Sequential Phased Estimation of Ionospheric Path Delays for Improved Ambiguity Resolution over Long GPS Baselines

by

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ABSTRACT

Satellite-based navigation systems make it possible to determine the relative positions of points on the earth with centimetre or even millimetre level accuracy over baselines of up to several thousand kilometres. The highest possible accuracy can only be achieved if the carrier phase integer ambiguities can be resolved. In order to resolve the L1 and L2 integer ambiguities over long GPS baselines, the double difference residual ionospheric errors must be estimated for every satellite, every epoch. The resulting number of parameters is usually too large for estimation using ordinary least squares to be practical due to the time or computing resources needed for the processing. The technique currently used to efficiently estimate the parameters is known as pre-elimination. Pre-elimination divides the unknowns into parameters of interest (the coordinates and ambiguities) and nuisance parameters (the ionospheric path delays). The nuisance parameters are treated as stochastic variables and modelled as process noise, avoiding the need for them to be explicitly estimated. Whilst this approach is highly efficient, it makes assumptions about the stochastic behaviour of the residual ionospheric error that are not necessarily valid. The effectiveness of pre-elimination can be increased through the use of a deterministic model of the ionosphere. It is the hypothesis of this research that the ionospheric error can be more effectively estimated than is possible with pre-elimination, leading to more reliable ambiguity resolution for long baseline precise positioning.

This research proposes a new solution based on sequential phased adjustment, a technique for rigorously estimating parameters in batches. The main advantage of this approach, over pre-elimination, is that it is not necessary to assume that the expected value of the ionospheric path delays is zero. Also, since the ionospheric path delays are explicitly estimated, the residuals can be calculated enabling further analysis of the results. The disadvantage of phased adjustment is that it is computationally less efficient than pre-elimination (though it is efficient enough to be practical for this application). In order to compare and contrast the sequential phased adjustment approach with pre-elimination, a set of eight medium to long (636 to 4,080km) baselines were processed and evaluated in terms of the accuracy of the float solution, the number of ambiguities that could be resolved and the reliability of the ambiguity
fixing. Data from the Australian Regional GPS Network (ARGN), collected during 2002, a period of high ionospheric activity, were used for the testing. The Bernese GPS Software v4.2 was used for the pre-processing and the ambiguity float solution estimates based on pre-elimination and the subsequent Quasi Ionosphere Free (QIF) ambiguity resolution and fixed solutions. An external test application, which interfaces to the Bernese GPS Software, was developed and used for the ambiguity float solutions using sequential phased adjustment and the associated ambiguity resolution. The resulting coordinate and ambiguity estimates show that sequential phased adjustment is a more effective technique for estimating ionospheric path delays for long GPS baselines. Using sequential phased adjustment, the float and fixed solutions are more accurate and more ambiguities can be resolved.
DECLARATION

This is to certify that

(i) the thesis comprises only my original work except where indicated in the preface,
(ii) due acknowledgement has been made in the text to all other material used,
(iii) the thesis is less than 100,000 words in length, exclusive of tables, maps, bibliographies and appendices.

Neil Brown
November 2006
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LIST OF SYMBOLS

Quantities:

- $x$ The scalar quantity $x$
- $\hat{x}$ The real valued estimate of $x$
- $\bar{x}$ The integer valued estimate of $x$
- $\bar{x}$ The vector $\bar{x}$
- $A$ The matrix $A$

GPS Symbols:

- $\varphi$ Raw phase measurement (in cycles)
- $\Phi$ Code pseudorange or phase range (in metres), $\Phi = \lambda \varphi$
- $\Delta$ Between receiver single difference operator
- $\nabla$ Between satellite single difference operator
- $\nabla \Delta$ Double difference operator
- $\lambda$ Wavelength
- $c$ Speed of light in a vacuum
- $\alpha$ Satellite elevation angle
- $z$ Satellite zenith angle

Mathematical Symbols:

- $A$ (First) Design matrix
- $C$ Variance-covariance (or simply covariance) matrix
- $I$ Identity matrix
- $0$ Zero matrix
- $N$ Normal equations matrix
- $b$ Right hand side of the normal equations
- $Q$ Cofactor matrix
- $v$ Vector of residuals
- $\Sigma$ Summation
- $\text{diag()}$ Diagonal elements of a matrix
- $D\{\}$ Digital filter
- $\forall j, n$ For any value of $j$ and $n$
- $\hat{x}$ Estimate of $x$
- $\mathbb{R}^n$ The space of all real numbers of dimension $n$
- $\mathbb{Z}^n$ The space of all integers of dimension $n$
Statistical Symbols:

- $E[]$ Expected value (first moment)
- $D[]$ Dispersion or expected squared value (second moment)
- $Pr[]$ Probability
- $\sigma$ Standard deviation or covariance
- $\varepsilon$ Measurement noise (random error)
- {...} Sequence of variables
**LIST OF ABBREVIATIONS**

<table>
<thead>
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<th>Description</th>
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<tbody>
<tr>
<td>AEST</td>
<td>Australian Eastern Standard time</td>
</tr>
<tr>
<td>AF</td>
<td>Adaptive Filter</td>
</tr>
<tr>
<td>AltBOC</td>
<td>Alternate Binary Offset Carrier</td>
</tr>
<tr>
<td>APC</td>
<td>Antenna Phase Centre</td>
</tr>
<tr>
<td>ARGN</td>
<td>Australian Regional GPS Network</td>
</tr>
<tr>
<td>ARP</td>
<td>Antenna Reference Point</td>
</tr>
<tr>
<td>AS</td>
<td>Anti-spoofing</td>
</tr>
<tr>
<td>BOC</td>
<td>Binary offset carrier</td>
</tr>
<tr>
<td>BPSK</td>
<td>Binary phase-shift keying</td>
</tr>
<tr>
<td>CDMA</td>
<td>Code Division Multiple Access</td>
</tr>
<tr>
<td>CNR</td>
<td>Carrier to noise ratio</td>
</tr>
<tr>
<td>CODE</td>
<td>Centre for Orbit Determination Europe</td>
</tr>
<tr>
<td>CS</td>
<td>(Galileo) Commercial Service</td>
</tr>
<tr>
<td>DCB</td>
<td>Differential Code Bias</td>
</tr>
<tr>
<td>DD</td>
<td>Double-difference</td>
</tr>
<tr>
<td>DGPS</td>
<td>Code based differential GPS positioning</td>
</tr>
<tr>
<td>DIA</td>
<td>Detection, Identification and Adaptation</td>
</tr>
<tr>
<td>DLL</td>
<td>Delay Lock Loop</td>
</tr>
<tr>
<td>DOP</td>
<td>Dilution Of Precision</td>
</tr>
<tr>
<td>EGNOS</td>
<td>European Geostationary Overlay Service</td>
</tr>
<tr>
<td>EPE</td>
<td>Estimated Position Error</td>
</tr>
<tr>
<td>FDMA</td>
<td>Frequency Division Multiple Access</td>
</tr>
<tr>
<td>FIR</td>
<td>Finite Impulse Response filter</td>
</tr>
<tr>
<td>FOC</td>
<td>Full Operational Capability</td>
</tr>
<tr>
<td>GDOP</td>
<td>Geometric Dilution Of Precision</td>
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<tr>
<td>GF</td>
<td>Geometry-free linear combination</td>
</tr>
<tr>
<td>GIM</td>
<td>Global Ionosphere Maps</td>
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<tr>
<td>GLONASS</td>
<td>Russia’s Global Navigation Satellite System</td>
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<tr>
<td>GNSS</td>
<td>Global Navigation Satellite System</td>
</tr>
<tr>
<td>GPS</td>
<td>United States Global Positioning System</td>
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<tr>
<td>HDOP</td>
<td>Horizontal Dilution Of Precision</td>
</tr>
<tr>
<td>IERS</td>
<td>International Earth Rotation Service</td>
</tr>
<tr>
<td>IF</td>
<td>Ionosphere-free linear combination</td>
</tr>
<tr>
<td>IGS</td>
<td>International GPS Service</td>
</tr>
<tr>
<td>IIR</td>
<td>Infinite Impulse Response filter</td>
</tr>
<tr>
<td>IONEX</td>
<td>Ionosphere Exchange Format</td>
</tr>
<tr>
<td>ITRF</td>
<td>International Terrestrial Reference Frame</td>
</tr>
<tr>
<td>JD</td>
<td>Julian Day (day of year)</td>
</tr>
<tr>
<td>LHS</td>
<td>Left Hand Side</td>
</tr>
<tr>
<td>LSE</td>
<td>Least Squares Estimation</td>
</tr>
<tr>
<td>Abbreviation</td>
<td>Description</td>
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<td>--------------</td>
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<tr>
<td>MA</td>
<td>Moving Average</td>
</tr>
<tr>
<td>MCS</td>
<td>Master Control Station</td>
</tr>
<tr>
<td>MINQUE</td>
<td>Minimum Norm Quadratic Unbiased Estimation</td>
</tr>
<tr>
<td>MW</td>
<td>Melbourne-Wübben linear combination</td>
</tr>
<tr>
<td>NDGPS</td>
<td>Network DGPS</td>
</tr>
<tr>
<td>NGS</td>
<td>United States National Geological Survey</td>
</tr>
<tr>
<td>NIMA</td>
<td>United States National Imagery and Mapping Agency</td>
</tr>
<tr>
<td>NMF</td>
<td>Niell Mapping Function for the neutral atmosphere from Niell (1996)</td>
</tr>
<tr>
<td>NRTK</td>
<td>Network RTK</td>
</tr>
<tr>
<td>OS</td>
<td>(Galileo) Open Service</td>
</tr>
<tr>
<td>OTF</td>
<td>On-the-fly (ambiguity resolution)</td>
</tr>
<tr>
<td>PCV</td>
<td>Antenna Phase Centre Variations</td>
</tr>
<tr>
<td>PDOP</td>
<td>Position Dilution Of Precision</td>
</tr>
<tr>
<td>PLL</td>
<td>Phase Lock Loop</td>
</tr>
<tr>
<td>PPP</td>
<td>Precise Point Positioning</td>
</tr>
<tr>
<td>PPS</td>
<td>Precise Positioning Service</td>
</tr>
<tr>
<td>PRN</td>
<td>Pseudorandom noise code</td>
</tr>
<tr>
<td>PRS</td>
<td>(Galileo) Public Regulated Service</td>
</tr>
<tr>
<td>QIF</td>
<td>Quasi-ionosphere-free ambiguity resolution strategy</td>
</tr>
<tr>
<td>QZSS</td>
<td>Japanese Quasi-Zenith Satellite System</td>
</tr>
<tr>
<td>RF</td>
<td>Radio frequency</td>
</tr>
<tr>
<td>RHS</td>
<td>Right Hand Side</td>
</tr>
<tr>
<td>RINEX</td>
<td>Receiver Independent Exchange format</td>
</tr>
<tr>
<td>RMS</td>
<td>Root Mean Square</td>
</tr>
<tr>
<td>RTK</td>
<td>Real Time Kinematic</td>
</tr>
<tr>
<td>SA</td>
<td>Selective Availability</td>
</tr>
<tr>
<td>SD</td>
<td>Single Difference</td>
</tr>
<tr>
<td>SLM</td>
<td>Single layer (ionosphere) model</td>
</tr>
<tr>
<td>SoL</td>
<td>(Galileo) Safety of Life Service</td>
</tr>
<tr>
<td>SNR</td>
<td>Signal to noise ratio</td>
</tr>
<tr>
<td>SPS</td>
<td>Standard Positioning Service</td>
</tr>
<tr>
<td>SV</td>
<td>Space Vehicle (satellite)</td>
</tr>
<tr>
<td>TEC</td>
<td>Total Electron Content</td>
</tr>
<tr>
<td>TECU</td>
<td>Total Electron Content in Units of $10^{16} \text{ m}^{-2}$</td>
</tr>
<tr>
<td>UERE</td>
<td>User Equivalent Range Error</td>
</tr>
<tr>
<td>USDoD</td>
<td>United States Department of Defence</td>
</tr>
<tr>
<td>USNO</td>
<td>United States Naval Observatory</td>
</tr>
<tr>
<td>UTC</td>
<td>Universal Time Coordinated</td>
</tr>
<tr>
<td>VDOP</td>
<td>Vertical Dilution Of Precision</td>
</tr>
<tr>
<td>VTECU</td>
<td>Vertical TECU</td>
</tr>
<tr>
<td>WAAS</td>
<td>Wide Area Augmentation System</td>
</tr>
<tr>
<td>WL</td>
<td>Widelane linear combination</td>
</tr>
<tr>
<td>ZHD</td>
<td>Zenith Hydrostatic (dry) Delay</td>
</tr>
<tr>
<td>ZWD</td>
<td>Zenith Wet Delay</td>
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1. INTRODUCTION

