfactory for their specific purpose. However, they do not lend themselves to a wider use because, as published, they are limited to the solution of a particular design problem.

Given the widespread use of the digital computer, it is appropriate that there should be a general solution that will solve all problems. Such a program is now available and can be downloaded from the University of Melbourne web site:

[http://repository.unimelb.edu.au/10187/2415](http://repository.unimelb.edu.au/10187/2415)

This will open up new fields of design that hitherto have been beyond the students' capacity if not beyond the designers' imagination. In terms of research, while this program will not totally eliminate the need for experimental work, it will provide guidance about the relative effects of the different variables and so guide the research along the most efficient lines.

If it is argued that we do not have sufficiently good data on material properties to justify the use of such a program, then the writer would respond that perhaps this is because, up till recently, our computational capacity did not justify their measurement. This program will provide a readily available but powerful tool to aid in the design of new machines and insights into their operation that would justify a new round of data collection to improve its accuracy.

Following the introductory review in Chapter 1, the interaction of bodies and fluids moving with a relative velocity is considered in Chapter 2. This is illustrated by the drag coefficient-Reynolds number relationship for bodies of various shapes and for some agricultural materials. In Chapter 3 the concepts of terminal and floating velocity are introduced and their application as a basis for the separation of two fractions in a mixture are discussed. Chapter 4 introduces the two-dimensional, general solution to the fluid-particle trajectory problem and explains the basis of the algorithm on which it is based. It includes a brief section validating results against published work. Chapters 5 to 16 detail a number of applications of the program, mainly in agricultural engineering. Chapters 9 and 13 are kept for future use.

Readers who are familiar with fluid-particle mechanics may move directly to Chapter 4. The program has an associated Users Manual (at the above site) to assist in its use.

The author wishes to acknowledge the published work on his topic. He also acknowledges the contribution of his agricultural engineering graduate students, the Computer Science students who developed an early version of the program, and AIMTEC who developed the current Windows-based version.

The author would value any comments on the work and a report on any errors.

Ross H. Macmillan  April 2007  r.macmillan@devtech.unimelb.edu.au
CHAPTER 1

INTRODUCTION

CONTENTS
1.1 BACKGROUND 1
1.2 EXAMPLES 1
1.2.1 Velocity fields
1.2.2 Force fields
1.2.3 System boundaries and orientation
1.2.4 Limitations
1.3 FLUID – PARTICLE SYSTEMS - AN OVERVIEW 2
1.3.1 Introduction
1.3.2 Basic studies
1.3.3 Formal studies of agricultural materials and processes
1.4 CONCLUSION 5
1.5 REFERENCES 5

CHAPTER 2

FLUID - PARTICLE INTERACTION

CONTENTS
2.1 INTRODUCTION 3
2.2 NEWTONS LAW 3
2.3 PARAMETERS 3
2.3.1 Drag coefficient
2.3.2 Reynolds number
2.3.3 Flow conditions
2.4 DRAG FORCE ON PARTICLES 5
2.5 ASSUMPTIONS 5
2.6 DRAG COEFFICIENTS FOR SPHERES 6
2.6.1 Laminar flow
2.6.2 Turbulent flow
2.6.3 Transition flow
2.7 DRAG COEFFICIENTS FOR NON-SPHERICAL SHAPES 7
2.7.1 Defining parameters
2.7.2 Orientation
2.7.3 Shape
2.7.4 Size
2.7.5 Summary
2.8 CROP PRODUCTS 9
2.9 OTHER SECONDARY EFFECTS 9
2.9.1 Effect of turbulence in the motion of the fluid
2.9.2 Effect of acceleration of the particle
2.9.3 Effect of the presence of other particles
2.9.4 Effect of particle roughness
2.9.5 Conclusion
2.10 CONCLUSION 12
2.11 REFERENCES 12
CHAPTER 7

GROUND SPRAYING

CONTENTS
7.1 INTRODUCTION
7.2 SPRAYING OF LIQUID DROPS
  7.2.1 Variables of interest
7.3 WORKED EXAMPLE
  7.3.1 Data
  7.3.2 Results
  7.3.3 Conclusions
7.4 WORKED EXAMPLE
  7.4.1 Data
  7.4.2 Geometry
  7.4.3 Calculations of ideal position
  7.4.4 Calculation of actual position using TPS
  7.4.5 Results
  7.4.6 Conclusions
7.5 EXAMPLE WHERE TPS CANNOT BE USED
7.6 EXAMPLE
7.7 EXAMPLE
7.8 REFERENCE

CHAPTER 8

AERIAL SPRAYING

CONTENTS
8.1 INTRODUCTION
8.2 AERIAL SPRAYING OF LIQUID DROPS
8.3 WORKED EXAMPLE
  8.3.1 Introduction
  8.3.2 Data
  8.3.3 Results
  8.3.4 Conclusions
8.4 EXAMPLE
8.5 REFERENCES

CHAPTER 9

Not used – kept for future use.
PART III
SEPARATION PROCESSES

CHAPTER 10
WINNOWING IN THE WIND

CONTENTS
10.1 INTRODUCTION ........................................... 10.1
10.2 VARIABLES OF INTEREST ......................... 10.2
10.3 WORKED EXAMPLE ............................... 10.3
   10.3.1 Data ........................................ 10.3
   10.3.2 Results ................................... 10.3
   10.3.3 Conclusion ................................ 10.3
10.4 REFERENCES ......................................... 10.7

CHAPTER 11
TUNNEL WINNOWING

CONTENTS
11.1 INTRODUCTION ........................................ 11.1
11.2 VARIABLES OF INTEREST .......................... 11.2
11.3 WORKED EXAMPLE .................................. 11.4
   11.3.1 Data ........................................ 11.4
   11.3.2 Results ................................... 11.4
   11.3.3 Conclusions ................................ 11.4
11.4 WORKED EXAMPLE .................................. 11.6
   11.4.1 Data ........................................ 11.6
   11.4.2 Results ................................... 11.6
   11.4.3 Conclusions ................................ 11.6
11.5 EXAMPLE ............................................. 11.8
11.6 EXAMPLE ............................................. 11.8
11.7 EXAMPLE ............................................. 11.8
11.8 REFERENCES ......................................... 11.8

CHAPTER 12
COLUMN WINNOWING

CONTENTS
12.1 INTRODUCTION ........................................ 12.1
12.2 ANALYSIS .............................................. 12.2
   12.2.1 Variables of interest ...................... 12.3
12.3 WORKED EXAMPLE .................................. 12.4
   12.3.1 Data ........................................ 12.4
   12.3.2 Results ................................... 12.4
   12.3.3 Conclusions ................................ 12.4
12.4 REFERENCES ......................................... 12.5

CHAPTER 13

Not used – kept for future use.
PART IV
MISCELLANEOUS PROCESSES

CHAPTER 14
RAIN

CONTENTS
14.1 INTRODUCTION 14.1
14.2 BEHAVIOUR OF LARGE DROPS 14.2
14.3 WORKED EXAMPLE 14.3
14.3.1 Introduction 14.3.1
14.3.2 Data 14.3.2
14.3.3 Results 14.3.3
14.3.4 Conclusions 14.3.4
14.4 REFERENCE 14.4

CHAPTER 15
FIRE!

CONTENTS
15.1 INTRODUCTION 15.1
15.2 BEHAVIOUR OF BURNING EMBERS 15.2
15.2.1 Plume 15.2.1
15.2.2 Ambient wind 15.2.2
15.3 WORKED EXAMPLE 15.3
15.3.1 Introduction 15.3.1
15.3.2 Data 15.3.2
15.3.3 Calculations 15.3.3
15.3.4 Results 15.3.4
15.3.5 Conclusions 15.3.5
15.4 REFERENCES 15.4

CHAPTER 16
MINERAL SEDIMENTATION

CONTENTS
16.1 INTRODUCTION 16.1
16.2 ANALYSIS 16.2
16.2.1 Traditional 16.2.1
16.2.2 Using TPS 16.2.2
16.2.3 Variables of interest 16.2.3
16.3 WORKED EXAMPLE 16.3
16.3.1 Data 16.3.1
16.3.2 Results 16.3.2
16.3.3 Conclusions 16.3.3
16.4 REFERENCES 16.4

APPENDIX I List of symbols
APPENDIX II Two dimensional analysis of a trajectory
APPENDIX III Physical properties of air and water
APPENDIX IV References
APPENDIX V Other on-line publications by the same author
PART I

CHAPTERS 1 – 4

GENERAL THEORY

It is not enough to harvest knowledge by study; the wind of talk must winnow it, and blow away the chaff; then will the clear, bright grains of wisdom be . . . . . .

William Mathews (1818 – 1909)
CHAPTER 1

INTRODUCTION

CONTENTS

1.1 BACKGROUND 1.2

1.2 EXAMPLES 1.2
  1.2.1 Velocity fields
    (a) Particles dropped in still air
    (b) Particles projected in still air
    (c) Particles dropping in an air-stream
    (d) Particles projected into an air-stream
  1.2.2 Force fields
  1.2.3 System boundaries and orientation
    (a) 'Environmental' boundary
    (b) 'Machine' boundary
  1.2.4 Limitations

1.3 FLUID – PARTICLE SYSTEMS - AN OVERVIEW 1.4

  1.3.1 Introduction
  1.3.2 Basic studies
  1.3.3 Formal studies of agricultural materials and processes
    (a) Practical / experimental
    (b) Theoretical

1.4 CONCLUSION 1.7

1.5 REFERENCES 1.7
1.1 BACKGROUND

Many natural processes such as seeds blowing in the wind, raindrops falling to the earth or sand grains carried in a stream involve fluid - particle interactions. Seeds that use a 'parachute' to float in the wind are examples of the exploitation of this interaction to achieve a beneficial outcome.

Many traditional agricultural processes such as spreading seed by hand or winnowing grain and chaff in the wind are human examples that are as old as civilization. However many modern operations in agriculture also involve the relative movement of particles in a fluid and require a detailed analysis if the equipment is to be designed on a more analytical basis.

The analysis of these processes, which is based on what is termed 'fluid - particle mechanics', is an important aspect of agricultural engineering. However it is only in relatively recent times that it has been possible to do this analysis with the use of the digital computer. With such programs it is possible for students and engineers to solve a range of typical problems or at least gain insight into these processes and hence into the machine and environmental elements that involve them.

The computer program, which is associated with this monograph, plots the trajectories of particles released with a defined (including zero) velocity into a fluid stream with a defined (including zero) velocity. While the program will work with any fluid (as defined by its density and absolute viscosity) most processes involve air, hence it is the aerodynamic properties and processes that are of most interest; the following introductory discussion in Chapters 2 to 4 inclusive will mainly be limited to these.

1.2 EXAMPLES

Typical situations and illustrative examples are given in the following:

1.2.1 Velocity fields

(a) Particles dropped in still air

This is the simplest example and is illustrated by example by the falling of rain drops in no-wind conditions. Traditionally the parameter that is of interest in that field is the terminal or maximum velocity since this is a measure of the erosive power of the rain.

(b) Particles projected in still air

This example is approximated by the distribution of particles as in spreading fertilizer or seed by hand or machine. Many sports using balls such as golf also involve this situation. Here the parameter that is usually of interest is the range, ie, the distance travelled by the particle from its release to the end of its trajectory (path). Where multiple particles with a range of diameters are involved, the success of the process might be represented by their final mass distribution.

1 'Fluid' includes liquids and gases
The simplest example is illustrated by the traditional cleaning (winnowing) problem where particle mixtures (fractions) are dropped into an air-stream and are separated according to their respective trajectories that arise from their different physical and aerodynamic characteristics. Grain cleaning in winnowers and complete harvesting machines also involve this situation.

This is the most general form of the problem - all of the above are particular examples of it. The usual objective is to obtain the trajectory and to read appropriate parameters from it.

Many real life problems involve combinations of such 'simple' situations; these can usually be solved sequentially. Also many secondary effects may be significant. For example it is often assumed that the particles are spheres or that trajectory of a stream of particles may be predicted by the trajectory of a single particle. Also the assumption of uniform fluid conditions may not be valid, for example, in natural wind where there may be a boundary layer, ie, a variation in velocity near to the earth's surface.

1.2.2 Force fields

(i) Gravity field which is the most common and applies to most problems near the earth's surface.

(ii) Electrostatic field where an electric potential may be superimposed on the gravity field to attract the very small particles (usually dust or liquid droplets) to the target, as in electrostatic spraying.

(iii) Centrifugal field where the particle is caused to rotate as on the disc of a rotary fertilizer distributor or in a centrifugal separator (cyclone) where there is a complex interaction with the gravity, fluid and solid friction.

The later two fields are not considered further in this volume.

1.2.3. System boundaries and orientation

Two forms of boundary can be identified:

(a) 'Environmental' boundary

Here the air-stream (if present) is the natural wind and the usual boundary is the surface of the earth, which of course may not be horizontal. The situation may be complicated by the presence of a velocity profile or boundary layer where the velocity close to the ground varies in magnitude. Here the problem is to allow for and, where appropriate, use the wind to achieve the objective of the process. Typical examples within the environmental boundary include rain, distributing particles onto a crop or ground surface and in many sporting activities using balls.

(b) 'Machine' boundary

Here, for example, the air-stream is produced by a fan and (perhaps initially) constrained within the boundaries of a 'machine' of some type. The design problem is to choose the air velocity and physical and geometrical arrangement of the components, relative to the gravity direction, to achieve some particular objective such as the separation of two or more fractions or components in a mixture of particles.
While we may identify these simple situations, real life problems may involve a combination of them. For example, in air blast spraying droplets are entrained in the discharge of air from a fan (the machine) and are blown into a tree which is the target or boundary. Similarly crop components may be injected into an air-stream then, after partial or complete separation, may pass out into the natural environment.

1.2.4 Limitations

It ought to be admitted that many crop components (such as leaves or stems) are very unlike 'particles'. Hence while the following general discussion is generally valid it is likely that the program will only predict the trajectory of such particles in a very approximate, but perhaps still useful way.

1.3 Fluid – Particle Systems - An Overview

1.3.1 Introduction

Fluid - particle systems are present in a wide range of natural and human situations. There has presumably been a wide range of informal observations of these processes over the centuries, the results of which informed the observer and guided the evolution of many useful developments. However any formal conclusions, if drawn, are usually lost to us in the mists of history.

The observation of seeds being carried in the wind or soil materials being carried in water were, no doubt, observed by the ancients. Further observation of these phenomena probably then raised the question of why some particles were transported further than others and this ultimately led to the use of these processes to achieve desirable goals such as the separation of chaff from grain in the wind or of one mineral material from another in a natural or constructed water stream.

The earliest reference to using fluid – particle interactions for useful purposes in agriculture is perhaps the biblical reference to winnowing in the book Ruth (3:2) dated some 10 to 12 centuries BC. Here presumably the chaff was carried away from the grain when the mixture was thrown or dropped through the wind. Other biblical references include Psalm 1:4 and Isaiah 41:16.

Perhaps the earliest reference to a machine involving fluid – particle principles is related to winnowing with a hand cranked rotary fan that occurs in a pottery diorama from the Han Dynasty in China ((206 BC – 221 AD) (Needham, J. (1984)). This also includes a tilt hammer and grist mill for processing rice. A further review of the development of winnowing machines is given in Quick and Buchele, (1978).

The hand and later machine harvesting of seeds initially involved the cutting, plucking or stripping of heads and their later threshing to detach the chaff from the grain. The grain and chaff were then separated in an air-stream (usually combined with a sieving or riddling process on a perforated sheet) in a separate winnowing machine. With the further development of the harvesting processes it was logical to include this winnowing or similar cleaning process in one mobile machine. Hence the working principle and the elements of many modern mobile machines, for harvesting a wide range of crops, were settled.
1.3.2 Basic studies

The foundations of the study of fluid – particle interactions were laid by Newton (1726) and although his analysis has been significantly modified, its basic quadratic form is still used to represent the drag force. Stokes (1851) provided a theoretical analysis for low velocity flow in the viscous regime. The work by Reynolds (1883) on the conditions for flow in pipes was extended to apply to particles moving relative to a fluid. This general analysis, which accommodates a full range of flow conditions, is now universally used and is the basis for the analysis in this work.

Later experimental studies of mineral particles by Schiller (1932) in Germany and Wadell (1934) in USA showed the parameters by the various shapes of such might be specified and the relationships that these bear to the settling velocity.

These studies formed the basis for the application of fluid – particle systems to one and later two dimensional processes in agricultural engineering as discussed in Chapter 3 and 4 respectively.

1.3.3 Formal studies of agricultural materials and processes

(a) Practical / experimental

(i) Aerodynamic properties A number of early workers undertook experimental studies of the aerodynamic properties of agricultural materials as a preliminary to, and often in conjunction with, more detailed study of associated aerodynamic processes.

These one dimensional studies usually involved the measurement of the terminal velocity and associated drag coefficient, values which characterise the particles at their terminal velocity condition. These were mainly used in studying separation in a vertical air-stream


(ii) Crop product cleaning The early application of fluid - particle systems in agriculture usually involved, for example, the study of the performance of the cleaning systems in complete machines. Typical of these include studies by Persson, (1957(a) and (1957(b) in Europe and Cooper, (1966) and Rumble and Lee, (1970) in USA.

Later studies sought to understand the fundamental parameters of the cleaning and separation processes, mainly by studying their behaviour in a vertical air-stream. These included Gilfillan and Crowther, (1959), Uhl and Lamp, (1966), Kiker and Ross, (1966) and Farran Macmillan, (1979).

One general purpose of these studies was to improve the design of cleaning systems involving a sieve of some kind by aerodynamically removing a significant proportion of the light fraction, so improving the sieve performance.

Kashayap and Pandya, (1965(a) and 1966) sought to clarify the general principles of the winnowing process and conducted an extensive series of experiments in a horizontal rectangular duct.
(iii) *Fertilizer spreading* The other significant area of practical study involved the distribution of agricultural materials such as granular fertilizer and seeds. Early studies were usually limited to the formal testing of existing machines and the reporting of the distribution of particles when they reach the ground. The purpose of this work was to guide in the appropriate setting and efficient operation of these machine rather than to provide a formal basis for their design.

Later workers, such as Patterson and Reece (1962), Inns and Reece (1962), Hollmann and Mathes, (1962 and 1963) and Dobler and Flatow (1968), investigated the effect of design parameters, particularly for spinning disc ground machines and Yates, Stephenson, Lee and Akesson, (1973) for aircraft on the resulting on-ground distribution.

(b) Theoretical

The publication of a general solution for the prediction of particle trajectories was provided by Lapple et al (1940) in the context of chemical engineering. This enabled the application of the general fluid–particle theory to the two dimensional analysis of a wide range of processes and arrangements that are of interest in agriculture. The hand calculation of trajectories is a time consuming process and initially this was replaced by analyses using the analogue and later the digital computer.

(i) *Grain cleaning* Following their earlier experiments noted above, Kashayap and Pandya (1965(b)) made a qualitative theoretical analysis of winnowing in a horizontal duct using a digital computer. This was extended to include an upward sloping duct.

Later work included fundamental studies for predicting grain and fertilizer particle trajectories by Reints and Yoerger (1967) in USA and for grain by Persson (1967) in Europe both using an analogue computer. Several other workers made theoretical and associated experimental investigations of various parameters associated with cleaning grain in a vertical wind tunnel (Farran and Macmillan (1979)) and in a header (combine) (Down 1978)) and separating good and bad wall nuts on a horizontal wind tunnel (Mueller (1967).

Macmillan (1999) illustrated the use of a general computer program (similar to that associated with this monograph) for calculating particle trajectories and the separation of light and heavy fractions by the analysis of materials dropped and thrown at various angles etc, in a horizontal wind.

(ii) *Fertilizer spreading* Prediction of trajectories for particles leaving mechanical distributors was carried out by Mennel and Reece, (1963), Reints and Yoerger (1967), Dobler and Flatow, (1968) each with an appropriate approximation to the calculation of the aerodynamic drag force.
1.4 CONCLUSION

The interaction of particles and fluids is one which arises in many aspects of nature and in agriculture, as well as in many other industries not mentioned above. The analytical basis for the study of these applications, here termed 'fluid – particle mechanics', is clearly one which has a significant literature that goes back to early developments in science in the 17th Century. However, in relation to modern needs, most of this analytical work suffers from one or two weaknesses.

Firstly it uses an approximate solution to the analysis which, while satisfactory for some situations, may be in significant error for others.

Secondly it is limited to a particular application and frequently to one dimension, for example, separating materials in a vertical air-stream. While this may be entirely satisfactory for the particular application, it is not suitable for a wider range of problems to which a more general program could be applied.

The computer program, which is associated with this monograph, is an attempt to provide a general solution to overcome these limitations and encourage the gathering of a more extensive range of data. It should also be emphasized that the examples presented in the following Chapters are illustrations of the use of the program and are not intended as a full analysis of the problems concerned.

Problems involving three dimensions and particle spin are beyond the present work and are not considered further. Problems with variable mass can however be solved (albeit slowly) with a stepwise approach if the rate of change of mass is known. (see Chapter 15).

1.5 REFERENCES

The references for Chapters 1 – 4 are given in Appendix IV.
CHAPTER 2

FLUID - PARTICLE INTERACTION

CONTENTS

2.1 INTRODUCTION 2.2
2.2 NEWTONS LAW 2.2
2.3 PARAMETERS 2.3
  2.3.1 Drag coefficient
  2.3.2 Reynolds number
  2.3.3 Flow conditions
      (a) Laminar flow
      (b) Turbulent flow
2.4 DRAG FORCE ON PARTICLES 2.6
2.5 ASSUMPTIONS 2.6
2.6 DRAG COEFFICIENTS FOR SPHERES 2.7
  2.6.1 Laminar flow
  2.6.2 Turbulent flow
  2.6.3 Transition flow
2.7 DRAG COEFFICIENTS FOR NON-SPHERICAL SHAPES 2.8
  2.7.1 Defining parameters
  2.7.2 Orientation
  2.7.3 Shape
      (a) Regular isometric shapes
      (b) Irregular shapes
  2.7.4 Size
      (a) Sieving
      (b) Direct measurement
      (c) Volume measurement
  2.7.5 Summary
2.8 CROP PRODUCTS 2.15
2.9 OTHER SECONDARY EFFECTS 2.15
  2.9.1 Effect of turbulence in the motion of fluid
  2.9.2 Effect of acceleration of particle
  2.9.3 Effect of the presence of other particles
  2.9.4 Effect of particle roughness
  2.9.5 Conclusion
2.10 CONCLUSION 2.16
2.11 REFERENCES 2.16
2.1 INTRODUCTION

It is a common experience that when a body that is wholly or partially immersed in a fluid moves relative to the fluid, it experiences a force that opposes the motion (Rayleigh (1876)). This will be true whether the fluid is at rest and the body moves or the body is at rest and the fluid moves or some combination of these. In each case the force is usually termed the 'resistance' or, as used in the following, the 'drag' force.

Common experience also suggests that the magnitude of the drag force depends on:
(i) the relative velocity of the body and fluid – higher velocity means greater drag.
(ii) the size of the body – a larger body means a greater drag.

also on the properties of the fluid including:
(iii) the density of the fluid – the force is greater for a 'heavy' fluid (greater mass per unit volume) such as a liquid than for a 'lighter' fluid such as a gas.
(iv) the 'thickness' of the fluid – the force is greater for a 'thick' liquid such as a cold syrup than for a 'thinner' liquid such as hot syrup.

2.2 NEWTON'S LAW

Newton, et. al. (1803) expressed these common experiences in terms of a 'resistance law' in which he wrote (as quoted above):

\[
\text{Drag force, } D \propto A \rho W^2 \tag{2.1}
\]

Where:
- \(A\) = cross sectional area of the body perpendicular to the relative velocity
- \(\rho\) = density of the fluid
- \(W\) = magnitude of the relative velocity
- \(\propto\) here means 'proportional to'

The drag calculated by Newton's equation was based on the idea of the change in momentum of the column of fluid as it impacts on the body. However the drag force calculated by Newton's Law did not agree with the experimental results because it only took into account the conditions on the front of the body; the conditions on the back are important as well.

---

1 'duplicate' = 'square'
The detailed explanation is complex and is discussed in most books on fluid and aerodynamics. Suffice to say that, as shown in Figure 2.1, the drag force is the sum of all the components in the direction of relative motion:

(i) of the pressure forces on the surface of the body. This is known as 'pressure' or 'form' drag and arises due to the pressure forces as a result of the inertia of the fluid (in effect the density mentioned above).

(ii) the friction forces on the surface of the body. This is known as 'friction' or 'surface' drag and arises due to the sliding of fluid elements over each other as a result of the viscosity of the fluid (in effect the 'thickness' mentioned above).

These separate effects dominate and make the major contribution to the drag according to the flow conditions, as explained below.

However, it can be shown (Prandtl and Tietjens, (1957)) that the form drag (due to inertia of the fluid) for a body in a completely frictionless fluid (viscosity equals zero) will be zero. Hence it is clear that, although the direct effect of viscosity may be small and can be neglected, in many situations its indirect effect is significant as it influences the way the fluid flows around the body and hence the form drag force.

2.3 PARAMETERS

2.3.1 Drag coefficient

Notwithstanding the problems with Newton's explanation, the form of his equation has been retained. The complicated effect of the flow conditions around the body and their effect on the drag is taken into account by the parameter called the 'drag coefficient' (C).

Thus in the development of his equation we can write Equation 2.1 as:

\[ D = C \cdot A \frac{\rho \, W^2}{2} \]  

(2.2)

where C is the constant or function to be determined.

Experimental study has confirmed that the drag force depends on the flow pattern around the body and can be characterised by a single number which shows the relative effects of inertia and viscosity.

