COMPUTER SIMULATION MODELS FOR
THE GRAVITY FLOW OF ORE IN
SUBLEVEL CAVING
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SUBLEVEL CAVING

By

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ABSTRACT

In recent years a number of research centres have endeavoured to provide a mathematical model that indicates the chief characteristics of ore flow in the sublevel caving mining method. Optimization of the design parameters for sublevel caving has ensued with the objective of maximizing ore recovery while minimizing waste dilution. Past studies have been confined by two simplifying assumptions: that the region of flow be approximated by a simple mathematical function, and that the flow analysis is static, i.e., extraction is calculated by employing an idealized ore-waste boundary position for each ring.

Further investigation into models for ore flow has been stimulated on two counts.

1. full scale tests have indicated that a more complex description is required;

2. as a design tool, a dynamic analysis that monitors the displacement of the broken rock mass during extraction would yield more realistic recovery predictions under operating conditions, and enable variations on the method to be evaluated.

This study reviews the various formulations currently available, or being developed to describe the flow of granular material. A viable solution in a mining context would be three dimensional, time dependent, and incorporate stress conditions. Although workers in diverse fields have analyzed the problem no solution exists at present. The majority of studies are either empirical in nature, or deficient in one or other of the above criteria.

Two approaches are developed and implemented on a digital computer:

(i) stochastic flow model

(ii) empirical flow model based on the results of modelling studies, and full scale test.
Although a number of concepts introduced in these models remain to be verified, the validity of these models would be measured by their success or failure as a simulation tool in a mining environment. Typical results from these models are presented; the limitations of the models clarified, and the application of the models in mine design expounded.
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CHAPTER 1: INTRODUCTION

In the extraction of ore deposits, the increasing scale, and complexity of mining operations, and the uncertainty of mineral commodity markets have forced the mining industry to put greater reliance on computer based mathematical models.

A reflection of this trend is the development of models for mining systems that enable the mine planning engineer to simulate various mining methods, and evaluate and optimize significant decision variables.

Papers presented at the annual International Symposium on the Application of Computers and Operations Research in the Mineral Industries (APCOM) witness the growing acceptance and application of models for mining systems. Comparatively cheap computing equipment, and interactive graphics systems make design and simulation analyses possible on the mine site. Progress in this field will benefit from computer education for both mining undergraduates and mine management, but also the support of fundamental research into mathematical models, and computer applications at educational institutions. Specification of system parameters, and evaluation of system performance and prediction, by mine site personnel is an integral part of system development. The role of experience and human judgement should not be undervalued for the adage 'Garbage in, garbage out' rules, and computer results must be carefully scrutinized to establish their validity.

This study investigates the development and application of mathematical and numerical models for sublevel caving operations at Mt Isa Mines Ltd, the sponsors of this research project.

The project goals were:

Establish a mathematical model to simulate the entire "cave" of fragmented material produced by the extraction process at successive stages for a pillar extraction i.e

Derive particle flow relationships to describe the movement of particulate material
Consider the effect of extraction layouts, and the effect of a variety of extraction strategies eg changing drawpoint, cut-off grade

Produce recovery and dilution contours for various stages of extraction

Where possible, establish the validity of the approach taken using scale model or full scale tests where they are documented, or else conduct model tests.

Apply the model to mine scheduling, as a tool for establishing grade control, and drawpoint procedures.

After a review of the existing literature on sublevel caving and attempts to build on the established mathematical models, it was concluded that more fundamental research was needed to produce an adequate model for the flow of particulate materials. This thesis explores various solutions to this problem, although to date no comprehensive theory for flow has been established. This can be attributed to the lack of a quantitative understanding of ore characteristics, and flow behaviour under test conditions, and the difficulty in formulating a flow model for granular materials.

This does not imply a lack of research interest because a solution could be applied in a variety of scientific and industrial processes. Little research has been carried out with materials as coarse and lumpy as the fragmented ore characteristic of mining systems. Baldwin (1966) aptly summarizes the general situation:

"... research workers have generally been reluctant to embark on fundamental studies when the possibility of making any substantial advance in a reasonable time is remote".

The major preoccupation of this thesis is numerical modelling of the flow of ore, specifically in a sublevel caving context. Insufficient time was available to pursue in depth all the topics that are relevant to this work. However brief notes are incorporated for the benefit of any investigator who pursues
the line of approach developed in this thesis. Although computer programs* have been developed to model fragmentation and flow of ore, validation of the results has not been completed. The scope and limitations of existing validation techniques are reviewed.

The approach followed in the remaining chapters is:

a survey of literature reviewing previous attempts to formulate models for sublevel caving
(Chapter 2)

a survey of literature relating to the development of continuum, and discontinuum material models; stochastic flow concepts and solution methods; kinematic models and flow mechanisms for granular materials
(Chapters 3, 4, 5)

a model to simulate material displacement and density changes during blasting and fragmentation
(Chapter 6)

implementation of Monte Carlo Method for simulation of stochastic flow
(Chapter 7)

extension of previous sublevel caving mathematical models with the introduction of 'zones of loosening' to form an empirical flow model
(Chapter 8)

an evaluation of existing model validation techniques, their scope and usefulness for the computer models developed
(Chapter 9)

the implementation of computer programs developed; for mine design and simulation
(Chapter 10)

conclusions, and suggestions for further research
(Chapter 11)

*The computer programs developed were considered too bulky to include in this thesis, but are available as separate research reports.
CHAPTER 2: TRENDS IN SUBLEVEL CAVING ANALYSIS

2.1 Introduction

2.2 Basic Flow Notation (after Janelid & Kvapil)
   Extension of basic notation to sublevel caving

2.3 Sublevel Caving Mining Design Principles

2.4 Mathematical Model Studies

2.5 Simulation Studies

2.6 Full Scale Mine Testing

2.7 Scale Modelling Studies

2.8 Blasting and Fragmentation

2.9 Mine Operating Problems

2.10 A New Basis for Simulating Sublevel Caving
The aim of this chapter is to review the progress that has been made in research investigations into the gravity flow of ore in a sublevel caving mining operation, and to enquire into the methodology developed in these analyses.

2.1 INTRODUCTION

Sublevel caving has evolved as a mining method since the first applications in the U.S.A. in the early 1900's. Considerable improvements in design, and a better understanding of flow behaviour have been achieved in the past decade.

In essence, the method consists of driving a series of sublevels commencing at the top of the orebody. A starting slot is cut and then a series of ring patterns drilled and blasted, the broken ore being drawn off after each blast. The overlying waste capping must cave and follow the broken ore in the drawing off procedure. The method allows development, drilling, blasting and loading operations to be carried out on separate sublevels simultaneously.

Mechanization has greatly increased the capacity of the method, and rationalized mine design by allowing larger and fewer development headings.

Sublevel caving has been applied to orebodies, both strong and weak, massive, steeply inclined and tabular. The chief disadvantage is the inevitable waste dilution, and incomplete ore recovery, though these effects are minimized by careful mine design, and draw control procedures.

2.2 BASIC NOTIONS OF FLOW

Until recently mine design relied upon trial and error methods.
Janelid and Kvapil (1966) presented the first clear account of gravity flow of ore in sublevel caving and provided a theoretical basis for sublevel caving design.

The basic notions upon which 'laws of gravity flow' were established derive from their observations of the flow of granular material in a bunker.

Free discharge of material from the outlet at the base of the bunker sets part of the material in the bunker in motion (termed the active zone), while the remainder of the material is immobile (termed the passive zone).

Figure 2.1 illustrates the flow of cohesionless material in a bunker. When a volume of material \( V_N \) runs out, a volume of material \( E_C \) is disturbed. In addition the volume of material \( V_N \), originated from the volume \( E_N \). Janelid approximated these shapes as elongated ellipsoids of revolution, and hence coined the terminology 'ellipsoid of motion' \( (E_N) \), and 'limit ellipsoid' \( (E_C) \). The ratio of volumes of ellipsoids is then a measure of the loosening that accompanied flow.

Extensive model testing indicated that the eccentricity (which characterizes the shape of the ellipsoid) was a function of

1. particle size
2. size of discharge opening
3. height of ellipsoid of motion, and hence volume extracted
4. velocity or rate of discharge.

The modelling material, although it may be 'lumpy' is assumed to be without sufficient cohesion to arch or form cavities, and thus flows freely and symmetrically.

If discharge of material is stopped, then the granular material would consolidate and diminish the degree of loosening due to flow.
Zones of motion during run-off of granular material

Particle Trajectories  Particle Displacements  Ellipsoid of like velocity

Figure 2.1  Flow notation (after Janelid and Kvapil, 1966)
Qualitative observation of the displacement of particles during flow drew the following conclusions:

1. The trajectories of particles are curved, not straight, though the curvature is small.

2. Particle velocity increases as the particle displaces towards the outlet. Velocity of particles along the centre was measured to be inversely proportional to the ratio h/c, where h is the height above the opening and c is the outlet width.

3. The velocity distribution was observed using time lapse photography. On the surface of the limit ellipsoid velocity is zero. Within the limit ellipsoid, lines connecting points with velocities of equal magnitude could be interpolated at any instant in time. Because these lines approximate ellipsoids, Janelid defines 'ellipsoids of like velocities'.

Extensions of basic notation to sublevel caving

Janelid observed that the analysis of flow in sublevel caving was in all respects similar to that observed in bunkers, despite the different geometry at the 'flow outlet' in sublevel caving.

The ellipsoid was observed to be truncated by the ring face, so that the major axes of the ellipsoids were oriented at an angle η to the ring face. Implicit in the analysis is the coincidence of the major axis of the limit ellipsoid, with that of the ellipsoid of motion.

The eccentricity now becomes a function of parameters that are particularly associated with gravity flow in a mining environment. These include:
$e$ - effective heading flow width
$\eta$ - angle of deviation
$\alpha$ - ring (slice) gradient
$\kappa$ - inclination of axis of draw

Figure 2.2  Flow notation for sublevel caving geometry
(after Janelid and Kvapil, 1966)
1. properties of the material eg particle size, shape, size distribution, the mechanical properties of the intact rock that constitutes the granular material after fragmentation, moisture content,

2. the loosening process, and the effect of external forces and pressures developed due to self-weight of the material, consolidation effects,

3. design geometry - heading width and height, sublevel height and width, ring burden and gradient,

4. wall roughness,

5. position of loader, width of loader and digging depth, which determines the run-off width e (Figure 2.1).

In these initial investigations, considerable effort was directed towards establishing relationships, both as trends and in a quantitative fashion, between eccentricity and the factors listed above. The work depended on scale modelling to predict relationships, and depict flow behaviour. Attention was primarily focussed on the ellipsoid of motion, for the character of the material within this volume dictates the recovery and dilution that can be expected in a particular modelling situation. The ellipsoid of motion is easily detected in modelling if a series of markers are embedded in the granular material. The ellipsoid of motion is defined as the surface connecting points that reach the outlet simultaneously. Consequently, those markers that are recovered simultaneously indicate the current position of the ellipsoid of motion.

Not only is the mining environment differentiated from bunker studies by geometry and loading techniques, but also by the presence of waste rock, usually of a different nature from the fragmented ore. Janelid has carried out studies to indicate the influence of this factor, not only on eccentricity, but also the angle of deviation of the major axis from the ring gradient. In addition, the dilution that is generated during extracted can be complicated by mixing and percolation processes.
Kvapil (1965a, 1965b) introduces some pertinent points in the description of flow for coarse materials in mining environments.

1. crushing and comminution can be significant in changing the size distribution during flow by increasing the proportion of fines. This effect increases with the distance material flows,

2. the variability of properties of coarse materials increases with a broader size distribution, and with greater variety of substance of the individual particles,

3. coarser materials tend to transmit forces, creating arching effects by particles mechanically interlocking, creating voids and cavities.

Consequently, although one could expect disimilarity in flow between bunkers, and mining environments, the work of Janelid and Kvapil was of significance as the first attempt to systematically quantify and investigate flow behaviour in sublevel caving.

2.3 SUBLEVEL CAVING MINING DESIGN PRINCIPLES

Successful implementation of sublevel caving as a mining method relies upon:

(i) correct planning of the mining geometry

(ii) adequate supervision, and direction of mining activities to ensure that activities are properly carried out

(iii) effective draw control procedures to monitor, and predict flow behaviour.

Assuming (ii) and (iii), the essence of sublevel caving is to optimize the design geometry factors, for any particular mining operation to maximize recovery and minimize dilution, by inducing the most favourable flow behaviour. Additional constraints may well be the need to impose a cut-off grade to satisfy mill requirements, or blend ore from different locations, or set a level of maximum dilution.
The optimization problem is often tackled on a pragmatic basis — "What works best in our situation". This may well be attributed to the unpredictable nature of flow in a particular area, but more than likely it is due to a poor knowledge of the variables involved and their influence, or the lack of an adequate model on which to base an optimization technique.

The goal of sublevel caving research is the need to rationalize the decisions that face the mine planner, so that a minimal amount of field experimentation is necessary, and to guide the assessment of how inefficiencies and poor performance can be restricted.

Additional constraints may well complicate the situation. Lack of foresight, or poor planning of adjacent mined out stopes, or the demands of management to step beyond the planned production schedule may leave the mine planner with few easy decisions.

In the sublevel caving environment, flow behaviour is determined by a complex set of inter-relationships between variables, some of which are quantifiable and controllable, others are not quantifiable, and often unpredictable.

These variables can be classified and detailed:

1. Design Geometry
   - Extraction heading height
   - Extraction heading width
   - Sublevel interval
   - Sublevel spacing
   - Ring inclination
   - Ring burden

2. Mining Practice
   - Drilling — accuracy
   - Blasting — fragmentation, size distribution
   - Loading — digging depth, positioning of loader, evenness of draw
3. Flow characteristics
   - phenomena such as frequency of arching
   - material description for both ore and waste rock
     eg particle size distribution, shape, density
     frictional properties
   - stress distributions in broken rock mass
   - influence of time, consolidation

4. Mathematical model or description of flow

   Following Janelid's theory, this involves selection
   of eccentricity for ellipsoid of motion.

The choice of an optimum design geometry in the past has
been based on:

1. previous mining practice
2. scale modelling studies
3. specification of a mathematical model for flow
4. mining equipment technology

This choice is dictated not only by an optimal flow behaviour,
but also by rock mechanics considerations eg stability of openings,
support requirements etc. Although mindful of the significance of
rock mechanics studies, this thesis attempts to examine questions
related to the description and prediction of flow behaviour.

Useful summaries of the evolution of design procedures are

Paralleling research investigations into gravity flow behaviour
and significant developments have been made in mining technology.
Figure 2.3 illustrates the development of sublevel caving design
from induced caving to controlled drilling and blasting, the fan
drill pattern being superseded by the silo drill pattern with the
advent of long-hole drilling.
Figure 2.3 Development of the sublevel caving method and blasthole pattern
Considerable insight has been gained through the application of the ellipsoidal flow pattern in determining the optimal design geometry.

2.4 MATHEMATICAL MODEL STUDIES

A number of studies have attempted to quantify the inter-relationship between the design variables, to provide a basis for computer simulation and mine design, by mathematical modelling techniques.

The primary problem was the determination of the geometrical shape of draw at various degrees of extraction, yielding ore recovery and waste dilution for different system designs (Bishop, 1971).

Initially the analysis was two dimensional, (Free, 1970), but three dimensional models were subsequently derived.

By making the following assumptions, no basic changes in flow behaviour for sublevel caving, from that of granular flow in bunkers, were pre-supposed (Figure 2.2):

(i) the solid ring intersects but does not distort flow behaviour

(ii) the major axis of the ellipsoid deviates an angle \( \eta \) from the ring

(iii) sources of waste dilution are derived from the intersection of draw zones

(iv) ore and waste rock assumed to have similar flow characteristics, or if they differ Bishop indicates that a change in the angle of deviation accounts for this
(v) the sources of dilution taken into account are
   - from the heading above
   - from the intersection with another previously
     extracted ellipsoid
   - from the heading at the side
   - from the back of the blasted ring

(vi) the notion of an effective heading width

(vii) the eccentricity was found to be a function of height
      (Just and Free, 1971).

Calculation of dilution for a particular design geometry was
obtained by reducing the ellipsoid to an equivalent sphere (similarly
for draw ellipsoids at adjacent headings) with dilution at the back
being calculated from the intersection of a plane (the back of the
blasted ring). Cullum (1972) assumed values for the percentages of
waste material from the top, side and back of ring due to adjacent
drawn rings and headings. (Figure 2.4).

Various design charts were published (Cullum, 1972) to
illustrate the value of this model of intersecting ring-ellipsoid-
heading geometry for determining the effect of any minor, or major
design change on ore recovery and waste dilution.

It is significant that the parameter used by Cullum, extraction
efficiency, was unable to differentiate designs because all layouts
evaluated recorded similar maximum efficiencies.

It is suggested that the mathematical model for flow presented
here is deficient at a number of points

1. The analysis is static, i.e., in calculating recovery and
dilution it must be assumed that adjacent rings have been
completely drawn.

2. No progressive dilution or interaction between adjacent
rings during draw can be modelled.
Waste Dilution Source:

- behind the ring slice
- intersection with heading at the side
- intersection with heading above
- intersection with a previously extracted ellipsoid

Figure 2.4  Sources of waste dilution (after Cullum, 1972)
3. The evaluation of sources of dilution from top, side and back of the ring could have been evaluated more precisely.

4. The shape of the ellipsoid only approximately simulates the draw shape. This point is elaborated in Chapter 9.

2.5 SIMULATION STUDIES

Just (1972) indicated that the purpose of simulating sublevel caving operations is to choose the best of a number of design layouts having similar efficiencies, but which could be differentiated on the basis of cost of unit operations.

Chatterjee & Ham (1975) present a dynamic simulation program for modelling and evaluating a sublevel caving system. The discrete event method is used to model the sequence of unit operations at each heading viz. drilling, charging, firing, loading and transporting taking into account stochastic elements eg equipment breakdowns, delays, hangups each having a known statistical occurrence, and deterministic factors eg equipment availability, development locations etc.

The goals of their study can be itemized:

1. Inter-relationship of variables, and their significance in the ultimate design.

2. Comparison of alternative designs. The model does not optimize, but can be used to compare the performance of different designs, and the engineer can choose the best design given the merits of each design tested.

Although the model is not the answer to all planning, operating or control problems, it is particularly useful for understanding the performance of a mine given decision-making rules for machine allocation and scheduling of operations.
If the main criterion of the model is the prediction of maximum recovery and minimum dilution, then the model becomes very dependent on the functional relationships established between design geometry and draw shapes.

Although the simulation model is based on dynamic concepts, extraction calculations use the static mathematical model, based on the work of Bishop and Cullum to estimate dilution and recovery for the design geometry under consideration. This is the weak link in the simulation model.

The model does not simulate the behaviour of the broken rock mass during draw, but instead assumes that extraction at each heading can be evaluated independently, and values predicted are based on an idealized draw being completed at each ring. Multiple drawpoint interaction is not considered.

2.6 FULL SCALE MINE TESTING

Being unable to model blasting effects, and material flow processes in sublevel caving with scale models necessitates full scale controlled testing to verify any physical or mathematical model used as a basis for design or simulation.

The only published test is that of Professor Janelid, conducted at Grangesburg Mine, Sweden. A well documented report was presented at the International Sublevel Caving Symposium, Stockholm (Janelid, 1975b).

Markers were placed in boreholes located on planes within each ring before blasting, and recovered during loading. Manual and computer methods were used to interpret the 'volumes of motion' (a more general term currently used by Janelid). The computer method assumes an ellipsoidal shape, whereas the manual method more closely approximates the actual shape. Parallel model tests (scale 1:20) were carried out to compare the results for full scale, and model testing.
The manually interpolated volumes of motion for full scale testing reveal a distorted shape, that lacks symmetry and evenness.

Janelid ascribes this irregular flow behaviour to:

1. hangups and boulders
2. loosening of rock at higher sublevel drifts
3. non-symmetrical loading practices, especially at bends
4. the weight of overburden waste rock, that was gravity fed to one side of the test area, setting up differential pressures in the rock mass, inclining volumes of motion
5. back break

Although further analysis and discussion of Janelid's results is included in Chapter 9, the following comments should be made:

1. Although no simple mathematical description of the manual volumes of motion, and consequently non-comparable term for eccentricity is available, Janelid finds a definite connection between degree of loading and the maximum height, width and depth of the volumes of motion for the test area (Figure 2.5).

2. Parallel model tests yielded a more even volume of motion (more closely approximating the ellipsoidal shape), but wider because of the greater mobility of the scaled material.

3. Higher charge density was used to increase fragmentation in the full scale test but because of back break, markers which would have indicated commencement of waste dilution from the back of the ring were not available, because they were drawn out in the previous ring. Back break increases the waste dilution (Figure 2.6).

4. The tests were carried out in an iron orebody with ore being much denser than waste rock. Bishop has noted the significant effect of relative densities of ore and waste rock in modelling, and consequently some caution is needed in applying Janelid's results to base metal deposits.
Figure 2.5 Results of full scale and model test. Height, width and depth of volume of motion as a function of degree of loading (after Janelid, 1975b)

Figure 2.6 Notation for parameters in sublevel caving (after Janelid, 1975b)
2.7 **SCALE MODELLING STUDIES**

Given the expense of full scale testing, modelling studies have provided a simple alternative. Modelling has the inherent limitations that it cannot simulate expansion and compaction process in blasting, nor the density changes due to loosening during flow, nor the pressure distribution present in the broken rock mass.

The majority of past modelling studies have focussed on the estimation of the ellipsoid of motion and assigning values to the eccentricity.

Modelling materials have a significant influence on flow behaviour, initial studies used single sized material (sand, glass beads), but recent studies (Bamford, 1975; Panczakiewicz, 1977) have employed crushed rock scaled down from observed size distributions of mine samples. The presence of fine size fractions have modelled the hangups observed in mine operations.

Cullum (1974) reports investigations into the shape of draw, and tentatively concludes that 'a flat or slight curve topped ellipsoid' might more closely resemble the draw shape.

2.8 **BLASTING AND FRAGMENTATION**

The nature and effects of confined blasting in an underground mine have received little attention. The consequent fragmentation, frequency of hangups, and gravity flow mechanism for any particular ring ie loosening of blasted ore or swelling must be evaluated, and where significant incorporated into a mine model.

There are obvious difficulties, and small scale model experiments are of limited value.

Cullum (1974) carried out scale model tests to gain an appreciation of the relationship of blast confinement, particle size, and swell factor, given that in blasting, particle fragments must be small enough not to produce arching, and loosening (or swell) must be adequate to permit flow.
2.9 MINE OPERATING PROBLEMS

An accurate simulation should account for those problems encountered in operating mines that induce irregular flow behaviour.

From experience at existing operations these appear to include

(i) poor drilling, and hence excessive burdens at the toe
(ii) poor fragmentation with unbroken 'bridges' of ore
(iii) excessive fragmentation
(iv) loading technique (Meramec)
(v) brow failure (Craigmont)
(vi) hangups — causing delays and permitting waste rock to flood (Craigmont)
(vii) detection of cut-off point (Craigmont)
(viii) overburden pressures.

2.10 A NEW BASIS FOR SIMULATING SUBLEVEL CAVING

Sublevel caving analysis to date has focussed primarily on static concepts of flow ie defining the volume from which extracted material originated. No existing mathematical model predicts particle flow relationships, or recovery and dilution for various stages of extraction.

Subsequent chapters investigate different models that could provide a basis for the description of the dynamical flow of ore in sublevel caving.
CHAPTER 3 : MATERIAL MODEL FORMULATION FOR GRANULAR MATERIALS

3.1 Introduction

3.2 Fluid Flow Analogies

3.3 Numerical Models for Discontinua

3.4 Plasticity Theory

3.5 Continuum Material Models

3.6 Conclusion
3.1 INTRODUCTION

In geomechanics, the primary concern in dealing with earth or rock structures has been the prediction of stability, collapse loads, and more recently deformation under an applied loading system. The solutions for stress distribution and consequent deformation are obtained by analyzing a continuum material model (elastic, plastic or viscous or a combination of these) satisfying the conditions of equilibrium, compatibility of deformation, together with the appropriate constitutive laws suitable for the medium under consideration. The objective is typically to determine safety factors and design to prevent failure.

In this study the primary interest is in the post-yield flow of a granular material involving large and permanent strains. There are ways of descriptively characterizing granular material (eg size grading), but a literature search was initiated to find ways of mathematically describing the essential characteristics of a granular medium and how this mathematical idealization could be applied to predicting the flow of granular material.

A great deal of research is being carried out in the related fields of bulk solids, particulate mechanics, powder technology and applied rheology, and also the study of discontinuous rock masses and behaviour of soils and rockfill in the disciplines of soil and rock mechanics.

This is not to imply that an understanding of granular material behaviour for one scale (eg powders) would imply a knowledge for other scales. Obviously the forces under examination for powders vary markedly from rockfill, but certain general propositions apply for all particulate materials, but not for other materials eg solids and liquids.

A material model that characterizes the material constituting the granules and one that characterizes the assemblage of particles should be distinguished. To quote Müller (1974) :
"In any discussion of rock characteristics or rock behaviour a distinction must be made between rock substance and rock mass ... The presence of discontinuities ... makes its behaviour different from that of the constituent rock."

Considerable effort has enabled the properties of solid rock to be defined. However in a mining environment it is the inter-particle forces that determine the flow mechanism.

Only a few monographs endeavour to draw together the knowledge relating to flow of granular materials (Jenike, 1970; Reisner et al., 1971), and many of the papers reviewed contained formalized mathematical symbolism, with little foreseeable application to practical engineering problems, or alternatively presented a realistic material mode without a tractable mathematical or numerical solution technique.

A brief description of approaches to the formulation of granular material models that were surveyed follows. Attempts are made to evaluate their applicability to flow in mining environments, the availability of solution techniques and possible implementation.

3.2 FLUID FLOW ANALOGIES

Because of the availability of powerful numerical techniques the possibility of employing a fluid analogy was considered.

Examples reported in the literature handle:

1. variable density
2. different materials bounded by an interface
3. three dimensional formulation
4. cavities.
The mathematical solution of fluid dynamics problems involves equations that specify conservation of mass, momentum, and energy (which are independent of the material) and an equation that specifies the properties of the fluid itself, the equation of state, usually relating pressure, density and heat energy per unit mass. A basic outline of the principles involved, and the range of numerical techniques is available (Harlow et al., 1971).

To date no approximate fluid analogy has been found. Newtonian fluids in general disqualify themselves on several counts, but one theory - that of polar fluids (Cowin, 1974c) does treat those fluids whose microstructure is mechanically significant eg suspensions, but only applications of flow in straight pipes, channels were illustrated.

Jenike makes the following comparisons between granular materials and fluids:

(i) Granular materials can transfer shearing stresses under static conditions - they have a static angle of friction greater than zero. Consequently they form piles whereas liquids maintain a horizontal free surface.

(ii) Many granular materials when consolidated under certain conditions possess cohesive strength and retain their shape. They form arches and piping effects whereas fluids cannot.

(iii) The shearing stresses which occur in a slowly deforming (flowing) granular material are considered independent of the shear rate and dependent on the mean pressure acting in the solid. Shearing stresses in a fluid are dependent on shear rate and independent of mean pressure.

In addition further comparisons are:

(iv) fluids are incompressible at ordinary temperatures.
Reynolds principle of dilatancy states that granular materials must expand (increase in voidage) if they are to flow (Brown et al., 1965). In addition displacements in a fluid are propagated through the medium contrary to the observed behaviour of finite propagation of disturbances in experimental model studies for granular materials. It should be noted however that Jenike assumes incompressibility for his design calculations.

(v) the inability of fluids to sustain shear stresses is evidenced in model silos where the pressure at the outlet is dependent on the head of fluid. However granular materials transmit forces and unstable arches form across the width of the silo to partially support the weight of overlying material: flow of granular materials is usually independent of the head of material.

Suzuki et al. (1971) have utilized a fluid analogy (ie steady and incompressible Newtonian flow) in the vicinity of a hopper outlet to predict flow rate; an assumption shared in the work of McDougall et al. (1965), Bosely et al. (1969), Holland et al. (1969), and Carleton (1972).

3.3 NUMERICAL MODELS FOR DISCONTINUA

More recently geomechanical engineers have taken into account more of the features of rock masses eg:

1. Rock structure ie the presence of discontinuities or planes of weakness like faults, joints and bedding planes

2. Joint fabric ie — location and orientation of joints
   — joint spacing and degree of joint continuity
   — opening, filling material and roughness of the joint surface
3. Water in pores and joints

4. Primary state of stress

5. The time dependent behaviour of rock substances, and appropriate creep laws.

As well as observing the response of rock masses with equivalent material models, modes of behaviour and failure (sliding, toppling) have been investigated using numerical methods of simulating the influence of joints and discontinuities.

a. The Finite Element Method

Different finite elements have been devised to simulate the strength and deformation of a joint, and so model discontinuities directly (Goodman et al., 1968, 1975; St John, 1971).

Burman (1971, 1974), neglecting the deformation within the block could describe the displacement of blocks in terms of components for rigid body motion i.e. rotation and translation, with normal and shear stiffnesses applied to adjacent blocks.

The chief restriction at present with regard to these finite element models is that the geometrical relationship between blocks cannot be disrupted or conveniently severed, although an application of the finite element method to the meshing of gears (FrancaVilla et al., 1975) offers the potential for this application.

b. Cundall Discrete Block Method

A discontinuous rock mass is in reality a finite number of irregularly shaped particles or blocks, in contact with each other at corners or edges, where forces are transmitted. If the particles can be assumed to be rigid, and inter-particle forces are localized at contact points, then simple
relationships can be devised to describe the shear and normal forces acting at contact points (ie joint stiffnesses) and laws describing shear and tensile failure of contacts or gross movement across contact edges, are incorporated.

Using this simple model Canadell (1971, 1974, 1976) has applied a relatively new technique termed Dynamic Relaxation. This is an iterative method involving the finite difference analogue of the force-displacement equations that can be derived from Newton's law of motion.

In each successive time increment the following calculations are performed for each block:

(i) detect all blocks with which the block under consideration is in contact

(ii) by virtue of the geometric position of the block relative to those with which it is in contact, analyze the incremental contact forces from the stiffness relations, and calculate an equivalent force and moment acting at the block centroid.

By assuming a law of motion, the consequent, incremental particle movement (in terms of translation and rotation) in the time increment, can be evaluated by integrating the accelerations successively to obtain velocities and incremental displacements.

With a small time increment, the motion of particles under applied loads and defined boundary conditions is modelled in a stepwise shuffling motion, as forces are redistributed amongst the particles.

Because the motion of particles is dependent on the movement of adjacent particles, the problem is non-linear and path dependent, and so explicit finite difference schemes were preferred to implicit.
An example provided by Cundall (Figure 3.1) shows the results of simulation of flow of particles out of a hopper.

Cundall's programs have been recently translated into Fortran, and adapted to run interactively on a terminal (FitzGerald, 1976) with the force-displacement relationships, and the contact point relationships expanded.

The program in its current form offers the following advantages:

1. Particles are free to rotate and translate, and contact with any other particle, and large displacements are modelled.

2. Randomly shaped particles can be generated.

3. The actual intergranular forces are being modelled in a simple fashion and yet complex material and geometric non-linearities are modelled.

4. Boundary conditions are easily implemented.

5. Multi-material definitions can be made.

The program has been used successfully to model rock slopes, and offers the greatest potential for modelling stability of discontinua, but also for modelling flow processes similar to those encountered in mining situations. However the method is not without its limitations.

1. The basic calculation cycle involves an updating for each particle of:

(a) — particle contacts, detecting new contacts between blocks as they arise due to changing geometry, and deleting contacts.

(b) — particle position, contact point forces, equivalent forces and moments at the particle centroids for each degree of freedom.

In addition the particle shape (ie position of corners) material type must be stored.
Figure 3.1(1) Discrete Block Method flow sequence
(after Cundall, 1974)
Figure 7(II)

Note: particles have been removed when they fall below a certain level.
All together 32 pieces of data for each particle are required. Several thousand particles appears a practical limit.

2. The time increment must be small, and consequently a compromise between problem time, and problem size is inevitable.

3. At present, the program has only a two dimensional capability and apart from using spheres or some other easily defined shape, three dimensional problems appear limited to small particle numbers because the particle data to be stored is proportionately larger.

4. Considerable effort is required to optimize data storage, calculation cycle, and search procedures.

Despite the limitations of problem size the method appears to be a realistic material model given the current knowledge of friction laws in a rock discontinuum. The chief limitation would be the coarseness of particles necessary to model a sublevel caving situation in two dimensions; the ratio of average particle diameter to heading width, would accentuate arching and particle interlocking unless artificial friction laws were introduced. This would weaken the rationale for a simulation to investigate the real behaviour and flow mechanisms of broken ore in an underground environment if it was intended to thereby circumvent the inherent problems of scale modelling.

