CREATING THE CONDITIONS FOR RICH TEACHER-LED WHOLE-CLASS DISCUSSIONS

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Submitted in partial fulfilment of the requirements of the degree of Master of Education (Stream 150) Major Thesis

August 2013
Melbourne Graduate School of Education
Teacher-led whole class discussion is an important pedagogical tool that is still widely used in classrooms today. This research analysed video recordings of four actual mathematics classrooms to look for segments where rich discussions in mathematics were taking place in order to understand how the teachers created the conditions for those rich discussions. By providing an empirical foundation for the construct of a ‘rich’ discussion, this research hopes to contribute towards greater understanding of the nature, enabling conditions and possible outcomes of a rich discussion. The findings of this research project suggest that teachers play a critical role in creating conditions favourable to the occurrence of rich discussions by their actions or non-actions towards student responses. It is hoped that the results of this research will contribute towards informing both teaching practice and programmes of teacher education.
Declaration

This is to certify that:

i. the thesis comprises only my original work towards the masters except where indicated,
ii. due acknowledgement has been made in the text to all other material used,
iii. the thesis is less than 22 000 words in length, exclusive of tables, maps, bibliographies and appendices.

Siew Hoon Sing
I would like to thank my supervisor, Professor David Clarke, for all the support he has given me in the undertaking of this research project. As a result of this project, we have had many rich discussions about ‘rich’ discussions and I appreciate the hard questions he posed to me along the way that helped clarify my thinking.
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CHAPTER 1

INTRODUCTION
1.1 The learning of mathematics

Mathematics learning is highly valued in most countries. A good grasp of the subject is often necessary for entry to university courses, and subsequently to jobs. Hence, students start learning mathematics at a young age. However, for many generations, students have graduated from schools hating the subject for reasons such as it is boring, irrelevant to daily life and difficult to learn (Boaler, 2009). Boaler (2009) argues that the mathematics taught in many schools today is a narrow “mutated” version of what real mathematics is all about. Real mathematics, to her, “involves problem solving, creating ideas, discussing methods and many different ways of working”, not just “copying methods teachers demonstrate and reproducing them accurately, over and over again”. This way of teaching mathematics is often referred to as traditional mathematics instruction (Baxter & Williams, 2010).

For years, mathematics instruction is monotonous, with the method of delivery consisting mostly of teachers lecturing or teaching definitions, formulas and computation procedures, and students learning passively (Boaler, 2009). Student questions are not particularly welcome as it disrupts the flow of the lesson (Stolp, 2005). As a result of this didactic approach to teaching mathematics, many students actually learn mathematical definitions, procedures and formulas by rote without real understanding (Skemp, 1976). In fact, the learning of mathematics has often been likened to learning a foreign language (Pimm, 1987).

The belief in Asian societies is practice makes perfect (熟能生巧) (Li, 2004). When this belief is applied to the learning of mathematics, it means by imitating what the teacher does and practising repeatedly, one will eventually become a highly skilled problem solver. A cross-cultural study on the differences between Asian and American beliefs in the learning of mathematics also shows that Asians believe that effort, more than innate ability, is the key to one’s level of mathematics achievement (Uttal, 1997). As a result, teaching in Asian classrooms, including Singapore, reflects this view and is “predominantly content oriented and examination driven” (Leung, 2006), with teacher-dominated instruction and minimal student involvement (Yeo & Zhu, 2005). In some classrooms, teaching is more procedurally directed than conceptually directed (Leung, 2006).

My personal experience in observing elementary mathematics classes in Singapore also shows that some teachers tend to emphasize procedure over concept. In whole-class discussions, when teachers asked students to explain what the area of a rectangle means, the first response given by students was typically the formula for finding the area of a rectangle (i.e. length multiplied by breadth), and such a response was often accepted as the correct answer by the teachers. Upon reflection, the teachers might realize that the response did not indicate an understanding of the concept of area, and that it was a simply a recitation
of the formula for area. I believe, however, that even mathematically knowledgeable teachers occasionally fall into this trap. It is this belief that has motivated me to carry out this research into how we can create the conditions for rich teacher-led whole-class discussions of mathematics.

1.2 Mathematics as a language

My interest in discussions also arises from the metaphor of “mathematics as a language”. To understand this proposition, we first have to understand what it means to know a language. There are several obvious aspects to language fluency: aural comprehension, speech, reading, writing, “knowledge of spelling, pronunciation, syntax, the possession of a vocabulary, detailed knowledge of its structure” (Pimm, 1987, p. 3). Some linguistic competencies are more subtle. For example, to understand a language, one has to know where “to segment a continuous outpouring of sound into individual words”, that is, to impose “a structure of words onto a flood of sound” (syntax) (Pimm, 1987, p. 4, italics as in original). One also has to be aware of the “particular, conversational or written, context-dependent conventions operating, how they influence what is being communicated, and how to employ them appropriately according to context” (semantics) (Pimm, 1987, p. 4). Most crucial, Pimm (1987) writes, is the ability to “assign meaning to what is heard or read, and to convey one’s intentions through the spoken and written channels” (p. 5). Halliday (1978) (as cited in Pimm, 1987, p. 6) succinctly summarizes knowing a language to comprise “access to and mastery of three interlocking systems, namely the forms, the meanings and the functions” (italics as in original).

Mathematics does have many of the characteristics of a language (i.e. it functions as a semiotic system). Firstly, students have to make meaning of the terms used in mathematics. Sometimes, they get to see these terms in the context of a mathematical problem, but often these terms are simply names given to special objects for easy reference. The terms, fractions and decimals, for example, are such examples. There are also many everyday words that differ significantly in meaning when used in a mathematical context. Students have to know the meaning of the words when used in a mathematical context, and assign its mathematical meaning to understand the situation. It is not unusual for anyone to attempt to find meaning by going back to what they already know of the words in everyday contexts, especially if it is the first time these words are encountered in an academic context. There have been many instances when students transfer the everyday meaning of the words into mathematical settings and end up with unexpected responses. A well-known mathematics joke resulting from this error is shown in Figure 1.
In Figure 1, the student takes the term “find” literally to mean “to look for” or “to locate”, instead of its mathematical meaning which has more to do with “to calculate”. Other terms include difference and odd. These confusing terms also occur in higher level mathematics (see Gough, 2007). For example, the term normal does not mean ordinary or usual when used in a mathematical context (Barton, 2008). When used in questions, such as “what is the difference between 2 and 5?”, students may become confused and use the everyday meaning, not the mathematical meaning, to interpret the question (Adams, 2003; Raiker, 2002).

To make sense of mathematics, students also have to be aware that much mathematical content is used in a very specific context. However, in a study by Nesher, students seem to pay scant attention to the plausibility of contexts (as cited in Pimm, 1987, p. 12). In another study by Swan, students come up with decimal problems such as “James had 4.6 sweets. His best friend gave him 5.3 sweets and he has 9.9 sweets altogether.” (as cited in Pimm, 1987, p. 12). Students are either blind to the subtleties of contextual relevance (Bell, Fischbein, & Greer, 1984) or “they have learnt to ignore context and work only with numbers” (Boaler, 2009). The most frequent student response to the question “An army bus holds 36 soldiers. If 1128 soldiers are being bussed to their training site, how buses are needed?” is 31 remainder 12, which clearly does not make sense in real-world context (Boaler, 2009, p. 47).

While mathematics content is always logical, students frequently apply a rule, legitimate in one domain, in a different mathematical context where it is no longer correct (Bell et al., 1984). For example, many students wrongly assume that the multiplication of two decimals will result in a larger answer as multiplying two whole numbers gives a larger whole number. Another example can be found in the naming of numbers. Young students often write 61 as the number for “sixteen” because all numbers above twenty are identified by name, first by the digit in the higher place value, and then followed by the subsequent digits. The exception lies only with the “-teen” numbers (Carter & Quinnell, 2012). Other languages, such as
Chinese, do not have this problem, and have number names that follow the same sequential pattern for all numbers above ten (Ng & Rao, 2010).

Mathematical symbols, and sometimes the representations used to show relationships among mathematical ideas are also confusing (Rothman & Cohen, 1989). To know mathematics, students have to understand how the symbols and representations are employed, and not read too much into the form itself. A common example can be found in algebra; students often find it hard to reconcile that $x$ represents a number when it is actually a letter (Pimm, 1987). The teaching of algorithms is often reduced to the teaching of rules such as “invert and multiply” or “to multiply by ten means to add a zero”, which does not facilitate the understanding of the algorithms (Chapin, O’Connor, & Anderson, 2003). The power of using mathematical symbols and representations is that a complicated problem can be reduced simply to a set of symbols, and work can continue without the distraction of the context until the final result of calculation is obtained. Connections of the result to the problem (i.e. making sense) then have to be made in order to solve the problem. When such representational shifts are not communicated to students, students will only see mathematics as even more arbitrary and abstract, and therefore more difficult (Chazan & Ball, 1999).

Lastly, there are also “rules” in mathematics that can be likened to syntax in a language. Algebraic expressions governing the transformation from one expression to another can be one such example (MacGregor & Stacey, 1999). If these rules are also taught superficially, that is, reduced to a set of procedures, over-generalized applications are common. For example, many students will expand $(a + b)^2$ as $a^2 + b^2$ and reduce $\sqrt{a+b}$ to $\sqrt{a} + \sqrt{b}$ (Pimm, 1987).

1.3 The call for more oral communication in the classroom

As with any language, communication can take place orally, by gestures or through written forms. The call for more oral communication in classrooms seems to be targeted at addressing the perceived imbalance between the two modes of communication in the classroom (Alexander, 2004, p. 9; Edwards & Westgate, 1994, p. 12; Pimm, 1987). It is without doubt that the learning of mathematics has been heavily dependent on written communication. The way through which mathematics assessment is conducted probably contributed to this development as well (Clarke, 1996).

The call for more oral communication in the classroom is greatly influenced by social-constructivist theories of learning. Piaget’s work on how a person deals with new concepts or
ideas through the process of accommodation or assimilation was one such example. In Piagetian theory, learning was largely perceived to be something personal, an individual activity. The emergence of Vygotsky’s work on how people learn through social interaction changed this view of learning. Vygotsky’s notion of how knowledge is developed by a child through interaction with more competent others led many to revise the organization of teaching and learning in the mathematics classroom (Moll & Whitmore, 1993). His idea of how socialized speech is later internalized by the child as part of his thinking routines similarly emphasizes the importance of providing a child with opportunities to interact with others during the learning process. The word ‘interact’ indicates a dialogic exchange between individuals rather than a one-way communication, so communication is seen to be more of a “socially constructed” phenomenon (Stephens, 1990). Our personal experience with learning can probably attest to Vygotsky’s idea of how learning is first verbalised before turning inwards. In my experience, some of us may find ourselves talking to ourselves as we think through a problem. Pimm (1987) reports the same habit. He explains that by talking to oneself, one can gain greater access to, and control over one’s thoughts.

Bruner’s research in 1996 converges on the principle that “children must think for themselves before they truly know and understand, and that teaching must provide them with those linguistic opportunities and encounters which will enable them to do so” (as cited in Alexander, 2004, p. 12). Many researchers have argued strongly for the promotion of communication in the mathematics classroom. They have cited cognitive, social, psychological and affective benefits to being able to communicate well.

First and foremost, communication assists in clarification of thinking (Chapin et al., 2003; Pimm, 1987; Weissglass, Mumme, & Cronin, 1990) which facilitates greater understanding of concepts. When student talk is incorporated into lessons, it means students have the opportunity to check if they have understood a concept, problem, procedure or problem solving approach clearly or deeply enough to articulate that understanding in words. If students are not given the opportunity to talk or write about the concept, they may not realise they do not have a clear understanding yet and where the gap in their understanding is. It is highlighted by Chapin et al. (2003) that talking aids students in remembering as well. This is particularly important in the learning of algorithms as students need to understand how algorithms work and not apply them blindly. Talking about the computations helps to clarify the process and strengthen students’ understanding and memory of the procedures (Chapin et al., 2003). As mentioned in earlier paragraphs, one of the difficulties associated with the learning of mathematics is the learning of mathematical terms and symbols. These terminologies have to be used precisely, so creating opportunities for students to clarify and discuss “the complexities associated with the appropriate words, phrases and symbols”, and
to use them in different contexts will help to alleviate this problem (Chapin et al., 2003) As Clarke (2010) has shown, this belief in the importance of student speech varies between countries.

The need for more oral communication in the classroom is further supported by findings in neuroscientific research. It has been found that “talk is necessary not just for learning but also for the building of the brain itself as a physical organism, thereby expanding its power” (Alexander, 2004) (cf. Kim cited in Clarke, 2010). Petty (2009) too makes a convincing argument of how the brain communicates in a language called ‘mentalese’ and the language of instruction has to be translated into the language the brain understands. The translation of a concept into ‘mentalese’ has personal meaning to the learner based on his past experiences with the concept. These past experiences also affect where the brain stores this new concept, and the connecting neurons. Hence, the process of creating meaning is often one of trial and error. Misconceptions are not unusual as students’ meaning making is based on what they think has been communicated, and how the new knowledge is related to prior knowledge or experience. Petty (2009) cites the example of a girl who had a misconception of what a ‘ball’ was. As she encountered objects that looked like a ‘ball’ such as a balloon, an egg or a round stone, she learnt from feedback what a ball really was. Similarly, students need opportunities to explore their understanding of mathematical concepts and ideas through talk or written work, so that their understanding can be clarified or extended (Chapin et al., 2003).

Secondly, oral communication enables children to have increased access to, and greater control of their thoughts (Zwier & Crawford, 2011). This assists children to become independent learners. Classroom talk can be used to give students “time and space to consider more deeply the content we expect them to learn and to revisit what has just been said” (Chapin et al., 2003). It also supports the development of expert thinkers as it creates opportunities for students to practise reflecting on their thinking processes, i.e. monitoring their own thinking and immediately address any confusion by asking a question or clarifying (Chapin et al., 2003).

Having students talk more has the added advantage of giving teachers greater insights into students’ understanding, and the opportunity to sieve out any misconceptions they have (Chapin et al., 2003). Teachers can then follow up with appropriate activities or situations to correct or expand their understanding (Holton & Clarke, 2006).

Incorporating talk in the classroom also means students get to work on their communication skills (Zwiers & Crawford, 2011), an important twenty-first century skill (Ananiadou & Claro, 2009). In the classroom, students can get to observe and hear how ‘more expert’ others
support their claims or make their arguments, and learn gradually how to make sound arguments themselves. As time goes by, students will also feel less stressed and more confident of presenting their views in front of others. As they communicate their views and arguments with one another, it is as though they are working in the world beyond the classroom. The shared understanding of mathematical definitions, terms and symbols in the discipline of mathematics is after all a product of communication within the mathematics community.

However, studies in the 1990s such as CICADA, ORACLE II have shown that reforms in education did not have much impact on classroom talk (Alexander, 2004; Edwards & Mercer, 1987; Myhill, Jones, & Hopper, 2006). An important finding from the studies was that teachers were still dominating the talk in the classrooms. It should be noted that these studies were conducted in Western classrooms, nonetheless, Clarke (2010) suggests that the relative proportions of teacher and student talk in non-Western classrooms is likely to be even more skewed towards the teacher. Despite this, Clarke (2010) reports great variation from country to country in the dominance and character of teacher talk.

There must be a reason why teachers are still dominating the talk in the classrooms. According to Stodolsky (1988), the nature of the knowledge, the structure and sequence of the discipline, and the desired goals all affect the pedagogy teachers use. Similarly, value placed on the subject, such as whether it is considered basic or enrichment, whether performance is assessed externally, and the degree of definition of the subject all have impact on pedagogy (Stodolsky, 1988). The factors affecting communication in the classroom have been summed up by a discussion group at ICME: institutional and cultural factors, teachers’ and students’ beliefs about the appropriateness of their own actions and roles, and beliefs about nature of school mathematics and the way it is to be taught and learned (Stephens, 1990). There may be times when teachers are forced “to compromise, or even abandon, their educational ideals and pedagogic intentions” due to practical issues such as “shortage of time, limited resources and large numbers of children” (Edwards & Mercer, 1987, p. 26).