1.1 BACKGROUND

Satellite based navigation systems, such as the well known Global Positioning System (GPS), have revolutionised the collection of spatial information. The global coverage, all-weather continuous operation, ease of use and high accuracy of satellite based navigation systems, collectively referred to as Global Navigation Satellite Systems (GNSS), are attractive to a wide variety of professionals and non-professionals. Navigators, geodesists, surveyors, meteorologists, geologists and recreational users alike have readily taken up GNSS technology. Satellite-based navigation systems not only offer unprecedented high accuracy/low cost solutions to many traditional positioning and navigation activities, but they have also been applied to, and indeed created, many new applications.

Over the past twenty years, beginning with the launch of the first few GPS satellites, there has been a desire to use GNSS technology to achieve positioning accuracies at the centimetre and millimetre levels (hereafter referred to as precise positioning). During this time there have been significant advances in the use of GNSS technology in the fields of positioning, navigation and time transfer. At one end of the spectrum, GPS receivers are becoming smaller, cheaper and are increasingly being incorporated into consumer electronics such as watches, mobile phones and personal digital assistants (PDA) offering positioning accuracies at the metre level. At the other end, GPS precise positioning offers increasingly greater precision, whilst simultaneously improving the flexibility and efficiency of the system through technical and theoretical developments such as on-the-fly (OTF) ambiguity
resolution. Such is the accuracy of GPS that it can be used to measure small scale, long term motion of the earth’s tectonic plates. It is the precise, long distance (>100km) type of application of GNSS technology that is the particular focus of this dissertation.

As with all other navigation technologies, position determination using GPS requires measurements to be recorded and used to estimate the parameters of interest (e.g. position, heading, velocity, time). These measurements are inevitably biased. The bias or error in a measurement can be subdivided into two components:

1. a *systematic* part which can be modelled by a mathematical function and is the result of scientifically explainable and repeatable processes, and
2. a *random* part which comprises non-systematic measurement errors arising from imperfect measurement techniques or equipment. Random errors tend to occur in an ad hoc (non-repeatable) fashion.

Improved mathematical modelling and estimation of the systematic error inherent in measurements from GNSS has resulted in greater accuracy of the estimated parameters (Schaer 1999; Rothacher et al. 1997; Euler and Ziegler 2000; Abidin 1994; Wang 1999; Wieser 2001; Wübben 1985; Melbourne 1985; Hopfield 1969; Niell 1996). Of particular interest has been the issue of ambiguity resolution, whereby efficient and robust algorithms have been developed to allow the unknown integer cycle ambiguities to be reliably estimated based on a minimum number of measurements. These advances have led to developments such as real time precise positioning, commonly known as Real Time Kinematic (RTK) positioning. Advances have also been made in stochastic modelling, which is the means of dealing with the random component of the measurements, further improving parameter estimation and ambiguity resolution (Han 1997; Radovanovic et al. 2000; Radovanovic 2001a; Wang 1998; Schaffrin and Bock 1988; Wang et al. 1998a; Wang et al. 2002; Brunner et al. 1999; Lau and Mok 1999).

Precise positioning with GNSS typically involves the use of two receivers where one receiver (the roving receiver) is positioned relative to the other (the reference receiver). As the separation between the reference and roving receiver (the
Baseline length) increases more residual error remains after differencing and, as a result, precise positioning becomes more difficult. A large proportion of GPS research has been directed towards real-time, surveying-type precise positioning over short (<10km) to medium (10-100km) baselines. Such applications have a greater market potential than geodetic applications, like monitoring tectonic plate movement. In contrast to surveying, geodetic applications of GNSS generally require long baselines (100km and beyond) using static receivers and often a network of reference stations.

1.2 PROBLEM STATEMENT

A fundamental requirement of high precision positioning is to resolve the integer ambiguities. To do so over long baselines requires that particular care be given to both the mathematical and stochastic models. The most problematic error source for long baseline ambiguity resolution is the ionosphere. It is the contention of this thesis that current models used for long baseline static positioning can be improved to allow more ambiguities to be resolved and with higher reliability. The focus of this thesis is the treatment of the ionosphere, in particular the efficient estimation of ionospheric path delays using both mathematical and stochastic techniques.

1.3 HYPOTHESIS

The hypothesis to be tested by this research is:

Improved estimation of ionospheric path delays will enable enhanced ambiguity resolution when undertaking long baseline, high precision GPS positioning.

Accordingly, the aims of this thesis are to:

1. Review the fundamental technology of GNSS in general and the Global Positioning System (GPS) in particular.
2. Review current functional and stochastic error modelling and processing techniques used for medium to long GPS baselines.
3. Describe the limitations of current modelling techniques.
4. Identify the mathematical and stochastic models most suitable for precise positioning over long static baselines.
5. Develop a suitable algorithm to efficiently and effectively estimate station coordinates and, as required, ambiguities, tropospheric delays, ionospheric delays and other errors using the selected models.
6. Test and refine the models and estimation routines to prove or disprove the research hypothesis.

1.4 Thesis Outline and Research Approach

The research approach and the structure of the thesis has been designed to sequentially address the aims presented in section 1.3.