A similar approach has been initiated by Serrano et.al. (1973) where they too examine contact forces and displacements, to analyze the "grading-porosity relationships of heterogeneous packings together with the spatial and numerical distribution of contacts". The incremental displacements of the particles needed to restore equilibrium after each external load or deformation increment can then be evaluated.
3.4 PLASTICITY THEORY

Application of plasticity theory to problems in soil mechanics and hopper flow has developed from the theory of metal plasticity.

Jenike (1967, 1970) and co-workers have pioneered and developed this continuum approach to the storage, and flow of "bulk solids", though considerable disagreement still exists on fundamental assumptions and the limitations of simplifications that are employed in the analysis.

Jenike employed the term "bulk solids" to differentiate these materials from soils, because the concepts of plasticity stated for soil mechanics "undergo subtle but significant changes when applied to bulk solids". Two recent symposia summarize research in the area of soil mechanics (Parry, 1972; Palmer, 1973), while Jenike's work appears in numerous publications.

In plasticity theory, granular material is represented by an idealized substance that deforms elastically up to some state of stress at which slip or yield occurs.

In order that plasticity theory may be applied to the analysis of flow, the following conditions need to be specified:

(i) A yield (failure) criterion specifying combinations of stresses which satisfy the failure condition of the material, and post yield behaviour i.e. strain hardening, strain softening or perfect plastic behaviour.

(ii) Equations of motion or equilibrium.

(iii) The geometry of strain, relating strain rates to velocity.

(iv) Flow rule to relate the directions of principal stresses and strains (and strain rates) in the failure state.

(v) Strain-density relationship.

(vi) Sufficient appropriate boundary conditions, including specification of surface and body forces initial stress distribution and free surfaces, and alterations of boundary conditions with time.
It is beyond the scope of this thesis to review in detail the progress that has been made in developing appropriate yield criteria, and flow rules. Pariseau (1968a, 1969a, 1969b, 1972) has outlined some of the specific problems in relation to hopper flow. More general reviews of developments in plasticity theory are given by Kurtay et al. (1970), Radenkovic (1974), and Nikolaeskii (1973), Drescher (1976).

To date only two dimensional plain strain or axisymmetric hopper sections (i.e., converging channels) have been analyzed with either static or quasi-static solutions. The work of Chen, Deutsch, Jenike, Johanson, Horne, Mroz, Pariseau, Pemberton, Savage, Spencer is noted here (see specific references in the Bibliography). Stress and velocity fields have been evaluated under steady state flow conditions.

Solutions have been obtained by:

(i) solving the appropriate differential equations by the method of characteristics

(ii) finite element method (Pariseau, 1971; Reddy et al., 1976).

These techniques have required considerable simplifications to be made in the original governing equations, and very idealized flow geometries are used. Geniev (1958) is the only reported study to deal with time dependent dynamic flow. The development of a time dependent plasticity theory solution, supported by experimental evidence for the theoretical model, is needed to analyze the initiation and progressive development of flow in hopper geometries.

3.5 CONTINUUM MATERIAL MODELS

Numerical models for discontinua enable the prediction (better 'simulation') of a collection of finite blocks or granules by postulating simple intergranular laws.
Several authors (Rowe, 1962, 1972; Horne 1965, 1969; Mogami, 1969; Oda, 1974a) have constructed a physical model based on an ensemble of idealized particles and then proceed to deduce the laws describing the mechanical behaviour of the entire ensemble.

An alternative approach has been to develop continuum material models, analogous to the classical continuum models for solids and fluids, but extended to incorporate concepts that characterise granular media e.g. void ratio, particle size and shape. Notable success has been achieved by Goodman et al. (1971, 1972) and Cowin (1974a, 1974b).

Incompressible granules experience dilatancy (expansion of voidage) during deformation. The cohesion and angle of internal friction are dependent on the voidage of the granular material, as has been shown by experiment. Although the theory predicts behaviour significantly different from fluids, and the constitutive equations imply a generalized Mohr-Coulomb failure criterion, the theory is presently limited to limiting equilibrium states, where in the flow regime the stress level is less than 70kPa. Cowin (1974a) also examines possible ways of characterizing the microstructure of granular materials (e.g. size and shape of particles) with kinematic variables termed 'packing measures'. This model is reviewed by Jenkins (1975). Aguirre-Ramirez et al. (1972) report a similar approach, while Passman (1977) extends Cowin's model to mixtures of granular materials.

The non-linear field theories of mechanics are a specialized subject (appearing vague and abstract in content) but a number of workers (Maml et al. 1970; Cudehus 1969, 1970; Romano 1974) are endeavouring to develop a continuum model that agrees with experimental observations.

3.6 CONCLUSIONS

Of the material models that have been pursued in this investigation that of Cundall's appears to have the most potential.
Because of the necessity to conclude this project no further work could be carried out using this technique, as a means of predicted gravity flow behaviour in mining environments.

As a technique for completing a simulation of sublevel caving operations it was obviously too restrictive, but the model could be useful on two levels:

1. As suggested by Cundall, "numerical experiments could be carried out on a simulated granular medium in order to derive constitutive equations and flow rules from the known properties of individual particles" to use in the various numerical continuum techniques available, because of the lack of experimental data currently available to support hypotheses on complex granular material behaviour.

2. As a tool to indicate the dominant mechanisms of flow in a mining environment, that presently cannot be adequately modelled.
CHAPTER 4 : STOCHASTIC FLOW MODELS

4.1 Introduction to Stochastic Models

4.2 Stochastic Models for Granular Material
   4.2.1 Fletcher
   4.2.2 Sweet
   4.2.3 Imenitov
   4.2.4 Mullins
   4.2.5 Litwiniszyn
       Steady State Flow
       Unsteady State Flow
   4.2.6 Limitations of analytical solution
       Methods

4.3 Moving Boundary Problems
   4.3.1 Finite Difference Methods
   4.3.2 Finite Element Methods

4.4 Monte Carlo Method
4.1 **INTRODUCTION TO STOCHASTIC MODELS**

The complexity of natural phenomena precludes the description and mathematical modelling for every event that transpires. Yet each event can be considered to be governed by deterministic laws. For example, in soil erosion the fall of each raindrop is governed by Newton's Law of motion and atmospheric friction. When the drop hits the ground and erodes some soil particles, again the appropriate laws of mechanics determine the sequence of events.

Nevertheless, although the knowledge of individual events can never be complete enough to deduce their result, certain average relationships can be deduced from the incomplete knowledge of individual processes, because the net effect of the many individual events is the same as if the individual events were to occur at random, although the small-scale events are, strictly speaking, entirely predetermined.

Stochastic models, involving a deterministic process that can be conceived as combining random components, have been introduced in many scientific disciplines eg

(a) models of cavern development (Curl, 1959)

(b) dispersion in porous media (Scheidegger, 1964; Fried et al 1971)

(c) soil creep and development of hillside slopes (Culling, 1963; Scheidegger, 1967)

(d) alluvial fan deposition (Price, 1976)

In the following sections it is established that the gravity flow of granular material can be understood as the upward migration of voids through a granular mass, inducing downward particle displacement.

To provide a mathematical model of the flow process, an adequate description must be provided for:
(a) the dispersing phase
(b) the phase into which the material disperses
(c) the mechanism of diffusion.

This chapter reviews and critiques the published literature on stochastic flow models for granular materials. The models have arisen in the context of research into subsidence (Litwiniszyn, Fletcher, Sweet) and flow in bunkers (Mullins).

The stochastic models are developed from idealized granular ensembles, and mathematical formulae are deduced to predict particle displacements. Although diffusion equations are introduced, the focus is not the concentration of the diffusing phase with time, but the displacement induced by the passage of the diffusing phase in the medium.

Three different solution methods are reviewed:
(a) closed form analytical solution
(b) discretization methods -- finite differences, finite elements
(c) digital simulation using Monte Carlo methods

4.2 STOCHASTIC MODELS FOR GRANULAR MATERIAL

4.2.1 Fletcher

Fletcher (1972) investigated the response of granular material under externally applied forces and displacements. The granular material was modelled as a two dimensional random arrangement of identical spheres, for which a probability density function of contact angles between spheres was developed. By generating paths of contacting particles, deflections due to displacement of a single surface particle were predicted. The vertical component of deflection occurring at a specified depth, was hypothesized to be the average of the probable maximum path deflections to that depth.
4.2.2 *Sweet*

Sweet's model had its primary application in the prediction of subsidence of a granular medium into a mined out cavity. The mathematical analysis was conducted in two dimensions, and proceeded in two stages (Sweet et al, 1965a, 1965b).

a. **Discrete model**

The granular medium was characterized by layers of circular disks, diameter d, in a regular packing with λ the vertical distance from particle to particle centre (Figure 4.1).

With the origin of the coordinate system as the sink for particles, then as the particle occupying position (0,0) is removed, each of the particles at (1,1) and (-1,1) is given an assumed probability of 0.5 of moving to position (0,0). The void formed as a result of the displacement is filled by one of the two particles directly above it.

Under the action of gravity, and assuming no arching, the void moves upwards through the lattice until it reaches the free surface.

Because a void cannot move to a lattice point already occupied by a void, the latter acts as a reflecting barrier to approaching voids. After a large number of particles have been removed from (0,0), the trough will be V-shaped with θ being the angle of repose (Figure 4.2).

The motion of any void through the lattice is an example of a one dimensional random walk ie a Markov chain with non-stationary transition probabilities. A dependence on the history of other random walk particles exists. Because of the difficulty in deriving an analytical solution, a simplification is introduced. If the incremental volume extracted is small and delayed, then the assumption that the motion of each void is independent of the motion of previous voids is valid.
Figure 4.1 Stochastic Medium (after Sweet and Bogdanoff, 1965)

Figure 4.2 V shaped subsidence profile (after Sweet and Bogdanoff, 1965)
b. Continuous model

By taking the limit
\[ d \rightarrow 0 \]
a transition from discrete random variables \( \left( \frac{X}{w}, \frac{h}{\lambda} \right) \) to continuous random variables \((z,h)\) can be made, with the displacement of the free surface (initially a distance \( h \) above the origin):
\[ s(z,h) = \frac{A}{\sqrt{2\pi} \sigma} \exp\left(-\frac{z^2}{2\sigma^2}\right) \]
where \( \sigma \) is the variance of the subsidence.

\( A \) is the area enclosed by the subsidence profile.
The average subsidence has the equation of a normal Gaussian curve for a point sink. An equation for the variance of the subsidence was also derived.

4.2.3 Imenitov

Imenitov et al (1971) investigated a similar problem to Sweet. The medium was considered as ore overlaid by waste rock, and the probability of waste rock arriving at the sink was determined as a function of the ore released through the sink, and the vertical extent of the ore.

4.2.4 Mullins

Mullins (1972) has developed an analytical method for determining the flow under gravity of cohesionless particles in a semi-infinite bed. Solutions representing a point orifice, and a finite orifice in the floor of the bed have been given. Particle flux lines, and marker motion and exit times are defined and calculated, firstly for steady state flow, and secondly for non-steady state intermittent flow.

The basic method employed is outlined below, although a detailed treatment of the mathematical theory of probability distributions that underlie the model, are beyond the scope of this thesis.
Steady-state flow (Mullins, 1972, 1974b)

The analysis is that of migration of discrete voids, which retain their identity during migration, originating from a point source and displacing upwards under the action of gravity (and hence biased).

It is assumed the concentration of voids is small, to the extent that consecutive void jumps can be considered independent, and hence uncorrelated because voids do not interfere with the motion of others. The medium can be envisaged as a lattice which can be occupied by particles, or else occupied by voids, and hence vacant.

If \( n \) = number of lattice sites per unit volume
\( c \) = number of voids per unit volume
\( c' \) = number of particles per unit volume

then \( c + c' = n \)
\( c' = n \) for \( \frac{c}{n} \ll 1 \)

Based on Chandrasekhar’s exposition of biased random flight, Mullins deduces equations which are equivalent to diffusion of voids according to Fick’s Law, in each of the three cartesian directions \( x, y, z \) (with unit vectors \( i, j, k \) respectively), and a convective term. The flux \( \dot{z} \) is defined:

\[
\dot{J} = -D_x \frac{\partial c}{\partial x} i - D_y \frac{\partial c}{\partial y} j + (V - D_z \frac{\partial c}{\partial z}) k
\]

Applying the equation of continuity, and setting \( D = D_x = D_y \) yields:

\[
\frac{\partial c}{\partial t} = V \cdot \dot{J} = D \frac{\partial^2 c}{\partial x^2} + D \frac{\partial^2 c}{\partial y^2} + D_z \frac{\partial^2 c}{\partial z^2} - V \frac{\partial c}{\partial z}
\]

\( = 0 \) for steady state flow.

The standard solution for this equation for a point source of strength \( Q \):

\[
c(x,y,z) = \frac{Q}{4\pi D z[1+(D_z/D)(x^2+y^2)/z^2]^2} \exp \left\{ -\frac{V}{2D_z} \left[ \frac{D}{D_z} \frac{x^2+y^2}{z^2} \right]^{1/2} \right\}^2 - 1\right\}
\]
can be simplified by neglecting several terms:

\[ c(x,y,z) = \frac{Q}{4\pi Dz} \exp\left(-\frac{x^2 + y^2}{4\alpha z}\right) \quad \alpha = \frac{D}{V} \]

For radially symmetric flow

\[ c(r,z) = \frac{Q}{4\pi Dz} \exp\left(-\frac{r^2}{4\alpha z}\right) \]

or \[ \frac{D\delta c}{\delta x^2} + \frac{\partial}{\partial y} \frac{\delta c}{\delta y^2} - V \frac{\delta c}{\delta z} = 0 \]

The previous equation is equivalent to the time dependent diffusion equation in two dimensions (x and y) in which the time coordinate has been replaced by the z-coordinate, and the diffusivity has been replaced by \( \alpha \).

The fraction \( f \) of the total flux in the z direction that passes a circular area

\[ f = \frac{1}{Q} \int_0^R \frac{Q}{4\pi Dz} \exp\left(-\frac{r^2}{4\alpha z}\right) = 1 - \exp\left(-\frac{r^2}{4\alpha z}\right) \]

A paraboloid of revolution defined by \( r = 4(\alpha z)^{1/2} \) encompasses 98% of the flux. Flow outside this radius could be neglected.

Material flux lines are defined to be lines where the tangent at each point coincides with the local flux direction; for radially symmetric flow:

\[ J_r = -D \frac{\delta c}{\delta r} \]
\[ J_z = \nu c \]

so that the defining equation for a flux line becomes:

\[ \frac{dr}{dz} = \frac{J_r}{J_z} = -\alpha \frac{\delta c/\delta r}{c} = \alpha \frac{\delta}{\delta r} (\text{Inc}) = \frac{r}{2z} \]

Integrating this result yields:

\[ r = Kz^2 \quad \text{ K = 4} \alpha^{1/2} \]

describing a family of parabolic flux lines tangent to the plane \( z = 0 \) at the orifice.
Defining a marker as an entity that moves along a fixed flux line with a speed equal to the mean particle speed at any given point, marker speed on a flux line is given by:

\[
\frac{ds}{dt} = u = (u_r^2 + u_z^2)^{\frac{1}{2}} \quad ds^2 = dr^2 + dz^2
\]

\[
u_r = -\frac{1}{n} J_r = \frac{Dc}{n \partial r} \quad U_z = -\frac{1}{n} J_z = -\frac{V}{n} c
\]

\[
\Rightarrow \frac{ds}{dt} = -\frac{V}{n} c [1 + (\frac{c}{c})^2]^{\frac{1}{2}} = -\frac{V}{n} c [1 + \left(\frac{dr}{dz}\right)^2]^{\frac{1}{2}} = -\frac{V}{n} c \frac{ds}{dz}
\]

To calculate the time for a marker to move along a flux line from position \((r_0 = Kz_0^{\frac{1}{2}}, z_0)\) to \((r = Kz^{\frac{1}{2}}, z)\)

\[
t = \int \frac{dz}{c} = -\frac{2\pi m}{Q} \exp\left(\frac{K^2}{4\xi}\right)(z^2 - z_0^2)
\]

Hence the exit time for a marker is obtained by setting \(z = 0\):

\[
T(r_0, z_0) = \frac{2\pi m}{Q} \exp\left(\frac{K^2}{4\xi}\right) z_0^2
\]

\[
\Rightarrow t = -T(r_0, z_0) \frac{z^2}{z_0^2} + T(r_0, z_0)
\]

\[
\Rightarrow z = z_0 (1 - \frac{t}{T(r_0, z_0)})^{\frac{1}{2}}
\]

and

\[
r = r_0 (1 - \frac{t}{T(r_0, z_0)})^{\frac{1}{2}}
\]

substituting \(r = Kz^{\frac{1}{2}}\)

**Unsteady state flow (Mullins, 1974a)**

As Mullins notes in a subsequent extension of the stochastic theory, to evaluate the time dependent development of the flow zone, more realistic assumptions can be postulated that are in accord with experimental results.

1) the geometry is retained as a point source in a semi-infinite bed

2) the material that has begun to flow occupies a flow zone, assumed to have a uniform density \(\rho_{ss}\) while the
undisturbed material has a density $\rho_o$ where

$$\rho_{ss} = \rho_o - \Delta \rho \quad \Delta \rho > 0$$

iii) the particle flux $J$ in the flow zone is given at all points by the steady state theory. Crank (1975, p.310) states that this assumption introduces minimal error, and is necessary to achieve an approximate analytical solution.

iv) the boundary of the flow zone is discontinuous, and advances at each point with a velocity:

$$V_n = -\frac{1}{\Delta \rho} J_n$$

where $V_n$ and $J_n$ are the normal components of velocity and flux. The voids accumulate at the flow zone boundary, and effect a density change.

Derivation of marker displacements in unsteady state flow.

An orifice, idealized as a point sink of $Q$ particles/sec is opened at $t=0$. The flow zone boundary will be generated by a set of points which start simultaneously from the orifice and follow a parabolic flux line. The time required for a point, generating the boundary, to reach $(r,z)$ is:

$$\tau(r,z) = \frac{\Delta \rho}{\rho_{ss}} T(r,z)$$

where $T(r,z) = \frac{2\pi \rho_{ss}}{Q} \exp\left(\frac{r^2}{4\Delta z}\right) z^2$

Given $\tau$ and $z$, the width of the flow zone $r$ can be obtained. Consequently the exit time $T^*(r_o, z_o)$ or time required for a particle at $(r_o, z_o)$ to reach the orifice is

$$T^*(r_o, z_o) = \tau(r_o, z_o) + T(r_o, z_o) = (1 + \frac{\Delta \rho}{\rho_{ss}}) T(r_o, z_o)$$

$$= (1 + \frac{\Delta \rho}{\rho_{ss}}) \frac{2\pi \rho_{ss}}{Q} \exp\left(\frac{r_o^2}{4\Delta z}\right) z_o^2$$
To calculate the position of a marker at $t: t \geq \tau(r_0, z_0)$ and initially at $(r_0, z_0)$

$$z = z_0 \left(1 - \frac{t - \tau(r_0, z_0)}{T(r_0, z_0)}\right)^k$$

$$r = r_0 \left(1 - \frac{t - \tau(r_0, z_0)}{T(r_0, z_0)}\right)^k$$

These equations were incorporated into a computer program to display the displacement of (initially) horizontal lines of markers as a function of time, with the flow zone limits plotted (Figure 4.3).

Mullins (1974) obtained good agreement between the experimental results of Gardner (1966), and predictions of his theory by the appropriate choice of parameters $\Delta \rho/\rho_{ss}$ and $\alpha$. The parameter $\alpha$ was found to correspond closely with the particle size in the test.

The finite orifice width case is also dealt with by Mullins.

4.2.5 Litwiniszyn

The most comprehensive and original contribution to the development of stochastic flow concepts is due to Litwiniszyn and coworkers.

Litwiniszyn has introduced a general mathematical formulation to describe the displacements in a granular medium. Several detailed expositions of the theory are available (Litwiniszyn 1958, 1964, 1969, 1974). A review article by Berry (1964) is useful for understanding the basic mathematical theory, and the physical interpretation, but several misunderstandings and errors have been pointed out in this article (Litwiniszyn, 1965). Experimental investigations (with dry sand) have confirmed the validity of the theory for small displacements using isotropic and anisotropic layered models. Several proposals to correct the theory for the case of large displacements are under consideration (Litwiniszyn, 1974).
Figure 4.3(1) Flow sequence for Mullins' analytical solution depicting flow zone and marker layer displacements
Although the theory recognizes the discontinuous nature of a granular medium (in contrast to the classical elastic continuum models), no applications of this theory, or comparisons with field studies by other researchers, have been noted. The terse and abstract presentation of Litwiniszyn's ideas reinforce scepticism.

Scheidegger (1966) indicates that Litwiniszyn's random walk theory can be justified from general principles of statistical mechanics.

4.2.6 Limitations of Analytical Solution Methods

Carslaw et al (1959), and Crank (1975) provide general solutions of the diffusion equation for a variety of initial and boundary conditions, using standard mathematical functions. In all cases, the medium must be assumed infinite, or semi-infinite or else of simple geometry eg spheres or cylinders, where symmetry can be assumed. Analytical solutions are not adaptable to the irregular boundary geometry of sublevel caving.

Similar restrictions apply to analytical solutions of flow problems based on the theory of Markov chains that are reviewed in previous sections. The remainder of this chapter reviews alternative solution strategies.

4.3 MOVING BOUNDARY PROBLEMS

The stochastic flow model proposed by Mullins for unsteady state flow bears strong resemblance to 'moving boundary' or 'Stefan' problems. In the Stefan problem, diffusion is accompanied by an instantaneous and irreversible immobilization of a limited number of diffusing particles, so that a sharp boundary surface moves through the medium. The situation is depicted in Figure 4.4. The boundary separates a region where all (absorbing or immobilizing) sites are occupied from region where the concentration of freely diffusing molecules is zero. Typical applications include chemical reactions where the diffusing molecules are precipitated or form a new immobile chemical
Figure 4.4 Diffusion of particles which are immobilized
compound, or heat flow in a medium undergoing a phase change at some fixed temperature accompanied by the absorption or liberation of latent heat, or diffusion with a discontinuity in the gradient of the concentration-distance curve.

An extensive review of solution methods is given in a recent symposium (Ockendon et al, 1975).

4.3.1 Finite Difference Methods

Several numerical methods, based on the finite difference replacements of the diffusion equation, have been proposed. They differ in the way the moving boundary and the grid on which the numerical values are treated. Solutions are generally confined to one dimensional problems. Multi dimensional solutions do not appear to have been explored with the exception of Lazaridis (1970), and Meyer (1970).

A tentative proposal, has been formulated (Alford, 1976), based on the method of Crank et al (1972). It employs an explicit finite difference scheme with Lagrange polynomials used to represent the concentration near the boundary.

The displacement of material within the flow zone is of primary interest, and the displacement of representative markers are evaluated from the void fluxes of the surrounding grid nodes at each time increment.

Further research is needed to implement this proposal, and examine the potential of finite difference methods.

4.3.2 Finite Element Methods

Recently several authors (Aral et al, 1974; Fisher et al, 1974; Bonnerot et al, 1974) have proposed that the finite element method be applied to moving boundary problems. No comparisons of the finite difference and finite element schemes have been published to enable a comparison. Less sophisticated methods, described in the next section have been adopted to solve the stochastic flow pattern.
4.4 Monte Carlo Method

An alternative method to the analytical and numerical methods discussed in previous sections for the solution of differential equations, is that of probability methods (Haji-Sheikh et al, 1967). This technique is generally described as the Monte Carlo method, denoting the use of statistical sampling techniques to approximate the solution of the mathematical problem.

Integral to the solution is the "random walk" in which the diffusing substance is conceived of as particles that migrate in random fashion on a fixed grid from the source. Alternatively, a "floating random walk" has been proposed by Haji-Sheikh et al (1966), where the step length and direction can be variable. After a sufficient number of individual steps have been taken in this random manner, the observable properties of the system eg concentration can be summarized.

The advantage of the Monte Carlo Method is that complicated systems with interacting components and irregular boundary geometries can be studied, though usually at some computational expense.

Where the behaviour of the components as they interact, is governed by known or postulated probability laws, and in effect the phenomena being studied is being simulated, the Monte Carlo method is an "analog" or "direct simulation" process. This approach is pursued in Chapter 7.
CHAPTER 5: KINEMATIC FLOW MODELS BASED ON MODELLING STUDIES

5.1 Introduction

5.2 Characterizing Granular Materials

5.3 Methods of Recording Modelling Results
   External measuring methods
   Internal measuring methods
   Application of data recording techniques

5.4 Flow Patterns, Flow Mechanisms and Particle Trajectories

5.4.1 Steady State Flow Patterns and Flow Mechanisms

5.4.2 Unsteady State Flow Patterns

5.4.3 Flow Regime Dimensions

5.4.4 Particle Trajectories
   Steady state flow
   Unsteady state flow

5.5 Scaling Laws

5.6 Conclusion
5.1 INTRODUCTION

If the search for a respectable material model has borne fruit within the last five years, this has not deterred numerous investigators from modelling studies to formulate the kinematic flow behaviour for gravity flow of granular materials.

Studies have mainly focussed on prediction of flow rates, flow patterns, design of bin geometries, and flow promoting devices. These topics are reviewed in four substantial monographs (Richards, 1966; Brown et al, 1970; Reisner et al, 1971; Jenike, 1970).

Because no functional mathematical model was available to model the dynamic flow of granular material those studies that deal with the prediction and measurement of pressures and stresses in bin flow (Perry et al, 1967a, 1970; Handley et al, 1965; Deutsch et al 1967, 1969) are not included in this review. However these studies are significant for correlating stress distributions to flow mechanisms, and should be used to validate any dynamic material flow model.

For the purposes of this study, which is concerned to simulate the flow (primarily particle displacements), and loosening mechanisms in a mining environment the following topics are reviewed:

1. Methods of data recording from model studies
2. Flow patterns and flow mechanisms
3. Formulae to predict particle trajectories
4. Scaling laws.

5.2 CHARACTERIZING GRANULAR MATERIALS

At this stage, it should be indicated that the flow behaviour of granular material is considerably influenced by its material properties or characterization.
A number of helpful categorizations are supplied by Richards (1966), Reisner & Rothe (1971), Kvapil (1965a, 1965b). From these and other studies it is deduced that coarse granular materials, present in a mining environment can be characterized by:

1. Particle size grading, and relative proportions of size fractions
2. Particle shape (i.e., angularity)
3. Porosity/Void ratio
4. Particle density
5. Packing
6. Surface properties
7. Compressibility and strength to withstand crushing and fracture
8. Physical and chemical changes with time
9. Environmental factors e.g., moisture, foreign chemical constituents present, temperature.

Obviously from a descriptive viewpoint, a granular material can be exceedingly complex in nature, and consequently it is equally difficult to prescribe the flow behaviour as a function of the listed parameters.

In addition, the forces operating within the granular assemblage will vary in different contexts, altering the flow mechanism. For instance, the significance of the surface compared with the rest of the particle decreases with increasing particle size as the gravitational effects on the particles are increased with increased particle size and the relative contribution of the cohesive, adhesive and frictional effects derived from the physical and chemical structure of the surface begin to diminish.

One must proceed with caution in adapting model results and extrapolating behaviour into mining environments. The choice of material for model testing can cause vastly different flow behaviour
dependent on the dominant flow mechanism. For instance, not only fine materials show cohesive properties, and tend to cohesive arching, but also mixtures of fine particles with coarser ones; only coarse materials can cause mechanical interlocking.

Flow in hoppers reproduces effects in a mining environment that are not conducive to regular flow. Although our purpose is initially to formulate a flow model that represents the regular flow situation, it is recognized that these undesirable features will be part of the capability of a complete general flow model. These include:


(ii) Piping (Jenicke, 1970; Saperstein et al, 1971a)


Many studies, and especially mathematical derivations approximate a granular material as a Mohr-Coulomb solid with bulk properties of cohesion and angle of internal friction \( \phi \), which are estimated by shear cell tests of material samples. Since the early work of Jenike, considerable debate has focussed on the method and interpretation of shear cell tests, to predict the yield strength of materials.

Although shear testing on a large scale has been reported for rockfill in dam structures (Marachi et al, 1972; Marsal, 1967) scale problems arise for testing broken ore, if Jenike's design methods were to be applied to draw down in an effort to predict conditions for favourable ore flow.

Because of the scarcity of data relating to scientifically controlled mine tests, and the lack of scaling laws (see Section 5.5) model tests are the remaining source of data for
postulating a flow mechanism as a basis for a flow model. If the flow model devised is sufficiently general it can be adapted to results obtained from full scale mine tests, as they become available.

Modelling studies, for gravity flow of granular solids, derive from two sources

(i) bunker flow studies, where the objective has generally been a pragmatic one of guaranteeing adequate flow-through in the bin. More scientific studies are being carried out at Cambridge applying the techniques developed under Roscoe for the analysis of the stress-strain behaviour of soils (Bramsby et al, 1975).

(ii) sublevel caving studies, where an attempt has been made to come to terms with the essential features of a sublevel caving environment, viz.

a. irregular boundary geometry
b. multi-material models (ore and waste)
c. densification of ore in the ring (by compaction techniques)

The chief preoccupation of these studies has been the determination of the limit ellipsoid, ellipsoid of motion, or more recently the volume of motion, which is essentially an imaginary surface, useful for design methods, but giving little information on the flow mechanism or particle trajectories necessary for understanding a kinematic model. This does not however dismiss this valuable accumulation of data, relating to the influence on general flow behaviour of the parameters peculiar to the sublevel caving system. It is recognized however that the methods for determination of the features of kinematic flow, in the past applied to bunker flow studies, have rarely been applied to sublevel caving studies. An important distinguishing feature of sublevel caving studies is that flow is interrupted. An equilibrium is reached after the formation of the rill heap, before the next increment is extracted from the model.
5.3 METHODS OF RECORDING MODELLING RESULTS

Recording methods basically fall into two categories depending on whether the model is constructed with transparent walls, for flow visualization.

A External Measuring Methods

(i) Grid Technique (Saperstein et al, 1971b)

The surface layer of the material is impregnated with a light sensitive dyazo dye, and grid lines are defined by the shadow of a wire screen. The intersection of the grid lines define movement, and no foreign material is introduced to change the granular material properties.

(ii) Marker Placement

Small visible markers, usually stained material particles or foreign bodies, are placed strategically to provide marker trajectories for representative points. Ideally markers should not influence flow. One disadvantage is that markers may recede back into the material. Alternatively the granular particles may be distinct enough to be recognized and traced.

(iii) Layers (Reisner, 1967; Toyama, 1970)

As an alternative to (ii), the general behaviour is apparent when layers of the same granular material are dyed, or material of different colour is placed in alternate layers.

(iv) Time Lapse Photography (Bosely et al, 1969; Kotchanova, 1972; O'Callaghan, 1960)

By using time exposures of various durations blurred pictures can be produced. The velocity distribution can be inferred from the degree of blur.
Each of these techniques assumes that material motion is confined to a plane, and the wall is sufficiently smooth to eliminate boundary effects. Photographic recording is often employed, interrupting flow to take still photographs, or else employing motion picture filming.

B Internal Measuring Methods

(i) Setting Agents

A technique employed in several studies (Saperstein et al, 1971b; Martin et al, 1965, Novosad et al, 1968) involves "freezing" the flow using setting agents, particularly paraffin wax but also chemical cementing agents. The model is cut open and the material which is distinguished by coloured layers or squares, can be examined for displacement. The disadvantage is that only one examination of flow conditions can be made for each model run. Another variation (Ciunta, 1969) tilts the model 90°, the top half of the bin frame is removed and excess material removed with a vacuum shovel to reveal flow conditions on the midplane of the model.

(ii) Radio Pill

Several investigators report the development of radio pills (Perry et al, 1975, 1976; Rao et al, 1973). The radio pill is a miniature pressure transducer and integral radio transmitter originally developed for use in medical research. Earlier experiments (Perry et al, 1967a, 1968, 1970) measured interrupted flow, while a later study investigated uninterrupted steady flow. The radio pill location is determined using separate ferrite aerials wound at regular intervals over the height of the model frame. The radio signals are recorded and analyzed to provide information on particle velocities given the time interval to cross successive aerials, and an assumed particle trajectory for the mass flow bunker. Alternatively, if flow was interrupted, the position of the pill could be obtained from the
signal strength but only over comparatively short distances. The chief limitations of the technique are
a. uncertainty as to whether the pill is following the flow paths of the surrounding data
b. the limited data that can be gathered from each model run. The use of more than one pill in a model run has not been reported.

(iii) Radiographic Techniques

X-ray radiography is a recent modelling technique that has been adapted to measuring the kinematics of granular materials. Roscoe et al (1963) investigated soil strains in earth pressure models.