1.4 The rationale for a closer look at teacher-led whole-class discussions

Despite the many educational reforms over the years and the call for more progressive teaching methods, teacher-led whole-class discussion is still a dominant feature in many classrooms (Alexander, 2004). Since teachers are still widely using whole-class discussions as an instructional strategy, there is a need for a closer examination of this strategy to see how it can be implemented more effectively for better learning outcomes, that is to say it
goes beyond a question-and-answer session. For lack of a better word to describe better whole-class discussions, the word ‘rich’ has been used to describe these episodes in the classroom. By ‘rich’ whole-class discussions, I refer to classroom conversations in which students are cognitively engaged and teachers are not the sole provider of subject-related information. This definition is discussed in greater detail in Chapter 2. It would be beneficial to observe skilled teachers conducting whole-class discussions to distill the features of rich discussions, and attempt to identify the conditions necessary for rich whole-class discussions to occur. An examination of the actions or moves undertaken by these teachers to ensure that they achieve rich whole-class discussions can also serve to inform practice.

1.5 Research Questions

My research questions are thus as follows:

1. What are the conditions conducive to ‘rich’ whole-class discussions in mathematics classrooms?

2. What teacher actions or strategies stimulate ‘rich’ discussions in mathematics classrooms?

1.6 Overview of Chapter

In this chapter, I have argued for a more communicative approach to the teaching and learning of mathematics in light of the metaphor “mathematics as a language”. Classroom discussions can become the medium through which teachers help students negotiate the meanings of mathematics terminology, symbols, and representations. In initiating this study, I defined rich discussions as those for which students are cognitively engaged and teachers are not the sole provider of subject-related information. In this thesis, I attempt to give an empirical foundation to our understanding of the nature of good questions and the conditions likely to lead to their occurrence. A better understanding of the key features of rich discussions and teacher actions or strategies leading to them will contribute towards enhancing teacher pedagogy and better supporting student initiation into the language and practice that is mathematics.
CHAPTER 2

LITERATURE REVIEW
2.1 Introduction

The notion of a 'rich' discussion is vague and needs to be further defined. While any conversation in the classroom between teacher and students may be taken as a discussion, this seems to be more of a layman’s understanding. Classroom interaction or talk is both the medium of teaching and learning (Burns & Myhill, 2004). The first part of this chapter investigates the construct of a ‘discussion’ as a form of classroom interaction and using that as a stepping stone, extend the understanding to the construct of a ‘rich discussion’. The second part of this chapter reviews factors that impact the quality of discussions, particularly teacher moves as they form part of my second research question.

2.2 Types of classroom interactions

To understand 'discussion', we first look at the different types of classroom interaction, and identify the ones which have the potential for rich discussions. Teaching and learning take place amidst classroom interactions. From studying the enactment of teaching in different countries, Alexander (2004) concludes that there are five kinds of talk in the classroom, namely rote, recitation, instruction/ exposition, discussion and dialogue, with the first three kinds being more frequently used by teachers. The first two kinds of talk fit the description that is referred to as the ‘recitation’ script. The recitation script has been cited as the most dominant pattern of interaction observed in classrooms (Wells & Arauz, 2006).

We refer to Dillion (1988) for a more detailed description of the ‘recitation’ script. According to Dillion (1988), it is easily distinguished from other classroom interactions in that the teacher asks question after question requiring answers that are predominately correct or incorrect. Given the nature of the questions, the recitation script is often fast paced as student responses are typically brief and teacher evaluation of the response is simple, just an indication of whether the response is correct or wrong. In a recitation script, even though the teacher and students speak in turn, it is generally observed that other than to evaluate a response, the teacher speaks to ask a question while the students speak only to answer the teacher’s questions. Beyond that, the students do not say anything else, and they do not talk to anyone else but the teacher (Dillion, 1988).

The way to identifying a recitation script is to look at the exchange between teacher and students. In a recitation script, a distinctive and predictable discourse structure can be observed. The discourse structure, also known as IRE (initiation-response-evaluation), coined by Mehan in 1979, starts with a question by the teacher, followed by a student response and then the teacher concludes the sequence with an evaluation of the student response.
response (Godinho & Wilson, 2004). A slight variation to this structure is the IRF (initiation-response-feedback) in which the teacher’s move after the student’s medial responding move is modified to include a follow-up action (Sinclair & Coulthard, 1975).

Recitation as a mode of teaching serves many purposes. Many teachers use it to get the more reserved students in their class to talk, and hence stimulate their participation in class activities. It is also used for purposes such as review and assess “what students know from what they have just studied or been taught”, check if “students have grasped some particular point of importance to this and future lessons”, “keep students’ attention”, “see if they are following” (Dillion, 1988). Recitation can occur at any level and in any subject matter. It has also been observed in supposedly more student-centred activities such as guided discovery, Socratic method, and so on (Dillion, 1988).

The third kind of talk, instruction/expository teaching is characterized by the teacher “telling the pupil what to do, and/or imparting information, and/or explaining facts, principles or procedures” (Alexander, 2004). From this description of expository teaching, it seems students’ involvement is minimal so it also does not have the potential to bring about rich discussions.

Alexander (2004) describes a discussion to be “the exchange of ideas with a view of sharing information and solving problems”, which does not give much details about its characteristics. From Dillion (1994), we are able to get more details on the characteristics of a discussion. He defines a discussion as a form of group interaction with the participants talking back-and-forth with one another, and the issue they are discussing is one which they have some questions about, not something they already know and understand, actions they are resolved to undertake, or experiences whose meaning is plain to them. It differs from the recitation script because of the following characteristics. Firstly, the teacher does not speak at every turn. He or she poses a question at the start to define the issue for discussion but otherwise redirects the question to other students. Meanwhile, students speak considerably longer and they can refer to one another’s contributions, and introduce other topics or materials when they speak. Essentially, the exchange of talk in a discussion will not be a series of question and answer from teacher and student, but consists of “a mix of statements and questions from a mix of students and teacher” (Dillion, 1988). The turn at talk is also not as predictable as in a recitation as:

“although the teacher still retains the right to speak at any and every turn, the cycle may begin with a student instead of a teacher and it may continue with student or with teacher; while the speaker, teacher or student, may ask a question or not, give an answer or not, evaluate or not the previous
Discussion features a mix of moves, a mix of speakers plus moves, and a mix of cycles with mixed speakers and moves” (Dillion, 1988, p. 123).

Hence in a discussion, there may be a student speaking after another student has just spoken for the purpose of addressing a student, referring to a student’s contribution, or evaluating a student’s contribution.

The above characteristic of discussion has a catch: even if the teacher speaks at every turn for she reserves the right to do so, it does not indicate that a discussion is not taking place. This is just how the teacher has chosen to organize the discussion. What is more important is what is said – the quality and the content of the talk. The video recordings of the teachers in this research project indicate that teachers do tend to speak at every alternate turn. That is, it has the form of a social ritual as well as an instructional act. This aspect will be discussed in greater detail in the discussion chapter.

The last kind of talk identified by Alexander (2004) is dialogue, which he describes as the building of “common understanding through structured, cumulative questioning and discussion which guide and prompt, reduce choices, minimize risk and error, and expedite ‘handover’ of concepts and principles” (Alexander, 2004, p. 30). From his description, dialogue also encompasses discussion.

In conclusion, the classroom discussions we will be looking for in this project should be of the last two kinds of talk described by Alexander (2004), and that of a discussion as described by Dillion (1988, 1994). It is obvious from the description of the recitation script and expository teaching that the participation of students in these interactions is very restricted due to the way discourse is structured and they are not conducive to producing quality talk in the classroom.

2.3 Defining a ‘rich’ discussion

The quality of talk in the classroom depends on the type of interactions that goes on in the classroom. Teachers can choose to deliver the curriculum to students using many formats or methods. To promote more talk in the classroom, teachers will have to deploy methods that will provide students with more opportunities to talk. However, more talk does not imply better talk. I am of the same view as Chapin et al. (2003) that the ultimate goal of teachers should be to “increase the amount of high quality talk” amongst the students. However, what would constitute high quality talk? To help me articulate the essence of rich discussions, the fundamental question I keep going back to is: What does one have to observe, hear or feel in the classroom in order to say that a rich discussion has taken place?
To distill the essence of a rich discussion, I refer to literature on math-talk learning communities (Hufferd-Ackles, Fuson, & Sherin, 2004) and effective teacher practices that promote “mathematically productive talk” (Chapin et al., 2003; Henning, 2005; Russell & Corwin, 1993; Stein, Engle, Smith, & Hughes, 2008; Zwiers & Crawford, 2011). It seems that the closest terminology I can find to describe a rich discussion is what these researchers jointly refer to as “productive” talk. I hope to use the description of a high-level math-talk community and examples of productive talk to regress to the criteria for a rich discussion.

Chapin et al. (2003) give four narrative examples, or cases to illustrate what productive talk looks like. They are of the view that the criteria for judging whether classroom talk is productive should not be based solely on feelings but the reason why the classroom talk presented in their examples are considered productive is not clearly articulated. For each example, the focus seems to be on understanding the rationale of discursive moves made by teachers. In fact, they have found through their research five teacher talk moves (which will be elaborated on in later paragraphs) which are effective in moving discussions forward. Zwiers and Crawford (2011) similarly recommend five “core conversation moves” that will focus and deepen conversations.

The importance of teacher actions is no doubt because “children do not typically engage in high-quality discourse unless they are required to provide reasons for their conclusions” (Rasku-Puttonen, Lerkkanen, Poikkeus, & Siekkinen, 2012). Dillon (1988) also has a take on how a good discussion can be achieved: through the teacher “using a mix of these [non-questioning] alternatives together with the occasional self-perplexing question and the single well-chosen question for discussion” as a combination of these will serve the purpose of “enhancing students’ cognitive, affective, and expressive processes while modeling appropriate discussion behaviours for students to imitate” (p. 119). As highlighted, the focus of the research seems to be on effective teacher moves. Based on this, I infer that a rich discussion is dependent on teacher moves but the reason for the teacher moves is more pertinent to my study.

The literature on math-talk community provides another pair of lens to look at rich mathematics talk. Hufferd-Ackles et al. (2004) have identified four distinct but related key components that capture the development of a math-talk community. The components are questioning, explaining mathematics thinking, source of mathematical ideas and responsibility for learning. As the math-talk community grows, shifts or “action trajectories” in teacher and student actions in each component are observed. For example, in the area of questioning, as the math-talk community grows, the teacher will not be the only one observed to be asking questions in class, students too will be responsible for asking questions. The changes in student behaviour depict a growing involvement in the learning of
mathematics. Hence, in a rich discussion, student actions or behaviour should be of essence. They should exhibit some form of involvement with the subject matter, and not be passively taking in information. I associate this behaviour with cognitive engagement.

Going back to the purpose of teacher moves, I infer from the readings (e.g. Hufferd-Ackles et al. 2004; Russell & Corwin, 1993) that math-talk is of a higher level when the teacher is not the only one providing the information in the classroom, that is, students also make contribution to the discussion. There seems to an implied relationship between the degree of teacher control during discussions and the richness of a discussion. A graphical representation of that relationship is shown below.

![Figure 2: Relationship between Richness of Discussion and Degree of Teacher Control](image)

The reason for this inverse relationship is as follows: The more control the teacher exercises, the narrower the window for students to express themselves during discussions, thereby affecting the quality of discussions. Conversely, when the teacher is less controlling, students will have greater opportunities to take part in or lead the discussions, thereby leading to rich discussions. On the surface, the argument seems plausible but on closer consideration, the idea that increased opportunities for students to take part in discussions will result in richer discussions is faulty. More student talk is not equivalent to a rich discussion. Rather, it should be the quality of student talk and interactions that will determine if a discussion is rich. I therefore propose that the teacher moves are required to improve the quality of student talk and interactions.

The criteria for judging if a rich discussion has taken place should be dependent on the quality of student talk and interactions. If during the course of discussion, there is good interaction between the teacher and students, and students are involved in quality talk such as providing explanations, justifications or asking questions, a rich discussion has occurred. I will further summarize the criteria used to decide if rich whole-class discussions have taken place by the satisfaction of three conditions:

1. **The student condition:** Students show cognitive engagement.
2. **The teacher condition**: The teacher acts to improve interaction with students and the quality of student talk. The teacher is not the sole information provider.

3. **The subject condition**: The discussion meets at least one educational objective of the subject.

The student condition addresses students’ behaviour during a discussion. It means that during a rich discussion, what students do or say should indicate an involvement with the subject matter. The teacher condition addresses how a teacher acts in a discussion. During a rich discussion, the teacher can be heard to seek the improvement of classroom interactions by getting students involved in the discussion. This includes asking questions to tap on their prior knowledge, scaffold their understanding, clarify their ideas and so on. The teacher can also be heard asking students for elaboration, clarification, explanation, reformulation or justification for the purpose of improving the quality of student talk. It also includes the demeanor of the teachers as they have to be personable to encourage student participation. The last condition means that a rich discussion has to be relevant to the subject matter.

Establishing these criteria also led to the question of whether all these conditions need to be met before we can say that a rich discussion has taken place. To help us make the decision, we imagine three scenarios.

**Scenario 1**: Students do not show cognitive engagement (i.e. student condition not met).

**Scenario 2**: Students show engagement in what is being discussed (i.e. student condition met). Yet, the teacher does not attempt to get students to participate more in the discussion, and is the only one sharing the ideas or providing the answers to questions (i.e. teacher condition not met). The discussion meets content objectives (i.e. subject condition met) but I suggest that a rich discussion has not taken place.

**Scenario 3**: Students show engagement in what is being discussed (i.e. student condition met). The teacher is getting students to share their ideas (i.e. teacher condition met) but the discussion is not related to any subject taught by the teacher. I suggest that a rich discussion has not taken place.

In the case of the first scenario, the failure to meet the first condition implies the failure of the second condition, and therefore the discussion cannot be rich. In the case of the second scenario, the failure of the second condition implies the communication taking place in a discussion is a one-way transfer of information. There is little interaction between student and teacher. Therefore, the discussion cannot be considered rich. This is consistent with our understanding that the quality of interactions affects the quality of discussions. The third scenario illustrates the necessity of the subject condition. It ensures that the content of the
discussion stays with the subject matter and focus the attention on what the discussion has achieved. As these scenarios illustrate, all three conditions pertaining to student, teacher and subject have to be satisfied before a rich discussion can be said to have occurred.

Having clarified the construct of a rich discussion, I look to research to frame my second research question: teacher actions that will affect the quality of discussions. With the conditions for rich discussions in mind, the second part of this chapter reviews the factors affecting the quality of discussion, and tries to make links to impact on student engagement and classroom interactions.