---

1 The density and the relative velocity are expressed in the term \( \frac{\rho \, W^2}{2} \) known as the 'dynamic pressure' of a moving liquid; it is included in this form because it frequently appears as a general term in fluid mechanics.
2.3.2 Reynolds number

The flow conditions are represented by the ratio of the inertia forces to the viscous forces and is termed the Reynolds number, R:

\[ R = \frac{\text{inertia forces}}{\text{viscous forces}} = \frac{\rho Wd}{\mu} \]  

(2.3)

Hence the drag coefficient \( C \) is not, in general, constant but is a function of the Reynolds number and it must be determined experimentally for a particular shape. Then knowing the drag coefficient – Reynolds number relationship, the drag force can be calculated for any body of that shape using Equation 2.2.

However it should be noted that this equation cannot, in general, be solved directly because the drag force on the particle at any instant depends on its velocity (relative to the fluid). Any acceleration of the particle depends on the force hence in any subsequent motion, the velocity of the particle also depends on the force. An iterative solution may therefore be required.

The drag coefficient has been determined experimentally for various ideal and miscellaneous shapes; see Section 2.6 and following. Rather than just use the graphical representation, empirical equations of the drag coefficient - Reynolds number relationships have also been developed for various shapes and are specified for various ranges of the latter.

2.3.3 Flow conditions

(a) Laminar flow

For very small bodies at very low velocities (for example the settling of soil or mineral particles in water or in air cleaning) the flow is 'laminar' or 'viscous' or 'streamlined' and the resistance is primarily due to viscosity.

(b) Turbulent flow

Where the Reynolds number is large, as is common in many situations in everyday life and in agriculture, the flow around the particle is often turbulent and the inertia forces are much larger than the viscous forces.

For example, consider a wheat grain with an equivalent diameter of 4 mm, dropped in air with a density of 1.2 kg/m\(^3\) and viscosity of 0.000018 Pa s. At its terminal or maximum velocity of fall of approximately 8 m/s:

\[ R = \frac{\rho Wd}{\mu} = \frac{1.2 \times 8 \times 0.004}{0.000018} = 2130 \]

It is clear from this that here the inertia effect is over 2000 times the viscous effect; hence in this particular situation the direct effect of the viscous forces can be neglected without serious error.
Figure 2.1: Form and friction drag on particle moving relative to fluid.

Figure 2.2: Forces on bodies in relative motion; (a) building; (b) aeroplane wing both with 'controlled' attitude; (c) particle in free flight.
2.4 DRAG FORCE ON PARTICLES

So far in the discussion it has been implied that the drag force on the body acts in the direction of relative motion. However for unsymmetrical bodies this is not true and the total drag force in general is not in the direction of relative motion.

Thus we may resolve the total drag force:

(i) in the direction of relative motion — the drag force.¹
(ii) perpendicular to the relative motion — the lift force.

Hence, in general, two types of problem may be identified:

(i) where the attitude (or angle) of the body is fixed (or known) in relation to the air-stream. This is conventional fluid mechanics applied to relatively large ('fixed') bodies such as a building or an aeroplane wing; Figure 2.1 (a) and (b). This is not of interest in fluid - particle mechanics and will not be considered further.

(ii) where the body (hereafter called a 'particle') is not restrained and is free to take up an attitude determined only by the fluid and gravitational forces acting. In general this is an extremely complex situation and, as the fluid force may not pass through the centre of mass of the particle, non - uniform motion and rotation may occur.

It is therefore necessary to make certain simplifying assumptions as follows.

2.5 ASSUMPTIONS

For simplicity in analysis it will be assumed that:

(i) The particle is subject to fluid and gravity forces only - other forces such as electrostatic forces are neglected. Buoyancy can be allowed for although its effect for common particles in air is small.

(ii) The presence of the particle does not sensibly alter the fluid stream. Where the analysis is then extended to a particle stream it is assumed that it behaves in the same way as the ideal single particle(s). This may be a reasonable assumption but see Sections 2.9 and 3.5.

(iii) The particle is symmetrical (usually spherical) or that it moves with a constant attitude, i.e, its area perpendicular to the relative velocity is constant.

Where the shape is not symmetrical the fluid force may not pass through the centre of mass and hence, under some orientations, the particle will swerve and may spin at high speed. While this will have an influence on the trajectory at the micro level, an analysis of such motion would be exceedingly complex. The usual analysis only treats the overall trajectory (in effect, of the centre of gravity) and assumes an equivalent spherical shape having the same volume as the particle.

(iv) The fluid force is opposite the relative velocity, i.e., the particle orientates itself in such a way and there is only a drag force and no lift force.

(v) The mass is constant although, if liquid is involved, the mass may be changing due to evaporation.

¹ It is necessary to distinguish which of the uses of 'drag force' is meant. In the following, since the lift is assumed to be zero, the 'drag force' and the 'total drag force' are identical.


2.6 Drag Coefficients for Spheres

2.6.1 Laminar flow

Stokes (1851) developed his law for the drag force on a sphere from theoretical
considerations. It applies to laminar or streamlined conditions (\( R<1 \) approx) where inertia
forces are negligible and the drag is composed mainly of frictional or viscous drag.

Stokes Law gives:

\[
F = 3\pi \mu Wd
\]

Equating this with \( D \) from Equation 2.2 above gives:

\[
3\pi \mu Wd = CA \frac{\rho W^2}{2}
\]

But

\[
A = \frac{\pi d^2}{4}
\]

\[
3\pi \mu Wd = C \frac{\pi d^2 \rho W^2}{4}
\]

\[
C = \frac{2A\mu}{Wd\rho} = \frac{24}{R}
\]

On the drag coefficient – Reynolds number plot on logarithmic scale this appears as a straight
line with a slope -1 through the point \( C = 24 \) when \( R = 1 \).

2.6.2 Turbulent flow

For turbulent flow for which \( R >500 \), the drag coefficient is substantially constant and is
usually given as:

\[
C \approx 0.44
\]

2.6.3 Transition flow

As indicated in Figure 2.3 the transition from laminar to turbulent flow is gradual. In this
region of so called 'transition flow' ie for \( 1 < R < 500 \) the drag coefficient is given as:

\[
C = \frac{18.5}{R^{0.8}}
\]

These data for particles (in the Reynolds number range of interest in agriculture) are shown
replotted from Schiller (1932) for spheres and rounded bodies in Figure 2.3.
Figure 2.3: Drag coefficient versus Reynolds number for spheres and rounded bodies
Replotted and reprinted, from Schiller (1932); see references for details.

2.7 DRAG COEFFICIENTS FOR NON-SPHERICAL SHAPES

2.7.1 Defining parameters

Spheres, which always present the same area and shape to the fluid, are a special and more simple case of the general particle shape. The regular isometric or geometric shapes may be useful approximations to natural shapes and are of interest because their shape can be calculated mathematically.

Irregular shapes, of which there is an infinite variety, tend to arise in nature and particularly in processes where bodies have been crushed or ground and have broken, sharp edges, etc.

Of more interest in this work are the characteristics of the agricultural products (many of which tend to be 'rounded bodies') and crop components which have a wide variety of shapes.

Three aspects associated with defining non-spherical shapes can be identified:

(i) orientation
(ii) shape
(iii) size

These arise and interact in various ways and even vary according to how they are measured and specified.
2.7.2 Orientation

As discussed in Section 2.2 above, the parameters for the analysis of the forces on 'particles' moving relative to a fluid have been defined for bodies where the orientation is known and or controlled. However for particles of non-spherical shapes in free motion it is impossible to allow for their orientation since this is not controlled and hence their 'size' and 'shape' relative to the fluid velocity will not be constant. Presumably most particles spin or tumble in a random way and have, at a micro-level, a non constant velocity. Such motion, which is observed when studying particles in a vertical wind tunnel, would be extremely difficult to predict and perhaps of little value in the solution of everyday problems.

Superimposed on this is the gross or 'macro' motion represented by the overall trajectory of the centre of mass. The success of the analysis and prediction of this motion, that can be undertaken with traditional fluid mechanics, will be measured by its comparison with experimental results.

At the level of analysis represented by the trajectory plotting system (TPS), we can only determine the macro effects; those at the micro level are beyond our analysis and will not be considered further.

2.7.3 Shape

One method of specifying shape is to classify non-spherical particles into groups. This limits the number of shapes for which drag coefficient – Reynolds number graphs have to be drawn, The method is to specify a 'shape factor' termed the 'sphericity'.

Because, for a given volume, a sphere has the minimum surface area, then any increase in surface area of a particle represents its divergence from the 'ideal' spherical shape. Thus sphericity has been defined as:

\[
\text{Sphericity } \psi = \frac{\text{Surface area of sphere with same volume as particle}}{\text{Actual surface area of particle}} \tag{2.7}
\]
We can apply this concept to particles with a range of different shape forms.

(a) Regular isometric (geometric) shapes

Drag coefficients have been determined theoretically and/or experimentally for a range of such particles:

(i) **Cubical shapes** This includes shapes varying from spheres through polyhedra (many faces) to tetrahedron (a triangular pyramid with four faces).

Drag coefficient values were measured by Pettyjohn and Christiansen (1948) for a range of geometrically shaped, non-spherical particles. In the laminar range ($R < 200$) the coefficients are virtually identical and slightly greater than the values for spheres. Figure 2.4 shows the minimum values in the turbulent range, also a plot of the following equation which they fitted to these data.

$$ C = 5.31 - 4.88 \Psi $$

where $\Psi$ is the sphericity defined in Equation 2.7 above.

![Figure 2.4: Minimum drag coefficient versus sphericity for regular, isometric shapes](image)

Plot, with permission, from Pettyjohn and Christiansen (1948); see references for details.

![Figure 2.5: Drag coefficient versus aspect ratio for rectangular plates and long cylinders.](image)

Plot, with permission, from Coulson & Richardson (1983); see references for details.
(ii) **Elongated shapes**  Other regular shapes are also of interest in this work because they may approximate the shape of natural materials and, for example, plant components such as stems and leaves. For some of these, sphericity is not a good measure and the drag coefficient of 'thin' particles is often correlated against a dimensionless number called the 'aspect ratio'.

For thin plates with flow perpendicular to their plane:

\[
\text{Aspect ratio} = \frac{\text{Length}}{\text{Width}}
\]

For long cylinders with flow perpendicular to their length:

\[
\text{Aspect ratio} = \frac{\text{Length}}{\text{Diameter}}
\]

Figure 2.5 shows a plot of drag coefficient in the turbulent range (assumed to be a constant minimum) versus aspect ratio. The values plotted at an aspect ratio of 100 are assumed to be infinite.

(b) Irregular shapes

Irregular shapes arise in nature and also as the result of processing procedures such as crushing and grinding. Again sphericity may be used to classify such shapes although it should be remembered that particles may have the same sphericity but a very different shape. Hence sphericity alone may not give a unique measure of shape.

**Figure 2.6:** Drag coefficient versus sphericity for irregular bodies with various sphericities. Replotted and reprinted, with permission, from Wadell (1934) and from Brown (1950); see references for details.
The experimental study of irregular shapes again suggests that the difference between these and spheres is small for streamlined flow with low Reynolds numbers (say R less than 1) but large for turbulent flow with Reynolds number greater than say 200. Hence in any discussion of non-spherical particles, the flow conditions (as specified by the R value) are also significant.

Figure 2.6 shows a plot of drag coefficient – Reynolds number relationship for bodies with various sphericities, successively plotted from various European authors by Wadell (1934) and then idealised by Brown (1950). Figure 2.7 shows a plot of the minimum drag coefficient versus sphericity in the turbulent region with data from Figure 2.6.

![Figure 2.7: Plot of minimum drag coefficient versus sphericity plotted with data from Figure 2.6.](image-url)
2.7.4 Size

Size is needed in the trajectory equations to define the frontal area for calculating the drag coefficient and the 'size' (usually diameter or equivalent diameter) for calculating the Reynolds number.

For non spherical particles, size is clearly related to orientation and therefore to how the size is measured. The measurement of very fine particles involves a range of techniques which are described in specialised text books on chemical engineering or mineral processing. Methods for the determination of size of relatively large particles relevant to agriculture include:

(a) Sieving

For particles without extreme aspect ratios (length approximately equal to the other dimensions) sieving will give a diameter corresponding to the least cross sectional area.

(b) Direct measurement

A common and relatively simple, if tedious, method that is used for specifying the size of larger particles is to calculate the geometric mean of the three major lengths $l_1, l_2, l_3$ of each piece. Thus the mean diameter,

$$ d = \sqrt[3]{l_1 \cdot l_2 \cdot l_3} $$

(c) Volume measurement

Here size (area and diameter) is inferred from the measurement of volume by liquid displacement. Thus diameter and area of the particle are the diameter and area of a sphere having the same volume as the particle.

Figures 2.8 (a) and (b), replotted with permission from Hawk, Brooker and Cassidy (1966), shows the effect of two different methods of measuring size on drag coefficients at terminal velocity for three grains; (a) shows the diameter calculated from the major lengths; (b) shows the diameter of a sphere of equal volume.

It also shows the significant change for corn compared with the two other grains. This may be as a result of the different way in which the authors treated the calculation of those parameters for corn in (a).

2.7.5 Summary

A summary of parameters for spheres, discs and cylinders, adapted from Lapple (1956), is shown in Table 2.1.
Figure 2.8: Data showing effect of length measurement on Reynolds Number and drag coefficient for three grains. Replotted and reprinted, with permission, from Hawk, Brooker and Cassidy (1966); see references for details.

<table>
<thead>
<tr>
<th>Form-&gt;</th>
<th>Sphere, any direction</th>
<th>Thin disc, perp. to face</th>
<th>Long circular cyl., perp. to axis</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frontal area</td>
<td>( \frac{\pi d_p^2}{4} )</td>
<td>( \frac{\pi d_p^2}{4} )</td>
<td>( d_pL )</td>
<td></td>
</tr>
<tr>
<td>Mass</td>
<td>( \rho_p \frac{\pi d_p^3}{6} )</td>
<td>( \rho_p \frac{\pi d_p^3}{4}L )</td>
<td>( \rho_p \frac{\pi d_p^3}{4}L )</td>
<td></td>
</tr>
<tr>
<td>Reynolds number</td>
<td>( \frac{\rho_p Ud_p}{\mu} )</td>
<td>( \frac{\rho_p Ud_p}{\mu} )</td>
<td>( \frac{\rho_p Ud_p}{\mu} )</td>
<td></td>
</tr>
<tr>
<td>Streamlined, N&lt;1</td>
<td>( 3\pi \mu Ud_p )</td>
<td>( 8\mu Ud_p )</td>
<td>( \frac{4\pi}{K} \mu UL )</td>
<td>( K=2 \ln N )</td>
</tr>
<tr>
<td>Terminal velocity</td>
<td>( 0.44 )</td>
<td>( 1.12 )</td>
<td>( 1.2 )</td>
<td></td>
</tr>
<tr>
<td>Drag force</td>
<td>( 24 )</td>
<td>( 64 \pi )</td>
<td>( \frac{8\pi}{K} )</td>
<td>( K=2 \ln N )</td>
</tr>
<tr>
<td>C, average</td>
<td>( 1x10^3 - 2x10^5 )</td>
<td>( &gt; 1000 )</td>
<td>( 1x10^2 - 2x10^5 )</td>
<td></td>
</tr>
<tr>
<td>R, range</td>
<td>( 1x10^3 - 2x10^5 )</td>
<td>( &gt; 1000 )</td>
<td>( 1x10^2 - 2x10^5 )</td>
<td></td>
</tr>
<tr>
<td>Term. velocity</td>
<td>( \frac{4gd_p(\rho_p - \rho_f)}{3C_i\rho_f} )</td>
<td>( \frac{2gL(\rho_p - \rho_f)}{C_i\rho_f} )</td>
<td>( \frac{\pi gd_p(\rho_p - \rho_f)}{2C_i\rho_f} )</td>
<td></td>
</tr>
</tbody>
</table>

Table 2.1: Summary of parameters for spheres, discs and cylinders (Adapted and reprinted from Lapple (1956); see references for details)
2.8 CROP PRODUCTS

Drag coefficients have been measured for various crop products such as seeds, fruits and tubers but usually only at their maximum fall or 'terminal' velocity (defined in Chapter 3). Hence we cannot, on the basis of these data, plot the drag coefficient values over a wide range of Reynolds numbers; the only variation is that associated with the natural variation in the size, shape and weight of the individual particles as shown for wheat grains (Shellard and Macmillan, 1978) and for walnuts (Mueller, Brooker and Cassidy, 1967) in Figure 2.9. See also Figure 2.8 showing drag coefficients for three grains.

The limitation that this places on the usefulness of this data, for example, in plotting particle trajectories, will be discussed in Chapter 3.

![Figure 2.9](image)

**Figure 2.9** Drag coefficient versus Reynolds number showing natural variation for: wheat grains from Shellard and Macmillan, (1978) and for walnuts from Mueller, Brooker and Cassidy, (1967) with permission; spheres and rounded bodies, from Schiller (1934); see references for details.

2.9 OTHER SECONDARY EFFECTS

2.9.1 Effect of turbulence in the motion of the fluid

The drag force on the particle will be a scalar sum of the velocity of the fluid and of the particle, both relative to the earth for one dimension situation as in Chapter 3 and a vector sum for two dimensions as in Chapter 4.

The presence of fluid velocity will in most practical situations introduce some extra turbulence into the fluid compared to a still fluid. This will be the equivalent of having a higher Reynolds number and hence a lower drag coefficient. This will not be allowed for in the trajectory calculations.
2.9.2 Effect of acceleration of the particle

The drag coefficient depends on the velocity of the particle relative to the fluid (as discussed above) but also on the acceleration of the particle. This effect is complex and is likely to be more so where the acceleration is variable as for conditions of interest in the present work. However studies have been somewhat inconclusive but suggest that the effect is likely to be small for conditions corresponding to higher Reynolds numbers. (Coulson and Richardson 1983)

2.9.3 Effect of the presence of other particles

As noted above the usefulness of the prediction of the trajectory of single particles depends in many applications on the assumption that a stream of particles will behave in the same way. This may, depending on the circumstances, be a reasonable assumption. However it is clear that the particles in the stream will to some extent affect each other by making the fluid flow more turbulent than it would otherwise be. Thus the drag coefficient will be lower than it would be in undisturbed air. However, for particles with a Reynolds number greater than about 1000, the difference is likely to be small because the drag coefficient is substantially constant under these conditions. See a comparison of such results in Chapter 12.

2.9.4 Effect of particle roughness

Particle roughness has a minor effect on the drag coefficient for the range of Reynolds numbers encountered in agriculture. It is only for large particles and high speed conditions that occur, for example with sports balls, (giving Reynolds numbers greater than about $10^4$) that roughness has a significant effect on the drag coefficient.

2.9.5 Conclusion

For the Reynolds numbers of interest in this work, each of the above is likely to cause a small but ill-defined reduction in the drag force. In the relatively simple analysis that follows it is not possible to allow for these effects and they will not be considered further.

Further discussion of these secondary effects will be found in more specialised fluid mechanics and chemical engineering text books.

2.10 CONCLUSION

Elementary fluid mechanics theory that is taught in second or later year engineering courses provides a firm foundation for the prediction of the trajectory of particles moving relative to a fluid.

Chapters 3 reviews the published material that is based on the simple one dimensional analysis and highlights the weakness of that approach. Chapter 4 provides the background that enables the two-dimensional theory to be extended to a wide range of problems and presents a validation that confirms its ability to predict a trajectory from 0.2 to 20 metre.

2.11 REFERENCES

The references for Chapters 1 – 4 are given in Appendix IV.
CHAPTER 3

ONE DIMENSIONAL ANALYSIS

CONTENTS

3.1 Introduction 3.2

3.2 Terminal Velocity 3.2

3.2.1 Analysis 3.2

3.2.2 Terminal velocity of spheres

(a) R less than 1
(b) R greater than 1000,
(c) R between 1 and 1000

3.2.3 Terminal velocity from time - distance relationship 3.2

3.3 Measurement of Terminal Velocity 3.8

3.3.1 Introduction 3.8

3.3.2 Dropping the particles in a still fluid 3.8

3.3.3 Floating individual particle in a vertical fluid stream 3.8

3.3.4 Limitations 3.8

3.4 Examples of Measurement 3.10

3.4.1 Ideal shapes 3.10

3.4.2 Crop components 3.10

(a) Floating technique 3.10

(b) Dropping particles 3.10

3.4.3 Rain-drops 3.10

3.5 Applications 3.13

3.5.1 Separations of crop fractions 3.13

3.5.2 Effect of multiple particles 3.13

3.6 Conclusion 3.14

3.7 References 3.14
CHAPTER 3
ONE DIMENSIONAL ANALYSIS

You may watch falling bodies for an eternity but without mathematics mere watching will yield no law of gravitation.

Johann Wolfgang Von Goethe (1749 – 1832)

3.1 INTRODUCTION

A simple, one dimensional illustration of fluid–particle mechanics is where a particle (with greater density than the fluid) is dropped in a stationary fluid—or the equivalent—the particle is supported in an upward moving stream.

In the former, when the particle is dropped, it will fall downwards and accelerate under the action of the resultant force, i.e., the difference between the net weight and the upward force viz, the fluid drag. As it falls the fluid drag increases and the particle ultimately reaches an equilibrium or constant velocity condition known as the ‘terminal velocity’ at which these two forces are equal.

This is the one and only condition that has been amenable to simple analysis and for which the drag coefficient can be readily determined. It has not in general been possible to determine the drag coefficient for agricultural materials for a wide range of Reynolds numbers; see Section 2.8.

3.2 TERMINAL VELOCITY

3.2.1 Analysis

Consider a particle dropped in a stationary fluid. The particle will attain this constant or terminal velocity, V_t, at which condition the net gravitational force (i.e., corrected for buoyancy due to the displacement of the fluid (according to Archimedes principle)) F_g will equal the drag force, D.

Note that the
* particle moves downward if the particle density is greater than the fluid density.
* particle moves upward if the particle density is less than the fluid density.

\[ F_g = D \]

For body immersed in a fluid:

Nett gravity force \( F_g \)

\[ = mg - upthrust \]

\[ = mg - weight \text{ of fluid} \]

\[ = mg - V_p \rho_f g \quad \text{where } V_p \text{ is the volume of particle} \]

\[ = V_p \rho_f g - V_p \rho_f g \]

\[ = V_p \rho_f g (1 - \frac{\rho_f}{\rho_p}) \]

\[ = mg \frac{\rho_f - \rho_f}{\rho_p} \]
Equating this with the fluid drag (Equation 2.2) gives:

\[ mg \frac{\rho_b - \rho_f}{\rho_f} = C_t A \frac{\rho_f U_t^2}{2} \]

where \( U_t \) is the terminal velocity and \( C_t \) is the drag coefficient at terminal velocity.

\[ U_t^2 = 2mg \frac{\rho_b - \rho_f}{C_t A \rho_b \rho_f} \quad (3.1) \]

This equation cannot be used to calculate \( U_t \) directly because it requires the value \( C_t \) which in turn depends on \( U_t \); an iterative process may be used to reach an accurate solution.

### 3.2.2 Terminal velocity of spheres

For spheres, diameter, \( d_p \), density, \( \rho_b \) and projected area, \( A = \frac{\pi}{4} d_p^2 \)

\[ U_t^2 = 2mg \frac{\rho_b - \rho_f}{C_t A \rho_b \rho_f} = \frac{2\pi d_p^3 \rho_b g}{6} \frac{\rho_b - \rho_f}{C_t} \frac{\pi}{4} d_p \rho_b \rho_f = \frac{4}{3} \frac{d_p g}{C_t} \frac{\rho_b - \rho_f}{\rho_f} \]

\[ (3.2) \]

This can be further analysed if the values of \( C_t \) are known in terms of the Reynolds number.

(a) \( R \) less than 1

\[ C = \frac{24}{R} \]

\[ U_t^2 = \frac{4d_p g}{3} \frac{\rho_b - \rho_f}{\rho_f} \frac{\rho_f U_t d_p}{24 \mu} \]

\[ U_t = \frac{d_p g}{18 \mu} (\rho_b - \rho_f) \quad (3.3) \]

This is a form of Stokes Law (see Section 2.6.1) which is useful for determining the equivalent diameter for very small particles.

Thus:

\[ d = \sqrt{\frac{18 \mu U_t}{g (\rho_b - \rho_f)}} \quad (3.4) \]

As will be seen in Figure 2.3 and 2.4 the shape of the particle is not too significant in laminar flow and hence in using Equation 3.4 the shape of the body is usually neglected and is, in effect, assumed to be spherical.
(b) \( R \) greater than \( 10^3 \),
In this range \( C_D \approx 0.44 \), and the flow is turbulent.
\[
U_t^2 = \frac{4d_p g (\rho_p - \rho_f)}{3 \times 0.44 \rho_f^2} = 3.03d_p g \frac{\rho_p - \rho_f}{\rho_f^2} \tag{3.5}
\]

(c) \( R \) between 1 and \( 10^3 \)
In this range the flow is transitional; \( C_d \) can be obtained from graph Figure 2.3 or the equation:
\[
C = \frac{18.5}{R_{Re}} \tag{3.6}
\]
The terminal velocity can then be calculated from Equation 3.1 by a direct insertion of Equation 3.5 or by an iterative process.

Experimental data for spheres (and rounded bodies) are shown plotted in Figures 2.3 and 2.4.

