SUMMARY

A short literature survey outlining recent developments in the modeling of wall turbulence is presented. One of the most recent developments, the A-vortex model of Perry and Chong (1982), which was based on Townsend's (1976) attached eddy hypothesis, is discussed.

The use of hot-wires, in particular X-wires, for measuring turbulence quantities, has been found to have several limitations especially over rough surfaces. Some methods of overcoming these shortcomings are proposed, tested and discussed. Mean flow, broad-band turbulence intensity and spectral measurements were carried out over smooth and rough surfaces in zero pressure gradient boundary layers.

The spectra measured in the turbulent wall region were found to follow the scaling laws put forward by Perry and Abell (1977) and the A-vortex model of Perry and Chong (1982) for both smooth and rough walls. The A-vortex model of Perry and Chong was used to generate spectral distributions corresponding to some flow cases tested and agreed well with the measured spectral distributions. Strong evidence is presented for the existence of an inertial subrange, where the small scale motions are locally isotropic. The spectral laws put forward by Perry and Abell (1977) are used to infer the broad-band turbulence intensity distributions and reasonable agreement was shown with the measured distributions.
The measured broad-band turbulence intensity distributions strongly support the Townsend (1956) Reynolds number similarity hypothesis and the extended form of this hypothesis for rough surfaces of Perry and Abell (1977), at least for the limited range of Reynolds numbers tested here.

The results also strongly support the theory that the structure of wall turbulence consists of eddies "attached" to the wall, and that their distribution in the boundary layer follows that proposed in the model of Perry and Chong (1982).
"The greater our knowledge increases, the greater our ignorance unfolds".

John F. Kennedy
September 1962
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INTRODUCTION AND AIMS

Mean Flow

In 1956, Townsend put forward his Reynolds number similarity hypothesis, which applied in the fully turbulent wall region and states that "...the mean (relative) motions and the motion of the energy containing components of the turbulence are dependent on the boundary conditions alone, and are independent of the fluid viscosity, in so far as a change in the fluid viscosity may change the boundary conditions...", provided that the direct effect of the viscous stresses are negligible and the viscous wall region is thin compared to the characteristic length scale of the fully turbulent region. This hypothesis has been extended to include rough-wall flows by Perry and Abell (1977) and Henbest (1983) by the assumption that the roughness only influences the boundary conditions of the flow, by changing $U_\tau$, the friction velocity.

In the wall region, the mean flow depends on the kinematic viscosity $\nu$ and the wall variables $U_\tau$ and $z$, where $z$ is the distance normal to the wall. By dimensional analysis, this leads to the relationship

$$\frac{\overline{U}}{U_\tau} = g_1 \left[ \frac{2U_\tau}{\nu} \right]$$

(0.1)

where $g_1$ is a universal function. This relationship was first derived by Prandtl in 1926, using a momentum transport model which assumed that the momentum of a particle of fluid is conserved over a characteristic mixing length.
He obtained that

$$\frac{\bar{U}}{U_\tau} = \frac{1}{\kappa} \ln \frac{zU_\tau}{v} + A \tag{0.2}$$

where $\kappa$ and $A$ are universal constants. This law is now generally known as Prandtl's logarithmic law of the wall and can be derived by dimensional arguments (eg. see Millikan (1938) and Rotta (1962)).

For the fully turbulent region of the flow, the mean relative motions are dependent only on $U_\tau$, $z$, and the boundary layer thickness $\delta_H$. By dimensional analysis, this gives

$$\frac{U_\tau - \bar{U}}{U_\tau} = g_2 \left[ \frac{z}{\delta_H} \right] \tag{0.3}$$

where $g_2$ is a universal function that depends on the large scale flow geometry and $U_\tau$ is the free stream velocity. This relationship was first derived by von Kármán and is known as von Kármán's velocity defect law.

The verification of these laws by experiment in smooth pipes provided the basics for which these laws could also be applied to flows over rough surfaces. This was done by Nikuradse in 1933 and the comprehensive and detailed nature of his experiments has made it a logical starting point for all work on rough wall flows. In rough wall flows, the velocities also depend on the surface roughness and hence equation 0.1 becomes

$$\frac{\bar{U}}{U_\tau} = g_3 \left[ \frac{zU_\tau}{v}, \frac{kU_\tau}{v} \right] \tag{0.4}$$
where \( k \) is a roughness scale and \( g_3 \) is a universal function for a given roughness. The law for smooth walls (equation 0.2) can be modified to apply to flow over rough walls by subtracting off from the right hand side a roughness function \( \Delta U/U_\tau \), which is a universal function of \( ku_\tau/\nu \) for a given roughness geometry (see Section 5.1). This notation was first used by Clauser (1954) and Hama (1954) to show that \( \Delta U/U_\tau \) is a measure of the decrement in non-dimensional velocity due to the effect of the roughness. Both Clauser's and Hama's experiments were on flat plate rough-wall flows in zero pressure gradients, and the mean flow laws that were previously applied to pipe flows were also shown to apply to flat plate flows in zero pressure gradients. The work of Perry and Joubert (1963) showed that equation 0.4 was also applicable in adverse pressure gradients. Work done by Perry, Schofield and Joubert over rough walls in zero and adverse pressure gradients drew attention to another "type" of roughness, which they called "d"-type roughness, which had a roughness function that depended on \( \delta, U/\nu \) (see Section 5.1).

**Broad-band turbulence intensities**

In the fully turbulent region, the extended form of Townsend's Reynolds number similarity hypothesis states that the energy containing components of the turbulent motions are independent of the fluid viscosity and surface roughness, and depend only on the wall shear velocity, boundary layer thickness and distance from the wall. Viscosity and surface roughness are only involved in the sense that they determine the shear velocity. This predicts that the broad-band turbulence
intensity distribution should be a function of $z/\delta_H$, ie.

$$\frac{\overline{u_i u_j}}{U_\tau} = g_4 \left[ \frac{z}{\delta_H} \right] \quad (0.5)$$

where $\overline{u_i u_j}$ is the mean of the correlation between $u_i$ and $u_j$ at a point distance $z$ from the wall. For the turbulent wall region, Townsend (1976) proposed his attached eddy hypothesis that stated that the main energy containing motions are made up of contributions from eddies that are in a sense attached to the wall. These attached eddies were assumed to be geometrically similar and to have the same characteristic velocity scale, and that in the turbulent wall region, the Reynolds shear stress is approximately constant with $z$. The assumption was also made that the normal velocity at the wall is zero, but the other two components of velocity are not, and this "slip at the wall" is allowed for by the viscous wall region. Townsend's analysis led to the formulation of a set of laws for the broad band turbulence intensity distributions and these are given in Chapter 3.

**Turbulence Spectra**

Kolmogoroff (1941) proposed that turbulent flows could be separated into large scale anisotropic motions and small scale isotropic motions, and that the turbulent energy is transferred from the large scale motions to slightly smaller motions and so on by a "energy cascade" process. A small amount of energy is converted into heat in each of these energy transfers until the eddies are of a size where viscosity dominates, and

$u_i$ is a turbulent velocity fluctuation in the $i$ direction ($i=1,2,3$) and similarly for $u_j$. 
the remainder of the energy is converted into heat by viscous dissipation. This theory led to the relationship that for \( k_n \ll l \) and \( k_l \ll l \), that

\[
\frac{E(k)}{\eta u^2} = \frac{C}{(k_n)^{5/3}}
\]

where \( k \) is the modulus of the 3-dimensional wavenumber, \( \eta \) and \( u \) are the length and velocity scales of the fine scale motions, \( \ell \) is the length scale of the large scale motions, \( C \) is a universal constant and \( E(k) \) is the three dimensional turbulence energy spectrum of \( u' \), where \( u' \) is a velocity fluctuation. The region where this law applies is known as the inertial subrange. However, the existence of an inertial subrange can be shown using a region of overlap argument and dimensional analysis (eg. see Tennekes and Lumley (1972), Townsend (1976), Hinze (1975) and Perry and Abell (1977)), and does not require the assumption of a cascade model.

Tennekes and Lumley (1972) point out that an inertial subrange will exist only if the turbulence Reynolds number \( \sqrt{\frac{u'^2 \eta}{\eta}} \) of the energy containing motions is greater than 200,000 while Bradshaw (1967) obtained that the condition required is \( 10^4 k_1 < \eta \) while \( k_1 \) is the longitudinal one-dimensional wavenumber.

Perry and Abell (1977) interpreted the one dimensional longitudinal spectral data of Abell in smooth and rough pipes by dividing the flow into different wavenumber regions of influence. By a region of overlap argument and dimensional analysis, they derived a set of spectral scaling laws for the longitudinal spectra. By integrating the contributions from
each of the various spectral ranges, they predicted the turbulence intensities in a smooth pipe, and by applying the extended form of Townsend's Reynolds number hypothesis, to a rough pipe as well.

In (1982), Perry and Chong proposed their A-vortex model for wall turbulence, which was based in part on Townsend's (1976) attached eddy hypothesis. They refer to this model as the A-vortex model but the vortices can be of any general hairpin-type shape; it is referred to here as the A-vortex model for convenience only. This model has been further developed to accurately predict mean flow, broad-band turbulence intensity and spectral distributions (Perry, Henbest and Chong (1984)).

**Hot-wire Anemometry**

King (1914) put forward a law for the variation of heat loss with velocity and his law in various forms is still being used in hot-wire anemometry today. The potential of hot-wires for measurement of turbulence in fluid flows was not realised until Dryden and Kuethe (1928) devised a method for compensating for thermal inertia and thus greatly extended the frequency response of the hot-wire anemometer.

Most of the work into the behaviour of hot-wires has centred on its calibration for measurement of fluctuating flow quantities. The formulation of the cosine cooling law by Champagne, Schleicher and Wehrmann (1967) was a step forward in the understanding of hot-wire behaviour and many calibration schemes at present assume this law. Two
basic calibration schemes are generally in use, static calibration and dynamic calibration. Static calibration is the traditional method and is often based on a fit to an empirical cooling law, which are variations of Kings law, and calibrations using this scheme have been refined over the years by many workers. Other schemes have been developed which do not rely on any specific cooling law, eg. look up tables, local Taylor series expansions etc., eg. see Willmarth (1977). Perry and Morrison (1971) introduced "dynamic" calibration where no cooling law has to be assumed and the hot-wire sensitivities are obtained directly.

The difficulty of obtaining accurate measurements of turbulence quantities with hot-wires is well known, and there are many pitfalls for the inexperienced. One of these shortcomings occurs in the measurement of high-intensity turbulence where the velocity vectors approaching the probe do so at large angles of incidence and cause the calibration of the probe to become invalid. This phenomenon was shown by Tutu and Chevray (1975) and Willmarth and Bogar (1977), and more recently by Kawal, Shokr and Keffer (1983), who have shown the effect of this phenomenon on turbulence intensity measurements and also the effect of the third turbulence component that is not being measured. This problem can also be encountered in flow over rough walls as is shown later. This aspect of hot-wire behaviour has recieved little attention to date.

Hot-wires are also limited by their spatial resolution capabilities when measuring in regions where fine scale motions are present. The limitations of the spatial resolution of hot-wires and the effect of this problem on turbulence measurements has been investigated by Wyngaard (1968).
The Structure of turbulence

The attached eddy hypothesis of Townsend and the model of Perry and Chong require the existence of eddies attached to the wall. The existence of horse-shoe shaped attached eddies was postulated by Theodorsen (1952), who also anticipated that different scales of these eddies existed, and he suggested that they were a fundamental building block of the structure of turbulence. Willmarth and Tu (1977) also anticipated the existence of these horseshoe shaped attached eddies in detailed measurements of the fine scale structure of flow close to a wall. Kline et.al.(1967) observed that these attached eddies "roll up" from the viscous sublayer and undergo stretching, and also established the lateral scaling of these eddies at the wall. Detailed flow visualisations by Kim et.al (1971), Grass (1971), Head and Bandyopadhyay (1981) Perry, Lim and Teh (1981) and others have also confirmed their existence and identified their structure in greater detail.


In fully developed circular pipe flow there are no streamwise derivatives of the mean flow and other statistical quantities. The hairpin, horseshoe or A-shaped vortices attached to the wall in a pipe according to the Perry and Chong model all point radially inwards, and as their scale increases they are probably distorted by the curvature of the
pipe wall. The largest eddies probably extend from one side of the pipe to the other, giving quite a tangled mess of vortex filaments. The model of Perry and Chong was developed for flow over flat surfaces and intrusion of eddies from other boundaries was not taken into account. In a circular pipe, the model is valid only in the wall region.

It was therefore decided to see if this model or variations of it were applicable to smooth and rough flat plate boundary layer flows with a zero pressure gradient. In a sense, these flow cases are closer to the conditions for which the model was intended except for the possible effect of variations with streamwise development. In the absence of other boundaries, the model may also be applicable beyond the wall region and may extend to the outer edge of the layer.

To pursue this study it is necessary to have a convenient and reliable method of measuring the surface shear stress. This is particularly a problem on a rough wall. Unlike pipe flow, a simple pressure drop measurement cannot be used. There are also no reliable mean flow similarity techniques for rough wall boundary layers which are analogous to the Clauser-chart method, Preston-tube method or surface heating element method. Momentum integral methods are known to be inaccurate because of having to find the derivative of a curve defined by sparse discrete data points. These methods are also sensitive to any secondary flow. The existence of the "flying hot-wire" as described in Watmuff, Perry and Chong (1982) enabled some of these difficulties to be overcome. The ability of this method to collect a large number of closely spaced experimental data points in the streamwise direction would
enable more accurate estimates to be made of streamwise variations in the flow. As it turned out, the "flying hot-wire" was also used to impose an additional bias velocity on the hot-wires, the reasons for which are discussed in detail later. Pressure tapped roughness elements are very difficult to manufacture and their use requires large elements for determining the form drag and hence the skin friction coefficient of the layer (eg. see Perry, Schofield and Joubert, (1969)). The skin friction coefficient, \( C_f' \) is defined as \( C_f' = \frac{\tau_0}{(\rho U_1^2)} \), where \( \tau_0 \) is the shear stress at the wall and \( U_1 \) is the mean free stream velocity. Floating element techniques are extremely inconvenient and require a large amount of technical development and care must be taken to account for, or else eliminate, momentum transfer due to leakage around the edges of the element.

It was therefore decided to attempt to use X hot-wire Reynolds shear stress measurements to infer the wall shear stress. On smooth surfaces this is not really necessary since the Clauser chart and Preston tube methods have been proven to be reasonably reliable. However, if measured Reynolds shear stresses showed agreement with the Clauser chart method on a smooth surface, then this would lend credibility to the other measured turbulence quantities. On rough surfaces, it has been known for some time that the Reynolds shear stress falls off as the rough wall is approached. This implies that there might exist a stationary mean flow "wave" above the elements which transports momentum. This layer, if it exists would have to be taken into account.
The aims of this thesis are therefore

1) To establish a reliable method for determining Reynolds shear stress.

2) To explain the fall off in measured Reynolds shear stress close to a rough surface and to see firstly whether this was a consequence of a stationary wave of mean velocity immediately above the roughness elements.

3) To use the measured Reynolds shear stresses to infer the wall shear stress.

4) To measure mean flow, broad-band turbulence intensities and spectra above smooth-and rough-wall boundary layers and make comparisons with recently developed models of wall turbulence.
CHAPTER 1

Experimental Techniques and Apparatus

1.1 CO-ORDINATE SYSTEM

The co-ordinate system used throughout this thesis follows that of Townsend (1976). This is best shown in figure 1.1. \( \bar{U}, \bar{V} \) and \( \bar{W} \) denote mean velocities in the \( x, y \) and \( z \) directions and \( u', v' \) and \( w' \) denote the fluctuating velocities.

1.2 WIND TUNNELS

Two wind tunnels were used for all the tests performed. They will be referred to as the large wind tunnel and the small wind tunnel.

1.2.1 The Large Wind Tunnel

This tunnel has working section inlet dimensions of 940mm (width) \( \times \) 410mm (height) and is 6.7m long and the free stream turbulence level is less than 0.3%. A cross-section of the large wind tunnel is shown in figure 1.2.
This wind tunnel has an adjustable roof and this was set to give a zero pressure gradient for all the tests. The tunnel is of the closed return type and is powered by a thyristor controlled D.C. motor which drives a centrifugal fan. This wind tunnel is equipped with a dynamic calibrator based on a Murray's cycloidal drive. It is also equipped with the flying hot-wire system as used by Watmuff (1979) and Watmuff, Perry and Chong (1983).

Boundary layers were measured over the following types of surfaces in this wind tunnel.

1) A Wavy Wall
2) A Smooth Wall
3) Mesh type roughness

1.2.2 The Small Wind Tunnel

The small wind tunnel is also of the closed return type and is powered by a thyristor controlled D.C. motor driving an axial flow fan. The working section has dimensions of 1345mm (length) X 270mm (height) X 345mm (width). Figure 1.3 shows the cross-section of the working section of this tunnel. The pressure gradient cannot be readily adjusted. A micrometer traverse is used to traverse the probe in a direction normal to the wall and it is possible to mount the traverse at different streamwise locations along the working section. This tunnel also has a facility for calibrating hot-wires dynamically.
Figure 1.1 Co-ordinate system

Figure 1.2 Cross-section of Large wind tunnel

Figure 1.3 Cross-section of Small wind tunnel
Boundary layers measured in this tunnel were over

1) The Small Wavy Wall
2) The Mesh Type Roughness

Some of the investigations into X-hot wire behaviour were also carried out in this wind tunnel.

In both these tunnels, it was possible to traverse the probe laterally in order to measure a transverse mean velocity profile ($\bar{V}$ vs $y$).

1.3 PRESSURE MEASUREMENTS

1.3.1 Large Wind tunnel

The pressure coefficient, $C_p$, along the working section of the tunnel was measured by mounting a static pressure tube on the flying hot-wire sled "sting" so that the static probe could be moved to pre-determined points along the working section. The pressure coefficient is defined as $C_p = (P - P_r)/\frac{1}{2} \rho U_r^2$, where $P_r$ and $U_r$ are the reference static pressure and reference velocity respectively and $P$ is the static pressure at the point in question. The reference pressure and velocity were measured with a pitot-static tube mounted at the start of the working section. (These were also used during calibration of hot-wires).
The mean velocity profiles were measured using a pitot-static tube mounted on the flying hot-wire sled "sting" and consisted of the static tube used in the pressure coefficient measurements and a flattened total-head tube. The pitot-static tube could thus be moved to any position along the working section and movement through the boundary layer was achieved using the sled stepping motor traverse.

Both pitot-static tubes were constructed from 2mm hypodermic tubing and were calibrated against an N.P.L pitot-static tube.

1.3.2 Small Wind Tunnel

The measurement of $C_p$ and mean velocity profiles in the small wind tunnel followed the same basic procedure as in the large wind tunnel, with the exception that the boundary layer pitot-static tube was mounted on a movable micrometer traverse. A screw thread was used to traverse the probes normal to the wall and the traverse itself could be moved to any streamwise position between 400mm and 1000mm from the start of the working section.

All pressure differences were measured using a Datametrics Barocel pressure transducer model 1014A. The output voltage of the transducer was then integrated on an EAI TR-20 analogue computer in three bursts of 20 seconds each. Additional bursts were taken if significant variation was noted between the first three bursts.
1.4 THE FLYING HOT WIRE

The flying hot-wire method was pioneered by Cantwell (1975) who used hot-wires mounted on whirling arms which sampled at closely spaced points in the wake of a circular cylinder. A variation of this method where the hot-wires were "flown" in a straight line was developed and used by Watmuff, Perry and Chong (1983). This system consists of a "sled" running on four almost frictionless air bearings on a monorail mounted above the working section of the wind tunnel. The hot-wire probe is mounted on a sting which passes through a slot in the roof of the working section. The vertical position of the hot-wire probe is controlled by a stepping motor which screws the probe up or down. By counting the voltage pulses sent to the stepping motor, the position of the probe is known to 1/200mm. A luffing mechanism acts as a bridge carrying the air supply, power and signals to and from the sled.

The flying hot wire system used here is basically the same as that used by Watmuff et al. However, the propulsive power for the air bearing sled is now provided by a pneumatic cylinder operating at 130 p.s.i. The use of pneumatics has reduced the complexity of the system enormously. The system now is more reliable, quieter and does not transmit as much shock to the air bearing sled due to the cushioning effect of the air in the cylinder. Other minor changes were made to the system in the interests of reliability. The logic control system remains as before.

Figure 1.4 is a schematic view of the flying hot-wire system, showing the relative positions of the working section, sled, propulsion...
system and the luffing mechanism. Figure 1.5 shows a more detailed end view of the sled on the monorail above the working section and the probe carrying system can be seen.

A film strip with alternate dark and transparent vertical bands runs alongside the monorail. By scanning this film strip with a LED-phototransistor pair through another film strip, a Moire fringe is produced, and the position of the sled is known by counting the number of fringes which pass the LED-phototransistor pair from a known position. This signal is frequency demodulated into a voltage which can be calibrated to give the velocity of the sled. The position of the film strip is shown in figure 1.5.

1.5 THE HOT-WIRES AND ANEMOMETERS

Hot-wire measurements were taken using the normal and X-wire probes with 5\(\mu\)m diameter etched Wollaston wires constructed by the author. Etched lengths of the hot-wires were typically 1.2mm unless where stated otherwise and the X-wire angles were nominally \(\pm45^\circ\) to the streamwise direction and sometimes \(\pm60^\circ\) to the streamwise direction. The hot-wires were annealed for at least 24 hours before use and were operated at a nominal resistance ratio of 2.0.

The constant temperature hot-wire anemometers were built by the author and follow the design of Perry and Morrison (1971a) and further details can be found in Perry (1982).
Figure 1.4
Schematic view of the flying hot-wire system
(From Watmuff, 1979)

Figure 1.5
More detailed view of the sled, rail and sting.
(From Watmuff, 1979)
1.6 DYNAMIC CALIBRATION

Here it is necessary to define a number of velocities. \( U \) is the velocity relative to a stationary observer, \( U_{\text{ref}} \) is an arbitrarily chosen reference velocity, (usually close to the middle of a calibration range), \( \bar{U} \) is a time averaged velocity, \( u \) is a velocity measured relative to \( U_{\text{ref}} \) and \( u' \) is a velocity perturbation measured relative to \( \bar{U} \) and has zero mean. Similarly for the other components.

In static calibration, the usual procedure is to assume a heat transfer law which is taken to be applicable over an appreciable velocity range. This gives a relationship between \( E_u \) and \( U \) where \( E_u \) is the voltage output from the anemometer due to the velocity \( U \). This relationship is then used to infer the hot-wire sensitivity \( \partial E_u / \partial U \) by differentiation. This leads to inaccuracies. Using the dynamic calibration method of Perry and Morrison (1971b), the sensitivity \( \partial E_u / \partial U \) can be found directly with good accuracy since the wires are given an accurately known sinusoidal velocity perturbation about a known mean velocity \( \bar{U} \). This method has been extended to include X-wires and has undergone considerable refinement in recent years by Watmuff (1979), Tan (1983) and the author. This method enables accurate measurement of the longitudinal, normal and lateral velocity fluctuations with X-wires. In this method the hot-wire sensitivities \( \partial E_u / \partial u \) and \( \partial E_w / \partial w \) are curve fitted locally about an operating point by an appropriate Taylor series expansion.

Figure 1.6 shows the signal processing circuit used to "match" the X-wires and for signal conditioning. To do this, two signals \( E_u \) and \( E_w \) must be obtained from the anemometer outputs \( E_0 \) and \( E_1 \), where \( E_u \) is sensitive only to \( u \) and \( E_w \) is sensitive only to \( w \). This process is
performed electronically and is known as matching. The sensitivities of
the X-wires to velocity perturbations in the U and W directions can now
be determined by shaking the X-wire probe first in the U direction and
then the W direction respectively in a steady stream where $U = U_{\text{ref}}$ and
$W = 0$. The details of the matching and dynamic calibration procedures can
be found in Perry (1982, pp. 122-128). The voltages $E_u$ and $E_w$ are
sampled on-line by a digital computer via an A-D converter.

A dynamic calibrator based on a Murray's cycloidal drive was built,
with which the probe could be given a sinusoidal perturbation at any
angle between the horizontal and vertical reference axes. Because of the
matching procedure, it was possible during calibration to shake the wires
at an angle of $45^\circ$ to the free stream, imparting both a horizontal and
vertical component of velocity on them at the same time. This enabled
simultaneous calibrations to be made for $u$ and $w$. These are the
velocities measured relative to the reference velocity $U = U_{\text{ref}}$ and
$W = 0$. The curves fitted to the calibration data obtained in this way were
compared with those obtained by separate horizontal and vertical shaking
and were found to be virtually identical.

The same calibration scheme could be used to calibrate the X-wires
used to measure $v$ by calibrating in the U-W plane and then simply rolling
the probe so that the wires lay in the U-V plane.

It was necessary to transfer the probe from the calibrator to the
measuring traverse and care had to be taken to ensure that the probe
was held with exactly the same orientation to the free stream in both
cases. This "pitch alignment" was done according to a scheme by Abell
(1976) and was checked at regular intervals.
Figure 1.6

Signal processing circuit used for bucking, matching and calibration.
1.7 EXPERIMENTAL DETAILS

Matching and Calibration.

The hot-wire probe is mounted on the calibrator sting and the tunnel is set at the intended free stream velocity for the experiment and allowed to run until the temperature has stabilised. This usually takes about 2-3 hours. As the ambient temperature in the laboratory is thermostatically controlled, once steady state is reached, the temperature in the tunnel usually only varies by ±0.25°C, and no temperature corrections are needed. If the tunnel temperature did change appreciably during the experimental run, the experiment was aborted and repeated.

Once the temperature had stabilised, the square wave response of the hot-wires and anemometers was set for optimum damping at the lowest instantaneous velocity expected to be encountered by the hot-wires, and their frequency response was typically better than 20kHz (refer Perry 1982). A velocity sweep was made to ensure that the wire response was not unstable anywhere in the velocity range expected during the run.

The tunnel is set at the reference velocity, which is a velocity mid-way between the maximum and minimum velocities expected to be experienced by the hot-wires. The 4th order Butterworth response output Krohn-Hite filters shown in figure 1.6 are set to 30Hz low pass, ie., a decade above the maximum perturbation frequency of 3Hz. Potentiometers $P_0$ and $P_1$ shown in figure 1.6 are set such that the outputs from
amplifiers $A_0$ and $A_1$ are zero volts. The probe is given a horizontal perturbation and potentiometer $K_2"$ in figure 1.6 adjusted such that $E_u$ is independent of $u$. The amplitude of $E_w$ is then typically less than 5% of $E_u$. The probe is then shaken vertically and potentiometer $K_2'$ in figure 1.6 is adjusted such that $E_u$ is independent of $w$. The hot-wires are now said to be matched. The tunnel velocity is changed to check that the matching holds over the experimental velocity range and the output voltages $E_u$ and $E_w$ do not exceed the ±5V limit of the A-D converter voltage window. $E_u$ and $E_w$ are then reset to be 0 V at the operating point if necessary. This does not affect the matching.