Chapter 2 introduces GPS/GNSS including existing and planned satellite navigation systems, the signals that they transmit and the measurements that can be derived from them. This chapter provides an extensive review of the error sources influencing the measurements. An outline of the general types of positioning techniques used with GNSS data is given.

Chapter 3 expands on Chapter 2 by providing a detailed review of phased-based relative positioning and the related error modelling strategies. Following the distinction made in section 1.1 between systematic and random errors, the discussion on error modelling is split into two sections: mathematical modelling and stochastic modelling. The numerous options regarding measurement differencing and the creation of new observables are examined in terms of their ability to help eliminate errors and resolve ambiguities. A wide range of stochastic models proposed in the literature are reviewed in terms of their effectiveness and practicality. This chapter also reviews fundamental theory pertaining to parameter estimation using least squares and ambiguity resolution.

Chapter 4 reviews the effectiveness of current mathematical and stochastic models with respect to each of the error sources identified in Chapter 2 with a particular emphasis on positioning based on long baselines and long sessions. Tests are conducted to determine the impact of various errors and error models. The
ionosphere is shown to be the error that is most problematic in resolving integer ambiguities over long baselines.

Chapter 5 explores the Quasi-Ionosphere-Free processing strategy, the technique identified as being most appropriate for long baseline phase positioning. The theoretical and practical limitations of the model are explored and new approaches are proposed and developed. The novel approach of using sequential phased adjustment in the ambiguity float solution is introduced.

Chapter 6 introduces a testing procedure to validate the developed algorithm by comparing it to the industry accepted solution given by the Bernese GPS Software v4.2. The test datasets are described and the results from empirical tests are presented. An analytical comparison of the results is used to measure the effectiveness of the proposed ionospheric estimation technique in terms of the number of ambiguities resolved and the reliability of the ambiguity resolution.

Chapter 7 presents conclusions, recommendations and suggestions for further development of this research. A list of publications, software and awards derived from this research is given in Appendix D.
2. SATELLITE POSITIONING

2.1 INTRODUCTION
Satellite based systems were first developed in the 1960s to facilitate the determination of positions anywhere on the earth’s surface. Of the existing systems, the Global Positioning System (GPS) is the most widely used. There are similar complementary but independent satellite navigation systems such as Russia’s Global Navigation Satellite System (GLONASS) and the proposed European Union’s Galileo system. In addition, there are both satellite and ground based augmentation systems designed to improve the availability, integrity, reliability and accuracy of GPS. Examples of such augmentation systems are the European Geo-stationary Overlay Service (EGNOS), the United States Wide Area Augmentation System (WAAS), and the proposed Japanese quasi-zenith satellite system (QZSS), dubbed the Japanese Regional Advanced Navigation Satellite (JRANS). Whilst this thesis focuses specifically on GPS, much of the general theory presented in this Chapter is applicable to these other systems.

2.2 GLOBAL POSITIONING SYSTEM (GPS)
The United States Department of Defence (USDoD) operates and maintains the Navigation Satellite Timing and Ranging Global Positioning System (NAVSTAR GPS) or simply GPS. Full Operational Capability (FOC) was declared on 17 July 1995, when a constellation of 21 satellites with three operational spares was achieved. GPS enables 24 hour, all weather positioning and navigation anywhere on the surface of the earth.
GPS is composed of three segments; the space segment, the ground segment and the user segment. The space segment refers to the constellation of GPS satellites. Each satellite transmits ranging signals that are used by the ground and user segments. The composition of the ranging signals transmitted by the GPS satellites will be examined in section 2.3.

The GPS ground segment includes the ground infrastructure required to operate and maintain the space segment. In the ground segment, a network of five globally distributed ground stations continuously tracks the satellites (Figure 2.1). Data is transmitted from each of these monitor stations to a Master Control Station (MCS) at Colorado Springs where the orbits, clock parameters and health of the satellites are determined. This information is fed to three of the monitor stations that also act as uplink sites for uploading data to the satellites. The satellites then re-transmit the (broadcast) navigation message to users.

Figure 2.1 GPS Ground Segment

Another critical component of the ground segment is the United States Naval Observatory (USNO), which has responsibility for maintaining the precise time standard used by GPS. GPS time is directly related to Universal Coordinated Time (UTC) as used by the civilian community. GPS time was synchronised to the USNO realisation of UTC on 1 January 1980. However, GPS time is a continuous time scale
that is not corrected by the leap seconds that are applied to UTC to keep it close to sidereal time. Thus GPS time and UTC are offset by an even number of seconds. The size of the offset depends on the current leap second correction applied to UTC.

The GPS user segment comprises the receivers used by civilians and the military for positioning, navigation, time transfer and numerous other applications. Due to constraints imposed by the US Congress, the USDoD was required to make part of the GPS service freely available to civilians, albeit at a reduced accuracy. Two service levels were instigated, the Standard Positioning Service (SPS) based on a special civilian code and the Precise Positioning Service (PPS) based on protected military codes. The degradation of service to civilian and non-allied military users was achieved through the process of Selective Availability (SA), whereby the satellite clock could be dithered and the Keplerian orbit elements in the navigation message modified to reduce the accuracy of the resulting position. SA was permanently removed at midnight 2 May 2000, greatly increasing the commercial potential of GPS by improving the positioning accuracy from approximately ±100m to ±10-20m at 95% confidence.

2.3 **FUNDAMENTAL GPS OBSERVABLES**

Each satellite in the GPS constellation transmits signals on two sinusoidal carrier waves, designated L1 and L2. These waves are generated using frequency multiplication from a fundamental (base) frequency of an onboard precise atomic oscillator (clock). The fundamental frequency is 10.23MHz and the L1 and L2 multipliers are 154 and 120 respectively, giving frequencies of \( f_{\text{L1}} = 1575.42\) MHz and \( f_{\text{L2}} = 1227.60\) MHz (IS-GPS-200D 2006).

Modulated onto the L1 carrier are two pseudo-random noise (PRN) codes, the coarse/acquisition or C/A code, and a protected (P) code. Subsequently these PRN codes will be referred to as C1 and P1 respectively. The frequencies of the C1 and P1 codes are \( f_{\text{C1}} = 1.023\) MHz and \( f_{\text{P1}} = 10.23\) MHz. A second P-code (P2) is modulated onto the L2 frequency, which also has a frequency of 10.23 MHz. The PRN codes are generated from the same fundamental frequency as the carrier waves. Each of the PRN codes has superimposed on a satellite message that contains clock
synchronisation, orbit and health information for the satellite, amongst other things. The PRN codes provide the basic ranging signals that are required for a user to determine position. Many receivers are only able to track signals on the L1 carrier and are referred to as single frequency receivers. In geodesy and other applications requiring high precision, dual-frequency receivers, which are able to track both the L1 and L2 signals, are generally used.

Under the USDoD anti-spoofing policy (AS), which was implemented to prevent jamming and generation of false GPS signals, the P-codes are encrypted by the W-code creating Y-codes. Receivers without the necessary military decryption capability must use a codeless technique to track and make measurements on L2 when AS is turned on. Several such techniques have been used in commercial GPS receivers such as Squaring, Cross-Correlation, Code Correction and Squaring and Z-Tracking. All of the techniques used to work around the encryption are inferior in tracking and measurement quality to direct P-code correlation.

2.3.1 Code Pseudorange Measurements

Code pseudorange measurements may be obtained using the C1, P1, and P2 PRN codes. During the measurement process, a GPS receiver replicates the PRN codes. Assuming the clocks in the satellite and receiver are synchronised to GPS time, the signal travel time can be determined by measuring the time shift between the internal and incoming version of the code. The PRN codes are designed to have low auto correlation allowing the time shift to be measured precisely and unambiguously. The range between the satellite and receiver is a function of the measured time shift and the speed of transmission. Since the positions of the satellites are known from the navigation message, a user is able to calculate receiver position if the signals from three or more satellites are available. In reality, a number of errors influence the measurement process. In particular, imperfect clock synchronisation biases the measured range, leading to the term pseudorange. Ignoring all influences apart from the receiver and satellite clock errors, the travel time \( \tau_i \) can be written as

\[
\tau_i = t_i^r(GPS) - t_i^s(GPS)
\]  

(2.1)
\[ \tau'_i = (t' - \delta t') - (t_i - \delta t_i) \]
\[ \tau'_i = (t' - t_i) - (\delta t' - \delta t_i) \]
\[ \tau'_i = \Delta t - (\delta t' - \delta t_i) \]

where \( t' \) (GPS) is the signal transmission time in true GPS time and \( t_i \) (GPS) is the signal reception time in true GPS time. The terms \( \delta t' \) and \( \delta t_i \) are respectively the offsets of the satellite and receiver clocks from GPS time.