**Worked Example 3.1**

The following data applies to a seed falling at its terminal velocity in water. Determine the equivalent diameter and check the flow conditions.
- Mass = 0.000458 g
- Terminal velocity = 4 mm/s
- Viscosity of water = 1.0 mPa.s
- Density of water = 1000 kg/m³
- Density of particle = 1100 kg/m³

From Equation 3.4:
\[
d_p = \sqrt[3]{\frac{18 \mu U_t}{g (\rho_p - \rho_f)}}
\]
\[
= \sqrt[3]{18 \times 0.001 \times 0.004 \over 9.8 (1100 - 1000)} = 0.00027 m = 0.27 mm
\]

\[
R = \frac{\rho_f U_t d_p}{\mu} = \frac{1000 \times 0.004 \times 0.00027}{0.001} = 1.1
\]
The Reynolds number is close to 1 hence the flow will be laminar and Equation 3.4 is applicable.
**Worked Example 3.2**

The following data applies to a near spherical seed.

(a) Estimate its terminal velocity falling in air and check the flow conditions.

- Mass = 0.5 g
- Equivalent diameter = 10 mm
- Viscosity of air = 0.018 mPa.s
- Density of air = 1.2 kg/m³
- Density of particle = 800 kg/m³

Using Equation 3.1, the following results were obtained:

From first iteration:
- \( C_t = 0.8 \) (guess)
- \( U_t, \text{m/s} = 11.4 \) (from Equation 3.1)
- \( R = 7600 \) (from Equation 2.3)

At this stage it is clear that the seed is falling in the turbulent range and that the above guess for \( C_t \) for a near spherical seed is too high.

From second iteration:
- \( C_t = 0.6 \) (assume from Figure 2.4 at \( R = 7600 \))
- \( U_t, \text{m/s} = 13.2 \) (from Equation 3.1)
- \( R = 8780 \) (from Equation 2.3)

Without data on the sphericity the data is not sufficiently accurate to warrant another iteration. The terminal velocity is approximately 13.2 m/s and the flow is turbulent.

(b) If the terminal velocity is later measured as 13.5 m/s estimate the sphericity \( \Psi \) of the seed.

\[
C_t = 2mg \left( \frac{\rho_s - \rho_f}{\rho_f} \right) \frac{U_t}{A \rho_s \rho_f} \quad \text{(from Equation 3.1)}
\]

\[
= 0.57
\]

\[
R = \frac{\rho U_t d}{\mu}
\]

\[
= 9000
\]

The flow is turbulent.

Interpolating in Figures 2.6 or 2.7 for the sphericity \( \Psi \) at \( C = 0.57 \) gives \( \Psi \approx 0.95 \).
3.2.3 Terminal velocity from time - distance relationship\(^1\)

On a plot of the time – distance data the terminal velocity will appear as a straight line with slope equal to the terminal velocity.

Based on Equation 2.2 we can write the equation of motion as:

\[
m \frac{dU}{dt} = mg - \frac{CA \rho f U^2}{2}
\]

\[
\frac{dU}{dt} = g \left(1 - \frac{CA \rho f U^2}{2g}\right)
\]

For convenience this can be written as

\[
\int \frac{dU}{(1 - a^2 U^2)} = g \int dt
\]

\[
\frac{1}{2a} \ln \frac{1 + aU}{1 - aU} = gt + C_1
\]

But \(U = 0\), when \(t = 0\), hence \(C_1 = 0\)

\[
\frac{1 + aU}{1 - aU} = e^{2agt}
\]

\[
U = \frac{1}{a} \left( e^{agt} - 1 \right)
\]

Dividing by \(e^{agt}\)

\[
U = \frac{1}{a} \left( e^{agt} - \frac{1}{e^{agt}} \right)
\]

But

\[
U = \frac{dS}{dt} = \frac{1}{a} \tanh agt
\]

\[
\int dS = \frac{1}{a} \int \tanh agt \, dt
\]

\[
S = \frac{1}{a^2 g} \ln \cosh agt + C_2
\]

But \(S = 0\) when \(t = 0\) hence \(C_2 = 0\)

\[
S = \frac{1}{a^2 g} \ln \cosh agt \quad \text{(3.7)}
\]

\[
S = \frac{2m}{CA \rho f} \sqrt{\frac{CA \rho f g}{2m}} t \quad \text{(3.8)}
\]

\(^1\) Adapted from Mohensin, N. N. (1970); see references for details.
Equation 3.8 plots the distance the particle has fallen from rest versus time. This may not be as useful as it appears since it is necessary to know the drag coefficient (which is a function of the velocity) any time before the distance can be determined.

However if the terminal velocity is known we can determine the distance – time relationship as follows.

If the particle is much more dense than the fluid we can, without significant error, write Equation 3.1 as:

\[ U_t^2 = \frac{2mg}{CA\rho_f} \]

But from the definition of \( a^2 \) above:

\[ \frac{1}{a^2} = \frac{2mg}{CA\rho_f} = U_t^2 \]

Hence from Equation 3.7

\[ S = \frac{U_t}{g} \ln \cosh \frac{g}{U_t} t \]

(3.9)
3.3 MEASUREMENT OF TERMINAL VELOCITY

3.3.1 Introduction

Terminal velocity of a falling particle has been identified because it is recognised as a naturally occurring phenomena and a characteristic of any fluid - particle combination. It is also an 'equilibrium' condition, albeit a 'dynamic' one. It is of special interest because only in this condition do we know exactly what the fluid drag force is - equal to the net gravity.

The equivalent of terminal velocity where a particle is supported in a rising fluid stream is the so called 'floating velocity'. It has (nominally) the same velocity relative to the fluid but is stationary relative to the earth and its use is a convenient experimental technique.

Ideally the velocities achieved in drop tests and in floating tests are the same. However secondary effects (as discussed below) are likely to mean that there will be differences associated with lack of symmetry and the inevitable, if low level of turbulence in the air stream, in the floating method.

In aerodynamic tests on ideal shapes, parameters such as size and fluid velocity are varied and values of drag coefficient are obtained over a wide range of Reynolds numbers. Non ideal shapes, such as many agricultural and industrial materials and liquid droplets, are often so small that drag coefficients cannot be measured directly in this way.

Terminal velocity has therefore been used because it is the only characteristic that is easily measured (relatively, with the right equipment) to characterise such materials, particularly in relation to their shape. But because the terminal velocity allows the drag coefficient to be determined for one condition only, its use is limited.

A knowledge of the terminal velocity (or floating velocity) is however useful in designing a system for separating one fraction of a mixture from another in a vertical column (sometimes termed elutriation). However the terminal velocity cannot, for example, be used for designing a horizontal cleaning (termied winnowing) system.

The terminal velocity is also a characteristic of rain and other droplets which are subject to a gravity field. It defines the kinetic energy with which rain droplets impact the soil and is therefore a significant parameter in erosion studies.

The idea that a particle has a single value of terminal velocity implies symmetry. When the particle is not symmetrical the terminal velocity will vary with the orientation. Where a particle is highly asymmetrical, such as with straws, the whole concept of a single aerodynamic characteristic becomes problematic. This applies to both drag coefficient and to terminal velocity.

With liquid droplets their shape is a function of the size and the velocity of the particle as they fall to earth. Larger droplets, where drag forces are more significant than surface tension, take up a flattened shape and, incidentally, not the 'stream-lined' shape as commonly supposed. (Laws, 1941); see Chapter 14.

There are essentially two ways of measuring terminal / floating velocity corresponding to the two concepts.

1 Fraction is the component of one type in a mixture.
3.3.2 Dropping the particles in a still fluid

This technique measures the true terminal velocity. By timing the passage of the particle in a tube that is large with respect to the particle size it is possible to plot the time - distance or time - velocity graph as shown in Figure 3.2 below. A near linear portion of the former or the maximum of the latter gives a close approximation to the terminal velocity.

Again unsymmetrical particles will not behave in an ideal manner; they may adopt different attitudes and so fall at different velocities. They may also not fall vertically but slip sideways or even behave like a falling leaf and slip first in one direction and then in another. The other problem with this technique is that for heavy and/or small particles, a considerable height of drop may be required before the terminal velocity is approached.

Notwithstanding the problems discussed below, this approach has been widely used to study many crop components and rain-drops. This is due to the fact that still air is easier to achieve than a uniform and constant air velocity required in the floating technique; modern electronic techniques can be used to give very accurate timing of the fall.

3.3.3 Floating individual particle in a vertical fluid stream

Here an 'open' (non-recirculating) wind tunnel is usually used to create a uniform vertical air-stream using appropriate wind tunnel techniques. Variation of the fan speed or the by-passing some of the air are used to provide a continuously variable air speed.

The walls of the tunnel are usually slightly divergent which causes a slight reduction in the air velocity up the tunnel. This helps to avoid the problem of particles moving rapidly to the screen at the top of the tunnel, if the air speed is increased to the terminal velocity, and falling to the bottom when it is decreased slightly. By varying the air speed it is possible to float the particle at a constant level.

The difficulties with this technique arise mainly where there is lack of symmetry in the particles. If it is remembered that the drag force depends on the projected area presented to the air-stream, then it is clear that orientation will have a significant effect on terminal velocity for unsymmetrical particles. Such a particle may be made to float when it is presented with its largest area across the air-stream but slip down through the stream when its smaller area is presented to it.

Bilanski et al (1965) reported a particle may adopt secondary motion such as rotating at high speed about a vertical axis. This will of course alter the drag on the particle with respect to the main air-stream and cause it to float at a higher or lower terminal velocity.

3.3.4 Limitations

From the above it is clear that the concept of terminal velocity, while useful for near symmetrical particles, is likely to be of limited value in many practical situations where a range of non-symmetrical shapes are involved as in processing crop components.

As discussed in Part 3 and Chapter 12 below the terminal velocity does provide some guidance in the choice of a velocity for separation of one or more fractions in a mixture of particles being fed into a vertical air-stream.
3.4 EXAMPLES OF MEASUREMENT

3.4.1 Ideal shapes

The early work studying fluid particle mechanics, which involved measuring the settling velocity (as it was called) of particles in a liquid, has been reviewed and reported by Schiller (1932), Wadell (1934) and Lapple and Sheppard, (1940).

3.4.2 Crop components

(a) Floating technique

Various workers have used this technique including Bilanski Collins and Chu, (1962), Hawk Brooker and Cassidy (1966), Mueller, Brooker and Cassidy,(1967), Law and Collier (1973) and Shellard and Macmillan (1978).

The results of Shellard and Macmillan who measured the terminal velocity for individual components of a wheat crop taken from a harvester are plotted in Figure 3.1. This is based on the mean and standard deviation of the results assuming a normal distribution.

They reported the problems discussed in Section 3.3.2 above, particularly for pieces of ear and straw.

![Figure 3.1](image-url)  

**Figure 3.1** Distribution of the terminal velocity of components of wheat crop when tested singly in a vertical wind tunnel. Replotted and reprinted from Shellard and Macmillan (1978); see references for details.
(b) Dropping particles

Many workers have reported using this technique in studying grain, fruit and even clods and stones. These include, Garrett and Brooker (1965), Keck and Goss (1965) and Bilanski, Collins and Chu (1962) and Shellard and Macmillan (1978) for wheat chaff.

Results are usually reported as terminal velocity or as drag coefficient at terminal velocity.

![Graph showing distance-time graphs for particles dropped in still air.](image)

**Figure 3.2**: Distance-time graphs for particles dropped in still air. Replotted and reprinted with permission from Bilanski, Collins and Chu, (1962); see references for details.
3.4.3 Rain-drops

One natural occurrence of terminal velocity is in the fall of rain-drops. This is relevant to the calculation of the erosion caused by rain and also in the design of rainfall simulators that are used for erosion and other studies.

Figure 3.3 shows the results of the photographic measurement of the velocity of fall of drops of water of various equivalent diameters\(^1\) for various heights of fall re-plotted, with permission, from Laws (1941). It will be seen that the droplets (like all particles) reach their terminal velocity gradually (asymptotically) after falling a large distance.

\[\text{Figure 3.3: Velocity versus height for water drops of different diameter dropped in still air.}\
\text{Re-plotted and reprinted, with permission, from Laws, J.O. (1941);}\
\text{see references for details}\]

The inset in Figure 3.3 shows how, as the equivalent drop diameter increases, the terminal velocity tends to a constant value.

Further analysis and prediction of the terminal velocity of rain drops is presented in Chapter 14.

---

\(^1\) Equivalent diameter is the diameter of a sphere of equal volume
3.5 APPLICATIONS

3.5.1 Separation of crop fractions

Notwithstanding the problems mentioned above, differences in terminal velocity have been frequently used as a basis for separation of mixtures of particles into their component fractions. Thus if the two fractions have different terminal velocities, it may be possible to choose a velocity between the two that will lift one and not the other.

This is relatively easy where the fractions have widely differing terminal velocities and each of these is in a narrow band as, for example, for the chaff and grain shown in Figure 3.1. However many naturally occurring particles have a distribution (even a wide distribution) of terminal velocity values as a result of the natural variation in their area and mass; see the results for the partly threshed ears in Figure 3.1.

Where there are two or more fractions, and the distribution of velocities overlap, then it is unlikely that it would be possible to find one velocity that will totally separate one fraction from the remainder. A multi stage process may then be necessary – the first to separate fraction(s) with the extreme (highest or lowest) velocities and a second or even third to separate those nearest to the desired fraction.

It may also be useful to use an aerodynamic separation to remove the extreme fraction (having the highest or lowest terminal velocity) and a different form of separation (for example sieving) for the other fractions; this is common in cereal harvesters.

3.5.2 Effect of multiple particles

The measurement of terminal velocities discussed above are made under ideal conditions where there is a constant or zero air velocity according to the method of measurement. This constancy is not only defined over the measurement space but also in time.

However many practical applications involve the separation of particles in a mixture that is presented to the air as a stream. In this respect we are far from this aerodynamic ideal in which the terminal velocities of the individual particles were measured. The presence of other particles in both fractions in the particle stream will clearly alter the conditions in the air-stream; the latter will be constant neither in space or in time.

In general the presence of many particles in the air-stream will increase the turbulence; this is the equivalent of increasing the Reynolds number which has the effect of reducing the drag on the particles. This suggests, for example, that we would need a higher air velocity to lift the light fraction in a continuous separation process than the terminal velocity measured for single particles under ideal conditions.

The confirmation of this conclusion is illustrated in Figure 3.6 where the results of a continuous separation process of grain and chaff equivalent (polystyrene balls) have been plotted from results by Farran (1978). The balls which were closely spherical had a terminal velocity in the range 2.08 to 2.17 m/s.

The results show clearly that even the presence of spheres alone (3.9 kg/min) affects the velocity required to lift them. The presence of increasing the amount of grain in the mixture (3:1 and 6:1, grain to spheres) influences the separation even further and an increasing air velocity is required to separate all of the balls.

This behaviour, requiring a greater velocity than the terminal value to lift the lighter fraction, is consistent with the discussion in Section 2.6.5 where it was noted that the presence of turbulence will cause a lower drag coefficient than under ideal conditions.
Figure 3.4: Separation of polystyrene balls (chaff equivalent) from a wheat grain-ball mixture when injected (under optimum conditions) at various rates into a vertical wind tunnel. Re-plotted from data in Farran (1978).

Other workers in this field such as (Persson (1957c), and Uhl and Lamp (1966)) have used a stepwise separation where the air velocity is increased in steps and the quantity of the various fractions that were not separated at a lower air velocity were measured. A range of velocities is required to separate all of the particles in each fraction – probably wider than the natural variation in terminal velocity associated with the fractions if measured as individual particles under ideal conditions. These latter data were not reported.

3.6 CONCLUSION

Terminal velocity is a simple, fundamental concept that can be used to characterize many fluid-particle systems. However it only provides one point on the drag coefficient – Reynolds number relationship and hence it cannot be applied to the process leading up to this equilibrium condition.

Nor can it be applied to the much more common and interesting problem of determining particle trajectories in two dimensions. This inability is the main reason why the Trajectory Plotting System was developed. It is explained in Chapter 4 and applied in the later Chapters of this work.

3.7 REFERENCES

The references for Chapters 1 to 4 are given in Appendix IV.
CHAPTER 4
TWO DIMENSIONAL ANALYSIS

CONTENTS

4.1 INTRODUCTION

4.2 ANALYSIS

4.3 EQUATION OF MOTION

4.4 COMPUTER SOLUTION TO TRAJECTORY PROBLEMS

4.5 WORKED EXAMPLE

4.6 VALIDATION

4.7 REFERENCE
CHAPTER 4
TWO DIMENSIONAL ANALYSIS

Why should that apple always descend perpendicularly to the ground? (thought Sir Isaac Newton to himself). Why should it not go sideways or upwards, but constantly to the earth's centre?

William Stukeley (1687 – 1765)

4.1 INTRODUCTION

Most agricultural processes that involve a fluid - particle interaction are two-dimensional in nature and the simple one-dimensional theory presented above is inadequate for a complete analysis.¹

In these circumstances it is necessary to undertake a more general two-dimensional analysis that allows the plotting of the particle trajectories for any combination of fluid and particle conditions. These are useful for analysis and design purposes or for selection of parameters for improved operation of machines and other fluid – particle systems.

The Trajectory Plotting System (TPS), which accompanies this monograph, plots the trajectory for any specified initial particle conditions. An end line or lines must be specified where the calculation of the trajectory calculation stops and the trajectory is considered to be complete.

The program draws the trajectory as (x, y), (x, t), or (y, t) and specifies the results as:

(i) the co-ordinates of the end point relative to a defined origin.
(ii) the magnitude and direction of the final velocity.
(iii) the duration of the trajectory.

The program solves two types of problems:

(a) The 'environmental' problem

Here there are usually no boundaries except the surface of the earth (which is assumed planar but not necessarily horizontal); it is usually the end line. The air velocity (if present) is parallel to the earth's surface and may be specified in layers of different velocity and thickness. Typical problems include spreading of agricultural materials, sporting activities with balls and any other outdoor processes such as rainfall, etc.

(b) The 'machine' problem

Here the fluid flows within a 'duct' or 'machine' of some kind; this might be at any angle to the horizontal. The end lines might be the boundaries of the machine and include some other line, for example, the end of a duct where one or more of the fractions passes to the atmosphere. Problems of this type include the sorting / cleaning of many particle mixtures and gas cleaning, etc.

¹ There are also many three dimensional problems where the fluid velocity does not lie in the vertical plane through the release point of the particle. These are not considered in this work and cannot be solved by the trajectory plotting system.
Other problems may involve both aspects and require the problem to be solved in two parts; the result of the first part becomes the input to the second. Dropping materials through air onto a liquid stream is such an example.

The program provides a powerful tool for the analysis of a wide range of design situations and will reduce the need for at least some exploratory experimental work. It will also allow a wide range of design problems to be introduced for student use that have not been possible up to this time.

### 4.2 Analysis

The general two-dimensional trajectory problem involves the solution of Newton's equations expressed in two perpendicular, usually vertical and horizontal, directions.

As explained in Chapter 2 the drag force depends on the total velocity of the particle relative to the fluid and is calculated according to Equation 2.1. Its horizontal component will give the mass – acceleration in the horizontal direction and its vertical component, together with the net gravity force, will give the mass – acceleration in the vertical.

The velocity of the fluid and the initial velocity of the particle are both specified in vector form relative to the earth or the frame, such as the machine boundary. However while we are interested in the velocity of the particle relative to the earth / frame (\(U_{pe}\)) (which represents the trajectory), to calculate the drag force on the particle at any instant we need to also calculate its velocity relative to the fluid, \(W_{pf}\).

Hence we have to solve the vector equation represented in Figure 4.1:

\[
W_{pf} = U_{pe} + V_{ef} = U_{pe} - V_{fe}
\]

(4.1)

where

\[p = \text{particle} \]
\[f = \text{fluid} \]
\[e = \text{earth / frame} \]

This is usually written in the following form where both the vector and the 'relative' relationships are understood:

\[W = U - V\]

The graphical vector representation of this equation is shown in Figure 4.1
4.3 EQUATIONS OF MOTION

As shown in Figure 4.1 we can write the separate equations of motion in the horizontal and vertical directions. Thus:

\[ m \frac{dU_h}{dt} = -D \cos \alpha \]  \hspace{1cm} (4.2(a))

\[ m \frac{dU_v}{dt} = mg \frac{\rho_h - \rho_f}{\rho_f} - D \sin \alpha \]  \hspace{1cm} (4.2(b))

From Equation 2.2

\[ D = CA \frac{\rho_f W_v^2}{2} \]  \hspace{1cm} (4.3)

The details of the numerical integration of these equations are shown in Appendix II

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1 Equations 4.2 and 4.3 reproduced with permission from Lapple and Shepherd (1940). see references for details.
4.4 COMPUTER SOLUTION OF TRAJECTORY PROBLEMS

4.4.1 Background to the program

The use of the computer program which is available with this monograph will allow the solution of a range of problems associated with the movement of particles relative to moving fluids.

Note that the calculation is in two dimensions only. The plane of the fluid velocity must be in the vertical plane through the trajectory of the particle. Any effects due to the boundaries perpendicular to this plane are considered negligible. It should be realised that the program is able to solve problems which it would be difficult or impractical to set up or which are approximations to the physical conditions.

The first step is to set up the physical arrangement with the horizontal and vertical (the direction of gravity) corresponding to the x and y axes respectively. All angles specifying the direction of the fluid and the particle are measured anticlockwise from the positive x axis.

The point at which the particle is released is at the condition x = 0, ie, on the y axis. This point may be at, above or below the origin, ie, the y value at release may be zero, positive or negative.

In order to specify the completion of the calculation of the trajectory it is necessary to specify an end line or lines forming the boundary within which the trajectory is of interest and will be plotted.

As noted above, two types of problems might be envisaged; other more complex problems might be combinations of these two.

(a) The ‘environmental’ problem

Where the trajectory is in open air (although it may be indoors), the trajectory usually terminates when the particle hits the ground or the equivalent and this is specified as the end-line; see Figure 4.2(a).

(b) The ‘machine’ problem

Were the trajectory (or trajectories if a mixture is being studied) are constrained within boundaries (eg, the walls of a machine), those boundaries form the area of interest and these will usually form the end-lines; see Figure 4.2(b). Where, for example, two fractions are being separated in a wind tunnel three end-lines may be identified:

(i) the edge of the tunnel opposite the point where the particles are introduced.
(ii) the lower limit of the tunnel where the heavier fraction may pass out of the tunnel.
(iii) the upper limit of the tunnel where the lighter fraction may pass out of the tunnel.

Note that the fluid velocity may not necessarily be parallel with the boundary or end line but, for the purposes of maintaining the generality of the solution, may be assumed to pass through it.

---

1 Velocities are relative to the earth / frame unless otherwise specified.
Figure 4.2(a): Variables for analysis of trajectory in the 'environment'.

Figure 4.2(b): Variables for analysis of trajectories in a 'machine'; only those for heavy fraction shown.
4.4.2 Primary variables

The following properties and parameters must be specified:

(i) Particle diameter: mm, millimetre. This is an actual or equivalent diameter (as discussed in Section 2.7 above).

(ii) Particle mass: g, gram.

(iii) Fluid density: kg/m$^3$, kilogram per cubic metre.

(iv) Fluid viscosity: mPa.s, milli Pascal second (= mN.s/m$^2$, milli Newton second per square metre).

(v) Fluid velocity, V, m/s, metre per second. The default condition for this velocity is that it is uniform over the entire trajectory. However it is possible to specify up to 10 layers each of chosen but constant thickness and velocity to simulate a boundary layer at the surface. See Figure 4.3.

(vi) Particle velocity, U, m/s metre per second (relative to the earth).

---

**Figure 4.3:** Specification of fluid velocity profile with five layers of constant thickness to simulate the velocity profile shown.
4.4.3 Setting up a scenario

In setting up a scenario it is necessary for the user to define the magnitude and direction of the fluid velocity which, at that stage, is assumed to be unbounded.

Over that is placed:
(i) the physical boundaries which will, to some extent, define the space in which the trajectory will be drawn.
(ii) the end-lines which may define further limits within which the trajectories will be drawn.

These may be coincidental where a solid boundary exists. However, end-lines may be used to define other limits which are not physical boundaries but at which the particle passes out of the area of interest. For example, in a wind tunnel for separating two fractions of a mixture, end-lines will coincide with the physical walls. Other end-lines must be placed at the ends of the tunnel - at the top where the light fraction passes out into the atmosphere (or to another process) and at the bottom where the heavy fraction passes out to where it is caught.

Where a fluid velocity profile is specified, at least one of the layers will usually coincide with a boundary or end-line or both.

4.4.4 Program details

Further details of setting up and using the program are given in the User Manual which accompanies the program. Both may be accessed at:

http://eprints.unimelb.edu.au/archive/00001513/

4.4.5 Default screens

For preliminary inspection the two default screens are shown in Figure 4.5. The computer program should open with the splash screen followed automatically by the default screen shown in Figure 4.5(a). If the use layers box is then selected, the screen shown in Figure 4.5(b) should appear.

The numbers which appear in the properties and parameters boxes are necessary for the program to run to produce the default screens.

It is suggested that the user become familiar with the operation of the program before proceeding.
Figure 4.5 (a): Default screen for constant fluid conditions

Figure 4.5 (b): Default screen for variable fluid conditions
4.5 WORKED EXAMPLE

Consider a particle, dropping at its terminal velocity, that passes into a horizontal air stream. The following data apply:

- Mass of particle, \( m = \) 0.001 g
- Diameter of particle, \( d = \) 1.0 mm
- Velocity of air stream, \( V_{fe} = \) 15 m/s
- Angle of air stream, \( \theta = \) 0 degrees (default)
- Terminal velocity of particle, \( U_{pe} = \) 5.87 m/s
- Density of air, \( \rho_f = \) 1.2 kg/m³ (default)
- Viscosity of air, \( \mu_f = \) 0.018 mPa.s (default)

Figures 4.5 and 4.6 show the results of the use of the program to plot the trajectory from the time when the particle enters the air-stream to when it reaches its new, steady state condition.