As mentioned before, calibration was performed by shaking the probe at an angle of 45° to the flow, thus simultaneously imposing $u'$ and $w'$ velocity perturbations on the X-wires. (A normal wire is calibrated by shaking horizontally only). Voltages are sampled at 120 points during the calibrator cycle and phase averaged on the computer. The sampling is controlled by a slotted "chopper" disc mounted on the calibrator. The sensitivities $\frac{\partial E_u}{\partial u}$ and $\frac{\partial E_w}{\partial w}$ are determined at six or seven free stream velocities so that a curve can be fitted. At each calibration point, the frequency of shaking is adjusted such that the maximum perturbation velocity does not exceed 10% of the free stream velocity, a condition necessary for linearity about that operating point (see Perry (1982) pp.118-122). The computer samples the voltages $E_u$ and $E_w$ and determines the RMS values of $E_u$, $E_w$, $u'$ and $w'$. The free stream velocity at each calibration point is also measured. Perry (1983, pp 122-128)
has shown that the velocities \( u \) and \( w \) can be related to \( E_u \) and \( E_w \), by

\[
u = x_1 + Q_1 E_u + S_1 E_u^2 + Z_1 E_u^3 + ... \tag{1.1}
\]

\[
w = x_2 + Q_2 E_u + R_2 E_w + W_2 E_u E_w + Z_2 E_u^2 E_w + ... \tag{1.2}
\]

and by differentiation, the sensitivities are

\[
\frac{3u}{3E_u} = Q_1 + 2S_1 E_u + 3Z_1 E_u^2 \tag{1.3}
\]

\[
\frac{3w}{3E_w} = R_2 + W_2 E_u + Z_2 E_u^2 \tag{1.4}
\]

From quadratic least squares curve fits to the sensitivities, \( Q_1, S_1, Z_1, R_2, W_2, \) and \( Z_2 \) can be found. \( x_1 \) in equation 1.1 is obtained from a plot of \( u \) vs \( E_u \). With the probe stationary, \( w=0 \) in equation 1.2 and from a plot of \( E_u \) vs \( E_w \) for \( w=0 \), \( x_2 \) and \( Q_2 \) (which are usually very small) can be obtained.

After the calibration procedure is completed, a reduction program based on the above is used to perform a curve fit to the anemometer output voltages and hot-wire sensitivities, and the calibration constants are obtained. The plots of the anemometer voltages and sensitivities are obtained on a storage oscilloscope and checked for spurious points. (The reduction program also gives the maximum deviation of the experimental points from the curve fit). If any spurious points are noticed or if the deviation from the curve fit is greater than 1%, the calibration is repeated. The probe can now be transferred to the sled sting.
The calibration constants obtained are inserted into a sampling program which converts hot-wire voltages into velocities and calculates $\bar{U}$, $\bar{W}$, $-u'w'$, $u'^2$, and $w'^2$ and so the method accounts for the non-linear characteristics of the hot-wires. The wind tunnel is set at the experimental run velocity and a burst of data is sampled and analysed to check that $\bar{U}$ and $\bar{W}$ are correct ($\bar{W}$ should be extremely small in the free stream if the probe is properly pitch aligned). If $\bar{W}$ was large, the pitch alignment of the probe is checked and realigned if necessary and the calibration repeated. The Krohn-Hite filters are opened up to 10kHz and the probe is traversed into the boundary layer for measurement.

The sampling program samples in bursts of 4000 samples at a predetermined rate (usually set at 200Hz) and determines the quantities mentioned earlier. Bursts of 4000 samples are measured and after each burst the cumulative average of the turbulence quantities are printed out. This is repeated until the data converge such that subsequent values do not change by more than 0.5%. This usually takes about 6-8 bursts of 4000 samples. During the experimental run, the mean velocity at each point, $\bar{U}$, sampled by the hot-wires, is monitored and compared with those obtained earlier with a pitot-static tube, to obtain an indication of drift. If the hot-wires are seen to drift, the run is aborted and the hot-wires re-calibrated. As most of the experimental runs took 3-4 hours, drift was not usually a problem. However, some experimental runs took up to 18 hours, eg. with the flying hot-wire, and in this case, the calibration was repeated at the end of the run, and if the calibration constants had changed, the data was corrected using the assumption that drift was linear with time.
1.8 MEASUREMENT OF SPECTRA

A fast Fourier transform algorithm was used to calculate the discrete Fourier transform of the hot-wire signal to the digital computer. An uncalibrated normal hot-wire was used to obtain the $u'$ turbulence spectra and dynamically matched (but uncalibrated) X-wires used to obtain the $v'$ and $w'$ components of the turbulence spectra.

The relevant hot-wire signal was bucked to reduce any d.c. offset and for maximum resolution, the signal was amplified to fill as much of the ±5V window of the A-D converter as possible. The signal was sampled at three sampling rates to improve the frequency bandwidth of the spectrum at low frequencies and was low-pass filtered at half the sampling rate to avoid aliasing. The overall frequency range obtained was 0.1Hz to 10kHz. The three resulting spectral files were matched, joined and smoothed to form a single spectral file.

The spectral argument was converted from frequency, $f$, to one-dimensional longitudinal wave number $k_1$ using Taylor's hypothesis of frozen turbulence, i.e., $k_1 = \frac{2\pi f}{U_c}$, where $U_c$ is the local convection velocity and was assumed to be equal to the local mean velocity at that point in the flow. The validity of this assumption is discussed later in Chapters 4 and 5 when the results are presented.
The spectra were smoothed and normalised such that

\[ \int_{0}^{\infty} \phi_{11}(k_l) \, dk_l = u'^2 \]  

(1.5)

\[ \int_{0}^{\infty} \phi_{22}(k_l) \, dk_l = v'^2 \]  

(1.6)

\[ \int_{0}^{\infty} \phi_{33}(k_l) \, dk_l = w'^2 \]  

(1.7)

where \( \phi_{11}, \phi_{22}, \) and \( \phi_{33} \) are the power spectral densities for \( u', v' \) and \( w' \) respectively. The quantities \( u'^2, v'^2, \) and \( w'^2 \) were obtained from the turbulence intensities measured with the dynamically calibrated hot-wires.

The dissipation rate \( \epsilon \) was calculated using the relationship (Townsend 1976) which assumes the existence of isotropic flow, ie.

\[ \epsilon = 15v \int_{0}^{\infty} k_l \phi_{11}(k_l) \, dk_l \]  

(1.8)
Chapter 2

Hot-Wire Measurement Techniques

In all boundary layer work it is important to determine the local skin friction coefficient, \( C_f' \), accurately. As was pointed out in the introduction, in the case of smooth walls, the Clauser chart and Preston tube methods provide proven techniques which rely on the existence of wall similarity laws. In the case of rough walls however, difficulties arise because the "effective origin" of the boundary layer, i.e. the apparent position of \( z=0 \) relative to the roughness elements which would yield the expected logarithmic velocity distribution, is not known (eg. see Perry and Joubert (1963)).

By measuring the Reynolds shear stress, \( -u'w' \), close to the rough surface, it is theoretically possible to obtain \( C_f' \) directly, without relying on a wall similarity law. In this Chapter, attempts to obtain \( C_f' \) with X-wires are discussed and comparisons are made with other accepted methods.

2.1 THE WAVY WALL

A schematic view of the wavy wall is shown in figure 2.1(a). The wall was constructed of five sheets of corrugated iron roofing material laid on the floor of the large wind tunnel. The corrugations ran laterally across the tunnel. Adjacent sheets overlapped and the joins
Figure 2.1 (a)
Schematic view of the wavy wall

Figure 2.1 (b)
Definition of flow quantities over the wavy wall
were faired and smoothed. The edges were cut so that they butted into the 45° corner fillets of the tunnel. The streamwise length of the wall was 3.5 metres, the wavelength, \( \lambda \), of the corrugations was 76mm and the amplitude "h" of the waves was 17mm. Figure 2.1(b) defines the flow quantities above the wavy wall. The flexible roof of the tunnel was adjusted to give a zero pressure gradient over the roughness length.

2.1.1 Mean Flow Measurements

Mean flow measurements were taken at seven stations along the length of the roughness (above the crests) at downstream distances of \( x = 991, 1372, 1754, 2135, 2517, 2899, \) and 3281mm from the leading edge. Because of the position of the flying hot-wire rail and sled, both the mean velocity and flying hot wire measurements were taken 35mm from the longitudinal centreline of the tunnel. The nominal free stream velocity, \( U_1 \), of the experimental runs was 16 m/s. The boundary layer thickness varied from approximately 100mm at the first station to approximately 180mm at the last.

Figure 2.2 shows a plot of the pressure coefficient, \( C_p \), measured in the free stream, versus the downstream distance, \( x \), along the plate. The pressure gradient was set such that the pressure gradient term \( ((H+2)\theta/U_1)du_1/dx \) in the von Karman momentum integral equation (Equation 2.1) was less than 10% of the skin friction coefficient term \( C_f'/2 \).
Figure 2.2
Pressure coefficient over wavy wall

Figure 2.3
Typical mean velocity profile over wavy wall
The von Kármán momentum integral equation is given by

\[
\frac{C_f'}{2} = \frac{d\theta}{dx} + (H+2) \frac{\theta}{U_1} \frac{dU_1}{dx}
\] (2.1)

Here \( \theta \) is the momentum thickness and \( H \) is the form factor which equals \( \delta*/\theta \), where \( \delta^* \) is the displacement thickness.

Figure 2.3 shows a typical mean velocity profile measured over the wavy wall with a pitot-static tube, where \( z_T \) is measured from the tops of the roughness elements. This profile was also verified with stationary hot-wire measurements. The odd shape of the mean velocity profiles meant that the usual log-law fit to determine the local skin friction coefficient \( C_f' \) could not be used and the von Kármán momentum integral equation had to be relied upon to obtain an estimate of \( C_f' \). This odd shape of the profile may have been due to the large value of \( h/\delta \) and hence lack of sufficient development length. Here \( \delta \) is the boundary layer thickness.

2.1.2 Hot-Wire Measurements

Hot-wire measurements were taken both with the flying X-wire and a stationary X-wire. By using the flying X-wire, a field of mean \( \bar{U} \) and \( \bar{W} \) velocity pairs were obtained, and from these, a set of 1100 mean velocity profiles spaced 2mm apart over a 2.2 metre section of the wavy surface could be obtained. Convergence of the mean flow data required approximately 600 sled passes. With such a large number of data points, the derivatives of the streamwise quantities could be determined more accurately than with a few discrete points. Reynolds shear stress profiles were obtained with a stationary X-wire at 3 stations. An
attempt was also made to obtain a set of Reynolds shear stress profiles by flying the X-wire. This attempt was unsuccessful as too many sled passes (about 24,000-30,000) were necessary for convergence of $-\bar{u}'w'$ because single position averaging was being employed. This difficulty was overcome in later work of this thesis. $\theta$ and $\delta^*$ were computed using the flying X-wire mean velocity profiles and from the plot of $\theta$ vs $x$, $(u_1 vs x)$, $((H+2)\theta/U_1)du_1/dx$ could be calculated and $C_f'$ obtained.

Figures 2.4(a) and (b) show plots of $\theta$ vs $x$ and $\delta^*$ vs $x$ respectively over a section of the sampling length. The value of $d\theta/dx$ was determined by smoothing the $\theta$ vs $x$ plot and then applying a polynomial curve fit to the smoothed data.

With the set of mean velocity profiles obtained, it was possible to investigate a theory proposed by Perry, Schofield, and Joubert (1969), which anticipated a fall off in the Reynolds shear stress close to a periodic array of two-dimensional roughness elements due to transport of momentum by a "stationary wave". This fall off has also been observed by Mulhearn (1977). A simple two-dimensional analysis into the mechanism of the stationary wave can be made using a momentum balance and this is shown in detail in Appendix 1. The shear stress at the wall, $\tau_0'$, is given by

$$\tau_0' = -\rho \frac{\bar{u}'(t,x)w'(t,x)}{\bar{u}(x)w(x)}$$

(2.2)

where the quantities are defined in figure 2.1(b). The symbols $<\xi>$ denote that the quantity $\xi$ has been spatially averaged over one roughness wavelength. For the rough surface tested, it was found that the first
Figure 2.4
(a) $\theta$ vs $x$
(b) $\delta^*$ vs $x$
term in equation 2.2 was invariant with $x$ for the smallest values of $z$ attainable with the X-wire probe. Therefore

$$
\tau_0 = -\rho \langle u'(t)w'(t) \rangle - \rho \langle u(x)w(x) \rangle
$$

(2.3)

The first term is the Reynolds shear stress as would be seen by a stationary X-wire probe and the second term represents the contribution to the momentum transport by the stationary wave above the elements. Close to the wall, the sum of the two terms should be constant in zero pressure gradient boundary layers. This would explain why the first term in equation 2.3 drops off as the wall is approached since it is expected that the second term would increase. Figure 2.5 shows a typical Reynolds shear stress profile measured over the wavy wall with a stationary X-wire. The fall off in the Reynolds shear stress close to the wall is obvious.

Figure 2.7(a) and (b), shows typical plots of $\overline{U}(x)$ and $\overline{W}(x)$ measured using a flying wire at different distances from the wall. It can be seen that $\overline{U}(x)$ and $\overline{W}(x)$ attenuate away from the wall and that they are approximately 90° out of phase. Because of this phase difference between $\overline{U}(x)$ and $\overline{W}(x)$, the spatially averaged quantity $\langle \overline{u(x)}\overline{w(x)} \rangle$ was negligible. Adding this small stationary wave term did not make any difference to the predicted shear stress. (Had $\overline{U}(x)$ and $\overline{W}(x)$ been in phase, however, then the quantity $\langle \overline{u(x)}\overline{w(x)} \rangle$ could have accounted for the fall off in the Reynolds shear stress). The shear stress predicted by the momentum integral equation is also indicated on figure 2.5 and a large discrepancy between the shear stress predicted by these two methods (hot-wire and
Figure 2.5

Typical Reynolds shear stress profile over the wavy wall

Figure 2.6

"Blowing" at the wall caused by secondary flow
Figure 2.7
Plots of $\overline{U}(x)$ and $\overline{W}(x)$ above the wavy wall obtained with the flying hot-wire
(note: origins of $\overline{W}$ have been shifted in (b))
(note also that (b) is on expanded scale)
momentum integral method) is obvious. Secondary flow in the tunnel is shown schematically in figure 2.6 and the presence of secondary flow would greatly complicate the momentum integral equation as derived in Appendix 1. One effect of the secondary flow would be to cause flow to occur in the grooves which would converge towards the plane of symmetry giving an effective suction or blowing across a horizontal plane just above the crests of the elements.

A smaller, geometrically similar version of the wavy wall was tested in the small wind tunnel to investigate the cause of secondary flow. The curve (i) in figure 2.8 shows the transverse mean velocity profile, \( \bar{V}/\bar{U} \) versus \( y \) in this tunnel. It was thought that the difference in drag between the wall with roughness and the other three smooth walls caused this secondary flow. A transverse mean velocity profile was then measured with roughness on all four walls of the working section and the result is shown as curve (ii) in figure 2.8. The reduction of secondary flow can be clearly seen. However, a transverse mean velocity profile in the large wind tunnel with the wavy wall in place showed that the secondary flow was small in this case (figure 2.9). In view of the fact that secondary flows can occur naturally, it is advisable to check for secondary flows whenever taking boundary layer measurements, particularly on rough walls with lateral strip type roughness.

Because the log-law could not be used to obtain \( C_f' \), it was felt that this large discrepancy may have been due to inaccuracies in the application of the von Kármán momentum integral equation. The von Kármán
Figure 2.8
Secondary flow in the small wind tunnel over small wavy wall

Figure 2.9
Transverse mean velocity profile, wavy wall, large wind tunnel
momentum integral equation is known to be highly sensitive to any three dimensionality of the flow and also has the undesirable feature of requiring the values of streamwise derivatives of various mean flow quantities. Errors in the determination of these derivatives would lead to a corresponding error in $C_f'$. The wavy wall case studied had an unusually large $h/\delta$. This large value was used so as to expand the region where the stationary wave might occur. Most measurements in the past on rough walls have been confined to small $h/\delta$. It was therefore decided to repeat this series of measurements using a "mesh type" roughness shown in figure 2.10 where $h/\delta$ was small. Here at least, it is expected that if an appropriate $\varepsilon$ (the "error in origin", figure 2.11) can be found from which to locate the effective origin for $z$, then the Hama velocity-defect law is applicable. This law is also valid on smooth walls. A scheme similar to that used by Perry and Joubert (1963), where by iteration $C_f'$ and $\varepsilon$ can be determined, was then developed, giving an independent means for determining $C_f'$. The details of this method are given below.

21.3 Hama Velocity-Defect Law Method Of Determining $C_f'$

The Hama (1954) velocity-defect law is given by

$$\frac{U_1 - U}{U_\tau} = f(\eta) \quad (2.4)$$

where $U_1$ is the free stream velocity, $U_\tau$ is the wall shear velocity
Figure 2.10
The "mesh" roughness

Figure 2.11
Definition of the "error in origin", $\varepsilon$
and \( f(n) \) is defined by

\[
f(n) = \begin{cases} 
-\frac{1}{.41} \ln(n) + 2.309 & \text{if } n < 0.15 \\
9.6(1-n)^2 & \text{if } 0.15 < n < 1 
\end{cases}
\]

where \( \eta = \frac{z}{\delta_H}, \delta_H = \frac{\delta z}{C_1} \) and \( z = \sqrt{\frac{2}{C_f'}} \)

For a zero pressure gradient boundary layer (Clauser 1954), \( C_1 = 3.3715 \).

Figure 2.12 shows a typical mean velocity profile measured over the mesh type roughness plotted as \( \bar{U}/U_1 \) vs \( \log(z_T + \varepsilon) \). As mentioned earlier, unlike the smooth wall case, an additional parameter \( \varepsilon \) (see figure 2.11) must be determined before \( C_f' \) can be found. Unfortunately, the slope of the log-law line, (which gives \( C_f' \)) is sensitive to the value of \( \varepsilon \) used. In the method described here, \( \varepsilon \) is judged to have to correct value when the mean velocity profile, plotted as \( \bar{U}/U_1 \) vs \( \log(z_T + \varepsilon) \) best fits the form defined by the Hama velocity defect law. This is done as follows.

An approximate value of \( \varepsilon \) is chosen and the mean velocity profile is plotted as \( \bar{U}/U_1 \) vs \( \log(z_T + \varepsilon) \). The slope of the log-law line is

\[ \frac{1}{\kappa} \sqrt{(C_f'/2)} \]

where \( \kappa \) is the so called von Kármán mixing constant (=0.41), and for \( \bar{U}/U_1 \) vs \( \log_{10}(z_T + \varepsilon) \) the slope is generally given by

\[ 5.616 \log_{10}(C_f'/2) \]

as shown in figure 2.12. The maximum deviation of the wake from the log-line is \( L/(C_f'/2) \), where \( L \) is a universal constant equal to 2.56 for zero pressure gradient boundary layers according to the Hama velocity defect law formulation. From the slope of the log-line, an initial
Figure 2.12
Typical mean velocity profile over the mesh roughness.
estimate of \(\sqrt{C_f'/2}\) and \(L\) can be obtained. Depending on this estimated value of \(L\), (whether it is larger or smaller than 2.56), another value of \(\varepsilon\) can be chosen and the process repeated. Figure 2.13 shows the mean velocity profile in figure 2.12 plotted for various values of \(\varepsilon\), and figures 2.14(a) and (b) show plots of the resulting values of \(L\) and \(\sqrt{C_f'/2}\) respectively versus \(\varepsilon\). The correct value of \(\varepsilon\) occurs at \(L=2.56\) and thus the correct value of \(\sqrt{C_f'/2}\) can also be found.

### 2.2 THE MESH ROUGHNESS

The mesh type roughness was installed on the floor of the large wind tunnel and its shape and geometry is shown in figure 2.10. A zero pressure gradient was set and mean velocity profiles were measured with a pitot-static tube at three streamwise stations (\(x = 700\text{mm}, 1100\text{mm}, 2500\text{mm}\)) at a nominal free stream velocity, \(U_1\), of 10 m/s, and at 7 streamwise stations (\(x = 500\text{mm}, 600\text{mm}, 1100\text{mm}, 1500\text{mm}, 1900\text{mm}, 2500\text{mm}, 3000\text{mm}\)) along the roughness at a nominal free stream velocity of 20 m/s. Reynolds shear stress profiles were measured at the \(x=600\text{mm}, x=1900\text{mm}\) and \(x=2500\text{mm}\) stations for each nominal velocity.

Figure 2.12 shows a mean velocity profile measured at \(x=1900\text{mm}\) over the mesh roughness, plotted as \(\bar{U}/U_1\) vs \(\log (z_T+\varepsilon)\). \(C_f'\) and \(\varepsilon\) were determined using the Hama velocity defect method. The values of \(C_f'\) determined by this method agreed fairly well with the values obtained using the momentum integral equation. A comparison of \(C_f'\) values obtained by the two methods is shown in figure 2.15. Secondary flow was checked and found to be negligible (see Chapter 5).
Figure 2.13

Mean velocity profiles over mesh roughness with different values of $\varepsilon$
Figure 2.14
Obtaining $C_f'/2$ using the Hama velocity defect law method
Figure 2.16 shows a Reynolds shear stress profile measured at $x=1900\text{mm}$ above the mesh-type roughness. The fall off in $-u'w'$ as the wall is approached is still obvious. The shear stress predicted by the Hama velocity defect law is also shown. It was then thought that the fall off may be due to inadequate spatial resolution of the X-wires, i.e. the length of the etched portions of the wires and their distance apart was too large to adequately resolve the finer scale eddies that exist close to the rough surface. The etched portions of the X-wires were typically $1.2\text{mm}$ long and the two wires were approximately $1.0\text{mm}$ apart. Another set of X-wires was constructed where the etched portions were $0.5\text{mm}$ long and the distance between the two wires was $0.5\text{mm}$. The same profile was obtained with these wires as with the first set and this seems to indicate that inadequate spatial resolution is not the cause of the fall off in the Reynolds shear stress close to the rough surface.

2.3 HOT WIRE TECHNIQUE INVESTIGATION

From a comparison of the $C_f'$ values over the mesh roughness obtained by the Hama (1954) velocity defect method and the Reynolds shear stress profile, it was obvious that either one or both of these methods was not giving the correct values. The universality of the Hama velocity defect law has yet to be verified by independent means and was also believed to be sensitive to the previous 'history' of the shear layer. This was shown by Coles for smooth walls and figure 2.17 which is from Coles (1962) shows this. The quantity $\Delta U/U_\tau$ shown in figure 2.17 is not the roughness function mentioned earlier but is related to $L$, the departure of the wake from the logarithmic profile. Also, mathematical predictions by Perry (1968) did not show this fall off. These mathematical predictions were based on the
Figure 2.15 Comparison of $C_f'/2$ using different methods

Figure 2.16 Typical Reynolds shear stress profile over mesh roughness
Hama velocity-defect law, the usual rough wall laws, the momentum equation and the momentum integral equation. Thus the fall off in the Reynolds shear stress near the rough surface in a zero pressure gradient boundary layer also casts doubts on results obtained using hot-wires. An investigation was then initiated into the behaviour of X-wires, and the calibration and sampling techniques.

2.3.1 Check on Calibration and Sampling

The dynamic calibrator enabled the probe to be shaken with a known velocity perturbation at any angle, $\alpha$, to the flow (figure 2.18). By shaking the probe at an angle $\alpha$ to the flow in the free stream, a perturbation in velocities $u'$ and $w'$ could be artificially imposed on the X-wires and these perturbations can be accurately determined as the stroke of the calibrator and the frequency of the shaking are known. The aim of this exercise was to compare the calculated values of $u'^2$, $w'^2$, and $-u'w'$ with the values obtained from the hot-wires. The values obtained by the two methods agreed very well, the error being of the order of one percent. This discrepancy was attributed to the calibrator speed varying slightly over the long sampling time. Therefore, within the limited conditions of this test, the X-wires were giving the correct values indicating that the calibration and sampling techniques were operating correctly.
Fig. 17—Approach to Equilibrium in Terms of the Wake Component

Figure 2.17
From Coles, 1962.

Figure 2.18
Direction of shaking of probe in calibrator
2.3.2 Time Delay in the Analogue to Digital Converter

For on-line sampling, a PDP 11/10 computer with LPS system is used. The A-D converter in the LPS samples sequentially. Thus, if two channels are being sampled, the second channel is sampled approximately 40 μs after the first. It had been noticed that the correlation $\phi$, where

$$\phi = \frac{-u'w'}{\sqrt{u'^2 \cdot w'^2}}$$

fell off close to the wall instead of asymptoting to a constant value as the wall is approached, as would be expected (e.g., see Klebanoff 1954). It was suspected that the time delay between the sampling of $E_u$ on channel "0" and $E_w$ on channel "1" of the A-D converter may be the cause of the fall off in Reynolds shear stress close to the wall. It was then proposed to use a pair of Krohn-Hite filters to delay one of the sampled voltages to compensate for the time delay in the A-D converter.

In order to determine the correct filter setting to achieve the required time delay between channels, a sine wave was fed simultaneously into Channel 0 and Channel 1 of the A-D converter and the voltages were sampled using the same sampling program used to sample the hot-wire signals. The mean square of the input signals to channel 0 and channel 1, $E_0^2$ and $E_1^2$, and their correlation $E_0 E_1$ were calculated. As the voltages going into channel 0 and channel 1 were in phase, any fall off
in $E_0/E_1$ would show up as a fall off in $\phi$, where

$$\phi = \frac{E_0 E_1}{\sqrt{E_0^2 + E_1^2}}$$

(2.7)

and where $\phi=1$ for zero time delay. For each set of trial filter settings, plots of $\phi$ vs $f$ were obtained, where $f$ is frequency of the input sine wave. This is shown in figure 2.19. As can be seen, the effect of the time delay becomes significant as the frequency increases. From the filter settings tried, low-pass settings of 15 kHz and 7 kHz for channels 0 and 1 respectively were chosen as the most appropriate. The X-wires were then re-calibrated and the Reynolds shear stress profile measured with and without the time delay in the line but the same profile was obtained in both cases. This seemed to indicate that most of the energy contributing to the Reynolds shear stress is contained in frequencies low enough for the time delay in the A-D converter to have no effect.

