"A STUDY OF THE DRILLING PROCESS"

by

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<tr>
<td>$\alpha_f$</td>
<td>Reference rake angle</td>
</tr>
<tr>
<td>$\alpha_{f0}$</td>
<td>Reference rake angle at periphery</td>
</tr>
<tr>
<td>$\alpha_e$</td>
<td>Effective rake angle</td>
</tr>
<tr>
<td>$\alpha_n$</td>
<td>Normal rake angle</td>
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<tr>
<td>$\beta_e$</td>
<td>Effective friction angle</td>
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<td>Relief angle</td>
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<td>Relief angle at periphery</td>
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<td>$\delta$</td>
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<td>$\eta_c$</td>
<td>True chip flow angle</td>
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<td>$\eta_1$, $\eta_2$, $\eta_3$, $\eta_4$</td>
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<td>$\omega$</td>
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<tr>
<td>$\Delta A$</td>
<td>Elemental cross-section area of cut</td>
</tr>
<tr>
<td>$M$</td>
<td>Total torque for full drill</td>
</tr>
<tr>
<td>$N$</td>
<td>Angular velocity of drill</td>
</tr>
</tbody>
</table>
Maximum radius of drill
Inside and outside radius of Annulus
Total thrust for full drill
Angular velocity of chip and workpiece
Elemental cutting forces acting on \( b \)
Total thrust for either cutting edge or chisel edge only
Total torque for either cutting edge or chisel edge only
Elemental width of cut in oblique cutting
Constants
Diameter of drill
Feed rate
Cutting Forces per unit radius
Thrust per unit radius
Tangential force per unit radius
Constants
Elemental length of cut
Constants
Half point angle
Chisel edge length
Elemental radius corresponding to \( \Delta b \)
Radius of drill in question
Half web-thickness
t'  Undeformed chip thickness
U  Specific energy
V_C  Chip velocity
V_F  Feed velocity
V_W  Work velocity
INTRODUCTION

The modern two flute twist drill has been in existence for more than fifty years and the drilling process has become one of the most widely used manufacturing operation. There has hardly been any major alteration in the general appearance of the twist drill despite the various studies on the effect of geometry on drill performance such as tool life and cutting forces. Nevertheless a better understanding of the capabilities and limitations of the twist drill has been made possible by these investigations.

The twist drill generates holes by the cutting action of the lips and the indenting action of the chisel edge. The geometry of the lips has been generalized to that of a lathe tool and the cutting action of the lips was reported as similar to single edged oblique cutting. A cutting model for drilling has not been developed possibly because of the complex geometry of the lips and the extruding action of the chisel edge, which is difficult to study. As a result many empirical equations for the prediction of forces have been proposed.

In this project the geometry of the various size drills will be studied. Analyses of the mechanics of drilling will be attempted in the hope that a cutting model may be constructed based on established principles of orthogonal and oblique cutting. This is desirable since in the design and application of drilling, like in other topics of metal cutting, it is useful to be able to predict the deformation and cutting forces.
SUMMARY

From a survey of drilling it was found that the geometry of the twist drill and consequently its specifications are very complex. Much experimental work has been done in searching for a drill geometry for optimum performance. The "one-variable-at-a-time" approach was usually adopted and it was shown that the value of each of the drill parameter influenced the drill performance in one way or the other. This approach cannot be considered as strictly valid because of the inter-dependence of the various drill angles and elements.

Equations for the fundamental cutting angles in drilling were derived by various workers and the chip flow condition along the cutting lips was studied. Some investigators applied Merchant's thin shear zone model to drilling but their approach was not strictly from first principles. While some means of force prediction was necessary, this was usually done by applying the many empirical equations which were established with little or no consideration given to the geometry of the individual drills.

In this project it was decided to construct a drilling model for predicting forces of the cutting edges. The necessity for such a model can be shown by the presence of the numerous empirical force equations, some of which were established as early as 1900. By simulating some simpler oblique cutting cases to that of drilling, a cutting model was developed for the flat rake drills, whose
cutting efficiency was expected to be higher than those of the standard drills due to the geometric modification to the point geometry. A cutting model for the standard drill was also derived using a similar approach. The predicted thrust using this model was unrealistic and the possible reason for its failure may be due to the lack of understanding of the deformation occurring on curved raked surface.

Due to the failure of the model for standard drills, the basic geometry of the drills was re-considered. The thought that drills might be the scaled version of each other was investigated. It was found that a geometric inter-relationship existed between the various size drills and that they could be considered similar to each other if they satisfied certain geometric requirements such as equal ratio of web-thickness to pitch length. This similarity concept led to the derivation of two equations inter-relating the thrust and torque of various similar drills. From these equations the thrust and torque of drills can be estimated from the known values of other similar drills. Even though a cutting model for the standard drills cannot be derived yet, the similarity concept would certainly help in the design of drills and minimize empirical testing.

Experiments were planned and run to study the force relationship and chip formation process for similar standard and flat rake drills. Statistical analysis such as the analysis of variance were applied and it was found that these force relationship and
identical deformation did exist as expected.

The cutting efficiency of the modified drills was proved, statistically, higher than that of the standard drills especially so for the thrust force.

The force and chip length ratio distributions predicted by the cutting model were comparable to the observed ones. The difference between the predicted and observed thrust was within 5% and about 10% for torque.
Drills are among the oldest manufacturing tools. The early types of drills were the flat drills made from a round metal bar flattened and sharpened as in Fig. 1-1(a). These drills had long been a source of dissatisfaction in the workshop due to their primitive design and construction leading to poor performance.

The next stage of development was the invention of a kind of twist drill as illustrated in Fig. 1-1(b). Its performance as compared with the flat type was not very much better mainly because there was not enough space in the body for the smooth removal of chips. Eventually the idea of introducing spiral grooves in the drill body to convey the chips away from the drill point was put forward. In about 1860 (1), a twist drill design approximating closely that of the modern type of drills came into existence. These drills were crudely manufactured from ordinary carbon steel which broke rather easily under ordinary drilling conditions. High cost and their capabilities not being fully realized, prevented the twist drill from being widely used in the
workshop for some years.

With the introduction of high speed steel, consequent design improvement and lower manufacturing costs, twist drills became indispensable tools in industry.
Specification of twist drill

Fig 2-1
GENERAL GEOMETRY OF TWIST DRILLS

The twist drill, though simple in appearance is geometrically very complex. Fig. 2-1 shows the end and side views of a conventional drill together with the usual standard nomenclature (2). In this section the general geometry of the twist drill will be considered with particular emphasis on those features which will in one way or another control drill performance. Galloway (3) suggested four criteria for assessing drill performance, namely: rate of penetration, drill life, accuracy and surface finish of the drilled hole, and drilling efficiency (expressed in terms of thrust and torque).

The body of the drill has two milled or rolled helical grooves or flutes. The web is the central column separating the two flutes. The general purpose drills, and drills designed for drilling shallow holes have a constant increase in web-thickness from point to shank for higher torsional rigidity. Web construction in deep hole drills is almost always parallel since chip removal has been proved to be better over parallel web-surface (4). The web-thickness in some drills can be as much as 65% of the drill diameter. With general purpose drills, the web-thickness lies in the range of 11% to 22% of the diameter, depending on drill sizes. The general trend is that the larger the drill the smaller is this percentage (4) (5). The web-thickness has been a subject of discussion as early as 1917 by Benedict and Lukens (6), and in subsequent years by many others.
Oxford Jr. (7) and Galloway (8) established a number of geometric expressions for some drill angles in terms of the web-thickness. The conclusions reached by these workers are that the web is a very significant element of the drill: it affects drill life, controls hole accuracy and contributes approximately half of the total thrust.

The chisel edge angle is formed by the intersection of the two flank faces and consequently the chisel edge angle is measured as shown in Fig 2-1(b). The chisel edge and invariably the chisel edge angle are important parameters in the performance of the drill (7-11). At the exact centre of the drill, the only motion is axial, so the action of the drill is similar to an indenting punch. As the radius increases the rotation of the drill becomes important and the chisel edge appears to have a combined action of cutting and extruding, yielding a highly deformed ribbon chip which eventually escapes into the flute.

Benedict and Lukens (6) and Oxford Jr. (7) ran a series of tests to illustrate the effect of chisel edge length and the chisel edge angle on thrust and torque. They concluded that the chisel edge is responsible for as much as 50% of the total thrust.

The point angle is one of the drill features which can be altered by subsequent grinding in the workshop. In general practice the flute form of the drill is such that the cutting edges will be straight when a certain predetermined point angle is ground. Hilliam and Gibson (12) reported that straight cutting edges give optimum drill performance. Benedict and Lukens (6) on the other hand claimed
it was only traditional to have straight cutting edges. The performance of any drill is very markedly affected by the point angle \((8)-(13)\). The magnitude of the point angle has been determined by various workers from performance tests based on one or more of the criteria mentioned earlier. If the point angle is too large plus the fact that the drill has no true point due to the chisel edge, the drill has centring difficulties \((10)-(11)\). If the point angle is too small, there is a tendency to weaken the drill point and more power is required due to the additional length of the cutting edges. A larger point angle will produce narrower and thicker chips, which is desirable in deep hole drilling \((4)\), since this type of chip can be more readily removed.

The helix angle is the acute angle between the leading edge of a land and the drill axis as illustrated in Fig. 2-1(b) and is considered similar to a rake angle at the periphery. Closely related to this helix angle is the pitch length. The helix angle is in fact a function of the pitch length and the radius in question. For a given drill the helix angle will vary along the drill lips, being largest at the periphery. From experimental results the helix angle has been shown to influence the performance of the drill \((4)-(6)-(8)\). In drilling deep holes larger helix angles are recommended \((4)\) because of their improved chip removal characteristics. Smaller helix angles are associated with the extremely heavy duty drills required for drilling very hard materials used in the aircraft and the missile industries. The magnitude of the helix angle can only be determined
in conjunction with those of the point angle and the flute form if a straight cutting edge is to be obtained.

In the early stage of drill point geometry investigation one clearance angle was specified and its function was to prevent rubbing against the workpiece. Benedict and Lukens (6) and Shaw (14) proposed a way of measuring the clearance angle. Their method of measuring this angle at the periphery is by wrapping a sheet of paper around the drill and marking it as shown in Fig. 3-1(a). The clearance angle is then given by angle \( \angle ABC \).

Galloway (8) in attempting to study this angle quantitatively, defined what he called the relief flank and the clearance flank; the relief flank is that part of the flank immediately adjacent to the cutting edge and the clearance flank is that part of the flank extending behind the relief flank. Galloway thus defined the relief angle as the angle between the relief flank and a plane perpendicular to the drill axis and perpendicular to a radius. Galloway made a corresponding definition for the clearance angle. These two angles are shown in Fig. 3-1(b).
From Galloway's definitions, both the relief angle and the clearance angle vary along the cutting edges, as compared with Benedict and Luken and Shaw's proposal of measuring the clearance angle at the periphery only. Galloway was able to derive an equation for the relief angle in terms of other drill parameters, and he also suggested a visual method for inspecting this relief angle. While defining the clearance angle as for the relief angle, Galloway did not show how it could be measured or be expressed in terms of other parameters.

It is apparent that the geometry of the twist drill is more complex than those of the conventional cutting tools. The varying rake angle, relief angle and angles (to be discussed later) makes the drilling action difficult to study.
The web, the helix angle, the point angle and the relief angle have been examined in some detail, each as individual features. But in fact they are all inter-related as will be shown in the next section. Changing one of them, such as the point angle or the web-thickness will change the entire geometry at the drill point. Therefore in the design of twist drills, the geometry as a whole must be considered, bearing in mind that each angle or element has to satisfy certain performance criteria. Thus if the web is too small, reduction of thrust is achieved at the expense of loss of torsional rigidity.

**METHOD OF DRILL POINT GRINDING**

Twist drills have been made and used for more than half a century and a number of investigators have suggested various "angle combinations" for drilling tests in the hope of improving the drilling performance. There are various types of drill point grinding machines available on the market (15) (16), yet there has not been many publications dealing with the grinding technique. Galloway (8) claimed that the relative motion between the grinding wheel and the drill point is very complex, but in most cases it was equivalent to a conical motion. The flank surface produced by such motion was regarded as a blending of portions of the surface belonging to a large number of different grinding cones. He considered that in some grinding machines, the relative motion of the grinding wheel and the drill point at any instant was reducible
to a pure conical grinding plus a sliding motion along the line of contact between the drill point and the wheel face. This sliding motion provided the relief and clearance angles.

Assuming conical grinding and using vectorial analysis, Galloway derived the following equations:

\[
\tan \gamma = (\sin \omega - \cos \omega \cot \theta) \cot \, \rho \quad (1-1)
\]

Where \( \gamma \) and \( \omega \) are the relief angle and web-angle respectively at radius \( r \) of the drill whose half point angle is \( \rho \). "\( \theta \)" is not the actual chisel edge angle as appears on the ground drill point. It is only known if the relative geometry of the grinding wheel and the drill point is specified and therefore Galloway found it more convenient to express the relief angle in terms of the relief angle at the periphery:

\[
\tan \gamma = \frac{T}{P} \left[ 1 - \left( \frac{P^2 - t^2}{1 - t^2} \right)^{\frac{1}{2}} \right] \cot \, P \quad + \quad \frac{1}{P} \left( \frac{P^2 - t^2}{1 - t^2} \right)^{\frac{1}{2}} \tan \gamma_0 \quad (2-1)
\]

where \( T = \frac{t}{R} \), \( P = \frac{r}{R} \)

\( t \) is half web-thickness of the drill with outside radius \( R \).

Equation 2-1 indicates that when the nominal relief angle is specified, the relief angle at any radius along the lips are automatically fixed.

The chisel edge angle \( \theta' \) was expressed as:

\[
\cos \theta' = \cos \lambda - \frac{t}{a} \quad (3-1)
\]
where:

\[
\cot \alpha = \frac{1}{(1 - \tan^2 \theta)^{\frac{1}{2}}} (\tan \beta - \tan \phi \tan \gamma) \tag{4-1}
\]

and "a" is the approximate value of the curvature of the radius of that section of the cone in question. Galloway considered "a" to be a constant.

From equations 1-1 and 3-1 it is seen that both the relief angle and the chisel edge angle are functions of the web-thickness, the point angle and the nominal relief angle \( \gamma_0 \). Changing any one of these parameters will invariably change the relief and chisel edge angles. This is the first comprehensive evidence which shows how drill point parameters are inter-related, and that they will make up a large number of drill point angle combinations.

Equations 3-1 represents the chisel edge angle. The web-thickness for a particular drill is a constant, "a" is a constant according to Galloway, and \( \lambda \) is a constant once the nominal relief angle is specified. This means that the chisel edge is a straight line passing through the centre of the drill point.

**PERFORMANCE TESTS AS CRITERIA FOR SELECTION OF DRILL POINT GEOMETRY**

The Metal Cutting Institute of America (5) reported that for optimum drill performance, the drill geometry should be changed for different metals. The various geometries were arrived at mainly from performance tests. A comprehensive series of experiments were
carried out by Benedict and Lukens (6). Later Galloway and Morton (1) tested the effect of rate of penetration on drill life while Cowie and Pegler (13) demonstrated how the point angle influenced drill life. The aims of all these research workers were to achieve some drill geometry for optimum performance.

Galloway studied the effect of the nominal relief $\gamma_o$ angle and the helix angle on drill life. He concluded that the nominal relief angles between $9^\circ$ and $15^\circ$ were the optimum valves for most metals, and the helix angle of $25^\circ$ would give longer drill life. Cowie and Pegler (13) demonstrated that for different materials, there were different point angles which would give longest drill life.

Oxford Jr. (18) reported that the flute length was very important in drill life, and in one case the drill life increased by $80\%$ when the flute length was reduced from $2\frac{1}{2}''$ to $1\frac{3}{8}''$. From these experimental results Oxford Jr. concluded that some critical valves of torsional rigidity (torque/unit angular deflection/unit length) was required for satisfactory drill life. The three methods for increasing the drill rigidity as suggested by Oxford Jr. are: reduction of overall length, reduction of flute length and increased web-thickness.

The other performance criterion used was the thrust force and torque. As for any other cutting tools, the drill geometry will control the thrust and torque required. Benedict and Luckens (6) were among the first to study these effects. Galloway (8) in 1957
made another detailed experimental study of the same problem. They tested the effect of point angle, helix angle, clearance angle, relief angle (Galloway only), chisel edge angle (Benedict and Lukens only) and the chisel edge.

Both Benedict and Lukens and Galloway concluded that increasing the helix angle would decrease the torque and thrust. They also claimed that these decreases were more pronounced at higher feeds. The range of helix angle tested was from 15° to 45°. Galloway suggested that the helix angle should only be determined in conjunction with other considerations, e.g. increasing the helix angle will reduce the amount of metal supporting the cutting edge, which will chip easily, especially at large feeds. The recommended helix angle for drilling manganese steel and the extra hard material was about 15° (4), whereas drills for ordinary steel would have helix angles of about 30°.

Benedict and Lukens showed that the variations in torque when drilling with point angle of 90° to 150° was very small. Thrust increased as the point angle increased. Galloway confirmed these results.

The effect of clearance angle on thrust and torque was also shown to be insignificant. Galloway found that the relief angle between 4° and 20° at the periphery did not change the thrust and torque much.

Keeping the point angle and the clearance angle constant, Benedict and Lukens showed that the torque variation was very small
for various chisel edge angles between 110° and 140°, but there was a marked decrease in thrust at about 130°. Related to the chisel edge angle is the chisel edge length, the effect of which was also considered by Benedict and Lukens. When the diameter of a pilot hole was equal to the chisel edge length, the thrust decreased by 59%. Decreasing the pilot hole further only decreased the thrust proportionally. The pilot hole hardly affected the torque at small diameters and it was only when the pilot hole became about 50% of the drill diameter that the decrease was noticeable.

These workers did not specify the percentage of the chisel edge length in terms of the diameter, therefore it is difficult to accept their result as a general guide, since this percentage changes from drill to drill. Oxford Jr. (7) ran a series of tests to demonstrate this effect of chisel edge length.

![Graphs showing thrust and torque vs. chisel edge length](image)

\[\text{Dia} = 0.5", \quad \text{Feed} = 0.001"/\text{rev}, \quad \text{Point angle} = 118^\circ, \quad \text{Cut dry} \]

\[\text{SAE 3245 steel}, \quad B_H = 200\]

Fig 4-1
Fig. 4-1 shows that for a half inch diameter drill with chisel edge length of about \(0.08\)" , which is common for many drills, the chisel edge was responsible for about 50% of the total thrust and about 5% of the torque. But in a heavy duty drill of the same size with a chisel edge length of \(0.20\)", the thrust due to the chisel edge length was about 75% of the total thrust and about 40% of the total torque. From Oxford Jr.'s results it is evident that the total thrust and torque are not only functions of the diameter, but also functions of the chisel edge length. The total thrust and torque of a heavy duty drill are higher than those of a standard drill of the same diameter.

These investigators did not elaborate how the drills were ground to give the specified geometry. Since all the angles are inter-related, changing one of these would change one or more of the other angles. One example of this is according to Galloway's equation for the relief angle. In this equation, the relief angle is a function of the point angle, the web-angle and the chisel edge angle. If the web-angle and the point angle are kept constant, then varying the chisel edge angle will vary the relief angle. In this case all these angles can only have "sets" or "combinations" of values and cannot be treated as single features.

In all these investigations, the angles concerned were specially ground to the values required. Micheletti and Levi (19) queried this "one variable at a time" approach and they demonstrated
how the magnitudes of the point angle, the rake angle, the clearance angle and the chisel edge angle could affect each other. They decided to study the effect on forces of the small variations in the drill point geometry of a number of nominally similar drills (about \( \frac{3}{8} \) diameter drills). The following seven parameters were considered as the major variables in the drill point geometry: web-thickness, chisel edge angle, rake angle, lip clearance, point angle, web-off-set and relative lip height (the last parameter will be discussed later). Fourteen nominally similar drills were selected and all the above variables measured. For each variable a mean and a standard error were estimated. Multiple regression technique was employed to study the effect of each variable on forces and the correlation between these variables. The form of the regression equation was given as:

\[
y = b_0 + b_1 x_1 + b_2 x_2 + \cdots + b_i x_i + \cdots + b_n x_n
\]

where \( y \) is either thrust or torque and \( b_i \) is the coefficient of the \( i \)th variable.

Tests were run at various feeds (one feed at a time) with all the drills. From the observed forces they were able to determine the significance of each variable. Only those variables whose significance exceeded a value in the form of an F level were retained in the regression equation. Using three feeds and two different materials, they obtained six thrust equations and six torque equations in terms of
the significant variables. An assessment of the effects of the variables was made using these equations, and they concluded that the chisel edge angle affected the thrust and torque more than the other variables. They also claimed that the high correlation between the clearance angle and forces turns out to be accounted for almost completely by the chisel edge angle.

An interesting and most surprising feature of this analysis was that the forces could be doubled by slight deviation of the drill point geometry. Although the chisel edge angle seemed to have the greatest effect on thrust and torque it also appeared to be the one with the largest deviation from the nominal values.

The effect of web-thickness and the various angles on drill life, torque and thrust have so far been discussed. Hole accuracy is another performance criterion proposed by Galloway. Ideally drills should have symmetrical flutes forms and finally ground with perfect symmetry. In practice this condition is not always achieved. The effect of this are over-sized holes and the shortening of drill life. It has often been reported that holes produced by drilling are always over-size. Oxford Jr. (7), Galloway (8) and Spur (20) looked into the problem of dimensional and geometrical accuracy of the holes due to asymmetrical drill geometry. The Metal Cutting Institute of America (5) conducted a number of tests in order to determine the amount of hole over-size that may be expected under normal workshop drilling conditions.
Fig. 5-1 shows the maximum, minimum and the mean values of oversize for six different drill sizes:

![Graph showing the maximum, minimum, and mean values of oversize for different drill sizes.]

Oversize produced by twist drills when drilling steel and cast iron

Fig 5-1

The range (i.e., between maximum and minimum) is more important than the mean value of oversize. In general, a smaller drill will not give the required hole size because of the spread of oversize, i.e., to compensate for oversize holes by reducing the drill diameter.

While the Metal Cutting Institute did not elaborate on the causes of oversize holes, Spur (20) made a study of the problem from force considerations.

![Diagram showing force components acting on the cutting lips at corresponding radii.]

Force components acting on the cutting lips at corresponding radii

Fig 6-1
In Fig. 6-1 Spur designated $P_{Z1}$ and $P_{Z2}$ as the two cutting forces acting on the two cutting edges. He then resolved these two forces into 3 main forces - the horizontal force (giving rise to torque), the vertical force (giving rise to thrust) and the radial forces $P_{P1}$ and $P_{P2}$. Spur claimed that ideally the radial forces should cancel each other since they were acting along the same line and in opposite direction. Experiments have shown that $P_{P1}$ and $P_{P2}$ would not always balance, thus there would be a resultant radial force. This radial force would rotate at the same speed as the drill spindle, causing a static bending of the drill and there would be a concentric increase in the hole diameter. Spur attributed the causes of radial forces to one or more of the following:

1) Dissimilar length of lips.
2) Dissimilar point angle.
3) Dissimilar clearance angle.
4) Web off-set.
5) Non-symmetric flutes.
6) Dissimilar cutting edges and sharpness.

Galloway (8), Landberg (9) and Haggerty (11) went further to point out the importance of asymmetrical drill point geometry on hole accuracy. Galloway made a thorough study of the problem of relative lip height (unequal lip length, but equal point angle). He expressed the relative lip height $H$ in terms of the displacement $E$
from the true axis of the drill by:

\[ E = \frac{1}{2} H \tan P \]

The hole oversize due to this type of error was considered equal to \(2E\) approximately.

Fig. 7-1 illustrates the effect of relative lip height on hole accuracy. The experimental results are seen to be in good agreement with the predicted results (represented by the full line). Another effect of asymmetrical drill point geometry is the possible shortening of drill life as claimed by Spur. Galloway showed that the drill life could be increased by three times when the relative lip height was reduced from \(.015"\) to \(.001"\) for a 5/8" diameter drill.
Modification of Drill Point

To Improve Drill Performance

It has been shown that the web constitutes the greater portion of the total thrust. Furthermore most drills are made with webs which increase in thickness towards the shank for higher rigidity. As the drill is reground, the web-thickness will increase and so does the thrust. This will make drilling more difficult so that it puts an increased load on the drilling machine and workpiece. Therefore it is obvious that some method of reducing the chisel edge length or modifying this region is desirable. This is usually achieved by what is known as point-thinning and point-relieving. Landberg (9), Oxford Jr. (18) and the Production Engineering Research Association of Britain (PERA) (21) proposed various forms of point-thinning or point-relieving. The latter also recommended values for the chisel edge length below or above which the drill should be point-thinned or relieved, and the minimum chisel edge length remaining after the operation. Fig. 8-1 shows three types of point-relieving and point-thinning:

![Drill point modifications](image)

Fig 8-1
When the chisel edge length is less than a certain value, PERA (21) recommends point-relieving as shown in Fig. 8-1(a).

The actual grinding method is illustrated in Fig. 9-1(a). Using this method the chisel edge length is not shortened but part of the metal near the chisel edge region is removed. Fig. 8-1(b) represents a type of point-thinning. The amount of the chisel edge length to be reduced depends on the drilling condition and the material being drilled. Fig. 9-1(b) is the method recommended by PERA. Fig. 8-1(c) is what Oxford Jr. called "split point" thinning. By this method a new pair of cutting edges are created in the chisel edge region as shown in Fig. 8-1(c). It is expected that the thrust due to the web will be a minimum because of the zero chisel edge length. The configuration of the final point would depend on the size of the grinding wheel and the amount of metal being ground from each lip. The force reduction would be controlled by these factors.

Armarego (22) tried a further modification. He ground a flat rake face on the drill lips such that the rake angle was increased at all radii. This method also led to a form of point relieving as
given in Fig. 8-1 (d). It was thought that the increased rake and the point relieving would result in lower thrust and torque. Some evidence of such reduction was given, but he suggested further investigation into the performance of these drills is necessary.

Haggerty (11) introduced a new drill point design - the spiral point. He claimed that these drills gave better performance than the conventional drills, i.e. better hole accuracy, lower cutting force and longer drill life. He compared the hole oversize effect of both types of drills with various relative lip heights and concluded that the amount of oversize is less for spiral point drills. It was claimed that another advantage of the spiral point drill was the reduction of thrust and torque due to the smaller negative normal rake angle in the chisel edge region, and larger chip space for easier chip removal. The test results showed that thrust for the spiral point drill ranged from 15 to 30 per cent lower than those of the chisel point drill. Haggerty also conducted drill life tests for both types of drills. His results indicated that the spiral point drill would last just as long or even longer than the chisel point drill under the same drilling conditions.

Drill point modifications are aimed at improving the performance of the drill. Most methods suggested have been successful to a lesser or greater extent. A difficulty with point-thinning or point relieving is that the drill point symmetry may be destroyed unless the modification operation is carefully controlled. The
consequence of this may be the shortening of drill life and the inaccurate hole size. The possible improvement in performance by point-thinning or point-relieving can be verified by testing the conventional drills of identical geometry. But it is rather difficult to compare different types of drill e.g. spiral point and conventional, since this will depend on the geometry of the drills being compared. Thus the spiral point drills need not always be better in performance than the conventional drills. Kinman (23) in discussing Haggerty's paper claimed that better performance of the conventional drill could be achieved by grinding the chisel edge angle sufficiently large to give adequate clearance in the chisel edge region, and thus eliminate the crowding of the chips. Kinman pointed out that the geometry around the chisel edge region could be controlled by using a drill grinder of new design. But he did not elaborate on the mechanics of grinding of this grinder. Therefore any comparison between two types of drill is only fair when their optimum geometries are used.

CHIP FORMATION IN DRILLING

While the indenting action of the chisel edge is most difficult to study, there are also very few reports on the mechanics of chip formation of the cutting edges based on thin shear plane model. The lack of such research could possibly be due to the complex geometry of the drill. Such complex geometry has led to the investigation
whether the cutting action of the drill lips is the same as other single edge tools. Oxford Jr. (7) using the "quick stop" technique was able to examine the chip formation process along the drill lips. He observed that the drill lips had basically the same cutting action as other single edge tools. Shaw (24), Rohlke (25) and Cowie and Pegler (13) suggested that the twist drill was analogous to a conventional single point tool, and that the cutting edges behaved like single edge oblique tools whose geometry was characterized by the normal rake angle and the angle of obliquity.

i) Fundamental Cutting Angles.

The presence of web-thickness, the point angle and the varying work velocity would certainly complicate the visualization and consequently the definition of the angle of obliquity $i$ and the normal rake angle $\alpha_n$. For drilling Oxford Jr. (26) defined the angle of obliquity as the angle between the work velocity vector $V_w$ and a normal to the cutting edge lying in a plane containing the cutting edge and the work velocity vector. Following this definition, Oxford Jr. derived an expression for this angle in terms of the web angle and the point angle, namely:
\[
\sin i = \sin \omega \sin P \tag{5-1}
\]

As can be seen from Fig 10-1 the feed velocity \( V_F \) will alter the direction and magnitude of the work velocity \( V_W \), and consequently the magnitude of the angle \( i \). Galloway's (8) equation for this feed angle is given by:

\[
\tan \beta = \frac{f}{2\pi r} \tag{6-1}
\]

where \( r \) is radius at which this feed angle is considered.

Colwell (27) went further to derive an equation for "i" including the effect of feed:

\[
\sin i = \sin \omega \sin P \cos \beta + \sin \beta \cos P \tag{7-1}
\]
Cowie and Pegler (13) indicated that the maximum error in \( i \) was about 1 -2% when the feed angle was neglected. Williams (28) also showed that the effect of feed on the geometry of the cutting edges was negligible. However he demonstrated that the geometry of the chisel edge region is dependent on the feed velocity since the rotational velocity of the drill decreased towards the centre until this velocity is zero at the centre of the drill and the cutting velocity is equal to the feed velocity.

The normal rake angle \( \alpha_n \) in drilling is defined as the angle between the normal to the work velocity and the tangent to the rake face in a plane perpendicular to the cutting edge at the point in question. Oxford Jr. (26) derived an expression for this angle in terms of the reference rake angle \( \alpha_f \) and the projected work velocity angle \( \psi \). These two angles are given by:

\[
\tan \alpha_f = \frac{\tan \delta \cos \omega}{\sin \varphi - \cos \varphi \tan \delta \cos \omega} \quad (8-1)
\]

and

\[
\tan \psi = \tan \omega \cos \varphi \quad (9-1)
\]

As illustrated in Fig. 10-1, the normal rake angle is related to \( \psi \) and \( \alpha_f \) by:

\[
\alpha_n = \psi - \alpha_f \quad (10-1)
\]

Manipulating equations 8-1, 9-1 and 10-1, the normal rake angle is finally represented as:

\[
\tan \alpha_n = \frac{\tan \delta}{\sin \varphi} \left[ \cos \omega + \sin \omega \tan \varphi \cos^2 \varphi \right] - \tan \omega \cos \varphi \quad (11-1)
\]
Galloway (8) derived an equation for what he called the rake angle:

$$\tan \alpha = \frac{(p^2 - t^2)^{\frac{1}{2}}}{p \cot \delta \sin P - t \cot P}$$  \hspace{1cm} (12-1)

where

$$p = \frac{r}{R} \quad \text{and} \quad t = \frac{t}{R}$$

Substituting $p$ and $t$ into equation 12-1, gives:

$$\tan \alpha_f = \frac{(r^2 - t^2)^{\frac{1}{2}}}{r \cot \delta \sin P - t \sin P}$$

$$= \frac{\cos \omega \tan \delta}{\sin P - \cos P \sin \omega \tan \delta}$$  \hspace{1cm} (13-1)

Comparison of equations (8-1) and (13-1) shows that they are in fact identical. Therefore what Galloway called the rake angle is actually the same angle which Oxford Jr. earlier defined as the reference rake angle.

In order to illustrate how the rake angle varies along the cutting edge and the chisel edge, Oxford Jr. sectioned through a chisel point drill and the partly formed chip at various points. Each of these sections were taken in a plane perpendicular to the cutting edge at the point. Fig. 11-1 is a series of photomicrographs which have been orientated so as to demonstrate the normal rake angles.
Photomicrographs of sections of twist drills and partly formed chips at successive points along the cutting edge

Fig 11-1

Fig. 12-1 illustrates the variations of the angle of obliquity $i$ and the normal rake angle $\alpha_n$ with radius.

Variations of $\alpha_n$ and $i$ with radius

Fig 12-1

Oxford Jr. (26) in attempting to study the cutting action of the drill lips found that the chip flow angles varied widely along the lips. This led him to a further study of the possible relationship between drill geometry and this angle. He measured the projected chip flow angle $\eta_p$, which was defined as the angle between the chip velocity vector normal to the cutting edge, measured in a plane containing the cutting edge and parallel to the axis of the drill. He showed that this angle varied along the cutting edge. The true chip flow angle $\eta_c$ was related to this angle by the following equation:

$$\tan \eta_c = \tan \eta_p \cos \alpha_f$$

where $\alpha_f$ is the reference rake angle.

Since drilling was shown to be similar to the oblique cutting Oxford Jr. decided to study the relationship between the chip flow angle $\eta_c$ and the angle of obliquity $i$. The chip flow angle $\eta_c$ could be found by measuring $\eta_p$. In Fig. 13-1 the chip flow angle $\eta_c$ for two materials are plotted against the angle of obliquity $i$. 
Oxford Jr. noted that Stabler's (29) (30) chip flow rule was a first degree approximation with the exception of one case, in which \( \eta_c = i \) for all radius. It appeared that he abandoned his attempts to study the chip flow angle because it was difficult to construct a model to account for the varying geometry of the cutting edges.

iii) Application of Merchant's Thin Shear Plane Equations to Drilling.

Pal, Bhattacharyya and Seng (31) attempted to analyze the drilling process by generalizing the geometry of the drill lips to that of a lathe tool. The complex indenting action of the chisel edge compelled them to restrict their analysis to the cutting edges only. They did not study the thrust due to the lips. Their total torque \( M_T \) due to the cutting edge was made up of two parts:
\[ M_T = M_C + M_F \]

where \( M_C \) is due to cutting and \( M_F \) is due to friction. \( M_C \) is represented by:

\[ M_C = F_p \cdot p \]

Mean Moment Arms

where \( F_p \) is the tangential force analogous to the power force in turning.

They considered an elemental area \( \Delta A \) as shown in Fig. 14-1.

In one revolution, the vertical distance covered by one lip is \( \frac{f}{2} \), therefore \( \Delta A \) can be represented as:

\[ \Delta A = \frac{f}{2} \sin P \cdot \frac{\Delta r}{2 \sin P} = \frac{f \cdot \Delta r}{4} \]

and they represented the elemental power force \( \Delta F_p \) in oblique cutting by:

\[ \Delta F_p = \frac{\Delta b \cdot t' \cdot r' \cos (\beta_e - \alpha_e)}{\sin \phi_e \cdot \cos (\phi_e + \beta_e - \alpha_e)} \]
where \( \alpha_e \) is the effective normal rake angle

\( \phi_e \) is the effective shear angle

\( \beta_e \) is the friction angle

\( t' \) is undeformed chip thickness \( \left( \frac{f}{2 \sin P} \right) \)

\( \Delta b \) is width of cut \( \left( \frac{\Delta r}{\sin P} \right) \)

\( \Delta r \) is elemental radius corresponding to \( \Delta b \)

\( \tau' \) is shear stress

Substituting \( \frac{\Delta r}{2 \sin P} \) and \( \frac{f}{2 \sin P} \) into equation 15-1, the elemental power force \( \Delta F_p \) due to one lip is given by:

\[
\Delta F_p = \frac{\Delta r \cdot f \tau'}{4} \cdot \frac{\cos (\beta_e - \alpha_e)}{\sin \phi_e \cos (\phi_e + \beta_e - \alpha_e)}
\]

This was approximated to:

\[
\Delta F_p = \frac{\Delta r \cdot f \tau'}{4} \left( A - B \alpha_e + \frac{1}{r_1} \right) \quad (16-1)
\]

Where \( A \) and \( B \) were constants and \( r_1 \) was the chip length ratio.