### 2.4 Factors affecting quality of discussions

#### 2.4.1 Nature of mathematical tasks

Teachers use mathematical tasks as part of classroom instruction for the purpose of focusing students’ attention on a particular mathematics concept. Cognitively-demanding tasks have been linked to high level mathematical thinking and reasoning in students (Stein, Grover, & Henningsen, 1996). However, variations in the difficulty of task content and task form can have impact on students’ interest or motivation in completing the task (Blumenfeld, Mergendoller, & Swarthout, 1987; Henningsen & Stein, 1997). Positive (or negative) reactions to task content are influenced by student beliefs about factors such as the attractiveness and difficulty of the content, and familiarity with the content. Blumenfeld et al. (1987) gave the example of how students may show less persistence in pursuing answers to mathematics and science tasks as they believe that these are harder subjects and require higher ability and effort. The impact of such negative reactions to content is such that even if tasks requiring complex problem-solving skills are given, they may be reduced to guess work on the part of the students. To ensure student engagement throughout the duration of the task, it has been recommended that teachers provide assistance or scaffolds to help students make connections among important ideas, model thinking strategies or processes explicitly and “encourage students to engage in self-monitoring or self-questioning” (Anderson, 1989; as cited in Henningsen & Stein, 1997, p. 527). Henningsen and Stein (1997) are careful to point out that the scaffolds provided should not reduce the overall complexity or cognitive demands of the task. This may pose a challenge to some teachers if they have to provide these scaffolds on the spot. Tasks also have to be carefully selected and structured so that students remain engaged throughout. It will be less daunting if teachers can “anticipate likely student contributions, prepare responses they might make to them”, and make “decisions about how to structure students’ presentations to further their mathematical agenda for their lesson” before the actual lesson (Stein et al. 2008, p. 321).
2.4.2 Student Engagement

Student engagement is no doubt another factor affecting the quality of discussions from the above discussion. How then can we tell that students are engaged in a discussion? Helme and Clarke (2001a, 2001b) have identified student behaviours that they believe can be associated with cognitive engagement by analysing videotape and interview data from four mathematics classrooms. Cognitive engagement is defined as the deliberate task-specific thinking that a student undertakes while participating in a classroom activity, and hence what students say (linguistic) or do (behaviour) can tell the story of whether they are engaged. In particular, they also identified the indicators of cognitive engagement of students in whole-class interactions with a teacher. These indicators will be helpful towards identifying rich discussions in the data set. Table 1 shows a summary of the indicators found.

Table 1: Indicators of cognitive engagement in whole-class interactions with teacher

<table>
<thead>
<tr>
<th>Linguistic Indicators</th>
<th>Behavioural Indicators</th>
</tr>
</thead>
<tbody>
<tr>
<td>asking and answering questions</td>
<td>persistence in completing tasks</td>
</tr>
<tr>
<td>making evaluative comments</td>
<td>active participation</td>
</tr>
<tr>
<td>contributing ideas</td>
<td>resistance to interruptions</td>
</tr>
<tr>
<td>completing teacher utterances</td>
<td>gestures</td>
</tr>
<tr>
<td></td>
<td>eye contact</td>
</tr>
<tr>
<td></td>
<td>body orientation</td>
</tr>
</tbody>
</table>

I am mindful however that even if students are not observed to exhibit linguistic or behavioural indicators of cognitive engagement, it does not mean that students are not involved in the discussion (see Inagaki, Hatano, & Morita, 1998). Behavioural indicators of cognitive engagement thus take on great significance.

2.4.3 Teacher Questioning

Teachers play a critical part in discussions in that they can plan the questions for discussion, and in a way direct the flow of the discussion. Questioning is the most common move made by teachers. It has been reported that teachers ask between 300 to 400 questions in a day, and many of these questions are considered low-level (Alexander, 2004; Almeida, 2012; Dillion, 1988).

The provision of good questions by a teacher seems to be an obvious key to a rich discussion. Godinho and Wilson (2004) believe that “asking questions is pivotal to learning
how to learn and becoming a lifelong learner” and that an effective teacher will ask questions that are suited to the purpose and appropriate to the cognitive level of the students. They also claim that “when a question engages students and motivates them to ask further questions or challenge their ideas, it has the potential to take students beyond their current thinking and engage them in higher-order thinking” (italics inserted for emphasis). This statement puts a lot of leverage on questions to push students to a higher level of thinking, and makes one wonder about the type of questions that will engage students or challenge their ideas. There are also various researches that look at the classification of questions (e.g. Bloom’s revised taxonomy) so that teachers can ask ‘better’ questions.

To stimulate discussion, the teacher has to ask questions but it is not just about high-level or open questions. More than that, the teacher has to be aware of the purpose of her questions “in the context of the lesson”, and what she hopes to achieve with them (Myhill, 2002). The questions have to serve the purpose of engaging students and improving the quality of student responses.

2.4.4 Chapin et al. (2003)’s five key teacher moves

A discussion will not be rich if teachers only ask questions. It will make the discussion seem more like an interrogation, and make it into the recitation script. Besides questioning, teachers can use non-questioning moves to handle student responses, as Dillon (1988) recommends, for example, adopting a neutral position in the discussion, and refraining from supporting a position. Chapin et al. (2003) have identified five teacher moves that are conducive to supporting productive talk. The five moves are: revoicing, asking students to restate someone else’s reasoning, asking students to apply their own reasoning to someone else’s reasoning, prompting students for further participation and using wait time. Non-questioning moves, in general, serve to increase student participation in a discussion. Using Chapin et al. (2003)’s recommendations as a guide, this section reviews each of these moves, the benefits of using them, and for what purpose they may be used by the teacher.

1. Revoicing (e.g. “So you are saying…?”)

Revoicing is particularly useful when there is a lack of clarity in student contributions. When students first talk about a mathematics concept, it may be difficult for them to put their thoughts into words. Hence it makes it difficult sometimes even for the teacher to understand them. In such a case, revoicing, in the form of hazarding a guess of what the student might be saying, gives the student a chance to clarify. Revoicing can also be used effectively to ascertain if the other students in the class understand what has been said. Revoicing, according to Chapin et al. (2003), “provides more thinking space and can help all students track what is going on
mathematically." An earlier research also shows that revoicing can be used strategically to “(1) position students in differing alignments with propositions and allow to claim or disclaim ownership of their position; (2) share formulations in ways that credit students with teachers' warranted inferences; (3) scaffold and recast problem-solution strategies of non-native-language students” (O'Connor & Michaels, 1993, p. 318).

2. Asking students to restate someone else's reasoning (e.g. “Can you repeat what he has just said in your own words?”)

This is another move to clarify what has been said. However, it is now done by a student. The teacher can ask a student to repeat or paraphrase what another student has said, instead of doing so himself. This move of asking students to restate another person's reasoning allows the class to hear the first student's contribution presented in another way, and gives them more time to process the contribution, thus increasing the chances of them following and understanding the discussion. This move also checks if other students can hear what the first student has said, thus ensuring that they can participate in the discussion. Lastly, this move affirms the first student's contribution, and shows the student that his thinking is taken seriously.

3. Asking students to apply their own reasoning to someone else's reasoning (e.g. “Do you agree or disagree and why?”)

By asking students if they support another student's claim and providing a reason for their stand, teachers can have a better understanding of students' thinking as they are forced to “make explicit their own reasoning by applying their thinking to someone else’s contribution” (Chapin et al. 2003).

4. Prompting students for further participation (e.g. “Would someone like to add on?”)

This move serves to increase the participation in the discussion, and increase the inputs to the discussion. Over time, students will be more willing to contribute to the discussion.

5. Using wait time (e.g. “Take your time...we will wait...”)

This move is the art of using silence to allow students to organize their thoughts to contribute to the discussion. If the teacher makes the effort to wait patiently for a number of students to think through the question posed, students will soon realize that everyone has a chance of responding to the teacher's question, not just the ones who can think superfast. Adequate wait time is recommended after the question has been posed (e.g. McCullough & Findley, 1983) but it has been reported that many teachers are not comfortable with lengthy silence, or pre-occupied with time factor and hence wait time is not adopted consistently (Chapin et al., 2003; Hollingsworth, 1982).
2.4.5 Teacher Noticing

Although researchers differ in their conceptualization of this construct, it has been used to encompass the processes through which the teachers manage and make sense of the overwhelming amount of data or information bombarding his senses during instruction (Sherin, Jacobs, & Philipp, 2011). Teachers choose to (or not to) attend to particular events happening in the classroom depending on how and what they make sense of or interpret the events. The interpretations teachers make are highly influenced by their prior experience in teaching (Erickson, 2011). A teacher, for example, may choose to attend to a particular student while filtering out others, based on his knowledge of that student. A distinction between expert teachers and beginning teachers may be their ability to notice. Hence, the ability of teachers to notice (and interpret) may lead to different actions being taken, and therefore different consequences.

Teacher noticing thus affects how a teacher listens and responds to his students, thereby impacting the quality of discussions. Wallach and Even (2005)'s investigation on what a teacher notices with or without being prompted suggest that there may be five types of hearing characteristics: over-hearing, compatible hearing, under-hearing, non-hearing and biased-hearing. Their findings suggest that the teacher's expectations of the students, her knowledge of her students and her conception of the mathematics they were doing impact her interpretation of what she hears.

2.4.6 Norms or ground rules for participation

For a discussion to take place, there have to be rules as to how the participants are supposed to behave. In the classroom, the rules and expectations governing the behaviour of students and teacher can be referred to as classroom social norms. From the psychological perspective, they can also refer to student and teacher beliefs about their own role, others' roles, and the general nature of mathematical activity in school (Cobb, Gravemeijer, Yackel, McClain, & Whitenack, 1997). These rules can vary very significantly between countries (Clarke, 2010). In their year-long study of classroom microculture, Cobb et al. (1997) were able to "describe the evolution of mathematical practices established by the classroom community." Norms in the classroom can in fact be put into three main categories: classroom social norms, sociomathematical norms and classroom mathematical practices. While classroom social norms can be applied to almost any subject area, sociomathematical norms are specific to the subject. They include “what counts as a different solution, a sophisticated solution, an efficient solution, and an acceptable solution” (Cobb et al., 1997). Classroom mathematical practices refer to practices involving interpretations, symbols, arguments and validations which the classroom community share,
and that which have been established as norms. These practices evolve as the class grows in the mathematical learning. For example, for the class studied, the mathematical practice of solving by counting by ones slowly evolved into solution methods that involved the conception of numbers as tens and ones. As students and teachers become aware of what is expected of them or their roles in the classroom, they can better contribute to the discussion, thereby enhancing its richness.

2.5 Role of teachers in mathematical discussion

One of the goals of mathematics instruction is to initiate students into the mathematical practices of the wider society. “Because of his expertise in using the conventions of the discipline, the teacher is in a powerful position to shape those interactions in ways that enhance the possibility that students will attach meanings to their mathematics that are congruent with those that are accepted by other users of mathematical language and symbols” (Lampert, 1990a, p. 255). However, it has been found that mathematical practices have evolved across time and mathematical practices differ across communities. Hence, mathematics “does not consist of timeless, ahistorical facts, rules, or structures but is continually negotiated and institutionalized by a community of knowers” (Cobb, Wood, & Yackel, 1993). With this view, it seems mathematics teaching should be more about helping students negotiate the fundamental meaning of mathematical concepts and ideas, and not be overly obsessed with precise use of rules or conventions.

Cobb et al. (1993) quoted the work of Voigt in which a microanalysis of classroom mathematical discourse showed that “dialogues typically degenerate into social guessing games when teachers attempt to steer or funnel students into a procedure or answer” teachers have in mind. According to the researchers, students learn during the course of such dialogues “highly contextual strategies” that enable them to act in accordance with the teacher’s expectations. To them, the teacher’s role should be “to initiate and guide a genuine mathematical dialogue between the students” so that students get to say how they actually interpreted and solved tasks and to “influence the course of the dialogue by capitalizing on students’ contributions” (Wood, Cobb, & Yackel, 1990). This perception of the role of teacher is still relevant after more than a decade. Schifter (2001)’s perception of a teachers’ role in mathematical discourse is not dissimilar. She stresses that a teacher’s role has changed in that it is no longer just about demonstrating the next algorithm to be committed to memory, they also have “to elicit and build on their students’ mathematical insights and conjectures” by providing them with challenging problems. The difficulty with this perception is teachers have to teach students differently from the way they themselves have been taught, and
hence therein lies the challenge of influencing teachers to change their behaviour so as to facilitate greater student understanding (Boaler, 2009).

In conclusion, to carry out rich discussions, the teacher has to be aware of the role he or she plays in the discussion. This will have direct impact on the teacher actions by way of teacher moves and the setting-up of norms of participation, and indirectly impact student actions.

### 2.6 Overview of Chapter

In this chapter, review of the literature suggests that a rich discussion is defined by the quality of student talk and classroom interactions. For the purposes of this research, it is proposed that a rich discussion can be judged to have occurred if three conditions are satisfied. The conditions relate to student engagement, the actions taken by the teacher to improve classroom interactions during the course of the discussion, as well as the educational objective achieved through the discussion. Research has been reviewed in relation to factors affecting the quality of discussions, and in particular how such factors impact the quality of student talk and classroom interactions.
CHAPTER 3

METHODOLOGY
3.1 Rationale, benefits and issues with secondary data analysis

The research project makes use of existing data collected for the Alignment Project so it is actually a secondary data analysis. The rationale for conducting a secondary data analysis instead of collecting raw data from scratch is because of the availability of a large set of data (23 lessons by four mathematics teachers). Timeframe, budget and resource (e.g. recording equipment) constraint for this research project means it is only possible for the researcher to conduct the research in one classroom. As opposed to conducting the research in one mathematics classroom and not being certain if ‘rich’ discussions will be observed in that classroom, it makes sense to study the data available. The teachers identified for the Alignment Project were also selected based on their teaching competence hence the probability of observing ‘rich’ discussions taking place is assumed to be greater. The advantage of using secondary data has also been highlighted by many researchers (Andrews, Higgins, Andrews, & Lalor, 2012; Trochim, 2006). In particular, these researchers view the re-use of existing data as “an efficient way of conducting research as it eliminates the need to spend time recruiting and gaining access to participants” (Trochim, 2006); it also minimizes “the time and financial expense associated with data collection, e.g. recording device, transport and transcription costs” (Corti, 2008, as cited by Andrews et al., 2012).

While there are benefits with secondary data analysis, there are also issues associated with its use. Heaton (2008) identifies three key issues with secondary data analysis. The first issue pertains to whether the data collected for one (primary) purpose is suited to be re-used for another (secondary) purpose. The second issue is whether there can be an accurate interpretation of the data that were collected by other researchers. The last issue has to do with “whether the results of qualitative research can or should be verified in the same ways as studies using statistical methods” (Heaton, 2008). As the research project falls within the field of qualitative research and does not intend to make use of statistical methods of data analysis, the last issue is not relevant. The teacher participants selected for the Alignment Project, however, match the participants needed for the research project as they are teachers who have been identified as ‘good teachers’ by their school leaders. The data available for re-analysis is also rich as it captures these teachers conducting a sequence of lessons for a particular topic instead of a one-off lesson. As the topic taught by the teachers is the same for the same educational level, the research will also be able to examine how different teachers approach the same topic and at the same time, examine if teachers at different levels (elementary and high school) conduct discussions differently. With regard to accuracy in interpretation, when there are doubts, there is opportunity to verify with the primary researchers as the Alignment Project is still on-going.
3.2 Procedure and Method used for Data Collection

A case study as defined by Freebody (2003) “focus[es] on one particular instance of educational experience and attempt[s] to gain theoretical and professional insights from a full documentation of that instance”. As the research project wishes to gain insights into ‘rich’ discussions in mathematics classrooms, it is appropriate to make use of a case study. However, as the number of cases being examined is four, and not one, it falls into the category of a collective case study (Stake, 1995; as cited by Creswell, 2008). The use of multiple cases to help understand ‘rich’ discussions is better than one case study as it may surface more ‘rich’ discussions segments, and allow for the in-depth analysis of the ‘rich’ discussion segments and the possibility of overlapping or distinct themes emerging from the study of the segments.

Even though the research project does not involve the collection of data, there is assurance that the method utilized by the ICCR in data collection is rigorous and has been “developed in an attempt to study learning in legitimate classroom settings, while minimizing researcher inference regarding participants’ thought processes and maximizing the richness of the research database” (Clarke, 2001).

