Seismic Performance of Limited-Ductile RC Columns in Moderate Seismicity Regions

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‘Produced on archival quality paper’
TO MY FATHER AND MOTHER
who raised me and taught me perseverance in the face of difficulties

& MY WIFE
who stood by me all the way throughout this journey
ABSTRACT

Many existing buildings in Australia and other places of the world with low to moderate seismicity, particularly old buildings, are not designed for credible seismic actions. Consequently there are considerable uncertainties over the safety of such buildings and the suitable risk management strategies that should be adopted. This study focuses on gravity collapse drift modelling and seismic collapse assessment of limited ductile RC columns which are commonly found in soft-storey buildings in low to moderate seismicity regions.

The RC columns of interest are typically lightly reinforced & poorly confined with vertical spacing of transverse reinforcements as large as the column depth. Such columns are categorized as shear critical and automatically deemed unsafe by the measures originally developed for the conditions of high seismic regions. Importantly, by such measures, collapse is conventionally defined at the point of nominal shear failure; that is the point of 20% degradation in peak lateral strength. This criterion however, does not take into account the properties of the projected ground motions in the low to moderate intra-plate seismicity region of Australia.

In Australia, the seismically induced displacement demand does not increase indefinitely with the effective period of vibration of the structure. Instead, the displacement demand is constrained to a limit once the period of the structure reaches the corner period of excitations ($T_2 \leq 1.5$ second). In conditions of displacement controlled behaviour, the actual displacement capacity of a structure prior to overturning/ gravity collapse is a more direct and realistic measure of the acceptance limit than the limit defined at the point of nominal shear failure.

Considering the outlined condition and objectives, four lightly-reinforced and poorly-confined concrete columns were tested up to gravity collapse. The columns were subjected to constant axial load and reversed cyclic displacements of increasing amplitude. An iterative equilibrium/compatibility-based shear-slip model was developed to simulate the commonly observed rotational deformation (in relatively slender columns) caused by the opening of the critical shear crack prior to gravity
collapse. The developed model was simplified and contributed to the development of Model (I) which was proposed for a deterministic estimation of total drift capacity at gravity collapse. Model I is limited to the limited-ductile RC columns with the flexural-shear mode of failure.

Based on the analysis of the experimental results corresponding to a dataset of 56 limited ductile RC columns, two additional probabilistic drift capacity models have also been developed (at nominal shear failure and the limit of gravity collapse). The developed models are mainly functions of axial load ratio which address the need for minimizing input requirements while providing realistic predictions.

Realistic backbone curves were constructed and incremental dynamic analyses were conducted for probabilistic seismic collapse assessment of a practical range of limited ductile RC columns. Axial load ratio, longitudinal and transverse reinforcement ratios, column size and aspect ratio and soil type were parameterized. A simplified probabilistic solution was proposed for collapse assessment based on the fragility curves of the columns analysed.
DECLARATION

This is to certify that:

(i) the thesis comprises only my original work towards the PhD,
(ii) due acknowledgement has been made in the text to all other material used,
(iii) the thesis is fewer than 100 000 words in length, exclusive of tables, maps, bibliographies and appendices

Mohammad Fardipour

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1 Introduction

1.1 Background

Buildings possessing soft-storey features are commonly found in low to moderate intra-plate seismicity regions including Australia. Typical soft-storey buildings in Australia range from 3 to 30 stories. Figure 1.1 shows an example of such buildings. The ground floor of these buildings features an open plan to allow for the flexible use of space. In contrast, the upper floors typically feature pre-cast structural wall panels resulting in a lateral stiffness significantly higher than that of the ground floor. These buildings are hence described as soft-storey owing to the expected concentration of displacement demand on the ground floor while the superstructure above translates horizontally as a relatively rigid block (when subjected to ground shaking). Consequently, the seismic performance of such buildings depends primarily on the performance of the columns supporting the superstructure.

The majority of soft-storey buildings in Australia are relatively old and were constructed prior to 1979 when the first seismic standard was introduced for Australia (AS 2121). No seismic provisions were considered for the design of the columns in such buildings. Such columns are typically lightly-reinforced and poorly-confined as found in a field reconnaissance survey of existing soft-storey buildings in Australia (Rodsin 2007). Importantly, the vertical spacing of transverse reinforcements in these columns could be as large as the column depth. These columns are automatically classified as shear critical and deemed unsafe by guidelines from regions of high seismicity (ATC40 1996; FEMA356 2000).
1.2 Problem Statement

Despite the aforementioned concerns regarding the lack of design provisions, soft-storey buildings in Australia continue to house a significant number of people and/or are occupied by organizations with post-disaster functions (Rodsin 2007). This raises the concern regarding the safety of such buildings and emphasizes the need for realistic performance assessment of them against the extreme seismic events credible in the region. It is acknowledged that soft-storey buildings are banned in high seismic regions by codes of practice due to their inherent vulnerability (ASCE/SEI 2007; IBC 2000).

Seismic performance of the columns supporting soft-storey buildings may be evaluated using the well-known capacity spectrum method. Alternatively, and more rigorously, the seismic collapse performance of the columns may be assessed employing the inelastic time history analysis. Central to these methods (from the capacity point of view) is a force-displacement response simulated up to a point which can be regarded as the realistic limit of failure.

Realistic Limit of Failure for Limited-Ductile RC Columns in Australia

The ultimate limit of failure for a column is conventionally defined at the point of notional lateral failure or shear failure where the peak lateral strength is degraded by 20% (Priestley et al. 2007). This limit is commonly considered to ensure that there is sufficient energy absorption capacity for a column while a trade-off between strength and ductility is considered. This strength-based criterion is however relevant in high
seismic regions where the displacement demand imposed on a structure can increase with the period of structure to a level which could be much higher than the capacity of typical structures.

The 20% strength reduction limit of failure as described above is believed to be too restrictive for the limited-ductile columns in Australia considering the ‘displacement controlled behaviour’ introduced by (Lam and Chandler 2005; Lam and Wilson 2004) for intra-plate seismicity regions. In the low to moderate seismicity intra-plate region of Australia, the characteristics of ground motions are such that the velocity demand imposed on a single-degree-of-freedom (SDOF) structure by an earthquake subsides with increasing natural period of vibration beyond a certain period limit. Consequently, the energy demand imposed on the structure is not sustained and the displacement demand levels off to a value constrained by the peak ground displacement (PGD). Displacement controlled behaviour is schematically illustrated in Figure 1.2 and presented in more details in Section 2.1.2. In this figure, the RSV and RSD represent the response spectral velocity and displacement of elastic SDOF systems respectively. The peak displacement demand is denoted by RSDmax (or PDD). Under displacement-controlled conditions and in contrast to the conditions of high seismic regions the sensitivity of structures to residual strength or the energy absorption capacity of structure is very low. Under displacement-controlled conditions, the failure is defined, more realistically, by the capacity of structure to deform or displace prior to gravity collapse.

![Figure 1.2: Schematic response spectra for low to moderate seismicity regions (Lam and Chandler 2005)](image-url)
Realistic Drift Capacity of Limited-Ductile RC Columns

Recent experimental studies (Rodsin 2007; Sezen and Moehle 2006; Wibowo et al. 2010b; Yoshimura and Yamanaka 2000) have shown that lightly-reinforced, poorly-confined columns could have drift capacities appreciably greater than the limits predicted by major assessment guidelines such as ATC (1996). This is particularly the case for lightly reinforced columns that are expected to experience the yielding of longitudinal reinforcement prior to shear failure (i.e. for the columns with flexural-shear mode of failure). This source of ductility is not available to columns with a brittle mode of shear failure (which is an abrupt shear failure without the yielding of longitudinal reinforcement). The experimental program conducted in this study (on lightly-reinforced, poorly-confined RC columns) revealed additional sources of ductility for columns with flexural-shear mode of failure. Importantly, the deformation caused by the opening of critical shear crack (up to the limit of gravity collapse) was found to be a common source for the columns under investigation.

Under the conditions of displacement-controlled behaviour, all the sources of ductility contributing to the deformation capacity of columns (up to the limit of gravity collapse) can be counted on, irrespective of the residual lateral strength of the columns. It is noted that strength degradation is accompanied by stiffness softening and period lengthening in structures. However, the seismic displacement demand in the region of interest can become insensitive to the period once the period of structure exceeds the dominant period of excitation as suggested before. This property of ground motions in Australia have been effectively used by Wilson and Lam (2006), Kafle et al. (2011) and others. Therefore the effective management of limited resources available for retrofitting and strengthening plans requires that all reserved deformation capacity of columns (up to the limit of gravity collapse) be taken into account when seismic performance of columns are assessed at the limit state of collapse.

Force-Displacement Simulation of the Columns Up to the Limit of Gravity Collapse

The lateral force-displacement relation of limited-ductile RC columns can be simulated with acceptable accuracy approximately up to the point of peak lateral
strength using the existing knowledge. For this purpose, standard analytical solutions could be employed coupled with the available models for estimating the shear and the strain penetration components of deformation. The main component of deformation (i.e. the flexural deformation in relatively slender columns) is obtained by non-linear moment-curvature analyses of the critical section. The flexural deformation is obtained by the double integration of the curvature (e.g. the yield curvature) assuming a linear distribution of curvature along the column height. The inelastic component of flexural deformation is usually estimated considering an empirical plastic hinge length at the column base over which a constant maximum curvature is considered. The curvature limit is usually controlled by a strain limit in the extreme concrete fiber in compression and/or the buckling of reinforcement.

The post peak region of the backbone curve can be simulated with the knowledge of two points: (1) the point of notional shear failure (or lateral failure) at which the peak lateral strength is degraded by 20%; and (2) the point at which the gravity load carrying capacity of the column is compromised (the point of gravity collapse). While the two points are important for accurate modelling of the backbone curves, the latter is critical for the seismic collapse assessment intended within the scope of this study. It should be mentioned that there exist a number of models in the literature which have been proposed to model the drift at the limit of gravity collapse (Elwood and Moehle 2005; OUSALEM et al. 2004; Rodsin 2007; Wibowo et al. 2012; Zhu et al. 2007). While these models are valuable solutions significantly contributing to the state of the art and the insight into the problem at hand, their application is hindered by certain factors some of which are listed below:

1) Existing models do not address the actual physical phenomenon corresponding to the commonly observed rotation at the level of critical shear crack as outlined previously.

2) Existing models are essentially empirical. The application of the empirical models is strictly restricted to the range of the columns included in model development.

3) Existing models require a comprehensive set of input data which is not generally available when old existing RC columns are to be assessed.

4) Existing models (except the one by Zhu et al. (2007)) are deterministic despite the presence of several uncertainties in relation to material properties and
highly complex and non-linear mechanisms of failure up to the limit of gravity collapse.

1.3 The Aim and Objectives of Study

The ultimate aim of this study is to provide a simplified yet realistic probabilistic seismic collapse assessment solution for a practical range of limited-ductile, cast-in-situ RC columns commonly found in soft-storey buildings in Australia. For this purpose the full deformation capacity of the columns (up to the limit of gravity collapse) need to be taken into account.

The ‘limited-ductile’ term is intended to describe the behaviour of lightly-reinforced, poorly-confined columns. These columns are mainly characterized (in this study) by largely–spaced transverse reinforcements. The vertical spacing of transverse reinforcement \(s\) in the columns of interest may be as large as the column depth \(d\) and as low as \((d/2)\) (i.e. \(d \leq s \leq d/2\)) which is generally insufficient for a ductile mode of failure controlled by flexure.

Squat columns with the shear span to depth ratio of approximately less than 2.5 are also beyond the scope of this study and have not been modelled here. These columns however, have been included for the purpose of comparisons and discussions.

In pursuit of the overall aim of the study the following objectives have been set:

1. To review the state of the art literature mainly: a) to gain an insight into the force-displacement response behaviour and the typical collapse mechanisms of limited-ductile RC columns as reported to date; b) to evaluate existing tools and models for drift modelling particularly at the limit of gravity collapse for the columns of interest; and c) to gain an insight into the seismicity of Australia in order to identify the key characteristics that need to be taken into account for a realistic seismic collapse assessment of the columns.

2. To conduct an experimental program on lightly-reinforced, poorly-confined concrete columns representing limited-ductile RC columns. The results and insight obtained from this experimental program is complementary to the
existing knowledge and essential for model development and verifications purposes.

3. To develop realistic/practical model(s) for predicting the drift capacity of limited-ductile RC columns at the limit of gravity collapse. The required model(s): 1) should rationally model the actual response behaviour and collapse mechanisms typically observed for the columns of interest; 2) should aim to deviate from existing empirical models to avoid strict limitations corresponding to the conditions of empirical calibrations; 3) should take into account that a complete set of input data is not generally available when old existing columns are to be assessed; and 4) should account for the uncertainties associated with the complex and highly non-linear problem at hand.

4. To employ the developed/recommended models and tools for force-deformation and dynamic modelling of limited-ductile RC columns. Simulated backbone models, the recommended hysteresis rule and calibrated stiffness parameters are to be validated against experimental results.

5. To conduct a parametric study on probabilistic seismic collapse assessment of a practical range of representative RC columns (employing incremental dynamic analyses). Effects of some parameters such as column size, axial load ratio, aspect ratio and etc. on the collapse probability of columns are to be investigated. Corresponding fragility curves need to be constructed to correlate a measure of ground motion intensity to the probability of collapse of the columns under analysis.

6. To provide a simplified probabilistic model to evaluate the vulnerability of the limited-ductile RC columns of interest to gravity collapse (accepting a 5% probability of collapse). The required simplified model should eliminate the need for rigorous analyses when similar columns need to be assessed for risk of collapse at the required return period.

An independent study on drift demand modelling for multi-storey buildings in low to moderate seismicity regions was also conducted. Corresponding results and findings are presented separately in Appendix B.
1.4 Thesis Layout

The present thesis comprises 8 chapters with an organisation as follows:

Chapter 1 presents the background, the aim and objectives of this study. A general overview of the thesis layout is provided in this chapter.

Chapter 2 presents a summary of the literature review covering a wide range of required concepts considering the aim and objectives of the study. This includes: (a) the nature and characteristics of projected earthquakes in Australia including a key trend which is known as displacement-controlled behaviour; (b) the background and typical characteristics of limited-ductile RC columns and the soft-story buildings in Australia; (c) existing tools and capacity models for force-deformation modelling of limited-ductile RC columns; and (d) existing seismic assessment procedures. Some critical evaluations of the existing capacity models and a comprehensive collection of relevant experimental results (from previous studies) were integrated in subsequent chapters when more appropriate.

Chapter 3 presents the results and insight obtained from an experimental program on lightly-reinforced, poorly-confined concrete columns. The programme consisted of four quasi-static reversed cyclic tests on columns subjected to medium to high axial load levels and minimal longitudinal and transverse reinforcement ratios. The tested specimens were intended to represent the columns found in existing soft-storey buildings, particularly old buildings, in Australia and some developing countries. The programme was collaboratively conducted by 3 organizations as follows:

1) The University of Melbourne, Melbourne, Australia
2) Swinburne University of Technology, Melbourne, Australia
3) Chulalongkorn University, Bangkok, Thailand

A comprehensive summary and discussions on reported test observations on limited-ductile RC columns as conducted by previous investigators is also presented in this chapter. The results and conclusions obtained in Chapter 3 were employed in Chapter 4 where new capacity models were developed.
Chapters 4 and 5 present solutions for the deformation modelling of limited-ductile RC columns of interest with the focus on predicting the drift at the limit of gravity collapse. The adequacy of the available models for post-peak deformation modelling is investigated in these chapters and three new models were developed to meet the criteria and requirements of the study (Model I, II and III).

1. Model I (Chapter 4) is an analytical deterministic model for predicting the drift capacity (at the limit of gravity collapse) of limited-ductile RC columns with flexural- shear mode of failure. The final model is a simplified solution (based on a rigorous procedure) focusing on modeling the actual failure mechanism of the column and minimizing empirical calibrations.

2. Model II (Section 5.1) is a practical probabilistic capacity model which predicts the drift capacity at the limit of gravity collapse for limited-ductile RC columns of interest. This model focuses on minimizing the required input parameters without compromising the reliability of estimates. The developed model is recommended to be incorporated in the intended seismic assessment solution (to be presented) considering that the detailed input parameters are not generally available for the old columns under assessment.

3. Model III (Section 5.2): The approach and criteria proposed for the development of Model II was extended to predict the drift capacity of columns at the limit where the peak lateral strength is degraded by 20% (this point is frequently referred to as the point of shear or lateral failure).

Chapter 6 brings together all the tools, models and input parameters required for inelastic dynamic modelling and analysis of limited-ductile RC columns of interest (within the scope of this study). It demonstrates the application of the capacity models developed in chapter 4 coupled with other standard solutions for simulating the full backbone curve of the columns of interest. This chapter provides recommendations on constructing linearized backbone models compatible with the input parameter requirements of classical dynamic programs such as RUAUMOKO. Further recommendations were provided regarding representative unloading and reloading stiffness parameters/values to be considered for the dynamic modelling of columns. This is followed by an alternative dynamic modelling solution within the program ‘OpenSees’ and corresponding discussions. This chapter is concluded by presenting
representative ground motions simulated for the purpose of incremental dynamic analyses. The results obtained in this chapter were employed in the parametric study presented in Chapter 6.

Chapter 7 presents the results of probabilistic seismic collapse assessments obtained from incremental dynamic analyses of a practical range of limited-ductile RC columns. The effects of key parameters namely column size, column aspect ratio, axial load ratio, longitudinal and transverse reinforcement ratios and soil conditions on the probability of collapse of the columns are investigated. This chapter presents the development of a simplified probabilistic seismic collapse assessment model based on the constructed fragility curves. The proposed simplified solution practically eliminates the need for repeating a rigorous approach (corresponding to incremental dynamic analyses and constructing fragility curves for seismic assessment) when similar columns are to be assessed.

Chapter 8 summarizes the findings and conclusions obtained in this study. Some recommendations are also provided for future research work and the extension of this study.
2 Literature Review

The literature review presented in this study covers a wide range of concepts required for the ultimate aim of this study (which is the seismic collapse evaluation of the limited-ductile RC columns supporting soft-storey buildings in Australia). A review of the nature and characteristics of the ground motions in Australia is presented in Section 2.1. The background of soft-storey buildings and the general characteristics of the limited-ductile RC columns of interest are presented in Section 2.2. Existing tools and models as required for the force-deformation modelling of limited-ductile RC columns are presented in Section 2.3. Section 2.4 presents existing seismic assessment procedures. Some critical evaluations of the existing capacity models and a comprehensive collection of relevant experimental results (from previous studies) were integrated in later chapters when more appropriate.

2.1 Seismic Activity in Australia

The majority of earthquakes, worldwide, occur along tectonic plate boundaries, which are categorized as inter-plate earthquakes. Less frequent but potentially destructive earthquakes can occur far away from tectonic boundaries, which are referred to as intra-plate earthquakes.

In the last 110 years, 20 earthquakes of magnitude 6 (M6) or greater have taken place in continental Australia which is wholly within the Indo-Australian plate. There are on average 2-3 earthquake of M5 or greater per year in Australia as reported by McCue et al. (1995).

Figure 2. 1 represents the economic loss from each Australian damaging earthquake as reported by Daniell and Love (2010). The most damaging Earthquake in the last 200 years was a moderate magnitude 5.6 earthquake in Richter scale near Newcastle NSW, which occurred in December 1989. This earthquake claimed several lives and resulted in $AUS 1.2 billion damage (McCue 1999). The largest known earthquake within continental Australia was a magnitude 7.2 event in 1906 off the central west coast of Western Australia (WA). The magnitude 6.8 Meckering WA earthquake in October 1986 left 35 km long surface rupture or fault. At the surface, the ground on one side of the fault overrode the other side by more than 2 m. Following this event
engineers accepted that even in the middle of the Australian plate earthquake hazard could not be neglected.

The first Earthquake Building Code (AS2121) was introduced in 1979, incorporating a number of regional hazard studies with the underlying assumption that earthquake activity will recur where it was observed in the past.

Gaull et al. (1990) adopted Cornell’s (1968) suggestion, to consider the random nature of seismicity in both time and space, in developing a probabilistic hazard map for Australia. That was to improve the recognized deficiencies in the previous hazard analyses, which merely considered probabilistic recurrence times of seismic activity. For this purpose, Gaull et al. introduced a series of source zones with uniform spatial and temporal probability of earthquake occurrence. The zones were chosen with the aid of geological, tectonic and geophysical data from 1900 to 1986. Seismicity was then modelled in these zones resulting in contour maps in terms of peak ground acceleration, velocity and intensity with 10% probability of being exceeded in 50 years.

![Figure 2.1: A representation of the economic loss from each Australian damaging earthquake (Daniell and Love 2010)](image_url)
The developed hazard map was employed as the basis for peak ground acceleration prediction (PGA) which is referred to as ‘Hazard Factor’ (Z) in Australian Earthquake loading standards (i.e. AS1170.4-1993, AS1170.4:2007). A small-scale overview of the earthquake hazard map of Australia is depicted in Figure 2.2. The hazard factor is 0.08g for major cities of Melbourne, Sydney and Canberra and ranges between 0.05g to 0.11g across Australia, at 10% probability of exceedance in 50 years (AS1170.4:2007).

Figure 2.2: Earthquake hazard map of Australia-1991 (McCue 1991)

The probabilistic seismic hazard approach is, however, considered to be more suitable in regions where there is sufficient knowledge of cause of seismicity and individual fault sources (Lam and Wilson 2008). Source uncertainties in intra-plate
regions are expressed in some other studies as well. Leonard (2008) for example, concluded that large earthquakes can occur anywhere in Australia despite that seismicity varies significantly across this continent. Therefore, the observed spatial distribution of historical events would not necessarily be indicative of potential future destructive events in intra-plate regions due to the random nature of accumulation and release of energy. Thus, following the occurrence of each major isolated earthquake (conceivably outside the identified source zones), significant changes to seismic hazard map is required. This concern is addressed in a number of studies (Lam et al. 2000b; Lam et al. 2000a) which is briefly presented next.

2.1.1 Synthetic Ground Motions and Response Spectrum for the Low to Moderate Seismicity Region of Australia

Damage potential of earthquake ground motion has traditionally been characterized using a number of approaches that can be broadly classified into qualitative and quantitative measures. One approach is the use of qualitative scales such as Mercalli and Modified Mercalli Intensity (MMI). The MMI scale is the most widely used scale for the felt intensity. This scale expresses the ground motion severity in accordance with the qualitative descriptions of felt intensity using a grade of I to XII. This scale however, is subjective and is affected by local conditions as well as the vulnerability of the affected structures.

Moment magnitude is a frequently used measure for indicating the size or magnitude of the energy released during an earthquake event. Moment magnitude is independent of the place of observation and is related to the fracture area on a fault. The larger the fractured area, the higher the energy released, the longer the duration of shaking and the higher the ground motion magnitude. Moment magnitude is often used as a quantitative parameter to classify ground motion records for engineering purposes.

Peak ground parameters such as peak ground acceleration (PGA), velocity (PGV) and displacement (PGD) have also been used as indicators of damage potential of earthquake ground motions and may also be related to the response parameters of structures.
The most commonly used tool for characterising the effects of the ground motion is the response spectrum. It is known that the damage potential of an earthquake is not only a function of ground motion properties but also a function of structural properties, particularly the natural period of vibration. Response spectrum can be presented in the acceleration, velocity or displacement formats and is usually constructed to show the elastic response of single-degree-of-freedom (SDOF) systems for 5% damping, of different natural periods, to a given ground motion accelerogram. The derivation of response spectrum (on rock) from first principles requires the analysis of a large number of strong motion accelerograms. In low to moderate seismicity intra-plate regions such as Australia, insufficient accelerograms have been recorded to satisfactorily undertake an empirical derivation of response spectrum as in (Newmark and Hall 1982).

Recognising the outlined difficulty a stochastically based seismological model was developed to generate synthetic accelerograms that are considered representative of intra-plate earthquake events recorded on rock (Lam et al. 2000a). For this purpose ground shaking was resolved into “source”, “path” and “local” components and their effects were incorporated separately. The source model was adopted from a source model previously developed (Atkinson 1993) for Central and Eastern North America (CENA) assuming that the averaged source properties are generic for different intra-plate regions across the globe. Path and local effects were incorporated by taking into account the following attenuation and amplification properties:

1. Geometrical spread of seismic energy
2. Dissipation of energy along the wave transmission path over long distances
3. Amplification of upward propagating waves due to impedance contrast
4. Dissipation of energy in the upper crust

Remarkable reduction of the complex ground shaking modelling problem to its achievable components together with a set of representative estimations of the aforementioned components (based on local geological and seismological information for averaged condition of Eastern Australia (EA) or Western Australia (WA)) led to a reliable seismological model that has been adopted in this study. The model was verified by comparing the average response spectra derived from the simulated accelerograms with the respective response spectra corresponding to locally recorded accelerograms.
The hybrid seismological model as outlined above was employed to develop a response spectral attenuation model comprising a source factor and several crustal component factors. The developed model was therefore termed ‘component attenuation model (CAM)’ (Lam et al. 2000b). CAM provided simple expressions (Equations (2.1) to (2.9)) for predicting maximum response spectral acceleration ($RSA_{\text{max}}$), velocity ($RSV_{\text{max}}$), and displacement ($RSD_{\text{max}}$) of elastic 5% damped SDOF systems, as functions of the source and regional component parameters for a given magnitude–distance combination. The procedure adopted in the study for model development could be summarized as follows:

1) Incorporating representative seismological parameters for Eastern and Western Australia (EA & WA respectively) into the program GENQKE (Lam 1999) for stochastic simulation of representative accelerograms.

2) Calculating the average elastic response spectra of the ensemble of simulated accelerograms corresponding to each magnitude-distance combination.

3) Developing CAM by curve-fitting the mean of the calculated results.

\[ \Delta = 0.78 \Delta^* \alpha(M)G(R)\beta(R)\gamma(\text{crust}) \quad (2.1) \]

\[ \alpha(M) = \alpha_1 + \alpha_2(M - 5)^{\alpha_3} \quad (2.2) \]

where

$\Delta$ = the response spectral parameter of interest

$\Delta^*$ = value for the reference event of $M = 6, R = 30$, employing the generic Eastern North America (ENA) intra-plate source, and hard rock crustal condition

$\alpha(M)$ = source function

$M$ = moment magnitude of the earthquake

The value of the term $0.78 \Delta^* \alpha(M)$ corresponding to $RSA_{\text{max}}$, $RSV_{\text{max}}$ and $RSD_{\text{max}}$ is estimated by Equations (2.3) to (2.5):

\[ \text{for } RSA_{\text{max}} : 0.78 \Delta^* \alpha(M) = 5.7[0.4 + 0.60(M - 5)^{1.5}] \ (\text{m/s/s}) \quad (2.3) \]

\[ \text{for } RSV_{\text{max}} : 0.78 \Delta^* \alpha(M) = 70[0.35 + 0.65(M - 5)^{1.8}] \ (\text{mm/s}) \quad (2.4) \]

\[ \text{for } RSD_{\text{max}} : 0.78 \Delta^* \alpha(M) = 10[0.20 + 0.80(M - 5)^{2.3}] \ (\text{mm}) \quad (2.5) \]

$G(R)$ = geometrical attenuation factor and $R$ is the source-site distance in km.
\[ G(R) = \frac{30}{R} \quad R < 45\text{km} \tag{2.6} \]
\[ G(R) = \frac{30}{45} \quad 45 \leq R \leq 75\text{km} \tag{2.7} \]
\[ G(R) = (\frac{30}{45})\sqrt{\frac{75}{R}} \quad R > 45\text{km} \tag{2.8} \]

\[ \beta(R) = \text{anelastic whole path attenuation factor that is estimated as follows} \]

\[ \beta(R) = \left(\frac{30}{R}\right)^{c_1} \quad R \leq 50\text{km} \tag{2.9} \]

where

\[ c_1 \text{ is taken equal to 0.003 and 0.005 and 0.015 for } RSD_{\text{max}}, RSV_{\text{max}} \text{ and } RSA_{\text{max}} \text{ respectively.} \]

\[ \gamma(\text{crust}) = \text{Combined crustal factor which can be taken (for } RSV_{\text{max}} \text{) as approximately 1.3 in Western Australia and between 1.6 to 2 for Eastern Australia} \]

The values 1.6 and 2 correspond to \( \kappa = 0.05 \text{ and 0.03 respectively where } \kappa \text{ is a parameter indicating the rate of attenuation or energy dissipation in the upper crust (Lam and Wilson 2008).} \]

Referring to the basic principles of dynamics of structures, response spectrum parameters are related to each other by the following equations where \( T \) is period of vibration of the SDOF system.

\[ RSD = RSV\left(\frac{T}{2\pi}\right) \tag{2.10} \]
\[ RSV = RSA\left(\frac{T}{2\pi}\right) \tag{2.11} \]

The period that separates the velocity and displacement controlled regions is referred to as second corner period, \( T_2 \), (see Figure 2.3). Structures with periods smaller than \( T_2 \) are velocity controlled (i.e. constant velocity demand) and those with periods greater than \( T_2 \) are displacement controlled (i.e. constant displacement demand) irrespective of their natural periods. \( T_2 \) is the controlling parameter in constructing the response displacement spectrum and could be obtained by rearranging Equation (2.10) as follows:

\[ T_2 = 2\pi \left(\frac{RSD_{\text{max}}}{RSV_{\text{max}}}\right) \tag{2.12} \]
$T_2$ can alternatively be estimated from the simplified magnitude dependent equation of (2.13) proposed by Lam et al. (2000c).

\[ T_2 \approx 0.5 + 0.5(M - 5) \text{ for } M \geq 5 \quad (2.13) \]

With the knowledge of the first and the second corner periods ($T_1 \approx 0.3-0.35$ sec & $T_2$ respectively) and response parameters of $RSD_{\text{max}}$, $RSV_{\text{max}}$, and $RSA_{\text{max}}$, idealized response spectra on rock can be constructed as illustrated in Figure 2.3.

![Figure 2.3: displacement, velocity and acceleration response spectrum](image-url)
The seismic demand can also be represented in the form of an acceleration-displacement response spectrum (ADRS) or capacity spectrum, which is schematically presented in Figure 2.4. The vertical and horizontal lines in the ADRS diagram represent the displacement and acceleration controlled actions respectively. The curve connecting the vertical and horizontal lines represents the velocity-controlled actions. Period independent $RSV_{\text{max}}$ would result in a constant kinetic energy demand (imposed on SDOF systems) which can be calculated as follows:

$$E = \frac{1}{2} m RSV_{\text{max}}^2$$ \hspace{1cm} (2.14)

The demand or absorbed energy imposed by seismic excitation could also be defined by the area under force ($F$) –displacement ($D$) curve in accordance to Equation (2.14).

$$E = \frac{1}{2} FD = \frac{1}{2} m RSA \cdot RSD$$ \hspace{1cm} (2.15)

Equating Equations (2.14) and (2.15) and cancelling $\frac{1}{2} m$ yields:

$$RSV_{\text{max}}^2 = RSA \cdot RSD$$ \hspace{1cm} (2.16)

Equation (2.16) can be used for construction of the velocity-controlled curve. This can be interpreted as a trade between acceleration and displacement demand which results in constant area equal to $\frac{1}{2} RSV_{\text{max}}^2$ for different triangles (Wilson and Lam 2003).

![Figure 2.4: Capacity Spectrum or ADRS diagram](image-url)
2.1.2 Peak Displacement Demand and Displacement Controlled Behaviour

Current seismic design and assessment guidelines for structures take into account the possible trade-off between strength and ductility ensuring that the structure has adequate energy dissipation capacity. A structure is deemed seismically unsafe when its energy dissipation capacity is exceeded by the energy demand imposed by an earthquake. This approach is mainly developed and justified for the condition of high seismicity regions. However, in low and moderate seismicity intra-plate regions such as Australia, the characteristics of ground motions are such that the velocity demand imposed on a SDOF structure by an earthquake subsides with increasing natural period of vibration beyond a certain limit. Consequently, the energy demand imposed on the structure is not sustained. This results in displacement demand, imposed on the SDOF system, reaching a limit termed as the peak displacement demand (PDD) (defined on a limiting natural period of 5 s). This trend is referred to as ‘displacement control behaviour’ and was introduced by (Lam and Chandler 2005) and further supported by the findings of a detailed study undertaken by (Lam and Wilson 2004).

Lam and Chandler developed a theoretical fault-slip model to generate single displacement pulses expected from earthquakes in stable continental regions (SCRs) such as Australia (Figure 2.5). Corresponding displacement spectra were then calculated using dynamic analyses of the single pulses (Figure 2.6) resulting in a simplified magnitude-dependent expression for prediction of PDD for reference hard rock crustal conditions, at the fixed notional source-site distance of 30km ($U_{\text{max}}$).

$$U_{\text{max}} (\text{mm}) = 10^{M-5}$$  

(2.17)

Various modification factors, similar to those introduced in Equation (2.1), were then employed enabling the PDD to be predicted for a range of distances and geological conditions as follows:

$$PDD = U_{\text{max}} (\gamma_{mc} \gamma_{uc}) G.B.S$$  

(2.18)

where

- $\gamma_{mc} = $ a factor for mid-crustal amplification
- $\gamma_{uc} = $ a factor for upper crustal modification (Lam et al. 2003)
$G \& B =$ factors for whole path attenuation
$S =$ a factor for soil amplification arising from resonance

The predicted PDD values were compared and justified against the results obtained from numerous empirical and stochastic models consolidating the previous findings by (Lam et al. 2000b, 2000c; Lam et al. 2000a) and establishing the displacement control behaviour in the intra-plate region of Australia. These findings are relevantly adopted and employed in this study.

As can be seen in the bi-linear model of displacement response spectrum, schematically shown earlier in Figure 2.3, the flat (plateau) part is associated with displacement-controlled behaviour having constant displacement demand at the PDD level. In contrast, the rising part of the spectrum is associated with ‘velocity-controlled’ behaviour and is sensitive to variations in the peak ground velocity (PGV), as inferred from the following relation (Wilson and Lam 2003):

$$RSV = 1.8PGV \frac{T}{2\pi} \quad \text{for} \quad 0 < T \leq T_2$$  \hspace{1cm} (2.19)
Figure 2.6: Displacement spectra obtained from dynamic analyses of theoretical ground displacement pulses

The second corner period \( (T_2) \) is the transition period separating the velocity-controlled and displacement-controlled parts of the displacement response spectrum and is dependent on the frequency content of the earthquake excitations as expressed previously by Equation (2.13).

For displacement-controlled structures, the predicted notional PDD could be compared with the displacement capacity of a structure, or component, for the purposes of seismic stability assessment. This is in contrast with the P-\( \Delta \) effect governing the stability requirement in high seismic regions.

Displacement-controlled behaviour has been further demonstrated by the analysis of non-ductile systems such as rigid free-standing components and soft-story buildings (Kafle 2011; Wilson et al. 2009). If yielding of the structure is represented by period-shift and increased damping, the displacement demand of an inelastically responding system can be tracked by the displacement spectrum. Thus, by definition, yielding will not increase displacement demand in displacement-controlled conditions. The stability of such structural systems could be assessed by comparing its displacement
capacity with the respective PDD limit of the applied excitations, irrespective of whether yielding has occurred (Lam and Chandler 2005; Wilson and Lam 2006).

2.1.3 Soil Amplification Factor and Site Specific Response Spectrum

Experiences obtained from previous earthquakes around the world emphasise the important influence of local geological deposits on the amount of damage and resultant loss of life. In general, damage and loss of life is concentrated in areas underlain by deposits of soft soil and hence the great amount of research within the last few decades to classify and quantify this effect. Soil amplification factors currently used in most codes of practice for earthquake actions around the world including AS1170.4 (2007) originate mainly from the combined empirical and analytical work of (Borcherdt and EERI 1994; Crouse et al. 1996).

The former work is based on the recordings of the Loma Prieta, California earthquake in 1989. Recordings were obtained at 35 sites of various geological deposits in close proximity, ranging from very soft clays to hard rock. Borcherdt and EERI used mean shear wave velocity to a depth of 30m as the basis for the definition of site classes (soil type) and for distinguishing the recording sites. In that study a series of Fourier spectral ratio (spectral amplification) with respect to nearby rock sites (with peak ground acceleration of 0.1g) was computed first. The spectral ratios represented average values over the short-period band (0.1-0.5s) intermediate period band (0.5-1.5s), long period band (1.5-5.0s), mid period band (0.4-2.0s) and the entire period band (0.1-5.0s). Regression curves for “average” horizontal spectral amplification as a function of mean shear wave velocity of soil deposits ($v$) for the above period bands revealed that the observed general trend of increasing amplification with decreasing shear velocity is distinctly less for the short period band (0.1-0.5s) than for other considered bands. Based on this observation it was suggested that the site response be characterized by two factors, one for the short component of motion (0.1-0.5s), $F_s$ and one for all other period bands represented by mid period band (0.4-2.0s), $F_v$. These amplification factors derived from the Loma Prieta strong-motion data are appropriate for input ground–motion levels less than 0.1g (PGA on firm to hard rock of the Franciscan formation). The corresponding regression curves developed in Borcherdt and EERI (1994) are given in Equations (2.20) and (2.21).
The amplification factors for higher ground motions up to 0.4g as introduced in the same paper are based on: (1) the laboratory and numerical modelling results by (Dobry et al. 1994; Seed et al. 1994); and (2) the assumption that the straight line trend (i.e. The straight line trend for the functional relation between the logarithms of amplification and mean shear-wave velocity as inferred from the Loma Prieta strong motion data) remains a straight line at higher levels of ground motion (e.g. for PGA>0.1g).

Although empirical soil amplification factors inferred from the Loma Prieta earthquake have been adopted by many seismic codes worldwide, the soil amplification issue is still a matter of controversy amongst researchers. Some critical arguments refer to the need for considering the fundamental principles of Physics as opposed to relying on mere empirical soil amplification factors.

When seismic waves pass through the bedrock and enter the overlying soil deposits, they get trapped between the soil surface and the soil-bedrock surface due to high impedance contrast and inability of the shear waves to travel through fluids. This culminates in multiple reflections of waves or periodic excitations. A periodic excitation is then expected to excite the vibrating system (soil) the most at its natural frequency or natural period \( T_g \). However, empirical seismological models typically do not express soil amplification as a function of \( T_g \) or soil depth. Instead shear wave velocity of soil columns averaged over a certain soil depth is used in their predictions as suggested in Equations (2.20) and (2.21). This averaging process has effectively smeared the periodicity (soil resonance) effects into a period band rather than focusing on distinct site periods. In the study carried out by Lam et al. (2001) a simplified (bi-linear) displacement response spectrum (RSD) on soil was suggested. Typical actual and simplified displacement response spectra on soil are schematically shown in Figure 2.7. The typical shape of the soil displacement response spectrum (with a peak at site natural period) was supported by inelastic dynamic soil analyses using the well-established SHAKE program (Idriss and Sun 1992) and the 1989 Loma Prieta recorded input ground motions. In the same paper, the authors drew an
interesting analogy between dynamic behaviour of soil and that of multi-storey moment resisting frames (with rigid girders) to manually estimate the soil amplification factor as a function of site period. This simulation strategy is named ‘frame analogy soil amplification (FASA)’ model.

A more recent research work on non-linear site response by Tsang et al. (2006) provided essentially a similar argument as the previous work outlined above with additional objectives and criteria which follows:

1. Non-linear site response should recognize the importance of site natural period and resonance phenomenon as well as soil damping and degradation (or period shift associated with inelastic soil response)
2. Characteristics and frequency content of seismic waves should be taken into account as they are known to be different for low and high seismicity regions
3. The important influence of ‘impedance contrast’ at the soil-bedrock interface should also be included.

Considering the above objectives, Tsang et al (2006) proposed a method named ‘single period approximation’. The single period approximation method consisted of three key components namely 1) soil damping ratio formula, 2) period shift ratio formula and 3) non-linear site response formula that collectively led to an estimator expressed by Equation (2.22). This equation provides the spectral ratio (SR) which is the ratio of spectral displacement on soil (\(RSD_{\text{max}}\)) to that of rock (\(RSD_{Tg}\)) at site period \(Tg\).

![Figure 2.7: Schematic displacement response spectrum on bedrock and soil](image-url)
\[
SR = \frac{RSD_{\text{max}}}{RSD_{\text{ref}} = f(\alpha) \frac{2\alpha}{1+\alpha} \sqrt{\frac{\beta}{1-R^4\beta^4}}}
\]

(2.22)

Where \( \alpha \) and \( f(\alpha) \) are the impedance ratio and resonance factor respectively and estimated as follows:

\[
\alpha = \frac{\rho RV_k}{\rho SV_s}
\]

(2.23)

\[
f(\alpha) = \alpha^{0.3} \leq 2.3
\]

(2.24)

\( \rho \) and \( V \) are the weighted-average density and shear wave velocity of respective layers and \( R \) and \( S \) represent rock and soil respectively.

\( \beta \) is the half-period damping factor and represents the hysteretic damping (also known as anelastic attenuation) of seismic wave energy in the soil layer for every half cycle of wave travel, which is defined by:

\[
\beta = \exp(-\pi\zeta)
\]

(2.25)

where \( \zeta \) is the soil damping ratio (as a proportion of critical damping) and \( R \) is the reflection coefficient which describes the amplitude ratio of the upwardly propagating reflected wave and the downwardly propagating incident wave within the soil layer and is calculated as follows:

\[
R = \frac{1-\alpha}{1+\alpha}
\]

(2.26)

2.2 Limited-Ductile RC Columns in Australia - Background and Seismic Vulnerability

2.2.1 Vulnerable RC Columns Reported from Field Reconnaissance Survey of Existing Soft-Storey Buildings in Australia

Buildings possessing soft-storey features are commonly found in low to moderate seismic countries such as Australia. A survey of existing soft-storey buildings in the Melbourne Metropolitan Area (MMA) was carried out by Rodsin (2007) to gather information concerning typical structural details of such buildings. These buildings are often occupied by organizations with post-disaster functions or house a significant number of people such as apartment, office, and hospital etc. The ground floors of these buildings have been designed with an open plan at ground floor to allow for
flexible utilisation of space. In contrast, the upper floors of these buildings feature pre-cast structural wall panels supported by a series of portal frames on the first floor. By using this type of construction, the inter-storey shear stiffness of the upper storeys greatly exceeds that of the ground floor. These buildings are usually described as soft-storey owing to the expected concentration of displacement demand on the ground floor, when subjected to ground shaking, while the superstructure translates horizontally as a relatively rigid block. It is noted that the concentrated displacement is to be sustained essentially by unbraced RC columns (at ground floor) as a result of the weak column, strong beam construction practice in Australia. The soft-storey buildings found in the MMA are representative of construction practice of many buildings in Australia and internationally in areas of low to moderate seismicity. Figure 2. 8 shows typical low, medium and high-rise soft-storey buildings in the MMA. These buildings range from 4 to 30 storeys. Most of the building blocks are rectangular in plan and supported at the first floor level by reinforced concrete portal frames at approximately 2.5 – 4.0m centres. Figure 2. 9a illustrates typical portal frames featuring a strong beam-weak column configuration. Part ‘b’ of the same figure illustrates the portal and spandrel directions. These terms are used hereafter without further explanation.

Figure 2. 8: Typical low, medium and high-rise soft-storey buildings in Australia. Photos (a) and (b) are from (Rodsin 2007)
The following is a brief summary of Rodsin’s observations of typical detailing and properties of the RC columns responsible for the lateral resistance of soft storey buildings.

1. Construction drawings revealed that, most soft story buildings were constructed during the 1960’s and hence columns and other components were designed without any seismic provisions. Consequently, these columns were considered to be of low ductility properties.

2. In areas of low seismicity, transverse reinforcement was often designed to resist code-specified shear forces resulting in widely spaced stirrups (i.e. $s \approx d$). The widely spaced stirrups are considered inadequate to provide an effective confinement for concrete and to prevent longitudinal reinforcing bars buckling in potential plastic hinge regions.

3. In some cases inadequate stirrups and low aspect ratios of the columns were observed. Such cases raise the concern regarding the possibility of brittle shear failure and highlight the need for more insight into the actual deformation capacity and seismic performance of the columns with different combinations of the key parameters.

4. Longitudinal reinforcement quantity was designed to resist the moments obtained from the code-specified lateral forces. This is reported to result in the columns which are weaker than the beams in the portal frame plane direction.
This may lead to an undesirable concentration of deformation and damage in the columns as opposed to the beams.

5. It was found that generous anchorage length was provided at both ends of the column. Therefore, anchorage slip of longitudinal reinforcements embedded in the footings and beams were judged unlikely.

6. It was reported that longitudinal bar lap-splices were commonly located in beams and footings rather than within the clear height of the columns.

7. Typical column-footing connections were found to be best described/idealized as fixed connections. This was concluded based on the observations that strong ground beams were commonly used for connecting the footings in both the portal and spandrel directions. The footings are therefore effectively restrained against rotations due to the use of strong ground beam and piled foundation.

8. Table 2.1 provides basic properties of columns in low-rise soft storey buildings in Melbourne. Table 2.2 provides similar information for medium to high-rise soft storey buildings in Melbourne. These information are extracted from the results of the field reconnaissance survey reported by Rodsin (2007).
Table 2. 1: Basic properties of columns in low-rise soft storey buildings in Melbourne

<table>
<thead>
<tr>
<th>Building Code</th>
<th>Structural System</th>
<th>Column Width (mm)</th>
<th>Column Depth (mm)</th>
<th>Clear Storey Height (mm)</th>
<th>Shear Span Ratio-Portal Direction</th>
<th>Shear Span Ratio Spandrel Direction</th>
<th>Axial Load Ratio</th>
<th>Transverse Reinforcement Ratio (%)</th>
<th>Longitudinal Reinforcement Ratio (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 I</td>
<td>OMRF²</td>
<td>460</td>
<td>460</td>
<td>2000</td>
<td>2.2</td>
<td>2.2²</td>
<td>0.1³</td>
<td>N.A.⁸</td>
<td>N.A.⁸</td>
</tr>
<tr>
<td>5 II</td>
<td>OMRF²</td>
<td>380</td>
<td>530</td>
<td>2500</td>
<td>2.4</td>
<td>3.3³</td>
<td>0.26¹</td>
<td>N.A.⁸</td>
<td>N.A.⁸</td>
</tr>
<tr>
<td>4 I*-5 I (four cases)</td>
<td>Portal Frame</td>
<td>300-380</td>
<td>380</td>
<td>2600</td>
<td>3.4</td>
<td>6.8⁶-8.7⁶</td>
<td>0.13⁴-0.15⁴</td>
<td>N.A.⁸,0.3⁸</td>
<td>N.A.⁸,1.05⁸</td>
</tr>
</tbody>
</table>

Notes:

1. Building Code: The digit represents number of storeys; the roman letter identifies building function, Type I = Apartment, Type II = Car park, Type III = Office
2. OMRF = Ordinary Moment Resisting Frame
3. Assumed concrete strength=35MPa.
5. Shear span ratio is calculated based on assumption that both ends of the column are fixed.
6. Shear span ratio is calculated based on assumption that only the bottom end is fixed and the top end is free.
7. Overturning effect is not included in the analysis
8. N.A. = Information not available
Table 2.2: Basic Properties of columns in medium to high-rise soft storey buildings in Melbourne

<table>
<thead>
<tr>
<th>Building Code</th>
<th>Structural System</th>
<th>Column Width (mm)</th>
<th>Column Depth (mm)</th>
<th>Clear Storey Height (mm)</th>
<th>Shear Span Ratio-Portal Direction&lt;sup&gt;3,5&lt;/sup&gt;</th>
<th>Shear Span Ratio Spandrel Direction&lt;sup&gt;6&lt;/sup&gt;</th>
<th>Axial Load Ratio&lt;sup&gt;3,4,7&lt;/sup&gt; (%)</th>
<th>Transverse Reinforce. Ratio&lt;sup&gt;3&lt;/sup&gt; (%)</th>
<th>Longitudinal Reinforce. Ratio&lt;sup&gt;3&lt;/sup&gt; (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>12 I-13 I</td>
<td>Portal Frame</td>
<td>410-640</td>
<td>610-850</td>
<td>3800-4800</td>
<td>2.2-3.0</td>
<td>6.3-11.7</td>
<td>0.16-0.21</td>
<td>0.43-0.8</td>
<td>1.13-1.57</td>
</tr>
<tr>
<td>Four cases</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>21 I-21 III</td>
<td>Portal Frame</td>
<td>450-457</td>
<td>610-914</td>
<td>3000-3800</td>
<td>1.9-3.1</td>
<td>6.7-8.3</td>
<td>0.18-0.27</td>
<td>0.41-0.49</td>
<td>1.7-2.54</td>
</tr>
<tr>
<td>Three cases</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>30 III</td>
<td>Portal Frame</td>
<td>450</td>
<td>870-1040</td>
<td>4000</td>
<td>1.9-2.3</td>
<td>8.9</td>
<td>0.16-0.19</td>
<td>N.A.</td>
<td>N.A.</td>
</tr>
</tbody>
</table>

Notes:
1. Building Code: The digit represent number of storeys; the roman letter identifies building function, Type I = Apartment, Type II = Car park, Type III = Office
2. Variations in depth is partly associated with the presence of taper columns in the data set. The depth at the base is typically smaller than that of the top of the taper column
3. The provided range include the effect of cross section change in taper columns
5. Shear span ratio is calculated based on assumption that both ends of the column are fixed.
6. Shear span ratio is calculated based on assumption that only the bottom end is fixed and the top end is free.
7. Overturning effect is not included in the analysis
8. N.A. = Information not available
Displacement behaviour in the portal directions may be significantly influenced by the overturning moment imposed from the upper storeys (by the pull-push action on columns) particularly in medium to high rise buildings. This effect could be studied using force-based approach or displacement-based considerations. Rodsin (2007) discussed that the stiffness properties of the column could be very sensitive to changes in the value of axial compression and consequently, columns on either side of the same portal frame may possess very different stiffness properties. The differential stiffness in the columns could result in an uneven sharing of horizontal forces between the columns within the same portal frame. Such differential load-sharing between the columns is typically not modelled by computer structural analysis procedures particularly in the elastic range and could lead to un-conservative results for seismic assessment.