Both velocity and porosity (ie bulk density) fields can be determined. Radiation absorption is proportional to the amount of material present, hence the porosity of the volume of material through the model can be determined and recorded by exposing radiographic film (attached to the back of the model) to the radiation transmitted from a source through the volume of the granular material. The position or velocity of particles can be determined by embedding small lead shot in the granular material. For interrupted flow the images of the shot on the film locate position, and for uninterrupted flow an exposure time of appropriate duration gives local velocities.

The method has the advantage that the velocity and porosity is determined everywhere at an instant, the results being restricted however to two dimensional models of limited thickness.

Initial investigations employing this method were carried out by Athey et al (1966), Cutress et al (1967), and more recently by Blair-Fish et al (1973), Bransby et al (1973, 1974) and Lee et al (1974).
McCabe (1974) used a gold radioactive isotope as a marker, and with an isotope scanner the positions of the markers were plotted on neutron sensitive scanner paper.

In another study on porosity profiles during flow, Van Zuilichem et al (1974) employed a caesium gamma source and a detector to pass γ radiation through their model silo and measure the absorption. Typical values over a selected area were measured by translating the source-detector unit. Bosely et al (1969) applied a similar technique.

(iv) Internally Placed Markers

Although internally placed markers can be recovered they do not indicate marker displacements or velocities. They do provide an indication of the origin of extracted material.

McCabe (1974) suspended small diameter horizontal disks on fine steel wire within a flexible cable supported at the top of the model. The velocity of descent during discharge at any instant was measured from the movement of the wire.

Application of data recording techniques

At present sublevel caving studies have been restricted to those of Section A(ii), (iii) and B(iv). Because of the unusual boundary geometry of a sublevel caving model little visual data on flow behaviour is available. To overcome this problem two dimensional model studies of front and side view geometry have been common. More recently, a new design, being basically a three dimensional model, but truncated along the heading centre line, was constructed by Alford. Both front and side views are available for analysis. Model testing by Panczakiewicz (1977) using this design seems to indicate that this half heading model replicates the behaviour of the full heading model.
However for a full understanding of the three dimensional flow model additional data beyond that of internally placed markers seems necessary. The only alternatives from this review of data recording techniques are those of Section B(i), (ii) and (iii).

Setting agents are not helpful for determining marker trajectories, while considerable time and expense are involved in applying radiography and radio pill techniques, both having limitations.

5.4 FLOW PATTERNS, FLOW MECHANISMS AND PARTICLE TRAJECTORIES

The results of experiments reported in this section apply to two, or three dimensional axisymmetric bin-hopper models. In these experiments the flow pattern that develops is usually classified as being one of two types:

(i) Plug flow where only granular material in the central core of the bin displaces, leaving material close to the hopper walls at rest.

(ii) Mass flow which occurs when the width of the flowing material coincides with the walls of the hopper.

Mass flow regimes have been investigated extensively because in industrial applications of bulk materials handling they provide the optimal mode of material transfer, both because no dead zones are created and also favourable mixing behaviour develops. In sublevel caving, plug flow is the dominant flow pattern.

In addition, steady state flow conditions predominate in experimental work because the mathematical equations that have been developed for bin design neglect time as a parameter. Sublevel caving models essentially must incorporate a time parameter. The flow regime is progressive and time dependent, and the particle trajectory as a function of extraction history should be modelled.
5.4.1 Steady State Flow Patterns and Flow mechanisms

Free (1970) reviewed the flow patterns from six studies:

1. Kvapil (1965a, 1965b)
2. Woehlbier et al (1963)
5. Deutsch et al (1967)

More recently flow patterns have been observed by


Of more significance are studies which apply Internal Measuring methods to the study of mass flow bunkers. Although steady state flow conditions prevail, these studies yield data on displacement, velocity and porosity fields. These studies include:


Studies 9-11 indicate more clearly a flow mechanism. At the transition between the junction of the bin section and the converging hopper, narrow bands (called rupture zones) are observed. These bands of sharp change in velocity and porosity arise because the rigid-body-like motion that translates the material in the bin section must now adjust to the dimensions of the converging hopper. Basically 4 regions are evident (Figure 5.1):

1. Free fall zone (C)
   In this zone each particle moves as a free body in a gravity field. Void ratio is the greatest at this stage.
Figure 5.1 Flow behaviour in converging hopper (after Bransby et al 1973; Lee et al 1974)
2. Plug flow zone (D)
   Here the granular material moves as a rigid body with uniform downward velocity, with little change in void ratio.

3. Core feed zone (B)
   In this zone the granular material is almost undeformed from its original configuration.

4. Rupture zones (A)
   These are bands of large relative deformation. This shear deformation causes the material to dilate and the void ratio reaches a critical value.

As a consequence of this flow mechanism, Lee et al (1974) argues against a number of generally accepted kinematic assumptions of flow:

1. Incompressibility
2. Radial flow towards the opening in the converging bin section
3. Steady flow. The cyclic formation of rupture zones alternately at each bin-hopper transition implies unsteady flow.

Lee does not indicate the degree of error attributable to these assumptions.

It may be objected that this analysis introduces unnecessary complexity into the flow pattern. It is significant the parallel results were obtained by Deutsch (although porosity was not measured) for a wider range of materials including wheat, sand and gravel. The same phenomena occur for continuous as well as uninterrupted flow (Cutress et al, 1967).

The scale of the flow patterns is dependent on the boundary geometry and properties of the granular material.
Van Zuilichem (1974) measured porosity profiles for flowing and static bulk solids in a silo. Although no discontinuities in the porosity patterns were detected, isoporosity lines were contoured from 8 degree polynomials fitted to the density data. They indicate that density decreases towards the orifice. It should be noted that only agricultural materials and polythene prills were used for this study.

Brown et al (1970) postulate the movement of a dilation wave. As flow is initiated, Reynolds principle of dilatancy gives expression to the observed behaviour of packed granular materials. Before the granular material can deform the original particle packing must be distorted. To achieve this a local dilation or expansion of the granular assembly must be effected. Granular assemblies fail by a sliding movement (shear deformation) which is resisted by internal friction and the mechanical structure of the granular assembly.

Not all materials exhibit this dilation effect to the same extent. Most materials can assume a variety of states depending on whether the particle packing is loose or dense, and the degree of compaction that can be effected. Cohesive powders, containing a high proportion of fine material will often deform as a series of rigid body motions that deform within a narrow dilated region (called "surfaces of sliding" or "line of slip"). This description of behaviour does not apply to regions (usually near the hopper outlet) where the material is unconstrained and falls freely under gravity.

In an experiment with steel balls Brown et al (1965) observed dilatant waves being generated and passing up through the balls in a "trumpet shaped cone" implying a fluctuation of voidage during flow. Irregularly shaped granules do not pack together in a relatively ordered tight configuration possible for equally sized balls. Brown comments that although rolling and sliding of balls is discernably different from that of a granular mass in a bunker it could be expected that a dilatant wave plays a part in the flow of a normal granular system.
In a theory to predict flow rates, in converging hoppers, Brown introduces a number of concepts that simplify the flow regime.

(i) Free Fall Arch or "dynamic arch" Immediately above the opening and in the transition from constrained to unconstrained flow, an unstable arch or dome forms. This forms, then immediately breaks.

(ii) Stream Tubes. Flow is confined to a volume for which mass is conserved at two sections enclosing the volume.

(iii) Radial Flow. Within the converging hopper section flow direction is radial towards the hopper opening. The flow of particles is independent of their origin i.e. they have no memory. As the flow tubes change direction as they approach the wedge section, the material must be loosened.

5.4.2 Unsteady State Flow Patterns

Few studies specifically draw attention to the progressive development of the flow pattern. The extent of the flow pattern is of course confined to material that is in motion, and consequently sensitive measuring techniques are necessary to detect the flow pattern boundary, with displacements of marker layers more frequently reported. Studies in this context include:

1. Chatlynne et al (1973)
2. Kvapil (1965a, 1965b)
3. Panczakiewicz (1977)
4. Mullins (1974a)

The work of Panczakiewicz clarifies the wide variance in behaviour between free flowing materials, and compacted cohesive materials that tend to arch and create cavities.
5.4.3 Flow Regime Dimensions

A number of investigators have endeavoured to define flow by indicating the maximum proportions of the flow regime, defining the region of flowing material. Usually these are expressed as maximum flow width as a function of material properties. Typical is Popowich (1969) who found that the core of flowing material tends to a circular cross-section irrespective of the shape of orifice, but was dependent on particle size and orifice dimensions. The transition from non-circular to circular shape occurring within a height of 150 times average particle diameter at a width of approximately 90 times the average for particle diameter for cohesionless material (Figure 5.2a).

Janelid (1975a) reports that in underground tests the "dividing layer" (an imaginary surface between broken rock in motion and at rest) is approximately 75-85°, and that this approaches 90° at a certain vertical distance, estimated to be of the order of 30 metres above the drawpoint (Figure 5.2b).

Based on theory developed by Johanson, Giunta (1969) predicted that for a flat bottom bin, the boundary of the flow pattern (i.e., the flow regime) could be determined from the equation (Figure 5.2c):

\[ 2V = D + 2 \tan \theta \left( \frac{H - A \frac{D}{2}}{1 + A \tan \theta} \right) \]

where the values \( \theta \) and A are dependent on the effective angle of friction (\( \phi \)) given from shear tests and available in charts supplied by Giunta.

5.4.4 Particle Trajectories

A number of attempts have been formulated for determining flow path trajectories.
Figure 5.2 Flow regime dimensions
A  Steady state flow

In this case velocity fields are determined. The radial flow assumption, was first employed by Jenike, for converging hoppers. Amongst others who have recorded velocity fields are: Pariseau (1969b), Lee et al (1974), Ko chanova (1972), Laforge et al (1964), Novosad et al (1968) and McCabe (1974). Brown et al (1965) concluded that flow was radial and velocity was inversely proportional to the radius from the apex of the hopper.

The only study to depart from the radial flow assumptions, and derive equations describing flow trajectories is Novosad et al (1968). They propose that successive positions of a layer of markers can be simplified to straight line segments for the flow in the lower half of the hopper. Conservation of mass determines the rate of displacement of these layers from the flow rate. An infinitesimal thickness is assigned to the initial marker layer. The position of the marker on the layer is determined from the new marker layer thickness, given that the volume between the wall and the marker is constant.

B  Unsteady state flow

Particle positions at various stages of the extraction history are recorded by Lee (1974), Panczakiewicz (1977). Chatlynne et al (1973) record displacements of marker layers but not individual trajectories. In their most recent study which applies to the initiation of flow, implying only small displacements, Bransby et al (1975) have produced displacement contours, and found that the radial assumption was valid. It is also observed that further deformation generates rupture surfaces. For the initial phase of deformation they found that the displacement followed a power law relationship.
\[ U = Ar^{-n}(\cos 2\theta)^{n+1}/2m \]

where \( A, m \) and \( n \) are constants and \( r, \theta \) are polar coordinates relative to the apex of the hopper.

Panczakiewicz (1977) has manually plotted the successive positions of markers placed in the granular mass, and estimated the active zone by interpolating a contour between particles that have displaced, and those that are unmoved at the particular stage of extraction. The flow of markers can be approximated by radial flow towards the centre of the heading floor.

In summary, each of the studies reported either plots or derives an empirical relationship for particle trajectories that relates to the modelling frame geometry and materials used in the particular test. In the conceptualization of the basic assumptions for an empirical flow model it was the work of Ko chanova that should be acknowledged as the genesis.

5.5 SCALING LAWS

If the performance of a prototype is to be correctly predicted from the observation of a model, then sufficient degree of similarity in the behaviour of model and prototype must be confirmed or else scaling factors must be introduced before the results can be applied with certainty.

Dimensional analysis is the mathematical expression of the requirements for similarity between model and prototype.

It requires that behaviour of the prototype be specified by some as yet unknown relationship between a finite number of measurable physical quantities, symbolically:
\[ \phi(q_1, q_2, \ldots, q_n) = 0 \quad (5.1) \]

or in an equivalent form

\[ q_1 = \phi(q_2, q_3, \ldots, q_n) \quad (5.2) \]

where \( q_1 \) has been selected as the dependent variable.

Equation (5.1) must include all the parameters \( q \) that define the behaviour of the prototype. Because of the complex nature of granular materials a simplified mathematical model or law must be postulated. It ought to be completely general because parameters \( q \) have different relative influences at different scales. Any variable that has no influence can be excluded.

Palacios (1964) introduces 3 rules for the application of dimensional analysis.

FIRST RULE : It is a necessary condition that the problem considered be in the domain of a physical theory already established and capable of complete solution if one could overcome the mathematical difficulties.

SECOND RULE : Apart from the unknown quantity \( (q_1) \), all those appearing in the list must be either constants or independent variables.

THIRD RULE : All the characteristic and universal constants belonging to the theory of the phenomenon considered must be taken into consideration.

The process of dimensional analysis is one of grouping the original quantities into "dimensionless ratios" \( \pi \) to form a new relationship,

\[ \phi'(\pi_1, \pi_2, \pi_3, \ldots, \pi_m) = 0 \quad (m < n) \quad (5.3) \]

which contains all the information already expressed in (5.1).
Equation (5.2) expresses a functional relationship between the parameters, and is established by one of several techniques which obtain dimensional homogeneity for each of the \( \pi \) terms.

The number of dimensionless terms is related to the number of fundamental dimensions \( k \). This is expressed by Buckingham's Pi Theorem which states:

"A complete dimensionally homogeneous equation relating \( n \) physical quantities which are expressible in terms of \( k \) fundamental quantities can be reduced to a functional relationship between \( n-k \) dimensionless products."

The fundamental dimensions are typically mass, length, time, and force. If \( n > k \), then equation (5.2) will be an unknown function of 2 or more dimensionless ratios. The values of these unknown functions can only be determined experimentally. Techniques are available to eliminate the unknown functions, but in any case the choice of fundamental dimensions can be critical for selection of a meaningful set of dimensionless groups.

The resultant dimensionless groups can be examined, and any groups that do not have practical or physical significance must be manipulated one with another to produce groups having relevance. Otherwise it is necessary to re-examine the parameters for completeness or change the set of fundamental dimensions.

To maintain similarity between model and prototype, then the \( \pi \) terms in equation (5.3) must have the same values in both systems.

Depending on the fundamental dimensions chosen for the dimensional analysis, one or more of the following conditions will apply:

1. Geometric similarity. This involves shapes and dimensions, and there is a point to point correspondence between model and prototype geometry.

2. Kinematic similarity. This involves similarity of motions, so that homologous points have similar components of velocity and acceleration.
3. Dynamic similarity. If in addition to geometric similarity, similar forces act at homologous points, dynamic similarity is achieved.

The relations between corresponding quantities in the two systems are termed scale factors. Scale factors can relate to the fundamental dimensions, or the initial quantities.

Discrepancies between the results for the model and the prototype due to lack of similarity are termed scale effects. These arise from two sources:

1. failure to obtain equality for the dimensionless groups
2. incomplete description of behaviour, i.e. quantities having insignificant influence in the prototype become significant in a scale model, or vice versa.

Scale effects may be unavoidable, implying that the "distorted model" must be reinterpreted by distortion factors (Schafer et al, 1973).

The primary task in the application of dimensional analysis to the flow of granular materials is the choice of quantities to describe the behaviour.

Fowler et al (1959) showed that the coefficient of friction is a function of particle shape, diameter and roughness. Subsequently Fowler et al (1959) used the following parameters to identify a dry granular solid flowing from a bin:

(i) orifice diameter \( D \), and orifice shape
(ii) container diameter \( D_{C} \)
(iii) particle diameter \( d_{s} \)
(iv) head of material above orifice \( H \)
(v) true density of solid \( \rho_{s} \)
(vi) bulk density of solid packing \( \rho_{B} \)
They derived a dimensional equation for the flow rate

$$\frac{W}{\rho_s D^2 g D} = f\left(\frac{D}{ds}, \frac{H}{ds}, \frac{\rho_s}{\rho_B}, S, \frac{Dc}{ds}\right)$$

Johanson (1971) considers that if particles are small enough then "solids flow theory based on the mechanics of a continuum" can be used to determine material behaviour. Johanson can then include the following quantities:

(i) Effective angle of internal friction $\delta$
(ii) Angle of internal friction $\phi$
(iii) Angle of sliding friction of bin wall $\phi'$
(iv) Unconfined yield strength $f_c$
(v) Compressibility factor $\beta$
(vi) Bulk density $\gamma$

The only dimensioned term $f_c$ is made dimensionless by introducing the major principle pressure $\sigma_1$. Therefore

$$\phi(\delta, \phi, \phi', \frac{\sigma_1}{f_c}, \beta, \gamma) = 0$$

Because flow is assumed incompressible (except for high flow rates), as an approximation

$$\phi(\delta, \phi, \phi', \frac{\sigma_1}{f_c}) = 0$$

Each of these dimensionless terms is pressure dependent so that a similar pressure range must prevail in both model and prototype. Johanson presents equations for the pressure distribution in deep and shallow bins.

The theory of Jenike predicts that for cohesive arches to be unstable, the necessary condition is

$$f_c < G \gamma B$$
G — geometry factor dependent on obstruction shape
B — characteristic dimension of outlet
γ — material density.

If for instance, arches were not observed in the prototype but the model was scaled by a factor of K with similar test material then

$$B' = \frac{B}{K}$$

therefore to ensure the equality still exists

$$\gamma' > K\gamma$$

This increase in density to compensate for the reduction in the span of a cohesive arch is implausible and so one could expect arching in the model.

Brown et al (1970), following Cutress et al (1966) advise that the use of scale models to reproduce arching necessitates a selection of a different material. For similarity the dimensionless ratio $\frac{B\gamma}{f_c}$ must be maintained. A suitable material is then one for which the ratio of the density to the strength is increased by the geometry scale factor.

A number of investigators have commented on the application of scaling laws to sublevel caving.

Janelid (1965) noted that although complete similarity is desirable, models were unable to reproduce:

(i) angle of friction of solid boundaries in the model
(ii) scaling of coarse ore, does not guarantee the reproduction of flow behaviour in the prototype
(iii) density changes in ore due to blasting.

As a consequence modelling has been restricted to making qualitative not quantitative predictions of behaviour trends. This does not
detract from the value of modelling for predicting the general
principles of flow behaviour as Janelid's more recent published
work evidences.

Sandstrom (1972) argues that "conversion of test model results
is sufficiently well understood to justify their use" as predictive
tools. Sandstrom lists the basic factors for conversion of results
as:

(i) Pressure distribution
   The pressure relationship between mine prototype
   and model is a linear function of the length scale.
   For cohesive solids, this pressure contrast will
   alter the strength properties of the model material

(ii) Kinematic angle of friction between wall slope and
     granular solid.

(iii) Consolidation
     The model does not reproduce the cohesive forces
     due to water, blasting and time delay, nor the consi-
     derable consolidation pressure due to blasting.

(iv) Density Difference
     Because expansion of model ore cannot be modelled
     a volume conversion factor is introduced.

The situation could be summarized by Free (1970) that:

"... little consideration has been given to the dynamic and
kinematic similarity between the designed model and the
full scale operational prototype. At present due to limited
knowledge available in the fields of stresses acting within
the caving ore and lateral pressures exerted around an
excavation heading, it is impossible to determine these
similarities accurately".

The need for a mathematical flow theory that incorporates stress
is evident.
Applications of dimensional analysis to modelling of soils is discussed by Roscoe (1968) and Freitag et al (1970), while Mandl (1968) and Pariseau et al (1968b) examine the problems of similitude for other rock masses.

Despite the use of centrifuge modelling for geotechnical structures (Rowe, 1975; Beasely et al, 1976; Hoek, 1968) for a decade, the method has only recently been applied to hopper design experiments (Molerus et al, 1977) to overcome scale effects, given the right choice of modelling materials.

5.6 CONCLUSION

Although the flow mechanisms and flow patterns outlined in this review do not produce a consistent picture, this lack of unanimity is attributed to the infancy of theoretical studies and the complexity of the subject.

No distinct mathematical relations are evident as a basis for an unsteady kinematic flow model that can identify flow patterns and particle trajectories as a function of extraction history. Previous modelling studies have not focussed on this goal.

However in the unsteady plug flow situation, two components are identified:

(i) A surface which encloses the material that has begun to move (Janelid's active zone).

(ii) Particle trajectories

    The magnitude and direction of the particle velocity will in general be a function of original position, flow mechanism and extraction history.

No satisfactory general relation has been established to relate these two components.
Although many mathematical studies have used the incompressibility assumption to advantage, recent experimental studies indicate this assumption is invalid. In particular, the studies of unsteady flow indicate that a loosening process dictates the extent of the active zone. The finite rate of propagation of the "loosening front" indicates that the average density within the active zone is less than that of unloosened material. No study indicates the degree of loosening as a function of position or extraction history.

Based on concepts from modelling studies reviewed in this Chapter a simple kinematic model will be developed in Chapter 8.

It will be evident that to justify the assumptions made in this model, a modelling programme is needed that investigates variables that to now have not received attention. Because the major focus of this study is the development of numerical and computer models for sublevel caving, insufficient time was available to implement a modelling programme that investigates the assumptions that the development of the computer models necessitated.

Appendix I indicates the necessary scope of the modelling programme and indicates a suitable method for data recording and analysis.

It is observed that little development of scaling laws exists for predicting mine behaviour from models, and to complicate the picture, scale models cannot model some of the basic phenomena (blasting, density changes). Consequently, incomplete similitude, being primarily geometric scaling of model and test materials, must be relied upon.
CHAPTER 6: BLASTING MODEL

6.1 Introduction

6.2 Application of Blasting Model

6.3 Descriptions of Blasting Models
   6.3.1 Model I
   6.3.2 Model II

6.4 Stages in Blasting Calculations
   Calculation of density function in variable density region
   Determination of relationship between position in blasted and unblasted zones
   Determination of relationship between position in compacted and uncompacted zones
   Detection of appropriate displacement calculation

6.5 Results Produced by the Blasting Model

6.6 Assessment of Blasting Models
This chapter examines the role of blasting in sublevel caving practice, reviewing relevant research and operating practice, and introduces simple models to simulate blasting that are subsequently programmed for use in an overall simulation program.

6.1 INTRODUCTION

A fundamental observation in the flow of granular material is the influence of particle size distribution and shape on the gravity flow behaviour. In sublevel caving the primary determinant for these factors is rock fragmentation due to blasting, and secondarily, comminution during flow.

In any mining operation the aspects of blasting design are well established:

a. blast pattern and geometry
b. blast hole diameter
c. explosive charge type and composition
d. charge depth and burden
e. hole stemming
f. sequence and delay between detonation of holes.

To say this is not to imply that the results of blasting are predictable. As remarked by Harries (1977), "in spite of apparent simplicity, the practical problem of relating explosive properties, the pattern and firing order used, and the rock properties to the blasting results, particularly fragmentation, is still obscure."

Sublevel caving introduces further variables to be considered in blasting design:

i) the irregular geometry providing no free face, forcing blasting to be confined and so restricting fracturing and loosening processes

ii) fragmentation must be even to prevent large boulders producing "hangups", and so delaying production; or introduce unevenness in flow, causing premature dilution
iii) the slice of ore is blasted against loose rock, and so a density gradient is established within the waste rock, as well as vertically within the ring, with the rill heap being completely loose.

Janelid (1972) estimated rock swell to be about 20% based on literature, and his observations. This compares with 60% swell in unconfined blasting.

The interaction between blasting design and ore flow proves difficult to study for a variety of reasons:

1) blasting in full scale tests is hidden from the observer limiting descriptive analyses

i) scale model tests introduce problems in establishing theoretical scaling laws for rock crystal size and gravity, with some explosives detonating unreliably for small diameters (Harries and Hengst, 1977).

Flow of ore, although significant for the success of sublevel caving, is not the only factor in blast design. Blasting is one phase of an overall mining system, and the effects of fragmentation on the costs of drilling, loading, hauling and crushing must be considered together with the recovery and dilution that can be achieved.

Rock fragmentation has been studied at various levels.

1) full scale testing, where particle size distribution can be measured using photographic techniques, point count methods, or full screen analysis (though difficult for coarse rock). (Janelid 1972, Bamford 1975, Brady 1971).
(ii) scale model tests by using screen analysis, and various empirical relationships for size distribution, equations have been formulated. (Just 1973, Just and Henderson 1971, Da Cama 1971, Bhandari 1975).

(iii) computer simulations of crack propagation, and strains (Harries 1977, Hattori and Mizutani 1971).

but our review of the available literature concurs with Hagan (1975) that "formidable barriers restricting calculation of optimum blast parameters ... for any conditions" exist.

This comment is evidenced by the tendency of mine operators, in determining blasting design, first to repeat procedures in previous mining operations and if results prove unsatisfactory, to use trial and error methods to make improvements (eg McMurray 1976).

The particular blast geometry employed at Mt Isa Mines Ltd is illustrated in Figure 6.1.

6.2 APPLICATION OF BLASTING MODEL

Subsequent chapters outline the assumptions and development of two flow models: stochastic and empirical. In each of these models the effects of blasting must be incorporated. A literature review indicates that, at present, no substantial relationships exist between ring design (ie sublevel caving geometry) drilling and blasting practice, subsequent fragmentation and ore flow characteristics. Consequently the simulation program retains blasting effects as separate and independent routines, where the input assumptions and data are as general as possible to allow predictions of the models to be compared with actual mining operations until a satisfactory correlation is found.

In order to apply to both flow models, the blasting model must predict:

(1) the grade, and density at any point \((x,y,z)\) after blasting
Sublevel interval : 35 ft.
Extraction heading spacing : 35 ft.
height : 12 ft.
width : 12 ft.

Ring burden : 6 ft.
gradien t: 70°

Figure 6.1 Sublevel caving geometry at Mt Isa Mines Ltd, 1976
(ii) the displacement of a point \((x,y,z)\) of known grade due to blasting, and its new density.

Consequently the function of the blasting model is to relate

(i) displacements due to expansion of ore, and compaction of surrounding waste

(ii) density changes in the blasting process.

The effect of fragmentation on rock flow is implicitly defined in the input assumptions to the flow models. As little research has been completed, and it was beyond the scope of this research to initiate research into blasting theory, the models to be outlined are an initial attempt to simulate the processes taking place, and are based on intuition and observation. Future research should focus on more realistic blasting models, that quantify the dynamic processes in blasting, and predict the rock fragmentation and size distributions equations for in situ rock conditions.

6.3 DESCRIPTION OF BLASTING MODELS

Basic length parameters were introduced to describe the expansion (or swell) of the ring, and the region of waste that is compacted. Parameters \(A1-A9\) and \(RA1-RA5\) describe the dimensions of the unfired ring, and rill heap, assumed symmetrical about a vertical plane bisecting the heading (Figure 6.2). These parameters can be used to describe 4 polyhedra, called ZONES with dimensions:

\[
A_{ij} = A_{ij} \times KE_{ij} \quad i = 1,9 \quad j = 1
\]

After blasting each of these polyhedra expand, with the possibility of displacement of zone centroids, or slump of ore downwards into lower zones as the ore expands and loosens. The blasted ore occupies zones where the new dimensions:

\[
A_{ij} = A_{i} \times KE_{ij} \quad i = 1,9 \quad j = 2
\]

\[
RA_{ij} = RA_{i} \times KE_{i+9,j} \quad i = 1,5 \quad j = 2
\]
(a) blasted zone dimensions and numbering convention

(b) compacted zone numbering convention

Figure 6.2 Ring geometry notation for blasting models
Surrounding the unfired ring, 9 other zones can be constructed by assuming that these zones contain the waste rock that is compacted when the ring is fired.

The geometry of these zones can be determined by making the simplifying assumption that the extent of the affected waste is related to the ring dimensions by the relationships.

\[ A_{ij} = A_{ij} \times K_{EXP_{ij}} \quad i = 1,9 \quad j = 3 \]

\[ R_{ij} = R_{i} \times K_{EXP_{i+9,j}} \quad i = 1,5 \quad j = 3 \]

The expansion factors \( K_{EXP_{ij}} \) are determined by data generating routines for two basic models for blasting which will be outlined.

It is assumed that displacements in blasting have a component parallel to the ring face, and a horizontal component back into the waste rock due to the influence of the solid ring or else in the direction of rill angle for flow in the rill heap.

6.3.1 Model I

The explosive charge fragments the ring (ie Zones II,III,IV) and the ore in each zone swells. Subsequently gravity flow takes over and broken ore fills the heading. This ore slump is effected by applying the empirical flow model routines to simulate the movement of the zone of loosening into the blasted ore, and calculating the displacements generated.

To calculate the effect of blasting the zones are initially expanded.

Zone II

The ore is confined by the solid ore pillars on both sides, the ring face, and the waste rock. The zone expands in three directions with the limitations (Figure 6.3)

\[ \frac{\Delta D_1}{\Delta 1} = \frac{\Delta D_2}{\Delta 2} = \frac{D_2 - D_1}{D_3} \quad \text{by similar triangles} \]
Figure 6.3 Blasting Model I
(ii) the ratio of vertical displacements

\[ S = \frac{A_1}{A_2} \]

\[ = \left( \frac{D_1}{D_2} \right)^* 2 \quad \text{(is assumed)} \]

(iii) expansion into the waste rock is given by KLS.

Consequently an expansion factor K in the vertical direction on the ring plane can be defined, if the density of blasted ore in zone II is defined to be DENSOS, by solving a polynomial expression, K being the unknown variable.

\[
\text{OLDMAS} = \text{DENSOS} \left( D_3 D_1 D_4 + \frac{D_3^2}{2} [D_2 D_4 + D_5 D_1] + \frac{D_3^3}{3} [D_2 D_5] \right)
\]

\[
\text{VOLNEW} = \frac{\text{OLDMAS}}{\text{DENSOS}}
\]

\[
D_3' = K.D_3
\]

\[
D_2' = D_2 + \frac{D_3 (K-1)}{1 + S} \cdot \frac{D_21}{D_3} = D_2 + L(K-1)
\]

\[
D_1' = D' - \frac{S D_3 (K-1)}{1 + S} \cdot \frac{D_21}{D_3} = D_1 - M(K-1)
\]

\[
D_4' = D_4 \times KLS
\]

\[
D_5' = D_5 \times KLS
\]

\[
\text{VOLNEW} = (D_3' D_1' D_4' + \frac{D_3^2}{2} [D_21' D_4' + D_5' D_1'] + \frac{D_3^3}{3} [D_21' \cdot D_5'])
\]

On substitution, a polynomial expression in K is found, which is solved (Appendix 4).

Zones III, IV

The expansion back into the waste is again determined by KLS, and the following expansion factors are introduced

- KBL the vertical expansion
- KLL the lateral expansion.

The consequent expanded ore density in these zones DENSCE can be obtained from a mass balance for the volumes of these zones before and after blasting.
An alternative would be to express DENSCE (=DENCS) and the ratio KBL/KLL and set up a calculation to determine KBL, KLL so that the blasted ore density in the ring is uniform.

In keeping with Model II we assign

\[ \text{DENSO} = \text{DENCS} \]
\[ \text{DENSOE} = \text{DENSCE} \]

**Compacted zones**

The dimensions of the compaction zone are linearly related to the dimensions of the blasted zone, using the proportionality factor KCZ.

**6.3.2 Model II**

Model II differs from Model I, in that the expansion factors are specified, and the blasting routine assigns a density distribution that is consistent with the expansion factors. An error flag indicates a lack of consistency in the input data.

The assumptions concerning the density distribution are:

(i) the density function is dependent only on height, not lateral position, and has the form \( \rho = Ah^2+Bh+C \) where the ratio \( A/B (=\text{SHAPEF}) \) is defined in input data.

(ii) the minimum density, i.e. the density of completely loosened ore (DENSO), and the density of blasted, but not loosened ore (DENSOE), are specified.

(iii) the height of loose ore is specified.

(iv) the variable density region is a quadratic function. A typical density function is shown in Figure 6.4.

In this model the ring is blasted, and the explosive mechanism forces ore into the heading, being the only free face, yielding a partially expanded ring.
Figure 6.4 Blasting Model II
6.4 STAGES IN BLASTING CALCULATIONS

Once the blasting model is chosen, the expansion factors and the form of the density function are defined. It remains to determine particle displacements and material densities. The logical steps and calculations involved, with their assumptions follow.

(a) **Calculation of the density function in the variable density region**

The unfired ring can be conveniently described by 3 polyhedra, (II,III,IV) for which the mass of each polyhedra can be evaluated using the density of unblasted rock. These polyhedra are expanded on blasting and one polyhedron is added to identify the rill heap.

A new table of blasted zones is constructed, where the material in each zone can be characterized as either:

(i) loose ore, density DENSOL
(ii) variable density ore where $\rho = Ah^2 + Bh + C$
     is the density function
(iii) blasted ore, but not loosened, density DENSOE.

A further restriction on these new zones is that the zones must be completely described by the five parameters.