The lessons have been recorded using three cameras, namely a teacher camera, a student camera and a whole class camera. Audio data has also been captured using multiple microphones together with the visual data. The teacher and two students selected by the teacher wore the microphones throughout the lessons so that their voices were distinctly captured on tape. Before the actual data was captured, there was a familiarization period of one week to ensure that students and teachers become comfortable with the presence of cameras and microphones. The data collected consisted of videotaped records of lessons, students’ written assignments, teacher interviews, school unit plans, teacher lesson plans, and teachers’ reflection after each lesson.

3.3 Purposeful Sampling

In this research project, although there is no need to gain access to sites or participants as it makes use of pre-existing data, whether the teacher participants of the Alignment Project fitted the purpose of the project was still taken into consideration. As highlighted in earlier paragraphs, purposeful sampling has been done for the Alignment Project as only teachers who are competent in their teaching have been selected for study. In all, the Project sampled a total of four Australian mathematics teachers (two from elementary schools and two from high schools). The schools these teachers are teaching in have also been purposefully selected based on their demographics. For the purpose of studying ‘rich’ discussions,
teaching competence would also be the criteria for the selection of study teachers if data were to be gathered. Hence, the sample from the Alignment Project fitted this project’s purpose perfectly. With respect to sampling of data, ‘rich’ discussion segments were also purposefully selected from among the 23 lessons. Only classroom segments that met the criteria defined for ‘rich’ discussions in Chapter 2 were transcribed and examined.

### 3.4 Role of the researcher

This being a secondary data analysis, the researcher is a complete observer, rather than a participant observer, as she was not present during the recording. Her role is entirely limited to observation and nothing more. However, it is noted that the presence of other participant observers (i.e. the primary researchers) may have some influence on the teacher participants and students.

### 3.5 Data analysis Approaches

The data being analyzed were collected for sequences of consecutive lessons in two grade 5 classrooms and two grade 9 classrooms. To differentiate the data sets, they were coded MEL1 to MEL4. The grade 5 classes were given the codes MEL1 and MEL3 while the grade 9 classes were coded MEL2 and MEL4. It follows that the teachers from each school were coded accordingly, that is the teacher teaching MEL1 was coded as T1, the teacher teaching MEL2 was T2 and so on. Lesson one from MEL1 was coded as MEL1_LO1 and the rest of the lessons followed on. The grade 5 lessons were on ‘Fractions’ while the grade 9 lessons were on ‘Trigonometric ratios’. As each teacher used a different number of lessons to complete teaching the above-mentioned topic, the lessons were coded accordingly. A total of twenty-three lessons had been videotaped and were available for analysis. Table 2 summarizes the details of the available data set.

<table>
<thead>
<tr>
<th>School code</th>
<th>Teacher code</th>
<th>Grade level</th>
<th>Topics</th>
<th>Number of lessons</th>
<th>Lesson code</th>
</tr>
</thead>
<tbody>
<tr>
<td>MEL1</td>
<td>T1</td>
<td>Grade 5</td>
<td>Fractions</td>
<td>4</td>
<td>MEL1_LO1 to MEL1_LO4</td>
</tr>
<tr>
<td>MEL2</td>
<td>T2</td>
<td>Grade 9</td>
<td>Trigonometric ratios</td>
<td>7</td>
<td>MEL2_LO1 to MEL2_LO7</td>
</tr>
<tr>
<td>MEL3</td>
<td>T3</td>
<td>Grade 5</td>
<td>Fractions</td>
<td>5</td>
<td>MEL3_LO1 to MEL3_LO5</td>
</tr>
<tr>
<td>MEL4</td>
<td>T4</td>
<td>Grade 5</td>
<td>Trigonometric ratios</td>
<td>7</td>
<td>MEL4_LO1 to MEL4_LO7</td>
</tr>
</tbody>
</table>
All twenty-three lessons were initially viewed to obtain a sense of how the teachers conducted the lessons and how they interacted with the students. This first step as described by Creswell (2008) is exploring the data. During the first phase, the lesson segments when the teachers made use of whole class instruction were identified. These lesson segments were then reviewed and where a ‘rich’ discussion was thought to have taken place, the criteria for judging if a ‘rich’ discussion has occurred are applied to ascertain that it is indeed so, i.e. the three conditions of students, teacher and subject are met. The entire activity leading up to the rich discussion and the rich discussion segments were then transcribed for further analysis. Transcribing the rich discussion segments posed some difficulty. Although student responses were generally clear and easily decipherable in the grade 5 classes, they were sometimes too soft to be captured by the teacher’s voice recorder as students were seated at their desks when responding to the discussion. In the MEL4 classroom, it was particularly challenging to capture student responses as the teacher did not have the habit of asking specific students to respond. The students in the class responded as a whole and the teacher would respond selectively to whichever response was heard. Whenever student responses were indecipherable, teacher questions and responses were referred to as a guide as to how students might have responded and therefore, as a deciding factor as to whether the discussion had been rich. The ‘rich’ discussion segments were then assigned the codes, RD1, RD2 and so on, RD1 being Rich Discussion 1.

While preliminary ideas of the characteristics of ‘rich’ discussions were formed from the initial viewings, once the video segments had been transcribed in detail, the video segments together with the transcript were examined together for greater clarity. These preliminary ideas were then further developed and built upon. The transcripts of ‘rich’ discussions allowed the lesson segments to be reviewed repeatedly and facilitated greatly in the articulation of the strategies deployed or the moves made by the study teachers to make the discussions rich. It is assumed in this research project that competent teachers will be able to lead students in ‘rich’ discussions and that the researcher will be able to observe ‘rich’ discussions in their classrooms. However, it is also possible that these teachers may not engage students in ‘rich’ discussions very often and the examples of ‘rich’ discussions in the data sets may be few and rare. It was therefore heartening to be able to identify a total of eleven rich discussions segments in the twenty-three lessons. An example of a rich discussion segment is shown below.

**Excerpt 1: Example of rich discussion segment**

<table>
<thead>
<tr>
<th>Time</th>
<th>T1</th>
</tr>
</thead>
<tbody>
<tr>
<td>07:45</td>
<td>What about just these two? [circling ‘denominator’ and ‘numerator’] Why did we...why did they pop into our heads? Raise your hand if you had these on your</td>
</tr>
</tbody>
</table>

Teacher invited students to explain how certain words are associated with fractions.
list. Oh…I would have like a few more.
But, J, why do you think we had those
popped up?

2  J  Those are like main things in a fraction.

3  T1  Main things in a fraction?
Teacher revoiced student response.

4  J  Like they are numbers.
Student clarified response.

5  T1  [nodding] Hmm, yeah they are [pointing
to the words] parts of a fraction, aren’t
they?
Teacher provided the proper
term to use.

6  J  Yeah.

7  T1  What about…who said “common
denominator”? [circling the words]
Teacher recognised the input
of this student.

8  S13  I did

9  T1  Really glad you said that. Why did that
pop into your head? Not just
denominator on its own.
Student was not afraid of
saying he did not remember
what common denominators
were.

10 S13  Uhm, I just remember learning about
this, that’s all.

11 T1  Do you remember anything about it?

12 S13  Not really.

13 T1  [laughs] That’s good. [pointing to another
student]
Teacher signalled another
student to help.

14 S14  The common denominator is a…there’s
two fractions and one’s different…you
have to find the one that is the same.
Another student attempted to
explain common
denominators.

15 T1  Yep. You have to make them both the
same. That’s finding the common
denominator. Uhm, why do I put these
two together? [circling ‘improper’ and
‘proper’] Oh mainly because they were
said together but why else, A?
Teacher rephrased student
response and then invited
reasons for
how ‘improper’
and ‘proper’ are related to
fractions.

16 A  Uhm, because they are like opposites
and have to do with like probably the
same thing like ‘improper and proper’,
‘proper’…
[T listening and nodding]
A student attempted to
explain what the terms
meant.

17 T1  [interrupting A] They do both have to do
with the same thing. They do…they are
opposites…
Teacher affirmed the
response.

18 A  Yep

19 T1  …they are two types of fractions, aren’t
they? Any more? Yep? [signaling to a
student]
Teacher built on student
response and invited another
student to add on.

20 S15  Uhm, improper is when the numerator is
bigger than the denominator so…
Another student attempted to
explain the term ‘improper’.

21 T1  [nodding] and a proper one?
Teacher prompted student to
explain ‘proper’.

22 S15  And a proper one is when the numerator
is smaller than the denominator.
Student explained the term
‘proper’ in relation to fraction.
The rich discussion segments were also reviewed with particular attention paid to teacher actions in the ‘rich’ discussions to identify key actions that stimulated the discussions. This step involved identifying, categorizing and coding the teacher actions according to their purpose. This involved the development of another scheme. It also involved an interpretation of the intentions of the teachers when the actions were done. It is hoped that having been immersed in the data for an extended period of time will reduce the risk of wrong interpretations in this step. Literature was referred to explain why teachers act in a particular way and what these actions can accomplish, for example, in facilitating student understanding or encouraging student participation.

Finally, thematic analysis was also used in this research project to provide insights to ‘rich’ discussions and teacher actions. As the research project involves the study of four cases, commonalities between them may exist and can be drawn out by the in-depth study of each classroom. Counting the number of times teachers make certain moves does not really aid in understanding what a rich discussion entails. What is needed are rich examples of rich discussions, and the conditions and teaching acts that led to their occurrence.

3.6 How rigour and trustworthiness is maintained

As the research project is an analysis of existing data, it is not possible to conduct interviews with the participants to confirm the findings. However, steps are still taken to ensure the rigor and trustworthiness of the research project. First of all, to triangulate, the other materials (i.e. lesson plans and so on) collected were referred to for evidence to support findings. Where there were doubts, the primary researchers were consulted to ensure that the interpretations seemed appropriate. The researcher’s supervisor also acted as an external auditor to assess if a thorough investigation of the research problem has been conducted and if the findings have been accurately and fairly reported.

3.7 Ethical Considerations

Ethical considerations in data collection are not relevant in this research project as no actual data is collected. However, the confidentiality of the participants, i.e. the identity of the teachers and students, are still of utmost importance. Hence, analysis of the videotapes are conducted within the confines of ICCR and in the coding of data, names of teachers and students are replaced by generic codes, such as T1 to present teacher from MEL1 and, pseudonyms are used to represent the students in the “rich” discussions segments. Under
no circumstances are the names of schools and teachers involved in the project mentioned to or discussed with outsiders. For ease of writing, they will be referred to singularly as ‘he’.

3.8 Overview of Chapter

In describing the research design for this study, the relevance and advantages of using secondary data analysis have been highlighted in this chapter. With respect to the process of data analysis, the detailed discussion is intended to demonstrate that this study has been rigorously designed and undertaken, and thereby ensure the trustworthiness of its findings.
CHAPTER 4

FINDINGS AND ANALYSIS
4.1 Introduction

From the identified rich discussion segments, the moves made by teachers to stimulate discussion are studied in greater depth. The excerpts shown in this chapter are extracted from within the rich discussion segments. Excerpt 2, for example, is extracted from Rich Discussion 6 (RD6) which is conducted by the teacher of classroom MEL3, identified by T3. The number of rich discussion segments identified for each study teacher is shown in the table below.

<table>
<thead>
<tr>
<th>Teacher</th>
<th>Level teaching</th>
<th>Number of RD segments</th>
<th>RD code</th>
</tr>
</thead>
<tbody>
<tr>
<td>T1</td>
<td>Grade 5</td>
<td>3</td>
<td>RD1 to RD3</td>
</tr>
<tr>
<td>T2</td>
<td>Grade 9</td>
<td>1</td>
<td>RD4</td>
</tr>
<tr>
<td>T3</td>
<td>Grade 5</td>
<td>6</td>
<td>RD5 to RD10</td>
</tr>
<tr>
<td>T4</td>
<td>Grade 9</td>
<td>1</td>
<td>RD11</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>11</strong></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

It is interesting to note that there are fewer rich discussion segments identified for the grade 9 study teachers compared to the grade 5 classrooms. For each of the grade 9 teachers, only one rich discussion has been identified. Both these teachers asked questions which required students to think a little deeper about the mathematical concepts they encountered, and to provide observations and reasons. However, in these classrooms, student responses were inclined to be short and more effort seemed to be required on the part of the teacher to get students to elaborate.

To facilitate the reading and understanding of the excerpts, the topic of discussion in each of the rich discussion segments are listed in Table 4.

<table>
<thead>
<tr>
<th>Rich Discussion Segment</th>
<th>Discussion Topic</th>
</tr>
</thead>
<tbody>
<tr>
<td>RD1</td>
<td>Words associated with fractions</td>
</tr>
<tr>
<td>RD2</td>
<td>How to add fractions and how to convert an improper fraction to a mixed number</td>
</tr>
<tr>
<td>RD3</td>
<td>Ways to add fractions to one whole and then to 2 wholes</td>
</tr>
<tr>
<td>RD4</td>
<td>Sources of error resulting in differences in ratios of sine, cosine, tangent</td>
</tr>
<tr>
<td>RD5</td>
<td>How fractions relate to percentage</td>
</tr>
<tr>
<td>RD6</td>
<td>Position of fractions on a number line</td>
</tr>
<tr>
<td>RD7</td>
<td>How to find the lowest common denominator</td>
</tr>
</tbody>
</table>
While the number of study teachers is too small to enable a generalization of their behaviour, it is observed that there were common moves executed by the study teachers to get the discussion going. To facilitate the elaboration of the teacher moves, they have been organised into three broad categories, which reflect the broader intentions of the teacher moves. These broader intentions have been identified as the promotion of student agency, the facilitation of student reflection, and the continual refinement of the teacher’s own practice. While each category is discussed on its own, it cannot be denied that there are some aspects which overlap, that is, some teacher moves may have multiple effects.

4.2 Intentional moves\(^1\) by the teachers to stimulate discussion

4.2.1 Promotion of student agency

In the promotion of student agency, the strategies used by the study teachers were mainly to solicit more information from students as they responded to questions. Some of the strategies were targeted at having a more inclusive discussion by encouraging more students to take part. Hence, to a large extent, the teacher moves in this category were focused on getting students to articulate their understanding and to show students that their contributions were valued.

(a) Focus on eliciting student understanding

- *Explaining or justifying ideas*

  In all the discussions, one common thing the teachers did is to ask the students, “Why do you say that?” or “Can someone explain this?” Students were required to give more than one-word responses. There were many opportunities for students to give reasons for their answers or talk about how they would solve a problem.

  Besides getting students to give the next step in solving a problem, the teachers also required the students to give the reason for their saying so. If their explanations were not clear or incorrect, the teachers either asked more questions to get them to clarify their ideas [e.g.

\(^1\) Order does not indicate frequency of use by teachers.
Chapter 4: Findings and Analysis

Excerpt 2, words in italics] or direct the question to other students [e.g. Excerpt 4, words in italics].

Excerpt 2: MEL3 LO3 RD6

1. T3 [pointing to the position of half] This is one half. Let’s start by having a look at one half. I’m going to follow it down. Using finger to trace down the number lines] I have two quarters, I have three fifths, three sixths and four sevenths. Would anyone like to make a comment on the location and position of those fractions? S1?

2. S1 Three fifths (...) adjust a bit.

3. T3 Yes, because is three fifth greater or smaller than one half?

4. Ss* Greater.

5. T3 S2, thanks for putting your hand up.

6. S2 Uhm, greater.

7. T3 Greater. So where should…where should I move it to?

8. S2 Just a bit more up…

9. T3 A little bit… that way?

   [moves three fifth to the right of number line]

10. S2 Yep.

*Ss refers to students

(…) indicates indecipherable speech.