Figure 4.5 (not to scale) shows typical velocity vector relationships. Note that in each case the drag force is opposite to \( W_{pf} \). The diagrams show the conditions:

(a) at the start, where the particle is moving vertical downward, will begin to move to the right under the action of the drag force that acts upward to the right.

(b) at a typical instant, 0.1 sec after the start. Here the particle is clearly moving in a downward direction and to the right as \( W_{pf} \) and the drag force rotates anti-clockwise.

(c) near the ‘end’, 2.0 seconds after the start when terminal (constant) conditions are again approached. The particle is moving in a straight line trajectory to the right. The horizontal velocity of the particle relative to the fluid is zero. \( W_{pf} \) is now again vertical.

![Vector diagrams of conditions during the trajectory in Example 4.1](image)

\( V_{ef}=-V_{fe}=-15 \)

\( \alpha = 201^\circ \)

\( \phi = 270^\circ \)

\( W_{pf}=16.1 \)

\( U_{pe}=5.87 \)

Start

Time = 0 sec

\( R = 1074 \)

\( C = 0.46 \)

\( \theta = 0^\circ \)

\( \alpha = 205^\circ \)

\( \phi = 308^\circ \)

\( W_{pf}=12.1 \)

\( U_{pe}=6.55 \)

Intermediate

Time = 0.1 sec

\( R = 805 \)

\( C = 0.49 \)

\( \theta = 0^\circ \)

\( \alpha = 270^\circ \)

\( \phi = 354^\circ \)

\( W_{pf}=5.83 \)

\( U_{pe}=15.7 \)

Near end

Time = 2 sec

\( R = 389 \)

\( C = 0.61 \)

\( \theta = 0^\circ \)

\( \alpha = 251^\circ \)

\( \phi = 308^\circ \)

\( W_{pf}=16.1 \)

\( U_{pe}=5.87 \)

\( \theta = 0^\circ \)

\( \alpha = 201^\circ \)

\( \phi = 270^\circ \)

\( W_{pf}=16.1 \)

\( U_{pe}=5.87 \)

Figures 4.5 and 4.6 show the results of the use of the program to plot the trajectory from the time when the particle enters the air-stream to when it reaches its new, steady state condition.

\[ V_{ef}=-V_{fe}=-15 \]

\[ \theta = 0^\circ \]

\[ \alpha = 201^\circ \]

\[ \phi = 270^\circ \]

\[ W_{pf}=16.1 \]

\[ U_{pe}=5.87 \]

\[ \theta = 0^\circ \]

\[ \alpha = 205^\circ \]

\[ \phi = 308^\circ \]

\[ W_{pf}=12.1 \]

\[ U_{pe}=6.55 \]

\[ \theta = 0^\circ \]

\[ \alpha = 270^\circ \]

\[ \phi = 354^\circ \]

\[ W_{pf}=5.83 \]

\[ U_{pe}=15.7 \]

This example is based on an idea given in Lapple, C.E. and Shepherd, C.B. (1940)
Figure 4.6: Graphs showing variation in velocities with time for Example 4.1

Note the following features in the individual graphs:

(a) the trajectory (x-y) of the particle; note that the angle of movement reaches a constant value.
(b) the velocity of the particle relative to the earth Upe and its angle $\phi$ approach constant values.
(c) the velocity of the particle relative to the fluid Wpf and its angle $\alpha$. Note in graphs (b) and (c) how, as the velocity of the particle with respect to the earth increases, the velocity with respect to the fluid decreases; they both reach their constant values together as do the respective angles. A new 'terminal' condition has been reached now at a constant angle to the horizontal as shown in graph (a). The small variation shown in graph (c) is due to the limit in the calculation of W based on the values of the drag coefficient – Reynolds number relation.
(d) the horizontal and vertical components of the particle relative to the earth, Upe as in (b). Here it will be seen that the immediate effect of the air blast and the associated increase in the total velocity of the particle with respect to the fluid is to cause a drop in the vertical component of the particle velocity. This occurs because, as will be seen from Equations 4.6 (a) and (b), both the vertical and the horizontal velocity are affected by the total velocity and the associated drag coefficient. The particle does not attain its vertical (terminal) velocity again until its horizontal component has equalled that of the air-stream.
4.6 VALIDATION

4.6.1 Introduction

In later Chapters the TPS program is used to analyse a number of different systems in which particle trajectories are of interest. However, before doing that, it is necessary to validate the program against published results and thereby increase the confidence in using it in new situations. It is of course worth remembering that, as acknowledged by some of the authors quoted, their work is based on approximate methods (particularly in relation to the drag coefficient – Reynolds number relationship) and may therefore be less reliable than the results obtained by the more computational intensive method used by TPS.²

The method of specifying the drag coefficient – Reynolds number relationship in the published work falls into two groups:

(i) Interpolation within the C-R data as for TPS:
   * Lapple and Shepherd (1940)
   * Winters (1970)
   * Farran and Macmillan (1979)

(ii) Fitting of equations to the C-R relationship:
   * Reints and Yoeger (1967)
   * Yates, Stephenson, Lee and Akesson, (1973)
   * Mennel and Reece (1963)
   * Kashayap and Pandya (1965b)

The basis for comparison between the published results and those obtained using TPS was the range (maximum distance); other parameters such as final velocity and angle and elapsed time could not be compared as they are usually not specified in published papers. The results were not separated according to the method used for specifying the drag coefficient – Reynolds number relationship.

4.6.2 Results

Range as calculated and reported in the various papers has been plotted against that obtained for TPS using the same particle and fluid properties. Where these were not available the standard conditions as in the TPS program were used. The results are shown in Figure 4.7 (a) and (b). A departure from the 45 degree line shows a difference in the two sets of values. Points above the line show values from TPS that are greater than the reported values and vice versa. The 5% error lines are also shown.

The results suggest that there is an excellent correlation between the published values and those from TPS. This conclusion is not wholly conclusive since the published values and those from TPS are based on the same theory. They may of course both be in error by similar amounts and still be well correlated. Nor does this result imply anything about the validation of either sets of results against what might be obtained in a practical situation. The reader is referred to papers by Reints and Yoeger, Kashayap and Pandya and Farran and Macmillan where detailed correlations between theoretical and experimental results are considered.

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² It is no criticism of Lapple and Shepherd (1940) to note that they used about 26 steps to calculate one trajectory (probably taking an hour or more) whereas the TPS program did the same in a few milliseconds. The important value of their work in providing a general solution to this and other trajectory problems has only been fully realized with the advent of the digital computer some 65 years later.
Figure 4.7(a) Correlation of reported and TPS values of range
- Lapple and Shepherd - horizontal projection in still air
- Winters – horizontal projection in still air
- Reints and Yoerger – angled projection in still air
- Mennel and Reece – horizontal projection in still air
- Yates et al – horizontal projection in still air

Figure 4.7(b) Correlation of reported and TPS values of range
- Kashayap and Pandya - vertical drop in horizontal air stream then still air.
- Farran and Macmillan – projection at various angles in vertical air stream.
4.6.3 Conclusion

The Preface and Chapter 1 stated the objective of providing (within the limitations noted) a program which would give a general solution to 'all' two dimensional problems involving particle trajectories.

Considering the range of air and particle velocities and directions evaluated (giving ranges with a ratio of >100:1) it is considered that this objective has been met and that the use of the program to analyse the various situations as presented in the following Chapters is fully justified.

4.7 REFERENCES

The references for Chapters 1 to 4 are given in Appendix IV.
One of the oldest groups of traditional agricultural processes are those which might be grouped under the title of 'distribution'. These include the traditional processes of sowing of seeds and crop promotion such as the spreading of manures and fertilizer; more recently it has included processes associated with crop protection with pesticides and weedicides.

These processes have their own name according to the physical form of the materials and the agricultural purpose for which they are being distributed.

(i) solids
   * sowing - seeds
   * spreading - fertilizers
   * planting - seedlings

(ii) liquids
   * spraying - insecticides / weedicides
   - fertilizers

Most of these processes involve particulate materials - grains, granules, pellets, droplets and seedlings. The aim in the processes is usually to have the materials distributed as uniformly as possible which usually implies uniform over an area or, if the release process is appropriately controlled, uniformly down a row.

It will be clear from the theory presented above that the variation in size of the particles, which may arise naturally in their production or growth, will have a significant effect on the resulting distribution.

Most of these materials are distributed 'en masse', ie, not individually and so particles are only treated in approximately the same way. Depending on the form of the distributor they will probably not be all thrown with the same velocity or in the same direction. Hence the variation in size and velocity will cause a significant variation in trajectory and in the resulting distribution. When referring to size, this variation is sometimes spoken of as 'segregation', a process which may even commence as the larger particles flow out of the hopper at a different time to the smaller ones.
In some ways this variation in properties of the materials and in the way that they are projected is a useful feature. If their physical properties were identical and they were, for example, subject to exactly the same velocity and projection angle then they would tend to be distributed in the same way and reach the same place on the soil. It would be necessary to introduce a controlled variation so that all the particles would be spread out in an appropriate way.

Given the materials are particulate in nature, they will all, to a greater or lesser degree, be subject to the forces and resulting motion described above. However the value of trying to analyse the distribution of those materials, which are markedly non-symmetrical, will of course depend to some extent on the accuracy of the data defining them. Notwithstanding this, some useful insights may be gained by an analysis based on what are clearly approximate assumptions.

Where the particles are assumed to be spheres (which is a reasonable assumption for many natural materials (eg, small droplets) or manufactured materials (eg, granules)) the actual materials (liquid or solid) will not be relevant to the analysis, provided of course that the appropriate physical properties are used.

In Part II the following are considered:

(i) Chapter 5 - Spreading solid granular fertilizers from ground machines. Here the trajectories and the effect of increasing the velocity and the angle of projection velocity are shown.

(ii) Chapter 6 - Spreading solid granular fertilizers from aircraft. Here the distribution of the material by weight is determined from the particle size distribution and the lateral spread of the particles which is derived from their range read from the trajectory plots. The effect of velocity and the angle of projection (relative to the velocity and direction of travel of the aircraft) on the resulting distribution are also evaluated.

(iii) Chapter 7 - Spraying drops from a fan nozzle close to the ground. The effect of a vertical wind, a constant and variable horizontal wind and rake of the nozzle is determined. The trajectories of drops within the fan (with and without rake) are compared with the ideal values for two drop sizes and drop velocities.

(iv) Chapter 8 - Drift from aircraft nozzles. The effect of a constant and variable wind profile on the drift of drops dropped and projected from a nozzle are evaluated. A detailed analysis of a small drop ejected from a nozzle with rake is presented.

(v) Chapter 9 – is reserved for future use.

It should be emphasised that these examples are not intended as a complete analysis of the subject concerned. Rather they are chosen to illustrate the range of problems to which the TPS program can be applied.
CHAPTER 5
GROUND SPREADING

CONTENTS
5.1 INTRODUCTION 5.1

5.2 THE SPINNING DISC 5.2
5.2.1 Mechanical action 5.2
5.2.2 Aerodynamic action 5.2
5.2.3 Trajectory processes 5.2
5.2.4 Design factors 5.2

5.3 WORKED EXAMPLE 5.4
5.3.1 Data 5.4
5.3.2 Results 5.4
5.3.3 Discussion 5.4
5.3.4 Conclusion 5.4

5.4 EXAMPLE 5.6

5.5 EXAMPLE 5.6

5.6 REFERENCES 5.6
CHAPTER 5

GROUND SPREADING

Dung is no saint, but where it falls it works miracles.
Spanish proverb

5.1 INTRODUCTION

The spreading of agricultural materials by hand and ground based machines are traditional processes, the techniques for which suggest a number of important variables that determine the trajectory of the particles being spread and hence the overall performance of the process.

Granular fertilizers varying in diameter from about 1 to 6 mm are one of the common materials that are spread by hand and various types of spreading devices including:

(i) spinning disc.
(ii) oscillating spout.
(iii) air blast from a fan.

To illustrate the use of the trajectory plotting program an analysis of some of the variables associated with the spinning disc machine is considered. A similar analysis can be applied to the oscillating spout type of machine and even hand spreading since the trajectory process similar for all of the above.

5.2 THE SPINNING DISC

The spinning disc is a widely used device for spreading granular agricultural materials such as fertilizer, seed, etc.

It is usually in the form of a flat or slightly ‘dished’ plate as shown in Figure 5.1. It rotates around a vertical axis at speeds of several hundred revolutions per minute giving corresponding peripheral speeds of the rim of 10 to 30 m/s. The upper surface usually has blades of some type arranged in a radial pattern.

Figure 5.1: Velocity diagram for particle leaving the spinning disc

The granular materials are fed in a stream onto the disc and are thrown outwards in a swath behind the machine as shown in Figure 5.2. Three factors contribute to this distribution process.
5.2.1 Mechanical action

As the particles come in contact with the blades they are forced to rotate with them with a peripheral speed corresponding to the peripheral speed at their radius at the particular instant. They are thus subject to centripetal forces which cause them to also move radially outwards across the surface of the disc. When they leave the disc they will have:

(i) a tangential velocity relative to the frame of the machine equal to the peripheral velocity of the outer radius of the disc.

(ii) a radial velocity relative to the disc which depends among other factors on the time that they have been on the disc which, in turn, depends on the radius at which they were fed on.


5.2.2 Aerodynamic action

By virtue of its shape and the associated blades, the disc acts as a mixed flow fan, drawing air onto it and expelling it in a more or less horizontal plane. Hence the particles, whose movement was described above in mechanical terms will, to some extent, be entrained in the air and so undergo an aerodynamic motion along with the air.

It is clear therefore that the movement of the particles will involve a complex interaction of the mechanical - particle and the air - particle dynamic factors described above.

In considering the resultant particle trajectory it is clear that the particles are not projected simply into still air. However as they move away from the disc the velocity of the air in which they are entrained will decrease. Also as they begin to drop toward the ground they will move out of the plane of the disc and into a region in which the air is sensibly stationary.

A further factor is the movement of the machine in the forward direction. This imposes an additional horizontal velocity (relative to the earth) on the particles as they leave the disc. This will be small but, if desired, it could be vectorially added to the velocity of the particle relative to the disc.

5.2.3 Trajectory processes

For consideration of the earlier work on the prediction of particle trajectories see Mennel and Reece (1963) and Dobler and Flatow, (1968).

5.2.4 Design factors

The use of the TPS to plot the trajectory of particles is based on the total velocity of particles relative to the earth. For the purposes of these examples typical velocities may be assumed to be in the range 10 to 30 m/s.

Various assumptions may be made as follows:
(i) still air in which the aerodynamic action of the disc described above is neglected.
(ii) an entrainment velocity equal to the particle velocity for a certain depth of fall below the plane of the disc and then still air for the remainder of the fall.

In each case the passage of the particles through the air has a significant effect on the trajectory and hence on:
(i) the range that the particles travel - their distance from the disc.
(ii) the spread of the particles - the distance from the particle with the minimum range to that with the maximum.
The design factors suggested in Figure 5.2 influence the trajectory and hence the spreading process. These include:

(i) the magnitude of the velocity of projection.
(ii) the angle (to the horizontal) of projection
(iii) the height of the spreading device.

The effect of these will be evaluated in the following example.

**Figure 5.2:** Parameters for ground spreading of fertilizer

### 5.3 WORKED EXAMPLE

#### 5.3.1 Data

Consider a machine spreading a mix of fertilizer particles in still air with a spinning disc at a height of 1.0 metre. Assume that the fertilizer particles have the following properties. The mass values were calculated from an assumed particle density of 0.0018 gram/cubic millimetre.

<table>
<thead>
<tr>
<th>Dia. of particle, mm</th>
<th>1.0</th>
<th>2.0</th>
<th>3.0</th>
<th>4.0</th>
<th>5.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass of particle, g</td>
<td>0.00092</td>
<td>0.0074</td>
<td>0.0249</td>
<td>0.0589</td>
<td>0.115</td>
</tr>
</tbody>
</table>

(i) For selected combinations of the following parameters determine the effect on range and spread of:

* particle diameter
* particle velocity
* projection angle

#### Parameter

* Particle velocity, m/s  
  10, 20, 30,
* Projection angle, degree  
  0, + 5, + 10,

(ii) Hence discuss the features of the design of spinning disc spreaders that influence the range and spread of particles.
5.3.2 Results

Figure 5.3 shows the trajectories for two conditions listed above.

![Figure 5.3](image)

**Figure 5.3:** Trajectories for two conditions. The legend shows:
Particle diameter (mm) / particle projection velocity (m/s) / particle projection angle (degrees)

![Figure 5.4](image)

**Figure 5.4:** Range for two projection conditions as in Figure 5.3 (also with 6 mm dia): 30 m/s, 0 degrees and 20 m/s, 10 degrees
5.3.3 Discussion

It is clear from the above results that there is considerable segregation by particle size for any given velocity and angle combination. While larger particles have a smaller drag coefficient, because of a greater Reynolds number, the drag force is significantly greater because of their greater size as represented by their projected area increasing with the square of the diameter.

The comparison of 30 m/s and zero angle with 20 m/s and 10 degree angle suggests that throwing of particles at angles above the horizontal has a more significant effect of increasing the range and the spread of the particles than increasing the velocity.

5.3.4 Conclusion

There is clearly a significant segregation in the range of particles due to their size. Any variation in the angle of throw due to the aerodynamic or mechanical action of the disc or pitching of the machine will have a significant effect on range and spread.

5.4 Example

For one of the conditions in Example 5.1 compare the effect of increasing:

(i) the throwing speed by 50%.
(ii) the disc height to:
   * 2.0 m
   * 3.0 m

5.5 Example

Consider the disc in Problem 5.1 above with a projection angle of 0 degrees to the horizontal. For one of the velocity and height conditions compare the distribution obtained there (for still air) with that obtained when, as the particle leaves the disc, there is an air velocity in the direction of and equal to the particle velocity. Assume that after the particle passes out of this band it falls in still air. Assume the depth of this band is:

(i) 100 mm
(ii) 200 mm

5.6 References


CHAPTER 6
AERIAL SPREADING

CONTENTS
6.1 INTRODUCTION 6.2
6.2 SPREADING OF SOLID PARTICLES 6.2
   6.2.1 Variables of interest
      (a) Variables associated with the aircraft
      (b) Variables associated with the material
      (c) The measure of performance
   6.2.2 Velocity relationships
6.3 WORKED EXAMPLE 6.5
   6.3.1 Data
   6.3.2 Calculation
   6.3.3 Results
   6.3.4 Conclusion
6.4 WORKED EXAMPLE 6.8
   6.4.1 Data
   6.4.2 Results
   6.4.3 Conclusion
6.5 EXAMPLE 6.8
6.6 REFERENCE 6.8
6.2

CHAPTER 6

AERIAL SPREADING

If we worked on the assumption that what is accepted as true really is true, then there would be little hope for advance.

Orville Wright (1871-1948)

6.1 INTRODUCTION

Aircraft are now commonly used for the distribution of agricultural materials. This use of aircraft allows rapid application of the material often in places where ground application is impossible.

The distribution of solid granular materials (small particles), called spreading, can be used for seeds and fertilizer and even tree seedlings. The distribution of liquids, called spraying, as fertilizer application or for pest and disease control, is also now common from aircraft; see Chapter 8.

The distribution of solid particles requires that the material be spread laterally. Liquids can be pumped to outlets (nozzles) along a boom (pipe) attached to the aircraft; the width of the swath or path is then more or less equal to the length of the boom. However the similar distribution of solid particles from individual outlets is not practical because particles will not flow unless entrained in a fluid such as air. Hence the material is usually spread laterally by being thrown from a central point on the aircraft.

The distribution of material from the air also depends on the form of aircraft. Fixed wing aircraft, which operate at a high forward speed, require solid material to be ejected at a high lateral speed to achieve a significant spread. This contrasts with helicopters which can operate at a lower forward speed but also have different design problems due to the downdraft from the main propeller.

6.2 SPREADING OF SOLID PARTICLES

6.2.1 Variables of interest

The significant variables involved in the spreading of granular materials from a fixed wing aircraft are:

(a) Variables associated with the aircraft:

(i) travel velocity.
(ii) height above the target.
(iii) ejection of material.
        * velocity of ejection.
        * direction of ejection relative to the direction of the aircraft.
        * direction of ejection relative to the horizontal.

The author acknowledges the insights for this Chapter provided by Yates, et al (1973).
(b) Variables associated with the material:

(i) diameter distribution of particles.
(ii) mass distribution of the particles.

(c) The measure of performance

The measure of performance is the lateral distribution of the particles as represented by the weight of the fertilizer material per unit ground area.

6.2.2 Velocity relationships

Figure 6.1 shows the general arrangement with the velocities that are relevant to an analysis of the problem where the aircraft is flying in still air.

It may appear that this needs to be considered as a three dimensional problem but this is not so. Consider the following velocities:

* $V_{ce}$ - the velocity of the aircraft relative to the earth.
* $V_{ca}$ - the velocity of the aircraft relative to the (assumed) still air.
* $V_{pc}$ - the velocity of the particle relative to the aircraft; this is the so called, ejection or projection velocity. This need not necessarily be perpendicular to the direction of the aircraft nor horizontal; it is shown at an angle of $\psi$ to the former and at an elevation of $\gamma$.

![Figure 6.1: Velocities associated with the lateral spread of particles from an aircraft](image-url)
Then

* \( V_{pe} \) - the velocity of the particles leaving the aircraft relative to the earth
  is the vector sum of the velocity of the particle relative to the aircraft \( V_{pc} \) plus the velocity of the aircraft relative to the earth \( V_{ce} \)

\[
V_{pe} = V_{pc} + V_{ce}
\]

And

* \( V_{pa} \) - the velocity of the particles leaving the aircraft relative to the (assumed) still air is the vector sum of the velocity of the particle relative to the aircraft \( V_{pc} \) plus the velocity of the aircraft relative to the air \( V_{ca} \).

\[
V_{pa} = V_{pc} + V_{ca}
\]

But with still air

\( V_{ca} = V_{ce} \)

Hence

\( V_{pa} = V_{pe} \)

We are then able to analyse the trajectory in two dimensions since the velocity of the particle relative to the air lies in the same plane as the velocity of the particle relative to the earth, i.e., as the vertical plane of the trajectory at an angle \( \delta \) to the direction of the aircraft as shown in Figure 6.2. When we have defined the velocities in this way, we can 'remove' the aircraft from consideration; the trajectory depends only on \( V_{pa} \) and the properties of the particle and the air as listed above.

**Figure 6.2:** Three dimensional view of the trajectories for three different particles.
As shown in Figure 6.2 we can calculate the range where each size of particle reaches the ground using the TPS. Then knowing the angle of the plane of the trajectories (δ) to the direction of the aircraft (calculated from the respective velocities), we can calculate the lateral spread for each particle.

The assumption that the particles are projected (by the combined effect of the aircraft velocity and lateral injection) into still air might be questioned if the particles pass through the wake of the aircraft. Yates et al (1973) discuss this matter briefly and conclude that particles with a greater diameter than about 0.5 mm are little affected by the wake.

### 6.3 WORKED EXAMPLE

#### 6.3.1 Data

An aircraft flying in still air is spreading granular fertilizer with a range of equivalent diameters from 1 to 6 mm.

(i) Determine the distribution of the fertilizer particles as represented by their lateral spread (perpendicular to the flight path) if the ejection velocity is perpendicular to that path and horizontal, ie, ψ and γ are = 0?

(ii) What is the resulting mass distribution of the fertilizer particles?

The data are as follows:

- Aircraft speed, Vca = 40 m/s,
- Aircraft height above ground = 10 m
- Ejection velocity of particles from aircraft, Vpc = 10 m/s
- Particle diameters = 1, 2, 3, 4, 5, 6 mm
- Particle density = 0.00176 gram/mm³.

#### 6.3.2 Calculation

(i) Velocity of the particles relative to the air; refer Figure 6.1.

\[
V_{pa} = \sqrt{V_{pc}^2 + V_{ca}^2}
\]

\[
= \sqrt{10^2 + 40^2}
\]

\[
= 41.2 \text{ m/s}
\]

(ii) Angle δ, between the path of the aircraft and velocity of the particle relative to the air, Vpc.

\[
\delta = \tan^{-1}\left(\frac{V_{pc}}{V_{ca}}\right) = \tan^{-1}\left(\frac{10}{40}\right) = 14.0°
\]

This is the angle of the plane of the trajectory to the direction of the aircraft; refer Figure 6.2.