2.4 THE SMOOTH WALL

It was then decided to repeat the experiment on a smooth wall as the Clauser (1954) chart could be used as another independent method of determining the wall skin friction. The Reynolds shear stress on a smooth wall obtained using X-wires gave the same value of $C_f'$ as determined from the mean velocity profiles on a Clauser chart and the shape of the Reynolds shear stress profile was also correct as seen in figure 2.20.
Filter settings (Hz)
- Ch'0' Ch'1'
+ 10k 10k
• 10k 7k
• 15k 7k

Figure 2.19
Effect of time delay

Figure 2.20
Reynolds shear stress profile over smooth wall
Having established that $C_f'$ could be obtained correctly with a X-wire over the smooth wall, the mesh roughness experiment was repeated with an extra check as follows. A smooth plate was constructed to cover the rough surface and a mean velocity profile was obtained at one station over this smooth plate. $C_f'$ at this station was determined both with the Clauser chart and with a X-wire, and agreement was obtained. The smooth plate was then quickly removed, uncovering the mesh roughness and a Reynolds stress profile measured. The same "incorrect" profile was obtained. As these X-wires had just given the correct Reynolds shear stress profile over a smooth wall, it was suspected that for some reason the X-wires were not able to determine the correct values of $-\bar{u}'\bar{w}'$ over a rough surface.

2.5 RESOLUTION OF THE MYSTERY

In view of the above, a reason was sought for the misbehaviour of the X-wires in measuring the Reynolds shear stress above a rough surface. Tutu and Chevray (1975), Willmarth and Bogar (1977) and Kawal, Shokr and Keffer (1983) have shown that the use of X-wires in regions of very high turbulence intensity can result in large errors in measurement. Tutu and Chevray showed this by turbulence measurements in a turbulent jet and Willmarth and Bogar by turbulence measurements very close to a smooth wall. The reason for these large errors was due to the large angles made by the velocity vectors as they approached the X-wires. One of the major differences between flow over a smooth surface and that over a rough surface is that for a given $\delta_u U_\tau /v$, where $\delta_u$ is the Hama boundary layer
thickness as defined in Section 2.1.3 and \( \nu \) is the kinematic viscosity,
the mean velocity at a given non-dimensional distance from the wall \( z/\delta_H \)
has a much lower value on a rough surface. In fact,

\[
\frac{U}{U_R} = \frac{U}{U_S} - \frac{\Delta U}{U_t} \tag{2.8}
\]

where \( \Delta U/U_t \) is the Hama (1954) roughness function; the suffix \( R \)
signifies a rough surface and the suffix \( S \) signifies a smooth surface. It
will be seen later that a number of models for wall turbulence predict
that \( w'^2/U_t^2 = f(z/\delta_H) \), where for large \( \delta_H U_t/\nu \), \( f \) is a universal
function. Hence for a fixed \( z/\delta_H \), \( w'^2 \) scales with \( U_t^2 \), and since \( U/U_t \)
for a rough wall is less than that for a smooth wall, it stands to reason
that the instantaneous velocity vectors approaching the X-wires in the
\( x-z \) plane will be undergoing a larger change in angle over the rough
wall. One could imagine that all velocity vectors are contained in an
elliptical cone and the cone angle experienced by the X-wires above a
rough surface will exceed that on a smooth surface.

Figure 2.21 defines \( \theta \), the cone angle of the velocity vector
measured in the \( x-z \) plane and \( \beta \), the included angle of the X-wires.

It is obvious that if \( \theta \) approached or exceeded \( \beta \), then any
measurements taken with such a set of X-wires would be completely
erroneous. To investigate this phenomenon, a computer program was
written to sample \( u-w \) velocity pairs. Figures 2.23(a) and (b) show two
such plots for the \( \beta=90^\circ \) X-wires at different values of \( z \) over the mesh
roughness with \( U_1 = 10 \text{ m/s} \). The data in these plots show only velocity
vectors "seen" by the X-wires and it is possible that velocity vectors approaching the X-wires at larger angles exist but are not seen by the wires. As can be seen from the figures, θ did not even get close to the critical value of θ=90°.

At the same time, a computer simulation of the velocity vectors seen by a mathematically modelled set of X-wires was carried out. The Λ-vortex model of Perry and Chong (1982) was used to generate the velocities seen by the modelled X-wire. This was done by assuming that the eddies above a rough surface follow the same scaling laws as those above a smooth surface, except that the scale of the smallest eddy now scales with the scale of the roughness instead of the Kline scaling. It was also assumed that the rough wall eddies have the same geometry as the smooth wall eddies and their geometry is shown later in figure 3.1. At a distance z above a boundary, eddies of scale z contribute mainly to the Reynolds shear stress at that point. Assuming that their characteristic velocity scales with $U_\tau$, the parameters were adjusted such that $-\frac{w'}{U_\tau} = 0(1)^\dagger$. By using the Biot-Savart law for an isolated Λ-shaped vortex, some likely velocity signatures for the velocity components a X-wire might experience close to a boundary (at half the eddy height) were computed. These computations were carried out for different values of $\overline{U}/U_\tau$. The X-wires were modelled according to the Champagne (1967) cooling law and the model simulated the X-wires, anemometers and calibration scheme. In spite of the fact that the calibration scheme took into account the X-wire non-linearities (see Perry, 1982 pp. 122-127), significant errors occurred in the inferred velocities observed by the modelled X-wires under certain flow conditions.

$^\dagger$O(ξ) means "of order ξ" throughout this thesis
Figure 2.21
Definition of wire angle $\beta$, and cone angle $\theta$

Figure 2.22
Expected error in measured Reynolds shear stress at different $\overline{U}/U_T$ (simulation)
Figure 2.23
Velocity vectors "seen" by a $\beta=90^\circ$ probe

$U_1 = 10\text{m/s}$

(a) $z=3\text{mm}$

(b) $z=8\text{mm}$
Figure 2.22 shows the expected error in Reynolds shear stress at different values of $\overline{U}/U_T$ for $\beta=90^\circ$ and $\beta=120^\circ$ X-wires according to the computer simulation. In these calculations, the effect of $v'$, the lateral velocity fluctuation, was also observed to be significant and the effect of $v'$ on the measured Reynolds stress is also shown.

The simulation described above should be equally valid for flow over smooth surfaces and some simplifying assumptions in the simulation should be pointed out. Only one eddy scale was used in the simulation and figure 2.22 is only meant to show trends. If more eddy scales are included, then the predicted error in the measuring of Reynolds stress would even be larger. From the figure it can be seen that for $\overline{U}/U_T=4$ (the mesh roughness), the $\beta=90^\circ$ X-wires give an error in the Reynolds shear stress of approximately 30% even though this was not expected from the indicated experimental cone angles of approximately $\theta=60^\circ$ (See figures 2.23(a) and (b)). Figure 2.22 also shows that with $\beta=120^\circ$ X-wires, the error is significantly reduced.

A set of $\beta=120^\circ$ X-wires was then constructed and the Reynolds shear stress above the mesh roughness was measured again. Figure 2.24 shows the Reynolds stress profiles measured above the mesh roughness with the $\beta=90^\circ$ and $\beta=120^\circ$ X-wires. The difference in the measured Reynolds shear stress using the two different sets of X-wires is obvious. Figure 2.25(a) and (b) show the velocity vector cone angles seen by the $\beta=120^\circ$ X-wires under the same conditions as in figures 2.23(a) and (b). As can be seen, the velocity vector cone angle $\theta$ recorded by the X-wires is up to $30^\circ$ larger in this case. However, given this, $\theta$ was still less than
Figure 2.24
Reynolds shear stresses measured with $\beta=90^\circ$ and $\beta=120^\circ$ X-wires
Figure 2.25
Velocity vectors "seen" by a $\beta = 120^\circ$ probe

$U_1 = 10\text{m/s}$

(a) $z = 3\text{mm}$

(b) $z = 8\text{mm}$
Thus, the limiting value of the allowable cone angle $\theta$ is not $\beta$, the included angle of the X-wires, but a value somewhat lower. As can be seen in figure 2.24, the measured Reynolds shear stress still falls short of the Reynolds shear stress predicted by the Hama (1954) velocity defect law and the discrepancy encouraged further investigation into X-wire behaviour.

A simple test was performed by tilting the X-wire probe at different angles in the x-z plane and obtaining plots of $E_U$ vs $\bar{w}$ and $E_W$ vs $\bar{w}$ with $\bar{U}$ kept constant. This was done by tilting the dynamically matched X-wires at different angles to the horizontal in the free stream and at the same time adjusting the tunnel speed such that the resultant mean velocity $\bar{U}$ along the probe axis remained constant at each angle of tilt. The resulting value of $\bar{W}$ could be calculated at each angle of tilt and the results are shown in figures 2.26(a) and (b). It can be seen that the $\beta=120^\circ$ X-wires exhibit linear behaviour up to larger velocity vector cone angles, supporting the trends shown by previous work. This linear behaviour between $E_W$ and $\bar{W}$ is in accordance with the calibration scheme used (see section 1.7). Note the similarity with the predicted X-wire behaviour according to the simulation described earlier (figures 2.27(a) and (b)).

A dynamic test was also performed by shaking the X-wire probe to impose an artificial Reynolds shear stress on the X-wires. This is similar to the test described earlier, but instead of shaking at $45^\circ$ to the flow, the probe was shaken at $70^\circ$ to the flow. By shaking the probe at different frequencies and at different free stream velocities, large, known, velocity vector cone angles could be imposed on the X-wires. At
Figure 2.26 (a), (b)

Effect of tilting the probe in the free stream.
Figure 2.27 (a), (b)

Computer simulation of the X-probe response to W
each measuring point, the calculated and measured Reynolds shear stress could be compared. Figure 2.28 shows the breakdown in the measured Reynolds stress at increasingly large values of cone angle. The improvement in behaviour of the $\beta=120^\circ$ X-wires over the $\beta=90^\circ$ X-wires is obvious. Also note that the $\beta=90^\circ$ X-wires start to deviate from correct behaviour at $\beta=20^\circ$ and that the $\beta=120^\circ$ X-wires deviate from ideal behaviour at $\beta=55^\circ$ (although less severely). It is thus conceivable that over the mesh roughness, the maximum allowable cone angle using $\beta=120^\circ$ X-wires was still being exceeded since the measured cone angle was larger than $55^\circ$. In view of the fact that the Reynolds shear stress measured with the $\beta=120^\circ$ X-wires still does not agree with the Hama velocity defect law prediction, this would seem to be the case. It was then decided to attempt to measure the Reynolds shear stress using the flying hot wire. It will be recalled that the earlier attempt to do this over the wavy wall failed, the problem being the excessive number of sled passes required. Flying the X-wires would have the following advantages

1. Reduce the velocity vector cone angles seen by the X-wires by imposing an additional $\overline{U}/U_t$ of about 2.5.
2. The effect of $v'$ would be reduced.
3. The contribution of the stationary wave could be obtained.

To reduce the required number of sled passes for convergence of $-u'w'$, the assumption was made that the Reynolds shear stress variation was linear over a $100\text{mm}$ streamwise distance. By sampling every millimetre along a $100\text{mm}$ sampling length, and taking the average value of $-u'w'$ to be equal to that at the mid-point in the sampling window, it was
Figure 2.28
Breakdown in measured Reynolds shear stress at large cone angles
found that convergence occurred with 300 sled passes (equivalent to 30,000 individual sled passes using the single point sampling method mentioned earlier). Figure 2.29 shows the Reynolds shear stress measured by flying the $\beta=90^\circ$ and $\beta=120^\circ$ X-wires and also earlier profiles measured with stationary wires so a comparison can be made between the different methods. The difference in Reynolds shear stress at the wall can be clearly seen. However, note that the values obtained with the $\beta=120^\circ$ X-wires seem to be independent of bias velocity and this would seem to indicate that the $\beta=120^\circ$ stationary X-wires are not being affected by excessive velocity vector cone angles over the mesh roughness. The effect of $v'$ appears to be insignificant in this case and the effect of the stationary wave was found to be negligible. Figures 2.30(a) and (b) show velocity vector cone angles seen by the $\beta=120^\circ$ X-wires as they are being "flown" and the actual cone angles as would be seen by a stationary X-wire, i.e. with the bias velocity subtracted.

Some cone angle probability density functions (histograms normalised to unit area) have also been obtained and some relevant cases are shown in figures 2.31(a) and (b). A large number of velocity vector cone angles were sorted into $5^\circ$ cone angle "intervals" for the different cases and their frequency of occurrence in these "bands" was counted. The data in figure 2.31(a) correspond to figures 2.30(a) and (b). The $\beta=120^\circ$ wires appear to be giving the correct readings whether they are stationary or flying and figure 2.31(a) shows how the spread in vector cone angle decreases as $\overline{U}/U_\tau$ is increased from 4.0 to 6.5. Figure 2.31(b) shows that the inferred spread in cone angles for the stationary $\beta=90^\circ$ wires is incorrect if the $\beta=120^\circ$ wire results are taken to be correct.
Figure 2.29
Reynolds shear stress profiles (see symbols)
Figure 2.30

Velocity vectors "seen" by a "flying" θ=120° probe

$U_1 = 6\text{m/s}$

Bias (flying velocity) = 2.5m/s

(a) vectors relative to sled

(b) vectors relative to wall, i.e. bias velocity subtracted
Figure 2.31 (a), (b) Cone angle p.d.f's for different cases.
Again, investigation into time delay and spatial resolution showed that these factors had no effect on the measured Reynolds shear stress.

If it is proposed that the $\beta=120^\circ$ X-wires are giving the correct Reynolds shear stress then some other reason has to be found for the discrepancy (15%) with the Hama velocity defect law prediction. It must be said that although the stationary $\beta=120^\circ$ X-wires are adequate in measuring the Reynolds shear stress over this mesh roughness, they may not be adequate over other types of roughness and a check should always be made by flying the X-wires.

A further check was performed by using the $\beta=120^\circ$ X-wires to obtain the Reynolds shear stress over a smooth wall, although the $\beta=90^\circ$ X-wires shear stress agreed with the Clauser chart. The $\beta=120^\circ$ X-wires gave the same Reynolds shear stress profile as the $\beta=90^\circ$ X-wires (figure 2.32), and the cone angle p.d.f's were the same, indicating that the $\beta=90^\circ$ X-wires were adequate for the smooth wall. These are shown in figure 2.33. The figure shows clearly how much narrower the spread in cone angle is over a smooth surface where $U/U_1 > 10$.

The author's supervisor, Professor A.E. Perry, approached Dr. M. Escudier of BBC, Zurich, at a recent turbulent shear flow conference (Karlsruhe 1983) where some of these results were published (Perry, Lim, Henbest and Chong (1983). From discussions, Dr. Escudier decided to repeat the author's stationary wire experiments with the use of an additional independent means of obtaining $C_f'$, i.e. the floating element drag balance. Dr. Escudier used both $\beta=90^\circ$ and $\beta=120^\circ$ X-wires and his results (Acharya
Figure 2.32
Reynolds shear stress profiles, smooth wall - see symbols

Figure 2.33 Cone angle p.d.f.'s - smooth wall
and Escudier, 1984) verified the findings stated here, i.e. that the $\beta = 120^\circ$ X-wire Reynolds shear stresses are correct (agreement with the drag balance). He also found the $\beta = 90^\circ$ X-wires to be in error and that Hama velocity defect law fit to the data gave a value of $C_f'$ about 20% too high.

It goes without saying that the X-wires should always be inspected for bowing, preferably with the X-wire anemometers switched on. This is because wires that appear straight "cold" have been observed to bow markedly when they are "hot". Another precaution that should be taken, especially in measurements over smooth walls (as the shearing rates in $U$ are very high close to the surface) is that the "X" formed by the two hot wires is centred. The Reynolds shear stress profile obtained using non-centred X-wires is also shown in figure 2.32 as crosses. It can be seen that the error made by the non-centred X-wires is large close to the surface but reduces as $z$ increases and the shearing rate in mean velocity is reduced.

2.6 THE ROLL ANGLE PROBLEM

Another problem that came to light during these investigations is the roll angle problem. Because the plane of the hot wires is not exactly at right angles to the $v$ direction, the velocities seen by the X-wires can be contaminated by $v'$ fluctuations, which can be relatively large in a turbulent boundary layer. Fortunately this does not appear to affect $-u'w'$ significantly as $-u'w'$ and $v'$ are uncorrelated.

Unfortunately, $w'^2$ measurements can be significantly contaminated by $v'$
fluctuations. As the X-wires can theoretically never be aligned to be perfectly perpendicular to the V direction, this problem will always be present to some extent. The only thing that can be done is to try to minimise this effect, by careful construction, inspection and alignment of the X-wires. The measured normal turbulence intensity \( \frac{v'^{2}}{U_{\tau}^{2}} \) over a smooth wall (Chapter 4), had a value of order 1.5 close to the wall and increased as the wall was approached. It had also been noticed that \( \frac{w'^{2}}{U_{\tau}^{2}} \) close to the wall measured with apparently similar X-probes differed inexplicably. A survey was conducted of the published data on normal turbulence intensities and it was found that the value of \( \frac{w'^{2}}{U_{\tau}^{2}} \) close to the wall was generally between 1.0 and 1.3, although slightly lower and much higher values are also recorded. A tabulation of this survey is presented in Table 4.2. At the time, it was felt that the value of 1.5 was high, possibly due the fact that the normal turbulence intensities measured by the author were being contaminated by lateral velocity fluctuations. It was thought that the reason for this was that the X-wires were sensitive to velocity fluctuations in the V direction when measuring in the W direction, possibly due to "rolling" of the X-wires about a longitudinal axis. The fact that the normal turbulence intensities increased as the wall was approached seems to support this theory, as \( v'^{2} \) increases towards the wall whereas \( w'^{2} \) should become constant or fall off due to the effects of viscosity close to the wall according to the theories of Townsend (1976), and Perry and Chong, (1982). A series of tests was performed to determine the cross sensitivity of the X-wires to fluctuations in the V direction when the X-wires were rolled.
If the X-wires used to measure $w'$ fluctuations are sensitive to lateral velocity fluctuations $v'$, then $e_w$, the output voltage of the hot-wire for $w'$-fluctuations can be expressed as

$$e_w = K_w w' + K_{v2} v'$$  \hspace{1cm} (2.9)

where $K_w$ is the sensitivity of the wires to $w'$ fluctuations and $K_{v2}$ is the sensitivity of $e_w$ to $v'$ fluctuations.

If the inferred normal velocity $w_I'$ is given by $w_I' = e_w / K_w$ then

$$w_I' = w' + \frac{K_{v2}}{K_w} v'$$ \hspace{1cm} (2.10)

and

$$\frac{\overline{w_I'^2}}{\overline{w_w^2}} = 1 + \frac{K_{v2}}{K_w} \frac{\overline{v'^2}}{\overline{w'^2}}$$ \hspace{1cm} (2.11)

It can be seen that any contribution from $v'$ fluctuations adds to the magnitude of $\overline{w_I'^2}$.

A rough calculation can be made to show the magnitudes of the errors in $\overline{w_w^2}$ due to different values of $K_{v2}/K_w$, using rough estimates of $\overline{w_I'^2}/U_w^2 = 1.0$ and $\overline{v'^2}/U_w^2 = 3.0$. For this equation 2.11 becomes

$$\frac{\overline{w_I'^2}}{\overline{w_w^2}} - 1 = 3 \left[ \frac{K_{v2}}{K_w} \right]^2$$ \hspace{1cm} (2.12)

The result is plotted in figure 2.34.
It was then decided to measure $w'^2$ and $-u'w'$ over the smooth wall at $x=3000\text{mm}$ and at a free stream velocity of 30 m/s, and to measure the sensitivity ratio of the X-wires used by shaking the wires to induce a known $v'$ fluctuation. This was done by first calibrating the X-wires in the $U-W$ plane and then rolling the probe through $90^\circ$ about its axis and shaking normal to the plane of the X-wires. The sensitivity of $e_w$ to $v'$ fluctuations will then be known and can then be compared with the sensitivity to $w'$ fluctuations. Knowing $K_{v'/w'}$, it was hoped that by referring to the plot of figure 2.34, the values of $w'^2$ could be corrected. The result of this test was that $K_{v'/w'}$ was always of order 0.1 and from the figure, the expected error in $w'^2$ would be 3.0%. This did not explain why the values of $w'^2/u'^2$ measured with apparently similar probes varied by up to 20%.

The unfortunate thing about the shaking test is that all motions are of a large scale. Close to a boundary, the scale of the motions are of the scale of the wires, which may give spurious results for $w'^2$ due to wire bowing, quite apart from the spatial resolution problem. Two other tests were performed. One was to shake the X-wires in the $u-v$ plane and to roll the probe to artificially introduce a roll angle, and the other was to measure $w'^2$ and $-u'w'$ in the boundary layer with the probe at a known roll angle. It was found that the X-wires were fairly insensitive to roll misalignment (figure 2.35(a),(b)). Microscopic inspection of the X-wires and probe showed that there was up to $4^\circ$ error in roll alignment of the X-wire filaments used in the measurements, and from the figure, this would only cause a small error in $w'^2/u'^2$. Measuring the fluctuating quantities in the boundary layer with this particular probe
Figure 2.34
Effect of cross-sensitivity on $\omega_1^2$.

Figure 2.35 (a)
Effect of probe roll on measured $\omega_1^2$ near the wall.

Figure 2.35 (b)
Effect of probe roll on measured Reynolds shear stress near the wall.
gave a value of $\frac{\overline{w'^2}}{U_1^2} = 1.2$, but interestingly, $\frac{-u'w'/U_1^2}$ close to the wall had a value of exactly 1.0, as with the X-probe which gave $\frac{\overline{w'^2}}{U_1^2} = 1.5$, seeming to confirm the idea that the Reynolds shear stresses are uncorrelated with the lateral fluctuations and are thus unaffected by any cross sensitivity.

Consider the case where $e_u$, the voltage output of the hot-wire for $u'$-fluctuations is also sensitive to $v'$-fluctuations. A similar equation to equation 2.10 can then be formulated, ie,

$$u_1' = u' + \frac{K_{vl}}{K_u} v'$$  \hspace{1cm} (2.13)

where $K_{vl}$ is the sensitivity of $e_u$ to $v'$-fluctuations. The inferred Reynolds shear stress $\overline{-u_1'w_1'}$ will then be given by

$$\overline{-u_1'w_1'} = -\overline{u'w'} + R_2 \overline{v'w'} + R_1 \overline{u'v'} + R_1 R_2 \overline{v'^2}$$  \hspace{1cm} (2.14)

where $R_1 = \frac{K_{vl}}{K_w}$ and $R_2 = \frac{K_{v2}}{K_w}$

As the shear stress in the $V-W$ and $U-V$ planes is expected to be zero, then,

$$\overline{-u_1'w_1'} = -\overline{u'w'} + R_1 R_2 \overline{v'^2}$$  \hspace{1cm} (2.15)

The value of $R_2$ has been found to be of order 0.1 and $R_1$ was even smaller, hence it can be seen that the Reynolds shear stress is relatively insensitive to $v'$-fluctuations.
It is conceivable that the $v'^2$ measurements are also being contaminated by $w'$-fluctuations. However, as $\frac{v'^2}{U^2}$ is much larger than $\frac{w'^2}{U^2}$, the errors in $\frac{v'^2}{U^2}$ caused by $w'$-contamination would be much smaller in proportion to the errors in $\frac{w'^2}{U^2}$ by $v'$-contamination.

All sets of X-wires used in the tests were examined under the microscope with current flowing and any that exhibited more than very slight bowing were discarded. It is inevitable that some bowing will be present due to filament expansion (even if they were straight when cold) especially as care was taken not to tension the filaments when the X-wires were constructed to avoid introducing spurious strain gauging signals. The difficulty with bowing is that it is unpredictable and the bow may change direction or position during use or the filament may even whirl (Perry and Morrison 1971(c)). Careful examination of a filament judged to be only "slightly" bowed showed that local deviations of the filament from the ideal direction could be quite large (eg. 20°). Although a bowed X-wire filament may show only slight cross sensitivity in the shaking test, when the velocity fluctuation length scales are of the order of the wire filament length scale, the apparent roll angle and cross sensitivity is much greater, and this effect will only show up when measuring the boundary layer close to the wall. The conclusion reached was that roll misalignment of the X-wire probe itself was insignificant compared with the "apparent roll" due to filament bowing when the X-wires are exposed to velocity fluctuations that are of the scale of the length of the filament. According to the attached eddy hypothesis of Townsend (1976) and the theory of Perry and Chong (1982), only eddies of scale of order $\frac{z}{U}$ contribute to $w'^2$ at a distance $z$ from the wall. As the wall is
approached, the eddies that contribute to $w'^2$ are those of increasingly finer scale and thus the local properties of the hot-wire filament become important. Some evidence of this can be seen in figure 2.36, where $w'^2/\bar{u}_t^2$ profiles measured with three apparently similar sets of X-wires are shown. The profiles do not agree close to the wall but seem to collapse onto one another for $z/\delta_H>0.3$. Therefore although care should always be taken in aligning the probe relative to the measuring surface, it is much more important that the alignment of the filaments themselves is checked, and the filaments examined carefully for bowing.

It should be pointed out that the reasons given here for the misbehaviour of the X-wires when measuring $w'$ fluctuations is still highly conjectural at the moment and further work is required to confirm this hypothesis. However, the "circumstantial evidence" presented here does seem to lend some support to this hypothesis.

2.7 SPATIAL RESOLUTION OF HOT-WIRES

It has been shown by Wyngaard (1968) that the highest wave number resolvable by a hot-wire filament of length $l$ is $k_1=O(l/l)$. Thus if the scale of the fine scale motions of the flow is less than $O(l)$, these motions will not be recorded by the hot-wire.
In X-wire probes, the hot-wire filaments are a finite distance $\delta_x$ apart. The hot-wire signal sensitive to $w'$-motions depends both on $E_0$ and $E_1$ (Section 1.7). Thus if the fine scale motions are of the order of $\delta_x$ or smaller, the two hot-wires in the X-probe are likely to be seeing two different features of the flow. Quite apart from the errors caused by the hot-wires not being able to resolve the fine scale turbulent motions of the flow, another form of spatial resolution problem is encountered when the X-probe is rolled to measure $v'$-fluctuations. The two filaments are then at different $z$ distances from the wall and each filament is therefore at a different operating point. This problem becomes much more serious close to the wall, where the shearing rates in velocity are high. This is especially the case close to a smooth surface, where the velocity shearing rates are higher than over a rough surface at the same $z/\delta_H'$. 
Figure 2.36 Normal turbulence intensities measured with apparently similar probes
CHAPTER 3

Recent Developments in the Modelling of Wall Turbulence

INTRODUCTION

The experimental data in this thesis are analysed and presented in the classical manner as well as in the light of Perry and Chong's (1982) model of wall turbulence, which was itself based in part on Townsend's (1976) attached eddy hypothesis. Initially the model of Perry and Chong was developed for zero pressure gradient boundary layer flows (and pipe or duct flows) and was only applicable in a region close to the wall where the Reynolds shear stress is approximately constant. The model of Perry and Chong (Perry, Henbest and Chong (1984)) has been further developed recently and can now be applied to the entire fully turbulent region. These latest developments are presented here.