Thus an elemental torque was given by: \( 2 \) lips

\[
\Delta M_C = \frac{f \tau'}{2} \left( A - B \alpha_e + \frac{1}{r_1} \right) \cdot \Delta r \cdot r \quad (18-1)
\]

To be able to determine the \( \Delta M_C \) and consequently the total torque due to cutting Pal, Battacharyya and Shen had to determine the shear stress \( \tau' \), the effective shear angle \( \phi_e \) and the chip
length ratio \( r_1 \) in addition to the two constants \( A \) and \( B \).

The shear stress is provided with Abulade's stress strain relationship:

\[
\tau' = 0.74 \sigma_\mu 6.6^\Delta
\]  

(19-1)

where \( \sigma_\mu \) is ultimate shear stress and \( \Delta \) is % of elongation.

Then they assumed Stabler's chip flow rule to hold and by comparing the drill lips to a lathe tool, the effective shear angle was expressed in terms of the radius \( r \), i.e.:

\[
\alpha_e = 2r + 3.6
\]  

(20-1)

Using Thomsen et al's (32) empirical chip ratio equation for turning as a guide, they ran drill tests on ductile materials with pilot hole and obtained the following chip length ratio equations:

\[
t_1 = \frac{r \cdot 7 \cdot 112}{14}
\]

(21-1)

Substituting equations 19-1 — 21-1 into equation 17-1 and assuming \( A = 1.25 \) and \( B = 0.022 \), Pal and others were able to derive a torque equation due to cutting, i.e. \( M_C \) by method of integration.

The torque due to friction was given by:

\[
M_F = \frac{F (R_2^2 - R_1^2)}{\sin \varphi}
\]

where \( F \) is specific friction force per unit contact length.
Later Bera and Bhattacharyya (33) attempted the same problems with a slightly different approach. They used equation 16-1 instead of 17-1 to represent the elemental power force and they did not generalize the drill lip geometry to that of a lathe tool as did Pal and others.

![Geometry of drill lips](image)

**Fig 15-1**

Fig. 15-1 shows some of the drill angles Bera and Bhattacharyya used in their derivation. The normal rake angle was given by

\[
\tan \alpha_n = \tan \alpha_0 \tan i
\]  

(22-1)

where \(i\) is the angle of obliquity and \(\alpha_0\) is what they called the orthogonal rake angle, and it is represented by:

\[
\tan \alpha_o = \frac{\left(\frac{L}{L}\right) \tan \delta - \tan (\sin i) \cos \rho}{\sin \rho}
\]  

(22-1)

where \(\delta\) is the helix angle. The orthogonal rake angle \(\alpha_o\).
appeared to be identical to the reference rake angle $\alpha_3$ as defined by Oxford Jr. and Galloway.

Assuming Stabler's rule and substituting equation 22-1 with the equation for the effective shear angle $\phi_e$, namely:

$$\sin \alpha_e = \sin^2 i + \cos^2 i \sin \alpha_n$$

they evaluated the effective shear angles. They observed close correlation between $\alpha_o$ and $\alpha_e$, i.e. $\alpha_o \approx \alpha_e$

Using the chip length ratio equation obtained by Pal and others earlier, i.e.:

$$r_1 = \frac{.7 \cdot 112}{r \cdot f \cdot 14}$$

and replacing $\alpha_e$ by $\alpha_o$, Bera and Bhattacharyya obtained the effective shear angle according to the following equation:

$$\tan \phi_e = \frac{r_1 \cdot \cos \alpha_e}{1 - r_1 \sin \alpha_e}$$

Lee and Shaffer's shear angle equation, i.e.:

$$\phi_n + \beta_e = \frac{\pi}{4} + \alpha_n$$

was used to calculate the friction angle $\beta_e$. Their next step was the substitution of the effective shear angle $\phi_e$, the friction angle $\beta_e$ and the orthogonal rake angle $\alpha_o$ into equation 16-1. From there they were able to obtain the total torque due to cutting...
along the lips. They determined the friction torque using the same approach as Pal and others.

Relating the thrust force due to the cutting edge to the established torque equation Bera and Bhattacharyya obtained a thrust equation for the cutting edges.

They went further to investigate the thrust due to the chisel edge. Assuming the chisel edge acted as an indenting wedge and applying the slipline analysis to a fully plastic material, the thrust due to the chisel edge was represented by:

\[ T_{C.E.} = 18.3 \int \mu \cdot 6.6 \cdot q \cdot f \]

where \( q \) is the chisel edge length.

They claimed that the experimental results conformed very well with the predicted values.

A number of doubtful steps have been used in their analyses. In Pal, Bhattacharyya and \( n \)'s work it is difficult to see the justification in replacing the term:

\[
\frac{\cos (\beta_e - \alpha_e)}{\sin \phi_e \cos (\phi_e + \beta_e - \alpha_e)}
\]

by \((A - B \cdot \alpha_e + \frac{1}{r_1})\). In doing so they eliminated \( \beta_e \) as one of the unknowns. They also called equation 15-1 the modified power force equation for oblique cutting. In fact this equation was derived by Merchant (34) (35) for a thin shear zone model in orthogonal cutting, namely:
Yet this power force equation was used to account for the more general oblique cutting case. It also appears that the power force equation was applied without due consideration given to the basic mechanics of the thin shear zone model. While recognizing drilling as oblique cutting they neglected the possible effect on torque and thrust of the other force components, namely the radial force $F_r$. They assumed Stabler's chip flow rule to hold, the applicability of which was queried by Oxford Jr. (26). Despite the various approximation and suspect step in their analyses, these equations did predict thrust and torque reasonably well.

These investigators' work is interesting to those attempting to study the mechanics of drilling from first principles. They took into consideration the varying geometry and paid attention to the expected chip length ratio variation, which was suggested by Pal and others. Their experimental results indicate that it is possible to apply the thin shear zone model to drilling with some success. With this type of approach a better understanding of the mechanics of drilling may be achieved. A drilling model is necessary since it will help to minimize experimental testing and possibly help at arriving at an optimum drill geometry.
PREDICTION OF CUTTING FORCES IN DRILLING

In order to provide suitable equipment for any drilling operations with satisfactory efficiency it is necessary to determine the thrust and torque required to operate the drill at some pre-determined speed and feed. Torque and speed control the power requirement whereas thrust directly affects the machine rigidity and strength (3). Therefore it is useful to be able to predict these forces in the design and application of the drills and the machine.

(i) Empirical Methods.

Boston and Oxford Sen. (36) reported that as early as 1901, Smith and Poliakoff (37) put forward two sets of equations for torque and thrust based on results from a large number of experimental results. These equations are reproduced below:

Cast iron:  \[ T = 35,500 \cdot 7 f \cdot 75 \]
\[ M = 740 \cdot 8 f \cdot 7 \]

Medium hard steel:
\[ T = 33500 \cdot 7 f \cdot 6 \]
\[ M = 1640 \cdot 8 f \cdot 7 \]

Following these investigations, Boston and Oxford Sen. carried out an intensive experimental programme in an attempt to establish some empirical equations of the same form. The drills they used in the test were standard high-speed-steel drills with diameter ranging from 0.5" to 1.5".

The material used for testing were different types of cast iron and steel widely used in industry. The speed and feed range were
74 to 441 r.p.m. and 0.009 to 0.015 in/rev. respectively. The observed thrust and torque were plotted against diameter and then against feed on log-log scale. Boston and Oxford were, then, able to suggest the following equations, which are similar to those of Smith and Poliakoff:

Cast iron: \[ T = C_1 d f^{0.6} \]
\[ M = C_2 d^2 f^{0.6} \]

Steel: \[ T = C_3 d f^{0.6} \]
\[ M = C_4 d L f^{1.8} f^{0.78} \]

They did not obtain any values for the constants \( C_1, C_2, C_3 \) and \( C_4 \), but claimed that these were functions of the material and could be determined experimentally. While Boston and Oxford expressed the forces in terms of diameter and feed only, they drew attention to the possibility that the forces were not only functions of the outside diameter but also of the drill geometry. The effect of drill geometry has been discussed in the previous section of this survey.

Kronenberg (38) derived an equation for torque using a different approach. He made use of the cutting pressure (specific energy) equation for turning, which he experimentally found and reported earlier (39). This equation was given by:

\[ U = \frac{1}{\xi} C \left(1,000 A\right)^{1 - \frac{1}{\xi}} \]

(23-1)
where: $U$ is the total cutting pressure
$Z$ is the number of lips
$A$ is the cross-section area of the chip ($= \frac{fd}{2}$).
$f$ is the feed rate
$d$ is the diameter of the drill
$C$ and $\xi$ are constants.

Assuming that torque was a function of this cutting pressure and diameter of the drill, and substituting $A = \frac{fd}{2}$ into equation 23-1, the torque became a function of feed and diameter only, i.e.

$$ M = g(f, d^2) $$

(24-1)

By experiment $\xi$ was determined and $C$ was given as $-1765$ for steel. The final form of his torque equation became:

$$ M = 2020 \cdot d^{1.803} f^{.803} $$

(25-1)

(ii) Dimensional Analysis.

The prediction of thrust and torque to this time had been based mainly on experimental results. There was hardly any analytical approach taking into consideration the basic geometry of the drill. Shaw and Oxford Jr. (4) making use of the concept of specific energy $U$, were able to derive the force equation using dimensional analysis. They demonstrated that a change in feed rate $f$ in drilling would change the specific energy $U$ according to the equation:

$$ U = K_1 f^a $$

(26-1)
where $K_1$ is a constant and "$a"$ is close to .2 for the normal range of feeds. They expressed the torque as a function of a number of variables:

$$M = E_1 \left( H_B, q, f, d, S \right)$$

where:
- $H_B$ is Brinnel hardness
- $q$ is chisel edge length
- $d$ is diameter of drill
- $S$ is main spacing of atomic imperfection in the metal.

Based on dimension analysis, the torque function was converted to the form:

$$\frac{M}{d^3H_B} = \varepsilon_2 \left( \frac{f}{d}, \frac{q}{d}, \frac{S}{d} \right)$$

The total torque was given by:

$$M = M_L + M_C \quad (27-1)$$

where $M_L$ and $M_C$ are torque due to the lips and the chisel edge respectively.

The energy per unit volume associated with the cutting edges and the chisel edge were given by:

$$U_L = \frac{2 \, \pi \, M_L}{4 \, \pi \, f \, (d^2 - q^2)} \quad (28-1)$$
and \[ U_L = \frac{2\pi M_C}{\pi/4 \cdot f \cdot C^2} \] (29-1)

Re-writing equations 28-1 and 29-1, it follows that:

\[ \frac{M_L}{d^3} = \frac{U_L}{8} \cdot \frac{f}{d} \sqrt{1 - \left( \frac{q}{d} \right)^2} \] (30-1)

and \[ \frac{M_C}{d^3} = \frac{U_C}{8} \cdot \frac{f}{d} \left( \frac{q}{d} \right)^2 \] (31-1)

thus:

\[ \frac{M}{d^3} = \frac{M_L}{d^3} + \frac{M_C}{d^3} \]

\[ = \frac{1}{8} \cdot \frac{f}{d} \left\{ U_L \sqrt{1 - \left( \frac{q}{d} \right)^2} + U_C \left( \frac{q}{d} \right)^2 \right\} \] (32-1)

Shaw and Oxford Jr. then represented the specific energy by the following equation:

\[ U = K_2 H_B \left( \frac{S^2}{f \cdot d} \right)^a \] (33-1)

where \( K_2 \) is a constant and \( H_B \) is Brinnel hardness. The term was obtained by considering the number of imperfections encountered in one revolution of drilling.

Thus for cutting along the lips:

\[ U_L = K_3 H_B \left( \frac{S^2}{f \cdot (d + q)} \right)^a \] (34-1)

and \[ = K_4 H_B \left( \frac{S^2}{f \cdot q} \right)^a \] (35-1)
Substituting equations 34-1 and 35-1 with 28-1,
gives:

\[
\frac{M}{d^3H_B} = -\frac{S^2a f^{1-a}}{d^{1+a}} K_5 \left[ \frac{1 - (\frac{q}{d})^2}{(1 + \frac{q}{d})^2} + K_6(-\frac{q}{d})^{2-a} \right] 
\]

For a given metal \( S \) was considered constant and "a" was assumed to be \( \cdot 2 \) as before (26-1). The form of the torque equation became

\[
\frac{M}{d^3H_B} = K_7 f^{1.2} \frac{1}{d^{1.2}} \left[ \frac{1 - (\frac{q}{d})^2}{(1 + \frac{q}{d})^2} + K_8(-\frac{q}{d})^{1.8} \right] 
\]

Again using dimensional analysis the thrust equation was represented by:

\[
\frac{T}{d^2H_B} = \varepsilon_3 \left( \frac{f}{d}, \frac{q}{d}, \frac{S}{d} \right) 
\]

The total thrust was given by:

\[
T = T_L + T_C + T_e 
\]

where \( T_L \) and \( T_C \) are thrust due to cutting at the lips and chisel edge respectively, and \( T_e \) is due to extrusion at the chisel edge. Following the same approach as before, the equation for thrust was given by:

\[
\frac{T}{d^2H_B} = K_9 f^{1.2} \frac{1}{d^{1.2}} \left[ \frac{1 - (\frac{q}{d})^2}{(1 + \frac{q}{d})^2} + K_{10} \left( \frac{q}{d} \right) + K_{11} \left( \frac{q}{d} \right)^2 \right] 
\]
Shaw and Oxford Jr. used $\frac{q}{d} = 0.18$ which they considered to be the average value for most drills, and finally obtained the following equations:

$$T = 0.195 H_B d^8 + 0.002 H_B d^2$$

$$M = 0.087 H_B d^{1.8} f$$

The equations established by all the workers are comparable, even though the approaches were different. Boston and Oxford's method was purely experimental, Kronenberg's method involved the application of the empirical force equation for turning to drilling and Shaw and Oxford Jr's method was semi-analytical. The exponential terms in Kronenberg's and Shaw and Oxford Jr's torque equations were practically equal despite the difference in the approaches. The magnitude of these exponential terms are controlled by "a" and "ε", which had to be determined experimentally.

**CONCLUDING REMARKS**

Because of its complex geometry the twist drill specification is more complex than for many other cutting tools. According to Galloway (8), drill specifications consist of twenty elements such as the flute, body clearance and the chisel edge, thirteen measurements, such as the diameter and web-thickness, and eight angles. All these add up to forty-one definitions (c.f. 35 definitions by the B.S 328 - 1959). Galloway's specification and those of the
B.S. 328 - 1958 are nevertheless very similar. Galloway defines the relief flank and the clearance flank and consequently the relief angle and the clearance angle, which is different from the clearance angle as defined by the B.S. 328 - 1959.

Both the B.S. 328 - 1959 and Galloway do not specify the appropriate or acceptable values for web-thickness, helix angle (or the pitch length), chisel edge angle and other such angles. One of the reasons for not specifying these parameters is that they should vary according to the material being drilled, depth of the holes and the hole accuracy required. Thus it is left to the users to specify and the manufacturer to recommend these parameters in each particular application.

Another difficulty in specifying the drill geometry is that all angles and elements are inter-related. It is possible that some combinations of specified angles and elements are actually not practical. Therefore understanding the grinding technique will facilitate any specification of the drill point geometry in quantitative terms.

The specification of twist drills do not include the fundamental cutting angles \( \alpha_n \) and \( i \). It therefore appears that the specification of twist drills have been based on criteria such as ease of grinding and inspection.

Perhaps because of the varying geometry, the chip formation process in drilling has not been fully studied. Oxford Jr. made a rather detailed study of the chip flow angle and went as far as
to check if Stabler's chip flow rule is applicable in drilling. Pal and others demonstrated that chip length ratio is a variable along the cutting lips. By generalizing the drill lip geometry to that of a lathe tool and applying Merchant's orthogonal cutting equation, they obtained an expression for torque. Bera and Bhattacharyya later supplemented on this work using approximately the same approach. The validity of their analysis is open to doubt and can only be considered as an application of Merchant's cutting equation to drilling but not as an attempt to develop a thin shear zone analysis to drilling. Their results however suggests that it is possible to predict forces in drilling from Merchant's thin shear zone model. Further research is necessary in order to develop a more realistic cutting model for drilling, from which the chip formation and forces may be readily predicted.

In the absence of a cutting model for drilling, the prediction of thrust and torque is well provided for by empirical and semi-analytical equations. These equations are supposed to be applicable to all sizes of drills. However it appears that when these equations were experimentally obtained, the difference in geometry between the various drills was not studied. The possible variation in thrust and torque due to a change in geometry of the same size drill (e.g. different manufacturer specifications) was not accounted for. Therefore in order to be able to understand under what conditions these empirical equations can be applied it is necessary to study the geometry of the various drills, and the possible geometric
relationship between them.

In conclusion, there appeared to have been a large volume of experimental work done on drilling and valuable information about the performance of the conventional drills has been gained from these investigations. On the other hand the analytical approach has not been followed by many workers. The analysis of mechanics of drilling has not been attempted from first principles and any approach in this direction should prove interesting.
II. ANALYSES OF SIMULATED AND CONVENTIONAL DRILLING OPERATIONS

It has been shown that the normal rake angle $\alpha_r$, the angle of obliquity $\theta$, and the work velocity $V_w$ all vary along the cutting edges (lips) of the drill. For these reasons, apart from the curvature of the drill rake face (flute) and the indenting action of the chisel edge, the chip formation model for drilling is expected to be more complex than in other types of metal cutting operations. Complex as it may be, if a better understanding of the drilling process is to be achieved, attempts must be made to develop analyses, which will qualitatively and quantitatively describe the chip formation in drilling, and enable the cutting forces to be predicted. While it is appreciated that the chisel edge action, which has been thought similar to "extrusion", will complicate the chip formation even further, there is no doubt that the cutting occurs on the drill lips. Therefore the cutting mechanics of the lips will be studied as a first step in the analysis of the drilling operations.

CUTTING MODELS FOR SIMULATED DRILL LIPS

Before any formal attempt is carried out to derive a cutting model for drilling, studies will be made on some simple cutting cases which can be simulated to the drilling process to a lesser or greater
Simulated drilling (drill lips)

Fig 1-2
Fig. 1-2(a) depicts a cutting tool with a flat rake face ABCD cutting a tubular element, which rotates about its longitudinal axis. The cutting edge AB is in line with the radius of the element. This case can be considered as one of drilling with a drill having zero web-thickness, i.e. \( 2t = 0 \) and half point angle \( P \) of \( 90^\circ \). The peripheral speed or the work velocity \( V_w \) varies from the centre to the outside radius, according to the relation:

\[
V_w = 2\pi N r
\]  

(1-2)

In the literature survey, the angle of obliquity \( \psi \) and the normal rake angle \( \alpha_n \) were given by the following equations when the feed velocity \( V_f \) was ignored:

\[
\sin \psi = \frac{t}{r} \sin P
\]

(2-2)

\[
\alpha_n = \alpha_f - \psi
\]

(3-2)

and

\[
\tan \psi = \tan \omega \cos P = \frac{t}{(r^2 - t^2)^{1/2}} \cos P
\]

(4-2)

where \( \alpha_f \), \( \psi \), \( \omega \), \( P \), \( t \) and \( r \) have the usual meaning as in the survey.

Using equations 2-2, 3-2 and 4-2, where \( 2t = 0 \), it follows that \( \psi = 0 \) and \( \alpha_n = \alpha_f \).

In this case the angle \( \alpha_f \) is made a constant by grinding.
a constant rake angle on the tool. Actually this model can be considered as orthogonal cutting with a variable work velocity \( V_w \), since \( V_w \) is perpendicular to the cutting edge at any radius \( r \). Thus the effect of a variable velocity as occurs in drilling can be readily simulated.

The model shown in Fig. 1-2(b) is a closer representation of the drillings process. The half point angle \( \beta \) is \( 90^\circ \) but the cutting edge is no longer in line with the centre of rotation of the work-piece. This off-set of the tool can be considered comparable to the half web-thickness of the drill \( t \). From equations 2-2, 3-2 and 4-2, \( \alpha_n \) is a constant and is equal to \( \alpha_f \), but \( i \) is now a varying parameter.

In Fig. 1-2(c), the cutting edge \( AB \) is tilted to make the half point angle \( \beta \) less than \( 90^\circ \). From the same equations as before, it is noted that the normal rake angle \( \alpha_n \), the angle of obliquity \( i \) and the work velocity \( V_w \) are all varying. The angle of obliquity increases and the normal rake angle decreases as the radius is increased.

The model depicted in Fig. 1-2(c) represents a form of drilling, e.g. masonry drills or the modified drill suggested by Armarego (22). The basic difference between the case in Fig. 1-2(c) and the conventional twist drills is that in the former the rake face is a flat plane. Thus the reference rake angle \( \alpha_f \), as used in drilling terminology, is constant. In a conventional drill
the rake face is a curved surface and the reference rake angle varies (26). The mechanics of metal cutting has been studied by many workers (41 - 48), all of which have concentrated on cutting with flat rake faced tools. Furthermore deformation was usually based on thin shear plane or zone models. There appears to be no work on the cutting mechanics for tools with curved rake faces.

In each of the above cases the principles of orthogonal or oblique cutting may be applied, at least when individual points along the cutting edge are considered independently. Some means of accounting for the variable geometry should be found.

In view of the possible practical advantages of modifying conventional drills to give a flat rake face (22), it was decided to consider this type of drills first. The technique of grinding such drills is given in Appendix A. An attempt to develop an analysis for conventional drills will also be made.

THIN SHEAR ZONE CUTTING MODEL FOR FLAT RAKE DRILL

Fig. 2-2 shows the flat rake drill in four views. Considering two points L and M along the cutting edge, the geometry of these two points can be described by the cutting angles \( i_L \), \( i_M \), \( \alpha_{nL} \) and \( \alpha_{nM} \), where \( i \) and \( \alpha_n \) are the angle of obliquity and the normal rake angle respectively. Assuming the
True view of chip velocity and chip flow angle

True end view of flat rake face

Drill point geometry and deformation parameters (flat rake drill)

Fig 2-2
thin shear zone or plane model applies and studying these two points independently, the velocity triangle must close in order to maintain continuity. Using the usual oblique cutting equations, it follows that:

\[
\frac{V_{C_L}}{V_{W_L}} = \frac{\sin \phi_{n_L} \cdot \cos \alpha_{n_L}}{\cos (\phi_{n_L} - \alpha_{n_L}) \cos \gamma_{c_L}} \tag{5-2}
\]

\[
\frac{V_{C_M}}{V_{W_M}} = \frac{\sin \phi_{n_M} \cdot \cos \alpha_{n_M}}{\cos (\phi_{n_M} - \alpha_{n_M}) \cos \gamma_{c_M}} \tag{6-2}
\]

where \(\gamma_{c_L}\) and \(\gamma_{c_M}\), \(\phi_{n_L}\) and \(\phi_{n_M}\) are the chip flow angles in the rake face and normal shear angles at L and M respectively, \(V_C\) and \(V_W\) are the chip and work velocities.

The chip formation at L and M has to be related if a cutting model is to be developed for the cutting edge as a whole. Since the deformation is considered to occur on a thin shear plane at each point of the cutting edge, deformation will not occur on either side of the shear zone. Therefore it is reasonable to expect the chip to flow as a rigid body along the drill flute. This condition is satisfied when the relative chip velocity between these two points L and M is zero, i.e.:
\[ V_{c_L} \sin \eta_{c_L} - V_{c_M} \sin \eta_{c_M} = 0 \]

or
\[ V_{c_L} \sin \eta_{c_L} = V_{c_M} \sin \eta_{c_M} \] (7-2)

According to this equation, the chip formation at one point will be affected by that at other points. There is further qualitative evidence to substantiate the applicability of equation 7-2. In Fig. 2-2, point M is nearer to the axis of the drill so that the work velocity \( V_w \) is smaller than \( V_w \) i.e.
\[ V_{w_M} < V_{w_L} \]
and the angle of obliquity \( \eta_{c_M} \) is larger than \( \eta_{c_L} \). If Stabler's rule is to apply, even to first approximation, then
\[ \eta_{c_M} > \eta_{c_L} \]
or
\[ \sin \eta_{c_M} > \sin \eta_{c_L} \]
From these two inequality expressions:
\[ V_{c_M} > V_{c_L} \]
and
\[ \sin \eta_{c_M} > \sin \eta_{c_L} \]
it may then be possible that the equality:
\[ V_{c_M} \sin \eta_{c_M} = V_{c_L} \sin \eta_{c_L} \]
be satisfied.

Manipulating equations 5-2 and 6-2, gives:
\[
\begin{align*}
\frac{V_{cL}}{V_{cM}} &= \frac{V_w \sin \phi_n \cos i \cos (\phi_n - \alpha_n) \cos \eta_c}{V_w \sin \phi_n \cos i \cos (\phi_n - \alpha_n) \cos \eta_c} \\
\frac{V_{cL}}{V_{cM}} &= \frac{\sin \eta_c}{\sin \eta_c} \\
\frac{V_w \sin \phi_n \cos i \cos (\phi_n - \alpha_n) \cos \eta_c}{V_w \sin \phi_n \cos i \cos (\phi_n - \alpha_n) \cos \eta_c} &= \frac{V_{cL}}{V_{cM}} \frac{\sin \eta_c}{\sin \eta_c} \\
\text{and from equation 7-2:} \\
\frac{V_{cL}}{V_{cM}} &= \frac{\sin \eta_c}{\sin \eta_c} \quad (9-2) \\
\text{combining equations 8-2 and 9-2, it follows that:} \\
\frac{\sin \eta_c}{\sin \eta_c} &= \frac{V_w \sin \phi_n \cos i \cos (\phi_n - \alpha_n) \cos \eta_c}{V_w \sin \phi_n \cos i \cos (\phi_n - \alpha_n) \cos \eta_c} \\
\frac{V_{cL}}{V_{cM}} &= \frac{\sin \eta_c}{\sin \eta_c} \quad (9-2) \\
\text{Since the work velocity } V_w \text{ is proportional to the radius, then:} \\
\frac{V_{wL}}{V_{wM}} &= \frac{2 \pi N r_L}{2 \pi N r_M} = \frac{r_L}{r_M} \quad (11-2) \\
\text{Arranging equations 10-2 and 11-2, provides} \\
\frac{\sin \eta_c}{\sin \eta_c} &= \frac{r_L \sin \phi_n \cos i \cos (\phi_n - \alpha_n) \cos \eta_c}{r_M \sin \phi_n \cos i \cos (\phi_n - \alpha_n) \cos \eta_c} \\
\text{or}
\end{align*}
\]
For a drill of known diameter and web-thickness, and with a pre-ground constant reference rake angle \( \alpha_f \), the angle of obliquity \( \alpha_i \) and \( \alpha_n \), the normal rake angles \( \alpha_{nL} \) and \( \alpha_{nM} \) can be readily found. \( \alpha_{nL} \) and \( \alpha_{nM} \) can be any points on the cutting edge. What are yet to be determined in equation 12-2 are the normal shear angles \( \phi_{nL} \) and \( \phi_{nM} \), and the chip flow angles \( \gamma_{cL} \) and \( \gamma_{cM} \). As for oblique cutting, the chip flow direction and the shear angle describe the deformation configuration, the problems of predicting these two parameters will also occur in drilling. The difficulties in experimentally determining the chip flow angle have been briefly discussed in the literature survey. Although there is no theoretical justification for Stabler's chip flow rule, there is experimental evidence suggesting that \( \gamma_{c} \) depends mainly on \( i \) and is also influenced by \( \alpha_n \) (49) (50). It may be assumed that the chip flow angle obeys either one of the following relations:

\[
\gamma_{c} = f(i) \quad \text{(13-2)}
\]

or

\[
\gamma_{c} = f(i, \alpha_{n}) \quad \text{(14-2)}
\]
The first relation can be represented as Stabler's chip flow rule which states:

\[ \eta_c = i \]

The second relation was reported by Armarego and Brown (49) and Russell and Brown (50).

Accepting either one of these two functions as representing the chip flow angle, the angles \( \eta_{c_L} \) and \( \eta_{c_M} \) at all radii are determined. The two remaining unknowns in equation 12-2 are the normal shear angle \( \phi_{n_L} \) and \( \phi_{n_M} \). If either one of these two angles can be estimated, then the other one will follow according to equation 12-1. In fact the shear angle distribution along the cutting edge can be found provided one value is known. In oblique cutting, the normal shear angle is given by:

\[
\tan \phi_n = \frac{r_1 \cos \alpha_n \cos \eta_c / \cos i}{1 - r_1 \sin \alpha_n \cos \eta_c / \cos i} \quad (15-2)
\]

where \( r_1 \) is the chip length ratio. If the chip length ratio at a radius, say point L, is determined experimentally, the normal shear angle can be obtained from equation 15-2. Substituting \( \phi_{n_L} \) into equation 12-2, the other normal shear angle \( \phi_{n_M} \) is evaluated. M can be any point on the cutting edge and this process can be repeated until the normal shear angle and the chip length ratio distributions along the cutting edges are established using both
equations 12-2 and 15-2.

The deformation along the drill lips can therefore be predicted provided the chip length ratio at one radius is known. One of the major aims of building up this drilling model is to enable the prediction of thrust and torque of the flat rake drill. Understanding of the deformation process is the first step in this direction. Fig. 3-2(a) and 3-2(b) show a simple oblique cutting model and a drilling model. Since drilling is basically oblique cutting, the elemental power force $\Delta F_p$ is in the view showing the true work velocity $V_w$ and the true cutting edge. The elemental force component $\Delta F_Q$ is perpendicular to the work velocity in the view containing the normal rake angle, and the elemental radial force $\Delta F_R$ is mutually perpendicular to $\Delta F_p$ and $\Delta F_Q$. From oblique cutting (49) the equations for $\Delta F_p$, $\Delta F_Q$ and $\Delta F_R$ can be expressed as:

$$\Delta F_p = \frac{T' \Delta b t'}{\sin \phi_n} \cdot \frac{\cos (\beta_n - \alpha_n) + \tan i \tan \gamma c \sin \beta_n}{\sqrt{\cos^2 (\phi_n + \beta_n - \alpha_n) + \tan^2 \gamma c \sin^2 \beta_n}^{1/2}} - \ldots$$

$$\Delta F_Q = \frac{T' \Delta b t'}{\sin \phi_n \cos i} \cdot \frac{\sin (\beta_n - \alpha_n)}{\sqrt{\cos^2 (\phi_n + \beta_n - \alpha_n) + \tan^2 \gamma c \sin^2 \beta_n}^{1/2}} - \ldots$$

$$\Delta F_R = \frac{T' \Delta b t'}{\sin \phi_n} \cdot \frac{\cos (\beta_n - \alpha_n) \tan i - \tan \gamma c \sin \beta_n}{\sqrt{\cos^2 (\phi_n + \beta_n - \alpha_n) + \tan^2 \gamma c \sin^2 \beta_n}^{1/2}} - \ldots$$
Forces and undeformed chip thickness in drilling

(a)

(b)

Fig 3-2
where: 
' \tau \prime \text{ is the shear stress of the material}

b \text{ is the width of cut}

t' \text{ is the undeformed chip thickness}

\beta_n \text{ is the normal friction angle.}

In Fig. 3-2(b), the elemental width of cut \( \Delta b \) at radius \( r \) and the corresponding elemental length \( \Delta l \) of the cutting edge are related by the equation:

\[ \Delta l = \frac{\Delta b}{\cos \theta} \]  \hspace{1cm} (19-2)

Also the corresponding element radius \( \Delta r \) is related to \( \Delta l \) by:

\[ \Delta r = \Delta l \sin \theta \cos \omega \]  \hspace{1cm} (20-2)

Combining equations 19-2 and 20-2, gives:

\[ \Delta r = \frac{\sin \theta \cos \omega}{\cos \theta} \]  \hspace{1cm} (21-2)

The forces per unit radius are thus given by:

\[ f_p = \frac{\Delta F_p \cos \theta}{\Delta b \sin \theta \cos \omega} \]  \hspace{1cm} (22-2)

\[ f_q = \frac{\Delta F_Q \cos \theta}{\Delta b \sin \theta \cos \omega} \]  \hspace{1cm} (23-2)

\[ f_r = \frac{\Delta F_R \cos \theta}{\Delta b \sin \theta \cos \omega} \]  \hspace{1cm} (24-2)
Following the definition of undeformed chip thickness in oblique cutting, the undeformed chip thickness \( t' \) in this model is the perpendicular distance between two projected work velocities in the view showing the normal rake angle as illustrated in Fig. 3-1(b). \( V_w' \) is the projected worked velocity exactly \( \frac{1}{2} \) revolution ahead of \( V_w \). Considering one cutting edge only, thus undeformed chip thickness \( t' \) can be expressed in terms of the feed penetration \( f \):

\[
t' = f \sin \phi \cos \psi
\]

(25-2)

Substituting \( \Delta F_p, \Delta F_q, \Delta F_r \) and \( t' \) into equations 22-2, 23-2, and 24-2, the following equations of force per unit radius are obtained:

\[
f_p = \frac{T' f \sin \phi \cos \psi \cos \iota}{2 \sin \phi_n \sin \phi \cos \omega} \cos (\beta_n - \alpha_n) + \tan \iota \tan \gamma_c \sin \beta_n \]

\[
f_q = \frac{T' f \cos \psi}{2 \sin \phi_n \cos \omega} \sin (\beta_n - \alpha_n)
\]

\[
f_r = \frac{T' f \cos \psi \cos \iota}{\sin \phi_n \cos \omega} \cos (\beta_n - \alpha_n) \tan \iota - \tan \gamma_c \sin \beta_n
\]

It is noted that \( f_p, f_q \) and \( f_r \) are functions of \( t', \phi_n, \alpha_n, \iota, \gamma_c, \beta_n \) and \( f \). \( t' \) is the shear stress and a known property of the material and may be considered constant.
At a particular radius the angle $\alpha_n$ and $\iota$ can be readily determined using equations 2-2, 3-2 and 4-2. The chip flow angle $\eta_c$ has to be assumed and the normal shear angle $\phi_n$ can be evaluated when investigating the deformation process. The remaining unknown in these force equations is the normal friction angle $\beta_n$. Assuming the force and velocity are collinear, i.e.:

$$\eta_s = \eta'_s$$
$$\eta_c = \eta'_c$$

where: $\eta'_s$ is the shear force angle in the shear plane
$\eta'_c$ is the friction force angle in the rake face

The normal friction angle is related to the other parameters by (30):

$$\tan (\phi_n + \beta_n) = \frac{\tan \iota \cos \alpha_n}{\tan \eta_c - \sin \alpha_n \tan \iota}$$

$\beta_n$ can thus be found provided $\phi_n$, $\iota$, $\eta_c$ and $\alpha_n$ are known.