(iii) The ranges of the particles which were calculated using the TPS are presented in Table 6.1. Using these the lateral spread of the particles are calculated from:

Lateral spread = Range sin δ.
<table>
<thead>
<tr>
<th>Parameters</th>
<th>Part. dia. mm</th>
<th>Part. vol. cu.mm</th>
<th>Part. mass g</th>
<th>Range m</th>
<th>Lateral spread m</th>
<th>Width m</th>
<th>Area %</th>
<th>Weight part. %</th>
<th>Rel. distrib</th>
<th>For info. only</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aircraft vel. rel. to air, Vca, m/s</td>
<td>40</td>
<td>1</td>
<td>0.52</td>
<td>0.0009</td>
<td>11.2</td>
<td>2.72</td>
<td>2.23</td>
<td>31.3</td>
<td>30.0</td>
<td>0.96</td>
</tr>
<tr>
<td>Part. vel. rel. a'craft, Vpc, m/s</td>
<td>10</td>
<td>2</td>
<td>4.19</td>
<td>0.0074</td>
<td>20.4</td>
<td>4.95</td>
<td>1.82</td>
<td>25.5</td>
<td>25.0</td>
<td>0.98</td>
</tr>
<tr>
<td>Part vel. rel. air, Vpa, m/s</td>
<td>41.2</td>
<td>3</td>
<td>14.13</td>
<td>0.0249</td>
<td>26.2</td>
<td>6.35</td>
<td>1.19</td>
<td>16.7</td>
<td>20.0</td>
<td>1.20</td>
</tr>
<tr>
<td>Angle, part vel. to a'craft dim, deg.</td>
<td>14.0</td>
<td>4</td>
<td>33.49</td>
<td>0.0589</td>
<td>30.2</td>
<td>7.32</td>
<td>0.82</td>
<td>11.6</td>
<td>13.0</td>
<td>1.12</td>
</tr>
<tr>
<td>Particle density, g/cu mm</td>
<td>0.00176</td>
<td>5</td>
<td>65.42</td>
<td>0.1151</td>
<td>33.0</td>
<td>8.00</td>
<td>0.58</td>
<td>8.2</td>
<td>8.0</td>
<td>0.98</td>
</tr>
<tr>
<td>Aircraft height, m</td>
<td>10</td>
<td>6</td>
<td>113.04</td>
<td>0.1990</td>
<td>35.0</td>
<td>8.49</td>
<td>0.49</td>
<td>6.8</td>
<td>4.0</td>
<td>0.59</td>
</tr>
<tr>
<td><strong>Totals</strong></td>
<td>7.13</td>
<td>100</td>
<td>100</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Table 6.1**: Data and calculation for Example 6.1

**Figure 6.3**: (a) Range and lateral spread for various fertilizer particle sizes
(b) Weight (%) of fertilizer for various particle sizes (assumed)
(c) Relative distribution (weight / unit area) for various lateral spreads
(iv) These values of the lateral spread of the particles need to be interpreted in terms of the areas (effectively the width) over which they are assumed to fall. For example, the width of the strip for the 3 mm particle is assumed to be from midway between the spread for 2 and 3mm particles to midway between the 3 and 4 mm particles. The % of the area for each particle size is calculated based on the total width of coverage.

(v) On the basis of a known weight distribution by particle size for the fertilizer then we can calculate the distribution of weight of fertilizer per unit area. These are relative values because the actual numbers of particles distributed are not specified.

Summary of calculations:
* the mass is based on an assumed density and a spherical shape.
* the range (x) is as obtained from the trajectory plots.
* the spread (s) is calculated as shown in Figure 6.1 = x sin δ. (See (iii) above)
* the width for any particle is calculated from the width between adjacent spread values (s).
  For the 3 mm diameter particle,
  \[ \text{width (3)} = \frac{s(4) - s(2)}{2} \]
  * the area values are the width values expressed as a % of the total width.
  * the weight values are an assumed distribution of the total weight of the various sized particles expressed as a % of the total weight of all particles.
  * the relative distribution is the weight % divided by the area %.

6.3.3 Results

The data and calculations are shown in Table 6.1 and Figure 6.3.

6.3.4 Conclusion

As noted in 5.3 Worked Example and as shown in Figure 6.3(a) which was plotted from the trajectory graphs, as the diameter is increased the drag force increases and hence it becomes increasingly difficult to distribute larger particles compared to smaller ones.

It is also clear from this analysis that the size and mass distribution of the particles (the % in each class) has a significant effect on their uniformity of mass distribution on the ground. Note, for example, that at the outer extremity the relative distribution is only 60% of a perfectly uniform distribution. A larger proportion of 6 mm particles and a smaller proportion of 3 and 4 mm particles would improve the mass distribution.

In 6.3 Worked Example it was assumed that the particles were ejected from the aircraft perpendicular to the flight path (ψ = 0) and horizontally (γ = 0) (See Figure 6.1). The question arises as to whether ejecting the particles at different angles would significantly increase the lateral spread. The former is considered in 6.4 Worked Example following and the latter is suggested in 6.5 Example below.
6.4 WORKED EXAMPLE

Consider the aircraft as in 6.3 Worked Example. Determine the range and lateral spread for the 4 mm diameter particle ejected horizontally ($\gamma = 0$) at various angles relative to the flight path, i.e., $\psi$ from 0 to 180 degrees.

6.4.1 Data

The data are as follows:

| Aircraft velocity relative to the air, $V_{ca}$ m/s | (i) 60 | (ii) 40 |
| Particle ejection velocity relative to the aircraft, $V_{pc}$ | (i) 20 | (ii) 60 |
| Particle diameter, mm | 4 |
| Particle ejection velocity angle, $\gamma$ | 0 |

6.4.2 Results

Lateral spread values calculated from range data taken from trajectory plots are shown in Table 6.2 and plotted in Figure 6.5; (see page 6.8).

The values of aircraft and ejection velocities and ejection angle have been chosen and plotted to illustrate the effect that these variables have on the spread and not as a basis for selecting optimum values.

It will be seen that the maximum spread is achieved for ejection angle of about 110 degrees, i.e., the particles ejected slightly rearwards relative to aircraft flight path. However it should be noticed that the angle of the particles relative to the air ($\delta$) under these conditions is approximately 70 degrees, i.e., forward relative to the aircraft path. The angle of ejection to cause the particles to move perpendicular to the flight path is approximately 130 degrees.

6.4.3 Conclusion

The analysis provides a useful illustration of the influence of aircraft and particle velocity on spreading and suggests that ejecting the particles perpendicular to the aircraft flight path achieves a near maximum spread and no doubt simplifies the design of the spreading mechanism.

6.5 EXAMPLE

(a) Consider the aircraft spreading fertilizer as in 6.4 Worked Example above. Compare the performance obtained there with that which would be achieved if:

(i) the ejection angle $\gamma$ was 20 degrees above the horizontal
(ii) the aircraft height was 15 metre

(b) Repeat for other

(i) aircraft speeds
(ii) ejection speeds

6.6 REFERENCE

### 6.4.2 Results (continued)

#### Table 6.2: Results of trajectory analysis for various ejection angles for 4 mm particle with

(a) $V_{ca} = 60$ m/s and $V_{pc} = 20$ m/s

(b) $V_{ca} = 40$ m/s and $V_{pc} = 60$ m/s

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Eject angle deg</th>
<th>Part. vel. rel. air, $V_{ca}$, m/s</th>
<th>Traj. Range m</th>
<th>Angle traj. deg.</th>
<th>Lat. spread m</th>
<th>Long. range m</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aircraft velocity rel. to air, $V_{ca}$, m/s</td>
<td>60</td>
<td>0</td>
<td>80.0</td>
<td>41.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Part. eject. vel. rel. a'craft, $V_{pc}$, m/s</td>
<td>20</td>
<td>20</td>
<td>79.1</td>
<td>40.9</td>
<td>5.0</td>
<td>3.5</td>
</tr>
<tr>
<td>Particle density, g/mm$^3$</td>
<td>0.00176</td>
<td>40</td>
<td>76.4</td>
<td>40.3</td>
<td>9.7</td>
<td>6.8</td>
</tr>
<tr>
<td>Particle diameter, mm</td>
<td>4</td>
<td>60</td>
<td>72.1</td>
<td>39.4</td>
<td>13.9</td>
<td>9.5</td>
</tr>
<tr>
<td>Particle mass, g</td>
<td>0.0589</td>
<td>80</td>
<td>66.5</td>
<td>38.0</td>
<td>17.2</td>
<td>11.3</td>
</tr>
<tr>
<td>Height, m</td>
<td>10</td>
<td>100</td>
<td>59.9</td>
<td>36.3</td>
<td>19.2</td>
<td>11.9</td>
</tr>
</tbody>
</table>

| Aircraft velocity rel. to air, $V_{ca}$, m/s | 40              | 0                                  | 100.0         | 44.6             | 0.0           | 0.0           | 44.6          |
| Part. eject. vel. rel a'craft, $V_{pc}$, m/s | 60              | 20                                 | 98.5          | 44.4             | 12.0          | 9.2           | 43.4          |
| Particle density, g/mm$^3$      | 0.00176         | 40                                 | 94.2          | 43.7             | 24.2          | 17.9          | 39.9          |
| Particle diameter, mm           | 4               | 60                                 | 87.2          | 42.4             | 36.6          | 25.3          | 34.0          |
| Particle mass, g                | 0.0589          | 80                                 | 77.7          | 40.6             | 49.5          | 30.9          | 26.4          |
| Height, m                       | 10              | 100                                | 66.1          | 37.9             | 63.4          | 33.9          | 17.0          |

Table 6.2: Results of trajectory analysis for various ejection angles for 4 mm particle with

(a) $V_{ca} = 60$ m/s and $V_{pc} = 20$ m/s

(b) $V_{ca} = 40$ m/s and $V_{pc} = 60$ m/s

#### Figure 6.5: Effect of ejection angle on longitudinal range and lateral spread for 4 mm particle

(a) Lateral spread versus longitudinal range; $V_{ca} = 60$ and 40 m/s and $V_{pc} = 20$ and 60 m/s; numbers show ejection angles. X marks values for 90 degree ejection.

(b) Lateral spread and longitudinal range versus ejection angle; $V_{ca}=40$, $V_{pc}=60$ m/s.
CHAPTER 7
GROUND SPRAYING

CONTENTS

7.1 INTRODUCTION 7.2
7.2 SPRAYING OF LIQUID DROPS 7.2
7.2.1 Variables of interest

7.3 WORKED EXAMPLE 7.4
7.3.1 Data
7.3.2 Results
7.3.3 Conclusions

7.4 WORKED EXAMPLE 7.7
7.4.1 Data
7.4.2 Geometry
7.4.3 Calculations of ideal position
7.4.4 Calculation of actual position using TPS
7.4.5 Results
7.4.6 Conclusions

7.5 EXAMPLE WHERE TPS CANNOT BE USED 7.11

7.6 EXAMPLE 7.12

7.7 EXAMPLE 7.12

7.8 REFERENCE 7.12
7.1 INTRODUCTION

Agriculture requires the distribution of various forms of pesticides, weedicides, fertilizers and other liquid materials for growth promotion and protection. Much of this work is done by humans or machines working on the ground, distributing these as finely divided materials. The trajectory of the drops is one aspect of the distribution process and hence it is appropriate to treat this topic as an example of the application of the TPS. Aerial spraying, which has much in common with ground spraying and aerial spreading, will be considered in Chapter 8.

While the use of many of the above materials is undesirable from an ecological point of view, the production of food and fibre requires these materials to be used at the present time and until such time as more benign materials and processes become available. There is of course need to optimise their use and this involves the study of various aspects of their distribution. Many of these are not related to their trajectory and will not be considered further in this work. The reader is referred to the more specific books.

The significance of drop diameter highlights the conflicting requirements in all spraying operations – the need to use large drops which are less subject to drift and evapouration and the need to use small drops to widely distribute the spray over the target. (Courshee 1959)

7.2 SPRAYING OF LIQUID DROPS

The distribution of liquid drops is similar in many ways to the spreading of solid particles. They are however usually very much smaller than granules and behave more like dusts.

Owing to their small size and mass, liquid drops (and solid dusts) used in spraying (and spreading) are subject to:

(i) a small gravity force (weight) hence their trajectory is only influenced to a small extent.
(ii) minor fluid turbulence and updraughts in the air near the ground.
(iii) natural wind and hence they may drift (be blown) away from the target.
(iv) evaporation unless materials to limit this are added to the carrier material.

These characteristics mean that:

(i) there is a complex interaction between the drops, the physical boundaries and the air movement, including any turbulence. Hence, their actual trajectory may be highly complex and, except in ideal situations, a theoretical analysis is impossible.
(ii) surface tension forces are large and hence the drops are near spherical in shape and behave as spheres.
(iii) the psychrometric conditions of the atmosphere are significant in determining the evaporation of the drop.
Notwithstanding the complex nature of the distribution process, the use of the TPS program for plotting droplet trajectories in ideal situations is justified because of the insights that it gives to the ways drops behave and hence ways in which the process may be improved.

Various forms of spray nozzle are available, some of which (eg, the common so called 'hydraulic' types) produce a range of droplet sizes; other devices such as the spinning disc produce drops in a narrow size range. It is not appropriate to discuss the dynamics of these various devices; again the reader is again referred to more specific books.

For the purposes of illustrating the use of the TPS the analyses will be limited to a small range of drops of specified diameter without specifying the device which produced them (although hereafter it will be referred to as a nozzle). The mass of the drops is based on the density of water - 0.001 gram/cubic millimetre.

7.2.1 Variables of interest
The significant variables involved in the spraying drops are:

(a) Variables associated with the nozzle:
(i) forward speed.
(ii) height above the target.
(iii) ejection of material from the nozzle.
   * velocity of ejection.
   * direction of ejection relative to the direction of the nozzle velocity.
   * direction of ejection relative to the vertical.

(b) Variables associated with the material:
(i) diameter of the drops.
(ii) evapourative characteristics of the drops.

(c) Variables associated with the atmosphere:
(i) wind velocity.
(ii) direction of wind relative to the direction of nozzle velocity.
(iii) the atmospheric conditions related to the evapouration of the drops.
7.3 Worked Example

7.3.1 Data

Assume that a stationary nozzle can be set to give the drops with the following dimensions:

<table>
<thead>
<tr>
<th>Dia. of drop, mm</th>
<th>0.05</th>
<th>0.1</th>
<th>0.2</th>
<th>0.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass of drop, g</td>
<td>0.000 000 0654</td>
<td>0.000 000 524</td>
<td>0.000 0041</td>
<td>0.000 0654</td>
</tr>
</tbody>
</table>

Table 7.1: Data for Worked Example 7.1

Determine the trajectory of drops under the following conditions; see also Figure 7.1

- Droplet ejection velocity, m/s: 10
- Nozzle height above ground, m: 1.0
- Alternative wind conditions:
  - Horizontal wind velocity, m/s: 2
  - Horizontal wind velocity profile, m/s: 0.2 (layer No 1) to 2.0 (layer No 10)
  - Vertical (downward) air velocity, m/s: 0.5 m/s
- Alternative nozzle angles:
  - Inclined nozzle at 45 against the 2.0 m/s horizontal wind (not shown).

Figure 7.1: Velocities and parameters associated with Worked Example 7.1 (not to scale)
Where it is assumed that there is a horizontal wind velocity of 2.0 m/s and a downward air velocity of 0.5 m/s the combined air velocity is given by:

\[ V = \sqrt{V_h^2 + V_v^2} \]
\[ = \sqrt{2.0^2 + 0.5^2} \]
\[ = 2.06 \text{ m/s} \]

Angle \( \theta \), between the direction of the wind velocity and the horizontal is given by:

\[ \theta = \tan^{-1} \left( \frac{V_h}{V_v} \right) = \tan^{-1} \left( \frac{0.5}{2} \right) = 14.0^0 \]

Note that the angle to be used in TPS will be \( 360 - 14 = 346^0 \)

**7.3.2 Results**

Figure 7.2 shows:

(i) the trajectories for the 4 drop sizes (0.05, 0.1, 0.2, and 0.5 mm) in a constant 2.0 m/s horizontal wind. The effect of increasing drop diameter (for example by a factor of 10 (from 0.05 to 0.5 mm) which increases the area by a factor of 100 and mass by 1000) is seen to be highly significant in reducing the drift.

(ii) the effect of a boundary layer with reducing air velocity near the ground (w/- layers) is seen in the comparison with the constant wind for the 0.1 mm drop.

Figure 7.3 shows the trajectories for a 0.2 mm diameter drop where it will be seen that, relative to vertical ejection in a horizontal wind (horizontal):

(i) inclining the nozzle forwards by 45 degrees (angle) has little effect on the trajectory.

(ii) the presence of a boundary layer (layers) has a significant effect on the reduction in drift (as can be seen for 0.1 mm drop in Figure 7.2 above).

(iii) the presence of a small 'down draft' (vertical) combined with the 2.0 m/s horizontal wind has a noticeable but not over-riding effect on the drift.

**7.3.3 Conclusions**

The program illustrates the major effect that size and to a lesser extent wind velocity have on the drop trajectory.
Figure 7.2: Trajectories for various drop diameters in a constant and layered wind velocity profile (See Figure 7.1)

Figure 7.3: Trajectories for 0.2 mm dia. drop with various conditions (See Figure 7.1) (i) 'Horizontal' wind, 2.0 m/s. (ii) 'Layers' - triangular wind velocity profile. (iii) 'Vertical' – wind velocity 2.06 m/s at 14 degrees to horizontal. (iv) 'Angle' – drops ejected at 45 degrees against horizontal (2.0 m/s) wind.
7.4 WORKED EXAMPLE

7.4.1 Data

Determine the distribution of drops from a fan nozzle moving forwards in still air and compare this with the geometric (ideal) distribution. The following data apply:

- Drop velocity, m/s: (i) 5, (ii) 10
- Fan angle, degrees: 120
- Forward velocity, m/s: 2.0
- Angle of rake, degrees: (i) 0, (ii) 30

7.4.2 Geometry

Figure 7.4 shows the nozzle on a pipe or boom. The motion of the drops is defined with respect to two vertical planes, one through the boom and the other perpendicular to this in the direction of motion. In general the plane of the trajectory in which the drops move is an oblique vertical plane between these two.

This plane is defined by:

(i) the forward velocity of the drops which is the sum of the forward component of drop velocity arising from the fact that the nozzle is raked forward and the velocity of the nozzle itself in the forward direction.

(ii) the lateral (sideways) velocity of the drops which is the component of the drop velocity in the lateral direction; this will be zero for drops in the centre of the fan.

7.4.3 Calculations of ideal position

Using the vector equations given below we can calculate the total drop velocity relative to the air; this is equal to the velocity relative to the earth since we are considering still air. This vector will define the magnitude and direction of the initial velocity of ejection from the nozzle at various angles within the fan. Then for a chosen height of nozzle we can calculate the points on the ground where the drops would ideally hit the ground (according to geometric considerations alone) and calculate the positions in the forward and sideways directions as shown plotted in Figure 7.5.

From Figure 7.4, we can write the vector equations for a general drop trajectory:

\[ \text{Velocity of drop r. t. earth} = \text{Velocity of drop r. t. machine} + \text{velocity of machine r. t. earth} \]

\[ U_{de} = J_{dm} + H_{me} \]

We can also write:

\[ \text{Velocity of drop r. t. the air} = \text{Velocity of drop r. t. the earth} + \text{velocity of earth r. t. the air} \]

\[ W_{da} = U_{de} + V_{ea} \]

\[ W_{da} = U_{de} - V_{ae} \]

But for still air, \( V_{ae} = 0 \) hence \( W_{da} = U_{de} \)

\[ W_{da} = J_{dm} + H_{me} \]

Thus the velocity of the drop relative to the air is its velocity relative to the earth, ie, the sum of their velocity relative to the machine and the machine relative to the earth.
Figure 7.4: Velocity diagrams for spray nozzle moving in still air.
(a) General oblique view; trajectory not shown
(b) End elevation with angle in fan = 0

Definitions:
- $U_{de}$ = Velocity drop relative to earth
- $W_{da}$ = Velocity drop relative to air
- $J_{dm}$ = Velocity drop relative to machine
- $H_{me}$ = Velocity machine relative to earth
- $V_{ae}$ = Velocity air relative to earth
7.4.4 Calculation of actual position using TPS

The above velocity and direction was used in the TPS program to calculate the actual range for drops at various angles within the fan; the same angles were used as for the ideal calculation above. Again the distances in the forwards and sideways directions are calculated and plotted in Figure 7.5. Since the ideal and actual points are in the same plane they plot in a straight line through the origin.

7.4.5 Results

Results for some of the values chosen from 7.4.1 above, are shown in Figure 7.5. Note that the diagrams are symmetrical; negative values represent the left hand side of the nozzle.

Various comparisons can be made between drop velocity (5 and 10 m/s), rake (0 and 30 degrees) and drop diameter (0.2 and 0.5 mm).

(i) (a) and (b) show the effect of decreasing drop velocity for a constant rake and diameter. The increase in drop velocity has a small effect on forward distance but not on sideways spread. The actual distance increases due to the influence the ejection velocity has on the range. Under these conditions the final motion of the drop is vertical for both velocities.

(ii) (a) and (c) show the effect of increasing drop diameter and associated mass for constant velocity and rake. The ideal is the same for both conditions (as expected) but the larger and heavier particle moves with a near straight trajectory and reaches a large proportion of the ideal.

(iii) (b) and (d) show the effect of decreasing the rake for constant velocity and diameter. Both ideal and actual distances are reduced (as expected) due to the ejection being more nearly vertical.

7.4.6 Conclusions

The spread and forward range falls far short of the ideal particularly for small droplets. Presumably the presence of a range of drop sizes adequately fills in the spaces between the nozzles.

The use of raked nozzles moves the ideal and actual spray pattern forward of the boom but has little effect on the width of spread.
Figure 7.5: Results of analysis of fan nozzle moving in still air with various conditions; height 0.5 m for all graphs.
(a) Drop: velocity 10 m/s; rake 30 deg; dia 0.2mm. (b) Drop: velocity 5 m/s; rake 30 deg; dia 0.2mm.
(c) Drop: velocity 10 m/s; rake 30 deg; dia 0.5mm. (d) Drop: velocity 5 m/s; rake 0 deg; dia 0.2mm.
7.5 **EXAMPLE WHERE TPS CANNOT BE USED**

It should be noted that in relation to 7.3 and 7.4 Worked Examples it is not possible to solve the general case where there is a wind present.

For example, consider the case of a fan nozzle with the drops moving in a vertical plane, i.e., a nozzle without rake as shown in Figure 7.6. Assume that there is a wind velocity equal to and in the same direction as the travel velocity of the machine. In effect this is the equivalent of having a fully shielded nozzle.

At the instant of ejection the velocity of a general drop (in the fan) relative to the earth, $U_{de}$, is in the oblique plane defined as in Figure 7.6 but the velocity of the drop relative to the air, $W_{da}$, (and relative to the machine $J_{dm}$), are in the vertical plane of the fan. That is, the motion of the particle relative to the earth and that relative to the air are not in the same plane; the TPS program cannot be used to solve this three dimensional problem.

![Figure 7.6: Velocity diagrams for spray nozzle moving in air with nozzle velocity:](image)

- (a) General oblique view; trajectory not shown.
- (b) End elevation with angle in fan = 0.
7.6 Example

Examine the effect of the interaction between rake (from positive (forward) to negative (backwards)), forward speed, drop diameter and height of nozzle on drop trajectories.

7.7 Example

Examine the conditions of rake (from positive (forward) to negative (backwards)), forward speed, drop diameter and height of nozzle that will give the final part of the drop trajectory in a vertical direction.

7.8 Reference

## 8.1 INTRODUCTION

## 8.2 AERIAL SPRAYING OF LIQUID DROPS

(a) Variables associated with the nozzle  
(b) Variables associated with the spray material  
(c) Variables associated with the atmosphere

## 8.3 WORKED EXAMPLE

8.3.1 Introduction
8.3.2 Data
8.3.3 Results  
(a) Drift behaviour  
(b) Dynamic behaviour of drops  
8.3.4 Conclusions

## 8.4 EXAMPLE

## 8.5 REFERENCES
8.2

CHAPTER 8

AERIAL SPRAYING

For I dipt into the future, far as the human eye could see,
Saw a vision of the world, and all the wonder that would be;
Saw the heavens filled with commerce, argosies of magic sails,
Pilots of the purple twilight, dropping down with costly bales;

Alfred Tennyson (1809-1892)

8.1 INTRODUCTION

Like aerial spreading, aerial spraying as been adopted in recent years in broad scale agriculture and plantation forestry. The impetus for this is usually the need to spray large areas quickly, the need to spray when access for ground based machines is not possible (due to terrain or soil conditions) and the difficulty of reaching tall plants and trees from the ground.

Aerial application of sprays is not as uniform as ground based spraying but is suitable for many applications particularly where there is a mobile pest. The variability of the natural environment and the disturbance caused by the passage of the aircraft limit any theoretical analysis to rather idealised situations. Notwithstanding these complications we can gain an insight into the trajectory aspects of the spraying process by applying the TPS for some limited situations.

8.2 AERIAL SPRAYING OF LIQUID DROPS

Aerial spraying involves many of the same variables and problems as in ground spraying. It also involves some additional ones associated with the atmosphere and the effect that the aeroplane itself has on the distribution process.

One significant factor is that aerial spraying is done several metres from the ground. The drops are therefore much more subject to the twin problems of drift and evapouration than in ground based spraying. Such drift occurs as a result of atmospheric winds which, in the context of fine drops, are significant even on the 'stillest' day. Evapouration depends on the psychrometric (humidity) conditions of the atmosphere. Both of these processes increase significantly as the drop diameter decreases.

As noted above the significance of drop diameter highlights the conflicting requirements in all spraying operations – the need to use large drops which are less subject to drift and evapouration and the need to use small drops to widely distribute the spray over the target (Courshee (1959)).

To illustrate the problem we can compare 0.2 mm, 0.4 mm and 1.0 mm diameter drops as shown in Table 8.1. The number of drops represents how finely the spray material is divided and hence how effective the distribution of the material is. The terminal velocity represents how quickly the drops will fall to the target - the faster they fall the less liable they are to drift.
The significant variables involved in spraying drops are:

(a) Variables associated with the nozzle:
   (i) travel velocity.
   (ii) height above the target.
   (iii) ejection of material from the nozzle.
      * velocity of ejection.
      * direction of nozzle axis (rake).
      * direction of ejection relative to the nozzle axis.

(b) Variables associated with the spray material:
   (i) diameter of the drops.
   (ii) evaporation characteristics of the drops.

(c) Variables associated with the atmosphere:
   (i) wind velocity.
   (ii) direction of wind relative to the direction of nozzle velocity.
   (iii) the atmospheric conditions related to evaporation of the drops.