The pull-push effect may be taken into account reliably using displacement principles considering possible range of axial compression fluctuations as induced from gravity loads and credible overturning moments on the columns of a portal frame. The column deformation capacity corresponding to controlling level of axial compression could then be estimated and compared against the imposed deformation demand (provided displacement controlled conditions hold). It is generally observed that the ultimate deformation capacity of column is inversely related to the imposed axial load level. In other words, the higher the imposed axial compression on a column, the lower the drift capacity at the limit of gravity collapse (refer Sections 3.2 and 3.3.4).

Limited ductile reinforced concrete columns are mainly characterized by largely spaced-poorly detailed stirrups and to a more limited extent by the presence of (inadequate) lap-splices within the column clear height and excessive axial load. The limited ductile columns featuring largely spaced and poorly detailed confinements are the focus of this study. These columns are sometimes referred to as non-ductile by the measures of high seismic regions.

### 2.2.2 Ductility Properties and Failure Criteria

Ductility is defined as the ratio of the force or deformation capacity of a structural component at a given limit state to corresponding yield parameter. By this definition, deformation ductility of a column at collapse prevention limit state is calculated by
the ratio of the deformation at the point of instability divided by the value of deformation at the point of yielding of the longitudinal reinforcement.

Ductility property is sometimes linked to the mode of failure of the columns. Shear dominated columns are generally believed to exhibit non-ductile behaviour, while flexure-dominated columns are usually categorized as ductile components in a number of the existing documents such as (Park and Paulay 1975; Priestley et al. 2007). Recent studies further subdivide shear failure into brittle and ductile modes. The former is associated with the case in which an abrupt shear failure occurs without yielding of the longitudinal reinforcements, whereas the latter addresses the case in which shear failure occurs subsequent to the yielding of the longitudinal reinforcements. The drift ductility value for a given column could be influenced by the definition of the drift capacity (or acceptance criterion) at a given limit state, such as collapse prevention limit state. That is because the acceptance criteria should realistically be specified considering the level of seismicity and characteristics of the ground motions in the region (as discussed later in this section). The Acceptance criteria for a given limit state and column properties may also change depending on whether the column is a secondary or primary structural component.

The modelling parameters and numerical acceptance criteria specified by ASCE/SEI41 (2007) standard employs a combination of failure modes and other criteria (as listed below) for classification of reinforced concrete columns. This classification is in fact based on the expected ductility property of columns.

1. Columns controlled by flexure
2. Columns controlled by shear
3. Columns controlled by inadequate development or splicing along the clear height
4. Columns with axial load exceeding $0.7P_0$

Each of the four groups given above stipulates separate estimates of drift capacity for primary and secondary columns. Flexure-dominated columns are further categorized into conforming (C) and non-conforming (NC) types. A component is conforming if, within the flexural plastic hinge region, hoops are spaced at $s \leq d/3$, and if, for component of moderate and high ductility demand, the strength provided by the hoops


\( v_i \) is at least three-fourth of the design shear, otherwise, the component is considered as non-conforming.

The general trend of the acceptance criteria stipulated in this most recent standard is tailored for high seismic regions in which lateral strength requirement is as critical of the deformation ductility requirement. This is evident comparing the permissible deformation capacity for primary and secondary columns, for instance. It could be seen that acceptance criteria is significantly higher for secondary components compared to primary components which is reasonable for stability requirement in such regions. This however does not seem appropriate for intra-plate low seismic regions such as Australia (the focus of this study) where displacement controlled behaviour limits the displacement demand for most structures irrespective of their lateral strength (refer to Section 2.1.2). Under the conditions of displacement controlled behaviour all deformation components contributing to extra drift capacity before gravity failure could be counted on for stability assessment while the same components may be considered unreliable for high seismic regions if they are accompanied by considerable strength loss, particularly beyond the accepted limit of failure in these regions (i.e. 20% reduction in peak lateral strength). Examples of such unconventional deformations are the ones provided by rotation at the level of lap-splices (Refer Section 3.3.5) and at critical shear crack 4.2). Influence of the lap-splices could be tracked in the recommendations stipulated in Table 6.8 of the standard (ASCE/SEI 41-6). Evidently, the acceptance criteria is significantly reduced for the cases in which stiffness softening is likely owing to inadequate development or splicing along the column height. This is again unreasonably conservative for the low to moderate seismicity regions such as Australia. In such regions column stability could be realistically assessed and ensured if the drift capacity reserved before gravity collapse exceeds the drift demand imposed by projected ground motions (Lam and Chandler 2005; Wilson and Lam 2006).
2.3 **Existing Tools and Models for Force-Deformation Modelling of Limited-Ductile RC Columns**

This section presents some of the empirical and analytical approaches available for simulating the force-deformation envelope curve of limited-ductile RC columns subjected to axial and lateral loading. A force-deformation curve which is also referred to as backbone curve may be illustrated schematically using the generalized form shown in Figure 2.10. As can be seen in this figure, such a curve can be constructed using the coordinates of the five points of A, B, C, D and E.

![Generalized full backbone curve](image)

**Figure 2.10: Generalized full backbone curve**

where

Point ‘A’ = the first cracking point
Point ‘B’ = the point of longitudinal reinforcements yielding
Point ‘C’ = the point of peak lateral force capacity
Point ‘D’ = the point of notional lateral failure (or shear failure, at 20% reduction in peak lateral strength)
Point ‘E’ = the point of gravity failure

Points A to C mark the onset of major decreases in the lateral stiffness value. The first appreciable stiffness reduction occurs at point A when concrete cracks usually at the vicinity of the critical flexural section. The reduced stiffness remains practically unchanged up to Point B when longitudinal reinforcements yield (in tension or compression). Longitudinal reinforcement yielding causes a considerable reduction in lateral stiffness which may be idealized as linear up to Point C where maximum lateral strength is developed. Tension yielding of longitudinal reinforcements is
typically expected for under-reinforced columns subjected to low to moderate axial compression. Similar stiffness softening may occur as a result of gradual softening of concrete in compression which is typically anticipated for the heavily reinforced columns and/or the columns subjected to high axial compression.

Imposed deformation beyond Point C is accompanied by both strength and stiffness degradations. Such degradations are attributed to the decline of material capacity in transferring the flexural and/or shear stresses. This is in turn attributed to the yielding of steel and/or softening of concrete in compression.

Point D is a notional point which marks 20% reduction in the peak lateral strength. This point is usually considered as the column failure point which is generally accepted in high seismic regions (refer to the discussion provided in Section 02.2.2). Displacing columns beyond Point D may result in several severe damages such as buckling and debonding of the longitudinal reinforcements, formation of critical shear cracks, and considerable concrete spalling. However, Point E which corresponds to the actual loss of gravity load carrying capacity marks the ultimate failure point in low to moderate seismic regions such as Australia. This concept is discussed elaborately in Sections 2.1.2 and 2.2.2. Reliable and practical prediction of the columns drift capacity at gravity collapse is one of the key objectives of this study which is addressed in Chapter 4.

The key coordinate points of A to E are generally required for characterizing the backbone curve for any given column. Such curves could be used in conjunction with a representative capacity spectrum for seismic performance assessment. These coordinate points are also required for classical dynamic modelling of the columns.

The generalized back bone model as illustrated in Figure 2. 10 may be simplified into a tri-linear backbone model as shown in Figure 2. 11. Such a model is believed to provide a balance between simplicity and accuracy and is therefore recommended in this study. Section 6.2 of this thesis provides specialized recommendations on constructing tri-linearized backbone models for the lightly reinforced, poorly confined columns of interest.
The following sections (Sections 2.3.1 to 2.3.4) present a number of analytical tools for section analysis and force/deformation modelling of columns. Deformation modelling is subdivided into component modelling (flexural, shear and strain penetration components) when appropriate. Existing approaches for lateral and particularly gravity failure modelling are critically reviewed and compared together in Chapter 4 where new improved models have also been proposed for predicting the column drift capacity at lateral and gravity failure limit states.

2.3.1 Moment-Curvature Analysis

Flexural behaviour of the column can be predicted accurately up to the peak lateral strength using nonlinear moment-curvature analysis (Park and Paulay 1975). Moment–curvature analysis ($M - \phi$) takes into account the interaction between flexural and axial stresses utilizing equilibrium. such analysis could be used to obtain section strengths, tangent (initial) and secant (yield) stiffness, cracking and yield curvatures ($\phi_{cr}, \phi_y$ respectively) and corresponding deformations $\Delta_{cr}$ and $\Delta_y$. Yield deformation $\Delta_y$ is considered as the elastic component of total flexural deformation. Underlying assumptions made for moment-curvature analysis are as follows:
1. Navier Bernoulli “plane-sections” holds. In other words, the strain profile is linear at all stages of loading.

2. Strain compatibility is assumed between steel and concrete. (i.e. perfect bonding between steel and concrete is assumed)

3. Concrete tension strength is ignored in the analysis.

4. Axial load is applied at the section centroid.

In moment curvature analysis the section is divided into a number of elements. Strain in the extreme concrete fibre in compression is then incremented starting from the lowest value. This is followed by assuming a neutral axis location at each strain increment and calculating the corresponding concrete and steel stresses at the centre of each layer, and hence the concrete and steel forces in each layer. The equilibrium of all forces including the axial force on the column is then checked using the following equation:

\[
N = \int f_{c(y)} b_{(y)} dy + \sum_{i=1}^{n} (f_{st(y)})_{i} A_{d}
\]  

(2.27)

where

\( N = \) axial force

\( f_{c(y)} \) and \( f_{st(y)} \) are the concrete and steel stresses respectively at a layer which is a distance ‘\( y \)’ away from the centroidal axis.

The concrete or steel stress at each layer is in turn a function of the corresponding strain at the layer \( \varepsilon_{(y)} \), thus:

\[
f_{c(y)} = \Psi_{c}(\varepsilon_{(y)})
\]  

(2.28)

\[
f_{st(y)} = \Psi_{s}(\varepsilon_{(y)})
\]  

(2.29)

If the equilibrium of the forces is not satisfied with the assumed neutral axis position, this position is modified in an iterative procedure until the equilibrium is satisfied. The whole procedure is then repeated for the next increment of the strain in extreme compression fibre. At each satisfied strain increment, moments of the stress resultants about the section centroid can be taken as follows:

\[
M = \int f_{c(y)} b_{(y)} y dy + \sum_{i=1}^{n} (f_{st(y)})_{i} y_{i} A_{d}
\]  

(2.30)
where \( A_i \) is the cross section area of the \( i^{th} \) steel bar which is a distance \( y_i \) away from the centroidal axis. The corresponding curvature is given by:

\[
\phi = \frac{\varepsilon_c}{c}
\]  

(2.31)

where \( \varepsilon_c \) is the extreme-fibre compression strain and \( c \) the neutral axis depth measured from this fibre.

Yield force \( F_y \) is then calculated for a cantilever column by:

\[
F_y = \frac{M_y}{L}
\]  

(2.32)

where \( L \) is the shear span length which is the length from the critical section to the point of contraflexure.

The yield deformation for a cantilever can be approximated using the equation below:

\[
\Delta_y = \frac{\phi_s L^2}{3}
\]  

(2.33)

Equations similar to that of (2.32) and (2.33) could be used to estimate cracking force \( F_{cr} \) and corresponding deformation \( \Delta_{cr} \) where subscript ‘cr’ refers to the limit state of cracking.

It is noted that the above formulae for estimating yield deformation \( \Delta_y \) are based on the simplifying assumption of linear distribution of curvature up the column height. This assumption is a reasonable assumption particularly for slender columns (Priestley et al. 2007).

Central to moment-curvature analysis is the definition of stress-strain relationship for steel and concrete materials, which is discussed in the next section. Employing a representative material relationship is crucial for a realistic section analysis. Material stress-strain relations may also be required as direct inputs for dynamic modelling of the structural components. In this latter case not only the deformation capacity but also the deformation demand can be sensitive to the accuracy of the input material.
2.3.2 Uniaxial Stress-Strain Relationship for Concrete and Steel

2.3.2.1 Stress-Strain Relations for Concrete

Over the past three decades, several studies have been conducted to model the stress-strain relation of concrete under uniaxial compression. These studies lead to a number of important empirical stress-strain relationships for concrete. The stress-strain relationship of concrete under compression is influenced by a number of parameters including the concrete compressive strength, strain rate, monotonic/cyclic loading, and the passive confinement provided by transverse reinforcements. Among all these parameters, the passive confinement of transverse reinforcement has received most of the attention. That is probably attributed to the early recognition of the significant effects of such confinement on the concrete stress-strain relationship and the variety of confinement configuration possible. These studies, in general, show that confinement can significantly enhance the strength and ductility properties of concrete (Kent and Park 1971; Mander et al. 1988; Park et al. 1982; Scott et al. 1982).

Kent and Park (1971) investigated the confining effect of rectangular transverse reinforcements based on the test results reported by Roy and Sozen (1965) and others available at the time. This early model neglected the increase in concrete strength but modelled the enhancement in strain ductility. A decade later the Kent and Park model was modified by Park et al. (1982) mainly to address the aforementioned deficiency. The proposed modified model was named ‘Modified Kent and Park model’.

The modified Kent and Park model took into account both the ductility and strength enhancement of concrete. However, the model was still limited to rectangular confining steel. The proposed model was based on tests at quasi-static or slow strain rate of compressive strain in concentrically loaded columns. The following empirical equations were proposed by (Park et al. 1982) defining the confined stress-strain relationship for concrete under uniaxial compression.

\[
f_c' = Kf_c \left[ 2\varepsilon_c - \left( \frac{\varepsilon_c}{0.002K} \right)^2 \right] \quad \text{for} \quad \varepsilon_c \leq 0.002K \quad (2.34) \\
\]

\[
f_c' = Kf_c' [1 - Z_m (\varepsilon_c - 0.002K)] \leq 0.2Kf_c' \quad \text{for} \quad \varepsilon_c > 0.002K \quad (2.35) \\
\]
\[ K = 1 + \frac{\rho_s f_{yh}}{f_c} \]  

\[ Z_m = \frac{0.5}{3 + 0.29 f'_c + 0.75 \rho_s \sqrt{\frac{h}{S_h}} - 0.002 K} \]  

where

\( \varepsilon_c \) = strain in concrete  
\( f_c \) = stress in concrete (MPa)  
\( f'_c \) = cylinder compressive strength of concrete (MPa)  
\( f_{yh} \) = yield strength of transverse reinforcement (MPa)  
\( \rho_s \) = ratio of volume of hoop reinforcement to volume of concrete core measured to outside of the hoops  
\( h' \) = width of concrete core measured to outside of the peripheral hoop (mm) and  
\( S_h \) = centre-to-centre spacing of hoop (mm)

Equation (2. 37) suggests that an increase in concrete compressive strength \( f'_c \) results in an increase in the slope \( Z_m \) of softening region of stress-strain curve if all other parameters remain constant. This, in fact, is translated to a reduction in the strain capacity or strain ductility.

Scott et al (1982) further extended the modified Kent and Park model to take into account the effect of strain rate on stress-strain curve. The strain rate at which their column specimens were loaded ranged from 0.0000033 to 0.0167/sec. The upper bound of strain rate was considered as representative of the rate expected during the response of reinforced concrete to earthquakes. Scott et al observed that in the concentrically loaded units the increase in the peak concrete stress due to the high strain rate was typically 25%. Based on the observations Scott et al proposed that, for high-strain rate, a multiplying factor of 1.25 be applied to the peak stress, the strain at the peak stress, and the slope of the falling branch. Thus for the high strain rate, the stress-strain relation is given by Equations (2. 34) to (2. 37), where the values of \( K \) and \( Z_m \) have been modified as follows:
Later in (1988), Mander et al. developed a very comprehensive model for concrete confined by transverse reinforcement with any general type of steel: either spiral or circular hoops; or rectangular hoops with or without supplementary cross ties. The influence of various types of confinement was taken into account by defining an effective lateral confining stress, which is dependent on the configuration of the transverse and longitudinal reinforcement. These cross ties could have either equal or unequal confining stresses along each of the transverse axes.

In the study by Mander et al. (1988) the effects of cyclic loading on the stress-strain relation of confined and unconfined concrete was investigated. Results of experiments conducted at different strain rates showed that the envelope curve under cyclic loading was almost identical to the curve obtained from monotonic loading.

Figure 2.12 shows the stress-strain curve for lightly confined (at high strain rate) and unconfined concrete (at low strain rate) estimated from the first three models described above. The maximum compressive strain of the confined concrete was estimated based on Equation (2.41). Input parameters employed in Equations (2.34) to (2.41) correspond to those of the tested column (Column S4) with the volume ratio of hoops being of the minimal order of $\rho_s = 0.0016$ (6mm@300mm transverse reinforcements) and cylinder compressive strength of $f'_c = 24$ MPa. As could be seen in this figure, at such minimal level of concrete confinement, estimated improvement in concrete compressive stress is negligible. This is concluded comparing the unconfined stress-strain curve (based on the Kent and Park Model) with the confined curve at low strain rate (based on the Modified Kent and Park Model). However, comparing the same curves, it is evident that even minimal extent of concrete confinement could enhance the strain ductility rather considerably. The confined stress-strain curve at high strain rates (based on Scott et al. model) suggests

\[
K = 1.25\left(1 + \frac{\rho_s f_{sh}}{f'c}\right)
\]

\[
Z_m = \frac{0.625}{3 + 0.29f'_c + 0.75 \rho_s \sqrt{\frac{h}{S_h}} - 0.002K}
\]
significant improvement in peak concrete compressive stress compared to other curves obtained at low strain rates.

Figure 2. 12: Stress-strain relation for lightly confined and unconfined concrete at low strain rate derived from leading models

Figure 2. 13 compares the confined stress-strain curves at slow and high strain rates based on the leading models presented in this section.

At slow strain rates, the main discrepancy between stress-strain curves is the slope of falling branch or the rate of concrete strength degradation. The slope of the softening branch could have a considerable effect on the force-displacement simulation of inelastic structures. However, in this study, consequence of such uncertainty is not investigated. This is explained considering the aim of the present study and the relevance of high strain rates to seismic events. It should be noted that, in general, in seismic events high levels of compressive strain, approximately of the order 0.017/sec, are expected (Scott et al. 1982). The demand rate of compressive strain is even intensified for high frequency structures.

At high strain rates, the two stress-strain models have resulted in approximately the same slope for their softening branch (see Figure 2. 13). The two models however,
resulted in rather different ascending parts, slightly different peak concrete compressive strength and appreciably different strain estimates (at peak strength). It should be noted that all the input details for producing the stress-strain curves in Figure 2.13 refer to the same case. Therefore the differences in various stress-strain curves originate merely from their respective models and/or the strain rate.

Section 6.4 studies potential effects of some of the uncertainties discussed above on the force-displacement simulation of columns.

![Stress-strain curves](image)

Figure 2.13: Confined concrete stress-strain curves at slow and high strain rates estimated based on the leading models available in literature

2.3.2.2 Ultimate Compressive Strain in Concrete

For unconfined concrete, the conservative value of 0.003 is generally accepted as the ultimate compressive strain limit. This is the strain often considered at the extreme compression fibre when calculating the flexural strength of a section. However it is known that the actual ultimate strain value is higher and of the order of 0.004-0.006 (Priestley et al. 2007). This is the range at which cover concrete spalling is usually observed during experimentations.
Ultimate compressive strains in confined concrete could be significantly higher compared to that of unconfined concrete. The extra strain available to the former is provided by the confining effects of the transverse reinforcements. Therefore the ultimate strain capacity of confined concrete is reached when the transverse reinforcement fracture or opening occurs. Mander et al. (1988) estimated the confined concrete strain limit based on the total energy reserved in transverse reinforcing bars before fracture. Equation (2. 40) was accordingly proposed in the study as follows:

\[ \varepsilon_{ccu} = 0.004 + 1.4 \rho_s \frac{f_{sh}}{f_{cc}} \]  \hspace{1cm} (2. 40)

Where

\[ \varepsilon_{ccu} = \text{the ultimate compressive strain of confined concrete} \]
\[ \varepsilon_{sm} = \text{the steel strain at maximum tensile stress} \]
\[ f_{cc} = \text{the compressive strength of confined concrete} \]
\[ f_{sh} = \text{the yield strength of the transverse reinforcement} \]

Equation (2. 40) was similar to an empirical equation developed earlier by Scott et al. (1982) which is given by Equation (2. 41).

\[ \varepsilon_{max} = 0.004 + 0.9 \rho_s \left[ \frac{f_{sh}}{300} \right] \]  \hspace{1cm} (2. 41)

Where

\[ \varepsilon_{max} = \text{the ultimate compressive strain of confined concrete} \]
\[ f_{sh} = \text{yield strength of transverse reinforcement (MPa)} \]
\[ \rho_s = \text{ratio of volume of hoop reinforcement to volume of concrete core measured to outside of the hoops} \]

Equation (2. 41) applies to both high and low strain rates though it is generally more conservative for the later. That is because an increase in strain rate results in a decrease in the level of ductility as reported in their study.

The proposed maximum compressive strain is in fact the strain at which the first hoop fractures. It was proposed that the estimated maximum strain be used as the strain limit needed in ductility calculations. It should be noted that the strain limit as proposed by Scott et al was based on the test data on both eccentrically and concentrically loaded units with volume ratio of rectangular hoops ranging from
0.0134 to 0.0309. The hoop bars were anchored by a 135° bend around a longitudinal reinforcing bar plus an extension beyond the bond of at least eight hoop bar diameters embedded in the concrete core.

The lower of the two strain values obtained from Equations (2.40) and (2.41) may be accepted as the limiting compressive strain ($\varepsilon_{\text{max}}$) of confined concrete provided that the hoop volume ratio is less than 0.0134. This assumption may be considered adequate until more specific test results (i.e. results from a dedicated study) are made available in the future. It is noted however that such assumption is not likely to introduce any critical uncertainties in the seismic assessment of poorly-confined concrete columns. In flexure-shear dominated columns, the flexurally-induced strain at the critical section is alleviated by the occurrence of other failures (such as shear failure, bar buckling, and etc.) elsewhere along the column height. Importantly the expected ease in strain level occurs at a stage before the peak axial compressive strain capacity ($\varepsilon_{\text{max}}$) is reached.

2.3.2.3 Stress-Strain Relations for Reinforcing Steel

Figure 2.14 shows the characteristics of typical monotonic and cyclic stress-strain response of reinforcing steel (Priestley et al. 2007).

![Figure 2.14: Reinforcing Steel Stress-Strain Characteristics](image-url)
Point A \((\varepsilon_y, f_y)\) on the curve represents the end of linearly elastic stress-strain relation and beginning of the yield plateau. Point B \((\varepsilon_{sh}, f_y)\) marks the start of strain hardening on the curve. Point C \((\varepsilon_u, f_u)\) marks the ultimate stress and Point D \((\varepsilon_{uu}, f_u)\) marks the ultimate stage in which local necking occurs (i.e. the reliable ultimate strain). The monotonic response can be represented mathematically for the three regions of the response are:

\[
\text{Elastic:} \quad 0 \leq \varepsilon_s \leq \varepsilon_y \quad f_s = E_s \varepsilon_s \leq f_y \quad 2.42
\]

\[
\text{Yield plateau:} \quad \varepsilon_y \leq \varepsilon_s \leq \varepsilon_{sh} \quad f_s = f_y \quad 2.43
\]

\[
\text{Strain-hardening} \quad \varepsilon_{sh} \leq \varepsilon_s \leq \varepsilon_{su} \quad f_s = f_u \left[1 - \left(\frac{f_u - f_y}{f_u - f_{sh}}\right) \left(\frac{E_u - E_s}{E_u - E_{sh}}\right)^2\right] \quad 2.44
\]

Where \(\varepsilon_s\) and \(f_s\) are the reinforcing steel strain and stress respectively; \(E_s\) is the modulus of elasticity of steel and the other symbols are as defined earlier.

Typical stress and strain values defining Points A to D depend on the type and grade of reinforcement used. The following values may be seen as approximate values indicative of the typical range expected.

For the grade 60 steel normally used in the American continent

\[
f_y = \sim 400MPa
\]

\[
\varepsilon_{sh} = 0.008
\]

\[
\varepsilon_{su} = 0.1 \text{ to } 0.12
\]

\[
f_u / f_y = 1.35 \text{ to } 1.5
\]

For the tempcore reinforcement used in Europe

\[
f_y = \sim 500MPa
\]

\[
\varepsilon_{sh} = \text{No pronounced yield plateau}
\]

\[
\varepsilon_{su} = 0.09
\]

\[
f_u / f_y = 1.2
\]

As illustrated in Figure 2. 14, the cyclic and monotonic characteristics of reinforcing steel are appreciably different. On unloading and reloading, the stress-strain curve softens early, due to the Bauschinger effect, without any pronounced yield plateau. However it is found that force-displacement response predicted from moment-curvature based on the monotonic stress-strain curves for both concrete and
reinforcement provide a good envelope to measured cyclic response (Priestley et al. 2007).

2.3.3 Shear Strength/Deformation

2.3.3.1 Concrete Contribution to Shear Resistance

The contribution of concrete in resisting shear stresses is usually considered taking into account the presence of reinforcing bar and the shear resistance mechanisms in different RC structural members. For RC columns, the combination of different parameters including column aspect ratio and axial load ratio could result in two rather different shear resistance mechanisms and failures namely diagonal tension and compression failures.

The diagonal tension failure is anticipated following the formation of inclined shear cracks. Whereas the diagonal compression failure can occur either before or after the formation of shear cracks and is characterized by crushing of the concrete along idealized compression struts.

Diagonal compression failure is primarily expected when the column aspect ratio is low particularly if the axial load ratio or longitudinal and confining reinforcement ratios are high. For lightly reinforced, poorly confined columns of interest with shear span to depth ratio \((a/d)\) over 2.5 and axial load near or below the balance point, a diagonal tension failure is expected.

Diagonal shear cracks appear when principle tension stresses \(\sigma_i\) exceed the tensile capacity of concrete. The \(\sigma_i\) can be related to normal tension stresses in the x and y directions \((\sigma_x \text{ and } \sigma_y\) respectively) and the shear stress \((\tau)\) acting on the faces having normal in the x and y directions using equilibrium as follows:

\[
\sigma_i = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2} \tag{2.45}
\]

Assuming \(\sigma_x = 0\) for poorly confined columns and \(\sigma_y = -\frac{P}{A_g}\) (which is the average normal compressive stress in the longitudinal direction of column), Sezen and
Moehle (2004) rearranged Equation (2.45) to obtain an estimation of the shear stress when \( \sigma_1 \) equals the nominal tensile strength of concrete (where \( \sigma_1 = 0.5\sqrt{f_c} \) MPa).

\[
\tau = 0.5\sqrt{f_c} \sqrt{1 + \frac{P}{0.5\sqrt{f_c} A_g}}
\]  

Equation (2.46)

This equation reflects the well-known trend that shear stress values at the onset of shear cracking increases with the axial load level.

Equation (2.46) however was found to constantly overestimate the shear resistance contributed by concrete. Reasons for this could be summarized as follows:

1. Concrete is not a homogeneous material and the state of stress is more complicated than what has been assumed in Equation (2.46).

2. Presence of flexural and bond cracks reduce the shear transfer capacity of concrete by disturbing the aggregate interlock. The extent of this disturbance varies depending on the combination of parameters contributing to the amount of flexural and or bond cracking. Aspect ratio \( a/d \) was found to be one of the main contributor as inferred from the recommendations made by Sezen and Moehle (2004). It was recommended, in the study, that the shear strength be modified to vary with the inverse of the aspect ratio, limited by \( 2 \leq a/d \leq 4 \).

The following equation was ultimately suggested by Sezen and Moehle (2004) for estimating the shear resistance contributed by concrete \( (V_c) \) and was reasonably supported by a data set comprising poorly-confined reinforced concrete columns.

\[
V_c = \frac{0.5\sqrt{f_c}}{a/d} \sqrt{1 + \frac{P}{0.5\sqrt{f_c} A_g}} 0.8A_g
\]  

Equation (2.47)

The exact contribution of concrete to shear resistance of RC members, however, is difficult to determine. This is inferred from the number and evolution of the models proposed by different investigators and guidelines (ACI 318 2008; ATC40 1996; Wibowo et al. 2010b). Existing models are also different in terms of the approach of model development and the nature and characteristics of the columns included in the supporting database. The following examples could be useful in demonstrating such variations.

The relatively recent standard of ASCE/SEI 41 (2007) recommends the use of the relevant provisions of ACI 318(2002) for calculating the shear strength of RC columns, unless modified by the respective standard. According to ACI318 (2002),
the shear strength contributed by concrete, for the RC columns with low ductility demands, is calculated by Equation (2.48).

\[
V_c = 0.17 \left(1 + \frac{N_u}{14A_g}\right) \sqrt{f_c} b_u d
\]  

(2.48)

where

\( N_u = \) axial load

\( A_g = \) gross concrete cross sectional area (\( \frac{N_u}{A_g} \) shall be expressed in MPa)

\( b_u \) and \( d = \) the breadth and effective depth of the column respectively

Priestley et al. (1994) proposed a different equation, Eq. (2.49), for estimating the concrete contribution based on a study investigating the shear strength of mostly ductile columns under cyclic lateral load.

\[
V_c = k \sqrt{f_c} (0.8A_g)
\]  

(2.49)

where \( f_c \) is expressed in MPa.

Parameter \( k \) depends on the member ductility and varies linearly from a maximum value of 0.29 (for a ductility level equal to or smaller than 2) to a minimum of 0.1 (for a ductility level equal to or greater than 4) relevant to columns subjected to uniaxial deformation.

Significance of ductility may also be inferred from the general classifications made by ACI by giving separate shear provisions for members with low, and medium to high ductility demands. Sezen and Moehle (2004), conversely, concluded that shear strength is only mildly sensitive to the imposed ductility demand based on the lightly reinforced columns analysed in the study.

The ductility dependence of shear strength could have important implications in relation to the expected mode of failure for columns, as shown in the conceptual model of Figure 2.15. If the shear force corresponding to flexural strength is less than the residual shear strength, ductile flexural response is ensured. If it is greater than the initial shear strength, a brittle shear failure results. If the shear force is between the initial and residual shear strength, then shear failure occurs at a ductility corresponding to the intersection of the strength and force-deformation curves.
2.3.3.2 Transverse Reinforcement Contribution to Shear Resistance

Contribution of transverse reinforcement to shear resistance is conventionally taken into account using truss analogy originally developed by Ritter and Morsch about a century ago. The truss analogy proposes that a cracked reinforced concrete member such as a beam or column acts similar to a truss when subjected to shear. It should be emphasised that such analogy is only valid upon the formation of a series of inclined shear cracks. The web of the notional truss is composed of diagonal concrete struts (forming in between the inclined cracks) and transverse steel ties. When shear is applied to this truss, the concrete struts are placed in compression, while tension is produced in the transverse ties and in the longitudinal chords. The force component in each member can be calculated from equilibrium. By truss analogy the total shear force that can be transferred is limited by (assuming a diagonal tension failure)

\[ V_s = A_v f_y d \cot \alpha / s \]  \hspace{1cm} (2.50)

where

\[ A_v = \text{the area of the web reinforcement spaced at a distance } s \text{ along the column} \]
\[ d = \text{the effective depth of the column and } \alpha = \text{the angle of inclination of concrete struts to the vertical direction} \]
\[ f_y = \text{the yield stress of the web reinforcement} \]

As for the angle of inclination of concrete struts \( \alpha \), Morsch concluded that it was mathematically impossible to determine the slope as reported by (Vecchio and Collins...
However, it was suggested that the 45 degree assumption could result in conservative estimations.

Equation (2.50) as obtained from the truss analogy (coupled with $\alpha = 45^\circ$) was found to underestimate the observed shear capacity that can be transferred by a cracked RC columns and hence the introduction of a correcting term which is named as the ‘concrete contribution’ at the onset of shear cracking (refer Section 2.3.3.1). Therefore, total or nominal shear resistance of RC columns ($V_n$) is usually calculated as the sum of contributions by the steel and concrete as follows:

$$V_n = V_s + V_c$$  \hspace{1cm} (2.51)

The format represented by Equation (2.51) with the assumption $\alpha = 45^\circ$ is seen in most of the existing models and is adopted by some current guidelines for design and assessment of shear strength of columns (ACI318 2002; ACI 318 2008; Sezen and Moehle 2004).

Priestley et al. (1994) proposed two corrections to compensate for the underestimations associated with truss analogy that follows:

$$V_n = V_s + V_c + V_p$$  \hspace{1cm} (2.52)

where

$V_p = $ shear force that can be resisted by arch mechanism associated with the axial load action. The $\alpha$ value proposed by Priestley et al., to be used in Equation (2.50) for estimating $V_s$, was 30 degree.

2.3.3.3 Modified Compression Field Theory (MCFT)

Vecchio and Collins (1986) developed an analytical model which is capable of predicting the load-deformation response behaviour of reinforced concrete elements subjected to in-plane shear and normal stresses (membrane element). Such an element and corresponding deformation is shown in Figure 2.16a. In this model cracked concrete is treated as a new material with its own stress-strain characteristics. Equilibrium, compatibility, and stress-strain relationships are formulated in terms of average stresses and average strains. This model also takes into account tensile stresses in the concrete between the cracks. According to this model, the principal
Figure 2.16: a) Membrane element and corresponding normal and shear deformations b) stress-strain relationship for cracked concrete in compression c) Average stress-strain relationship for cracked concrete in tension; reprinted from Vecchio and Collins (1986)
-compressive stress in the concrete ($f_{c2}$) is a function of not only the principal compressive strain $\varepsilon_2$ but also the co-existing principal tensile strain $\varepsilon_1$. Thus, cracked concrete subjected to high tensile strains in the direction normal to the direction of compression is softer, and weaker, than concrete in a standard cylinder test (see Figure 2. 16b). This finding was based on the observations from an extensive series of well-designed membrane-element tests and resulted in the following stress-strain relationships for softened concrete in compression.

$$f_{c2} = f_{c2max} \cdot \left[ 2 \left( \frac{\varepsilon_2}{\varepsilon'_c} \right) - \left( \frac{\varepsilon_2}{\varepsilon'_c} \right)^2 \right]$$

$$\frac{f_{c2max}}{f'_c} = \frac{1}{0.8 - 0.34 \varepsilon_1/\varepsilon'_c} \leq 1.0$$  \hspace{1cm} (2.53, 2.54)$$

where $\varepsilon'_c$ is a negative quantity (usually -0.002).

The relationship between the average principal tensile stress in the concrete and average principal tensile strain is nearly linear prior to cracking and then shows decreasing values of $f_{c1}$ with increasing values of $\varepsilon_1$ (see Figure 2. 16c). The following relationship is suggested prior to cracking (i.e., $\varepsilon_1 \leq \varepsilon_{cr}$)

$$f_{c1} = E_c \varepsilon_1$$  \hspace{1cm} (2.55)$$

where $E_c$ is the modulus of elasticity of the concrete which can be taken as $2f'_c / \varepsilon'_c$.

The relationship suggested after cracking (i.e., $\varepsilon_1 > \varepsilon_{cr}$) is:

$$f_{c1} = \frac{f_{cr}}{1 + \sqrt{200\varepsilon_1}}$$  \hspace{1cm} (2.56)$$

The modified compression field theory is in fact a biaxial model that takes into account the interaction between shear and axial load. In this model, degradation of concrete corresponding to the interaction of shear and axial load appears in a softened average stress-strain relationship for concrete in compression and stiffened average stress-strain relationship for concrete in tension. The MCFT was later employed in numerous studies for predicting the shear response of RC concrete members such as
in (Vecchio and Collins 1988). This model however is of little interest when the RC member is not fully/well cracked. Such situation is expected in the case of lightly reinforced columns. In this study the truss analogy as briefly introduced in Section 0 has been employed for developing a simplified model for predicting the shear deformation expected in lightly reinforced columns (refer Section 4.3.2).

2.3.4 Strain Penetration Deformation

Strain penetration deformation ($\Delta_{ext}$) is the deformation caused by the extension of longitudinal reinforcing bars at the column base ($\delta_{ext}$) (Figure 2.17). The extension of longitudinal bars results from the accumulation of strain over the embedded length of longitudinal bars. Strain penetration deformation is extensively studied in different studies over the past decades (Elwood and Eberhard 2006; Park and Paulay 1975; Priestley et al. 2007; Wibowo et al. 2012).

![Figure 2.17: Strain penetration deformation ($\Delta_{ext}$) produced by reinforcing bar extension ($\delta_{ext}$) at the column base](image)

Priestley et al. (2007), for example derived an empirical equation for estimating an effective strain penetration length ($L_p$) as expressed by Equation (2.57)

$$L_p = 0.022 f_{y} d_{bl} \quad (2.57)$$
where

\( f_{ye} \) (in MPa) and \( d_{bl} \) (in mm) are the yield strength and diameter of longitudinal reinforcement respectively.

The strain penetration length was then incorporated into Equation (2.58) for estimating the plastic hinge length (\( L_p \)) (see Figure 2.18). With this approach, the extra deformation expected from strain penetration is indirectly taken into account by increasing the plastic component of flexural deformation.

\[
L_p = KL_x + L_{sp} \geq 2L_{sp}
\]

for beams and columns

\[
K = 0.2 \left( \frac{f_u}{f_y} - 1 \right) \leq 0.08
\]

\[
\Phi_p = \Phi_u - \Phi_y
\]

Figure 2.18: Plastic hinge and strain penetration length idealisation incorporated into the idealized flexural curvature distribution for columns

The empirical solution proposed by Priestley et al. is mainly developed for well reinforced columns exhibiting ductile flexural behaviour. Thus, such solution cannot be automatically extended to treat limited-ductile columns.

The extension of the longitudinal reinforcing bar at the base may be calculated in accordance to the recommendation made by Alsiwat and Saatcioglu (1992). Longitudinal reinforcement extension is the result of accumulation of strain over the embedded length of the reinforcement as mentioned previously. Alsiwat and
Saatcioglu proposed that the strain experienced by the embedded reinforcement be divided into elastic, yield-plateau, strain hardening and pullout cone regions. Elastic region is the portion of reinforcement over which stress remains elastic. The length of this region can be found by equilibrium consideration and the knowledge of average elastic bond stress $U_e$, which could be estimated using Equation (2.60).

Elastic length $L_e$ is estimated by Equation (2.62).

$$\begin{align*}
U_e &= \frac{f_y d_b}{4d} \text{ MPa} \quad (2.60) \\
I_d &= \frac{0.37 A_b f_y}{d_b \sqrt{f_c}} \geq 300 \text{ mm} \quad (2.61) \\
L_e &= \frac{f_y d_b}{4U_e} \quad (2.62)
\end{align*}$$

where

$f_y$, $d_b$ & $A_b$ = the yield stress (MPa), diameter (mm) and cross sectional area (mm$^2$) of the longitudinal reinforcing bar respectively

$I_d$ = development length (mm);

Yield plateau region is the portion of reinforcement with approximately constant stress while in the strain hardening region; the reinforcing bar is further stressed into the strain-hardening range and up to the ultimate stress. In the later case, the reinforcing bar has already yielded and the concrete keys between the lugs have already been crushed. Therefore, according to the recommendations made by Alsiwat and Saatcioglu, it is reasonable to use average frictional bond stress $U_f$ in the last two regions as expressed by Equation (2.63).

$$U_f = (5.5 - 0.07 \frac{S_L}{H_L}) \sqrt{\frac{f_c}{27.6}} \text{ MPa} \quad (2.63)$$

where the clear spacing and height of lugs on the reinforcing bar are $S_L$ & $H_L$ respectively. $S_L$ & $H_L$ may be obtained based on the recommendations of AS1302 - 1977 regarding the maximum average spacing and minimum average height of the reinforcing bars’ deformation. In accordance with this standard, the maximum
average lug spacing ranges from 8.4 to 35mm and the minimum average lug height ranges from 0.5 to 2.5mm for reinforcing bars of 12 to 50mm diameter.

The length of the yield plateau region \( L_{yp} \) can then be calculated from equilibrium of forces by Equation (2.64):

\[
L_{yp} = \frac{\Delta f_{yp} d_b}{4U_f}
\]  

(2.64)

\( \Delta f_{yp} \) is the incremental stress between the beginning and end points of the yield-plateau region.

\[
\Delta f_{yp} = f_{sh} - f_y
\]  

(2.65)

The length of the strain hardening region \( L_{sh} \) can similarly be calculated from equilibrium of forces by Equation (2.66)

\[
L_{sh} = \frac{\Delta f_{sh} d_b}{4U_f}
\]  

(2.66)

where \( \Delta f_{sh} \) is the incremental stress between the beginning and end points of the strain hardening region.

\[
\Delta f_{sh} = f_s - f_{sh}
\]  

(2.67)

Pullout cone is formed when the cover concrete at the loaded end of the embedded longitudinal reinforcement spalls forming a zone of constant stress and strain. This region is assumed to extend a distance equal to the net concrete cover of the column which may be taken as \( L_{pc} = d_b \geq 25.4 \) mm if the exact information is not available.

Total bar extension \( \delta_{ext} \) is estimated by Equation (2.68):

\[
\delta_{ext} = 0.5(\varepsilon_y)L_c + 0.5(\varepsilon_y + \varepsilon_{sh})L_{yp} + 0.5(\varepsilon_{sh} + \varepsilon_s)L_{sh} + \varepsilon_s L_{pc}
\]  

(2.68)

where

\( \varepsilon_y \) = yield strain

\( \varepsilon_{sh} \) = strain at the onset of strain hardening;

\( \varepsilon_s \) = strain at maximum stress or at the end of strain hardening.

Strain penetration deformation (SPD) can be obtained with the knowledge of neutral axis depth \( d_1 \) (Figure 2.17) and the value of \( \delta_{max} \). The value of \( d_1 \) is traditionally obtained using moment-curvature analysis.
Wibowo et al. (2012) proposed a “closed form” iterative procedure, based on equilibrium and compatibility considerations, for simulating strain penetration deformation (SPD). They questioned the use of the classical moment-curvature analysis, proposed by some of previous investigators, for SPD modelling owing to the violation of the plane section assumption (i.e. the linear strain profile assumption). The proposed iterative procedure was applied (by Wibowo et al.) to the limited-ductile RC columns tested in this study (i.e. columns S1-S4). The simulated force-SPD curves were found to match well with the results obtained from experimental observations. This solution is recommended for incorporation into computer programs when detailed SPD modelling is required.

2.4 Existing Seismic Analysis/Assessment Procedures

Existing seismic analysis procedures may be categorized into:

1. the Linear Static Procedure (LSP)
2. the Linear Dynamic Procedure (LDP)
3. the Nonlinear Static Procedure (NSP)
4. and the Nonlinear Dynamic Procedure (NDP)

Linear procedures employ traditional linear stress-strain relationship for estimating the elastic response of buildings, but incorporate adjustments to overall building deformations and material acceptance criteria to account for the probable nonlinear characteristics of seismic response. These procedures are typically used when the building is not expected to experience significant non-linear response. This may be judged, for example, by ‘Demand Capacity Ratio (DCR)’ as introduced in Section 2.4.1.1 of ASCE/SEI 41 (2007).

Non-linear procedures are used when significant inelastic behaviour is expected for the components of the building or the building possess considerable irregularities in plan or vertical direction. Such properties, in extreme cases, may result in strength/deformation discontinuity of structural components and are ideally expected to be detected by non-linear analyses.
Dynamic procedures are employed for the taller buildings when higher mode effects are significant and cannot be ignored, while the static procedures are used when the behaviour of the system is dominated by the first mode of vibration.

The following sections provide a brief account of the linear and nonlinear static procedures and provide brief discussions over the difficulties and uncertainties involved in these procedures. This is intended to partly justify the choice of inelastic time history analysis as employed in this study for the purpose of seismic collapse assessment of lightly reinforced concrete columns.

2.4.1 Linear Static Procedure

When the linear static procedure (LSP) is selected for seismic analysis, an equivalent static force (also known as base shear or pseudo-lateral force) is to be determined. The LSP is based on the notion that the seismically induced strength demand is a function of the spectral acceleration defined at the initial natural period of the building. The magnitude of the base shear is also selected with the objective that when applied to the linearly elastic model of the building, it will result in design displacement amplitudes that represent the maximum displacement expected during the design earthquake.

The magnitude of pseudo-lateral force in a given horizontal direction of a building may be calculated using Equation as stipulated by (AS1170.4 2007).

\[ V = [C(T_i)S_p / \mu]W_i \]  

(2.69)

where

\[ C(T_i) = \text{spectral acceleration at fundamental natural period of building (} T_i \text{)} \]

The parameter \( C(T_i) \) is a function of seismicity of region and site (soil type).

\[ W_i = \text{seismic weight of the structure} \]

\[ S_p = \text{structural performance factor} \]

\[ \mu = \text{structural ductility factor} \]

The fundamental natural period of building \( (T_i) \) may be approximated using

\[ T_i = 1.25K_i h_w^{0.75} \]  

(2.70)

where
\( K \) = a factor which is a function of the building structural system

This factor ranges between 0.05 to 0.11 for different systems.

\( h_n \) = the height of the building

The underlying concept of Equation (2.69) is to allow damage but prevent collapse under the design earthquake events. For this purpose, the elastic earthquake forces are modified (reduced) by the factor \( S_p / \mu \) to allow for inelastic behaviour. The factor \( \mu \) represents the available structural ductility and \( 1/S_p \) represents the available over-strength which is different for different structural systems. The lower the \( S_p / \mu \) value, the lower the base shear \( V \) (or force demand) and the higher the ductility demand. \( S_p / \mu \) value ranges between 0.17 to 0.38 for steel and concrete structures and between 0.38 to 0.77 for masonry structures (AS1170.4 2007). The lower limit of the \( S_p / \mu \) value is specified for systems with higher displacement ductility capacity.