In drilling, these forces, i.e. $f_p$, $f_q$ and $f_r$, are not the forces of direct interest. It is the thrust and torque that are of prime importance. The thrust per unit radius $f_{th}$ at radius $r$ and the tangential force per unit radius $f_{tang}$ can be expressed in terms of $f_p$, $f_q$ and $f_r$. They are given as:

$$f_{th} = \int f_p \sin i - f_r \cos i \int \cos P$$
$$- \int (f_p \cos i + f_r \sin i) \sin \psi - f_q \cos \psi \int \sin P$$
\[ f_{\text{tang}} = \int (f_p \sin i - f_r \cos i) \sin \psi \cos \omega \]
\[ + \int (f_p \cos i + f_r \sin i) \sin \psi - f_q \cos \psi J \cos \phi \sin \omega \]

Algebraic manipulation shows that some of the coefficients of the terms of \( f_p \), \( f_q \) and \( f_r \) are equal to zero. Thus the two equations can be simplified to the following forms:

\[ f_{\text{th}} = f_q \cos \psi \sin \phi - f_r (\cos i \cos \phi + \sin i \sin \psi \sin \phi) \quad (30-2) \]
\[ f_{\text{tang}} = f_p \quad (31-2) \]

The total thrust \( F_{\text{th}} \) and torque \( F_{\text{torq}} \) are equal to the integrations of \( f_{\text{th}} \) and \( f_{\text{tang}} \cdot r \) with respect to the radius. The exact integrations procedure is impossible to be carried out due to the complicated terms in equation 30-2 and 31-2. Instead the total force can be obtained using numerical method of summation. Thus the total thrust \( F_{\text{th}} \) and torque \( F_{\text{torq}} \) can be represented as:

\[ F_{\text{th}} = 2 \cdot \sum_{r = R_1}^{R_2} f_{\text{th}} \cdot \Delta r \quad (32-2) \]
\[ F_{\text{torq}} = 2 \cdot \sum_{r = R_1}^{R_2} f_{\text{tang}} \cdot r \cdot \Delta r \quad (33-2) \]

where \( R_1 \) and \( R_2 \) are two radius limits over which the \( F_{\text{th}} \) and \( F_{\text{torq}} \) are required.

Equation 25-2 represents the undeformed depth of cut \( t' \) as a function of feed \( f \), the point angle \( 2 \phi \) and the projected velocity angle \( \psi \). Since \( \psi \) is a variable \( t' \) is expected to
vary along the cutting edge as well. It is necessary to check if this is the correct function for $t'$. The elemental area of cut $\Delta A$ over an elemental width $\Delta b$ at radius $r$ is given by:

$$\Delta A = \frac{f}{2} \sin \phi \cos \psi \cdot \Delta b$$

Therefore the total area of cut $A$ is given by:

$$A = \int_{b_1}^{b_2} \frac{f}{2} \sin \phi \cos \psi \cdot db$$

It has been established that:

$$\frac{\Delta b}{\Delta r} = \frac{\cos i}{\sin \phi \cos \omega}$$

therefore:

$$A = \int_{R_1}^{R_2} \frac{f}{2} \cdot \frac{\cos i \cos \psi}{\cos \omega} \cdot dr$$

Where the fraction $\frac{\cos i \cos \psi}{\cos \omega}$ can be proved equal to unity, hence:

$$A = \frac{f}{2} \int_{R_1}^{R_2} 1 \cdot dr$$

$$= \frac{f}{2} \left[ R_2^R_1 \right]$$
This is in fact the true area of cut and thus the function for $t'$ is reasonable. The volume of material removed is therefore correctly accounted for.

i) Numerical Testing of Cutting Model For Flat Rake Drills:

In order to check if the model will predict the expected chip formation process and forces, it is decided to test the model numerically. Two parameters $r_1$ and $\kappa_c$ have to be determined before using the model. The chip length ration $r_1$ is selected at 0.53 at the outside radius of a 1.00" diameter drill, whose normal rake angle $\alpha_n$ at this radius is about 30°. The value of 0.53 roughly corresponds to that of orthogonal cutting with the same normal rake angle. A chip flow condition must also be selected and the first one used is Stabler's chip flow rule, i.e.:

$$\eta_c = i$$

the second chip flow condition is:

$$\eta_c = k i$$

where $k$ is a function of $\alpha_n$. From data supplied by Armarego and Brown (49), it is noted that $K$ may be represented by:

$$k = C_1 - C_2 \alpha_n$$

$C_1$ and $C_2$ are constants to be determined from the data. For testing purpose, $C_1$ and $C_2$ are taken as 0.90 and 0.02 respectively.
The data obtained by Armarego and Brown was from tests run on mild steel. Therefore the values of $C_1$ and $C_2$ and consequently $k$ may be different for a different material. However as a first investigation of the applicability of the model, these two values of $C_1$ and $C_2$ can be considered as satisfactory. The third chip flow condition was proposed by Russell and Brown (50) based on some experimental evidence, i.e.:

$$\tan \gamma_c = \tan i \cos \alpha_n$$

Figs. 4-2(a) - 4-2(f) show the predicted chip length ratio, shear angle and forces. The only difference between the three cases is the input chip flow condition as shown in Fig. 4-2(a). The subsequent variations in the predicted chip length ratio, the shear angle and the forces are depicted in Fig. 4-2(b) - 4.2(h). The small variation in the input chip flow angles causes large variations among the other parameters except for $f_r$. This is especially obvious in the cases of $f_p$ and $f_q$. The trend of chip length ratio when using the relationship $\gamma_c = i$ is rather unexpected. In this instant the model suggests variation in chip length ratio at the various radii. Because of the decreasing normal rake angle towards the centre of the drill, the chip length ratio is expected to decrease as predicted by the other two cases.

The trends of $f_p$, $f_q$ and $f_r$ are in agreement with those in orthogonal and oblique cutting that they increase from the outside radius inwards.
Chip flow angle ($\eta_c$) distributions

Shear angle ($\phi_n$) distributions

Fig 4-2
Chip length ratio ($r_l$) distributions

\[ \eta_c = i \]
\[ \eta_c = (C_1 - C_2 \alpha_n)i \]
\[ \tan \eta_c = \tan i \cos \alpha_n \]

\[ f_r \] (lb)

\[ \eta_c = i \]
\[ \eta_c = (C_1 - C_2 \alpha_n)i \]
\[ \tan \eta_c = \tan i \cos \alpha_n \]

Fig 4-2
Fig 4-2

\[ \gamma_c = (C_1 - C_2 \cos \alpha_n) i \]

\[ \tan \gamma_c = \tan \cos \alpha_n \]

\[ \gamma_c = i \]
Thrust ($f_{th}$) distributions

Torque ($f_{torq}$) distributions

Fig 4-2
Fig. 4-2(g) and 4-2(h) represent the predicted distributions of \( f_{th} \) and \( f_{torq} \). While the trends of the predicted torque appears to be reasonable and are in agreement with each other, the trends of the predicted thrust are quite different.

In view of the facts of a constant chip length ratio, a negative \( f \) and finally a negative \( f \) at all radii, the model, when using \( \gamma_c = i \), is considered as unsatisfactory. The trends for the other chip flow conditions are reasonable. It appears at this stage that the cutting model is inherently sound in principle and that it can be used to qualitatively predict the deformation and forces in drilling. It is difficult to conclude which chip flow condition, i.e. \( \gamma_c = k_i \) or \( \gamma_c = \tan i \cos \alpha_n \), represents the true chip flow condition in the drilling, since the trends of the predicted thrust are different in these two cases. Also the chip length ratio cannot be theoretically determined, experiments must be run to test which model can quantitatively predict the forces on the cutting edges. The summation process was carried out by computer and the programme is in Appendix 1 - C.
THIN SHEAR ZONE CUTTING MODEL FOR

CONVENTIONAL DRILLS

In view of the promising predicted trends obtained for the flat rake drill, a similar analysis is attempted for the conventional drill.

Since basically the conventional drill cuts like the flat rake drill, i.e. oblique cutting, the idea of rigid body flow may still be applied. Fig. 5-2 shows the chip flow angles \( \gamma_1, \gamma_2, \gamma_3, \gamma_4 \) and \( \gamma_c \) at one point Q on the cutting edge of a conventional drill. \( \gamma_c \) is the true chip flow angle and as in the previous case, these chip flow angles are arbitrarily fixed. The chip flow angle \( \gamma_4 \) is the angle between the normal to the work velocity and the tangent to the rake face at this point, and in effect it is equal to the reference rake angle \( \alpha_f \) at the same point, i.e.:

\[
\gamma_4 = \alpha_f
\]  

(34-2)

Equally true is:

\[
\gamma_1 = \delta
\]  

(35-2)

where \( \delta \) is the helix angle at point Q. From geometry of Fig. 5-2, \( \gamma_4 \) is given by:

\[
\tan \gamma_4 = \frac{x \cos (\gamma_2 - \omega) / \cos \gamma_2}{y \sin p + x \sin (\gamma_2 - \omega) \cos p / \cos \gamma_2}
\]  

(36-2)
Drill point geometry and chip flow angles
(conventional drill)

Fig 5-2
and:

\[ x = y \tan \eta_1 \]  \hspace{1cm} (37-2)

Substituting equation 37-2 into equation 36-2 gives:

\[ \tan \eta_4 = \frac{\tan \eta_1 \cos (\eta_2 - \omega) / \cos \eta_2}{\sin p + \tan \eta_1 \sin (\eta_2 - \omega) \cos p / \cos \eta_2} \]

From equations 34-2 and 35-2:

\[ \tan \alpha_f = \tan \eta_4 \]

\[ \tan \delta = \tan \eta_1 \]

Therefore equation 38-2 can be re-written as:

\[ \tan \alpha_f = \frac{\tan \delta \cos (\eta_2 - \omega) / \cos \eta_2}{\sin p + \tan \delta \sin (\eta_2 - \omega) \cos p / \cos \eta_2} \]

Equation 39-2 represents the reference rake angle and Oxford Jr's (26) equation for this angle is given as:

\[ \tan \alpha_f = \frac{\tan \delta \cos \omega}{\sin p - \tan \delta \sin \omega \cos p} \]  \hspace{1cm} (40-2)

Equations 39-2 and 40-2 can only be equal if and only if \( \eta_2 = 0 \), which implies that the chip flow direction must be collinear with the input work velocity \( V_w \) in a view normal to the drill axis as shown in Fig. 6-2.
Fig. 6-2 shows the work and chip velocity at a particular radius \( r \). To satisfy the conditions of rigid body flow, the relative velocity between two points must be equal to zero. This condition can be represented by the following equations:

\[
V_C \cos \delta = \text{constant} = C \quad (41-2)
\]

and

\[
V_C \sin \delta = \mathcal{L}_c r \quad (42-2)
\]

so that the chip travels in a helical path about the drill axis. \( \mathcal{L}_c \) is the angular velocity of the chips.

From the velocity triangle:

\[
V_C = \frac{\sin \phi_e}{\cos (\phi_e - \delta)} \cdot V_w \]

\[
= \frac{\sin \phi_e}{\cos (\phi_e - \delta)} \cdot \mathcal{L}_w r \quad (43-2)
\]

where \( \phi_e \) is the effective shear angle in the velocity plane and \( \mathcal{L}_w \) is the angular velocity of the work-piece and \( \delta \) is equivalent to the effective rake angle \( \phi_e \) \( (49) \).

For equations 41-2 and 42-2 to apply simultaneously:

\[
\tan \delta = \frac{\mathcal{L}_c r}{C} \quad (44-2)
\]

or

\[
\frac{2 \pi r}{L} = \frac{\mathcal{L}_c r}{C} \quad (45-2)
\]

\[
\frac{2 \pi}{L} = \frac{\mathcal{L}_c}{C}
\]
Drill point geometry and deformation parameters
(conventional drill)

Fig 6-2
where \( L \) is the pitch length of the drill.

Manipulating equations 41-2 and 45-2, give:

\[
C = V_C \cdot \cos \delta = \frac{\sin \phi_e}{\cos (\phi_e - \delta)} \cdot V_w \cdot \cos \delta
\]

\[
= \frac{V_w}{\cot \phi_e + \tan \delta}
\]

or

\[
\cot \phi_e = \frac{V_w - C \tan \delta}{C}
\]

\[
= \frac{\frac{L}{\cos \phi_e}}{2 \pi L} \cdot \frac{(L - 2 \pi J)}{L}
\]

The effect of feed is usually very small and is neglected in the analysis.

The true chip flow angle is given by:

\[
\sin \gamma_c = \frac{y \cos p + y \tan \delta \sin \omega \sin p}{V_c}
\]

\[
= \frac{V_c \cos \delta \cos p + V_c \sin \delta \sin \omega \sin p}{V_c}
\]

\[
= \cos \delta \cos p + \sin \delta \sin \omega \sin P
\]

And the reference shear angle \( \phi_s \) in the shear plane is given by:

\[
\tan \phi_s = \frac{y \sin p + \sin \omega \cos p}{\cos}
\]

\[
= \frac{\tan \phi_e \sin p}{\cos \omega} + \tan \omega \cos P
\]
\[
\tan \phi_n = \frac{\tan \phi_e \sin p}{\cos \omega} + \tan \psi
\]

(48-2)

From the geometry of the figure:

\[
\phi_n = \phi_s - \psi
\]

the following equation is obtained:

\[
\tan \phi_n = \tan (\phi_s - \psi)
\]

\[
= \frac{\tan \phi_e \sin P}{\cos \omega} \frac{\cos P - \sin \omega \sin P}{\tan \psi \tan \psi}
\]

(49-2)

The direction of shear velocity in the shear plane is given as:

\[
\tan \gamma_s = \frac{\tan \phi_e \cos P - \sin \omega \sin P}{\cos \omega}
\]

\[
= \cot \phi_e \left[\tan \phi_e \cos P - \sin \omega \sin P\right]
\]

(50-2)

In oblique cutting (49) the shear force angle \( \gamma_s \) is relative to the friction force angle \( \beta_n \) by:

\[
\tan \gamma_s' = \frac{\sin \beta_n \tan \gamma_c}{\cos (\phi_n + \beta_n - \alpha_n)}
\]

(51-2)

Assuming \( \gamma_c = \gamma_c' \) and \( \gamma_s = \gamma_s' \), and noting that \( \phi_n - \alpha_n = \phi_s - \alpha_s \), equation 51-2 can be re-arranged to give:

\[
\tan \beta_n = \frac{\tan \gamma_s \cos (\phi_n - \alpha_n)}{\tan \gamma_c' + \tan \gamma_s \sin (\phi_n - \alpha_n)}
\]

(52-2)
The only unknown in equation 52-2 is the normal shear angle. By measurement of the deformed chip length and using equation 15-2, the normal shear angle can be found. All the parameters i.e. $i$, $\gamma_c$, $\alpha_n$, $\phi_n$, $\omega$, $\nu$ and $\beta_n$, necessary for calculating the $f_p$, $f_q$ and $f_r$ can be evaluated. By similar reasoning as before the thrust and torque per unit radius are given by equations 32-2 and 33-2.


The model has been subjected to numerical testing to check if it provides reasonable trends. The torque appeared to be satisfactory - its magnitude increased with increase in radius. The thrust is negative at all radius implying that the drill is being pulled into the workpiece. This is obviously not true and consequently the analysis is suspect.

One of the possible reasons which leads to the failure of this model is the consideration that the chip flow direction must be coincident with the tangent to the rake face at the point in question, i.e.

$$\alpha_f = \eta_4$$

and

$$\delta = \eta_1$$

In order to satisfy this condition the chip flow angle $\eta_2$ must be equal to zero. Thus the chip has to flow in a direction collinear
to the work velocity $V_W$. This theoretical chip flow angle is contrary to the expected chip flow angle originally assumed. Also as a consequence of meeting the above requirement, the true chip flow angle is given by equation 47-2, of which all the angles are known values. Therefore the chip flow angle is a known function of the geometry of the point in question and one of the difficulties in oblique cutting, i.e. estimating the chip flow angle, is eliminated. But this is somewhat rather hard to understand.

The other unexpected trend of result is represented by equation 46-2. The effective shear angle increases as the radius decreases, even though the normal rake angle decreases.

The chip flow angle $\eta_2$ was not assumed, but made equal to zero to satisfy some geometric definition of the drill. In doing so the chip flow angles appear unlikely and the predicted results are also unreal. It is reasonable to state that any drilling model based on chip flow direction in tangent planes will lead to unsatisfactory results.
GEOMETRIC SIMILARITY OF CONVENTIONAL DRILLS

In the last sections, the cutting action of both flat rake and conventional drills were studied from first principles. The chip formation model for the flat rake drill showed reasonable trends when tested numerically. The second model for the conventional drills was found to be unsatisfactory since it gave unrealistic results such as negative thrust force. The possible reason for its failure was that the chip was forced to flow in some unrealistic direction when approximating the curved surface of the drill flute to a series of planes. Furthermore, little is known about the effect of rake face curvature on the chip formation process. However, conventional drills are the drills mainly used in industry. Therefore it is felt that more information and understanding about these drills is most desirable. In fact much work has been carried out by various drill investigators to establish thrust and torque equations. This point has been discussed in some detail in the literature survey. Most of these equations were established empirically, giving thrust and torque in terms of feed and diameter. These equations were represented as exponential functions. The effect of the basic geometry of the drill in most cases was not considered quantitatively. Boston and Oxford in presenting their empirical equations did mention the possible effect of drill geometry on forces. The change in geometry when the drill diameter is varied has not been clearly or deeply reported.
Therefore, at this point, it is decided to study the basic geometry and to search for some geometric inter-relationship among the various drill sizes.

Drills are usually specified in terms of parameters such as point angle, chisel edge angle and diameter. Considering a large drill simply as a magnification of a small drill, it is obvious that these two drills must satisfy some basic requirement in their geometry, such as equal point angle, equal normal rake angle and equal angle of obliquity at some corresponding radii. These are conditions for geometric similarity. Conventional drills will be considered in two parts: conditions for similarity of the cutting edges followed by condition for similarity of the chisel edge.

For two points on the cutting edges of two different size drills to be similar in geometry, these two points at least have to satisfy the simultaneous requirement of equal normal rake angle $\alpha_n$ and equal angle of obliquity. These two angles $\alpha_n$ and $\iota$ are given by:

$$\tan \alpha_n = \frac{\tan \delta}{\sin P} \left[ \cos \omega + \sin \omega \tan \omega \cos^2 P \right]$$

$$- \tan \omega \cos P$$

$$\sin \iota = \sin \omega \sin P$$

(53-2)  

(54-2)
Considering two drills 1 and 2, let \( i_1, i_2, \alpha_{n1} \) and \( \alpha_{n2} \) be the angle of obliquity and normal rake angle at radii 1 and 2. Also let \( t_1 \) and \( t_2 \) be the half web-thickness of drill 1 and drill 2 respectively.

**Fig 7-2**

From Fig. 7-2, it is noted that:

\[
\cos \omega = \frac{(r^2 - t^2)^{\frac{1}{2}}}{r} \\
\sin \omega = \frac{t}{r} \\
\tan \omega = \frac{t}{(r^2 - t^2)^{\frac{1}{2}}}
\]

The angle of obliquity \( i_1 \) and \( i_2 \) at their corresponding radii \( r_1 \) and \( r_2 \) are:

\[
\sin i_1 = \frac{t_1}{r_1} \sin \phi_1 \quad (56-2) \\
\sin i_2 = \frac{t_2}{r_2} \sin \phi_2 \quad (57-2)
\]
\[
\begin{align*}
\sin \frac{i_1}{2} &= \frac{t_1 r_2 \sin P_1}{\sin \frac{i_2}{2} t_2 r_1 \sin P_2} \quad (58-2)
\end{align*}
\]

Similarly the normal rake angles at these two radii \( r_1 \) and \( r_2 \) are as follows:

\[
\tan \alpha_{n_1} = \frac{\tan \delta}{\sin P_1} \left[ \cos \omega_1 + \sin \omega_1 \tan \omega_1 \cos P_1 \right] - \tan \omega_1 \cos P_1
\]

\[
= 2 \frac{\pi r_1}{L_1 \sin P_1} \left[ \frac{\left( r_1^2 - t_1^2 \right)^{\frac{1}{2}}}{r_1} + \frac{t_1}{r_1} \cdot \frac{t_1}{\left( r_1^2 - t_1^2 \right)^{\frac{1}{2}}} \cos^2 P_1 \right]
\]

\[
- \frac{t_1}{\left( r_1^2 - t_1^2 \right)^{\frac{1}{2}}} \cos P_1 \quad (59-2)
\]

and

\[
\tan \alpha_{n_2} = 2 \frac{\pi r_2}{L_2 \sin P_2} \left[ \frac{\left( r_2^2 - t_2^2 \right)^{\frac{1}{2}}}{r_2} + \frac{t_2}{r_2} \cdot \frac{t_2}{\left( r_2^2 - t_2^2 \right)^{\frac{1}{2}}} \cos^2 P_2 \right]
\]

\[
- \frac{t_2}{\left( r_2^2 - t_2^2 \right)^{\frac{1}{2}}} \cos P_2 \quad (60-2)
\]

Equations 58-2, 59-2 and 60-2 are functions of \( t, r, L \) and \( P \).

To satisfy simultaneous conditions for the two points \( r_1 \) and \( r_2 \):

\[
P_1 = P_2
\]

\[
i_1 = i_2
\]

\[
\alpha_{n_1} = \alpha_{n_2}
\]
It follows from equation 58-2 that

\[
\frac{t_2}{t_1} \frac{r_1 t_2}{r_2 t_1} = 1
\]

or

\[
r_2 = \frac{t_2}{t_1} r_1
\]  

(61-2)

Substituting equation 61-2 with equation 60-2, the normal rake angle can be represented as:

\[
\tan \alpha_{n_2} = \frac{t_2}{L_2} \frac{r_1 t_2}{t_1 \sin P_2} \left( \frac{r_1^2 - t_1^2}{r_1} \right)^{\frac{1}{2}} + \frac{t_1}{r_1} \frac{t_1 \cos^2 P_2}{(r_1^2 - t_1^2)^{\frac{1}{2}}}
\]

(62-2)

For \( \alpha_{n_1} = \alpha_{n_2} \), equations 59-2 and 62-2 can be equated, from which:

\[
\frac{2 \pi t_2}{L_2 t_1 \sin P_2} r_1 = \frac{2 \pi}{L_1} r_1
\]

i.e.

\[
\frac{t_1}{t_2} = \frac{L_1}{L_2}
\]  

(63-2)

where \( L_1 \) and \( L_2 \) are the pitch lengths of the two drills.

The normal rake angles for two points are equal if the web-thickness ratio is equal to the pitch length ratio. Another way of
arriving at equation 63-2 is by studying the web angle $\omega$ and the helix angle at two corresponding radii $r_1$ and $r_2$. For similarity the following conditions must hold, i.e.:

$$
P_1 = P_2
$$

$$
\omega_1 = \omega_2
$$

$$
\delta_1 = \delta_2
$$

hence:

$$
\sin \omega_1 = \frac{t_1}{r_1} \sin P_1
$$

$$
= \frac{t_2}{r_2} \sin P_2
$$

$$
= \sin \omega_2
$$

or

$$
\frac{t_1}{r_1} = \frac{t_2}{r_2}
$$

and

$$
\tan \delta_1 = \frac{2}{r_1} \frac{\pi}{L_1}
$$

$$
= \frac{2}{r_2} \frac{\pi}{L_2}
$$

$$
= \tan \delta_2
$$

or

$$
\frac{r_1}{r_2} = \frac{L_1}{L_2}
$$
Therefore combining these two equations:

\[
\frac{t_1}{t_2} = \frac{L_1}{L_2} = \frac{r_1}{r_2}
\]

Galloway's equation for the relief angle \( \gamma \) is given by:

\[
\tan \gamma = \sqrt{\sin \omega - \cos \omega \cot \theta} \cot P
\]

where \( \theta \) is the chisel edge angle. For \( \gamma_1 \) and \( \gamma_2 \) to be equal the following conditions must hold:

\[
P_1 = P_2
\]

\[
\omega_1 = \omega_2
\]

and

\[
\theta_1 = \theta_2
\]

Therefore this is a further requirement that the chisel edge angle must be equal for similarity.

The idea of similarity can be extended over a position or the whole of the cutting edges.

Similarity over portions of cutting lips of similar drills

Fig 8-2
Fig. 8-2 shows the end view of two drills which satisfy the conditions for similarity. If:

\[
\begin{align*}
    r_1 &= \frac{t_1}{t_2} \times r_2 \\
    r_1' &= \frac{t_1}{t_2} \times r_2'
\end{align*}
\]

the geometry of drill 1 between \( r_1 \) and \( r_1' \) is similar to that of drill 2 between \( r_2 \) and \( r_2' \). The condition for complete similarity of the cutting edges is satisfied if \( r_1' \) and \( r_2' \) are the maximum radii of the two drills respectively, i.e.:

\[
\frac{R_1}{R_2} = \frac{t_1}{t_2}
\]

or

\[
\frac{d_1}{d_2} = \frac{t_1}{t_2}
\]

The basic requirement for complete similarity at the lips is thus given by:

\[
\frac{d_1}{d_2} = \frac{t_1}{t_2} = \frac{L_1}{L_2}
\]

(64-2)

where \( d_1 \) and \( d_2 \) are the drill diameters.

When drilling with the same spindle speed, the peripheral speed would be higher at \( r_2 \) and \( r_2' \) than at \( r_1 \) and \( r_1' \). If speed is considered to have an effect on the deformation process and forces,
then it is important to ensure that equal speed as well as similar geometry at corresponding points is maintained. The peripheral speed at \( r_1 \) and \( r_2 \) can be made equal by either increasing the spindle speed of drill 1 or reducing the speed of drill 2. In doing so, the peripheral speed at other corresponding points may also be equal.

If \( N_1 \) and \( N_2 \) are the two spindle speeds of the two drills, to main equal peripheral speed at \( r_1 \) and \( r_2 \), i.e.:

\[
2 \pi N_1 r_1 = 2 \pi N_2 r_2
\]

or

\[
\frac{N_1}{N_2} = \frac{r_2}{r_1}
\]  

(64-2)

If the peripheral speed at \( r_1' \) and \( r_2' \) are to be equal, it is necessary that:

\[
2 \pi N_1 r_1' = 2 \pi N_2 r_2'
\]

or

\[
\frac{N_1}{N_2} = \frac{r_2'}{r_1'}
\]  

(65-2)

Equations 64-2 and 65-2 can only be satisfied simultaneously if:

\[
\frac{r_2}{r_1} = \frac{r_2'}{r_1'}
\]
Earlier it had been shown that:

\[
\frac{r_2}{r_1} = \frac{t_2}{t_1}
\]

and

\[
\frac{r'_2}{r'_1} = \frac{t_2}{t_1}
\]

Therefore it is possible to have identical speed point for point and the spindle speed ratio is given by:

\[
\frac{N_1}{N_2} = \frac{t_2}{t_1} = \frac{d_2}{d_1} = \frac{L_2}{L_1}
\] \hspace{1cm} (65-2)

Provided that two drills will satisfy equations 64-2 and 65-2, they can be considered as similar in geometry as well as in speed.

So far only the similarity among the cutting edges has been studied. The chisel edge has not been discussed in any way. Any attempt to do so would demand some understanding of the geometry of the chisel edge and its vicinity. A study was made on how the chisel edge was formed during the grinding process using the grinding machine in the workshop of this university. The general conclusions from the study are: the chisel edge is not perfectly straight in the end view and side view of the drill; it is possible to obtain two equal chisel edge angles for two different drills, and one chisel edge can be considered as the magnification of another.
For two chisel edges to be similar, the two chisel edge angles must be equal. In fact this is one of the conditions for similarity of the cutting edges. For two drills with half web-thickness $t_1$ and $t_2$ respectively, then their corresponding chisel edge length are:–

\[
q_1 = \frac{2t_1}{\cos \theta_1}
\]

and

\[
q_2 = \frac{2t_2}{\cos \theta_2}
\]

since

\[
\frac{q_1}{q_2} = \frac{t_1}{t_2}
\]

Full similarity for both the cutting lips as well as for the chisel edges is achieved when the following requirement is met:

\[
\frac{t_1}{t_2} = \frac{L_1}{L_2} = \frac{d_1}{d_2} = \frac{q_1}{q_2}
\]

\[(66-2)\]

i) Thrust and Torque Relationship For Similar Drills.

In the section of chip formation models, the total thrust and torque acting on an annulus of inside radius $R_1$ and outside
radius $R_2$ are given by

$$F_{th} = 2 \cdot \sum_{r = R_1}^{R_2} f_{th} \cdot \Delta r$$

$$F_{torq} = 2 \cdot \sum_{r = R_1}^{R_2} f_{tang} \cdot \Delta r \cdot r$$

where $f_{th}$ and $f_{tang}$ are thrust and tangential force per unit radius as defined previously.

Let $F_{th} (1)$ and $F_{th} (2)$ etc. be the thrust of annulus 1 and annulus 2 respectively, then:

$$F_{th} (1) = 2 \cdot \sum_{r = R_1(1)}^{R_2(1)} f_{th} (1) \cdot \Delta r (1)$$

$$F_{th} (2) = 2 \cdot \sum_{r = R_1(2)}^{R_2(2)} f_{th} (2) \cdot \Delta r (2)$$

In order to maintain similarity for each increment in the summation process it is necessary from equation 61-2 that:

$$\Delta r (2) = \Delta r (1) \cdot \frac{t_2}{t_1}$$

$$r_1 (2) = r_1 (1) \cdot \frac{t_2}{t_1}$$

$$r_2 (2) = r_2 (1) \cdot \frac{t_2}{t_1}$$
In equations 67-2 - 71-2, $f_{th(1)}$ is equal to $f_{th(2)}$ for each consecutive increment of $\Delta r_{(1)}$ and $\Delta r_{(2)}$. Equation 68-2 can be re-written as:

$$F_{th(2)} = 2 \cdot \sum_{r = R_{1(1)}}^{R_{2(1)}} f_{th(1)} \cdot \frac{t_2}{t_1} \cdot \Delta r_{(1)}$$

$$= \frac{t_2}{t_1} F_{th(1)}$$

$$= \frac{d_2}{d_1} F_{th(1)} \quad (72-2)$$

Equation 72-2 states that the thrust per unit diameter of all similar drills is equal. The practical application of this equation is that it can be used to estimate the thrust of a drill if that of a second similar drill is known.

The torque acting on drill 1 and drill 2 are given by:

$$F_{torq(1)} = 2 \cdot \sum_{r = R_{1(1)}}^{R_{2(1)}} f_{tang(1)} \cdot r_{(1)} \cdot \Delta r_{(1)}$$

$$= \quad (73-2)$$

$$F_{torq(2)} = 2 \cdot \sum_{r = R_{1(2)}}^{R_{2(2)}} f_{tang(2)} \cdot r_{(2)} \cdot \Delta r_{(2)}$$

$$= \quad (74-2)$$
Applying the same reasoning as for thrust, the two torque equation can be related:

\[
F_{\text{torq}}(2) = 2 \cdot \sum_{r = R_1(1)}^r f_{\text{tang}}(1) \cdot \frac{t_2}{t_1} \Delta r(1) \cdot \frac{t_2}{t_1} \cdot r(1)
\]

\[
= \left(\frac{t_2}{t_1}\right)^2 \cdot 2 \cdot \sum_{r = R_1(1)}^r f_{\text{tang}}(1) \cdot r(1) \cdot \Delta r(1)
\]

\[
= \left(\frac{t_2}{t_1}\right)^2 \cdot F_{\text{torq}}(1)
\]

\[
= \left(\frac{d_2}{d_1}\right)^2 \cdot F_{\text{torq}}(1)
\]

(75-2)

By considering the basic geometry of the drills, the torque can be related as shown in equation (75-2). This equation can be used for prediction of any drills provided they satisfy the requirement of similarity.

It is reasonable to believe that the thrust due to the chisel edge is a linear function of its own length. Therefore the thrust of two drills can also be related by:

\[
F_{\text{th}(\text{C.E.})}(2) = \frac{q_2}{q_1} F_{\text{th}(\text{C.E.})}(1)
\]

Substituting \( \frac{q_2}{q_1} = \frac{d_2}{d_1} \), gives:
Similarly the effect of chisel edge on torque can be found from:

\[ F_{\text{torq}(C.E.)(2)} = \left( \frac{q_2}{q_1} \right)^2 F_{\text{torq}(C.E.)(1)} = \left( \frac{d_2}{d_1} \right)^2 F_{\text{torq}(C.E.)(1)} \]  \hspace{1cm} (77-2)

The total thrust and torque are made up of that due to the cutting edge and the chisel edge, i.e.:

\[ T = F_{\text{th}} + F_{\text{th}(C.E.)} \]  \hspace{1cm} (78-2)

\[ M = F_{\text{torq}} + F_{\text{torq}(C.E.)} \]  \hspace{1cm} (79-2)

Manipulating equations 72-2, 75-2, 76-2, 77-2, 78-2 and 79-2, the thrust and torque for two fully similar drills are given by:

\[ T(2) = \left( \frac{d_2}{d_1} \right)^1 T(1) \]  \hspace{1cm} (80-2)

\[ M(2) = \left( \frac{d_2}{d_1} \right)^2 M(1) \]  \hspace{1cm} (81-2)

Equations 80-2 and 81-2 represent the theoretical relationship.
between the thrust and torque of two similar drills. The forms of the empirical thrust and torque equations discussed in the literature section are:

\[
T = C_1 f^{n_1} d^{n_2}
\]

\[
M = C_2 f^{n_3} d^{n_4}
\]

where \(C_1, C_2, n_1, n_2, n_3\) and \(n_4\) are constants \(f\) is feed rate and \(d\) is diameter of the drill. From these two equations the empirical relationship of \(T\) and \(M\) for two drills are:

\[
T(2) = \left(\frac{d_2}{d_1}\right)^{n_2} T(1)
\]

\[
M(2) = \left(\frac{d_2}{d_1}\right)^{n_4} M(1)
\]

Comparing equations 80-2 - 83-2, the pattern of the equations are identical, but the exponentials \(n_2\) and \(n_4\) are only roughly in the vicinity of 1 and 2 (36) (37) (38) (40) respectively. The discrepancies between these equations is likely to be due to the lack of complete similarity between drills used to obtain the empirical equations. From information supplied by a leading Australian drill manufacturer, it was noted that the ratios of \(t/d\) and \(t/L\) are not constant for drills between '5'' and 1·5'' dia. as illustrated in Fig. 9 -2(a) and 9 -2(b).
Variations of "t to d" and "t to L" ratios for different drills

For reason of torsional rigidity, the ratio of $t/d$ for smaller drills is larger and this can be represented by the following inequality:

$$\frac{t_1}{d_1} > \frac{t_2}{d_2}$$

where $t_1$, $t_2$, $d_1$, and $d_2$ are the half web-thickness and diameter of the smaller and larger drills respectively. Due to this inequality, part of the cutting edges of the larger drill near the periphery is not matched by the cutting edges of the smaller drill. This gives rise to what can be called partial similarity of the cutting edges.