(i) width of base D1
(ii) width of top D2
(iii) vertical height between base and top D3
(iv) depth of base D4
(v) depth of top D5.

In addition each zone is characterized by

(vi) height of top relative to HREF
(vii) height of base relative to HREF
(viii) HREF, the base of blasted polyhedra 2
(ix) HREF, the origin for variable density function
(x) the coefficients A,B,C of the density function
(xi) mass of ore in zones below this zone
(xii) mass of ore in this zone.
(a) polyhedra (1-6) defined by density profile

(b) linear transformation to determine position $P'$ after blasting

Figure 6.5 Blasting model calculation sequence
Referring to Figure 6.5, a typical calculation has the sequence

1. if volume of rill heap (I) > 0, and HL > HREF, zone 1 is defined

2. because HL lies in polyhedra II, zone 2 being that part of polyhedron II below HL, is defined

3. evaluate mass contained in the remainder of polyhedron II, with III and IV if assume density DENSOE (MASOE), and evaluate mass of unfired ring not accounted for in blasted zones (MASREM).

   If MASOE > MASREM no density function consistent with the ring geometry and expansion factors is possible

   MASOE = MASREM all the remaining ore is of density DENSOE (as is the case for Model I)

   MASOE > MASREM continue to 4

4. Using the limits of polyhedron II, III, IV to define density functions C_{II}, C_{III}, C_{IV}, and hence evaluate the mass in these upper polyhedra (MASVD) for increasing variable density region interval. The polyhedra above the variable density polyhedra are of density DENSOE (and sum to MASOE).

   If (MASOE + MASVD_{C_I}) < MASREM < (MASOE + MASVD_{C_{I+1}}) the upper limit of the variable density region lies in polyhedra i and using an iteration procedure the upper limit of variable density region (HVD) is located to effect the mass balance

   \[ \text{MASOE} + \text{MASVD}_{C_{\text{VVD}}} = \text{MASREM} \]

   If MASVD_{C_{IV}} < MASREM again no consistent density function is obtainable.

   In the example illustrated 6 zones were obtained.

(b) Determination of relationship between position in blasted zones and the unblasted zones

Given the height of a point \((x,y,z)\) lying in the blasted
zones, it is possible using the density function $\rho = Ah^2+Bh+C$ and the geometry of the blasted zones to calculate the mass below the point, by integration (see Appendix 4).

Given this mass it is possible to calculate the height of the particle in the unblasted zones by solving for the roots of a polynomial expression (see Appendix 4).

Two assumptions are involved in calculating the unblasted position:

(i) all material on a horizontal plane in the blasted zones derives from a horizontal plane in the unblasted zones

(ii) the width and depth of these horizontal planes can be determined from zone geometry for the appropriate heights, by assuming a linear coordinate transformation relative to these planes (Figure 6.5b).

(c) Determination of relationship between position in compacted and uncompacted zones

The expansion factors defined 9 zones surrounding the unfired ring, which suffer compaction in the blasting process.

The direction of compaction is approximated by specifying the compacted zone surface facing the ring to be that same surface of the ring after blasting (Figure 6.6).

It should be noted that if the expansion factors indicate the compaction zone increases in size, then loosening of overlying waste rock is indicated.

To calculate the displacements, and resultant densities in the compaction process, a local coordinate system $(u,v,w)$ is defined by mapping the compaction zone into a unit cube (see Figure 6.6).

During compaction the point $P(x,y,z)$ is displaced to $P'(x'y'z')$ with the limitation that the only local coordinate change is in $w$. 
Figure 6.6 Displacement in compaction zones
To determine the change in $w$, a densification process must be postulated.

The planes $w=0$, $w=1$ will not generally be parallel for the compaction zones, and so assuming the compacted density is constant across the plane $w=0$, and constant across the plane $w=1$, it is convenient to make the compacted density a function of the local coordinate $w$, and independent of $u,v$ and of quadratic form: $\rho = Aw^2 + Bw + C$. Consequently the mass in the compaction zone

$$\text{Mass} = \int_0^1 \int_0^1 \int_0^1 \rho(w) \frac{\delta(x,y,z)}{\delta(u,v,w)} \, du \, dv \, dw$$

$$= \int_0^1 \int_0^1 \int_0^1 \text{DENSWA} \frac{\delta(x,y,z)}{\delta(u,v,w)} \, du \, dv \, dw$$

with $\rho = \text{DENSWA}$ for $w=1$, and $A/B = \text{SHAPEF}$.

To evaluate the displacement of a point $\rho(u,v,w)$, the mass between planes $w=0$ and $w=w'$, is compressed to $(0, w')$.

$$\text{Mass} = \int_0^{w'} \int_0^{v'} \int_0^{u'} \rho(w) \frac{\delta(x,y,z)}{\delta(u,v,w)} \, du \, dv \, dw$$

for which $(u'v'w')$ must be evaluated. This would require numerical integration. A simpler approach to obtaining $(u'v'w')$ was investigated, to avoid the complications in solving for $(u'v'w')$.

Assuming that the planes $w = 0$, and $w = 1$ were of approximately equal dimensions and area, a simplification is to assume a one dimensional compaction path.

Therefore mass between planes $w=0$ and $w=w'$

$$\text{Mass} = \text{DENSWA} \ast \text{W}$$

$$= \int_0^{w'} (Ah^2 + Bh + C) \, dh$$
Given \( \rho = \text{DENSWA} \) at \( w=1 \), ie \( A+B+C = \text{DENSWA} \)

\[ A/B = \text{SHAPEF} \quad \text{ie} \quad A/B = \text{SHAPEF} \]

\[ \int_0^1 (Ah^2+Bh+C) \cdot dh = \text{DENSWA} \quad \text{ie} \quad \frac{A}{3} + \frac{B}{2} + C = \text{DENSWA} \]

These equations yield:

\[ A = \frac{6 \cdot \text{SHAPEF}}{4 \cdot \text{SHAPEF} + 3} \]

\[ B = \frac{6}{4 \cdot \text{SHAPEF} + 3} \]

\[ C = \text{DENSWA} - \frac{6}{4 \cdot \text{SHAPEF} + 3} (1 + \text{SHAPEF}) \]

\[ W = \frac{1}{\text{DENSWA}} (A \frac{w^3}{3} + B \frac{w^2}{2} + Cw) \]

Therefore the uncompacted position \( P(u,v,w) \) is obtained directly if the compacted position \( P'(u'v'w') \) is known, or if the uncompacted position is known, \( P(u,v,w) \) can be obtained by solving the polynomial expression for \( w' \).

(d) Detection of appropriate displacement calculation

The blasting model yields

(i) unblasted or uncompacted position of a point \( P'(x,y,z) \) so that grade at the position \( P'(x,y,z) \) could be assigned after blasting

(ii) the displacement of a point \( P(x,y,z) \) after the blasting process

(iii) the density at \( P'(x,y,z) \) after blasting.

The calculation of the displacements for blasted and compacted zones have been outlined. It remains to outline the selection of the type of zone the point \( P(x,y,z) \) or \( P'(x',y',z') \) lies in.
The expansion factors define a solid volume that specifies the outer dimensions of the
(i) unblasted ring
(ii) blasted ring
(iii) compacted material.

In addition the directions of displacement are distinguished.
(i) the displacement from \( P(x,y,z) \) to \( P'(x',y',z') \)
(ii) the original undisplaced position of \( P'(x',y'z') \), i.e. \( P(x,y,z) \).

Referring to Table 6.1, the appropriate action for the five possible cases is outlined. In case 2, although blast expansion has occurred, the slump into the heading induces a loosening (instead of the usual compaction) in the compaction zone. This provides a special situation (Figure 6.7).

Three possible courses of action are available depending on the combination of IC1, IC2, IC3 and K

1. If IC1 = 0, then the point lies outside the region of influence of blasting process and no action is taken (ITYPE = 0) since there can be no displacement

2. If ITYPE = 1, then the height, and corresponding width and depth for \( P \) and \( P' \) are read from interpolation tables set up at Stage (a) and a linear transformation relates \( P \) to \( P' (K = 2) \) or \( P' \) to \( P(K = 1) \)

3. If ITYPE = 2, then the local coordinates of the point \((u,v,w)\) are obtained by searching the polyhedra defined by planes \((1,3)\) for \( K=2 \) or planes \((1,2)\) for \( K=1 \). The search is shortened by selecting several height intervals, and selecting those polyhedra that intersect each interval and ranking them according to the likelihood of a point in each interval being found in each of the possible polyhedra. Given the Cartesian coordinates of the
Figure 6.7 Case of loosening within compaction zone

<table>
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<th>IC3</th>
<th>K</th>
<th>Action (ITYPE)</th>
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<td></td>
<td>2</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 6.1 Determination of displacement calculations to be applied

**CODE:**

IC1 = 1 point lies inside limits of compacted zone, otherwise IC1=0.
IC2 = 1 point lies inside limits of blasted zone, otherwise IC2=0
IC3 = 1 point lies inside limits of unblasted zone, otherwise IC3=0
K = 1 find position after blasting, otherwise prior to blasting (K=2).
point and the corners of a polyhedron, the local coordinates \((u,v,w)\) can be evaluated by the Newton Raphson Method, and the new position found by equations outlined in Appendix 4.

6.5 RESULTS PRODUCED BY THE BLASTING MODEL

Appendix 4 illustrates the density functions, and resultant displacements that are produced by MODELS I and II after the blasting process.

6.6 ASSESSMENT OF BLASTING MODELS

The key assumptions in the models are:

1. Mechanism of blasting process yielding
   - expansion of fired ore (swell)
   - flow into heading
   - compaction of broken rock surrounding ring

2. Density variations with blasted ore and compacted rock

3. Mechanism of compaction indicating extent of compacted material.

Only simple models have been postulated, that indicate direction and magnitude of displacements, without investigating the physics of blasting and fragmentation. The model relies on simple geometrical shapes, and consequently backbreak, and brow collapse have not been represented though the presence of these can be implied in the empirical and stochastic flow models.
CHAPTER 7: MONTE CARLO SIMULATION MODEL

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7.3 Representation of Flow of Granular Material

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7.1 INTRODUCTION

The Monte Carlo Method is frequently being used to model flow phenomena. Some recent examples are the diffusion of ideas (Hagerstrand, 1968) seepage in jointed rock (Kelly, 1971) formation of salt domes (Howard, 1968), and alluvial fan deposition (Price, 1976).

The Monte Carlo Method can be applied to two types of problems, either probabilistic or deterministic according to whether the problem is directly concerned with the behaviour and outcome of random processes (Hammersely, 1964).

Often a flow problem cannot be formulated or expressed in formal mathematical equations, although components of the process can be identified but not linked because of mathematical complexity. Frequently however no solution technique is available. Although a problem may be deterministic, in the sense that it has no direct link with random processes, the underlying structure of the problem may have parallels with some unrelated random process and so the deterministic problem can be solved by Monte Carlo simulation of the probabilistic analog. A simple example is the solution of Laplace's equation for heat flow

\[ \frac{\delta^2 \phi}{\delta x^2} + \frac{\delta^2 \phi}{\delta y^2} = 0 \]

where heat conduction is simulated by particles diffusing randomly in a region bounded by absorbing barriers (Barakat, 1966).

7.2 MONTE CARLO METHODS IN A MINING CONTEXT

The only published study of the application of Monte Carlo Methods to ore flow is that by Jolley (1968). The purpose of his study was the evaluation of the ore flow in a large underground pillar that had been fragmented by longhole drilling and blasting.
The level of simulation was very crude, with the pillar represented by a three dimensional model 12x12x14 cubes, however the model is reported to have been useful to make qualitative comparisons between drawdown strategies that involved different combinations of

(i) position and number of drawholes beneath the pillar
(ii) sequence of drawdown
(iii) the ore/waste cutoff ratio

By assigning ore grade to each cube, an economic appraisal was made. Imenitov et al (1971) enumerated the following deficiencies —

1. mechanical properties of the ore (arching and confinement of flow to narrow columns)
2. inhomogeneity in the density of the ore after blasting
3. loosening after extraction
4. size of drawhole Bio relative size of blocks to mine (ie geometry)
5. irregular boundaries of pillar.

In addition, further deficiencies are:

6. choice of lattice. Diffusion processes can be more accurately and realistically simulated by decreased block size
7. mixing and percolation processes

7.3 REPRESENTATION OF FLOW OF GRANULAR MATERIAL

The mechanism of flow of granular material can be conceptualized as an upward diffusion of voids that loosen and displace rock granules (Mullins, 1972, 1974a, 1974b).

Although analytical and finite difference methods are known for the solution of simple diffusion equations, a third method, Monte Carlo simulation is applied to the problem because of its
flexibility in setting up initial and boundary conditions, and the ability to represent factors that influence ore flow.

In the limit, as block size becomes infinitesimally small, the solution obtained by the Monte Carlo Method would approximate the closed form solution. A particular example of this would be the problem of granular flow from a two dimensional hopper analyzed by Mullins. An illustration of the flexibility of the Monte Carlo Method is the ability to introduce loosening processes (ie density changes); while Mullins presentation is only able to retain a solution by introducing mathematical approximations.

In the sublevel caving, the flow of ore into the drawpoint is the flow of a collection of irregularly shaped particles falling in a sequence where the magnitude and direction of displacements are completely determined by particle-particle, particle-boundary, and gravity forces, ie flow is completely deterministic.

By modelling the diffusion of voids the Monte Carlo method solves the deterministic problem of granular ore flow, by solving the probabilistic analog.

It is observed that the draw in sublevel caving is uneven and not symmetric, as if flow is irregular due to random effects. It will be observed from simulation results that this erratic flow is easily reproduced. This effect is due to the finite block size. Decreasing block size eliminates this erratic flow behaviour. An optimum must be sought, balancing realistic results with reasonable computational times, attempting to approximate deterministic behaviour.

A range of results is obtained for ring extraction depending on known factors eg deviation of drill holes from the planned shape of blasted ring, and also currently unpredictable factors eg fragmentation due to change in rock character, joint spacing etc. Although it is possible to postulate numerous factors that govern flow behaviour, few quantitative mathematical relationships exist. To represent this range of behaviour a probabilistic model
could be developed where a random component is applied to these factors that dictate ore flow. These random components have not been incorporated into the current stochastic model. The model does not take into account all the factors influencing flow, and remains partly intuitive, however it provides a basis for research and experimentation.

The flow process can be conceptualized as the upward displacement of discrete voids, which induce the downward displacement of discrete particles. The process continues until all the voids reach an equilibrium (i.e. by reaching the upper free surface or else by being trapped within the rock mass by what would be arching phenomena or particles interlocking in the real situation).

Each void performs a "random walk", but the numerous techniques available to rapidly solve Monte Carlo problems are not available in this case, because the problem is of a nonlinear type, (as opposed to linear as most of the techniques assume). This important point can be further elaborated.

In the usual applications of the Monte Carlo technique, one or more quantities pertaining to a given system are computed for each of a reasonably large number of independent particle histories. The desired result is obtained by averaging all of the particle histories.

However in the nonlinear transport phenomenon, the motion of individual particles is influenced by the motion of surrounding particles. Gravity flow of ore obviously falls into the latter category because particle flow disturbs the nature of the rock mass, and hence motion of voids cannot be considered irrespective of the stage of extraction. For example, a void cannot displace another void, but only a rock block, and the probability of surrounding blocks displacing a void will depend on the flow properties of each block, some possibly being more mobile than others.
This means that it is not possible, due to the nature of the quantities to be evaluated, to calculate the average behaviour of the system from the aggregate of individual void path histories. At all stages of the system, every event in the flow system must be recorded. This has the advantage of completely monitoring the flow process but at the expense of computational time, and computer storage facilities, providing a significant restriction for simulations using computing systems currently available, especially for three dimensional problems. Similar problems are, of course, experienced in large three dimensional finite element.

7.4 DATA STRUCTURE AND DISCRETIZATION OF PROBLEM

A computer program to perform simulations in two or three dimensions was designed to examine the validity of Monte Carlo Method.

The model is that of a single heading, though a multiple heading would be the next stage of development, and be available through the application of extraction procedures at a sequence of heading locations. The choice of a single heading allowed examination of the ramifications of assumptions in the program methodology.

The program was designed to allow specification of the flow problem to be:

(i) two dimensional – parallel to the heading
(ii) two dimensional – parallel to the ring face
(iii) fully three dimensional

Problem (iii) has been investigated least of the three, because considerable modifications in the program have been necessary to yield reasonable processing times and computer storage requirements.

The present program structure has facilitated the investigation of a number of assumptions, and mechanisms, with the modular nature of the program logic being an advantage for developing large scale simulation problems.
For the purpose of this simulation, the rock mass (broken, and unbroken) is treated as a continuum, and the discretization process is achieved by superimposing a rectangular three dimensional grid over the problem domain.

**Data Structure**

The region to be analyzed is divided into discrete elements of dimensions ELX x ELY x ELZ units. These elements can be referenced in three dimensional space relative to a Cartesian coordinate grid where successive elements in each coordinate direction are assigned an integer element number (I,K,J).

The problem domain is defined by considering elements that have element numbers in the range (Figure 7.1):

\[
\begin{align*}
I : & \quad ILXL - ILXU \\
K : & \quad ILYL - ILYU \\
J : & \quad ILZL - ILZU
\end{align*}
\]

By defining an origin, the coordinates of any point in the problem domain can be ascertained by knowing

(i) position relative to element
(ii) origin
(iii) element number (I, K, J)

Because computer memory must be allocated to storing variables and program, only limited core space is available to store the contents of each element. Consequently, as well as overlaying the program, a file structure must be created. The nature of the problem dictates that the file will be read and written to in non-sequential fashion, necessitating the use of random access files. An index array resides in core to store the address of each record on the disc file. Only those elements that are under immediate consideration are held 'in core', and are written onto the file, while other elements are read off the file as the area of interest changes.
Range of elements in each coordinate direction: 
(ILXL,ILXU), (ILYL,ILYU), (ILZL,ILZU)

Number of elements within block: 
(IBX,IBY,IBZ)

Number of blocks within problem domain: 
(INNX,INNY,INNZ)

Figure 7.1 Element reference system
At this point a trade off occurs. Obviously the length of the index array cannot equal the number of elements or a memory overflow occurs. One may as well attempt to hold all elements in core. Elements must be amalgamated into subsets of the problem domain, called blocks. A block comprises:

\[ IB3 = IBX \times IBY \times IBZ \text{ elements.} \]

It is noted that:

(i) Increasing the number of elements in a block decreases the number of blocks for a given problem, and the consequent length necessary for the index array.

(ii) If the elements of immediate interest occupy an irregular volume in space then smaller blocks give the best approximation to that volume.

(iii) Larger blocks will tend to be swapped in/out of core more frequently than a larger number of smaller blocks occupying an equivalent volume, because the former will tend to increase the number of elements that are not of immediate interest but must still reside in core because blocks are transferred as complete entities.

(iv) An upper and lower limit on block size is determined by allocating available memory so as to minimize length of index array, and maintain one block in core; or to maximize the length of the index array maintaining an integral number of elements in each block.

(v) The efficiency of transfer of data to random access files is a function of the size of data records and the frequency of data transfer.

The individual element with element coordinates \((I, K, J)\) can then be referenced by two parameters:

\[ IBL : \text{ indicating the block in the problem domain within which the element is stored} \]

\[ IEL : \text{ indicating the position of the element with the block IBL.} \]
The element referencing system describes geographical location within the problem domain. Conceptually the broken rock displaces through the grid as voids migrate upwards. In the notation of fluid dynamics, the Eulerian reference frame is employed where the state of the system is described at fixed locations as a function of time. The Lagrangian reference frame that describes the coordinates of moving particles as a function of time does not form the basis of this model, though this data is included with a representation for the flow of markers within the computer simulation.

The characteristics for the discretization chosen are:

(i) continuity — as material displaces from one element to another, continuity is maintained. The non-material space is defined as void.

(ii) material representation — the content of each element could be seen as a collection of particles, or as a continuum, but no direct relation between average particle size and element dimensions should be inferred (Figure 7.1).

(iii) artificial flow — individual void migrations (and consequent material transfers) can only be vertical and diagonal, implying an artificial restriction in movement, although this effect is "smoothed", over a large sequence of movements.

(iv) grid shape — any grid representation, curvilinear or orthogonal might have been chosen. An orthogonal cartesian grid was chosen for simplicity of element referencing, although Figure 7.3 indicates another possible option which would determine the possible directions of void migration.
<table>
<thead>
<tr>
<th>Material type</th>
<th>Density assigned</th>
</tr>
</thead>
<tbody>
<tr>
<td>solid ore</td>
<td>DENSOS</td>
</tr>
<tr>
<td>blasted (expanded)ore</td>
<td>DENSOE</td>
</tr>
<tr>
<td>loose ore</td>
<td>DENSOL</td>
</tr>
<tr>
<td>waste</td>
<td>DENSWA</td>
</tr>
<tr>
<td>void</td>
<td></td>
</tr>
</tbody>
</table>

*Figure 7.2 Material types*

*Figure 7.3 Potential grid models, and void movements*
(v) material characterization — in superimposing a grid it is assumed elements are so small that the character of material at the element centre is representative for the whole element. After flow is initiated the number of materials in an element (initially one) may increase. (Figure 7.2)

(vi) boundary representation — the representation of domain boundaries and multi-material interfaces is obviously restricted by the coarseness of the grid chosen.

(vii) material quantification within elements — of all possible representations for the quantification of different material types and location within an element, the simplest scheme was chosen. If more than one material occurs in an element the vertical order:

VOID
WASTE
ORE

is assumed, and all ore is assumed to be one density. The proportion of different materials is determined by the vertical dimension.

(viii) flowability — the contents of an element can be flagged as flowable or nonflowable to restrict flow direction.

7.5 FLOW ALGORITHMS

Once the problem domain has been set up, the next phase is to simulate the dynamic motion of element contents into the heading.
Figure 7.4 outlines the basic flowchart to understand the programming of upward migration of voids and downward displacement of broken rock.

In addition other processes that are taking place eg mixing and percolation, must be included.

The flow mechanism as represented in the simulation program has four stages

A. Selection of void to be displaced
B. Choice of direction of motion of the void
C. Material transactions that occurs with displacement of the void
D. Updating void queue.

Before the program structure is outlined, these assumptions will be examined.

7.5.1 Void Selection Process

In this section three questions are asked.

1. From where do the voids originate?
2. What is the volume of void compared to elemental volume?
3. In what order are voids displaced?

At the drawpoint, falling rock forms a well defined rill heap. This rill heap is approximated by elements in the heading. It is these elements that contain the slumped rock. Two situations are possible (see Figures 7.5 and 7.6). After blasting, ore that was previously a solid ring slumps into the heading. All those elements in the rill heap that were void before the blast slump are called the void blast queue and total ICTB elements. Similarly during extraction, a finite number of elements are removed by the loader and these are referred to as the void extraction queue and total ICTB elements. The extraction queue will be a function of the loading position in the heading and digging depth.
Figure 7.4 Flowchart for void flow – material displacement in the stochastic program
In addition to specifying the number of elements that are void, and into which material can slump, an order must be assigned to the sequence in which voids will migrate upwards, because no two mass transactions can be investigated simultaneously.

Although it might seem consistent to choose the void order randomly, voids will only displace upwards if material can slump downwards. In the queue, voids with greatest Z coordinate will displace first. Voids along the heading centreline displace, being equivalent to an arch collapsing at the centre.

The voids are ordered layer by layer, by x-coordinate and lastly by y-coordinate, as detailed in Figure 7.7.

The program takes each void in the queue in turn. Each pass through the void queue is termed a cycle. The pass is repeated until all voids have been eliminated, or until none of the voids displace in a cycle, indicating that equilibrium positions have been established, or until a maximum number of cycles NCYC is completed.

7.5.2 Direction of Void Displacement

The void is restricted to move only to an adjacent element, and it is also constrained to move upwards, yielding 9 potential sites to which it might migrate (Figure 7.8). The inclusion of purely horizontal movement would yield 17 potential sites, but this was not incorporated because it doubles computation time for little advantage.

To each of these sites a likelihood of void migration is assigned.

In the usual diffusion context, individual probabilities can be derived from the diffusion coefficients which are potentially concentration, time, location or material dependent. In the study of Nullins, coefficients were independent and constant. The probability measures the likely direction of void migration.
Figure 7.5 Location of void elements in blast queue

Figure 7.6 Location of void elements in extraction queue

Figure 7.7 Ordering of void elements in queue

Figure 7.8 Sites for void displacement
Any number of factors might influence the probability calculation. The most likely are selected:

(i) **Position relative to solid boundaries**

In the sublevel caving environment, the presence of solid boundaries directly influences the direction of flow. A void can only migrate to surrounding loose rock.

In addition, flow could be inhibited or promoted along or in the vicinity of a solid boundary surface, depending on frictional forces that develop.

(ii) **Material type**

Different materials can be assigned different mobilities depending on the physical characteristics of the material. In addition, loose ore or waste flows more freely than compacted material.

(iii) **Flow regime**

It has been observed in model experiments that flow is restricted to a well defined flow regime.

If voids are migrating from a source, there is a finite probability that voids will deviate well away from the flow regime. Using an orthogonal grid and assuming only vertical or diagonal displacement, flow is physically restricted to lie above a diagonal line radiating from the void source. A flow regime defined on horizontal sections within which voids are constrained to flow is postulated (Figure 7.9).

It is noted that the coarser the grid, the more pronounced would be the deviations, if an artificial flow regime were not incorporated. The flow regime boundary complements the notion of flow direction probability outlined in the next section. Justification for this choice of "cone column" flow regime is found in Section 8.5.
Figure 7.9 Definition of "cone-column" flow regime to confine flow
(iv) Flow direction

Irrespective of these other factors, a probability can be assigned to the vertical and diagonal motion. The ratio of these values determines the spread of voids. Because the basic column flow regime shape is employed, voids above the transition are more likely to migrate vertically upwards. Hence flow direction probabilities are defined to be height dependent, and to facilitate evaluation, are defined as a step function.

For each site, the individual probabilities $P_i$ are evaluated as the product of the factors, so that the vertical and diagonal directions of motion may be selectively weighted to take into account the relative influence of each factor, in order to emphasize preferred directions of movement.

Sites that lie outside the flow regime or within solid rock have zero probability, but other positive, non-zero probabilities are generated.

The cumulative probability is calculated by summing the individual probabilities for each potential site, and a random number $R$ is generated to determine the direction of migration.

$$0 \leq R \leq \sum_{i=1}^{9} P_i$$

such that

$$\sum_{i=1}^{n-1} P_i \leq R \leq \sum_{i=1}^{n} P_i \quad 1 \leq n \leq 9$$

indicating the void migrates to site $n$.

Stationary (time independent) probability distributions are utilized for simplicity. New flow probabilities are calculated for each successive void movement that is simulated, but probabilities are not retained after the void movement has been simulated because they are time dependent.
7.5.3 Mass Transaction Process

To increase the effective number of random walks, a limit VDMAX is set for the void volume that can migrate in any mass transaction. (If all blocks that will potentially contain voids are held in computer memory, no substantial increase in computational time will be expected). The process that takes place during mass transaction assumes the case where any combination of one or all the possible element components is present. The element contents retain the descending order,

VOID
WASTE
ORE

or otherwise not only the contents but the order of the contents would need to be stored for the case where the same material component appears twice, separated by another material component.

As the void migrates from element (I, K, J) to (II, KL, JL) two processes can potentially take place.

(i) Loosening mechanism:
    if (I, K, J) contains dense ore, this can be loosened as it displaces downwards.

(ii) Mixing and percolation mechanism:
    if the elements contain different material types then mixing and percolation are possible. Although particles may be displaced relative to each other when only one material is present, this is quantified only when different materials mix.

These mechanisms simulate what is a continuous dynamic process, in a stepwise discontinuous fashion, and consequently only a limited degree of realism is attained.

In both mechanisms a mass transaction between the adjacent elements occurs with no reference to history of the random walk of void, and in that sense the process is "Markovian".
Loosening Mechanism

The simplest loosening mechanism assumes that complete loosening takes place once flow is initiated. Consequently ore could be solid, blasted or loose.

Waste is currently assumed to be completely loose, and consequently any void will continue to migrate upwards to the surface of the broken rock. Further research should allow for waste compaction by the ring blast action adjacent to the blasted ring ore.

If a progressive loosening were allowed, a mechanism would need to be postulated, for example: loosening takes place over a specified distance, or in discrete density steps. The problem parallels the mixing one in that a dynamic process is being simulated, but also that no laboratory, or on-site test data exists for comparison with simulation results.

Assume loosening takes place over a step function density curve with a finite distance for each step. Yet loosening may be directionally dependent! In the light of these difficulties the simplest choice has been retained — complete loosening, if any. If the situation arises where a void is not large enough to completely loosen the overlying element during material slump, the remaining unslumped material is assumed to remain dense.

Consequently no intermediate densities are retained. In most practical cases loosening during a mass transaction will be complete.

Mixing and Percolation

As the void migrates through the ore/waste interface, the process of percolation (depending on relative size, density and shape of the ore and waste materials), and mixing that takes place by a "turbulent" mechanism, is simulated.
Because of the discretization of stochastic flow simulation, it is necessary to model the penetration of waste into ore in a number of discrete steps, subsequently determining the relative proportions of ore and waste in each element.

Although a number of studies have attempted to model the mixing and percolation of particle aggregates (Cahn, 1967; Masliyah, 1974; Moore, 1973) no direct resemblance to the current problem was established.

The following illustrations indicate the intuitive development of a model for mixing that yields the approximate mixing effects:

1. If the contents of an element are represented by horizontal slices, and slices are assumed to slump in their vertical order, then no mixing can ever take place.

2. Mixing about an interface position could span a number of elements vertically. To make the mixing process a function of the character of elements in a local region, would seem to imply that a mass transaction cannot take place in isolation from the contents of neighbouring elements. Yet this restriction would require a forward looking scan that considered the interaction of multiple (>2) elements; a computational complexity that would be grossly undesirable.

It is easy to construct a situation where no mixing would occur (Figure 7.10). A void migrates through an ore/waste interface but no mixing occurs, because each element retains only one material type. This situation indicates that mixing could be of two types:

(i) where elements contain one material type, but relative location of ore and waste elements resembles a "chess board" effect

(ii) multi material elements are generated, and mixing takes place within elements.
Figure 7.10  Mixing Illustration 1

No mixing achieved at interface when mass transactions are considered between adjacent elements and void has element volume.
Situation (i) is rejected because of the coarseness of the element grid and (ii) has been pursued.

In effect what is happening in mixing and percolation is that waste is moved preferentially to ore. In a mixed region both ore and waste exist in one element. An obvious method of achieving this is by floating voids of size $VD < ELZ$, so that overlying waste enters "ore" elements. Once the element contains ore and waste; all the ore or all the waste, or a combination of the two can slump into a partial void.

From this elementary model it is concluded that the initial waste penetration, and the subsequent waste penetration and mixing, define a mixed region that is dependent on two factors:

(i) size of void $VD$ and the ratio $VD/IELSZ = VDR^*$
(ii) the ratio of waste to ore that fills a void, $R$.

Consequently if waste is taken preferentially to ore:

(i) depth of penetration is related to the number of voids migrating through the waste material, not their size. Depth of penetration can be limited by placing a lower limit on waste volume for which penetration can be effected in a mass transaction.

(ii) extent of mixing is diminished for high $VDR$, low $R$

(iii) for the same $VDR$, $R$, element grid spacing has no effect except on the geometric scale of mixing.

To summarize; mixing is achieved when $VDR < 1$ $R > 0$.

Irregularities in mixing are introduced due to discretization at the waste/ore interface, particularly if it is inclined to the element grid. These effects are minimized with smaller grid spacing and void size $VDR$, so that more void walks are generated to achieve the same flow, permitting better flow definition.

*Terms $VD$, $IELSZ$, $R$ are defined in subsequent paragraphs.
If the interface fell within an element then the mixing result would be initially obscured, but would be established after a few mass transactions.

To illustrate the effect of these parameters VDR, R on mixing, a number of one dimensional problems are illustrated (Figure 7.11). These help to indicate the appropriate choice for program data input.

The representation of mixing and percolation in this fashion has the advantage of simple calculations, only adjacent elements are considered in the mass transaction, and calculations are simply incorporated into the mass transaction algorithm.

Other factors, for example the density of ore prohibiting mixing and percolation are incorporated because no waste passes through an element till ore is completely loosened.

It appears that an adequate discretized process to represent what would be continuous in reality has been presented. In interpreting the contents of an element, (unless R = 0 when mixing is eliminated), the presence of both ore and waste in an element is indicative of a mixed state, although the significance of mixing will be determined by R. (Even if the occasional element happens to contain approximately equal amounts of ore and waste – these will generally only be at the interface unless mixing is dominant.)