Newmark and Hall (1982) related the structural ductility factor (denoted as \( R_\mu \) hereafter) to the natural period of the system based on the concept of equal displacement, velocity and acceleration demands as follows:

\[
R_\mu = 1 \quad \text{for } T < 0.1 \text{sec} \tag{2.71}
\]

\[
R_\mu = \sqrt{2\gamma - 1} \quad \text{for } 0.1 \leq T \leq 0.5 \text{sec} \tag{2.72}
\]

\[
R_\mu = \gamma \quad \text{for } T > 0.5 \text{sec} \tag{2.73}
\]

\[
\gamma = \frac{\Delta_u}{\Delta_y} \tag{2.74}
\]

Where

\( \Delta_u \) = ultimate displacement of a building

\( \Delta_y \) = yield displacement of a building

Newmark and Hall concluded that the elastic and inelastic systems with very short natural period \( (T < 0.1 \text{sec}) \) appear to experience the same acceleration demand and hence the design base shear should not be reduced for inelastic responses \( (\text{i.e. } R_\mu = 1) \).
For relatively long period range \( (T > 0.5 \text{ sec}) \), Equation (2.73) implies that both elastic and inelastic systems experience the same displacement demand and hence the design base shear could be scaled down, providing that the ductility capacity is scaled up (enhanced) by the same factor.

For systems with intermediate natural period of \( 0.1 \leq T \leq 0.5 \text{ sec} \), the structural ductility factor \( R_\mu \) increases with ductility capacity \( \gamma \) (Equation (2.72)) but with a slower rate as compared to the systems with \( T > 0.5 \text{ sec} \). This recommendation reflects the relevance of equal energy principle for such intermediate period range systems. In other words, for velocity-controlled structures, a trade off between the force and displacement capacity is possible provided the resultant energy absorption capacity of the system remains constant.

The base shear is vertically distributed in proportion to the seismic weight of the structure at each floor and with a pattern which is affected by the period of the building. The following equation is proposed by (AS1170.4 2007) for vertical distribution of seismic forces.

\[
F_i = K_{F,i}V = \frac{W_i h_i^k}{\sum_{j=1}^{n} [W_j h_j^k]} V
\]

(2.75)

Where

\( K_{F,i} \) = seismic distribution factor for the \( i \)th level

\( W_i \) = seismic weight of the structure at the \( i \)th level, in kilo newtons

\( h_i \) = the height of level \( i \) above the base of the structure, in metres

\( k \) = a period dependent factor ranging from 1 to 2

\( n \) = number of levels in a structure

The horizontal equivalent static earthquake shear force \( V_i \) at storey \( i \) is the sum of all the horizontal forces at and above the \( i \)th level \( F_i \) to \( F_n \).

LSP is highly dependent on the accuracy of the estimated fundamental natural period of the system. Natural period, however, is known to be difficult to determine with good accuracy particularly for the systems experiencing (cyclic) inelastic deformation.

Considering a SDOF system, the period is calculated with the knowledge of stiffness and mass of the system as given by Equation (2.76). The mass and stiffness
parameters are not generally constant for a given system/structural component and
tend to change during the course of dynamic response. This results in uncertainties
and/or complexities in obtaining accurate estimates for natural period of the system
/component.

\[ T = 2\pi \sqrt{\frac{m}{k}} \quad (2.76) \]

where

- \( m \) = mass
- \( K \) = lateral stiffness

Lateral stiffness of a column for instance, is reduced as a result of flexural cracking
which generally increases with increasing displacement amplitude. Stiffness is also a
function of the number of cycles experienced at a given displacement amplitude.
Therefore stiffness is drastically affected by the history of response.

Stiffness is not even a constant parameter, at a given point of response, along the
height of a reinforced concrete wall/column (FENWICK and BULL 2000; FENWICK
and BULL 2001; PRIESTLEY and PAULAY 2002). Fenwick explains that the
amount of stiffness reduction in a cracked concrete depends in part on the relative
extent of flexural and axial stresses acting on a section which in turn defines the
position of the neutral axis at the section. As a consequence, the effective moment of
inertia and therefore the effective stiffness of the section vary with height (with the
lowest value occurring at the base).

The external compressive forces (or the equivalent mass) acting on a column may also
change due to pull-push action caused by seismic overturing moments. Such push-pull
actions can dramatically change the stiffness and period characteristics of a column in
view of the significant effect of axial load level on the lateral stiffness.

2.4.2 Nonlinear Static Procedure

Nonlinear static procedure (NSP), also known as ‘static push over analysis (POA)’
may be viewed as a method for predicting seismic force and deformation demands,
which takes into account (in an approximate manner) the redistribution of internal
forces occurring when the structure experiences inelastic behaviour under inertia
forces. For such analysis a mathematical model directly incorporating the nonlinear load-deformation characteristics of individual components of the building is required. The structure is then subjected to monotonically increasing lateral loads representing inertia forces in an earthquake until a target displacement (TD) is exceeded (ASCE/SEI 2007).

Static push over analysis is based on the assumption that response of the structure can be related to the response of an equivalent single degree of freedom (SDOF) system. This implies that the response of structure is controlled by a single mode of vibration (the first mode) and that the assumed mode shape remains unchanged throughout the time history of the response. With this approach, the deflected shape of the MDOF system is represented by a shape vector which remains constant irrespective of the consequences of the strength deterioration of individual elements on the dynamic behaviour of the structural system. These assumptions are known to result in acceptable predictions of the maximum seismic response of multi degree-of-freedom (MDOF) systems, under certain conditions as specified in (ASCE/SEI 2007) and provided that the structural response is truly dominated by a single mode (Krawinkler 1998). The force and displacement requirements obtained from the NSP can be used in conjunction with a series of force and displacement acceptance criteria to determine the seismic performance of different structural component.

Nonlinear force-displacement relationship between base shear and displacement of the control node (taken at the centre of mass at the roof of a building) can also be obtained form POA. Such curve is replaced by an idealized tri-linear curve to estimate the effective lateral stiffness, \( K_e \), and effective yield force \( V_y \) (Figure 2.19). The idealized force-displacement curve is constructed using an iterative graphical procedure in order to get a balance between area under the actual and idealized curves up to \( \Delta_f \). For this purpose, the effective yield strength and deformation (\( V_y \) and \( \Delta_y \) respectively) may be located/moved slightly by judgment. However, care should be exercised to have a reasonable match in the pre-yield ascending region as well as a positive post yield slope representing the base shear- displacement curves.

The effective stiffness, \( K_e \), is taken as the secant stiffness calculated at a base shear force of 60% of the effective yield strength. Effective fundamental period, \( T_e \), can
then be obtained with the knowledge of \( K \), (e.g. in accordance with Provision 3.3.3.2.6 of ASCE/SEI41 (2007)).

![Idealized Force-Displacement Curves](ASCE/SEI 41-06)

Figure 2.19: Idealized Force-Displacement Curves (ASCE/SEI 41-06)

### 2.4.2.1 Target Displacement and Coefficient Method

The target displacement (TD) at the design level of an earthquake can be obtained using an approach which is appropriately named as the coefficient method (e.g. by Equation (2.77) as specified by ASCE/SEI41 (2007)). The TD value represents an estimate of the mean inelastic displacement likely to be experienced by a MDOF system at design level of earthquakes. This value is obtained from elastic spectral displacement of an equivalent SDOF system (at effective natural period) of the structure, modified with a series of modification factor \( C_0, C_1, C_2 \) as given below.

\[
\delta_t = C_0 C_1 C_2 \delta_e
\]

(2.77)

where

- \( \delta_t \) = target displacement at the control node
- \( \delta_e \) = spectral displacement of an equivalent SDOF system obtained in accordance to Equation (2.78)
- \( C_0 \) = modification factor to relate the spectral displacement of a SDOF system to the roof displacement of the MDOF system

This factor is a function of number of storeys, imposed load pattern, and building type.
$C_1 =$ modification factor to relate the maximum inelastic displacements to displacements calculated for linear elastic response
This parameter is a function of: a) $R$ that is a ratio of elastic strength demand to a measure of yield strength; b) site class; and c) effective fundamental period of the building.

$C_2 =$ modification factor to represent the effect of pinched hysteresis shape, cyclic stiffness degradation and strength deterioration on maximum displacement response
Modifications $C_0, C_1, C_2$ can be obtained from Provision 3.3.3.3.2 in (ASCE/SEI 2007).

$$\delta_e = S_0 \frac{T_e^2}{4\pi^2} g$$

(2.78)

where

$T_e, S_0 =$ effective fundamental period of the building and corresponding elastic spectral acceleration respectively
$g =$ gravitational acceleration $\text{m/s}^2$

(ASCE/SEI 2007) standard stipulates that the push over analysis be carried out to at least 150% of the specified TD, recognizing that the recommended TD is the mean value of the expected displacement and that there is considerable scatter about the mean. Primary and secondary components shall have expected deformation capacities not less than maximum deformation demand calculated at the increased target displacement.

2.4.3 Capacity Spectrum Method

The capacity spectrum method was developed by Freeman (Freeman 1998; S. A. Freeman et al. 1975). This method utilizes a graphical procedure to compare capacity of a structure with the demands of earthquake ground motion on the structure. The capacity of the structure is represented by a force- displacement curve, obtained by non-linear static (pushover) analysis. The base shear forces ($V_b$) and roof displacements ($\delta$) need to be converted to the spectral accelerations and spectral displacements of an equivalent Single-Degree-Of-Freedom (SDOF) system, respectively (Fajfar 1999) as expressed by Equations (2. 79) to (2. 82).
\[ S_a = \frac{V_b}{M_1} \]  

(2.79)

where

\( S_a \) = spectral acceleration
\( V_b \) = base shear
\( M_1 \) = the effective modal mass for the fundamental mode of vibration calculated in accordance with Equation (2.80)

\[ M_1 = \left( \frac{\sum_{j=1}^{n} m_j \phi_{j1}}{\sum_{j=1}^{n} m_j \phi_{j1}^2} \right)^2 \]  

(2.80)

where

\( m_j \) = lumped mass at the \( j^{th} \) floor level
\( \phi_{j1} \) = the mode shape element corresponding to the first mode and \( j^{th} \) floor
\( n \) = the number of storeys

\[ S_d = \frac{\delta}{\Gamma_1 \phi_{el}} \]  

(2.81)

where

\( S_d \) = spectral displacement
\( \Gamma_1 \) = modal participation factor for the fundamental mode which is calculated by Equation (2.82)

\[ \Gamma_1 = \frac{\sum_{j=1}^{n} m_j \phi_{j1}}{\sum_{j=1}^{n} m_j \phi_{j1}^2} \]  

(2.82)

The demand of the earthquake ground motion is defined by Acceleration-Displacement Response Spectrum (ADRS) as shown in Figure 2.4. In the ADRS diagram, spectral accelerations are plotted against spectral displacements, with the periods represented by radial lines. The intersection of the capacity spectrum and the demand spectrum provides an estimate of the inelastic acceleration (strength) and displacement demand. Figure 2.20 illustrates the procedure involved in capacity spectrum method. Figure 2.20c schematically shows two different capacity diagrams (i.e. CD1 and CD2) superimposed on the elastic ADRS diagrams constructed
corresponding to 5% and 10% of critical damping. As can be seen in this figure the CD1 intersect the two ADRS diagrams indicating that energy absorption capacity of the structure is greater than the energy demand imposed by the respective earthquake ground motions. It is noted that in this case the assumed level of damping ratio is not critical for the purpose of collapse assessment. However, the damping ratio level seems to be critical for collapse assessment of the structure represented by the capacity diagram of CD2. This structure is expected to collapse if the damping ratio is taken as 5% but is likely to survive if damping ratio is 10%.

According to Krawinkler (1994) there are two fundamental flaws that render the quantitative use of the capacity spectrum method questionable. First, there is no physical principle that justifies the existence of a stable relationship between the hysteretic energy dissipation of the maximum excursion and equivalent viscous damping, particularly for highly inelastic systems. The second flaw is that the period associated with the intersection of the capacity curve with the highly damped spectrum may have little to do with the dynamic response of the inelastic system.

1. Conversion of push over curve to capacity diagram
2. Conversion of elastic response spectrum from standard format to ADRS format

3. Estimating the force and displacement demands imposed on the structure

Figure 2.20: Illustration of the process required for capacity spectrum method

As an attempt to partly address the aforementioned shortcomings, Fajfar (1999) proposed a capacity spectrum method based on inelastic demand spectra. However, such spectra are typically constructed based on the analysis of bi-linear elastic-plastic SDOF systems with certain pre-defined unloading and reloading stiffness rules under particular ensemble of ground motions. These specializations introduce limitations and also uncertainties in application of the method if any of the parameters changed.

The capacity spectrum method is not employed in this study in view of the difficulties and uncertainties discussed above. This study will instead rely on inelastic time history analysis for the intended seismic performance assessment that will be presented in Chapters 5 and 6.
2.5 Summary

Seismic Activity in Australia

- Even in the middle of the Australian plate, far away from tectonic boundaries, earthquake hazard could not be neglected as is evident from the available records such as the magnitude 7.2 event in 1906 off the central west coast of Western Australia (WA) or the 1989 Newcastle (NSW) event which claimed several lives and left $AUS 1.2 billion damage. Another example could be the magnitude 6.8 Meckering (WA) earthquake in October 1986 which left 35 km long surface rupture.

- In low to moderate seismicity intra-plate regions such as Australia, insufficient accelerograms have been recorded to be reliably employed in different seismic assessment procedures. The scarcity of the recorded strong ground motion data could introduce difficulties with seismic analysis/assessment of the buildings in such regions. This issue has been possibly addressed relying on representative synthetic accelerograms.

- Generated synthetic accelerograms based on stochastic simulations of the seismological model are representative of intra-plate earthquake events recorded on rock (Lam et al. 2000a). These accelerograms were employed in the present study for seismic collapse assessment of lightly reinforced RC columns.

- In the low to moderate seismicity regions of Australia, the characteristics of ground motions are such that the velocity demand imposed on a SDOF system by an earthquake subsides with increasing natural period of vibration beyond a certain period limit. Consequently, the energy demand imposed on the structure is not sustained and the corresponding displacement demand is limited to a peak displacement demand (PDD) of engineering interest irrespective of the natural period of the system ($T$). This trend is referred to as ‘displacement control behaviour’ for the systems with the natural period greater than the second corner period of $T_2$ (Lam and Chandler 2005; Lam and Wilson 2004).
Under the condition of displacement controlled behaviour, the PDD value could be compared with the displacement capacity of a structure, or component, for the purposes of seismic stability assessment. This is in contrast with the P-Δ effect governing the stability requirement in high seismic regions.

Soil amplification factors currently used in most codes of practice for earthquake actions around the world including AS1170.4 (2007) originate mainly from the combined empirical and analytical work of (Borcherdt and EERI 1994; Crouse et al. 1996; Dobry et al. 1994; Seed et al. 1994) based on the recordings of the Loma Prieta, California earthquake in 1989. Several other approaches have also been proposed for estimating the soil amplification factor such as the frame analogy (Lam et al. 2001) and the single period estimations method (Tsang et al. 2006). The latter approaches account, directly, for the site natural period, resonance phenomenon, soil damping and characteristics & frequency content of seismic waves and etc. However, in this study the soil inelastic (dynamic) effects are included at the level of ground motions considering the need for incremental dynamic analysis and the objectives of study. For this purpose the well-established SHAKE program by Idriss and Sun (1992) has been employed.

Limited-Ductile RC Columns in Australia

Buildings possessing soft-storey features are commonly found in low to moderate seismicity regions of Australia. Most of the soft story buildings were constructed during 1960’s (prior to the introduction of the first seismic code in this region) and hence the columns and other components were designed without any seismic provisions. These buildings range from 4 to 30 storeys.

The transverse reinforcement of the columns in such buildings was often designed to resist code-specified shear forces resulting in wide-spaced stirrups (i.e. s ≈ d). The widely spaced stirrups are considered inadequate to provide an effective confinement for concrete and to prevent longitudinal reinforcing bars buckling in potential plastic hinge regions.
Longitudinal reinforcement quantity was typically designed to resist the moments obtained from the code-specified lateral forces. This has led to weak column – strong beam construction practice in Australia increasing the uncertainties in regard to the safety of such buildings.

Under the conditions of displacement controlled behaviour all deformation components contributing to extra drift capacity before gravity failure of columns could be counted on for stability assessment purpose. Examples of such (unconventional) deformations are the ones provided by rotation at the level of lap-splices or by rotation at critical shear crack before shear friction failure. The same deformation components may be considered unreliable in high seismic regions particularly when the column strength falls below 80% of the peak lateral strength.

Existing Tools and Models for Force-Deformation Modelling

Sections 2.3.1, 2.3.3 and 2.3.4 present a number of analytical tools for section analysis and force/deformation modelling of RC columns. Deformation modelling is subdivided into component modelling (flexural, shear and strain penetration components) when appropriate.

Some of the existing models for uniaxial stress-strain relationship of steel and concrete are reviewed in Section 2.3.2

Existing approaches for drift modelling at lateral and gravity failure are critically reviewed and numerically compared in Chapter 4 where new improved models have also been proposed for these two performance levels.

Existing Seismic Analysis/Assessment Procedures

Existing seismic analysis procedures may be categorized into the linear and non-linear procedures each of which could be conducted statically or dynamically.
Linear procedures employ traditional linear stress-strain relationship for estimating the elastic response of buildings, but incorporate adjustments to overall building deformations and material acceptance criteria to account for the probable nonlinear characteristics of seismic response. These procedures are typically used when the building is not expected to experience significant non-linear response.

The linear static procedure is highly dependent on the accuracy of the estimated fundamental natural period of the system. Natural period, however, is known to be difficult to determine with good accuracy particularly at inelastic range. The reasons include 1- the inherent difficulties in determining the actual stiffness particularly for cracked RC columns and walls (FENWICK and BULL 2000; PRIESTLEY and PAULAY 2002), 2- stiffness deterioration associated with inelastic response resulting in period shift, and 3- fluctuations in the applied axial load or corresponding mass caused by seismic overturning moments (i.e. by push and pull action).

The Nonlinear static procedure (NSP), such as the capacity spectrum method’ may be viewed as a technique for predicting seismic force and deformation demands, which accounts in an approximate manner for the redistribution of internal forces occurring when the structure experiences inelastic behaviour under inertia forces.

There are two fundamental flaws that render the quantitative use of the capacity spectrum method questionable. First, there is no physical principle that justifies the existence of a stable relationship between the hysteretic energy dissipation of the maximum excursion and equivalent viscous damping, particularly for highly inelastic systems. The second flaw is that the period associated with the intersection of the capacity curve with the highly damped spectrum may have little to do with the dynamic response of the inelastic system (Krawinkler 1994).

This study relies on inelastic time history analysis, in view of the objectives of the study and difficulties involved in static procedures particularly at ultimate inelastic dynamic response of columns (up to gravity collapse). With time history analysis, site effects and uncertainties associated with ground motions can be addressed.
more directly. Time history results are then presented in the form of a simplified collapse assessment procedure as proposed in Chapters 5 and 6.
3 Laboratory Tests on Limited-Ductile Reinforced Concrete Columns

An experimental program was designed and conducted to provide an insight into ultimate drift capacity and cyclic force-displacement behaviour of the concrete columns featuring minimal amount of longitudinal and lateral reinforcement ratios. The program was collaboratively conducted by 3 organizations as follows:

1) The University of Melbourne, Melbourne, Australia
2) Swinburne University of Technology, Melbourne, Australia
3) Chulalongkorn University, Bangkok, Thailand

The experimental program was conducted at Chulalongkorn University.

Lightly-reinforced, poorly-confined concrete columns are found in existing buildings, particularly old buildings, in Australia and some developing countries. The behaviour of these columns, particularly at post-yield stage up to gravity collapse, is not well understood causing considerable uncertainties regarding their seismic performance and safety. This study and the following experimental program are committed to address these uncertainties, building on the findings contributed by previous investigators.

Test setup and details of the four specimens tested (i.e. Columns S1-S4) is presented in Section 3.1. Corresponding test observations obtained during the experimental program of this study is given in Section 3.2. A summary and discussion on reported test observations on limited ductile RC columns as conducted by previous investigators is presented in Section 3.3 followed by a summary/conclusions section (Section 3.4).

3.1 Specimens Details and Test Setup

The experimental program conducted in this study consisted of four cantilever RC columns (Columns S1-S4) subjected to nominally constant axial compression and reversed cyclic lateral deformation of increasing amplitude. The tests continued until loss of gravity load carrying capacity in all the cases.
Table 3.1 presents details of the four columns tested. All columns had cross sectional dimensions of 270 mm by 300mm, the later being in the direction of lateral loading, and height of 1200mm (measured from the base to the centre-line of the lateral action). The only parameters changed among the specimens were:

1) Axial load ratio (20% or 40%) – This range was selected to simulate the typical axial load values imposed on RC columns in common medium-rise, soft-storey buildings in Australia.

2) Longitudinal reinforcement ratio (0.56% or 1%) - This range was selected to allow for the study of response behaviour of columns which may just comply or even fail to comply with the minimum reinforcement requirement.

3) Transverse reinforcement ratio was minimal for all specimens and of the order of 0.07%. This minimal value was selected to represent the worst case scenario in regard to shear reinforcement.

Longitudinal reinforcing bars were continuous over the column length. The average yield strength of the vertical reinforcements was 527 MPa and 515 MPa respectively for the 12 and 16 millimetre diameter bars. The average yield strength of the 6mm diameter bar used in transverse reinforcements was 365MPa. The yield values were obtained experimentally.

Vertical spacing of the stirrups was 300mm and the first stirrup was 150mm above the base in all specimens. Stirrups had 135° hook with 60 mm extension which did not meet current seismic detailing requirements.

The cover concrete was 25mm to longitudinal reinforcing bars.

Table 3.1: details of the column specimens prepared for experimentation

<table>
<thead>
<tr>
<th>Sp. No.</th>
<th>Dimensions d×b×h (mm)</th>
<th>a</th>
<th>n</th>
<th>(\rho_l) (%)</th>
<th>(\rho_t) (%)</th>
<th>(L.B.) config.</th>
<th>(s &amp; T.B.) config.</th>
<th>(f_c) (MPa)</th>
<th>(P) (KN)</th>
<th>(f_{yt}), (f_{yt}) (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>300<em>270</em>1200</td>
<td>4</td>
<td>0.2</td>
<td>0.56</td>
<td>4Y12</td>
<td>0.07</td>
<td>R6@300, 135 deg. hooks</td>
<td>20.3</td>
<td>329</td>
<td>527,365</td>
</tr>
<tr>
<td>S2</td>
<td></td>
<td>0.2</td>
<td>1</td>
<td>4Y16</td>
<td></td>
<td></td>
<td></td>
<td>21</td>
<td>340</td>
<td>515,365</td>
</tr>
<tr>
<td>S3</td>
<td></td>
<td>0.4</td>
<td>1</td>
<td>4Y16</td>
<td></td>
<td></td>
<td></td>
<td>18.4</td>
<td>596</td>
<td>515,365</td>
</tr>
<tr>
<td>S4</td>
<td></td>
<td>0.4</td>
<td>0.56</td>
<td>4Y12</td>
<td></td>
<td></td>
<td></td>
<td>24.2</td>
<td>784</td>
<td>527,365</td>
</tr>
</tbody>
</table>

Notes: Shear span ratio \(a = h / d\); axial load ratio \(n = P / A_g f_c\); longitudinal reinforcement ratio \(\rho_l = A_{sl} / bd\); transverse reinforcement ratio \(\rho_t = A_{st} / bs\); \(L.B.\) & \(T.B.\) = longitudinal and transverse reinforcement configuration respectively; s=spacing of stirrups; \(f_c\)=concrete cylinder compressive strength; \(P\)=axial load; \(f_{yt}\) and \(f_{yt}\) are the yield strength of the vertical and transverse steel bars respectively.
The test setup is illustrated in Figure 3.1. Axial compressive load was applied using a hydraulic jack and maintained approximately constant during the test (with about 5% fluctuations). The hydraulic jack was connected to a roller support at the top (allowing for horizontal movement of the jack together with column) and had a hinged interface (between the jack and the column tip) ensuring the verticality of the applied axial load throughout the cyclic test.

Lateral load was provided by a 1000 KN actuator. The loading was applied in displacement-control mode with drift increments of 0.25% (3mm) up to 2% drift (24mm) followed by drift increments of 0.5% (6mm) up to the axial load carrying failure. Two complete cycles of reversed displacement were applied at each drift level. Figure 3.2 illustrates the applied lateral loading scheme.
Figure 3. 2: Displacement-controlled loading scheme- 2 cycles at each drift level

Figure 3. 3: Test set up- horizontal, vertical & diagonal LVDTs to measure total & components of lateral deformation (flexure & shear components)
Figure 3.3 shows the instrumentations employed for the tested specimens. Horizontal LVDTs were mounted at 5 different locations along the column height to measure and monitor lateral deformation of the column at each location throughout the experiment. Six LVDTs were mounted vertically at two column sides to provide the data required for constructing experimental curvature profiles at different drift limits (see Figure 4.17). Flexural component of deformations could be obtained from these profiles (Wibowo et al. 2010a). Diagonally arranged LVDTs were mounted to acquire the data necessary for calculating the shear component of deformation and its height-wise distribution along the column height at different drift limits (see Figure 4.19). Axial deformation was also monitored and recorded from the very first stage of loading up to gravity failure. Several LVDTs and dial gauges were also employed to monitor any possible displacement of the footing.

Apart from LVDTs, a number of strain gauges were mounted on vertical reinforcing reinforcements (at three levels) and on stirrups (on the first two stirrups from the base). The strain data was acquired by a data logger and was recorded at every 0.5 second intervals. Attached strain gauges on vertical reinforcements (at the level of the column-footing interface and 50mm below the column base) were mainly intended to monitor the strain penetration while the rest were primarily intended to reflect possible yielding at the vicinity of the base.

3.2 Test Observations Obtained in This Study

3.2.1 Column S1

Specimen S1 with only 0.56% vertical reinforcement ratio and 0.07% confinement reinforcement ratio exhibited a surprising drift capacity of 5% before complete gravity collapse (Figure 3.5). First horizontal crack was observed when 3mm displacement was imposed to the column (corresponding to the drift ratio of 0.25%). With increasing the amplitude of cyclic displacements a few more widely-spaced flexural cracks were formed within approximately 45cm of the column length measured from the base. Tensile yielding of vertical reinforcements was recorded at a few locations in the vicinity of the column base between 0.75% to 1% drifts. Further increase in lateral displacement caused most of the flexural cracks to propagate with an inclined pattern indicating shear distresses. At 2.5% drift, the horizontal crack at
the column base was the widest one amongst all cracks (about 3mm). Such a wide crack may be explained considering inelastic strain of vertical reinforcement at the base coupled with bar pullout caused by strain penetration both upward (into the column) and downward (into the footing). At 3% drift, buckling of a vertical reinforcement at the base of the column led to spalling of cover concrete at the relevant corner as shown in Figure 3.4. The spalled length was less than 15 cm from the base. Reversing the direction of the imposed deformation, the previously buckled bar was stretched back into the straight condition and continued to contribute to flexural resistance of the column. By the end of the two cycles at 3.5% drift ratio, similar localized bar buckling and spalling of concrete, occurred at all other corners of the column. The buckling and stretching mechanism of longitudinal reinforcements were observed during all cycles beyond first buckling (i.e. 3%). It is interesting, however, to note that the vertical bar buckling did not trigger gravity failure. The column could maintain 100% of the applied axial load and 50% of the maximum lateral capacity at 4% drift ratio.

The bar-buckling and cover concrete spalling at the base, however, caused some redistribution of compressive forces (or stresses) relying increasingly on the inner core confined concrete. The trend was evident by the formation of discrete and continues vertical crack followed by core concrete spalling, within the 100 to 400 mm of the column height measured from the base, (refer Figure 3.5).

![Figure 3.4: Spalling of concrete caused by bar buckling at 3% drift ratio](image)
Surprisingly no opening of the hooks was observed although the hook extension was shorter than the recommended length stipulated by current seismic codes. In summary despite the formation of plastic hinge and buckling of vertical reinforcements at the bottom 100 mm of the column, propagation of concrete spalling to the core concrete, above the buckled area, resulted in ultimate axial failure.

Figure 3. 6 illustrates the recorded hysteresis loops for Column S1. Strength degradation started at the drift ratio of approximately 1.7% (~0.02m). The residual lateral strength reached 80% of the peak strength at the drift ratio of 3.3% (~0.04m). This point marked the point of notional lateral failure or shear failure.

Figure 3. 7 shows selected positive drifts, taken from the cyclic excursion of Column S1, versus corresponding axial deformations. It can be seen that the measured axial deformations remained relatively small (i.e. less than 1mm) up to 80% of the ultimate drift value. Within this range of drift the axial load carrying capacity of the column was practically unaffected.

Figure 3. 5: Column S1 at: (a) 4% drift ratio; and (b) complete collapse (5% drift ratio)
Figure 3.6: Force-displacement hysteretic response of Column S1

Figure 3.8 shows the complete history of the measured axial deformation versus corresponding drift ratio.

Figure 3.7: Column S1-Axial deformation versus drift ratio
3.2.2 **Column S2**

Column S2 featured 1% vertical reinforcement, 0.56% lateral reinforcement, and 20% axial load.

The observed behaviour and failure process of this column could be summarised as follows:

1. The first flexural (horizontal) crack was observed at 3mm lateral displacement corresponding to 0.25% drift ratio.
2. Few more flexural cracks were formed with increasing deformation. Cracks were approximately equally spaced occurring at 15 cm intervals.
3. Starting from about 0.75% drift, flexural cracks grew in length with increasing deformation and turned into inclined shear cracks (Figure 3. 9a).
4. Tensile yielding of longitudinal reinforcements was recorded at the drift ratio of 0.75% to 1% at the vicinity of the critical flexural section (i.e. the column base).
5. Maximum lateral capacity was reached at a drift ratio of about 1.7%. Formation of inclined shear cracks did not seem to interrupt the lateral strength development. Beyond this point (i.e. 1.7% drift ratio) the lateral strength was degraded, showing a 20% reduction in peak capacity at the drift.
ratio of 2.1%. Figure 3. 9 illustrates cracking and damage conditions at 1.25% and 2% drifts.

6. From about 1% drift ratio, discrete vertical cracks appeared along the main bars (beginning at the level of each horizontal crack). Discrete cracks grew in length with the imposed lateral displacement forming critical vertical cracks as can be seen in Figure 3. 9b. Such cracks were clear indication of debonding of the longitudinal bars.

7. Debonding of the longitudinal bars, once completed, interrupted the flexural resistance mechanism of the column and resulted in large spalling of cover concrete (Figure 3. 10a), leaving the column more vulnerable to shear failure.

8. Imposing 2.5% drift, Column S2 lost its integrity and gravity load resistance. Figure 3. 10 shows the damage state at this stage. It could be seen that opening of the critical shear crack resulted in slippage of the column upper part with respect to the lower part leading to disintegration of the core concrete. Buckling of the longitudinal bars was mainly a by-product of the rotation and slippage at critical shear crack.

It is interesting to note that, in contrast to the first specimen (S1), no conventional plastic hinge was formed in Column S2. The bar buckling in S2 also occurred over a considerable length, approximately 300 mm, as opposed to the localized bar buckling observed in the Specimen S1 which was about 100mm long at the base.
Figure 3.9: Column S2 at: (a) 1.25%; and (b) 2% drift ratios; small vertical cracks merged together indicating the debonding of L.B. over a considerable length of the column.

Figure 3.10: Column S2 at failure (2.5% drift)
Figure 3. 11: Force-displacement hysteretic response of Column S2

Figure 3. 12: Axial deformation history up to gravity failure-Column S2

Figure 3. 12 shows the column axial deformation history versus drift ratio. As can be seen in this figure, positive axial deformation (column shortening) gradually increases with increasing drift ratio approximately up to the drift ratio of 2.25%. Past this point, the axial shortening is abruptly increased marking the actual gravity failure for this column.
3.2.3 **Column S3**

Column S3 featured 1% vertical reinforcement, 0.56% lateral reinforcement, and 40% axial load (more details can be found in Table 3.1). The main observations from the initial loading stage to gravity failure can be summarized as below:

1. First crack and longitudinal bar yielding occurred at the drift ratio of 0.25% and 1% respectively. The cracks were predominantly of horizontal (flexural) type and were widely-spaced up to 0.5% drift.
2. Within the drift ratios ranging from 0.5% to 1.25%, inclined shear cracks were mainly formed extending from the existing flexural cracks. The shear distress did not seem to prevent the flexural capacity development as the maximum capacity was reached at the drift ratio of 1.12%.
3. By imposing 15 mm displacement (corresponding to the drift ratio of 1.25%) measured at the top of the column, discrete vertical cracks were appreciably appeared. These cracks rapidly increased in number with further cycles of the same drift level and merged together forming critical vertical cracks (see Figure 3.13). Discrete vertical cracks could be attributed to high axial strains resulted from axial and flexural compressive stresses.
4. The final axial failure was accompanied by bar buckling within the critical unconfined region, rotation at the level of the critical shear crack, and slippage of the upper part of the column with respect to the lower part (Figure 3.14).

Figure 3.15 shows the measured lateral load-displacement hysteretic response for Column S3. Figure 3.16 shows the history of the measured axial deformations versus corresponding drift ratios for this column.
Figure 3.13: Column S3: (a) at 1.25% drift ratio; and (b) at the initial cycle corresponding to 1.5% drift ratio

Figure 3.14: Column S3 after completion of: (a) the first quarter cycle at 1.5% drift ratio; and (b) the second quarter cycle at the same drift level (marking complete failure)
Figure 3. 15: Hysteretic response of Column S3

Figure 3. 16: Axial deformation history up to failure-Column S3

3.2.4 Column S4

The longitudinal reinforcement and axial load ratios in Column S4 were 0.56% and 40% respectively. The vertical reinforcement ratio in this column was about half of the ratio implemented in Column S3. Nevertheless, the experimental results revealed
similar behaviour and collapse mechanism for the two specimens suggesting that longitudinal reinforcement ratio is not a critical parameter when the axial load ratio is high.

Buckling of vertical bars and spalling of concrete at the buckled region can be seen in Figure 3. 17. The buckling is caused by the resultant contribution of the axial and flexural stresses within the unconfined length of the column (i.e. 150 to 450mm from the base). The bar buckling and therefore concrete spalling reduced the concrete area contributing to shear resistance. The remaining concrete was also totally unprotected against cyclic shear stresses and deterioration. This made the specimen to end up in abrupt shear failure as can be seen in this figure.

Figure 3. 17: Column S4 at gravity load collapse (at 1.5% drift ratio) shown from different view points

Column S4 continued to carry the axial load up to 1.5% drift. This may be verified by the axial deformation data recorded during the test (see Figure 3. 19). It can be seen that the axial deformation is relatively small and of the order of 0.5 mm up to first reaching 1.5% drift. This point is circled in Figure 3. 19. Completing the cycle(s) at the same drift level was accompanied with slippage of the column upper part along the critical shear crack resulting in abrupt axial deformation for this column.
Table 3.2 presents a summary of the experimental observations for Columns S1-S4. Observed lateral forces (in KN) and corresponding drifts (drift ratios in %) are tabulated for the important points of force-displacement history, namely: 1) longitudinal bar yield point; 2) point of peak lateral strength; 3) nominal point of shear failure; and 4) ultimate gravity load failure. The last two columns in this table provide abbreviations to briefly describe the observed failure modes and the key mechanisms contributing to failures. It should be mentioned that all the flexure-shear dominated columns denoted by (FS), ultimately developed critical shear crack and their gravity failure was marked by slippage of the upper part of the column with respect to the lower part when frictional resistance at critical shear crack fell below the level required to maintain equilibrium.

![Experimental force-displacement hysteretic response](image)

Figure 3.18: Hysteretic response of Column S4
However, the mechanisms experienced before this very ultimate stage was different from one column to another. For example, debonding of longitudinal bars initially enhanced the drift capacity in Column S2 by providing temporary relief in compressive stresses in critical flexural sections (referred to as averaging process).

Table 3. 2: A summary of the experimental results for Specimens S1 to S4

<table>
<thead>
<tr>
<th>Parameter</th>
<th>S1</th>
<th>S2</th>
<th>S3</th>
<th>S4</th>
</tr>
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<tr>
<td>(V_y) (KN)</td>
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<td>66.7</td>
<td>79.4</td>
<td>76.0</td>
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<td>(\delta_y) (%)</td>
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<td>0.75</td>
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<td>1.00</td>
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<td>(V_{max}) (KN)</td>
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<td>78.4</td>
<td>82.9</td>
<td>77.0</td>
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<td>(\delta_{max}) (%)</td>
<td>1.71</td>
<td>1.73</td>
<td>1.12</td>
<td>1.01</td>
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<td>(V_{80%}) (KN)</td>
<td>47.8</td>
<td>62.7</td>
<td>67.0</td>
<td>64.0</td>
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<td>3.30</td>
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<td>1.5</td>
<td>1.5</td>
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<td>(V_{collapse}) (KN)</td>
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<td>(60)*22</td>
<td>67.0</td>
<td>64.0</td>
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<tr>
<td>(\delta_{collapse}) (%)</td>
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<td>(2.25)*2.5</td>
<td>1.5</td>
<td>1.5</td>
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<td>Mode of failure</td>
<td>F</td>
<td>FS</td>
<td>FS</td>
<td>FS</td>
</tr>
<tr>
<td>The main contributor to failure</td>
<td>PSC after BB</td>
<td>DLB</td>
<td>Excessive compressive forces-LCC</td>
<td>Excessive compressive forces-LCC</td>
</tr>
</tbody>
</table>

Notes:
- \(V_y, V_{max}, V_{80\%}\) & \(V_{collapse}\) is the observed lateral capacity of the tested columns at yield, peak lateral strength, shear and gravity failure limit states respectively (in KN).
- Corresponding drift ratios (%) are denoted by \(\delta\) values.
- *The values in parentheses represent conservative points of gravity failure judged based on axial deformation records (Figure 3.8 and Figure 3.12)
- The abbreviations used in the last two rows are: Flexure-dominated column (F); Flexure-shear dominated column (FS); Progressive spalling of concrete (PSC); Debonding of longitudinal bars (DLB); Longitudinal bar buckling (BB), and Local crushing of concrete (LCC)
Debonding, once completed, interrupted the flexural load transfer mechanism and led to spalling of cover concrete over a considerable length letting the column be relied on frictional shear resistance at the critical shear crack. Columns S3 and S4, mainly suffered from excessive compressive stresses and strains associated with high axial load and flexural deformation. Excessive compressive stresses led to bar buckling and local crushing of concrete (LCC) over the critical unconfined length letting the equilibrium of forces, similar to Column S2, be dependent on frictional shear resistance at critical shear crack.

3.3 Test Observations on Limited-Ductile RC Columns (Previous Studies)

Effects of longitudinal reinforcement ratio and axial load ratio on drift capacity and collapse mechanism of lightly-reinforced, poorly-confined columns were presented with reference to the experimental results obtained in this study. Some effects associated with other key parameters (i.e. aspect ratio, transverse bar ratio, axial and lateral loading schemes, and longitudinal bar lap-splices) on limited-ductile RC columns were collected from previous studies and are summarized in this section. This is intended to facilitate a broader understanding of the concept which is both helpful for deformation modelling and also crucial when conservative assumptions or informed engineering decisions need to be made. Along with this objective a data set comprising 56 limited-ductile RC columns (all tested up to gravity collapse) were collected from 10 studies as given in Table 3.3. The referenced studies are (Ho 2003; Lam 2003; Lynn et al. 1996; Nakamura and Yoshimura 2002; Nakamura and Yoshimura 2003; OUSALEM et al. 2004; Rodsin 2007; Sezen and Moehle 2006; Yoshimura et al. 2004; Yoshimura and Yamanaka 2000). This table provides basic information as well as the measured lateral and gravity failure drifts for the 56 columns listed. The compiled data set has been used for the discussions provided in this section and also for the probabilistic lateral and gravity load failure models developed in Sections 5.2.5 respectively.
3.3.1 Effects of Column Aspect Ratio on Gravity Failure Drift Capacity

The columns stated in the above mentioned literature cover a wide range of aspect ratio \( n = L/d = \text{shear span to depth ratio} \) ranging from 1 to as high as 5 (see Table 3.3).

Rodsin (2007), specifically investigated the effects of aspect ratio on gravity failure drift capacity of limited-ductile columns. The three columns tested in the study (i.e. S1, S2 & S3) featured different aspect ratios (i.e 3.75, 2.75 & 2.25 respectively) while all other parameters were kept unchanged amongst the three columns. Experimental results showed that the value of drift at the limit of gravity collapse could increase or decrease with an increase in the aspect ratio value. This behaviour was reported to depend on the resultant mechanism of failure and the deformation components contributing to the total drift capacity prior to gravity load failure. The observed superior deformation capacity for Column S2, for example, as compared to the other two columns (i.e. S1 & S3) was attributed to the additional deformation provided by rotation (or opening) at the level of the critical shear crack.

Investigations considered in this thesis, including the 56 columns collected from the literature, suggest that the ultimate drift capacity is not appreciably sensitive to the aspect ratio value \( n \) when \( n \) is approximately greater than 2.5 (refer to Figure 5.1). For columns with aspect ratio smaller than about 2.5, the drift capacity expressed in terms of \% of the column height at gravity load failure is generally significantly higher than columns with higher aspect ratio.

3.3.2 Effects of Transverse Reinforcement Ratio on Gravity Failure Drift Capacity

Effect of transverse reinforcement on concrete confinement and control of the lateral strength deterioration is well-established particularly up to the limit of lateral failure (typically taken at 20% reduction in peak lateral strength). It is generally observed that the greater the transverse reinforcement ratio and the smaller the spacing of stirrups, the greater the ductility characteristics of columns. Importantly, this effect was found to be generally true at later stages of loading up to gravity failure (as suggested previously). For example, considering the experimental program conducted by OUSALEM et al. (2004) and comparing the first two shear-dominated specimens
(i.e. C1 & C4), it can be seen that increasing the transverse reinforcement ratio from the low value of 0.08% to the moderate value of 0.28% could improve the drift ratio at gravity failure by a factor of 4. This trend however cannot be generalized as the drift capacity also depends on many other parameters and resultant mode of failure.

To clarify this statement, results from the testing of Specimen C4 considered above is compared with results from the flexure-dominated specimen as tested in this study (i.e. Column S1). As shown, Column S1 with 0.07% transverse reinforcement ratio had a gravity failure drift capacity 25% greater than that of C4.

The investigations conducted in this thesis and also in a recent study by Wibowo et al. (2012) confirmed that no one single design parameter could adequately constrain the ultimate drift limits. Nevertheless, it was observed that the transverse reinforcement ratio and the axial load ratio were far more influential compared to the other parameters: aspect ratio and longitudinal bar ratio.

3.3.3 Effects of Lateral Loading Scheme on Gravity Failure Drift Capacity, (Monotonic vs. Cyclic)

A review of the existing literature revealed that only a very limited number of column specimens have been tested monotonically up to the limit of gravity collapse while the majority were subjected to reversed-cyclic loading. The ultimate drift capacity reported for similar columns with different lateral loading schemes were found to be closely comparable. For instance, specimens 2CLD12 & 2CLD12M tested by Sezen and Moehle (2006) or Specimens FS1 & FS0 tested by Yoshimura and Yamanaka (2000) clearly displayed such behaviour. Some reported observations, within the scope of discussion, can be summarized as follows:

1. Maximum lateral resistance in monotonically and cyclically tested specimens were similar.
2. The rate of lateral force degradation with increasing lateral deformation was slightly less in monotonically tested specimens (i.e. 2CLD12M & FS0) as compared to cyclically tested counterpart specimens (i.e. 2CLD12 & FS1 respectively).
3. Axial shortening at gravity collapse was generally comparable for each column pairs subjected to different lateral loading conditions (i.e. monotonic or reversed-cyclic).
It is noted that the above observations are associated with relatively slender columns with flexure-shear mode of failure. Relevant observations on shear–dominated short columns as reported by Nakamura and Yoshimura (2002) showed some discrepancies with the observations summarized above. Detailed discussions could be found in the respective studies.

Based on the observations from the limited tests reported it can be assumed that strength and stiffness properties at a given drift limit, of the limited-ductile RC columns with the aspect ratio (shear span to depth ratio) of approximately greater than 2.5, are not significantly affected by the number of cycles experienced previously.

3.3.4 Effects of Axial Loading Condition on Gravity Failure Drift Capacity

As for the axial loading scheme, only few limited-ductile specimens were tested under variable compressive-tensile axial load (up to gravity collapse) while the majority were subjected to nominally constant axial compression. Variable loading is intended to simulate the push and pull action in perimeter/exterior columns. Sezen and Moehle (2006) reported two similar columns, one subjected to constant compressive load of 2670KN (i.e. Column 2CHD12) and the other one with variable compressive-tensile axial load, ranging from 2670 KN to -250 KN (i.e. Column 2CVD12). All other parameters for the two columns were kept constant.

Experimental results showed 50% higher drift capacity for columns subjected to varying axial load level. Importantly, it was reported that Column 2CVD12 suffered more strength degradation in the increasing compressive direction than in the increasing tensile direction. Based on this observation it is hypothesized/explained that such less-demanding cyclic deformations could apparently provide the opportunity for additional deformation components to contribute towards the ultimate drift capacity before gravity load collapse. This is consistent with the reported observations corresponding to this column (i.e. extensive longitudinal reinforcement debonding, widening of inclined and vertical cracks, concrete crushing and spalling as well as longitudinal bar buckling in this column).
Referring to the discussion above and based on the limited data available to date, it is concluded that a drift capacity model developed with reference to a database comprising of columns subjected to constant axial compression, could be expected to result in acceptable estimations for perimeter columns which are subjected to push and pull actions.

3.3.5 Effects of (Insufficient) Lap-Splices on Gravity Failure Drift Capacity

Lynn et al. (1996) reported the results for eight poorly-confined reinforced concrete columns tested up to the limit of gravity load collapse. The columns had either continuous or lap-spliced longitudinal bars (at the base) denoted by letters ‘C’ and ‘S’ respectively in the designated column names. The tested columns were subjected to moderate or low axial loads identified by Letters ‘M’ and ‘L’ respectively. The following observations are of interest herein:

1. Presence of lap-splices in longitudinal bars has the potential to improve the drift capacity reserve up to the limit of gravity failure (e.g. compare Columns 3SLH18 with 3CLH18 in Table 3. 3). The improvement is associated with vertical bar debonding and sliding, at the level of the lap-splices, which are linked to inadequate passive confinement/pressure on vertical bars as expected in poorly-confined RC columns.

2. The desirable effect of lap-splices on the ultimate drift capacity of columns is further enhanced by reduction in the longitudinal reinforcement ratio (e.g. compare the measured ultimate capacity of Columns 3SLH18 and 2SLH18 with 3 and 2 percent longitudinal bar ratio respectively); and finally an increase in the axial load ratio could compromise the potential effects mentioned above (e.g. compare Columns 3SLH18 and 3SMD12 listed in Table 3. 3).

3. Lynn et al. (1996) summarized the above observations by concluding that “If a column has light axial load, and failure initiates in the lap-splices, the column may simply hinge at the splice, temporarily relieving shear demands, and leaving a column capable of sustaining gravity loads until splice failure degenerates to shear failure.”
Based on the experimental observations discussed above, it is conservative to ignore additional drift capacity that might be available to poorly-confined columns due to the presence of vertical bar lap-splices.