3.1 THE A-VORTEX MODEL OF PERRY AND CHONG

Although it is not the author's intention to describe the model of Perry and Chong in detail, a short outline of the history of the development of the model is necessary and is presented here.

† This region will be referred to as the turbulent wall region and will be tentatively defined as \( z/\delta_H < 0.15 \) and \( z_+ > 150 \) for smooth wall flows. \( (z_+ = zU_1/\nu) \)
Perry and Chong (1982) proposed that wall shear flows can be modelled using a "forest" of hairpin or A-shaped vortices. This proposal was based on Townsend's (1976) attached eddy hypothesis and the flow visualisation experiments of Head and Bandyopadhyay (1981) and Perry, Lim and Teh (1981). They proposed that these A-vortices are formed due to an instability in the viscous sublayer which causes sublayer material (which contains cross-stream vorticity) to roll up and lift up from the surface to form a vortex loop. At the completion of roll up this vortex loop has a lateral scale $\lambda_0$ at the wall and a height $h_0$. $\lambda_0$ scales with the Kline scaling, i.e. $\lambda_0 \sim \nu^{2/5} U_{\tau}$, (Kline et al 1967). Throughout this thesis $\sim$ means "scales with".

By applying the Biot-Savart law to an isolated A-vortex and its image in the wall, (see figure 3.1), they showed that the vortex stretches in a "plane-strain-like" manner ie.

$$\lambda h = \lambda_0 h_0$$ (3.1)

where $\lambda$ is the distance between the legs at the base and $h$ is the height of the vortex loop. The vortex also convects itself upstream relative to the surrounding fluid. They went on to show that if a straight vortex rod underwent local uniform axisymmetric stretching, i.e. where a marked length of vortex rod increases uniformly with time, viscous diffusion ultimately dominates the stretching process. Thus the diameter of the vortex rod ultimately increases with time. This, together with the fact that the legs of the vortex are approaching each other, leads to vorticity cancellation and the death of the eddy. Thus the maximum
Figure 3.1
A Λ-vortex and its image in the wall
height of the eddy is limited. In fully developed pipe flow, where $U_t$ is constant, the total population of eddies must be constant, i.e. the rate of eddy death must equal the rate of eddy generation. Perry and Chong ignored the debris of dead eddies and assumed that the fluid surrounding the attached eddies was irrotational in the mean sense, and that the attached eddies do not interact strongly with each other. In order to obtain a logarithmic mean velocity distribution for arbitrarily large $zU_t/\nu$ as $\delta U_t/\nu^{+\infty}$, Perry and Chong found it necessary to introduce the concept of a range of scales of geometrically similar attached eddies or hierarchies where the length scales follow a geometrical progression-like distribution if the velocity scale of all attached eddies is assumed to be fixed. The smallest hierarchy height is assumed to scale with the Kline scaling on a smooth surface and with the roughness scale, $k$, on a rough surface. The largest hierarchy scale is assumed to scale with the boundary layer thickness.

Perry and Chong used the notation $\delta_E$ for the boundary layer thickness (or length scale of the largest eddy) but here $\delta_H$ is used to mean the same thing.

Initially Perry and Chong assumed a discrete distribution of hierarchy length scales, where the length scales doubled from one hierarchy to the next. In this system, the vortex rolls up at the wall, undergoes plane-strain-like stretching until its height is the maximum for the first hierarchy, then either dies by vorticity cancellation or pairs with another similar vortex to form the smallest vortex in the next largest hierarchy, which undergoes stretching, and the process is
repeated. To maintain the geometrical progression like distribution, half the eddies must die and the other half pair. This pairing process is consistent with all hierarchies having the same characteristic velocity scale. On pairing, the length and circulation doubles. In real flows however, it is postulated that randomness about each discrete hierarchy height leads to "smearing" of the hierarchy scales and a continuous inverse power law distribution for the probability density function (p.d.f.) of hierarchy scales results. With this continuous distribution it was found that regardless of the shapes of the eddies and the manner of stretching, provided all hierarchies are geometrically similar and have the same characteristic velocity scale, a logarithmic distribution of mean velocity always results.

Within a single hierarchy, there will exist eddies of various shapes and sizes at different stages of stretching. However for some purposes such an assemblage of eddies will be replaced by a single "representative eddy" shape. Such a representative eddy has a meaning only in the statistical sense and may not bear any resemblance to a single physically realizable eddy. However for some simple case studies to be outlined here, this representative eddy will be taken to represent a single simple realizable vortex. It can be shown by dimensional analysis that the contribution to the mean cross stream vorticity \( \eta' \) from a single hierarchy is given by

\[
\eta' = \frac{U_0}{\delta} f(z/\delta)
\]  

(3.2)

where \( U_0 \) is the velocity scale of a hierarchy of scale \( \delta \). The velocity
scale $U_0$ will be taken to be $U_\tau$ for all hierarchy scales and the function $f(z/\delta)$ will be referred to as the vorticity intensity function. The vorticity intensity function is related to the representative eddy geometry.

Consider a range of hierarchy scales from $\delta_1^+$ to $\delta_H$. Then at a fixed distance $z$ from the wall, it can be shown that the total mean cross-stream vorticity is given by

$$\overline{\eta} = \frac{d\overline{U}}{dz} = \int_{\delta_1}^{\delta_H} \frac{U}{\delta} f(z/\delta) P_H(\delta) \, d\delta$$  \hspace{1cm} (3.3)

Here $P_H(\delta)$ is the p.d.f. of hierarchy scales. For a continuous p.d.f. Townsend (1976) and Perry and Chong (1982) assumed that $P_H(\delta)$ is given by

$$P_H(\delta) = \frac{M}{\delta}$$  \hspace{1cm} (3.4)

where $M$ is a disposable constant.

Thus

$$\overline{\eta} = \frac{d\overline{U}}{dz} = \int_{\delta_1}^{\delta_H} \frac{U}{\delta} f(z/\delta) \frac{M}{\delta} \, d\delta$$  \hspace{1cm} (3.5)

$\delta_1^+$ is the scale of the smallest hierarchy and scales with $\nu/U_\tau$. 

$\delta_1$ is the scale of the smallest hierarchy and scales with $\nu/U_\tau$. 

$\delta_H$ is the scale of the largest hierarchy.
3.1.1 The Hama Velocity Defect Law

An attempt is made here to obtain the form of the Hama velocity defect law using the model presented above. From equation 3.5, the mean vorticity can be expressed as

$$\frac{d\bar{U}}{dz} = \frac{U \cdot \tau}{z} f_0(\lambda) e^{-\lambda} d\lambda$$

where \(\lambda_E = \ln(\delta_H/z)\), \(\lambda_1 = \ln(\delta_1/z)\), \(\lambda = \ln(\delta/z)\)

and \(f_0(\lambda) = f(z/\delta)\)

Integrating equation 3.6 with respect to \(z\) using a suitable change of variable, it can be shown that

$$U_D^* = \frac{U_{1U} - \bar{U}}{\tau} = M \int_0^\lambda E \left[ f_0(\lambda) e^{-\lambda} d\lambda \right]$$

\(U_D^*\) is the mean velocity at the edge of the boundary layer and \(U_D^*\) will be recognised as the non-dimensional velocity defect.

The Hama (1954) velocity defect law for zero pressure gradient turbulent boundary layers, equation 2.5, can be expressed in terms of \(\lambda\) as

$$U_D^* = \frac{1}{\kappa} \lambda_E + c \quad \lambda_E > \ln(1/0.15) \quad (3.8(a))$$

$$U_D^* = K(1 - e^{-\lambda_E})^2 \quad \lambda_E < \ln(1/0.15) \quad (3.8(b))$$

where \(K = 9.6\) and \(c = 2.309\).
Figures 3.2(a), (b) show equations 2.5 and 3.8 respectively for the Hama velocity defect law. Consider a Π-shaped eddy as shown in figure 3.3(a), (b). Its vorticity intensity function \( f(z/\delta) \) is given in figure 3.3(c) and is a Dirac delta function at \( z/\delta = 1.0 \). The corresponding \( f_0(\lambda) \) is also a delta function and is shown in figure 3.3(d).

The method of performing the inner integral of equation 3.7 is shown schematically in figure 3.3(e). Imagine that the integrating window shown, which has limits \( \lambda_1 \) and \( \lambda_1 \), moves from left to right as \( z \) decreases. As the right hand edge of the integrating window passes through the delta function given by \( f_0(\lambda)e^{-\lambda} \) at the origin, the inner integral has a finite value and is constant until \( \lambda_1 \) passes through the origin. The result is shown in figure 3.3(f). Integrating this with respect to \( \lambda_1 \) between the limits of 0 to \( \lambda_1 \) will then give \( U^*_D \). For a Π-eddy \( U^*_D \) is logarithmic for all \( \lambda_1 > 0 \) as shown in figure 3.3(g). It has been assumed that \( \lambda_1 \) is sufficiently negative so that it does not enter the calculation. The value of \( M \) is obtained using the relation

\[
\frac{dU^*_D}{d\lambda_1} = \frac{1}{\kappa} \quad \lambda_1 \gg 0 \quad (3.9)
\]

However this formulation when compared with the Hama velocity defect law has the incorrect intercept at \( \lambda_1 = 0 \) and the wrong form for \( \lambda_1 < \ln(1/0.15) \). Similarly, the same procedure can be performed for a Λ-shaped eddy and the results are shown in figure 3.4(a)-(g). Notice that for \( \lambda_1 \) sufficiently large a logarithmic distribution of \( U^*_D \) results but again has the incorrect intercept.
Figure 3.2 (a),(b) The Hama velocity defect law
Figure 3.3 (a)-(g) Obtaining $U_D^*$ from a II-eddy
Figure 3.4 (a)-(g) Obtaining $U_D^*$ from a A-eddy.
So far, vortex stretching has not been considered. For interest, the vorticity intensity function for a hierarchy consisting of a group of A-vortices undergoing plane strain like stretching is shown in figures 3.5(a) - 3.5(c) and for a hierarchy consisting of \( n \)-eddies undergoing stretching in figures 3.6(a) - 3.6(c). The velocity defect law with stretching taken into consideration is shown in figure 3.8. It should be noted that the velocity defect law for a vortex with stretching lies below that without stretching.

An eddy shape that gives the correct form of the Hama velocity defect law can be found by determining the vorticity gradient necessary to give the correct form of this velocity defect law. The vorticity gradient from equation 3.7 is given by

\[
\frac{d^2 u^*_D}{d\lambda_E^2} = M f_0(\lambda_E) e^{-\lambda_E} \quad \lambda_E << 0 \quad (3.10)
\]

and from equation 3.8(b) by

\[
\frac{d^2 u^*_D}{d\lambda_E^2} = 2K (e^{-\lambda_E} + 2e^{-2\lambda_E}) \quad (3.11)
\]

Equating equations 3.10 and 3.11 gives

\[
f_0(\lambda) = f(z/\delta) = 2K(2z/\delta - 1) \quad (3.12)
\]

This vorticity distribution is shown in figure 3.7(b). This requires an eddy with some negative vorticity, and this condition is satisfied by an eddy with parabolic legs, or a bow legged eddy, shown in figure 3.7(a).
Figure 3.5 (a)-(c)
\( \Lambda \)-eddy with stretching

Figure 3.6 (a)-(c)
\( \Pi \)-eddy with stretching
The resulting $U_D^*$ distribution predicted by the various shaped eddies is shown in figure 3.8. It can be seen that the bow-legged eddy gives the closest approximation to the Hama velocity defect law. However, the existence of bow-legged eddies is unlikely as it requires the existence of negative vorticity close to the wall.

To obtain the Hama velocity defect law using representative eddy shapes with only positive cross-stream vorticity, it is necessary to assume that there are more large scale hierarchies present in the boundary layer than expected. If this were the case, the p.d.f. of hierarchy scales would have a "bump", somewhat like that shown in figure 3.9(a). The p.d.f. $P_H(\delta)$ is now modified so that

$$P_H(\delta) = \frac{M}{\delta} W(\delta/\delta_H)$$

(3.13)

Equation 3.7 may now be written as

$$U_D^* = M \int_0^{\lambda_1} \int_{\lambda_1}^{\lambda_E} f_0(\lambda) e^{-\lambda} w(\lambda - \lambda_E) d\lambda d\lambda_E$$

(3.14)

where $w(\lambda - \lambda_E) = W(\delta/\delta_H)$ and $w(\lambda - \lambda_E)$ would possibly have the form shown in figure 3.9(b).
Figure 3.7 (a)
"Bow-legged" eddy

Figure 3.7 (b)
Vorticity distribution for "bow-legged" eddy

Figure 3.8 $U_D^*$ distributions
(a) Hama velocity defect law and bow-legged eddy,
(b) H-eddy without stretching
(c) A-eddy without stretching
(d) A-eddy with stretching
Figure 3.9  (a) Modified inverse power law p.d.f.
(b) Weighting function
The form of the "weighting function", \( w(\lambda - \lambda_E) \) can be determined for an assumed eddy geometry as the exact form of the Hama velocity defect law is known. The overpopulation of the larger eddies in the boundary layer should also be seen in the spectra of motions parallel to the wall, and the spectra would have more energy at low wavenumbers than expected. Some evidence of this is presented later.

3.1.2 The Weighting Function

Consider that the inverse power law form of \( P_H(\delta) \) has been modified to the form shown in figure 3.10(a). Then on \( \delta/\delta_H \) axes the weighting function \( W(\delta/\delta_H) \) would have the form shown in figure 3.10(b). Figure 3.10(c) shows the form of the weighting function \( w(\lambda - \lambda_E) \) and this form is used in the integration scheme demonstrated earlier. Figure 3.10(d) - 3.10(f) shows this weighted function used in the evaluation of \( U_D^* \) for a H-eddy and the resulting \( U_D^* \) distribution. This gives a \( U_D^* \) more in agreement with the Hama velocity defect law and with vortex stretching included the weighting function can be modified to give agreement.
Figure 3.10 (a)-(f) $U_D^*$ distribution obtained from a $\Pi$-eddy using a weighted p.d.f.
3.2 BROAD-BAND TURBULENCE INTENSITIES

Perry and Chong showed that the broad band turbulence intensity distribution for a range of scales of attached eddies is given by

\[
\frac{\overline{u_i u_j}}{U^2} = \int_{\delta_1}^{\delta_H} I_{ij}(z/\delta) P_H(\delta) \, d\delta
\]

(3.15)

where \( I_{ij}(z/\delta) \) is the Townsend eddy intensity function for a hierarchy of scale \( \delta \) and is a measure of the non-dimensional contribution this hierarchy makes to the area averaged correlation between \( u_i \) and \( u_j \) in a plane distance \( z \) from the boundary. Townsend (1976) has shown that if the attached eddies are assumed to have inviscid behaviour (ie. slip is allowed at the boundary), then for small \( z/\delta \)

- \( I_{11} \) and \( I_{22} \) are constant and finite
- \( I_{13} \) varies linearly with \( z/\delta \)
- \( I_{33} \) varies with \( (z/\delta)^2 \)

Figures 3.11(a), (b) and (c) show typical forms of the eddy intensity functions. As can be seen in figures 3.12(a) and (b) a hot-wire probe situated at a distance \( z \) from the boundary will see only contributions to \( w' \) from eddies of height of order \( z \) whereas contributions to \( u' \) and \( v' \) will be made by all eddies of height \( z \) and
Figure 3.11 (a)-(c)
Typical eddy intensity functions
larger. Using a similar scheme as that used in Section 3.1 to evaluate the velocity defect law and assuming that \( P_H(\delta) \) is of the form given by equation 3.13, equation 3.15 can be written as

\[
\frac{u_i u_j}{U^2} = M \int_{\lambda_1}^{\lambda_E} I_{ij}(\lambda) w(\lambda - \lambda_E) \, d\lambda
\]  

(3.16)

This integral is evaluated using a similar scheme to that used to evaluate \( U_D^* \) and typical distributions of the broad-band turbulence intensities are shown in figures 3.13(a), (b) and (c).

The forms of the broad-band turbulence intensity distributions as given by Townsend for \( \lambda_E \gg 0 \) and \( \lambda_1 \) sufficiently less than 0 are

\[
\frac{\overline{u'^2}}{U^2} = A_1 \lambda_E + B_1
\]  

(3.17)

\[
\frac{\overline{v'^2}}{U^2} = A_2 \lambda_E + B_2
\]  

(3.18)

\[
\frac{\overline{w'^2}}{U^2} = A_3
\]  

(3.19)

\[
\frac{-\overline{u'w'}}{U^2} = 1
\]  

(3.20)

More details are given later in this Chapter. \( A_1, A_2, A_3 \) are universal constants and \( B_1, B_2 \) are constants characteristic of the large scale geometry of the flow, i.e. they depend on whether boundary layer, pipe or duct flow is being considered. Such constants will be referred to as characteristic constants. The weighting function changes
Figure 3.13 (a), (b), (c)

Typical broad-band turbulence intensity distributions
the shape of the curves in the outer part of the boundary layer and this weighting function is also characteristic of the large scale geometry.

3.3 SPECTRAL DISTRIBUTIONS

Consider a $\Lambda$-vortex in non-dimensional coordinates $x/\delta$, $y/\delta$, $z/\delta$ with its image in the wall as was shown in figure 3.1.

From the Biot-Savart law, it is possible to compute

$$\frac{U_1}{U_\tau} = f_1(x/\delta, y/\delta, z/\delta)$$  \hspace{1cm} (3.21)

Taking the Fourier transform in the $x$-direction in terms of $x/\delta$ gives

$$\frac{F_1(k_1 \delta, y/\delta, z/\delta)}{U_\tau^2} = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-i(k_1 \delta)z/\delta} d(x/\delta)$$ \hspace{1cm} (3.22)

The power spectral density (p.s.d.) of a random array of $\Lambda$-vortices is then

$$\frac{Q_{ii}(k_1 \delta, y/\delta, z/\delta)}{U_\tau^2} = \frac{|F_1(k_1 \delta, y/\delta, z/\delta)|^2}{U_\tau^2 K_X}$$ \hspace{1cm} (3.23)

where $K_X$ depends on the average non-dimensional streamwise spacing of the eddies. The p.s.d. over an infinite plane of constant $z$ is then

$$\frac{P_{ii}(k_1 \delta, z/\delta)}{U_\tau^2} = \frac{1}{K_Y} \int_{-\infty}^{\infty} \frac{Q_{ii}(k_1 \delta, y/\delta, z/\delta)}{U_\tau^2} d(y/\delta)$$ \hspace{1cm} (3.24)
where \( k_y \) depends on the average non-dimensional cross-stream spacing. 

\( P_{ii}(k_1 \delta) \) is the power spectral density per unit non-dimensional wave number \( k_1 \delta \). Note that the repeated suffix notation does not imply a summation. The spectral argument will be used to indicate the unit quantity over which the energy density is to be measured. Therefore,

\[
P_{ii}(k_1 \delta) = P_{ii}(k_1) / \delta
\]

\[
k_1 \delta P_{ii}(k_1 \delta) = k_1 \delta P_{ii}(k_1) / \delta = k_1 P_{ii}(k_1)
\]

\[
k_1 P_{ii}(k_1) = k_1 P_{ii}(k_1 z)
\]

\[
= k_1 \delta_H P_{ii}(k_1 \delta_H)
\]

Let \( k_1 \delta P_{ii}(k_1 \delta, z/\delta) = \psi_{ii}(a, \lambda) \)

\[
(3.25)
\]

\[
(3.26)
\]

\[
(3.27)
\]

where \( a = \ln k_1 \delta, \quad \lambda = \ln \delta/z \)

The function \( \psi_{ii}(a, \lambda) \) is called the premultiplied spectral function of \( u_i \) for a representative eddy of scale \( \delta \).
Therefore for a range of scales of hierarchies of scale $\delta_1$ to $\delta_H$ with a continuous p.d.f. given by equation 3.13, the summed p.s.d. at a fixed value of $z/\delta_H$ can be expressed as

$$k_1 z\mathbf{f}_{ii}(k_1 z, \frac{\lambda_E}{\delta_H}, \lambda) \rightarrow \frac{U_2}{\tau} \int_{\lambda_1}^{\lambda_E} \frac{L_{ui}(a, \lambda)}{U_2} \omega(\lambda - \frac{\lambda_E}{\delta_H}) \, d\lambda \quad (3.28)$$

where $\phi_{ii}$ is the p.s.d. of the $u_i$ fluctuations. The eddy intensity function can be determined from $\psi_{ii}(a, \lambda)$ as follows

$$I_{ii}(\lambda) = \int_{-\infty}^{\infty} \frac{\psi_{ii}(a, \lambda)}{U_2} \, d\lambda \quad (3.29a)$$

$$I_{ii}(z/\delta) = \int_{0}^{\infty} \frac{P_{ii}(k_1, z/\delta)}{U_2} \, d(k_1 \delta) \quad (3.29b)$$

Perry, Henbest and Chong (1984) have numerically computed the form of the function $\psi_{ii}(a, \lambda)$ for a single representative $\Lambda$-shaped eddy as shown in figure 3.1 and it can be represented as a "ridge" for the $\psi_{11}$ and $\psi_{22}$ functions, and a "hill" for $\psi_{33}$. See figures 3.14(a) and (b).
Figure 3.14 (a), (b)

Computed premultiplied spectral functions for a \( \Lambda \)-vortex
3.3.1 Pre-multiplied Spectra

If the spectra are plotted as $k_1 \phi_{ii}(k_1)$ vs log ($k_1$), where the area under the curve is non dimensionalised such that

$$\int_{0}^{\infty} \phi_{ii}(k_1) \, dk_1 = \bar{u}_i^2$$

(3.30(a))

or

$$\int_{0}^{\infty} k_1 \phi_{ii}(k_1) \, dlnk_1 = \bar{u}_i^2$$

(3.30(b))

then such a plot is known as a premultiplied $u_1$ spectrum. The pre-multiplied spectrum gives information about the energy contained in a given wavenumber range since the area under the curve within a given wavenumber range is proportional to the energy contained in that range.

3.3.2 The $k_1z$ Spectra (Inner flow scaling)

Equation 3.28 is an integral with respect to $s$ at a fixed value of $k_1z$. On a plot of $\psi_{ii}(a,\lambda)$, a line of constant $k_1z$ (or $\beta$) is a straight line of slope +1 with intercept $\beta=ln(k_1z)$ on the $a$-axis. The integral of $\psi_{ii}/U_\tau^2$ in equation 3.28 at a fixed value of $k_1z$ is shown schematically in figure 3.15.

Here, the cut a plane of constant $k_1z$ perpendicular to the $a-\lambda$ plane makes through the spectral ridge $\psi_{ii}/U_\tau^2$ is projected onto the $\psi_{ii}/U_\tau^2-\lambda$
plane and weighted with \( w(\lambda - \lambda_E) \). The area under this projection between \( \lambda_1 \) and \( \lambda_E \) is equal to the non-dimensional energy contribution at that particular \( k_1 z \). Allowing \( k_1 \) to vary with \( z \) fixed, \( k_1 z \delta_1 (k_1 z, z/\delta_1, z/\delta_H)/U_\tau^2 \) can be obtained. (See figure 3.15). The resulting premultiplied spectrum then has the form shown in figure 3.16. If \( z \) alone is changed, then the values of the limits of integration \( \lambda_1 \) and \( \lambda_E \) will also change but \( L=(\lambda_E - \lambda_1) \) remains fixed. It should be noted that \( L \) depends on \( \delta_H U_\tau/\nu \) (the Karman Number) since

\[
L = \ln(k_1 \delta_H) - \ln(k_1 \delta_1)
\]

\[
= \ln \frac{\delta_H}{\delta_1} - \ln \frac{\delta_H U_\tau}{\nu}
\]

(3.31)

as \( \delta_1 \sim \nu/U_\tau \) and \( \delta_H/\delta_1 \) is fixed.

Figure 3.17(a) shows the resulting premultiplied spectra for varying values of \( z/\delta_H \) where \( \lambda_E >> 0 \) and \( \lambda_1 \) is sufficiently negative. Note that at high \( k_1 z \), the premultiplied spectra collapse to a universal curve and at low \( k_1 z \), peel off at a value of \( k_1 z = Fz/\delta_H \), where \( F \) is a constant. A region exists where \( k_1 z \phi_{11}(k_1 z)/U_\tau^2 \) is a constant provided that \( \lambda_E >> 0 \) and \( \lambda_1 \) is sufficiently negative (ie. in the turbulent wall region). Therefore in this region,

\[
\frac{\phi_{11}(k_1 z)}{U_\tau^2} = \frac{A_1}{k_1 z}
\]

(3.32)

\( A_1 \) is a universal constant for a given representative eddy geometry. This gives an inverse power law for \( \phi_{11}(k_1 z)/U_\tau^2 \). The corresponding plot of
Figure 3.15
Integration scheme for the longitudinal spectral function
(inner flow scaling)

Figure 3.16
Resulting premultiplied spectrum from integration scheme shown in figure 3.15
log $\phi_{11}(k_1 z)/U_2^2$ vs log $(k_1 z)$ is shown in figure 3.17(b). In this plot the spectra peel off at low wavenumbers from the inverse power law at a value of $k_1 z = \delta_H/\lambda_1$.

Figure 3.18 shows the above procedure repeated with the $\nu_3$ "hill", and the resulting premultiplied spectra are shown in figure 3.19(a) for varying values of $z/\delta_H$, where $\lambda_1 > 0$ and $\lambda_1$ sufficiently negative, i.e. only in the turbulent wall region.

Note that the $\phi_{33}$ spectra, figure 3.19(a) and (b), are universal for all $k_1 z$ and are independent of $z/\delta_H$ in the turbulent wall region, whereas the $\phi_{11}$ spectra are universal only at high $k_1 z$ and depend upon $z/\delta_H$ at low $k_1 z$. Also note that no inverse power law exists for the $\phi_{33}$ spectra.

The spectral distribution of $\phi_{22}(k_1 z)/U_2^2$ is similar to that of $\phi_{11}(k_1 z)/U_2^2$ by virtue of the fact that they have similar premultiplied spectral functions. This is a consequence of the similar Townsend boundary conditions.