With the assumptions outlined in these two mechanisms and the two elements depicted in Figure 7.12, the following definitions are introduced:

- **IELSZ** — the vertical (Z) dimension of element
- **MINVOL** — tolerance on calculations
- **MAXVD** — maximum allowable void migration in one mass transaction
- **MIXRO** — mixing rate relating proportion of waste/ore
- **MIXRW** — that slumps into VD
- **VD** — the volume of void under consideration
(a) $VDR = 1$  \  $R = \frac{1}{2}$  

no mixing

(b) $VDR = \frac{1}{2}$  \  $R = \frac{1}{2}$  

mixed region defined and persists

(c) $VDR = \frac{1}{2}$  \  $R = \frac{3}{4}$  

mixed region increases until percolation is restricted by MINMAX.

Figure 7.11  Mixing Illustration 2

Effect of the two parameters $VDR$, $R$ on the mixing process
MINMAX — minimum amount of waste that can slump in a mass transaction. This sets a limit on the rate of percolation.

Because computation accuracy is not at a premium, a number of variables can be represented within each computer word by assigning variable fields within each 60 bit word (on CDC CYBER). A restriction on this technique is that negative values cannot be represented. Values are assigned, and retrieved by the use of shift and mask operations on the binary arguments. Integer arithmetic is employed for all computations.

The current variable definition is outlined in Table 7.1. To complete the mass transaction the following calculations are made:

Check size of void if VD > MAXVD 
VD = MAXVD

1. Calculate expanded ore
   MOLE = M01 * DENSOE/DENSOL if ore density = DENSOE
   MOLE = M01 if ore density = DENSOL

2. Calculate maximum proportions of ore, waste that would be slumped if available
   MOSM = VD * (1 - R)
   MWSM = VD * R
   R = \[\frac{MIXRW}{MIXRO + MIXRW}\]

Consequently the actual material slumped
   MOS = MOSM if MOLE ≥ MOSM
   MOLE if MOLE ≤ MOSM
   MWS = MWSM if MW1 ≥ MWSM
   MW1 if MW1 ≤ MWSM

If insufficient ore or waste to make up mixed proportions, then any remaining waste, or ore fills the void.

To prevent excessive percolation effect,
   If MWSM ≤ MINMAX 
   MWSM = 0
<table>
<thead>
<tr>
<th>Field (in multiples of 3 bits)</th>
<th>Variable</th>
<th>Max. Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 - 2</td>
<td>Material type (MAT)</td>
<td>7B</td>
</tr>
<tr>
<td>3</td>
<td>Flow regime flag (IFLO)</td>
<td>7B</td>
</tr>
<tr>
<td>4</td>
<td>Density type (IDENS)</td>
<td>7B</td>
</tr>
<tr>
<td>5 - 10</td>
<td>Marker number (IMK)</td>
<td>777777B</td>
</tr>
<tr>
<td>11 - 15</td>
<td>Ore content (NO)</td>
<td>77777B</td>
</tr>
<tr>
<td>16 - 20</td>
<td>Waste content (MW)</td>
<td>77777B</td>
</tr>
</tbody>
</table>

Table 7.1  Convention for variable assignment in each computer word to store element data
Figure 7.12  Element contents before mass transaction (general case)

Figure 7.13  Marker displacement in mass transaction
3. Calculate new contents of elements.

\[
\text{In (I, K, J)}
\]
\[
\begin{align*}
\text{MON} & = \text{MO} + \text{MOS} \\
\text{MWN} & = \text{MW} + \text{MNS} \\
\text{IVDN} & = \text{IELSZ} - (\text{MON} + \text{MWN})
\end{align*}
\]

\[
\text{In (II, KI, JI)}
\]
if the remaining swelled ore and waste
\[
\begin{align*}
\text{MOIN} & = \text{MOI} - \text{MOS} \\
\text{MWIN} & = \text{MWI} - \text{MNS}
\end{align*}
\]
if (MOIN + MWIN) > IELSZ
then assume the ore is dense
hence MOIN = MOIN * DENSOL / DENSOS
\[
\text{IVDIN} = \text{IELSZ} - (\text{MOIN} + \text{MWIN})
\]

Because loose ore cannot fall into an element already containing dense ore, a restriction that voids cannot pass out of an element till the ore is completely loosened is applied.

It should be noted that the contents of a block are relative to a standardized scale 0 – IELSZ, and to facilitate handling of element data all calculations are carried out in integer arithmetic. To calculate the element's contents

volume of element, \( \text{VOL} = \text{ELX} \times \text{ELY} \times \text{ELZ} \)

... ore volume = \( \text{VOL} \times \text{DENS/DENSOL} \times (\text{MO/IELSZ}) \)

waste volume = \( \text{VOL} \times (\text{MN/IELSZ}) \)

No volume change in mixing is assumed.

7.5.4 Marker Displacements During Mass Transaction

A marker is assumed to lie within the material contents of the element. To calculate the displacement of an arbitrarily placed marker the effects of:
(i) expansion of dense ore
(ii) effects of mixing and percolation preferentially displacing marker through ore, as it is entrained in waste must be considered.

The calculation sequence for new marker positions (Figure 7.13):

1. Calculate position relative to expanded ore reference frame
   if $P < \text{MOI}$
   
   $$P = P \times \frac{\text{DENSOE}}{\text{DENSOL}}$$

2. Test if marker displaces out of $(I, K, J)$
   if $P < \text{MOS}$
   
   $$P'' = \text{IVDN} + \text{MWS} + (\text{MOS} - P)$$
   if $P > \text{MOLE}$ and lies in waste and
   
   $$(P - \text{MOLE}) < \text{MWS}$$
   
   $$P'' = \text{IVDN} + \text{MWS} - (P - \text{MOLE})$$

3. Test if marker remains in element $(I, K, J)$
   if $P < \text{MOLE}$ and $P > \text{MOS}$
   
   $$P' = P - \text{MOS}$$
   if remaining ore is assumed dense
   
   $$P' = P' \times \frac{\text{DENSOL}}{\text{DENSOS}}$$
   if $P > \text{MOLE}$ and $(P - \text{MOLE}) > \text{MWS}$
   
   $$P' = (\text{MOLE} - \text{MOS}) + (P - \text{MOLE}) - \text{MWS}$$
   if remaining ore is assumed dense
   
   $$P = (\text{MOLE} - \text{MOS}) \times \frac{\text{DENSOL}}{\text{DENSOS}} + P - (\text{MOLE} - \text{MWS})$$

4. Calculate new marker position in Cartesian coordinate system. If more than one marker is displaced into an element, then the marker numbers are stored in an auxiliary array and instead of the marker number the element records
   
   $$\text{IMK} = \text{IMKREF} + I$$
   where
   
   IMTMP$(I, 1)$ = number of markers in element, $n$
   IMTMP$(I, 2 ... n + 1)$ are the element numbers.
7.5.5 Updating Void Queues

As a result of the mass transaction, the void queue which registers each void and its current position (ie element location) must be updated. One of the following situations prevails as a result of the mass transaction:

1. Void (I, K, J) disappears with no void appearing in (I1, K1, J1)
2. Any of void (I, K, J) passing to (I1, K1, J1) is absorbed by expansion of ore
3. Void in (I1, K1, J1) disappears with no void remaining in (I, K, J)
4. Void in (I1, K1, J1) disappears with partial void remaining in (I, K, J)
5. Void in (I, K, J) passes to (I1, K1, J1), previously holding no void
6. Part of void (I, K, J) passes to (I1, K1, J1), previously holding no void
7. Increased void in (I1, K1, J1) with no void remaining in (I, K, J)
8. Increased void in (I1, K1, J1) with void remaining in (I, K, J)
9. No possible transaction
10. Eliminate void from queue because $VD < MINVOL$

Because material can only expand or slump, no extra voids beyond the initial queue total can be generated unless $MAXVD < IELSZ$, in which case the void queue can potentially increase beyond its initial size by the ratio $IELSZ/MAXVD$.

7.6 PRESENTATION OF RESULTS

A number of effective methods to display flow simulation results have been developed.
Line printer plots were implemented with the first two-dimensional test program, and have been modified to allow presentation of line printer maps for any specified horizontal or vertical section through the problem domain, or an inclined plane parallel to the ring face for 3 map types:

1. Material contents, or dominant rock type in each element
2. Flow regime dimensions
3. Marker displacements.

This technique has subsequently been complemented with three-dimensional perspective views of marker planes, using a "hidden line" plotting algorithm. This has the advantage of removing scale distortions of the line printer maps, and yields a better appreciation of the irregularity, or smoothness of flow that can be induced with suitable choice of simulation parameters.

The interval between line printer maps and plots is defined in the input data. Typical simulation results are presented in Appendix 5.

In addition a typical simulation run generates the following data:

(i) stage of extraction when markers are recovered
(ii) new marker positions
(iii) tonnage and dilution of material extracted, and cumulative extraction results
(iv) according to the level of diagnostic output specified, timing data, current extraction queue contents, etc.

7.7 EVALUATION OF PROGRAM PERFORMANCE

The simulation program as it is currently available reflects a substantial amount of modification and improvement, and evaluation of ideas reported here.
In brief the stages for development and their rationale will be enumerated.

1. **Choice of loosening mechanism**

   Although loosening was initially programmed as a multi-stage process, making loosening a one step process eliminates a variable and simplifies graphic output because fewer rock type symbols are used.

2. **Computation mode**

   Although the first version of the program used floating point arithmetic, the subsequent versions employ integer arithmetic because precision is not critical, and the elimination of the floating point assisted storage by ENCODE/DECODE, and SHIFT/MASK techniques.

3. **Block transfer onto mass storage**

   Although initial two-dimensional test runs with large elements could be run "in core", the introduction of smaller elements, and three-dimensional problems necessitated the introduction of mass storages devices, and rationalization of rules for block swapping.

4. **Flow probabilities**

   Flow direction probabilities were initially interpolated as a function of height but subsequently the probabilities have been calculated using step functions between the horizontal sections on which probabilities are specified.

5. **Mass transaction algorithm**

   A region of mixed material within an element is no longer explicitly defined, and only the proportion of ore and waste in the element is retained.
6. **Queue changes**

Because a minimum void volume can be specified for the mass transaction, the queues have been modified to allow for more voids to exist than existed at the start of void cycling, and a technique for putting new voids generated, at the top of the queue.

7. **Storage techniques**

As larger problems were tested it became evident that in terms of computation time the most critical aspect was data storage and transfer. Three stages of evolution have been formulated for storage of element data.

(i) direct storage. Element data occupied ten computer words

(ii) ENCODE/DECODE statements. Element data occupied three computer words

(iii) SHIFT/MASK operations. Element data occupied one computer word.

Encode/decode was employed for floating point arithmetic and has not been compared to SHIFT/MASK operations for integer arithmetic where it would have an identical storage efficiency as (iii).

8. **Timing routines**

To give a general indication of time elapsed in each routine a timing routine was incorporated.

In terms of program development, and optimization, further research on the following problems is recommended:

(i) Detection of the most inefficient pieces of code eg queue searches, element referencing and investigation of the benefits of programming in assembler code.
(ii) Re-examination of ENCODE/DECODE routines for integer variable storage.

(iii) Incorporation of blasting routines to simulate fragmentation and expansion during ring firing.

(iv) Specification of a driving program for large scale simulation

(v) Identification of realistic computation times for application of this technique to mine simulation problems.

(vi) Whereas the current program aims at simulating a deterministic flow pattern, the potential and desirability of incorporating probabilistic elements into the overall flow behaviour could be pursued.

(vii) Effect of block dimensions on frequency of block transfer and overall computation times.

7.8 **SIMULATION RESULTS**

The approach and programming reported herein can be deployed in at least in two ways:

(i) as a complement to the so-called "empirical approach" ie by providing data on shapes of "zones of loosening" and a method for specification of marker displacements.

(ii) as an independent and different method of simulating sublevel caving or any other mining method dependent on gravity flow of particulate materials. Sublevel caving provides a difficult case for program development because of the irregular geometries inherent, whereas other methods may allow a coarser analysis, and yet yield valid results.

Apart from general restructuring of the program, and correction or improvement of computer algorithms, the majority of this research work has been concerned with development of efficient techniques for storage and transfer of data, this critical problem being exposed with three-dimensional simulation results, and the desire to decrease element dimensions, hence increasing problem size.
The output display routines have been developed to better present the simulation results.

The results presented in Appendices indicate in a qualitative fashion the validity of this particular approach to modelling granular flow phenomena.

7.8.1 General Comment on Results

(i) The smoothness of flow is improved by increasing the number of discrete steps (ie void migrations) in a simulation. This can be achieved by
   - decreased element size
   - decreased void flow IVDMAX
   - increasing time lapse between results plotted.

(ii) The flow of voids can be congested, or freed up according to the order in which voids are selected.

(iii) Flow is (artificially) geometrically limited to lie above a grid diagonal radiating from the void source, dictating a minimum rill angle dependent on element dimensions.

7.8.2 Specific comments on simulation results

Included in Appendix 5 respectively are selected maps, and plots from: side view, front view, and three-dimensional simulation runs.

The results presented should be viewed as indicative of the status of program development. No attempt is made to quantify the effect of model parameters on the resultant flow behaviour, and so an optimal combination of model parameters has not been achieved.

Each simulation run models:

(i) the slump of ore into the heading after blasting (NBUCK=0)

(ii) flow after extraction of 3 "buckets" ("bucket" being an artificial term that implies full heading width).
The two-dimensional results explicitly portray the gradual development of the "zone of loosening". Loosening is instantaneous, and no intermediate density states, between blasted and completely loose ore are generated.

The initial maps in the sequence depict:

(i) initial material state before flow
(ii) initial marker positions
(iii) flow regime dimensions.

It will be observed that:

(i) in the side view results, because waste rock is assigned a higher mobility than blasted ore, voids preferentially migrate through the waste.

(ii) because of the high mobility assigned to waste, exaggerated waste rock flow into the drawpoint has been effected in the results presented. A large mixing effect was generated because the MINMAX parameter has not limited waste penetration.

(iii) a coarser flow analysis is obtained for the three-dimensional simulation because the element dimensions are doubled. Planes of markers replace the single line of the two dimensional results.

The plot scale is in units of ft/in, and the mass extracted has units of ft$^3$/in, (no tonnage factor being applied).

To date, through lack of time, no specific comparison has been made between simulation results and either laboratory tests or full scale test results reported by Professor Janelid.

However a number of points should be made:

(i) the results obtained by Professor Janelid indicate an irregularity of flow in full scale tests not previously
indicated in the published literature, implying that past analyses will be somewhat artificial.

(ii) the stochastic flow program requires confirmation at two levels:

I. that acceptable simulation run times will produce realistic simulation results for the discretization chosen

II. that a parametric analysis correlates the flow probabilities with flow behaviour, and mine design parameters.

Both objectives need to be tackled together, but it is envisaged that the stochastic flow program, built as it is, on simple flow concepts can be adapted to reproduce any specified result or range of results.

(iii) development of criteria to measure the precision of simulation models needs research in the light of (i).

At present measurement of flow behaviour in full scale tests is dependent on the recovery of markers which indicate the end result of flow, but not the "path dependence" or the "flow mechanism" of gravity flow. Because of this, only partial quantitative verification for the flow concepts used in any simulation model will ever be available to the mine planner in sublevel caving. Correlation of actual mine results with simulation results is only a partial solution, until more accurate definition of the gravity flow environment in mines is available, to verify new numerical models for flow.

To date, the computer simulation of gravity ore flow has been applied to single heading extraction in the sublevel caving method. Application of the model to multiple heading, multiple ring three-dimensional problems has been envisaged, and the program is configured to handle this. At the present stage of development, simulation run times would be a function of:
(i) problem size

(ii) level of discretization, and hence the coarseness of the simulation

(iii) core storage, and consequently the demand on mass storage devices

(iv) program optimization, for instance isolating key routines and programming these in assembler code.

Further research is required to assess the application of this simulation technique to large scale mining problems but currently computational times indicate batch processing to be more suitable than interactive design using a graphics terminal.
CHAPTER 8: EMPIRICAL FLOW MODEL

8.1 Introduction

8.2 Previous Static Analyses of Sublevel Caving

8.3 Basic Concepts in a Dynamic Analysis

8.4 Flow Mechanisms

8.5 Formulation of Flow Behaviour in Sublevel Caving

8.6 Specification of Zone of Loosening Shape and Flow Path Placement

Prototype zone of loosening

8.7 Basic Description of Flow Grid Elements and their Properties

8.8 Further Comments on Representation of Flow Grid

8.9 Simulation of Single Heading Draw

8.9.1 Introduction

8.9.2 Contents of flow grid and loosening mechanism

Position of zone of loosening
Zone of extracted material
Recovery and dilution

8.9.3 Prediction of marker displacement

8.9.4 Results of simulation program

8.10 Data Structures to Represent Rock Mass in Empirical Flow Model

Orthogonal grid data structure
Alternative data structures

8.11 Conclusions
8.1 INTRODUCTION

The basic notions in the empirical model relate to, and develop ideas based on results from scale modelling. It can be inferred that during extraction a specific volume of material is loosened. Particles within this loosened zone displace, and the magnitude and direction of movement can be measured by particle trajectories. The loosened zone was initially approximated by an ellipsoid, but recent studies have indicated that a better approximation to the loosened zone is required. This model incorporates a more realistic approximation.

8.2 PREVIOUS STATIC ANALYSIS OF SUBLEVEL CAVING

One of the essential characteristics of the past research into sublevel caving has been the preoccupation with static concepts of flow. Studies were concerned with the description of material extracted and its origin within the broken rock mass (hence 'volume of motion') and with the extent of loosening (hence 'limit ellipsoid') to enable prediction of recovery and dilution. Calculations presumed a knowledge of the redistribution of rock mass after extraction so that extraction at adjacent headings could be analyzed.

To date, particle trajectories, although acknowledged, have not been incorporated into the attempts to simulate sublevel caving.

Sublevel caving layouts were designed by superimposing draw ellipsoids, and optimizing the design to minimize overlap; overlap implying that material drawn from an adjacent ring involved drawing off waste rock. Optimization of mine design involved selecting the eccentricities of draw ellipsoids, selecting a range of values for the ring geometry variables, and choosing the combination which produced optimal results for recovery and dilution for a single sublevel caving unit.

The primary disadvantages were that complete extraction had to be assumed in these calculations, and progressive recovery, and multiple heading draw interaction could not be simulated. The
assumption that only waste rock slumped into the draw ellipsoid is only partly correct (as evidenced by modelling results by Sandstrom (1972) and Panczakiewicz (1977) for infiltration of waste into the heading).

8.3 BASIC CONCEPTS IN A DYNAMIC ANALYSIS

In view of the basic inadequacies outlined, the problem of constructing a dynamic flow model was investigated. Because of the lack of adequate experimental data, and the nature of the problem of verifying any assumptions made, a simplified flow model was introduced.

Recognizing the deficiencies of representing the flow with ellipsoidal surfaces, a more generalized surface was defined.

The basic goal of the model is the prediction of displacement of representative "particles" (or markers) in the broken rock mass as a function of the extraction history. The model analyzes flow for a single sublevel caving unit, but adoption of the model to pillar simulation is discussed.

The model is based on three fundamental notions:

(i) an expanding series of 'zones of loosening', where the volume between successive zones is defined as a shell. Essentially, the shape of zones are as observed in model and full scale tests.

(ii) a radiating series of flow paths, modelling flow trajectories of particles, see Figure 8.1.

(iii) that a discontinuous granular medium can be modelled by a continuum characterized by local density.
It will be observed that any location near the heading can then be referenced relative to a particular shell, and within the shell, relative to flow paths. Defining the intersection of these predefined flow paths and zones of loosening as grid points, any location can be referenced relative to a select number of adjacent grid points that define an element (defined in Section 8.7).

The basic flow model is now examined. As material is loaded from the heading the overlying material loosens. At any stage of extraction, the volume of loosened material coincides with a zone of loosening or a surface interpolated between successive zones of loosening. After the zone of loosening* passes position \( P(x,y,z) \) the particle \( P \) begins to displace toward the heading. A number of significant simplifying assumptions are introduced to define the magnitude and direction of motion.

1. The direction of motion is defined by the series of radiating flow paths. In general, the direction of motion would be a function of position, and time (ie within the extraction history of the ring). An exceptionally complicated empirical function would be needed to identify direction. Instead an approximation is introduced. For steady state flow in bunkers, particle velocity can vary with position but is independent of time. Approximating unsteady state flow direction by the corresponding steady state flow direction ie by "steady state flow paths", the direction of motion at point \( P(x,y,z) \) corresponds to the direction of the flow path through \( P(x,y,z) \). If no flow path is explicitly defined, an interpolated flow path locates the direction (Figure 8.1)

*"zone of loosening" is a term used to describe the surface enclosing loosened materials and also the individual surfaces that are specified as input data for the empirical model. The context will imply the particular meaning.
Figure 8.1 Zones of loosening and flow paths on front and side sections. Grid points define an element (shaded) that encloses particle \( P(x, y, z) \)
A number of corollaries follow:

1. the direction of motion of any particle at location \((x,y,z)\) is unique, and is independent of its original location in the rock mass.

2. adjacent flow paths form a "flow tube" which is a small imaginary tube or conduit bounded by flow paths. There is no velocity perpendicular to the flow tube walls, and so flow is confined to the flow tube.

3. because the motion of granular material is uneven, in that the local interaction of particles during flow causes an irregular motion of individual particles, the flow path represents a local average behaviour.

4. flow paths are characterized as path lines and not stream lines in the language of fluid dynamics (Daily, 1966) though for steady state flow they correspond.

(ii) Although simplifying displacement direction to be independent of time, the magnitude of particle displacement is dependent on the extraction history. Instead of particle velocity, the rate of displacement as a function of mass extracted (a monotonically varying quantity replacing time) is defined. A surface (or zone of loosening) \(S_1\), can be interpolated through \(P(x,y,z)\), Figure 8.2. The displacement due to an increment of mass extracted will be a function of:

1. magnitude of mass extracted
2. the position relative to the heading, and hence the time before the zone of loosening incorporates \(P\)
3. the loosening mechanism which defines the density within the loosened zone, and hence the rate at which the zone advances.
Figure 8.2 Application of shell and flow tube continuity requirements for empirical flow model
Section 8.9.2 takes into account these factors, but for now, particles on $S_1$ will displace to another surface $S_2$ which can be monitored in model tests.

The model introduces the following simplifying assumptions:

1. The surface $S_2$ corresponds to an interpolated zone of loosening. Particles on a zone of loosening will displace simultaneously.

2. Consider a slice of infinitesimal thickness $\delta t_1$ about $P(x,y,z)$. The particle $P$ displaces to $P'(x',y',z')$. If mass is to be conserved, then the slice at $P'$ will be of thickness $\delta t_2$ where:

$$
\int_{S_1} \delta t_2 = \int_{S_2} \delta t_2
$$

This rule is referred to as "shell-wise" continuity. This implies that one shell displaces into another.

It should be remarked that shell thickness variables $t_1$, $t_2$ are normalized relative to shell thickness $T_1$, $T_2$.

3. It will be observed that by the previous definition of flow tubes, that continuity of mass must be applied within flow tubes, as well as "shell-wise". If this is applied rigorously then $\delta t_2$ will be a discontinuous thickness depending on the individual flow tube dimensions. It is obvious that to maintain mass continuity through the flow grid two options are available:

a. retain shell-wise continuity, allowing local density fluctuations within each flow tube to maintain flow path continuity, or assume an even density within the shell, retain flow paths to locate direction, and tolerate the errors in direction of particle trajectory.

b. retain flow tube continuity, allowing local density fluctuations within each flow tube to maintain zone surface continuity, or assume even density within the flow tube, and introduce discontinuities in the zone surface, or else tolerate the errors in magnitude of particle trajectory.
Both methods potentially incorporate errors because the unsteady state flow grid is being approximated by a steady state flow grid but approach (a) is employed for reasons that will be clear in the case of subsequent developments of the model.

As indicated in Figure 8.2, approach (a) could be corrected to yield a more consistent result, but the improvements are not warranted, because part of the error is also due to the surface S2 of slumped material in reality not corresponding exactly to the zone of loosening shape as loosening propagates outwards from the heading.

The above empirical model is an attempt to specify a dynamic flow model, based on observed behaviour, and in that sense is not predictive (but a simulator). The model incorporates a geometrical grid that has proved comparatively simple to develop, based on techniques of curve and surface interpolation, and volume interpolation to be outlined in later sections.

8.4 FLOW MECHANISMS

From the review of current modelling studies, (Chapter 5) it will be observed that different flow mechanisms (describing or characterizing the particle motions) occur in bunkers depending on the cohesive and frictional properties, and to some degree the particulate nature of the rock mass, and the boundary geometry.

It is instructive to examine how the concepts of flow paths and zones of loosening could be applied in the two extreme cases.

(i) **freely flowing materials** (Figure 8.3)

This mechanism is exemplified in dry, cohesionless sand where loosening propagates rapidly through the medium, represented by a series of flow paths radiating from the opening, and zones of loosening that continuously expand about the opening and completely enclose previous zones.
Figure 8.3 Empirical model flow pattern for freely flowing material

Figure 8.4 Empirical model flow pattern for plug flow material
(ii) frictional-cohesive materials (Figure 8.4)

The material fails in shear, and piping or plug flow is the dominant flow mechanism. Arching and cavitation are likely but this can be represented as a continuum, by assuming that arches are unstable and collapse to reinitiate flow. For this mechanism the horizontal component of flow is minimal compared to freely flowing materials. The zone of loosening is seen to be a surface that does not enclose previous zones of loosening because a shear surface develops in the material.

8.5 FORMULATION OF FLOW BEHAVIOUR IN SUBLEVEL CAVING

Given the basic strategy for defining a flow grid, a number of extensions to the theory are made in the empirical flow model proposed for sublevel caving. These are:

1. definition of stages of flow
2. a loosening mechanism.

Essentially these describe the irregular boundary geometry that applies in sublevel caving, and how the ring, fragmented by blasting, but still compacted is loosened during flow.

Up till breakthrough of the zone of loosening into the loose rock mass surrounding the blasted ring, flow is simplified to the formation of a series of unstable arches which create a void, as material is extracted from the heading. The arches collapse so that dense ore (due to blasting) is loosened, and the process is at present idealized so that all blasted material that flows is considered to be in a loosened state.

Depending on the ore characteristics, the shape of the expanding zone of loosening, and the geometry of the blasted ring, the zone of loosening will initially break through into the surrounding compacted and loose rock, at the back, side or top of ring.
Accordingly three different mechanisms could be postulated. Whether flow after breakthrough is significantly different for each case would depend on the influence of, and interaction between the loose waste rock and the surrounding dense ore. The subsequent flow could be significantly different if there was preferential flow of blasted ore or loose waste, the latter being more likely because of its greater mobility. Whichever mechanism applies, the implication is that the zone of loosening is distorted towards the region of preferential flow so that more of this material is contained within the zone of loosening and less of the less mobile material.

These three mechanisms require further evaluation in a qualitative fashion in a three-dimensional scale modelling program, by varying ore and waste characteristics, compaction and ring burden with the same modelling frame geometry.

Following the theory expounded for flow in bunkers, the flow is seen to be due to a series of expanding zones of loosening that are contained within a well-defined flow regime, which would normally be characterized by a cone surmounted by a vertical circular column, where the width and height of the cone are determined by material properties. The normal, even symmetric flow regime observed in bunkers is distorted by the presence of the solid ring face. The consequent shapes of the zones of loosening can be envisaged by considering the effect of pressing a hexagonal plate (the shape of the solid ring face) into a long elongated balloon.

The width of the flow regime of any relatively free flowing material is determined by the characteristics of the material, but it increases with height above the opening until a maximum value that can be correlated to the strength of arches forming in the material, where upon flow occurs in a vertical column (Janelid, 1975a). Although the dimensions of the cone formed in broken ore are still largely speculation, they provide an initial geometrical shape for the flow regime. It should be noted that poor fragmentation or variability in the mobility of the ore could distort the shapes.
Consequently four basic stages in the progressive development of the zone of loosening are modelled:

STAGE A : Flow within blasted ring
STAGE B : Breakthrough into surrounding loose rock
STAGE C : Formation of cone
STAGE D : Flow in vertical column.

In the loosest single size granular material, the zone of loosening is still observed (in model tests) to propagate upwards at a finite rate, and for this reason (and that the flow grid defines magnitude and direction of motion) the zone of loosening concept is retained for flow in overlying loose material.

Overburden pressure would imply a degree of reconsolidation with time in the overlying material. The effect of the zone of loosening is to re-orientate and so loosen this material, implying a slight density decrease. Consequently the overlying material will begin to displace a finite time after breakthrough, and the higher the material's position above the ring, the greater the time lapse before flow is initiated. Over a period of time the displaced (and hence loosened) material would reconsolidate.

In regard to a loosening mechanism, the simplest assumption available is that all material that has begun to move (and hence lies within the zone of loosening) is considered completely loosened, is made. A more realistic model would be that the mechanical action of loosening takes place as fragmented rock re-orientates, and hence over a finite distance.

A phenomenon of arching is often observed in a sublevel caving operation creating cavities (ie "hangups"). It is usually a combination of mechanical interlocking and a cohesive arch. This doming effect can be precipitated by the energy of an explosive force ("bombing"), though with delay in the mining schedule. The flow behaviour returns to free flow and the cavity is eliminated. Over the long term, flow can be represented as a continuum, even though
denotes maximum zone width.

Figure 8.5 Typical zone of loosening shapes for each stage
(front, ring and maximum width; side mid-heading section)

<table>
<thead>
<tr>
<th>Region</th>
<th>Stage A</th>
<th>Stage B</th>
<th>Stage C</th>
<th>Stage D</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 rill heap</td>
<td>(\checkmark)</td>
<td>(\checkmark)</td>
<td>(\checkmark)</td>
<td>(\checkmark)</td>
</tr>
<tr>
<td>2 zone bisected by ring face</td>
<td>(\checkmark)</td>
<td>(\checkmark)</td>
<td>(\checkmark)</td>
<td>(\checkmark)</td>
</tr>
<tr>
<td>3 ring influence region - below top of ring</td>
<td>(\checkmark)</td>
<td>(\checkmark)</td>
<td>(\checkmark)</td>
<td>(\checkmark)</td>
</tr>
<tr>
<td>4 - above top of ring</td>
<td>(\checkmark)</td>
<td>(\checkmark)</td>
<td>(\checkmark)</td>
<td>(\checkmark)</td>
</tr>
<tr>
<td>5 completion of cone</td>
<td></td>
<td></td>
<td>(\checkmark)</td>
<td>(\checkmark)</td>
</tr>
<tr>
<td>6 formation of column</td>
<td></td>
<td></td>
<td></td>
<td>(\checkmark)</td>
</tr>
<tr>
<td>7 cap(^b)</td>
<td>(\checkmark)</td>
<td>(\checkmark)</td>
<td>(\checkmark)</td>
<td>(\checkmark)</td>
</tr>
</tbody>
</table>

Table 8.1 Flow regions for each zone stage

Notes:

a. For stage B, the initial breakthrough could occur at the side, top or back or ring. Simultaneous breakthrough at the top and side is shown.

b. Four caps corresponding to each stage (i.e. \(C_1, C_2, C_3, C_4\)) are defined.
the mechanism is the formation and collapse of unstable arches, with the occasional stable arch forming (the question of whether particle trajectories are the same for continuous and discontinuous flow has not been investigated in this thesis).

In summary, the sublevel caving zone of loosening model has the following characteristics:

1. successive zones of loosening must lie beyond or coincide with the previous zone of loosening surface,
2. each zone of loosening stage is characterized by a combination of surface section curves that identify the interaction of basic flow forms (cone, column, cap) and the ring geometry. These are outlined in Figure 8.5 and Table B.1. Typical horizontal sections are depicted in Figure 8.6.
Regions 3 and 4 are defined as "ring influence" regions with the horizontal section segment obscured in side view by the outer ring surface, and defined as lying in the "ring shadow".
3. the height of the ring influence region H3 increases during flow as the zone of loosening broadens,
4. a shearing action is assumed to take place during flow along the ring face, hence zones do not enclose each other until region 4 where the model pictures zones "tied" to the top of the ring.

Flow paths are defined to have the following properties:

1. a flow path is defined to intersect every defined zone of loosening,
2. a flow path cannot intersect or overlap another flow path.
Figure 8.6  Typical horizontal section shapes for zone regions
(plan view)
8.6 SPECIFICATION OF ZONE OF LOOSENING SHAPE AND FLOW PATH PLACEMENT

Because of the lack of adequate experimental data, only model data observed through transparent plates is available, although the recorded full scale testwork of Janclid details the recovery of internally placed markers. The use of model and full scale test data is discussed in Chapter 9.

The typical flow behaviour for front and mid-side sections of the sublevel caving geometry can be posited with more certainty than the internal flow behaviour between these two sections.