Occasionally, another student would be called upon to make sense of another student’s response as in the example below. The teacher (T3) has asked students the following question “Can you get to 1 in any other way than using the same denominators?”. Student P gave a correct equation but T3 posed the question which will help to justify the equation, to another student, N.

Excerpt 3: MEL3 LO4 RD8

1. P 1/3 + 4/6

2. T3 [writes down the fractions as child dictates] Well done, because what is 4/6 equal to? I’ll pop that in brackets here [draws brackets next to 4/6] What is 4/6 equal to, N?

3. N Uhm

4. T3 How many thirds is it equal to?

5. N (...) 

6. T3 Two thirds so… P knows that 1/3 plus 2/3 is a whole so he’s changed the 2 thirds to 4/6 and then you’ve got a whole new sum using different fractions.

RD6 involved a discussion of the position and location of fractions on a number line after students had worked together in groups to place different fractional parts (e.g. halves, thirds, quarters, sixths) on a number line.
• **Redirecting question back to problem student**
  Sometimes, the teachers would redirect the question back to the same student to respond again. In the excerpt below, the student, B, was not very sure when asked how to convert an improper fraction to a mixed number. The teacher got another student to help her with the initial step but in the end, the question was redirected back to her and she had to figure out what the final answer was. [See Excerpt 4, Line 11].

Excerpt 4: MEL1 LO1 RD2

1. T1 Now [circling 6/4] is this the correct answer?
2. B No
3. T1 No, we’ve got to change it around, don’t we?
4. B [Nods]
5. T1 And do you know how we do that?
6. B Uhm…take away three, take away four?
7. T1 Good try, not quite. *Who can help her out?* What’s the next step, S1?
8. S1 Uhm… four quarters is one whole…
9. T1 Four quarters [writes 4/4 below 6/4] is the same as one whole. [writes 1 next to 6/4 and adds “=” sign]
10. S1 And then how many left over from that is…
11. T1 Okay, stop [puts hand up] cause I want B to tell me the answer to that. B, if I have used four quarters out of six quarters [circling each fraction] to make a whole [hand goes over 1], so this four makes me the whole [draws arrow from 4/4 to 1], how many are left over? [indicates 6/4 and 1]
   I’ve got six quarters altogether [circling 6/4] I’ve used four of them up [circling 4 ] how many is left?
13. T1 Two… [writes 2 as numerator next to 1 whole]
15. T1 Not fourths, two…
16. B Two quarters.
17. T1 [writes 4 as denominator] Good girl. Two quarters.

• **Using proper and precise mathematical language**
  While engaging in discussions, not only do students get to practise their communication skills, teachers got the opportunity to check students’ understanding as well as ensure that students were able to use proper and precise mathematical language to communicate their ideas. In the elementary classrooms, especially as the topic being taught was fractions, students often made mistakes naming fractions, such as quarters and fifths,
and mixed numbers. Many students also seemed to have the habit of giving single-word responses and needed prompting from teachers to elaborate further. Excerpt 4, lines 11 to 17, illustrates such a case. In response to the teacher question of the number of quarters left, the student just gave a one-word response. If the teacher had not prompted her to elaborate, it might not have been discovered that she did not know the proper name for fractions with denominator ‘4’.

(b) Valuing student contribution

There are many instances to illustrate how the teachers attempted to show that they value student contribution. Firstly, the teachers often referred to individual student’s responses in a discussion. At other times, they would record a student’s response even though it was partially correct [see Excerpt 5]. They would also address concerns raised by students.

Excerpt 5: MEL3 LO2 RD5

1 T3 [writes the word “fraction” on the board] Who can give me the definition of a fraction? What is a fraction? Have a think.

*...

2 L You can add and subtract fractions.

3 T3 You can also multiply and divide but I’ll just put your idea down. That is fantastic, well done.

* This notation denotes omitted discourse which is irrelevant to the issue being discussed

In the above excerpt, even though the student had only mentioned two operations related to fractions, it does not indicate that she did not know that it was possible to multiply or divide fractions. Perhaps, the class has not learned to multiply and divide fractions. It hence seems appropriate for the teacher to add to the student’s idea and accept her contribution.

In particular, the elementary school teachers set tasks that leveraged on student thinking. One such example involves a teacher (T1 from MEL1) getting his students to come up with words associated with ‘fractions’. After the brainstorming session, the teacher proceeded to write down the student responses while at the same time, categorizing them. He then asked students to look at the categories and think about how their responses had been grouped. Similarly, the teacher, T3, gave each group of students a number line and had them position their fractions on the number line. Then the class had a
discussion on how appropriately the fractions had been positioned on each number line.

Through their actions, these teachers sent a consistent message to all students that their ideas and comments were important, and worth listening to. As a result, in the elementary classrooms, it is evident that the students were more vocal and willing to share ideas and comments. When it was time to comment or respond to a question, more hands were raised.

4.2.2 Facilitation of student reflection

(a) Incorporating wait time or thinking time into the discussions

In the video segments identified, it is evident that even in discussions, the teachers gave plenty of time for students to consider how best to respond to the questions asked. This can be in the form of wait time or as some of these teachers would call it ‘thinking time’. One particular strategy used by one of the elementary school teacher (T1) is ‘brainstorming’. Others might say “Have a think, everyone” or “Any ideas?” Teachers also used long pauses to give students’ time to think. In one instance, the teacher, T3, first asked students to indicate if they knew all the decimals along a number line that had been divided into ten parts. He then paused to give them some time to think and a few more hands were raised. To further encourage students, he asked them to raise their hands if they knew at least one decimal on that number line. Along the way as he got the students to contribute their decimals, he drew their attention to the pattern emerging on the number line so that those students who did not have a clue initially were also able to join in the discussion. Hence, wait time is vital to rich discussions in that, as researchers have reported, it will increase both the quality and quantity of student responses (McCullough & Findley, 1983).

(b) Putting mathematics language or concepts in a context

The elementary teachers tried to make connections between mathematical language and what students had learnt in English. For example, students in the MEL1 class were asked what they thought the word ‘common’ in the phrase ‘common denominator’ might mean whereas students in the MEL3 class were asked if they knew another meaning for ‘quarters’. The same teacher, T3, also drew the students’ attention to the prefix ‘im-’ before the term ‘improper’ to help students interpret what improper fractions meant. In the high school classroom, MEL4, the students obviously had a little bit of history lesson when they learnt
about Pythagoras’ theorem as the teacher questioned them about other discoveries made by Pythagoras and his so-called “minions”.

(c) **Having discussions after students worked on a task**

Most of the rich discussion took place after students have worked on a task. The way the discussions were structured after students had time to work on the task helped to stimulate the discussions. By working on the tasks, students had time to think about what they were doing and what they have learnt. The teacher, T3, occasionally, had students work through tasks together on the spot. However, during these occasions, he had manipulatives, in the form of fraction discs or bars, on the board to assist students in their thinking.

### 4.2.3 Iterative refinement of the teachers' practice

(a) **Scaffolding questions to assist in discussions**

Often, when the students could not respond to the broader questions asked, it was observed that the teachers narrowed the questions so that students’ attention was focused on one aspect first. This strategy helped those students who were unsure get started. For example, when students were not sure of the comments to make about the position of different fractions on various number lines, the teacher had students focus on the position of halves first [see Excerpt 6].

*Excerpt 6: MEL3 LO3 RD6*

1. **T1** While I am putting these up, you can start having a look at them and see if you would like to make any comments about the position of some fractions.

2. **T1** [pointing to the position of half] This is one half. Let’s start by having a look at one half. I’m going to follow it down. [using finger to trace down the number lines] I have two quarters, I have three fifths, three sixths and four sevenths. Would anyone like to make a comment on the location and position of those fractions? N?

Teachers have been encouraged to ask higher order questions so that students can perform at a higher cognitive level (Hollingsworth, 1982). The argument makes sense. However, if the students in the class are not ready to perform at a higher level, the teacher has to help them bridge the gap somehow, and breaking a challenging problem into smaller parts is an option (McCullough &

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3 Prior to the discussion, students worked in groups to place groups of fractions along a blank number line 0 to 1. The groups of fractions students worked with were halves, thirds, quarters, fifths, sixths and sevenths.
Findley, 1983). Teachers should nonetheless be mindful not to oversimplify a challenging problem too much or exercise this option too often such that students become reliant on teachers to help them think through a more challenging problem. In this instance however, I think, the focusing question by the teacher did not give away anything. In fact, students who did not understand equivalence might even jump to the wrong conclusion that one-half is equivalent to two quarters, three fifths, three sixths and four sevenths since they were at the same position on the number lines.

(b) Checking student progress
Teachers would constantly check to ensure that no child was left behind. For example, within one lesson, a high school teacher (T4) would ask the class several times if they were keeping up with the discussion [see Excerpt 7].

Excerpt 7: MEL4 LO1 RD11

<table>
<thead>
<tr>
<th></th>
<th>T4</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>[writes $80 = c^2$] ssshhhh, sssshhh, so, sssshhhh, is everyone okay with how I got from the second line to the third line?</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Ss</td>
<td>Yes</td>
</tr>
<tr>
<td>3</td>
<td>T4</td>
<td>Eyes up so you can see. Boys? Is everyone okay, A, with how I got to the first, from the second line to the third line? Where, where did the eighty come from? Where did the eighty come from? [nodding to a student] [points to board] Where did it come from?</td>
</tr>
</tbody>
</table>

(c) Monitoring one's thoughts as a teacher
From the videos, there were times when teachers stopped in the midst of asking a question or making a statement, and rephrased it [e.g. Excerpt 8 and Excerpt 9, italicized lines]. In Excerpt 8, the teacher (T3) realized immediately that starting the question with ‘where’ would limit the type of student responses, and switched to a more open question. In Excerpt 9, the teacher (T1) intended to illustrate with an example that not all equations would be laid out nicely so that the first two fractions would add up to one whole, as in the first example he gave. However, once he wrote the equation on the board, he realized immediately that it did not fit with his instructional goal for that day, which was to get students to convert improper fractions to mixed numbers. The equation he wrote out could be easily solved to a mixed number by recognizing that five fifths is equivalent to one whole. These instances provided evidence to show that the teachers were consciously monitoring their own thinking as they facilitated the discussions.
4.3 Conditions for rich discussions

In order for rich discussions to take place, it is observed that certain conditions have to be in place. These conditions can be broadly classified into four categories, namely physical context, social-organisational context, affective and cognitive conditions. The categories are arranged such that we first take the broader perspective of looking at the physical aspects of the classroom that may facilitate rich discussions before moving on to look at the conditions governing interactions in a group context. Finally, we focus on the aspects relating to individual students, that is the cognitive and affective aspects [see Figure 3].

![Figure 3: Categories of factors that facilitate rich discussions](image-url)
4.3.1 Physical context

The elementary school students seemed more participative in discussions than the high school students. A possible reason for this could be attributed to differences in the physical layout of the elementary and high school classrooms. In the elementary classrooms (MEL1 and MEL3), the teachers had the students seated on the floors in front of the board for their discussions. The proximity to their teachers could have contributed to their greater attention or engagement to the topic of discussion. The high school students on the other hand sat at their tables so some students who were a further distance from their teachers could allow their attention to wander off for a while without the teachers noticing. While there may be other reasons accounting for the differences in behaviour, physical layout is the one of the most visible difference observed from the videos.

4.3.2 Socio-organisational context

(a) Norms or rules to facilitate discussion

It was observed that rules to facilitate discussion had been established in all the classrooms. Common rules included students raising hands to indicate that they wanted a chance to talk and taking turns to talk. Nonetheless, the teachers still had to remind students sometimes when the rules were broken. One classroom (MEL4) is an exception. In the MEL4 classroom, the students responded to the teacher’s questions at the same time as the teacher did not always indicate to whom her questions were directed at. However, there was still a structure within the class that enabled the teacher to conduct discussions. Reminding students of the norms is one strategy used by teachers. Another strategy observed is the use of praise. The teacher (T3) praised students for exhibiting the right behaviours, i.e. raising their hands to talk, listening well and showing good thinking (e.g. Excerpt 2, line 9, p. 36).

(b) Lesson Structure

For MEL2 and MEL4 (the high schools), the structure of the lessons typically proceeded in three phases: (1) teacher introducing a task, (2) students working on the task individually or in groups while teacher walks around monitoring, and lastly (3) a whole-class discussion which involves going through the task to talk about the difficulties students encountered or the answers they reached which get them to think more deeply about the task. MEL1 used the same structure but for each activity. So whole-class discussions can take place several times in a lesson. MEL3 also used the same structure for each activity with a slight variation. For MEL3, the last activity in each lesson involved groups of students working on
different tasks individually, and the time allocated for students to share their ideas was seriously limited at the end of the lesson.

4.3.3 Affective conditions

There are three types of affective interactions connected with student learning: the student’s relationship with the teacher, the subject and other students. We will start by considering the teacher-student relationship.

(a) Teacher-student relationship

It was observed that the teachers attempted to build a positive relationship with the students through various strategies. This could be influenced by their knowledge of the impact of positive teacher-student relationship on engagement in the classroom (Marsh, 2012; Roorda, Koomen, Spilt, & Oort, 2011).

- **Use of self-referential (personal) statements by the teacher**
  
  During the course of the discussion, the teachers were observed making statements with “I”. For example, after students shared words related to fractions which they came up with within the time frame of three minutes, the teacher (T1) said, “They are certainly the ones that I would have… uhm have popped into my head first.” The statement can serve two purposes. One would be to tell students they have done well as the words they came up with are what the teacher would have thought of as well. Another reason could be to try to close the gap between the teacher and student so that the students view the teacher as one of them. Another teacher (T4), when discussing how to solve a mathematical problem with the class, shared a personal strategy, saying, “I always like to start with writing whatever theorem or formula that I’m using.”

- **Use of positive remarks or gestures by the teacher**
  
  The teachers were skillful at making positive remarks or gestures in response to students. One teacher (T3) was particularly adept at it. When a child could not put his thoughts on how to make the denominators of two fractions the same in words, he smiled and praised the student for his effort [see Excerpt 10]. He then got another child to rephrase what the first student had said.

Excerpt 10: MEL3 LO4 RD7

<table>
<thead>
<tr>
<th></th>
<th>T3</th>
<th>Now, N, how do you think we could do that?</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>N</td>
<td>Uhm, see what if 3 x 8 can go inside is 24 and 8 x 3 is 24, so, 3 and 8 have to greater or equal…no, I can’t explain.</td>
</tr>
</tbody>
</table>
| 3 | T3     | [writes denominators of 24 under the 2 fractions] [smiles and
On another occasion [see Excerpt 11], when a child failed to predict which fraction circle he was going to put up next, he simply accepted the response and carried on, after letting him know it was not the correct one.

**Excerpt 11: MEL3 LO2 RD5**

1. T3  We have one whole, halves, thirds, quarters, what might be next? A, have a try?
2. A   Anything...sixths.
3. T3  Sixth? They are in here. [searches and takes out fraction circle] Not in that order but they are in here. Sixths, sixths, it's the hardest one to say. You might end up spitting a lot on somebody when you say that.

There was another occasion when he got a wrong response from a student. The class was asked to come up with an estimated percentage for a third and the student said it was seventy-five percent. When the student seemed disappointed his was not the correct answer, T3 was quick to encourage him [see Excerpt 12].

**Excerpt 12: MEL3 LO2 RD5**

1. T3  So, let's see if we can work it out for thirds. One-third [writes 1/3 beneath thirds] as a percentage. Obviously, we will take one whole which is one hundred percent and we will divide it by three. Roughly. Third is tricky because you have decimals but roughly, about...L?
2. L   Uhm, seventy-five percent.
3. T3  Not for thirds.
4. L   Aw...
5. T3  You are thinking of a different fraction but remember that, we'll talk about that too.