3.3.3 The $k_1 \delta_H$ Spectra (Outer flow scaling)

If $\lambda_1$ is fixed, then a line of constant $k_1 z$ is also a line of constant $k_1 \delta_H$. Let $\gamma = \ln(k_1 \delta_H)$

Then, $\gamma = \ln(\delta_H/z) + \ln(k_1 z)$ i.e. $\gamma = \lambda_1 + \delta$
Figure 3.17 (a), (b)
Predicted spectra for varying values of $z/\delta_H$
(inner flow scaling)
Figure 3.18
Integration scheme for the normal spectral function
(inner flow scaling)

Figure 3.19 (a), (b)
Resulting spectra from integration scheme in figure 3.18 (for varying values of $z/\delta_H$)
By varying $k_1 \delta_H$ with $L_e$ fixed, $k_1 \delta_H \phi_{11}(k_1 \delta_H, z/\delta_H, z/\delta_H)/U_t^2$ can be obtained using a similar procedure to that used for the $k_1 z$ spectra and the resulting premultiplied spectral distribution for varying $z/\delta_H$ ($\delta_H < z < \delta_H$) are shown in figures 3.20(a), (b). The resulting spectral distribution shows that the spectra are universal at low wavenumbers and at high wavenumbers peels off from an inverse power law region at $k_1 \delta_H = P \delta_H / z$.

For the $\phi_{33}$ spectra, the process can be repeated with the "hill" shaped spectral function, and the result is shown in figures 3.21(a), (b). Note that no collapse of the $\phi_{33}$ spectra when scaled with outer flow scaling coordinates exists.

Figures 3.22(a), (b) and (c) show the forms of the $\phi_{11}$, $\phi_{22}$ and $\phi_{33}$ spectra respectively for different numbers of observed hierarchies of A-shaped eddies obtained by Dr. M.S. Chong by numerical simulation from the respective spectral functions. (N.B. The weighting function was not included in this calculation.) Note how the length of the inverse power law increases with the number of observed hierarchies in the $\phi_{11}$ and $\phi_{22}$ spectra and is not apparent if an insufficient number of hierarchies is observed. The slight difference shape between the $\phi_{11}$ and $\phi_{22}$ spectra in the low wavenumber region is a consequence of the geometry of the representative eddy. In the $\phi_{33}$ spectra, collapse is obtained only in the turbulent wall region.

Figure 3.23(a) shows a premultiplied spectrum measured over the smooth wall (Chapter 4) at $z_+=140$ and $z/\delta_H=.03$ at the highest Re tested, and figure 3.23(b) shows the corresponding $\phi_{11}(k_1)$ spectrum. It was
Figure 3.20 (a), (b) Predicted $\phi_{11}$ spectra for varying values of $z/\delta_H$ (outer flow scaling)

Figure 3.21 (a), (b) Predicted $\phi_{33}$ spectra for varying values of $z/\delta_H$ (outer flow scaling)
Figure 3.22 (a), (b)
(For caption see next page)
Figures 3.22 (a), (b), (c)

$\frac{\phi_{33}(k_{1}z)}{U^2/\tau}$

number of hierarchies observed

$\phi_{11}$, $\phi_{22}$ and $\phi_{33}$ spectra for different numbers of observed hierarchies computed from the spectral functions for a $\Lambda$-vortex.
Figure 3.23 (a), (b)

Measured spectra over the smooth wall
Figure 3.24(a), (b)

Spectra predicted by the model under similar conditions to the measured spectra shown in fig. 3.23

(weighting function used, \(b=3.5, \ c=1.7\))
calculated that about 7 hierarchies would exist. The "bump" at low wavenumbers is clearly visible. Dr. M.S. Chong obtained the spectra in figures 3.24(a),(b) by numerical simulations using the spectral function \( \psi_{ll}(a,A) \) generated for 7 hierarchies of A-shaped eddies and a weighting function with \( b=3.5 \) and \( c=1.7 \) where \( b \) and \( c \) are defined in figure 3.10(a). These values of \( b \) and \( c \) gave the Hama velocity defect law distribution for mean velocity. The similarities are obvious.

3.4 A DIMENSIONAL ANALYSIS APPROACH TO SPECTRA

Perry and Abell (1977) used a dimensional analysis approach to obtain the form of the \( u' \)-spectra in the turbulent wall region for fully developed flow. This approach was based on Townsend's attached eddy hypothesis and their approach for the \( u' \)-spectra is extended below. Imagine an eddy of scale \( \delta \) attached to the wall. At \( z<<\delta \), there will be no contribution to \( w' \) and the \( u' \) and \( v' \) motions are invariant with \( z \). At \( z=O(\delta) \), this eddy contributes to \( u' \), \( v' \) and \( w' \). At \( z>>\delta \), no contributions are made to \( u' , v' \) or \( w' \). This is a consequence of the Townsend (1976) boundary condition. In this analysis the flow is assumed to consist of A-shaped attached eddies as proposed by Perry and Chong, and that these attached eddies are surrounded by fine scale isotropic motions which are thought to result from the debris of dead eddy material being convected away from the near wall region.

The following analysis is applicable for \( z<<\delta_H \) and for \( z>>v/U_t \) on a smooth wall and for \( z<<\delta_H \) and \( z \) sufficiently greater than the roughness scale on a rough wall.
The behaviour of the $u'$-spectra for $z<<\delta_H$ can be separated into three different wavenumber regions.

Large scale attached eddies of height of order $\delta_H$ will contribute energy at low wave numbers and these contributions should be invariant with $z$. The expected "outer flow" spectral scaling law will then be

$$\frac{\Phi_{ll}(k_1 \delta_H)}{U^2} = F_1(k_1 \delta_H)$$  \hspace{1cm} (3.33)

The fine scale attached motions are controlled by dissipation and viscosity (and in the region where local isotropy is expected) and this occurs at very high wavenumbers. The scaling of the $u'$-spectra in this region would be expected to follow the Kolmogoroff scaling law, ie.

$$\frac{\Phi_{ll}(k_1 \eta)}{\nu^2} = F_3(k_1 \eta)$$  \hspace{1cm} (3.34)

where $\eta=(\nu^3/\epsilon)^{1/4}$, the Kolmogoroff length scale

and $u=(\nu\epsilon)^{1/4}$, the Kolmogoroff velocity scale.

Between the regions defined by equations 3.33 and 3.34, ie. at moderate to high wavenumbers, would be a region where universal wall motions are present and these depend on $z$. The expected "inner flow" scaling law would then be expected to have the form

$$\frac{\Phi_{ll}(k_1 z)}{U^2} = F_2(k_1 z)$$  \hspace{1cm} (3.35)
The expected regions of validity of equations 3.33, 3.34 and 3.35 are shown in figure 3.25. The two expected regions of overlap are also shown in the figure. In region of overlap I, equations 3.33 and 3.35 are simultaneously valid. This is possible only if

\[
\frac{\phi_{ll}(k_1)}{U^2_\tau} = \frac{A_1}{k_1}
\]

ie.

\[
\frac{\phi_{ll}(k_1z)}{U^2_\tau} = \frac{A_1}{k_1z}
\]  \hspace{1cm} (3.36)

and

\[
\frac{\phi_{ll}(k_1\delta_H)}{U^2_\tau} = \frac{A_1}{k_1\delta_H}
\]  \hspace{1cm} (3.37)

where \( A_1 \) is a universal constant.

Thus in this region the spectra should collapse onto an inverse power law region for both inner and outer flow scaling.

In region of overlap II, equations 3.34 and 3.35 are simultaneously valid. This is only possible if \( v \) is not explicitly involved in equation 3.34. This leads to \( F_3 \) having the form

\[
\frac{\phi_{ll}(k_1\eta)}{u^2} = \frac{K_0}{(k_1\eta)^{5/3}} = F_3
\]  \hspace{1cm} (3.38)

where \( K_0 \) is the universal Kolmogoroff constant. To relate equation 3.38 and 3.34 in this region, use must be made of Townsend's (1961) assumption that close to the wall in the log-law region, turbulent energy production
Motions which are independent of viscosity (i.e. Townsend's Reynolds number similarity hypothesis) ~ dependent motions

\[
\frac{\phi_{11}(k_1 \delta_H)}{U^2} = F_1(k_1 \delta_H)
\]
("Outer flow" scaling)

\[
\frac{\phi_{11}(k_1 \eta)}{U^2} = F_3(k_1 \eta)
\]
(Kolmogoroff scaling)

Overlap region I

\[
\frac{\phi_{11}(k_1 z)}{U^2} = F_2(k_1 z)
\]
("Inner flow" scaling)

Overlap region II

or \( k_1 z = M k^{-1/4} z_+ ^{3/4} \)

\( z_+ = z U_t / v \)

Figure 3.25

Wavenumber regions for the longitudinal spectra showing regions of overlap
(p) equals turbulent dissipation (ε), ie.

\[ \varepsilon = p = -u'w' \frac{\partial U}{\partial z} \]  

(3.39)

and that in the log-law region, \( -u'w' = u' \tau^2 \). Thus it can be shown that

\[ u = \left[ \frac{\nu U \tau}{Kz} \right]^{1/4} \]  

(3.40)

\[ \eta = \left[ \frac{3 \nu Kz}{U \tau} \right]^{1/4} \]  

(3.41)

and equation 3.38 can be written as

\[ \frac{\phi_{11}(k_1, \eta)}{U \tau^2} = \frac{K_0}{\kappa^{2/3}} \frac{1}{(k_1 \tau z)^{5/3}} \]  

(3.42)

Thus the u'-spectra at very high wavenumbers when plotted with inner flow scaling or Kolmogoroff scaling would be expected to collapse to a -5/3 power law. Region of overlap II is referred to as the inertial subrange.

The dissipation range must start at a universal value of \( k_1 \eta = M \). From equation 3.41, this corresponds to a value of \( k_1 \tau z \) given by

\[ (k_1 \tau z)^p = \frac{M}{K^4} z_+^{3/4} \]  

(3.43)

where \( z_+ = z U_\tau / \nu \).

† Note: This is a different M from that defined in equation 3.4.
Thus the high wavenumber viscous peel offs of the spectrum depart from the -5/3 power law at a value of \((k_1 z)_p\) which is proportional to \(z_+^{3/4}\) when scaled with inner flow scaling coordinates. If a plot was to be made of \(\log(k_1 z)_p\) vs \(\log(z_+)\), then the expected slope of the line will be +3/4. By extrapolating the line of slope +3/4 to \((k_1 z)_p=1.0\), the value of \(M\) is then \(\kappa^{1/4} / \left[ (z_+)(k_1 z)_p=1.0 \right]^{3/4}\). Perry, Henbest and Chong (1984) plotted a family of curves for different \(\ell\), the length of the hot-wire filament, and obtained a value of \(M=0.085\). This is the value of \(M\) used by the author in the determination of the broad-band turbulence intensity distributions from the spectral laws by integrating over the various spectral regions as shown later.

As mentioned in section (2.7), the the high wavenumber cut off of a hot-wire filament of length \(\ell\) is \(k_1=0(\ell/\ell)\). If the spatial resolution of the hot-wire is inadequate to resolve the fine scale motions of the flow, then on the spectral plot, the high wavenumber cut off of the spectrum will no longer depend on \(z_+\) but instead will depend on \(\ell\), i.e. \((k_1 z)_p\propto (z/\ell)\). As \(\ell\) is a constant, then \((k_1 z)_p\propto (z)\). Thus on a plot of \(\log(k_1 z)_p\) vs \(\log(z_+)\), the slope of the line will be +1 instead of the expected +3/4.

Ideally, the expected \(u'-\)spectra plotted with "inner flow scaling" axes will have the form shown in figure 3.26(a), (b). The low wavenumber peel offs of the spectra from the inverse power law region occur at a point where \(k_1 z=F(z/\delta_H)\) (equation 3.36.) The corresponding boundaries of the various regions shown in figure 3.25 are indicated.
Figure 3.27(a), (b) shows the expected form of the spectra plotted with outer flow scaling.

Note that the form of the spectral plots predicted by dimensional analysis corresponds to those predicted using the A-vortex model of Perry and Chong with the exception that the A-vortex model does not take into account the existence of a Kolmogoroff region with an inertial subrange.

The expected form of the $v'$-spectra will be similar to that of the $u'$-spectra for reasons discussed earlier.

Figure 3.28 shows the scaling regions for the $w'$-spectra in the turbulent wall region. From earlier discussion, no outer flow scaling law is expected and thus there is only one region of overlap, the inertial subrange. By a similar analysis to the $u'$-spectra, it can be shown that at very high wavenumbers, the $w'$-spectra peel off from a $-5/3$ power law region at a value of $k_1 z$ proportional to $z_+^{3/4}$. Thus the $w'$-spectra with inner flow scaling would collapse at low to high wavenumbers, with the collapse following a $-5/3$ power law at the high wavenumber end, and then to ultimately peel off due to viscosity from this $-5/3$ power law region at a point proportional to $z_+^{3/4}$. This is shown in figure 3.29. The $w'$-spectra are shown with outer flow scaling in figure 3.30(a), (b).
Figure 3.26 (a), (b)
Predicted u'-spectra - inner flow scaling

Figure 3.27(a), (b)
Predicted u'-spectra - outer flow scaling
Viscous motions which are independent of viscosity (i.e. Townsend's Reynolds number similarity hypothesis)

\[ \frac{\phi_{33}(k_1 \eta)}{u^2} = G_3(k_1 \eta) \]

(Kolmogoroff scaling)

\[ k_1 z = N \]

\[ k_1 n = M \]

or \[ k_1 z = M k^{1/4} z^{3/4} \]

Overlap region

\[ \frac{\phi_{33}(k_1 z)}{U^2_{\tau}} = G_2(k_1 z) \]

("inner flow" scaling)

Figure 3.28

Wavenumber regions for the normal spectra showing regions of overlap
\[ \log \frac{\phi_{33}(k_1 z)}{U^2} = \frac{-1}{4} k_1 z \]

\[ \log N \]

Figure 3.29 (a), (b)

Predicted $w'$-spectra - inner flow scaling

\[ \log \frac{\phi_{33}(k_1 \delta_H)}{U^2} \]

\[ \log(k_1 \delta_H) \]

(a) log(k_1 \delta_H)

(b) log(k_1 \delta_H)

Figure 3.30 (a), (b)

Predicted $w'$-spectra - outer flow scaling
3.4.1 Broad-band turbulence intensities

The broad band turbulence intensity distribution laws in the turbulent wall region can be obtained from the deduced spectral laws by integrating over the various spectral regions.

For the $u'$-motions,

$$\frac{u'^2}{U_\tau} = \int_0^P F_1(k_1 \delta_z) \, d(k_1 \delta_z) + \int_0^P F_2(k_1 z) \, d(k_1 z) + \int_0^N F_2(k_1 z) \, d(k_1 z)$$

$$- \frac{1}{4} (z+) \frac{3}{4}$$

$$+ \int_0^P F_2(k_1 z) \, d(k_1 z) + \frac{u^2}{U_\tau} \int_0^\infty F_3(k_1 \eta) \, d(k_1 \eta)$$

ie.

$$\frac{u'^2}{U_\tau} = I_1 + I_2 + I_3 + I_4 + I_5$$

(3.44)

where

$$I_1 = \int_0^P F_1(k_1 \delta_z) \, d(k_1 \delta_z) = R$$

a characteristic constant

(3.45)

$$I_2 = \int_0^P \frac{A_1}{k_1 z} \, d(k_1 z) = A_1 \ln(k_1 z)$$

$$= -A_1 \ln(z/\delta_H) - A_1 \ln(F) + A_1 \ln(P)$$

(3.46)

$$I_3 = \int_0^N F_2(k_1 z) \, d(k_1 z) = S$$

a universal constant

(3.47)

$$I_4 = \frac{3}{2} \frac{K_0}{\kappa^{3/2}} \frac{1}{N^{2/3}} - \frac{3}{2} \frac{K_0}{\kappa^{1/2}} \frac{1}{M^{2/3}} z^{-1/2}$$

(3.48)

$$I_5$$ is assumed to be negligible

(3.49)
$I_1$ is obtained from the data, $I_2$ can be determined analytically provided $A_1$ is fixed from a straight line fit to the inverse power law region, $I_3$ is obtained from the data and $I_4$ is analytically obtained using values of $K_0$ and $M$ obtained from the data. The value of $K_0$ was found to be 0.5, and this is the accepted value for the Kolmogoroff constant (Townsend 1976). The value of $M$ used was 0.085, the value obtained for $t=0$. $I_5$ is assumed to be negligible. The spectra measured at the highest Reynolds number and furthest downstream were used when determining $I_1$ and $I_3$.

$P$ and $N$ can be thought of as universal constants which are associated with the ends of the various spectral regions. However the final integral for the broad band turbulence intensities are independent of the values of $F$, $P$ and $N$ chosen, provided $F$ and $P$ correspond to values of $k_1 \delta_H$ and $k_1 z$ respectively which lie in the inverse power law region. The value of $N$ corresponds to a chosen value of $k_1 z$ which lies in the inertial subrange. Figure 3.25, 3.26 and 3.27 show the various regions representing $I_1$, $I_2$, $I_3$, $I_4$ and $I_5$ and the positions of $F$, $P$ and $N$.

This gives

$$\frac{\overline{u'w'}}{U_2^{1/2}} = B_1 - A_1 \ln(z/\delta_H) - C_1 z^{-1/2}$$  \hspace{1cm} (3.50)

where

$$C_1 = \frac{3}{2} \frac{K_0}{\kappa^{1/2}} \frac{1}{M^{2/3}}$$ \hspace{1cm} (3.51)

and

$$B_1 = R - A_1 \ln(F) + A_1 \ln(P) + S + \frac{3}{2} \frac{K_0}{\kappa^{2/3}} \frac{1}{N^{2/3}}$$

$A_1$ and $C_1$ are universal constants and $B_1$ is a characteristic constant.
For the $v'$-turbulence intensities, a similar result is obtained.

$$\frac{v'^2}{\bar{u}_r^2} = B_2 - A_2 \ln(z/H) - \frac{4}{3} C_1 z_+^{-1/2}$$

(3.52)

where $A_2$ is a universal constant and $B_2$ is a characteristic constant.

The use of $\frac{4}{3} C_1$ is necessary to satisfy the conditions for the existence of isotropic motions (see Townsend, 1976 and Batchelor, 1953).

For the $w'$-turbulence intensities,

$$\frac{w'^2}{\bar{u}_r^2} = \int_0^N \frac{G_2(k_1 z) \, d(k_1 z)}{G_2(k_1 z) \, d(k_1 z)} + \int_{N}^{\infty} G_3(k_1 \eta) \, d(k_1 \eta)$$

$$+ \frac{v^2}{\bar{u}_r^2} \int_{M}^{\infty} G_3(k_1 \eta) \, d(k_1 \eta)$$

(3.53)

ie. $\frac{w'^2}{\bar{u}_r^2} = J_1 + J_2 + J_3$

$J_1$ is obtained from the data

$$J_2 = \frac{2K_0}{\kappa^{2/3}} \frac{1}{N^{2/3}} + \frac{2K_0}{\kappa^{1/2}} \frac{1}{M^{2/3}} z_+^{-1/2}$$

(3.54)

$J_3$ is negligible.
Thus,
\[
\frac{\omega'^2}{U^2 \tau} = A_3 - \frac{4}{3} C_1 z_+^{-1/2}
\]  
(3.55)

Where $A_3$ is universal for the wall region.

If $z/\delta_H \rightarrow 0$, and $z_+ \rightarrow \infty$, the Townsend (1976) result is obtained, ie
\[
\frac{u'^2}{U^2 \tau} = B_1 - A_1 \ln(z/\delta_H)
\]  
(3.56(a))

\[
\frac{v'^2}{U^2 \tau} = B_2 - A_2 \ln(z/\delta_H)
\]  
(3.56(b))

\[
\frac{w'^2}{U^2 \tau} = A_3
\]  
(3.56(c))

which is equivalent to equations 3.50, 3.52 and 3.55 for $z_+ \rightarrow \infty$.

These equations are also equal to equations 3.17, 3.18 and 3.19 which were derived using the $A$-vortex model given earlier.
CHAPTER 4

The Smooth Wall Results

In this chapter, experimental results of the mean flow, broad band turbulence intensity and spectral distributions of flow over a smooth wall with zero pressure gradient are presented. Attempts are made to view the results in terms of the Townsend (1956) Reynolds number similarity hypothesis, the Townsend (1976) attached eddy hypothesis, and Perry and Chong's (1982) model of wall turbulence.

4.1 THE MEAN FLOW

The smooth wall was made up of a single sheet of acrylic laminate bonded to a 12mm thick chipboard sheet and was screwed to the floor of the large wind tunnel. Every effort was made to ensure that the wall was flat and the leading edge was faired into the working section with a wooden ramp. The flow was tripped with a 0.9mm diameter stainless steel rod. This diameter was a compromise between the two optimum diameters (the trip wire diameter was chosen according to a formula by Gibbings, 1959) appropriate for use at the two intended free stream velocities, 10m/s and 30 m/s. A 100mm wide strip of 40 grade sandpaper was also tried as a tripping device and the result was very similar to that obtained with the 0.9mm rod. All experimental results presented here were taken with the rod as a tripping device.
The tunnel roof was adjusted to give a zero pressure gradient and figure 4.1 shows plots of $C_p$ vs $x$ for free stream velocities of 10m/s and 30m/s. A spanwise mean velocity profile was also measured at about $x=1750\text{mm}$ and this is shown plotted as $\bar{V}/\bar{U}$ vs $y$ in figure 4.2. The secondary flow can be seen to be very small. Mean velocity profiles were measured at 5 stations along the smooth surface, at $x$-distances of 500mm, 1125mm, 1750mm, 2375mm, and 3000mm, (where $x$ is measured from the tripping device). Broad band turbulence intensities and spectra were measured at $x$-distances of 500mm, 1750mm, and 3000mm. $Re$ for the relevant cases is shown in Table 4.1.

<table>
<thead>
<tr>
<th>$x$ (mm)</th>
<th>$U_1$ (m/s)</th>
<th>$Re$</th>
</tr>
</thead>
<tbody>
<tr>
<td>500</td>
<td>10</td>
<td>1203</td>
</tr>
<tr>
<td>1750</td>
<td>10</td>
<td>2791</td>
</tr>
<tr>
<td>3000</td>
<td>10</td>
<td>4245</td>
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<tr>
<td>500</td>
<td>30</td>
<td>5111</td>
</tr>
<tr>
<td>1750</td>
<td>30</td>
<td>8661</td>
</tr>
<tr>
<td>3000</td>
<td>30</td>
<td>12059</td>
</tr>
</tbody>
</table>

Table 4.1

The mean velocity profiles are shown plotted as $\bar{U}/U_\tau$ vs $zU_\tau/\nu$ in figure 4.3. The values of $U_\tau$ were obtained from a Clauser chart (Clauser 1954). The profiles can be seen to collapse onto the classical logarithmic law of the wall, ie.

$$\frac{\bar{U}}{U_\tau} = \frac{1}{\kappa} \ln \frac{zU_\tau}{\nu} + A$$

(4.1)

where $\kappa = 0.41$ and $A = 5.0$. 
Figure 4.1 $C_p$ vs $x$

Figure 4.2 Lateral mean velocity profile
The mean velocity profiles are also shown plotted as a velocity defect in figure 4.4. The collapse is reasonable except for the profiles at the two lowest values of $R_\theta$. For these two cases, it is thought that the flow may have been underdeveloped. This is supported by the fact that their corresponding values of $R_\theta$ are well below 5000. Coles (1962) has shown that below $R_\theta=5000$ the defect law breaks down (see Chapter 2). Also for the well developed (higher $R_\theta$) profiles, Hama's velocity defect law fitted the data well. Use of this law for deducing $U_\tau$ gave values in good agreement with the Clauser chart values at the higher $R_\theta$'s (>5000).

4.2 BROAD-BAND TURBULENCE INTENSITIES

4.2.1 The Reynolds Shear Stresses

Reynolds shear stresses over the smooth wall were measured using a $\beta=90^\circ$ X-wire probe at the three stations and two free stream velocities specified earlier.

A $\beta=120^\circ$ X-wire probe was used to check if a cone angle problem existed when measuring the Reynolds shear stress with a $\beta=90^\circ$ probe. The Reynolds shear stress profiles obtained were identical and confirmed that the $\beta=90^\circ$ X-wires were adequate in this case. This is attributed to the fact that at similar values of $z/\delta_H$, $\bar{U}/U_\tau$ is higher for a smooth wall than a rough wall at the same $\delta_H/U_\tau/\nu$ (see Chapter 2).
Figure 4.3 Mean velocity profiles

Log-law of the wall (eqn. 4.1)

<table>
<thead>
<tr>
<th>Symbol</th>
<th>$R_e$ Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>+</td>
<td>1203</td>
</tr>
<tr>
<td>□</td>
<td>5111</td>
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<tr>
<td>×</td>
<td>2791</td>
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<td>△</td>
<td>4245</td>
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<tr>
<td>○</td>
<td>12059</td>
</tr>
</tbody>
</table>
**Figure 4.4** Mean velocity profiles
Defect law plot
The Reynolds shear stresses are shown plotted as $-\frac{\overline{u'w'}}{\tau} U^2$ vs $z/\delta_H$ and $-\frac{\overline{u'w'}}{\tau} U^2$ vs $\log z/\delta_H$ in figures 4.5(a) and (b) respectively. The values of $U_\tau$ were obtained using a Clauser chart. As can be seen in the plots, the Reynolds shear stress profiles at the higher values of $R_\theta$ collapse and asymptote to a value of approximately 1.0. At the lower values of $R_\theta$, the data show a weak variation with $R_\theta$ close to the wall, similar to that also observed in the mean flow velocity defect plots. Some of the variation could be due to the fact that the value of $U_\tau$ used to non-dimensionalise the Reynolds shear stresses may be in error. This error may have been due to the inadequate recovery of the flow after tripping, leading to an error in inferred $U_\tau$ from the Clauser chart. The suspicion that this may have been the case is supported by the fact that in the turbulence intensity plots, (and in the velocity defect plots as well) the measurements taken at the station closest to the leading edge (at both free stream velocities) do not seem to collapse with the data at higher $R_\theta$'s.