Table 3.3: 56 limited -ductile RC columns tested up to gravity failure collected from 10 studies

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<th>Specimen Designation</th>
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<th>Exp. D.R.</th>
<th>F.M.</th>
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<td>d x b x h (mm)</td>
<td>n</td>
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<td>300</td>
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<td>600</td>
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</tbody>
</table>

Notes:
- d and b = the depth and breadth of column;
- n = shear span ratio
- \( LBR = \frac{A_y}{bd} \) (longitudinal reinforcement ratio)
- \( TBR = \frac{A_y}{ds} \) (transverse reinforcement ratio)
- \( ALR = \frac{P}{A_y f'c} \) (axial load ratio)
- \( f'c \) = concrete cylinder compressive strength
- SF & GF= observed drift ratio at shear (point of 20% drop in peak lateral strength) and gravity load failure limits respectively
- F.M.= failure mode
- F= flexure dominated column
- FS= flexure-shear dominated column
- S= shear dominated column

### 3.4 Summary

► An experimental program consisting of cyclic tests on four lightly-reinforced, poorly-confined concrete columns was undertaken. The only parameters which varied amongst the cantilever specimens were the axial load ratio (i.e. 20% or 40%) and the longitudinal reinforcement ratio (i.e. 0.56% or 1%). The transverse reinforcement ratio was minimal and the same for all specimens (i.e. 0.07%). The main outcomes from these tests (Column S1-S4) are summarized in below.
**Column S1**

- Specimen S1 with only 0.56% vertical bar ratio, 0.07% confinement reinforcement ratio, and 20% axial load ratio exhibited a surprisingly high drift capacity \( \delta_{\text{collapse}} \) of 5% where complete gravity collapse occurred. The column could maintain 100% of the applied axial load and 50% of the peak lateral capacity at 4% drift ratio.
- The failure mechanism in Column S1 was dominated by flexure.
- The buckling and re-stretching mechanism of longitudinal bars at the base were observed during all cycles beyond first buckling (i.e. 3%). Bar buckling did not cause abrupt gravity failure.
- Bar buckling caused redistribution of compressive forces relying increasingly on the core concrete. This gradually led to spalling and disintegration of the core concrete which compromised column stability.

**Column S2**

- Doubling the longitudinal bar ratio in Column S2 reduced the ultimate drift capacity in this column by a factor of 2 (i.e. \( \delta_{\text{collapse}} = 2.5 \)) when compared to Column S1. This behaviour is strongly attributed to the change in the failure mode and collapse mechanism. This observation suggests that a rational theoretical capacity model to be developed in this study should take into account the common mode and mechanism of failure for the lightly reinforced, poorly confined columns of interest (refer Chapter 4).
- Column S2 (as well as S3 & S4) exhibited a flexure-shear mode of failure, that is the disintegration of the core concrete along the critical shear crack, subsequent to flexural development (yielding of longitudinal bars).
- Critical shear cracks are conventionally linked with squat columns. It is however important to note that such cracks are commonly observed for the relatively slender columns as tested in this study as well as previous studies. Critical shear cracks act as a source of additional deformation capacity after the yielding of longitudinal reinforcement and prior to gravity collapse. This source of ductility was not explicitly/comprehensively modeled in any of the existing capacity models. The shear-slip model developed in this study addresses the gap identified in the literature (refer Section 4.2).
Excessive debonding of longitudinal bars was observed in this specimen. Debonding was shown to be a rather common feature in poorly-confined columns particularly when the axial load level was low to medium. That is because the passive confining pressure expected from transverse bars is not adequate to prevent debonding and relative movement of the longitudinal bars with respect to the adjacent concrete.

Debonding of longitudinal bars initially enhanced the drift capacity in Column S2 by providing temporary relief in compressive stresses in critical flexural sections. Debonding however, once completed, interrupted the flexural load transfer mechanism and led to the spalling of the cover concrete over a large length. This resulted in the column being supported by the frictional shear resistance developed across the critical shear crack.

Column S3 and S4

Columns S3 and S4 showed a mode of failure characterized by the formation of a critical shear crack subsequent to the yielding of longitudinal reinforcement (similar to that of Column S2).

Doubling the imposed axial load ratio in Columns S3 and S4 (i.e. from 20% in Columns S1 and S2 to 40% in S3 and S4) further reduced the drift capacity of these columns to ($\delta_{\text{collapse}}=1.5$) irrespective of the longitudinal reinforcement ratio of these columns. This observation suggests that axial load ratio is an important parameter to be considered in the capacity models to be developed within the scope of this study.

Doubling the longitudinal bar ratio (from about 0.5% in Column S4 to 1% in Column S3) did not affect the observed drift capacity of the columns at the limit of gravity collapse. This observation suggests that longitudinal reinforcement ratio could be a secondary or trivial parameter when estimating the drift capacity of limited-ductile RC columns at the limit of gravity collapse. This finding however may be limited to columns with flexural-shear mode of failure.

Excessive compressive stresses caused buckling of longitudinal bars and spalling of cover concrete over the critical unconfined length of the columns (which was above the base) resulting in the columns being supported by the frictional shear resistance developed across the critical shear crack (prior to gravity collapse).
FINDINGS FROM THE STUDY OF PREVIOUS RESEARCH

Effects of Column Aspect Ratio on Gravity Failure Drift Capacity

- The investigations conducted in this thesis, including the 56 columns collected from the literature, suggest that the ultimate drift capacity is not appreciably sensitive to the value of the aspect ratio \( n \) where \( n \) is approximately greater than 2.5 (refer to Figure 5. 1). For columns with aspect ratio approximately smaller than 2.5, the drift capacity at gravity load failure is generally significantly higher compared to the former group of specimens.

Effects of Transverse Reinforcement Ratio on Gravity Failure Drift Capacity

- Increasing the transverse reinforcement ratio could generally improve the drift ratio at gravity failure. This trend however cannot be readily generalized as the drift capacity also depends on many other parameters and the mode of failure. Investigations considered in this thesis and also in a recent study by Wibowo et al. (2012) confirmed that no single design parameter could be adequately correlated to the measured ultimate drift limits. Nevertheless, it was observed that the transverse reinforcement ratio and the axial load ratio are far more influential than the other two parameters: aspect ratio and longitudinal bar ratio.

Effects of Lateral Loading Scheme on Gravity Failure Drift Capacity, (Monotonic vs. Cyclic)

- Based on the observations from the limited tests reported it can be assumed that strength and stiffness properties at a given drift limit, of the limited-ductile RC columns with the aspect ratio of approximately greater than 2.5, are not significantly affected by the number of cycles experienced previously.

Effects of Axial Loading Condition on Gravity Failure Drift Capacity

- Based on the limited data available to date, it is concluded that a drift capacity model developed with reference to a database comprising columns subjected to constant axial compression, could be expected to result in acceptable estimations for perimeter columns which are subjected to push and pull actions.
Effects of (Insufficient) Lap-Splices on Gravity Failure Drift Capacity

- Presence of lap-splices in longitudinal bars has the potential to improve the drift capacity reserve up to the limit of gravity failure (but high axial load ratio could compromise this beneficial effect).

- Based on the experimental observations discussed in Section 3.3.5, it is prudent to ignore this reserve drift capacity (that might be available to poorly-confined columns due to the presence of vertical bar lap-splices).
4 Deterministic Drift Capacity Modelling for Limited Ductile RC Columns (at the Limit of Gravity Collapse)

The previous chapter (Chapter 3) presented the results of an experimental program on lightly-reinforced, poorly-confined concrete columns as conducted in this study. It also provided a summary and discussion over the relevant results reported by previous investigators to establish the required insight into the response behaviour of limited-ductile RC columns. It was importantly found that axial load ratio is the key parameter influencing the column drift capacity at the limit of gravity collapse.

Chapter 4 is concerned with deformation modelling of limited-ductile RC columns of interest at the limit of gravity collapse. The columns of interest are mainly characterized by widely spaced and/or poorly detailed transverse reinforcement. The transverse bar spacing may be as large as the column depth and the longitudinal reinforcement ratio could be as low as 0.5% (as were implemented in the tested columns described in Chapter 3). Poorly confined columns are usually expected to exhibit a mode of failure controlled by shear either prior to flexural yielding (which is referred to as brittle shear failure) or subsequent to flexural yielding of the longitudinal reinforcement (which is referred to as flexural-shear mode of failure). The scope of this study is limited to the columns with a flexural-shear mode of failure. Consequently this study is more concerned with the columns with a rather high aspect ratio (approximately greater than 2.5).

Chapter 4 starts with evaluating the adequacy of existing models for estimating the lateral drift capacity of RC columns at the limit of gravity collapse (refer Section 4.1). This is followed by presenting the development of a theoretical/deterministic model (referred herein as Model I) for predicting the drift capacity of limited ductile RC columns of interest at the limit of gravity collapse.

Model I specializes for a mechanism of failure which is commonly seen in lightly reinforced concrete columns (characterized by the formation of critical shear crack subsequent to the flexural yielding of longitudinal reinforcements). Model I is a simplified solution based on: A) an analytical shear-slip deformation model as
developed in Section 4.2; and B) the specialized solutions for estimating the conventional components of deformation (i.e. the flexural, shear and strain penetration components of deformation) as presented in Section 4.3. Model I is verified against the experimental results of a comprehensive dataset in Section 4.4.

4.1 Existing Models for Predicting Drift at the Limit of Gravity Collapse

The most relevant existing models for predicting drift at the limit of gravity collapse are summarized in Table 4.1. The last column in this table provides a brief description of each model. Table 4.1 is followed by a detailed description and analysis of the well-known model by Elwood and Moehle (2005).

A common attribute of the existing capacity models is their empirical nature, which is embedded in the model development either explicitly (Wibowo et al. 2012; Zhu et al. 2007) or implicitly (Elwood and Moehle 2005; OUSALEM et al. 2004). While empirical models are valuable tools in treating complex problems, as the problem at hand, their application is limited as is evident by the need for model refinement each time new or rather different experimental data becomes available.

Figure 4.1: Formation of critical shear crack in flexure-shear dominant columns: (a) Specimen 2CLD12M (Sezen and Moehle 2006); (b) Columns S3; and (c) Column S4 (described in Chapter 3)
Table 4.1: A summary of models available for estimation of the drift limit at gravity collapse

<table>
<thead>
<tr>
<th>Model</th>
<th>Proposed equations</th>
<th>Comments</th>
</tr>
</thead>
</table>
| Elwood & Moehle (2005)       | \[ \mu = \frac{P - A_{xx} f_{y} d_{c} / s}{P / \tan \theta + A_{xx} f_{y} d_{c} \tan \theta / s} \] \hspace{1cm} (4.1) | 1. For this model the coefficient of friction (\( \mu \)) developed at the critical shear crack was related to loading, reinforcement and section parameters with equilibrium considerations as given by Eq. (4.1)  
2. A \( \mu \) value was then calculated for each column in their data set.  
3. Estimated \( \mu \) values were empirically correlated with recorded drift values at gravity failure (\( \delta_{u} \)). This resulted in Eq. (4.2) which relates \( \delta_{u} \) to column and reinforcing parameters. |
| Ousalem et al (2004)          | \[ \delta_{u} = \frac{4}{100} \frac{1 + (\tan \theta)^{2}}{\tan \theta + P \left( \frac{s}{A_{xx} f_{y} d_{c} \tan \theta} \right)} \] \hspace{1cm} (4.2) | 1. For this model a similar approach as above adopted.  
2. A different estimation was made for the shear crack angle \( \theta \), (i.e. \( \tan \theta = \sqrt{K_{1}} \)).  
3. Aspect ratio of the additional columns employed for refining the previous correlation was very low (i.e. \( a = 1 \ or \ 1.5 \)). |
| Zhu et al (2007)              | \[ \delta_{u} (\%) = \frac{1}{k} \left( \frac{0.97 - 1.33k}{\sqrt{0.03 + 1.33k}} \right)^{1/0.36} \] \hspace{1cm} (4.3) | 1. Eq. (4.5) was obtained using regression analysis.  
2. The \( \mu \) parameter as proposed by Elwood and Moehle (2005) (i.e. Eq. 4.1) was incorporated in regression analyses. |
A total of 46 specimens from 12 studies were employed for developing Equation (4.6).

\[
\delta_a = 5(1 + \rho_v) + 7 \rho_h + 0.2/n \tag{4.6}
\]

\[
n_b = \frac{P_{ab}}{f_y bd} \tag{4.7}
\]

\[
P_{ab} = 0.85 f'_c \gamma_s k_{ub} b d + f_y (A_{st} - A_{se}) \tag{4.8}
\]

Notes: See the next page for the definition of the symbols

In relation to Equations (4.1), (4.2) and (4.5) (refer to Figure 4.2 for illustrations):

- \( \mu \) = coefficient of friction at critical shear crack surface
- \( \delta_a \) = drift capacity at gravity load failure (%)
- \( \theta \) = angle of inclination of critical shear crack to the horizontal (degree)
- \( P \) = imposed axial compressive load (N)
- \( s \) = centre to centre spacing of transverse reinforcing bars in vertical direction (mm)
- \( A_{st} \) = total cross sectional area of transverse bars confining the column side perpendicular to lateral loading direction (mm²)
- \( f_{yt} \) = yield strength of transverse reinforcement (MPa)
- \( d_c \) = centre to centre spacing of longitudinal reinforcing bars parallel to the direction of lateral loading

In relation to Equations (4.3) and (4.4):

- \( \rho_{ws} \) = transverse reinforcement ratio (%)
- \( f_{ws} \) = transverse reinforcement yield strength (MPa)
- \( F_c \) = concrete compressive strength (MPa)
- \( \eta \) = axial load ratio

In relation to Equations (4.6) to (4.8):

- \( \rho_v \) = longitudinal reinforcement ratio (in %)
- \( \rho_h \) = transverse reinforcement ratio (in %)
- \( n \) = axial load ratio
- \( nb \) = balance axial load ratio (N)
- \( P_{ab} \) = balance axial load (N)
- \( f'_c \) = concrete cylinder compressive strength (MPa)
- \( f_y \) = longitudinal reinforcement yield strength (MPa)
- \( b \) and \( d \) = column breadth and depth (mm)
• \( A_{st} \) = area of longitudinal reinforcement in tension (\( mm^2 \))
• \( A_{sc} \) = area of longitudinal reinforcement in compression (\( mm^2 \))
• \( \gamma_d = 0.85 - 0.007(f'c - 28) \quad , \quad 0.65 \leq \gamma_d \leq 0.85 \)
• \( K_{ub} = \epsilon_{cu} / (\epsilon_{cu} + \epsilon_{ub}) \) neutral axis multiplication factor

4.1.1 Discussion on the Model Proposed by Elwood and Moehle (2005) for Estimating the Drift at the Limit of Gravity Collapse

The model by Elwood and Moehle is selected herein for a detailed description and discussions. The reasons include:

1) This model was one of the elegant early models to consider the equations of equilibrium at the critical shear crack for modelling drift capacity at the limit of gravity collapse.

2) To demonstrate that the equations of equilibrium have been inadequate for an analytical shear-slip or capacity modelling. This is partly attributed to the fact that no compatibility equation was employed at the critical shear crack to take into account the actual interaction of the crack width at the critical shear crack & clamping forces with aggregate interlock & developed frictional shear forces.

3) To demonstrate that the provided solution (by Elwood and Moehle) mainly relied on the recorded drift values (at the limit of gravity collapse) of a dataset comprising 12 lightly reinforced concrete columns. Empirical calibrations introduce limitations. Therefore a model which is primarily based on a theoretical approach was warranted.

The approach proposed by Elwood and Moehle for model development can be summarized as follows:

1) Equations of equilibrium in horizontal and vertical directions (i.e. Equations (4. 9) and (4. 10) respectively) were written to incorporate resolved components of forces acting on the critical shear crack (see Figure 4. 2). The parameters involved in these equations can be categorized into: a) known,
assumed or estimated input parameters; and b) unknown parameters as listed in Table 4.2.

\[ P = N \cos \theta + V_{sf} \sin \theta + n_{bars} P_s \]  

\[ N \sin \theta + V = V_{sf} \cos \theta + \frac{A_{sl} f_{yt} d_c}{s} \tan \theta + n_{bars} V_d \]  

where

- \( P \) = axial load in Newtons and \( N \) = clamping force acting normal to shear crack plane
- \( V_{sf} \) = shear force resisting the slip of the upper section of the column relative to the lower section
- \( \theta \) = angle of inclination of the critical shear crack relative to the horizontal
- \( V \) = external shear force which is set to zero considering the instant when lateral capacity has dropped to zero
- \( n_{bars} \) = number of vertical reinforcing bars;
- \( P_s \) = axial force contributed by each of the vertical bars
- \( V_d \) = dowel resistance of a vertical bar
- \( s \) = spacing of the stirrups
- \( A_{sl} \) = cross sectional area of one transverse bar
- \( f_{yt} \) = yield strength of the stirrups and
d\(_c\) = centre to centre of the longitudinal bars. Refer to Figure 4.2 for illustration of the parameters introduced.

Figure 4.2: Free body diagram of column after shear failure (Elwood and Moehle 2005)
Table 4.2: Overview of the parameters involved in the equations of equilibrium (i.e. Equations (4.9) and (4.10)) as treated in the approach proposed by (Elwood and Moehele 2005)

<table>
<thead>
<tr>
<th>Known, assumed or estimated input parameters</th>
<th>Unknown parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P$, $\theta$, $P_t$, $V$, $V_d$, and transverse reinforcement parameters</td>
<td>$V_{sf}$ and $N$</td>
</tr>
</tbody>
</table>

2) With reference to the classical shear friction model, the unknown shear resistance force $V_{sf}$ was replaced by the product of an unknown effective coefficient of friction $\mu$ and unknown clamping force at the instant of gravity collapse (i.e. $V_{sf} = \mu N$). Table 4.3 reflects this substitution as well as the simplifying assumptions made in relation to the input parameters.

Table 4.3: Analysing the parameters involved in the equations of equilibrium (i.e. Equations (4.11) and (4.12)) as treated in the approach proposed by (Elwood and Moehele 2005)

<table>
<thead>
<tr>
<th>Known, assumed or estimated input parameters</th>
<th>Unknown parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P = P_{\text{imposed}}$, $\theta$, $P_t = 0$, $V = 0$, $V_d = 0$, and transverse reinforcement parameters</td>
<td>$\mu$ and $N$</td>
</tr>
</tbody>
</table>

The value of the external shear force ($V$) was assumed zero based on the assumption that lateral capacity is negligible at the instant of gravity collapse. Axial load carrying resistance and dowel resistance of vertical bars (i.e. $P_t$ and $V_d$ respectively) were judged to be negligible and subsequently removed from the final model proposed. With these modifications the equations of the equilibrium (i.e. Equations (4.9) and (4.10)) were reduced to the forms given below:

\[ P = N \cos \theta + \mu N \sin \theta \quad (4.11) \]

\[ N \sin \theta = \mu N \cos \theta + \frac{A_{st} f_{sf} d_c}{s} \tan \theta \quad (4.12) \]

3) Combining Equations (4.11) and (4.12) and algebraic manipulations of the combined equation resulted in an expression for estimating $\mu$ as functions of: (a) the transverse reinforcement parameters; (b) the axial load ($P$); and (c) the critical crack angle ($\theta$) as given in Equation (4.1). It is noted that the unknown parameter $N$ was eliminated in the combined equation.
The approach proposed by Elwood and Moehle as described so far does not take into account the actual correlation of the forces, acting on the plane of the shear failure, with the width of the critical shear crack and resulting rotational deformation $\Delta_{\text{rot}}$. Instead, the ultimate deformation at the limit of gravity collapse ($\Delta$) was related to the rest of parameters using an empirical approach as described next.

4) Equation (4.1) was evaluated for each of the 12 columns included in the dataset and a $\mu$ value was obtained for each column.

5) The calculated $\mu$ values were then plotted against recorded drift ratios at the limit of gravity failure ($\Delta/L$) forming the empirical core of their model (see Figure 4.3). The linear trend fitted to the data points was expressed by Equation (4.13).

$$\mu = \tan \theta - \frac{100}{4} \left( \frac{\Delta}{L} \right)_{\text{Axial}} \geq 0$$ (4.13)

6) The empirical expression for estimating the critical coefficient of friction ($\mu$) as obtained above was substituted back into Equation (4.1) to derive the final expression for estimating drift at the limit of gravity collapse (as given by Equation (4.2)). This expression is believed to have the limitations of an

Figure 4.3: Recorded drift values at the limit gravity collapse are plotted against evaluated $\mu$ values to obtain an empirical correlation
empirical model due to the important empirical feature employed for estimating the critical coefficient of friction ($\mu$).

The remainder of this chapter focuses on developing/specializing models (with considerably reduced reliance on empirical features) for estimating the deformation components that can be superimposed for predicting total drift at the limit of a gravity collapse.

An analytical shear-slip model (which is developed in this study) is presented in Section 4.2. This model is employed to estimate the component of deformation caused by rotation at the critical shear crack. Section 4.3 focuses on other components of deformations (i.e. flexure, shear and strain penetration deformations) for lightly-reinforced and poorly-confined columns and provides specialized/simplified models based on existing theoretical solutions.

### 4.2 Shear-Slip Model for Estimating the Deformation Resulting from Rotation at Critical Shear Crack ($\Delta_{rot}$)

Critical shear cracks are found to be common in shear and flexure-shear dominated columns. Critical shear cracks are predominantly responsible for ultimate disintegration of the core concrete leading to gravity load collapse. Such cracks are observed in a number of experimental tests as reported by different researchers (Lynn et al. 1996; Rodsin 2007; Sezen and Moehle 2006) and also observed during the experimental program of this study (see Figure 4.1).

Rotation at the critical shear crack may be a considerable source of deformation contributing to the total deformation prior to the loss of gravity load carrying capacity. For instance based on experimental results Wilson et al. (2009) reported that a flexure-shear dominated column had about 40% more drift capacity (up to the limit of gravity collapse) as compared to a similar but flexure dominated column. This extra deformation observed was attributed to the rotational deformation at a critical shear crack which was experienced in this column. This source of deformation, however, is not considered in most theoretical models (Wibowo et al. 2012).
This section presents a rigorous procedure leading to a simplified equation for estimating the aforementioned component of deformation taking into account: (a) the equilibrium of all forces including the applied axial load, clamping effect of the reinforcements crossing the shear crack and the developed frictional shear force; and (b) compatibility condition which provides the relationships between the shear across the crack, the crack width and required compressive stress on the crack. It is noted that an iterative procedure is required for satisfying the equilibrium and compatibility conditions for estimating the limiting crack width beyond which shear-slip will occur. This is because the state of stress acting on the crack is not constant and changes with the crack width. The developed procedure is described next.

**Equilibrium and Compatibility Equations at the Critical Shear Crack**

Equilibrium condition at the critical shear crack can be considered employing Equations (4. 9) and (4. 10) as proposed by Elwood and Moehle (2005) (see Figure 4.2). These equations are repeated here for the ease of reference.

\[
P = N \cos \theta + V_s \sin \theta + n_{bars} P_s
\]

**vertical equilibrium**

(4.14)

\[
N \sin \theta + V = V_s \cos \theta + \frac{A_{st} f_y d}{s} \tan \theta + n_{bars} V_d
\]

**horizontal equilibrium**

(4.15)

where

- \( P \) = axial load in Newtons and \( N \) = clamping force acting normal to shear crack plane
- \( V_s \) = shear force resisting the slip of the upper section of the column relative to the lower section
- \( \theta \) = angle of inclination of the critical shear crack relative to the horizontal
- \( V \) = external shear force which is set to zero considering the instant when lateral capacity has dropped to zero (following the recommendation made by Elwood and Moehle (2005))

- \( n_{bars} \) = number of vertical reinforcing bars;
- \( P_s \) = axial force contributed by each of the vertical bars
- \( V_d \) = dowel resistance of a vertical bar
- \( s \) = spacing of the stirrups
- \( A_{st} \) = cross sectional area of one transverse bar
\( f_{yd} \) = yield strength of the stirrups and
\( d_c \) = centre to centre of the longitudinal bars. Refer to Figure 4.2 for illustration of the parameters introduced.

Table 4.4 provides an overview of the parameters involved in the equations of equilibrium (i.e. Equations (4.14) and (4.15)) in the context of the shear-slip model developed in this study (scope of section 4.2).

<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Known, assumed or estimated input parameters</td>
<td>Unknown parameters</td>
<td>Control parameter</td>
</tr>
<tr>
<td>( \theta, P_f, V, V_d ), and transverse reinforcement parameters</td>
<td>( V_{sf} ) and ( N )</td>
<td>( ** P = f(N,V_{sf}) ) to be checked against ( P_{imposed} )</td>
</tr>
<tr>
<td>( V_{sf} = f(w,N,f_{c}^{'},a) )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes:
* \( V_{sf} \) is an unknown parameter and in fact a function of: (a) crack width \( (w) \); (b) clamping force normal to the critical shear crack \( (N) \); (c) concrete compressive strength \( (f_{c}^{'}) \); and (d) maximum concrete aggregate size \( (a) \)

** \( P \) is treated as an unknown parameter. In other words, the value of axial load that can be transferred by friction (i.e. \( P = f(N,V_{sf}) \)) is evaluated and compared against the imposed axial load (\( P_{imposed} \)).

This table categorizes the parameters involved in the equations of equilibrium (i.e. Equations (4.14) and (4.15)) into 3 different groups: (1) the input parameters which are either known, or can be estimated or conservatively assumed; (2) the unknown parameters namely the clamping force \( (N) \) normal to the critical shear crack and corresponding shear force \( (V_{sf}) \) that is developed across the critical shear crack; and (3) a control parameter namely: the resultant axial load \( (P) \) which is a function of \( N \) and \( V_{sf} \) corresponding to a given state of rotation experienced by the critical shear crack. Parameter \( P \) needs to evaluated, at each iteration, and be compared against the imposed axial load (\( P_{imposed} \)) to decide whether further opening of crack can be accommodated without compromising the column capacity to carry the imposed axial load.

The equations of equilibrium (i.e. Equations (4.14) and (4.15)) are required to evaluate the 2 unknown parameters as well as the control parameter as outlined above.
So there are 3 unknowns and only 2 nonlinear equations. Furthermore these equations do not provide the relationship between the crack width ($w$ which is referred to as compatibility parameter herein), $N$, $f'_c$ & $a$ and the resulting frictional shear resistance ($V_{sf}$) developed across the critical shear crack. Therefore a compatibility equation is required in addition to the equilibrium equations to address this requirement. Equation (4. 16) is proposed to address the need for a 3rd equation and the compatibility requirement discussed above.

Equation (4. 16) is adopted from Vecchio and Collins (1986) and is originally based on the findings by Walraven (1981). Equation (4. 16) was also considered by Rodsin (2007) in an attempt to model shear–slip. This equation is based on sufficiently large number of experimental results concerning the relationship between the parameters mentioned above (see Figure 4. 4). In this equation the resisting shear force provided by friction $V_{sf}$ (or corresponding shear stress $V_{c}$) is related to normal force $N$ (or corresponding normal stress $f'$) and also to the crack width ($w$) and other key parameters through $V_{c_{\text{max}}}$.

$$V_{c} = 0.18V_{c_{\text{max}}} + 1.6f'_{c} - 0.82\frac{f'_c^2}{V_{c_{\text{max}}}}$$

Compatibility equation

where $V_{c_{\text{max}}}$ is the maximum shear stress that a crack of given width can resist which is estimated by Equation (4. 17).
where

\( a \) = maximum concrete aggregate size

\( f'_c \) = concrete compressive strength

\( w \) = the width of critical shear crack

The frictional shear stress \((V_{ci})\) can be converted to shear force \((V_{sf})\) if Equation (4.16) is multiplied by the effective contact area \((A_c)\) as follows:

\[
V_{sf} = A_c (0.18V_{cmax} + 1.64f'_c - 0.82 \frac{f^2_{ci}}{V_{cmax}})
\]  

where \( A_c \) is calculated from Equation (4.19).

\[
A_c = bd_c / \cos \theta
\]
Where
\[ d_c = \text{Centre to centre distance of longitudinal reinforcements along the depth of the section} \]
\[ b = \text{Breadth of the section} \]
\[ \theta = \text{Angle of inclination of the critical shear crack relative to the horizontal axis} \]

4.2.1 Proposed Iterative Procedure to Solve the Equilibrium and Compatibility Equations

The problem at hand is to employ nonlinear Equations (4.14), (4.15) and (4.18) to determine the limiting crack width beyond which the equilibrium of forces in the vertical direction is compromised. The proposed iterative procedure for estimating the limiting crack width is conceptually summarized as follows:

(i) An average crack width is assumed for the critical shear crack.
(ii) Clamping force \((N)\) and frictional shear resistance \((V_f)\) corresponding to the assumed crack width are estimated by solving two non-linear equations (i.e. Equations (4.15) and (4.18)) using an iterative procedure. Estimated values will be forwarded to the next step.
(iii) The resultant vertical load that can be transferred \((P)\) is evaluated using equilibrium (i.e. Equation (4.14)) and is compared against the imposed axial load \((P_{\text{imposed}})\).
(iv) The crack width is refined if the resultant axial load does not equate the imposed axial load. The steps outlined above are repeated for the refined crack width.
(v) The crack width which satisfies equilibrium and compatibility conditions (i.e. Equations (4.14) to (4.18)) is accepted as the answer. The obtained value can be employed for estimating corresponding rotational deformation \((\Delta_{\text{rot}})\) using Equation (4.33). Rotational deformation is illustrated in Figure 4.9.

In the above procedure one should bear in mind that the amount of shear force that can be transferred across a crack by friction is reduced with increasing crack width. Increasing crack width is therefore translated into reduced capacity in transferring the
axial load. The details corresponding to each of the steps outlined above are elaborated next:

**Step (I):**
The iterative procedure is started by assuming an initial value for the unknown parameter of crack width at the limit of gravity collapse \( w \). This assumption is refined iteratively until equilibrium and compatibility equations, defined at the critical shear crack, are satisfied.

The initial crack width value may be taken as a fraction of maximum concrete aggregate size \( a \) as suggested by Equation (4. 20). It was found that Equation (4. 20) provides a sufficiently small value to start. Alternatively an initial trial value could be selected based on engineering judgment but not greater than \( a/5 \).

\[
w_1 = a/20
\]

**(4. 20)**

**Step (ii):**
Assumed crack width affects the extent of shear force that can be developed across the crack \( V_{sf} \) as suggested by Equations (4. 17) and (4. 18). To check the condition of equilibrium at the critical shear crack \( V_{sf} \) should be evaluated. Evaluating \( V_{sf} \) using Equation (4.18), however, requires the knowledge of clamping stresses acting normal to critical shear crack \( f_{c} \) or corresponding normal force \( N \). Normal force \( N \) on the other hand, appears in the equations of equilibrium defined at the shear crack (e.g. Equation (4. 15)). It can be seen that evaluating the value of \( N \) is not possible using this equation unless the value of \( V_{sf} \) is known. One solution to this problem could be an iterative procedure as proposed below:

1. The iterative procedure may be started by an initial estimate of \( N \) (which is denoted as \( N_0 \)) by considering only the clamping effect of the imposed axial load \( P_{imposed} \) according to Equation (4. 21).

\[
N = N_0 = P_{imposed} \cos \theta
\]

**(4. 21)**

where
The component of the imposed axial load resolved in the direction normal to the critical shear crack.

Given the value of $N_0$ and the effective contact area ($A_c$), initial clamping stresses can be calculated by Equation (4.22)

$$f_{ci} = f_{ci0} = \frac{N_0}{A_c}$$  \hspace{1cm} (4.22)

Where

$f_{ci0}$ = The initial estimate of clamping stresses acting normal to the critical shear crack

$A_c$ = Effective contact area at the plane of shear crack which is calculated by Equation (4.19)

2. Based on the assumed value of crack width and the initial estimate of the clamping normal stresses (i.e. $f_{ci} = f_{ci0}$) as suggested above it is possible to evaluate shear and normal forces corresponding to first iteration (i.e. $V_{sf1}$ and $N_{j=1}$) using Equations (4.18) and (4.15) respectively. The subscript ‘$j=1$’ represents the values obtained in the $j^{th}$ iteration.

3. In this step, the initial value of the clamping stresses ($f_{ci0}$) can be updated using Equation (4.23). The shear and normal forces corresponding to the next iteration (i.e. $V_{sf2}$ and $N_{j=2}$) are evaluated based on the updated value (i.e. $f_{cij}$).

$$f_{ci} = (f_{ci})_j = \frac{N_j}{A_c}$$  \hspace{1cm} (4.23)

where

$N_j$ = the normal clamping force obtained from Equation (4.15) in the $j^{th}$ iteration

$(f_{ci})_j$ = Clamping stresses normal to the critical shear crack obtained based on the information available in the $j^{th}$ iteration.

4. the clamping stresses normal to the critical shear crack is refined iteratively until the following criterion is satisfied.

$$N_j - N_{j-1} \leq 0$$  \hspace{1cm} (4.24)

where

$j$ = the counter of iteration and $N$ as defined above

Steps (iii) and (iv):

The converged $N_j$ value and corresponding $V_{sf}$ are brought forward from the iterative procedure described in Step (ii). These parameters can be substituted into the
equation of equilibrium concerning the component of forces in the vertical direction (i.e. Equation (4.14)). The axial load which can be calculated using this equation is denoted as $P_{\text{cap}}$ (the subscript ‘1’ represents the axial load capacity corresponding to the assumed initial crack width ($w_1$)). It is noted that the axial load carrying capacity is reduced with increasing crack width. The required final outcome will be the limiting crack width ($w$) beyond which the developed frictional shear resistance is not adequate to maintain the equilibrium of forces in the vertical direction. This requires systematic refinements of the assumed value of the crack width (i.e. repeating Steps (i) to (iv)) until the following criterion is met.

$$P_{\text{imposed}} - P_1 \equiv 0$$

(4.25)

where

$P_{\text{imposed}}$ = the imposed axial load

$P_1$ = as defined above

The procedure described above (i.e. Steps (i) to (v)) is also presented in the form of an algorithm as illustrated in Figure 4.5.

The remainder of this section (i.e. Section 4.2) will focus on describing the input parameters required and assumptions made for the developed shear-slip model (refer Section 4.2.2 and 4.2.3). Section 4.2.4 presents a worked example to clarify the application of the proposed shear-slip model for estimating the additional rotational deformation of a column due to the opening of the critical shear crack. The proposed iterative procedure was then incorporated in Excel spreadsheet and subsequently employed in a number of sensitivity analysis presented in Section 4.2.5.
Figure 4.5: Proposed iterative procedure for estimating the rotational deformation provided by opening of critical shear crack.

1. Assume a value for the critical shear crack width \( w_i \) beyond which the frictional resistance is inadequate to maintain equilibrium in vertical direction.

2. Obtain an initial estimate for the clamping force normal to the critical shear crack \( N_0 \) using Eq. (4.21) and set \( N = N_0 \).

3. Calculate resulting shear provided by friction \( V_{sf} \) using Eq. (4.18).

4. Calculate an updated value for clamping force (e.g. \( N_i \)) using equilibrium, Eq. (4.15).

5. Check if \( N_i - N_{i-1} \approx 0 \).
   - Yes: Update the clamping force by setting \( N = N_i \).
   - No: Refine the crack width.

6. If \( N \) and \( V_{sf} \) are brought forward & substituted in Eq. (4.14) to calculate resultant vertical load \( P_i \) corresponding to the assumed crack width \( w_i \).

7. Compare \( P_i \) with the imposed axial load - Check if \( P_{\text{imposed}} - P_i \approx 0 \).
   - Yes: Assumed crack width \( w_i \) is the answer. Set \( w = w_i \) & estimate rotational deformation using Eq. (4.33).
   - No: Refine the crack width.

8. Repeat steps 3-7 until convergence is achieved.
4.2.2 Critical Shear Crack Angle

The angle of inclination of critical shear crack (θ) as illustrated in Figure 4.2 is one of the input parameters required for the equations of equilibrium (i.e. Equations (4.14) and (4.15)). The equations of equilibrium are employed in the developed shear-slip model for estimating rotational deformation (the scope of Section 4.2). Rotational deformation is a deformation component that could be experienced by a column due to the opening of the critical shear crack. For columns with the flexural-shear mode of failure (the typical of lightly reinforced, poorly confined columns with aspect ratio greater than 2.5), rotational component is added to other components of deformation (i.e. to shear, flexure and strain penetration components) for estimating total deformation experienced up to the limit of gravity collapse.

Elwood and Moehle (2005) proposed an angle of 65 degrees relative to the horizontal axis which is an experimental averaged value representing the inclination of the critical shear crack for lightly reinforced/poorly confined columns. They also proposed an alternative empirical equation (4.26) for estimating the crack angle as a function of the normalized axial load \( P / P_0 \) which follows:

\[
\theta = 55 + 35P / P_0 \tag{4.26}
\]

where

\( P_0 = \) axial capacity of the undamaged column given by \( P_0 = 0.85f_c(A_g - A_{sl}) + f_{ys}A_{sl} \)

\( f_c = \) concrete cylinder compressive strength

\( A_g = \) gross concrete area

\( A_{sl} = \) area of longitudinal steel

\( f_{ys} = \) yield strength of longitudinal reinforcement.

Application of Equation (4.26) for the columns tested in this study showed that the proposed equation over-estimates the critical angle for the columns with high axial load ratios \( P / P_0 \), i.e. for Columns S3 and S4 with \( P / P_0 \) is in the order of 0.35 and 0.41 respectively.

Figure 4.6 shows the experimental critical shear crack angles as reported by Elwood and Moehle (2005) and the relevant data corresponding to Columns S2 to S4 as tested in this study. The solid line is the trend line defined by Equation (4.26).
Figure 4.6: The change in critical shear crack angle with axial load ratio $P/P_0$ (the figure includes 12 data points as reported in Elwood and Moehle study and 3 data points as obtained experimentally in this study).

It can be seen that both experimental data sets are generally consistent in spite of the fact that vertical reinforcement ratio of the columns tested in this study were at least 2 times smaller than those of the other columns. It is however, evident that the critical crack angles do not increase with axial load ratios beyond the mid-range values of $P/P_0$, which is in contrast to what the proposed linear trend line suggests. It is therefore suggested that Equation (4.26) be limited to $P/P_0 \leq 0.25$ as modified in Equation (4.27). For axial load ratios greater than 0.25, the crack angle may be assumed as a constant value equal to 60 degree which is supported by experimentation of this study.

$$\theta = 55 + 35P/P_0 \quad \text{for} \quad P/P_0 \leq 0.25$$  \hspace{1cm} (4.27)

$$\theta = 60^\circ \quad \text{for} \quad P/P_0 > 0.25$$
4.2.3 Assumptions Made in the Model Development

This section presents the assumptions made in regard to the input parameters required for the developed shear-slip model for estimating limiting crack width and corresponding rotational deformation caused by the opening of the critical shear crack (the scope of section 4.2). The input parameters addressed in section 4.2.3 are introduced previously and summarized in Table 4.4. The assumptions made and the recommended approaches are as follows:

A. The angle of inclination of the critical shear crack ($\theta$) remained an empirical feature in the developed model and was estimated according to Equation (4.27) as proposed in the previous section (i.e. Section 4.2.2).

B. The upper bound of the axial load carried by a longitudinal reinforcing bar ($P_t$) may be taken as proportional to the transformed area of compression steel according to Equation (4.28).

$$P_t = P \frac{h A_p}{A_g} \tag{4.28}$$

where

- $P =$ imposed column axial load
- $A_p =$ the cross sectional area of one longitudinal bar
- $A_g =$ the gross cross sectional area of the column

And $n = \frac{E_s}{E_c}$ where $E_s$ & $E_c$ are the modulus of elasticity of steel and concrete respectively.

Equation (4.28) is usually employed when the elastic response of column and therefore strain compatibility between the concrete and longitudinal reinforcement can be reasonably assumed. At the ultimate stage of loading, however, there are uncertainties over the exact contribution of longitudinal reinforcements in resisting the imposed axial load (i.e. the exact value of $P_t$). Such uncertainties are mainly attributed to the fact that poorly-confined columns are prone to debonding and buckling of longitudinal reinforcements particularly along the unconfined length of the bar crossing the inclined shear crack (see Figure 4.7). In Figure 4.7,
the white curves show the observed deformed shape of longitudinal bars in Column S4 after complete collapse. It should however be noted that a major part of buckling and lateral deformation of the bars are in fact the by-product of the collapse. In other words, such pronounced deformation and considerable buckling of longitudinal reinforcement do not exist at the verge of collapse (just before collapse). This may be clear in Figure 4.8 in which Column S3 is shown after the first and the second cycle of the ultimate drift was completed (both at 1.5% drift level).

Elwood and Moehle (2005) recognized the outlined uncertainties and conservatively concluded that for lightly-reinforced and poorly-confined columns, at collapse limit state, the longitudinal bars are ineffective and hence their axial load carrying contribution may be neglected (i.e. $P_s = 0$).

The results of experimental program conducted in this study further supported the conclusion made by Elwood and Moehle as mentioned above. This was partly inferred from the recorded drift capacity for Columns S3 and S4. The two columns showed the same drift capacity up to the limit of gravity collapse (1.5%). This was the case while Columns S3 and S4 had different longitudinal reinforcement ratios (i.e. 1% and 0.5% for S3 and S4 respectively- all other parameters were the same). In other words, doubling the longitudinal reinforcement ratio in S3 as compared to S4 did not affect the ultimate capacity. This trend, however, cannot be generalized and assumed for all columns with any possible combinations of loading and reinforcing details (refer Section 3.2 for further discussions).

Based on the discussions given above, the conservative assumption made by Elwood and Moehle is adopted in this study (i.e. $P_s = 0$). However an upper bound and lower bound range will still be considered for $P_s$, as given by Equation (4.29), for the purpose of sensitivity analysis presented in Section 4.2.5. This is to study the possible effects of the inaccuracies of the assumed $P_s$ value on the estimated limiting crack width developed at the critical shear crack.
\[ 0 \leq P_l \leq P \frac{nA_d}{A_g} \]  

(4. 29)

Figure 4. 7: Column S4 front and back view- longitudinal reinforcement experienced: (a) buckling; and (b) a double curvature sway at the ultimate loading stage

Figure 4. 8: Column S3 after completion of: (a) the first cycle of 1.5% drift; and (b) the second cycle of 1.5% drift (complete collapse)
C. The dowel resistance of longitudinal reinforcing bar was calculated according to recommendations made by Park and Paulay (1975) using Equation (4.30)

\[
V_d = \frac{2M}{l} = \frac{1.33d_b A_d f_{ct}}{\pi l} \tag{4.30}
\]

D. Critical shear crack length \( L_{cr} \) (see Figure 4.9) was calculated using Equation (4.31).

\[
L_{cr} = \frac{d}{\cos \theta} \tag{4.31}
\]

where \( d \) is the column depth and \( \theta \) is the angle of inclination of the critical shear crack with respect to the horizontal axis.

E. The rotation \( \tan \alpha \) at critical shear crack (refer Figure 4.9) was calculated as follows:

\[
\tan \alpha = \frac{2w}{L_{cr}} \tag{4.32}
\]

It is noted that the obtained critical crack width \( w \) based on the proposed iterative procedure represents an average value. The average crack width was assumed to correspond to mid-length (i.e. at \( L_{cr} / 2 \))

F. The inclined shear crack was assumed to start at a distance \( d \) from the base. Therefore, the \( \Delta_{rot} \) value caused by opening of the critical shear crack is estimated by Equation (4.33):

\[
\Delta_{rot} = (L - d) \tan \alpha \tag{4.33}
\]

The drift ratio \( \theta_{rot} \) in % as contributed by the opening of the shear crack is then given by Equation (4.34).

\[
\theta_{rot} = 100(L - d) \tan \alpha / L \tag{4.34}
\]

The parameters included in Equations (4.31) to (4.34) are illustrated in Figures 4.2 and 4.5 or as defined previously.
4.2.4 Worked Example-Estimating Rotational Deformation Caused by the Opening of Critical Shear Crack

This section presents a worked example to estimate the component of deformation resulting from rotation at the critical shear crack employing the proposed shear-slip model (as presented in Section 4.2 so far). In flexure-shear dominated columns, rotational deformation is added to other components of deformation (e.g. flexural and shear components which are addressed in Section 4.3 for predicting the total drift capacity of a column up to the limit of gravity collapse (the approach adopted for the development of Model I).

**Question:** Consider Column S2 as tested in this study (see Figure 4.10). Estimate the component of lateral deformation (up to the limit of gravity collapse) caused by the opening of critical shear crack.

**Solution:** Part A and B (as presented below) provide the required details for Column S2 and some basic input parameters respectively. The actual solution is presented in Part C.
Figure 4.10: Column S2 as tested in this study (the reinforcements shown in the footing do not show the actual details)

A. Details and properties of Column S2:

Imposed axial load: \( P = 340000 \text{KN} = 340 \text{KN} \)

Dimensions: \( b = 270 \text{mm} \) (breadth), \( d = 300 \text{mm} \) (depth), \( L = 1200 \text{mm} \) (length)

Concrete cover to the longitudinal bars: \( c = 25 \text{mm} \)

Maximum concrete aggregate size: \( a = 25 \text{mm} \)

Concrete compressive strength: \( f_c = 21 \text{MPa} \)

Longitudinal reinforcement: \( 4\#16 \text{mm} \)

Longitudinal reinforcement yield strength: \( f_y = 515 \text{MPa} \)

Transverse reinforcement: \( 1\#6 \text{mm} @ 300\text{mm} \)

Transverse reinforcement yield strength: \( f_{yt} = 365 \text{MPa} \)

A full description of test set up, loading regime, response behaviour and experimental observations corresponding to this column can be found in Chapter 3.

B. Basic input parameters:

The required basic and input parameters are obtained in Part B. These parameters are employed within the solution presented in next part (Part C).
Gross cross sectional area of the column:
\[ A_g = 300 \times 270 = 81000 \text{ mm}^2 = 0.08 \text{ m}^2 \]

Total cross sectional area of longitudinal reinforcements:
\[ A_{sl} = 4 \times \left( \frac{16}{2} \right)^2 \times \pi = 803 \text{ mm}^2 \]

Total cross sectional area of transverse reinforcement:
\[ A_{sl} = 2 \times \left( \frac{6}{2} \right)^2 \times \pi = 56.5 \text{ m}^2 \]
\[ P_0 = 0.85 \times 21 \times (81000 - 803) + 515 \times 803 = 1845061(N) \equiv 1845(KN) \]
Using Equation (4. 26)
\[ \frac{P}{P_0} = \frac{340}{1845} = 0.18 \]

0.18 = The axial load ratio required for estimating the angle of inclination of the critical shear crack (\( \theta \)) (see Figure 4.2)
\[ \theta = 55 + 35 \times 0.18 = 61.3^\circ \] Using Equation (4. 27)

Total contact area (\( A_c \)) at the plane of shear failure:
\[ A_c = 270 \times 300 / \cos 61.3^\circ = 168671 \text{ mm}^2 = 0.17 \text{ m}^2 \]

The dowel resistance of longitudinal reinforcing bar
\[ V_d = \frac{1.33 \times 16 \times 200 \times 515}{3.1415 \times 300} = 2.3(KN) \] Using Equation (4. 30)
\[ d_c = d - (2 \times c + d_k) = 300 - (2 \times 25 + 16) = 234 \text{ mm} \] (see Figure 4. 2)
\[ n = \frac{E_s}{E_c} = \frac{200000}{5055 \sqrt{21}} = 8.6 \] Modulus ratio

\[ P_s = \text{Axial contribution of one longitudinal reinforcement- Equation (4. 29)}: \]
\[ 0 \leq P_s \leq \frac{340 \times 10^2 \times 8.6 \times 803 / 4}{270 \times 300} \]
\[ 0 \leq P_s \leq 0.7 \text{KN} \]

**C. Estimating Limiting Crack Width:**

The problem at hand is to employ the nonlinear equations of equilibrium (Equations (4. 14), (4. 15)) and the compatibility Equation (4. 18) to determine the limiting crack width (\( w \)) (at the critical shear crack) beyond which the equilibrium of forces in the vertical direction is compromised. Rotational deformation (\( \Delta_{rot} \) as illustrated in Figure 4. 9 ) will then be estimated using Equation (4. 33).
Solution: Let’s assume that the required limiting crack width is \( w_1 \) (i.e. \( w = w_1 \)). \( w_1 \) may be taken as a fraction of maximum concrete aggregate size \( a \) as follows:

\[
w_1 = a / 20 = 25 / 20 = 1.25 \text{ (mm)}
\]

The maximum shear stress \( (V_{ci,\text{max}}) \) that can be developed across the crack corresponding to: 1) the assumed crack width \( (w_1 = 1.25 \text{ (mm)}) \); 2) the maximum aggregate size of concrete; 3) and the compressive strength of concrete can be estimated according to Equation (4.17) as given below:

\[
V_{ci,\text{max}} = \frac{\sqrt{f_c}}{0.31 + 24w/(a + 16)}
\]

\[
V_{ci,\text{max}} = \frac{\sqrt{21}}{0.31 + 24*1.25/(25 + 16)} = 4.4 \text{ MPa}
\]

The actual shear force developed across the crack \( (V_{gf}) \) is to be calculated next. Referring to Equation (4.18), \( V_{gf} \) is not only a function of \( V_{ci,\text{max}} \) (and therefore the assumed crack width) but also a function of the clamping stresses \( (f_{ci}) \) acting normal to the critical shear crack (or corresponding normal force \( N \) as shown in Figure 4.2). Normal force \( (N) \), on the other hand, cannot be evaluated unless \( V_{gf} \) is known as suggested by the equation of equilibrium (i.e. Equation 4.15). To resolve this issue, \( V_{gf} \) may be evaluated (in the first iteration) based on an initial estimation of \( N \). This initial estimation is denoted \( N_0 \) and is calculated taking into account the clamping effect of imposed axial load only (i.e. the component of \( P_{\text{imposed}} \) resolved in a direction normal to critical shear crack). The clamping force will be later updated and refined using an iterative procedure.

\[
N = N_0 = P_{\text{imposed}} \cdot \cos \theta
\]

\[
N_0 = 340 \cdot \cos 61.3 = 163 \text{ KN}
\]

With the knowledge of \( N_0 \) and the contact area \( (A_c) \), corresponding clamping stress \( (f_{ci0}) \) can be calculated as follows:

\[
f_{ci} = f_{ci0} = N_0 / A_c
\]

\[
f_{ci0} = 163 \times 10^3 / 0.17 \times 10^6 = 0.96 \text{ MPa}
\]

An initial estimation of \( V_{gf} \) can be obtained now using Equation (4.18) as follows:

\[
V_{gf} = A_c \left( 0.18V_{ci,\text{max}} + 1.64f_{ci} - 0.82 \frac{f_{ci}^2}{V_{ci,\text{max}}} \right)
\]
Having estimated an initial value for the developed shear force $V_{gf}$, it is possible to obtain an updated value for the clamping force normal to the crack ($N$) using Equation (4.15). This updated value is denoted $N_1 = N_0$.

\[ N \sin \theta + V = V_{gf} \cos \theta + \frac{A_{sl} f_{sl} d_c}{s} \tan \theta + n_{pars} V_d \]

\[ N \sin 61.3 + 0 = 373 \times 10^3 \times \cos 61.3 + \frac{56.5 \times 365 \times 234}{300} \tan 61.3 + 4 \times 2.3 \times 10^3 \]

$N_1 = 248 \text{ KN}$

The procedure presented above is repeated and the normal clamping force is refined until convergence is obtained. In other words, until:

\[ N_j - N_{j-1} \equiv 0 \]

Comparing $N_1$ with $N_0$ in this example shows that the convergence is not achieved yet (i.e. 248 – 163 ≠ 0). Therefore a new clamping stress ($f_{cil}$) will be calculated based on $N_1$ and the resulting shear forces will be updated.