How a teacher respond to students is crucial as it serves as a type of feedback to the students as a learner and to their learning. Feedback, as defined by Hattie and Timperley (2007), is the information an agent (e.g. one’s teacher, peer, parent, oneself or an experience) provides regarding aspects of one’s performance or understanding and is “one of the most powerful influences on learning and achievement.” As such, if use correctly, it can greatly influence student learning and achievement. Students also form identities of themselves as learners based on a teacher’s response to their contribution (Solomon & Black, 2008). If students feel that their contributions
are valued, it fuels their confidence as learners and encourages them to make more contributions in future.

- **Use of empathetic remarks by the teacher**

Some teachers used remarks such as “This is a tricky one, isn’t it?” to show empathy towards the challenges the students were facing. ‘Tricky’ seemed to be the word that have been adopted by two of the study teachers (the elementary school teachers) to describe the more challenging mathematical problems or questions. While these remarks may seem, to some, to convey sympathy, and indirectly teacher expectations, especially when addressed to specific students, the teachers observed used them when addressing the whole class. The purpose of using these remarks (e.g. Excerpt 12, line 1, word in bold) to the whole class may serve to downplay the difficulty of the problems (i.e. the problem is not that difficult, it is just tricky) and increase student engagement (i.e. motivation to resolve to solve the ‘tricky’ problem), thereby giving students a greater sense of accomplishment (i.e. increase their confidence) when they are able to solve the ‘tricky’ problems posed by the teacher.

As shown, teachers use varied strategies to bond with their students and draw students closer to them. The purpose they all hoped to achieve is to reduce the boundary between teacher and students so that students may be more willing to participate in the classroom activities or ask questions to clarify their doubts. In the classrooms observed, the teachers seemed to have established good rapports with their students. Sometimes they even joked with one another, which break the monotony of the class time [see Excerpt 13]. As there were no unpleasant repercussions, the students were not hesitant to speak their mind when asked if they had comments or ideas. Neither were they afraid of telling the truth. In one instance, when a teacher (T1) asked a student if he remembered anything of ‘common denominator’, the student was able to be completely honest and said that he did not remember much of the concept; he merely remembered the term [see Excerpt 14].

**Excerpt 13: MEL1 LO1 RD1**

1. T1 Alright, tuck your chairs in, come down to the floor with me.  
   [Students moving to the floor]
2. T1 You can come a bit closer, I don’t bite…much.
3. S Ah…you bit me.
Chapter 4: Findings and Analysis

Excerpt 14: MEL1 LO1 RD1

1  T1  What about…who said "common denominator"? [circling the words]
2  S   I did.
3  T1  Really glad you said that. Why did that pop into your head? Not just denominator on its own.
4  S   Uhm, I just remember learning about this, that's all.
5  T1  Do you remember anything about it?
6  S   Not really.

Students also felt safe to express themselves when they have a different opinion or something different. This was observed particularly in one classroom (MEL3) when a child put up a hand to indicate that she disagreed with what another child was saying about how to make the denominators of fractions the same [see Excerpt 15, italicized line]. Similarly, in another classroom, after a brainstorming session on “\( \frac{2}{7} + \frac{3}{7} + \frac{2}{7} = 2 \)”, a child was not hesitant in sharing an equation he came up with although he knew it was different from the ones the other students had shared [see Excerpt 16].

Excerpt 15: MEL3 LO4 RD7

1  T3  What should I do now? I have got 24. Somehow, I have got to make that 1/3 into twenty-fourths.
     And somehow I have got to make 3/8 into twenty-fourths? C?
2  C   Use what times 8…
3  T3  What times 8? And then what will I write up here? [pointing to the fraction with denominator 24]
4  C   Uhm, 8
5  T3  *Are you putting your hand up to disagree?* Okay, we will listen to what Z has to say and we might decide which way to go, C, alright? Z?
6  Z   24 divided by 8 is 3. 3 times 3 is 9. And if you divide 24 by 3 is 8 so 1 time 8 is 8.
7  T3  That’s exactly what you said, C. A different way to get there. So, 3, we talked a bit about this yesterday.

Excerpt 16: MEL1 LO2 RD3

1  S   I don’t know if you’d appreciate this [teacher laughs] but …
2  T1  Okay?
3  S   Nine, Nine eighths plus one eighth plus six eighths. Basically from the first one, I get one whole and… [teacher writes equation]
4  T1  I like that.

It turned out he was the only student who used an improper fraction as one of the missing fractions and as a result, the other students were able to benefit from seeing a different way to complete the equation.
(b) Student-subject relationship

Some teacher actions can be interpreted as trying to make the subject less intimidating to students. For instance, in the excerpt below, after introducing the idea of division to convert fractions into decimals, the teacher (T3) asked students to guess the decimal for a ‘trickier’ fraction, that is $\frac{379}{800}$. The use of the term ‘tricky’ and asking students for guesses, which in essence is to estimate, seemed to make the activity more fun than procedural.

**Excerpt 17: MEL3 LO5 RD10**

So let’s do a tricky one. [Writes a fraction] Three hundred and seventy-nine over eight hundred? Anyone want to have a guess before Q tells us what that would be? Hmmm, E?

Similarly, in the high school class MEL4, the students learnt about Pythagoras and his ‘minions’. The teacher’s approach suggested that the intention was to help students like the subject. It was however not obvious what strategies the teachers might have used to help students develop particular attitudes or dispositions to learning mathematics, such as persistence and risk-taking.

(c) Student-student relationship

How students interact with one another is initially governed by classroom rules that dictate how they should behave toward one another. Fostering student-student relationship is important, as positive and supportive peer relationships mean students will help and support one another in times of need. This relationship can be a source of support when faced with challenging questions posed by teachers. Providing opportunities for students to work in groups may be a way of fostering this relationship and in all the classrooms, it was observed that students had opportunities to work in groups. The evidence of this effort to foster student-student relationship can be observed whenever teachers asked students to help another who could not respond to the questions posed by the teachers (e.g. Excerpt 4, line 7, p. 36). It was also observed when students feel comfortable enough to disagree with one another (see Excerpt 15, p. 47).

- **Accountability to other students**

  This refers to showing respect for other people’s views. It also means that students “cannot purposely ignore the relevant work of others without justification” (Resnick & Hall, 2001; as cited in Engle & Conant, 2002). In two classrooms (MEL1 and MEL3), it was observed that teachers tried to foster students’ responsibility in this aspect by getting students to rephrase what
others had said (see Excerpt 10, p. 42). In this way, the children in the class not only had to think about the question posed by the teachers, they also had to pay attention to what their classmates were saying. Another example was found in RD8 when students had to make sense of another student’s response.

4.3.4 Cognitive conditions

(a) Use of a task to anchor the discussion

In all cases of rich discussions identified, it is observed that the teachers used tasks to engage students in discussions. These tasks can be part of textbook exercises or something teachers came up with. High school teacher, T2, got the students to work independently to draw right-angled triangles with the same \( \theta \) but with varying side lengths and then to calculate the ratios of the side lengths, i.e. sine \( \theta \), cosine \( \theta \) and tangent \( \theta \). In theory, given that \( \theta \) is the same for all the triangles, sine \( \theta \), cosine \( \theta \) and tangent \( \theta \) for all the triangles should be identical. However, students did not obtain that result. That prompted a discussion on the possible sources of error that could have contributed to the inaccuracies in their answers.

Using tasks to facilitate or anchor discussions has emerged as an area of significant research interest as conceptions of good practice move from teacher-centred to student-centred (Stein et al., 2008). However, the tasks which researchers were interested in were those which were “challenging and open-ended” (e.g. Schifter, 2001), “cognitively demanding” (e.g. Henningsen & Stein, 1997) or had “multiple solutions” (e.g. Lampert, 2001; as cited in Stein et al., 2008) as these were perceived to “promote conceptual understanding and the development of thinking, reasoning and problem solving skills” (Stein et al., 2008).

While I see the value of using such ‘challenging’ tasks in the classroom, the tasks which the four study teachers used could not be put into the same category. Their tasks were not consistently open-ended, for example, or even consistently cognitively demanding. Only in the elementary school classes were open-ended tasks actually used at all. There can be many ways to view this. It can mean that teachers rarely use open-ended tasks or the topic they were teaching (i.e. fractions for the elementary schools and trigonometric ratios for the high schools) did not facilitate the use of such tasks. If teachers rarely use challenging tasks, then we should investigate how they use more routine tasks to stimulate rich discussion on a day-to-day basis. I feel that there is value in having routine tasks which require students to talk about mathematical concepts and procedures, as
talking about them can assist students “in organizing what they already know into larger and more powerful conceptual structures”, i.e. make connections (Chapin et al., 2003).

(b) Teacher knowledge

It was noted in this study that teachers asked very perceptive questions related to the tasks, which required a little extra effort or thinking on the part of the students. The questions seemed to indicate that the teachers had good knowledge of the tasks they had presented to their students; and had anticipated the student responses. Examples of such questions are “What could have caused the error in the ratios?” and “Between what whole numbers would we have no mixed numbers?” In some instances, from some of the students’ responses and the relatively long pauses after the questions were asked, it would appear that the students had previously never considered such issues. The questions allow teachers to check if students have understood the content presented and get them to think beyond the information presented in the textbooks. Students had to understand what they had been doing thus far, and synthesize various pieces of information to respond to the questions.

Effective questioning has always been associated with open questions so teachers are encouraged to ask more open questions as opposed to closed ones in their classrooms (Godinho & Wilson, 2004). However, the questions asked by the study teachers were not always open. In fact, they vary in difficulty according to context, and build from lower level questions up to the discussion questions. Hence, questions that trigger discussions need not be open but they were of a level above students’ current understanding so that students needed to exercise their reasoning skills to close that gap. It takes a skillful teacher, who is attuned to curriculum goals and the level of student understanding, to sieve out the pertinent questions to ask. The ability of the teacher to ask the ‘right’ questions, and hence facilitate a rich discussion, would depend on factors such as his knowledge of the subject matter (Ma, 1999; Schifter, 2001), his pedagogical content knowledge (Barnett, 1991; Baumert et al., 2010) and his understanding of curriculum goals (Sherin, 2002).

4.4 Overview of Chapter

In this chapter, taking incidents of rich discussion as the focus of the investigation, the moves used by the study teachers were analysed. These moves reflect the broader
intentions of the teachers to promote student agency, facilitate student reflection and continually refine their own practice. While these moves have been discussed with respect to one teacher’s intentions, it is important to note that some moves may serve multiple purposes, and therefore their impact on discussions may be greater than they seem and more complex. In making the moves, the teachers created a learning environment that provided particular physical and socio-organisational contexts, and addressed the affective and cognitive needs of the learners. The findings suggest particular teacher moves that were conducive to the occurrence of rich discussions.
CHAPTER 5
DISCUSSION AND CONCLUSION
5.1 Introduction

This research project set out to look into how discussions can be successfully implemented in mathematics classrooms so that to whoever seated in the classroom, the discussion can be said to be rich. Hence, teacher moves during rich discussion segments were examined, and from these, the conditions for generating rich whole-class discussions emerged. This chapter discusses the factors responsible for creating the conditions, namely the teachers, the students and the classroom environment, and how these factors can be utilized to best effect so that rich discussions are always achievable. At the same time, issues that may impact the quality of discussions are also discussed.

5.2 Levels of rich discussions

In this study, after reviewing the transcripts of mathematics talk presented in literature and those that occurred in the study classrooms, one thing that stood out is the richness of mathematics discussions can vary greatly in classrooms. If mathematics discussions can be systemically described in terms of levels of ‘richness’, educators will find it easier to navigate the classroom discussions to higher levels.

Ultimately, rich discussions are dependent on many factors, other than student responses, as discussed in Chapter 2, and it may be difficult to distill the impact of individual factors’ contribution to the richness of discussions, as these factors are distinct yet inter-related. The rich discussions presented in this study may not be as ‘rich’ as those generated in experimented classrooms (e.g. Cobb et al., 1997), but what has been presented here is authentic and reflects the reality in the classrooms. If rich discussions can be classified into levels with distinctive features, and specific teacher moves and strategies for each level can be highlighted, what teachers have will be a practical and doable guide to help them make their discussions richer. This study has identified some of the characteristics of rich discussion, but further research is required to establish possible levels of rich discussion.

5.3 The teacher factor

In creating the conditions for rich whole-class discussion, the teacher plays a pivotal role. A lot depends on how the teacher chooses to respond after the students have responded, and how much the teacher understands of the student responses. Hence, I would like to discuss how the teacher impacts the discussion.
5.3.1 Teacher moves: Questioning or non-questioning?
While research has identified many questioning and non-questioning moves teachers make (see Chapter 2), there is a tendency for teachers to only make use of a small number of these moves. For example, adopting a neutral stand when students gave an answer was not a move popular with the study teachers. This is inevitable as everyone has a preferred style. More often than not, teachers used questioning moves rather than non-questioning ones to direct the flow of the discussion, and to get students to respond. In order for teachers to start using different moves, they have to first become aware of their preferred moves. Only then can they make refinements to their teaching, and get students engaged in rich discussions.

5.3.2 Scaffolding: How much?
This is a major dilemma faced by teachers, possibly every day. When students seem incapable of responding to questions, how much assistance or scaffolding should teachers provide? Study teacher T2, for example, made the decision to narrow his initial question of factors that could account for differences in the ratios of opposite/adjacent, adjacent/hypotenuse and opposite/hypotenuse obtained by students (see Excerpt 18) to one which seemed more manageable by students, i.e. asking them to recall the instructions that he gave them for the activity (refer to italicized lines 1 and 9).

Excerpt 18: MEL2 LO2 RD4

1  T2  What could we have, er, what could have introduced errors into our measurements here? Do you have any ideas, D? B?
2  B   Huh, like when you measure them, you might, you might not be accurate.
3  T2  What might not have been accurate?
4  B   The opposite
5  T2  The…
6  B   The opposite
7  T2  The…What, what would have been inaccurate about it? I mean, are we talking about you can’t read a ruler or are we talking about something else, about how you make the measurement?
8  S1  How you read it.
9  T2  I mean, I’m gonna…I’m gonna work on the assumption that you know how to use the ruler… so what about how we measure it then? What about a ruler that meant that we are not getting a good enough measurement? (long pause)

Any ideas? When I said to measure these [referring to first 2 columns of table], what, what instructions did I give you when I said to measure these?

---

4 This discussion took place after students constructed three similar right-angled triangles with the same angle θ, calculated the ratios of opposite/adjacent, adjacent/hypotenuse and opposite/hypotenuse for each one of them, and tabulated the results in a table.
From the flow of the discussion, it can be seen that he made several attempts to get students to clarify their responses, and hence it is highly likely that he decided to break down the initial question to a simpler one as students did not seem to be able to provide clear or the expected correct responses. The pauses in his speech and the way he kept correcting himself indicated that he was monitoring his actions and thinking about how to help students make connections with what they have done and his question. In this case, has the simpler question resulted in the lowering of student understanding? Perhaps, it has but it would be difficult for the lesson to proceed if students were not able to participate in the discussion. What this example shows is, at times it is very challenging for teachers to stimulate discussions when students do not have the prior knowledge (e.g. at the start of a new topic), and when students are giving vague responses. Rather than thinking on the spot, prior preparation by teachers would be more beneficial. Stein et al. (2008) advocates that teachers spend time anticipating likely student responses as one of the five practices that will help teachers facilitate discussions around cognitively demanding tasks. In the same respect, teachers too should anticipate how students are likely to contribute to a discussion, and prepare the responses they might make. This will make teachers feel better prepared for discussions and students are then more likely to benefit from the discussions.