### 4.2.2 Longitudinal Turbulence Intensities

Figure 4.6 shows a plot of the longitudinal turbulence intensities plotted as $\overline{u'^2}/U^2$ vs $\log z/\delta_H$ measured with a normal hot-wire. It can be seen that in the turbulent wall region there seems to be an increase in the turbulence intensity with $R_\theta$ although the trend is weak and scatter is present at very low values of $z/\delta_H$. This scatter may have been due to inadequate spatial resolution of the normal wire. Although the Reynolds number trend is not as strong as that observed by Henbest
Figure 4.5 (a) Reynolds shear stress profiles
Figure 4.5 (b) Reynolds shear stress profiles
Figure 4.6 Longitudinal turbulence intensities
(1982-pipe flow) it does seem that the longitudinal turbulence intensity distributions are asymptoting at the highest values of $R_{\theta}$. The data at these $R_{\theta}$'s appear to collapse to a semi-logarithmic region, in agreement with the distributions predicted by Townsend and by Perry and Chong for the turbulent wall region. As shown in Chapter 3, the $u'$-turbulence intensity distributions can be obtained by integrating the $u'$-spectral laws. The distribution for the $u'$-turbulence intensities is given in equation 3.50 and the values of the constants $A_1$, $B_1$ and $C_1$ have been determined from the $u'$-spectra as 1.03, 2.48 and 6.08 respectively. $K_0$ was found from the spectra to have a value of 0.5014 and this is close to the accepted universal value for the Kolmogoroff constant of 0.5. Thus,

$$\frac{\overline{u'^2}}{U'_{\tau}^2} = 2.48 \cdot 1.03 \ln(z/\delta_H) - 6.08 \cdot z_+^{-1/2} \quad (4.2)$$

The predicted distributions for the asymptotic case (ie. $z_+ = \infty$) and for the highest and lowest $R_{\theta}$'s are shown in figure 4.6. From these predicted distributions it can be seen that the data appear not to have reached the asymptotic values but the fit is reasonable.

4.2.3 Normal Turbulence Intensities

Normal turbulence intensities were measured using a dynamically calibrated $\beta=90^\circ$ X-wire and their distributions are shown in figure 4.7(a) plotted as $w'^2/U'_{\tau}^2$ vs $\log z/\delta_H$ for various values of $R_{\theta}$. There is reasonable collapse of the data in the wake region but in the turbulent wall region there is some scatter and a weak $R_{\theta}$ dependence.
Figure 4.7 (a) Normal turbulence intensities
However the main point of concern is not the scatter but the value of $w' / U^2$ close to the wall. In this region $w' / U^2$ has a value of approximately 1.5 which is higher than expected from the results of other workers. According to Townsend's attached eddy hypothesis and the A-vortex theory of Perry and Chong, the normal turbulence intensity distributions should be approximately constant in the turbulent wall region provided that $z_+$ is sufficiently large. A survey of the literature for similar data mentioned in Chapter 2 shows that measurements by other workers indicates that in general $w' / U^2$ has a value that lies between 1.0 and 1.3 in the turbulent wall region although there are some exceptions (see Table 4.2), and the value of 1.5 obtained here seems too high. As mentioned in Chapter 2, the author repeated some of these experiments using different probes and values of $w' / U^2$ in the turbulent wall region of about 1.2 were obtained. For reasons given in Chapter 2 it would appear that the X-wire probe used to measure the data shown in figure 4.7(a) may have suffered from excessive cross contamination from the lateral ($v'$) fluctuations. As mentioned in Chapter 2, it was because of this phenomenon that a series of experiments was initiated to determine the cross sensitivities of the X-wire used to measure the normal turbulence intensities to lateral velocity fluctuations.

The data of the later experimental run where $w' / U^2 = 1.2$ in the turbulent wall region is also shown in figure 4.7(b), which includes the $R^* = 12059$ profile from figure 4.7(a). By integration of the spectral laws, $A_3$ was found to have a value of 1.90 and the predicted normal turbulence intensity distribution is given by
\[
\frac{\overline{w'w'}}{\frac{U^2}{\tau}} = 1.90 - 8.1 z_+^{-1/2}
\]  
(4.3)

This distribution for the highest \( R_e \) case is superimposed on the data in figure 4.7(a). As can be seen, the cutoff due to the viscous term is significant, up to quite high values of \( z_+ \). A survey of the literature in Table 4.2 yielded very little information about the behaviour of \( \overline{w'w'}/U^2 \) at very low values of \( z_+ \) (ie. \( z_+ < 100 \)). However Grass (1971) and Bremhorst and Walker (1973) obtained results which showed that \( \overline{w'w'}/U^2 \) fell off when the value of \( z_+ \) fell below about 60. According to equation 4.3, the asymptotic value of \( \overline{w'w'}/U^2 \) in the wall region will only be attained at very high values of \( z_+ > 10,000 \) but unfortunately, in all the data obtained in wind tunnels so far, the upper limit of the wall region \((z/\delta_H=0.15)\) severely limits the value of \( z_+ \) achievable in this region. To obtain a true measure of the asymptotic value of \( \overline{w'w'}/U^2 \), measurements are needed at much higher Reynolds numbers.

According to the theory of Perry and Chong, \( \delta_H = 100 v/U \). The number of hierarchies in the boundary layer thus depends on \( \delta_H U/\nu \), which is the Kármán number. At a fixed value of \( z/\delta_H \), \( z_+ \) also depends on the Kármán number, thus the Kármán number also gives an indication of how much energy is cut off by dissipation, and this applies to both rough and smooth walls. It was therefore decided to plot equation 4.3 with \( \gamma_3 = 1.0, 1.5 \) and 2.0 on a plot of \( \overline{w'w'}/U^2 \) at \( z/\delta_H = 0.1 \) vs \( \delta_H U/\nu \) (figure 4.8), and the values of \( \overline{w'w'}/U^2 \) of some of the workers in Table 4.2 at \( z/\delta_H = 0.1 \) are also shown in this figure (where enough information was available to enable the Kármán number to be determined).

\* Only for smooth walls. On rough walls the number of hierarchies depends on \( \delta_H/k \) or \( \delta_H/\delta_1 \). Some rough wall results are also shown in figure 4.8.
All cases $R_e=12059$

- $\times$ cross contamination from $v'$ suspected
- $\circ$ bowed wires
- $\square$ Wires with no obvious defects.

All three X-wires gave the same correct value of Reynolds shear stress

Figure 4.7 (b) Normal turbulence intensities measured with apparently similar X-wires
<table>
<thead>
<tr>
<th>Name</th>
<th>Year</th>
<th>Measurement</th>
<th>Method used to get $U_t$</th>
<th>$\frac{\delta U_t}{v}$</th>
<th>$\frac{w'^2}{U_t^2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Klebanoff</td>
<td>1954</td>
<td>Flat Plate, zero P.G.</td>
<td>Hot-wires Reynolds shear stress</td>
<td>3118</td>
<td>1.14</td>
</tr>
<tr>
<td>Laufer</td>
<td>1954</td>
<td>Pipe</td>
<td>Hot-wires dP/dX</td>
<td>1116</td>
<td>1.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>9695</td>
<td>1.2</td>
</tr>
<tr>
<td>Antonia, Luxton</td>
<td>1971</td>
<td>Smooth, rough flat plate. Step change in r'ness.</td>
<td>Hot-wires Clauser Chart, pressure tapped element</td>
<td>Smooth-1.1</td>
<td>Rough-1.0</td>
</tr>
<tr>
<td>Grass, A.J.</td>
<td>1971</td>
<td>Flat plate (water) Hydrogen bubble</td>
<td>Hot-wires Clauser chart</td>
<td>1.0</td>
<td></td>
</tr>
<tr>
<td>Kim, Kline, Reynolds</td>
<td>1971</td>
<td>Flat Plate, (water) Hydrogen bubble</td>
<td>Hot-wires Clauser chart</td>
<td>1.2</td>
<td></td>
</tr>
<tr>
<td>Lawn</td>
<td>1971</td>
<td>Pipe</td>
<td>Hot-wires dP/dX</td>
<td>1010</td>
<td>0.80</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>5469</td>
<td>1.0</td>
</tr>
<tr>
<td>Gupta, Kaplan</td>
<td>1972</td>
<td>Flat plate</td>
<td>Hot-wires Clauser chart</td>
<td>768</td>
<td>1.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>2160</td>
<td>1.0</td>
</tr>
<tr>
<td>Bremhorst, Walker</td>
<td>1973</td>
<td>Smooth pipe</td>
<td>Hot-wires dP/dX</td>
<td>1823</td>
<td>1.2</td>
</tr>
<tr>
<td>Abell</td>
<td>1974</td>
<td>Smooth, rough, pipes</td>
<td>Hot-wires Clauser ch., dP/dX</td>
<td>Smooth-1.0</td>
<td>Rough-0.9</td>
</tr>
<tr>
<td>Wood, Antonio</td>
<td>1975</td>
<td>Flat plate</td>
<td>Hot-wires Clauser ch., Preston tube</td>
<td>0.75</td>
<td></td>
</tr>
<tr>
<td>Sabot, Comte-Bellot</td>
<td>1976</td>
<td>Smooth pipe</td>
<td>Hot-wires dP/dX</td>
<td>2600</td>
<td>1.25</td>
</tr>
<tr>
<td>Shivaprasad, Ramaprian</td>
<td>1978</td>
<td>Developing curved duct</td>
<td>Hot-wires Preston tube convex concave</td>
<td>1.90 1.11</td>
<td></td>
</tr>
<tr>
<td>Sabot, Saleh Comte-Bellot</td>
<td>1977</td>
<td>Rough pipe</td>
<td>Hot-wires dP/dX</td>
<td>6200</td>
<td>0.81</td>
</tr>
<tr>
<td>Willmarth, Bogar</td>
<td>1977</td>
<td>Flat plate, zero P.G. very small X-probe</td>
<td>Hot-wires Drag balance</td>
<td>15,000</td>
<td>1.95</td>
</tr>
<tr>
<td>Mulhearn</td>
<td>1978</td>
<td>Very rough flat plate</td>
<td>Hot-wires Drag balance</td>
<td>15,000</td>
<td>1.95</td>
</tr>
<tr>
<td>Hunt, Joubert</td>
<td>1979</td>
<td>Developing straight duct</td>
<td>Hot wires Clauser ch., Preston tube</td>
<td>1.0</td>
<td></td>
</tr>
<tr>
<td>Kreplin, Eckelmann</td>
<td>1979</td>
<td>Channel, oil</td>
<td>Hot-film $\delta U/\delta z$, hot-film on wall</td>
<td>1.05</td>
<td></td>
</tr>
<tr>
<td>Hooper, Harris</td>
<td>1982</td>
<td>Pipe</td>
<td>Hot-wires dP/dX, Preston tube</td>
<td>1.2</td>
<td></td>
</tr>
<tr>
<td>Murliss, Tsai, Bradshaw</td>
<td>1982</td>
<td>Flat plate, zero P.G.</td>
<td>Hot-wires Preston tube</td>
<td>1.05</td>
<td></td>
</tr>
<tr>
<td>Hancock, Bradshaw</td>
<td>1983</td>
<td>Flat plate, zero P.G.</td>
<td>Hot-wires Preston tube</td>
<td>1.28</td>
<td></td>
</tr>
<tr>
<td>Watson, Witt, Joubert</td>
<td>1984</td>
<td>Developing b/l, zero P.G.</td>
<td>Hot-wires Preston tube</td>
<td>700 1.18</td>
<td></td>
</tr>
</tbody>
</table>
Figure 4.8 Normal turbulence intensity vs Karman number - data of various workers
At this fixed value of $z/\delta_H$, the value of $z_+$ is a tenth of the Karman number. As can be seen in the figure, Karman numbers in excess of 100,000 are required before an asymptotic value is approached. As $\delta_l = 100v/\tau$, this means that at least 10 hierarchies of attached eddies are required for $\overline{w'^2}/\overline{u'^2}$ to reach its asymptotic value. It can be seen from the figure that:

a) Because of the large amount of scatter between the results of different workers, it is impossible to put a precise value to $A_3$. However, the trend suggests that $A_3$ has a value between 1.5 and 2.0.

b) Almost all the measurements in the survey lie at Karman numbers less than 10,000, and as such, an asymptote has yet to be reached. This is a limitation when measuring in wind tunnels and the only way to achieve high enough Karman numbers may be by measuring atmospheric boundary layers.

The large variation in $\overline{w'^2}/\overline{u'^2}$ between the results of the various workers is understandable in view of the limitations of X-hot wire anemometry at the present, especially when used to measure $w'$-turbulence. The present state of the art in measuring $\overline{w'^2}/\overline{u'^2}$ with hot-wires is inadequate, and this is a major obstacle in the formulation of a set of similarity laws for the turbulence quantities, especially the normal broad band turbulence intensities. As such a proposed set of similarity laws now exists, i.e., those proposed by Townsend and Perry and Chong, this factor now seems to be an obstacle to the verification of these laws.
The measurement of \( \frac{\overline{w'}^2}{U'_\tau} \) is particularly difficult to perform accurately, especially close to the wall in a boundary layer, due to various factors mentioned earlier. These factors are discussed in more detail here.

Spatial resolution

With a normal hot-wire, the spatial resolution is limited by the length of the filament, (and is in the lateral direction only), whereas with X-wires, where the inferred value of \( w' \) depends on the output from both hot-wires in the X-probe, the spatial resolution problem is compounded by the fact that it is now restricted by both filament length and the spacing of the filaments. According to Townsend's attached eddy hypothesis and the model of Perry and Chong, the main contribution to the \( w' \)-turbulence intensities at a distance \( z \) from the wall is from eddies of scale of order \( z \). As the wall is approached, the eddies that contribute directly to \( \overline{w'}^2 \) become smaller in scale, and thus the closer to the wall measurements are made, the more important is the spatial resolution of the hot-wires. As the fine scale detached eddies make a fractionally greater contribution to \( \overline{w'}^2 \) than the other components, it is especially important for the X-wires to be able to resolve these motions correctly when measuring \( \overline{w'}^2 \).
Cross contamination from other velocity fluctuations

It has also been shown earlier that X-contamination of the w' signal by v'-fluctuations can cause gross inaccuracies. Again, this problem is worse close to the wall as u' and v' are large compared to w'. Also u' and v' increase towards the wall, whereas w' is expected to be constant close to the wall, and therefore u' and v' become increasingly larger in magnitude compared with w' with decreasing z. Thus the errors due to cross contamination of w' increase towards the wall, and the fractional error to w' is much greater than the other components.

Cone Angle

In the measurements of rough wall boundary layers, the cone angle problem limits the accuracy of the w' measurements.

Prong effects

Prong thermal effects may cause a variation in hot-wire frequency response with frequency especially in the moderate to high wavenumber range, and there are significant contributions to $w'^2$ in this range (Perry 1982).

Therefore, to obtain accurate, asymptotic values of $\frac{w'^2}{U'_r^2}$, measurements must be made at very high Reynolds numbers (Kármán number in excess of 100,000) with perfect hot-wires that do not suffer from any of the problems mentioned above (and any other problems that may yet be discovered). Laser-doppler anemometry may be useful. In view of all that has been said, it is probably surprising that there is not even more scatter in the results presented in figure 4.8. For the present however,
it is proposed that the value of $A_3$ is tentatively 1.5 (although the reason for the choice of this value is hardly scientific but based on a consensus of data of various workers) and that the normal turbulence intensity distribution is given by

$$\frac{w'^2}{u'^2} = 1.5 - 8.1 z_+^{-1/2}$$

(4.4)

It must be noted that $-u'w'$ measurements seem to be relatively insensitive to the problems in measuring $w'$, and this was shown to be due to the fact that $-u'w'$ is relatively insensitive to cross contamination in section 2.6. Contributions to $-u'w'$ are also thought to be at wavenumbers low enough for spatial resolution not to be a serious problem (see section 2.3.2).

4.2.4 Lateral Turbulence Intensities

Figure 4.9 shows a plot for the lateral turbulence intensities plotted as $v'^2/U_\tau^2$ vs log $z/\delta_H$ for various values of $R_\theta$. The collapse of the data is fair. At the higher values of $R_\theta$ and in the turbulent wall region the data appear to collapse to a semi-logarithmic line. This distribution was predicted earlier in Chapter 2 and agrees with the predictions of Townsend and of Perry and Chong. However on close examination it can be seen that the profiles which depart from the expected semi-logarithmic line are those with low values of $R_\theta$ (and are close to the trip).
Figure 4.9 Lateral turbulence intensities
The predicted \( v' \)-turbulence intensity distribution are given by equation 3.51 where \( A_2 \) and \( B_2 \) were found to have values of 0.73 and 1.12 respectively. Thus

\[
\frac{v'^2}{U'^2} = 1.12 - 0.73 \ln(z/\delta_H) - 8.1 z_+^{-1/2} \quad (4.5)
\]

The distributions for the asymptotic case and for the highest and lowest \( R_\theta \)'s is also shown in figure 4.9. The agreement is fair although the data can be seen not to have reached an asymptotic value.

4.3 TURBULENCE SPECTRA

4.3.1 Longitudinal Turbulence Spectra

From Townsen's Reynolds number similarity hypothesis, the spectrum of the longitudinal energy containing components of the turbulent \( u' \)-motions in the fully turbulent region is given by

\[
\frac{\Phi_{11}(k_1 z)}{U'^2_\tau} = F_0(k_1 z, z/\delta_H) \quad (4.6)
\]

where \( \Phi_{11}(k_1 z) \) is the energy per unit non-dimensional wave number \( k_1 z \).

Longitudinal turbulence spectra were measured above the smooth wall at six (or more) values of \( z/\delta_H \) at each \( R_\theta \). These are shown plotted with "inner flow scaling" in figures 4.10(a)-(f).
Figure 4.10 (a), (b) $u'$-spectra, inner flow scaling
Figure 4.10 (c), (d) $u'$-spectra, inner flow scaling
Figure 4.10 (e),(f) $u'$-spectra, inner flow scaling
The data in figures 4.10(a)-(f) at low to moderate \( k_1 \cdot z \) are similar to the distributions predicted by the \( \Lambda \)-vortex model. For \( z/\delta_h \) sufficiently small, the experimental data and the predicted spectra each collapse to an inverse power law region at moderate wavenumbers. The low wave number cut-off of the energy containing region depends on the size of the largest attached eddy, which scales with \( \delta_h \), and at high wavenumbers, an inertial subrange is apparent in the experimental data. Such a region does not exist in the spectra predicted by the \( \Lambda \)-vortex model as this model did not take into account the fact that the attached eddies are surrounded by fine scale motions. The existence of an inertial subrange at these high wavenumbers can be derived only from dimensional arguments at the present state of knowledge. This dimensional argument also predicts that at very high wavenumbers, the spectra scaled with inner flow scaling peel off from the -5/3 power law at a point where \( k_1 \cdot z \) is proportional to \( z_+^{3/4} \). It can be seen from the figures that an inertial subrange is also present in the \( u' \)-spectra measured in the wake region scaled with inner flow scaling. The dimensional analysis argument of Perry and Abell (1977) was originally developed to be applied to flow in the turbulent wall region. Later work, (Perry, Henbest and Chong (1984)) has led to the belief that the wake region consists of fine scale detached eddies, the remains of what were once attached eddies, which have been stretched, distorted and convected away from the wall region. These fine scale motions would be expected to follow the Kolmogoroff scaling laws and thus an inertial subrange would be expected for the \( u' \)-wake-spectra (ie. region of overlap II). Figure 4.11 shows all \( u' \)-spectra measured in the wake region scaled with inner flow scaling. An inertial subrange is
Figure 4.11

$u'$-spectra, wake region

inner flow scaling
vivid.

Figures 4.10(a)-(f) show that the experimentally determined spectra follow the expected trends. At low $R_\theta$ the trends are not very obvious as $\zeta_H$ is small (and the smallest value of $z/\zeta_H$ at which spectral measurements are taken is limited) but at higher $R_\theta$'s the inverse power law region becomes more obvious as the number of hierarchies observed by the hot-wires increases. The inertial subrange and the various peel-offs also become more obvious as $z_+$ becomes sufficiently large.

Figures 4.12(a)-(f) show the spectra plotted as $\phi_{11}(k,\eta)/u^2$ vs $k_1\eta$ where $\eta$ and $u$ are the Kolmogoroff length and velocity scales respectively. The data collapse at very high $k_1\eta$ and this includes an inertial subrange.

4.3.2 Longitudinal Spectra - The Turbulent Wall Region

Over the smooth wall, the turbulent wall region has been tentatively defined as the region where

\[ z_+ > 150 \]
\[ z/\zeta_H < 0.15 \]

Figure 4.13 shows all spectra measured in the turbulent wall region scaled with inner flow scaling coordinates and figure 4.14 shows the same spectra scaled with outer flow scaling coordinates.
Figure 4.12 (a), (b) $u'$-spectra, Kolmogoroff scaling
Figure 4.12 (c), (d) $u'$-spectra, Kolmogoroff scaling
Figure 4.12 (e), (f) u'-spectra, Kolmogoroff scaling
With inner flow scaling, at low to moderate \( k_z \) the spectra collapse onto an inverse power law region. At low wave numbers, the spectra peel off from the inverse power law region at \( k_z \) proportional to \( z/\delta_H \) to a region of constant energy. At higher wave numbers the inertial subrange becomes evident by the existence of a region of slope \(-5/3\). The subsequent peel-offs from this region are expected to occur at a value of \( (k_z)^p \) which is proportional to \( z_+^{3/4} \) as was shown by dimensional arguments in Chapter 3. Figure 4.15 shows a plot of \( \log(k_z)^p \) vs \( \log(z_+) \) and the slope of the line seems to be \( 3/4 \) lending support to the result of the dimensional argument. The exact peel off point of the spectra from the \(-5/3\) power law is difficult to determine accurately due to the fact that the spectrum is not a "smooth" function and the peel off occurs gradually. Also the spectra are plotted on log-log axes and this makes it difficult to accurately read the value of \( (k_z)^p \). The peel off points in figure 4.15 were determined from spectra which were plotted individually on greatly expanded axes, but even so scatter is present. It should be mentioned here that the fact that the hot-wires have finite length can also influence the peel off point. As mentioned earlier, Wyngaard (1968) has shown that the highest wave number that can be resolved by a hot-wire filament of length \( \ell \) is given by \( k_\perp = O(1/\ell) \). Thus, if the fine scale motions present in the flow have scales of order less than \( \ell \), these will not be observed by the hot-wire and the high wavenumber peel off point will instead be at a wavenumber \( k_\perp z = z/\ell \). If this were the case, then the plot of \( \log(k_z)^p \) vs \( \log(z_+) \) will have a slope \(+1\). Figure 4.15 seems to suggest that the resolution limit of the 1.2mm hot-wire used is not being exceeded in this case.
Figure 4.13
$u'$-spectra, the turbulent wall region
inner flow scaling

Figure 4.14
$u'$-spectra, the turbulent wall region
outer flow scaling
With outer flow scaling, the spectra again collapse onto a region of slope \(-1\). The spectra also collapse in the very low wavenumber (constant energy) region. At higher wavenumbers the spectra peel off from the inverse power law region at a value of \(k_1 z\) proportional to \(\delta_H / z\). There is some scatter of the data at low wavenumbers, and this is thought to have been caused by the fact that Taylor's hypothesis of frozen turbulence (which was used to infer the convection velocity of the eddies) leads to errors in the inferred wavenumbers at low wavenumbers. This is because the large scale attached eddies are thought to have a higher convection velocity than the attached eddies of scale \(O(z)\). This will not affect the region of collapse to a line of slope \(-1\) as an error in inferred wave number would shift the spectra along a line of slope \(-1\).

Perry and Abell (1977) showed that given a spectrum function \(W(k,c)\)\(^\dagger\) in non-dimensional phase velocity-wavenumber space,

\[
W(k,c) = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{u'^2}{R(\delta,\tau)} e^{ik(\delta-\tau)} \, d\delta \, d\tau \tag{4.7}
\]

(where \(R(\delta,\tau)\) is the two point space-time correlation coefficient, \(\delta\) is the streamwise spacing of the sensing element and \(\tau\) is a time shift, eg. see Wills, 1964), the relationship between this function measured by an observer in laboratory coordinates, \(W_0(k,c)\), and that measured by an observer moving at velocity \(U\), is given by

\[
W_U(k,c) = W_0(k,c - U) \tag{4.8}
\]

\(^\dagger\) Here \(k\) is the longitudinal wavenumber and \(c\) is the phase velocity.

(Notation of Perry and Abell)
This means that the $W(k,c)$ contours translate without distortion for a change in velocity of the observer. They went on to show that the wall similarity hypothesis can be expressed as

$$\frac{W}{zU} = q(kz, \frac{c-U}{U})$$

Given that the small scale motions are convected at the local mean velocity $\overline{U}$, whereas the large scale motions are convected at a velocity nearer $U_1$, then, at a certain value of $kz$, a spread in convection velocity of order $(U_1 - \overline{U})/U_\tau$ would exist, and it can be seen that the larger $U_1/U_\tau$, the less the fractional spread in convection velocities. This fractional spread is the spread in convection velocities at low wavenumbers compared with the local mean velocity. This is shown schematically in figure 4.16. As $U_1/U_\tau$ approaches $\infty$, for example on a smooth wall at very high Reynolds numbers, Taylor’s Hypothesis is totally valid.

A crude simulation was carried out by Perry, Henbest and Chong (1984) to find out if the lack of collapse is caused by this spread in convection velocity of the attached eddies. Using their $\Lambda$-vortex model, a spectrum was calculated by Fourier decomposing the spatial variation in velocity. This spectrum is independent of the convection velocity and can be regarded as the "true" spectrum. They then recalculated the spectrum using the assumption that the convection velocity of a hierarchy of eddies of scale $\delta$ is invariant with $z$ and equal to the local mean velocity at $\delta/2$. The local mean velocity was calculated using equation 4.1 and the frequency spectrum as seen by a stationary probe at $z$ was
Figure 4.15 \( u' \)-spectra, \( (k_1z)_p \) vs \( z_+ \)

Figure 4.16
Conjectured contours of \( W/2U_\tau \) (From Perry and Abell, 1977).
then calculated using Taylor's hypothesis of frozen turbulence to obtain
the wavenumber. They found that this changed the shape of the "true"
spectrum and also caused the low wavenumber "shift", consistent with the
observed trends. This result is shown in 4.17(a) and (b).

Figure 4.18(a) and (b) show premultiplied $u'$-spectra in the
turbulent wall region scaled with inner and outer flow scaling
respectively. With inner flow scaling the spectra can be seen to
collapse at high wavenumbers and the low wavenumber peel off is
proportional to $z/\delta_H$. With outer flow scaling, the spectra collapse at
low wavenumbers and the peel off at high wavenumbers is proportional to
$\delta_H/z$. This is in accordance with the trends predicted by the extended
A-vortex theory of Perry and Chong (Section 3.3). The premultiplied
spectra also show the "bump" at low wavenumbers, lending support to the
theory that there are more large scale eddies present than are expected from the simple inverse power law p.d.f. of eddy scales, and justifying
the use of the weighting function (Section 3.1.2).