To allow the greatest flexibility in the early stages of development and experimentation, flow paths and zones of loosening are specified independently on both front and side sections (Figure 8.7). To simplify preparation of data for the computer model, the placement of flow paths on front and side sections is determined by the spacing of data points defining the zone of loosening shape on each section. With the zone of loosening defined to be a three dimensional surface and flow paths defined for front and side sections, it remains to establish a working hypothesis to locate internal flow paths between these two sections on each zone of loosening.

At present intermediate grid points are interpolated by defining a curve of intersection; being the line defining the intersection of the zone of loosening with a plane defined by the grid points on front and side sections (Figure 8.8).

By defining the same number of grid points on each curve of intersection, a line connecting corresponding grid points on successive zones of loosening defines the internal flow path, in similar fashion to front and side sections.

The location of flow paths on the curve of intersection is determined by a spacing rule. Placement is currently achieved by the simplest rule: equal arc length segments.
Figure 8.7 Front and side section flow paths and zones of loosening.

Figure 8.8 Location of curve of intersection, interval grid points, and consequent internal flow paths.
The hypotheses for internal flow path placement:

(1) planar section
(2) spacing rule on curve of intersection

prove significant because they define the internal flow behaviour. The hypotheses intuitively appear a reasonable approximation of flow until breakthrough, but becomes a more complex problem to visualize after breakthrough.

The hypotheses are significant not for when a particle or marker is recovered in the heading, because all points on a zone of loosening are recovered simultaneously (by definition) but in the direction the motion takes. If material is not extracted at one heading but is being drawn to an adjacent heading then the trajectory (both magnitude and direction) of the marker due to the first heading becomes important for determining at what stage the marker is recovered at the second heading.

It will be observed that the scheme for placement of internal flow paths, although applicable to Stage A, must undergo modification for Stages B-D, where a more complex zone of loosening shape exists.

Two exceptions to the above scheme apply:

(1) for a point $f_n$, below breakthrough height, where the slope $f_n - S_n$ is greater than the angle of ring inclination, and for which the zone of loosening may not be defined. In practice this situation has not arisen (Figure 8.9).

(2) for a point $f_n$ above the lower ring influence height, and below the upper ring influence height, where the slope is greater than the angle of ring inclination, or even below it, where the planar section is non existent. A non-planar section must be interpolated. To overcome this inconsistency the curve of intersection is assumed to be divided into two segments: $f_n - B$, where $B$ lies on the curve $AE$, and $B - S_n$ where the slope of $f_n - B$
Figure 8.9 Possible inconsistency in curve of intersection definition (i)

URIH: upper ring influence height
LRIH: lower ring influence height

zone surface nonexistent for curve of intersection

Figure 8.10 Possible inconsistency in curve of intersection definition (ii)
is chosen to be constant unless $B > E$ when the first curved segment (that lies in the ring shadow) is defined by $f_n - E$ (Figure 8.10). More effective non-planar sections require investigation.

The zone of loosening is a three dimensional surface. Front and side section data have been specified, and the nature of a curve of intersection locating internal grid points is defined. Grid points are assumed equally spaced along the curve of intersection. Alternative functions could have been defined to relate the ratio of distances between grid points.

Provided the $(x,y,z)$ coordinates of points on the curve of intersection can be generated, then each zone of loosening will be defined by a matrix of points that when connected will form flow tubes.

The problem posed is to establish the equation of the curve of intersection, and then deduce the arc length function. The solution to the problem is dictated by the method of defining the shape of each zone of loosening surface, and the necessity for a relatively simple and flexible method for data input. Any surface shapes chosen would have the status of "informed guesses", and the verification of the model would necessitate easy manipulation of trial shapes.

An initial attempt required each zone of loosening to be defined by points on horizontal sections that defined the proportions of the zone of loosening on the section. A curve that passed through these points defined the section shape, and sections could be interpolated between defined sections. This technique was not pursued when it became obvious that more points were needed on the sections to control the section shape, and applied for each zone of loosening (say 10), the data entry time would be prohibitive.
A subsequent attempt (described in the next section), allows the proportions of each zone of loosening to be defined on horizontal sections. With this data the zone of loosening is mapped onto a single prototype zone of loosening, where the section shape is adequately defined by increasing the density of data points. In this way the proportions of each zone of loosening can be manipulated easily with the prototype zone of loosening controlling the shape between data points. This scheme works well where basic zone of loosening shapes have been identified.

Because the zone of loosening shape is defined in a piecewise fashion, and no global mathematical function applies, the equation of the curve of intersection is not known a priori.

Instead the curve function is approximated by selecting points \((y_i, z_i)\) on the curve, and estimating the \(x\)-coordinate \((x_i)\) using a mapping relationship.

The basis for determining the \(x\)-coordinate values of the curve of intersection grid points is now discussed. There is no one unique method of solution and the one presented provided the best shape definition for a minimal amount of input data specification, given little data is available on experimental shapes for zones of loosening. The technique allows the basic proportions of the zone of loosening at Stages A-D to be specified by parameters on particular horizontal sections (Figure 8.11):

(a) width of zone of loosening on ring face, \(S_4\)

(b) maximum width of zone of loosening in front view, \(S_5\)

(c) width of zone of loosening on side section by \(y\)-coordinates, \(S_1S_2\)

(d) where appropriate, \(S_3\), the lower \(y\) coordinate of zone of loosening in ring influence region

(e) height of each of 7 characteristic surface sections defined by the cone-column shape, and the ring influence region, and floor of heading \(H_{\text{BASE}}, H_1, H_2, H_3, H_4, H_5, H_6, H_7\) (Figure 8.11). For some of Stages A-D, these surface
<table>
<thead>
<tr>
<th>SECTION TYPE</th>
<th>Y-MAPPING</th>
<th>X-MAPPING</th>
</tr>
</thead>
<tbody>
<tr>
<td>Point P lies outside regions 3, 4. ( IV=)3</td>
<td>( y_p = \frac{y - S_1}{S_2 - S_1}(P_2 - P_1) + P_3 )</td>
<td>( x = \frac{x_p}{P_5} \cdot S_5 )</td>
</tr>
<tr>
<td>Point P lies inside region 3 or 4, and within ring shadow ( IV=)1</td>
<td>( y_p = \frac{y - S_3}{S_2 - S_3}(P_1 - P_3) + P_3 )</td>
<td>( x = \frac{x_p - P_4}{P_5 - P_4}(S_5 - S_4) + S_4 )</td>
</tr>
<tr>
<td>Point P lies inside region 3 or 4, but outside ring shadow ( IV=)2</td>
<td>( y_p = \frac{y - S_3}{S_2 - S_3}(P_2 - P_3) + P_3 )</td>
<td>If ( y_p \geq P_6 ) ( x = \frac{x_p}{P_5} \cdot S_5 ) If ( y_p &lt; P_6 ) ( x = \frac{x_p - P_4}{P_5 - P_4}(S_5 - S_4) + S_4 )</td>
</tr>
</tbody>
</table>

Table 8.2 Mapping functions for prototype zone of loosening

Figure 8.11 Parameters for zone definition
will be non-existent. Consequently, the relative height of a point \( P(x,y,z) \) lying in region \( n \) will be

\[
\Delta H = \frac{z - H_{n-1}}{H_n - H_{n-1}} \quad \text{where } H_0 = H_{\text{BASE}}
\]

Using these proportions of the zone of loosening section, appropriate \( x \) values for any \( y \) value at height \( z \) on the zone of loosening can be defined.

**Prototype Zone of Loosening**

To allow calculation of \( x \) values, a prototype zone of loosening is defined comprising a series of predefined horizontal sections, that are coded for whichever section interval (1-7) they pertain to (termed "regions"), or else which of the 4 possible caps (Figure 8.5).

Each section is defined by IPPPS points, and a total of IPSEC sections are defined, with the qualification that in regions 3,4 that ISHAD points are used to define the ring shadow portion of the section.

Accordingly to define a prototype zone of loosening for any of Stages A-D, a search is made of all prototype sections, and if the zone of loosening region or cap has a region of non-zero height, all prototype sections for that region or cap are included in the prototype zone of loosening for the Stage relevant to the zone of loosening being interpolated.

Because each section is defined to have IPPPS points a new section at height \( H_p \)

\[
H_p = \Delta H \times (H_{p_n} - H_{p_{n-1}}) + H_{p_{n-1}} \quad H_{p_0} = H_{\text{BASE}}
\]

\( H_{p_n} = \text{height of prototype region } n \)
is evaluated by fitting a curve through each of the corresponding points on each prototype section and interpolating the \((x,y)\) coordinates at \(H_p\).

Two points that must be considered are:

(i) at the top of ring if sections are defined above this a discontinuity occurs because sections of different shape are defined for the same height. If this situation arises, only those sections above or below the point are used to interpolate a section, with only a possible sacrifice of slope continuity for shape definition at this height.

(ii) the proportions of the prototype zone of loosening at height \((z)\) are established: \(P_1, P_2, P_3, P_4, P_5\) corresponding to the zone of loosening limits. In addition, \(P_6\) is the \(y\)-coordinate corresponding to the \(x\)-coordinate \(P_5\).

Further definition of section shapes could be implemented, but was not warranted at this stage. The technique could certainly be generalized. Minimal definition follows from the assumption of symmetry about the \(x=0\) plane, so that transformations are carried out for all \(P(x,y,z)\) by calculating the corresponding positive \(x\)-coordinate.

For each of the points \(P(y,z)\) it is necessary to map the zone of loosening section into the prototype zone of loosening section, and reverse the mapping to obtain the \(x\)-coordinate. A piecewise linear mapping is employed. Table 8.2 details the mapping relations.

The stages involved are:

1. Map into the prototype : \(y \rightarrow y_p\)

2. Given the points on the prototype section are interpolated, calculate polygonal arc length, \(s\). Given the curve of intersection segment (ie IV = 1, 2 or 3, Table 8.2) search
for the interval on the segment bounding $y_p$, form a cubic polynomial using an adaption of a curve fitting routine INTRPL (Akima, 1970), and solve for $s$ at $y_p$, and hence interpolate $x_p$.

3. Map out of prototype: $x_p \rightarrow x$.

With $P(x,y,z)$ defined at a suitable number of points on the curve of intersection, calculate polygonal arc length and interpolate the internal grid points by the spacing rule.

A considerable number of algorithms (Barnhill et al, 1974) exist for surface interpolation, although predominantly for interpolation of gridded data in two dimensions. The related techniques of Coons (1967) and Ferguson (1964) can be applied to surface fitting on non-gridded unequally spaced data, provided the data points are distributed so as to be topologically equivalent to a rectangular grid. A number of applications have been published. It requires that the coordinates $(x,y,z)$ on the grid be given a parametric representation $(s_1s_2s_3)$ where at least two parameters are monotonically increasing with the assigned order of the data points.

Because parts of the surface are multivalued in $x$ and $y$, no convenient parametric representation is available. However parametric representation of the curve at height $z$, where $x = f(s)$ $y = y(s)$, $s$ being polygonal arc length has been extensively employed, and is readily applied to this application because each of the IPPPS points on the curve can be specified as single valued functions of $z$.

Having completed the interpolations for internal flow behaviour for each zone of loosening the coordinates of the flow grid can be assembled and stored.
8.7 BASIC DESCRIPTION OF FLOW GRID ELEMENTS AND PROPERTIES

Using the techniques outlined in the previous section, the "flow grid", for flow simulation has been established. Before elaborating the method for calculating marker displacements, recovery and dilution it is necessary to define the relationship between a point P(x,y,z) and its position relative to the flow grid.

In common with the literature of finite elements, the concept of an element is introduced that will have the following characteristics:

1. it describes the volume bounded by consecutive zones of loosening, and four flow paths, whose intersections are defined by the three dimensional flow grid

2. in addition to the global cartesian, coordinate system (x,y,z) and paralleling the isoparametric formulations, a local coordinate system (r,s,t) that relies on the element geometry for its definition and whose coordinates range between zero and unity within the element, termed a natural coordinate system, is defined.

This application of finite element methodology is particularly advantageous in the present application, because the three dimensional flow grid is then topologically equivalent to a square orthogonal grid. That is, the flow grid elements can be stretched into natural coordinates to yield (IRULL * ICULL * IZULL) cubes (Figure 8.12).

A flow path is now defined to be the locus of all points within the element having identical (r,s) coordinates. Successive elements in the flow tube defined the total particle trajectory. A zone of loosening is defined by all points with the element having an identical (t) coordinate. The complete zone of loosening surface is defined by connecting the surface through all adjacent elements.

Consequently the position of P with this grid can be defined by six parameters:
Figure 8.12 Flow grid in natural and cartesian coordinates

Figure 8.13 Hexahedron definitions
(IR, IC, IZ) — define the position of the element containing P relative to the grid

(r, s, t) — define the position of P relative to the element in natural coordinates.

More specifically, P is defined to lie at a shell number K:

$$K = (IZ - 1) + \frac{r + 1}{2}$$

with the position relative to the shell being (r, s) in the (IR, IC) element of the shell, because of the correspondence of (r, s, t) and (IR, IC, IZ) coordinate directions.

The simplest element that could be formed is the linear hexahedron, formed by connecting adjacent grid points with straight lines. Higher order elements are common in the literature of the finite elements, and the cubic hexahedron allows the element to be defined by curved surfaces. Consequently the complex, arbitrarily defined flow net can be accurately modelled by an assemblage of geometrically simple element shapes.

The elements employed in this study are shown in Figure 8.13. The element is defined by exterior nodes (i.e., corner or midside) lying on the boundary of the element.

The local and global coordinate systems are related by "mapping" (or "shape", or "interpolation") functions (Cook, 1974, p.100) which relates a linear transformation from the unit cube in isoparametric coordinates to the hexahedron in (x, y, z) coordinates.

For the linear hexahedral element

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \sum_{i=1}^{n} \begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix} \begin{bmatrix} N_i \end{bmatrix}$$
where \( N_i = \frac{1}{8} (1 + r_i^2)(1 + s_i^2)(1 + t_i^2) \)

\((r_i, t_i, s_i)\) are the coordinates of the corner nodes defining the element.

The shape functions for the cubic hexahedral element are published (Huebner, 1975, p.184).

The inverse transformation \((r, s, t)\) for \([x, y, z]\) can be obtained by solving the three simultaneous non-linear equations by any of a number of established methods (see Appendix 3).

The volume of each hexahedral element is evaluated

\[
V = \iiint dx
dy
dz
\]

\[
= \int_{-1}^{+1} \int_{-1}^{+1} \int_{-1}^{+1} \frac{\delta(x, y, z)}{\delta(r, s, t)}
\, dr
ds
dt
\]

where \( \frac{\delta(x, y, z)}{\delta(r, s, t)} \) denotes the Jacobian matrix of partial derivatives,

\[
\frac{\delta(x, y, z)}{\delta(r, s, t)} = \begin{bmatrix}
\frac{\delta x}{\delta r} & \frac{\delta x}{\delta s} & \frac{\delta x}{\delta t} \\
\frac{\delta y}{\delta r} & \frac{\delta y}{\delta s} & \frac{\delta y}{\delta t} \\
\frac{\delta z}{\delta r} & \frac{\delta z}{\delta s} & \frac{\delta z}{\delta t}
\end{bmatrix}
\]

This definite integral is evaluated using Gaussian Quadrature.¹

\[
V = \sum_{i=1}^{IGP} \sum_{j=1}^{IGP} \sum_{k=1}^{IGP} W(i) W(j) W(k) \frac{\delta(x, y, z)}{\delta(r, s, t)_{ijk}}
\]

¹ It should be noted that other effective quadrature rules have been employed in finite elements (Helen, 1972; Irons, 1971).
With the order of integration being determined by the form of the Jacobian (Cook, 1974, p.104, 114). The IGP point formula is exact when the function (in this case, the Jacobian) is a polynomial of degree $2 \times (IGP - 1) + 1$ or less. Consequently to evaluate the volume of linear element, we use $2 \times 2 \times 2$ point Gaussian quadrature (see Appendix 3).

Isoparametric finite elements have been employed in this model to describe the space coordinates of an element in terms of the coordinate values at boundary nodes.

Three properties of the element assemblage are now outlined:

(i) **Continuity requirements**


(a) only nodes at the interface between elements influence the value of the function of the interface. Adjacent elements use the same nodes to describe the boundary implying a common interface.

(b) the polynomial terms of the shape function are such that the variation of the function across the interface are of such order that the number of nodes determines this uniquely.

Hence the complete volume of the flow grid is represented without interelement gap or overlap because continuity of contiguous elements in local coordinates ensures continuity of contiguous distorted elements in global coordinates. However slope continuity cannot be presumed because derivatives have not been incorporated into the shape functions.
(ii) **Uniqueness of mapping**

The mapping functions that are employed ensure a unique relation between local and global coordinates, but the reverse mapping need not necessarily apply if the element is too distorted.

If the Jacobian exists for the element, then the mapping will be unique and acceptable provided the sign of the Jacobian does not change in the element domain. A check is made on element geometry during volume calculations.

Strang et al (1973) derive the conditions for non-uniqueness. While general rules are not available, corner angles should be less than 180°.

(iii) **Degeneration of elements**

The shape functions are still applicable even when adjacent nodes coalesce eg when a quadrilateral becomes a triangle.

8.8 **FURTHER COMMENTS ON REPRESENTATIVE OF FLOW GRID**

It has been observed that it is possible to have flow paths and zones of loosening implicitly defined by forming the three dimensional flow grid, defined by the intersection points of the flow paths and zones of loosening.

More accurate specification of the flow net (ie increasing the density of points in the flow net) can be obtained by increasing the number of points that are interpolated in Section 8.6, and also increased input data definition. To retain accuracy in the specification of zone of loosening shape, extra points are generated (to form cubic hexahedra) during mappings from the prototype zone of loosening.
The current scheme is to create 32-node cubic hexahedral elements, and then each element can be subdivided into \((IEXR \times IEXC \times IEXZ)\) 8 node linear hexahedral elements.

As in finite element formulations (Huebner, 1975, p.235) where the question of which elements give the best accuracy per unit computation time is discussed, so here computation time and degree of element definition must be optimized.

A very significant reduction in calculation time for operations employing the mapping functions is obtained by employing linear in preference to cubic hexahedral elements (Appendix 3).

In addition subdividing the cubic elements, creates a longer search time to locate the element containing the marker but increases the likelihood of the element lying in smaller straight sided linear elements.

Precision in modelling curved segments by straight sided segments, without sacrificing accuracy in modelling the flow grid, is evidenced in Appendix 6.

It will be observed that the flow grid could be optimally modelled by having fewer elements where smaller volumes are to be mapped (for instance in the ring), and increasing the element density where poor representation of flow grid is envisaged.

The possible criteria for substructuring the flow grid would be based on

(i) volume of element
(ii) ratio of length : width : height
(iii) measure of element distortion.

This technique would then remove the restriction on equal numbers of flow paths through all zones of loosening, and in the light of the considerable computational times would present significant savings when implemented.
8.9 SIMULATION OF SINGLEheading DRAW

8.9.1 Introduction

The basic method of representation of the flow process by the empirical model can now be outlined. The following input data is assumed:

(i) sublevel caving design geometry

(ii) density profile and displacements produced in blasting have disturbed and redistributed the blasted ring and surrounding waste rock, with density and grade known as a function of position

(iii) specification of the appropriate zones of loosening and flow paths for this broken rock mass configuration for extraction at this single heading.

A flowchart for the sequence of calculations involved in the empirical model is given in Figure 8.14. The blasting model is described in Chapter 6.

The basic unit for modelling flow that is examined here is the single ring, single heading component. Flow calculations can proceed until flow at a neighbouring heading indicates that a point is being drawn to both headings indicating that one ring must be given a priority (or precedence in time) to perform calculations for displacement of that point.

In general, a multiple heading draw can be conceptualized as a sequence of draws from headings in a predefined order, by superposition of the application of single heading unit, to effect displacements for each draw.

8.9.2 Contents of Flow Grid and Loosening Mechanism

To evaluate the dilution and recovery figures, and predict the rate of loosening,
Figure 8.14 Flow Chart for Basic Empirical Model
(i) ore
(ii) waste
(iii) density

for the contents of the grid are evaluated.

As was elaborated in Section 8.3, the flow of broken rock is assumed to proceed in a "shell-wise" fashion, so that if loosening has proceeded to the Kth shell (K being a non-integer parameter expressing the number of shells or part thereof), then all material beneath the Kth shell can be considered completely loosened.

**Position of zone of loosening**

To evaluate the extent of loosening the mass of material within each shell, must be known, or more exactly to define mass:

\[ M = f(\text{local density,flow grid geometry}) \]

Because in general the position of the zone of loosening will not correspond to the input zone of loosening positions but be intermediate, mass must be evaluated not as a shell-wise step function but as a continuous function of K

\[ . \cdot \ M = \int \rho \, dv \]

This equation cannot be evaluated exactly because numerical quadrature is used to evaluate volume in each element. Consequently mass is evaluated incrementally:

\[ M^k = M^{k-\Delta k} + \frac{\text{IRUL1 ICUL1}}{k} \Delta k \sum_{ir=1}^{k} \sum_{ic=1}^{k} \rho \, dV_{\text{element}} \]

Defining \( n_o \) as the number of sampling points recorded as ore in the element and \( n_w \) the waste, the mass of ore (MO) and waste (MW) are evaluated (Figure 8.15):
\[ M_{0k} = M_{0k-\Delta k} + \sum_{ir=1}^{IRUL} \sum_{ic=1}^{ICUL} \frac{n_o}{n} \cdot \rho_o \int_{k-\Delta k}^{k} dV_{element} \]

\[ M_{nk} = M_{nk-\Delta k} + \sum_{ir=1}^{IRUL} \sum_{ic=1}^{ICUL} \frac{n_w}{n} \cdot \rho_w \int_{k-\Delta k}^{k} dV_{element} \]

\[ \rho_o = \frac{1}{n_o} \sum_{i=1}^{n_o} \rho_i \]

\[ \rho_w = \frac{1}{n_w} \sum_{i=1}^{n_w} \rho_i \]

To facilitate these calculations, the volumes of each element are calculated beforehand, and stored, with the progressive volume being evaluated with increasing shell number for interpolation:

\[ \int_{k-\Delta k}^{k} dV_{element} = (V_k - V_k - \Delta k)_{ir, ic} \]

The results obtained for the data used are presented in Section 8.9.4. It should be noted that all masses are expressed as equivalent volumes, by dividing the mass by the loose density of the material under consideration, ore or waste.

As long as the same flow grid is employed, the stored volumes need not be recalculated, but for every new increment of mass extracted, the masses in the flow grid must be reassessed.

In each calculation, a test is made on the volume and mass of the elemental slice, and if either exceeds a specified upper limit, \( \Delta k \) is appropriately decremented to minimize errors in the approximate integration.
Figure 8.16 Marker extraction definitions

Figure 8.15 Placement of sampling points to evaluate contents of elemental slice
If the elemental slices are large enough to conceivably allow significant proportions of ore and waste, the density of sampling points would be incremented to adequately represent the ore and waste fractions.

Alternatively if elemental slices are small enough they can be designated either ore or waste with minimal error.

To evaluate the extent of loosening, a mass balance is evaluated; loosening progresses until

\[
\begin{align*}
\text{Volume of mass extracted} & \quad + \quad \text{Volume to the } K_{ZL} \text{th shell} \\
\text{Total (equivalent) mass of} & \quad \text{expanded ore and waste to the } K_{ZL} \text{th shell}
\end{align*}
\]

The calculation procedure iterates and converges to a value \( K_{ZL} \) where the mass balance can be effected within a specified tolerance. Hence the current position of the zone of loosening, for the mass extracted is evaluated.

**Zone of extracted material**

Although loosening has propagated to the \( K_{ZL} \)th shell, the mass extracted originates from within the \( K_E \)th shell, the shell containing a total mass of ore and waste equivalent to the mass extracted.

**Recovery and dilution**

Because the zone of loosening \( (K_{ZL}) \) always contains the zone of extracted material \( (K_E) \), in calculating the contents of flow grid, the contents of shell \( K_E \) are implicitly defined (Figure 8.16).
8.9.3 Prediction of Marker Displacements

The flow of ore in sublevel caving is visualized readily by observing the behaviour of a line of markers in cross-section, or a plane of markers in three dimensional perspective view.

To evaluate marker displacements, the stages are:

1. **Definition of initial marker positions**

   Markers are defined on lines or planes by specification of marker densities and the limits of marker line or plane positions at various heights.

2. **Flow grid search**

   Having defined the mass increment extracted from the ring, and evaluated mass since material contents of flow grid were last updated, the current position of the zone of loosening is available.

   Consequently any material within the zone of loosening would have displaced, and any marker lying in a hexahedra within or below the $K_{ZL}$th shell will be displaced.

   The search is taken shell-wise, searching each of the hexahedra to locate if any of the markers are located within the hexahedron boundaries.

3. **Location of hexahedron containing marker**

   It is obvious that for an eight point linear hexahedron that by selecting the upper and lower values ($XL, XU, YL, YU, ZL, ZU$) from the coordinates at the corner nodes, any point that lies outside these values cannot lie within the volume defined by corner nodes.

   For the thirty-two point cubic hexahedron, the corner and midside nodes do not completely define the limits of the $(x,y,z)$ values of the hexahedron surface.
Consequently for hexahedron (IR,IC,IZ)

\[ X_L \leq X_P \leq X_U \]
\[ Y_L \leq Y_P \leq Y_U \]
\[ Z_L \leq Z_P \leq Z_U \]

it is assumed the marker \((X_P, Y_P, Z_P)\), potentially lies within the volume defined by the hexahedron, otherwise a new marker is tested.

The shape functions define a transformation from \((r,s,t)\) coordinates, relative to the hexahedron nodes, to Cartesian coordinates \((x,y,z)\). The inverse transformation can be used to obtain \((r,s,t)\) coordinates for \((x,y,z)\) coordinates, as proposed in section 8.7.

Consequently if

\[ -1.0 \leq \begin{bmatrix} r \\ s \\ t \end{bmatrix} \leq +1.0 \]

the marker lies in the hexahedron \((IR,IC,IZ)\).

To narrow the marker search, if all but one of \((r,s,t)\) lie within the limits, then the parameter \((IR,IC,IZ)\) corresponding to the coordinate direction is incremented (if \(>1.0\)) or decremented (if \(<-1.0\)) to test if the marker lies in an adjacent hexahedron.

The marker search is faster if opposite bounding surfaces of the hexahedron parallel planes in the Cartesian coordinates, and is extended for hexahedron exhibiting none of these characteristics.
At present no technique exists for mapping the complete flow grid into a unit cube, and bypassing the local mapping functions with a global mapping function, preferable as this would be.

After the marker search, a marker status is assigned:

- N2 marker lies outside flow grid as defined
- N1 marker extracted
- 0 marker not yet located in a hexahedron
- N1 initial hexahedron location is defined
- N2 marker displaced for the mass extracted

where N > 0, defines the ring at which marker hexahedron was located.

4. Calculation of magnitude of marker displacement

The marker position within the shell is defined by \( K_A \). Referring to Figure 8.17, it can be seen that if the material within the zone of loosening is defined to be completely loosened then

\[
\begin{align*}
\text{Total mass in } K_A \text{th shell} &= \text{MAS} \\
\text{Total mass extracted} &= \text{MAST} \\
\text{Volume in } K_A \text{th shell} &= \text{VOLA} \\
\text{Mass extracted before marker first moves} &= \text{MAS} - \text{VOLA} \\
&= \text{MASA} \\
\text{Mass extracted after marker moves} &= \text{MAST} - \text{MASA} \\
&= \text{DELME}
\end{align*}
\]

Consequently position of marker is available because the mass extracted since marker movement was initiated is the volume of material removed from beneath the marker causing it to slump towards the heading.

\[
\therefore \text{VOLB} = \text{VOLA} - \text{DELME} + K_B
\]
Figure 8.17 Marker displacement definitions
If VOLB < 0, the marker lies outside the flow grid and is considered extracted, otherwise the marker retains the hexahedron parameters (IR, IC), but (IZB) may be redefined if the marker passes into a lower hexahedron as a result of its displacement.

Given (r, s, Kp) the new marker position can be evaluated relative to the hexahedron (IR, IC, IZB).

Other loosening rules could be incorporated at this stage by assuming that loosening took place over a defined number of shells, or over a defined volume. An extension of loosening rules is not warranted at this stage.

5. **Updating of marker data**

To facilitate validation of the model, the marker displacement routines operate in modes, that indicate the type of calculations completed at the ring.

**Mode 0**: No extraction at ring

**Mode 1**: Temporary marker displacement. For the mass increment specified, the new marker positions are not updated.

**Mode 2**: Calculate new marker positions for the mass increment and update old marker positions. This mode assumes that material representation of the ring is also updated, so that the material contents of flow grid must be freshly evaluated. Mode 2 is employed when extraction at an adjacent ring would affect the contents of the flow grid.

Consequently for each marker there are associated parameters

\[
\begin{align*}
(IR, IC, IZ) & \quad \text{original marker position, or else the} \\
(x, y, z) & \quad \text{permanently updated positions} \\
(r, s, K) & \quad \text{temporary marker positions} \\
\pm Ni & \quad \text{marker status.}
\end{align*}
\]
6. **Graphical output of marker data**

For visual evaluation of flow behaviour the positions of markers after extraction of each mass increment can be plotted. If the path of a marker was traced during flow it would describe an interpolated flow path.

8.9.4 **Results of Simulation Program**

The algorithms presented in this Chapter have been implemented in a series of computer programs that follow the flowchart given in Figure 8.14.

The data set implemented employed 10 zones of loosening (the base zone having no volume, but locating the flow path termination), and 15 flow paths on the front section.

Included in Appendix 6 are computer plots of

(i) zone of loosening proportions for front and side sections

(ii) curves of intersection for each zone of loosening, with grid points connected by straight lines for the linear hexahedra grid

(iii) cubic hexahedra grid sections

(iv) linear hexahedra grid sections.

Typical results produced in the extraction simulation are also presented.

It should be noted that the flow algorithm is not path dependent ie if the same total mass is extracted but in different increments then the simulation yields the same result for both sequences.
Experience with the simulation programs indicated:

(i) when using straight lines to approximate the curved flow grid, inconsistencies can arise eg intersecting flow paths or zones of loosening

(ii) too few linear hexahedra elements, implying large and distorted elements can introduce non-uniqueness of mapping ie some markers are not found to have \((r,s,t)\) coordinates to locate them in the hexahedral elements expected. No quantitative test has been implemented to test distortion of elements.

8.10 DATA STRUCTURES TO REPRESENT ROCK MASS IN THE ZONE OF LOOSENING MODEL

The extraction sequence calculations outlined in the previous sections presume a knowledge of grade and density at any point \(P(x,y,z)\) in the vicinity of the heading after blasting. For examination of single ring simulation, this requirement is simply met by the existing blasting routines, provided that all broken rock outside (ie above, behind, beside) the blasted ring is assumed waste.

In the multi-ring, multi-drawpoint simulation, the character of the broken rock surrounding the ring will be dependent on the draw down behaviour of previously extracted rings. The present model has the capability of simulating single heading-single ring draw down, predicting recovery, dilution and marker displacements.

Orthogonal grid data structure

Several routines were written to store ore grade and density on random access files. The orthogonal grid employed used similar file structure and indexing procedures to the stochastic flow computer program data structure, Section 7.4.
To update the material identity after a blasting, a search for all grid points lying with the region of blasted and compacted material was effected. New values of grade and density were defined by the procedures outlined in Chapter 6.

The data structure provides material characterization at unit spacing (say 1 foot), and could be developed to give more detailed characterization by decreasing grid spacing, or alternatively increasing grid spacing in areas of little interest (Figure 8.18).

Reasonable computational times were achieved for redistribution of material through the grid after the blasting process.

This data structure has yet to be implemented for the extraction phase. Longer computational times could be expected due to:

1. For each increment of extraction under consideration, new grade and density must be defined for each grid point lying within the current zone of loosening—considerably greater volume than the ring itself.

2. To calculate position of each grid point before the extraction increment, a search of all the hexahedral elements within the zone of loosening is involved—considerably more hexahedra are required to define the flow grid compared to the blasting grid.

It is anticipated that the orthogonal grid may well prove an unattractive data structure, although the larger the extraction increment, the less frequent updating is necessary.

Alternative data structures

Other sparser data structures are available. The criterion for acceptability would be the adequacy of representation of parameters of interest.
Figure 8.18 Grid resolution decreased in areas of little interest (not to scale)
An alternative, proposed by Raftery (1976) involves defining the nature of the rock mass by representative markers that indicate specific features eg ore grade contours, ore waste interface. If the markers are linked geometrically, the extraction (or blasting) involves updating the positions of markers on a three dimensional surface.

The position of those markers lying within the current zone of loosening could be updated, and the recovery and dilution evaluated by interpolating material characterization relative to the marker surfaces, or else presuming that markers recovered for the mass increment are representative of material in the increment. Increased marker populations give a better estimation of recovery and dilution.