$N_1 = 248 \text{ KN}$

\[ f_{cil} = 248 \times 10^3 / 0.17 \times 10^6 = 1.46 \text{ MPa} \]

\[ V_{gf} = 0.17 \times 10^6 \times (0.18 \times 4.4 + 1.64 \times 1.46 - 0.82 \times \frac{1.46^2}{4.4}) = 474 \text{ KN} \]

\[ N_2 = (474 \times 10^3 \times \cos 61.3 + \frac{56.5 \times 365 \times 234}{300} \tan 61.3 + 4 \times 2.3 \times 10^3 \) / \sin 61.3 = 303 \text{ KN} \]

This procedure should be repeated until convergence is obtained. For the case of Column S2, 8 iterations were required as given in Table 4.5. Circled values shown in Table 4.5 are brought forward. Theses values can satisfy two nonlinear Equations of (4.18) and (4.15).

With the knowledge of frictional shear resistance and clamping force acting on the critical shear crack (i.e. $V_{gf} = 587 \text{ KN}$ and $N = 365 \text{ KN}$ respectively), total transferrable axial load can be evaluated employing the equation of equilibrium written in vertical direction (i.e. Eq. (4.14)) as follows:

\[ P = N \cos \theta + V_{gf} \sin \theta + n_{pars} P_s \]

\[ P = 365 \times \cos 61.3 + 587 \times \sin 61.3 + 4 \times 0 = 690 \text{ KN} \]
Table 4.5: Iterative procedure for estimating $N$ and $V$ corresponding to $w_1$

<table>
<thead>
<tr>
<th>Iteration number ($j$)</th>
<th>$N_{j-1}$ (KN)</th>
<th>$V_g$ (KN) Eq. (4.18)</th>
<th>$N_j$ (KN) Eq. (4.15)</th>
<th>$N_j - N_{j-1}$ (KN)</th>
</tr>
</thead>
</table>
| 1                      | 163 (from axial load only) | 373                  | 248                  | 85  
|                        | 85            |                      | 85                  |
| 2                      | 248          | 474                  | 303                  | 55  
|                        | 303          | 531                  | 335                  | 55          
| 3                      | 303          | 531                  | 335                  | 32          
| 4                      | 335          | 561                  | 351                  | 16          
| 5                      | 351          | 575                  | 359                  | 8          
| 6                      | 359          | 582                  | 363                  | 4          
| 7                      | 363          | 585                  | 365                  | 2          
| 8                      | 365 (assumed) | 587                  | 365 (calculated)     | 0          |

Estimated axial load (i.e. $P = 690$ KN) corresponds to the assumed crack width (i.e. $w_1 = 1.25$ mm). This value is much greater than the required (imposed) axial load (i.e. $P_{imposed} = 340$ (KN)). This suggests that the limiting crack width must be greater than the assumed initial value (i.e. $w_1 = 1.25$ mm). Therefore the procedure described above is repeated for a new trial value of the crack width. Trial crack width values should be systematically refined (e.g. $w_i = w_{i-1} + 0.25$ mm) until the estimated axial load equates the imposed axial load.

The results corresponding to the final trial value of crack width (i.e. $w_i = 3.7$ mm) are presented in Table 4.6:
Table 4. 6 Iterative procedure for estimating $N$ and $V$ corresponding to $w_i$

For the $i^{th}$ trial value of limiting crack width ($w_i = 3.7\text{mm}$):

$V_{ci,\text{max}} = 1.85(\text{MPa})$ (using Equation (4. 17))

<table>
<thead>
<tr>
<th>Iteration number ($j$)</th>
<th>$N_{j-1}$ (KN)</th>
<th>$V_{sf}$ (KN)</th>
<th>$N_j$ (KN)</th>
<th>$N_j - N_{j-1}$ (KN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>365</td>
<td>308</td>
<td>212</td>
<td>153</td>
</tr>
<tr>
<td>2</td>
<td>212</td>
<td>287</td>
<td>201</td>
<td>11</td>
</tr>
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<td>1</td>
</tr>
<tr>
<td>5</td>
<td>197</td>
<td>278</td>
<td>196</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>196</td>
<td>278</td>
<td>196</td>
<td>0</td>
</tr>
</tbody>
</table>

The two values of $V_{sf} = 278\text{KN}$ and $N = 196\text{KN}$ are brought forward.

$P = 278\cos 61.3 + 196\sin 61.3 + 4*0 = 339(\text{KN})$ (using Equation(4. 14))

$P_{\text{imposed}} - P = 340 - 339 = 0$ ($w_i = 3.7\text{mm}$ is accepted as the answer)

The axial load difference calculated above ($P_{\text{imposed}} - P$) is less than 0.5% of the imposed axial load. This however, can be further reduced, if required, by further refinement of the trial crack width ($w$). $w_i = 3.7\text{mm}$ is accepted here as the answer and is brought forward for estimating corresponding rotational deformation which follows:

D. Rotational Deformation:

The critical crack width as estimated above (i.e. $w_i = 3.7\text{mm}$) should now be translated into corresponding rotational deformation ($\Delta_{rot}$) as illustrated in Figure 4. 9.

Total contact length at critical shear crack can be estimated as follows:

$L_{cr} = \frac{300}{\cos 61.3} = 625(\text{mm})$ Using Equation (4. 31)

Deformation caused by column rotation at the critical shear crack is estimated:

$\tan \alpha = \frac{2*3.70}{625} = 0.012$ Using Equation (4. 32)
\[ \Delta_{\text{rot}} = 0.012 \times (1200 - 300) = 11 \text{ (mm)} \quad \text{Equation (4.33)} \]
\[ \theta_{\text{rot}} = 100 \times \frac{11}{1200} = 0.92\% \quad \text{Equation (4.34) (rotational drift ratio in \%)} \]

4.2.5 Sensitivity Analyses on Critical Crack Width (w) - Simplified Expression

The developed iterative procedure for estimating the limiting crack width (in Sections 4.2.1 to 4.2.4) was incorporated into Excel spreadsheet and employed in a number of analyses including some sensitivity analyses. The objectives of the analyses were:

A. To develop a simplified expression that eliminates the need for a detailed iterative procedure for estimating the limiting crack width. The required simplified expression will be incorporated in Model I (together with shear and flexural deformation components presented in Section 4.3 for estimating the ultimate drift capacity of lightly-reinforced, poorly-confined concrete columns with flexure-shear mode of failure.

B. To provide a better insight into the effects of the uncertainties (involved with some input parameters- e.g. \( \theta \), \( s \)) on the estimated limiting crack width.

A - Simplified Expression for Estimating the Limiting Crack Width:

Table 4.7 summarizes the information corresponding to 21 lightly-reinforced, poorly-confined columns collected from literature (including Columns S2-S4 as tested in this study). A flexure-shear mode of failure is common amongst all these columns although the exact response behaviour and mechanism of failure is generally slightly different from one specimen to another. Some discussions and more information regarding these columns can be found in Chapter 3.

The iterative procedure developed in Sections 4.2.1 to 4.2.4 was employed to estimate limiting crack width values for the 21 columns analysed. Estimated values versus corresponding axial load ratios are plotted as a series of data points as can be seen in Figure 4.11. Estimated crack width values are mainly based on the actual input parameters as reported in the literature for the columns in the considered dataset. Maximum concrete aggregate size was taken 25 mm if not reported. Angle of
inclination of the critical shear crack was assumed 60 degree (i.e. \( \theta = 60^\circ \)) for all columns. Axial contribution of the longitudinal reinforcement (\( P_L \)) was conservatively taken as zero for all cases as discussed in Section 4.2.3. Uncertainties in relation to the assumed \( \theta \) and \( P_L \) values (as well as those attributed to other input parameters) are addressed later in this section.

To develop a conservative simplified estimator for the typical columns of interest a generally lower bound curve was fitted to the data points in Figure 4.11. This curve is mathematically defined by Equation (4.35).

\[
w = 0.45 \times ALR^{-1.05}, \quad ALR \geq 0.05
\]

where

- \( w \): the limiting crack width (in mm) beyond which the developed frictional shear force and clamping force are inadequate to maintain the imposed axial load.
- \( ALR = \frac{P}{A_g f_c} \): column axial load ratio

![Figure 4.11: Estimated crack width values corresponding to the columns included in the data set](image)

The fitted lower bound curve suggests that the limiting crack width increases with decreasing axial load ratio particularly for axial load ratios smaller than 0.1. At some axial load ratios, however, some scatters of the estimated data points are seen in Figure 4.11 as expected. Such scatters are attributed to variations of the contributing parameters corresponding to different columns of similar aspect ratios.
The simplified Equation (4. 35) may replace the detailed iterative procedure developed earlier for estimating the critical crack width. Rotational drift ratio ($\theta_{\text{rot}}$ in %) caused by opening of the critical shear crack can therefore be simplified into Equation (4. 36). This Equation is obtained by combining Equation (4. 35) and Equations (4. 31) to (4. 34) with ($\theta = 60^\circ$).

$$\theta_{\text{rot}}(\%) = \frac{45(L - d)}{Ld} \cdot ALR^{-1.05}$$

Equation (4. 36)

Parameters are as defined earlier
Table 4.7: Details of the columns analysed for verification purpose of the proposed theoretical drift capacity model

<table>
<thead>
<tr>
<th>Reference</th>
<th>Specimen designation</th>
<th>Dimensions</th>
<th>Clear Height</th>
<th>n=L/d</th>
<th>f’c (Mpa)</th>
<th>Spacing S (mm)</th>
<th>LBR (%)</th>
<th>TBR (%)</th>
<th>ALR (%)</th>
<th>Hook details (deg.)</th>
<th>Lateral loading scheme</th>
<th>Axial loading scheme</th>
<th>Observed drift ratio at gravity collapse %</th>
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<tbody>
<tr>
<td><strong>This study (Collaborative)</strong></td>
<td></td>
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<td>300</td>
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<td>C</td>
<td>2Cycles Comp.</td>
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</tbody>
</table>

Notes: \(d\) and \(b\) are the depth and breadth of column; \(n=\text{shear span ratio}\); \(LBR = A_{lb}/bd\) \((\text{longitudinal bar ratio})\); \(TBR = A_{ts}/ds\) \((\text{transverse bar ratio})\); \(ALR = P/A_{x}f'_{c}\) \((\text{axial load ratio})\); \(C\) and \(LS\) in Column 13 stand for continuous or lap-splice longitudinal bar respectively; Column 14: lateral loading scheme 2 or 3 Cycles at each drift limit.
B - Sensitivity Analyses

There are some assumptions involved in the development of the simplified expression for estimating the limiting crack width (i.e. Equation (4. 35)). These assumptions include: (a) the angle of inclination for the critical shear crack was set to 60° for all columns (i.e. $\theta = 60^\circ$); (b) the axial load carrying contribution of longitudinal reinforcement which was assumed to be negligible (i.e. $P_r = 0$); and (c) the maximum aggregate size of concrete which was taken 25mm if not reported (i.e. $a = 25mm$). In a general case, particularly in the case of old columns, it is likely to have additional uncertainties in relation to the exact content of vertical and transverse reinforcements, the vertical spacing of transverse reinforcement and also concrete compressive strength.

This section presents the results of a number of sensitivity analyses conducted to investigate the effect of the aforementioned uncertainties on the estimated limiting crack width. The result of this section provides the insight required to make reasonable assumptions when necessary and to verify that no unconservative assumptions are made in relation to the simplified Equation (4. 35).

For the purpose of sensitivity analyses a typical column of interest with varying axial load ratio was initially selected. This column is referred herein as the ‘reference column’. The parameters corresponding to the reference column could be found in Table 4. 8. A practical range of 0.1 to 0.6 was selected as the lower and upper bound of the imposed axial load ratio respectively. The limiting crack width corresponding to the reference column was then estimated (at different axial load ratios) employing the iterative procedure developed in Sections 4.2.1 to 4.2.4. Estimated limiting crack width values were plotted against corresponding axial loads. The obtained data points were connected together to construct a curve which is referred herein as the ‘reference trend’ (see Figure 4. 12 and Figure 4. 13).

To study the sensitivity of the estimated crack width (represented by the reference trend) to possible uncertainties in the input parameters, one parameter was changed at a time and a new trend was obtained. The altered parameters include:
- 25% change in the concrete compressive strength (i.e. $f'_c = [27, 34 \text{MPa}]$)
- 50% change in the maximum concrete aggregate size (i.e. $a = [25, 38 \text{mm}]$)
- 100% change in the longitudinal reinforcement ratio (i.e. $LRR = [1\%, 2\%]$)
- 100% change in transverse reinforcement ratio (i.e. $TRR = [0.1\%, 0.2\%]$)
- The Angle of inclination of the critical shear crack was reasonably altered considering the result from the limited experimental results reported to date (i.e. $\theta = [55^\circ - 65^\circ]$)
- 25% change in transverse bar spacing (i.e. $s = [300, 375 \text{mm}]$). The larger value corresponds to section depth.
- Change in the considered level of $P$, or the axial contribution of longitudinal reinforcements prior to gravity collapse (i.e. $0 \leq P_s \leq P_{nA_r} / A_g$) (refer to section 4.2.3 for relevant discussions)

### Table 4.8: The parameters considered for the reference column subjected to variable axial load.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Section dimensions (breadth &amp; depth)</td>
<td>$b = d = 375 \text{mm}$</td>
</tr>
<tr>
<td>Concrete compressive strength</td>
<td>$f'_c = 27 \text{MPa}$</td>
</tr>
<tr>
<td>Maximum aggregate size</td>
<td>$a = 25 \text{mm}$</td>
</tr>
<tr>
<td>Concrete cover to longitudinal bars</td>
<td>25 mm</td>
</tr>
<tr>
<td>Longitudinal reinforcement ratio</td>
<td>$LRR = 1%$</td>
</tr>
<tr>
<td>Longitudinal reinforcement yield strength</td>
<td>430 MPa</td>
</tr>
<tr>
<td>Transverse reinforcement ratio</td>
<td>$TRR = 0.1%$</td>
</tr>
<tr>
<td>Transverse reinforcement yield strength</td>
<td>380 MPa</td>
</tr>
<tr>
<td>Vertical spacing of transverse reinforcement</td>
<td>$s = 300 \text{mm}$</td>
</tr>
<tr>
<td>Angle of inclination of the critical shear crack</td>
<td>$\theta = 60^\circ$</td>
</tr>
<tr>
<td>Axial contribution of longitudinal reinforcements ($P$)</td>
<td>$P_s = 0$</td>
</tr>
</tbody>
</table>
Each of the graphs provided in Figure 4.12 and Figure 4.13 presents the reference trend as well as a trend which reflects the effect of the considered change in only one parameter (i.e. $f_c, a, LRR, TRR, \theta, s$ and $P_s$). These figures show that, in general, the greater the axial load ratio the smaller the limit of crack width beyond which stability is compromised. The results obtained from sensitivity analysis also suggest that the concrete compressive strength ($f_c$) and the concrete maximum aggregate size ($a$) have greater effects on frictional resistance and therefore on the estimated crack width than other parameters (e.g. $LRR$ and $TRR$). It is seen that increasing aggregate size from 25 to 38 mm (~50% increase) can considerably enhance the ultimate crack width particularly at smaller axial load ratios (see Figure 4.12b). Similar trends are observed for concrete compressive strength (see Figure 4.12a), and less significantly for $LRR$ and $TRR$ (see Figure 4.13b & c).

Importantly, it was observed that a reasonable change in the angle of inclination of the critical shear crack (i.e. $\theta = [55^\circ - 65^\circ]$) has insignificant effect on the estimated crack width (see Figure 4.13a) at all levels of axial load ratio. The results obtained from the analyses of this study also showed that the change in vertical spacing of transverse reinforcement has negligible effect on crack width estimations (when the reinforcement ratio is kept constant) (see Figure 4.12c).

Figure 4.14 reveals the sensitivity of estimated critical crack width values to $P_s$ (the axial contribution of longitudinal reinforcements prior to gravity collapse). It can be seen that the uncertainty in $P_s$ has a considerable effect on the estimated crack width values when the axial load ratio is low (approximately when it is less than 0.2). This effect however is negligible for axial load ratios greater than 0.2 and approaches zero at about the axial load ratio of 0.5. It is suggested, in this study, that a conservative value of $P_s = 0$ be taken into account as utilized in the development of Equation (4.35).
Figure 4. 12: Trend of estimated critical crack width vs. axial load ratio when (a) concrete compressive strength $f_c$; (b) maximum aggregate size; and (c) vertical spacing of transverse reinforcement ($s$) are parameterized.
Figure 4.13: Trend of estimated critical crack width vs. axial load ratio when (a) critical crack angle $\theta$; (b) transverse reinforcement ratio (TRR); and (c) longitudinal reinforcement ratio (LRR) are parameterized.
Figure 4.14: effect of the assumed axial contribution of the longitudinal reinforcement \( (P_s) \) on the estimated critical crack width

4.3 Specialized Models for Estimating Conventional Components of Lateral Deformation for Limited-Ductile RC Columns

Section 4.2 presented the developed shear-slip model (i.e. a detailed iterative procedure and a simplified model) for estimating the rotational component of deformation caused by the opening of critical shear crack (addressing the column response behaviour at an ultimate loading stage prior to gravity collapse). This component, however, is not the only deformation component experienced by a column with flexural-shear mode of failure (This mode of failure is seen to be very common for lightly-reinforced concrete columns particularly those with a shear span to depth ratio of greater than 2.5-refer Chapter 3). Such columns usually yield prior to the development of critical shear crack as promoted by light content of longitudinal reinforcements. Total deformation experienced by the columns of interest up to the limit of gravity collapse is expected to be obtained by superposition of the following deformation components (i.e. by Equation (4.37)):

1) Rotational deformation (refer Section 4.2)
2) Flexural deformation (refer Section 4.3.1)
3) Shear deformation (refer Section 4.3.2)
4) Strain penetration deformation (refer Section 4.3.3)

\[
\Delta_{\text{collapse}} = \Delta_{\text{rot}} + \Delta_f + \Delta_s + \Delta_{\text{eff}}
\]  \hspace{1cm} (4.37)

where
While the developed shear-slip model presented in Section 4.2 provides a new rational solution for the gap identified in the literature, the models presented in Section 4.3 are primarily based on the existing models. Section 4.3 focuses on specializing and simplifying the existing analytical models aiming at developing a theoretical deterministic model (Model I) for estimating total deformation capacity of lightly reinforced-poorly confined concrete columns, in accordance with Equation (4.37).

### 4.3.1 Flexural Deformation

Flexural deformation of columns is composed of two components namely, elastic and plastic deformations. The former component is typically assumed up to the yielding of longitudinal reinforcement or reaching a certain strain limit in the extreme concrete fibres in compression (i.e. \( \epsilon_c = 0.003 \)). The relevant deformation experienced beyond this limit is the plastic component of the flexural deformation. The plastic component of flexural deformation is addressed in 4.3.1.2.

Flexural deformation of a RC column is generally well-understood and can be calculated reliably provided the column is not interrupted by shear failure prior to yielding of its longitudinal bars. A shear failure is usually expected if shear span ratio is very low (i.e. \( \leq 2.5 \)) and/or the shear force index is equal to or greater than one (i.e. \( V_p / V_n \geq 1 \)). In this index, \( V_p \) is the shear demand corresponding to flexural strength of section and \( V_n \) is the nominal shear strength of reinforced concrete section that can be obtained using available models (Sezen and Moehle 2004; Wibowo et al. 2012). Considering the focus of the study which is lightly-reinforced columns of medium to high aspect ratio, flexural yielding and strength development is usually anticipated though the final failure is generally governed by shear owing to
inadequate lateral confinement. For flexural deformation modelling, a nonlinear moment-curvature analysis is usually employed to define the section strength, yield curvature ($\phi_y$) and the elastic stiffness.

In this study an Excel Based Nonlinear Section Analysis program (EBNSA) was developed. Figure 4.15 shows the input page of this program. The moment-curvature analysis procedure as outlined in Section 2.3.1 was incorporated in EBNSA.

Unconfined stress-strain relationship is used for cover concrete. The limited confinement effect of the core concrete is taken into account, for a given set of reinforcing and section details, in accordance to the concrete stress-strain models proposed by Mander et al. (1988) or Scott et al. (1982). The program also gives the option for the rate of straining in concrete.

Figure 4.15: Parameter input page of the developed Excel Based Nonlinear Section Analysis program (EBNSA)

The section yield curvature $\phi_y$ and corresponding yield moment $M_y$ are among the outputs of the program. The yield deformation for a cantilever column can then be approximated using Equation (4.38):
where \( L \) is the shear span length that is the length from critical flexural section to the point of contraflexure.
Equation (4.38) is based on the well known assumption of linear distribution of curvature up the column height. This is a reasonable assumption for non-squat columns as is suggested by previous investigators (Priestley et al. 2007) and further verified in the experimentations conducted in this study.
The EBNSA program was utilized for developing a simplified expression (to be presented in the next section) for estimating yield curvature which is in turn required for estimating the flexural component of deformation in the intended theoretical axial capacity model.

4.3.1.1 Simplified Yield Curvature

Priestley et al (2007) suggested a simplified expression for estimating the yield curvature as follows:

\[
\phi_y = \frac{k \varepsilon_y}{l/d}
\]  \hspace{1cm} (4.39)

where \( \varepsilon_y \) is the longitudinal steel yield strain and \( d \) is the depth of section.

Priestley derived an average value (i.e. \( k = 2.1 \)) for rectangular RC columns recognizing that, in general, coefficient \( k \) (yield strain factor) is not significantly sensitive to axial load ratio and reinforcement ratio within the ranges studied.

Evaluating the adequacy of this equation, it was found that the limited variation of the yield strain factor especially as a function of axial load ratio could be relatively significant for the lightly reinforced-poorly confined columns of interest.

Figure 4.16 shows variation of \( k = \phi_y d / \varepsilon_y \) calculated for three different lightly reinforced sections (i.e. A, B & C) with the dimensions and reinforcing details as given in Table 4.9.
Table 4.9: Details of the sections included in a parametric study on yield strain factor (k) for sections with light vertical reinforcement ratios and a minimal confining reinforcement

<table>
<thead>
<tr>
<th>Section designation</th>
<th>Cross section dimensions (mm)</th>
<th>Axial load ratio</th>
<th>Vertical reinforcement ratio (%)</th>
<th>Transverse reinforcement ratio (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>270*300</td>
<td>[0.1-0.4]</td>
<td>1</td>
<td>0.08</td>
</tr>
<tr>
<td>B</td>
<td>360*360</td>
<td>[0.5-1.5]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>540*540</td>
<td>[0.5-1.5]</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

As can be seen in this table, the considered transverse reinforcement ratio was very light and of the order of 0.08% for the 3 cases. The thick dashed line, in this figure, represents the average trend and is defined by Equation (4.40).

\[
K = 3 \times \rho + 1.4, \quad 0.1 \leq \rho \leq 0.4
\]  

(4.40)

where

\[
\rho = \frac{P}{A_x f_c'}
\]

(4.41)

It should be noted that for longitudinal bar ratio greater than 1.5% and/or transverse bar ratio greater than 0.1% the parameter \( K \) may be taken as suggested by Priestley (i.e. \( K = 2.1 \)).

Figure 4.16: Yield strain factor \( (k = \phi_y d / \varepsilon_y) \) for lightly reinforced-poorly confined sections (refer to Table 4.9)
4.3.1.2 Inelastic Flexural Deformation

The moment-curvature analysis as discussed in preceding section can be rationally applied to calculate the maximum moment capacity of the section $M_{max}$ and the corresponding curvature $\phi_{max}$. The plastic capacity of the section ($V_p$) is given by:

$$V_p = F_{max} = \frac{M_{max}}{L} \quad (4.42)$$

where $L$ is the shear span length of a cantilever column.

Table 4.10 provides the values for $F_y$ and $F_{max}$ for Columns S1-S4 as measured experimentally and the corresponding values obtained using the moment-curvature analyses. The calculated yield curvature ($\phi_y$) and the curvature at maximum flexural strength ($\phi_{max}$) of the columns are also given in this table which compare well with experimental values (refer to Figure 4.17).

Table 4.10: Yield and plastic force and corresponding curvatures for columns S1 to S4

<table>
<thead>
<tr>
<th></th>
<th>Column S1</th>
<th>Column S2</th>
<th>Column S3</th>
<th>Column S4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measured values</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$F_y$ (KN)</td>
<td>51.2</td>
<td>66.7</td>
<td>79.4</td>
<td>76</td>
</tr>
<tr>
<td>$F_{max}$ (KN)</td>
<td>59.7</td>
<td>78.4</td>
<td>82.9</td>
<td>77</td>
</tr>
<tr>
<td>Calculated values</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$F_y$ (KN)</td>
<td>52.4</td>
<td>68.3</td>
<td>76.6</td>
<td>75.8</td>
</tr>
<tr>
<td>$F_{max}$ (KN)</td>
<td>55.1</td>
<td>72.5</td>
<td>81</td>
<td>77.6</td>
</tr>
<tr>
<td>$\phi_y$</td>
<td>1.69E-05</td>
<td>1.73E-05</td>
<td>2.00E-05</td>
<td>1.98E-05</td>
</tr>
<tr>
<td>$\phi_{max}$</td>
<td>5.78E-05</td>
<td>6.26E-05</td>
<td>2.76E-05</td>
<td>2.69E-05</td>
</tr>
</tbody>
</table>

Referring to the tabulated data, for the columns with moderate axial load ratio (i.e. Columns S1 and S2 with 20% ALR) the curvature values at flexural strength are considerably greater than the corresponding yield curvatures (by a factor of up to 3.6). Conversely, for columns with high ALR of 40%, the yield and maximum curvature values are relatively close (i.e. $\phi_{max} / \phi_y = 1.4$). It is noted that in the former case the section behaviour is governed by yielding of the longitudinal bars in tension while in the latter case the softening of concrete controlled the section strength.
The drift at the plastic capacity of a section (\(\Delta_{\text{max}}\)) cannot be generally calculated using the assumption of the linear curvature distribution up the column height as assumed for estimating \(\Delta_y\). This is due to the fact that the curvature distribution tends to have a highly nonlinear pattern at post yield stage. This results in a region of concentrated damage at the vicinity of the column base which is known as plastic hinge region. The plastic hinge concept is originally considered for ductile columns where a distinct and concentrated flexural damage is expected at the base. In this context the core concrete is expected to maintain its integrity and the buckling of longitudinal bars is controlled.

Plastic hinge definition in non-ductile columns is however different from that of the ductile columns in a number of ways as follows:

1) The plastic hinge region in non-ductile columns, in general, is not as distinct and concentrated as that of the ductile columns.

2) A non-ductile column may sustain some concentrated damage caused by other sources than just the flexural rotation at the base. The damage, for instance, can occur due to rotation at the level of the critical shear crack above the base or by extra rotation due to vertical cracking and debonding of longitudinal bars.

Figure 4. 17 illustrates the measured and idealised yield and plastic curvature distribution up the height of Columns S1 to S4. The proposed curvature distribution (at the limit of maximum flexural capacity) is constructed by assuming \(\phi_{\text{max}}\) at the base linearly reduced to the value of elastic curvature at the height of \(2L_p\) (\(L_p\) value was assumed equal to tie spacing). It can be seen that the idealized variable plastic curvature matches reasonably well with the measured experimental curves. This idealization is approximately equivalent to the recommendation made by Rodsin (2007). For limited-ductile columns, Rodsin recommended an approximate plastic hinge length equal to tie spacing (\(s\)) coupled with a constant plastic curvature over the hinge length.

Based on the outlined observations, a plastic hinge equal to tie spacing (i.e. \(L_p = S\)) with a constant curvature of \(\phi_{\text{max}}\) over the hinge length is adopted for estimating the
Figure 4.17: measured and idealised yield and plastic curvatures for Columns S1 to S4. $\phi_{\text{max}}$ is taken as the smallest curvature resulting in maximum section capacity.
maximum flexural contribution to total deformation at peak lateral strength and also at
the limit of gravity collapse. In other words, it is conservatively assumed that in
poorly-confined columns, flexural deformation is interrupted by localised stiffness
softening along the column height after the peak strength is developed. Such stiffness
softening may be resulted from debonding of longitudinal bar or hinging at the level
of lap-splices, for instance, as observed in experimental observations (e.g. Lynn et al,
1996). Deformations beyond peak flexural point must therefore be provided by other
sources as mentioned above or as quantified previously (Section 4.2). Identification
and quantification of all sources of deformation beyond peak strength require further
studies to be conducted in future.

Given the values of $L_p$, $\phi_{max}$ and $\phi_y$, plastic rotation $\theta_p$ is calculated by:

$$ \theta_p = (\phi_{max} - \phi_y)L_p $$

(4.43)

where $\phi_{max}$ is the curvature corresponding to maximum flexural strength of the section.

The total flexural deformation $\Delta_f$ can be estimated by:

$$ \Delta_f = \Delta_y + \theta_p (L - L_p / 2) $$

(4.44)

In which $\Delta_y$ and $\theta_p$ are obtained from Equations (4.38) and (4.43) respectively. The
parameter ‘$L$’ is the shear span length that is the length from the critical flexural
section to the point of contraflexure.

Table 4.11 summarizes the predicted flexural deformation values (i.e. $\Delta_y$ and $\Delta_f$ )
for the tested columns (S1 to S4). The plastic hinge length was taken equal
to $s = 300mm$ and the required curvature values (i.e. $\phi_y$ and $\phi_{max}$ ) were obtained from
moment-curvature analyses from Table 4.10.

The calculated values can be compared against the observed total deformations at
yielding and maximum lateral capacity limits (refer to Table 4.11). These
comparisons are justified considering the negligible contribution of shear deformation
in pre-peak region for the relatively slender columns considered (i.e. S1-S4). The
extra deformation expected from longitudinal bar extension at the base is
compensated by the assumed linear curvature distribution up the column height
(Priestley et al. 2007). Tabulated data in Table 4.4 suggests that the estimated values
are reasonably representative of the observed values.
Table 4. 11: Measured and estimated values of the drift at yield and peak strength limits

<table>
<thead>
<tr>
<th></th>
<th>Column S1</th>
<th>Column S2</th>
<th>Column S3</th>
<th>Column S4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measured values</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \Delta y ) (mm)</td>
<td>9</td>
<td>9</td>
<td>12</td>
<td>12</td>
</tr>
<tr>
<td>( \Delta_{\text{max}} ) (mm)</td>
<td>20.5</td>
<td>20.8</td>
<td>13.4</td>
<td>12.1</td>
</tr>
<tr>
<td>Calculated values</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \Delta y ) (mm)</td>
<td>8.1</td>
<td>8.3</td>
<td>9.6</td>
<td>9.5</td>
</tr>
<tr>
<td>( \Delta_{\text{max}} ) (mm)</td>
<td>21.0</td>
<td>22.6</td>
<td>12.0</td>
<td>11.7</td>
</tr>
</tbody>
</table>

4.3.2 Shear Deformation \( \Delta_s \)

Test observations revealed that in limited-ductile columns with largely-spaced stirrups most shear deformation is experienced at vicinity or beyond the peak flexural strength (Wibowo et al. 2010a; Wibowo et al. 2010b) when potential inclined shear cracks are almost fully developed. Opening of stirrups, however, was barely found in any specimens after the test at gravity collapse. It is therefore conservative to estimate the potential shear deformation (for a general relevant case) using the conventional truss model assuming the maximum force that can be developed by the largely spaced-stirrups (i.e. assuming yielding condition). The following sections presents a procedure leading to simplified expressions (Equations (4. 52) and (4. 53)), for estimating the corresponding potential shear deformation.

Truss analogy is conventionally used to represent the shear behaviour of a reinforced concrete beam or column subsequent to the formation of diagonal cracks. By truss analogy, transverse stirrups and diagonal concrete struts (as formed in between the diagonal cracks) are assumed to form the web members (Figure 4. 18). It is further assumed that the chord members are infinitely rigid for the purpose of determining the web distortion only(Park and Paulay 1975). Denoting the elongation of the stirrups as \( \Delta_s \) and the shortening of the compression struts as \( \Delta_c \), the shear distortion \( \Delta_s \) can be found by applying Williot’s principles as given by Equation (4. 45). For simplicity the cracks are assumed to be at 45 degree to the horizontal (Park and Paulay 1975).

\[
\Delta_v = \Delta_s + \sqrt{2}\Delta_c
\]  

(4. 45)

where the elongation of transverse bars is calculated by:
\[ \Delta_s = \frac{f_s}{E_s} d \]  \hspace{1cm} (4.46)

where \( f_s \) is the tensile stress in transverse bar which may be taken equal to corresponding yield strength \( f_{yt} \) for the purpose of this study; \( d \) is the effective depth from extreme concrete compression fibre to the tensile steel and \( E_s \) is the modulus of elasticity of steel.

The shortening of diagonal concrete struts is calculated by:

\[ \Delta_c = \frac{f_{cd}}{E_c} \sqrt{2d} \]  \hspace{1cm} (4.47)

where

\[ E_c = \text{the modulus of elasticity of concrete} \]

\[ f_{cd} = \text{the diagonal compression stresses due to the truss mechanism that can be approximated by Equation (4. 51).} \]

Number of stirrups crossing a diagonal crack \( (n) \) is calculated by:

\[ n = \frac{d \cot \alpha}{s} \]  \hspace{1cm} (4.48)

where \( \alpha \) is the angle of inclination of the compression struts relative to the longitudinal bars and \( s \) is the centre to centre spacing of the stirrups along the column height (see Figure 4. 18).

Figure 4. 18: Parameters involved in shear deformation modelling
Resultant of all stirrup forces across the diagonal crack \( T_s \) is given by

\[
V_s = T_s = n f_{ys} A_{st}
\]  

(4.49)

where \( A_{st} \) is the total cross sectional area of the transverse bars perpendicular to the lateral load direction (at a single level) and \( f_{ys} \) is the stirrup yield strength; \( V_s \) is the maximum external shear force that can be transferred through the notional truss model at the yield condition of stirrups.

Average shear stress acting on column section is estimated by:

\[
v_s = \frac{V_s}{b_n d}
\]  

(4.50)

where \( v_s \) is the shear stress corresponding to the shear force \( V_s \).

For horizontal stirrups \( \beta = 90^\circ \) and compression diagonals at \( \alpha = 45^\circ \)

\[
f_{sd} = 2v_s
\]  

(4.51)

The shear distortion \( \Delta_v \) can be estimated using Equations (4.45) to (4.51). The estimated values for Columns S1 - S4 are plotted in Figure 4.19 (a to d). These figures also show the experimental shear displacement up the height of Columns S1 to S4. By comparison a reasonable agreement can be seen between the estimated and maximum measured values of shear displacement. It is however noted that the experimental measurements had to be stopped at certain stage (before gravity collapse) due to the high risk of damage to the instrumentation.

The shear distortion per unit of the cracked length (\( \theta_v \)) is given by:

\[
\theta_v = \frac{\Delta_v}{d}
\]  

(4.52)

Where \( d \) is the effective depth of the column (i.e. extreme concrete compression fibre to the centre of the longitudinal bar in tension)

The outlined procedure was employed to the columns given in Table 4.7. The results revealed that the change in estimated or expected shear distortion per unit length is slight, with a mean value of \( \theta_v = 0.00245 \) and a standard deviation of less than 15% of the mean value (see Figure 4.20). This is probably explained by the fact that the analysed columns mainly feature light and widely spaced stirrups (i.e. 0.5d<s<d)
which limits the maximum force that can be carried by these members of the notional truss. This in turn controls the resultant force on other web members of the truss (i.e. on inclined concrete compressive struts). In other words, compressive strain and resultant shortening of inclined compressive struts is dependant on the yield force that is expected to be put into the truss system by stirrups. On the other hand, the deformation expected from stirrups themselves is limited to yield deformation and hence the potential for suggesting an approximately constant shear deformation per unit length for limited ductile columns.

It is noted that the full length of column is not generally cracked (particularly in lightly reinforced columns) and therefore the total shear deformation ($\Delta_s$) depends on the cracked portion of the height. It is emphasised that the truss analogy is only applicable when the member develops a series of inclined cracks. The shear deformation for flexural dominant, non-ductile columns of interest in this study may therefore be estimated using the following simplified expression.

$$\Delta_s = 0.0025KL \quad (4.53)$$

where $L$ is the column clear height and $K$ is a factor defining the cracked portion of the height. Based on experimental observations of this study and other relevant studies, a set of values for $K$ can be suggested as follows:
Figure 4. 19: Column S1-S4, measured and estimated shear displacements at yield and peak lateral strength
\[ K = 0.5 \quad \text{for} \quad ALR \geq 0.4 \quad (4.54) \]

\[ K = 0.75 \quad \text{for} \quad ALR \leq 0.2 \]

where

\[ ALR = \frac{N}{A_g f_c} \quad (4.55) \]

where \( ALR \) is the axial load ratio; \( A_g \) is the gross cross sectional area of the column and \( f_c \) is the cylinder compressive strength of concrete.

For axial load ratios between 0.2 and 0.4, the \( K \) value may be estimated by a linear interpolation.

The drift ratio \( (\theta, \text{in } \%) \) contributed by the shear deformation is given by:

\[ \theta_s = 0.0025KL^*100/L = 0.25K \quad (4.56) \]

Figure 4.20: Generalized shear distortion per unit length
(Applicable to the cracked portion of limited-ductile RC columns)
4.3.3 Deformation Caused by Extension of Longitudinal Bar at the Base $\Delta_{\text{ext}}$

Deformation due to longitudinal bar extension at the column base (which is also referred to as strain penetration deformation) is addressed by a number of researches (Elwood and Eberhard 2006; Park and Paulay 1975; Priestley et al. 2007; Wibowo et al. 2012). Wibowo et al for example, provided a useful discussion over the previous models and proposed a “closed form” iterative procedure which was shown to be accurate in modelling the aforementioned deformation component. The simulated and measured curves (as provided in the study) showed that the yield penetration drift generally increases with the lateral load resisted by column (the curves were simulated up to the peak lateral strength). Such trend is also inferred from other models such as the simplified expression proposed by Elwood and Eberhard (2006), as given in Equation (4.57). As can be seen in this equation, the deformation corresponding to bar extension at the base $\delta_{\text{slip}}$ is directly related to the stress in the tension reinforcement $f_s$.

$$\delta_{\text{slip}} = \frac{d_b f_s \phi_y}{8u}$$

where

$$f_s = f_y \text{ if } \frac{p}{A_y f_c} < 0.2 \text{ and } f_s = 0 \text{ if } \frac{p}{A_y f_c} \geq 0.5$$

$$u = 0.5 \sqrt{f_c} \text{ MPa}$$

where $d_b$ is the diameter of the longitudinal reinforcement and $u$ is the average bond stress between the longitudinal reinforcement and the beam–column joint concrete.

The test observations made in this study and similar observations made by previous investigators (e.g. Lynn et al 1996) clearly show either a sudden drop or gradual reduction of stress demand on tension reinforcement at post-peak region of deformation. In other words, gradual strength degradation caused by reversed cyclic loading or sudden stiffness degradation caused by buckling of rebar or by formation of critical shear crack along the column height, for example, could result in stress redistribution and significant reduction of stress demand on tension reinforcement at
the base. Figure 4. 21 is taken from the study by Lynn et al. (1996) and is included here as an example of the experimental evidence in support of the provided discussion. In this figure longitudinal bar strain profiles are plotted for different stages of loading up to gravity failure for specimen 3SLH18. As is evident, tensile (positive) strain on longitudinal bar at the base considerably reduces with the increase of imposed lateral displacement beyond peak strain/stress demand.

Based on the observations discussed above, it is expected that elastic strain penetration and corresponding drift to be recovered or to be redistributed following the release of the tension stress on longitudinal reinforcing bar at the base. The accumulated plastic strain (if any) is assumed sufficiently small to be neglected. This assumption is justified given the relatively small contribution of strain penetration to the total deformation in lightly reinforced-poorly confined columns even at peak lateral strength limit. The measured drift ratios at the peak lateral strength were found to be of the order of 0.3% and 0.04% for the tested columns (i.e. S1-S4) which had axial load ratio of 0.2 and 0.4 respectively.
4.4 Verification and Discussions on the Developed Capacity Model

In previous sections (i.e. Sections 4.2 and 4.3) different components of deformation experienced by a RC column up to the limit of gravity collapse were studied. A shear-slip model was developed for estimating the rotational deformation caused by the widening of the critical shear crack prior to gravity collapse (Section 4.2). The other components of deformation (e.g. flexural and shear) were dealt with in Section 4.3 and simplified expressions were developed corresponding to these components. Superposition of all deformation components results in a simplified model which is referred to as Model I. This model is in fact a specialized form of Equation (4.37) and can be used for estimating the total drift capacity of lightly-reinforced, poorly-confined concrete columns of interest up to the limit of gravity collapse. Model I is represented by Equation (4.60) as follows:

\[
\theta_{\text{collapse}}(\%) = \frac{45(L-d)}{Ld} ALR^{-0.05} + \left( \frac{\phi_y L^2}{3} + (\phi_{\text{max}} - \phi_y) L_p (L - L_p / 2) \right) / \left( L \times 100 + 0.25K \right)
\]

Where:

\( ALR = P/A_g f'_c \) = axial load ratio

\( L (\text{mm}) = \) shear span length that is the length from the critical flexural section to the point of contraflexure

\( d (\text{mm}) = \) column depth in mm

\( L_p (\text{mm}) = \) plastic hinge length which may be taken equal to tie spacing but not less than half of the section depth

\( \phi_y = \) yield curvature which may be obtained employing Equations (4.39) and (4.40)

\( \phi_{\text{max}} = \) curvature corresponding to peak flexural capacity of the section ideally obtained from moment curvature analysis (but not greater than \( 5\phi_y \) considering the uncertainty in plastic hinge length).

\( K \) is a factor defining the cracked portion of the column height and may be obtained using Equation (4.54).

The first term of Equation (4.60) represents the drift caused by rotation at the level of critical shear crack (refer Equation (4.36)). Equation (4.36) provides conservative
lower bound estimates of the rotational drift ratio in %. This equation is derived from the proposed equilibrium based iterative solution presented in Sections 4.2.

The second term of Equation (4.60) represents the yield and plastic components of flexural deformation. This term is based on the standard solution for predicting the flexural component of deformation (refer to Equation (4.44)). In this study however, the applicability of this solution for the limited ductile columns of interest is examined and verified against the relevant experimental records for the tested columns (i.e. Column S1-S4 tested in this study) (refer to Table 4.11). Importantly the proposed plastic hinge length for limited ductile RC columns was verified against experimental results (refer to Figure 4.17). And finally the simplified model for estimating the yield curvature proposed by Priestley et al. (2007) was specialized for lightly-reinforced, poorly-confined concrete columns, based on a parametric study presented in Section 4.3.1.1.

The third term of Equation (4.60) (i.e. $0.25K$) represents the drift ratio (in %) due to shear deformation. The corresponding simplified expression for estimating the shear drift per unit of cracked length (Equation (4.53)) is derived from specialization and numerical evaluations of the classical truss analogy for the limited-ductile RC columns of interest (refer to Section 4.3.2). The proposed expression was verified against the relevant experimental results of this study (refer to Figure 4.19).

The deformation caused by the extension of longitudinal reinforcement at the base was excluded as discussed in Section 4.3.3 for the case of drift capacity at gravity failure.

Equation (4.60) was employed to estimate the drift capacity of the columns included in Table 4.7. Figure 4.22 shows estimated versus observed drift ratios at the limit of gravity collapse for the columns analysed. The ratio of estimated to observed drift ratios were found to have a mean and standard deviation of 0.89 and 0.22 respectively.

Figure 4.23 shows the estimated versus observed drift ratios (for the same data set as analysed above) based on the empirical equation proposed by Zhu et al (2007) (i.e.
Equations (4.1) and (4.5)). The calculated mean and standard deviation of the estimated to measured values were 0.83 and 0.22 respectively which suggest a similar accuracy and dispersion as can be obtained from Equation (4.60) developed in this study. It is however believed that Equation (4.60) offers improvements over the previous models by the considerable elimination of empirical calibrations for model development. It is therefore believed that the objective set in the present chapter (that is developing a theoretical gravity collapse capacity model) is reasonably met. Further studies are required to predict and quantify any other deformations which might contribute to total drift capacity at the limit of gravity load failure. For instance, unbonding of longitudinal reinforcement has been observed in a number of specimens to be a source of temporary relief of concrete compressive stress in critical sections resulting in additional drift capacity prior to gravity failure. Unbonding is seen to be mainly associated with poor transverse confinement coupled with low axial load ratio.

Figure 4.22: Estimated vs. observed drift ratios at the limit of gravity load failure based on Equation (4.60) developed in this study.
4.4.1 Worked Example for Model I-Gravity Collapse Estimation

Estimate the drift capacity of Column S2 at the limit of gravity collapse assuming a flexural-shear mode of failure.