As noted above, this work is limited to consideration of the ideal trajectory of drops associated with drift. In principle it would be possible to use the TPS (in a stepwise fashion) to consider evaporation if a model for that process is available. However the effect of evaporation is only significant with respect to the trajectory of the drop if or when its diameter is very small, for example, less than 0.1 mm. (Elliot and Wilson 1983).

Thus for medium to large drops the influence of evaporation is small, initially at least, and this has a negligible effect on the trajectory. The TPS program was not designed to handle the evaporation and resulting trajectory of such very small drops and will not be considered further here.

<table>
<thead>
<tr>
<th>Diameter of drop, mm</th>
<th>Mass of drop, gram</th>
<th>Number of drops / mL</th>
<th>Terminal velocity, m/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>0.000 0041</td>
<td>244 000</td>
<td>0.76</td>
</tr>
<tr>
<td>0.4</td>
<td>0.000 0335</td>
<td>29 800</td>
<td>1.65</td>
</tr>
<tr>
<td>1.0</td>
<td>0.000 524</td>
<td>1 900</td>
<td>3.00</td>
</tr>
</tbody>
</table>

Table 8.1: Characteristics of drops
8.3 WORKED EXAMPLE

8.3.1 Introduction

In this example it is assumed that the drops are being ejected from an aircraft flying horizontally into a horizontal head wind. The plane of the trajectories lie in the vertical plane in the wind and in the aircraft flight path, hence the TPS can be used.

The problem is to determine the distance that the drops drift and their impact velocity. Two alternative velocity conditions are assumed:

(i) constant for the full depth of fall.
(ii) a velocity profile, with a linear reduction in velocity from a maximum in the top layer to a minimum in the lowest.

![Diagram](image)

Figure 8.1: Velocities and parameters associated with 8.4 Worked Example (not to scale)

8.3.2 Data

Assume the following data; see also Figure 8.3.

- Drop diameters, mm = 1.0, 0.8, 0.6, 0.4, 0.2, 0.1, 0.05
- Initial vertical drop velocity, m/s = 10.0
- Nozzle travel velocity, m/s = 40.0
- Nozzle height above ground, m = 3.0
- Wind velocity, m/s = 1.5
- Layer data
  - Wind velocity profile (max), m/s = 1.5 (layer 6)
  - Wind velocity profile (min), m/s = 0.25 (layer 1)
  - Number of layers = 6
  - Depth of each layer, m = 0.5

It can be assumed that the drop is ejected with an initial velocity relative to the earth Ude equal to the vector sum of its velocity relative to the aircraft, Jdc and the velocity of the aircraft relative to the earth Hce.
\[ U_{de} = J_{de} + H_{ce} \]

The total particle velocity is given by
\[ U_{de} = \sqrt{J_{de}^2 + H_{ce}^2} \]
\[ U_{de} = \sqrt{10^2 + 40^2} \]
\[ = 41.2 \text{ m/s} \]

Angle \( \theta \), between the direction of the drop velocity and the negative horizontal is given by:
\[ \delta = \tan^{-1} \frac{J_{de}}{H_{ce}} \]
\[ = \tan^{-1} \frac{10}{40} \]
\[ = 14^\circ \]

### 8.3.3 Results

(a) Drift behaviour

![Figure 8.2: Trajectories for drops of diameter shown in 8.3 Worked Example](image)

The Mechanics of Fluid Particle Systems  R.H. Macmillan
Trajectories were plotted for drops with diameters in the range 0.05 to 1.0 mm. in a constant wind (Figure 8.2) and wind with linear velocity profile.

From these the drift distances for all drops are shown in Figure 8.3. These distances have all been calculated from the point where their respective trajectories without wind reached the ground surface as shown at point A in Figure 8.1. This basis for specifying the drift distance was desirable as this gives a true reflection of the effect of the wind and was necessary for large drops to avoid negative values when plotted on a log scale on the Y axis.

Figure 8.3: (a) Drift distance (log scale) for various drop diameters in a constant 1.5 m/s wind (constant) and with a linear wind velocity profile (layers). (b) Impact velocity (log scale) for above conditions; terminal velocities are plotted for comparison.

Small drops very rapidly assume a velocity dependent on the wind and behave almost as if they were simply dropped in the wind. The difference in drift distances between those released with zero velocity (not shown) and those projected downward and against the wind as in Figure 8.5 is small.

When measured as specified above, the drift distances of the larger drops are also dominated by the effect of the wind.
(b) Dynamic behaviour of drops

**Figure 8.4:** (a) Trajectory of 0.4 mm diameter drop projected at 41.2 m/s and 14 degrees to horizontal negative axis in 1.5 m/s wind. (b) Drop velocity for above conditions (c) Velocity components for above conditions. (d) Acceleration of drop for above conditions.

Figure 8.4 shows the detailed analysis of the behaviour of the dynamic behaviour of a medium drop (0.4 mm diameter) ejected into a uniform wind as discussed above.

The trajectory is shown in (a) where it will be seen that its motion reverses and it reaches the ground down wind of the release point. During this initial part (the first 0.25 sec) the horizontal velocity (relative to the earth) rapidly decreases as shown by the plot of the velocity (b).

This is also shown by the plot (d) where it is seen that the drop undergoes a very large negative acceleration (retardation). The maximum (not shown) is some 145 times that due to gravity.
Graph (c) shows the components of the velocity during the early part of the trajectory and the unusual behaviour also seen in 4.6 Worked Example. Here when the horizontal component of the velocity is zero, at about 0.25 second after release, the drop is moving vertically at a velocity of 1.39 m/s. It is then subject to a period of positive acceleration during which time the total velocity and its angle increase and reach new constant 'terminal' values as the drop moves towards the ground. This only occurs when the horizontal velocity of the drop reaches the velocity of the wind, i.e., the horizontal velocity component relative to the wind has disappeared.

Note that the corresponding velocity components are not exactly equal and in general would not be so; they just happen to be close in this example.

**8.3.4 Conclusions**

The initial conditions with which drops are ejected have a relatively small effect on drift distances; their drift tends to be dominated by the effect of the wind. The presence of a velocity profile has a significant effect on the drift distances for small drops.

This behaviour is a result of the fact that drop ejected at high velocity are initially subject to very large fluid drag forces. The latter motion of the drops is dominated by the gravity force (weight) and the fluid drag due to the wind.

**8.4 Example**

Examine the effect of changing some of the parameters in 8.3.2 above. For example, for a typical drop, consider other values of travel velocity, drop diameter and height of nozzle also nozzle rake angle.

**8.5 References**


PART III

CHAPTERS 10 – 12

SEPARATION PROCESSES

'What has straw in common with wheat', says the Lord.

The Bible: Jeremiah 23:28

Another group of traditional agricultural processes are those which might be classified under the title of 'separation'. These separation processes have various names including cleaning\(^1\), winnowing, sorting, sieving, sifting, fanning etc depending on their traditional origins and methods.

The traditional crop separation process termed 'winnowing' involves the use of natural or induced wind to blow away the lighter part or 'fraction' of the crop. To achieve a more controlled process, a perforated sheet known as a screen, sieve or riddle is now frequently used in conjunction with an air-stream.

These processes may be mainly mechanical as in sieving or mainly aerodynamic as in winnowing, although in both a mechanical and aerodynamic component are involved to a greater or lesser degree. In some machines an aerodynamic separation is performed first to remove the bulk of the non product material before the mixture reaches a second, or even a third, more mechanical separation stage.

All separation processes use the differences in the properties of the fractions to cause a physical separation. If there is a difference in the size of the particles in the fractions this causes a difference in the drag force due to:

(i) the difference in the drag coefficient; the larger particle has a higher Reynolds number hence smaller drag coefficient (see Figure 2.3).
(ii) the cross sectional area of the particle based on the square of the diameter; the larger particle (the simulated chaff in the following examples) has almost twice the area and hence twice the drag force compared to that of the grain.

As discussed in Chapter 2 above, few of the components are near ideal shape and in some cases the characteristics (size, drag coefficient) of the product are similar to those of the non-product material.

\(^1\) The term 'cleaning' makes a judgement about what is 'clean' and hence 'useful' and what is (presumably) 'dirty' or 'useless' and hence of no or lesser value. The use of the term cleaning may be more justified where it is required to separate non-crop components (eg, contaminants such as weed seeds, insects, soil particles) from the useful component(s).
In separation processes the usual aim is to have the fractions separated completely. Again it will be clear from the above theory that the variation in size of the particles which may arise naturally in their production or growth will have a significant effect on the resulting separation.

Hence it is unlikely that this ideal of complete separation will be achieved and for many separation processes each of the fractions will be 'contaminated' with the other fraction. In statistical terms there will be some overlap of the distributions as shown in Figure III.1

Again most of these materials are separated 'en masse', ie, not individually, and so they are all treated in approximately the same way although this may depend to some extent on the form of the separator. While the variation in properties of the materials was seen as a useful feature in the distribution processes, here the variation is a complication to the design of suitable arrangement. It will be the variation in size and shape that will cause a significant variation in trajectory and in the resulting separation.

![Figure III.1: Distribution of quantities of three fractions in a separation process](image)

Again, given the materials are particulate in nature, they will all, to a greater or lesser degree, be subject to the forces and resulting motion described in the theory given above. The success in trying to design a system to separate those materials which are markedly non-symmetrical will, of course, depend on the accuracy of the data defining them. Notwithstanding this, some useful insights may be gained by an analysis based on what are clearly approximate assumptions.
Further, the following examples concentrate on grain and chaff separation mainly because this is the type of product for which the aerodynamic properties are available. Now that a more powerful analysis and trajectory plotting program is available, it may be found worthwhile to measure the properties of a wider range of products.

In the following Chapters various materials and a range of processes are considered; no doubt the reader could find local examples to which the program could be applied.

It should be remembered that the trajectory plotting program deals with single particles only. The influence of a stream of particles and their non-spherical shape will, as discussed briefly in Chapter 12, influence the results obtained in any practical situation.

It should also be emphasised that the purpose of the examples presented below is not to critically evaluate the previous published work (although that might be an interesting exercise) but rather to illustrate the capacity of the TPS program and to suggest further lines of investigation that hitherto have not been possible.

In Part III, the following are considered:

(i) Chapter 10: Winnowing in the wind where the effect of wind and particle velocity and direction are investigated.

(ii) Chapter 11: Winnowing in a horizontal tunnel where the effect of air velocity and depth of tunnel are investigated, also the angle of the tunnel and the injection process.

(iii) Chapter 12: Winnowing in a vertical column where the effect of injection angle and velocity on range are investigated.

(iv) Chapter 13: Reserved for future use

For the purposes of providing a small set of examples of separation in Chapters 10, 11 and 12, it has been assumed that the materials to be separated consist of wheat grain and 'simulated chaff' – hereafter called 'chaff'. Following Farran and Macmillan (1979), the particles are assumed to be spherical with the following values as measured by those authors.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Grain</th>
<th>Chaff</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass, gram</td>
<td>0.04</td>
<td>0.0035</td>
</tr>
<tr>
<td>Equivalent diameter, mm</td>
<td>4.0</td>
<td>5.5</td>
</tr>
</tbody>
</table>
There are also a number of other fields in the chemical and mineral industries in which physical separation processes can be identified. The trajectory plotting program associated with this monograph may be useful in these but, apart from the examples given in Section IV, these will not be considered further.

Again it should not be assumed that these examples are a complete analysis of the subject concerned. Rather they are chosen to illustrate the range of problems to which the TPS can be applied.
CHAPTER 10
WINNOWING IN THE WIND

<table>
<thead>
<tr>
<th>CONTENTS</th>
<th>10.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>10.1 INTRODUCTION</td>
<td>10.2</td>
</tr>
<tr>
<td>10.2 VARIABLES OF INTEREST</td>
<td>10.3</td>
</tr>
<tr>
<td>10.3 WORKED EXAMPLE</td>
<td>10.3</td>
</tr>
<tr>
<td>10.3.1 Data</td>
<td>10.3</td>
</tr>
<tr>
<td>10.3.2 Results</td>
<td>10.3</td>
</tr>
<tr>
<td>(a) Introduction</td>
<td>10.3</td>
</tr>
<tr>
<td>(b) Separation in the wind</td>
<td>10.3</td>
</tr>
<tr>
<td>(c) Separation without wind</td>
<td>10.3</td>
</tr>
<tr>
<td>10.3.3 Conclusion</td>
<td>10.3</td>
</tr>
</tbody>
</table>

| 10.4 REFERENCES | 10.7 |
CHAPTER 10
WINNOWING IN THE WIND

Take a straw and throw it up into the air; you may see by that which way the wind is.
John Selden (1584 - 1654)

10.1 INTRODUCTION

Winnowing in the wind is the process of separating chaff from grain in a natural air-stream, the origins of which process are lost in the mists of history. However it is clear from Figure 10.1, showing (a) winnowing in China during the Sung dynasty in the 12th Century (Needham, 1984) and (b) in the middle East at the end of the 20th Century (WFP) that little has changed in 800 years. The process is still commonly used in the Third World today but as a problem in fluid - particle mechanics, it appears to have received little attention (Macmillan (1999))

The process involves tipping or throwing a mixture of grain and chaff / straw in the wind, a process which allows the latter, lighter fractions to be blown further than the grain. If the fractions are not completely separated then further winnowing stages are necessary to achieve total separation and a clean grain sample.

The failure to achieve this in one operation is presumably due to the variation in the properties of the particles and the presence of many particles in the mixture stream which causes turbulence in the air stream and a reduction in the fluid drag on the individual particles; see the discussion in Section 2.9 above.

Figure 10.1 (a) Winnowing in the wind in China in 12th Century (Needham 1984) (b) Winnowing in the wind in the Middle East in the 20th century (WFP)
Reprinted with permission from Needham Research Institute and World Food Program

1 Anecdotal evidence and the results of analysis using the trajectory plotting system would suggest that the effectiveness of separation shown in Figure 1(a) is greatly exaggerated.
10.2 VARIABLES OF INTEREST

The following variables may be used to define the system appropriate to the two alternative winnowing methods described above:

(i) Tipping the mixture:
   * height of fall.
   * velocity of wind (magnitude).

(ii) Throwing the mixture:
   * height of release.
   * velocity of throw (magnitude).
   * direction of throw in the vertical plane parallel to the wind.
   * velocity of wind (magnitude).

It is assumed that the throw of the particles is in the vertical plane in the direction of the wind; otherwise the TPS cannot be used.

The program is run with a set of input parameters for a single grain and then with the same set for a single chaff particle. The horizontal positions of the particles at ground level relative to the release point are obtained from the program. Negative distances represent a final position 'up wind' of the release point. The success of the process is represented by the separation, the distance between where the grain and the chaff reach the ground – sometimes called the (horizontal) resolution.

10.3 WORKED EXAMPLE

Investigate the effect of varying wind velocity, throw velocity, height of release and angle of throw for a mixture of wheat grain and chaff.

10.3.1 Data

Grain and chaff properties (as per Table III.1, Part III Separation Processes)

| Grain - mass, g | 0.04 | diameter, mm | 4.0 |
| Chaff - mass, g | 0.0035 | diameter, mm | 5.5 |

Velocity of wind, m/s 0, 1, 2, 3, 4, 5
The zero value will give the separation of the particles in still air due to 'segregation'.

Throw velocity of mixture, m/s 0, 1, 2, 3
The zero value represents dropping the mixture.

Height of release of throw, m 1, 2, 3

Angle of throw, degrees 0, 30, 60, 90, 120, 150, 180
The zero and 180 degrees represent a throw horizontally with and against the wind respectively.
10.3.2 Results

Figure 10.2 Trajectories of (a) grain particles and (b) simulated chaff particles for angles of throw shown (degrees). Height of release = 2 m; particle velocity, 2 m/s; air velocity 2 m/s. Similar results were published in Macmillan (1999). See references for details.
Figure 10.3: Further results of similar trajectory plots to those shown in Figure 10.2
10.3.2 Results (continued)

(a) Introduction

See the general discussion of separation processes in Section III above.

(b) Separation in the wind

Figures 10.2 (a) and (b) show typical trajectories for a range of angles of throw in the wind, while Figure 10.3(a) shows the data taken from these and the resulting separation.

(i) Chaff thrown at various angles has a smaller spread (approximately one metre) compared to grain where the spread is approximately 2.5 m.

(ii) The separation is largely determined by the trajectory of chaff because it is more affected by the wind than is the grain, particularly when the mixture is thrown against the wind. When thrown at about 100° all of the separation is due to the movement of the chaff.

Figure 10.3 (b) to (e) shows typical results for various particle velocities, angles of throw, air velocity and heights of throw.

(i) Increasing the particle velocity causes a linear increase in the separation (b) and (c) for higher angles but a decrease at low angles has no effect on the optimum angle of throw (c).

(ii) Increasing angle of throw up to about 150° increases separation for all particles velocities. At small angles (<80°) low particle velocities give greater separation than higher velocities (c) whereas the opposite is true at higher angles of throw.

(iii) Increasing air velocity (d) has a near linear effect on separation for all angles of throw and for dropping particles.

(iv) Increasing height of release (e) also has a near linear increase in separation for all angles of throw. This occurs because throwing the particles effectively increases the height of 'fall'.

(c) Separation without wind

Some degree of separation of grain and chaff (and any particles with different masses and diameters) will occur even when they are projected in still air. This is sometimes called segregation. As shown in Figure 10.3(f) it is clear that the grain with smaller diameter and greater mass moves further than the chaff. This occurs particularly as the height of fall increases. The angle of throw for maximum distance is approximately 20 deg for both particles but there is little difference in the separation for angles from 0 to 20 degrees.
10.3.3 Conclusion

It is clear that the simplest method of increasing the separation is to increase the air velocity and / or throw the mixture against the wind although the effectiveness of this will depend on the constancy of the wind. If a greater wind cannot be easily accessed, for example between buildings, the cheapest, if perhaps the most labour intensive method of increasing the separation, is to increase the height of fall.

10.4 References


CHAPTER 11
TUNNEL WINNOWING

CONTENTS

11.1 INTRODUCTION

11.2 VARIABLES OF INTEREST
(a) Angle of the tunnel to the horizontal (and air-stream)
(b) Depth (and height) of the tunnel
(c) Velocity of air
(d) Velocity of injection of mixture
(e) Angle of injection
(f) Fall after leaving the tunnel
(g) Other variables

11.3 WORKED EXAMPLE
11.3.1 Data
11.3.2 Results
(a) Air velocity
(b) Tunnel depth
(c) Tunnel angle
11.3.3 Conclusions

11.4 WORKED EXAMPLE
11.4.1 Data
11.4.2 Results
(a) Air velocity
(b) Particle injection angle
(c) Particle injection velocity
(d) Tunnel angle
11.4.3 Conclusions

11.5 EXAMPLE

11.6 EXAMPLE

11.7 EXAMPLE

11.8 REFERENCES
11.2

CHAPTER 11
TUNNEL WINNOWING

Winnow not with every wind . . . .
Apocrypha, Ecclesiasticus, 5:9

11.1 INTRODUCTION

In Chapter 10 we considered the process of winnowing in the wind by dropping or throwing the grain - chaff mixture in a natural air stream. While that process is simple and inexpensive (except in time) it can only be effectively used when suitable weather and particularly wind conditions are available. Improved conditions for winnowing could be obtained by taking advantage of higher wind velocity at elevated or exposed sites or by creating such a site by standing on a ladder or similar device. Another means of increasing the wind velocity using an embryonic wind tunnel is to drop the mixture in the air-stream created by two converging walls or partially opened doors as suggested by Handley (1953).

Given the difficulties in relying on fine weather and natural wind for winnowing it is therefore not surprising that mechanical devices were developed to create an artificial wind and so make the process independent of weather conditions. Needham (1965) shows a pottery model of a mechanical rotating fan being used to winnow grain in the time of the Han Dynasty in China (206 BC to 221 AD). There is also evidence that winnowing devices or fanning mills as they were then known (probably following Chinese designs) had reached Europe by the 16th Century (Quick and Buchele (1978)).

Apart from the work by Kashayap, and Pandya (1965 (a) & (b), 1966) there is little analytical or experimental evaluation of wind tunnel concepts although no doubt there has been considerable study of the process for the commercial design harvesting machines.

In designing a 'stand alone' tunnel winnower it is clear that there are many variables open to choice; the TPS program will enable a significant and rapid evaluation of the effect of these variables on the performance of such a unit.

11.2 VARIABLES OF INTEREST

The following variables may be used to define a tunnel winnowing system; see Figure 11.1

(a) Angle of the tunnel to the horizontal (and air stream), \( \theta \)

There is, conceptually at least, no limit on this variable in the range 0 to 90 degrees although for larger values the operation will approach that of a vertical tunnel where the fractions pass out of the ends of the tunnel rather than through the bottom or wall. (See Chapter 12).

(b) Depth (and height) of the tunnel

The depth is the dimension perpendicular to the tunnel centre line. The height will correspond to the depth for a horizontal tunnel but will be equal to depth/cos \( \theta \) for the tunnel at an angle to the horizontal; see Figure 11.1. The depth, together with the width of the tunnel, will define its cross sectional area which, for a given fan capacity, will define the air velocity.

(c) Velocity of air, \( V \)

Values are likely to be related to the materials being separated.
Variables for tunnel winnowing

(d) Velocity of injection of mixture, $U$

Values need to be chosen in relation to the air velocity and the direction of the injection. If desired, the values of injection velocity can be interpreted in terms of the height of fall of the mixture before entering the tunnel. However this will, of course, be limited to the terminal velocity of the different fractions. This raises the question of whether in dropping material into the tunnel this difference in injection velocity of the fractions would be an advantage.

(e) Angle of injection, $\phi$

Conceptually at least, the mixture could be injected or dropped at any angle in the range (for a horizontal tunnel) from 0 (against the air) through 90 degrees (perpendicular to the tunnel) to 180 degrees (with the air).

(f) Fall after leaving the tunnel

A separation is achieved when there is a difference in trajectory between the two fractions (in the horizontal direction within the tunnel). However it may be possible to achieve a greater separation (without increasing the size of the tunnel) if we allow the particles further opportunity to continue this separation process by a free fall outside it.

(g) Other variables

The width of the tunnel is not considered to be a variable at this stage but it will determine the throughput capacity of the tunnel; the wider the tunnel the 'thinner' can be the stream of mixture and the greater is likely to be the tunnel capacity.

Conceptually at least, other variables could also be considered. For example:

(i) increasing or decreasing velocity gradient down the depth of the tunnel.

(ii) increasing or decreasing the velocity along the length of the tunnel (by tapering it).

(iii) injecting the mixture through the bottom of the tunnel as in winnowing in the wind.

(iv) using a curved tunnel which is convex up or concave up. These would, in some respects, correspond to varying of the angle of the tunnel as in (a) above.
The success of the winnowing process may be measured, as with winnowing in the wind, in terms of the separation (resolution) of the fractions. In other situations, if the light fraction passes outside the tunnel, it may be sufficient to use the TPS program to determine the values of the above variables which will bring this about.

11.3 WORKED EXAMPLE

Consider a wind tunnel in which grain and chaff particles are dropped through the top with zero initial velocity. Determine the variation in separation with the parameters given in 11.3.1 below.

11.3.1 Data

Grain and chaff properties (as per Table III.1, Part III Separation Processes.)

<table>
<thead>
<tr>
<th></th>
<th>Grain - mass, g</th>
<th>0.04</th>
<th>diameter, mm</th>
<th>4.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chaff - mass, g</td>
<td>0.0035</td>
<td></td>
<td>diameter, mm</td>
<td>5.5</td>
</tr>
<tr>
<td>Air velocity, m/s</td>
<td>0.5  1.0  1.5  2.0  3.0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Depth of tunnel, m</td>
<td>0.5  1.0  1.5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Angle of tunnel, degrees</td>
<td>0  ±15  ±30  ±45</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: A positive (negative) tunnel angle is an angle of elevation (depression) above (below) the horizontal.

11.3.2 Results

The results taken from the trajectory plots are shown in Figure 11.2 for various conditions.

(a) Air velocity

Here it is seen that separation is almost proportional to air velocity irrespective of the height of the tunnel. The trajectories show that this is mainly due to the increasing movement of the chaff as the air velocity is increased.

(b) Tunnel depth

Again the separation is nearly proportional to the depth for all air velocities, mainly as a result of the increasing movement of the chaff as the depth is increased.

(c) Tunnel angle

Note that here the direction of injection for the particles is vertical for all tunnel angles.

For a tunnel with a constant depth, increasing the tunnel angle above the horizontal gives a small increase in the separation and below the horizontal gives a small decrease.

Increasing the tunnel angle (with a variable depth to keep the height constant) gives a decrease in separation for angles both above or below the horizontal.
Figure 11.2: Variation in separation of grain and chaff dropped into wind tunnel with:
(a) air velocity; tunnel angle = 0 degrees
(b) height of tunnel; tunnel angle = 0
(c) angle of tunnel; air velocity = 1 m/s with (i) constant height = 1 m
and (ii) constant depth = 1 m

11.3.3 Conclusions
The separation of a grain – chaff mixture dropped into a wind tunnel is largely determined
by the tunnel height and air velocity. There is a slight benefit if the tunnel with a given
depth is operated above the horizontal.
11.4 WORKED EXAMPLE

11.4.1 Data

Consider a wind tunnel in which grain and chaff particles are now injected through the top with various initial velocities. Determine the variation in separation with a selection of the following parameters.