A simplification that would be desirable in all data structures is the elimination of density. Rock is then only characterized as ore (grade > 0) or waste (grade = 0). This is obtained by assuming that all rock outside the blasted ring is completely loose. [This assumption is presently used in the computer model for extraction.]

8.11 CONCLUSIONS

A flexible empirical flow model has been developed to simulate dynamic flow behaviour during draw down in a sublevel caving environment.

The model has been implemented in a computer program that evidences the application of the model for supplying accurate estimates of recovery and dilution, and description of displacements in the broken rock mass as a function of extraction history. The model currently models single drawpoint extraction. The advantages over previous static analyses are evident.
The model does not directly incorporate material properties and consequently relies on the skill and experience of the engineer to specify the input data for position of zones of loosening and flow paths, for any mining environment to be realistically simulated.

The model is predictive if the input data correlates well with actual mine behaviour. The validation of the flow model is discussed in Chapter 9, while utilization of the model in mine design is discussed in Chapter 10.

Although a specific flow mechanism for flow in sublevel caving has been modelled, it is apparent that the methodology of specifying a flow grid by surfaces and connecting lines allows a range of flow mechanisms to be implemented. It is anticipated that the general technique could have application in other caving methods.

Although the model has been intuitively developed from the results of past modelling programmes, a number of assumptions incorporated in the program require verification:

1. Accurate definition of the limits of the flow regime and shape of zones of loosening at each stage of flow on horizontal section.

2. Placement of internal flow paths, or in terms of the method developed, the validity of the "planar section" and "spacing rule" assumption.

3. The approximations inherent in calculating unsteady state flow from a steady state flow grid.

It is recommended that implementation of the model be refined in the following directions:

a. Input data be assembled with a digitizer, instead of manual plotting, and reading of data points to specify the flow grid shape.
b. That simple techniques be developed to apply transformations to the flow grid to alter flow behaviour or introduce a new ring geometry in preference to assembling new data sets. Techniques currently exist in computer aided design and computer graphics to manipulate data in this fashion for complex shapes.

In updating the data set a number of situations may arise:

(i) distort the complex flow grid by some linear or non-linear function of height, depth or width

(ii) retain flow path location but alter zone of loosening

(iii) retain zones of loosening but relocate flow paths.

c. Techniques to assess the distortion of individual elements in the grid in regard to uniqueness of mapping.
CHAPTER 9: VALIDATION OF COMPUTER MODELLING CONCEPTS FROM TEST DATA

9.1 Introduction

9.2 Results from Full-scale and Model Tests (Janelid)
   Manual evaluation of volumes of motion
   Computer evaluation of volumes of motion
   Comments on Janelid's results

9.3 Spatial Data Analysis: Techniques
   Mathematical analysis

9.4 Spatial Data Analysis: Results and Interpretation

9.5 Application of Results to Computer Models

9.6 Future Development of Validation Techniques
9.1 INTRODUCTION

In the construction of a model to simulate real physical phenomena five basic stages are followed

1. Isolation of the phenomena
2. Enumeration of the key variables, and construction of a mechanism to describe the significant behaviour
3. Implementation of the model to gain numerical solutions
4. Comparison of predicted results with actual behaviour, with a measure of error and consideration of random components in the behaviour
5. Improvement of the model.

In evaluating the performance of the model, one has the opportunity to test the validity of the assumptions employed in the development of the model. However one needs an appreciation of the accuracy required. Gerrard (1975) states that:

"... the desired degree of accuracy in the prediction of the full scale performance provides an important constraint in the development of an appropriate mathematical model. Models are by their very nature approximate representations of material response, and hence a proof of inaccuracy of a model is merely a statement of the obvious. The real questions are how inaccurate the prediction is and what the consequences of this will be. Two extreme approaches to the problem of definition of desired accuracy can be thought of as that of the 'engineer' and that of the 'scientist'.

The 'engineer's' approach, when extrapolated to its logical conclusion, is to employ within the time scale of the project that level of sophistication in modelling that will minimize the total cost ... on the other hand, the scientist is not constrained to a time scale and seeks to understand all aspects of behaviour to an extremely high level of prediction".
Such is the tension in mathematical modelling in a mining context. Each level of model sophistication has to be balanced by the economic advantage.

The modelling of sublevel caving suffers from a number of shortcomings that have been alluded to at various stages in this study:

(i) difficulty in monitoring drawdown behaviour on a mine site

(ii) the variability, and hence unpredictability of factors in the mining environment ie mining methods and material response

(iii) the failure to incorporate parameters characterizing the broken ore in the flow models.

As a result of these factors some difficulty can be expected in carrying out Stages 4,5. These can be itemized as:

(i) determination of the level of reliability of the model ie when the predicted results are meaningful: over one ring? one sublevel?

(ii) elucidation of which aspects of the model contribute significantly to the errors. To quote Gerrard (1975) again: "... the accuracy of the predictions of a mathematical model depend very largely on the adequacy and relevance of the input data".

As an introduction to the problems of validation of the computer model, a study of the results obtained by Professor Janelid in monitoring full scale and model tests was initiated, in an endeavour to validate the stochastic and empirical computer models for a single ring extraction from available data.

9.2 RESULTS FROM FULL SCALE AND MODEL TESTS (JANELID)

One of the chief disadvantages of modelling programmes was that no established laws had been obtained to relate model results to full scale mine results.
Professor Janelid supervised a series of tests to investigate:

(i) scaling factors between model tests and full scale tests

(ii) gravity flow subsequent to blasting against loose rock at the Grangesberg mine, with parallel model tests being conducted at the Royal Institute of Technology, Stockholm.

The results obtained for extraction at Drift 9, Round 3 were supplied by Professor Janelid for analysis. They assist in the validation of the computer models developed in this thesis. Professor Janelid reported that this ring gave typical results for the full scale test.

Markers were inserted into the ore before mining started, by drilling fans of holes perpendicular to the sublevel drifts, the rows of holes being 30cm apart, yielding 5 planes of markers per ring. Markers (of plastic hose) were distributed and their position coordinates surveyed. No markers were placed in the bottom half of the ring (see Figure 9.1).

The recovery of markers was recorded by examining the contents of each bucket loaded. Weight was measured, and dilution estimated. About 75% of all markers were recovered.

The particle size distribution curve for fragmentation after blasting was estimated by manual screening, and comparing results with photographs of the rill heap using two procedures to evaluate the portions of coarser size fractions, namely photodominance, and photo-frequency methods. This curve was employed to simulate the tests in scale models. Further details of the testing are elaborated in a recent report (Janelid, 1972).

The data supplied consisted of

1. marker-code
2. associated volume
3. x,y,z coordinates relative to the ring.
Figure 9.1 Vertical section through drill holes for marker insertion (after Janelid 1972)
These values resulted from applying the following calculation steps:

(i) for full scale test results

a. The marker coordinates were corrected from the original surveyed positions by assuming 20% swell during confined blasting against loose rock, perpendicular to the ring face.

b. Volume associated to each marker was calculated:

\[
V_{os_i} = \frac{(1 - P_i) \cdot Q_i}{P_i \gamma_{ww} + (1 - P_i) \gamma_{os}}
\]

"; volume of ore associated with markers of nth wagon

\[
V_n = V_{OD} + \sum_{i=1}^{n} V_{os_i} - \frac{1}{2} V_{os_n}
\]

\[
P_i = \text{waste fraction by volume in } i\text{th wagon (determined visually as 0, 0.25, 0.50, 0.75, 1.00)}
\]

\[
Q_i = \text{weight of ore and waste in } i\text{th wagon}
\]

\[
\gamma_{ww} = \text{bulk density of waste (1.6 ton/m}^3\text{)}
\]

\[
\gamma_{os} = \text{bulk density of swelled ore after blasting}
\]

\[
\text{Density of intact ore = 4.2 ton/m}^3
\]

\[
\text{Swell factor = 1.2 (estimated)}
\]

\[
\gamma_{os} = 4.2/1.2 = 3.5 \text{ ton/m}^3
\]

\[
V_{OD} = \text{volume of ore that slumps into rill heap during blasting, but is not extracted (13.5m}^3\text{)}
\]

A loosening factor of 1.6 was obtained for complete loosening (in rill heap).

(ii) for model tests

A similar equation for the volume of ore associated with markers of nth wagon applies, but with \( V_{os_i} \) redefined
\[ V_{os_i} = \frac{Q_{w_i} - Q_{ww_i}}{\gamma_{os}} \]

- \( Q_{w_i} \) = weight of ore and waste in \( i \)th wagon
- \( Q_{ww_i} \) = weight of waste in \( i \)th wagon (obtained by magnetic separation)
- \( \gamma_{os} \) = bulk density of ore in model (= 3.24 g/cm\(^3\))

It should be noted that the associated volume relates the volume of ore only, considered to be in its blasted state. The volume of motion is not defined to be a surface enclosing all the material removed for a specified loaded volume as is our zone of loosening. However due to backbreak, portion of the volume of motion will be waste for which no markers exist, so that only that part of the volume of motion lying in ore can be evaluated. A significant problem will be isolating sources of dilution for full scale testing.

Small errors in the data can be attributed to
1. surveying errors
2. error in estimate of swell factor, and due to extra swell in backbreak
3. associated volume can only be evaluated to within an interval of one wagon load
4. uneven blasting and fragmentation over the ring.

Other variables that affect the result are:
1. loading position, and digging depth.

A number of techniques were employed by Professor Janelid to analyze the resultant data.

1. **Manual evaluation of volumes of motion**

   For each plane of markers, the recovered markers are plotted with different symbols for different volume intervals on scaled drawings. To interpolate the volume of motion section on each marking plane, the total area of
Figure 9.2 Manual evaluation of volumes of motion
(after Janelid, 1972)
all sections is $\frac{V}{a} \text{ m}^2$, where $V$ is the volume of motion and $a$ the distance between marking planes. This area is represented by circular metal discs in hexagonal dense packing distributed over the marking planes in the ring (including overbreak planes), until the best overall approximation can be made for each section. The method is repeated for each successive volume interval.

Typical results obtained by Professor Janelid are reproduced in Figure 9.2.

2. **Computer evaluation of volumes of motion**

Because initial modelling theories predicted that the zone of motion could be approximated by a rotational ellipsoid, a program was developed by Janelid to fit a polynomial surface of 2nd degree $P(x,y,z)$ to the marker data, by the method of least squares.

A number of restrictions on the surface were employed.

(i) the volume enclosed by the surface that corresponded to ore in the blasted ring should approximate the associated volume

(ii) assuming that gravity flow was not uniform for different degrees of loading, the markers were partitioned according to several criteria so that the surface could be fitted to markers lying in an optimal volume interval about the loaded volume desired.

The criteria for selecting markers $V_i$ for volume UTGRAD were:

a. $0 \leq V_i \leq (C \cdot \text{UTGRAD} + B) = \text{UTMAXI}$

for which the standard deviation (STDV) for deviation of markers from the resultant fitted surface is calculated
b. Markers deviating sufficiently are eliminated according to two criteria

\[ |V_i - P(x_i y_i z_i)| \geq \frac{\text{REL} \times V_i}{100} \]

\[ |V_i - P(x_i y_i z_i)| \geq \text{AVMAX} (=\text{AVMIX.STDV}) \]

c. With the new marker partition obtained, the surface is fitted, and a new standard deviation calculated. The ore volume defined by the surface \( P(x,y,z) = \text{UTGRAD} \) and the plane \( y = 0 \) is obtained by approximate integration of horizontal slices. If \( |\text{obtained volume} + \text{REST} - \text{UTGRAD}| \leq \frac{\text{ACCEPT} \times \text{UTGRAD}}{100} \) the polynomial is accepted.

d. If the polynomial is still unsatisfactory, a new volume control is established

\[
\text{UTMAXI}_{\text{new}} = \text{UTMAXI}_{\text{old}} - \text{DIFF.C}
\]

\[
\text{DIFF} = \text{UTGRAD} - (\text{obtained volume} + \text{REST})
\]

providing

\[
\text{UTMAXI}_{\text{new}} \leq \text{VMAX}
\]

\[
\geq \text{VMIN}
\]

\[
\leq \text{UTGRAD}
\]

In addition, if the maximum height of the curve is greater than a specified value (STOP), a new partition is chosen.

\[
\text{UTMAXI}_{\text{new}} = \text{UTMAXI}_{\text{old}} - \text{CONST.}(=1000\text{cm}^3)
\]

The overall criteria is to eliminate or add markers to the initial partition in an endeavour to satisfy the volume criterion, with suitable choice of constants \( C, B, \text{REL}, \text{AVMIX}, \text{REST}, \text{ACCEPT}, \text{VMIN}, \text{VMAX}, \text{STOP} \).

Because no markers are recovered from the lower part of the ring, fictitious markers are introduced. These are placed on a horizontal semi-circle with radius, height and loaded volume specified in the input data. When the markers within a partition are removed, only those fictitious markers that have a volume \( V > \text{UTGRAD-CONST} \).
are retained. Basically these markers are intended to force the behaviour of lower portions of the surface. Typical results from model test data are reproduced in Figure 9.3.

Some relevant conclusions were detailed:

1. Surfaces with higher order than 2 did not give a reasonable result.

2. For model test values of ACCEPT in range 5-10% were necessary, and for full scale tests, 25-50%.

3. A mathematical function is fitted with markers removed from the marker partition, if they deviate markedly from the surface. These are referred to as "obviously mislocated ones".

4. The manual and computer curves were compared at 120% degree of loading by superimposing sections at the marking planes. Positive deviations (where manually evaluated volumes of motion are larger) and negative deviations (where manually evaluated volumes of motion are larger) and negative deviations (computer evaluated volumes of motion are larger) were defined. Results for Drift 9 Round 3 are reproduced in Figure 9.4. To measure how the shapes compare, Janelid defined

\[
\text{Absolute deviation} = \frac{\sum \text{pos. dev.} + \sum \text{neg. dev.}}{\sum \text{computer surfaces}} \times 100\% 
\]

To measure how the volumes compare

\[
\text{Total Deviation} = (100 - \frac{\sum \text{manual surface}}{\sum \text{computer surface}}) \times 100\%
\]

Typical results for absolute deviation were recorded as 75%, with total deviation as 90-98%, with better results for the model tests compared to full scale tests, and better comparisons for higher degrees of loading. In addition the manually designed shapes were narrower and higher than computer designed curves.
Figure 9.3 Computer evaluation of volumes of motion
(after Janelid, 1972)
Figure 9.4 Deviation between manual and computer curves
(after Janelid, 1972)
3. value of test data in validating computer flow models was initiated.

9.3 SPATIAL DATA ANALYSIS TECHNIQUES

The placement of internal markers, and the recording of their subsequent recovery yields a sequence of data points

\[ P_1 \ldots P_i \ldots P_n \]

where \((x_i, y_i, z_i)\) are the initial marker coordinates, and \((V_i)\) is the loaded volume when the marker is recovered.

The fact that markers are recovered in no way implies a knowledge of marker trajectories. In fact the volume of motion defined by Janelid is only an imaginary surface, useful for design but only of secondary importance for simulation studies, except that the simulation should reproduce the recovery of markers with reasonable accuracy.

Janelid defines the manually and computer defined surfaces as volumes of motion. The loaded volume \(V_i\) relates the volume of ore, and does not include waste rock. To distinguish this analysis from that of Janelid, the extraction surface is defined as the surface interpolated through the data points at a specific value \(V_i\). An extraction contour is the intersection of the surface with a plane. If the surface is assumed continuous (a bold assumption), the surface will exist for regions of waste rock for which no markers could be recovered (Figure 9.5). Extrapolation of surface behaviour into regions where no data exists invites error. This is accentuated because the waste rock has a different flow behaviour from ore (Janelid). Because the volume extracted is the volume contained by the extraction surface, a more detailed analysis would compare not only the volume of ore recovered with the volume of intersection of the blasted ring and the extraction surface (as Janelid has done) but also the volume of waste with the remaining volume of the extraction surface. The necessary test data was not available to perform the latter analysis.
Figure 9.5 Location of markers, waste and backbreak in relation to the extraction surface
5. Later published work illustrates the relationship between height, width and depth of manually designed curves, and hints that the ellipsoid is an inadequate representation for the volume of motion (hence the disposal of the term "ellipsoid of motion"). These results are reproduced in Chapter 2.

**Comments on Janelid's Results**

1. Because Janelid could only conceive the volume of motion to be of a globally explicit mathematical form, i.e., an ellipsoid, investigation into other interpolation algorithms seemed necessary, especially in the light of more appropriate methods of surface definition that had evolved in the development of the empirical model. Local surface definition instead of global also seemed desirable to eliminate the tedious manual methods.

2. Janelid's choice of marker partitions could not do justice to the data, because any marker that deviated from the idealized surface was removed to permit a surface that satisfied the volume control. Those markers that questioned the nature of the surface most were rejected first.

3. The significance of the basic assumption behind marker partitioning i.e., that the surface shape was dependent on degree of loading had not been verified. For example the deviation of the major axis of ellipsoid showed minor deviations of only a few degrees except for initial loading. For the full scale tests, the manual curves indicate a marked deviation that is not represented in the computer curves.

Consequently using the data supplied by Janelid, an investigation of:

1. alternative mathematical forms

2. validity of Janelid's techniques for representing the data, involving a repetition of manual and computer evaluations
It is suggested that the computer evaluation of a global surface fit (ie taking into account all the markers) superimposes a form on the resultant surface and that the manual methods, although efficient, have no sound mathematical basis for the interpolation.

**Mathematical Analysis**

To gain an insight into the type of results that could be obtained from full scale and model test data, results of one ring (Drift 9, Round 3), supplied by Professor Janelid were evaluated with a range of interpolation algorithms.

Having no prior knowledge of the functional form of the surface that should be fitted, it is necessary to approximate the unknown function with one of arbitrary nature eg polynomial, and evaluate the goodness of fit. There is no guarantee that the markers will fit a readily definable function (Agterberg, 1974).

A number of inherent limitations hinder the analysis due to the data available.

(i) markers are located mainly within the upper half of the ring (Figure 9.1). Janelid placed fictitious markers in the lower part of the ring to define the surface shape in this area.

(ii) markers are clustered, and not evenly distributed over the volume to be interpolated.

The problem is to define a surface \( V = F(x,y,z) \) at \( V = V_1 \). Various interpolation algorithms are available for data in this context.

A. Predefine the global nature of the surface by fitting all points simultaneously.

A polynomial of sufficient degree and order is usually prescribed, and the data fitted by a least squares fit

\[
V = F(x,y,z) = a_0 + a_1x + a_2y + \ldots + x^m y^n z^m + e
\]
The variable e represents residuals whose sum of squares is being minimized in the multiple regression analysis. Considerable experience has developed this technique in the area of trend surface analysis (Davis, 1973, Agterberg, 1974).

B. Predefine the local nature of the surface by fitting the surface over subsets of the data.

In general this requires definition of a number of domains within which a local surface is fitted. Typical procedures are

(i) apply a weighted or unweighted least squares polynomial surface
(ii) define the markers to be topologically equivalent to a rectangular or triangular grid and fit bicubic spline surface patches
(iii) apply moving averages, weighted or unweighted to data points in the domain
(iv) although no specific applications were found, the geostatistical theory of Matheron (1971) could potentially be applied to this problem.

Because limited time was available to implement the algorithms, and because the surface \( V = V_i \) potentially would pass through only some or none of the data points methods A(i), B(i), (iii) were employed. Method A(i) is essentially Janelid's method for computer evaluation, where only those markers within the partitions indicated in the previous section were used.

The computer program plots extraction contours (a term equivalent to loaded volume contours) on front \((z-x)\), side \((z-y)\) or horizontal \((y-x)\) sections. An orthogonal grid, defined on the section, is bounded by the maximum and minimum coordinates of markers in the subset \(N_5\) required by the interpolation algorithm. The interpolation algorithm is used to interpolate extraction values on the grid. The grid is then contoured with an algorithm
by Kalkani (1975). A flowchart for the program is given in Figure 9.6. No attempt was made to confine the extraction contours to the perimeter of the marker distribution on the section plotted.

The 4 interpolation algorithms are now detailed.

1. Algorithm NWLSA
   Performs a quadratic weighted least squares fit in 3 dimensional space. The weighting function, at present defined to be inversely proportional to distance squared, is optional. It is used for local surface fits.

2. Algorithm WLSFT
   Performs a quadratic weighted least squares fit in 2 dimensional space. Again it is intended for local surface fits, with optional weighting function.

3. Algorithm NVAL
   This algorithm is an extension to three dimensional space of an algorithm by Shepard (1968) that applied to two dimensional space. It is a sophisticated version of an inverse distance weighted average approach, with additional weighting functions to account for clustering and a slope correction factor. The algorithm has been extensively tested in a map production system SYMAP.

4. Algorithm RMULT
   This algorithm fits a general polynomial of degree IORD in m-dimensional space using the least squares regression analysis. The program has a capability of fitting a surfaces to degree 4, to 3 dimensions, with up to 700 data points generating a polynomial of up to 34 terms. The program follows the method recommended by Davis (1973) to evaluate the coefficients of the polynomial using a multiple regression analysis, and produces the variance table and goodness of fit to evaluate the suitability of the surface for fitting the data set. The program was designed to handle large
START

Read in Markers from file of model or full scale test data

Select interpolation algorithm RMULT, NWLSA, WLSFT, NVAL

Select plane to be interpolated
- front section Z-X
- side section Z-Y
- horizontal section Y-X
and position of plane in third coordinate direction

Select subset of markers ($N_s$)
- with coordinates $>DL, <DU$
  in 3rd coordinate direction in 2D space
- eliminating Janelid's fictitious markers

Calculate limits of orthogonal grid and coordinates from number of rows and columns specified

Apply interpolation algorithm at each grid point

Plot extraction contours on ring profile

Plot marker positions on ring profile, or print marker densities on line printer plot

Are further sections wanted

Are other interpolation algorithms wanted

STOP

Figure 9.6 Flowchart of Extraction Contouring Program
data sets, and is computationally more complex to improve accuracy, and generates statistics. It is only employed for global surface fitting.

The results obtained from small trial data sets were checked against those of program LLSQAR (International Mathematical and Statistical Library) which computes the least squares solution from an overdetermined set of linear equations, and found to agree.

It should be observed that interpolation using weighted (or unweighted) least squares polynomials is subject to a number of weaknesses:

1. 'Border Effect'
   Interpolation can be unreliable on the border or outside the region defined by the markers.

2. 'Clustering'
   Uniformity (ie equal spacing) of data point distribution is preferable to clustered data points, or the surface can be unreliable between the clusters.

3. 'Discontinuity'
   Discontinuities in the data, ie markers with similar or identical coordinates but markedly different extraction values, are not well modelled by a continuous function. In this case examination of the residuals becomes important.

4. Matrix inversion can prove difficult with large data sets, and a high order polynomial. Davis (1973) and Agterberg (1974) illustrate the likely conditions. The algorithm RMULT employs techniques that will improve computational accuracy.

5. The statistics produced from the residuals $e$, are technically only meaningful if the residual can be assumed to be a normally distributed random variable, with mean zero, and unknown variance $\sigma^2$ independent of the dependent variable (extraction count). In this case, the least
squares method will give coefficients which are the most likely estimate of the regression parameters. The F-test can be applied to determine if the polynomial model fitted to the data is statistically significant, at a specified level of confidence.

Following general practice, it is accepted that the statistical assumptions mentioned are approximately valid, noting however the reservations expressed by Agterberg (1974) and Agterberg and Cheung (1975).

9.4 SPATIAL DATA ANALYSIS : RESULTS AND INTERPRETATION

A sequence of plots was generated for the test data from Drift 9, Round 3. Figures 9.7 - 9.16 apply to the full scale test, and Figures 9.17 - 9.25 are the corresponding sequence for the model tests. Experimentation with interpolation algorithm parameters was pursued until suitable plot contours for comparison were obtained.

The extraction contour values remain as Janelid's tonnes loaded, because no other data was available to alter this parameter.

Of primary interest is the comparison between global and local interpolation methods, for model and full scale test data. Because the comparison is qualitative, in that we know local methods fit the marker data exactly whereas global methods are only a partial fit to the data, we note the way a global fit is able to obscure the irregularity of flow (where the contours depict material recovered simultaneously at the heading).

Both weighted average and least squares fits confirm that contours are more irregular than is evidenced by Janelid's manual evaluation, the weighted average being more greatly influenced by the nearest points than the (unweighted) least squares fit.

It should be noted that the global least squares fit does not create marker partitions but fits a global function to all the
Table 9.1 Extraction contour sequence for full scale and model test data*

<table>
<thead>
<tr>
<th>Figure</th>
<th>Markers selected</th>
<th>Extraction contour plane</th>
<th>Interpolation</th>
<th>Comments</th>
</tr>
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<tbody>
<tr>
<td></td>
<td></td>
<td></td>
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<td>-</td>
<td>-</td>
</tr>
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<td>9.8</td>
<td>$0.20 \leq y \leq 0.24$</td>
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<td>2D RMULT</td>
</tr>
<tr>
<td>9.9</td>
<td>9.19</td>
<td>all</td>
<td>$y = 0.22$</td>
<td>3D RMULT</td>
</tr>
<tr>
<td>9.10</td>
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<td>NVAL</td>
</tr>
<tr>
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<td>9.21</td>
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<td>9.12</td>
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<td>$0.20 \leq y \leq 0.24$</td>
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</tr>
<tr>
<td>9.13</td>
<td>9.22</td>
<td>all</td>
<td>$x = 0.0$</td>
<td>-</td>
</tr>
<tr>
<td>9.14</td>
<td>9.23</td>
<td>all</td>
<td>$x = 0.0$</td>
<td>3D RMULT</td>
</tr>
<tr>
<td>9.15</td>
<td>9.24</td>
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<td>NVAL</td>
</tr>
<tr>
<td>9.16</td>
<td>9.25</td>
<td>all</td>
<td>$x = 0.0$</td>
<td>NWLSA</td>
</tr>
</tbody>
</table>

* Model test data is sealed up to full size for analysis
POSITIONS OF MARKERS

Figure 9.7
EXTRACTION CONTOURS  FULL-SCALE  PLOT SECT. AT .22
RMULT  NO WEIGHTS GLOBAL FIT  2D

Figure 9.8
Marker code:

<table>
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<th>no.</th>
<th>loaded volume (m³)</th>
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<tr>
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<tr>
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<td>81-100</td>
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<td>141-160</td>
</tr>
<tr>
<td>8</td>
<td>161-180</td>
</tr>
<tr>
<td>9</td>
<td>181-200</td>
</tr>
</tbody>
</table>

Figure 9.12
SCALE: METRES

EXTRACTION CONTOURS FULL-SCALE PLOT SECT. AT 0.00
RMULT NO WEIGHTS GLOBAL FIT 3D

Figure 9.14
Figure 9.15

EXTRACTION CONTOURS  FULL-SCALE  PLOT SECT. AT 0.00
NVAL  WEIGHTS  LOCAL FIT

SCALE: METRES
SCALE: METRES

Extraction Contours  Full-Scale  Plot Sect. At 0.00

NWLSA  Weights  Local Fit

Figure 9.16
POSITIONS OF MARKERS

Figure 9.17
EXTRACTION CONTOURS

MODEL

PLOT SECT. AT 22

WEIGHTS LOCAL FIT

Figure 9.21
SCALE: METRES

EXTRACTION CONTOURS  MODEL  PLOT SECT. AT 0.22

RMULT  NO WEIGHTS/GLOBAL FIT  3D

Figure 9.23
SCALE: METRES

EXTRACTION CONTOURS MODEL PLOT SECT. AT 0.00
NVAL WEIGHTS LOCAL FIT

Figure 9.24
Figure 9.25

EXTRACTION CONTOURS  MODEL  PLOT SECT. AT 0.00
NWLSA  WEIGHTS  LOCAL FIT
<table>
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<th>Test</th>
<th>Figure</th>
<th>Goodness of fit</th>
</tr>
</thead>
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</tr>
<tr>
<td></td>
<td>RMULT 3D</td>
<td>9.9</td>
</tr>
<tr>
<td>Model Test</td>
<td>RMULT 2D FV</td>
<td>9.18</td>
</tr>
<tr>
<td></td>
<td>RMULT 3D</td>
<td>9.19</td>
</tr>
</tbody>
</table>

Table 9.2 Goodness of fit for global least squares fit to marker data
markers available. The addition of markers beyond the plane $y = 0.22m$, forces a narrower draw shape for the three dimensional least squares fit than the two dimensional case. Local interpolation methods do effectively create a marker partition, based on search radius, not loaded tonnage.

The model results in each case yield smoother, and narrower draw shapes than the full scale test results. This result is consistent with Janelid's results.

No attempt has been made to date to evaluate the volumes of the draw shapes that have been interpolated and contoured. The primary intent is to apply interpolation methods that yield a more consistent interpretation of the marker data that is available on one plane, or all the marker planes in the ring combined.

In Figure 9.10, 11 the shaded regions indicate contour regions that lie below the 60, 100, 140 tonne contour intervals that appear out of sequence. Apart from the possibility of an error in recording when markers were recovered, this indicates some form of preferential flow, possibly due to uneven fragmentation, or else a 'hangup'.

Figure 9.12 is a manually contoured section for the full scale test data of Figure 9.7. In the shaded portion, adjacent markers were recovered at irregular intervals. Markers are numbered according to the tonnage interval, indicating the basis for the lack of regularity of extraction contours for the full scale tests.

Table 9.2 indicates the goodness of fit of the global least squares surface fits, with model test fits being superior to full scale test.

Although the analysis presented is only of a preliminary nature it indicates that draw shapes are less regular than Janelid's published results would lead us to believe.

Only the data for one ring has been analyzed. Before an assessment of more realistic draw shapes can be made, further rings would need to be contoured, noting the behaviour on several
sections in each ring (for front, side and horizontal sections).

It is conceivable that Janelid's manual evaluations provide a realistic average draw shape, but the deviations in the draw shape for any specific ring should be noted in a drawdown simulation. Figure 9.26 indicates manually evaluated draw shapes for several adjacent rings, as an indication of the variability in flow behaviour for one test.

The ring analysis would be more effective if markers were located in the bottom portion of the ring, to better define flow behaviour rather than introduce arbitrary fictitious markers.

9.5 APPLICATION OF RESULTS TO COMPUTER MODELS

In the previous section, it was observed that some markers were extracted at unexpected times in the extraction history. This seems to imply an irregular and non-symmetrical flow for this particular ring. Other published results by Janelid indicate that the lack of symmetry in flow under normal conditions has no preferred direction, taking a large number of rings into account. Janelid does indicate that some causes can be elucidated and so preferred directions could be anticipated in some special cases.

In comparing the predicted results of computer model simulations with actual mine results, the difference in predicted recovery time should be calculated. If these differences then correlate with

(i) extraction tonnage
(ii) a direction vector relative to the ring face,

one has reason to suspect that the differences are attributable, not to random factors, but to errors in the assumption of the model.

The correction necessary then depends on the model:

(i) Stochastic model
15cm in front of mining front

--- DRIFT 9 ROUND 3
-.-- DRIFT 8 ROUND 3
--- DRIFT 7 ROUND 3

Figure 9.26 Range of draw behaviour for adjacent headings
(after Janelid, 1972)
The following would be checked:

a. if markers lay outside of the flow regime

b. if loosening mechanism forced markers to arrive too early or too late

c. if the differences are biased in a particular direction that the probability vectors be altered accordingly either to promote or discourage flow from that direction.

(ii) Empirical Model

In the empirical flow model attention was focussed on the progressive development of the zone of loosening. Markers were recovered simultaneously from a surface interpolated between zones of loosening. Marker recovery analysis specifies this surface as the extraction surface, and consequently it is of no value for verifying the shape of the initial zone of loosening as it propagates upwards, but can verify the shape of those surfaces interpolated within the moving zone of loosening. It cannot verify the assumption that these interpolated surfaces correspond to the initial zone of loosening shape as it passes through the region.

However the following would be checked

a. if the shape definitions for the zones of loosening did not approximate the contoured section shapes an error could be expected. Distortion of the shape definitions should be effected.

b. if the loosening mechanism forced markers to arrive too early or too late.

It is reiterated that marker recovery is not a technique for validating the placement of flow paths, or determining the flow behaviour of material that is not extracted. If the displacements of material
not extracted from a previous ring cannot be established, then no clear idea of their position relative to the current extraction heading will be known. The marker recovery analysis assumes a knowledge of precise marker position before extraction from the current ring.

Because the local surface fitting methods indicate a very distorted shape for the surface enclosing extracted material, the problem will be a decision concerning a tolerable error, and a realistic degree to which this surface ought to be represented within the computer model.

Once the surface is generated to fit the marker data, deviations from the surface could be regarded as random behaviour in a local region that generate "random errors". An accurate knowledge of the rock mass and flow mechanisms allows an accurate model of flow behaviour. The generation of input data to the computer models is an attempt to model this flow behaviour.