Multiplying (2.1) by the speed of light we get the basic pseudorange observation equation, expressed in units of metres

\[
c t_i = c \Delta t - c(\delta t' - \delta t_i) \quad (2.2)
\]
\[ \rho = \Phi - c(\delta t' - \delta t_i) \]
\[ \Phi = \rho + c(\delta t' - \delta t_i) \]

where \( \rho \) is the true range and \( \Phi \) is the pseudorange. Expanding to include other sources of error, we get the full pseudorange observation equation

\[
\Phi_i(t) = \rho'_i(t, t - \tau'_i) + c(\delta t' - \delta t_i) + I_i^s + T_i^s + O_i^s + M_i^s + \varepsilon_i^s \quad (2.3)
\]

where

- \( \rho'_i \) is the distance between the receiver at the time of measurement (\( t \)) and the satellite at time of transmission (\( t - \tau'_i \)), i.e.
  \[ \rho'_i(t, t - \tau'_i) = \| \chi'_i(t - \tau'_i) - \chi_i(t) \| \]
  where \( \chi'_i, \chi_i \) are the geocentric position vectors of the satellite and receiver electrical phase centres,

- \( I_i^s \) is the error in the range caused by the ionosphere,

- \( T_i^s \) is the error in the range caused by the troposphere,

- \( O_i^s \) is the orbital error along the line of sight resulting from imperfect knowledge of the satellite’s position,

- \( M_i^s \) is the multipath error,
is the random measurement noise, 
\[ c \] is the speed of light, and

Note that with code measurements the measurement noise is generally higher than the antenna phase centre offset and variations, so this term is neglected in equation (2.3).

### 2.3.2 Carrier Phase Range Measurements

It is possible to measure the phase of the L1 and L2 carriers more precisely than the PRN codes. However, determination of precise ranges using the carrier waves is complicated by the need to resolve the so-called integer ambiguity.

The full range (in cycles) between the receiver (r) and satellite (s) is given as (Hoffman-Wellenhof et al. 1994)

\[ \Phi^{s}_r(t) = \Delta \Phi^{s}_r + l^{s}_0 + N^{s}_t \] (2.4)

where \( \Delta \Phi^{s}_r \) is the instantaneous fractional phase at epoch \( t \) augmented by the number of full cycles since the initial epoch \( t_0 \). The term \( N^{s}_t \) is the number of full cycles between the receiver and satellite at the initial epoch and is known as the integer ambiguity. Note that the initial fractional phase at both the receiver and transmitter may be non-zero.

A phase pseudorange \( \Phi^{s}_r \) between receiver r and satellite s (in metres) at epoch \( t \) can then be expressed by multiplying (2.4) by the wavelength (\( \lambda \)):

\[ \Phi^{s}_r(t) = \lambda \Phi^{s}_r(t) \] (2.5)

\[ = r^{s}_r(t - \tau^{s}_r) + c(\delta t^{s} - \delta t^{r}) - I^{s}_r + T^{s}_r + O^{s}_r + M^{s}_r + D^{s}_r + \lambda N^{s}_t + \epsilon^{s}_r \]

where

\[ \lambda \] is the wavelength of the signal, and
is the antenna phase centre offset and variation along the line of sight resulting from the difference between the electrical phase centre of the receiving antenna and the receiver’s antenna reference point (ARP) and the electrical phase centre of the transmitting antenna and the satellite’s centre of mass.

Equation (2.5) for the carrier phase is essentially the same as equation (2.3) for the code pseudorange, but with a sign change in the ionospheric term and addition of the terms $D^s_R$ and $N^s_R$ to include the antenna phase centre variations and the integer ambiguity. In order to estimate precise positions from carrier phase pseudoranges, it is necessary to determine the integer ambiguity, a process known as ambiguity resolution. The important thing to note is that the integer ambiguities, unlike the other terms, are not time dependent. The integer ambiguities will be constant unless a break in tracking the signal occurs (referred to as a loss of lock). When loss of lock occurs there is a discontinuity in the phase measurement and a new integer ambiguity parameter is created unless the break can be repaired. Loss of lock may be caused by (Hugentobler et al. 2001):

- obstruction of the satellite signal by trees, buildings etc.,
- low signal to noise ratio due to attenuation from the atmosphere, multipath or receiver dynamics,
- failure of the receiver software, or
- malfunctioning of the satellite or receiver’s oscillator.

2.4 MODERNISED GPS

The USDoD is currently enhancing the ground segment through the addition of six additional monitor stations from the NIMA network and various modelling and processing improvements as part of the Legacy Accuracy Improvement Initiative (L-AII) (Bellows 2006; Ballenger 2006). The additional monitor stations will mean that all satellites are tracked by at least two stations at all times, increasing the ability of the system to detect failures and providing a small increase in accuracy to the user. The USDoD also has plans to update the GPS constellation with new Block IIR-M and Block IIF satellites that will offer a number of improvements over the existing
Block II/IIA/IIR satellites (Fontana et al. 2001). The new satellites will have a new military (M) PRN code on both L1 and L2. For civilian users, the L2 signal will have a new open access PRN code, known as L2C. The L2C code will allow civil users to have access to dual frequency code-based positioning without the need for proprietary algorithms for reconstructing the P-code. The Block IIF satellites will also include a third signal in the L5 band (1176.45MHz), with a civilian access code L5C. The L2C code will offer improved threshold tracking and measurement performance, increasing signal availability for navigation users (Fontana et al. 2001). L5 also enables development of smaller, lower power receivers for use in wristwatches and mobile phones (Fontana et al. 2001). The first Block IIR-M was launched on 5 September 2005 and the second on 23 September 2006. The first Block IIF is currently planned for launch in 2008. Due to a 90° phase shift (quadrature-phase) between the P2 and L2C signals (IS-GPS-200D 2006), the use of the new L2 civil signal is problematic for phase-based positioning, at least until there are more satellites transmitting the L2C signal. Practical use of these new L2C and L5 signals requires a constellation of at least 12 satellites, which realistically will not be reached for several years.

2.5 GLONASS

The GLONASS satellite navigation system was developed and deployed by the former Soviet Union. The nominal GLONASS space segment design consists of 21 satellites in 3 orbital planes, with 3 operational spares. The GLONASS satellites orbit the earth at an altitude of 19,100 km with an orbital period of approximately 11 hours, 15 minutes. GLONASS was declared operational on 24 September 1993, though the full constellation was not deployed until late 1995. Due to reliability and funding problems the system has been operating with a reduced constellation in recent years. Currently there are 12 operational satellites in orbit. A further six are scheduled for launch in 2007 followed by another five in 2008.

Similar to GPS, GLONASS is a dual frequency system with signals in the L1 and L2 bands. Unlike GPS, which uses CDMA (code division multiple access) technology, GLONASS uses FDMA (frequency division multiple access) to distinguish the signals from the different satellites. Each satellite has a frequency
number that is used to determine the L1 and L2 frequencies that it transmits using the
formulae (ICD 2002):

\[ f_{L1}^s = 1602MHz + K \times 5.625kHz \]  \hspace{1cm} (2.6)
\[ f_{L2}^s = 1246MHz + K \times 4.375kHz \]

where \( K \) is the integer frequency channel number of satellite \( s \). One consequence of
using multiple L1/L2 frequencies is that the signals of different GLONASS satellites
will take different paths through the receiver, leading to different hardware delays for
signals from different satellites (Rossbach 2000b). Another problem with the FDMA
technology is that the integer nature of the phase ambiguities is lost when double
differences (section 3.2.2) are formed. A number of techniques are available in the
literature to overcome this problem (Rossbach 2000a; Rossbach 2000b; Wang 1999)
such as introducing a single difference term or introducing an auxiliary wavelength.

The GLONASS system has some other important differences from GPS that
make dual-system phase-based positioning difficult. In particular the use of the
geodetic reference frame PZ-90 and the GLONASS system time are problematic.
When combining GPS and GLONASS, the GLONASS satellite coordinates are
generally rotated into WGS84 using a 7-parameter transformation using one of a
number of published parameter sets (Zinoviev 2005; Takac et al. 2005; Rossbach
2000b). GLONASS system time is coupled with UTC to an accuracy of
microseconds, but has a three hour time offset to adjust it to the Moscow time zone
resulting in the relationship,

\[ t(UTC) = t(GLONASS) + \tau_c - 3h \]  \hspace{1cm} (2.7)

where \( \tau_c \) is the variable offset between GLONASS system time and UTC.
Modernization of GLONASS is also planned by the Russian Federation, including
replenishment of the constellation with new, more reliable GLONASS-K satellites
and addition of a third frequency L3. A Presidential Directive issued on 18 January
2006 set the following objectives for GLONASS (Averin 2006):
To ensure GLONASS minimum operational capability (constellation of 18 NSV) by the end of 2007

To ensure GLONASS full operational capability (constellation of 24 NSV) by the end of 2009

To ensure GLONASS performance comparable with that of GPS and Galileo by 2010

To achieve these goals, the GLONASS system will be improved in the following ways (Averin 2006):

- GLONASS-M and GLONASS-K satellite launches planned through until 2012
- Modernization of the GLONASS time keeping system
- Improving ground segment processing techniques to achieve better ephemeris and clock accuracy based on combination of one-way and two-way measurements
- Improving stability of onboard satellite clock from $3 \times 10^{-13}$ to $1 \times 10^{-13}$ seconds (GLONASS-M onwards)
- Modulation of a civil code on the L2 signal (GLONASS-M onwards)
- Improving the agreement between PZ-90 and ITRF
- Introduction of the third civil signal in L3 in 2008 (GLONASS-K onwards)
- Providing GLONASS with Search and Rescue capability (GLONASS-K onwards)

2.6 GALILEO

Galileo is a joint initiative of the European Commission (EC) and the European Space Agency (ESA). This next generation satellite navigation system differentiates itself from GPS and GLONASS in the following ways:

1. It will be under civilian control.
2. It will be a commercial system with both free to air and pay for access services.
3. It will guarantee availability of the service under all but the most extreme circumstances.
4. It will inform users within seconds of a failure of any satellite.