Use the following additional data

<table>
<thead>
<tr>
<th>Particle injection angle, degrees</th>
<th>-45, -30, -15, 0, 15, 30, 45</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air velocity, m/s</td>
<td>0.5, 1.0, 2.0, 3.0, 4.0, 5.0</td>
</tr>
<tr>
<td>Tunnel angle, degrees</td>
<td>0, 15, 30, 45, 60</td>
</tr>
<tr>
<td>Depth of tunnel, m</td>
<td>0.5, 1.0, 1.5</td>
</tr>
<tr>
<td>Fall after tunnel, m</td>
<td>0.5, 1.0, 1.5</td>
</tr>
</tbody>
</table>

11.4.2 Results

Figure 11.3: Variation in separation of grain and chaff projected into a wind tunnel with:

- (a) air velocity; height = 1 m, tunnel angle = 0 deg.
- (b) Particle injection angle; height = 1 m, tunnel angle = 0 deg.
- (c) Particle injection velocity; height = 1 m, tunnel angle = 0 deg.
- (d) Tunnel angle; height and fall = 1 m, particle velocity = 1 m/s
11.4.2 Results (continued)

The results taken from the trajectory plots are shown in Figure 11.3 for various conditions. Note: A positive (negative) angle is an angle of elevation (depression) above (below) the horizontal.

(a) Air velocity

Increasing the air velocity increases the separation somewhat more than proportionally for angles both above and below the horizontal. This is mainly due to the increasing movement of the chaff as the air velocity is increased.

(b) Particle injection angle

Note: A positive (negative) angle is an angle measured anti-clockwise (clockwise) of the vertical. These are, respectively, with (against) the wind.

Projecting the particles downward but against the air has a small positive effect on separation (compared to with the air) for all air velocities. This is similar to, but much smaller than, the effect of injection angle when particles are projected in the upwards direction as shown in Figure 10.4(b). In the latter they would be in the air-stream for a longer period of time.

(c) Particle injection velocity

This had a negative effect on separation, confirming the suggestion by Kashayap and Pandya (1965) that the time during which the particles are subject to the air movement is the main determinant of separation.

(d) Tunnel angle

Note: A positive (negative) tunnel angle is an angle of elevation (depression) above (below) the horizontal.

For a constant depth of tunnel (corresponding to an increasing height) separation is almost constant with an increase in tunnel angle above the horizontal. Introducing a 'depth' of free fall (equal to the depth of tunnel) after the particles leave the tunnel gives a significant increase on separation. This is dependent on the horizontal velocity with which the particles leave the tunnel, hence on the air velocity. The decreasing effect of increasing the angle and hence depth occurs because the particles end falling in a near vertical direction.

11.4.3 Conclusions

The separation of a grain – chaff mixture injected into a wind tunnel is largely determined by the air velocity. There is a slight benefit if the tunnel is operated above the horizontal and the particles are projected against the air velocity.

The conclusion that we can draw from Figures (b), (c), (d) and height of the tunnel (not shown) is that the separation is also dependent (for a given air velocity) on the time which is available for the air to act on the particles and so cause a differential movement between them. This conclusion is consistent with the results of Kashayap and Pandya (1965, 1966).

Arrangements which increase the time for which the particles are in the air-stream are likely to increase the separation.
11.5 Example

The results presented above suggest that use of a free fall of the mixture before it enters a tunnel, which would allow the chaff to enter it with a lower injection velocity than the grain, would promote even greater separation between the fractions. Evaluate this concept. (Kashayap and Pandya (1965, 1966)).

11.6 Example

In the light of the results in Chapter 10, compare the separation in a tunnel with the mixture injected upward through the floor with that obtained by dropping the fractions through the roof. Consider the effect of air velocity, and tunnel angle as well as injection angle.

11.7 Example

For various separation processes test the thesis (idea) that separation (for a given air velocity) is significantly dependent on the time for which the particles are in the air stream. Use the x-t plotting feature of the TPS where appropriate.

11.8 References


Quick, G.R. & Buchele, W.F. The Grain Harvesters. (American Society of Agricultural Engineers, St Joseph, Michigan (1978)).
CHAPTER 12
COLUMN WINNOWING

CONTENTS

12.1 INTRODUCTION 12.1
12.2 ANALYSIS 12.2
12.2.1 Variables of interest
(a) Air velocity
(b) Velocity of injection of material into the column
(c) Angle of injection of material into the column
(d) Column width (in the plane of the trajectory)
(e) Other variables

12.3 WORKED EXAMPLE 12.3
12.3.1 Data
12.3.2 Results
(a) Air velocity
(b) Injection velocity and angle
12.3.3 Conclusions

12.4 REFERENCES 12.4
CHAPTER 12
COLUMN WINNOWING

12.1 INTRODUCTION

In Chapter 11 we considered winnowing in a horizontal and moderately sloping tunnel in which it was envisaged that the heavy particles, at least, left the tunnel through the floor. With appropriate choice of air velocity and tunnel length it is possible that the light fraction could be blown out of the tunnel with the discharging air. It could then pass into a device such as a cyclone to remove it from the air or directly to the atmosphere.

The above is essentially a two dimensional process. However, as discussed in Chapter 3, it also is possible to winnow in what could be considered as a one dimensional process in a vertical or near vertical tunnel or column. (Hassebrauck (1964), Farran and Macmillan (1979)).

Superficially, vertical winnowing is a somewhat simpler concept than horizontal and essentially relies on the fact that the light fraction has a higher drag force relative to it weight than does the heavy fraction. If the air velocity is appropriately chosen, the light fraction will move with and be discharged with the rising air stream while the heavy fraction will fall through the latter and be discharged at the base of the column.

The success of the process involves three aspects.

(i) The air velocity that will lift the light fraction but not the heavy. This matter was discussed in general terms in the introduction to Section III. The main problem here is the variability in the dimensions and other properties of the fractions, factors which are usually related to the attitude of the particles as the move in the air stream. In the simple theoretical analysis it was necessary to assume symmetrical particles.

(ii) The method of introducing the mixture into the air stream. The method of introduction itself modifies the air stream and so the process is by no means 'simple'.

(iii) The presence of the mass of particles. As discussed in Chapters 2 and 3 this complicates the process and will not be considered further here.

The trajectory plotting program can be used to study the above parameters in a winnowing process. 'Separation' as defined and used in tunnel winnowing cannot be used in this situation where the light fraction passes outside the tunnel. However Farran and Macmillan (1979) used a computer program similar to the TPS to calculate the particle trajectories and suggested that the optimum are those which will cause the light fraction to pass up the centre of the column and the heavy fraction to pass across the full width of the column. This would expose both fractions to the air stream and allow any light particles which are 'trapped' in the heavy the maximum chance of being lifted in the air stream.

These conditions are adopted in the following to illustrate the use of the TPS.
12.2 ANALYSIS

12.2.1 Variables of interest

The following variables may be used to define the column winnowing system:

(a) Air velocity

This value needs to be chosen relative to the terminal velocities of the fractions. In general a single value greater than the terminal velocity of the lighter and less than that of the heavier fraction is used. In practice a significantly greater value might be necessary if all of the light fraction is to be lifted (See Section 3.5.2).

(b) Velocity of injection of material into the column

Intuition suggests, and previous work confirms, that this will be an important factor in achieving the optimum conditions and will involve a significant interaction with the air velocity.

(c) Angle of injection of material into the column

This variable is related to the injection velocity and will also involve a significant interaction with the air velocity.

(d) Column width (in the plane of the trajectory)

If the column width is chosen independently then the injection conditions (b) and (c) should be such as to achieve the optimum described above, i.e., the light fraction to pass up the centre of the tunnel and the heavy fraction to pass across its full width.

(e) Other variables

The depth of the tunnel (perpendicular to the trajectory) is not considered to be a variable at this stage but it will determine the through-put capacity of the tunnel; the wider the tunnel the 'thinner' can be the stream of mixture and the greater is likely to be the tunnel capacity.

Conceptually at least, other variables could be considered. For example:

(i) increasing or decreasing velocity along the height of the column by tapering it.
(ii) forming a velocity gradient across the width of the tunnel.

For the purposes of evaluating column winnowing concepts the following properties of wheat grain and chaff are used (Farran and Macmillan(1979)).

In the following the TPS program is used to determine the values of the above variables which will optimise the process in terms given above.
12.3 WORKED EXAMPLE

Consider a vertical wind column in which grain and chaff particles are injected through a hole on the side. Using the parameters given in 12.3.1 below, determine the trajectories for chaff and grain and parameters which will cause the chaff to pass up the centre of the column.

12.3.1 Data

Grain and chaff properties (as per Table III.1, Part III Separation Processes.)

<table>
<thead>
<tr>
<th></th>
<th>Grain - mass, g</th>
<th>0.04</th>
<th>diameter, mm</th>
<th>4.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chaff - mass, g</td>
<td>0.0035</td>
<td></td>
<td>diameter, mm</td>
<td>5.5</td>
</tr>
</tbody>
</table>

| Air velocity, m/s | 2.5 |
| Width of tunnel, m | 0.5 |
| Injection velocity, m/s | 0.4  0.6  0.8  1.0  1.2  1.4 |
| Injection angle of particles, degrees | -60  -30  0   30  60 |
| End point for grain trajectory, m | 1.0 |

Note: For convenience the depression / elevation angle are plotted negative / positive respectively in Figures 12.2 and 12.3. In TPS all angles are specified in an anti-clockwise direction with reference to the positive x axis.
12.3.2 Results

The results, taken from the trajectory plots for various conditions, are shown in Figures 12.2 and 12.3.

(a) Air velocity

In this analysis the only requirement is that the air velocity should be greater than the terminal velocity of the chaff. Greater values will require greater injection velocities and shorten the time that the materials are in the air-stream – presumably an undesirable outcome.

(b) Injection velocity and angle

For a given air velocity the injection velocity and angle can be chosen to achieve the optimum conditions given above. Figure 12.2 shows the results taken from trajectories for grain and chaff injected into a column with air velocity = 2.5 m/s. The data are appropriate for small tunnels up to about 0.5 m wide. The range is linearly related to the injection velocity and has a maximum value for angles of about 10 to 15 degrees upward (with the air).

The plot injection velocity versus angle (Figure 12.3) shows the values of these variables which will give a chosen constant range for the chaff and, separately, for grain.

12.3.3 Conclusions

The results show that there are various combinations of injection velocity and angle that will give the conditions for optimum separation of a grain – chaff mixture injected into a wind column. There is a slight benefit if the mixture is injected some 10 to 15 degrees above the horizontal.

12.4 References


12.3.2 Results (continued)

Figure 12.2: Variation in range for particles injected into a vertical column; air velocity = 2.5 m/s. Grain range at 1.0 m below injection point; (a) & (b) chaff; (c) & (d) grain.

Figure 12.3 Lines of equal trajectory range for particles injected into a vertical column; air velocity = 2.5 m/s. Grain range at 1.0 m below injection point; (a) chaff and (b) grain.
PART IV

CHAPTERS 14 - 16

MISCELLANEOUS

All matter is comprised of four elements of water, fire, earth and air.

Greek philosopher, (Fifth Century B.C)
# CHAPTER 14

## RAIN

<table>
<thead>
<tr>
<th>CONTENTS</th>
<th>14.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>14.1 INTRODUCTION</td>
<td>14.2</td>
</tr>
<tr>
<td>14.2 BEHAVIOUR OF LARGE DROPS</td>
<td>14.2</td>
</tr>
<tr>
<td>14.3 WORKED EXAMPLE</td>
<td>14.3</td>
</tr>
<tr>
<td>14.3.1 Introduction</td>
<td></td>
</tr>
<tr>
<td>14.3.2 Data</td>
<td></td>
</tr>
<tr>
<td>14.3.3 Results</td>
<td></td>
</tr>
<tr>
<td>14.4 REFERENCE</td>
<td>14.4</td>
</tr>
</tbody>
</table>
CHAPTER 14
RAIN

God . . . . made a decree for the rain . . .
The Bible, Job 28:26

14.1 INTRODUCTION

As noted in Chapter 3, the determination of the terminal velocity of rain-drops is relevant to the calculation of the erosion caused by rain and also in the design of rainfall simulators that are used for erosion and other studies.

However TPS cannot be applied directly to the determination of the terminal velocity of such drops because, while smaller drops behave as spheres (due to the high surface tension which exists in them), successively larger drops (particularly over about 2 mm) become flattened and are therefore less spherical.1

The default drag coefficient – Reynolds number relationship used in TPS is that for spheres. However the program has the facility for the user to store other drag coefficient – Reynolds number relationships for future use. These can be determined from experimentally measured terminal velocities as in the following example.

14.2 BEHAVIOUR OF LARGE DROPS

As noted above, large drops falling at their terminal velocities are not spherical but become flattened and present an effective diameter2 to the air that is larger than their equivalent diameter3. Hence, for the same equivalent diameter (and mass), such a rain-drop has a lower terminal velocity than a spherical one. Further, as the equivalent diameter of a drop increases the effective diameter increases at a faster rate (becoming less spherical and more flattened) and hence the terminal velocity tends to a constant value.

As the equivalent and effective diameters increase still further the drop becomes unstable; the effect of the drag forces overcomes the surface tension and the drop breaks up into a number of smaller droplets. This aspect is not of interest here.

---

1 They certainly do not behave as frequently depicted with a streamlined shape.
2 Effective diameter is the diameter of a particle which would give the same terminal velocity as the actual particle.
3 Equivalent diameter is the diameter of a particle which has the same volume as the actual particle.
14.3 WORKED EXAMPLE

14.3.1 Introduction

Compare the terminal velocity of drops as calculated by TPS with published data and develop a drag coefficient – Reynolds number relationship appropriate for using TPS to calculate the terminal velocity of drops directly.

14.3.2 Data

Figure 14.1 shows the terminal velocity of water drops plotted against their equivalent diameter. The terminal velocity of drops as measured by Gunn and Kinzer (1949) has been replotted (G & K) and shows a maximum velocity of about 9 m/s. Also plotted is the terminal velocity of spherical drops as determined by the TPS using the drag coefficient – Reynolds number relationship for spheres. The divergence of the two graphs occurs for the reasons given above.

We can use Gunn and Kinzer's data to calculate the drag coefficients and Reynolds numbers since both use the terminal velocity.

This can be done using a transformed version of Equation 3.2 and Equation 2.3.

\[ U_t^2 = \frac{4d_g g}{3C_t} \frac{\rho_b - \rho_f}{\rho_f} \]  
\[ R = \frac{\rho_f Vd_b}{\mu} \]  

\[ \text{Figure 14.1: Terminal velocity versus equivalent diameter of water drops. TPS: as calculated for spherical drops. G & K: measured by dropping in still air. Plotted and printed, with permission, from Gun and Kinzer, (1949). See references for details.} \]
Therefore

\[ C_t = \frac{\Delta d \rho g \rho_p - \rho_i}{3 U_t^2 \rho_i} \]  \hspace{1cm} (14.1)

\[ R_t = \frac{\rho_i U_t d_p}{\mu} \]  \hspace{1cm} (14.2)

The data for particles of different equivalent diameters \( d_p \) are shown in Table 14.1 and plotted for two different Reynolds number ranges in Figure 14.2.

**14.3.3 Results**

The behaviour of falling drops described above in relation to Figure 14.2 is illustrated by the drag coefficient – Reynolds number relationship.

(i) Small drops (less than about 2 mm) behave as spheres and the graphs are virtually identical. Hence in Figure 14.2(a) the line for drops has, for clarity purposes, been omitted.

(ii) For larger drops (greater than about 2 mm) the divergence of their drag coefficient graph from that for spheres illustrates the behaviour described in analytical terms in 14.2 above and shown in empirical terms in Figure 14.1.

These larger drops begins to behave in a similar way to the 'rounded body' as shown in Figure 2.3.

**14.3.4 Conclusions**

The behaviour of falling water drops depends on their size; they become less like spheres and more like 'rounded bodies' as their size increases above about 2 mm. To allow for this it is possible to load (and store) a drag coefficient – Reynolds number table for water drops into the TPS program and so determine the terminal velocity directly from the results data.

The same technique can be used for other non-spherical particles if their drag coefficient – Reynolds number relationship can be determined. However, as discussed in Section 2.8 above, this may be difficult since, for a given type of particle, eg, a variety of seed, it is not easy to obtain the drag coefficient data for a range of Reynolds numbers. It can be achieved for drops because a range of sizes (diameters) can be defined and the resulting terminal velocities measured.

**14.4 Reference**

Table 14.1: Data for determination of drag coefficient – Reynolds number relationship for water drops. Data for spheres

<table>
<thead>
<tr>
<th>Equivalent Diameter (mm)</th>
<th>Mass (gram)</th>
<th>Water Drops</th>
<th>Spheres</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Ut (m/s)</td>
<td>C</td>
<td>R</td>
</tr>
<tr>
<td>0.20</td>
<td>0.72</td>
<td>9.6</td>
<td>4.205</td>
</tr>
<tr>
<td>0.25</td>
<td>0.95</td>
<td>15.8</td>
<td>3.019</td>
</tr>
<tr>
<td>0.30</td>
<td>1.17</td>
<td>23.4</td>
<td>2.389</td>
</tr>
<tr>
<td>0.35</td>
<td>1.40</td>
<td>32.7</td>
<td>1.946</td>
</tr>
<tr>
<td>0.4</td>
<td>1.62</td>
<td>43.2</td>
<td>1.661</td>
</tr>
<tr>
<td>0.5</td>
<td>2.06</td>
<td>68.7</td>
<td>1.284</td>
</tr>
<tr>
<td>0.6</td>
<td>2.47</td>
<td>98.8</td>
<td>1.072</td>
</tr>
<tr>
<td>0.7</td>
<td>2.87</td>
<td>133.9</td>
<td>0.926</td>
</tr>
<tr>
<td>0.8</td>
<td>3.27</td>
<td>174.4</td>
<td>0.815</td>
</tr>
<tr>
<td>0.9</td>
<td>3.67</td>
<td>220.2</td>
<td>0.728</td>
</tr>
<tr>
<td>1.0</td>
<td>4.03</td>
<td>268.7</td>
<td>0.671</td>
</tr>
<tr>
<td>1.5</td>
<td>5.43</td>
<td>543.0</td>
<td>0.555</td>
</tr>
<tr>
<td>2.0</td>
<td>6.49</td>
<td>865.3</td>
<td>0.518</td>
</tr>
<tr>
<td>2.5</td>
<td>7.42</td>
<td>1237</td>
<td>0.495</td>
</tr>
<tr>
<td>3.0</td>
<td>8.06</td>
<td>1612</td>
<td>0.503</td>
</tr>
<tr>
<td>3.5</td>
<td>8.51</td>
<td>1986</td>
<td>0.527</td>
</tr>
<tr>
<td>4.0</td>
<td>8.83</td>
<td>2355</td>
<td>0.559</td>
</tr>
<tr>
<td>4.5</td>
<td>9.05</td>
<td>2715</td>
<td>0.599</td>
</tr>
<tr>
<td>5.0</td>
<td>9.09</td>
<td>3030</td>
<td>0.660</td>
</tr>
<tr>
<td>5.5</td>
<td>9.15</td>
<td>3355</td>
<td>0.716</td>
</tr>
<tr>
<td>6.0</td>
<td>9.18</td>
<td>3672</td>
<td>0.776</td>
</tr>
</tbody>
</table>

Figure 14.2: Drag coefficient – Reynolds number relationship for water drops and spheres of different equivalent diameters; in (a) no line shown for drops. Water drops plotted with permission from Gunn and Kinzer,(1949). See references for details.
CHAPTER 15
FIRE!

CONTENTS

15.1 INTRODUCTION 15.2

15.2 BEHAVIOUR OF BURNING EMBERS 15.2
15.2.1 Plume 15.2
15.2.2 Ambient wind

15.3 WORKED EXAMPLE 15.4
15.3.1 Introduction
15.3.2 Data
15.3.3 Calculations
   (a) Velocities
   (b) Mass and area loss
15.3.4 Results
15.3.5 Conclusions

15.4 REFERENCES 15.7
CHAPTER 15

FIRE!

Behold, how great a matter a little fire kindleth!
The Bible: James 3:5

15.1 INTRODUCTION

Australia has large areas of Eucalypt species that are particularly subject to bush fires (or wild fires) which cause great damage to life and property. Such fires, when burning under the influence of a strong wind, create their own 'fluid – particle' system. The burning trees and 'under-story' material generates great heat and this causes a rising plume of hot air and gases which may have a velocity of 30 m/s and rise several hundred metres.

The rising gas and air plume may entrain and lift embers (sometimes called fire-brands) which may continue to burn for several minutes. As the air rises in the plume or convection column the influence of the fire decreases and that of the following ambient wind increases. The embers are blown forward, perhaps many kilometres, during which time they gradually fall back to the earth. If still burning they may start a new 'spot' fire far ahead of the main blaze. This cycle of fire may move forward at great speed and consume anything burnable in its path.

Various approaches have been used in the study of this system including work based on an approximate analytical solution of the equations of motion by Tarifa et al (1965) and a more empirical approach, based on the measured terminal velocity of burning embers by Ellis (2000). However the action of the fluid stream and entrained embers form a fluid - particle system to which a model based on predictive equations discussed in Chapter 4 and used in the TPS could be applied. The complication in the study of burning embers is that, as the embers are consumed, their mass and size are reduced.

For this reason it is not therefore possible to apply the present form of TPS directly but, for the purposes of illustrating its potential, it has been applied in a stepwise fashion in the following example; in each step it is assumed that the mass and size are constant. These steps can be as small as desired although the errors in the associated drag coefficient – Reynolds number data that are available at this stage would not warrant very small steps.

This material is therefore included here, not to provide a working system nor to validate published results but rather to indicate its usefulness and prompt the development of a version that allows for variable mass and size.

15.2 BEHAVIOUR OF BURNING EMBERS

The system which gives rise to spot fires has, for the purposes of analysis, two stages:

15.2.1 Plume

This plume of rising air and gas, which is driven by the fire, lifts the embers to their maximum height at which time they are assumed to pass out of the plume and enter the airstream (wind) which is usually assumed to be horizontal.
Figure 15.1: Trajectory of ember in plume and ambient wind stages. 
(Adapted from Tarfia, et. al. (1965); see references for details)

Tarfia et al (1965) identify two forms of plume, one vertical, the other inclined forward at an angle as shown in Figure 15.1 and used in the following analysis. The angle is determined by the velocity of air in the plume (relative to the earth) $V_{pe}$ and the horizontal wind velocity relative to the earth, $V_{we}$. Within the plume the embers are assumed to start at an initial height above the ground and to be carried upward and down-wind under the action of the total velocity of the air relative to the earth, $V_{ae}$.

### 15.2.2 Ambient wind

After the embers leave the plume under the increasing influence of the ambient wind they are blown forward and gradually fall back to earth. If they are still burning this will occur as their mass and size is further reduced. Whether they start a spot fire when they reach the earth will of course depend on their condition at that time and on the presence of appropriate fuel where they land.

The trajectories in both the above stages will depend on the magnitude of the wind and its variation with height. If this velocity profile is known it can be allowed for in the step-wise analysis by varying not only the mass and size of the ember but the wind velocity for each step.
15.3 WORKED EXAMPLE

15.3.1 Introduction
Consider a fire for which the following data apply:

15.3.2 Data

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial mass of ember, gram</td>
<td>2.0</td>
</tr>
<tr>
<td>Rates of loss in mass, gm/sec</td>
<td>(i) 0.036, (ii) 0.0036</td>
</tr>
<tr>
<td>Rate of loss of area, mm²/sec</td>
<td>1000 x mass loss rate</td>
</tr>
<tr>
<td>Equivalent diameter of ember, mm</td>
<td>50</td>
</tr>
<tr>
<td>Vertical air velocity in plume, Vₚₑ, m/s</td>
<td>(i) 16.8, (ii) 11.2</td>
</tr>
<tr>
<td>Ambient wind velocity, Vₖₑ, m/s</td>
<td>(i) 6.75, (ii) 4.5</td>
</tr>
<tr>
<td>Initial height of ember, m</td>
<td>25</td>
</tr>
<tr>
<td>Temperature of air in plume, °C</td>
<td>300</td>
</tr>
<tr>
<td>Density of air in plume, kg/m³</td>
<td>0.62</td>
</tr>
<tr>
<td>Viscosity of air in plume, mPa.s</td>
<td>0.028</td>
</tr>
<tr>
<td>Temperature of ambient air in the wind, °C</td>
<td>40</td>
</tr>
<tr>
<td>Density of ambient air, kg/m³</td>
<td>1.15</td>
</tr>
<tr>
<td>Viscosity of ambient air, mPa.s</td>
<td>0.019</td>
</tr>
</tbody>
</table>

It is assumed that:
(i) the drag coefficient - Reynolds number relationship for rectangular plates apply (see Figure 2.3).
(ii) the steps are defined by the vertical interval - to be chosen at each stage to suit the level of interest.

15.3.3 Calculations

(a) Velocities

The total velocity of air in the plume Vₑ is the vector sum of the vertical air velocity in the plume Vₚₑ and the horizontal wind velocity Vₖₑ, all relative to the earth.

\[ Vₑ = \sqrt{Vₚₑ² + Vₖₑ²} \]
\[ = \sqrt{16.8² + 6.75²} \]
\[ = 18.1 \text{ m/s} \]

The angle of the air velocity in the plume,

\[ \theta = \tan \left( \frac{Vₚₑ}{Vₖₑ} \right) \]
\[ = \frac{6.75}{16.8} \]
\[ = 68 \text{ degrees} \]

(b) Mass and area loss

Because the program assumes constant area and mass it must be used in a stepwise fashion over short steps where it can be assumed, without serious error, that the mass and area are constant. These steps can be short or long depending on the level of interest and the rate at which the variables are changing.
Using this method, the input data for each step in the calculation is the end data for the previous one.

For example (i) above the mass loss rate = 0.036 gram/second

Hence mass for a step = Mass for previous step - 0.036 x time for previous step

Area loss rate = 1000 x mass loss rate
= 1000 x 0.036
= 36 mm²/sec

Hence the area for a step = Area for previous step - 36 x time for previous step

Since the input size parameter for the TPS program is the equivalent diameter (as a measure of area)

\[ \text{Diameter for this step: } = \sqrt{\frac{4}{\pi}} \text{ Area of step} \]

However for the purposes of defining the drag coefficient - Reynolds number relationship the embers are assumed to be flat discs with values shown in Table 15.1 (Schiller, 1932).