The spectra in the wall region are shown plotted with Kolmogoroff
scaling in figure 4.19 and an inertial subrange is present at higher
values of $R_e$. 
Figure 4.17 Shift of spectra caused by error in assumed convection velocity
**Figure 4.18 (a)**
Premultiplied $u'$-spectra
Turbulent wall region, inner flow scaling

**Figure 4.18 (b)**
Premultiplied $u'$-spectra
Turbulent wall region, outer flow scaling
Figure 4.19

$u'$-spectra, the turbulent wall region

Kolmogoroff scaling
4.3.3 Normal Turbulence Spectra

Normal turbulence spectra were measured above the smooth wall at six values of $z/\delta_H$ at each of the $R_\theta$'s. Again, following Townsend's Reynolds number similarity hypothesis, the spectra of the normal energy containing motion is given by

$$\Phi_{33}(k_1z) = G_0(k_1z, z/\delta_H)$$

(4.10)

The spectra are shown plotted for each $R_\theta$ in figures 4.20(a)-(f) with inner flow scaling.

If figures 4.20(a)-(f) were superimposed it will be seen that the Townsend Reynolds number similarity hypothesis holds in the main energy containing region. At low $R_\theta$'s the Townsend Reynolds number similarity hypothesis breaks down at high $k_1z$ due to the effect of viscosity becoming important. At the higher $R_\theta$'s, it can be seen that the spectra ultimately collapse onto a line of slope $-5/3$ indicating the presence of an inertial subrange. At very high wavenumbers, the spectra peel-off from the $-5/3$ region due to viscosity. As $R_\theta$ decreases, the extent of the $-5/3$ region decreases and ultimately disappears and is replaced by a $-5/3$ envelope from which the spectra peel off. In accordance with the theory, no inverse power law region is present.

The normal spectra are shown plotted with Kolmogoroff scaling in figures 4.21(a)-(f). The data collapse onto an inertial subrange at the higher $R_\theta$'s but its extent decreases with decreasing $R_\theta$ until it eventually disappears at the two lowest $R_\theta$'s.
Figure 4.20 (a), (b) $w'$-spectra, inner flow scaling
Figure 4.20 (c), (d) $w'$-spectra, inner flow scaling
Figure 4.20 (e), (f) $w'$-spectra, inner flow scaling.
Figure 4.21 (a), (b) $w'$-spectra, Kolmogoroff scaling
Figure 4.21 (c), (d) $w'$-spectra, Kolmogoroff scaling

- $R_0 = 4245$
- $R_0 = 5111$

\[ \frac{\phi_{33}(k_1 n)}{u^2} \]
Figure 4.21 (e), (f) w'-spectra, Kolmogoroff scaling
4.3.4 Normal Spectra - The Turbulent Wall Region

All normal spectra measured in the turbulent wall region are shown plotted with inner flow scaling in figure 4.22.

From the predictions of Perry and Chong's $\Lambda$-vortex model (see section 3.3), the spectra should collapse onto a single universal curve when plotted using this scaling (with possible high wave number peel-offs from the inertial subrange due to the effects of viscosity as expected from the dimensional analysis arguments). As can be seen in figure 4.22 the spectra do not collapse well and it is felt that this is caused by cross contamination of the $w'$ signals by $v'$ fluctuations. This theory is substantiated by the behaviour of the normal broad-band turbulence intensities (see Section 4.2.3). Unfortunately, all of these spectral results were obtained before it was realised that possible hot-wire bowing and other probe geometrical imperfections could cause significant errors in $w'^2/U_2$. The region of slope $-5/3$ is short and of course no inverse power law collapse is present.

Figure 4.23 shows the normal spectra plotted with outer flow scaling in the wall region. No collapse is predicted by the theory and the data is consistent with this. At high wavenumbers, the spectra at each $R_\theta$ collapse, and this seems to indicate a spatial resolution problem, since for each $R_\theta$ case, $\delta_H/t$, (where $t$ is the hot-wire filament length) is constant. The spectra were then re-plotted scaled with $t$, and this is shown in figure 4.24. The improvement in collapse at high wavenumbers with this scaling seems to confirm that the spatial resolution problem is
Figure 4.22
$w'$-spectra, the turbulent wall region
inner flow scaling

Figure 4.23
$w'$-spectra, the turbulent wall region
outer flow scaling
present. Figure 4.25 shows the premultiplied $w'$-spectra scaled with inner flow scaling coordinates. The theory predicts that the spectra should collapse at low to high wavenumbers but the collapse in the figure is poor.

Figure 4.27 shows a plot of $\log(k z)$ vs $\log z_+$ and it appears to confirm that the X-wires used were suffering from a spatial resolution problem as the fit to the data has a slope of +1.

Figure 4.26 shows the normal spectra in the turbulent region plotted with Kolmogoroff scaling. The spectra collapse at high wave numbers but a region of slope $-5/3$ is not readily apparent. This could be due to the inadequate spatial resolution of the X-wires.

It should be pointed out here that the $w'$ spectra show fair collapse in spite of the fact that the normal turbulence intensities, $\frac{w'^2}{U_\tau^2}$ may have been in error. A slight error in $\frac{w'^2}{U_\tau^2}$ will not be readily apparent on log-log axes.

4.3.5 Lateral Turbulence Spectra

According to the theory of Perry and Chong and the Townsend attached eddy hypothesis, the lateral turbulence spectra should follow similar scaling laws to the longitudinal spectra (see Section 3.3). Figures 4.28(a)-(f) show the lateral spectra plotted according to inner flow scaling and confirms that the lateral spectra do in fact follow the same
Figure 4.24
\( \frac{\phi_{33}(k_1 \ell)}{u^2_\tau} \)

Wall region

Figure 4.25
\( \frac{k_1 z \phi_{33}(k_1 z)}{u^2_\tau} \)

Premultiplied \( w' \)-spectra
turbulent wall region, inner flow scaling
Figure 4.26
$w'$-spectra, turbulent wall region
Kolmogoroff scaling

Figure 4.27 $w'$-spectra, $(k_1 z)_p$ vs $z_+$
scaling laws as the longitudinal spectra. The peel offs of the spectra at low wavenumbers occur at $k_z \propto z/\delta_H$ and for $z/\delta_H$ sufficiently small, an inverse power law region becomes evident. At high wavenumbers all spectra (including those measured in the wake) collapse onto an inertial subrange.

It will be seen that the regions of $-1$ and $-5/3$ slope in the lateral spectra are not as distinct in the longitudinal spectra.

Figures 4.29(a)-(f) show the lateral spectra at each $R_e$ plotted with Kolmogorov scaling. The spectra collapse to an inertial subrange.

4.3.6 Lateral Spectra - The Turbulent Wall Region.

Figure 4.30 shows all lateral spectra measured in the turbulent wall region plotted with inner flow scaling, and figure 4.31 shows the same spectra plotted with outer flow scaling.

With inner flow scaling the lateral spectra show reasonable collapse onto a short inverse power law region. The reason for the limited extent of the inverse power law region may have been due to the fact that with X-wires, the lowest values of $z/\delta_H$ measured were larger than with the normal wire and thus the number of hierarchies observed by the X-wires would be less than in the case of the $u'$-spectra. As a consequence, the length of the inverse power law region would be limited by the minimum value of $z/\delta_H$ practical using the X-wires. At low wave numbers, there is
Figure 4.28 (a), (b) \( v' \)-spectra, inner flow scaling
Figure 4.28 (c), (d) $v'$-spectra, inner flow scaling
Figure 4.28 (e),(f) $v'$-spectra, inner flow scaling
Figure 4.29 (a), (b) $v'$-spectra, Kolmogoroff scaling.
Figure 4.29 (c), (d) $v'$-spectra, Kolmogoroff scaling
Figure 4.29 (e),(f) $v'$-spectra, Kolmogoroff scaling

\[ \frac{\phi_{22}(k_1 n)}{v^2} \]

\[ R_0 = 8661 \]

\[ R_0 = 12059 \]
a peel off proportional to $z/\delta_H$ to a region of constant energy, and at the highest wave numbers the spectra peel off from the inertial subrange.

With outer flow scaling, the lateral spectra show fair collapse to an inverse power law region. At low wavenumbers the collapse is not as good as that with the longitudinal spectra. This may have been due to errors in inferred convection velocity discussed earlier. The high wavenumber collapse seems to depend on $\delta_H$, and this suggests a spatial resolution problem, as with the $w'$-spectra. The spectra are shown scaled with $\ell$ in figure 4.32, and the improvement in collapse at high wavenumbers confirms that this problem exists. Figure 4.33 shows a plot of $\log(kz)$ vs $\log z$ for the lateral spectra and further confirms that a spatial resolution problem does exist.

Figure 4.34 shows the lateral spectra in the wall region plotted with Kolmogoroff scaling. The spectra collapse onto a universal region of slope $-5/3$ at high wavenumbers at the higher $R_e$'s indicating weakly the presence of an inertial subrange.

Figures 4.35(a) and (b) show premultiplied $v'$-spectra with inner and outer flow scaling respectively. With inner flow scaling, the spectra can be seen to collapse at high wavenumbers and to peel off at $kz$ proportional to $z/\delta_H$ at low wavenumbers, whereas with outer flow scaling, the collapse is at low wavenumbers with peel off at high wavenumbers. This behaviour is in accordance with the A-vortex theory, although the "bump" present at low wavenumbers in the $u'$-spectra is not obvious here. The shape of these spectra is influenced by many factors besides the
Figure 4.30
$v'$-spectra, turbulent wall region
inner flow scaling

Figure 4.31
$v'$-spectra, turbulent wall region
outer flow scaling
Figure 4.32
$v'$-spectra, turbulent wall region
scaled with wire length

Figure 4.33
$v'$-spectra, $(k_2 z)_{p}$ vs $z_{+}$
Figure 4.34

$v'$-spectra, turbulent wall region
Kolmogoroff scaling
Figure 4.35 (a)
Premultiplied v'-spectra
turbulent wall region, inner flow scaling

Figure 4.35 (b)
Premultiplied v'-spectra
turbulent wall region, outer flow scaling
weighting function $W(\delta/\delta_H)$. The velocity signature shape also plays a part and it is obvious from these pre-multiplied spectral results that there is insufficient spread in hierarchy scales to give a convincing inverse power law region, although on log-log plots, an inverse power law region is apparent. More work is required to justify the spectral laws proposed in Chapter 3, and data at much higher Reynolds numbers is needed.

\[\text{Also, as } z/\delta_H \text{ is insufficiently small, the inverse power law region ("flat" region on premultiplied spectrum) is short, making it difficult to verify the existence of a "bump". Measurements closer to the wall are required.}\]
CHAPTER 5

The Rough Wall Results

Measurements of the mean flow, broad-band turbulence intensities and their spectral distributions were carried out over the mesh roughness with zero pressure gradient. The geometry of the roughness has been described in Section 2.2. As described in Chapter 2, the values of $U_T$ obtained using the Hama velocity-defect-law fit to the mean velocity profiles did not agree with those obtained from the Reynolds shear stresses measured with $s=120^\circ$ X-wires. Some possible reasons for the discrepancy are discussed and some experimental evidence is presented to show that the values for the shear stress obtained with X-wires may be more correct under the conditions of these tests.

5.1 THE MEAN FLOW

The pressure gradient in the working section of the wind tunnel for free stream velocities of 10 m/s and 20 m/s are shown in figure 5.1 as a plot of $C_p$ vs $x$, where $x$ is the distance behind the leading edge of the roughness. A check for secondary flows over the roughness was also made and this is presented as a transverse mean velocity profile $\overline{V/U}$ vs $y$ at $x=1750$mm in figure 5.2. The secondary flow was found to be negligible.
Figure 5.1 $C_p$ vs $x$

Figure 5.2 Lateral mean velocity profile
Mean velocity profiles were measured at 3 stations (x=600mm, 1100mm and 2500mm) at a free stream velocity of 10 m/s and at 6 stations (x=500mm, 600mm, 1100mm, 1500mm, 1900mm, 2500mm, and 3000mm) at 20 m/s. This enabled $U_\tau$ to be also estimated using a momentum balance. Broad-band turbulence intensities and spectra were measured at the x=600mm and 2500mm stations at $U_\tau=10$ m/s and 20 m/s.

The relevant $R_\theta$ cases are

<table>
<thead>
<tr>
<th>x (mm)</th>
<th>$U_\tau$ (m/s)</th>
<th>$R_\theta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>600</td>
<td>10</td>
<td>2720</td>
</tr>
<tr>
<td>1100</td>
<td>10</td>
<td>4059</td>
</tr>
<tr>
<td>2500</td>
<td>10</td>
<td>7285</td>
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<td>5563</td>
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<tr>
<td>1100</td>
<td>20</td>
<td>7984</td>
</tr>
<tr>
<td>2500</td>
<td>20</td>
<td>14215</td>
</tr>
</tbody>
</table>

Table 5.1

The mean velocity profiles are presented in figures 5.3 and 5.4 as a velocity defect. In figure 5.3, $U_\tau$ has been obtained using the Hama velocity defect law fit to the data described in section 2.1.3. In figure 5.4, $U_\tau$ has been obtained from the extrapolated Reynolds shear stresses measured with $\beta=120^\circ$ X-wires.

In the case of figure 5.3, the collapse is expected as the values of $U_\tau$ and $\tau$ (see figure 2.11) have been adjusted to force the mean velocity profile to fit the Hama velocity defect law, equation 2.5. In figure 5.4
Figure 5.3 Mean velocity profiles - defect law plot

\[ \frac{u_1 - U}{U_\tau} \]

(Hama velocity defect law \( U_\tau \))

\[ \frac{z}{\delta_B} \]

\( + R_\theta = 2720 \)
\( \times R_\theta = 4059 \)
\( \diamond R_\theta = 7984 \)
\( \Delta R_\theta = 7285 \)
\( \circ R_\theta = 14215 \)
Figure 5.4 Mean velocity profiles - defect law plot
the collapse is also quite good, and ε was adjusted such that U_τ deduced from the extrapolated values of the Reynolds shear stresses led to a semi-logarithmic region with a slope of 1/κ, where κ=.41. However, the intercept at z/H =1 is 2.75 instead of 2.309 as given by the Hama law. If the β=120° X-wire U_τ's are to be believed, the larger intercept means that the wake is larger than expected (perhaps after being tripped by the leading edge of the roughness) and the flow has yet to recover to its "equilibrium state", which is a necessary condition for application of the Hama velocity defect law. Some evidence of this can be seen by the fact that \( \overline{-u'w'} \) agrees better with the Hama velocity defect law value of U_τ as the development length of the flow (x-distance) increases, and also by the fact that the values of \( \frac{\overline{-u'w'}}{U_τ^2} \) at each station are similar regardless of the free stream velocity. Here, U_τ was determined from the Hama velocity defect law. For an "equilibrium" profile on a smooth wall, the Kármán number, \( \frac{\delta U_τ}{v} \) must be sufficiently large and on a rough wall δ_H/k must be sufficiently large. Both these quantities are a measure of the number of hierarchies present. The larger the number of hierarchies, the more "well developed" the flow is and the closer the behaviour of the flow to these established similarity laws.

Figure 5.5 shows a comparison of U_τ's (shown as \( \sqrt{C_τ'/2} \)) obtained using three independent methods; momentum balance, Hama velocity defect law fit and the Reynolds shear stress measured with β=120° X-wires at a free stream velocity of 20 m/s. The momentum balance seems to agree quite well with the Reynolds shear stress. The shortcomings of the momentum integral method have already been pointed out in Chapter 2, one being the sensitivity of the method to secondary flows. However, in this
instance the result has some credibility as the secondary flow has been
found to be very small. As mentioned earlier a similar mesh roughness
has been tested by Acharya and Escudier (1984), who used \( \beta = 90^\circ \) and \( \beta = 120^\circ \)
X-wires, the Hama velocity defect law fit and a drag balance to estimate \( U_t \).
Their finding was that the value of \( U_t \) as determined by the \( \beta = 120^\circ \) X-wire
Reynolds shear stresses agreed with the values obtained with the drag
balance. They also found that the Hama velocity defect law fit to the
mean velocity profiles overestimated the value of \( U_t \) by about 20\%.
This is in complete agreement with the findings presented here.

Figure 5.6 shows the Hama (1954) roughness function \( \Delta U/U_t \) plotted
against \( kU_t/\nu \) and \( \delta U/H_t/\nu \), (Hama value of \( U_t \)) where

\[
\frac{\Delta U}{U_t} = 5.616 \log \frac{\delta U}{\nu} + A - \frac{\Delta U}{U_t} \quad (5.1)
\]

Perry, Schofield and Joubert (1968) classified roughnesses into "k"- and
"d"-type roughnesses, where "k"-type roughness gives a roughness function
for fully rough flow of

\[
\frac{\Delta U}{U_t} = 5.616 \log \frac{kU_t}{\nu} + B \quad (5.2)
\]

where \( B \) is a constant characteristic of the roughness geometry for the
fully rough regime, whereas for "d"-type roughness the roughness function
follows the law

\[
\frac{\Delta U}{U_t} = 5.616 \log \frac{\delta U}{H_t/\nu} + C \quad (5.3)
\]
where $C$ is a constant characteristic of the roughness geometry. Results have been correlated in both schemes and are shown in figure 5.6. It can be seen that the mesh roughness appears to be "k"-type roughness.

In all plots of broad-band turbulence intensities and spectra plotted in this Chapter, the distance normal from the mesh, $z$, is equal to $(z_T +t)$ (see figure 2.11).

5.2 BROAD-BAND TURBULENCE INTENSITIES

The broad band turbulence intensities $u'^2$, $v'^2$, $w'^2$, and $-u'w'$ were measured using dynamically matched and calibrated hot-wires at the two free stream velocities at the $x=600\text{mm}$ and $2500\text{mm}$ stations. A normal hot-wire was used to measure $u'^2$ and $\beta=90^\circ$ X-wires was used to measure the other quantities. Unfortunately the problem with the $\beta=90^\circ$ X-wires had not yet been discovered when these measurements were taken. Measurements of $-u'w'$ were later repeated with $\beta=120^\circ$ X-wires.

5.2.1 The Reynolds Shear Stresses

The Reynolds shear stresses are presented here non-dimensionalised with $U_T$ obtained from the Hama velocity defect law fit. Figure 5.7(a) shows the Reynolds shear stresses obtained using the $\beta=90^\circ$ X-wires and figure 5.7(b) shows the Reynolds shear stresses obtained with $\beta=120^\circ$ X-wires. The Reynolds shear stresses measured with $\beta=120^\circ$ X-wires are
Figure 5.5
Comparison of $\sqrt{C_f'/2}$ obtained with different methods

Figure 5.6
The roughness function for the mesh roughness
Figure 5.7 (a) Reynolds shear stress profiles
Figure 5.7 (b) Reynolds shear stress profiles

(\(\beta=120^\circ\) X-wire)

(Hama velocity defect law \(U_\tau\))
substantially higher than those measured with the $\beta=90^\circ$ X-wires, although still below the value given by the Hama velocity defect law. It is felt that the Reynolds shear stresses measured with $\beta=120^\circ$ X-wires give the correct value of $U_\tau$, and this is supported by independent tests performed by Acharya and Escudier (1984). (See Section 2.5).

5.2.2 Longitudinal Turbulence Intensities

Figure 5.8(a) shows a plot of $u'/U_\tau^2$ vs log $z/\delta_H$ for the four $R_8$'s measured over the mesh roughness, non-dimensionalised with the Hama value of $U_\tau$ and figure 5.8(b) shows a plot of $u'/U_\tau^2$ vs log $z/\delta_H$ where $U_\tau$ in this case is that obtained from the $\beta=120^\circ$ X-wires. The collapse of the data seems to be better in figure 5.8(b), lending further support to the claim that the Reynolds shear stresses give a more realistic estimate of $U_\tau$. The $u'$-turbulence intensity distribution predicted by integrating the $u'$-spectral laws is

$$\frac{u'/U_\tau^2}{2} = 2.01 - 1.26 \ln(z/\delta_H) - 7.50 z^+ - 1/2$$

(5.4)

where $B_1=2.01$, $A_1=1.26$ and $C_1=7.50$ were determined from the rough wall spectral data. A comparison of the smooth and rough wall constants in table 5.2 shows that there are slight differences between the smooth- and rough-flow constants.
Figure 5.8 (a) Longitudinal turbulence intensities

(Normal hot-wire)

(Hama velocity defect law $U_\theta$)
Figure 5.8 (b) Longitudinal turbulence intensities
The reasons for these slight differences is not known. Perhaps at these low values of $\delta_H/k$, the turbulence structure has not yet reached its asymptotic state and the eddy shapes are still being influenced by the roughness geometry. Much higher Kármán numbers are required for sufficient hierarchies to exist, and for the shape of the eddies on a rough wall to approach those on a smooth wall. At these high Kármán numbers the spectral laws for the smooth- and rough-wall flows would be expected to become universal. The predicted distribution is shown in figure 5.8(b) for the asymptotic case i.e. $z_+ = \infty$, the highest $R_e$ case and the lowest $R_e$ case. The predicted distribution seems to fit the $R_e = 14215$ case reasonably but the turbulence intensity deviates from the prediction when $z/\delta_H < 0.1$. This could be because $z$ is not sufficiently greater than $\delta_1$. In section 3.2 where the A-vortex model of Perry and Chong was used to determine the broad band turbulence intensities, it was always assumed that $\delta_1$ was sufficiently small so as not to enter the calculation, i.e., for very high Reynolds number smooth wall flow. In this case however, $\delta_1$, the scale of the smallest hierarchy will scale with the scale of the mesh roughness and may no longer be less than $z$ for values of $z/\delta_H < 0.1$. It is therefore conceivable that when measuring

<table>
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<th>Rough</th>
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<tbody>
<tr>
<td>$A_1$</td>
<td>1.02</td>
<td>1.26</td>
</tr>
<tr>
<td>$B_1$</td>
<td>2.48</td>
<td>2.01</td>
</tr>
<tr>
<td>$C_1$</td>
<td>6.08</td>
<td>7.50</td>
</tr>
<tr>
<td>$K_0$</td>
<td>0.50</td>
<td>0.62</td>
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</table>

Table 5.2
close to the wall, some non-geometrically similar eddies in the first hierarchy are being seen by the hot-wire. In the integration scheme of Section 3.2 the expected broad-band turbulence intensity will be correct as long as \( z > \delta_h \), i.e., \( \lambda_1 < 0 \). Figure 5.9 shows the \( u' \)-turbulence intensity distribution expected using the integration scheme of Section 3.2 when \( z < \delta_h \), i.e., when \( \lambda_1 > 0 \). As can be seen, for \( \lambda_1 > 0 \), the model predicts that the turbulence intensities will be constant with decreasing \( z/\delta_h \). In reality however, this will probably not be the case, but as the exact details of the non-geometrically similar eddies of height less than \( \delta_h \) are not known, the turbulence intensity distribution will probably lie somewhere in the shaded area shown in figure 5.9. In figure 5.8(b), it can be seen that the experimental turbulence intensities appear to be constant for \( z/\delta < 0.1 \). It is tempting to attempt to predict \( \delta_h \) from this "peel-off to a constant turbulence intensity" region, but other factors such as viscosity and inadequate spatial resolution of the hot-wire may also be involved. The usual plot of \( \log(kz) \) vs \( \log(z) \) in figure 5.16 seems to suggest that spatial resolution of the hot-wire is adequate, but as mentioned earlier, the determination of \( (kz) \) is notoriously inaccurate.

5.2.3 Normal Turbulence Intensities

Figures 5.10(a) and (b) show plots of \( w'^2/U_\tau^2 \) vs \( \log(z/\delta_h) \) over the mesh roughness. In figure 5.10(a), \( w'^2 \) has been non-dimensionalised with \( U_\tau \) obtained by the Hama velocity defect law fit and figure 5.10(b) shows the data non-dimensionalised with the \( \beta=120^\circ \) Reynolds shear stress \( U_\tau 's \). The data seem to collapse slightly better in figure 5.10(b). However, it
Figure 5.9

Longitudinal turbulence intensity distribution predicted by A-vortex model for $z < \delta_1$
Figure 5.10 (a) Normal turbulence intensities
Figure 5.10 (b) Normal turbulence intensities
should be pointed out that the $w'^2$ in this case were obtained using $\beta=90^\circ$ X-wires and errors due to excessive velocity vector cone angles can be large under the conditions of this test (see Chapter 3). Therefore the data in figure 5.10(b) should not be taken as absolutely correct but to show trends. Some $w'^2$ profiles were also measured using $\beta=120^\circ$ X-wires but unfortunately, experience has shown that the $\beta=120^\circ$ X-wires are much more sensitive to roll misalignment and subsequent cross-contamination from $v'$ fluctuations than the $\beta=90^\circ$ X-wires and the $w'^2$ measurements could (and did) vary wildly although they appeared to give the correct value for the Reynolds shear stress. However, for completeness, these results are shown in figure 5.10(c). Reasonable collapse is obtained only between data measured at the same station.

The $w'$-turbulence intensity distribution obtained from the integration of the spectral laws where $A_3$ was found to have a value of 1.78 is

$$\frac{w'^2}{U^2} = 1.78 - 10.0 \ z^+^{-1/2}$$

and this distribution is shown in figure 5.10(b).

However, as mentioned in section 4.2.3, $w'^2/U^2$ is difficult to measure accurately close to the wall, and when measuring over the mesh roughness, these X-wires are known to suffer from the cone angle problem. As shown later in section 5.3.4, the spatial resolution of this probe is also inadequate to resolve the fine scale motions in the flow. The Kármán number range of these measurements is also not high enough to
Figure 5.10 (c) Normal turbulence intensities
approach the asymptotic value for the $w'$-turbulence intensities. Therefore, it would be unwise to attempt to draw any conclusions regarding the $w'$-turbulence intensity distributions with these results. The data for this case is also shown in figure 4.8 with the results of other workers, and although known to be inaccurate, lies in approximately the right area on the plot.