Galileo will use a Walker constellation with 27 satellites in three orbital planes. Galileo will be interoperable with GPS and GLONASS (ESA 2006) providing a combined constellation of up to 75 satellites (if all systems are at FOC) thereby increasing coverage and reliability for users with multi-system receivers. The Galileo system will have a complicated signal structure designed to have high tracking performance, high accuracy, high robustness and low interference with GPS (Rodríguez et al. 2004). Presently ten signals, shown in Table 2.1 and Figure 2.2, are planned that will address the various needs of the open (OS), safety-of-life (SoL), commercial (CS) and publicly-regulated services (PRS) to be offered by Galileo.

From the ten signal components in Table 2.1 it is expected that there will be five observables E5a, E5b, AltBOC (E5a+E5b), E6 and L1. E5a has a frequency of 1176.45 MHz and is the equivalent of GPS L5. L1 has a frequency of 1575.42 MHz and is the equivalent of GPS L1. Alt-BOC (alternate binary offset carrier) tracking uses the whole 90 MHz-wide bandwidth of E5a+b signals, which results in a higher value of the effective modulation rate and a reduction in noise by a factor of 10 (Simsky and Sleewaegen 2005).

Compared with the current GPS signals, Galileo will offer the following advantages for geodetic applications (Simsky and Sleewaegen 2005):

1. Signal power higher by a factor of 2, leading to a reduction of tracking noise for both phase and code ranges.
2. Use of so-called “pilot” data-less components allowing for faster re-acquisition of phase after the loss-of-lock.
3. New modulation schemes will result in a significant reduction of both tracking and multipath noise for all the code ranges, especially with the E5-AltBOC signal.
4. A more robust 3-step coding scheme for navigation bits will be used significantly increasing the reliability of navigation message decoding in the presence of interference or with low signal power.

<table>
<thead>
<tr>
<th>N</th>
<th>Signal</th>
<th>Modulation</th>
<th>Carrier frequency (MHz)</th>
<th>Symbol/s</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>E5a-I*</td>
<td>BPSK(10)</td>
<td>1176.45</td>
<td>50</td>
<td>OS; open, no encryption</td>
</tr>
<tr>
<td>2</td>
<td>E5a-Q*</td>
<td>BPSK(10)</td>
<td>1176.45</td>
<td>-</td>
<td>No data (pilot)</td>
</tr>
<tr>
<td>3</td>
<td>E5b-I*</td>
<td>BPSK(10)</td>
<td>1207.14</td>
<td>250</td>
<td>OS, SoL; open, no encryption</td>
</tr>
<tr>
<td>4</td>
<td>E5b-Q*</td>
<td>BPSK(10)</td>
<td>1207.14</td>
<td>-</td>
<td>No data (pilot)</td>
</tr>
<tr>
<td>5</td>
<td>E6-A</td>
<td>BOC(10,5)</td>
<td>1278.750</td>
<td>Classified</td>
<td>PRS; encrypted code and data</td>
</tr>
<tr>
<td>6</td>
<td>E6-B</td>
<td>BPSK(5)</td>
<td>1278.750</td>
<td>1000</td>
<td>CS, encrypted code and data</td>
</tr>
<tr>
<td>7</td>
<td>E6-C</td>
<td>BOC(5)</td>
<td>1278.750</td>
<td>-</td>
<td>No data (pilot), code encrypted</td>
</tr>
<tr>
<td>8</td>
<td>L1-A</td>
<td>BOC(15,2.5)</td>
<td>1575.420</td>
<td>Classified</td>
<td>PRS; encrypted code and data</td>
</tr>
<tr>
<td>9</td>
<td>L1-B</td>
<td>BOC(1,1)</td>
<td>1575.420</td>
<td>250</td>
<td>OS, SoL; open, no encryption</td>
</tr>
<tr>
<td>10</td>
<td>L1-C</td>
<td>BOC(1,1)</td>
<td>1575.420</td>
<td>-</td>
<td>No data (pilot)</td>
</tr>
</tbody>
</table>

* The satellites will transmit the E5a and E5b signals as one wide-band modulation referred to as “Alt-BOC”, centered at 1191.795MHz.

Table 2.1 Galileo signal components (adapted from Simsly and Sleawaegen (2005))

![Figure 2.2 Galileo Frequency and Signal Baseline after the agreement between the EU and USA in June 2004 (from Rodríguez et al. (2004))](image)

The first Galileo test satellite GIOVE-A was launched on 28 December 2005 for in-orbit validation of the system. The second test satellite is scheduled for launch in
late 2006. The Galileo project has suffered numerous setbacks and the date of FOC is not known at the time of writing, but will probably be sometime after 2010.

## 2.7 ERROR SOURCES IN GPS

### 2.7.1 Preamble

There are numerous errors that affect GPS measurements, as illustrated by the error terms in equations (2.3) and (2.5). These errors come from a variety of sources, but may be categorised into three groups: satellite related errors, signal propagation errors and receiver related errors. A diagrammatic representation of GPS error sources is given in Figure 2.3.

![Figure 2.3 Error Sources in GPS](image)

**Figure 2.3 Error Sources in GPS**
The approximate (one-sigma) magnitude of the major errors affecting GPS measurements are given in Table 2.2. Note that the tabulated values assume that the satellite clock and ionospheric errors have been corrected with the models provided in the broadcast navigation message and that a standard atmosphere model has been used to correct for tropospheric delay. The achievable accuracy of positions derived from GPS is related directly to how the various errors are managed or modelled. Due to the different characteristics of the error sources, no single technique is able to adequately model all error sources in all operating conditions. Rather, combinations of strategies are often required to achieve high accuracies. Hence it is important to understand the nature of the errors before examining the techniques used to derive positions from the GPS signals.

<table>
<thead>
<tr>
<th>Error Source</th>
<th>One-sigma error (m) for SPS</th>
<th>One-sigma error (m) for PPS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Orbit</td>
<td>2.1</td>
<td>2.1</td>
</tr>
<tr>
<td>Satellite clock</td>
<td>2.1</td>
<td>2.1</td>
</tr>
<tr>
<td>Ionosphere</td>
<td>4.0</td>
<td>1.2</td>
</tr>
<tr>
<td>Troposphere</td>
<td>0.7</td>
<td>0.7</td>
</tr>
<tr>
<td>Multipath</td>
<td>1.4</td>
<td>1.4</td>
</tr>
<tr>
<td>Receiver noise</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>User equivalent range error (UERE)</td>
<td>5.1</td>
<td>3.3</td>
</tr>
<tr>
<td>Horizontal Accuracy (HDOP 2.0)</td>
<td>10.1</td>
<td>6.6</td>
</tr>
<tr>
<td>Vertical Accuracy (VDOP 2.5)</td>
<td>12.6</td>
<td>8.3</td>
</tr>
</tbody>
</table>

Table 2.2 Error Budget (adapted from Parkinson (1996))

The UERE is the square root of the sum of the squares of the individual errors (not including multipath) and is an estimate of the total error in the pseudoranges. The UERE together with a Dilution of Precision (DOP) factor can be used to compute the one-sigma Estimated Position Error (EPE) by

\[
EPE = DOP \times UERE
\]  

(2.8)

DOP factors are derived from the diagonal terms of the co-factor matrix resulting from the least squares estimation of receiver coordinates and receiver clock offset from code pseudorange observations and are reflective of the satellite geometry.
Horizontal (HDOP), vertical (VDOP), position (PDOP) and geometric (GDOP) dilution of precision values are commonly used to estimate positioning accuracy.

### 2.7.2 Satellite Related Errors

#### 2.7.2.1 Orbit

In order to obtain the position of a receiver using a satellite-based positioning system, it is necessary to know the position of the satellites. There is circularity in this problem because the positions of points on the ground must be known in order to calculate the positions of satellites. For this reason, GPS tracking networks have been established with well-known coordinates derived using alternate means in order to determine satellite orbit parameters. Two tracking networks that are important for GPS orbit modelling are the USDoD ground segment (refer to section 2.2) and the International GPS Service (IGS) network. The datum for satellite-based positioning calculation is based on the coordinates of the satellites and, hence, is directly related to the datum of the tracking network used to determine the satellite orbits in the first instance.