<table>
<thead>
<tr>
<th>DATA FOR TRAJECTORY OF BURNING EMBERS</th>
<th>DRAG COEFF - REYNOLDS NO. DISCS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ember</td>
<td>Figure 15.2 (a)</td>
</tr>
<tr>
<td>Ember</td>
<td>Initial mass, g</td>
</tr>
<tr>
<td>Mass burn rate, g/sec</td>
<td>0.036</td>
</tr>
<tr>
<td>Initial diameter, mm</td>
<td>50</td>
</tr>
<tr>
<td>Initial area, mm²</td>
<td>1963</td>
</tr>
<tr>
<td>Area burn rate, mm²/sec</td>
<td>36</td>
</tr>
<tr>
<td>Plume</td>
<td>Diameter for this step: = \sqrt{\frac{4}{\pi}} \text{ Area of step}</td>
</tr>
<tr>
<td>Plume</td>
<td>Vertical air velocity, m/s</td>
</tr>
<tr>
<td>Plume</td>
<td>Horiz. air velocity, m/s</td>
</tr>
<tr>
<td>Plume</td>
<td>Total air velocity, m/s</td>
</tr>
<tr>
<td>Plume</td>
<td>Temperature, C</td>
</tr>
<tr>
<td>Plume</td>
<td>Density, kg m⁻³</td>
</tr>
<tr>
<td>Plume</td>
<td>Viscosity, mPa.s</td>
</tr>
<tr>
<td>Plume</td>
<td>Angle, degrees</td>
</tr>
<tr>
<td>Plume</td>
<td>Ambient</td>
</tr>
<tr>
<td>Plume</td>
<td>Horizontal wind velocity, m/s</td>
</tr>
<tr>
<td>Plume</td>
<td>Temperature, C</td>
</tr>
<tr>
<td>Plume</td>
<td>Density, kg m⁻³</td>
</tr>
<tr>
<td>Plume</td>
<td>Viscosity, mPa.s</td>
</tr>
<tr>
<td>Plume</td>
<td></td>
</tr>
<tr>
<td>Plume</td>
<td></td>
</tr>
</tbody>
</table>

**Table 15.1:** Data used for determination of trajectories in Figure 15.2 (a) and (b)
Drag coefficient - Reynolds number data for discs reproduced from Schiller (1932)
Figure 15.2: Trajectories for burning embers
(a) burn loss rate 0.036 g/s; (b) plume velocity 18.1 m/s
15.3.4 Results

Figure 15.2 shows the trajectories for embers lifted in a plume and blown downwind. All show a linear result largely because the wind velocities have been assumed constant with height. The particle velocities (not shown) are virtually constant which confirms the conclusion of Tarifa et. al. (1965) that particles rapidly reach their terminal velocity (in a constant wind) and continue to fall at this value. This occurs notwithstanding the varying mass and area for the conditions chosen in this analysis.

Figure 15.2 (a) shows the significant effect that a 50% increase in plume velocity gives a 100% increase in range for a mass burn rate of 0.036 g/s.

Figure 15.2 (b) shows that a 90% reduction in burn rate has a rather small effect (13%) on range for a plume velocity of 18.1 m/s.

15.3.5 Conclusions

The results have not been validated but suggest that a modified version of the TPS could provide a useful means of predicting the trajectories of burning embers.

It would be desirable that such a program should provide for varying the mass and area during the flight and to allow the use of varying velocity as in the present program.

Again it should be emphasised that this example is meant to illustrate the use of the TPS and does not represent a significant analysis of the problem concerned.

15.4 References


Tarifa, C.S., del Notario, P.P., and Moreno, F.G. (1965) On the flight paths and lifetimes of burning particles of wood. *Tenth Symposium (International) on Combustion*, The Combustion Institute, 1021 – 1037. Figure 15.1 adapted and reprinted, with permission, from this article. Copyright 1965: the Combustion Institute.

Schiller, L (1932) Fallversuche mit kugeln und scheiben (Drop tests with spheres and discs) *Handbuch der Experimental – Physick*: IV, (2); 337 – 387. Data in Table 15.5 reproduced with permission, from this article. Copyright 1932; Akademische Verlagsgesellschap
CHAPTER 16
MINERAL SEDIMENTATION

CONTENTS

16.1 INTRODUCTION

16.2 ANALYSIS

16.2.1 Traditional
16.2.2 Using TPS

16.2.3 Variables of interest
   (a) Slurry velocity
   (b) Particle mass
   (c) Particle diameter
   (d) Slurry density
   (e) Slurry viscosity

16.3 WORKED EXAMPLE

16.3.1 Data
16.3.2 Results
   (a) Particle diameter
   (b) Slurry fluid fraction

16.3.3 Conclusions

16.4 REFERENCES
16.1 INTRODUCTION

One of the versatile and useful aspects of the traditional analysis of fluid - particle systems discussed in Section I above, and used by the TPS, is its ability to be applied to any particles in any fluid (gas or liquid). As an example of this, in this Chapter we consider sedimentation of mineral particles from a slurry mix as it flows through a settling pool. In many ways this is similar in principle to the problem of tunnel winnowing considered in Section III above except that here the lighter fraction leaves the pool with the out-flowing slurry.

A typical arrangement of a pool used in mineral sedimentation is shown as a longitudinal cross-section in a somewhat idealised form in Figure 16.1 adapted from Fitch (1962). Here it is assumed that the slurry enters and leaves through screens and flows with a uniform velocity across the full depth and length of the pool. This pool could also be considered as a segment of the most common sedimentation unit used in mineral processing, the cylindrical continuous thickener. In this system feed enters the thickener through a central feed well and the clarified liquor overflows into a launder (channel) around the periphery. The sludge, which settles to the bottom, is raked by slowly moving rakes into a central discharge point. Further description of such systems can be found in texts such as Kelly and Spottiswood (1982).

It is also assumed that the particles are uniformly distributed in the flow at entry to the pool and then settle at their respective terminal velocities soon after.

Figure 16.1 shows a conceptual view of the distribution of two sizes of particle at three positions along the pool. The limiting size of the heavier particles, shown as ‘ ’ is such that they will just reach the floor of the pond while the lighter ones, shown as ‘ ’ will be removed with the out - flow.

![Figure 16.1 Sedimentation of two particles in an ideal pool showing their distribution at, and smaller than, the limiting size. (Adapted and reprinted with permission from Fitch (1962) ; see references for detail.)](image)
16.2 ANALYSIS

Two methods of analysis are possible.

16.2.1 Traditional

The following variables are used to define the system:

(i) horizontal slurry velocity, V
(ii) length of the pool, L
(iii) depth of the pool H
(iv) terminal velocity of the heavy fraction in the slurry $U_t$

This analysis, given by Kelly and Spottiswood (1982), is as follows. If it is assumed that the heavy fraction, with terminal velocity $U_t$, will just settle out to a depth $H$ in length $L$, then

\[ t = \frac{H}{U_t} \]

and

\[ t = \frac{L}{V} \]

hence

\[ U_t = V \frac{H}{L} \] (16.1)

Thus the limiting (minimum) sized particle (expressed as its terminal velocity) that will settle out is the slurry velocity times the aspect ratio ($H/L$) of the pool. With TPS we can (for a known particle density) calculate or otherwise determine the size (diameter) of particles with this terminal velocity.

16.2.2 Using TPS

If the TPS is used the particle will be defined, not by its terminal velocity, but by the usual parameters given below.

With TPS we can determine the range (horizontal distance for a given pool depth (height)) for various particle sizes and identify the particle which just gives the range equal to the length of the pool. This method, which exploits the use of TPS more directly, will be followed in Worked Example 16.3 below.

However, because the terminal velocity is reached very quickly the difference between the results using these two methods is likely to be small.
16.2.3 Variables of interest

The parameters that are relevant to the process of sedimentation are as follows.

(a) Slurry velocity, \( V \)

This value will need to be chosen relative to the terminal velocities of the fractions.

(b) Particle mass

This will depend on the diameter of the mineral and density (as specific gravity); the latter is usually in the range 2 to 6.

(c) Particle diameter

This will vary with the application but is likely to be in the range of 0.1 to 0.5 mm. In the use of TPS the appropriate drag coefficient – Reynolds number relationship can be used if known; the default relationship is that for spheres which is likely to be accurate for the low Reynolds number experienced by particles settling in a slurry.

(d) Slurry density

This will depend on the density of the fluid but also on the volume proportions of the mineral fractions multiplied by their respective specific gravities (SG).

The appropriate density for a slurry based on water would therefore be:

\[
\text{Slurry density} = 1000 \left[ \text{Fluid volume fraction} + \text{mineral volume fraction (1)} \times \text{SG(1)} + \text{mineral volume fraction (2)} \times \text{SG (2)} + \ldots \right]
\]

where

\[
\text{Fluid volume fraction} + \text{mineral volume fraction (1)} + \text{mineral volume fraction (2)} + \ldots = 1.0
\]

(e) Slurry viscosity

This will also depend on the presence of the mineral particles. Steinour (1944) proposed the use of a viscosity multiplier based on the fluid volume fraction (X).

\[
\text{Slurry viscosity} = \frac{10^{1.82(1-X)}}{X} \times \text{fluid viscosity}
\]

There are likely to be several secondary effects as the sedimentation process proceeds. For example, the concentration of particles will increase down the depth of the pool and hence the density and viscosity of the slurry will not be constant; the following example does not take these secondary effects into account. The reader is therefore referred to texts which give a more detailed treatment of the subject.

16.3 Worked Example

Consider a pool used for the sedimentation of mineral particles of different diameters from water. It is assumed that the slurry moves with a uniform velocity along the pool and that the density and viscosity of the slurry are constant. The particles are assumed to be spherical.

Using the parameters given in 16.1.1 below, determine the lengths of pool which will just allow the various particles to settle out.
16.3.1 Data

<table>
<thead>
<tr>
<th>Slurry velocity, m/s</th>
<th>0.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Depth of pool, m</td>
<td>0.5</td>
</tr>
<tr>
<td>Particle diameter, mm</td>
<td>0.1 0.15 0.2 0.3 0.4 0.5</td>
</tr>
<tr>
<td>Particle SG</td>
<td>3.0</td>
</tr>
</tbody>
</table>

The density and viscosity of the slurry, which are calculated as discussed in (iv) and (v) above, are shown plotted in Figure 16.2.

![Figure 16.2: Density factor (a) and viscosity factor (b) for slurry in Worked Example 16.3, as a function of volume fraction of fluid in the slurry](image)

![Figure 16.3: Length of pool required to allow sedimentation of (a) particles of various diameter from slurries with (b) various fluid fractions](image)
16.3.2 Results

The results taken from the trajectory plots are shown in Figure 16.3.

(a) Particle diameter

Figure 16.3 (a) shows that length is inversely related to particle diameter. Because, for particles of these sizes, terminal velocity is linearly related to particle diameter, length of pool will also be inversely related to terminal velocity. Equation 16.1 would also confirm this relationship for a constant V and H.

(b) Slurry fluid fraction

Figure 16.3 (b) shows that length is inversely related to fluid fraction in the slurry and is very significant for smaller particles. Slurry density and viscosity are each inversely related to fluid fraction (See Figure 16.2) and hence length will be directly related to these variables.

16.3.3 Conclusions

The required length of the pool to achieve sedimentation is inversely related to particle diameter for a given fluid fraction and to the fluid fraction, particularly for small diameter particles.

It should be understood that the above analysis represents a very limited part of the topic of mineral sedimentation. Books such as Kelly and Spottiswood (1982) or later texts should also be consulted for further details.

16.4 References

Fitch, B., (1962) Why particles separate in sedimentation processes *Industrial and Engineering Chemistry* 54 (10), 44 – 51. Figure 16.1 is redrawn and reprinted, with permission, from this article. Copyright, 1962, American Chemical Society


The assistance of Dr. Donald F. Stewart during the preparation of this Chapter is gratefully acknowledged.
APPENDICIES

APPENDIX I   List of symbols
APPENDIX II   Two dimensional analysis of a trajectory
APPENDIX III  Physical properties of air and water
APPENDIX IV  References
APPENDIX V   Other on-line publications by the same author
### APPENDIX  I

### LIST OF SYMBOLS

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Explanation</th>
<th>Defining section</th>
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<tbody>
<tr>
<td>A</td>
<td>Frontal or projected area of particle</td>
<td>Eq. 2.1</td>
</tr>
<tr>
<td>C</td>
<td>Drag coefficient</td>
<td>Eq. 2.2</td>
</tr>
<tr>
<td>D</td>
<td>Drag force</td>
<td>Eq. 2.1, 2.2</td>
</tr>
<tr>
<td>F</td>
<td>Force in Stokes Law</td>
<td>Sect. 2.6.1</td>
</tr>
<tr>
<td>J</td>
<td>Velocity of drop relative to the machine</td>
<td>Sect. 7.4.3</td>
</tr>
<tr>
<td>H</td>
<td>Velocity of machine relative to the earth</td>
<td>Sect. 7.4.3</td>
</tr>
<tr>
<td>L</td>
<td>Length of pool</td>
<td>Sect. 16.2.1</td>
</tr>
<tr>
<td>R</td>
<td>Reynolds number</td>
<td>Eq. 2.3</td>
</tr>
<tr>
<td>S</td>
<td>Distance / range / spread</td>
<td>Eq. 3.7</td>
</tr>
<tr>
<td>V</td>
<td>Velocity of fluid relative to the earth</td>
<td>Fig. 4.1</td>
</tr>
<tr>
<td>W</td>
<td>Velocity of particle relative to the fluid</td>
<td>Fig. 4.1</td>
</tr>
<tr>
<td>X</td>
<td>Fluid volume fraction</td>
<td>Sect. 16.2.2</td>
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<table>
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<td>air</td>
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<tr>
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<td>drop</td>
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<td>plume</td>
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<td>vertical</td>
</tr>
<tr>
<td>w</td>
<td>wind</td>
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</table>
APPENDIX II
TWO DIMENSIONAL ANALYSIS OF A TRAJECTORY

Figure AII.1: Vector diagram for determination of particle velocity relative to the fluid using Equation AII.1 \( W_{pf} = U_{pe} - V_{fe} \)

AII.1 EQUATIONS OF MOTION

As shown in Figure 4.1 we can write the separate equations of motion in the horizontal and vertical directions. Thus:

\[
m \frac{dU_h}{dt} = -D \cos \alpha \quad (\text{AII.1(a)})
\]

\[
m \frac{dU_v}{dt} = mg \frac{\rho_f - \rho_p}{\rho_f} - D \sin \alpha \quad (\text{AII.1(b)})
\]

---

\(^1\) Equations AII.1 to AII.3 reproduced with permission from Lapple and Shepherd (1940). See references for details.
From Equation 2.2
\[ D = CA \frac{\rho_i W^2}{2} \]  
(AII.2)

where the drag force \( D \) is based on the velocity \( W_{pf} \) of the particle relative to the fluid.

\[
\cos \alpha = \frac{W_h}{W} = \frac{U \cos \phi - V \cos \theta}{W} = \frac{U_h - V_h}{W} \\
\sin \alpha = \frac{W_v}{W} = \frac{U \sin \phi - V \sin \theta}{W} = \frac{U_v - V_v}{W}
\]

Hence
\[
\frac{dU_h}{dt} = -CA \frac{\rho_i}{2m} W(U_h - V_h) 
\]  
(AII.3(a))

\[
\frac{dU_v}{dt} = g \frac{\rho_i - \rho_f}{\rho_f} - CA \frac{\rho_i}{2m} W(U_v - V_v) 
\]  
(AII.3(b))

Where
\[
W = \sqrt{W_h^2 + W_v^2} \\
= \sqrt{(U_h - V_h)^2 + (U_v - V_v)^2} \\
= \sqrt{U_h^2 + U_v^2 + V_h^2 + V_v^2 - 2U_h V_h - 2U_v V_v} \\
= \sqrt{U^2 + V^2 - 2(U \cos \phi V \cos \theta - U \sin \phi V \sin \theta)} \\
= \sqrt{U^2 + V^2 - 2UV(\cos \phi \cos \theta - \sin \phi \sin \theta)} \\
= \sqrt{U^2 + V^2 - 2UV(\cos \phi - \theta)} 
\]  
(AII.4)

We can write Equations 4.4 in incremental form:
\[
\Delta U_h = \left[ -CA \frac{\rho_i}{2m} W(U_h - V_h) \right] \Delta t 
\]  
(AII.5(a))

\[
\Delta U_v = \left[ g \frac{\rho_i - \rho_f}{\rho_f} - CA \frac{\rho_i}{2m} W(U_v - V_v) \right] \Delta t 
\]  
(AII.5(b))
By making $\Delta t$ small, we can also write:

$$U_h(n + 1) = U_h(n) + \Delta U_h(n + 1)$$

$$U_v(n + 1) = U_v(n) + \Delta U_v(n + 1)$$

The mean velocities over the interval $\Delta t$ in the horizontal and vertical directions are then:

$$U_h(n + 1) = \frac{U_h(n + 1) + \Delta U_h(n)}{2}$$

$$= \frac{2U_h(n) + \Delta U_h(n + 1)}{2}$$

$$= U_h(n) + \frac{\Delta U_h(n + 1)}{2}$$

And

$$U_v(n + 1) = \frac{U_v(n + 1) + \Delta U_v(n)}{2}$$

$$= \frac{2U_v(n) + \Delta U_v(n + 1)}{2}$$

$$= U_v(n) + \frac{\Delta U_v(n + 1)}{2}$$

And the increments in the trajectory are:

$$\Delta S_h(n + 1) = \left[U_h(n) + \frac{\Delta U_h(n + 1)}{2}\right]\Delta t \quad (AII.6(a))$$

$$\Delta S_v(n + 1) = \left[U_v(n) + \frac{\Delta U_v(n + 1)}{2}\right]\Delta t \quad (AII.6(b))$$
### II.2 Calculating the Velocity and Position of a Particle

The initial conditions at n=0 are, \( U(0) \), \( f(0) \), \( V \), \( q \), where:

\[
U_h(0) = V \cos \phi(0)
\]

\[
U_v(0) = V \sin \phi(0)
\]

\[
W(0) = \sqrt{U(0)^2 + V^2 - 2U(0)V \cos(\phi - \theta)}
\]

The initial particle position is \( S(0) \); also \( S_h(0) = 0 \) and \( S_v(0) = 0 \)

Calculate, in \( \Delta t \) steps, the velocity and the new position of the particle in the horizontal and vertical directions.

The change in velocity in the first \( \Delta t \) interval is:

\[
\Delta U_h(1) = \left[-CAf \frac{\rho_r}{2m} W(0) \{U_h(0) - V \cos \theta\}\right] \Delta t
\]

\[
\Delta U_v(1) = \left[g \frac{\rho_r - \rho_f}{\rho_r} - CA \frac{\rho_r}{2m} W(0) \{U_v(0) - V \sin \theta\}\right] \Delta t
\]

The velocity at the end of the first time interval \( t \) (\( n=1 \Delta t \))

\[
U_h(1) = U_h(0) + \Delta U_h(1)
\]

\[
U_v(1) = U_v(0) + \Delta U_v(1)
\]

The increments in the trajectory over the first time interval are calculated by:

\[
\Delta S_h(1) = \left[U_h(0) + \frac{\Delta U_h(1)}{2}\right] \Delta t
\]

\[
\Delta S_v(1) = \left[U_v(0) + \frac{\Delta U_v(1)}{2}\right] \Delta t
\]

The values for \( U \) and \( W \) after the first time interval are given by:

\[
U(1) = \sqrt{U_h(1)^2 + U_v(1)^2}
\]

\[
W(1) = \sqrt{U(1)^2 + V^2 - 2U(1)V \cos(\phi - \theta)}
\]

The change in velocity during the second interval is:
\[ \Delta U_h(2) = \left[ -CA \frac{\rho \partial_x}{2m} W(1) \{U_h(1) - V \cos \theta \} \right] \Delta t \]

\[ \Delta U_v(2) = \left[ g \frac{\rho \partial_y \partial_z}{\rho_x} - CA \frac{\rho \partial_z}{2m} W(1) \{U_v(1) - V \sin \theta \} \right] \Delta t \]

\[ U_h(2) = U_h(1) + \Delta U_h(2) \]

\[ U_v(2) = U_v(1) + \Delta U_v(2) \]

The increments in the trajectory over the second time interval are calculated by:

\[ \Delta S_h(2) = \left[ U_h(1) + \frac{\Delta U_h(2)}{2} \right] \Delta t \]

\[ \Delta S_v(2) = \left[ U_v(1) + \frac{\Delta U_v(2)}{2} \right] \Delta t \]

The values for \( U \) and \( W \) after the second time interval are given by:

\[ U(2) = \sqrt{U_h(2)^2 + U_v(2)^2} \]

\[ W(2) = \sqrt{U(2)^2 + V^2 - 2U(2) V \cos(\phi - \theta)} \]
APPENDIX III

PHYSICAL PROPERTIES OF AIR AND WATER

Figure AIII.1: Physical properties of air

Figure AIII.2: Physical properties of water
APPENDIX IV

LIST OF REFERENCES


Bilanski, W.K., Collins, S.H. and Chu, P. (1962) Aerodynamic properties of seed grains; their behaviour in free fall. *Agricultural Engineering*: 43(4), 216-219. Figure 3.2 re-plotted and reprinted, with permission, from this article. Copyright, 1962; American Society of Agricultural and Biological Engineers.

Brown, G.G. *et al* *Unit Operations* (J.Wiley and Sons, New York, 1950). Figure 2.6 re-plotted and reprinted from this book. Copyright, the authors.


Fitch, B., (1962) Why particles separate in sedimentation processes. *Industrial and Engineering Chemistry* 54 (10), 44 – 51. Figure 16.1 redrawn and reprinted, with permission, from this article. Copyright, 1962; American Chemical Society.


Handley, J.E. Scottish farming in the 18th Century. (Faber and Faber Ltd., London, 1953).


Hawk, A.L., Brooker, D.B. and Cassidy, J.J. (1966) Aerodynamic characteristics of selected farm grains. *Transactions of the American Society of Agricultural Engineers*: 9(1), 48-51. Figure 2.8 re-plotted, with permission, from this article. Copyright, 1966; *American Society of Agricultural and Biological Engineers*.


Lapple, C.E. and Shepherd, C.B. (1940) Calculation of particle trajectories. *Industrial and Engineering Chemistry*; 32, 605 – 617. Equations 4.2 and 4.3 are reproduced, with permission, from this article. Copyright 1940; American Chemical Society.


Laws, J.O. (1941) Measurements of the fall-velocity of water-drops and raindrops. *Transactions of American Geophysical Union* (Hydrology), 22; 709 – 721. Figure 3.3 replotted and reprinted, with permission, from this article. Copyright, 1941; American Geophysical Union.


Mueller, R.A., Brooker, D.B. and Cassidy, J.J. (1967) Aerodynamic properties of black walnuts: Application in separating good from bad walnuts. *Transactions of the American Society of Agricultural Engineers*:11 (1), 57 – 61. Figure 2.9 replotted and reprinted, with permission, from this article. Copyright, 1967, American Society of Agricultural and Biological Engineers.

Newton, I., Emerson, W. and Machin, J. (1803) The mathematical principles of natural philosophy Book II Section 7, 89.


Pettyjohn, E.S. and Christianson, E.B. (1948) Effect of particle shape on free-settling rates of isometric particles. *Chemical Engineering Progress*: 44(2), 157 – 172. Figure 2.4, reprinted, with permission, using data from this article. Copyright, 1948; American Institute of Chemical Engineers.


Reynolds, O. (1883) An experimental investigation of the circumstances which determine whether the motion of water shall be direct or sinuous and of the law of resistance in parallel channels. *Philosophical Transactions of Royal Society of London* CLXXIV, 935 – 982.

Reints, R.E.(Jr) and Yoerger , R.R. Trajectories of seeds and granular fertilizer. American Society of Agricultural Engineers: 10 (2), 213-216.


Schiller, L (1932) Fallversuche mit kugeln und scheiben (Drop tests with spheres and discs) *Handbuch der Experimental – Physik*: IV, (2); 337 – 387. Figures 2.3 and 2.9 replotted and reprinted and data in Table 15.5 reproduced with permission. from this article. Copyright 1932; Akademische Verlagsgesellschaft.


Tarifa, C.S., del Notario, P.P., and Moreno, F.G. (1965) On the flight paths and lifetimes of burning particles of wood. *Tenth Symposium (International) on Combustion*, The Combustion Institute, 1021 – 1037. Figure 15.1 adapted and reprinted, with permission, from this article. Copyright 1965: The Combustion Institute.


Wadell, H. (1934) The coefficient of resistance as a function of Reynolds Number for solids of various shapes. *Journal of the Franklin Institute*: 459 – 490. Figure 2.6 reprinted, with permission, from this article (as modified by Brown, (1950) which see). Copyright, 1934; Elsevier.

APPENDIX V

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<td></td>
</tr>
<tr>
<td>Locally Made Equipment for Teaching and Research in Agricultural Engineering</td>
<td>Information on the theory, construction, calibration and use of small equipment for teaching and research in agricultural engineering and associated technologies.</td>
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</table>

Note that no English language hard copies of these publications are available.
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Title: The mechanics of fluid - particle systems: with special reference to agriculture

Date: 2007


Publication Status: Unpublished

Persistent Link: http://hdl.handle.net/11343/34073

File Description: The mechanics of fluid - particle systems: with special reference to agriculture

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