5.2.4 Lateral Turbulence Intensities

Lateral turbulence intensities were measured using $\beta=90^\circ$ X-wires matched and calibrated dynamically. Figure 5.11(a) shows a plot of $v'^2/U^2_\tau$ (U\_\tau from Hama method) vs log $z/\delta_H$, and figure 5.11(b) shows $v'^2/U^2_\tau$ (U\_\tau from $\beta=120^\circ$ X-wire) vs log $z/\delta_H$. It is difficult to tell if any improvement in collapse of the data has occurred when scaled with the $\beta=120^\circ$ X-wire U\_\tau's. The predicted $v'$-turbulence intensity distribution using the spectral laws for the asymptotic case ($z_+ \to \infty$), and the highest and lowest $R_\theta$ cases is shown in figure 5.16. The values of $A_2=0.63$ and $B_2=1.08$ were determined by integration of the spectra and thus the $v'$-turbulence intensity distribution is given by

$$\frac{v'^2}{U^2_\tau} = 1.08 - 0.63 \ln(z/\delta_H) - 10.0 z_+^{-1/2}$$

(5.6)
Figure 5.11 (a) Lateral turbulence intensities

\( \frac{v^2}{u^2} \)

- \( R_0 = 2720 \)
- \( R_0 = 5563 \)
- \( R_0 = 7285 \)
- \( R_0 = 14215 \)

\( \beta = 90^\circ \) X-wire

(Hama velocity defect law \( U_1 \))

\( \frac{z}{\delta_H} \)
Predicted distributions

\[ \frac{v'^2}{u'^2} = \frac{1}{2} \left( \frac{R_0}{R} \right)^{2/3} \]

Figure 5.11 (b) Lateral turbulence intensities

\( \beta = 90^\circ \) X-wire

\( \beta = 120^\circ \) X-wire \( u' \)
behave in a similar manner to the $u'^2/u_t^2$ turbulence intensities, probably due to similar reasons as the $u'$-turbulence intensities, ie $z<\delta_1$.

5.3 TURBULENCE SPECTRA

As mentioned earlier, according to the theory of Perry and Chong, $\delta_1$, the scale of the smallest hierarchy, should scale with $k$, the roughness scale and it is expected that $\lambda_0$, the lateral spacing of the smallest hierarchy, scales with the lateral spacing of the roughness. Therefore the height of the smallest hierarchy over the mesh would be larger than over the smooth wall since the lateral scale of the roughness is large and only a small range of scales of hierarchies would be expected. For a fully rough surface, the height of the smallest hierarchy should be constant over the surface. Thus, the range of hierarchy scales present over any single rough surface would then depend on $\delta_H'$, the boundary layer thickness. In fact, at the $x=600\text{mm}$ station, where $\delta_H$ is approximately 38mm, one would expect only 1 or 2 hierarchies, (in terms of discrete hierarchies) and at the $x=2500\text{mm}$ station, where $\delta_H$ is approximately 95mm, only 2 or 3 hierarchies would be expected. Because of the small number of hierarchies, the length of the inverse power law region of collapse of the spectra scaled with inner flow scaling will be much shorter than that over a smooth wall. Also, in the case of a smooth wall, it is possible to take measurements at much smaller values of $z/\delta_H$ without $z$ becoming less than $\delta_1$ (when compared with the rough wall).
5.3.1 Longitudinal Turbulence Spectra

Following the extended form of the Townsend Reynolds number similarity hypothesis, the spectrum of the longitudinal energy containing motion in the fully turbulent region is given by

\[
\frac{\Phi_{11}(kz)}{U^2} = F_0(kz, z/\delta_H) \tag{5.7}
\]

The longitudinal spectra are presented in figures 5.12(a)-(d). The spectra are similar to the smooth wall spectra as expected in that there is a region of constant energy at low wavenumbers, a collapse onto a line of slope -1 for \( z/\delta_H \) small (a short region due to the small number of hierarchies), a region of collapse onto a line of slope -5/3 (the inertial subrange), and then peel-offs from the -5/3 line due to the effect of viscosity.

As the boundary layer thickness \( \delta_H \) at each station did not change significantly with a change in free stream velocity, i.e. the flow was fully rough, and hence the general features of the flow are independent of Reynolds number, the number of hierarchies at each station would also not be expected to change with a change in free stream velocity. This can be seen in the figures, where the spectra at different free stream velocities at each station are very similar and in fact, if superimposed, they are virtually indistinguishable from one another except at very high wavenumbers. At the \( x=600\text{mm} \) station (\( Re_9 = 2720, 5563 \)), the -1 collapse is barely discernible, whereas at the \( x=2500\text{mm} \) station (\( Re_9 = 7285, 14215 \)), it is noticeably longer because more hierarchies exist. This is consistent
Figure 5.12 (a), (b) $u'$-spectra, inner flow scaling
Figure 5.12 (c), (d) u'-spectra, inner flow scaling
with the theory of Perry and Chong. The spectra are shown plotted using Kolmogoroff scaling in figures 5.13(a)-(d) and the data collapse at high $k_1$ and this includes a region of slope $-5/3$ as expected.

5.3.2 Longitudinal Turbulence Spectra - The Turbulent Wall Region

From the analysis of the spectra on a smooth wall and assuming that attached eddies are also present over rough walls, the spectra over the rough surface would be expected to follow scaling laws similar to those over a smooth surface.

Over the mesh roughness the closest distance to the mesh measured with a normal wire probe was 2.0mm from the tops of the roughness elements and this corresponded to $z=3.5\text{mm}$ as the apparent origin was 1.5mm below the tops of the roughness elements. It is felt that at this distance from the wall, wall viscosity effects would be negligible and the turbulent wall region is tentatively defined for the mesh as

$$z > 3.5\text{mm}$$

$$z < 0.15\frac{H}{H}$$

All spectra measured in the wall region are plotted with inner flow scaling in figure 5.14. As the length of the $-1$ collapse depends on the number of hierarchies observed, it would be expected that the spectra measured at the $x=2500\text{mm}$ station would show a longer inverse power law region than that measured at $x=600\text{mm}$ regardless of the free stream
Figure 5.13 (a), (b) $u'$-spectra, Kolmogoroff scaling
Figure 5.13 (c), (d) $u'$-spectra, Kolmogoroff scaling
This can be seen in figure 5.14 where two groups of peel-offs occur at low wavenumbers. The region of slope -1 is short as expected due to the small number of hierarchies, and there is a region of -5/3 collapse indicating the presence of an inertial subrange. It should be noted also that the length of the -5/3 collapse of the $u'$-spectra is longer than that on a smooth surface, and this is consistent with the theory as $zU_t/\nu$ is larger on a rough wall for a fixed $z/\delta_R$. High wavenumber peel-offs due to the effects of viscosity can also be seen. Figure 5.17 shows a plot of $\log(k_{1z})_p$ vs $\log(z_+)$ for these spectra. The probe appears to be capable of resolving the fine scale motions in the flow.

At low $k_{1z}$, the inner flow scaling law must break down due to the large scale flow geometry and the spectra would then be expected to follow an outer flow scaling law.

The spectra are shown plotted with outer flow scaling coordinates in figure 5.15. Using this scaling, the low wavenumber collapse is not as good as that obtained over the smooth wall and this can be attributed to errors in inferred wavenumber. As mentioned earlier in section 4.3.2, Perry and Abell showed that the fractional spread in convection velocity, and hence the fractional error made in inferred wavenumber by the use of Taylor's hypothesis depends on $U_1/U_\tau$, and that as $U_1/U_\tau \to \infty$, Taylor's hypothesis is totally valid. On a rough wall, the roughness causes a decrement in $U_1/U_\tau$ ($=\Delta U/U_\tau$, the roughness function), and this causes the fractional error in inferred wavenumber on a rough surface to be greater than that over a smooth surface at the same Reynolds number. Increasing the Reynolds number increases $\Delta U/U_\tau$, and the error increases. (See figure 4.16)
Figure 5.14
\( u' \)-spectra, turbulent wall region
inner flow scaling

Figure 5.15
\( u' \)-spectra, turbulent wall region
outer flow scaling
Figure 5.16 shows the spectra in the wall region plotted using Kolmogorov scaling. The spectra collapse onto a region of slope $-5/3$ indicating the presence of an inertial subrange consistent with the Kolmogoroff theory.

5.3.3 Normal Turbulence Spectra

The spectrum of the main energy containing components in the fully turbulent region according to the Townsend Reynolds number similarity hypothesis is given by equation 4.10. Plots of the normal spectra with inner flow scaling are shown for each separate $R_e$ in figures 5.18(a)-(d). As expected from the theory, the spectra close to the roughness (the turbulent wall region), collapse over all wavenumbers except in the dissipation region. The form of the curve is exponential-like. The low wavenumber peel offs of the spectra in the wake region depend on $z/\delta_H$ and are independent of viscosity in the energy containing region, in accordance with the Townsend Reynolds number similarity hypothesis. At higher wave numbers, the spectra collapse to a $-5/3$ law with subsequent viscous controlled peel-offs. The region of slope $-5/3$ extends as $R_e$ increases.

The normal spectra are plotted with Kolmogoroff scaling in figures 5.19(a)-(d). The collapse of the spectra onto a line of slope $-5/3$ is obvious only in the higher Reynolds number cases.
Figure 5.16
\[ \frac{\phi_{11}(k_1 n)}{u^2} \]

\[ k_1 n \]

Figure 5.17
\[ (k_1 z)_p \text{ vs } z_+ \]
Figure 5.18 (a), (b) w'-spectra, inner flow scaling
Figure 5.18 (c), (d) $w'$-spectra, inner flow scaling
Figure 5.19 (a), (b) $w'$-spectra, Kolmogoroff scaling
Figure 5.19 (c), (d) $w'$-spectra, Kolmogoroff scaling
5.3.4 Normal Spectra - The Turbulent Wall Region.

The normal turbulence spectra measured in the wall region are presented in figure 5.20 with inner flow scaling.

The spectra show reasonably good collapse at low wave numbers, there is a region of constant energy, and the spectrum falls off in an exponential like manner. This is consistent with the theory of Perry and Chong. At high wavenumbers, the spectra appear to peel off due to viscous controlled effects from a short region of slope $-5/3$. It is surprising that these results show such good collapse to the appropriate spectral laws. The X-wires used were $\beta=90^\circ$ and would have suffered from the cone angle problem.

Due to the fact that a hot-wire probe at a distance $z$ from the wall sees only contributions to $w'$ from hierarchies of height of order $z$, the errors made in using the Taylor hypothesis to determine the convection velocity would not be expected to cause a lack of low wave number collapse as in the case of the longitudinal (and lateral) spectra.

The normal spectra in the turbulent wall region scaled with outer flow scaling are shown in figure 5.21. According to the theory, no collapse is expected with outer flow scaling and this is the case. However groups of spectra with the same $\delta_H$ do collapse at high wave numbers. The reason for this is that the X-wires are incapable of "seeing" the finest scale motions in the flow, i.e. the spatial resolution of the X-wires is inadequate. This could also be the reason for the
Figure 5.20
\( \frac{\phi_{33}(k_Lz)}{u^2} \) – spectra, turbulent wall region
inner flow scaling

Figure 5.21
\( \frac{\phi_{33}(k_L\delta_H)}{u^2} \) – spectra, turbulent wall region
outer flow scaling
unusually short -5/3 region in the spectra scaled with inner flow scaling as the high wavenumber cut off would not be dependent on viscosity but would scale with the wire length. This was verified by plotting the spectra scaled with the length scale of the hot-wire filaments, $l$, shown in figure 5.22. The spectra appear to collapse better at high wavenumbers.

Figure 5.23 shows a plot of $\log(k_z)_{lp}$ vs $\log(z_+)$ vs $\log(z_+)$). There is some scatter but the data appear to collapse onto a line of slope +1. This confirms that spatial resolution is a significant problem when measuring $w'$-velocity fluctuations.

The normal spectra in the turbulent wall region scaled with Kolmogoroff scaling are shown in figure 5.24. Collapse of the spectra onto a slope of -5/3 is very short, probably due to an anomaly of the X-wires. Due to the obvious spatial resolution problem, it is at first surprising that the Kolmogoroff scaling caused such good collapse of the data. However, since the energy dissipation was computed using equation 1.8, this appears to force the data to collapse whether the Kolmogoroff law is correct or not.

5.3.5 Lateral Turbulence Spectra

According to the theory of Perry and Chong and the Townsend attached eddy hypothesis, the lateral spectra should follow the same scaling laws as the longitudinal spectra.
Figure 5.22
w'-spectra, turbulent wall region
scaled with wire length

Figure 5.23
\((k_1 z)_p\) vs \(z_+\)
Figure 5.24

$w'$-spectra, turbulent wall region
Kolmogoroff scaling
Figures 5.25(a)-(d) show the lateral spectra plotted with inner flow scaling. Regions similar to those in the longitudinal spectra can be seen. There is a flat region at low wave numbers, a collapse onto an inverse power law region for values of $z/\delta_H$ small (which is more extensive at the $x=2500$mm station due to the larger number of hierarchies), and at high wavenumbers, the inertial subrange is obvious especially in the wake region with viscous dependent peel offs. The low wavenumber peel offs depend on $z/\delta_H$. The $-5/3$ collapse is more apparent at higher $Re$'s.

The spectra are shown plotted with Kolmogoroff scaling in figures 5.26(a)-(d). The lateral spectra can be seen to collapse onto a region of slope $-5/3$ only at higher $Re$'s, indicating the presence of an inertial subrange.

5.3.6 Lateral Spectra - The Turbulent Wall Region

Figure 5.27 shows all spectra measured in the turbulent wall region scaled with inner flow scaling. The collapse onto an inverse power law is not very convincing compared with the longitudinal spectra. Due to the short region of $-1$ collapse expected, factors such as the presence of mean shear over the wires, a cone angle problem or spatial resolution problem could effectively mask the $-1$ collapse. The low wavenumber peel off from the inverse power law region is proportional to $z/\delta_H$ for $z$ sufficiently greater than $\delta_1$. At higher wave numbers there is a $-5/3$ collapse with subsequent viscous peel offs.
Figure 5.25 (a), (b) $v'$-spectra, inner flow scaling
Figure 5.25 (c), (d) v'-spectra, inner flow scaling
Figure 5.26 (a),(b) $v'$-spectra, Kolmogoroff scaling

\begin{align}
\phi_{22}(k_1 \eta) &= \frac{\Phi_{22}(k_1 \eta)}{u^2} \\
R_0 &= 2720 \\

\phi_{22}(k_1 \eta) &= \frac{\Phi_{22}(k_1 \eta)}{u^2} \\
R_0 &= 5563
\end{align}
Figure 5.26 (c), (d) $v'$-spectra, Kolmogoroff scaling
Figure 5.28 shows the lateral spectra plotted with outer flow scaling. The collapse at low wavenumbers is only fair and this lack of collapse is thought to be due to errors made in the calculation of the inferred wave number assuming Taylor hypothesis. There is a short region of -1 collapse. However, compared with the spectra scaled with inner flow scaling, the low wavenumber collapse is significantly better in the outer flow scaling case and this fact supports the model of Perry and Chong. At the highest wavenumbers however, groups of $v'$-spectra at the same $H$ collapse, suggesting that the hot-wires used had inadequate spatial resolution. Plotting the $v'$-spectra scaled with the length scale of the hot-wire filaments seems to confirm this as the spectra collapse better at high wavenumbers with this scaling as shown in figure 5.29, even though the usual plot of $\log(k_z_H) vs \log(z_+)$ (figure 5.30) suggests that spatial resolution of this probe may have been adequate. However, it is thought that the spatial resolution problem is not as serious with $v'$-measurements as it is with $w'$-measurements.

Figure 5.31 shows the lateral spectra plotted with Kolmogoroff scaling. A short region of collapse onto a line of slope $-5/3$ can be seen, perhaps indicating the presence of an inertial subrange.
Figure 5.27
\( v' \)-spectra, turbulent wall region
inner flow scaling

Figure 5.28
\( v' \)-spectra, turbulent wall region
outer flow scaling
Figure 5.29
$v'$-spectra, turbulent wall region
scaled with wire length
Figure 5.31

$\frac{\phi_{22}(k_1^\eta)}{\nu^2}$

$v'$-spectra, turbulent wall region
Kolmogoroff scaling
CONCLUSIONS

The overall conclusion reached in this thesis is one of very strong support for the fact that the energy containing component of wall turbulence is in fact made up of eddies that are in a sense "attached to the wall" as proposed by Townsend (1976), and that the general distribution of the eddy scales follows the hierarchy structure proposed by Perry and Chong (1982) at least in the turbulent wall region.

The longitudinal \((u')\) spectra in the turbulent wall region collapse to an inverse power law region with low wavenumber "peel offs" proportional to \(z/\delta\) when scaled with "inner" flow scaling coordinates \((U_\tau, z)\), whereas the normal \((w')\) spectra collapse to a universal curve with no inverse power law region. This is in agreement with the Townsend (1976) eddy intensity functions which were derived assuming attached eddies, and implies that at a distance \(z\) above the wall, a probe sensitive to \(u'\)-motions sees contributions from eddies of height of order \(z\) and larger whereas a probe sensitive only to \(w'\)-motions sees contributions only from eddies of scale of order \(z\). According to the theory, the behaviour of the lateral \((v')\) spectra scaled with inner flow scaling in the wall region should follow the same scaling laws as the \(u'\)-spectra, and the measured \(v'\)-spectra seem to confirm this.

The existence of a \(-5/3\) power law collapse with inner flow scaling and Kolmogoroff scaling in the spectral plots seems to confirm the existence of an inertial subrange where fine scale locally isotropic motions exists. The \(\Lambda\)-vortex model of Perry and Chong does not predict
the existence of an inertial subrange, but it can be derived from a region of overlap argument (eg. see Perry and Abell, 1977). The inertial subrange is also evident in spectra measured in the wake region.

The behaviour of the u' and v' spectra with "outer" flow scaling (U, δH) where they collapse at very low wavenumbers, and then collapse to an inverse power law region at moderate wavenumbers also supports the model. There was some lack of collapse at the very low wavenumber end of the spectrum and this is attributed to the fact that the use of Taylor's hypothesis of frozen turbulence to calculate the convection velocity led to errors in the inferred wavenumber. These spectra peel off at high wavenumbers at a point proportional to δH/z and this is consistent with the theory of Perry and Chong.

At very high wavenumbers, the spectra scaled with inner flow scaling show viscous controlled peel offs from the inertial subrange, and Perry and Abell (1977) show by dimensional analysis that the wavenumber at which this peel off occurs should scale with (z+) 3/4. This was confirmed for the u'-spectra which were measured with a normal hot-wire. In the v'- and w'-spectra, where X-wires were used, the lack of spatial resolution of the X-wires possibly masked this result. It was found that the peel off point in the v'- and w'-spectra was then proportional directly to z+, and that the spectra at high wavenumbers collapse when scaled with λ, the length of the hot-wire filaments. This is in agreement with the findings of Wyngaard (1968) on the spatial resolution of hot-wires.
Perry and Abell (1977) showed that the $u'$-spectra should collapse to an inverse power law region with inner and outer flow scaling by a region of overlap argument, and the predictions of the $A$-vortex model are in agreement with their predictions.

According to the $A$-vortex model of Perry and Chong, the scale of the smallest attached eddy over a smooth surface should scale with the Kline scaling, $\nu/U_t$. They expect that the smallest attached eddy on a rough surface will scale instead with the size of the roughness, $k$. Thus the number of hierarchies present in the flow over a smooth wall will depend on the Kármán number $\delta_H U_t / \nu$, whereas over a rough wall it will depend on $\delta_H / k$. It would then be expected that the number of hierarchies present over a rough wall will be less than that over a smooth wall at similar values of $U_t$ and $\delta_H$. Measurements over a rough wall show that the spectra follow the same scaling laws as the smooth wall, the only difference is that the inverse power law region is shorter due to the smaller number of hierarchies and that the inertial subrange is longer, as $z_+$ is higher for a rough wall at the same $z/\delta_H$. This behaviour is also consistent with the extended form of the Townsend (1956) Reynolds number hypothesis proposed by Perry and Abell (1977).

It had been noticed in the $u'$- and $v'$-spectra that an unexplained "bump" occurred in the spectra at low to moderate wavenumbers. This bump could be explained if there were more eddies than expected in a certain wavenumber range in the flow. The distribution of eddy scales had hitherto been assumed to follow an inverse power law distribution as suggested by Townsend. In order to successfully predict the Hama
velocity defect law for the mean velocity distribution using the $A$-vortex model, Perry, Henbest and Chong (1984) found that they had to modify the distribution of eddy scales with a weighting function. This same weighting function when used in the prediction of the spectral distributions with the $A$-vortex model, gave the bump observed in the measured spectra.

Using the spectral laws proposed by Perry and Abell (1977), the spectra were integrated to give the broad-band turbulence intensity distributions. The formulations of Perry and Abell were an extension of the asymptotic predictions of Townsend (1976) and include the effect of viscosity. The predictions agreed well with the measured $u'$-turbulence intensities which were measured with a normal hot-wire, and with the $v'$-distributions. Problems in measuring the $w'$-turbulence intensity distributions with X-hot wires prevented the verification of these laws in the case of the $w'$-turbulence intensities. However, a $w'$-distribution is tentatively suggested, and using this distribution, the behaviour of the $w'$-turbulence intensities for a range of Karman numbers is plotted, where it was found that the $w'$-distributions would only asymptote at very high Karman numbers. The values obtained by various workers is also shown in this plot and the scatter in the data emphasises the difficulty in measuring $w'^2$ accurately with X-wires. This plot also shows that the work in wind tunnels to date are at Karman numbers that are well below those required for an asymptotic distribution to be achieved. From the spectral data, it can be seen that the higher the Karman number, (ie, the more hierarchies), the better the data fitted the predictions. Thus in terms of the models Townsend and Perry and Chong, the Karman number can be thought of as a measure of the state of development of the flow.
Reynolds shear stress measurements over a wavy surface showed a fall off close to the surface and this was at first attributed to the existence of a stationary wave close to the roughness elements as suggested by Perry, Schofield and Joubert (1969). Using a flying hot-wire to measure a set of mean velocity profiles 2mm apart along the surface showed that the influence of the stationary wave was negligible. Due to the large scale of the wavy roughness, the mean velocity profiles were of an unusual shape and the usual log-law fit to the data could not be used to obtain $C_f'$. Measurements over the mesh roughness, where the Hama velocity defect law fit to the mean velocity profiles were used to obtain $C_f'$, also showed this fall off in Reynolds shear stress although the effect of the stationary wave was again found to be negligible. From the findings of Tutu and Chevray (1975) and Willmarth and Bogar (1977), where velocity vectors approaching the probe at excessively large angles was found to cause errors in the measured quantities, an investigation was carried out and this in fact was found to be the cause of the fall off in Reynolds shear stress. A set of X-wires where the included angle of the filaments, $\beta$, was $120^\circ$ instead of $90^\circ$ was constructed and measurements with these wires did not show this fall off. However, the value of $C_f'$ predicted by the Reynolds shear stress still fell short of those predicted by the Hama law fit value. Measurements over a smooth surface with $\beta=90^\circ$ X-wires gave agreement with $C_f'$ values obtained from a Clauser chart. The reason for this is that the velocity vectors approaching the probe over a smooth wall are at smaller cone angles, as $\bar{U}/U_*$ is larger over a smooth wall at similar values of $z/\delta_H$ compared with a rough wall, due to the decrement in mean velocity ($=\Delta U/U_*$, the roughness function) caused by the roughness. It was thought that the allowable cone angle
was still being exceeded in the case of the $\beta=120^\circ$ X-wires. A scheme was devised to use a flying hot-wire to obtain the Reynolds shear stress as this would have the effect of increasing $\overline{U}/U_1$ and also to decrease the influence of $v'$-fluctuations on the measurements. The Reynolds shear stress values were the same as those obtained with stationary $\beta=120^\circ$ X-wires, indicating that these were adequate.

The Hama velocity defect law method used relies on the size of the wake, and this has been found to be sensitive to flow development (eg. see Coles 1962). It is felt that the flow has not yet reached its asymptotic state where the Hama velocity defect law would be expected to be valid, and that the values of $C_f'$ obtained using this law are in error. This is supported by the findings of Acharya and Escudier (1984) who repeated these experiments with an additional method of measuring $C_f'$, the drag balance. Their findings are in complete agreement with those reported here.

Thus, the attached eddy hypothesis of Townsend and the $A$-vortex model of Perry and Chong are upheld by the results presented in this thesis. The Townsend Reynolds number similarity hypothesis and the Perry and Abell's extended form of this hypothesis are also upheld, at least for the limited range of Kármán numbers tested here.
Consider the control volume ABCD shown in figure A.1. Assume that \( z \) is sufficiently small such that the momentum flux across AB has reached its wall value, i.e., the x-momentum flux across AD is equal to the x-momentum flux across BC, and thus all x-momentum flux occurs across the surface AB. It is also assumed that the effect of the externally applied pressure gradient does not contribute to the drag on the element. The condition necessary for this assumption to hold is

\[
\frac{dP}{dx} \lambda h \ll \tau_0 \lambda \tag{A.1(a)}
\]

As \( \lambda \) is of order \( h \), then this condition becomes

\[
\frac{dP}{dx} \lambda^2 \ll \tau_0 \lambda \tag{A.1(b)}
\]

ie.

\[
\frac{\lambda}{\tau_0} \frac{dP}{dx} \ll 1 \tag{A.1(b)}
\]

Let \( D \) be the drag force per unit width of flow acting on the wall. Hence the effective "wall shear stress" \( \tau_0 = D/\lambda \). (Here the effect of viscous shear has been assumed to be negligible). No mass flow can occur along the curved surface CD and a simple 2-dimensional mass and momentum balance on the control volume ABCD leads to the result that

\[
\tau_0 = -p\langle u'(t,x)w'(t,x) \rangle - \rho \langle u(x)w(x) \rangle \tag{A.2}
\]

using the notation given in figure 2.1(b)
Figure A.1

Control volume for momentum balance over the wavy surface
Consider now a momentum balance on the control volume ABFE in figure A.2 and assume that effectively, there is a shear stress $\tau_0$ acting along the face AB. The assumption is made that the velocity field patterns in neighbouring grooves are the same as in the groove under consideration, i.e. a similarity condition exists and $\tau_0$ does not vary significantly from one groove to the next. This means that the pressure pattern on the faces AE and BF are of a similar shape but incremented by an amount $\lambda dp/dx$ due to the imposed external pressure gradient.

A simple 2-dimensional mass and momentum balance gives a "finite difference" momentum integral equation of the form

$$\frac{\tau_0}{\frac{\lambda}{2} U_1^2} = \frac{C_f'}{2} = \frac{\Delta \theta}{\lambda} + (H+2) \frac{\theta}{U_1} \frac{\Delta U_1}{\lambda}$$

(A.3)

where $\Delta \theta$ is the change in momentum thickness over a length $\lambda$ and $\Delta U_1$ is the change in free stream velocity over a length $\lambda$.

If $\delta/\lambda$ is sufficiently large and if $z/\delta$ is sufficiently small, this finite difference equation can be written as

$$\frac{C_f'}{2} = \frac{d\theta}{dx} + (H+2) \frac{\theta}{U_1} \frac{dU_1}{dx}$$

(A.4)

which is the von Kármán momentum integral equation.
Figure A.2
Control volume ABFE
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