The GPS ground segment is responsible for determining and uploading the broadcast navigation message to the GPS satellites. The navigation message contains the Keplerian elements that describe the orbit of each satellite or space vehicle (SV). Deviations, or perturbations, from the Keplerian orbits are caused by the non-sphericity of the earth, gravitational attraction of other planetary bodies, earth tide potential, ocean tide potential, solar radiation pressure, earth albedo, atmospheric drag and residual acceleration of the satellites (Govind 1994). A study by Roulston et al. (2000) determined that the mean error in the broadcast orbit is approximately five metres with a standard deviation of three metres. A more recent estimate of the accuracy of the broadcast ephemeris given by the IGS is 160cm (IGS 2006a). The datum of the broadcast ephemeris is the World Geodetic System 1984 (WGS84), which is defined and maintained by the United States National Imagery and Mapping Agency (NIMA).
The IGS maintains a network of 333 active stations distributed across the globe (IGS 2006b). The weighted orbit solutions from up to seven analysis centres are combined to produce the IGS precise orbit products. The orbits contain X,Y,Z coordinates and velocities of the satellites, which must be numerically integrated using an orbit model to obtain accurate satellite positions. These orbits may be freely downloaded from the IGS website (igscb.jpl.nasa.gov) and are available in an open ASCII format known as SP3c. The IGS Final Orbit product enables computation of the satellite coordinates to an accuracy of about 50mm within the International Terrestrial Reference Frame (ITRF) (IGS 2006a). The ITRF is a civilian reference frame maintained by the International Earth Rotation Service (IERS). Importantly, WGS84 is maintained to be within two centimetres of ITRF (NIMA 2001) thereby making them essentially the same for most GPS applications.

Both broadcast and precise orbits are modelled relative to the centre of mass of the space vehicles, which may not be coincident with the electrical centre of the transmitting antenna. Therefore satellite specific corrections may need to be applied to ensure that calculated ranges are correct. These corrections are the satellite equivalent of the receiver antenna offsets, which are explained in section 2.7.4.2. The satellite specific corrections can be estimated using the GPS measurements themselves if the absolute receiver antenna corrections and station coordinates are known (Schmid and Rothacher 2003).

2.7.2.2 Satellite Clock
The difference between the satellite clock derived from the onboard oscillators and GPS time as maintained by the USNO causes a direct error in the measured range. The satellite clocks are adjusted on a regular basis to keep them close to GPS time, though the residual range error is still considerable. Both the broadcast satellite message and the IGS precise orbits contain satellite clock correction parameters that may be used to account for most of the satellite clock error. The estimated accuracies of the corrections are 7ns for the broadcast orbits and better than 0.1ns for the IGS Final products (IGS 2006a).
2.7.3 Signal Propagation Errors

2.7.3.1 Ionosphere

The upper, ionised part of the atmosphere, referred to collectively as the ionosphere, exists from about 50 to 1000km above the surface of the earth. The ionosphere is characterised by sufficient numbers of positive ions and free electrons to affect the propagation of radio waves. The ionosphere is a dispersive medium for radio waves at the frequencies of the GPS carrier signals. Ionospheric refraction results in phase advance of the L1 and L2 carriers and group delay of the PRN codes. The ionosphere is the dominant source of error in GPS point positioning (Table 2.2). The degree of ionisation and, hence, the influence of the ionosphere on GPS signals varies greatly due to changes in solar input. The state of the ionosphere is measured by the total electron content (TEC), which is the integrated electron density along the path of the signal. In geodetic applications the TEC is expressed in units of $10^{16}$ m$^{-2}$ (referred to as TECU). One TECU represents a 0.163m advance or delay (0.853 L1 cycles). The largest factor that influences the state of the ionosphere is solar activity and as such ionospheric activity typically follows the 11-year solar cycle (Schaer et al. 1998) (Figure 2.4).

![Sunspot activity (SIDC 2006)](image)

Figure 2.4 Sun spot activity (SIDC 2006)
A large spatial variation in TEC is found across the globe. TEC distribution is not only spatially inhomogeneous but is also temporally unstable, mainly due to changes in solar input. This spatial and temporal variability can be seen in Figure 2.5, which shows the global change in vertical TEC over a 24-hour period (blue is low VTEC and red is high VTEC). The parts of the globe where it is night are marked by a lower colour intensity. A clear increase in TEC is seen after sunrise and a gradual decrease after sunset. Typically, maximum TEC values occur during the solstices and minimums occur during the equinoxes (Schaer et al. 1998).

![Figure 2.5 Four-hourly global TEC maps starting 0:00 UTC 22 February 2003 (derived using data from CODE (2006))](image)

If the ionosphere is assumed to consist of equal numbers of positive ions and free electrons and the earth’s magnetic field is uniform, the refractive index of the ionosphere appropriate for carrier phase observations may be approximated by (Langley 1998b)
\[ n_e = 1 - \frac{\alpha N_e}{f^2} \]  

(2.9)

where \( \alpha \) is a constant \((4.028 \times 10^{17} \text{ms}^{-2} \text{TECU}^{-1})\), \( N_e \) is the electron density in units of electrons per cubic metre and \( f \) is the frequency of the signal.

The total path length of a carrier wave signal, made up of the geometric range \( \rho \) and the apparent range delay caused by the ionosphere \( t_i^s \), can be calculated as the integral of the refractive index along the signal path, written as (Langley 1998b)

\[ \rho_e = \int_\rho \left(1 - \frac{\alpha N_e(s)}{f^2} \right) ds = \rho - t_i^s \]  

(2.10)

The apparent range delay is given by

\[ t_i^s = \frac{\alpha E}{f^2} \]  

(2.11)

where \( E \) is the total electron content in TECU.

The dual frequency ionospheric delay \((L2 - L1)\) is (Klobuchar 1996)

\[ \delta t_i^s = \frac{\alpha}{c} \left( \frac{1}{f_2^2} - \frac{1}{f_1^2} \right) E \]  

(2.12)

Due to the dispersive nature of the ionosphere, dual-frequency measurements can be used to estimate or eliminate the first order ionospheric effects. Higher order ionospheric errors are caused by the earth’s geomagnetic field and ray path bending (which causes the L1 and L2 signals to pass through slightly different parts of the ionosphere) (Langley 1998b). A wider spacing between the L1 and L2 frequencies would produce more precise ionospheric modelling results, however if the frequencies were too far apart separate antennas and electronics would be required (Klobuchar 1996). If the L1 and L2 frequencies were more closely spaced the dual frequency ionospheric delay of equation (2.12) would be lost in the measurement noise.
Short-term signal (amplitude) fading and phase changes result from irregularities and short-term variations (scintillations) in the ionosphere. Very rapid but small changes in the TEC can make it very difficult for a receiver to maintain lock on the GPS signals, often causing cycle slips. The ionospheric amplitude scintillations have the greatest influence in equatorial regions from approximately one hour after local sunset to local midnight (Klobuchar 1996). Ionospheric amplitude scintillations also demonstrate patterns related to the season and the solar cycle.

2.7.3.2 Troposphere
The lower, non-ionised or neutral, part of the atmosphere is comprised of the troposphere and the stratosphere and is non-dispersive at the GPS signal frequencies. Because the contribution of the troposphere to the total path delay is much greater than that of the stratosphere, the combined delay is commonly referred to simply as tropospheric path delay. The troposphere extends from the surface of the earth up to about 9km above the poles and 16km above the equator (Langley 1998b). The tropospheric path delay can be written as (Hoffman-Wellenhof et al. 1994)

\[ T_r = \int (n - 1) ds = 10^{-6} \int N_{trop} ds \]  

(2.13)

where \( n \) is the refractive index, \( N_{trop} \) is known as the refractivity and the integration is made along the ray path from the satellite to the receiver. The typical vertical tropospheric delay is about 2.3 metres (Bock 1998).

Hopfield (1969) shows that the tropospheric delay can be broken into a dry (hydrostatic) and a wet component

\[ T_r = T_r^{\text{(dry)}} + T_r^{\text{(wet)}} = 10^{-6} \int N_{trop}^{\text{dry}} ds + 10^{-6} \int N_{trop}^{\text{wet}} ds \]  

(2.14)

The dry component results from dry gases in the atmosphere, and the wet component is due to water vapour. Delay due to liquid water from clouds and rain is typically much less than one centimetre and is generally ignored (Langley 1998b).
The main difficulty in modelling the troposphere is measuring the atmospheric parameters, especially water vapour, along the path of the signal. However, the dry component, which constitutes about 90% of the delay, is stable and is relatively easily modelled using standard atmosphere models (Hoffman-Wellenhof et al. 1994). Three standard atmosphere models commonly used in GPS are Hopfield (Hopfield 1969), Modified Hopfield (Goad and Goodman 1974) and Saastamoinen (Saastamoinen 1973). The wet delay, which contributes approximately 10% of the delay, is highly variable both spatially and temporally.