However, the mining environment is variable and so flow behaviour varies from ring to ring. This is difficult to predict and so a range of flow behaviour should be defined. Although the flow behaviour is completely deterministic if one could define all the variables, we consider the flow behaviour as probabilistic. Flow behaviour is selected at random from a range of likely flow results (Figure 9.26).

A consistent simulation model incorporates input data selected on a probabilistic basis from a range of possible data sets.

9.6 FUTURE DEVELOPMENT OF VALIDATION TECHNIQUES

Methods of validating the computer models should proceed to two levels.
1. Scale Modelling

Because of the change in emphasis, from modelling draw ellipsoids to modelling particle trajectories and flow patterns, confirmation of the validity of the assumptions in the computer models should be found in a modelling programme with a new direction to past studies. A tentative set of goals are outlined in Appendix 1, while Appendix 2 outlines a new method for extracting suitable data from model observations.

2. Mine Testing

Marker recovery and progressive dilution figures will not be conclusive evidence of the internal flow behaviour. A new method is required. Visual observation of internal flow is obviously precluded.
CHAPTER 10: SIMULATION AND MINE DESIGN

10.1 Application of Flow Models
10.2 Validation of Flow Model
10.3 Performance
10.4 Mine Monitoring and Feedback
10.5 Other Mining Methods
10.6 Conclusions
This chapter endeavours to indicate the place of system models in sublevel caving design, and to indicate the relevance and application of the various flow models that have been reported in this study.

10.1 APPLICATION OF FLOW MODELS

As indicated in Section 2.3, the implementation of sublevel caving as a mining method has the following phases:

(i) selection of design geometry that will maximize recovery and minimize dilution

(ii) blasting technique to achieve fragmentation (fine and even)

(iii) determination of a mining schedule to allocate machines, and sequence ring extraction for a multi-drawpoint situation

(iv) adequate draw control procedures to maintain an even draw

(v) contingency plans in the event of hangup, unexpected dilution etc.

If flow behaviour could be predicted as a function of fragmentation and design geometry, then assuming that ground stability and sound mining practice could be guaranteed, then the whole mining system could be optimized, possibly with the objective of maximizing profit subject to constraints eg mill grade etc.

In reality flow behaviour is very sensitive to local conditions eg rock joint patterns, brow conditions, and to that extent is unpredictable. As found in the digital simulation model of Chatterjee et al (1975) although the unit operations of drilling, blasting and loading at each heading are modelled, the level at which simulation results are validated is much higher, possibly in the order of weeks.
Because this study has focussed on the single drawpoint model, and no extensive simulation results are available, the level at which the new flow model results would be validated can only be estimated.

The flow models primarily focus on the prediction of marker displacements and a prediction of recovery and dilution during the ring extraction history, and in this role are an adjunct to Chatterjee's simulator.

The application of the flow model is directed towards modelling:

(i) partial extraction at adjacent headings
(ii) consequences of mining one area out of its normal sequence
(iii) the "cave" during the progressive extraction of a whole pillar.

In addition the following situations could be modelled:

(i) where low grade capping is employed as a buffer between ore and waste rock, and where the objective is to maintain the buffer zone on a horizontal plane. In this case the focus of attention will be the ore/waste interface, which is located high above the mining front. If the flow in Stage D (Section is essentially vertical then the validation of the model essentially involves determination of the width of the flow regime

(ii) optimize the extraction of ore, where the head grade must be maintained for the complete pillar extraction.

It is also envisaged that the models will be useful for illustrative purposes in instructing machine operators concerning:

(i) loading sequences and digging depth
(ii) over drawing
(iii) sources of dilution
and providing mine planning personnel with a tool to give an intuitive grasp of ore flow behaviour under various conditions.

The next stage in the application of these flow models is the application of the single drawpoint extraction methodology, to the multi-drawpoint situation, and then to a pillar simulation. This work would be subject to adequate solutions for the problems mentioned in the next sections.

10.2 VALIDATION OF FLOW MODEL ASSUMPTIONS

One of the great assets of previous sublevel caving analyses has been that the mathematical models were related to comparatively few parameters (eg eccentricity).

The advantage of the flow models here developed is their ability to more accurately model the flow behaviour known to occur, but at the cost of much more detailed specification of input data and consequently less flexibility in altering the flow behaviour. Admittedly previous mathematical models never modelled flow, but now no simple tabulations and graphs are available to summarize the dynamic ore flow.

The problem of validating each of the computer flow model assumptions remains to correlate:

(i) mine design geometry
(ii) material characterization
(iii) flow model variables (ie probability functions in the stochastic model, zone of loosening shapes and flow path placement in the empirical model)
(iv) loosening mechanism.
It is significant that the development of these models has indicated the necessity for more detailed scale modelling, in particular the modelling of the flow of granular materials around obstructions (as is the case in sublevel caving geometry).

The flow models have been developed with the intention of modelling deterministic ("average") flow behaviour. An extension would be to incorporate stochastic elements into the flow models. A comparison of several pillar simulations could then be used to predict the likely range of results, and hence measure the usefulness of the simulation results for evaluating a particular extraction sequence.

10.3 PERFORMANCE

Each of the computer based flow models is presently restricted by one or more of the following factors:

(i) reliance on mass storage devices for intermediate storage of data during simulations

(ii) large computing time requirements

(iii) definition of a data structure to describe grade and density of the rock mass.

The computer programs are only suitable for batch mode, and interactive modelling at a graphics terminal does not seem realistic at present though re-design and optimization of the computer programs may permit some improvement. In addition faster computational times would be expected if a coarser flow simulation would suffice.
10.4 MINE MONITORING AND FEEDBACK

In the event of a flow model being employed as an operating tool for decision-making at various stages in a pillar simulation, thought must be given to providing regular feedback by which the discrepancy between model predictions, and actual results can be corrected, viz:

(i) a marker placement and recovery program as some check on model predictions
(ii) accurate recording of mining operations in a log
(iii) some elementary scale to indicate mining conditions at the heading, eg fragmentation, brow conditions, moisture, digging conditions
(iv) adequate draw control supervision and a monitor for dilution.

Hopefully, some form of updating and learning capacity could be developed and incorporated into the flow model, to correlate mining conditions, flow behaviour and model assumptions.

10.5 OTHER MINING METHODS

As indicated in Chapters 7, 8, the techniques for numerical modelling developed for sublevel caving operations, offer the potential for application to other caving methods, where the boundary geometry and drawpoints define a volume of granular material free to flow under gravity.

10.6 CONCLUSIONS

The general thrust of this thesis is that computer based models, given a realistic set of assumptions, provide a meaningful tool for mine planners in their task.
It is to be hoped that continuing research into gravity flow of particulate materials will provide opportunity for further development, and validation of flow models to yield a design and operating tool with a convincing predictive capacity. Meanwhile it is true that in most engineering applications, complex materials can be represented by simple laws eg Coulomb's Law for soils, where the constants must be evaluated for each material, and cannot be determined from the physical structure of the materials; likewise, though granular material flow models remain empirical in nature, their usefulness is limited only by the skill with which they are applied.
CHAPTER 11 : CONCLUSIONS

This study set out to examine and extend previous mathematical models that had been applied to sublevel caving with a view to modelling the dynamical flow behaviour of broken ore.

A literature search revealed no working models to describe the multi-dimensional flow of broken ore using continuum material models derived from constitutive relations for granular material.

A numerical model for discontinua, developed by P. Cundall, although in an early stage of development, offers the possibility of modelling the interaction of individual particles. The chief limitation is problem size, and to date only two-dimensional problems have been attempted.

Two kinematic models are developed, and results for single drawpoint simulation are presented. Although lacking a stress description, they offer potential as simulation tools in mine design. In the stochastic model, the flow of broken ore into the drawpoint is modelled by the counterflow diffusion of voids upwards. Three methods of solution are outlined:

(i) closed form solution
(ii) finite difference schemes
(iii) Monte Carlo Simulation.

The last method has been implemented, while the finite difference approach to moving boundary problems should be investigated further.

Based on modelling studies, and past static mathematical models developed in Sweden, and Australia, a new dynamic empirical flow model is presented.

Each of the computer models has the capacity to predict:

(i) recovery and dilution
(ii) displacement of the broken rock mass during drawdown
(iii) displacement of markers placed in the rock mass.
By modelling rock flow, these computer models complement the mine production simulation model developed by P.K. Chatterjee for the sublevel caving method.

The application of these computer models to mine design problems, and the restrictions, present and potential, in modelling with the programs are discussed. Program efficiency remains low, and the definition of an optimal data-base to characterize the problem domain requires development before extensive simulations can be undertaken.

The accuracy with which flow behaviour can be modelled is assessed. Because of the difficulties in verifying the components and assumptions of each model individually, either in full scale or model tests, only limited confidence can be placed in the computer model results before extensive on-site application of the models. Detailed monitoring of mining operations and feedback to establish model parameters in a specific mining environment will be needed before evaluating model performance.

From the viewpoint of rock mechanics, the biggest deficiency of the two computer models is that stresses that develop in the broken rock mass, and their influence on draw behaviour cannot be modelled. The flow mechanism must remain essentially an intuitive development from modelling studies because established scaling laws are not available.

It is recommended that the work be developed in the following sequence:

1. **Cundall Method**
   Although only a two dimensional analysis appears feasible, an investigation of flow mechanisms, and stress distributions during flow, and the formulation of a constitutive equation for granular material could be pursued.

2. **Stochastic Method**
   A more efficient solution than Monte Carlo simulation may arise in the context of numerical methods for moving boundary problems. Either way the results from (1) could be used to generate realistic flow probabilities. From the stochastic results the
development of zones of loosening and flow path location on the zones could be investigated.

3. **Empirical Method**

Once the assumptions made in the empirical method are confirmed, this method could be used to predict flow of isolated markers or regions without explicitly calculating displacement of every point in the broken rock mass.

Depending on data-base development, and program optimization either of the three methods might assume superiority as the mine simulation tool.

Although this study proceeded in the context of simulating sublevel caving operations, these models would be equally applicable to other mining methods where ore flows under gravity.
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APPENDIX 1: A PROJECTED MODELLING PROGRAMME

Resulting from the assumptions developed in the computer models in this thesis, an extensive programme to analyze the significance of various factors is needed.

Apart from the development of a numerical model which will probably have limitations for the size of the problem that can be modelled, Chapter 9 indicated that, at present the notion of marker recovery programmes to confirm or validate the assumptions made was deficient. Apart from employing costly internal measuring methods, the understanding of internal flow behaviour will essentially remain an intuitive process.

However it is recommended that a new technique be applied to the analysis of modelling data. This is outlined in Appendix 2.

The modelling frame to investigate sublevel caving should be a multiple ring, multiple heading and three dimensional. Facilities to load the frame with crushed ore and waste rock, such that the ore is compacted to simulate the blasted ring (with templates separating successive rings for modelling extraction) should be developed.

The basic goals of the programme would be to investigate:

(i) The proportions of the flow regime at each of the 4 stages defined in Chapter 8 and horizontal section shapes, especially noting the influence of the solid ring.

(ii) The mechanism of breakthrough at the side, top and back of the ring.

(iii) Locating of positions of the zones of loosening and particle trajectories by the method outlined in Appendix 2 to establish the validity of approximating unsteady flow by the steady state flow grid.

(iv) The effect of mixing, percolation and segregation in flow.
(v) The effect of material properties (size grading, relative sizes and size fractions, and particle shape) on flow mechanisms.

(vi) Influence of roughness of internal surfaces of the model on flow.

(vii) Improvement of simulation of fragmentation and swelling of blasted ore.

Although data would mainly relate to the flow behaviour on front and side sections, it would be a step towards defining that which has been intuitive, and investigating not only the character of the ore that is recovered, but the flow behaviour of the material not extracted.

SIMULATION OF LOOSENING AFTER BLASTING

Because of the significance of simulating loosening mechanism after blasting an initial attempt was made to model this situation.

Hexagonal shaped flat discs were prepared and placed in closely packed formation in a two dimensional model frame. The attempt proved abortive due to

(i) arching due to geometrical packing

(ii) arching across opening because the ratio of hexagon diameter to opening width was too small.

It was hoped that the model would be effective in simulating loosening because randomly packed hexagons occupy approximate 25% greater area than the close packet counterpart. This is close to the estimate of Janelid, for swell after blasting and loosening.

However, with the potential development of the Cundall discrete rock block program which would provide a numerical model for this situation of hexagonal discs, further attempts were abandoned.
APPENDIX 2: STEREO PHOTOGRAMMETRY

The purpose of this note is to outline the basic details of a comparatively recent photographic technique that can be employed in model analysis to make a detailed investigation of flow behaviour.

Stereo photogrammetry is an accepted discipline within surveying for the production of plans and maps based on photographs taken from ground or air positions. Two photographs of an object taken from different vantage points, are placed in a stereo-comparator, and a stereoscopic relief is obtained which can be scaled and plotted. Applications in rock mechanics include stability of rock slopes and measurement of texture profiles on rock samples (Wickens & Barton, 1971).

The necessity to measure displacements and strain fields in the kinematics of soil materials has led to the adaption of stereo-photogrammetric techniques. If the model walls are transparent, and the granular material displaces in time, photographs can be taken at frequent time intervals, from one camera position. When successive photographic plates are viewed in a stereo-comparator, a stereo-image is created, where the granular material surface appears as elevations and depressions which can be scaled and contoured. The contours describe displacements in the direction of a line joining the centres of the photographic plates under view. Rotating the plates 90° yields displacements in the perpendicular direction. Further details of the method are available (Butterfield and Andrewes, 1970).

An initial examination, of this method, using equipment made available by the Surveying Department, University of Melbourne indicated the potential of this method.

Two methods are available for processing a sequence of photographic plates:

(1) Topocart which produces a stereoscopic image which can be manually contoured with output displayed on a plotting machine
(ii) Stecometer which again produces the stereoscopic image but only the placement of selected points are measured, and recorded using an on-line digitizer.

Method (i) appears particularly useful in that the displacement vectors of any point can be evaluated by interpolation of the two components of the contoured displacement field through successive plates. These vectors together model the particle trajectory. The zone of loosening is immediately defined by superimposing the perpendicular displacement fields so all points without a component of displacement lie outside the zone of loosening.

Although insufficient time was available to implement the method in a modelling programme, the trial analysis showed that the method was applicable to photographic results obtained from research by Panczakiewicz (1977).

The method has the advantages:

(i) no markers are necessary to indicate displacement, and hence no foreign material is present in the model.

(ii) all the requisite details of flow can be reconstructed from the photographic sequence, including
   a) marker trajectories
   b) zones of loosening
   c) flow behaviour within the loosened zone to check on the accuracy of assumptions made in the zone of loosening model to the effect that flow paths and zones of loosening are independent of extraction history.
   d) local density, and hence loosening mechanism.

(iii) only as much analysis as is desirable need be carried out, and as shown by Panczakiewicz, photographs are a necessary record of modelling.
However the method also needs to be used with caution:

(i) the granular particles must be discernible yet not so large as to blur the stereo image

(ii) the stereo image is improved when grain movement is minimized

(iii) high quality film negative is required that won't shrink in processing, and fine grain film paper is required to reproduce the texture of the granular media.

(iv) a considerable amount of data processing would be required for which a computer program should be developed.

The method, in common with most other modelling techniques surveyed, is unable to detect and predict internal flow behaviour, however further research is recommended, to obtain the data that is available, using transparent model frame components.
APPENDIX 3: SOLUTION OF SYSTEMS OF NON-LINEAR EQUATIONS, AND CALCULATION OF A HEXAHEDRAL VOLUME

Integral to the method of calculating displacements in both blasting and zone of loosening based flow simulations is the calculation of the local coordinates of a point relative to a defined volume where the cartesian coordinates of the point are defined.

As outlined in Chapter 8, the local coordinates \((r,s,t)\) are obtained by a mapping process that relates the cartesian and local coordinates of the point by a system of non-linear equations, which can be solved for \((r,s,t)\).

The finite element literature outlines a variety of mapping functions and two of these were tested: 32 point and 8 point hexahedral brick elements.

An interactive computer program to run on a remote terminal was developed to generate a test set of elements, calculate volumes and then solve for local coordinates \((r,s,t)\) for a specified marker location. Figure A3.1 illustrates the program flowchart. To perform the numerical integration standard Gaussian quadrature was used.

To perform the solution of the system of non-linear equations, three methods were investigated:

(i) Newton Raphson Iteration \((\text{Carnahan et al 1969})\)
(ii) Brown's Method \((\text{Brown, 1973})\)
(iii) Powell's Method \((\text{Powell, 1970})\)

In each case, the system of non-linear equations was set up using the shape functions elaborated in Chapter 8. In addition, for the case of the 8 point hexahedra the equations set out in Appendix 4 were solved using Newton Raphson iteration. Results for one data set used are given in Table A3.1. The trends and relative magnitudes of the solution times were confirmed in all data sets tested.
The results indicate that:

(i) 8 point hexahedra have considerable computational advantage over 32 point hexahedra

(ii) Brown's method is the fastest solution method

(iii) The use of shape functions is a slower method for defining the coefficients and derivatives of the system of equations if the equivalent algebraic equations can be derived as has been done in Appendix 4.

In conclusion, provided that the 8 point hexahedra yield an adequate description of the flow grid, they are to be preferred for flow simulations, in the light of computational experience which indicates that to locate the hexahedra within the flow grid that contains a specified marker, the local coordinates (r,s,t) in several adjacent hexahedra may need to be tested.
Figure A3.1  Flowchart for hexahedra test program
<table>
<thead>
<tr>
<th>Number of points in each coordinate direction</th>
<th>8 point hexahedra</th>
<th>32 point hexahedra</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Volume</td>
<td>Solution Time</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(sec)</td>
</tr>
<tr>
<td>2</td>
<td>159.160</td>
<td>.094</td>
</tr>
<tr>
<td>3</td>
<td>156.544</td>
<td>.323</td>
</tr>
<tr>
<td>4</td>
<td>159.161</td>
<td>.782</td>
</tr>
</tbody>
</table>

Table A3.1 Results for Gaussian Quadrature

<table>
<thead>
<tr>
<th>Solution Method</th>
<th>8 point hexahedra</th>
<th>32 point hexahedra</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shape function/Newton Raphson</td>
<td>.050</td>
<td>.317</td>
</tr>
<tr>
<td>Shape function/Brown</td>
<td>.043</td>
<td>.254</td>
</tr>
<tr>
<td>Shape function/Powell</td>
<td>.168</td>
<td>.338</td>
</tr>
<tr>
<td>Blast simulation equations/ Newton Raphson</td>
<td>.017</td>
<td>-</td>
</tr>
</tbody>
</table>

Table A3.2 Results for solution of system of non-linear equations
<table>
<thead>
<tr>
<th>ORDER</th>
<th>ORDINATE</th>
<th>WEIGHT</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>-0.57735</td>
<td>02691</td>
</tr>
<tr>
<td></td>
<td>+0.57735</td>
<td>02691</td>
</tr>
<tr>
<td>3</td>
<td>-0.77459</td>
<td>66692</td>
</tr>
<tr>
<td></td>
<td>+0.00000</td>
<td>00000</td>
</tr>
<tr>
<td></td>
<td>+0.77459</td>
<td>66692</td>
</tr>
<tr>
<td>4</td>
<td>-0.86113</td>
<td>63115</td>
</tr>
<tr>
<td></td>
<td>-0.33998</td>
<td>10435</td>
</tr>
<tr>
<td></td>
<td>+0.33998</td>
<td>10435</td>
</tr>
<tr>
<td></td>
<td>+0.86113</td>
<td>63115</td>
</tr>
</tbody>
</table>

Table A3.3  Gaussian integration weighting functions and sampling points
APPENDIX 4 : BLASTING MODEL RESULTS

A4.1 Derivation of Zone II Expansion Factor $K$

A4.2 Derivation of Volume of Polyhedron

A4.3 Derivation of Height of Mass in Polyhedron

A4.4 Derivation of Local Coordinates of Point in Polyhedra

A4.5 Model I Results

A4.6 Model II Results
A4.1 Derivation of Zone II Expansion Factor K

Substitution of the expressions for D1', D2', D3', D4', D5' into the equation

\[
\text{VOLNEW} = (D3' D1' D4' + \frac{D3'^2}{2}[D21' D4' + D54' D1']) + \frac{D3'^2}{3}[D21' D54'])
\]

= OLDMAS/DENCS

yields the equation:

\[
K^4\left(\frac{D3^3}{3}[L + M][D5.KLS - D4.KLS]\right)
\]

+ \[
K^3\left(\frac{D3^3}{3}[D2-L-D1-M][D5.KLS-D4.KLS] + \frac{D3^2}{2}[LD4 + 2M.D4-M.D5.KLS]\right)
\]

+ \[
K^2\left(\frac{D3^2}{2}D5.KLS[D1+M] + \frac{D3^2}{2}D4.KLS[D2-L-2D1-2M] - D3D4.KLS.M\right)
\]

+ \[
K (D3D4.KLS [D1 + M])
\]

- \[
\frac{OLDMAS}{DENCS} = 0
\]

A4.2 Derivation of Mass below Point in Polyhedron

Referring to Figure 4.1, the polyhedron can be divided into infinitesimal slices parallel to the upper and lower surfaces. The volume can be integrated between \( t = 0 \) and \( t = t' \):

\[
V_{t'} = \int_0^{t'} (D1 + t[D2 - D1]) (D4 + t[D5 - D4]) \, dt
\]

= \[
D1D4t' + \frac{t'^2}{2} (D21D4 + D54D1) + \frac{t'^3}{3} (D21D54)
\]

The volume of the polyhedron is then

\[
V = D1D4D3 + \frac{D3^2}{2} (D21D4 + D54D1) + \frac{D3^3}{3} (D21D54)
\]
Figure A4.1 Volume of polyhedron

Figure A4.2 Mass of polyhedron with variable density function
A4.3 Derivation of Height of Mass in Polyhedron

(a) Calculation of mass in polyhedron assuming variable density function (Figure A4.2):

\[ H_2 = H_2\text{ABS} - H\text{REF} \]
\[ H_1 = H_1\text{ABS} - H\text{REF} \]

Changing density function coefficients to the polyhedron reference frame:

\[ \rho = AH^2 + BH + C \]
\[ k = H\text{REF} - \text{HDREF} \quad h = h + k \]

\[ \therefore \rho = A(h + k)^2 + B(h + k) + C \]
\[ = h^2(A) + h(2Ak + B) + (Ak^2 + Bk + C) \]
\[ = h^2(AA) + h(BB) + (CC) \]

\[ \therefore \text{Mass} = \int_{H_1}^{H_2} \left( D_1 + \frac{h}{D_3} \right) \left( D_2 + \frac{h}{D_3} \right) \left( D_3 + \frac{5D_4}{D_3} \right) (AAh^2 + BBh + CC) \, dh \]

\[ = \int_{H_1}^{H_2} h^4 \left( AA \frac{D_21 D_54}{D_3^2} \right) \]
\[ + h^3 \left( BB \frac{D_21 D_54}{D_3^2} + AA \left( \frac{D_1 D_54}{D_3} + \frac{D_21 D_4}{D_3} \right) \right) \]
\[ + h^2 \left( CC \frac{D_21 D_54}{D_3^2} + BB \left( \frac{D_1 D_54}{D_3} + \frac{D_21 D_4}{D_3} \right) + AA[D1 D4] \right) \]
\[ + h \left( CC \left( \frac{D_1 D_54}{D_3} + \frac{D_21 D_4}{D_3} + BB[D1 D4] \right) + CC[D1 D4] \right) \]
\[ + \left( + CC[D1 D4] \right) \, dh \]

\[ = \int_{H_1}^{H_2} \left( C_1 h^4 + C_2 h^3 + C_3 h^4 + C_4 h + C_5 \right) \, dh \]

\[ = \frac{1}{5}(H_2-H_1)^5 C_1 + \frac{1}{4}(H_2-H_1)^4 C_2 + \frac{1}{3}(H_2-H_1)^3 C_3 + \frac{1}{2}(H_2-H_1)^2 C_4 + (H_2-H_1) C_5 \]
Figure A4.3 Local coordinates of a point in polyhedron
(b) To calculate the height for a mass above H1, the previous equation must be expanded and terms collected yielding:

\[ H^2 \left( \frac{C1}{5} \right) + H^2 \left( -\frac{5H1C1}{5} + \frac{C2}{4} \right) + H^2 \left( +\frac{10H1^2C1}{5} - \frac{4H1C2}{4} + \frac{C3}{3} \right) + H^2 \left( -\frac{10H1^3C1}{5} + \frac{6H1^2C2}{4} - \frac{3H1C3}{3} + \frac{C4}{2} \right) + H \left( \frac{5H1^4C1}{5} - \frac{4H1^3C2}{4} + \frac{3H1^2C3}{3} - \frac{2H1C4}{2} + C5 \right) + (-\frac{H1^5C1}{5} - \frac{H1^4C2}{4} - \frac{H1^3C3}{3} + \frac{H1^2C4}{2} - H1 C5) \]

- Mass = 0

This polynomial of degree 5 can be solved by standard methods to yield H2.

A4.4 Derivation of Local Coordinates (u,v,w) of a point in a polyhedron.

Referring to Figure A4.3 the compaction zone defined by planes i,j is mapped into a unit cube by a linear transformation.

\[
\begin{align*}
B_X &= P_{11X} + u(P_{12X} - P_{11X}) \\
B_Y &= P_{14X} + u(P_{13X} - P_{14X}) \\
B_Z &= P_{j1X} + u(P_{j2X} - P_{j1X}) \\
B_Y &= P_{j4X} + u(P_{j3X} - P_{j4X}) \\
B &= B_X + v(BR_X - BL_X) \\
C &= CL_X + v(CR_X - CL_X) \\
X &= B_X + W(C_X - B_X)
\end{align*}
\]
Sustituting and collecting terms:

\[
UV(P_{j3X} - P_{j4X} - P_{j2X} + P_{jiX} - P_{i3X} + P_{i4X} + P_{i2X} - P_{i1X})
+ UV(P_{j3X} - P_{i4X} - P_{i2X} + P_{i1X})
+ VW(P_{j4X} - P_{j1X} - P_{i4X} + P_{i1X})
+ WU(P_{j2X} - P_{j1X} - P_{i2X} + P_{i1X})
+ U(P_{i2X} - P_{i1X})
+ V(P_{i4X} - P_{i1X})
+ W(P_{j1X} - P_{i1X})
+ P_{i1X} - X = 0
\]

Similar equations in \((U,V,W)\) are obtained by substituting \(y\) and \(z\) for \(x\), yielding three non-linear simultaneous equations in three unknowns \((U,V,W)\) that can be solved by methods outlined in Appendix 3.

A4.5 Model I Results

In Model I, ore expands but no slump is simulated. The geometry of blasted and compacted zones is specified by the expansion factors, and the consequent density values are evaluated.

The sequence of plots, Figures A4.5 : (i) - (iv) depict:

(i) the density profile (within the blasted ore, parallel to the ring face) as a function of height

(ii) the displacement of selected points in the rock mass

(iii) the displacement of selected (initially) horizontal lines in the rock mass.
A4.6 Model II Results

In Model II, ore slumps into the heading. The densities of loose, expanded and blasted (but not loosened) ore are specified with the height of loose ore. Expansion factors KLS, KBL, KLL determine the region affected by expansion and compaction processes.

The plot sequence, Figures A4.6: (i) - (iv) parallel those of the previous section.
Figure A4.5 (iii)
Figure A4.5 (iv)
VERTICAL DENSITY PROFILE

Figure A4.6 (1)
APPENDIX 5: STOCHASTIC FLOW MODEL RESULTS

A5.1 Two Dimensional Front View
A5.2 Two dimensional Side View
A5.3 Three dimensional
NOTES ON STOCHASTIC FLOW MODEL RESULTS

Plot sequences are displayed for three stochastic flow simulations. For each flow simulation, the following data is presented:

(i) the initial rock type location.
(ii) the location of representative markers (denoted by "." on the plot).
(iii) the flow regime dimensions (denoted by "_" on the plot).
(iv) the flow sequence for the slump of ore into the heading after blasting, and the subsequent mining of ore in the heading. For each bucket (i.e., NBUCK), the flow is presented for several cycle numbers, with last cycle number for each NBUCK being the equilibrium state where no further flow was detected.
(v) for each cycle number in the previous sequence, the (initially) horizontal lines of markers are plotted.
(vi) the trajectories of individual markers for the duration of the simulation.

The element rock type is coded by the following symbols:

X solid ore
+ blasted ore
. loose ore
= waste — undisplaced
— — displaced
no contents
% mixed material — mostly ore
# — mostly waste
\ — mostly void.
A5.1 Two Dimensional Front View — Figures A5.1: (i) — (xiv)

Each element is denoted by a symbol (where relevant) and numbered within the problem domain. Elements dimensions are 1 ft x 1 ft, with the simulation being in only two dimensions.

A5.2 Two Dimensional Side View — Figures A5.2: (i) — (xiv)

These results parallel A5.1 for a two dimensional simulation for a section along the heading centreline.

A5.3 Three Dimensional — Figures A5.3: (iv) — (xv)

Element dimensions of 2 ft were selected, and the probability data and flow regime dimensions for A5.1, A5.2 employed with extra data supplemented for the third dimension. The resulting flow is more erratic, as would be expected.
Figure A5.1 (11)
NOTE: marker positions in heading not plotted.
NOTE: marker positions in heading not plotted.
2-D SIDE SECTION  SCALE: 0.06  BUCKET NO. 1
MASS EXTRACTED  96.0  CYCLE NO. 82

2-D SIDE SECTION  SCALE: 0.06  BUCKET NO. 2
MASS EXTRACTED  192.0  CYCLE NO. 48
Figure A5.3 (ii)
APPENDIX 6: EMPIRICAL FLOW MODEL RESULTS

A6.1 Zone of loosening shape definition for front and side section

A6.2 Curve of intersection for front and side section with grid points connected by straight lines for the linear hexahedra grid

A6.3 Cubic hexahedra grid sections

A6.4 Linear hexahedra grid sections

A6.5 Front view results

A6.6 Side view results

A6.7 Three dimensional perspective view results
NOTES ON EMPIRICAL FLOW MODEL RESULTS

A6.1 Zone of Loosening Shape Definition for Front and Side Section

For each of the 10 zones of loosening, the input data points were interpolated as a function of height (in the same manner as the flow grid definition algorithm) and plotted, Figures A6.1: (i) - (x). The first, the base zone of loosening, is flat and is located on the heading floor. Figures 8.5, 8.11 depict the essential features of the zone data included here. The shape data employed here was deduced from the examples in Chapter 8 together with the assistance of mine-site planning engineers.

A6.2 Curve of Intersection for Front and Side Section, with Grid Points Connected by Straight Lines for the Linear Hexahedra Grid

Figures A6.2: (i) - (x) depict the curves of intersection for each zone of loosening. In this model each 32-node cubic hexahedron was approximated by one 8-node linear hexahedron to form the flow grid. The curve of intersection is plotted here with flow grid points on the curve of intersection connected by straight lines.

A6.3 Cubic Hexahedra Grid Sections

In Figure A6.3 the curved edges of each cubic hexahedra on the heading centreline, and on the ring face are plotted. The location of corners for each hexahedron on a zone of loosening are dictated by the spacing of the input data for the zone.

A6.4 Linear Hexahedra Grid Sections

Figures A6.4 parallels Figure A6.3, except that the edges of the linear hexahedra are plotted.
A6.5 Front View Results

Four horizontal lines (of 20 markers) parallel to the ring face were defined and Figures A6.5 : (i) - (v) depict the flow sequence for the specified extraction increments. The path for individual markers would correspond to an interpolated flow path through the original marker position.

A6.6 Side View Results

The flow sequence for markers parallel to the heading is depicted in Figures A6.6 : (i) - (v).

A6.7 Three Dimensional Perspective View Results

The last sequence, Figures A6.7 : (i) - (ii), indicate the slump of (initially) horizontal planes of markers towards the heading. A hidden line plotting algorithm was used in this instance.
ZONE OF LOOSENING

Figure A6.1 (i)
ZONE OF LOOSENING

Figure A6.1 (ii)
Figure A6.1 (iv)
ZONE OF LOOSENING

Figure A6.1 (vi)
ZONE OF LOOSENING

Figure A6.1 (vii)
ZONE OF LOOSENING 10

Figure A6.1 (x)
Figure A6.2 (iii)
Figure A6.2 (v)
Figure A6.2 (viii)
Figure A6.2 (x)
Figure A6.4
Figure A6.5 (iii)
markers initially not detected.

Figure A6.6 (i)
Figure A6.6 (iii)
Figure 46.6 (v)

MARKER PLANE PLOT... VOLUME EXTRACTED=  4500.00

MARKER PLANE PLOT... VOLUME EXTRACTED=  5000.00
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