There appears to be little or no sound published work justifying the magnitudes of the variations in $t/d$ and $t/L$ with diameter. While these values may be common for general purpose drills, they are not specified in the standards, e.g. B.S. 328 Part 1, 1959.
GEOMETRIC SIMILARITY OF FLAT RAKE DRILLS

It is possible to check if the same similarity conditions can be extended to flat raked drills. For each flat rake drill, the reference rake angle is a constant and is equal to $\alpha_{f0}$, where $\alpha_{f0}$ is the reference angle at the periphery of the drill. At two corresponding radii of two drills the normal rake angle $\alpha_{n}$ and angle of obliquity $i$ are given by:

$$\alpha_{n1} = \alpha_{f01} - \gamma_1$$
$$\alpha_{n2} = \alpha_{f02} - \gamma_2$$

$$\sin i_1 = \sin \omega_1 \sin p_1$$
$$\sin i_2 = \sin \omega_2 \sin p_2$$

For $\alpha_{n2} = \alpha_{n1}$, the two reference rake angle $\alpha_{f02}$ and $\alpha_{f01}$ must be equal and this can be satisfied if $\frac{t_1}{L_2} = \frac{t_2}{L_2}$. Therefore as in the case of conventional drills, the requirement for similarity at two points are:

$$p_1 = p_2$$
and $\frac{t_1}{t_2} = \frac{L_1}{L_2} = \frac{r_1}{r_2}$.

Like for conventional drills, similarity can be extended over portions of the cutting edges and the whole cutting edges if
they satisfy the condition:

\[ \frac{t_1}{t_2} = \frac{L_1}{L_2} = \frac{d_1}{d_2} \]

For the case of when the reference rake angles \( \alpha_{f_01} \) and \( \alpha_{f_02} \) being equal and are greater than the original \( \alpha_{f_0} \), the ratio of \( L_1/L_2 \) need not be equal to \( t_1/t_2 \) for similarity. Similarity can be maintained if \( \alpha_{f_01} = \alpha_{f_02} > \alpha_{f_0} \), and:

\[ \frac{t_1}{t_2} = \frac{r_1}{r_2} \neq \frac{L_1}{L_2} \]

\[ \frac{t_1}{t_2} = \frac{d_1}{d_2} \neq \frac{L_1}{L_2} \]

The chisel edge length of the flat rake drill has not been altered by the modification, but there is some change of configuration in the chisel edge region as will be discussed later. The degree of modification can be properly controlled and its effect on thrust and torque is expected to be proportional to the size of the drill as for the conventional drills. Therefore the thrust and torque equations 80-2, 81-2 will still apply.
CONCLUDING REMARKS

The model for the flat rake drills appears to be promising judging by the trends obtained from numerical analysis. A further advantage is that the performance of these drills may surpass those of the conventional curved rake drills as suggested by Armarego (22).

With the introduction of a constant positive reference rake angle, the normal rake angle becomes less negative near the chisel edge corner as shown in Fig. 10-2 (22). From past experience in orthogonal and oblique cutting, the normal rake angle has been known to control the cutting forces. Therefore the flat rake drill is expected to have better cutting efficiency along the cutting lips than the conventional drills. Also in the process of modification portions of the chisel edge region have been ground away and this appears to have the same effect as point relieving, from which reduction in thrust is expected. In fact these flat rake drills can be considered as modified drills since their performance may be improved by these favourable alterations to its geometry.
The unsatisfactory results of the conventional drilling model led to the investigation of geometric relationship among the various size drills. Theoretical thrust and torque relationship for similar drills were derived. These force relationships would be most useful in the design and testing of drills, since according to these relationships the thrust and torque of any drills can be predicted provided those of a similar drill are known. At the same time there are practical problems to be considered in designing drills, one of these is that the $t/d$ ratio has to be larger for smaller drills. This point has been discussed by some investigators. For this reason the theoretical equations cannot be applied and every drill must be tested to determine its forces and other performance characteristics. It is most desirable to search for some compromise.
by which the theoretical equation can be applied as a guide in design and as a means of minimizing testing, and at the same time the rigidity requirement met. From drill specifications supplied by a drill manufacturer, it is noted that there are 126 different sizes of drills with diameter ranging from 0.0135" to 2.0" diameter. It seems reasonable to group these drills and within each group, say 21 drills, the \( \frac{t}{d} \) ratio be made constant. This idea is demonstrated in Fig. 11-2.

The continuous line shows the gradual reduction of \( \frac{t}{d} \) as diameter increases. The step function represents the proposed \( \frac{t}{d} \) ratio for each group of drills. Of course the number of groups and the step function in relation to the continuous function can be adjusted as necessary. If the \( \frac{t}{L} \) ratio is not equal, the same argument can be applied. The drills in each group are therefore similar to each other and their forces can thus be represented
by the theoretical equations. The full implication of this proposal can be an area of research in its own right.
III. EXPERIMENTAL INVESTIGATION, RESULTS AND DISCUSSIONS

A series of experiments was planned and run to verify the force relationship of the similar drills, to study the performance of conventional and modified drills and to investigate the applicability of the cutting model for flat rake drills.

EQUIPMENT AND EXPERIMENTAL PROCEDURE

The three major items of equipment used in this project were the drilling machine, the force recording instrument and the dynamometer.

The drilling machine used was the WEBO vertical variable speed machine with three feeds of \(0.005, 0.008\) and \(0.012\) in/rev. The lowest spindle speed was 150 rpm and the highest 2000 rpm.

The dynamometer was made up of two parts - the thrust measuring part and the torque measuring part, both of strain gauge type. The original dynamometer available in the laboratory consisted of three half-rings with strain gauges for measuring both thrust and torque. Preliminary testing indicated that the measurement of torque using this dynamometer was not very satisfactory. While keeping the same dynamometer for thrust measurement. A new element had to be designed for measuring torque. Appendix B gives a detailed description of this component as well as of the U.V. Recorder.
There were two force recording instruments: the amplifier and the recorder. The electrical output from the strain gauges were passed through the amplifiers where the desired attenuations were chosen. The rectified and filtered signals were registered on photo-sensitive paper via the U.V. Recorder galvanometer. Fig 1-3 shows the general set up of the equipment:

General set-up of equipment

Fig 1-3

In this experiment it was important to ensure that the vertical axis of the torque measuring component, and consequently the axis of the workpiece annulus, was in line with the axis of the drill as shown in Fig. 2-3.
A centring rod, which could be fitted to the spindle of the drill was specially designed and used to locate the axis of the dynamometer. The workpiece were machined to fit the central seating of the torque measuring element. The maximum position tolerance between the two axes was '001".

The material used in this project was 65 ST6 aluminium alloy. The shear stress of this material in orthogonal cutting, milling and form tool cutting has been intensely studied by Epp (51) and Armarego (52). They reported that the shear stress for this material is about 40000 p.s.i. The density of this material is 44.2 gm per cubic inch.
METHOD OF CHIP MEASUREMENT

In most of these experiments, chip length ratios were required. Reasonable length of chip from both cutting edges were collected and each was long enough (about 4") to be broken into three or four chips. Usually a feed of 1/8" of the spindle would give this length.

There are more problems associated with the measurement of chip length ratio in drilling than in simple orthogonal and oblique cutting. The undeformed chip length as well as the chip length may have to be measured, and their actual measurement demands special techniques. In these experiments the chip length ratios at both the inside and outside radii of an annulii are usually required. Figure 3-3(a) shows an annulus of inside radius $R_1$ and outside radius $R_2$.

![Diagram of annulus and measurement of chip length](image)

One of the methods of finding the undeformed chip length is by drilling a small hole in the wall of the annulus or putting
a groove on the side of the wall of the annulus. The undeformed chip length is obtained by counting the number of holes or grooves appearing in a corresponding length of chip. One of the practical difficulties with this method is that in some cases the wall-thickness of the annulus is only '07"., thus putting a mark on the wall of the annulus is a problem in itself.

Another possible method of determining the undeformed chip length is by weighing the chip. The volume V of material removed in N revolution by one cutting lip can be represented by:

\[ V = \frac{W}{\rho} \]

\[ = \frac{f}{2} \cdot \pi N (R_2^2 - R_1^2) \]

Where W is the weight of the chip, \( \rho \) is the density of the material and f is the feed rate. Manipulating N can be expressed as:

\[ N = \frac{2 W}{\pi f \rho (R_2^2 - R_1^2)} \]
The undeformed chip length at the inside and outside radius \( (UCL_1 \text{ and } UCL_2) \) are given by:

\[
UCL_1 = 2 \pi R_1 N \\
UCL_2 = 2 \pi R_2 N
\]

Substituting \( N \) into these two equations the two undeformed chip length equations become

\[
UCL_1 = \frac{4 W R_1}{f \rho \left( R_2^2 - R_1^2 \right)} \\
UCL_2 = \frac{4 W R_2}{f \rho \left( R_2^2 - R_1^2 \right)}
\]

Therefore knowing the weight of the chip, the undeformed chip length for a certain portion of chip can be calculated. This method was found to be more satisfactory, since any convenient length of chip could be selected for measurement.

The actual measurement of the deformed chip length is rather difficult because the chip is in the form of a helix. The method adopted in these experiments is by breaking the two chips (one from each cutting edge) into several convenient lengths. Each one of them is weighted, then flattened and pressed against a tape as shown in Fig. 3-3(b). A cotton thread is used to follow the inside
and outside arc of the chip. The total deformed and undeformed chip lengths are equal to the sums of these measurements, i.e.

\[
UCL_1 = \frac{4R_1 \text{(sum of weights of chips)}}{f \int P (R_2^2 - R_1^2)}
\]

\[
UCL_2 = \frac{4R_2 \text{(sum of weights of chips)}}{f \int P (R_2^2 - R_1^2)}
\]

\[
CL_1 = \text{Sum of individual chip lengths at inside radius.}
\]

\[
CL_2 = \text{Sum of individual chip lengths at outside radius.}
\]

where \(CL_1\) and \(CL_2\) are the total chip length at the inside and outside radii respectively.

The chip length ratio \(r_{11}\) and \(r_{12}\) at the inside and outside radius respectively are represented by:

\[
r_{11} = \frac{CL_1}{UCL_1} = \frac{CL_1}{4W R_1} \int P (R_2^2 - R_1^2)
\]

\[
r_{12} = \frac{CL_2}{UCL_2} = \frac{CL_2}{4W R_2} \int P (R_2^2 - R_1^2)
\]
SELECTION OF SIMILAR DRILLS AND WORKPIECE GEOMETRY.

For the standard drills to be similar they must satisfy the geometric requirements specified in the analysis section. It was found to be easier to select drills which would give partial similarity along the cutting edges. From a number of general purpose drills, supplied by a drill manufacturer, three drills were chosen with diameters of 0.75", 1.00" and 1.25". The criteria for selection were equal point angles, equal chisel edge angles and equal $t/L$ ratio. Table 1-3 contains the general information about these drills:

<table>
<thead>
<tr>
<th>Dia. (d&quot;)</th>
<th>w.T (2t&quot;)</th>
<th>Pitch Length (L&quot;)</th>
<th>2t/L</th>
<th>Chisel Edge Angle ($\theta$)</th>
<th>Point Angle (2P)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.75</td>
<td>0.109</td>
<td>4.37</td>
<td>2.5%</td>
<td>122°45'</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>119°00'</td>
</tr>
<tr>
<td>B</td>
<td>1.00</td>
<td>0.126</td>
<td>5.05</td>
<td>2.5%</td>
<td>125°35'</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>122°50'</td>
</tr>
<tr>
<td>C</td>
<td>1.25</td>
<td>0.15</td>
<td>6.35</td>
<td>2.40%</td>
<td>125°55'</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>120°00'</td>
</tr>
</tbody>
</table>

Partially Similar Drills

Table 1 - 3
It was not known if the drills were manufactured with equal \( t/L \) ratios. However in the process of selection it was noted that while the pitch length for a particular drill was fairly constant, the web-thickness variation could be as high as 10% from drill to drill of the same size. Therefore it was possible to select three drills having very close \( t/L \) ratio. The \( t/L \) ratio for drill C was slightly smaller but with the drills available for selection, this was the best possible combination.

The design of similar annulii followed the discussion in the analysis section. In this case, the size of the annulii for the 1.25" drills were fixed and then the corresponding sizes of annulii for the other two drills were calculated according to the equation:

\[
\frac{r_1}{t_1} = \frac{r_2}{t_2} = \frac{r_3}{t_3}
\]

In Table 2-3, the dimensions of the annulii for each of the three drills are shown:
<table>
<thead>
<tr>
<th>Dia. of Drill (in)</th>
<th>Annulus</th>
<th>Inside Rad. (in)</th>
<th>Outside Rad. (in)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.25&quot;</td>
<td>1A</td>
<td>.110</td>
<td>.210</td>
</tr>
<tr>
<td></td>
<td>1B</td>
<td>.210</td>
<td>.310</td>
</tr>
<tr>
<td></td>
<td>1C</td>
<td>.310</td>
<td>.410</td>
</tr>
<tr>
<td></td>
<td>1D</td>
<td>.410</td>
<td>.510</td>
</tr>
<tr>
<td></td>
<td>1H</td>
<td>.310</td>
<td>.510</td>
</tr>
<tr>
<td></td>
<td>1J</td>
<td>.110</td>
<td>.510</td>
</tr>
<tr>
<td>1.00</td>
<td>2A</td>
<td>.092</td>
<td>.175</td>
</tr>
<tr>
<td></td>
<td>2B</td>
<td>.175</td>
<td>.259</td>
</tr>
<tr>
<td></td>
<td>2C</td>
<td>.259</td>
<td>.342</td>
</tr>
<tr>
<td></td>
<td>2D</td>
<td>.342</td>
<td>.426</td>
</tr>
<tr>
<td></td>
<td>2H</td>
<td>.259</td>
<td>.426</td>
</tr>
<tr>
<td></td>
<td>2J</td>
<td>.092</td>
<td>.426</td>
</tr>
<tr>
<td>.75</td>
<td>3A</td>
<td>.079</td>
<td>.152</td>
</tr>
<tr>
<td></td>
<td>3B</td>
<td>.152</td>
<td>.222</td>
</tr>
<tr>
<td></td>
<td>3C</td>
<td>.222</td>
<td>.296</td>
</tr>
<tr>
<td></td>
<td>3D</td>
<td>.296</td>
<td>.368</td>
</tr>
<tr>
<td></td>
<td>3H</td>
<td>.222</td>
<td>.368</td>
</tr>
<tr>
<td></td>
<td>3J</td>
<td>.079</td>
<td>.368</td>
</tr>
</tbody>
</table>

Dimension of Annulii

Table 2-3
Annulii 1A, 1B, 1C and 1D formed one continuous annulus equivalent to 1J for the 1.25" drill. There were six annulii for each drill. The annulii 1A, 2A and 3A correspond to each other, and were considered as a set of similar annulii, whose geometry at the inside and outside radii are identical to each other.

The selection of standard drills to satisfy the requirement of complete similarity was more tedious. In this instance, the ratio of $t/d$ must also be equal. In general, smaller drills usually have a larger ratio of $t/d$. From another batch of drills, five were selected. Table 3 - 3 shows the dimensions and ratios of these drills.

<table>
<thead>
<tr>
<th>Drill</th>
<th>Dia. ($d''$)</th>
<th>w. T. (2t'')</th>
<th>Pitch Length (L'')</th>
<th>$2t/L$</th>
<th>$2t/d$</th>
<th>Chisel Edge Angle ($\theta$)</th>
<th>Point Angle (2p)</th>
</tr>
</thead>
<tbody>
<tr>
<td>D</td>
<td>.75</td>
<td>.093</td>
<td>4.25</td>
<td>2.20%</td>
<td>12.4%</td>
<td>125°15'</td>
<td>121°5'</td>
</tr>
<tr>
<td>E</td>
<td>1.00</td>
<td>.120</td>
<td>5.35</td>
<td>2.22%</td>
<td>12.0%</td>
<td>124°5'</td>
<td>120°50'</td>
</tr>
<tr>
<td>F</td>
<td>1.25</td>
<td>.145</td>
<td>6.35</td>
<td>2.28%</td>
<td>11.6%</td>
<td>124°00'</td>
<td>120°00'</td>
</tr>
<tr>
<td>G</td>
<td>.75</td>
<td>.093</td>
<td>4.25</td>
<td>2.20%</td>
<td>12.4%</td>
<td>125°00'</td>
<td>121°30'</td>
</tr>
<tr>
<td>H</td>
<td>1.00</td>
<td>.120</td>
<td>5.35</td>
<td>2.22%</td>
<td>12.0%</td>
<td>124°10'</td>
<td>120°20'</td>
</tr>
</tbody>
</table>

Completely Similar Drills

Table 3 - 3
Due to the large web-thickness variation of the drills, it was possible to select a set of drills (D, E, F) with approximately equal ratio of $\frac{t}{L}$ and $\frac{t}{d}$. These drills could be considered similar to a good approximation.

In order that the performance of the modified and standard drills could be compared, two drills of 0.75" and 1.00" diameter with equal dimensions to those listed in Table 3-3 were selected (G and H). The maximum reference rake angle of the 0.75" and the 1.00" drill were 34 and 35 degrees respectively. The small difference of one degree was due to the small variation of the ratios of $\frac{t}{L}$ and $\frac{t}{d}$. Using the grinding technique given in appendix A, these two drills were ground with the appropriate constant reference rake angles. The standard and the modified drills were similar in every respect except for the normal rake angle formed by the modification.

The two drills marked G and H were to be used to verify the geometric similarity of the modified drills, since they all satisfied the requirement of equal $\frac{t}{L}$ and $\frac{t}{d}$ ratios.

Either one of these two modified drills could be used to check the cutting model proposed in the analysis.
MACHINING WITH SIMILAR CONVENTIONAL DRILLS

The force relationship and chip formation process for similar conventional drills will be studied. Following the verification of the force relationship, two empirical equations will be established for these drills.

PRELIMINARY TESTS ON A CONVENTIONAL DRILL

The concept of similarity involves two or more drills. Before any verification of the force relationship or the comparison of chip formation by these drills can be carried out, some of the effects, such as speed and geometry, on a single drill must be studied first.

1) Effect of Speed

In drilling, for the same spindle speed, the peripheral speed is different for different sizes of drills or different sizes of annulii. It is therefore necessary to determine the effect of speed before checking similarity by comparing the observed forces. Thrust and torque measurement were used to study the effect of speed.

A series of tests was run with a 1.0" diameter drill and an annulus of .587" inside diameter and .989" outside diameter. The speed range was between 150 rpm and 400 rpm, within which all the other tests will be run. Table 4 - 3 shows these results:
Effect of Speed on Forces

Table 4 - 3

Linear regression was used to analyse the results. The low correlation coefficient of \(0.13\) for thrust and \(0.032\) for torque suggested that speed and forces were not correlated. These two correlation coefficients were small compared with \(0.5494\) at 90% confidence level with degrees of freedom of \(8\).

ii) Effect of Geometry on Force and Chip Length Ratio Distribution Along Drill Lips

To arrive at the force relationship for two similar annulii, i.e.:

\[
F_{th(1)} = \frac{\tau_1}{\tau_2} F_{th(2)} \quad (1-3)
\]

\[
F_{torq(1)} = \left(\frac{\tau_1}{\tau_2}\right)^2 F_{torq(2)} \quad (2-3)
\]

the following two equations have to be applied:
\[
F_{th} = \sum_{r = R_1}^{r = R_2} f_{th} \cdot \Delta r \quad (3-3)
\]

\[
F_{torq} = \sum_{r = R_1}^{r = R_2} f_{tang} \cdot \Delta r \cdot r \quad (4-3)
\]

where \(F_{th}\) and \(F_{torq}\) are the total thrust and torque respectively acting on an annulus of inside radius \(R_1\) and outside radius \(R_2\).

\(f_{th}\) and \(f_{tang}\) are the thrust and tangential force per unit radius at radius \(r\). Consequently \(f_{th} \cdot \Delta r\) and \(f_{tang} \cdot \Delta r \cdot r\) represent the elemental thrust and elemental torque acting on an elemental radius \(\Delta r\), which is shown in Fig. 4-3.

Before equations 1-3 and 2-3 can be verified experimentally, it is necessary to check the validity of equations 3-3 and 4-3. This can be done by comparing the observed forces of both sides of equations 3-3 and 4-3. The forces on the right hand side of these equations can be obtained by drilling a number of continuous annulii.
of wall-thickness $\Delta r$. The force on the left hand side is obtained by drilling an equivalent annulus of radii $R_1$ and $R_2$ as shown in Fig. 4-3. Ideally $\Delta r$ should be as small as possible, but there are practical difficulties associated with very small $\Delta r$. The chip length ratios are required for other comparison purpose. If $\Delta r$ is too small, the chip will break easily as observed in some preliminary tests, furthermore the chip measurement became very tedious. Therefore it was decided to use the 1.0" diameter drill together with the four continuous annulii 2A, 2B, 2C, 2D. Their dimensions and those of the equivalent annulus 2J are shown in Table 5-3. These four annulii were expected to give a fair force representation of the right hand side of equations 3-3 and 4-3. The sum of the forces from 2A, 2B, 2C and 2D will be compared with that found from 2J. To enable statistical comparison, a number of runs were taken from each annulus. The tests were run with feed rate of 0.08 in/rev. and at 150 rpm. The results are shown in Table 5-3.

<table>
<thead>
<tr>
<th>Annulus</th>
<th>Inside Rad. (in)</th>
<th>Outside Rad (in)</th>
<th>Mean Thrust (lb)</th>
<th>Mean Thrust (lb)</th>
<th>Var.</th>
<th>DOF</th>
<th>t-test ratio</th>
<th>Remark</th>
</tr>
</thead>
<tbody>
<tr>
<td>2A</td>
<td>0.092</td>
<td>0.175</td>
<td>51.0</td>
<td>( )</td>
<td>( )</td>
<td></td>
<td>.575</td>
<td>Passed at 95%</td>
</tr>
<tr>
<td>2B</td>
<td>0.175</td>
<td>0.259</td>
<td>40.6</td>
<td>( )</td>
<td>( )</td>
<td>10</td>
<td>.27</td>
<td></td>
</tr>
<tr>
<td>2C</td>
<td>0.259</td>
<td>0.342</td>
<td>29.8</td>
<td>( )</td>
<td>( )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2D</td>
<td>0.342</td>
<td>0.426</td>
<td>21.6</td>
<td>( )</td>
<td>( )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2J</td>
<td>0.092</td>
<td>0.426</td>
<td>147.0</td>
<td>147.0</td>
<td>21.4</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Comparison of Thrust

Table 5-3 (a)
The sum of the mean forces of the small annulii is compared with that of the large annulus by the t-test. The t-test carried out in this experiment is slightly different from the ordinary comparison of two means. In this case the first mean was made up of four individual means, each one of them was subject to variation and therefore had its own variance. A slight modification of the standard t-test was necessary in order that the above means could be compared. A description of this t-test appears in Appendix 2-C.

In Tables 5 - 3(a) and 5 - 3(b) it is observed that the sum of thrust and torque of the small annulii are equal to that of the large annulus, both passing the t-test at different levels of confidence. The conclusion from these results is that equations 3-3 and 4-3 are valid and thus it is logical to arrive at equations 1-3 and 2-3.
Fig. 5-3(a) displays the distribution of thrust when plotted against radius. The trend of the distribution was expected since the normal rake angle at the smaller radii is much less than those at the larger radii. The increase in torque (Fig. 5-3(b)) with increase in radius is reasonable, since torque is a function of the tangential force and radius.
Thrust and torque distributions of standard drill (drill B)  
Feed = 0.008"/rev

Fig 5-3
Similar drills can be considered as the scaled version of one another, and the implication of this is that they are expected to have identical chip formation, e.g. chip flow angles and chip length ratio. Also the forces of these drills can be related as discussed in the previous section. Before comparing the chip formation of two similar drills, it is informative to study the chip formation of a single drill first. A convenient parameter to observe and measure in the chip formation process of drilling is the chip length ratio. It was decided to investigate the distribution of this parameter at various radii and if it is only function of the geometry at the point in question and independent of other factors such as the thickness of the annulus. Chips were collected when the previous experiment was run. The chip length at the inside and outside radii were measured. The observed chip length ratio at radii .092", .175", .259", .342" and .426" are shown in Table 6 - 3.

<table>
<thead>
<tr>
<th>Anulus</th>
<th>Inside Rad. (in)</th>
<th>Outside Rad. (in)</th>
<th>Mean chip length Ratio at $R = .092&quot;$</th>
<th>Var.</th>
<th>DOF</th>
<th>t-test Ratio</th>
<th>Remark</th>
</tr>
</thead>
<tbody>
<tr>
<td>2A</td>
<td>.092</td>
<td>.175</td>
<td>.397</td>
<td>.0016</td>
<td>18</td>
<td>.567</td>
<td>Passed at 90%</td>
</tr>
<tr>
<td>2J</td>
<td>.092</td>
<td>.426</td>
<td>.393</td>
<td>.0016</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Chip Length Ratio At Rad. = .092"

Table 6 - 3(a)
<table>
<thead>
<tr>
<th>Anulus</th>
<th>Inside Rad. (in.)</th>
<th>Outside Rad. (in.)</th>
<th>Mean chip length Ratio at R = .175&quot;</th>
<th>Var.</th>
<th>DOF</th>
<th>t-test Ratio</th>
<th>Remark</th>
</tr>
</thead>
<tbody>
<tr>
<td>2A</td>
<td>.092</td>
<td>.175</td>
<td>.410</td>
<td>.0009</td>
<td>16</td>
<td>.537</td>
<td>Passed at 90%</td>
</tr>
<tr>
<td>2B</td>
<td>.175</td>
<td>.259</td>
<td>.404</td>
<td>.0009</td>
<td>16</td>
<td>.537</td>
<td>Passed at 90%</td>
</tr>
</tbody>
</table>

Chip Length Ratio at Rad. = .175"

Table 6 - 3 (b)

<table>
<thead>
<tr>
<th>Anulus</th>
<th>Inside Rad. (in.)</th>
<th>Outside Rad. (in.)</th>
<th>Mean chip length Ratio at R = .259&quot;</th>
<th>Var.</th>
<th>DOF</th>
<th>t-test Ratio</th>
<th>Remark</th>
</tr>
</thead>
<tbody>
<tr>
<td>2B</td>
<td>.175</td>
<td>.259</td>
<td>.430</td>
<td>.0004</td>
<td>20</td>
<td>.537</td>
<td>Passed at 90%</td>
</tr>
<tr>
<td>2C</td>
<td>.259</td>
<td>.342</td>
<td>.428</td>
<td>.0016</td>
<td>20</td>
<td>.537</td>
<td>Passed at 90%</td>
</tr>
</tbody>
</table>

Chip Length Ratio at Rad. = .259"

Table 6 - 3 (c)

<table>
<thead>
<tr>
<th>Anulus</th>
<th>Inside Rad. (in.)</th>
<th>Outside Rad. (in.)</th>
<th>Mean chip length Ratio at R = .342&quot;</th>
<th>Var.</th>
<th>DOF</th>
<th>t-test Ratio</th>
<th>Remark</th>
</tr>
</thead>
<tbody>
<tr>
<td>2C</td>
<td>.259</td>
<td>.342</td>
<td>.463</td>
<td>.0006</td>
<td>22</td>
<td>1.562</td>
<td>Passed at 90%</td>
</tr>
<tr>
<td>2D</td>
<td>.342</td>
<td>.426</td>
<td>.469</td>
<td>.0016</td>
<td>22</td>
<td>1.562</td>
<td>Passed at 90%</td>
</tr>
</tbody>
</table>

Chip Length Ratio at Rad. = .342"

Table 6 - 3 (d)
In each of the above tables, the mean chip length ratios at a particular radius for two different annulii are shown, e.g. in Table 6 - 3(d) the chip length ratio at the outside radius of 2C and the chip length ratio at the inside radius of 2D are tabulated. From the t-test to compare these means, the chip length ratios at a particular radius are proved to be equal statistically, so that the chip length ratio could be considered as independent of the size of the annulii but only dependent on the point (or radius) in question.

From the above results it is also noted that the chip length ratio is less at smaller radii. This trend was expected since the normal rake angle is much less at smaller radii. To confirm that the variation of chip length ratio with radius is statistically significant, an analysis of variance was carried out.

The chip length ratios at one radius for two different annulii were considered to be from one population and the mean chip length ratios at the different radii compared by an analysis of
The variance ratio of 46.65 indicates that the chip length ratios were significantly different.

**VERIFICATION OF SIMILARITY CONCEPT ON DRILL LIPS**

In Table 8 - 3, the three similar annulii 3H, 2H and 1H, corresponding to three similar drills of diameter .75", 1.00" and 1.25" respectively, are different in sizes, but similar in geometry at corresponding radii. The chip length ratios of these three annulii are expected to be equal at appropriate radii. For each annulus, six runs were taken at a feed rate of .008 in/rev. and speed of 150 rpm. Table 8 - 3 contains the observed chip length ratio at both the inside and outside radii of these three annulii:
<table>
<thead>
<tr>
<th>Annulus</th>
<th>Inside Rad. (in)</th>
<th>Outside Rad. (in)</th>
<th>Dia. of Drill (in)</th>
<th>Mean Chip Length Ratio</th>
<th>Var.</th>
<th>DOF</th>
<th>F-test Ratio</th>
<th>Remark</th>
</tr>
</thead>
<tbody>
<tr>
<td>3H</td>
<td>.222</td>
<td>.368</td>
<td>.75</td>
<td>.40</td>
<td>.0003</td>
<td></td>
<td></td>
<td>Passed at 90%</td>
</tr>
<tr>
<td>2H</td>
<td>.259</td>
<td>.426</td>
<td>1.00</td>
<td>.41</td>
<td>.0007</td>
<td>2,15</td>
<td>.54</td>
<td>Passed at 90%</td>
</tr>
<tr>
<td>1H</td>
<td>.310</td>
<td>.510</td>
<td>1.25</td>
<td>.41</td>
<td>.0005</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Chip Length Ratios at Inside Radii of Annulii

Table 8 - 3(a)

<table>
<thead>
<tr>
<th>Annulus</th>
<th>Inside Rad. (in)</th>
<th>Outside Rad. (in)</th>
<th>Dia. of Drill (in)</th>
<th>Mean Chip Length Ratio</th>
<th>Var.</th>
<th>DOF</th>
<th>F-test Ratio</th>
<th>Remark</th>
</tr>
</thead>
<tbody>
<tr>
<td>3H</td>
<td>.222</td>
<td>.368</td>
<td>.75</td>
<td>.56</td>
<td>.0010</td>
<td></td>
<td></td>
<td>Passed at 90%</td>
</tr>
<tr>
<td>2H</td>
<td>.259</td>
<td>.426</td>
<td>1.00</td>
<td>.53</td>
<td>.0004</td>
<td>2,15</td>
<td>2.57</td>
<td>Passed at 90%</td>
</tr>
<tr>
<td>1H</td>
<td>.310</td>
<td>.510</td>
<td>1.25</td>
<td>.55</td>
<td>.0004</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Chip Length Ratios at Outside Radii of Annulii

Table 8 - 3(b)

The chip length ratios at both the side inside and outside radii were compared by an analysis of variance and were found to be equal statistically. The analysis of variance programme is given in Appendix 3-C. The conclusion from these results is that the chip formation process is the same on corresponding radii by similar drills using the chip length
ratio as the main deformation parameter.

In order to define the chip formation process completely, it is also necessary to define the chip flow direction. There is no convenient means by which the chip flow angles of these drills can be compared. However there is every reason to suggest that the chip flow angles for these similar drills should be the same since chip flow condition is mainly controlled by geometry.

The indirect method of checking whether the chip flow angles are equal is by observing the cutting forces on the similar annulii. In the analysis section the thrust and torque relationship for the cutting lips were given as:

\[ F_{\text{th}}(1) = \frac{t_1}{t_2} F_{\text{th}}(2) \]  \hspace{1cm} (5-3)

\[ F_{\text{torq}}(2) = \left(\frac{t_1}{t_2}\right)^2 F_{\text{torq}}(2) \]  \hspace{1cm} (6-3)

In arriving at these two equations it was assumed that:

\[ f_{\text{th}}(1) = f_{\text{th}}(2) \]  \hspace{1cm} (7-3)

\[ f_{\text{tang}}(1) = f_{\text{tang}}(2) \]  \hspace{1cm} (8-3)

where \( f_{\text{th}} \) and \( f_{\text{tang}} \) are thrust and tangential force per unit radius at radius \( r \). For equations 7-3 and 8-3 to hold, the \( f_p \), \( f_q \) and \( f_r \) (see equations 26-2-28-2 in Section II) at corresponding radii must be equal as well. This is only true if the chip flow angles and
the chip length ratio are equal. Therefore if equations 5-3 and 6-3 are proved to be applicable, then the whole chip formation process can be considered as identical since the shear stress is considered a constant for a given material (51) (52).

In order to verify equations 5-3 and 6-3, extra tests were run in addition to the previous runs with specimen 3H, 2H and 1H. The present experiment involved three procedures: observation of forces, conversing of forces, according to equation 5-3 and 6-3, to give the estimated forces, and finally the comparison of observed and estimated forces. Table 9-3 shows the results of these comparisons:

<table>
<thead>
<tr>
<th>Annulus</th>
<th>Inside Rad. (in)</th>
<th>Outside Rad. (in)</th>
<th>Dia. of Drill (in)</th>
<th>Mean of obs. Fth (lb)</th>
<th>Est. Fth (lb)</th>
<th>Var.</th>
<th>DOF</th>
<th>F-test Ratio</th>
<th>Remark</th>
</tr>
</thead>
<tbody>
<tr>
<td>3C</td>
<td>.222</td>
<td>.296</td>
<td>.75</td>
<td>25.80</td>
<td>37.09</td>
<td>.30</td>
<td>2.15</td>
<td>2.40</td>
<td>Passed at 90%</td>
</tr>
<tr>
<td>2C</td>
<td>.259</td>
<td>.342</td>
<td>1.00</td>
<td>30.30</td>
<td>36.78</td>
<td>.70</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1C</td>
<td>.314</td>
<td>.410</td>
<td>1.25</td>
<td>37.59</td>
<td>37.59</td>
<td>.30</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Comparison of Thrust of Three Similar Annulii

Table 9-3(a)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>3C</td>
<td>.222</td>
<td>.296</td>
<td>.75</td>
<td>18.50</td>
<td>35.89</td>
<td>.10</td>
<td>2.15</td>
<td>3.83</td>
<td>Passed at 97.5%</td>
</tr>
<tr>
<td>2C</td>
<td>.259</td>
<td>.342</td>
<td>1.00</td>
<td>25.80</td>
<td>35.99</td>
<td>.60</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1C</td>
<td>.310</td>
<td>.410</td>
<td>1.25</td>
<td>36.75</td>
<td>36.75</td>
<td>.40</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Comparison of Torque of Three Similar Annulii

Table 9-3(b)
Comparison of Thrust of Three Similar Annulii

Table 9 - 3 (c)

<table>
<thead>
<tr>
<th>Anulus</th>
<th>Inside Rad. (in)</th>
<th>Outside Rad. (in)</th>
<th>Dia. of Drill (in)</th>
<th>Mean obs. Fth (lb)</th>
<th>Est. Fth (lb)</th>
<th>Var.</th>
<th>DOF</th>
<th>F-test Ratio</th>
<th>Remark</th>
</tr>
</thead>
<tbody>
<tr>
<td>3H</td>
<td>222</td>
<td>368</td>
<td>75</td>
<td>44.30</td>
<td>61.47</td>
<td>.3</td>
<td>2.15</td>
<td>2.07</td>
<td>Passed at 90%</td>
</tr>
<tr>
<td>2H</td>
<td>259</td>
<td>426</td>
<td>1.00</td>
<td>51.20</td>
<td>61.30</td>
<td>.7</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1H</td>
<td>310</td>
<td>510</td>
<td>1.25</td>
<td>62.53</td>
<td>62.53</td>
<td>.4</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Comparison of Torque of Three Similar Annulii

Table 9 - 3 (d)

The web-thickness of the three drills used in this experiment were \( 2t_1 = .109" \), \( 2t_2 = .126" \) and \( 2t_3 = .150" \) for the three drills of diameter \(.75"\), \(1.00"\) and \(1.25"\).