2.7.3.3 Multipath

Multipath occurs when the signal from the satellite arrives at the receiver via an indirect path. Multipath is caused by signal reflection and diffraction. Typically, reflecting surfaces in the vicinity of the receiver will mean that many reflected signals, in addition to the direct signal, will also reach the antenna. The multipath signals overlay with the direct signal and cause a change in phase and amplitude of the signal that translates into an apparent delay. In extreme cases multipath can cause the receiver tracking loops to lose lock on the signal. The maximum multipath range errors are one quarter of the wavelength for phase and one half the chip length for code measurements (Langley 1998b). This corresponds to maximum errors of approximately 5cm for L1 phase and 15m for P-code measurements.

Mora-Castro et al. (1998) give the multipath tracking loop error (phase delay) on the carrier phase as

\[ \tilde{\varphi}_m = \tan \left( \sum_{i=1}^{N} \alpha_i \sin(\varphi_{m,i}) \right) \left/ \left( 1 + \sum_{i=1}^{N} \alpha_i \cos(\varphi_{m,i}) \right) \right. \]  

(2.15)

where

\[ \tilde{\varphi}_m \] is the phase lock loop tracking error (in radians),

\[ \alpha_i \] is the relative amplitude of the \( i^{th} \) reflected or multipath signal,
\( \phi_{m,i} \) is the relative phase (difference in phase from the direct signal) of the \( i^{th} \) multipath signal, and 

\( N \) is the number of multipath signals.

The relative phase difference of a multipath signal is the result of the additional path length of the combined reflected and direct signals compared to the direct signal. The path length of a given multipath signal is a function of the satellite-reflector-antenna geometry and the reflectance properties of the reflectors. An almost limitless number of reflected signals may result from the receiver’s environment, making equation (2.15) very complex (or even impossible) to implement in practice.

The strength of the multipath signals reduces as the distance from the receiver to the reflecting object increases. Also, the further the object is from the satellite-antenna line of sight, the weaker the multipath signal (Braasch 1996). Most multipath comes from at or below the antenna horizon. To account for this, many GPS antennas are designed to have low gain for signals coming from these directions and may also use physical mitigation devices such as ground planes and choke rings. The amplitude of the multipath error will be the same on L1 and L2, however the relative phase will be different, as will the frequency, making it impossible to eliminate multipath using dual frequency observations (Ray and Cannon 1999).

### 2.7.4 Receiver Related Errors

#### 2.7.4.1 Receiver Clock

GPS receivers typically maintain time using a quartz oscillator. Quartz oscillators are far less stable than the atomic clocks used in the satellites. A receiver clock is used to generate internal copies of the PRN codes and to ensure observations by multiple receivers are recorded simultaneously. Due to the drift, the receiver must regularly correct the internal clock to keep it synchronised to within one millisecond of GPS time. Some receivers perform this correction continuously and others only make the correction once the clock drift reaches a predefined threshold. The difference between the receiver time and GPS time is called the receiver clock offset, or \( \delta t_r = t_{GPS} - t_r \). This offset translates directly into a range error during the measurement process and
will affect all simultaneous measurements equally. The receivers adjust their internal clock periodically to keep it synchronised with GPS time. Some receivers make this adjustment continuously using a technique called clock steering while other receivers allow the clock to drift until it reaches a certain limit (e.g. 1ms) before making the correction. The influence of the clock correction on the measured ranges can be expressed as

\[
\Phi(t_r + \Delta) = \Phi(t_r) + \Phi\Delta = \Phi(t_r) + \dot{\rho}\Delta - c\Delta
\]

(2.16)

where
\[
\begin{align*}
\Delta & \quad \text{is the correction to the receiver clock (in s),} \\
\dot{\Phi} & \quad \text{is the measurement rate of change (in m/s),} \\
\dot{\rho} & \quad \text{is the geometric satellite-receiver range rate (in m/s).}
\end{align*}
\]

Such a clock correction results in a clock jump \(c\Delta\) that is common to all satellites and a satellite specific jump \(\dot{\rho}\Delta\) that is proportional to the geometric range rate and which can be corrected for using Doppler frequency or carrier phase difference measurements.

Whilst the receiver clock offset is the largest source of error affecting the GPS measurements, it is less problematic than atmospheric errors because it may be readily solved for mathematically or eliminated by differencing (refer to section 3.2.2).

2.7.4.2 **Antenna Phase Centre Offset and Variations**

The electrical antenna phase centre (APC) of many GPS antennas does not coincide with the physical centre or the antenna reference point (ARP), which is the point of interest to the user. The vector between the ARP and the APC is known as the phase centre offset and may be different for each frequency. If ignored, an error of up to one decimetre (primarily in the height component) may be introduced. Additionally, the position of the phase centre varies with the elevation and azimuth of the received signal. In fact the phase centre offset is really the average variation and, hence, is a function of the elevation cutoff used in the antenna calibration process. Phase centre variations (PCVs) and offsets are determined empirically by a number of
organisations including the United States National Geological Survey (NGS) (Mader 1999) and Geo++ (Wübbena et al. 2000). The variations are described using either an elevation/azimuth grid or spherical harmonic coefficients. The antenna phase centre corrections are calculated as

\[
D = \sin(E) \cdot \Delta U + \cos(E) \cdot \cos(A) \cdot \Delta N + \cos(E) \cdot \sin(A) \cdot \Delta E - \Delta r_{pcv}
\]  

(2.17)

where

- \( D \) is the total antenna phase centre correction,
- \( \Delta N, \Delta E, \Delta U \) are the antenna phase centre offsets in the north, east and up direction respectively,
- \( \Delta r_{pcv} \) is the antenna phase centre variation, dependant on elevation and (possibly) azimuth, and
- \( E, A \) is satellite elevation and azimuth.

Due to the difficulty in measuring absolute antenna phase centre offsets, the offsets have often been measured relative to a reference Dorne/Margolin antenna mounted on a JPL designed model T choke ring (Mader 1999). Also it is common, due to the cost, to use calibrations based on a type mean rather than individual calibrations for each antenna. Absolute antenna calibration has some advantages over relative calibration such as the ability to estimate azimuth dependant PCVs and PCVs for elevations below 10\(^\circ\). With long baselines the curvature of the earth causes the satellite elevation to be significantly different for each antenna. Relative antenna calibrations are not able to correctly take this situation into account and introduce biases. With current techniques for absolute field calibration of antennas it has become more common to perform individual antenna calibrations. To address the limitations of relative antenna calibrations, the IGS has decided to switch to absolute antenna calibrations and since June 2005 the IGS Final products have been computed in parallel with relative and absolute antenna calibrations.

### 2.7.4.3 Antenna Orientation

The GPS carrier signals are right circularly polarised and, hence, the measured phase depends on the orientation of the satellite and receiver antennas (Wu et al. 1993).
Thus rotation of either the satellite or receiver antenna will introduce an error (sometimes referred to as phase windup) into the phase measurement. The error will generally increase with baseline length and was shown to reach 0.04 m on L1 in a 4,300 km baseline (Wu et al. 1993). Formulas have been derived by Wu et al. (1993) to correct observation data for the geometrical effects of antenna orientation.

### 2.7.4.4 Measurement Noise

Measurement noise arises because the GPS receiver is not able to perfectly track the GPS signals. Such imperfections in the tracking loops are the result of electrical current generated by thermal noise and signal loss due to the atmosphere, cables and interference (Langley 1998a).

It is common to measure the signal power compared to the noise power using the carrier to noise power-density ratio \((C/N_0)\) in units of dB-Hz. The carrier to noise ratio and signal to noise ratio \((S/N\) or \(SNR)\) are related by the relationship (Lau and Mok 1999)

\[
10\log \frac{S}{N} = 10\log \frac{C}{N_0} + 10\log K_R
\]

where \(K_R\) is the receiver gain. The \(C/N_0\) is directly related to the jitter (rapid fluctuations of a signal) in the common Costas-type phase lock loop (PLL) that is used to track the carrier phase. The noise in the PLL resulting from jitter is expressed by Ward (1994) as,

\[
\sigma_{PLL} = \sqrt{\frac{B_p}{c/n_0} \left[ 1 + \frac{1}{2.7 T c/n_0} \right] \frac{\lambda}{2\pi}}
\]

where

- \(B_p\) is the carrier loop noise bandwidth (Hz),
- \(c/n_0\) is the carrier to noise density ratio \((c/n_0 = 10^{(C/N_0)/10})\),
- \(T\) is the pre-detection integration time (sec), and
\( \lambda \) is the wavelength of the carrier.

The equivalent expression for the noise is a delay lock loop (DLL) used to track the PRN codes is (Ward 1994)

\[
\sigma_{\text{DLL}} = \sqrt{\frac{\alpha B_L}{c/n_0} \left[ 1 + \frac{2}{T_n c/n_0} \right]} \lambda_c
\]

(2.20)

where
\( \alpha \) is a dimensionless discriminator correlator factor,
\( B_L \) is the code loop noise bandwidth (Hz),
\( \lambda_c \) is the wavelength of the code (293.05m for C1 and 29.305m for P1).