5.3.3 Use of task: How challenging?

Excerpt 18 highlights another issue, that is, the importance of the selection of tasks. With the shift of agency from teacher to student, selection of tasks to build mathematical understanding has become a central activity for teachers preparing for lessons (Smith, 1996). From the rich discussion segments, it would appear that teachers can trigger discussions with the use of any task. However, from the questions the study teachers asked in the discussion of these tasks, it can be said with confidence that the teachers had given some thought to them. In so saying, it means that teachers have to have an intimate knowledge of the content of the tasks, and that includes possible student misconceptions in order to discern the right questions to ask. For instance, in Excerpt 19, the students in MEL4 had
computed using Pythagoras theorem and arrived at the answer of $\sqrt{35197}$. The teacher purposely asked students for the reason why some of them would stop there, knowing full well that this was a common mistake made by students. In bringing this common mistake to light, students could also reflect on whether they thought the same, and avoid such errors in future.

**Excerpt 19: MEL4 LO1 RD11**

<table>
<thead>
<tr>
<th></th>
<th>T4</th>
<th>S1</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Why would people assume that when they've got there, that they've answered the question? Often when you are under pressure, people gets there and go, &quot;yep, finish.&quot;</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Because there's no more…</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>[makes hand gesture to invite elaboration] things to plus or</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>yeah</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>because there's no more basic operations</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>because there's no more basic operations left. You've got one term on either side.</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>[responding to a student] No, you got it right. So what do I need to do? Someone tell me the square root of thirty-five thousand, one hundred and ninety-seven. (Writes $\sqrt{35197}$)</td>
<td></td>
</tr>
</tbody>
</table>

I would argue that with the current emphasis on student-centred pedagogies, teachers should expand their thinking beyond anticipating student responses to the tasks. They should also consider how they intend to attend to student responses in order to generate rich discussions. This goes beyond preparing responses to possible student responses. It is about how to leverage on anticipated student responses to get students to generate rich discussions. The activity whereby the teacher (T1) categorized student responses and got students to explain how their responses had been categorized led me to think about how important teacher actions are. I do not refute how important it is for teachers to anticipate student responses (see Stein et al. 2008). I just think it is equally important for teachers to think about how they intend to use student responses effectively for rich discussions. If the above-mentioned teacher had simply listed the students' responses one by one, there might not have been anything to discuss at all. This goes to show that a simple activity can generate rich discussions just by teachers using a different way to handle student responses. As "no rules can specify how to manage and balance among competing concerns, teachers must be able to consider multiple perspectives and arguments and to make specific and justifiable decisions about what to do” (Lampert, 1986; as cited in Ball, 1993). It would thus benefit teachers to engage in professional discourse with other teachers about how they would approach and structure a task so as to get students thinking.
5.3.4 Telling: When?

None of the study teachers began lessons by lecturing or teaching mathematical concepts explicitly. It is observed in the rich discussion segments that the study teachers tried as much as possible to elicit these ideas from the students (e.g. Excerpt 20). In Excerpt 20, the teacher (T3) tried to elicit how to convert a fraction to decimal from the students. However, after three failed attempts, the teacher had to tell students how it was done.

Excerpt 20: MEL3 L05 RD10

1. T3 Does anybody know...I’ll just finish these off now...does anybody know how you work out the decimal...let me...next time hand up, H. Just say for example, we have a really hard fraction. [writes a fraction, 327/900] Three hundred and twenty-seven nine hundredths, what’s the decimal for that? Does anyone know how you work out a decimal? From any fraction? O?

2. O Er, you take the numerator and then you pop that uhm behind the decimal on the right hand side.

3. T3 No, but good try, they don’t all work like that. Some of them do work like that. That’s why we get tricked. Nobody knows. M?

4. M I forgot

5. T3 [laughs] Okay. P?

6. P Don’t you put, put like zero point three two seven?

7. T3 No, that would have to be over [cleans away “9”] one thousand because one thousand has three zeros, so then you would have three decimal places. [writes 327 and adds decimal in front of two] Alright, well I’ll tell you how we do that. U, do you want to have a guess?

8. U Yep, uhm, isn’t it when you, when you’ve got two zeros or like, you count from two numbers from the like (...)?

9. T3 Yea, but we’ve only been looking at [writes 4/10=0.4] tenths and hundredths [writes 4/100]. What’s the decimal for four hundredths, W?

10. W Zero point zero four.

11. T3 So what we know is that if you have one zero, your decimal has one decimal place [underlines the zero and decimal place]. If you have two zeros [underlines zeros in a hundred and decimal places], your decimal has two decimal places. [writes 4/1000] What about four over one thousand, four thousandths? R?

Even though the study teachers created opportunities for students to participate in the discussions, they still talked a lot. By talking, I mean that after student responses, teachers tended to spend time elaborating on student responses, possibly because student responses were still not to their satisfaction in terms of clarity. There were parts in the discussions when teachers had to resort to telling the answers. These situations occurred usually when students failed to provide them with the expected answer [e.g. Excerpt 21]. In this instance, the teacher was looking for a particular answer and after three students failed to come up with that, he was compelled to tell the class the ‘correct’ answer. Similarly teachers had to ‘tell’ the answers when students experienced difficulty expressing their ideas.
[e.g. Excerpt 22]. In Excerpt 22, other students were not even given the opportunity to attempt answering. What if there were students who could make sense of the answer obtained? The teacher would never find out. Fortunately, teaching by telling is not a norm in the study classrooms as “when teachers teach by telling, students tend to assume the roles of passive recipients of information, and move away from the active involvement in mathematical discourse and practice that will supposedly help them learn mathematics with understanding” (Baxter & William, 2010, p. 8) (see also Lobato, Clarke, & Ellis, 2005). However, teachers need to be aware of their tendency to tell, if any, and consciously work to control the urge to do so, in order to give students the opportunity to contribute to discussions.

Excerpt 21: MEL1 LO1 RD1

1  T1  Now, in what way? Who can, in their own words, tell me, in what way percentages and decimals are like fractions? [3 hands come up] How are they like fractions? What do they both have...what do they all have in common? [More hands come up] J?

2  J  You can change...you can change them all... you can change them...you can change a fraction into decimal or into a percentage and it still won't change. It still stays the same.

3  T1  With...with some fractions, percentage and decimals, you can switch them around and change them into one another but that's not what they all have in common...You are absolutely right, J but that's not the main feature that they have in common. N?

4  N  They have like...they can have quarters and stuff.

5  T1  Oh, that's fractions can have quarters but yes, you can have quarters represented as a decimal and a percentage. One more and then I am going to clue you in. A? [pointing to student]

6  A  Uhm...

7  T1  I can see your hand up.

8  A  If I put them, tenths and hundredths, it's easier to...uhm...put them into, like turning fractions into decimals, percentage, it's easy.

9  T1  Okay, so they...you think they are similar because they are easy. Fair enough. What I would have said is that they are all...this is one of the first things we teach in grade 3 about fractions...they are all smaller parts of a whole. So a decimal is a smaller part of a whole. So is a fraction and so is a percentage. Okay?

Excerpt 22: MEL4 LO1 RD15

1  T4  Eight point nine. Alright, so what does that eight point nine mean?

2  S1  It is like...

3  T4  What is it? It's a number. It's not the gradient. It's the hypotenuse so it's the distance between this point and this point. [labels on triangle] Yeah? Is everyone okay with that? So how did I get the eight again? What was the eight? Shhhh. What was the eight? Where did I get the eight from?

5 In this part of the discussion, students and teacher applied Pythagoras theorem on two sets of coordinates and obtained the answer of 8.9. The teacher wanted to check if students were able to make sense of the answer they obtained.
5.3.5 Teacher Noticing: Sensitivity to new information?

When confronted with a “blooming, buzzing confusion of sensory data” in the classroom, teachers constantly have to sieve through the information and make split-second decisions on what to attend to (Sherin & Star, 2011). However, at the back of minds, they still carry the burden of achieving the curriculum goals. Hence, it is inevitable that teachers may miss or choose not to attend to some information. In highlighting situations where the teachers may have missed opportunities to deepen the discussion, it is not to fault the teachers. Rather, it is to emphasize the importance of increasing teachers’ sensitivity to such opportunities. The next excerpt is such an example.

Excerpt 23: MEL1 LO2 RD3

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>M</td>
<td>Ah, ninety-nine upon a hundred</td>
</tr>
<tr>
<td>2</td>
<td>T1</td>
<td>[writing] Ninety-nine hundredths</td>
</tr>
<tr>
<td>3</td>
<td>M</td>
<td>Plus ninety-nine hundredths</td>
</tr>
<tr>
<td>4</td>
<td>T1</td>
<td>[writing] Plus ninety-nine hundredths</td>
</tr>
<tr>
<td>5</td>
<td>M</td>
<td>Plus, plus another ninety-nine hundredths</td>
</tr>
<tr>
<td>6</td>
<td>T1</td>
<td>Alright, that won’t give us, that won’t give us erm two, two wholes [erasing the equation]</td>
</tr>
</tbody>
</table>

The teacher had asked the students to come up with equations that satisfy the following condition: \( \frac{?}{7} + \frac{?}{7} + \frac{?}{7} = 2 \). When the students gave their equations, he did not simply write them down. Certain equations were in the same column while one was not. His actions showed that he had noticed the similarity, and that was why they were put under the group. The two types of responses he received from the students are shown in Table 5.

Table 5: Types of student responses

<table>
<thead>
<tr>
<th>Type 1 response (involves first and second fraction adding up to one whole)</th>
<th>Type 2 response (involves use of improper fraction)</th>
</tr>
</thead>
<tbody>
<tr>
<td>37/100 + 63/100 + 100/100</td>
<td>9/8 + 1/8 + 6/8</td>
</tr>
<tr>
<td>10/30 + 20/30 + 30/30</td>
<td></td>
</tr>
<tr>
<td>3/4 + 1/4 + 4/4</td>
<td></td>
</tr>
<tr>
<td>10/100 + 90/100 + 100/100</td>
<td></td>
</tr>
</tbody>
</table>

In the midst of eliciting the equations from students, student M gave an equation that was very different. His equation was 99/100 + 99/100 + 99/100. However, his equation was constructed wrongly, and as a result, not accepted by the teacher. If he or the class had been given a chance to correct the equation, it could have expanded the class’s knowledge of ways to add up to 2. Hence, anticipating student responses prior to the lesson is important but during instruction, teachers have to remain open or sensitive to possibly other responses.
from students, so as to take the discussion to a higher level. This will also involve being flexible and willingness to make modifications to the instructional plan.

5.3.6 Teacher Content Knowledge in relation to Teacher Noticing

There are researchers (Ball, 1993; Schifter, 2001) who suggest that content knowledge is not everything; teachers need other skills or resources to help students learn with understanding. Schifter (2001) suggests that teachers need to learn to attend to the mathematics in what students say and do, assess the mathematical validity of their ideas, listen for the sense in students’ mathematical thinking even when something is amiss, and identify the conceptual issues on which they are working. Teachers need to develop such skills because as Ball (1993) stresses “it is very difficult to figure out what some students know or believe – either because they cannot put into words what they are thinking” or because the teacher cannot keep track. The skills they allude to are similar to the construct of noticing which has been discussed in Chapter 2, Section 2.4.5. Ball, however, does not deny the importance of content knowledge. The teacher with a high level of content knowledge is likened to a guide with an expert knowledge of the terrain, who will then be able to perceive what students say “both from the perspective of the expert (to know what there is to know and learn to do) and from the fresh perspective of the learner (to see familiar as strange)” and “students’ thinking will go unnoticed by someone who does not know the terrain” (Ball, 2011). As such, teacher education should still focus on increasing teachers’ content knowledge. This is particularly important for elementary school teachers, who are generalists, even though the mathematics content they are teaching is of a lower level. My personal experience with elementary teachers, for example, shows that many of them do not know how to attend to different solutions to a mathematical problem, beyond what is presented in the textbooks.

5.3.7 Assessment for Learning: Possibilities in whole-class discussions

In recent years, assessment for learning has gained in importance in many countries, partly due to the work of Black and Wiliam (see Black & Wiliam, 1998). The move has influenced a re-examination of schools’ assessment system, and in particular, the type of feedback students receive that will move them forward in their learning. Hattie’s synthesis of more than 900 meta-analyses has also shown that feedback has more impact on learning than any other general factor (Hattie, 2009, 2012b). Hence, there is a need to ensure that students are provided with good feedback.

Looking back at the study teachers’ feedback to students during the rich discussion segments, it was noted that most of the feedback given were very general. The teachers used words or short phrases like ‘good work’, ‘correct’, ‘well done’, ‘fantastic’, ‘wonderful’ and
‘clever’ to signal their approval of student responses. Sometimes, it was given at the end of the discussion as a signal to how well the class has done. When put in context, these comments seemed to be value judgments, which “rate, evaluate, praise, or criticize what was done” (Wiggins, 2012). There is also a danger that the seemingly encouraging remarks or gestures such as “Good girl” (e.g. Excerpt 4, p. 37) might be perceived negatively by students, especially as they seem to be expressing “evaluations and affects” about students as a person of a certain gender, which goes against the teachers’ intentions to boost students’ self-efficacy or self-identity through the use of positive remarks or gestures (Brophy, 1981; as cited in Hattie & Timperley, 2007).

Researchers have offered similar views with regard to effective feedback, that it should provide students with the following information: where the student is going (the goal), where the student is in relation to the goal (the gap), and finally what the student has to do next to reach the goal (the way forward) (Brookhart, 2008; Hattie, 2012a; Wiggins, 2012). In relation to these features, feedback should therefore be task specific, timely and actionable (Brookhart, 2008; Hattie & Timperley, 2007; Wiggins, 2012). As discussed, the feedback that have been observed during the rich discussions do not exhibit these features. Perhaps, the structure of whole-classroom discussions does not allow effective feedback to be given to students; rather the feedback would be more for teachers on the effectiveness of their instruction. Another reason can be the focus of teachers during whole-class discussions was on facilitation of the discussion more than on providing feedback to students. The flow of discussions might also be disrupted if more time was spent on providing feedback to an individual. Hence, I question if it is possible for teachers to provide quick effective individual feedback during rich discussions, and if it is possible, how teachers should phrase their feedback. More research will help to clarify how feedback can be maximized during whole-class discussions. With respect to providing feedback to students about how they are doing as a whole, it was observed that the rich discussions ended without a summary of what students have accomplished thus far. A short summary at the end of each activity segment, in this case a discussion, may help students focus on what they are supposed to have learnt.

5.4 The student factor

Students contribute to the discussions, and how they do so in a way impact the quality of the discussions. Hence, it will be beneficial to look at how students can be better supported so that there are richer discussions.
5.4.1 Student responses: Why are they so brief and uncertain?

In the rich discussion segments, it was observed that compared to students in the elementary classes, student responses in the high schools were noticeably briefer (cf. Excerpt 18, p. 55 and Excerpt 21, p. 59). Students also had difficulty expressing their ideas, let alone trying to justify them (e.g. Excerpt 22, p. 59). In their study of mathematics talk, Hufferd-Ackles et al. (2004) has found that mathematics talk usually fall to a lower level whenever new topics are introduced so it is possible that students had difficulty expressing themselves because the vocabulary was new, and they had not mastered them fully to use them with confidence. Hufferd-Ackles et al. (2004) also argue that “students must have a grasp of the domain of mathematics to carry on math talk both to describe one’s own thinking question or extend the work of others” (p. 111). This indicates that students need lots of opportunities to talk, not just in whole-class discussions, to gain mastery of mathematical language before the quality of their contribution can improve.