According to equation 5-3, the estimated thrust (Table 9 -3(a)) of the \(1.25"\) drill using the observed values of the \(.75"\) and \(1.00\) drills...
are given by:

\[
F_{th}(1.25) = \frac{0.150}{109} \times 25.80 \\
= 37.89 \text{ lb}
\]

\[
F_{th}(1.25) = \frac{0.150}{126} \times 30.30 \\
= 36.78 \text{ lb}
\]

These two estimated thrust are compared with the observed thrust of the 1.25" drill. Similar manipulation was applied to torque. These observed and estimated values were proved to be equal by analysis of variance as illustrated in Table 9-3. Two conclusions can be drawn from these results. If geometric similarity does exist between the cutting edges of the drills, then it is possible to relate their thrust and torque by equation 5-3 and 6-3. At the same time the chip flow conditions for similar annulii, say 3H, 2H and 1H, are proved to be identical, since the above results implies equal \( f_p, f_q \) and \( f_r \), which in turn implies equal chip flow angles.

**VERIFICATION OF SIMILARITY CONCEPT ON FULL DRILL**

The drills listed in Table 2-4 are considered as fully similar drills since they satisfied practically all the requirement of full similarity. In the analysis section, thrust and torque were considered as linear functions of the chisel edge length. Therefore these drills
are expected to follow the thrust and torque relationship, which are represented by the following equations:

\[
T(1) = \frac{d_1}{d_2} T(2) \quad (9-3)
\]

\[
M(1) = \left( \frac{d_1}{d_2} \right)^2 M(2) \quad (10-3)
\]

where \( T \) and \( M \) are total thrust and torque for the full drill.

For each of the three drills used in this experiment a number of tests were run at one speed for three different feeds. The speed was 150 rpm. According to equations 9-3 and 10-3, the observed force of any one of the three drills can be used to predict those of the other two drills. The observed thrust and torque are shown in Fig. 6-3. For comparison purpose the three forces must be converted to one level. In Fig. 6-3 the observed values of the .75" and the 1.25" drill were converted to that of the 1.00" drill and these three values at each feed were compared using an analysis of variance.
Fig 6-3

(a) F_{th} (lb)

A: .750" drill observed
B: 1.00" 
C: 1.25"

Feed (in/rev)

(b) F_{torq} (in-lb)

D: 1.00" drill estimated using A

Feed (in/rev)

E: " " " " B
<table>
<thead>
<tr>
<th>Drill Dia.</th>
<th>Obs. T</th>
<th>Est. T</th>
<th>Var.</th>
<th>DOF</th>
<th>F-test Ratio</th>
<th>Remark</th>
</tr>
</thead>
<tbody>
<tr>
<td>(in)</td>
<td>(lb)</td>
<td>(lb)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.75</td>
<td>216.6</td>
<td>279.41</td>
<td>1185</td>
<td>2.15</td>
<td>4.38</td>
<td>Passed at 97.5%</td>
</tr>
<tr>
<td>1.00</td>
<td>307.5</td>
<td>307.50</td>
<td>517</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.25</td>
<td>413.7</td>
<td>341.94</td>
<td>2326</td>
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</tr>
</tbody>
</table>

Comparison of Thrust of Three Similar Drills at \( f = 0.005 \) in/rev

Table 10 - 3(a)

<table>
<thead>
<tr>
<th>Drill Dia.</th>
<th>Obs. T</th>
<th>Est. T</th>
<th>Var.</th>
<th>DOF</th>
<th>F-test Ratio</th>
<th>Remark</th>
</tr>
</thead>
<tbody>
<tr>
<td>(in)</td>
<td>(lb)</td>
<td>(lb)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.75</td>
<td>379.2</td>
<td>489.2</td>
<td>627</td>
<td>2.15</td>
<td>1.93</td>
<td>Passed at 90%</td>
</tr>
<tr>
<td>1.00</td>
<td>497.5</td>
<td>497.5</td>
<td>105</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.25</td>
<td>618.8</td>
<td>511.4</td>
<td>442</td>
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</tr>
</tbody>
</table>

Comparison of Thrust of Three Similar Drills at \( f = 0.008 \) in/rev

Table 10 - 3(b)

<table>
<thead>
<tr>
<th>Drill Dia.</th>
<th>Obs. T</th>
<th>Est. T</th>
<th>Var.</th>
<th>DOF</th>
<th>F-test Ratio</th>
<th>Remark</th>
</tr>
</thead>
<tbody>
<tr>
<td>(in)</td>
<td>(lb)</td>
<td>(lb)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.75</td>
<td>537.5</td>
<td>693.4</td>
<td>137</td>
<td>2.15</td>
<td>0.04</td>
<td>Passed at 90%</td>
</tr>
<tr>
<td>1.00</td>
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<td>696.3</td>
<td>1247</td>
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<tr>
<td>1.25</td>
<td>843.1</td>
<td>696.8</td>
<td>301</td>
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Comparison of Thrust of Three Similar Drills at \( f = 0.012 \) in/rev.

Table 10 - 3(c)
<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>.75</td>
<td>52.4</td>
<td>87.0</td>
<td>6.8</td>
<td>2.15</td>
<td>.13</td>
<td>Passed at 90%</td>
</tr>
<tr>
<td>1.00</td>
<td>86.6</td>
<td>86.6</td>
<td>86.6</td>
<td>2.15</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.25</td>
<td>124.9</td>
<td>85.5</td>
<td>65.3</td>
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</tbody>
</table>

Comparison of Torque of Three Similar Drills at $f = .005$ in/rev.

Table 10 - 3(d)

<table>
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</tr>
</thead>
<tbody>
<tr>
<td>.75</td>
<td>87.2</td>
<td>144.7</td>
<td>84.7</td>
<td>2.15</td>
<td>.51</td>
<td>Passed at 90%</td>
</tr>
<tr>
<td>1.00</td>
<td>140.3</td>
<td>140.3</td>
<td>23.2</td>
<td>2.15</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.25</td>
<td>210.7</td>
<td>144.3</td>
<td>100.0</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Comparison of Torque of Three Similar Drills at $f = .008$ in/rev.

Table 10 - 3(e)

<table>
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<tr>
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<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>.75</td>
<td>121.2</td>
<td>201.2</td>
<td>122.5</td>
<td>2.15</td>
<td>1.66</td>
<td>Passed at 90%</td>
</tr>
<tr>
<td>1.00</td>
<td>193.2</td>
<td>193.2</td>
<td>22.4</td>
<td>2.15</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.25</td>
<td>286.4</td>
<td>196.1</td>
<td>32.6</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Comparison of Torque of Three Similar Drills at $f = .012$ in/rev.

Table 10 - 3(f)
In each case the validity of equations 9-3 and 10-3 was proved to hold, since the estimated and observed forces for the three drills were found to be equal. The observed and estimated forces are shown in Table 10-3.

**EMPirical Force Equations for Similar Conventional Drills.**

The thrust and torque for similar drills can be expressed as:

\[
T = f_1(f, d) \quad (11-3)
\]

\[
M = f_2(f, d^2) \quad (12-3)
\]

There are two possible relationship for equations 11-3 and 12-3. In the first instant thrust and torque can be considered as linear functions of the feed rate \( f \). Therefore the thrust and torque equations may take the forms:

\[
\frac{T}{d} = C_1 + C_2 f \quad (13-3)
\]

\[
\frac{M}{d^2} = C_3 + C_4 f \quad (14-3)
\]

where \( C_1, C_2, C_3 \) and \( C_4 \) are constants. \( C_1 \) and \( C_3 \) are possible cutting edge effect in drilling and \( C_2 \) and \( C_4 \) are dependent on the property of the material being drilled. Evidences of cutting edge effect for orthogonal cutting and form tools have been given.
by many workers (51) - (55). It is also possible to represent
the thrust and torque as some exponential function of the feed
rate \( f \), as has been suggested by various empirical equations,
i.e.:

\[
\frac{T}{d} = C_5 f^{n_1} \tag{15-3}
\]

\[
\frac{M}{d^2} = C_6 f^{n_2} \tag{16-3}
\]

Where \( C_5, C_6, n_1, \) and \( n_2 \) are constants. Taking the log of both
sides of the above equations:

\[
\log \left( \frac{T}{d} \right) = \log C_5 + n_1 \log f
\]

\[
\log \left( \frac{M}{d^2} \right) = \log C_6 + n_2 \log f
\]

The distinction between equations 13-3 - 16-3 is that in 15-3 and
16-3 the edge effect is neglected and the thrust and torque are made
to pass through the origin.

In order to determine the various constants in these equations
tests were run, at three feeds using three drills listed in Table 13 -3.
There were only three feed rates available on the drilling machine and
the estimated values for the constant terms would be most unrealistic
if only one observation was taken at each feed. Instead six cuts
at each feed were taken. The ordinary linear regression analysis could not be used to fit a straight line through these observed values of $T/d$ and $M/d^2$, since there were more than one such observations at each feed. In fact there were eighteen observations at each feed, six from each of the three drills. Appendix 4 - C is a linear regression analysis designed for the case of multi-observation at each feed. Considering the case where $T/d$ is linear to feed:

Equation 13-3 can be re-written as:

$$
\frac{T}{d} \text{ mean } = C_1 + C_2 f + k \sqrt{V_{\text{mean}}} \quad (17-3)
$$

$$
\frac{T}{d} \text{ ind } = C_1 + C_2 f + k \sqrt{V_{\text{ind}}} \quad (18-3)
$$

where $V_{\text{mean}}$ and $V_{\text{ind}}$ are the variances of the mean and individual observation of $T/d$.

By similar reasoning:

$$
\frac{M}{d^2} \text{ mean } = C_3 + C_4 f + k \sqrt{V_{\text{mean}}} \quad (19-3)
$$

$$
\frac{M}{d^2} \text{ ind } = C_3 + C_4 f + k \sqrt{V_{\text{ind}}} \quad (20-3)
$$

The 108 observations of thrust and torque, 36 for each feed were analysed by the special linear regression programme and the following information is obtained:
\[ \frac{T}{d} = 40 + 52762 \ell \quad (21-3) \]

\[ \frac{M}{d^2} = 12 + 15669 \ell \quad (22-3) \]

For thrust:

\[
\begin{align*}
V_{\text{mean}} &= 36.1 - 36453 \ell + 21872 \ell^2 \\
V_{\text{ind}} &= 1007.2 - 36453 \ell + 21872 \ell^2
\end{align*}
\]

For torque:

\[
\begin{align*}
V_{\text{mean}} &= 2.8 - 2718.4 \ell + 163106 \ell^2 \\
V_{\text{ind}} &= 75.3 - 2118.4 \ell + 163106 \ell^2
\end{align*}
\]

The \( V_{\text{mean}} \) and \( V_{\text{ind}} \) depended on the feed rate. At 97.5% confidence level and with 51 degrees of freedom \( K = 2.01 \). In the following tables the various mean and individual confidence limits for \( T/d \) and \( M/d^2 \) are presented:

<table>
<thead>
<tr>
<th>Feed (in/rev.)</th>
<th>V mean</th>
<th>( \sqrt{V_{\text{mean}}} )</th>
<th>k ( \sqrt{V_{\text{mean}}} )</th>
<th>( (-\frac{T}{d})_{\text{mean}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.005</td>
<td>42.286</td>
<td>6.503</td>
<td>13.2</td>
<td>304 ( \pm ) 13</td>
</tr>
<tr>
<td>0.008</td>
<td>18.227</td>
<td>4.269</td>
<td>8.6</td>
<td>462 ( \pm ) 9</td>
</tr>
<tr>
<td>0.012</td>
<td>47.398</td>
<td>6.884</td>
<td>14.0</td>
<td>674 ( \pm ) 14</td>
</tr>
</tbody>
</table>

Thrust - Mean

Table 11 - 3(a)
### Thrust Individual

Table 11 - 3(b)

<table>
<thead>
<tr>
<th>Feed (in/rev.)</th>
<th>V ind</th>
<th>( \sqrt{V_{\text{ind}}} )</th>
<th>k ( \sqrt{V_{\text{ind}}} )</th>
<th>( \frac{\mathbf{T}}{d^2} ) ind</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.005</td>
<td>1013.406</td>
<td>31.834</td>
<td>64.0</td>
<td>304 ± 64</td>
</tr>
<tr>
<td>0.008</td>
<td>989.436</td>
<td>31.454</td>
<td>64.0</td>
<td>462 ± 54</td>
</tr>
<tr>
<td>0.012</td>
<td>1018.509</td>
<td>31.914</td>
<td>64.0</td>
<td>674 ± 64</td>
</tr>
</tbody>
</table>

### Torque Mean

Table 11 - 3(c)

<table>
<thead>
<tr>
<th>Feed (in/rev.)</th>
<th>V mean</th>
<th>( \sqrt{V_{\text{mean}}} )</th>
<th>k ( \sqrt{V_{\text{mean}}} )</th>
<th>( \frac{\mathbf{M}}{d^2} ) mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.005</td>
<td>3.153</td>
<td>1.776</td>
<td>3.6</td>
<td>90.5 ± 3.6</td>
</tr>
<tr>
<td>0.008</td>
<td>1.359</td>
<td>1.166</td>
<td>2.2</td>
<td>138.0 ± 2.2</td>
</tr>
<tr>
<td>0.012</td>
<td>3.534</td>
<td>1.880</td>
<td>3.8</td>
<td>200.0 ± 3.8</td>
</tr>
</tbody>
</table>

### Torque Individual

Table 11 - 3(d)

<table>
<thead>
<tr>
<th>Feed (in/rev.)</th>
<th>V ind.</th>
<th>( \sqrt{V_{\text{ind}}} )</th>
<th>k ( \sqrt{V_{\text{ind}}} )</th>
<th>( \frac{\mathbf{M}}{d^2} ) ind.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.005</td>
<td>75.573</td>
<td>8.693</td>
<td>17.5</td>
<td>90.5 ± 17.5</td>
</tr>
<tr>
<td>0.008</td>
<td>73.778</td>
<td>8.589</td>
<td>17.0</td>
<td>138.0 ± 17.0</td>
</tr>
<tr>
<td>0.012</td>
<td>75.953</td>
<td>8.715</td>
<td>18.0</td>
<td>200.0 ± 18.0</td>
</tr>
</tbody>
</table>
Fig. 7 - 3(a) and 7 - 3(b) represent the two mean lines of \( \frac{T}{d} \) and \( \frac{M}{d^2} \) together with the confidence limits of the individual observations at 97.5\% confidence level.
Fig 7-3

(a) Mean values
Confidence limits for individual observations (at 97.5% c.l.)

(b) $M/\sigma^2$

Feed (in/rev) vs. $T/D$
The second type of empirical equations of $T/d$ and $M/d^2$ were assumed to have the following forms:

\[
\log \left( \frac{T}{d} \right) = \log C_5 + n_1 \log f \\
\log \left( \frac{M}{d^2} \right) = \log C_6 + n_2 \log f
\]

As before the mean and individual values of $\log \left( \frac{T}{d} \right)$ and $\log \left( \frac{M}{d^2} \right)$ were subject to variations, i.e.

\[
\log \left( \frac{T}{d} \right)_{\text{mean}} = \log C_5 + n_1 \log f \pm k \sqrt{V_{\text{mean}}} \\
\log \left( \frac{T}{d} \right)_{\text{ind}} = \log C_5 + n_1 \log f \pm k \sqrt{V_{\text{ind}}} \\
\log \left( \frac{M}{d^2} \right)_{\text{mean}} = \log C_6 + n_2 \log f \pm k \sqrt{V_{\text{mean}}} \\
\log \left( \frac{M}{d^2} \right)_{\text{ind}} = \log C_6 + n_2 \log f \pm k \sqrt{V_{\text{ind}}}
\]

For convenience of algebraic manipulation let $\log K = k \sqrt{V}$, then the above equation could be re-written as:

\[
\log \left( \frac{T}{d} \right)_{\text{mean}} = \log C_5 + n_1 \log f \pm \log K
\]

i.e. \[
\left( \frac{T}{d} \right)_{\text{mean}} = K_1 C_5 f^{n_1}
\]

or \[
\left( \frac{T}{d} \right)_{\text{mean}} = \frac{C_5 f^{n_1}}{K_1}
\]
Similarly

\[ \frac{T}{d}\text{ ind} = K_2 C_5 f^{n_1} \]

or

\[ = \frac{C_5 f^{n_2}}{K_2} \]

\[ \frac{M}{d^2}\text{ mean} = K_3 C_6 f^{n_2} \]

or

\[ = \frac{C_6 f^{n_2}}{K_3} \]

\[ \frac{M}{d^2}\text{ ind} = K_4 C_6 f^{n_2} \]

or

\[ = \frac{C_6 f^{n_2}}{K_4} \]

The general equation and the variances output by the linear regression analysis were:

\[ \frac{T}{d} = 42580 \cdot 936 \]

\[ \frac{M}{d^2} = 13176 \cdot 945 \]

For thrust:

\[ V_{\text{mean}} = 3.404 + 0.009 (\log f) + 0.001 (\log f)^2 \]

\[ V_{\text{ind}} = 3.411 + 0.009 (\log f) + 0.001 (\log f)^2 \]
For torque:

\[ V_{\text{mean}} = 3.404 + 0.005 \log f + 0.001 (\log f)^2 \]

\[ V_{\text{ind}} = 3.407 + 0.005 \log f + 0.001 (\log f)^2 \]

The means and individual confidence limits at 97.5% for thrust and torque are tabulated in the following tables:

<table>
<thead>
<tr>
<th>Feed (in/{per})</th>
<th>\sqrt{\langle V \rangle}</th>
<th>K \langle V \rangle</th>
<th>K_1</th>
<th>C_5 f</th>
<th>\frac{C_5 f}{K_1}</th>
<th>K_1 C_5 f</th>
</tr>
</thead>
<tbody>
<tr>
<td>\cdot 005</td>
<td>0.018</td>
<td>0.036</td>
<td>1.025</td>
<td>298</td>
<td>290</td>
<td>310</td>
</tr>
<tr>
<td>\cdot 008</td>
<td>0.011</td>
<td>0.022</td>
<td>1.025</td>
<td>468</td>
<td>456</td>
<td>450</td>
</tr>
<tr>
<td>\cdot 012</td>
<td>0.017</td>
<td>0.034</td>
<td>1.03</td>
<td>665</td>
<td>645</td>
<td>690</td>
</tr>
</tbody>
</table>

Thrust - Mean, \( n_1 = 0.936 \)

Table 12 - 3 (a)

<table>
<thead>
<tr>
<th>Feed (in/{per})</th>
<th>\sqrt{\langle V \rangle}</th>
<th>K \langle V \rangle</th>
<th>K_2</th>
<th>C_5 f</th>
<th>\frac{C_5 f}{K_2}</th>
<th>K_2 C_5 f</th>
</tr>
</thead>
<tbody>
<tr>
<td>\cdot 005</td>
<td>0.083</td>
<td>0.166</td>
<td>1.14</td>
<td>298</td>
<td>262</td>
<td>338</td>
</tr>
<tr>
<td>\cdot 008</td>
<td>0.082</td>
<td>0.164</td>
<td>1.14</td>
<td>468</td>
<td>412</td>
<td>532</td>
</tr>
<tr>
<td>\cdot 012</td>
<td>0.083</td>
<td>0.166</td>
<td>1.14</td>
<td>665</td>
<td>585</td>
<td>756</td>
</tr>
</tbody>
</table>

Thrust - individual, \( n_1 = 0.936 \)

Table 12 - 3(b)
Table 12 - 3(c)

<table>
<thead>
<tr>
<th>Feed (in/pt)</th>
<th>$\sqrt{V}$ mean</th>
<th>$\overline{V}$ mean</th>
<th>$K_3$</th>
<th>$C_6 f^{n_2}$</th>
<th>$C_6 f^{n_2} K_3$</th>
<th>$K_3 C_6 f^{n_2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>.005</td>
<td>.013</td>
<td>.026</td>
<td>1.02</td>
<td>88</td>
<td>86</td>
<td>91</td>
</tr>
<tr>
<td>.008</td>
<td>.008</td>
<td>.016</td>
<td>1.02</td>
<td>132</td>
<td>129</td>
<td>136</td>
</tr>
<tr>
<td>.012</td>
<td>.013</td>
<td>.026</td>
<td>1.025</td>
<td>202</td>
<td>197</td>
<td>209</td>
</tr>
</tbody>
</table>

Torque - Mean, $n_2 = 0.945$

Table 12 - 3(d)

<table>
<thead>
<tr>
<th>Feed (in/pt)</th>
<th>$\sqrt{V}$ ind</th>
<th>$k\sqrt{V}$ ind</th>
<th>$K_4$</th>
<th>$C_6 f^{n_2}$</th>
<th>$C_6 f^{n_2} K_4$</th>
<th>$K_4 C_6 f^{n_2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>.005</td>
<td>.062</td>
<td>.124</td>
<td>1.13</td>
<td>88</td>
<td>78</td>
<td>100.0</td>
</tr>
<tr>
<td>.008</td>
<td>.061</td>
<td>.122</td>
<td>1.135</td>
<td>132</td>
<td>116</td>
<td>150.0</td>
</tr>
<tr>
<td>.012</td>
<td>.062</td>
<td>.124</td>
<td>1.120</td>
<td>202</td>
<td>180</td>
<td>230.0</td>
</tr>
</tbody>
</table>

Torque - Individual, $n_2 = 0.945$

Table 12 - 3(d)

Table 12-3 shows the mean and individual observation limits of $T/d$ and $N/d^2$, assuming an exponential relationship with feed. Comparing these values with the ones shown in Table 11 - 3, a close agreement is noted. It is therefore reasonable to represent the thrust and torque by either set of equations, i.e. equations 13-3.
and 14-3 or 15-3 and 16-3. The constants C's and u's obtained from this experiment were only suitable for the material used - 65 ST6 Aluminium Alloy.

Equations 21-3 and 22-3 show the existence of edge effects of the drill on thrust and torque. The magnitude of these effects are proportional to the size of the drill since these two equations can be re-written as:

\[
T = (C_1 + C_2 f) \cdot d \\
M = (C_3 + C_4 f) \cdot d^2
\]

Thus these experimental results are consistent with those of other types of orthogonal cutting, in which edge forces are proportional to the cutting edge length.

When the observed data are represented by the equations:

\[
\frac{T}{d} = C_3 f^{n_1} \
\frac{M}{d^2} = C_4 f^{n_2}
\]

the relationship between the two parameters \(\frac{T}{d}\) or \(\frac{M}{d^2}\) and the feed rate is assumed to be represented by the dotted curve as shown in Fig. 8-3. The two unknowns C and n in each equation can be determined by taking the logarithm of both sides, i.e.:
\[
\log \left( \frac{T}{d} \right) = \log C_3 + n_1 \log f \\
\log \left( \frac{M}{d^2} \right) = \log C_4 + n_2 \log f
\]

It is obvious that the exponential terms \( n_1 \) and \( n_2 \) have to be less than 1 to satisfy the observed data. This was in fact what appeared in all the empirical equations presented by other investigators.
As for standard drills, the modified drills can also be considered similar provided that they satisfy the requirements specified in the analysis section. The two modified drills used in these experiments are 0.75" and 1.00" diameter (G and H) as listed in Table 3-3. Similarity of the cutting edges of the drills as well as similarity of the full drills will be studied.

VERIFICATION OF SIMILARITY CONCEPT FOR MODIFIED DRILLS

The first lot of tests was run with specimens with pilot holes. This was to eliminate the chisel edge effect when verifying similarity of the whole lips. The sizes of the pilot holes were made equal to the chisel edge length, being 0.114" and 0.149" for the 0.75" and 1.00" drill respectively. The tests were run at three feeds with six runs at each feed giving a total of 36 runs. The same experimental procedure as in the case of standard drills was employed except that in this case only two drills were used.
In Table 13-3 the observed thrust and torque of the 0.75" drill were used to estimate those of the 1.00 drill. At all the three feeds these estimated and observed thrust and torque were compared by the t-test. The t-test ratios all passed at different confidence limits with the exception of the thrust at 0.008 in/rev. In this case the ratio was passed at a higher level but still within statistical control. Figs. 9-3(a) and 9-3(b) show the observed and estimated forces plotted against feed.
A: 0.750" drill observed
B: 1.00" "
C: 1.00" " predicted

Similarity of modified drills
(with pilot hole)

Fig 9-3
A method of creating a flat rake face on a standard drill had been proposed by Armarego (22). The dimensions X and Y as shown in Fig. 10-3 must be carefully considered in the grinding process. If X is too large, the chisel edge region may be weakened excessively and collapse under high rate of penetration. Y must be greater than the expected contact length between the chip and the new rake face.

Various views of modified drill point geometry

Fig 10-3

Lo, Lodge and Armarego (56) established a relationship between the contact length h and the undeformed chip length t'.
in orthogonal cutting, i.e.:

\[ \text{h} = \cdot 6t \]

The maximum undeformed chip thickness in these experiments is about \( \cdot 012/2 = \cdot 006" \). Therefore the maximum possible contact length is \( \cdot 0036" \).

\( X \) is made equal to one-third of the total web-thickness (e.g. \( x = \cdot 04 \) for the 1\cdot 00" drill), which is comparable to the recommended value of point relieving (21). The relationship between \( X \) and \( Y \) is:

\[ X = Y \sin \alpha_f \]

where \( \alpha_f \) is the reference rake angle at the periphery and is equal to 35°. In this case \( Y \) exceeds the natural tool-chip contact length by a large margin.

The chisel edge length was not altered in any way with the exception of its surrounding region, which had been modified giving a form of point relieving. Since the magnitude of \( X \) was proportional to the chisel edge length, it was reasonable to assume that the two modified drills would still behave as similar drills when their chisel edges were included in the full consideration. The experimental procedure was the same as in the previous one except that tests were run with specimens without pilot holes. The t-test was again employed to compare the observed and estimated means. Table 14 - 3 contains these results:
The t-test ratio in Table 14-3 indicates that the observed and estimated thrust and torque can be considered statistically equal. This implies that the thrust and torque relationship derived for conventional drills can also be applied to the modified drills. The observed and estimated forces are shown in Fig. 11-3(a) and 11-3(b).
Similarity of modified drills
(without pilot hole)

Fig 11-3
EMPERICAL EQUATIONS FOR MODIFIED DRILLS

Since modified drills also satisfied the forces relationship as did the standard drills, empirical equations for these drills could be obtained based on the same reasoning as for the standard drills. Briefly, in the case of thrust, a mean line through the thrust per unit diameter \((T/d)\) was obtained by linear regression analysis. When force was considered as linear to feed rate the thrust and torque could be represented by the following equations:

\[
\frac{T}{d} = 27 + 25329 f \quad (23-3)
\]
\[
\frac{M}{d^2} = 12 + 13576 f \quad (24-3)
\]

The variances for these two equations were:

**Thrust:**

\[
V_{\text{mean}} = 18.6 - 15657 f + 111944 f^2
\]
\[
V_{\text{ind}} = 350.0 - 15657 f + 111944 f^2
\]

**Torque:**

\[
V_{\text{mean}} = 4.5 - 4320 f + 259247 f^2
\]
\[
V_{\text{ind}} = 81.2 - 4320 f + 259247 f^2
\]

The mean and individual values of \(T/d\) and \(M/d^2\) at each feed depends on the feed rate and the confidence level at which these values are required. Equations 19-3 - 20-3 illustrate how these confidence limits can be calculated. At 97.5% confidence level and with 33 degrees of freedom \(k = 2.04\). In the following Tables, the
various mean and individual values of $T/d$ and $T/d^2$ are provided:

![Image]

<table>
<thead>
<tr>
<th>Feed (in/rev)</th>
<th>$V_{\text{mean}}$</th>
<th>$k\sqrt{V_{\text{mean}}}$</th>
<th>$(T/d)_{\text{mean}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.005</td>
<td>21.6</td>
<td>9.3</td>
<td>152.4 ± 9.3</td>
</tr>
<tr>
<td>0.008</td>
<td>9.3</td>
<td>6.1</td>
<td>229.4 ± 6.1</td>
</tr>
<tr>
<td>0.012</td>
<td>24.3</td>
<td>9.8</td>
<td>330.7 ± 9.8</td>
</tr>
</tbody>
</table>

**Thrust - Mean**

*Table 15-3(a)*

<table>
<thead>
<tr>
<th>Feed (in/rev)</th>
<th>$V_{\text{ind}}$</th>
<th>$k\sqrt{V_{\text{ind}}}$</th>
<th>$(T/d)_{\text{ind}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.005</td>
<td>353.0</td>
<td>37.6</td>
<td>152.4 ± 36.6</td>
</tr>
<tr>
<td>0.008</td>
<td>340.7</td>
<td>36.9</td>
<td>229.4 ± 36.9</td>
</tr>
<tr>
<td>0.012</td>
<td>355.6</td>
<td>37.7</td>
<td>330.7 ± 37.7</td>
</tr>
</tbody>
</table>

**Thrust - Individual**

*Table 15-3(b)*

<table>
<thead>
<tr>
<th>Feed (in/rev)</th>
<th>$V_{\text{mean}}$</th>
<th>$k\sqrt{V_{\text{mean}}}$</th>
<th>$(M/d^2)_{\text{mean}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.005</td>
<td>5.0</td>
<td>4.5</td>
<td>89.5 ± 4.5</td>
</tr>
<tr>
<td>0.008</td>
<td>2.1</td>
<td>2.9</td>
<td>120.3 ± 2.9</td>
</tr>
<tr>
<td>0.012</td>
<td>5.6</td>
<td>4.7</td>
<td>174.6 ± 4.7</td>
</tr>
</tbody>
</table>

**Torque - Mean**

*Table 15-3(c)*
When $T/d$ and $M/d^2$ were considered as exponential functions of feed, the general information from the regression analysis were:

\[
\frac{T}{d} = 16731 f^{0.838}
\]

\[
\frac{T}{M^2} = 9444 f^{0.903}
\]

Thrust:

\[
V_{\text{mean}} = 5.10 + 0.012 \log f + 0.001 (\log f)^2
\]

\[
V_{\text{ind}} = 5.11 + 0.012 \log f + 0.001 (\log f)^2
\]

Torque:

\[
V_{\text{mean}} = 5.10 + 0.009 \log f + 0.001 (\log f)^2
\]

\[
V_{\text{ind}} = 5.11 + 0.009 \log f + 0.001 (\log f)^2
\]

The mean and individual confidence limits at each feed at 97.5% confidence level for $T/d$ and $M/d^2$ are shown in the Table 16 - 3.
<table>
<thead>
<tr>
<th>Feed (in/rev)</th>
<th>$V_{mean}$</th>
<th>$K\sqrt{V_{mean}}$</th>
<th>$n_1$</th>
<th>$K_1$</th>
<th>$C_5 f^{n_1} \frac{n_1}{K_1}$</th>
<th>$K_1 C_5 f^{n_1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.005</td>
<td>0.021</td>
<td>0.042</td>
<td>0.888</td>
<td>1.042</td>
<td>150</td>
<td>144</td>
</tr>
<tr>
<td>0.008</td>
<td>0.013</td>
<td>0.026</td>
<td>0.888</td>
<td>1.026</td>
<td>221</td>
<td>216</td>
</tr>
<tr>
<td>0.012</td>
<td>0.020</td>
<td>0.040</td>
<td>0.888</td>
<td>1.041</td>
<td>330</td>
<td>317</td>
</tr>
</tbody>
</table>

Thrust - Mean

Table 16-3(a)

<table>
<thead>
<tr>
<th>Feed (in/rev)</th>
<th>$V_{ind}$</th>
<th>$K\sqrt{V_{ind}}$</th>
<th>$n_1$</th>
<th>$K_2$</th>
<th>$C_5 f^{n_1} \frac{n_1}{K_2}$</th>
<th>$K_2 C_5 f^{n_1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.005</td>
<td>0.080</td>
<td>0.160</td>
<td>0.888</td>
<td>1.173</td>
<td>150</td>
<td>128</td>
</tr>
<tr>
<td>0.008</td>
<td>0.078</td>
<td>0.156</td>
<td>0.888</td>
<td>1.169</td>
<td>221</td>
<td>190</td>
</tr>
<tr>
<td>0.012</td>
<td>0.079</td>
<td>0.158</td>
<td>0.888</td>
<td>1.172</td>
<td>330</td>
<td>282</td>
</tr>
</tbody>
</table>

Thrust - Individual

Table 16-3(b)

<table>
<thead>
<tr>
<th>Feed (in/rev)</th>
<th>$V_{mean}$</th>
<th>$K\sqrt{V_{mean}}$</th>
<th>$n_2$</th>
<th>$K_3$</th>
<th>$C_6 f^{n_2} \frac{n_2}{K_3}$</th>
<th>$K_3 C_6 f^{n_2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.005</td>
<td>0.018</td>
<td>0.036</td>
<td>0.903</td>
<td>1.036</td>
<td>83</td>
<td>80</td>
</tr>
<tr>
<td>0.008</td>
<td>0.011</td>
<td>0.022</td>
<td>0.903</td>
<td>1.023</td>
<td>125</td>
<td>122</td>
</tr>
<tr>
<td>0.012</td>
<td>0.017</td>
<td>0.034</td>
<td>0.903</td>
<td>1.035</td>
<td>180</td>
<td>174</td>
</tr>
</tbody>
</table>

Torque - Mean

Table 16-3(c)
From the empirical equations representing the thrust and torque of the standard and modified drills, it is noted that the standard drills do not operate as efficiently as the modified drills. This is especially true in the case of thrust, where the modified drills only require about 60% of what the conventional drills require.

<table>
<thead>
<tr>
<th>Feed (in/rev)</th>
<th>V Ind</th>
<th>$k \sqrt{V \text{ Ind}}$</th>
<th>$n_2$</th>
<th>$K_4$</th>
<th>$C_6 f n_2^2$</th>
<th>$C_6 f n_2^2/K_4$</th>
<th>$K_4 C_6 f n_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.005</td>
<td>0.69</td>
<td>0.138</td>
<td>0.903</td>
<td>1.148</td>
<td>83</td>
<td>72</td>
<td>95</td>
</tr>
<tr>
<td>0.008</td>
<td>0.68</td>
<td>0.136</td>
<td>0.903</td>
<td>1.145</td>
<td>125</td>
<td>109</td>
<td>143</td>
</tr>
<tr>
<td>0.012</td>
<td>0.69</td>
<td>0.138</td>
<td>0.903</td>
<td>1.148</td>
<td>180</td>
<td>159</td>
<td>206</td>
</tr>
</tbody>
</table>

Torque - Individual

Table 16 - 3(d)
It was expected that the modified drills might have better performance than the standard drills, since the normal rake angle of the modified drill was vastly increased at the smaller radii, and the region near the chisel edge corner had been under-cut, which had the same effect as point relieving. To verify these results the relevant data were collected from the previous two experiments, in which six runs were taken with each of the four drills (two modified and two standard) at three different feeds, giving a total of 144 runs. The thrust and torque of the modified drills were compared with those of the standard drills.