Axial load ratio = $ALR = 0.2$

Shear span length = $L = 1200$ mm

$\phi_y = 1.73 \times 10^{-5}$ (From Table 4.10 obtained using moment-curvature analysis)

$\phi_{\text{max}} = 6.26 \times 10^{-5}$ (From Table 4.10 obtained using moment-curvature analysis)

$L_p = 300$ mm (Plastic hinge length= tie spacing, refer to Section 4.3.1.2)

$K = 0.75$ From Equation (4.54)

\[
\theta_{\text{collapse}} = \frac{45(200 - 300)ALR^{1.05}}{1200 \times 300} + \left( \frac{1.73E - 5 \times 1200^3}{3} + (6.26E - 5 - 1.73E - 5) \times 300 \times (1200 - 150) \right) / 12 + 0.25 \times 0.75
\]

$\theta_{\text{collapse}} = 0.61 + 1.88 + 0.19 = 2.68\%$

The predicted drift for Column S2 at gravity load collapse (i.e. $\theta_{\text{collapse}} = 2.68\%$) may be compared against the recorded value ($=2.5\%$) for this column.
4.5 Summary

1. In this chapter the adequacy of existing gravity collapse drift capacity models were evaluated. It was found that:
   a) Existing models (for estimating the drift capacity of RC columns at the limit of gravity collapse) are essentially empirical, which raises the concern regarding the generality of such models. This is evident by the history of model development each time additional or rather different experimental data were introduced (refer Table 4.1).
   b) The commonly observed rotational component of deformation caused by the opening of the critical shear crack (in lightly reinforced, poorly confined columns) is not explicitly/comprehensively modelled in any of the existing models.
   c) Most existing models require detailed input information to provide a deterministic estimation of the drift capacity. Such detailed input data are not generally available when existing old columns are to be assessed.

2. This chapter (Chapter 4) addressed the first two shortcomings as mentioned above by developing and verifying a drift capacity model (Model I). The third shortcoming is to be treated in the next chapter (Chapter 5) by developing probabilistic capacity models which require the least number of input parameters without compromising the reliability of estimations.

3. Model I is a theoretical deterministic model for predicting drift capacity at the limit of gravity collapse for limited-ductile RC columns with flexural-shear mode of failure. The main properties of Model (I) are as follows:
   a) The reliance of this model on empirical calibrations is significantly reduced as compared to that of the existing models presented in Table 4.1.
   b) A rigorous equilibrium & compatibility-based iterative procedure was developed to explicitly model the component of drift allowed by the opening of critical shear crack prior to gravity failure. This model was named ‘shear-slip model’ and presented in Section 4.2. A simplified expression was also developed eliminating the need for the iterative procedure (refer Section 4.2.5) to pursue the requirement for a simplified solution in Model I.
   c) Other components of deformation such as flexure and shear components were also included in Model I (refer Section 4.3). For this purpose existing standard
solutions were specialized for limited-ductile RC columns of interest. Relevant simplified expressions were developed and incorporated into the required simplified expression which predicts total drift up to the limit of gravity failure (Equation (4.60)).

d) Model (I) was verified against the experimental results of a comprehensive dataset including 21 limited-ductile columns all tested up to the limit of gravity collapse (refer Section 4.4)
5 Probabilistic Drift Capacity Modelling for Limited-Ductile RC Columns

Chapter 4 led to the development and verification of Model (I) which is a simplified model for predicting the drift capacity of lightly-reinforced, poorly-confined concrete columns up to the limit of gravity collapse. Model I is a theoretical deterministic model focused on the mode of failure and the major physical phenomena commonly reported for the columns of interest (i.e. a flexural-shear mode of failure with a distinct post-yield rotational deformation experienced prior to gravity collapse due to the opening of the critical shear crack). Model I is therefore recommended when the condition of the model can be ensured. This process however is accompanied with some uncertainties. For instance, the mode of failure can only be roughly predicted. For this purpose one may adopt the recommendations made by Zhu et al. (2007) or (ASCE/SEI 2007) which are based on estimated strength ratio \( \frac{V_p}{V_n} \) for a given column. With this approach a flexure-shear mode of failure is expected if the following condition is satisfied:

\[
0.7 \leq \frac{V_p}{V_n} \leq 1
\]  

(5.1)

where \( V_p \) is the shear demand corresponding to the plastic capacity of the section and \( V_n \) is the nominal shear strength of the section.

However, existing methods of classifications including the well-known strength-based approach as outlined above are inadequate to reliably determine the mode of failure particularly in the case of the flexure-shear mode of failure (Zhu et al. 2007).

This Chapter addresses some of the uncertainties as partly outlined above. This chapter presents two capacity models (Model II and III) with some improved/new aspects as compared to the existing counterparts.

Model II is a practical probabilistic capacity model (at the limit of gravity collapse) focusing on minimizing the required input parameters without compromising the reliability of the model. The developed model (presented in Section 5.1) is recommended to be incorporated in the intended probabilistic seismic assessment.
procedure (within the scope of this study) particularly if detailed input parameters are not available for estimating the ultimate drift capacity.

Section 5.2 presents Model III, which predicts the drift capacity of lightly-reinforced columns at the limit of 20% degradation in the peak lateral strength (frequently referred to as shear or lateral failure limit state). The criteria and approach proposed for the development of Model II was adopted for the development of this model. Models II and III are employed for simulating the degrading part of the force displacement backbone curve of limited ductile RC columns within the scope of this study. Such curves are required for inelastic dynamic modeling and analysis of the columns of interest.

5.1 Probabilistic Drift Modelling at the Limit of Gravity Collapse (Model II)

This section presents a probabilistic model (referred herein as Model II) for estimating the ultimate drift capacity of the limited ductile RC columns of interest at the limit of gravity collapse. A probabilistic model is required to deal with uncertainties in material behaviour and to address the difficulties in predicting and modelling the diverse mechanisms of failure.

The development of Model II started with establishing a comprehensive relevant dataset composed of 56 column specimens from 10 studies (including the 4 specimens tested in this study). All the columns were cyclically or monotonically tested up to gravity collapse. The data set may be regarded as a representative sample of the limited-ductile RC columns found in old buildings. The data set includes columns with a broad range of aspect ratios, reinforcement details, material properties and axial and lateral loading schemes. Some details of the specimens and key experimental observations are presented in Section 3.3.

For the development of Model II, the recommendations made by Aslani and Miranda (2005) was re-evaluated and subsequently adopted in this study. Aslani and Miranda collected a dataset of 23 limited-ductile columns and found that normalized axial load
defined by Equations (5.2) or (5.3) are best correlated with the observed drifts at the limit of gravity collapse (the two equations were found to have similar correlation strength). It should be noted that given similar correlation accuracy, the normalized parameter corresponding to the latter equation is preferred in this study considering the minimum number of input information involved.

\[ x' = \frac{P}{f_{cy} A_t \frac{d_c}{s}} \]  \hspace{1cm} (5.2)

\[ x = \frac{p}{A_g f_c \rho''} \]  \hspace{1cm} (5.3)

where \( x \) & \( x' \) are normalized axial load parameter, \( P = \) column axial load; \( d_c = \) horizontal spacing of ties; \( s = \) vertical spacing of stirrups, \( \rho'' = \) transverse bar ratio in \%; \( A_g = \) gross cross sectional area and \( f_c = \) cylinder compressive strength of concrete.

Figure 5.1 shows the observed ultimate drifts versus corresponding normalized axial load values as obtained using Equation (5.3). From this plot and several other trial plots (not shown here) the need for classification of specimens became evident as no single parameter could be reasonably correlated to the entire range of the columns. Therefore, it was found that columns may be conveniently classified into two types based on the aspect ratio of the columns as follows:

1. Type 1 columns with shear span to depth ratio smaller than 2.5 (i.e. \( L/d < 2.5 \)).
2. Type 2 columns with shear span to depth ratio equal to or greater than 2.5 (i.e. \( L/d \geq 2.5 \)). All the poorly confined columns fell in this group (with a total number of 23) were reported to have a flexural or flexural-shear mode of failure. These columns are used for the development of Equation (5.4) and further discussed in this study.

Normalized axial load index (NALI) defined by Equation (5.3) was selected herein coupled with a supplemented limit which excludes short columns with shear span to depth of less than 2.5. The NALI was found to result in the highest coefficient of correlation (i.e.) despite the least number of input parameters required. The identified parameter is consistent with findings of a study by Elwood and Moehle (2005) and also a more recent study by Wibowo et al. (2012). These findings (as discussed next) further support the proposed approach for the development of Model II.
Wibowo et al. collected a database comprising 46 limited-ductile RC columns and plotted the measured drift ratios (at gravity load failure) versus four different parameters namely axial load ratio, transverse reinforcement ratio, aspect ratio, and longitudinal bar ratio. Wibowo et al. concluded that the last two parameters have no clear relationship with axial drift failure as indicated by the very scattered plots obtained in the study. The first two parameters (i.e. axial load ratio and transverse reinforcement ratio) are both included in the considered NALI (i.e. \( x = \frac{P}{A_g f_c \rho''} \)).

Elwood and Moehle (2005) on the other hand, concluded that the effect of longitudinal bar ratios on ultimate drift capacity is negligible and this parameter was subsequently removed from their proposed estimator (i.e. Equation (4.2)).

Based on the observed correlations and the discussion above, the following equation is suggested for estimating the mean inter-storey drift ratio (\( \delta_a \) in %) at the limit of gravity collapse.

\[
\delta_a = 3.53x^{-0.53}
\]  

(5.4)

where \( x \) is the normalized axial load index (NALI) and is obtained using Equation (5.3).

![Figure 5.1: Observed and predicted drift ratios versus normalized axial load parameter-SSTDR=Shear span to depth ratio](image)

Figure 5.1: Observed and predicted drift ratios versus normalized axial load parameter-SSTDR=Shear span to depth ratio
Equation (5.4) was employed to obtain drift capacity estimations for the column specimens analysed. Figure 5.2 illustrates the observed versus estimated drifts at the limit of gravity collapse. The ratio of observed to estimated values were found to have a mean of 1.06 and a standard deviation of 0.4.

![Drift ratios at gravity collapse](image)

**Figure 5.2: Observed versus estimated drift ratios (estimated mean values employing Equation (5.4))**

In order to present the estimated values in a probabilistic format, which allows for deviating from the mean estimation given the required level of confidence, the procedure used by Berry and Eberhard (2003) was adopted herein. Berry and Eberhard used normalized fragility curves for a similar purpose. Such curves are constructed by plotting the ratio of the observed capacity to the capacity from a deterministic model in a cumulative frequency diagram. The procedure adopted and the assumptions made for generating the required fragility curves are as follows:

1. The ratio of observed to calculated drifts (Obs./Cal.) were sorted in an ascending order (refer to the 2nd column in Table 5.1). Sorted values ranged from a minimum of 0.61 to a maximum of 2.48.
2. Cumulative frequencies of the Obs./Cal. values were computed using Equation (5.5).

\[ C_f = \frac{(i - 0.5)}{n} \]  \hspace{1cm} (5.5)

Where

- \( C_f \) = the cumulative frequency of occurrence (Column 3, Table 5.1)
- \( i \) = the \( i^{th} \) value in the sorted list (Column 1, Table 5.1)
3. Obs./Cal. values were then plotted against their corresponding cumulative frequency as obtained above. (refer to Table 5.1 and Figure 5.3).

Table 5.1: Data processing for generating cumulative frequency diagram

<table>
<thead>
<tr>
<th>ith value in the sorted list</th>
<th>Obs./Cal.</th>
<th>Cumulative freq. distri.</th>
<th>Ln(Obs./Cal.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.61</td>
<td>0.02</td>
<td>-0.4945</td>
</tr>
<tr>
<td>2</td>
<td>0.68</td>
<td>0.06</td>
<td>-0.3886</td>
</tr>
<tr>
<td>3</td>
<td>0.68</td>
<td>0.10</td>
<td>-0.3839</td>
</tr>
<tr>
<td>4</td>
<td>0.68</td>
<td>0.15</td>
<td>-0.3820</td>
</tr>
<tr>
<td>5</td>
<td>0.73</td>
<td>0.19</td>
<td>-0.3180</td>
</tr>
<tr>
<td>6</td>
<td>0.73</td>
<td>0.23</td>
<td>-0.3091</td>
</tr>
<tr>
<td>7</td>
<td>0.76</td>
<td>0.27</td>
<td>-0.2696</td>
</tr>
<tr>
<td>8</td>
<td>0.80</td>
<td>0.31</td>
<td>-0.2230</td>
</tr>
<tr>
<td>9</td>
<td>0.81</td>
<td>0.35</td>
<td>-0.2136</td>
</tr>
<tr>
<td>10</td>
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<td>0.40</td>
<td>-0.1072</td>
</tr>
<tr>
<td>11</td>
<td>1.02</td>
<td>0.44</td>
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<td>1.06</td>
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<td>0.60</td>
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</tr>
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<td>1.08</td>
<td>0.65</td>
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</tr>
<tr>
<td>17</td>
<td>1.12</td>
<td>0.69</td>
<td>0.1100</td>
</tr>
<tr>
<td>18</td>
<td>1.18</td>
<td>0.73</td>
<td>0.1640</td>
</tr>
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<td>1.19</td>
<td>0.77</td>
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<tr>
<td>20</td>
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<td>0.2828</td>
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<td>21</td>
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<td>0.85</td>
<td>0.2898</td>
</tr>
<tr>
<td>22</td>
<td>1.47</td>
<td>0.90</td>
<td>0.3875</td>
</tr>
<tr>
<td>23</td>
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<td>0.94</td>
<td>0.4676</td>
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<tr>
<td>24</td>
<td>2.48</td>
<td>0.98</td>
<td>0.9088</td>
</tr>
</tbody>
</table>

Figure 5.3: Cumulative frequency distribution of the observed to calculated drift values and fitted lognormal curve- The ratio values (i.e. the fitted fragility curve values/ corresponding cumulative frequency values) had a mean of 1.01 and a standard deviation of 0.23.
Figure 5. 3 also shows a cumulative lognormal curve which has been fitted to the frequency data points. It was found that the ratio of values from the fitted fragility curve to corresponding cumulative frequency values had a mean of 1.01 and a standard deviation of the order of 0.23 which is reasonably accurate. The following assumptions are made for fragility curve fitting:

A The ratio of observed to calculated drift values (i.e. Obs./Cal.) are random and fit a log normal-distribution and therefore can be expressed using a lognormal cumulative distribution function (i.e. fragility curves).

B Referring to the fundamentals of statistics, if a random variable has a lognormal distribution, the natural logarithm of the variable is normally distributed. Thus, the mean and standard deviation of the natural logarithm of Cal./Obs. values were used for curve fitting purpose (see the 4th column in Table 5. 1).

C Assuming that the data set is large enough to represent the potential behaviour of the population (i.e. the gravity collapse capacity in poorly-confined columns with shear span to depth ratio of equal to or greater than 2.5) the results shown in Figure 5. 3 can be generalized and be expressed as shown in Figure 5. 4 (Berry and Eberhard 2003).

Figure 5. 4: Cumulative lognormal probability of gravity load failure
Figure 5.4 suggests that if the observed drift ratio (= accepted drift ratio in this new context) is only 0.75 of the drift capacity estimated by Equation (5.4), there is still 20% chance of failure due to uncertainty inherent in the estimated value. Equation (5.4) coupled with Figure 5.4 are recommended collectively as the required probabilistic gravity collapse capacity model (Model II).

The accuracy of the fitted lognormal curve at its tail part could have been developed more reliably had there been more experimental data points (i.e. cumulative frequency distribution data points) (see Figure 5.4). Nevertheless, it is noted that the available experimental-based values of probability of failure was as low as 6% and 2% corresponding to the Observed/Cal. values of 0.68 and 0.61 respectively. This suggests a decreasing trend and the level of probability was already below the usually adopted threshold of 5% for engineering design purposes. Although there are scopes for further justifications of the fitted curve, at probability levels below 5%, the current lognormal curve is still justifiable at levels of probability equal to and higher than 5%.

5.2 Probabilistic Drift Modelling at Nominal Shear Failure (Model III-The Drift at 20% Loss in Peak Lateral Strength)

The developed deterministic model (Model I, Chapter 4) and the probabilistic model (Model II, Section 5.1) were concerned with estimating the drift capacity of limited-ductile RC columns at the limit of gravity collapse. The output of these models can be employed to identify the relevant point on the required force-displacement backbone curve of a given column. Simulating a full backbone curve however, requires the knowledge of other points in addition to the point of gravity collapse.

Section 5.2 addresses the simulation of the point at which shear failure occurs (also known as the point of lateral failure). Shear failure in RC columns is commonly defined as the point of 20% drop in peak lateral strength (as opposed to the actual point of gravity collapse). The objectives of this section are to evaluate existing models and to recommend a suitable solution for estimating the point of lateral failure for the limited-ductile columns of interest. It is noted that developing a new lateral failure model is not warranted considering the scope of this study and the advanced
models available in the literature (Aslani and Miranda 2005; Elwood and Moehle 2005; Wibowo et al. 2012; Zhu et al. 2007). Instead the recommended existing model was further modified to fit the requirements of this study.

The following criteria were taken into account for recommending a lateral failure model.

1. A probabilistic model is required to quantify uncertainties associated with the estimated capacity. Using a probabilistic model, for instance, one should be able to obtain a scaled capacity corresponding to a given probability of failure.
2. The number of input parameters required for drift estimation should be a minimum in order to minimize unquantified uncertainties when such information is not available for the columns under assessment. Such situation is expected particularly for the old columns of interest.
3. The probabilistic solutions for estimating the drift at gravity collapse and lateral failure limit states should be ideally consistent in terms of the required input parameters and associated uncertainties.

Table 5.2 gives a summary of the main models available for estimating lateral drift capacity of limited-ductile RC columns. The third column of this table gives short comments about these models considering the context of comparison. The available models were evaluated against the same data set as used in the previous section (Table 3.3). It is noted that only columns with the shear span to depth ratio of greater than 2.5 were included in order to be consistent with the developed probabilistic axial capacity model (refer Section 5.1).

For model comparison, lateral failure drifts were estimated using Equations (5.6) to (5.15). Ratio of observed to estimated drifts and corresponding mean and standard deviation were computed for each model (refer to Table 5.3). The model by Elwood and Moehle (2005) was selected comparing the statistics of results (i.e. $\mu = 1.06$ & $\sigma = 0.33$ respectively) and simplicity of models. It should be noted that drift estimations using Equation (5.6) were made implementing a mean value of $\left( \frac{\nu}{\sqrt{f_c}} \right)_\text{mean} = 5.6$ for all specimens.
Table 5.2: A summary of the main shear failure models for limited-ductile RC columns

<table>
<thead>
<tr>
<th>Model</th>
<th>Proposed equations</th>
<th>Comments</th>
</tr>
</thead>
</table>
| Elwood & Moehle (2005) | \[
IDR_{90\%} = \frac{3}{100} + 4\rho' - \frac{1}{500} \sqrt{f_c} - \frac{1}{40} \frac{P}{A_g f'_c} \geq 0.01
\] (5.6) | Deterministic model (psi units) |
| & Based on the database of the study |
| Aslani and Miranda (2005) | \[
IDR_{90\%} = \frac{1}{0.26x + 25.4} \geq 0.01
\] (5.7) | Probabilistic model, least amount of input data required |
| | \[
x = \frac{P}{A_g f'_c - \rho'}
\] (5.8) | |
| Zhu et al. (2007) | \[
(\delta_f)_{\text{median}} = 0.049 + 0.716\rho_I + 0.120 \frac{\rho'' f_{sh}}{f_c}
\] - 0.042 \frac{s}{d} - 0.070 \frac{P}{A_g f_c} (5.9) | Probabilistic model requiring more input data compared to previous models |
| Wibowo et al. (2012) | \[
\delta_f = \frac{\delta_x}{k} [1 + k\alpha] - 0.8 V_{\text{max-failure}} \frac{V_{\text{tot-shear}}}{V_{\text{tot-shear}}}
\] (5.10) | Deterministic model, could be employed when detailed input data is available |
| | \[
V_{\text{tot}} = \frac{2}{3} A_{ct} \sqrt{f_t} + \frac{f_t P}{A_{ct}} + \frac{A_v f_{sh} d}{s}
\] (5.11) | |
| | \[
\delta_y = \frac{M_y L}{3E_i I_{\text{eff}}} \quad \text{if} \quad n \geq 0.6 \Rightarrow I_{\text{eff}} = I_g
\] \[
\delta_y = \frac{M_y L}{4E_i I_{\text{eff}}} \quad \text{if} \quad n \leq 0.2 \Rightarrow I_{\text{eff}} = 0.4 I_g
\] (5.12) | |
| | Linear interpolation \text{if} 0.2 < n < 0.6 |
| | \[
A_{ct} = 0.85(n_c \rho''_x)^{0.36} d, \quad n_c = \frac{E_i}{E_c}
\] (5.13) | |
| | \[
k = \frac{0.3e^{5.7n}}{(9 - a)}
\] (5.14) | |
| | \[
a = \frac{L}{D} = 2 - 4 \quad \text{and} \quad n = \frac{P}{A_g f'_c} = 0.1 - 0.4
\] (5.15) | |

Notes: List of symbols

**In relation to Equations (5.6) to (5.9)**

- \( IDR_{90\%} = \) inter-storey drift ratio at shear failure
- \( \rho'' = \) Transverse reinforcement ratio
\[ \rho_l = \text{longitudinal reinforcement ratio} \]
\[ n = \frac{V_{\text{max}}}{bd} = \text{average shear stress (MPa)} \]
\[ V_{\text{max}} = \text{measured or predicted maximum shear force that can be carried by a column (N)} \]
\[ f_c = \text{concrete cylinder compressive strength (MPa)} \]
\[ P = \text{imposed axial compressive load (N)} \]
\[ A_g = \text{gross cross sectional area of a column} \]
\[ \delta_f (\text{median}) = \text{median drift ratio at shear failure} \]
\[ f_{yt} = \text{yield strength of transverse reinforcement (MPa)} \]
\[ s = \text{centre to centre spacing of transverse reinforcing bars in vertical direction (mm)} \]
\[ d = \text{column depth (mm)} \]

**In relation to Equations (5.10) to (5.15)**

\[ \delta_{lf} = \text{drift at lateral load failure} \]
\[ \delta_y = \text{column yield drift} \]
\[ k = \text{strength degradation drift} \]
\[ \alpha = \text{the drift ductility when the shear strength commences to decline} \]
\[ = 1 \text{ for } a \geq 3 \]
\[ = 2 \text{ for } a < 3 \]
\[ V_{\text{tot}} = \text{total shear capacity} = V_c + V_s \]
\[ V_{\text{max}} = \text{column flexural strength} \]
\[ f_t = \text{concrete tensile strength} \]
\[ \rho_d = \text{ratio of the cross-sectional area of longitudinal reinforcement in the tension zone to the effective concrete area (} A_d / bd \text{)} \]
\[ V_s = \frac{A_t f_{sh} d}{s} = \text{the shear strength contribution of transverse reinforcement} \]
\[ E_s \text{ & } E_c = \text{modulus of elasticity of steel and concrete respectively} \]
\[ a = L/D = \text{column shear span to depth ratio} \]
\[ I_g \text{ & } I_{eff} = \text{gross and effective second moment of area respectively} \]

This value is experimentally obtained from the data set used in Elwood and Moehle study. The remaining input parameters for Equation (5.6), namely axial load ratio \(( P / A_g f_c )\) and transverse reinforcement ratio \(( \rho^* )\), are the same as those required for the proposed gravity collapse model by Equation (5.4). Therefore, the two models have approximately the same source of uncertainties as far as the input parameters are concerned.
Equation (5. 6) could be reduced to the form of Equation (5. 16) substituting the mean value of 5.6 for \( \left( \frac{v}{\sqrt{f'_c}} \right)_{\text{mean}} \).

Table 5. 3: A summary of the mean and standard deviation of the observed to estimated lateral drifts of the columns analysed.

<table>
<thead>
<tr>
<th>Shear failure models examined against the data set of this study</th>
<th>Mean of Obs. /Cal. (( \mu ))</th>
<th>Standard deviation of Obs. /Cal. (( \sigma ))</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elwood &amp; Moehle (2005)</td>
<td>1.06</td>
<td>0.33</td>
<td>Mean value of ( \left( \frac{v}{\sqrt{f'<em>c}} \right)</em>{\text{mean}} = 5.6 ) was used for all estimations. ( \rho^* = TBR )</td>
</tr>
<tr>
<td>Aslani &amp; Miranda (2005)</td>
<td>1.30</td>
<td>0.56</td>
<td>( \rho^* = TBR )</td>
</tr>
<tr>
<td>Zhu et al. (2007)</td>
<td>1.66</td>
<td>1.31</td>
<td>( \rho^* = TBR(%) ), ( \rho_t = LBR(%) ), This model underestimates the observed capacity, particularly for the tested Columns S1-S4</td>
</tr>
<tr>
<td>Wibowo et al. (2012)</td>
<td>1.29</td>
<td>0.35</td>
<td>Observed ( F_u ) values were employed for estimations</td>
</tr>
</tbody>
</table>

\[
IDR_{80\%} = 0.0188 + 4\rho^* - \frac{1}{40} \frac{P}{A_g f'_c} \geq 0.01 \tag{5.16}
\]

where

\( IDR_{80\%} \) = Estimated inter-storey drift ratio up to the point of nominal shear failure (the point at which the peak lateral strength is degraded by 20%) 

\( \rho^* \) = transverse reinforcement ratio 

\( A_g \) = gross cross sectional area of the column 

\( P \) = imposed axial load 

\( f'_c \) = concrete compressive strength

Equation (5. 16) however, is a deterministic predictor and provides median estimations corresponding to 50% probability of failure only. To obtain drift estimations corresponding to other confidence levels (say 90%), representative
distribution of failure probability should be constructed. For this purpose a similar approach as in Section 5.1 was adopted utilizing the dispersions in the obtained observed to calculated (Obs./Cal.) values and assuming that the data set is large enough to represent the potential trend of the model. Figure 5.5 shows the data points corresponding to the obtained cumulative frequency of the Obs./Cal. values. A cumulative lognormal curve is also shown which is fitted to the data points assuming that Obs./Cal. values are random variables with lognormal distribution following the work by Berry and Eberhard (2003). The reader may refer to Section 5.1 for more details regarding this approach.

\[ \text{Accepted/Cal. drift ratio} \]
\[ \text{Probability of lateral failure (20% drop of strength)} \]
\[ \text{Cumulative frequency distribution data points} \]
\[ \text{Fitted cumulative lognormal curve} \]

Figure 5.5: Cumulative lognormal probability of lateral load failure fitted on cumulative frequency data points

It is noted that the obtained curve is a normalized fragility curve. The term “normalized” refers to fragility curves that are plotted as a function of the ratio of the accepted drift to the median prediction (Zhu et al. (2007)). Interestingly the obtained normalized fragility curve matches well with the relevant curve developed by Zhu et al. (the dashed curve corresponding to ‘0.8V_{eff} ‘Zone F’ series in Figure 5.6).

For comparison purpose the relevant plots from the two studies were superimposed in Figure 5.6. The ‘Zone F’ curve in this figure corresponds to columns with a flexure or flexure-shear mode of failure while the ‘Zone S’ corresponds to columns experiencing a pure shear failure (the latter is beyond the scope of this study). It is acknowledged that the fragility curve developed by Zhu et al was based on a different
approach and uncertainties in material properties and applied load were accounted for by random variation of these parameters assuming a lognormal distribution and following coefficients of variation for the variables:

\[ COV[s] = 0.02; \quad COV[f_c] = 0.05; \quad COV[f_y] = 0.05; \quad \text{and} \quad COV[P] = 0.1 \]

This observation suggests that the lumped uncertainty associated with the dispersion of the observed to calculated (Obs. /Cal.) drifts could represent the resultant uncertainty linked with reinforcement configuration, material properties and axial load as were explicitly included by Zhu et al.

![Figure 5.6: Comparison of the normalized fragility curves as obtained in this study (the fitted cumulative lognormal curve) and by Zhu et al. (2007) (the dashed curve labelled 0.8V_{eff}, Zone F)](image)

Based on the discussion provided above, Equation (5.16) can be recommended to obtain the median drift prediction at the limit of 20% drop in peak lateral strength. Drift prediction corresponding to other probability limits may be obtained by scaling the median value in accordance to the normalized fragility curve given in Figure 5.5.

The fitted lognormal curve, shown in Figure 5.5, should be treated with caution at a probability level of below 5%, due to insufficient data points. The proposed curve, however, is justifiable at levels at least equal to or higher than 5%, considering the available experimental data points (i.e. cumulative frequency distribution data points)
5.3 Summary

This chapter presented two drift capacity models (Model II and III) to address:

A) the fact that a detailed set of input data is generally not available for old columns to be assessed against seismic risk.

B) the uncertainties involved in drift capacity estimations.

Model II is a practical probabilistic model which can be used for predicting the drift capacity of limited-ductile RC columns at the limit of gravity failure (refer to Section 5.1). The developed model requires the least number of input information and is developed based on the analyses of the test results of a comprehensive dataset of 56 limited-ductile RC columns.

It was found that the normalized axial load can provide a consistent correlation with observed drift capacity at the limit of gravity collapse. Fitting a curve to the observed correlation resulted in an expression which was shown to provide mean estimations corresponding to 50% confidence level (Equation (5.4)). The probabilistic feature of the model was developed based on the statistical analyses of the dispersion of the ratio of observed to estimated (mean values) drift capacity (see Figure 5.4). Employing the graph in Figure 5.4, it is possible to scale down the mean estimates, as can be obtained by Equation (5.4), to correspond to the required confidence level (say 95% confidence level).

Model III is a practical probabilistic model for estimating the drift capacity of limited-ductile RC columns at the limit of nominal shear failure (that is the point where a 20% reduction in peak lateral strength is reached). This model is based on the same dataset and requires the same input parameter requirements as in Model II.

In Model III, estimates of relevant mean drift capacity corresponding to 50% confidence level could be obtained by Equation (5.16) which is adopted from literature. This equation was shown to be statistically the most accurate compared to other existing solutions. The probabilistic feature of the model was developed based on the same approach as employed for the development of Model II (see the result in Figure 5.5). Employing the graph in Figure 5.5, it is possible to scale
down the mean estimates, as can be obtained by Equation (5.16), to arrive at the required confidence level (say 95% confidence level).

The applications of Model II and III will be demonstrated in Chapters 6 and 7. These models contribute, significantly, to constructing full backbone curves for the RC columns of interest. Such backbone curves are in turn required for dynamic modelling and analyses of columns.
6 Parameters for Dynamic Analysis of Limited-Ductile RC Columns

The previous chapter focused on developing two practical probabilistic models for predicting the drift capacity of limited-ductile RC columns at the limit of gravity collapse (Model II) and at the point where 20% decrease in peak lateral strength is experienced (Model III). The models developed in Chapter 5 are employed in Chapter 6 to determine the respective points on the force-displacement backbone curves to be simulated.

Chapter 6 brings together all the tools, models and input parameters required for dynamic modelling and analysis of limited-ductile RC columns (within the scope of this study). Section 6.1 demonstrates the application of the models developed in chapter 5 coupled with other standard solutions for simulating the full backbone curve of the columns of interest. Section 6.2 provides recommendations on constructing linearized backbone models compatible with the input parameter requirements of classical dynamic programs such as RUAUMOKO (Carr 2007). Section 6.3 provides recommendations in relation to unloading and reloading stiffnesses to be considered for inelastic dynamic analysis of limited-ductile RC columns. Section 6.4 presents recommendations for an alternative dynamic modelling solution within the program ‘OpenSees’ (Mazzoni et al. 2006). Applicable material models and elements available in this program (for modelling limited-ductile RC columns) are proposed. Corresponding sensitivities and limitations are discussed. Section 6.5 presents simulated ground motions (both on rock and soil) as required for the purpose of incremental dynamic analyses to be employed in the parametric study of Chapter 7.

6.1 Force-Displacement Backbone Simulation

Nonlinear dynamic analysis of a structural component requires a simplified model to be made within a dynamic analysis program (e.g. RUAUMOKO). Central to such a model is the definition of the relation between force and displacement of the model
from the early elastic stage to the ultimate inelastic stage of interest (which is gravity collapse in this study).

Lateral force-displacement relation of a column (or corresponding dynamic model) is typically characterised by a combination of: 1) elastic/secant stiffness of the column; 2) the yield strength; 3) post yield positive stiffness and the peak lateral strength; 4) post peak negative stiffness; 5) the point of ultimate deformation capacity which may be taken as the point of shear failure or gravity load failure; and 6) a set of hysteretic rules which define unloading and reloading stiffness degradation of the structural component or model. The data/parameters numbered 1 to 5 above could collectively be represented by a full backbone curve. Classical dynamic analysis programs (such as RUAUMOKO) require a direct input of the parameters.

The backbone curve of limited-ductile RC columns can be simulated using the standard nonlinear moment-curvature analysis and by considering other deformation components as recommended in Section 4.3. This could result in the appropriate simulation of the portion of the backbone curve approximately up to the point of 20% decrease in peak lateral strength (as can be seen in Figure 6. 1). This portion of the backbone curve is referred herein as the flexure-dominant curve.

Median drift values at the limit of nominal shear failure ($IDR_{80\%} =$ the point of 20% reduction in peak lateral strength) were estimated using Equation (5. 16) as presented in Section 5.2. The estimated values are shown by solid vertical lines in Figure 6. 1. The nominal point of shear failure (SF) could then be found as the intersection of the flexure-dominant curve and the relevant vertical line.

Drift values at the loss of gravity load carrying capacity were predicted using Equation (5. 4) for Columns S2 to S4. The predicted drift values are shown by vertical dashed-lines in Figure 6. 1. The nominal point of gravity failure (GF) was found assuming a 50% residual strength at this drift limit. As could be seen in these figures, the 50% residual strength assumption at the predicted drift limit compare well with the recorded force-displacement response of the tested columns.
Figure 6.1: Simulated backbone curves for: a) Column S2; b) Column S3; and c) Column S4. SF= nominal point of shear failure; GF=nominal point of gravity failure
It is noted that Equations (5. 4) and (5. 16) have not been calibrated to achieve the best results for the tested columns of S1-S4. These equations are statistically the most viable solutions for the columns analysed in the considered dataset (Table 3. 3) while minimizing the number of input information required for the models to work.

6.2 Idealized Tri-Linear Backbone Simulation

Rigorously simulated non-linear backbone curves such as those presented in the previous section cannot be incorporated directly in dynamic models. For this purpose a non-linear deformation model of a column is usually represented by an idealized piece-wise linear model. It was decided, in this study, that a tri-linear idealization of the simulated curves can provide a balance between accuracy and simplicity. Figure 6. 2 illustrates the linearized curves for the tested Columns S2-S4.

a. Ascending part of the idealized tri-linear curves

It was found that employing the effective flexural rigidity \((EI)_{eff}\), as stipulated by ASCE/SEI 41-06, in Equation (6. 2) for the tested cantilever columns can result in good estimates of the ascending part of the tri-linear curves. The effective rigidity according to this standard is a function of axial load ratio as follows:

\[
(EI)_{eff} = 0.7E_c I_g \quad \text{when} \quad P \geq 0.5Agf_c' \quad (6. 1)
\]

\[
(EI)_{eff} = 0.5E_c I_g \quad \text{when} \quad P \leq 0.3Agf_c' \quad (6. 2)
\]

where

- \(P\) = compression due to design gravity loads;
- \(I_g\) = second moment of area and \(A_g\) = gross cross sectional area of column
- \(f_c'\) = concrete compressive strength and \(E_c\) = modulus of elasticity of concrete

For columns with axial compression falling between the limits provided, linear interpolation is permitted.

\[
F = K\Delta = (3EI_{eff} / L^3)\Delta \quad (6. 3)
\]

where

- \(F\) = column lateral force
- \(\Delta\) = column lateral deformation
- \(K\) = lateral stiffness and \(L\) = column clear height
Figure 6.2. Idealized tri-linear backbone curves for Columns: (a) S2; (b) S3; & (C) S4
b. Post yield regions of the idealized tri-linear curves

It is suggested that the initial ascending section of the backbone curve be extended approximately to the point of yielding of the critical section. The second linear part, AB, may be determined by considering a horizontal line (at the level of the estimated column yield strength) limited to the vertical solid line which represents the predicted drift at the limit of nominal shear failure (obtained using Equation (5.16)). Point B is determined by the intersection of the horizontal and the vertical lines. Section AB may be shifted vertically based on equating areas under the linearized and the original backbone curves up to Point B. It was found that, however, the extra accuracy which might be obtained from such adjustments has negligible effects on the collapse probability of the columns analysed in this study (see Figure 7.18 to Figure 7.20).

The degrading section (BC) is obtained by connecting Points B to Point C which is the predicted limit at which gravity load carrying capacity is lost. Point C can be determined as described in the previous section.

6.3 Unloading and Reloading Stiffness Parameters

Classic dynamic modelling tools such as RUAUMOKO require the user to select from a series of built-in hysteresis rules or models such as the Modified TAKEDA model, Origin-Centred Bi-linear hysteresis, Degrading Bi-linear hysteresis, Sina Degrading Tri-linear hysteresis and etc. Unloading and reloading stiffness degradation can then be defined, for post yield region of the response, using parameters associated with each of the hysteresis models. These parameters are usually functions of the maximum ductility experienced by the structure and the history of displacement excursion.

Inspection of the available models in RUAUMOKO revealed that the Modified TAKEDA hysteresis model has the potential to represent the stiffness degradation of the limited-ductile reinforced concrete columns of interest in this study. Figure 6.3 illustrates the parameters involved in this model.
where

\( K_0 = \) elastic stiffness

\( K_u = \) post-yield unloading stiffness

\( \alpha = \) parameter affecting the rate of unloading stiffness degradation \((0.0 \leq \alpha \leq 0.5)\)

\( \beta = \) parameter affecting the rate of reloading stiffness degradation \((0.0 \leq \beta \leq 0.6)\)

\( d_p = \) plastic deformation

\( d_y = \) yield displacement

\( F_y = \) yield force

\( d_m = \) maximum displacement experienced

With the Modified TAKEDA hysteresis model, unloading stiffness degradation could be calculated using either ‘Emori rule’ or a rule known as ‘Drain-2D’ which are both defined in Figure 6.3. This model was employed to simulate the hysteretic force-displacement response of Columns S1-S4 to obtain representative set of \( \alpha \) and \( \beta \) values for each column analysed. The \( \alpha \) and \( \beta \) values reported in Figure 6.4 and Figure 6.5 (and also summarized in Table 5.4) represent the best matches, between
the simulated and experimental hysteretic results. The matches were obtained through systematic trial and error and utilizing Program ‘HYSTERES’.

Table 5.4: Calibrated hysteresis parameters representing unloading and reloading stiffness degradation of the tested columns

<table>
<thead>
<tr>
<th>Takeda Hyst. Parameters</th>
<th>Column S1</th>
<th>Column S2</th>
<th>Column S3</th>
<th>Column S4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha )</td>
<td>0.5</td>
<td>0.5</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>( \beta )</td>
<td>0.3</td>
<td>0</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>Unloading rule</td>
<td>Emori</td>
<td>Drain-2D</td>
<td>Drain-2D</td>
<td>Drain-2D</td>
</tr>
</tbody>
</table>

It is inferred from the results listed in Table 5.4 that \( \alpha \) and \( \beta \) values are controlled mainly by the axial load ratio than the longitudinal reinforcement ratio in lightly reinforced columns. It was seen that Columns S3 and S4 (with axial load ratio of 40\%) were both represented by a common set of values for the unloading and reloading stiffness despite the fact the longitudinal reinforcement ratio in Column S3 was double that of Column S4. The results given in Table 5.4 also suggest less severe stiffness degradation parameters for Columns S3 and S4 as compared to those of the other two columns. This is understood considering the fact that a column subjected to high axial load ratio tends to provide higher lateral stiffness and lower ductility properties. The lateral stiffness in such situation is better maintained up to a certain ductility limit followed by a sudden stiffness lost; whereas in Columns S1 and S2, with lower axial load ratio of 20\%, the rate of unloading and reloading degradation is higher. Nevertheless, the ductility properties of these columns (particularly Column S1) allowed for gradual loss of stiffness approaching zero at gravity load failure.
Figure 6.4: Calibration of unloading and reloading stiffness parameters (\( \alpha \) and \( \beta \) respectively) using program HYSTERES for: a) Column S1; b) Column S2
Figure 6. 5: Calibration of unloading and reloading stiffness parameters ($\alpha$ and $\beta$ respectively) using program HYSTERES for: a) Column S3; b) Column S4
6.4 Alternative Dynamic Modelling of Limited Ductile RC Columns Integrated in FE Program ‘OpenSees’

In Sections 6.1 to 6.3 specific recommendations were made for force-displacement backbone simulation of limited-ductile RC columns. Unloading & reloading stiffness degradation parameters were also calibrated with reference to the experimental results (Column S1-S4) to provide a typical range for the limited ductile columns of interest. Simulated response history could be used to obtain strength, stiffness and ductility parameters for a column. These parameters were required as input data for inelastic dynamic modelling of the RC column within a classical dynamic analysis program such as ‘RUAUMOKO’ (Carr 2007).

Force-displacement response modelling may be alternatively integrated (at least in part) within a finite element program using specialized elements and tools that might be available for particular structural response behaviour. In such a case the input information are usually at material level (i.e. stress-strain relation of steel and concrete). Additionally, geometry and reinforcing details of the structural element are usually required.

In this study several elements available in FE program ‘OpenSees’ (McKenna et al. 2000) were studied as part of an attempt for an integrated force-displacement simulation of limited-ductile RC columns. It was found that a nonlinear beam column element, beamWithHinges, has promising potential for force-displacement response simulation of the tested columns (i.e. Columns S1-S4) up to certain limits of inelastic response.

The beamWithHinges element is composed of three parts, one hinge at each end, and a linear elastic part in the middle. Plasticity is distributed within a user-specified hinge length with the aid of two integration points implemented at each of the hinge regions. A previously defined fibre section is also assigned to each hinge. This is required for the moment-curvature analysis which is implemented into the element using a finite element formulation. The element can therefore take into account axial and flexural stresses and their interaction (both in elastic and inelastic regions) when simulating the force-displacement response history of RC columns.
The required plastic hinge length in the aforementioned element may be obtained using available empirical equations. Parametric studies (not presented here) showed that simulated force-displacement response is not significantly sensitive to the input hinge length for the lightly reinforced columns of interest. It was found that employing the empirical hinge length, Equation (6. 3), as suggested by Priestley et al (2007) is adequate for this input, though this equation was not specifically developed for lightly reinforced columns. Alternatively the recommendations made in Section 4.3.1.2 may be adopted for this input.

\[ L_p = K L_c + L_{sp} \geq 2 L_{sp} \quad (6. 3) \]

\[ K = 0.2 \left( \frac{f_u}{f_y} - 1 \right) \leq 0.08 \quad (6. 4) \]

\[ L_{sp} = 0.022 f_y d_{bl} \quad (6. 5) \]

where

- \( L_p \) and \( L_{sp} \) = plastic hinge and strain penetration length respectively (both in mm).
- \( K \) = a factor which is a function of ultimate to yield strength \( (f_u/f_y) \) of longitudinal steel bar (in MPa)
- \( d_{bl} \) = diameter of the longitudinal bar
- \( L_c \) = length from the critical section to the point of contra flexure of the column (in mm)

The \textit{beamWithHinges} element has some limitations and modelling of RC columns with this element requires some manual interventions if seismic collapse assessment is to be undertaken. This element is not capable of capturing any abrupt changes in stiffness due to column shear failure. Similarly the ultimate drift at the loss of gravity load carrying capacity is not captured by this element. This element is generally more reliable when column lateral strength and stiffness degradations are mainly resulted from softening of material due to excessive axial stresses. In other words it is expected to be most accurate for relatively slender columns; particularly those
subjected to high axial load ratio. Based on the analysis of this study, application of this element is further limited to columns with longitudinal reinforcement ratio of the order of equal to or greater than 1, unless axial load ratio is high as in Column S4.

Figure 6.6 to Figure 6.8 show simulated cyclic force-displacement responses for Columns S2-S4 using the aforementioned element. Simulated responses are benchmark against their corresponding experimental records.

Figure 6.6: Column S2; axial load ratio=20%

Figure 6.7: Column S3; axial load ratio=40%
6.4.1 Concrete and Steel Material Models Used for Modelling of Lightly Reinforced Concrete Columns in OpenSees.

Representative concrete stress-strain relation/parameters should be implemented in OpenSees using or manipulating one of the built-in concrete models available in the program (e.g. Concrete 01, 02, 03).

Concrete-01 model is based on the uniaxial Kent-Scott-Park concrete material model (Scott et al. 1982) with degraded linear unloading/reloading stiffness according to the work of Karsan-Jirsa (1969). With this model no tensile strength is considered for concrete.

Concrete-02 features similar backbone curve as in concrete-01 with different unloading/reloading stiffness/rules. The concrete-02 model also allows for inclusion of tensile strength in concrete with linear softening branch for the stress-strain curve of concrete in tension. Figure 6. 10 shows typical hysteretic stress-strain relation of concrete-02 model (dotted lines) and compares with that of concrete-01 model (solid line). It can be seen that the bi-linear unloading rule implemented in concrete-02
introduces some level of energy dissipation corresponding to each unloading and reloading course.

Trial push over and cyclic push over analysis, for Columns S1-S4, with different material models, partly presented above, revealed that Concrete-02 is a reasonable option which provides a balance between simplicity and accuracy. The input parameters required for concrete-02 model are summarized in Table 6. 1 and illustrated in Figure 6. 10. The last column in Table 6. 1 provides some recommendations for each of the parameters based on the analyses of this study including the brief uncertainty analysis presented in Section 6.4.2. Application of these recommendations resulted in reasonably successful response simulations for Column S1-S4.

Figure 6. 9: Typical hysteretic stress-strain relation of Concrete_02 model (dotted lines) and its comparison with that of concrete-01 Model (solid lines)(Mazzoni et al. 2006)
### Table 6.1: Recommended parameters/values for Concrete-02

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
<th>Comments/Suggested values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_{pc}$ &amp; $\varepsilon_{pc0}$</td>
<td>Confined concrete peak compressive stress &amp; corresponding strain respectively</td>
<td>These parameters were obtained from stress-strain curve, at high strain rate, modelled in accordance with Mander et al. (1988) (refer Section 6.4.2).</td>
</tr>
<tr>
<td>$f_{pcU}$ &amp; $\varepsilon_{pcU}$</td>
<td>Confined concrete crushing stress &amp; corresponding strain respectively</td>
<td>The point corresponding to 50% reduction in peak compressive strength ($f_{pc}$) was read off the modelled stress-strain curve</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Ratio between unloading slope at $\varepsilon_{pcU}$ and initial slope</td>
<td>Recommended value = 0.1</td>
</tr>
<tr>
<td>$f_t$</td>
<td>Concrete tensile strength</td>
<td>Can be obtained by $f_t = 0.4 \sqrt{f_c}$ where $f_c$ = characteristics cylinder compressive strength of concrete (MPa)</td>
</tr>
<tr>
<td>$\varepsilon_{ts}$</td>
<td>Tension softening stiffness (absolute value)</td>
<td>$\varepsilon_{ts} = \frac{f_t}{0.002}$</td>
</tr>
</tbody>
</table>

**Figure 6.10: Concrete-02 material built in OpenSees (Mazzoni et al. 2006)**

There are a number of steel models available in the finite element program OpenSees (e.g. Steel01 and steel02). The common input parameters for definition of these models are: 1) steel yield strength; 2) initial elastic tangent stiffness; 3) strain hardening ratio (the ratio of the post-yield tangent to initial elastic tangent. These
parameters may be obtained from experimental results as in the case of Column S1-S4 or be set to typical nominal values.

Steel02 material allows for additional parameters controlling the transition from elastic to plastic branches of stress-strain curve. In this study Steel02 model was employed for two reasons: 1- the smooth transition from elastic to plastic branch is more realistic than sharp transition implemented in Steel-01 stress-strain curve; 2) the stress-strain model worked well in the FE model built for simulating the force-displacement response of the tested columns (Columns S1 to S4). Input values for elastic-inelastic transition parameters were set to default values as recommended in Mazzoni et al.(2006).

6.4.2 Uncertainty About Confined Concrete Peak Compressive Strength and Corresponding Strain Estimated at High Strain Rate

Confined concrete stress-strain relation (curve) is one of the sensitive inputs required to model RC columns in OpenSees using the element discussed in the preceding section. The stress-strain curve of interest should correspond to high rate of straining to represent the expected rate of straining imposed on typical structural members in seismic events (Mander et al. 1988).