Another observation is student responses seemed quite tentative (e.g. Excerpt 19, p. 57). In some instances, upon further questioning by teachers, students became unsure of their responses. One particular instance caught my attention. It was not the intention of the teacher (T1) to question the student’s response. The teacher simply had not caught what the student had said. A simple “pardon?” from the teacher caused the student to doubt his response and subsequently alter it. Even though the students had good rapport with their teachers, it seems they were still inhibited to speak freely, and fear making mistakes. Rowland (1995) refers to these uncertainties in student responses as ‘hedges’, and students deploy these hedges as a shield against being wrong in the face of intellectual ‘risk’. He suggests that the prevailing school-culture of “maths is about right and wrong answers, and it is much better to be right” to be responsible for students’ use of these hedges. Teachers being seen as the authority for knowledge, “the spirit in which the mathematics learning takes place” and the way teachers respond to student contributions (by language and body) contribute to building that culture (Rowland, 1995). In the case of the study teachers, I would think that the teachers need to move away from being the authority for knowledge in the classroom, and shift more of the responsibility to the students, for example, getting students to comment on one another’s responses. In this way, the class as a whole can also develop further as a community of mathematics learners.

Some researchers have also attributed the quality of student participation in group work to lack of conversation skills, and suggest that students need to be taught explicitly what they have to do in response to what their fellow students said (Zwiers & Crawford, 2011). These conversation skills or discursive moves are generic and therefore applicable in all classroom
situations. While teachers may model such skills, explicit discussion of such skills, I believe, is necessary so that students are aware that they can actively practise the skills.

5.4.2 Student questions: Is rapport and a safe environment enough?

Student questions during the rich discussion segments are few and rare. If there are, they are more related to how they should approach a task set by the teachers. This is consistent with research findings that indicate that teacher questions are still dominating classroom discourse (Almeida, 2012). We want more student questions because questions are indicators that students are trying to make sense of new information, that is, trying to accommodate or assimilate new information within their prior knowledge, and realizing that there are gaps or discrepancies in their knowledge. Student questions therefore “can be very revealing about the quality of students’ thinking and conceptual understanding, their alternative frameworks and confusion about various concepts, their reasoning, and what they want to know” (Almeida, 2012, p. 635). Besides this, student questions can also feed forward to teachers in terms of lesson preparation and the effectiveness of their instruction. The study teachers did open the floor for student questions, mostly at the end of the discussion when they have achieved their curriculum goals. However, the take-up rate was not high, and instances where students interrupted in the midst of a discussion to pose questions were not observed. Perhaps, whole-class discussion is not the best activity to see student questions for reasons such as students are shy and inhibited to highlight their ‘incompetency’ to others. To encourage student questions, Godinho and Wilson (2004) suggest that teacher and students work jointly to identify “behaviours associated with a classroom culture that values questioning.”

5.5 The environment factor – building a culture of questioning

By this, I am referring not to the physical environment but to the socio-emotional environment and to the culture of learning in the classroom. Even though students feel rapport and safe with the teacher, it does not necessarily mean students will ask more questions; it may just be an indicator that the students are ready to ask questions. Students may not know that asking questions is an expectation of the teacher. For example, in the case of the study classrooms, it is not clear student questioning is an expectation. In communicating teacher expectations, the teacher has to then nurture the behaviours that meet the expectations. For example, to encourage students to ask questions, positive acknowledgements have to be given consistently to students who raise questions, even if their questions divert the focus of the discussion, so that students know that their questions are valued (Godinho & Wilson, 2004). Creating spaces for student questions, like what the
study teachers did, is another possibility but when students are not responding in kind, teachers may have to explore other means to get students to raise their questions. Building a culture of any sort takes time and effort, and the responsibility rests on the teacher (Baxter & Williams, 2010). Hufferd-Ackles et al. (2004), for example, see their research classroom develop from a level 0 math-talk community to a level 3 math-talk community over a period of several months, and they attribute this ‘rapid change’ to be the result of intense support given to the teacher by the school administrators and the researchers.

5.6 Dilemmas in Teaching and Learning

I would like to address some of the issues I see in the classrooms. These issues have been discussed by many researchers so they have been around for a long time but I am not sure if we have found a way to resolve them. In reviewing these dilemmas, I want to highlight the complexity and the dynamism of teaching. Unlike assembling or operating a machine, there is no manual which teachers can refer to that will give them step-by-step instructions on how to facilitate a specific discussion topic, for example, and the right moment to do the right thing. What teachers have are generic guides with recommendations on what they can do, for example moves that will help them orchestrate discussions. Although there is prior preparation, teachers still have to be constantly on the alert to decipher the myriad of information they receive from students, and decide on the spot whether to proceed as planned. Hence, teachers struggle with these dilemmas each day, and often are left to wonder if they have made the right decisions. With the push for greater teacher accountability, teachers today have to know how to handle these dilemmas all the more.

5.6.1 Student agency versus discipline accountability

With the reforms in education, mathematics curriculum in many countries have been revised to recommend that students studying mathematics should be engaged in activities such as “making conjectures, abstracting mathematical properties, explaining their reasoning, validating their assertions, and discussing and questioning their own thinking and the thinking of others” (Lampert, 1990b). This leads to the dilemma of how much authority teachers should give to the mathematical conventions and knowledge. The issues teachers grapple with include to what extent students can participate in constructing and inventing mathematical knowledge and what they should do if students' constructions are wrong (Russell & Corwin, 1993). Ball (1993) has also documented her own struggles to develop “a practice that respects the integrity both of mathematics as a discipline and of children as mathematical thinkers”. In this age when student-centred instruction is widely encouraged,
the challenge for the teacher is therefore how to strike a balance between student authority and accountability to the discipline (Stein et al., 2008, p. 332).

5.6.2 Student agency versus teacher authority

The social constructivist viewpoint of learning means there is more emphasis on how to help students construct mathematical knowledge through engaging in mathematical practices (Baxter & Williams, 2010; Chazan & Ball, 1999; Lampert, 1990a, p. 256). Hence, mathematical discourse, in the form of whole-class discussions, is one of the ways to achieve this. However, with the social constructivist viewpoint, teachers have to assume new roles, that of facilitators of knowledge, rather than providers of knowledge. This gives rise to the dilemma of teachers - when to step in and tell students the answers, and when to let students struggle with a difficulty. Ultimately, it is the responsibility of teachers to ensure that their students learn. The study teachers provide us with a case where student agency is highly promoted and valued but they are still the authority figures of knowledge in the classroom; their students look to them as providers of the correct answers. In facilitating whole-class discussions, they also exercised a fair bit of control over the direction of the discussions. There is obviously a conflict in the two constructs, student agency and teacher authority (see Lobato et al., 2005). Cobb et al. (1993) has said that it is not surprising that the teacher actions are often in conflict as they often have to serve multiple functions. We do not know why the teachers chose to control the discussion. It may be due to time constraint or pressure to complete the syllabus. We also did not follow the teachers long enough to observe any change in their style of facilitating discussions. In view of the data collected, it is assumed that this is what the teachers usually do, and the question therefore is: How do we help teachers feel more comfortable to let go of the reins to pursue important mathematical ideas in light of student responses or interests? Teachers may also wish to make use of other means to nurture the development of student authority. Stein et al. (2008), for example, advocate the use of cognitively demanding tasks with more than one valid mathematical solution strategy to do so.

5.7 Strength of Study

The strength of this research project lies primarily in the fact that the analysis and findings for the project arise from the examination of real classroom footages and not from indirect proxy measures. While I have referred to literature to support my analysis and findings, I was mindful not to become overly dependent on or constrained by the literature so that I was not open to other observations or interpretations.
5.8 Limitation of Study

While research have shown that exposure to high-level tasks will enable students to develop high-levels of thinking, it is not possible in this study to say that the teachers do not present their students with high-level tasks as it only looks at teachers teaching only one topic within the mathematics curriculum. It can only be said that high-level tasks may not be used in every topic. There may be a need to look into the frequency at which teachers use high-level tasks in actual classroom settings. How often should students be exposed to high-level tasks for it to have an impact on their levels of thinking?

While I have tried my very best to give an accurate reading of the teachers’ intentions, it is possible that my interpretations may not be entirely correct. My status as a non-participant observer carries both advantages and disadvantages. Relative lack of familiarity with the norms of a community can restrict some interpretations while simultaneously provide a fresh outsider’s perspective that may reveal unnoticed norms of practice. As this is a secondary data analysis, it is also not possible to confirm with the teachers. However, given that teachers make split-second pedagogical decisions daily, unless they are stopped on the spot to question their intentions or purpose, any interviews after the lessons may also not fully capture or reveal their original intentions or purposes.

My Asian background may also have influenced my perceptions or interpretations of what went on in the classroom. As opposed to Asian classrooms, I expected Western classrooms to be more student-oriented, and therefore it is possible that I gave disproportionate weight to instances of student interaction, anticipating that these would characterize practice in those classrooms. Since ‘rich’ discussions are the focus of my analysis, the exaggeration of student interactive elements is not so much of a concern. I am not typifying classroom practice; I am identifying key features of rich discussion.

5.9 Implications for teacher education and teacher mentoring programmes

As highlighted in earlier sections, the teacher plays a critical role in stimulating rich discussions. However, much of the research findings in the area of mathematics talk, whereby the students were observed to make rapid transformations in their talk, come in the light of intensive support given to the project teachers. The methodology adopted for the research projects involved the observation of the classrooms before, during and after an intervention. Hence these teachers had workshop sessions to improve their content knowledge, the support of school administrators, and the practised eye of the researchers to help them improve their practice (see Hufferd-Ackles et al., 2004; Russell & Corwin, 1993). Not every teacher has the good fortune to participate in such research projects, and receive
expert advice on their practice when they are already in schools. The heavy demands of school work also do not allow teachers much time to reflect after every lesson. This is where the importance of teacher training comes in. Ensuring that beginning teachers leave with strong content knowledge should be the aim of every teacher preparation programme. This cannot be compromised. Equipping student teachers with the knowledge to stimulate discussions is another, but it is perhaps the simplest thing to do. The practical sessions in actual classrooms should be the most critical aspect of teacher education. During school practicum, feedback given to the student teachers should include how they leverage on student responses, and their preferred moves during whole-class discussions. Teaching supervisors or mentoring teachers can work with them to increase their repertoire of teacher moves, and question them after lesson observations as to why they make certain moves so as to increase their sensitivity to student responses and the opportunities for rich discussion. Based on the discussion, student teachers can be asked how they intend to follow up with their observations and interpretation of student thinking. A reflective journal can be kept to document their learning. Teacher education should focus on deepening student teachers’ thinking about their practice and heighten their sensitivity to student responses. Leinhardt and Steele (2005) share a similar view. Their study of how a teacher used instructional dialogues in the teaching of mathematics shows that reflections of the types described above were often the “impetus for activities, questions, or probes that drove the dialogues in the next lesson.”

In schools, professional development sessions can get teachers to reflect on issues such as teacher moves that can help them leverage on student responses during discussions and their approaches to deal with a topic. These conversations build teacher competency by making them more aware of their practice, and send key messages such as the importance of listening keenly to student responses, and interpreting what they actually mean, as well as how else they can structure learning tasks so as to generate rich discussions. If resources allow, schools should also explore the option of letting teachers videotape their lessons for professional development.

5.10 Conclusion

Rich discussions in the classroom can occur in every classroom. However, this research suggests that their occurrence is greatly dependent on a few critical factors. These factors, as we have discussed in this chapter, in turn, are very much dependent on the teacher. For rich discussions to take place, teachers have to consider how they intend to use mathematical tasks to build up to the discussion question so that the students have the
knowledge to participate in the discussion, how to elicit from students their ideas and understanding, and finally how to increase the quality of student responses. Creating a safe environment where students feel no fear of speaking up and building classroom cultures of questioning and accountability to other students are important tasks teachers have to embark on for rich discussions to have the chance of taking place. While researchers have surfaced teacher moves that can help to orchestrate discussions, teachers still have to pay keen attention to what they hear from their students and make on-the-spot decisions on when and how they should lead the discussions. These decisions may have impact on the quality of the discussions.

Here, I showcase a short lesson segment where all the features of rich discussion are clearly visible. This illustrates the type of classroom practice that this research has targeted and seeks to promote.

**Excerpt 24: Lesson segment with all features of rich discussion**

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| 15 | T | [smiles and nods] Ah but you are doing a great job though. I know what
you mean. So, what N is trying to say is? F?

F Uhm...you got to find, like, like the common number they can make together, the lowest...(interrupted by teacher)

T We call it a common denominator [writing the words next to the 24]

F And you got to see the lowest one you can make.

T Yes, and the lowest one. [writing 'lowest'] If you don’t find the lowest one, you can simplify the fraction at the end but it is helpful to find the lowest. One way you will find the common denominator is to multiply because that is looking at the factors of 3 and 8 (pause) but is that the lowest? 8 goes into 8 (pause) evenly. Does 3 go into 8 evenly?

S Not really

T No, so 8 can’t be our lowest common denominator. The next number that 8 will divide equally into is, P?

P 16

T 16. Does 3 go evenly into 16?

(murmurs heard from students)

T No! So I would say it probably is 24, yeah?

C Use what times 8...

T What times 8? And then what will I write up here? [pointing to the fraction with denominator 24]

C Uhm, 8

T Are you putting your hand up to disagree? Okay, we will listen to what Z has to say and we might decide which way to go, C, alright? Z?

Z 24 divided by 8 is 3. 3 times 3 is 9. And if you divide 24 by 3 is 8 so 1 time 8 is 8.

T That's exactly what you said, C. A different way to get there. So, 3, we talked a little bit about this yesterday...I will just wait for everyone to put their fraction on the floor again. Thanks, X.

Teacher education can prepare teachers with the theory of facilitating discussions, but greater focus should be given to teaching practicum when student teachers put into practice what they have learnt. These practical sessions in schools should focus on developing in student teachers greater sensitivity to student thinking, and get student teachers started on reflecting on their own practice. It should never be assumed that these student teachers will develop the skills as qualified teachers in schools. When these student teachers go into schools, they will be faced with many other pressures of a teacher’s job such as administrative duties, and continuing to reflect on and keep track of their practice will not be easy. They will also not have the practiced eye of their teaching supervisors to guide them. One possibility would be to have a third party observe lessons to give constructive feedback. While this may be welcomed by beginning teachers, experienced teachers in schools may not feel comfortable with other people in their class. For such a practice to be initiated
school-wide, it would be necessary to create a school culture of collegiality and trust among the teachers. Allowing teachers to videotape their lessons for their own reflection or for discussion with other teachers is an option schools can explore, if resources allow. There is a widespread recognition that professional development in schools should incorporate opportunities for all teachers to reflect on their practice; for example, to consider their preferences for certain moves or pedagogical approaches during professional development sessions. Ensuring that teachers have meaningful conversations related to their practice will ensure greater consistency in instructional practice and thereby in the types of learning experiences students are exposed to in the classrooms.

Last but not least, teachers are torn between progressive, child-centred methods and formal, traditional methods of instruction daily. School administrators should understand the dilemmas the teachers face. Perhaps, the issue should not be a choice of one over the other. Rather, it is about providing students with a balance of different kinds of opportunities and guidance as Mercer (1995) and Lobato et al. (2005) have rightly pointed out. As have been illustrated by the findings reported in this study, central to all the preceding dilemmas are the nature, conduct and purpose of classroom discussion. The characteristics of rich discussion that have been identified and empirically instantiated in this study can serve as benchmarks against which to evaluate one aspect of quality instructional practice. Further, it is hoped that the identification of the conditions by which such rich discussions were initiated will inform teacher education programs and, ultimately, teaching practice.
References


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Author/s: Sing, Siew Hoon

Title: Creating the conditions for rich teacher-led whole-class discussions

Date: 2013

Citation: Sing, S. H. (2013). Creating the conditions for rich teacher-led whole-class discussions. Masters Research thesis, Melbourne Graduate School of Education, The University of Melbourne.

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File Description: Creating the conditions for rich teacher-led whole-class discussions

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