<table>
<thead>
<tr>
<th>Feed (in/rev)</th>
<th>Modified Drill (lb)</th>
<th>Standard Drill (lb)</th>
<th>Modified std.</th>
<th>DOF</th>
<th>t-test Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>.005</td>
<td>52.8</td>
<td>91.5</td>
<td>57.8%</td>
<td>10</td>
<td>41.5 *</td>
</tr>
<tr>
<td>.008</td>
<td>77.5</td>
<td>137.9</td>
<td>56.5%</td>
<td>10</td>
<td>48.1 *</td>
</tr>
<tr>
<td>.012</td>
<td>101.0</td>
<td>178.7</td>
<td>57.0%</td>
<td>10</td>
<td>50.8 *</td>
</tr>
</tbody>
</table>

Thrust with Pilot Hole, .75" Dia.

Pilot Hole = .114" Dia.

Table 17 - 3(a)
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>.005</td>
<td>42.6</td>
<td>50.4</td>
<td>84.5%</td>
<td>10</td>
</tr>
<tr>
<td>.008</td>
<td>68.5</td>
<td>82.3</td>
<td>83.5%</td>
<td>10</td>
</tr>
<tr>
<td>.012</td>
<td>97.2</td>
<td>116.8</td>
<td>83.5%</td>
<td>10</td>
</tr>
</tbody>
</table>

**Torque with Pilot Hole, .75" Dia.**

Pilot Hole = .114" Dia.

Table 17 - 3 (b)

<table>
<thead>
<tr>
<th>Feed (in/rev)</th>
<th>Modified Drill (lb)</th>
<th>Standard Drill (lb)</th>
<th>Modified DOF</th>
<th>t-test Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>.005</td>
<td>68.3</td>
<td>114.0</td>
<td>60%</td>
<td>10</td>
</tr>
<tr>
<td>.008</td>
<td>95.2</td>
<td>177.0</td>
<td>54%</td>
<td>10</td>
</tr>
<tr>
<td>.012</td>
<td>126.6</td>
<td>226.2</td>
<td>56%</td>
<td>10</td>
</tr>
</tbody>
</table>

**Thrust with Pilot Hole, 1.00" Dia.**

Pilot Hole = .149" Dia.

Table 17 - 3 (c)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>.005</td>
<td>69.1</td>
<td>83.2</td>
<td>83%</td>
<td>10</td>
</tr>
<tr>
<td>.008</td>
<td>115.9</td>
<td>136.7</td>
<td>85%</td>
<td>10</td>
</tr>
<tr>
<td>.012</td>
<td>160.2</td>
<td>191.1</td>
<td>83.5%</td>
<td>10</td>
</tr>
</tbody>
</table>

**Torque with Pilot Hole, 1.00" Dia.**

Pilot Hole = .149" Dia.

Table 17 - 3 (d)
<table>
<thead>
<tr>
<th>Feed (in/rev)</th>
<th>Modified Drill (lb)</th>
<th>Standard Drill (lb)</th>
<th>Modifiedstd.</th>
<th>DOF</th>
<th>t-test Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>.005</td>
<td>120·0</td>
<td>223·8</td>
<td>53·5%</td>
<td>10</td>
<td>30·4 *</td>
</tr>
<tr>
<td>.008</td>
<td>184·8</td>
<td>351·0</td>
<td>52·0%</td>
<td>10</td>
<td>38·4 *</td>
</tr>
<tr>
<td>.012</td>
<td>263·4</td>
<td>497·0</td>
<td>53·0%</td>
<td>10</td>
<td>34·0 *</td>
</tr>
</tbody>
</table>

Thrust without Pilot Hole, .75' dia.

Table 17 - 3 (e)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>.005</td>
<td>47·7</td>
<td>59·2</td>
<td>80·5%</td>
<td>10</td>
<td>9·5 *</td>
</tr>
<tr>
<td>.008</td>
<td>76·9</td>
<td>94·5</td>
<td>81·5%</td>
<td>10</td>
<td>18·1 *</td>
</tr>
<tr>
<td>.012</td>
<td>107·1</td>
<td>130·6</td>
<td>82·5%</td>
<td>10</td>
<td>22·9 *</td>
</tr>
</tbody>
</table>

Torque without Pilot Hole, .75" Dia.

Table 17 - 3 (f)

<table>
<thead>
<tr>
<th>Feed (in/rev)</th>
<th>Modified Drill (lb)</th>
<th>Standard Drill (lb)</th>
<th>Modifiedstd</th>
<th>DOF</th>
<th>t-test</th>
</tr>
</thead>
<tbody>
<tr>
<td>.005</td>
<td>148·8</td>
<td>282·6</td>
<td>52·5%</td>
<td>10</td>
<td>29·8 *</td>
</tr>
<tr>
<td>.008</td>
<td>230·0</td>
<td>450·0</td>
<td>51·0%</td>
<td>10</td>
<td>25·8 *</td>
</tr>
<tr>
<td>.012</td>
<td>330·0</td>
<td>643·9</td>
<td>51·5%</td>
<td>10</td>
<td>34·9 *</td>
</tr>
</tbody>
</table>

Thrust without Pilot Hole, 1.00" Dia.

Table 17 - 3 (g)
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0.005</td>
<td>78.5</td>
<td>97.2</td>
<td>81%</td>
<td>10</td>
<td>13.9 *</td>
</tr>
<tr>
<td>0.008</td>
<td>120.9</td>
<td>151.0</td>
<td>80.5%</td>
<td>10</td>
<td>18.2 *</td>
</tr>
<tr>
<td>0.012</td>
<td>170.0</td>
<td>211.1</td>
<td>81.0%</td>
<td>10</td>
<td>17.6 *</td>
</tr>
</tbody>
</table>

Torque without Pilot Hole - 1.00" Dia.

(Table 17 - 3(h)

* significantly different at all confidence level.

From the large t-test ratios, it is seen that the thrust and torque of both drills differ. The modified drill gave only about 54 - 60% of thrust and about 80% of torque found from the conventional drill, when only the cutting edges were considered.

In the case of the full drills, the thrust by the modified drills is about 51 - 52% of the standard drills. This percentage is in general smaller than the one just quoted above. The significance of this is that the thrust due to the chisel edge of the modified drill must be about 50% of that by the standard drill. Table 18 - 3 shows the percentage of web-thrust due to the two drills.

<table>
<thead>
<tr>
<th>Feed (in/rev)</th>
<th>0.75&quot; Dia.</th>
<th>1.00&quot; Dia.</th>
<th>0.75&quot; Dia.</th>
<th>1.00&quot; Dia.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Web-thrust Modified (lb)</td>
<td>Web-thrust standard (lb)</td>
<td>%</td>
<td>Web-thrust Modified (lb)</td>
</tr>
<tr>
<td>0.005</td>
<td>67.2</td>
<td>132.3</td>
<td>51%</td>
<td>80.5</td>
</tr>
<tr>
<td>0.008</td>
<td>107.3</td>
<td>213.0</td>
<td>50%</td>
<td>134.8</td>
</tr>
<tr>
<td>0.012</td>
<td>162.0</td>
<td>319.0</td>
<td>51%</td>
<td>203.4</td>
</tr>
</tbody>
</table>

Comparison of Web-Thrust

Table 18 - 3.
The results in Table 18 - 3 indicate that the web-thrust of the modified drills is only about 50% of that of the standard drills. So the modification to the chisel edge region can be considered in effect similar to point relieving and on the whole the modified drills operated more efficiently than the standard drill as suggested by Armarego (22) earlier. In the following Fig. 12 - 3 some of these results are plotted:
By numerical testing, the thin shear plane analysis for the modified flat rake drill has been shown to predict reasonable trends. It is desirable to verify these trends with the experimental results in qualitative and quantitative terms. For this purpose, a 1.00" diameter modified drill and the following annulii were used:

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>2A</td>
<td>0.092&quot;</td>
<td>0.175</td>
</tr>
<tr>
<td>2B</td>
<td>0.175</td>
<td>0.259</td>
</tr>
<tr>
<td>2C</td>
<td>0.259</td>
<td>0.342</td>
</tr>
<tr>
<td>2D</td>
<td>0.342</td>
<td>0.426</td>
</tr>
<tr>
<td>2J</td>
<td>0.092</td>
<td>0.426</td>
</tr>
</tbody>
</table>

Table 19 - 3

For each annulus three runs were taken at each feed for three different feeds, giving a total of 45 runs and 90 chips (one for each cutting edge).
i) Observed Chip Length Ratio Distribution:

For each annulus, at each feed, there were six deformed chips, from which the chip length ratio at the inside and outside radius were evaluated and shown in the following tables:

<table>
<thead>
<tr>
<th>Feed (in/rev)</th>
<th>Inside Chip Ratio Rad = '092&quot;</th>
<th>Outside Chip Ratio Rad = '175&quot;</th>
</tr>
</thead>
<tbody>
<tr>
<td>.005</td>
<td>.463</td>
<td>.505</td>
</tr>
<tr>
<td>.008</td>
<td>.432</td>
<td>.473</td>
</tr>
<tr>
<td>.012</td>
<td>.432</td>
<td>.481</td>
</tr>
</tbody>
</table>

Annulus 2A
Table 20 - 3(a)

<table>
<thead>
<tr>
<th>Feed (in/rev)</th>
<th>Inside Chip Ratio Rad = '175&quot;</th>
<th>Outside Chip Ratio Rad = '259&quot;</th>
</tr>
</thead>
<tbody>
<tr>
<td>.005</td>
<td>.463</td>
<td>.522</td>
</tr>
<tr>
<td>.008</td>
<td>.487</td>
<td>.525</td>
</tr>
<tr>
<td>.012</td>
<td>.476</td>
<td>.518</td>
</tr>
</tbody>
</table>

Annulus 2B
Table 20 - 3(b)

<table>
<thead>
<tr>
<th>Feed (in/rev)</th>
<th>Inside Chip Ratio Rad = '259&quot;</th>
<th>Outside Chip Ratio Rad = '342&quot;</th>
</tr>
</thead>
<tbody>
<tr>
<td>.005</td>
<td>.523</td>
<td>.530</td>
</tr>
<tr>
<td>.008</td>
<td>.508</td>
<td>.527</td>
</tr>
<tr>
<td>.012</td>
<td>.519</td>
<td>.527</td>
</tr>
</tbody>
</table>

Annulus 2C
Table 20 - 3(c)
In the case of conventional drills, the chip length ratio has been shown to be independent of the size of the annulus and was only a function of the geometry on the radius concerned. The chip length ratio, in this experiment, were obtained for three different feeds. The feed rate in drilling controls the undeformed chip thickness and it is believed that the undeformed chip thickness has no effect on chip length ratio. This follows from reported results on orthogonal cutting, milling and form tool tests on the same material used in this project (51) (52).
At any radius, say \(1.75''\), there were six mean chip length ratios, three from annulus 2A (one for each feed), and three from 2B. An analysis of variance was performed to determine if these six means were statistically equal. Table 21-3 shows the test results:

<table>
<thead>
<tr>
<th>Rad.</th>
<th>Mean Chip Ratio</th>
<th>Var.</th>
<th>DOF</th>
<th>F-test Ratio</th>
<th>Remark</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.092&quot;</td>
<td>0.444</td>
<td>0.043</td>
<td>5.30</td>
<td>0.50</td>
<td></td>
</tr>
<tr>
<td>1.175</td>
<td>0.481</td>
<td>0.058</td>
<td>5.30</td>
<td>0.36</td>
<td>All passed at 90%</td>
</tr>
<tr>
<td>0.259</td>
<td>0.519</td>
<td>0.056</td>
<td>5.30</td>
<td>0.07</td>
<td></td>
</tr>
<tr>
<td>0.342</td>
<td>0.524</td>
<td>0.047</td>
<td>5.30</td>
<td>0.09</td>
<td></td>
</tr>
<tr>
<td>0.426</td>
<td>0.533</td>
<td>0.044</td>
<td>5.30</td>
<td>0.16</td>
<td></td>
</tr>
</tbody>
</table>

Chip Length Ratio at Various Radii

Table 21-3

*Fig 13-3* Chip length ratio distribution for 1.00" modified drill (drill H)
From the F-test ratios in Table 21 - 3 it can be concluded that the chip length ratio at any radius is a constant irrespective of feed rate and size of the annulus. Fig. 13 - 3 is the distribution of chip length ratio for the drill used in this experiment. The chip length ratio variation with radius was subject to another analysis of variance to determine if this variation was statistically significant at each radius.

<table>
<thead>
<tr>
<th>Rad.</th>
<th>Mean Chip Ratio</th>
<th>Var.</th>
<th>DOF</th>
<th>F-test Ratio</th>
<th>Remark</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.092</td>
<td>0.439</td>
<td>0.0011</td>
<td></td>
<td></td>
<td>All means are significantly different</td>
</tr>
<tr>
<td>0.175</td>
<td>0.476</td>
<td>0.0016</td>
<td>4,175</td>
<td>31.87</td>
<td></td>
</tr>
<tr>
<td>0.259</td>
<td>0.519</td>
<td>0.0022</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.342</td>
<td>0.523</td>
<td>0.0019</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.426</td>
<td>0.532</td>
<td>0.0018</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Variation of \( r_1 \) with radius

Table 22 - 3

The chip length ratio at one radius was compared with those at other radii. The F-test ratio indicates that these variations were significantly different as suggested by the large variance ratio of 31.87.
ii) Thrust and Torque Distributions

For each annulus, a number of tests was run at three different feeds. The thrust and torque were recorded. Assuming, and this had been proved reasonable for standard and modified drills, that the thrust and torque were linear to feed, then it was possible to obtain the linear equations relating these two parameters with feed. Using the special linear regression analysis as for the case of standard drills, the following equations were obtained:

Annulus 2A :-

\[
T = 10.8 + 2271f \\
M = 3 + 1459f
\]

Annulus 2B :-

\[
T = 8.4 + 2011f \\
M = 1.0 + 1243f
\]

Annulus 2C :-

\[
T = 7.4 + 1780f \\
M = 3.8 + 2172f
\]

Annulus 2D :-

\[
T = 6.5 + 1735f \\
M = 1.7 + 3297f
\]
Annulus 2J:

\[
T = 31.3 + 7666 f \\
M = 9.5 + 8570 f
\]

The reason to establish these equations was to enable the actual thrust and torque due to deformation to be calculated, i.e. to isolate any edge effects representing the intercept at \( f = 0 \). In the following tables these forces are shown:

<table>
<thead>
<tr>
<th>Feed (in/rev)</th>
<th>Thrust (lb)</th>
<th>Torque (in-lb)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Including Edge Effect</td>
<td>Excluding Edge Effect</td>
</tr>
<tr>
<td>.005</td>
<td>21.1</td>
<td>11.4</td>
</tr>
<tr>
<td>.008</td>
<td>28.9</td>
<td>18.2</td>
</tr>
<tr>
<td>.012</td>
<td>38.0</td>
<td>27.3</td>
</tr>
</tbody>
</table>

Annulus 2A, Inside Rad. = .092", Outside Rad. = .175"

Table 23 - 3(a)

<table>
<thead>
<tr>
<th>Feed (in/rev)</th>
<th>Thrust (lb)</th>
<th>Torque (in-lb)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Including Edge Effect</td>
<td>Excluding Edge Effect</td>
</tr>
<tr>
<td>.005</td>
<td>18.4</td>
<td>10.1</td>
</tr>
<tr>
<td>.008</td>
<td>24.4</td>
<td>16.1</td>
</tr>
<tr>
<td>.012</td>
<td>32.5</td>
<td>24.1</td>
</tr>
</tbody>
</table>

Annulus 2B, Inside Rad. = .175", Outside Rad. = .259"

Table 23 - 3(b)
<table>
<thead>
<tr>
<th>Feed (in/rev)</th>
<th>Thrust (lb)</th>
<th>Torque (in-lb)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Including Edge Effect</td>
<td>Excluding Edge Effect</td>
</tr>
<tr>
<td>.005</td>
<td>16.4</td>
<td>8.9</td>
</tr>
<tr>
<td>.008</td>
<td>21.7</td>
<td>14.2</td>
</tr>
<tr>
<td>.012</td>
<td>28.8</td>
<td>21.4</td>
</tr>
</tbody>
</table>

Annulus 2C, Inside Rad. = .259", Outside Rad. = .175"  
Table 23 - 3(c)

<table>
<thead>
<tr>
<th>Feed (in/rev)</th>
<th>Thrust (lb)</th>
<th>Torque (in-lb)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Including Edge Effect</td>
<td>Excluding Edge Effect</td>
</tr>
<tr>
<td>.005</td>
<td>15.1</td>
<td>8.7</td>
</tr>
<tr>
<td>.008</td>
<td>20.3</td>
<td>13.9</td>
</tr>
<tr>
<td>.012</td>
<td>27.3</td>
<td>20.8</td>
</tr>
</tbody>
</table>

Annulus 2D, Inside Rad. = .342", Outside Rad. = .426"  
Table 23 - 3(d)

<table>
<thead>
<tr>
<th>Feed (in/rev)</th>
<th>Thrust (lb)</th>
<th>Torque (in-lb)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Including Edge Effect</td>
<td>Excluding Edge Effect</td>
</tr>
<tr>
<td>.005</td>
<td>69.6</td>
<td>38.3</td>
</tr>
<tr>
<td>.008</td>
<td>92.6</td>
<td>61.33</td>
</tr>
<tr>
<td>.012</td>
<td>123.3</td>
<td>92.0</td>
</tr>
</tbody>
</table>

Annulus 2J, Inside Rad. = .092", Outside Rad. = .426"  
Table 23 - 3(e)
The important point noted in these results is that the thrust decreases as the mean radius increases, thus confirming what the cutting model was predicting earlier in the Analysis Section. Fig 14-3 shows the thrust and torque of Annulus 2J represented by a mean line, and the variance about this mean line and the individual points. Any prediction falling within the region bound by the variance could be considered as satisfactory statistically (at 97.5% confidence level).

![Diagram showing thrust and torque with mean lines and confidence limits](image)

*Fig 14-3* Mean values and confidence limits for individual observations (at 97.5% C.L.)
iii) Predicted Chip Length Ratio And Forces.

In order to theoretically predict the chip length ratio and forces distributions, two parameters have to be determined first. The chip length ratio at a particular radius and the chip flow relationship for the whole cutting edge must be known. The chip length ratio at a particular radius can be obtained from the observed chip length ratio distribution curve and therefore the only unknown is the chip flow relationship. When the model was tested numerically the relationship \( \eta_c = i \) had been discarded because it gave negative \( f_q \) and \( f_{th} \) and also a constant chip length ratio for all radii which contradicted what was expected and experimentally observed in the modified and conventional drills. There are still two other relationship left for consideration:

\[
\eta_c = ki \\
\text{and} \\
\tan \eta_c = \tan i \cos \alpha_n
\]

where \( k = c_1 - c_2 \alpha_n \)

As for Stabler's rule, there is no theoretical justification for either one of these two relationships. The first relationship is more flexible in that the two constants \( c_1 \) and \( c_2 \) can be arbitrarily adjusted, of course, based on experimental evidence as explained before. The second relationship is more rigid in that the angles of obliquity and the normal rake angle are all known quantities. Also this relationship does not apply when \( \alpha_n \) is negative assuming the published experimental results are true, i.e. \( \eta_c > i \) when \( \alpha_n < 0 \). The two relationship will be inputed into the model in turn. The
predicted chip length ratio distribution will be compared with the observed values.

The experimental chip length ratio of 0.533 at the outside radius of 0.426" was used as the starting point. The predicted chip length ratio when using the second relationship, i.e.:

\[
\tan \eta_c = \tan \iota \cos \alpha_n
\]

is represented in Fig. 15 - 3, together with the observed values. The two distributions appeared to agree fairly well except for some divergence at the small radii. The next consideration is how well does the model predict thrust and torque. The following Table 24 - 3 shows the predicted values at three feeds:

| Feed (in/rev) | Thrust (lb) | | | | | Torque (in-lb) |
|---------------|-------------|-----------------|-----------------|-----------------|-----------------|
|               | Observed | Predicted | Predicted Edge Effect | Observed | Predicted | Predicted Edge Effect |
| 0.005         | 69.9     | 5.2        | 36.5             | 51.4     | 42.5        | 52.0             |
| 0.008         | 92.6     | 8.3        | 39.8             | 78.0     | 67.2        | 76.7             |
| 0.012         | 123.3    | 12.5       | 43.8             | 112.3    | 101.0       | 109.5            |

Annulus 2J

Table 24 - 3
The edge effect must be added to the predicted thrust and torque when comparing with the observed values. The torque values agree extremely well. The difference between the two is less than 3%. But the predicted thrust only accounts for 52% of the observed thrust at 0.005 in/rev., 43% at 0.008 in/rev. and a mere 35% at 0.012 in/rev. So the cutting model with this chip flow relationship is rather unsatisfactory in the sense that it predicts the thrust poorly.

The first relationship was also tried. The constants $C_1$ and $C_2$ had to be determined first of all. From the results obtained by Armarego and Brown (49), $C_1$ and $C_2$ are in the vicinity of 0.9 and 0.002.
But these values were only for cutting steel in simple oblique cutting, and therefore they may not suitably apply in this cutting case. However these values can be used as guide lines. A large combination of $C_1$ and $C_2$ were fed into the cutting model programme and it was hoped that a satisfactory combination might be obtained by trial and error. Appendix E shows two thrust and torque distributions for a certain combination of $C_1$ and $C_2$. Table 25-3 shows the predicted thrust and torque when using the various values of $C_1$ and $C_2$. 
<table>
<thead>
<tr>
<th>No.</th>
<th>$C_1$</th>
<th>$C_2$</th>
<th>Chip Ratio at $R=0.426$</th>
<th>Chip Ratio at $R=0.098$</th>
<th>Diff</th>
<th>Predicted Thrust (lb)</th>
<th>Predicted Torque (in-lb)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.005 in/rev</td>
<td>0.008 in/rev</td>
</tr>
<tr>
<td>1</td>
<td>0.71</td>
<td>0.0010</td>
<td>0.533</td>
<td>0.434</td>
<td>0.003</td>
<td>42</td>
<td>67</td>
</tr>
<tr>
<td>2</td>
<td>0.71</td>
<td>0.0075</td>
<td>0.533</td>
<td>0.455</td>
<td>0.006</td>
<td>39</td>
<td>63</td>
</tr>
<tr>
<td>3</td>
<td>0.71</td>
<td>0.0050</td>
<td>0.533</td>
<td>0.475</td>
<td>0.020</td>
<td>36</td>
<td>58</td>
</tr>
<tr>
<td>4</td>
<td>0.72</td>
<td>0.0015</td>
<td>0.533</td>
<td>0.412</td>
<td>0.008</td>
<td>43</td>
<td>69</td>
</tr>
<tr>
<td>5</td>
<td>0.72</td>
<td>0.0010</td>
<td>0.533</td>
<td>0.435</td>
<td>0.003</td>
<td>40</td>
<td>64</td>
</tr>
<tr>
<td>6</td>
<td>0.72</td>
<td>0.0075</td>
<td>0.533</td>
<td>0.456</td>
<td>0.007</td>
<td>37</td>
<td>59</td>
</tr>
<tr>
<td>7</td>
<td>0.73</td>
<td>0.0010</td>
<td>0.533</td>
<td>0.418</td>
<td>0.007</td>
<td>41</td>
<td>66</td>
</tr>
<tr>
<td>8</td>
<td>0.73</td>
<td>0.0015</td>
<td>0.533</td>
<td>0.437</td>
<td>0.003</td>
<td>38</td>
<td>61</td>
</tr>
<tr>
<td>9</td>
<td>0.73</td>
<td>0.0075</td>
<td>0.533</td>
<td>0.457</td>
<td>0.007</td>
<td>35</td>
<td>56</td>
</tr>
<tr>
<td>10</td>
<td>0.74</td>
<td>0.0015</td>
<td>0.533</td>
<td>0.401</td>
<td>0.019</td>
<td>42</td>
<td>67</td>
</tr>
<tr>
<td>11</td>
<td>0.74</td>
<td>0.0012</td>
<td>0.533</td>
<td>0.420</td>
<td>0.007</td>
<td>39</td>
<td>63</td>
</tr>
<tr>
<td>12</td>
<td>0.74</td>
<td>0.0010</td>
<td>0.533</td>
<td>0.439</td>
<td>0.003</td>
<td>35</td>
<td>56</td>
</tr>
<tr>
<td>13</td>
<td>0.75</td>
<td>0.0015</td>
<td>0.533</td>
<td>0.403</td>
<td>0.017</td>
<td>39</td>
<td>63</td>
</tr>
<tr>
<td>14</td>
<td>0.75</td>
<td>0.0012</td>
<td>0.533</td>
<td>0.421</td>
<td>0.006</td>
<td>36</td>
<td>58</td>
</tr>
<tr>
<td>15</td>
<td>0.75</td>
<td>0.0010</td>
<td>0.533</td>
<td>0.440</td>
<td>0.003</td>
<td>33</td>
<td>53</td>
</tr>
</tbody>
</table>

Predicted Thrust and Torque for Various Combinations of $C_1$ and $C_2$.  

Table 25-3
The constants $C_1$ and $C_2$ had been chosen such that the predicted chip length ratio would follow closely the observed values. In order to obtain some indication of the difference between the two distributions, the sum of the square of the differences at radius $0.092''$, $0.175''$, $0.259''$, $0.342''$, and $0.426''$ were calculated and these are shown in the sixth column of Table 25 - 3. The smallest value is $0.0003$.

There were several combinations of $C_1$ and $C_2$, i.e. No. 2, 6, 8, 11 and 13 in Table 25 - 3, with which the model predicted nearly the same thrust as the observed ones. The predicted torque was almost constant for all $C_1$ and $C_2$. The predicted torque plus the observed edge effect was about 10% more than the observed torque at all feeds. In view of the fact that the model can predict very well the chip length ratio distribution, the thrust (less than 1% out) and fairly well the torque (10% out), the model using this chip flow relationship, i.e. $\eta_c = (C_1 - C_2 \alpha_n) \beta$, can be considered more satisfactory than that using $\tan \eta_c = \tan \beta \cos \alpha_n$.

The next check on the model's applicability was its predicted trends of thrust and torque as compared with the observed values of the smaller annulii. The following tables contain these informations:
Predicted And Observed Thrust

And Torque, Annulus 2A

using \( \eta_c = k_i \)

Table 26 - 3(a)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>+ Edge Effect</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.005</td>
<td>21.1</td>
<td>20.4</td>
<td>3.3%</td>
<td>7.6</td>
<td>8.1</td>
</tr>
<tr>
<td>0.008</td>
<td>28.9</td>
<td>28.4</td>
<td>1.7%</td>
<td>12.0</td>
<td>11.3</td>
</tr>
<tr>
<td>0.012</td>
<td>38.0</td>
<td>37.7</td>
<td>0.8%</td>
<td>17.8</td>
<td>17.9</td>
</tr>
</tbody>
</table>

Predicted And Observed Thrust

And Torque, Annulus 2B,

using \( \eta_c = k_i \)

Table 26 - 3(b)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>+ Edge Effect</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.005</td>
<td>18.4</td>
<td>17.3</td>
<td>6%</td>
<td>10.2</td>
<td>11.2</td>
</tr>
<tr>
<td>0.008</td>
<td>24.4</td>
<td>23.3</td>
<td>5.4%</td>
<td>15.8</td>
<td>17.3</td>
</tr>
<tr>
<td>0.012</td>
<td>32.5</td>
<td>30.9</td>
<td>4.3%</td>
<td>23.1</td>
<td>25.2</td>
</tr>
</tbody>
</table>
Predicted And Observed Thrust And Torque, Annulus 2C,

\[ \eta = k_i \]

Table 26 - 3(c)

<table>
<thead>
<tr>
<th>Feed in/rev</th>
<th>Observed</th>
<th>Predicted + Edge Effect</th>
<th>Diff.</th>
<th>Observed</th>
<th>Predicted + Edge Effect</th>
<th>Diff.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.005</td>
<td>15.1</td>
<td>15.3</td>
<td>-1.4%</td>
<td>14.7</td>
<td>17.4</td>
<td>-18%</td>
</tr>
<tr>
<td>0.008</td>
<td>21.7</td>
<td>20.6</td>
<td>5.1%</td>
<td>21.2</td>
<td>25.5</td>
<td>-20%</td>
</tr>
<tr>
<td>0.012</td>
<td>28.8</td>
<td>27.7</td>
<td>3.8%</td>
<td>29.9</td>
<td>36.4</td>
<td>-21%</td>
</tr>
</tbody>
</table>

Predicted And Observed Thrust And Torque, Annulus 2D,

using \[ \eta = k_i \]

Table 26 - 3(d)
<table>
<thead>
<tr>
<th>Feed in/rev</th>
<th>Thrust (lb)</th>
<th>Torque (in-lb)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Observed</td>
<td>Predicted + Edge Effect</td>
</tr>
<tr>
<td>0.005</td>
<td>69.6</td>
<td>69.3</td>
</tr>
<tr>
<td>0.008</td>
<td>92.6</td>
<td>92.3</td>
</tr>
<tr>
<td>0.012</td>
<td>123.3</td>
<td>123.3</td>
</tr>
</tbody>
</table>

Predicted And Observed Thrust and Torque, Annulus 2J,

using $\frac{\eta}{\sqrt{\tau}} = ki$

Table 26 - 3(e)
From the results contained in Table 26-3, it is noted that the predicted trend of thrust is in the expected direction. The predicted thrust compares very well with the observed values of the smaller annulii, i.e. 2A, 2B, 2C and 2D, as well as with those of the large annulus 2J. The largest variation in thrust for the small annulii was about 5%.

The variation between the predicted and observed torque was between 2 - 10% except annulus 2C (about 20%). The possible reason for this rather large discrepancy could be that the edge effect on torque had been over-estimated as indicated by the empirical torque equation in page 179. Using the observed edge effect on torque of the other three annulii 2A, 2B and 2D as guides, the edge effect for 2C should be about 1.5 in-lb instead of 3.8 in-lb. If this was the case, then the discrepancy was about 10%, which would be in comparable agreement with those of the other annulii. Fig. 16-3 represent all these predicted and observed results.

It was rather puzzling to find that the first chip flow relationship i.e. \( \tan \gamma_c = \tan \alpha \cos \alpha_n \) did not give reasonable thrust predictions. The only difference, in both cases, was the chip flow relationship being employed. The thrust was a function of \( f_q \) and \( f_r \) as given by equation 30-2, i.e.:

\[
 f_{th} = f_q \cos \gamma \sin \beta - f_r (\cos \alpha \cos \beta + \sin \alpha \sin \gamma \sin \beta) 
\]

- (25-3)
Predicted, Observed (in rev)

F_{th} (lb)

F_{torq} (in-lb)

Feed (in/rev)

Annulus 2A

Fig 16-3
Fig 16-3

(c) Annulus 2B

(d)
Observed

Predicted

Fig 16-3

Annulus 2C
Fig 16-3
Fig 16-3

Annulus 2J

(i) 

(ii)
Any variation between the predicted thrust must be due to the variation in these two parameters. Fig. 17 - 3(a) shows the difference of the chip flow angles when using either one of the two chip flow relationships. The deviation from the angle of obliquity was more when the relationship

\[ \gamma_c = (C_1 - C_2 \alpha_n) \]

was used. The difference was about 1° at the outside radius and about 4° at the inside radius. The deviation did not vary much for both relationship as shown in Fig. 17 - 3(b). But there was a very marked difference between the two \( f \)'s as could be seen in Fig. 17 - 3(c). The \( f \) with the relationship of \( \gamma_c = k \alpha \) was about 50 lb larger than the other \( f \) at all radii. According to equation 25-3, it became obvious why \( f \) was so small when using the relationship \( \tan \gamma_c = \tan \alpha \cos \alpha_n \). Because of the comparatively small values of \( f \) at small radii, the thrust predicted using this relationship was in the reverse trend to that of the other relationship as shown in Fig. 17 - 3(d). Thus the model with this relationship not only under-estimated the thrust, it also predicted a trend which was inconsistent with the observed one as shown in Fig. 18 - 3(b). The predicted torque using the two chip flow relationships is shown in Fig. 18 - 3(a).
Fig 17-3 (a)

Chip flow angle ($\eta_c$) distributions

Fig 17-3 (b)
Fig 17-3 (c)

**Thrust ($f_{th}$) distributions**

Fig 17-3 (d)
Torque $\tau_{\text{torq}}$ distributions

Fig 10-3 (a)

Thrust distribution of modified drill (drill H) $Feed = .023$/rev

Fig 10-3 (b)
CONCLUDING REMARKS

It was shown that speed does not affect the magnitude of the cutting forces. Thus, when comparing the force relationship of similar annulii or similar drills of different sizes, there is no need to adjust the cutting speed.

The force equation for a single drill, i.e.:

\[ F_{th} = \sum_{r=R_1}^{r=R_2} f_{th} \cdot \Delta r \]

and

\[ F_{torq} = \sum_{r=R_1}^{r=R_2} f_{tang} \cdot \Delta r \cdot r \]

based on which the force relationship for similar drills were derived, was shown to be valid. These results together with the comparison of chip length ratios showed that deformation and forces are only functions of the geometry at the point in question, and that they are significantly different at different radii.

Despite the variation in sizes, the drills used in these experiments were geometrically similar. In the case of the cutting edges only, the concept of similarity was proved by studying one of the chip formation parameter — the chip length ratio and then by comparing the observed forces of these similar drills.

The similarity concept for the full drills was also proved
by force comparison. When the value of \( X \) and \( Y \) as shown in Fig. 10 - 3 were properly controlled in the grinding process, modification to the rake face and to the chisel edge region did not destroy the geometric similarity for the modified drills. This was confirmed from the tests run.

The reduction in thrust due to modification to the rake face (i.e. when drilling with pilot hole) was about 40 - 45% of that of the standard drills. The reduction in torque was not so pronounced, only 20%. The reduction in thrust for the full modified drill was about 53%. The under-cutting of the chisel edge region could be considered as a type of point relieving, and was responsible for over 50% reduction in the chisel edge thrust.

The cutting model for flat rake drills is satisfactory when the chip flow angle \( \gamma_c = k_1 \) was inputed into the model. The trends of the predicted and observed chip length ratio, thrust and torque agree very well. Quantitative comparison of thrust show that the difference between the predicted and observed values is at the most 3%, whereas for torque it is about 10%.
While the geometry of the individual standard drill is very complex, it is possible to relate the geometry of different size drills. Based on geometric similarity the following force equations can be established:

\[ T_1 = \left( \frac{d_1}{d_2} \right)^1 T_2 \]

\[ M_1 = \left( \frac{d_1}{d_2} \right)^2 M_2 \]

These equations may be considered more realistic than the empirical equations since their derivations were based on the understanding of the basic geometry of the individual drills and their geometric inter-relationship.

The exponential terms in these two equations are 1 and 2, which are different from those of the empirical equations. This is probably due to the lack of similarity between the drills used in the empirical tests.

In the absence of a valid cutting model for the standard drills, these equations provide a convenient means of predicting cutting forces. These force relationships should prove to be most useful in the design and testing of drills. Once the optimum geometry of one drill is satisfied, there is no need to test every drill. But there are practical problems such as the larger ratio of t/d.
normally used for smaller drills. For this reason the theoretical force equations cannot be applied and every drill must be tested. A compromise can be reached by grouping together a number of similar drills, which will satisfy the following requirements:

\[ \frac{t_1}{t_2} = \frac{L_1}{L_2} = \frac{d_1}{d_2} \]

and

\[ \frac{p_1}{p_2} = \frac{\theta_1}{\theta_2} \]

Within each group the force relationship can be applied as has been shown by the results of statistical analysis of the data. The number of similar drills in each group was discussed in the analyses section. It is envisaged that the design of drills can be facilitated by this compromise and the number of performance tests would be minimized since the drills in each group are similar.