Given a lack of agreement on confined concrete compressive strength ($f_{cc}$) and corresponding strain ($\varepsilon_{cc}$) estimated by different models at high strain rate (refer Section 2.3.2.1 and Figure 2.13), it is appropriate to address these uncertainties by sensitive analyses/discussions as briefly presented next.

A. Uncertainty in $f_{cc}$

Figure 6.11 shows force-displacement response of Column S1 simulated by cyclic push over analysis in OpenSees (up to 2% drift ratio). The tested column (S1) had a vertical reinforcement ratio and axial load ratio of the order of 0.56% and 20% respectively. Transverse reinforcement ratio in this column was very low (0.07%).

The “grey” lines corresponds to Case 1 with $f_{cc}$ value obtained in accordance with the model by Mander et al. (1988) at high strain rate. For the cyclic response shown in
black, the $f'_{cc}$ value was increased by 20% reflecting an assumed maximum uncertainty in this parameter. All other parameters were kept constant in the two cases. The following conclusions could be drawn from the results of these analyses:

1. In lightly reinforced concrete columns, peak lateral strength is rather insensitive to moderate changes in the estimated $f'_{cc}$ value. In such columns the peak flexural strength is generally controlled by the longitudinal reinforcement capacity as opposed to concrete compressive strength unless the axial load ratio imposed on the column is high (say 40%).

2. Increasing the concrete compressive strength could result in a reduction in the amount of hysteretic energy absorption. This is understood as stronger concrete (Case 2) features lower ductility in comparison with Case 1.

3. Rate of strength degradation is slightly higher in Case 1 than in Case 2. In other words concrete with higher compressive strength undergoes less degradation at a given inelastic drift level.

![Figure 6.11: effect of confined concrete compressive strength on simulated force-displacement response; responses were obtained from cyclic push over analyses in OpenSees.](image)

From the above discussion it may be concluded that the difference in predicted confined concrete compressive strength as given by the Scott et al. (1982) and
Mander et al. (1988) model (which is in fact about 5% in the case shown in Figure 2. 13) has negligible influences when the flexural behaviour of the column is controlled by the tensile strength of steel. However, it is suggested to adopt the lower of the two confined concrete compressive strengths especially when the column is subjected to higher level of axial load. In such columns the flexural strength is highly affected by concrete strength and any un-realistic increase in concrete compressive strength may result in an un-conservative high flexural strength.

B. Uncertainty in $\varepsilon_{c,0}$

Referring to Figure 2. 13 and to the stress-strain curves at high strain rate (=0.017 strain/sec), it is evident that there are discrepancies between different models in the value of the confined concrete strain at peak stress ($\varepsilon_{c,0}$). The $\varepsilon_{c,0}$ value is 20% higher than corresponding unconfined value ($\varepsilon_0$) in accordance with the model by Scott et al. (1988), while it is 10% lower than $\varepsilon_0$ according to the model presented in Mander et al. (1988).

Mander et al. reported that concrete showed a significant increase in both the strength and stiffness when loaded at an increased strain rate. It was also reported in the same study that results of experiments by various investigators showed no consensus on the value of the strain at peak stress for high rates of strain. This suggests some level of uncertainty which warrants further investigations to be conducted in the future.

Application of the Mander model (at high strain rate), however, resulted in acceptable response simulations in this study as shown in Figure 6. 6 to Figure 6. 8. This model (coupled with recommendations made previously in relation to conservative $f_{cc}$ value) is therefore adopted in this study.
6.5 Ground Motion for Dynamic Analysis

Employing a representative set of ground motions (either generated or recorded) is known to be imperative in nonlinear response history analysis and realistic seismic collapse assessment of structures. Hines et al. (2011) called attention to the sensitivity of non-ductile structures (when compared to that of ductile structures) to the ground motions employed for seismic assessment particularly at the collapse limit state.

Section 6.5.1 presents incremental ground motions on rock which were generated based on the seismic conditions of Australia. Simulated ground motions on soil Class C and D were presented in Section 6.5.2. These motions were employed in the incremental dynamic analyses presented in this chapter.

6.5.1 Incremental Ground Motions on Rock

Accelerograms on rock were generated by stochastic simulations of the seismological models using Program GENQKE (Lam 1999). The generate accelerograms are considered to represent the seismic conditions of Australia (Lam et al. 2000a; Lam et al. 2005). Tables 6.2 and 6.3 provide a summary of basic parameters of the motions. These ground motions were obtained based on scenarios consisting of a constant magnitude (i.e. M=6 or M=7) and variable epicentres ranging from about 10 to 75 KM. Corresponding to each magnitude-distance scenario, 12 accelerograms were simulated in order to include some randomness inherent in ground motions. Figures 6.13 and 6.14 provide examples of the generated accelerograms; the former corresponds to (M=7 & R=45KM) and the latter to (M=6 & R=20 KM). Figure 6.12 plots average effective peak ground velocity (EPGV) versus site-source distance values corresponding M=6 and 7. As can be seen in this figure, for magnitude 7 ground motions, the EPGV values on rock range from 49 to 367 mm/s. For magnitude 6 scenarios, these figures range from 19 to 231 mm/s.
Table 6.2: Basic/elastic parameters of the simulated ground motions on rock (magnitude 6)

<table>
<thead>
<tr>
<th>Intensity level des.</th>
<th>M</th>
<th>R (KM)</th>
<th>No of acc. simulated</th>
<th>PGA m/s/s</th>
<th>EPGV mm/s</th>
<th>RSAmax m/s/s</th>
<th>RSVmax m/s</th>
<th>RSDmax mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6</td>
<td>70</td>
<td>12</td>
<td>0.30</td>
<td>19</td>
<td>0.95</td>
<td>0.03</td>
<td>3.8</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
<td>60</td>
<td>12</td>
<td>0.39</td>
<td>24</td>
<td>1.25</td>
<td>0.04</td>
<td>4.6</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
<td>50</td>
<td>12</td>
<td>0.54</td>
<td>32</td>
<td>1.71</td>
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<td>5.8</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
<td>40</td>
<td>12</td>
<td>0.78</td>
<td>44</td>
<td>2.45</td>
<td>0.08</td>
<td>7.7</td>
</tr>
<tr>
<td>5</td>
<td>6</td>
<td>30</td>
<td>12</td>
<td>1.19</td>
<td>64</td>
<td>3.74</td>
<td>0.12</td>
<td>10.9</td>
</tr>
<tr>
<td>6</td>
<td>6</td>
<td>25</td>
<td>12</td>
<td>1.52</td>
<td>81</td>
<td>4.79</td>
<td>0.15</td>
<td>13.3</td>
</tr>
<tr>
<td>7</td>
<td>6</td>
<td>20</td>
<td>12</td>
<td>2.04</td>
<td>106</td>
<td>6.41</td>
<td>0.19</td>
<td>17.2</td>
</tr>
<tr>
<td>8</td>
<td>6</td>
<td>15</td>
<td>12</td>
<td>2.92</td>
<td>148</td>
<td>9.15</td>
<td>0.27</td>
<td>23.1</td>
</tr>
<tr>
<td>9</td>
<td>6</td>
<td>10</td>
<td>12</td>
<td>4.75</td>
<td>231</td>
<td>14.81</td>
<td>0.42</td>
<td>36.0</td>
</tr>
</tbody>
</table>

Notes:
M=ground motion magnitude
R=site-source distance (KM)
PGA=peak ground acceleration (average of 12 accelerograms)
EPGV=effective peak ground velocity (average of 12 accelerograms) EPGV= RSVmax /1.8 (Lam et al. 2000c; Wilson and Lam 2003)
RSAmax= maximum response spectral acceleration (elastic 5% damped-average of 12 accelerograms)
RSVmax= maximum response spectral velocity (elastic 5% damped-average of 12 accelerograms)
RSDmax= maximum response spectral displacement (elastic 5% damped-average of 12 accelerograms)

Table 6.3: Basic/elastic parameters of the simulated ground motions on rock (magnitude 7)

<table>
<thead>
<tr>
<th>Intensity level des.</th>
<th>M</th>
<th>R (KM)</th>
<th>No of acc. simulated</th>
<th>PGA m/s/s</th>
<th>EPGV mm/s</th>
<th>RSAmax m/s/s</th>
<th>RSVmax m/s</th>
<th>RSDmax mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7</td>
<td>75</td>
<td>12</td>
<td>0.61</td>
<td>49</td>
<td>1.91</td>
<td>0.09</td>
<td>14.6</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
<td>60</td>
<td>12</td>
<td>0.89</td>
<td>68</td>
<td>2.78</td>
<td>0.12</td>
<td>19.4</td>
</tr>
<tr>
<td>3</td>
<td>7</td>
<td>50</td>
<td>12</td>
<td>1.19</td>
<td>87</td>
<td>3.71</td>
<td>0.16</td>
<td>23.3</td>
</tr>
<tr>
<td>4</td>
<td>7</td>
<td>45</td>
<td>12</td>
<td>1.40</td>
<td>100</td>
<td>4.35</td>
<td>0.18</td>
<td>27.2</td>
</tr>
<tr>
<td>5</td>
<td>7</td>
<td>40</td>
<td>12</td>
<td>1.67</td>
<td>116</td>
<td>5.18</td>
<td>0.21</td>
<td>31.0</td>
</tr>
<tr>
<td>6</td>
<td>7</td>
<td>30</td>
<td>12</td>
<td>2.50</td>
<td>166</td>
<td>7.71</td>
<td>0.30</td>
<td>42.3</td>
</tr>
<tr>
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<td>7</td>
<td>20</td>
<td>12</td>
<td>4.22</td>
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<td>12.98</td>
<td>0.48</td>
<td>67.1</td>
</tr>
<tr>
<td>8</td>
<td>7</td>
<td>15</td>
<td>12</td>
<td>5.98</td>
<td>367</td>
<td>18.36</td>
<td>0.66</td>
<td>90.9</td>
</tr>
</tbody>
</table>

Note: As given for Table 6.2.

Figure 6.12: Effective peak ground velocity of simulated motions (on rock) versus site-source distances (R)
Figure 6.13: Example time history acceleration on rock generated using GENQKE (PGV=70mm/s, M=7, R=45KM)

Figure 6.14: Example time history acceleration on rock generated using GENQKE (PGV=68mm/s, M=6, R=20KM)
6.5.2 Incremental Ground Motions on Soil

Ground motions could be significantly amplified at a period close to site natural period particularly if the frequency content of the motion is rich at the vicinity of the site natural period. To simulate representative ground motions on soil sites, the generated ground motions on rock (presented in Section 6.5.1) were amplified using Program SHAKE (Idriss and Sun 1992).

For ground motions on soil Class C, each of the 12 accelerograms on rock (corresponding to each magnitude-distance combination) was amplified (in a random fashion) by one of the two soil profiles shown in Figure 6. 15 to Figure 6. 18. The soil profiles utilized were classified as class C in accordance to AS1170.4 and had natural periods in the order of 0.44 sec and 0.59 sec respectively.

A similar approach as outlined above was adopted for simulating representative ground motions on soil class D. For this purpose an onerous soil deposit with initial natural period of 0.89 was employed. Corresponding borehole log is shown in Figure 6. 19.

![Borehole log Mel07](image)

Figure 6. 15: Borehole log Mel07, from Western City Link Project, Melbourne. This site is classified as class C according to AS1170.4 (2007)
Figure 6.16: Shear wave velocity profile of Borehole Mel07- Total depth 31.5 m & initial or low strain site period = 0.44 sec.

Figure 6.17: Borehole log Mel 091, from Western City Link Project, Melbourne. This site is classified as class C according to AS1170.4 (2007)
Figure 6. 18: Shear wave velocity profile of borehole Mel091- Total depth 31.5 m & initial or low
strain site period = 0.59 sec.

Figure 6. 19: Borehole log Me13, from Western City Link Project, Melbourne. This site is
classified as class D according to AS1170.4 (2007), initial or low strain site period = 0.89 sec.
Figure 6. 20 illustrates the variations of peak displacement demand values corresponding to magnitude 7 earthquakes on soil Class C and D (i.e. RSDmax) with a range of epicentral distances (R) considered.

Figure 6. 20: Peak displacement demand of M7 earthquakes (on soil Class C and D) at various epicentral distances (R)

6.6 Summary

It has been demonstrated that the full backbone curve up to the limit of gravity collapse for limited-ductile RC columns can be simulated by employing: (1) the developed drift capacity models (Models II and III) presented in Chapter 4; and (2) the specialized/standard solutions which were mainly presented in Section 4.3. Model II and III are particularly recommended when only a minimum number of input information is available for the old columns to be assessed.

The simulated backbone curves cannot be directly incorporated into dynamic programs such as RUAUMOKO. Section 6.2 proposed recommendations on constructing tri-linear backbone models for the limited-ductile RC columns of interest considering the need for consistency in translating the simulated backbone curves into relevant input parameters required for dynamic modelling (e.g. stiffness, strength and ductility parameters). The proposed Tri-linear model is believed to provide a balance between simplicity and accuracy.
Inelastic dynamic modelling of columns requires the knowledge of unloading and reloading stiffness values and their degradation trends with increasing deformation or ductility. Section 6.3 investigated these parameters and showed that the Modified TAKEDA hysteresis rule can represent the stiffness degradation history of the limited-ductile RC columns of interest. Unloading and reloading stiffness parameters were calibrated against the recorded results of the tested columns (Columns S1-S4) and were recommended to be employed in dynamic modelling of similar columns (see Table 5.4). This recommendation is adopted in the parametric study presented in Chapter 6.

An alternative approach and related recommendations for inelastic dynamic modelling of limited-ductile RC columns using the FE program ‘OpenSees’ have been proposed (see Figures 5.10 to 5.12). The recommendations address the choice of element, materials and corresponding input parameters. This section also discusses some uncertainties and limitations of the proposed modelling approach.

The ground motions on rock were generated using Program ‘GENQKE’ for the low to moderate intra-plate seismicity region of Australia. The generated motions correspond to a number of scenarios with magnitude 6 or 7 and various epicentral distances ranging from 10 to 75 km (see Table 6.2 and Table 6.3). Incremental accelerograms on soil were simulated using Program ‘SHAKE’ and employing three boreholes which represent typical soil deposits corresponding to Class C and D soil sites (based on the classification stipulated in AS1170.4).
Chapter 6 presented the results of a parametric study on the probabilistic seismic collapse assessment of limited-ductile RC columns in the low to moderate seismicity region of Australia. The most critical scenario of interest is the case in which the vertical spacing of transverse reinforcements is as large as the column depth.

The recommendations made and the capacity models developed in previous chapters (Chapter 4 and 5) are employed in this chapter for force-deformation modelling of the limited-ductile RC columns to be analysed within the scope of this parametric study. The recommended/specialized input data (e.g. incremental ground motions and stiffness parameters) as discussed in Chapter 5 were also employed for dynamic modelling and analyses of the columns of interest. The objectives of the present parametric study are as follows:

1) To utilize the models, tools and the recommended input data for analytical modelling and incremental dynamic analyses of a practical range of limited-ductile RC columns.
2) To provide fragility curves for probabilistic seismic collapse assessments of the analysed columns.
3) To study the effects of key parameters namely: 1) column size; 2) column aspect ratio 3) soil condition; 4) axial load ratio; and 5) longitudinal and transverse reinforcement ratios on the probability of collapse of the columns included in the parametric study.
4) To explore the possibility of the development of a simplified collapse assessment solution at a given probability of failure (e.g. 5%). Such a solution should eliminate the need for repeating the rigorous assessment procedure as presented in this thesis when similar columns are to be analysed.
7.1 General Modelling Parameters and Seismic Assessment Considerations

Selecting a representative scale or measure of intensity is vital for characterizing the intensity of ground motions employed in incremental dynamic analyses. For this purpose, elastic response parameters such as maximum response spectral acceleration, velocity or displacement of an elastic 5% damped single-degree-of-freedom system are commonly considered \( R_{SA_{\text{max}}} \), \( R_{SV_{\text{max}}} \), \( R_{SD_{\text{max}}} \) respectively. In an ideal case the response of a SDOF system, for instance, should show a steady increase with increasing intensity measure. An ideal or generic intensity measure however, does not generally exist as there is always a chance for a structure to have a response which does not correspond to the considered intensity measure. The reasons for this phenomenon include: (a) the random nature of earthquakes and their frequency contents at different period ranges; (b) the diversity of soil and structural natural periods and the period shift that is experienced due to inelastic response. Nevertheless, an approximate trend may be identified between the structural response and an appropriate measure of ground motion intensity if a large number of relevant ground motions are employed.

Results of the analyses conducted in this study and those reported in previous studies by Lumantarna et al. (2010) and Kafle (2011) suggested that \( R_{SD_{\text{max}}} \) is generally a representative measure of ground motion intensity (for the conditions of seismicity in Australia) particularly for the structures of medium to large natural periods on soil class C and D. Figure 7.1 illustrates the results of the dynamic analyses conducted in this study on Columns S2, S3 and S4 (refer Sections 3.1 and 3.2 for the details of the tested columns). Each column was subjected to ground motions of increasing intensity on soil Class C. The ground motions were based on a constant magnitude 7 and decreasing epicentral distances from 75 to 15 (mm) simulated to represent the conditions of seismicity in Australia (refer Section 6.5.2). Figure 7.1 shows that:

1) In general, the peak inelastic displacement demand (\( PIDD \)) values increase with the \( R_{SD_{\text{max}}} \) of the input ground motions.

2) The calculated \( PIDD \) values (related to Columns S2, S3, and S4) are generally limited to the corresponding \( R_{SD_{\text{max}}} \) value for any given input ground motions. This trend was rather perfect when magnitude 6 ground motions were employed.
A similar trend and scatter were also observed from the analyses of other columns (not shown here) regardless of the soil class (i.e. Class C or D) and magnitude of the ground motions.

![Graph showing RSDmax vs Peak Inelastic Dis. Demand for tested columns (M7, soil class C)](image)

**Figure 7.1: Incremental dynamic analyses results for tested columns (M7, soil class C)**

**Hysteresis Rule**

The modified TAKEDA hysteresis rule coupled with the calibrated unloading and reloading stiffness degradation parameters ($\alpha$ & $\beta$ respectively) were used to represent the stiffness degradation of the analysed columns (Column S2, S3 and S4). The calibrated $\alpha$ & $\beta$ values can be found in Table 5.4.

The TAKEDA hysteresis rule and the following values for the $\alpha$ & $\beta$ stiffness parameters are recommended conservatively for the dynamic modelling of limited-ductile RC columns included in this parametric study.

\[
\begin{align*}
(\alpha = 0.5) & \quad \text{for columns with } ALR = 20\% , LRR = 1\% \text{ and } TRR\% = 0.1 \\
(\alpha = 0.1, \beta = 0.5) & \quad \text{for columns with } ALR = 40\% , LRR = 1\% \text{ and } TRR\% = 0.1
\end{align*}
\]

where

$ALR$ = axial load ratio

$LRR$ = longitudinal reinforcement ratio

$TRR$ = transverse reinforcement ratio
This recommendation is based on the following facts:

1) The above $\alpha$ & $\beta$ values are obtained from calibrations against the experimental records of the tested columns (Column S1 to S4) (refer Section 6.3 for discussions and details). The tested columns are closely representative of the type and the range of the columns included in this parametric study.

2) The first set of the recommended values (i.e. $\alpha = 0.5$ & $\beta = 0$) represents the most damaging/critical scenario by minimizing the energy absorption capacity of the column. That is because the rate of unloading/reloading stiffness degradation increases with increasing $\alpha$ and decreasing $\beta$ values. The recommended values for stiffness parameters could also be assumed (conservatively) for the columns which feature reinforcement ratios higher than the minimal values implemented in Columns S1 to S4 (e.g. M40-4, M40-5, S20-2 and S20-3 as given in Table 7.1).

**Damping**

A 5% critical viscous damping was assumed as the damping value within the elastic stage of the response. Tangent-stiffness proportional damping was incorporated for the elastic portion of the damping within inelastic stage of the response. This followed the recommendations made by Priestley et al (2007) as tangent stiffness is updated corresponding to the current state of the structural deformation. This is to avoid an unrealistically high damping coefficient and therefore high damping forces based on the constant initial stiffness of the structure. An incorrectly high damping coefficient may result in an un-conservatively low displacement demand prediction (Priestley et al. 2007).
7.2 Column Size and Axial Load Effects on the Probability of Collapse

For the objectives outlined earlier in this chapter, six RC columns were initially considered and analysed. It was aimed for these columns to represent a practical range of the limited-ductile RC columns of interest.

The first three columns (i.e. Columns S20, M20 and L20 referred as Group I) had a constant axial load ratio of 20%, an aspect ratio of 7, and very low longitudinal and transverse reinforcement ratios in the order of 1% and 0.1% respectively. The only parameter which was varied amongst these columns was their sizes. The dimensions of the smallest of the three columns (i.e. Column S20) were 230mm×320mm×2250mm (width×depth×height). The height of the medium size (M20) and the large size (L20) columns were increased to 3000mm and 4000mm respectively. Corresponding depth values for the latter two columns were calculated considering the required aspect ratio (=7). The width of M20 and L20 were so calculated that the cross sectional area of each column is increased by a factor of 2 compared to that of a one-size-smaller column. The columns included in Group (I) are schematically shown in Figure 7.2. As can be seen in this figure, fixed-ends condition is assumed for these columns with only one dynamic degree of freedom in horizontal direction. These columns represent a critical case corresponding to the weak-column, strong-beam construction practice observed in most existing soft-storey buildings in Australia.

The next three columns (i.e. Columns S40, M40 and L40 referred as Group II) had the same aspect ratio, reinforcement ratios and sizes as those of the columns in Group I. The only altered parameter for this group was the axial load ratio. For these columns a constant axial load ratio of 40% was considered: (a) to study the seismic performance of the typical columns of interest when axial load ratio is increased from 20% to 40%; and (b) to study the column size effect (on the probability of collapse) at different axial load ratios (i.e. 20% and 40%). The columns included in Group II are schematically compared in Figure 7.3. Table 7.1 summarizes the details of the 6 columns introduced above. This table also provides the information corresponding to another 6 columns which will be addressed in the next section of parametric study.
Table 7. 1: Details of the columns included in parametric seismic collapse assessment of columns

<table>
<thead>
<tr>
<th>Group</th>
<th>Column designation</th>
<th>b (mm)</th>
<th>d (mm)</th>
<th>h (mm)</th>
<th>AR</th>
<th>P (KN)</th>
<th>ALR</th>
<th>LRR</th>
<th>TRR</th>
<th>s (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>S20</td>
<td>230</td>
<td>320</td>
<td>2250</td>
<td>7</td>
<td>294</td>
<td>0.2</td>
<td>1.0%</td>
<td>0.1%</td>
<td>230</td>
</tr>
<tr>
<td></td>
<td>M20</td>
<td>345</td>
<td>425</td>
<td>3000</td>
<td>7</td>
<td>587</td>
<td>0.2</td>
<td>1.0%</td>
<td>0.1%</td>
<td>345</td>
</tr>
<tr>
<td></td>
<td>L20</td>
<td>520</td>
<td>565</td>
<td>4000</td>
<td>7</td>
<td>1176</td>
<td>0.2</td>
<td>1.0%</td>
<td>0.1%</td>
<td>520</td>
</tr>
<tr>
<td>II</td>
<td>S40</td>
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<td>320</td>
<td>2250</td>
<td>7</td>
<td>589</td>
<td>0.4</td>
<td>1.0%</td>
<td>0.1%</td>
<td>230</td>
</tr>
<tr>
<td></td>
<td>M40</td>
<td>345</td>
<td>425</td>
<td>3000</td>
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<td>1173</td>
<td>0.4</td>
<td>1.0%</td>
<td>0.1%</td>
<td>345</td>
</tr>
<tr>
<td></td>
<td>L40</td>
<td>520</td>
<td>565</td>
<td>4000</td>
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<td>2351</td>
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<td>1.0%</td>
<td>0.1%</td>
<td>345</td>
</tr>
<tr>
<td></td>
<td>M20-5</td>
<td>345</td>
<td>425</td>
<td>3400</td>
<td>8</td>
<td>340</td>
<td>0.2</td>
<td>1.0%</td>
<td>0.1%</td>
<td>345</td>
</tr>
<tr>
<td>IV</td>
<td>M40-4</td>
<td>345</td>
<td>425</td>
<td>3000</td>
<td>7</td>
<td>596</td>
<td>0.4</td>
<td>1.0%</td>
<td>0.15%</td>
<td>345</td>
</tr>
<tr>
<td></td>
<td>M40-5</td>
<td>345</td>
<td>425</td>
<td>3000</td>
<td>7</td>
<td>780</td>
<td>0.4</td>
<td>1.0%</td>
<td>0.20%</td>
<td>345</td>
</tr>
<tr>
<td>V</td>
<td>S20-2</td>
<td>230</td>
<td>320</td>
<td>2250</td>
<td>7</td>
<td>294</td>
<td>0.2</td>
<td>1.5%</td>
<td>0.1%</td>
<td>230</td>
</tr>
<tr>
<td></td>
<td>S20-3</td>
<td>230</td>
<td>320</td>
<td>2250</td>
<td>7</td>
<td>587</td>
<td>0.2</td>
<td>2.0%</td>
<td>0.1%</td>
<td>230</td>
</tr>
</tbody>
</table>

Notes:
h, d and h are breadth, depth and the clear height of column respectively.

\[ P \] = axial load

\[ AR = \text{aspect ratio (} AR = \frac{h}{d}) \]

\[ ALR = \text{axial load ratio (} ALR = \frac{P}{f_c b d}) \]

\[ LRR = \text{longitudinal reinforcement ratio} \]

\[ TRR = \text{transverse reinforcement ratio (} TRR = \frac{A_w}{b s}) \]

\[ s = \text{vertical spacing of transverse reinforcement} \]

\[ A_w = \text{total cross sectional area of the transverse bars confining the concrete along the column depth} \]

\[ f_{yw} = \text{the yield strength of transverse reinforcement (} f_{yw} = 300 MPa \text{ for all columns}) \]

\[ f_{yl} = \text{the yield strength of longitudinal reinforcement (} f_{yl} = 400 MPa \text{ for all columns}) \]

\[ f'_c = \text{concrete compressive strength (} f'_c = 20 MPa \text{ for all columns}) \]

\[ c = \text{cover concrete to transverse reinforcement (} c = 25 mm \text{ for all columns}) \]

Figure 7.4 and Figure 7.5 illustrate the tri-linear force-deformation backbone models predicted for the columns included in Groups I and II respectively. The backbone models were obtained based on the recommendations made in Section 6.2. Estimated deformation capacity at the limit of gravity collapse (\( \Delta_{GF} \)) were based on Equation (7.2) which was in turn based on the drift capacity model developed in Section 5 (i.e. Equation (5.4)). Equation (7.2) was obtained by combining Equation (7.1) and (5.4).

\[ \Delta_{GF} = h \delta_a / 100 \quad (7.1) \]

Where

\[ \Delta_{GF} = \text{Column deformation capacity at the limit of gravity failure (} mm) \]

\[ \delta_a = \text{Column drift capacity at the limit of gravity failure (} \delta_a = \text{drift ratio in %}) \]

\[ h = \text{Column height} \]

\[ \Delta_{GF} = \frac{3.53 h}{100} \times \left( \frac{P}{A_g f_c \rho} \right)^{-0.53} \quad (7.2) \]

where

\[ P = \text{column axial load} \]

\[ \rho = \text{transverse bar ratio in %} \]

\[ A_g = \text{gross cross sectional area of the column} \]

\[ f'_c = \text{cylinder compressive strength of concrete} \]
Eight different magnitude-distance scenarios were considered for generating the ground motions required for incremental dynamic analyses (IDA). The scenarios consisted of a constant magnitude $M=7$ earthquakes with decreasing epicentral distances ranging from $R=75\text{km}$ to $15\text{km}$. Corresponding to each scenario, 12 accelerograms on rock were generated using Program GENQKE (Lam 1999) (refer 6.5.1Table 6.3). The generated ground motions were then amplified using the nonlinear program SHAKE (Idriss and Sun 1992) to simulate strong motions on typical Class C and D sites conforming to the site classifications specified in
AS1170.4. The details of the soil deposits employed in SHAKE could be found in Section 6.5.2.

Peak inelastic displacement demand ($PIDD$) imposed on each column (included in Groups I and II) by each of the ground motions on soil Class C and D sites (as outlined above) were estimated using the inelastic time history dynamic analyses of SDOF models. Calculated demand values for each column were then compared against the displacement capacity of the corresponding column at the limit of gravity collapse ($\Delta_{GF}$, estimated in accordance with Equation (7. 2)) to identify collapse cases. A collapse case is defined when the value of $PIDD$ exceeds the value of $\Delta_{GF}$.

Fragility curves were constructed (based on the results obtained above) to correlate the cumulative probability of collapse ($P$) with the selected measure of intensity ($RSD_{max}$) using Equation (7. 3). In this equation the parameter $i$ represents the analysis case corresponding to the $i^{th}$ ground motion. Such fragility curves assume that the result of seismic collapse assessment has a random nature with a log-normal distribution. Equation (7. 3) is defined by merely a mean ($\mu$) and a standard deviation value ($\sigma$) for constructing the required curve.

$$P(RSD_{max}^{i}) = \Phi\left(\frac{\ln\left(\frac{RSD_{max}^{i}}{\mu}\right)}{\sigma}\right)$$

(7. 3)

where

$\Phi(X)$ is the cumulative probability of collapse with a log-normal distribution.

The two controlling parameters (i.e. $\mu$ and $\sigma$) were obtained using the well-known Maximum Likelihood method as proposed by Shinozuka et al (2001).

Figure 7. 6 shows the fragility curves corresponding to the columns in Group (I) subjected to incremental ground motions on Class C sites. The fragility curves suggest that:

1- The collapse probability values of the analysed columns are less than 5% at $RSD_{max} = 60 mm$ irrespective of the column size. This value (60mm) is identified as the design displacement demand (at 500 years return period) of an elastic SDOF
system on class C sites as derived from AS1170.4 (2007) (refer Appendix A). This suggests that the analysed columns are likely to be safe at the conditions described above.

2- The probability of collapse of the analysed columns at any given level of ground motion intensity ($RSD_{\text{max}}$) decreases with increasing column size when all other parameters (e.g. the reinforcements and axial load ratios) are kept constant. This could be acknowledged, for instance, by comparing the fragility curve of the large column (L20) with that of the small column (S20) in Figure 7.6.

Figure 7.7 shows that the fragility curves corresponding to the columns of Group II, with 40% axial load ratio, have a trend (when subjected to the simulated ground motions on class C sites) similar to those of the columns in Group I. This suggests that the size effect described above is relevant irrespective of the axial load ratio imposed on the columns (i.e. 20% or 40%).

The $RSD_{\text{max}}$ values corresponding to 5% probability of gravity collapse ($RSD_{\text{max}}^{5\%}$) were extracted, for the six columns included in Group I and II, from the constructed fragility curves shown in Figures 7.6 and 7.7. Extracted values were plotted against the estimated displacement capacity of the corresponding columns (i.e. $\Delta_{GF}$ as predicted using Equation (7.2)). The plotted results (shown in Figure 7.8) suggest a steady increase of $RSD_{\text{max}}^{5\%}$ with the increasing values of $\Delta_{GF}$, irrespective of the column size and the axial load ratio. Interestingly, it was found that the obtained data points could be represented, with a good accuracy, by a polynomial of the second order as given in the same figure. This trend, if can be generalized for the parameters included in this study, has the potential to be employed as a simplified collapse assessment model which is of interest within the scope of this study.
Figure 7. 6: Fragility curves corresponding to the columns of Group (I) subjected to incremental ground motions on soil class C (M=7, R=[10-70Km]).

Figure 7. 7: Fragility curves corresponding to the columns in Group (II) subjected to incremental ground motions on soil class C (M=7, R=[10-70Km]).

Figure 7. 8: The $RSD_{max}$ at 5% probability of failure versus the ultimate displacement capacity of columns (Group I and II columns on Class C sites)
To further examine the credibility of the identified trend in Figure 7.8, other parameters were included and investigated. In the next stage, the six columns corresponding to Group I and II were analysed incorporating the ground motions of increasing intensity on class D sites (instead of Class C as employed initially). A procedure similar to that explained above was adopted to construct fragility curves corresponding to Columns S20, M20, L20, S40, M40 and L40. Obtained fragility curves are shown in Figure 7.9. A general trend as observed in Figure 7.6 and Figure 7.7 is also evident in this figure.

![Figure 7.9: Fragility curves corresponding to the columns in: (a) Group (I); and (b) Group (II) subjected to incremental ground motions on soil Class D (M=7, R= [15-75km]).](image-url)
To study the correlation of the required $RSD_{\text{max}}$ at 5% probability of failure with the estimated displacement capacity of columns ($\Delta_{GF}$), corresponding data were extracted from the fragility curves of Figure 7.9. These data were plotted as shown in Figure 7.10. Figure 7.10 revealed a polynomial correlation between $RSD_{\text{max}}^{5\%}$ and $\Delta_{GF}$ for the columns in Group I and II on Class D sites, similar to the trend identified previously for the same columns on class C sites (Figure 7.8).

Figure 7.11 allows a side by side comparison of the obtained polynomial trends corresponding to different soil classes (i.e. class C and D). As could be seen in this figure, for any given displacement capacity ($\Delta_{GF}$), the required ground motion intensity ($RSD_{\text{max}}$) to cause a 5% probability of gravity collapse is lower for the analysed columns on soil class D than soil class C. However the difference is generally small and does not warrant extra complications introduced in the required simplified solution. Thus the grey curve corresponding to class D sites are conservatively adopted for the analysed limited ductile RC columns on both class C and D soil sites. The remainder of this chapter will investigate whether the identified trend is remained relevant if the aspect ratio or the reinforcement ratios of the columns are altered from the values considered so far.

Figure 7.10: The $RSD_{\text{max}}$ at 5% probability of failure versus the ultimate displacement capacity of columns (Group I and II columns on Class D sites)
If the relevance can be proved, the seismic vulnerability of the limited ductile RC columns of interest can be significantly simplified using a two-step instruction as follows:

1. Estimating the column displacement capacity up to the limit of gravity collapse ($\Delta_{GF}$) in accordance with Equation (7.2). This equation is a function of the column height, the axial load ratio and the transverse reinforcement ratio.

2. Estimating the value of $RSD_{max}^{5\%}$ (the $RSD_{max}$ required to result in a 5% probability of gravity collapse) as a function of $\Delta_{GF}$ using the conservative polynomial trend developed in this study. The obtained trend is mathematically expressed by the following equation:

$$RSD_{max}^{5\%} = 0.0163\Delta_{GF}^2 - 1.13\Delta_{GF} + 77 \quad (7.4)$$

Once the value of $RSD_{max}^{5\%}$ is estimated for a given column, it can be compared against the expected $RSD_{max}$ value corresponding to the required return period, hazard factor and soil class (for instance the $RSD_{max}$ at design level of excitations on soil class C). Relevant $RSD_{max}$ values may be obtained from the table given in Appendix A.
7.3 **Effects of Aspect Ratio on the Probability of Collapse**

In the previous section, the effects of column size on the probability of collapse of limited-ductile RC columns were investigated within two groups (the first group with 20% axial load ratio and the second group with 40% axial load ratio). Corresponding fragility curves were constructed for the columns on both class C and D sites as functions of the $RSD_{\text{max}}$ of the incremental ground motions employed. Based on the obtained curves two correlations of polynomial type were drawn between: (a) the level of $RSD_{\text{max}}$ required to cause a 5% probability of gravity failure; and (b) the estimated displacement at the limit of gravity collapse (i.e. between $RSD_{5\%}^{\text{max}}$ and $\Delta_{Gr^\text{c}}$). The polynomial trend on soil class D (Equation (7.4)) was conservatively selected as a potential tool for simplified collapse assessment of the columns of interest on both Class C and D sites.

Although columns of different sizes were included in the parametric study as outlined above, the aspect ratio was kept constant for all the columns analysed so far (i.e. $\frac{h}{d} = 7$). It is therefore of interest to investigate the effects of column aspect ratio on the probability of gravity collapse. It is also important to verify the relevance of the observed correlation (Equation (7.4)), when the column aspect ratio deviates from the previously analysed value (i.e. 7).

To address the objectives set above, Columns M20-4 and M20-5 (Group III) were introduced based on the reference column M20 (with the aspect ratio of 7 as analysed previously). All parameters were kept constant for the three columns except their height. This resulted in different aspect ratios (i.e. 6 and 8 respectively for M20-4 and M20-5). The details of the new columns could be found in Table 7.1.

Figure 7.12 illustrates predicted backbone curve for the columns in Group III. The backbone curve corresponding to Column M20 is also included for comparison purposes. The backbone curves were obtained in accordance with the recommendations made in Section 6.2. As could be seen in this figure a change in the aspect ratio (or more specifically in the height of the columns with the same section
size) could result in changes in all elements of the force-displacement backbone curves including the ultimate displacement capacities at the limit of gravity collapse.

Figure 7.13 illustrates the fragility curves corresponding to the columns of Group III. These curves could be compared against the fragility of the reference column (M20) shown in the same figure. This figure shows that the probability of failure, at any given $RSD_{\text{max}}$, decreases with increasing aspect ratio (or height).

The $RSD_{\text{max}}$ values corresponding to 5% probability of collapse ($RSD_{\text{max}}^{5\%}$) were extracted from the fragility curves constructed for Columns M20-4 and M20-5 (to study the effects associated with the considered variations in the column aspect ratio). These values, coupled with corresponding $\Delta_{\text{GF}}$ values were employed to plot two new data points which were superimposed on previously obtained $RSD_{\text{max}}^{5\%} - \Delta_{\text{GF}}$ trend (see Figure 7.14). It could be seen that considered changes in aspect ratio of the columns does not violate the captured correlation/trend expressed by Equation (7.4).
Figure 7.12: Simulated tri-linear backbone curves (up to the limit of gravity collapse) for the columns of different aspect ratio (Group III)

Figure 7.13: Fragility curves corresponding to the columns in Group III

Figure 7.14: Verification of the validity of the identified trend (Equation (7.4)) when the aspect ratio is parameterized (as in Columns M20-4 and M20-5)
7.4 Effects of Transverse Reinforcement Ratio on the Probability of Collapse

In this section the effects associated with transverse reinforcement ratio on the collapse probability of limited-ductile RC columns are studied. The result of the analyses will importantly be used to further investigate the validity of the identified $RSD_{\text{max}}^{5\%} - \Delta_{GF}$ trend (Equation (7.4)) when transverse reinforcement ratio is parameterized.

For the objective outlined above, Columns M40-4 and M40-5 were generated based on the reference column M40 (as analysed in Section 7.2). The values of transverse reinforcement ratio were increased, from the 0.1% value considered for M40, to 0.15% and 0.2% for M40-4 and M40-5 respectively. The details of the new columns (referred to as Group IV) could be found in Table 7.1.

The force-displacement backbone models and corresponding fragility curves, for Columns M40-4 and M40-5, were obtained using the same approach as employed in the previous analyses of this chapter. The results are shown in Figure 7.15 and Figure 7.16 respectively. New data points $(\Delta_{GF}, RSD_{\text{max}}^{5\%})$ corresponding to Columns M40-4 and M40-5 were plotted and superimposed on the captured trend as could be seen in Figure 7.17. It was found that the captured trend (as expressed by Equation (7.4)) is valid irrespective of the changes applied in the ratio of the transverse reinforcement. The reasonable match obtained (between the plotted data points and the trend) further supports the previously identified polynomial trend which correlates the 5% probability of gravity collapse with the estimated displacement capacity of the columns.
Figure 7.15: Simulated tri-linear backbone curves (up to the limit of gravity collapse) for the columns in Group IV- TRR=transverse reinforcement ratio

Figure 7.16: Fragility curves corresponding to the columns in Group IV- TRR=transverse reinforcement ratio

Figure 7.17: Verification of the validity of the identified trend (Equation (7.4)) when the transverse reinforcement ratio is parameterized (as in Columns M40-4 and M40-5)
7.5 Effects of Longitudinal Reinforcement Ratio on the Probability of Collapse

A similar procedure as described above was employed to parameterize the ratio of longitudinal reinforcement (LRR). This was achieved by analysing Columns S20-2 and S20-3 (Group V) which had identical parameters as in Column S2 (as analysed previously) except for their LRR. Longitudinal reinforcement in S20-2 and S20-3 were increased to 1.5% and 2% respectively from the 1% value considered in column S20. Further details corresponding to these columns could be found in Table 7.1.

The force-deformation backbone models and corresponding fragility curves, for the columns outlined above were obtained as shown in Figure 7.18 and Figure 7.19 respectively. Based on these results, two new data points \((\Delta_{GF},\ RSD_{max}^5)\) were extracted and plotted as circled in Figure 7.20. It was found that the validity of the captured trend expressed by Equation (7.4) is not violated when the longitudinal reinforcement ratio is parameterized within the range considered [1% - 2%].

A number of columns as described above were analysed again (rechecked) employing ground motions of increasing intensities based on a constant magnitude of 6 (as opposed to the magnitude 7 scenarios considered so far). The results (not shown here) showed that Equation (7.4) provides a reasonable simplified solution irrespective of the change in the magnitude of ground motions.

Based on the results obtained from the analyses of this study and limited to the range of parameters investigated, Equation (7.4) is proposed for a significantly simplified and yet reliable seismic collapse assessment of the limited-ductile columns of interest (at 5% probability of collapse). The results presented in Sections 7.2 to 7.5 showed that the captured polynomial correlation is valid for the columns of different sizes, axial load ratios, aspect ratios and reinforcement ratios. The proposed trend can also be employed for the assessment of the columns on both class C and D sites accepting a slightly higher margin of conservatism for the former. It should be emphasized that Equation (7.4) does not rule out the possibility of other correlations that might exist between seismic collapse performance of the columns and their design or backbone...
Figure 7. 18: Simulated tri-linear backbone curves (up to the limit of gravity collapse) for the columns in Group V- LRR=longitudinal reinforcement ratio

Figure 7. 19: Fragility curves corresponding to the columns in Group V- LRR=longitudinal reinforcement ratio

Figure 7. 20: Verification of the validity of the identified trend (Equation (7. 4)) when the longitudinal reinforcement ratio is parameterized (as in Columns S20-2 and S20-3)
parameters. This trend should not be assumed for probability levels other than 5% without dedicated verifications.

7.6 Worked Example

Column E1 is a reinforced concrete cast-in-situ column located at the ground floor of an old soft-storey building in Melbourne. The building is on soil class C according to classifications stipulated in AS1170.4. The collapse vulnerability of this column is required due to projected local earthquakes at 500 years and 2500 years return periods (RP).

The purpose of this worked example is to evaluate the seismic vulnerability of Column E1 (with a 95% confidence level) to gravity collapse. The limited information available for this column is as given below:

\[ b = 400\text{mm}, \quad d = 400\text{mm} \quad \text{and} \quad h = 3200\text{mm} \quad (\text{the width, the depth the and the height of the column-measured}) \]

\[ P = 800\text{KN} \quad (\text{maximum imposed axial load –estimated}) \]

\[ f_c' = 25\text{MPa} \quad (\text{concrete compressive strength- estimated using a Schmidt hammer}) \]

\[ \rho^* = 0.1\% \quad (\text{transverse reinforcement ratio in } \% \text{- conservatively assumed}) \]

Solution:

Step 1: Estimate the displacement capacity of the column at the limit of gravity collapse (\( \Delta_{GF} \)) using Equation (7.2). Equation (7.2) is developed in this study and is recommended where minimal input information is available. This equation is applicable to limited-ductile RC columns with shear span to depth ratio greater than 2.5 (refer Section 5).

\[ n = \frac{h}{d} = \frac{3200}{400} = 8 \quad (n = \text{the column aspect ratio}) \]

\[ \Delta_{GF} = \frac{3.53 h}{100} \times \left( \frac{P}{A_{f_c'} \rho} \right)^{-0.53} \]

\[ \Delta_{GF} = \frac{3.53 \times 3200}{100} \times \left( \frac{800 \times 3}{400 \times 400 \times 25 \times 0.1} \right)^{-0.53} = 78\text{mm} \]

Step 2: Estimate the \( \text{RSD}_{\text{max}} \) required to result in a 5% probability of gravity collapse (i.e. \( \text{RSD}_{\text{max}}^{5\%} \)). This can be done employing Equation (7.4) as developed in this study.
and as a function of the estimated $\Delta_{GF}$ value of the column. Validity of this equation was investigated and verified for a range of key parameters (for limited ductile RC columns) throughout Sections 7.2 to 7.5.

\[
RSD_{\text{max}}^g = 0.0163\Delta_{GF}^2 - 1.13\Delta_{GF} + 77
\]
\[
RSD_{\text{max}}^s = 0.0163 \times (78)^2 - 1.13 \times 78 + 77
\]
\[
RSD_{\text{max}}^s = 88\text{mm}
\]

Table 7.2 summarizes the $RSD_{\text{max}}$ corresponding to elastic 5% damped SDOF systems estimated for Australian conditions on different soil sites (as per AS1170.4) (Fardipour et al. 2011). Considering the hazard factor for Melbourne ($Z = 0.08$) and the specified Class C soil, the relevant $RSD_{\text{max}}$ values at 500 and 2500 years return periods were circled in this table. It can be seen that the $RSD_{\text{max}}^g$ value required for a 5% probability of gravity failure ($= 88\text{mm}$) is greater than the expected $RSD_{\text{max}}$ values at the required return periods. This suggests that the probability of gravity collapse for Column E is less than the conservative limit considered (i.e. 5%). Therefore:

For Column E1 on soil Class C:
- $36 < RSD_{\text{max}}^g = 88\text{mm}$ (the probability of gravity collapse is less than 5% at 500 years RP)
- $65 < RSD_{\text{max}}^g = 88\text{mm}$ (the probability of gravity collapse is less than 5% at 2500 years RP)

The obtained value ($RSD_{\text{max}}^g = 88\text{mm}$) can also be compared against relevant $RSD_{\text{max}}$ values if the soft-story building was located on a soil Class D (as opposed to Class C).

For Column E1 on soil Class D:
- $58 < RSD_{\text{max}}^g = 88\text{mm}$ (the probability of gravity collapse is less than 5% at 500 years RP)
- $104 > RSD_{\text{max}}^g = 88\text{mm}$ (the probability of gravity collapse is greater than 5% at 2500 years RP).
Quick assessments of Column E1 using the proposed displacement based procedure suggests that:

1- This column is generally safe and highly unlikely to experience gravity collapse under the conditions analysed.

2- A follow up detailed assessment is warranted for the last case (i.e. the case at 2500 years RP on soil Class D). Such detailed seismic collapse assessment may be conducted following the recommendations made in this study for limited-ductile RC columns (as employed for the assessment of columns included in the parametric study of this chapter).

### 7.7 Summary

In this chapter, twelve limited-ductile RC columns were assessed for risk of collapse under the seismicity of Australia. The columns were designed to represent a practical range of limited-ductile RC columns typically found in low to moderate seismicity regions.

The columns were arranged in five different groups. The effects of a number of key parameters on the probability of gravity collapse of the columns were studied. The parameters investigated include magnitude-distance scenarios (i.e. M= [6 & 7], R= [15 to 75km] respectively), soil classes C and D, column aspect ratio = [6 to 8], longitudinal reinforcement ratio = [1% to 2%], transverse reinforcement ratio= [0.1% to 0.2%], axial load ratio= [0.2 to 0.4] and column size. In each group, only one parameter was changed at a time and the remaining of parameters were kept constant.

For each column, a representative backbone curve was simulated using the models and solutions presented in the previous chapters. This was followed by conducting

### Table 7.2: Estimates of RSD_{max} (mm) for Australian conditions as per AS1170.4 (Fardipour et al. 2011)

<table>
<thead>
<tr>
<th>Site</th>
<th>Fv</th>
<th>500 yr R_{p} (k_{p} = 1.0)</th>
<th>2500 yr R_{p} (k_{p} = 1.8)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Z = 0.06</td>
<td>Z = 0.08</td>
</tr>
<tr>
<td>Class B</td>
<td>1.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Class C</td>
<td>1.4</td>
<td>19</td>
<td>26</td>
</tr>
<tr>
<td>Class D</td>
<td>2.25</td>
<td>27</td>
<td>36</td>
</tr>
<tr>
<td>Class E</td>
<td>3.5</td>
<td>68</td>
<td>90</td>
</tr>
</tbody>
</table>
inelastic incremental dynamic analyses and constructing fragility curves (for each column) corresponding to different soil conditions and ground motion scenarios with different magnitudes.

It was found from the constructed fragility curves that the RSDmax value required to result in a 5% probability of failure \( RSD_{max}^5 \) in the analysed columns is well correlated with column drift capacity at the limit of gravity collapse \( \Delta_{GF} \). The observed correlation was found to be valid for the range of parameters included in this parametric study. Based on the observed correlation a simplified probabilistic seismic collapse assessment solution was proposed.

Constructed fragility curves also showed that for a given ground motion intensity represented by RSDmax:

a) the probability of gravity collapse increases with axial load ratio.

b) the larger the column size the lesser the probability of gravity collapse if all other parameters are kept constant (e.g. column aspect ratio, reinforcement ratios, and etc.).

c) the higher the column aspect ratio or the transverse reinforcement ratio the lesser the probability of gravity collapse (when axial load ratio and other parameters are kept constant)

d) the sensitivity of the probability of gravity collapse to longitudinal reinforcement ratio is very low. This is attributed to the fact that the effect of longitudinal reinforcement ratio on the ultimate drift capacity of poorly confined concrete columns is generally insignificant (refer Chapter 3).
8 Summary and Conclusions

This study investigated the seismic collapse performance of soft-storey buildings in the low to moderate seismicity region of Australia. Soft-storey buildings of 3 to 30 stories are wide-spread in this region. These buildings have open plans at the level of ground floor and are typically supported by cast-in-situ RC columns at this level. The lateral stiffness of the RC columns is found to be considerably lower than that of the superstructure supported on top of them. Consequently, it is expected that the seismically induced deformation demand on these buildings be concentrated at the level of soft-storey (i.e. at the columns). Importantly the majority of these buildings are old and have been constructed prior to the introduction of the first seismic code in Australia (1979). As a result, no seismic provisions were generally considered for the design of such buildings. The columns in these buildings (referred to as limited-ductile in this study) are typically lightly-reinforced and poorly-confined with the vertical spacing of transverse reinforcements being as large as the column depth. Such columns are categorized as non-ductile and automatically deemed unsafe by the measures of high seismic regions.

8.1 Key Conclusions from Literature Review

1) The literature reviewed in this study suggested that the seismic collapse assessment of soft-storey buildings could be misleading or overly conservative if the key characteristics of the ground motions in the region are not taken into account.

2) It was found that the limit of failure conventionally defined for the columns in high seismic regions (i.e. the point where the peak lateral strength is degraded by 20%) is too restrictive for the limited-ductile RC columns in Australia. This is attributed to ‘displacement controlled behaviour’ which is introduced to describe an important characteristics of ground motions in the low to moderate intra-plate seismicity region of Australia (Lam and Chandler 2005; Lam and Wilson 2004).
3) Under displacement-controlled behaviour, the velocity demand imposed on a SDOF structure by an earthquake subsides with increasing natural period of vibration beyond a certain period limit. Consequently, the energy demand imposed on the structure is not sustained and the displacement demand levels-off to a value constrained by the peak ground displacement (PGD).

4) Under the condition of displacement-controlled behaviour, the failure is more realistically defined by the capacity of structure to deform or displace prior to gravity collapse irrespective of the residual strength and the energy absorption capacity of the structure. Therefore realistic collapse assessment of soft-storey buildings requires that the deformation capacity of the columns up to the point of gravity collapse be simulated.

5) Reviewing the existing models for predicting the drift capacity of limited-ductile RC columns at the limit of gravity collapse revealed that existing models are essentially empirical. Such models are strictly limited to the conditions of calibrations employed for model development.

6) It was also found that existing models do not address the actual physical phenomenon corresponding to the commonly observed rotation at the level of critical shear crack.

7) It was further shown that existing models rely on a complete set of input data which is not generally available when existing RC columns are to be assessed.

8) Existing models (except the one by Zhu et al. (2007)) are deterministic despite the presence of several uncertainties in relation to material properties and highly complex and non-linear mechanisms of failure at the limit of gravity collapse.

9) Based on the review of the literature and the requirements for a realistic displacement-based collapse assessment of soft-storey buildings, it was concluded that: a) an experimental program is required to provide more insight into the force-displacement response behaviour (up to the limit of gravity collapse) of the RC columns featuring minimal reinforcements; b) the existing drift capacity models are not adequate for the practical estimation of drift capacity at the limit of gravity collapse; and c) a simplified probabilistic collapse assessment solution specialized for the limited-ductile RC columns of interest and the conditions of seismicity in Australia does not exist.
8.2 Key Conclusions from the Conducted Experimental Program

This study presented the results of a dedicated experimental program (including 4 lightly-reinforced, poorly-confined concrete columns—S1 to S4) to provide some insight into the response behaviour of the limited-ductile RC columns (refer Chapter 3). The columns were subjected to a constant axial load (i.e. 20% or 40%) and a reversed cyclic lateral displacement of increasing amplitude up to the point of gravity collapse. The columns had minimal longitudinal and transverse reinforcement ratios in the order of 0.07% and [0.5% or 1%] respectively. The vertical spacing of the columns was as large as the column depth (i.e. $s = d = 300mm$). These columns were considered to represent existing RC columns in Australia. Key observations and results from the conducted experimental program and from the study of past experimental programs are summarized next:

1) Lightly-reinforced, poorly-confined columns could have a considerable drift capacity up to the limit of gravity collapse. For instance Specimen S1 (as tested in this study) with only 0.56% vertical bar ratio, 0.07% confinement reinforcement ratio, and 20% axial load ratio exhibited a drift capacity of 5%. The column could maintain 100% of the applied axial load and 50% of the peak lateral capacity at 4% drift ratio.

2) It was observed that the buckling of longitudinal bar does not generally result in an immediate gravity collapse in lightly reinforced concrete columns. For instance for the column mentioned above the buckling of longitudinal reinforcement occurred at about 3% drift ratio. Bar buckling caused spalling of cover concrete. Nevertheless, the actual gravity failure occurred later (at 5% drift ratio) and due to disintegration of the core concrete.

3) Flexural-shear mode of failure was found to be a common mode of failure for non-squat lightly reinforced concrete columns. This mode of failure initiates with the yielding of longitudinal reinforcement prior to the formation of critical shear crack and the disintegration of the core concrete in shear.

4) Critical shear cracks were found to be a common source of ductility for lightly-reinforced, poorly-confined columns (prior to gravity collapse). Such columns could experience some rotational deformation caused by the opening of critical shear crack. Shear slip could occur once the resisting frictional shear
force along the critical shear crack is exceeded by the shear demand from axial and lateral loads imposed on columns.

5) A detailed summary of experimental observations obtained in this study and also the conclusions from the study of past experimental results could be found in Section 3.4.

8.3 Drift Capacity Modelling for Limited-Ductile RC Columns

Post-peak drift capacity modelling for limited-ductile RC columns is considered as the main objective of this study (particularly at the limit of gravity collapse). Chapters 4 and 5 were dedicated to this objective and provided a detailed description of the procedure considered for the development and verification of three capacity models (i.e. Models I, II and III).

Model (I):

The first model developed (Model I) is a theoretical deterministic model for estimating the drift at the limit of gravity collapse. This model focused on the modelling of the actual physical phenomena typically observed within the course of lateral response of limited ductile columns up to gravity collapse (as opposed to relying on empirical calibrations). Model I is focused on the limited-ductile RC columns with a flexural-shear mode of failure. This model led to a simplified expression for estimating the drift at the limit of gravity collapse ($\theta_{\text{collapse}}(\%)$) as proposed by Equation (4.60). This equation is repeated here for the ease of reference:

$$\theta_{\text{collapse}}(\%) = \frac{45(L-d)}{Ld} \text{ALR}^{-1.05} + \left( \frac{\phi_sL^2}{3} + (\phi_{\max} - \phi_s)L_p(L - L_p/2) \right) / L \times 100 + 0.25K$$

Some important points in relation to Equation (4.60) can be summarized as follows:

1) The first term of Equation (4.60) represents the drift caused by rotation at the level of critical shear crack. This simplified term was derived from a rigorous iterative model (shear-slip model) as developed in this study (refer Sections 4.2).
2) For the developed shear-slip model the following items were taken into account: (a) the equilibrium of all forces including the applied axial load, clamping effect of the reinforcements crossing the shear crack and the developed frictional shear force; and (b) compatibility condition which provides the relationships between the shear across the crack, the crack width and required compressive stress on the crack. It is noted that an iterative procedure is required for satisfying the equilibrium and compatibility conditions for estimating the limiting crack width beyond which shear-slip will occur. This is because the state of stress acting on the crack is not constant and changes with the crack width.

3) The second term of Equation (4.60) represents the yield and plastic components of flexural deformation. This term is based on the standard solution for predicting the flexural component of deformation (refer to Equation (4. 44)). In this study however, the applicability of this solution for the limited-ductile columns of interest is examined and verified against the relevant experimental records for the tested columns (i.e. Column S1-S4 tested in this study) (refer to Table 4. 11). Importantly the proposed plastic hinge length for limited ductile RC columns was verified against experimental results (refer to Figure 4. 17). And finally the simplified model for estimating the yield curvature proposed by Priestley et al. (2007) was modified to fit lightly-reinforced, poorly-confined concrete columns, based on a parametric study presented in Section 4.3.1.1.

4) The last term of Equation (4.60) represents the drift ratio (in %) corresponding to the shear component of deformation. This term is derived from the specialization and numerical evaluations of the classical truss model for the limited-ductile RC columns of interest (refer to Section 4.3.2). $K$ is a factor defining the cracked portion of the column height. Based on the experimental observations of this study and other relevant studies the value of $K$ is proposed to vary from 0.5 to 0.75 (depending on the level of axial load ratio (see Equation (4. 54)). This last term of Equation (4.60) was verified against the relevant experimental results of this study (refer to Figure 4. 19).

5) Equation (4.60) was employed to estimate the drift capacity of 21 columns included in a dataset collected (see Table 4. 7). The ratio of estimated to observed drift ratios were found to have a mean and standard deviation of 0.89
and 0.22 respectively. The proposed model is believed to have reasonably achieved the objectives set for this model (which were reducing the reliance on empirical calibrations and theoretical modelling of the actual mechanism of failure typically observed for limited-ductile RC columns of interest). Refer to Section 4.4 for more details regarding the verification of Model I.

Model (II):

The second model (Model II) is a probabilistic capacity model for estimating the drift capacity of limited-ductile RC columns at the limit of gravity collapse. This model was developed to minimize the input parameters required without compromising the reliability of the estimations. Model II addresses the case where there are uncertainties in relation to the actual content, configuration and reinforcement properties of the existing columns to be assessed. Model II also addresses the uncertainties in predicting the exact mechanism of failure. The proposed simplified estimator (Equation (5.4) which provides mean estimations) is repeated here for ease of reference. This equation is limited to columns with shear span to depth ratio greater than 2.5.

\[ \delta_a = 3.53x^{-0.53} \]

where \( x \) is the axial load ratio normalized by transverse reinforcement ratio in % (i.e. \( x = \frac{p}{A_k f, \rho} \)).

Some points regarding Model II can be summarized as follows:

1) Equation (5.4) was employed to estimate the drift values (at the limit of gravity collapse) for the relevant columns in a comprehensive dataset of 56 limited-ductile RC columns all tested to the limit of gravity collapse. The ratio of observed to estimated values was found to have a mean of 1.06 and a standard deviation of 0.4.

2) In order to present the estimated values in a probabilistic format, a normalized fragility curve was constructed in accordance to the recommendations made by Berry and Eberhard (2003). For this end, the proposed equation (5.4) was employed to calculate drift values at the limit of gravity collapse for the relevant columns of the aforementioned dataset. The ratio of observed to
calculated drift values were calculated and sorted in an ascending order. The cumulative distribution of the ratios was calculated. The fragility curve was then fitted to the obtained data points. The resulting fragility curve (originally presented in Figure 5.4 is shown below:

![Fragility Curve](image)

3) Figure 5.4 (above) suggests that, for instance, if the accepted drift ratio is only 0.75 of the drift capacity calculated by Equation (5.4), there is still 20% chance of failure due to uncertainty inherent in the estimated value.

**Model (III):**

The approach and criteria employed for the development of Model II were extended to develop a probabilistic model (referred as Model III) for predicting the drift capacity of limited-ductile RC columns at the limit of nominal shear failure (or lateral failure). Nominal shear failure is the point where peak lateral strength is degraded by 20%. Model III is based on the same experimental dataset (see Table 3.3) as utilized for the development of model II and shares the same minimal input parameters as in the former model (refer Equation (5.16) and Figure 5.6).

### 8.4 Force-Deformation and Dynamic Modelling of Limited-Ductile RC Columns

Chapter 6 combined the developed models (Models II and III) with other standard analytical solutions for constructing full backbone curves for the limited-ductile RC columns of interest. The obtained curves were validated against experimental results
Recommendations were made for constructing simplified tri-linearized backbone curves (see Figure 6. 2). Tri-linearized curves are required as part of the input data for the dynamic modelling of columns to be made within a classical program such as RUAUMOKO. Chapter 6 also provided other specialized parameters for inelastic dynamic analyses of limited-ductile RC columns. The provided data included: 1) a hysteresis rule and calibrated unloading and reloading stiffness parameters representative of the columns of interest (see Figure 6. 4 and Figure 6. 5); and 2) ground motions of incremental intensities simulated on rock and soil (both on Class C and D sites) (using programs GENQUAKE and SHAKE respectively). The generated ground motions are based on the conditions of the low to moderate seismicity of Australia.

8.5 Key Conclusions from Parametric Seismic Collapse Assessment of Limited-Ductile RC Columns

1) Chapter 7 presented a parametric study on the probabilistic seismic collapse assessment of a practical range of limited-ductile RC columns in the low to moderate seismicity region of Australia. Based on the results obtained a simplified collapse assessment solution (corresponding to 5% probability of gravity failure) was developed which is recommended for the range of parameters included.

2) Table 7. 1 presents the details of the columns included in the parametric study. The columns were analysed in 5 different groups. Each group investigating the effects of one parameter only on the probability of collapse of the columns. The parameters analysed were: 1) column size; 2) column aspect ratio; 3) axial load ratio; 4) longitudinal & transverse reinforcement ratios; and 5) soil classes (C and D as classified in accordance to AS1170.4).

3) Representative ground motions with incremental intensities were employed for the purpose of incremental dynamic analyses. The ground motions were based on Magnitude 6 and 7 scenarios and different epicentral distances from 15 to 75 km. The intensity of ground motions was represented by the peak response spectral displacement of elastic 5% damped SDOF systems ($RSD_{max}$). This was decided based on the results of the analyses conducted in this study (e.g.,
see Figure 7. 1) and similar trends relevantly reported by Lumantarna et al. (2010) and Kafle (2011).

4) Force-deformation backbone curves (up to the point of gravity collapse) were simulated for the columns included in the parametric study (using the models and the recommendations presented in the previous chapters). Inelastic incremental dynamic analyses were conducted and representative fragility curves were constructed to correlate the probability of gravity collapse of each column as a function of the intensity of ground motions represented by $RSD_{\text{max}}$.

5) Based on the results obtained from the analyses of this study and the constructed fragility curves it was found that the probability of gravity collapse is less than 5% for the analysed columns at the design level of earthquakes (i.e. $RSD_{\text{max}} = 60\text{mm}$) as considered relevant in major cities in Australia (such as Melbourne, Canberra, Sydney). Importantly this result could be verified regardless of the combination of parameters considered within the following ranges.

- Section size = [as small as (230*320) and as large as (520*565)] (units in mm)
- Column height = [from 2250 to 4000mm]
- Column aspect ratio = [from 6 to 8]
- Axial load ratio = [20% and 40%]
- Transverse reinforcement ratio = [from 0.1% to 0.2%]
- Longitudinal reinforcement ratio [from 1% to 2%]
- Vertical spacing of transverse bar = $d/2 \leq s \leq d$ where $d$ is the column depth
- Soil condition = [Class C and D sites]

6) Based on the results of probabilistic seismic collapse assessment represented by the constructed fragility curves (throughout Chapter 6) it was found that the $RSD_{\text{max}}$ required to cause a 5% probability of failure ($RSD_{5\%}^{\text{max}}$) is generally well correlated with the displacement capacity of the columns up to the limit of gravity collapse ($\Delta_{\text{GF}}$).

7) The obtained correlation was found to be of the type of a polynomial of the second order as introduced by Equation (7. 4). Importantly the captured correlation was shown to be valid regardless of the range and combinations of
parameters included in the parametric study (refer to Figures 6.8, 6.10, 6.14, 6.17 and 6.20). Equation (7.4) is repeated below for the ease of reference.

\[ RSD_{\text{max}}^{5\%} = 0.0163\Delta_{\text{GF}}^2 - 1.13\Delta_{\text{GF}} + 77 \]

where \( \Delta_{GF} \) can be obtained from Equation (5.4) (from Model II). It is noted that the predicted drift ratio at the limit of gravity collapse (\( \delta_i \) in \%) is expressed in terms of displacement capacity (\( \Delta_{GF} \)) in the equation given below.

\[ \Delta_{GF} = \frac{3.53h}{100} \times \left( \frac{P}{A_{g} f_{c} \rho} \right)^{-0.53} \]

where \( h \) = height of the column; \( P \) = axial load; \( A_{g} \) = gross cross sectional area of the column; \( f_{c} \) = cylinder compressive strength of concrete; and \( \rho \) = transverse reinforcement ratio in \%. A conservative lower bound value may be assumed for \( \rho \) if the actual value is not available (i.e. \( \rho = 0.1 \)).

Based on the developed simplified expressions (above) the vulnerability of the limited-ductile RC columns to gravity collapse can be assessed (with 95% confidence) by comparing the estimated \( RSD_{\text{max}}^{5\%} \) with the \( RSD_{\text{max}} \) corresponding to any given level of seismicity or return period. The worked example in Section 7.6 demonstrates the application of the simplified equations proposed in this study.

### 8.6 Applications and Significance of Findings in This Study

The findings of this study could be utilized by practicing engineers for the seismic performance assessment of limited ductile RC columns which are the key structural components in soft-storey buildings. Such assessments are vital to ensure the safety of the occupants of soft story buildings and to manage and sustain such infrastructures. Practical and reliable seismic assessment solutions are of particular interest to insurance companies, and to relevant pre and post disaster organisations.

The drift capacity models developed in this study are practically simplified while ensuring reliable estimations. The models recognize and address practical issues when old existing columns are to be assessed. The developed models could be used by
practicing engineers who may need to obtain quick and reliable estimations of ultimate drift capacity with minimal computational cost and complexity.

The simplified probabilistic seismic assessment solution developed in this study provides a practical tool for assessing the probability of gravity collapse of limited-ductile RC columns in the moderate seismicity region of Australia. The developed solution eliminates the need for rigorous and computationally demanding inelastic incremental dynamic analyses (IDA). More importantly, the solution eliminates the need for engineering judgments on detailed input parameters which are shown to require not only a high level of specialized expertise but also the knowledge which is still a matter of research (as obtained in this study). The simplified model is recommended within the range of parameters included in the parametric analyses of this study.

8.7 Recommendations for Future Studies

Based on the literature reviewed and the insight obtained within the course of this study, the following studies are recommended to be conducted in future:

1) Improving the existing knowledge for predicting the exact mode and mechanism of failure in RC columns.

The ultimate drift capacity of a column is immensely affected by the mode and mechanism of failure. The state of art has put a considerable emphasis on predicting/providing the conditions that ensure a fully ductile mode of response. The corresponding mechanism of failure (i.e. flexural failure) is also well-studied particularly up to the point of nominal shear failure. However, the state of art is not as developed in predicting other modes of failure such as flexural-shear mode of failure; and in predicting the failure mechanisms involved beyond the nominal point of shear failure. The prediction of mechanism and sequence of failure should ideally be related to engineering parameters such as reinforcement ratios, column dimensions and aspect ratio, and loading conditions.
2) Modelling other sources of ductility contributing to total deformation capacity of RC columns up to the limit of gravity collapse.

It has been observed that there exist more elements than the 3 components conventionally considered as having contributions to total deformation of a column (i.e. shear, flexure and strain penetration components of deformation). The current study modelled the observed rotation at the level of critical shear crack which occurs at loading stages beyond the 20% degradation in peak lateral strength. Other components are yet to be modelled, such as the debonding of longitudinal bars as observed in a number of tested specimens. It is noted that in intra-plate seismicity regions such as Australia all sources of deformation, up to the point of gravity collapse, can be counted on considering the recognised displacement-controlled behaviour in such regions.

3) Theoretical modelling of plastic hinge length and reducing the uncertainties introduced by empirical nature of existing models particularly for the case of limited-ductile RC columns.

The accuracy of deformation modelling is affected by the accuracy of the estimated plastic hinge which is needed for estimating the inelastic component of flexural deformation. The plastic hinge length, for limited-ductile RC columns with flexural-shear mode of failure, in this study was taken (assumed) equal to the spacing of transverse reinforcement as supported by limited test results and recommended by a number of past investigators. However, there is still room for an elaborate estimator.

4) Extending drift capacity modelling (at the limit of gravity collapse) to columns subjected to bi-axial bending. This study is limited to the columns subjected to axial load and uniaxial bending condition. Some additional studies are required to provide insight into the response behaviour and performance of columns subjected to axial load and bi-axial bending.

5) Extending drift capacity modelling to columns subjected to combined torsion and bending actions. In a credible scenario, a column is not only subjected to bending but also to torsion. For a comprehensive seismic performance assessment of existing columns, this scenario should also be investigated.

6) For probabilistic model development as in Figure 5.4, a goodness-of-fit test could be used to demonstrate the preference of the adopted lognormal distribution on other potentially relevant distributions such as normal
distributions. In this study, such goodness-of-fit is inferred from the reasonable statistical information such as corresponding mean and standard deviation values.

7) Developing elements to be implemented in open source dynamic analysis programs such as ‘OpenSees’ for an integrated and automated drift capacity modelling of columns up to gravity collapse.

8) Extending the parametric study (and therefore the resultant seismic collapse assessment solution) to include other parameters and conditions not addressed in this study
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Appendix A: Estimates of RSDmax (mm) for Australian Conditions

The value of RSDmax on rock sites as per the Australian Standard for Seismic Actions AS1170.4 (2007) can be estimated by Equation (A.1):

\[
RSD_{\text{max}} = 1.8(750R_pZ)F_v \frac{T_{\text{corner}}}{2\pi}
\]

(A.1)

where, \( Z \) is the seismic hazard factor, \( R_p \) is the return period factor (which is normalised to unity for return period of 500 years), \( F_v \) is the site amplification factor (which is normalised to unity for average rock conditions), \( T_{\text{corner}} \) is the second corner period (Lam et al., 2000) at which the linear part of the response spectrum intercepts the constant (flat) part of the response spectrum. AS1170.4 stipulates a second corner period (\( T_{\text{corner}} \)) value of 1.5 seconds representative of M7 earthquake events.

The estimated RSDmax values for 500 and 2500 year return period earthquakes using AS1170.4 (2007) and Equation (A.1) are presented in Table A.1.

<table>
<thead>
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<th>Site</th>
<th>500 yr ( R_p ) (( k_p = 1.0 ))</th>
<th>2500 yr ( R_p ) (( k_p = 1.8 ))</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( Z = 0.06 )</td>
<td>( Z = 0.08 )</td>
</tr>
<tr>
<td>Class B</td>
<td>0.10</td>
<td>0.14</td>
</tr>
<tr>
<td>Class C</td>
<td>0.14</td>
<td>0.19</td>
</tr>
<tr>
<td>Class D</td>
<td>0.23</td>
<td>0.30</td>
</tr>
<tr>
<td>Class E</td>
<td>0.35</td>
<td>0.47</td>
</tr>
</tbody>
</table>

Appendix B: Drift Demand Predictions in Low to Moderate Seismicity Regions

This section presents the results of a separate study (conducted within the course of this study) on drift demand modelling in low to moderate seismicity regions. The conducted research work resulted in a publication as included in this appendix.
Drift demand predictions in low to moderate seismicity regions

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ABSTRACT: This paper presents results obtained from a recent study that is aimed at assessing the drift demand on buildings for a range of projected earthquake scenarios in Australia. Parameters considered that may affect the response of buildings included building height, structural systems, and mass and stiffness distributions. It has been found that, for the range of buildings studied, the maximum angle of drift is 2.6-4.4 times the maximum response spectral displacement of the earthquake divided by the building height. This can be checked against the limiting drift capacity of the building to enable various levels of damage to be predicted for given earthquake scenarios.

1 INTRODUCTION

The displacement demand on a structure can become insensitive to its natural period. This can happen if the natural period of the structure has exceeded the dominant period of the applied excitations as illustrated by the displacement response spectrum shown in figure 1 (which depicts the response of the structure to a single pulse or a series of periodic pulses).

In earthquake scenarios up to M7 that are consistent with a design peak ground velocity (PGV) of 50-100 mm/s on rock, the dominant period of excitations is considered to be up to 1.5 s for both rock and soil sites (Standards Australia, 2007).

In this study, the displacement controlled phenomenon is used as the basis for estimating the maximum displacement demand on structures including non-ductile structures (Lumantarna et al, 2007; 2010; Bhamare et al, 2008) and structures that possess significant asymmetry in plan (Lumantarna et al, 2008). The developed methodology is simple to apply in that the maximum displacement demand of the structure is constrained by the peak level of an elastic response spectrum for 5% damping ($RSD_{max}$). The $RSD_{max}$ value is a function of the frequency content and intensity of ground shaking. Consequently, reasonable estimates of the maximum displacement (drift) demand on the structure can be made without prior knowledge of the natural period (which is well known to be difficult to predict with good accuracy).

Figure 1: Displacement-controlled behaviour (Lam & Chandler, 2005).
This paper is part of a research project that is aimed at enabling the state of damage to a range of infrastructure to be predicted for given earthquake scenarios. In this paper, the peak displacement demand of linear elastic systems associated with the seismic hazard of Australia as stipulated in the Australian Standard AS1170.4 (Standards Australia, 2007) is first presented (section 2). A model for predicting the drift demand on medium-rise buildings (which was developed from modal analysis results) is then presented in section 3. The developed drift predictors are evaluated using time history analysis in section 4. The methodology was further developed to account for inelastic behaviour of the building (section 5). Results from these investigations have been integrated with other information to develop a simple assessment procedure for practical applications (section 6).

2 PEAK DISPLACEMENT DEMAND

The maximum displacement demand of elastic single degree-of-freedom (SDOF) systems can be obtained directly from an elastic displacement response spectrum. Figure 2 presents the displacement response spectrum of an idealised bi-linear form. The displacement response spectrum is defined by two parameters: maximum response spectral displacement \( RSD_{\text{max}} \) and second corner period \( T_{\text{corner}} \).

The value of \( RSD_{\text{max}} \) on rock sites as per the Australian Standard for Seismic Actions AS1170.4 (Standards Australia, 2007) can be estimated by equation (1):

\[
RSD_{\text{max}} = 1.8(750R_pZ)F F_{\pi} T_{\text{corner}}
\]

where \( Z \) is the seismic hazard factor, \( R_p \) is the return period factor (which is normalised to unity for return period of 500 years), and \( F \) is the site amplification factor (which is normalised to unity for average rock conditions). \( T_{\text{corner}} \) is the second corner period (Lam et al, 2000a) at which the linear part of the response spectrum intercepts the constant (flat) part of the response spectrum. AS1170.4 stipulates a second corner period \( T_{\text{corner}} \) value of 1.5 s representative of M7 earthquake events.

The estimated \( RSD_{\text{max}} \) values for 500- and 2500-year return period earthquakes using AS1170.4 (Standards Australia, 2007) and equation (1) are presented in Table 1.

3 DRIFT DEMAND ON TALL BUILDINGS

Approximate estimation for the drift demand in multi-storey buildings has been addressed in a number of previous studies, such as Miranda (1999), Miranda & Reyes (2002), Miranda & Akkar (2006), Tsang et al (2009), Lumantarna et al (2009) and Fardipour et al (2010). Miranda & Akkar (2006) used a simplified continuous beam model consisting of cantilever shear and flexural beams with uniform mass and stiffness distribution up the height of the building. The modelled response behaviour of a cantilever shear beam can be used to represent the overall response behaviour of a moment resisting frame in which the beams would be much stiffer than the columns. Alternatively, the cantilever flexural beam can be used to represent slender shear walls or braced frames because such structural systems exhibit predominant flexural deformation behaviour. Combined modes of deformation (varying from those of a shear beam to those of a flexural beam) could also be modelled by including a parameter (\( \alpha \)) in the partial differential equation of the fourth order governing the first mode of response of the shear-flexural model.

The average and maximum drift demands on tall buildings can alternatively be estimated by modal analysis or by elastic time-history analysis if linear elastic behaviour is assumed. For this purpose an elastic dynamic analysis program has been
developed using an EXCEL spreadsheet (Lam et al., 2010). The developed spreadsheet program was used to perform modal analyses on simplified height-dependent building models (section 3.1). The simplified models are capable of representing wall structures deforming primarily in bending, frames deforming mainly in shear or any combined shear and flexural modes of deformation (i.e. wall-frame interaction). Rigid struts were also used to represent actions by the floor diaphragms (figure 3). The analysis tool and simplified building models used in this study provided advantages over previous studies through the inclusion of higher mode effects and variation of mass and lateral stiffness up the height of the structure.

Section 3.2 presents parametric modal analyses that are intended to examine the generic characteristics of the modal displacement profiles (MDPs). The findings are then used as the basis for developing simplified practical equations for estimation of the elastic drift demands of the multi-storey buildings.

3.1 Height-dependent building models

In order to have more realistic building models representing existing wall-structures (structures with lateral supports that are primarily provided by structural walls), the use of height-dependent macro models is considered. In a height-dependent building model, variations in the lateral stiffness of the building up its height are correlated with the total height of the building.

In low to moderate seismicity regions such as Australia, design of tall building structures (that are over 30 m high) is likely to be governed by wind actions. Therefore, the relative lateral stiffness was estimated from typical wind-induced demand bending moment profiles (for wall-structures of 20 to 100 m high). For this purpose, provisions in the Australian standard for wind actions, AS/NZS 1170.2 (Standards Australia, 2002), were considered. This resulted in four building models, which are schematically shown in figure 4, with the masses represented by lumped masses at each floor level. In this figure, the lateral stiffness in the first building model, has been reduced by 50% in the upper 70% of the building.

3.2 Generalisation of shapes of deflection

This section discusses MDPs and their sensitivity to various structural parameters of the building. Factors considered are: (i) curtailment in the lateral stiffness of the building up its height; (ii) total height (or total number of storeys) of the building; (iii) disposition of lumped masses up the height of the building; and (iv) degree of wall-frame interactions.

Building models that have been analysed consist of a moment frame and a shear wall, which are connected by pure diaphragm actions, as shown by the schematic diagram of figure 3. The curtailment of the lateral stiffness of the shear wall up the height of the buildings is shown in figure 4. The building models used in this study are essentially considered for approximate global analysis of the considered type of buildings. Such models may not be capable of capturing any sensitivity or variation in seismic demand caused by highly localised or specific structural details, configurations and interactions. Some other effects such as torsional effects and masonry in-fills were also considered beyond the scope of the present study. The building models,
However, are yet considered representative of the typical building types in Australia. It should be noted that the simplified building models are not intended to replace a detailed model when a rigorous analysis is warranted. Instead they are meant to identify vulnerable buildings in which more rigorous analyses may be required.

The flexural stiffness of the shear wall at the base level is defined by the value of $EI$, where $E$ is the Young’s modulus and $I$ is the second moment of area of the shear wall. The value of $EI$ for a building of a given height ($h$) is calibrated in order that the calculated fundamental natural period of vibration ($T_1$) becomes consistent with the predictions by equation (2) (Standards Australia, 2007), while conforming to the rate of curtailment as defined by figure 4.

$$T_1 = 0.0625h^{0.75}$$ (2)

The shape of deflection of buildings (which are supported by shear walls only) for the first three significant modes of vibration as obtained from modal analyses is presented in figure 5. It is noted that deflection profiles presented in the normalised form are not directly indicative of the individual modal contributions to the amplitude of deflection of the building in an earthquake. The MDPs as shown in figure 6 were obtained by scaling the normalised deflection profiles (mode shape) of figure 5 by the respective modal participation factor. The MDPs are more representative of the displacement amplitude than the mode shapes. Interestingly, the MDPs for buildings of different height ranges are remarkably similar when plotted against the normalised height. The actual deflection of the building as contributed by a mode of vibration is obtained simply by scaling the MDP by the respective period dependent ordinate value of the elastic displacement response spectrum (ie. $RSD$ value).

The deflection of the building is finally obtained by superimposing the modal contributions. There is clearly a scope for developing a generalised model for predicting the deflection response behaviour of buildings, which is the subject matter to be addressed in the later sections of the paper.

Results presented so far have been based on analyses of simple building models that feature equal lumped masses being uniformly distributed up the height of the building. Further analyses were undertaken on building models that featured a non-uniform mass distribution with height (figure 7).

Figure 5: Mode shapes of vibration.

Figure 6: Modal displacement profiles.
Initially, the $EI$ values that characterise the stiffness of shear walls at their base were calibrated in order that the fundamental natural period of the “wall only” building conforms to predictions by equation (2). Similarly, the $GA$ values that characterise the stiffness of the moment frames at their base were calibrated in order that the fundamental natural period of the “frame only” building conforms to predictions by equation (2). These are extreme cases of 100% wall and 100% moment frame, respectively. Two additional intermediate cases of (i) 75% wall and 25% moment frame, and (ii) 50% wall and 50% moment frame were considered. In each case, the $EI$ and $GA$ values were scaled in accordance with the respective stated percentage weightings (figure 4). Thus, four classes of wall-frame combinations with a uniform distribution of mass up the 40 m height of the building were analysed. The MDP obtained from the comparative analyses (figure 9) reveal little sensitivity to the response of buildings laterally stiffened with at least 50% structural walls.

Results changed very little between the assumptions of uniform and non-uniform distribution of mass and stiffness up the height of the building for the range of parameters investigated. It is noted that buildings with abrupt changes in lateral stiffness and mass, such as buildings featuring soft-storey and buildings with vertical set back, could behave differently to the buildings presented in this paper. The response behaviour of such buildings warrants further investigation and is outside the scope of this paper.

Comparison of the normalised mode shapes obtained from the two building models (with uniform and non-uniform distribution of masses) revealed insignificant differences as shown by figure 8. Results presented in figure 8 were based on models of a 10-storey (40 m high) building that corresponds to curtailment model type “3” of figure 4.

Sensitivity analyses have also been undertaken to investigate interactions between shear walls and moment frames in buildings, and the effects of the interactions on the potential seismic response behaviour. In the parametric study undertaken by the authors, models of shear walls (that are characterised by the $EI$ parameter) and moment frames (that are characterised by the $GA$ parameter, where $G$ is the shear modulus and $A$ is the cross-section area resisting shear) were incorporated into the same building and connected by struts representing diaphragm action.

Initially, the $EI$ values that characterise the stiffness of shear walls at their base were calibrated in order that the fundamental natural period of the “wall only” building conforms to predictions by equation (2). Similarly, the $GA$ values that characterise the stiffness of the moment frames at their base were calibrated in order that the fundamental natural period of the “frame only” building conforms to predictions by equation (2). These are extreme cases of 100% wall and 100% moment frame, respectively. Two additional intermediate cases of (i) 75% wall and 25% moment frame, and (ii) 50% wall and 50% moment frame were considered. In each case, the $EI$ and $GA$ values were scaled in accordance with the respective stated percentage weightings (figure 4). Thus, four classes of wall-frame combinations with a uniform distribution of mass up the 40 m height of the building were analysed. The MDP obtained from the comparative analyses (figure 9) reveal little sensitivity to the response of buildings laterally stiffened with at least 50% structural walls.

Results changed very little between the assumptions of uniform and non-uniform distribution of mass and stiffness up the height of the building for the range of parameters investigated. It is noted that buildings with abrupt changes in lateral stiffness and mass, such as buildings featuring soft-storey and buildings with vertical set back, could behave differently to the buildings presented in this paper. The response behaviour of such buildings warrants further investigation and is outside the scope of this paper.
Subsequent comparative analyses that have been undertaken involved building models of varying height (i.e., varying number of storeys). Figure 10(a) revealed insignificant influence of the building height on the deflection behaviour of the building. MDPs representing buildings of different height ranges were remarkably similar when plotted against the normalised height. Noticeably, rate of increase of the modal displacement coefficient value at the top of the building diminished rapidly for buildings greater than 8 storeys (figure 10(b)).

The roof level modal displacement coefficient (DC) values for buildings of 3, 10, 20 and 30 storeys are shown in figure 10(b) and summarised in table 2. The inter-storey DC values are presented in figure 11 (the difference in DC values between adjacent floors), from which the roof displacements can be estimated using the generalised expressions of equations (3) to (5) for buildings greater than 10 storeys. Roof level DC values for a 10-storey building were estimated to be 1.48, 0.7 and 0.34 for first, second and third modes of vibration, respectively, which are consistent with the results shown in table 2.

\[ DC_1(n) = 1.48 + \sum_{m=1}^{\infty} 0.762 \times m^{-1.882} \]  
(3)

\[ DC_2(n) = 0.70 + \sum_{m=1}^{\infty} 1.244 \times m^{-1.825} \]  
(4)

\[ DC_3(n) = 0.34 + \sum_{m=1}^{\infty} 0.831 \times m^{-1.701} \]  
(5)

The DC values can be combined with the respective response spectral displacement ordinates (RSD) for calculation of the roof displacement (\( \Delta \text{roof} \)) based on the square-root-of-the-sum-of-the-squares (SRSS) combination method as indicated in equation (6), next page.

\[ \Delta \text{DC}(n) = DC(n) - DC(n-1) \]

\[ \Delta \text{DC} \text{ at the top} = \sum_{n=1}^{\infty} \Delta \text{DC}(n) \]
\[ \Delta_{\text{roof}} = \sqrt{(DC_1 \times \text{RSD}(T_1))^2 + (DC_2 \times \text{RSD}(T_2))^2 + (DC_3 \times \text{RSD}(T_3))^2} \]  
\[ \Delta_{\text{roof}} = \sqrt{(1.54 \times \text{RSD}(T_1))^2 + (0.83 \times \text{RSD}(T_2))^2 + (0.46 \times \text{RSD}(T_3))^2} \]  
\[ \theta_{\text{ave}} = \frac{\Delta_{\text{roof}}}{h} \]  
\[ \theta_{\max} = \frac{2.3}{h} \sqrt{(\text{RSD}(T_1))^2 + 4(\text{RSD}(T_2))^2 + 3.7(\text{RSD}(T_3))^2} \]

Since the increase in the roof level DC values is insignificant when the number of storeys exceeds 10-20 (Table 2), a reasonably conservative estimate of the displacement at the roof level can be made by using the DC values for the 30-storey building as represented by equation (7), above.

The average angle of drift (\( \theta_{\text{ave}} \)) of the building can readily be obtained using equation (8).

\[ \theta_{\text{ave}} = \frac{\Delta_{\text{roof}}}{h} \]  

where \( h \) is the total height of the building.

The maximum drift angle \( \theta_{\max,i} \) (ie. rate of increase in drift at the roof level) attributed to vibration mode \( i \) is defined by the following equation:

\[ \theta_{\max,i} = \frac{(DC_i(1.0h) - DC_i(0.9h))_{\max}}{0.1h} \times \text{RSD}(T_i) \]  

where \( (DC_i(1.0h) - DC_i(0.9h))_{\max} \) is the modal drift angle, and \( DC_i(1.0h) \) and \( DC_i(0.9h) \) are, respectively, the modal displacement coefficients at normalised height of 1.0h and 0.9h for the \( i \)th mode of vibration, as illustrated in figure 10(a). \( \text{RSD}(T_i) \) is the response spectral displacement for the \( i \)th mode of vibration.

Parametric studies revealed that the calculated modal drift angle was not significantly affected by the building height (figure 12). Consequently, solutions to equation (9) can be simplified as follows:

\[ \theta_{\max,1} = 0.23/(0.1h) \times \text{RSD}(T_1) \] for Mode1  
\[ \theta_{\max,2} = 0.46/(0.1h) \times \text{RSD}(T_2) \] for Mode2  
\[ \theta_{\max,3} = 0.44/(0.1h) \times \text{RSD}(T_3) \] for Mode3

The calculated modal drift angles can be combined to estimate the maximum resultant storey drift (\( \theta_{\max} \)) using equation (11) based upon the SRSS combination rule.

\[ \theta_{\max} = \frac{2.3}{h} \sqrt{(\text{RSD}(T_1))^2 + 4(\text{RSD}(T_2))^2 + 3.7(\text{RSD}(T_3))^2} \]

Conservative estimates for the maximum storey drift can be obtained using the much simplified expressions of equations (12) to (13c).

\[ \theta_{\max} = \frac{1.8}{h} \times \text{RSD}_{\text{max}} \] for \( T_i \leq 1.5 \text{ s} \)  
\[ \theta_{\max} = \frac{2.6}{h} \times \text{RSD}_{\text{max}} \] for \( 1.5 \leq T_i \leq 3 \text{ s} \)  
\[ \theta_{\max} = \frac{4.4}{h} \times \text{RSD}_{\text{max}} \] for \( 3 \leq T_i \leq 5 \text{ s} \)

Equation (12) was derived from equations (7) and (8), and the conservative assumption that \( \text{RSD}(T_i) = \text{RSD}(T_j) = \text{RSD}(T_k) = \text{RSD}_{\text{max}} \) given that the factor of 1.8 is sensitive to contributions from the higher modes. Equations (13a) to (13c) were derived from equation (11) and the bi-linear displacement response spectrum models of figure 13, together with the assumption that \( \text{RSD}(T_i) = \text{RSD}_{\text{max}} \) and ratios \( T_i/T_1 = 0.23 \) and \( T_i/T_1 = 0.09 \). These ratios represent the mean values obtained from the range of analyses conducted in this study.

### 3.3 Comparison with time-history analysis

Time history analyses were conducted on a pilot wall-frame 10-storey building using the program OpenSees (McKenna et al, 2000) to examine the accuracy of the proposed SRSS equations (7) and (11), and also that of more simplified equations (ie. equations (12) and (13)).

In the pilot case, the structural wall, beams and columns were modelled explicitly. The beams featured infinite stiffness to promote pure shear behaviour in the frame. A representative estimation of the seismic mass (10 kN/m²) was considered for the arbitrary tributary areas of 68 and 40 m² for the wall and frame, respectively. The total lateral stiffness was determined in an iterative procedure to result in the first natural period of vibration of 1 s for the building (equation (2)). The wall and frame lateral stiffness was further proportioned in order to be consistent with the case of 50% wall and 50% frame. An ensemble of six synthetic accelerograms (S1-S6) was considered for the time history analyses. The accelerograms on rock were stochastically simulated using the programs GENQKE (Lam et al, 2000b) for magnitude 7 and episcopal distance.
other predictors. The robustness of the simplified equations in predicting the building drift demands have been demonstrated.

4 INELASTIC DRIFT DEMAND ON NON-DUCTILE STRUCTURES

The drift response of structures in the inelastic range can be approximated by combining the inelastic first-mode of response with the elastic response for higher modes, using the SRSS rule (section 3.2) as recommended by Priestley et al (2007) for cantilever walls and dual wall-frame structures. Comparison with results from inelastic time-history analyses have found the response behaviour of the multi-storey buildings under dynamic excitations to be reasonably represented by the approach based on modal analyses (Priestley et al, 2007). It is assumed that significant inelastic behaviour in the cantilever walls will occur at the base, which will lengthen the fundamental natural period of vibration of the building but not affect the modal displacement shape (section 3.2).

Non-linear time history analysis parametric studies have been undertaken on inelastic SDOF systems based on the well known Takeda hysteresis model (Lumantarna et al, 2010). The Takeda model is defined by parameters \( \alpha \) and \( \beta \), which model the stiffness degradation of the SDOF systems at unloading and reloading. It was found that the well known equal-displacement proposition can provide a reasonable estimate of the maximum displacement demand of systems in the displacement sensitive regions (ie. \( T > T_{\text{cor}} \)). However, there are uncertainties associated with the inelastic displacement demand on SDOF systems in the acceleration and velocity controlled regions (ie. \( T < T_{\text{cor}} \)). Importantly, the maximum inelastic displacement demands (shown by the symbols in figure 15(a)) can be represented conservatively by the highest point on the elastic displacement response spectrum (\( RSD_{\text{max}} \)), or the constant part of the response spectrum in the bi-linear form (figure 15(b)). Consequently, the average

![Figure 13](image-url) Bi-linear displacement response spectra conservatively used corresponding to (a) equation (13a), (b) equation (13b), and (c) equation (13c).

![Figure 14](image-url) Elastic displacement response spectra corresponding to accelerograms S1 to S6.

of 37. This earthquake scenario is consistent with the seismic hazard of Melbourne and other major cities in Australia for 2500-year return period (PGV = 110 mm/s or peak ground acceleration (PGA) = 1.8 \times 0.08 g on rock) as specified in the Australian Standard AS1170.4 (Standards Australia, 2007). The accelerograms were then used as inputs to the program SHAKE (Idriss & Sun, 1992) to include soil effect in the excitations (resulting in S1-S6). The soil featured a site period of 0.9 s approximately equivalent to site class D according to the Australian Standard AS1170.4 (Standards Australia, 2007). Figure 14 presents the elastic spectral displacement with 5\% damping calculated for S1 to S6 to be used in conjunction with equations (7) and (11) for drift demand predictions.

Table 3 compares the results obtained from time history analyses with equations (7), (8) and (11), and the simplified equations (12) and (13). The results suggest reasonably good drift estimation obtained from SRSS equations. The simplified equations (12) and (13) were found to be more conservative than
or maximum storey drift in a building experiencing inelastic behaviour at its base can be conservatively estimated using equations (12) and (13) with a lengthened fundamental natural period based on the secant stiffness.

5 APPROXIMATE SEISMIC ASSESSMENT

Seismic assessment of structures in regions of low to moderate seismicity can be based on comparison of the calculated drift demand with the drift capacity. Equations (12) and (13) may be used as conservative estimates of the drift angles irrespective of the fundamental natural period of the structure.

Table 4 summarises the maximum storey drift calculated for a 50 m tall building (with a natural period of 1.2 s), which is laterally supported by a combination of shear walls and moment resisting frames. This table is based on equation (13a), along with the values of $RSD_{max}$ listed in table 1. The maximum storey drift estimated for different site classes and levels of seismic hazard can be compared against the ultimate storey drift limit of 1.5% as stipulated by AS1170.4 (Standards Australia, 2007). It is shown that the maximum angle drift demands are within the stipulated drift limits even for the most onerous site conditions and extreme return periods.

6 CONCLUDING REMARKS

This paper presents findings from a research study investigating storey drift demands in tall buildings. Parametric studies were undertaken on a range of multi-storey buildings of different height, and laterally supported by a combination of shear walls and moment frames in varying proportions. The effect of height–wise distribution of mass and lateral stiffness was also investigated in the parametric studies. It was found that the normalised deflected shape of the first three vibration modes of a tall building were rather insensitive to the height, and the mass and stiffness distribution. The modal deflection
profiles were used to estimate the maximum building storey drift demand using simple expressions, which accounted for higher mode effects. A quick assessment of the building performance can be readily made by assessing the likely maximum storey drift and comparing with different damage states and the ultimate drift capacity.

It should be emphasised that the proposed simplified drift demand predictors are not intended to replace detailed and rigorous methods of analysis. Instead they are considered as supplement to the rigorous analysis methods, and may be most useful in quick preliminary check of the seismic demands both in early design stage of new buildings and also in seismic assessment of the existing building.

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