The modified drills have merits of their own. In addition to their higher cutting efficiency, the similarity concept is also applicable together with the above mentioned advantages. Furthermore the chip formation process of the modified drill lips can be described by a cutting model. Thus the forces of the modified drill can be predicted by using the force equations or the cutting model.

While the model predicts forces very well, it is only as good as other simpler orthogonal or oblique cutting analyses.
Difficulties such as the chip flow relationship and chip length ratio as in oblique cutting still remain. The chip flow angles in the model can only be estimated using other investigators' experimental data as guides. The chip length ratio has to be measured but only one value at a particular radius is necessary. This can be considered an advantage of the model since it would be impractical or even impossible to measure the chip length ratio at all radii.
FUTURE STUDIES

In the specification of twist drills, only the convenient parameters such as the point angle and clearance angle are specified. The fundamental cutting angles $\alpha_n$ and $i$ are not included. These are the two angles which control the cutting forces and chip flow directions. Therefore in order to be able to recommend the correct drill for a particular operation, based in principles of mechanics of metal cutting, it is necessary to investigate if these two angles can be incorporated in the drill specification. In fact this would fall in line with the attempts to specify single point lathe tool in terms of $\alpha_n$ and $i$.

The geometry of the chisel edge and its surrounding region has been shown to have a marked influence on thrust. Understanding the effect of the geometry on the mode of deformation of the metal in this region is a necessary step in arriving at an optimum geometry for the drill. The mechanics of grinding must be studied to enable a proper control of the geometry in this region.

While larger ratio of $t/d$ is recommended for smaller drills for reasons of higher torsional rigidity, a constant $t/d$ ratio is necessary for the force equation to apply. One possible solution to this problem is by shortening the overall length of the smaller drills so that their torsional rigidity is increased and at the same time the $t/d$ ratio can be made equal. It has been reported that the overall
length of the drill influences drill life. It seems probable that a tool life relationship can be established by studying the various overall length and the speed effect on tool life of these drills simultaneously. The speed effect must be considered since speed at corresponding radii of the similar drills is different and it has been shown to influence drill life. This approach can be considered as investigation into the dynamic similarity of drills, i.e. geometric similarity, rigidity similarity and tool life similarity.

A cutting model for the standard drill should prove useful in the design of these drills. The failure of the proposed model in the analysis section can be partly due to the fact that the chip was forced to flow in some unrealistic directions. Some re-thinking about the flute geometry and the effect of the rake face curvature may help eliminate the restriction imposed on the chip flow direction for standard drills discussed in the analysis presented. This approach may also lead to a cutting model which can qualitatively predict the forces and deformation. Quantitative prediction may largely depend on better understanding of the simpler cutting model of orthogonal and oblique cutting.

The modified drills have been proved successful in reducing thrust and torque. The extent to which the drill points should be modified to give optimum performance is still an area for further research.
REFERENCES

5. METAL CUTTING TOOL HANDBOOK, Metal Cutting Institute, N.Y. 1965, P.1.


APPENDIX A: Method of Grinding a Flat Rake Face on a Conventional Drill:

For this purpose, a tool and cutter grinder with the attachment of a universal vice is used. The drill grinder can be located in any position relative to the wheel. Fig. 1 - A(a), and (b) are the two original views of the drill relative to the grinding wheel. Axes 1, 2 and 3 are mutually perpendicular to each other. Axis 1 coincides with the longitudinal axis of the drill, which is perpendicular to the face of the grinding wheel at this instant. The next step is to rotate about Axis 2 by an amount equal to half the point angle of the drill, so that the grinding wheel face is parallel to the cutting edge ab as shown in Fig. 1 - A(c) and (d). Then the drill is turned about Axis 3 through an angle of $\alpha_{fo}$ as shown in Fig. 1 - A(e). The grinding wheel is moved towards the drill cutting edge until the wheel face is in contact with the cutting edge. Using this point as the reference point, the wheel is moved in further by Y, the value of which had been discussed in the experimental section. The grinding wheel is gradually lowered onto the rake face of the drill as shown in Fig. 1 - A(f). Grinding is completed when the lowest point of the grinding wheel just missed the point b. The drill is rotated 180° about its longitudinal axis, and the same grinding process is applied to the other cutting lip. Fig. 1 - A(g) shows the resultant geometry of this drill. The relationship between X and Y was given in the experimental section.
Fig 1-A
Various views of modified drill point geometry

Fig 1-A (9)
APPENDIX B: DESCRIPTION OF THE FORCE MEASURING SYSTEM
AND DESIGN OF TORQUE MEASURING ELEMENT.

Fig. 1-B is a block diagram of the force measuring system:

Carrier amplifier system—block-diagram

Fig 1-B

$V_1$ is the input voltage to the amplifier and it is given by:

$$V_1 = \frac{V}{4.0} \times g \times \text{(sum of strain)}$$

Where $V$ is the output voltage from the oscillator and is equal to 5 volt. $g = 1.91$ is the gauge factor. The
gauges are usually mounted in such a way (e.g. the 4 arm Wheatstone Bridge illustrated in Fig. 2-B) that the sum of strain is equal to the absolute value of strain $\varepsilon$ of the individual gauges. Therefore $V_1$ can be re-written as:

$$V_1 = \frac{a \times 5 \times 1.91}{4.0} \varepsilon \quad \text{-(1-B)}$$

where "$a$" is the number of active gauges.

From data supplied by the Handbooks, it was calculated that an amplifier voltage of 250 $\mu$V (i.e. $V_1 = 250 \, \mu V$) would give 568 mm deflection in the U.V. Recorder at minimum attenuation (i.e. at 0 db). In general this deflection is approximately halved for each successive increase in attenuation, i.e. $\frac{568}{2}$ mm at 6 db, $\frac{568}{4}$ mm at 12 db, $\frac{568}{8}$ mm at 18 db and $\frac{568}{16}$ mm at 24 db.

From preliminary tests and rough calculations based on the cutting model the expected torque was to be in the range of 10 - 150 in-lb. The width of the Recorder Photo-sensitive paper is 100 mm. In order to make full use of the width of the paper 1 in-lb should give a deflection of $\frac{1 \, \text{in-lb}}{150 \, \text{in-lb}} \times 100 \, \text{mm} = 0.65 \, \text{mm}$ at a certain attenuation.

The amplifier becomes difficult to balance as well as there is always a "shift" of the datum at low db. Therefore attenuation at 24 db is selected. According to equation 1-B, for $\chi / \mu \varepsilon$ in every gauge, the deflection in the Recorder is given by:
Deflection = \( \frac{a \times 5 \times 1.91}{16 \times 4} \times \frac{568}{250} \times \chi \text{ mm} \)

Therefore for 0.65 mm deflection at 24 db the required strain in each of the gauges is:

\[
\chi = \frac{16 \times 4 \times 0.65 \times 250}{4 \times 5 \times 1.91 \times 568} \approx 0.5
\]

For measuring torque a tubular (4 arm bridge) element together with a full Wheatstone Bridge was to be designed. (Fig. 2-B(a) shows the functional section of the element and the locations of the gauges:

**Fig 2-B**

---

**Gauge 1**

**Functional dimensions of element (a)**

**Resistance of gauge = 120 Ohm**

**Gauge factor = 1.91**

(b)
The four active gauges were mounted at 45° to the axis of the element to coincide with the direction of principal stress due to torsion. Also at this angle the cross-sensitivity due to compression is theoretically eliminated. The maximum stress $\tau'$ in torsion for a tubular element is:

$$\tau' = \frac{M_d}{2I_p}$$

Where $M$ is the torque ($= 1.0$ in-lb), $I_p$ is the polar moment of inertia and $d_0$ is the outside diameter of the tube.

The shear stress due to the strain $\chi$ in this direction is equal to:

$$\tau' = E \chi$$

where $E$ is Young's modulus. ($= 30 \times 10^6$ in/lb$^2$). Thus:

$$\tau' = 30 \times 10^6 \times 0.5 \times 10^6$$
$$\tau' = 15 \text{ lb}$$

Substituting this value into equation 2-B gives

$$15.0 = \frac{M_d}{2I_p}$$

or

$$I_p = \frac{M_d}{2 \times 15}$$

$$= \frac{\tau}{32} \left( d_0^4 - d_1^4 \right)$$
there are two unknowns $d_o$ and $d_i$ in the one equation. $d_o$ was selected at .8" and $d_i$ could then be calculated, i.e.:

$$\frac{1.0 \times .8}{2 \times 15} = \frac{\pi}{32} \left( .8^4 - d_i^4 \right)$$

$$d_i^4 = .8^4 - \frac{32 \times 1.0 \times .8}{2 \times 150 \times}$$

$$= .410 - .270$$

$$= .138$$

$$d_i = .607$$

$\approx .60$ in.

It was necessary to keep the length of the element as short as practical for reasons of lateral rigidity. At the same time the element must be long enough for easy mounting of the gauges. One inch was selected. The dimensions of the element were $L = 1.0"$, $d_i = .6"$, $d_o = .8"$. Fig. 3-b is a drawing of this element.

The calibrated sensitivity was .77 mm/in-lb, which agreed reasonably well with the expected sensitivity of .67 mm/in-lb. Fig. 4-3 is the calibration chart for torque.

When a torque of $T$ in-lb and a thrust of $P$ lb are applied to the element, the compressive stress $\sigma_C$ and shear stress $\tau_{xy}$ can be represented by:
Calibration chart — Torque

Fig 4-B
They can also be represented by a Mohr's circle:

From the Mohr's circle it follows that:

\[ \sigma_1 = \frac{\sigma_C}{2} + \sqrt{\left(\frac{\sigma_C}{2}\right)^2 + \tau_{xy}^2} \]
\[ = \frac{p}{2A} + \sqrt{\left(\frac{p}{2A}\right)^2 + \left(\frac{d_o}{2I_p}\right)^2} \hspace{1cm} (4-B) \]
\[ \tau_{\text{max}} = \sqrt{\left(\frac{p}{2A}\right)^2 + \left(\frac{d_o}{2I_p}\right)^2} \hspace{1cm} (5-B) \]

The maximum-shear-stress theory predicts that yielding begins when the maximum shear stress equals the shear stress corresponding to the yield strength \( \tau_{yp} \), i.e., yielding occurs at:
In order to calculate the failure strength of the element under the combined action of thrust and torque, it is necessary to have some knowledge about the proportion of $P$ to $T$ according to equations 4-B and 5-B. Therefore before checking this failure strength, the maximum thrust and torque, that may be applied individually are considered first. The compressive yield strength $y_p$ is about 40000 p.s.i., therefore

$$P = A \sum y_p$$

$$= \Pi (\cdot4^2 - \cdot3^2) \times 40000$$

$$= 8400\text{ lb}$$

and

$$\frac{T_d}{2 \frac{I}{P}} = \frac{T_{\text{max}}}{2}$$

$$= \frac{40000}{2}$$

$$T = 1400\text{ in-lb.}$$

Preliminary tests showed that one part of torque corresponded to two part of thrust. Therefore equation 5-B can be re-written as:

$$T_{\text{max}} = \sqrt{\frac{2T}{2A} + (\frac{T_d}{2 \frac{I}{P}})^2}$$

$$20000 = T \left[ \frac{1}{A^2} + \left( \frac{d_o}{2 \frac{I}{P}} \right)^2 \right]$$
Substituting $A = \left(4^2 - 3^2\right)$, $d_o = 8$ and $I_p = \frac{\pi}{32} \left(8^4 - 6^4\right)$ into the above equation, is found to be 1200.

Therefore the maximum allowable torque is 1200 in-lb and the corresponding thrust is 2400 lb.
APPENDIX I - C: Computer Programme of Cutting Model for Flat Rake Drills.

CUTTING FORCES IN DRILLING BY METHOD OF SUMMATION

DIMENSION TR2(50), TR3(50), TR1(50), FR2(50), FR3(50),
1TRamt(50), ETgmt(50), FFM(50), TANGM(50), FRACM(50),
DIMENSION SAFM(50), NETA(50), CRM(50), SHR1(50), SHR2(50),
1SHR3(50), CF1(50), CF2(50), CF3(50), CR1(50), CR2(50), CR3(50),
DIMENSION CRU(20), CRV(20),
REAL LHS, KL, NLR, NRR, NR, LAMDA, NAME, NETALR, NETAR, K, KK
REAL METAL, NETA

C TRAD -- RADIUS OF DRILL

C TT -- WEB THICKNESS

C P -- POINT ANGLE

C ALFA -- REFERENCE RAKE ANGLE

READ(5,10) TRAD, TT, P, ALFA

10 FORMAT(4F3.0)

C R1 -- INSIDE RADIUS OF ANNULUS

C R2 -- OUTSIDE RADIUS OF ANNULUS

C CR -- STARTING CHIP LENGTH RATIO

1 READ(5,11) R1, R2, FEED, CRR, MM

110 FORMAT(4F3.5,11)

R1O=R1
R2O=R2
CRR=CRR

5 READ(5,12) C1, C2, MM

120 FORMAT(2F8.5,11)

R1=R1O
R2=R2O
CRR=CRR

C RR(1) = .533

ALFA RL = ALFA * CF

C DR -- ELEMENTAL RADIUS

DR = (R2-R1) / 23.5

T=TT/2, 7
PR=P*CF

RADL=R2

C OR -- ANGLE OF OBliquity

C CR -- WEB ANGLE

C NR -- NORMAL RAKE ANGLE

ALR=AR SIN(T/RADL* sin(PR))

ULR=AR SIN(T/RADL)

PHIEL=ATAN(TAN(UMLR)*cos(PR))

NR=ALFA RL-PHIEL

NRL=NR/L*CF

LHS=1.0

J=1

74 J=J+1

WRITE(6,20) TRAD, P, TT, FEED

200 FORMAT(1H5, 9H RADIUS = , F7.3, X, 4H INCH, 3X, 14H POINT ANGLE = , 1F7.2, X, 4H INCH, 3X, 10H WEB THICKNESS = , F7.3, X, 4H INCH, 23X, 7H FEED = , F9.4, X, 7H IN/REV.)
WRITE(6,210) CRR,R1,R2

210 FORMAT(1H,23H CHIP REDUCTION RATIO = ,F7.3,3X,26H INSIDE RADIUS OF 1 ANNUlus = ,F7.3,X,4HINCH,3X,27H OUTSIDE RADIUS OF ANNUlus = ,F7.3,X, 24HINCH)

77 WRITE(6,300)

300 FORMAT(1H,23HTAN(META)=TAN(OR)=COS(NR))

79 WRITE(6,220)

220 FORMAT(1H,3H RADIUS,9H CRR,13H SA,7H PHI, 19H NR,8H CO,8H META,8H FP,8H EQ, 29H FR,9H FTRU,9H FTANG,9H FRAA,7H FTORU, 37H BETA)

RAD=R2
N=0
I=1

5 IF(I.EQ.1) RAD=RAD-DR/2.0
3 IF(I.GT.1) RAD=RAD-DR

5C CRR=ARSIN(T/RAD*SIN(PR))
CR=CR/W
CRR=ARSIN(T/RAD)

C PHI=--PROJECTED WORK VELOCITY ANGLE
C META=--CHIP FLOW ANGLE
C SA=--SHEAR ANGLE FOUND FROM GIVEN CHIP LENGTH RATIO
PHI=PHIER/CF
NRR=ALER-PHIER
RR=RR/W
META=ATAN(TAN(OR)*COS(NR))
META=META/W
META=ATAN(TAN(OBLR)*COS(NRRL))
META=META/W

75 SALR=ATAN(CRR*COS(META)*SIN(NRRL)/COS(OBLR))/
1(L+S*CR*COS(META)*SIN(NRRL)*CCS(OBLR))
SAL=SALR/CF

C ITERATION TO DETERMINE SHEAR ANGLE AT NEIGHBOURING POINT
80 SA=SAL-25.0

3 IF(SA.LT.5.0) SA=0.0
DO 30 I=1,3
DO 31 IA=1,50
NN=NN

3 SA=SA+10.0*NN+10.0
SAR=SA/CF

C=G2
A=A*TAN(META)*SIN(SAR)*COS(OBLR)*COS(SAR-NRR)
B=B*RAD*TAN(META)*SIN(SAR)*CCS(OBLR)*COS(SALR-NRRL)
RHS=A/B

41 IF(M.EQ.1) GOTO 41
GOTO 42

40 IF(M.EQ.1) GOTO 40
42 IF(I-V-1) 2,25,22
43 IF(LHS.6T.RHS) GOTO 15
GOTO 16
15 IV=1
20 IF((LHS-RHS).LT.5.0) GOTO 21
GOTO 31
16  IV=2
22  IF( (LHS-KHS).ST.0.* ) GOTO 21
    GOTO 31
21  SA=SA-1*0.**NN*.0
    GOTO 30
31  CONTINUE
30  CONTINUE
    SAFR=SA*CF
    SAF=SA
C  CHIP LENGTH RATIO FOUND FROM SHEAR ANGLE
CR=TAN(SAFR)*COS(OBR)/((COS(NETAR)*(COS(NRR)+TAN(SAFR)*SIN(NRR)))
U=TAN(CRR)*COS(NRR)
V=TAN(NETAR)-SIN(NRR)*TAN(OBR)
W=ATAN(U/V)
C  LAMDA--FRICITION ANGLE
LAMCAR=K-SAFR
LAMDA=LAMCAR/CF
CRR=CR
META=NETAR
NRR=NRR
UPL9=OBR
R2=RAD
AAA=SAFR+LAMDA-NRR
BB=LAMCAR-NRR
CCC=SQRT((COS(AAA)+2)*(TAN(NETAR))*2*(SIN(LAMCAR)*2)
DDD=SAFR
C  FP,FC,FR--FORCES PER UNIT RADIUS
FP=EEE*COS(PHIER)*COS(OBR)/((COS(OMR)+TAN(OBR)*TAN(NETAR))
1*SIN(LAMDA))/,(2.*COS(UMR)+SIN(DDD)*CCC)
FC=EEE*COS(PHIER)*SIN(OMR)/(2.*COS(OMR)*SIN(DDD)*CCC)
FR=EEE*COS(PHIER)*COS(OBR)/((COS(OMR)*TAN(OBR)-TAN(NETAR))
1*SIN(LAMDA))/,(2.*COS(UMR)+SIN(DDD)*CCC)
FF=(COS(2*PR)+TAN(OBR))*2*SIN(LAMDA))/CCC
C  FTNR=THRUST PER UNIT RADIUS
FTNR=FP*COS(PHIER)*SIN(PR)-FR*(COS(OBR)*COS(PR)+SIN(OBR)*
1*SIN(PHIER)*SIN(PR))
FTNR=FP
C  FTORC--TORQUE PER UNIT RADIUS
FTORC=FTNR/RAD
FRAD=FR*(SIN(OBR)*COS(PHIER)*SIN(OMR)+COS(OBR)*SIN(PR)*COS(OMR)-
1*SIN(OMR)*SIN(PHIER)*COS(PR)*COS(OMR))*FQ*COS(PHIER)*COS(PR)*
2*COS(OMR)+FQ*SIN(PHIER)*SIN(OMR)
FM(j,i)=FF
FTHRUM(j,i)=FTNR*DR*SS*.2
FTAUM(j,i)=FRAD*DR*SS*.2
FRAUM(j,i)=FRAD*DR*SS*.2
FTORM(j,i)=FTORC*DR*SS*.2
WRITE(6,240) RAD,CR,SAF,PHI,NN,DR,NETA,FP,FC,FR,FTNR,FTANG,
1FRAC,FTORC,LANDA
12FODO,240)
12F7.3,F7.2)
IF(RAD.GT.(X1+DR)) GOTO 70
SOS=0.0
DO 420 I=1,4
C  SUMF--SUM OF THRUST

C SUM7=SUM OF TORQUE
IF (1.EQ.1) SUMT2=FTCRQ(2,1)
IF (1.GT.1) SUMT2=SUMT2+FTCRQ(2,1)
420 CONTINUE
DO 450 I=1,4
IF (1.EQ.1) SUMF2=FTHRUM(2,1)
IF (1.GT.1) SUMF2=SUMF2+FTHRUM(2,1)
450 CONTINUE
WRITE(6,250)
250 FORMAT (1H,15H ,4X,15H TOTAL THRUST 2,4X,
115H ,4X,15H TOTAL TORQUE 2)
WRITE(6,260) SUMF2,SUMT2
260 FORMAT (1H ,F29.2 ,F37.2)
DO 440 I=1,7
IF (1.EQ.1) SUMFD2=FTHRUM(2,1)
IF (1.GT.2) SUMFD2=SUMFD2+FTHRUM(2,1)
IF (1.EQ.1) SUMTD2=FTHRUM(2,1)
IF (1.GT.1) SUMTD2=SUMTD2+FTHRUM(2,1)
440 CONTINUE
DO 430 I=8,14
IF (1.EQ.1) SUMFC2=FTHRUM(2,1)
IF (1.GT.2) SUMFC2=SUMFC2+FTHRUM(2,1)
IF (1.EQ.1) SUMTC2=FTHRUM(2,1)
IF (1.GT.1) SUMTC2=SUMTC2+FTHRUM(2,1)
430 CONTINUE
DO 500 I=15,21
IF (1.EQ.15) SUMFA2=FTHRUM(2,1)
IF (1.GT.2) SUMFA2=SUMFA2+FTHRUM(2,1)
IF (1.EQ.15) SUMTA2=FTHRUM(2,1)
IF (1.GT.15) SUMTA2=SUMTA2+FTHRUM(2,1)
500 CONTINUE
DO 510 I=22,28
IF (1.EQ.22) SUMFB2=FTHRUM(2,1)
IF (1.GT.2) SUMFB2=SUMFB2+FTHRUM(2,1)
IF (1.EQ.22) SUMTB2=FTHRUM(2,1)
IF (1.GT.22) SUMTB2=SUMTB2+FTHRUM(2,1)
510 CONTINUE
WRITE(6,290)
290 FORMAT (1H,11H THRUST OF A,4X,11H THRUST OF B,4X,11H THRUST OF C,4X,
11H THRUST OF D,4X,11H TORQUE OF A,4X,11H TORQUE OF B,4X,
11H TORQUE OF C,4X,11H TORQUE OF D)
WRITE(6,290) SUMFA2,SUMFB2,SUMFC2,SUMFD2,SUMTA2,SUMTB2,SUMTC2,
1SUMTD2
290 FORMAT (1H ,F8.2,F8.2,F8.2,F8.2)
IF (IXN.EQ.1) GOTO 5
IF (IXN.NE.1) GOTO 1
CALL EXIT
END
ENTRY
APPENDIX 2 - C: Relevant Statistical Equations and Computer t-test Programme.

The pooled variance of two estimated means $\bar{x}_1$ and $\bar{x}_2$, whose estimated variances are $S_1^2$ and $S_2^2$, calculated from $N_1$ and $N_2$ observations respectively is given by:

$$S^2 = \frac{(N_1 - 1) S_1^2 + (N_2 - 1) S_2^2}{(N_1 + N_2 - 2)} \quad (2-C1)$$

The second mean $\bar{x}_2$ in this experiment is the sum of a number of means ($\bar{x}_1$, $\bar{x}_2$, $\bar{x}_3$ ---- $\bar{x}_k$), i.e.

$$\bar{x}_2 = \sum_{i=1}^{k} \bar{x}_i$$

Each of these means is subjected to variation and therefore a variance of its own. Following the definition for estimated variance, the estimated variance of $\bar{x}_2$ should be equal to:

$$S_2^2 = \frac{\sum_{i=1}^{k} \sum_{i=1}^{n_k} (x_{ki} - \bar{x}_k)^2}{\sum_{k=1}^{k} n_k - k}$$

Where $k$ is the number of means and $n_k$ is the number of observations in estimating the $k^{th}$ mean.

The degree of freedom in estimating $\bar{x}_2$ would be:

$$N_2 - 1 = \sum_{k=1}^{k} (n_k - 1)$$
Substituting $\bar{x}_2$ and $N_2$ with equation $2 - C1$, the pooled variance is obtained.

The t-test ratio is thus given by:

$$ t = \frac{\bar{x}_1 - \bar{x}_2}{S \sqrt{\frac{1}{N_1} + \sum_{k=1}^{k} \frac{1}{n_k}}} $$

If $t$ is less than $t_\alpha$, where $t_\alpha$ is the value at $(1 - \alpha) \times 100\%$ confidence level with degree of freedom of $(N_1 + N_2 - 2)$, then the two sample means $\bar{x}_1$ and $\bar{x}_2$ are not significantly different.
**COMPARISON OF FORCES OF SINGLE DRILL BY T-TEST**

```plaintext
* $IBJOB NODECK
$IBFTC NODECK

DIMENSION F(10), SUM(10), Z(10), SM(10), DEVS(10), DEVT(10), VAR(10),
LSVAR(10)

1    REAC(5,100) N,(F(I), I=1,N),M,MM
100   FORMAT(11,3F8.0,2I1)
   D1=C*0
   J=J+1
   Z(J)=N
   DO 200 I=1,N
   SUM(I)=C1+F(I)
   D1=SUM(I)
200   CONTINUE
   SM(J)=D1/Z(J)
   D2=0*0
   DO 210 I=1,N
   CEVS(I)=(F(I)-SM(J))**2
   DEVT(I)=C2+CEVS(I)
   C2=DEVT(I)
210   CONTINUE
   SVAR(J)=C2
   VAR=VARANCE
   VAR(J)=D2/(Z(J)-1.0)
   IF (M.EQ.0) GOTO 1
   C3=C*0
   D4=0*0
   DO 220 J=2,NN
      ZZ=DEGREE OF FREEDOM OF SECOND MEAN
      ZZ=C3+Z(J)-1.0
   C3=ZZ
   TSVAR=C4+SVAR(J)
   D4=TSVAR
220   CONTINUE
   VARPT=VARANCE OF SECOND MEAN
   GVARP=POOLED VARIANCE
   VARPT=C4/D3
   D7=C*0
   DO250 J=2,NN
      PDRF=D7+(Z(J)-1.0)*VARPT
      D7=PCOF
250   CONTINUE
   GVARP=(SVAR(1)+PCOF)/(Z(1)-1.0+ZZ)
   TDF=TOTAL DEGREE OF FREEDOM
   TDF=Z(1)-1.0+ZZ
   D5=C*0
   DO 240 J=2,NN
   GS=SUM CF SECOND MEAN
   GS=C5+SM(J)
   D5=GS
240   CONTINUE
   D6=C*0
   DO260 J=2,NN
```

(C7)
SUMDF = C6 + 1.0/Z(J)
D6 = SUMDF

CONTINUE

C0M = ABS(C5 - SM(1))
SSCV = GVARP * SQRT(1.0/Z(1) + SUMDF)

C

TT = T-TEST RATIO
TT = DCM / SSCV

WRITE (6, 330)

FORMAT (1H0, 5X, 4HMEAN, 8X, 8HVARIANCE)
DO 270 J = 1, NN

WRITE (6, 310) SM(J), VAR(J)

FORMAT (1H0, 5F9.3, F14.3)

CONTINUE

WRITE (6, 320)

FORMAT (1HC, 5X, 5HMEAN1, 5X, 4HVAR1, 5X, 5HMEAN2, 5X, 4HVAR2, 5X,
13HDCF, 5X, 7HT-RATIC)
WRITE (6, 330) SM(1), VAR(1), GS, VAR(2), TDF, TT

IF(MM.EQ.0) GOTO 2
CALL EXIT

END

ENTRY
APPENDIX 3 - C: TABLE AND COMPUTER PROGRAMME OF ANALYSIS OF VARIANCE

ANALYSIS OF VARIANCE TABLE

<table>
<thead>
<tr>
<th>Source of Variance</th>
<th>Sum of Squares</th>
<th>Degree of Freedom</th>
<th>Mean Square</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between Group</td>
<td>$k \sum_i n_i (\bar{x}_i - \bar{x})^2$</td>
<td>$k - 1$</td>
<td>$S_2$</td>
<td>$\frac{S_2^2}{S_1^2}$</td>
</tr>
<tr>
<td>Within Group</td>
<td>$k \sum_i \sum_v (x_{iv} - \bar{x}_i)^2$</td>
<td>$(\sum_i n_i) - k$</td>
<td>$S_1$</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>$k \sum_i \sum_v (x_{iv} - \bar{x})^2$</td>
<td>$(\sum_i n_i) - 1$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

To test the null hypothesis that the $k$ population means are equal, the two mean squares $S_1^2$ and $S_2^2$ are compared. If the variance ratio of $\frac{S_2^2}{S_1^2}$ is not significant, i.e. $F$ is less than $F_{\alpha}$, where $F_{\alpha}$ is the value at $(1 - \alpha) \times 100\%$ confidence level with degree of freedom of $(k - 1)$ and $\sum_i n_i - k$, then under the null hypothesis, the $k$ means are all equal.
ANALYSIS OF VARIANCE - CHIP RATIO FOR ALL FEEDS

DIMENSION SUMFA(100), SUMFA2(100), SUM2FA(100), AVFA(100), SDFA(100),
IF2A(100), SIGMA(100), DOFF(100)
DIMENSION A(100), FA(50,50), UL(100), EL(100)
DIMENSION Y(50,50)

K = 1
K = K + 1
N = 0
READ(5,150) M, J, (FA(I,K), I=1,N)
150 FORMAT(I2,Il,12F0.0)
DO 111 I = 1,N
M = M + 1
FA(K,K) = FA(I,K)
111 CONTINUE
A(K) = M
IF(J .EQ. 0) GO TO 2
IF(J .EQ. 1) GO TO 3
K = K
PVAR = 0.0
TSUMFA = 0.0
SUMN = 0.0
SSUMF2 = 0.0
TCORFA = 0.0
DO 60 K = 1,KK
SUMFA(K) = 0.0
SUMFA2(K) = 0.0
N0 = A(K)
DO 50 I = 1,N
SUMFA(K) = SUMFA(K) + FA(I,K)
50 SUMFA2(K) = SUMFA2(K) + FA(I,K) * FA(I,K)
TSUMFA = TSUMFA + SUMFA(K)
SUMN = SUMN + A(K)
SSUMF2 = SSMUF2 + SUMFA2(K)
SUM2FA(K) = SUMFA(K) * SUMFA(K)
TCORFA = TCORFA + SUM2FA(K) / A(K)
IF (A(K), LE, 2.0) GO TO 60
FA2(K) = SUMFA2(K) - SUM2FA(K) / A(K)
SIGMA(K) = FA2(K) / (A(K) - 1.0)
AVFA(K) = SUMFA(K) / A(K)
SDFA(K) = SQRT(SIGMA(K))
TAV = TSUMFA/SUMN

C DOF = DEGREES OF FREEDOM
DOFF(K) = A(K) - 1.0
PVAR = PVAR + SIGMA(K) * DOFF(K)
60 CONTINUE
Z = KK
DOFB = (Z - 1.0)
DOFW = (SUMN - Z)
DOF1 = (SUMN - 1.0)
TSUMF2 = TSUMFA * TSUMFA
CORFA = TSMUF2 / SUMN

C TOTAL SUM OF ERRORS SQUARED
TOTAL = SSMUF2 - CORFA
C BETWEEN GROUPS' SUM OF ERRORS' SQUARED
   BETWN = TCEFA - CCEFA
C WITHIN GROUPS' SUM OF ERRORS' SQUARED
   WITHIN = TOTAL - BETWN
   POVAR = POVAR/DDOF
   VARFAB = BETWN / DDOF
   VAFAW = WITHIN / DDOF
C VARIANCE RATIO
   VRATIO = VARFAB / VAFAW
6 WRITE(6,116)
116 FORMAT(1H-,24X,12H DOF ,12H SUMSQ ,12H VAR ,12H VRATIO)
   WRITE(6,117)DOF3, BETWN, VAFAB, VRATIO
117 FORMAT(1H-,6X,12H SETWN GROUPS, 4X, F6.2, 2X, E12.5, E14.5, F7.2)
   WRITE(6,118)DOF3, WITHIN, VAFAW
118 FORMAT(1H-,6X,13H SETWN GROUPS, 3X, F6.2, 2X, E12.5, E14.5)
   WRITE(6,119)DOF3, TOTAL
119 FORMAT(1H-,6X,5HTOTAL, 11X, F8.2, 2X, E12.3)
   WRITE(6,120)
120 FORMAT(1H-,5X,9H K ,12H SIGMAF ,1CH AVFA ,1CH SDFA)
   WRITE(6,121)K, SIGMAF(K), AVFA(K), SDFA(K), DOFF(K)
121 FORMAT(1H-,9X,11, E15.7, F8.3, E12.3, F4.3)
   CONTINUE
   WRITE(6,122) DOF3, TAV, POVAR
122 FORMAT(1H-,8X, F6.2, F8.3, E13.4)
   CALL EXIT
   END
ENTRY
APPENDIX 4 - C: RELEVANT STATISTICAL EQUATION AND COMPUTER PROGRAMME OF LINEAR REGRESSION WITH MULTI-OBSERVATION AT EACH LEVEL.

A linear regression, which accounts for the multi-observations at each level, is presented by Brownlee (57). The following are the various regression coefficients and variances derived for this regression:

Regression equation to the line:

\[ Y = K_1 + K_2 \]

Regression coefficients:

\[ K_1 = \frac{\sum_{i}^{k} n_i \bar{y}_i}{\sum_{i}^{k} n_i} = \bar{y} \]

\[ K_2 = \frac{\sum_{i}^{k} n_i (x_i - \bar{x}) (\bar{y}_i - \bar{y})}{\sum_{i}^{k} n_i (x_i - \bar{x})^2} \]

Pooled Estimate \( S_1^2 \) of \( \sigma^2 \):

\[ S_1^2 = \frac{\sum_{i}^{k} n_i (\bar{y}_i - \bar{y}_i)^2 + \sum_{i}^{k} \sum_{i}^{n_i} (y_{iv} - \bar{y}_i)^2}{\sum_{i}^{k} n_i - k} \]
Variances:

\[
\begin{align*}
\text{Var} \left[ \bar{K}_1 \right] &= \frac{\sigma^2}{\sum_{i} n_i} \\
\text{Var} \left[ \bar{K}_2 \right] &= \frac{\sigma^2}{\sum_{i} n_i (x_i - \bar{x})^2} \\
\text{Var} \left[ \bar{Y} \right] &= 2 \left[ \left( \frac{1}{k} \sum_{i} n_i \right) + \frac{(x - \bar{x})^2}{\sum_{i} n_i (x_i - \bar{x})^2} \right] \\
\text{Var} \left[ \bar{Y} - \bar{Y} \right] &= 2 \left[ \left( \frac{1}{k} \sum_{i} n_i \right) + \frac{1}{k} + \frac{(x - \bar{x})^2}{\sum_{i} n_i (x_i - \bar{x})^2} \right]
\end{align*}
\]
Linear Regression Table

<table>
<thead>
<tr>
<th>Source of Variance</th>
<th>Sum of Squares</th>
<th>Degree of Freedom</th>
<th>Mean Squares</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slope of Line</td>
<td>$\sum_{i} n_i (y_i - \bar{y}_i)^2$</td>
<td>1</td>
<td>$S_3^2$</td>
</tr>
<tr>
<td>Variation of true group means about line a within group</td>
<td>$\sum_{i} \sum_{v} (y_{iv} - \bar{y}_i)^2$</td>
<td>$k - 2$</td>
<td>$S_2^2$</td>
</tr>
<tr>
<td>Total</td>
<td>$\sum_{i} \sum_{v} (y_{iv} - \bar{y})^2$</td>
<td>$k (\sum_{i} n_i) - k$</td>
<td>$S_1^2$</td>
</tr>
</tbody>
</table>

The linearity of the line can be tested by comparing the variance ratio $\frac{S_2^2}{S_1^2}$ with $F_d (k - 2, (\sum_{i} n_i) - k)$

If the variance ratio $\frac{S_3^2}{S_1^2}$ is larger than $F_\alpha (1, (\sum_{i} n_i) - k)$, then the gradient of the line is not equal to zero.

The confidence limits for the mean and individual observation at $(1 - \alpha)\%$ confidence level are given by:

$$Y_{\text{mean}} = a \pm t_\alpha \left( \sum_{i} n_i - 2 \right) \sqrt{\frac{\bar{y} - \bar{y}}{V}}$$

$$Y_{\text{ind}} = a \pm t_\alpha \left( \sum_{i} n_i - 2 \right) \sqrt{\frac{\bar{y} - \bar{y}}{V}}$$
**LINEAR REGRESSION - MULTIPLE OBSERVATIONS OF FORCES**

```
$IBJOB  NDCECK
$IBFTC  NDCECK

1 DIMENSION X(20), Y(20, 25), YYYY(20, 25), YYYY(20, 25), A(20), YM
2 DIMENSION YYYY(20, 25), YLSIG(20), YL(20), YLSIG(20)
3 DIMENSION YYYY(20, 25), YLSIG(20), YMEAN(20), YINC(20), YEEE(20)
4 REAL MTSES, MAGSUS, YYYY, NOSOL, MXTGM, LIMIT
5 READ(5, 150) W1, W2
6 150 FORMAT(2F3.0)
7 3 READ(5, 120) MM
10 120 FORMAT(11)
11 XX=X COORDINATE
12 READ(5, 110) LLL, (XX(I), I=1, LLL)
13 C=LLL
14 8=C-2.0
15 Z=Z+C
16 I=0
17 1 I=I+1
20 KK=0
C YYY=OBSERVED FORCES
21 15 READ(5, 110) N, M, MM, YYYY(I, K), K=1, N
22 110 FORMAT(311, 8F8.0)
23 IF(YMM, EQ, 1) DCF=W1/WT1
24 IF(YMM, EQ, 2) DCF=1.0
25 IF(YMM, EQ, 3) DCF=(W1/WT1)**2
26 IF(YMM, EQ, 1) DCF=1.0
27 IF(M, EQ, 1) CF=1.03
28 IF(M, EQ, 2) CF=3.63
29 IF(M, EQ, 3) CF=7.05
30 IF(M, EQ, 4) CF=1.26
31 IF(M, EQ, 5) CF=2.44
34 DO 610 K=1, N
35 KK=KK+1
36 YYYY(I, KK)=YYYY(I, K)* CF*DCF
37 610 CONTINUE
40 IF(K<LLT, 10) GOTO 15
41 A(I)=KK
42 CC=KK
43 Z=Z+CC
44 BB=Z-C
45 I=I+1
46 IF(I, LLT, 3) GOTO 1
47 J=0
50 600 J=J+1
51 IF(J, EQ, 1) GOTO 40
52 IF(J, EQ, 2) GOTO 41
53 DO 500 I=1, LLL
54 X(I)=XX(I)
55 500 CONTINUE
56 I=0
57 11 I=I+1
60 N=A(I)
```
DC 516 K=1,1
Y((I,K)=YY(I,K)
CONTINUE
IF(I.LT.II) GOTO 12
GOTO 10
DO 220 I=1,II
X(I)=ALG(XX(I))
CONTINUE
I=6
I=I+1
N=A(I)
DO 330 K=1,N
Y(I,K)=ALG(YY(I,K))
CONTINUE
IF(I.LT.II) GOTO 12
GOTO 10
C SUM OF SQUARES AT ONE LEVEL
DO 220 K=1,1
C =C1+Y(I,K)**2
C SUM OF FORCES AT ONE LEVEL
DO 220 I=1,II
Y(I)=C2/A(I)
CONTINUE
Y(I)=C2*Y(I,K)
IF(I.LT.II) GOTO 2
TS=TS+C2/2
C TOTAL SUM OF SQUARE
TSO=S1+YS
C PEAK TOTAL SUM OF SQUARE
MTS=MTS+TS/II
I=7
C3=0.0
C4=0.0
I=I+1
N=A(I)
DO 220 K=1,N
C3=C3+Y(I,K)
CONTINUE
C4=C4+C3*Y(I,K)
CONTINUE
IF(I.LT.II) GOTO 4
C WITHIN GROUP SUM OF SQUARE
WS=WS+C1-C4
C WITHIN GROUP SUM OF SQUARE
WS=WS/WS/BS
I=0
C5=0.0
C6=0.0
I=I+1
C \( DS = CS + X(1) \cdot (X(1) \cdot X(2)) \)
C \( DS = CS + X(1) \cdot X(2) \)
C \( IF(J.EQ.1) \quad YINTL = YINT \)
C \( IF(J.EQ.2) \quad YINTL = EXP(YINT) \)
C \( YM = 0 \cdot Y \)
C \( DO 2 = I = 1,LLL \)
C \( YEE = -Y \) COORDINATE EXCLUDING INTERCEPT
C \( YEE = X(1) \cdot X(2) \cdot \beta \)
C \( YV = \) VARIANCE ABOUT MEAN
C DCSOL = --DENOMINATOR FOR CALCULATION OF SLOPE OF LINE
DOSOL = DS - D6 * DS/Z
C XGO--NUMBERATOR FOR CALCULATION OF SLOPE OF LINE
SCSOL = DS - XX
C SQ = SUM OF SQUARE OF SLOPE OF LINE
SCSOL = N * SCSOL/GOCSL
C VCTOG --VARIATION OF TRUE GROUP MEANS ABOUT LINE
VCTOG = C4 + YY - SCSOL
C MVCTOG --MEAN VALUE OF VCTOG
MVCTOG = VCTOG/B
C LINT = MVCTOG * MVGOCSL
C \( \beta = \) SLOPE OF LINE
C YINT --Y INTERCEPT
YINTL = DS/Z - \\( \beta \) \* YINT/Z
C BETAXY = CORRELATION COEFFICIENT
BETAXY = SCSOL/GOCSL
C PEV --POOLED ESTIMATED VARIANCE
PEV = (GOCSL + VCTOG) / (N + B)
C GOCSL = PEV / DV
C SIGCA = SQRT(VCO)
C TCO = \( \beta \) \* QA / SIGCA
C SIGUA = SQRT(PEV/Z)
T1 = PEV/Z
T2 = PEV/Z + ((GO/Z) \* X) / GOCSL
T3 = PEV / DV
T4 = \( \beta \) \* QA + PEV / GOCSL
T5 = T1 + T2
T6 = PEV + T1 + T2
C IF(J.EQ.1) \quad YINTL = YINT
C IF(J.EQ.2) \quad YINTL = EXP(YINT)
YV(1) = PVY*(1.0/2 + ((X(I) - XM)**2)/DSCL)
YVSIG(I) = SQRT(YV(1))

C YL --- VARIANCE ABOUT INDIVIDUAL
YL(I) = PVY*(1.0/2 + ((X(I) - XM)**2)/DSCL)
YLSIG(I) = SQRT(YL(I))

YVSIGL(I) = 2.0 * YVSIG(I)
YLSIGL(I) = 2.0 * YLSIG(I)

IF (J.LT.1) YMEAN(I) = YVSIGL(I)

C YMEAN --- CONFIDENCE LIMITS OF INDIVIDUAL VARIATION
IF (J.LT.1) YMEAN(I) = YLSIGL(I)

C YMEAN --- CONFIDENCE LIMITS OF MEAN VARIATION
IF (J.LT.2) YMEAN(I) = EXP(YVSIGL(I))
IF (J.LT.2) YMEAN(I) = EXP(YLSIGL(I))

900 CONTINUE

IF (X**.5.E-1) GOTO 3
IF (X**.4.E-2) GOTO 9

8 WRITE (6,332)
532 FORMAT (1H-, 1X, 1HEXFUNCTION OF THRUST)
6 WRITE (6,333)
9 WRITE (6,328)
625 FORMAT (1H-, 1X, 1HEXFUNCTION OF TORQUE)
7 WRITE (6,365)
365 FORMAT (1H-, 1X, 1HY-INTERCEPT, 4X, 1HEXGRADIENT OF LINE, 6X, 1HEXCONSTANT)
375 WRITE (6,370) YINT, XINT, YINT, XINT
4 FORMAT (1H-, 1X, 1HY-VARIANCE =, F10.3, 4X, 3H-, F10.3, 3H =, F10.3, 3H+ ,
16, 3, 2X, 4X*2)
321 WRITE (6,327) T0, T4, T3
327 FORMAT (10.0, 12.0 LIMIT IF Y =, F10.3, 4X, 3H-, F10.3, 3H x, 4X, 13H+ , F10.3, 4X, 4X*2)
325 WRITE (6,320)
30.0 FORMAT (1H- =, 4X, 1HEESEE, 4X, 1HVARIANCE OF Y, 5X, 1HESIGMA, 3X, 1HLIMIT
1 Y, 3X, 1HESIGMA, 6X, 2HK, 1.5X, 2HK2, 1.3X, 4HYEEE)
324 DO 325 I=1,111
325 WRITE (6,333) YX(I), YV(I), YVSIG(I), YL(I), YLSIG(I), YMEAN(I), YMEAN(I)
3220 CONTINUE
323 IF (J.LT.2) GOTO 630
324 IF (X**.5.E-2) GOTO 3
325 CALL EXIT
326 END
APPENDIX 1 —D: RAW DATA — PRELIMINARY TESTS ON A CONVENTIONAL DRILL

1) THRUST AND TORQUE DISTRIBUTIONS

DRILL DIA. — 1.00" (DRILL B) FEED — .008 IN/REV. SPEED — 150 RPM

T — THRUST (LB) M — TORQUE (IN-LB)

<table>
<thead>
<tr>
<th>ANNULUS 2A</th>
<th>ANNULUS 2B</th>
<th>ANNULUS 2C</th>
<th>ANNULUS 2D</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>M</td>
<td>T</td>
<td>M</td>
</tr>
<tr>
<td>51.2</td>
<td>14.5</td>
<td>42.1</td>
<td>19.5</td>
</tr>
<tr>
<td>50.3</td>
<td>13.9</td>
<td>39.4</td>
<td>18.6</td>
</tr>
<tr>
<td>50.3</td>
<td>15.4</td>
<td>40.3</td>
<td>20.2</td>
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</table>

ANNULUS 2J

<table>
<thead>
<tr>
<th>T</th>
<th>M</th>
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</thead>
<tbody>
<tr>
<td>142.7</td>
<td>88.2</td>
</tr>
<tr>
<td>146.4</td>
<td>89.5</td>
</tr>
<tr>
<td>151.9</td>
<td>90.7</td>
</tr>
</tbody>
</table>
2) CHIP LENGTH RATIO DISTRIBUTIONS

<table>
<thead>
<tr>
<th>ANNULUS 2A</th>
<th>ANNULUS 2B</th>
<th>ANNULUS 2C</th>
<th>ANNULUS 2D</th>
</tr>
</thead>
<tbody>
<tr>
<td>R1 R2</td>
<td>R1 R2</td>
<td>R1 R2</td>
<td>R1 R2</td>
</tr>
<tr>
<td>.425 .410</td>
<td>.416 .419</td>
<td>.416 .445</td>
<td>.504 .553</td>
</tr>
<tr>
<td>.388 .437</td>
<td>.441 .461</td>
<td>.417 .454</td>
<td>.519 .619</td>
</tr>
<tr>
<td>.398 .442</td>
<td>.387 .415</td>
<td>.393 .447</td>
<td>.469 .502</td>
</tr>
<tr>
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<td>.348 .450</td>
<td>.414 .442</td>
<td>.449 .449</td>
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<tr>
<td>.425 .394</td>
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<td>.420 .520</td>
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<td>.444 .475</td>
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<td>.479 .444</td>
<td>.388 .454</td>
<td>.446 .527</td>
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</table>

<table>
<thead>
<tr>
<th>ANNULUS 2J</th>
</tr>
</thead>
<tbody>
<tr>
<td>R1 R2</td>
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<tr>
<td>.382 .510</td>
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<tr>
<td>.307 .500</td>
</tr>
</tbody>
</table>
APPENDIX 2 - D: RAW DATA VERIFICATION OF SIMILARITY CONCEPT ON CONVENTIONAL DRILL LIPS

1) OBSERVED THRUST AND TORQUE OF VARIOUS ANNULII.

FEED - .008 IN/REV.  SPEED - 150 RPM

<table>
<thead>
<tr>
<th>ANNULUS 1C (Drill C)</th>
<th>ANNULUS 2C (Drill B)</th>
<th>ANNULUS 3C (Drill A)</th>
</tr>
</thead>
<tbody>
<tr>
<td>T -- THRUST (LB)</td>
<td>M -- TORQUE (IN-LB)</td>
<td></td>
</tr>
<tr>
<td>36.6</td>
<td>31.8</td>
<td>26.8</td>
</tr>
<tr>
<td>37.5</td>
<td>31.8</td>
<td>26.8</td>
</tr>
<tr>
<td>37.9</td>
<td>31.2</td>
<td>26.8</td>
</tr>
<tr>
<td>37.9</td>
<td>30.7</td>
<td>26.8</td>
</tr>
<tr>
<td>37.5</td>
<td>30.7</td>
<td>26.8</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>ANNULUS 1H (Drill C)</th>
<th>ANNULUS 2H (Drill B)</th>
<th>ANNULUS 3H (Drill A)</th>
</tr>
</thead>
<tbody>
<tr>
<td>T -- THRUST (LB)</td>
<td>M -- TORQUE (IN-LB)</td>
<td></td>
</tr>
<tr>
<td>62.2</td>
<td>50.6</td>
<td>44.3</td>
</tr>
<tr>
<td>61.3</td>
<td>51.5</td>
<td>45.2</td>
</tr>
<tr>
<td>62.2</td>
<td>51.5</td>
<td>44.7</td>
</tr>
<tr>
<td>63.1</td>
<td>51.5</td>
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</tr>
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<td>62.2</td>
<td>52.5</td>
<td>45.2</td>
</tr>
</tbody>
</table>

2) OBSERVED CHIP LENGTH RATIO OF VARIOUS ANNULII

FEED - .008"/REV.  SPEED - 150 RPM

<table>
<thead>
<tr>
<th>ANNULUS 1H (Drill C)</th>
<th>ANNULUS 2H (Drill B)</th>
<th>ANNULUS 3H (Drill A)</th>
</tr>
</thead>
<tbody>
<tr>
<td>R1 -- INSIDE RADIUS</td>
<td>R2 -- OUTSIDE RADIUS</td>
<td></td>
</tr>
<tr>
<td>0.486</td>
<td>0.530</td>
<td>0.581</td>
</tr>
<tr>
<td>0.567</td>
<td>0.514</td>
<td>0.533</td>
</tr>
<tr>
<td>0.565</td>
<td>0.543</td>
<td>0.580</td>
</tr>
<tr>
<td>0.543</td>
<td>0.501</td>
<td>0.553</td>
</tr>
<tr>
<td>0.557</td>
<td>0.556</td>
<td>0.543</td>
</tr>
<tr>
<td>0.568</td>
<td>0.520</td>
<td>0.561</td>
</tr>
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</table>
APPENDIX 3 - D: RAW DATA - VERIFICATION OF SIMILARITY CONCEPT ON CONVENTIONAL DRILLS (FULL DRILL)

1) OBSERVED THRUST AND TORQUE
SPEED -- 150 RPM
T -- THRUST (LB) M -- TORQUE (IN-LB)

<table>
<thead>
<tr>
<th>DRILL DIA. -- .75&quot; (Drill D)</th>
<th>DRILL DIA. -- 1.0&quot; (Drill E)</th>
<th>DRILL DIA. -- 1.25&quot; (Drill F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>FEED--.005&quot;/REV.</td>
<td>FEED--.008&quot;/REV.</td>
<td>FEED--.012&quot;/REV.</td>
</tr>
<tr>
<td>T</td>
<td>M</td>
<td>T</td>
</tr>
<tr>
<td>-----</td>
<td>-----</td>
<td>-----</td>
</tr>
<tr>
<td>263.0</td>
<td>50.5</td>
<td>332.0</td>
</tr>
<tr>
<td>224.0</td>
<td>50.5</td>
<td>385.0</td>
</tr>
<tr>
<td>205.0</td>
<td>53.5</td>
<td>392.0</td>
</tr>
<tr>
<td>196.0</td>
<td>53.5</td>
<td>396.0</td>
</tr>
<tr>
<td>224.0</td>
<td>53.0</td>
<td>378.0</td>
</tr>
<tr>
<td>187.0</td>
<td>53.5</td>
<td>384.0</td>
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</table>
APPENDIX 4 - D: RAW DATA - VERIFICATION OF SIMILARITY CONCEPT ON MODIFIED DRILL LIPS

1) OBSERVED THRUST AND TORQUE
SPEED -- 150 RPM
T -- THRUST (LB)   M -- TORQUE (IN-LB)

<table>
<thead>
<tr>
<th>DRILL DIA. -- .75&quot; (Drill G)</th>
<th>FEED - .005&quot;/REV.</th>
<th>FEED - .008&quot;/REV.</th>
<th>FEED - .012&quot;/REV</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>M</td>
<td>T</td>
<td>M</td>
</tr>
<tr>
<td>54.0</td>
<td>42.8</td>
<td>77.5</td>
<td>68.0</td>
</tr>
<tr>
<td>57.5</td>
<td>47.9</td>
<td>81.0</td>
<td>63.0</td>
</tr>
<tr>
<td>50.5</td>
<td>39.0</td>
<td>78.5</td>
<td>72.0</td>
</tr>
<tr>
<td>52.5</td>
<td>41.0</td>
<td>68.5</td>
<td>65.5</td>
</tr>
<tr>
<td>52.5</td>
<td>41.0</td>
<td>73.5</td>
<td>68.0</td>
</tr>
<tr>
<td>47.0</td>
<td>42.8</td>
<td>77.5</td>
<td>74.5</td>
</tr>
</tbody>
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<table>
<thead>
<tr>
<th>DRILL DIA. -- 1.00&quot; (Drill H)</th>
<th>FEED - .005&quot;/REV.</th>
<th>FEED - .008&quot;/REV.</th>
<th>FEED - .012&quot;/REV</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>M</td>
<td>T</td>
<td>M</td>
</tr>
<tr>
<td>72.0</td>
<td>77.0</td>
<td>92.0</td>
<td>117.0</td>
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<td>122.0</td>
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<tr>
<td>72.0</td>
<td>69.0</td>
<td>98.0</td>
<td>115.0</td>
</tr>
<tr>
<td>63.0</td>
<td>69.0</td>
<td>88.0</td>
<td>107.0</td>
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<tr>
<td>63.0</td>
<td>68.0</td>
<td>90.0</td>
<td>112.0</td>
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<tr>
<td>72.0</td>
<td>63.0</td>
<td>99.0</td>
<td>122.0</td>
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APPENDIX 5 - D: RAW DATA - VERIFICATION OF SIMILARITY CONCEPT ON MODIFIED DRILLS (FULL DRILL)

1) OBSERVED THRUST AND TORQUE

<table>
<thead>
<tr>
<th>SPEED</th>
<th>T -- THRUST (LB)</th>
<th>M -- TORQUE (IN-LB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>150 RPM</td>
<td>109.0 46.6</td>
<td>194.0 75.5</td>
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<tr>
<td></td>
<td>130.5 45.5</td>
<td>187.0 78.0</td>
</tr>
<tr>
<td></td>
<td>137.5 48.0</td>
<td>177.0 80.0</td>
</tr>
<tr>
<td></td>
<td>109.0 45.0</td>
<td>209.0 74.5</td>
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<tr>
<td></td>
<td>105.5 50.5</td>
<td>166.0 74.5</td>
</tr>
<tr>
<td></td>
<td>113.0 50.0</td>
<td>177.0 78.0</td>
</tr>
</tbody>
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DRILL DIA. -- .75" (Drill G)

<table>
<thead>
<tr>
<th>FEED - .005&quot;/REV</th>
<th>T</th>
<th>M</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>109.0 46.6</td>
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<td></td>
<td>130.5 45.5</td>
<td>187.0 78.0</td>
</tr>
<tr>
<td></td>
<td>137.5 48.0</td>
<td>177.0 80.0</td>
</tr>
<tr>
<td></td>
<td>109.0 45.0</td>
<td>209.0 74.5</td>
</tr>
<tr>
<td></td>
<td>105.5 50.5</td>
<td>166.0 74.5</td>
</tr>
<tr>
<td></td>
<td>113.0 50.0</td>
<td>177.0 78.0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>FEED - .008&quot;/REV</th>
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<th>M</th>
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<tbody>
<tr>
<td></td>
<td>194.0 75.5</td>
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<tr>
<td></td>
<td>187.0 78.0</td>
<td>277.0 101.0</td>
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<tr>
<td></td>
<td>177.0 80.0</td>
<td>272.0 107.0</td>
</tr>
<tr>
<td></td>
<td>209.0 74.5</td>
<td>252.0 110.0</td>
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<td>166.0 74.5</td>
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<tr>
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<td>177.0 78.0</td>
<td>263.0 110.0</td>
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<table>
<thead>
<tr>
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<th>M</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>248.0 103.0</td>
<td>314.0 178.0</td>
</tr>
<tr>
<td></td>
<td>277.0 101.0</td>
<td>335.0 163.0</td>
</tr>
<tr>
<td></td>
<td>272.0 107.0</td>
<td>362.0 159.0</td>
</tr>
<tr>
<td></td>
<td>252.0 110.0</td>
<td>310.0 183.0</td>
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<td>266.0 112.0</td>
<td>295.0 183.0</td>
</tr>
<tr>
<td></td>
<td>263.0 110.0</td>
<td>295.0 153.0</td>
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DRILL DIA. -- 1.0" (Drill H)

<table>
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<th>M</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>155.0 83.0</td>
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<tr>
<td></td>
<td>137.0 73.0</td>
<td>220.0 118.0</td>
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<tr>
<td></td>
<td>137.0 77.0</td>
<td>245.0 111.0</td>
</tr>
<tr>
<td></td>
<td>162.0 82.0</td>
<td>212.0 101.0</td>
</tr>
<tr>
<td></td>
<td>158.0 75.5</td>
<td>224.0 131.0</td>
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</table>

<table>
<thead>
<tr>
<th>FEED - .008&quot;/REV</th>
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<th>M</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>220.0 120.0</td>
<td>314.0 178.0</td>
</tr>
<tr>
<td></td>
<td>238.0 111.0</td>
<td>335.0 163.0</td>
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<td></td>
<td>220.0 118.0</td>
<td>362.0 159.0</td>
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<td></td>
<td>245.0 111.0</td>
<td>310.0 183.0</td>
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<td></td>
<td>212.0 101.0</td>
<td>295.0 183.0</td>
</tr>
<tr>
<td></td>
<td>224.0 131.0</td>
<td>295.0 153.0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>FEED - .012&quot;/REV</th>
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<th>M</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>314.0 178.0</td>
<td>310.0 183.0</td>
</tr>
<tr>
<td></td>
<td>335.0 163.0</td>
<td>295.0 183.0</td>
</tr>
<tr>
<td></td>
<td>362.0 159.0</td>
<td>295.0 153.0</td>
</tr>
</tbody>
</table>
APPENDIX 6 - D: FORCE COMPARISONS OF MODIFIED AND CONVENTIONAL DRILLS

1) OBSERVED FORCES (CONVENTIONAL DRILL WITH PILOT HOLE)

SPEED -- 150 RPM
T -- THRUST (LB)  M -- TORQUE (IN-LB)

FORCES OF MODIFIED DRILLS LISTED IN APPENDIX 4 - D

<table>
<thead>
<tr>
<th>Drill Dia. -- .75&quot; (Drill D)</th>
<th>(CONVENTIONAL DRILL)</th>
</tr>
</thead>
<tbody>
<tr>
<td>FEED - .005&quot;/REV</td>
<td>FEED - .008&quot;/REV</td>
</tr>
<tr>
<td>T</td>
<td>M</td>
</tr>
<tr>
<td>93.5</td>
<td>46.6</td>
</tr>
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<td>91.5</td>
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</tr>
<tr>
<td>97.0</td>
<td>53.0</td>
</tr>
<tr>
<td>91.5</td>
<td>49.2</td>
</tr>
<tr>
<td>84.5</td>
<td>60.5</td>
</tr>
<tr>
<td>91.5</td>
<td>51.5</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Drill Dia. -- 1.0&quot; (Drill E)</td>
<td>(CONVENTIONAL DRILL)</td>
</tr>
<tr>
<td>-----------------------------</td>
<td>----------------------</td>
</tr>
<tr>
<td>FEED - .005&quot;/REV</td>
<td>FEED - .008&quot;/REV</td>
</tr>
<tr>
<td>T</td>
<td>M</td>
</tr>
<tr>
<td>119.0</td>
<td>82.0</td>
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<tr>
<td>108.0</td>
<td>81.5</td>
</tr>
<tr>
<td>119.0</td>
<td>81.5</td>
</tr>
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<td>108.0</td>
<td>82.0</td>
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<tr>
<td>115.0</td>
<td>83.5</td>
</tr>
</tbody>
</table>
2) OBSERVED FORCES (CONVENTIONAL FULL DRILL)

SPEED -- 150 RPM
T -- THRUST (LB)     M -- TORQUE (IN-LB)

FORCES OF MODIFIED DRILLS LISTED IN APPENDIX 5 - D

**DRILL DIA. -- .75" (Drill D)**
(CONVENTIONAL DRILL)

<table>
<thead>
<tr>
<th>FEED - .005&quot;/REV</th>
<th>FEED - .008&quot;/REV</th>
<th>FEED - .012&quot;/REV</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>M</td>
<td>T</td>
</tr>
<tr>
<td>220.0</td>
<td>60.6</td>
<td>342.0</td>
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<td>242.0</td>
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<td>360.0</td>
</tr>
<tr>
<td>220.0</td>
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<td>381.0</td>
</tr>
<tr>
<td>238.0</td>
<td>70.5</td>
<td>320.0</td>
</tr>
<tr>
<td>196.0</td>
<td>63.0</td>
<td>360.0</td>
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<td>227.0</td>
<td>55.5</td>
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**DRILL DIA. -- 1.0" (Drill E)**
(CONVENTIONAL DRILL)

<table>
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<th>FEED - .005&quot;/REV</th>
<th>FEED - .008&quot;/REV</th>
<th>FEED - .012&quot;/REV</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>M</td>
<td>T</td>
</tr>
<tr>
<td>310.0</td>
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<td>302.0</td>
<td>91.0</td>
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</tr>
<tr>
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<td>543.0</td>
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<tr>
<td>302.0</td>
<td>106.0</td>
<td>465.0</td>
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<tr>
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<td>91.0</td>
<td>444.0</td>
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APPENDIX 7 - D: MECHANICS OF CUTTING FOR MODIFIED DRILLS

1) OBSERVED CHIP LENGTH RATIO DISTRIBUTIONS

DRILL DIA. -- 1.0" (Drill H)    SPEED -- 150 RPM

CHIP LENGTH RATIO AT RAD. -- .092"

<table>
<thead>
<tr>
<th>FEED - .005&quot;/REV</th>
<th>FEED - .008&quot;/REV</th>
<th>FEED - .012&quot;/REV</th>
</tr>
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<tbody>
<tr>
<td>2A 2J</td>
<td>2A 2J</td>
<td>2A 2J</td>
</tr>
<tr>
<td>.395 .419</td>
<td>.483 .415</td>
<td>.452 .371</td>
</tr>
<tr>
<td>.454 .409</td>
<td>.415 .389</td>
<td>.419 .452</td>
</tr>
<tr>
<td>.420 .478</td>
<td>.406 .492</td>
<td>.416 .433</td>
</tr>
<tr>
<td>.403 .468</td>
<td>.400 .433</td>
<td>.428 .506</td>
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<tr>
<td>.520 .473</td>
<td>.473 .451</td>
<td>.421 .451</td>
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<tr>
<td>.589 .459</td>
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<td>.455 .410</td>
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CHIP LENGTH RATIO AT RAD. -- .175"

<table>
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<tr>
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<th>FEED - .008&quot;/REV</th>
<th>FEED - .012&quot;/REV</th>
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</thead>
<tbody>
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<td>2A 2J</td>
<td>2A 2J</td>
<td>2A 2J</td>
</tr>
<tr>
<td>.382 .547</td>
<td>.575 .527</td>
<td>.471 .528</td>
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<td>.522 .398</td>
<td>.464 .433</td>
<td>.500 .459</td>
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<td>.444 .416</td>
<td>.416 .555</td>
<td>.473 .466</td>
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<tr>
<td>.446 .492</td>
<td>.492 .438</td>
<td>.465 .489</td>
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<tr>
<td>.652 .517</td>
<td>.517 .476</td>
<td>.472 .436</td>
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<td>.583 .408</td>
<td>.408 .491</td>
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CHIP LENGTH RATIO AT RAD. -- •259"

<table>
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<tr>
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<th>FEED - •008&quot; /REV</th>
<th>FEED - •012&quot; /REV</th>
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<tbody>
<tr>
<td>2B</td>
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<td>2B</td>
</tr>
<tr>
<td>•621</td>
<td>•572</td>
<td>•543</td>
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<td>•447</td>
<td>•480</td>
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<tr>
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<td>•488</td>
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<tr>
<td>•514</td>
<td>•490</td>
<td>•552</td>
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<tr>
<td>•439</td>
<td>•520</td>
<td>•543</td>
</tr>
<tr>
<td>•610</td>
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<td>•502</td>
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CHIP LENGTH RATIO AT RAD. -- •342"

<table>
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<th>FEED - •008&quot; /REV</th>
<th>FEED - •012&quot; /REV</th>
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<td>2C</td>
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<td>•540</td>
<td>•529</td>
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CHIP LENGTH RATIO AT RAD. -- •426"

<table>
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<th>FEED - •012&quot; /REV</th>
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<tr>
<td>•570</td>
<td>•549</td>
<td>•570</td>
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2) OBSERVED THRUST AND TORQUE DISTRIBUTIONS

DRILL DIA. -- 1.0"  SPEED -- 150 RPM
T -- THRUST (LB) (Drill H)  M -- TORQUE (IN-LB)

### ANNULUS 2A

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<th>FEED - °012&quot;/REV</th>
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APPENDIX E: PREDICTED DISTRIBUTIONS OF ANGLES, CHIP LENGTH RATIO AND FORCES.

\[ \gamma_c = \left( \cdot730 - \cdot010 \cdot\alpha_h \right) \]

Drill Dia - 1.00"

Speed = 150 RPM

Feed - \cdot005 in/rev

\[ \text{Speed} = 150 \text{ RPM} \]

\[ \text{Feed} = \cdot005 \text{ in/rev} \]

\[ \begin{align*}
\text{r} & \quad \text{c} & \quad \phi_h & \quad \phi & \quad \alpha_h & \quad \beta & \quad \gamma_c \\
0.427 & 0.532 & 32.33 & 4.13 & 3.87 & 7.11 & 0.17 \\
0.433 & 0.532 & 32.32 & 4.23 & 3.75 & 7.31 & 0.11 \\
0.396 & 0.532 & 32.38 & 4.33 & 3.62 & 7.54 & 0.27 \\
0.384 & 0.531 & 32.29 & 4.52 & 3.48 & 7.77 & 0.44 \\
0.372 & 0.531 & 32.25 & 4.67 & 3.33 & 8.02 & 0.01 \\
0.362 & 0.532 & 32.21 & 4.83 & 3.17 & 8.29 & 0.03 \\
0.348 & 0.529 & 32.16 & 4.99 & 3.01 & 8.56 & 0.00 \\
0.337 & 0.528 & 32.11 & 5.16 & 2.82 & 8.82 & 0.08 \\
0.325 & 0.527 & 32.04 & 5.37 & 2.63 & 9.21 & 0.45 \\
0.313 & 0.526 & 31.97 & 5.58 & 2.42 & 9.57 & 0.73 \\
0.306 & 0.524 & 31.90 & 5.71 & 2.21 & 9.93 & 0.97 \\
0.289 & 0.523 & 31.73 & 6.06 & 2.19 & 10.10 & 0.21 \\
0.277 & 0.521 & 31.63 & 6.33 & 2.07 & 10.32 & 0.59 \\
0.253 & 0.519 & 31.50 & 6.63 & 1.94 & 10.59 & 0.33 \\
0.261 & 0.516 & 31.37 & 6.90 & 1.82 & 10.84 & 0.32 \\
0.241 & 0.514 & 31.28 & 7.28 & 1.70 & 11.10 & 0.74 \\
0.229 & 0.513 & 31.11 & 7.72 & 1.63 & 11.36 & 9.71 \\
0.218 & 0.511 & 30.93 & 8.18 & 1.60 & 11.62 & 9.75 \\
0.205 & 0.508 & 30.72 & 8.69 & 1.60 & 11.90 & 9.57 \\
0.193 & 0.499 & 30.72 & 9.20 & 1.60 & 11.84 & 9.81 \\
0.161 & 0.492 & 30.28 & 9.34 & 1.60 & 11.84 & 9.81 \\
0.17 & 0.493 & 29.57 & 10.33 & 1.60 & 11.84 & 9.81 \\
0.153 & 0.488 & 29.43 & 11.23 & 1.60 & 11.84 & 9.81 \\
0.146 & 0.479 & 29.30 & 12.78 & 1.60 & 11.84 & 9.81 \\
0.134 & 0.472 & 29.30 & 14.44 & 1.60 & 11.84 & 9.81 \\
0.122 & 0.461 & 28.56 & 15.82 & 1.60 & 11.84 & 9.81 \\
0.113 & 0.449 & 27.87 & 16.93 & 1.60 & 11.84 & 9.81 \\
0.099 & 0.437 & 27.43 & 18.82 & 1.60 & 11.84 & 9.81 \\
\end{align*} \]

\[ \text{SUM OF SQR. OF C.R. DIFF.} = 0.0003 \]

\[ \text{TOTAL THRUST} = 237.61 \]

\[ \begin{align*}
\text{THRUST OF A} & \quad \text{THRUST OF B} & \quad \text{THRUST OF C} & \quad \text{THRUST OF D} \\
15.89 & 9.39 & 8.83 & 8.70 \\
\end{align*} \]
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<th>( f_r )</th>
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<th>( f_{TANG} )</th>
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**TOTAL TORQUE** 2

\( 47.94 \)

**TORQUE OF A** 6.92

**TORQUE OF B** 13.21

**TORQUE OF C** 13.63

**TORQUE OF D** 17.14
NOTE 1: ALL HOLES & OUTSIDE DIA. OF TORQUE ELEMENT TO BE CONCENTRIC WITHIN .001"

NOTE 2: SURFACES S1 & S2 TO BE PARALLEL WITHIN .0005"

NOTE 3: OUTSIDE DIA. OF PLUG TO BE CONCENTRIC WITH TAPER WITHIN .005"

NOTE 4: SURFACE S3 TO BE PERPENDICULAR TO VERTICAL AXIS OF TAPER

NOTE 5: BOTH COMPONENTS TO BE FINE MACHINED

NOTE 6: REMOVE ALL SHARP EDGES