During the design of the receiver, a trade-off must be made in choosing \( B_p \) between having low noise and the ability to maintain lock during periods of dynamic antenna motion or rapid changes in the ionosphere. Using typical values for the loop parameters (Langley 1998a) shows that for a signal of nominal strength, \( C/N_0 = 45 \text{ dB-Hz} \), and with \( B_p = 2 \text{ Hz} \), the carrier phase noise on L1 is \( \sigma_{\text{PLL}} = 0.0002 \text{m} \). Equivalent values for the noise in a code-tracking DLL produce values of \( \sigma_{\text{DLL}} = 1.04\text{m} \) for C1 and \( \sigma_{\text{DLL}} = 0.104\text{m} \) for P1. This clearly illustrates the motivation in using carrier phase pseudorange measurements for precise positioning applications.

There are a number of other errors (with small magnitude) that affect the measurement precision of a GPS receiver including L1-L2 differential delays, inter-channel biases, local oscillator instabilities, cross-talk, drift and quantisation noise (Langley 1998a). Differential L1-L2 delays results from the signals of the different bands taking slightly different paths through the electronics of the transmitter and receiver. For the code measurements there are similar errors known as differential code biases. Gao et al. (1999) also shows that both constant and time-varying biases can exist between the C1 and P2 observables.
The two carrier waves (L1 and L2) generated by the satellites are carefully synchronised so that they will be sent at exactly the same time. The GPS receiver also carefully synchronises the internal copies of the signals it generates. However this synchronisation is not perfect resulting in satellite and receiver L1/L2 differential delays. These delays are normally absorbed by the satellite and receiver clock offsets and do not impact significantly on the position solution. The differential delays can become significant when dual frequency techniques are used to estimate TEC (Lin and Rizos 1996). Of most significance are the differential code biases (DCB). DCBs are caused by mis-synchronisation of the dual frequency signals by both the satellite and receiver, and may reach up to ten or more nanoseconds (~3 metres).

Modern receivers using separate channels for each satellite will show oscillator instabilities equally across all measurements (code and phase on all channels). Since this error cancels when taking between measurement differences it can be ignored here.

When anti-spoofing is enabled, receivers must apply reconstruction techniques to enable measurement of the P-code and carrier phase on L2. This can result in a correlation between the observables on the two frequencies (Bona 2000). Some receivers also filter the measurements in an effort to reduce noise and as a consequence introduce a time correlation into the observations (Bona 2000).

Inter-channel biases result when the path lengths of the different channels in a multiple channel receiver are different (Langley 1998a). In modern receivers the path lengths are calibrated to 0.1mm or better (Hoffman-Wellenhof et al. 1994). The paths length can also drift because of temperature variations, equally affecting all simultaneous measurements (Langley 1998a).

Quantisation noise is the result of imperfect analogue to digital conversion in digital receivers and makes a slight contribution to the measurement noise.

The significance of these additional sources of noise is that the measurement noise may have a substantially larger magnitude than equations (2.19) and (2.20)
would indicate. These problems are more pronounced in older receivers that use a squaring technique to make measurements on L2 under anti-spoofing conditions.

2.7.5 Other Errors

A number of other errors become significant when seeking the highest accuracy results from GPS. These errors are independent of the measurement system (satellites, signal propagation, user equipment) and are of particular importance in applications such as absolute sea level monitoring. Some errors that affect final station positions are tectonic plate motion, solid earth tides, ocean tide loading and atmospheric loading.

Movement of the earth’s tectonic plates results in movement in GNSS observing stations and permanent reference networks. In order to remove this affect and thereby relate coordinates from different epochs, a plate motion or velocity model is required. For example, the Geocentric Datum of Australia 1994 (GDA94) is fixed at the 1994.0 epoch of ITRF92. To obtain accurate coordinates at a different epoch of ITRF, Geoscience Australia has published parameters for a 14-parameter transformation model (three shifts, three rotations, one scale factor and their corresponding velocities) (Dawson and Steed 2002). Such an approach is possible for GDA94 because Australia is situated in the middle of the Australian plate where tectonic movement is approximately uniform across the Australian landmass. For New Zealand, which lies at the border of the Australian and Pacific plates, tectonic movement is much more complicated. To deal with this, New Zealand uses a grid-based deformation model with velocities to account for the non-uniform movement of the New Zealand land mass (Blick et al. 2003). Additionally, New Zealand’s deformation model must be updated from time to time to reflect changes caused by earthquakes and other seismic activity.

The gravitational attraction of the sun and the moon acting on the earth’s surface cause a radial displacement in station position, known as solid earth tides. Govind (1994) shows that neglecting to make corrections for this affect can cause errors of several centimetres in a station’s position. Similarly, rotational deformation results from polar motion, an effect known as pole tides. Corrections for station
coordinates to account for solid earth tides and pole tides are published by the IERS (McCarthy 1996).

The crustal compression and rebound that results from redistribution of the ocean mass caused by astronomical tides also influences station position, especially the height component. This effect is known as ocean tide loading and varies with location being most pronounced near the coast. Unless it is modelled, ocean tide loading may cause errors of several millimetres over long baselines (Govind 1994).

Another source of station displacement results from changes in atmospheric pressure. Variations in the atmospheric mass load cause deformation of the earth’s crust. Atmospheric loading has most impact on station height and, if left unmodelled, may cause a displacement of up to one centimetre (Govind 1994). However, a study by Chang (1999) showed that displacements due to atmospheric pressure loading for one permanent GPS reference station in Taiwan were 34 to 38 millimetres during summer and 26 to 29 millimetres during winter.

When examining coordinate times series, especially when monitoring sea level change, the effect of isostatic or postglacial rebound should be accounted for. Isostatic rebound causes a slow change in station height and is due to viscously-delayed rebound of the earth’s crust after removal of the ice load from the last glacial maximum. Scotland, Finland and Antarctica are examples of areas that are undergoing isostatic rebound which has been measured by GPS, see e.g. Tregoning et al. (2000), Scherneck et al. (2001) or Bingley et al. (2001).

2.7.6 Summary

The error sources affecting GPS measurements come from a wide range of independent physical processes. In most cases little or no information is available on the presence and magnitude of the errors besides what is provided by the GPS signals themselves. Thus, error-handling capabilities are greatly improved when redundant observations are available from many satellites and code and phase measurements are taken on both the L1 and L2 frequencies. In the following section the common methods that are used to derive positions from GPS measurements will be outlined.
2.8 POSITIONING TECHNIQUES

2.8.1 Preamble

A number of different measurement techniques are used in the field of satellite positioning. The techniques are characterised by the fundamental measurements they are based on and the methods of error handling that are used. All of the following techniques may be used to determine the position of both static and moving (kinematic) GPS receivers. A distinction is often made between positioning and navigation. In positioning only the receiver coordinates are of interest and high accuracy often is sought. On the other hand, navigation applications of satellite positioning require heading and velocity information in addition to position, whilst positioning accuracy is generally less critical.

2.8.2 Absolute Positioning

2.8.2.1 Single Point Positioning

Single point positioning (SPP) positioning is based on the PRN codes. SPP may be carried out using the standard positioning service (SPS), which is based on the C/A code available to civilians. Alternatively, SPP may be performed using the precise positioning service (PPS) based on the dual frequency P-code, which is only accessible to authorised users.

The SPS and PPS require simultaneous measurements of code pseudoranges to a number of satellites using a single receiver. Measurements to at least four satellites are required to calculate a three-dimensional (3D) position because the receiver clock offset must also be estimated. The number of satellites required for 3D positioning reduces to three if Doppler (the apparent change in wavelength of the signals due to the relative motion of the satellite and receiver) measurements are available (Hoffman-Wellenhof et al. 1994). The datum for positioning comes from the geocentric WGS84 coordinates of the satellites derived through the GPS broadcast ephemeris. As such, SPP positions are nominally relative to the centre of the earth, and are thus called absolute positions. Horizontal accuracy is typically about 20m
(95%) with the SPS (Parkinson 1996). Note that this is considerably less accurate than the values in Table 2.2 would indicate, due to the fact that higher DOP values are common.

2.8.2.2 Precise Point Positioning

Precise point positioning (PPP) is similar to SPP but uses phase ranges. Precise orbits and satellite clock offset estimates are required to achieve decimetre level accuracy. Cycle slip detection and repair is a difficult task because of problems in modelling and estimating the ionospheric and tropospheric errors, which would normally be substantially reduced by differencing. PPP is currently unable to resolve the integer ambiguities and is at best able to achieve the same accuracy as a float solution (integer ambiguities treated as real numbers) using relative positioning (section 2.8.3.2).

2.8.3 Relative Positioning

2.8.3.1 Code Based Differential Positioning

Code based differential GPS positioning (generally referred to as DGPS) uses C/A or P-code measurements to the same satellites from two or more receivers. One receiver, the base (or reference) receiver, is located on a point with known coordinates. The differences between the measured coordinates (or pseudoranges) and the known coordinates (or calculated pseudoranges) at the reference receiver are then applied as corrections at the roving receiver. These corrections are based on the assumption that errors in the measurements from two receivers to a single satellite will be spatially correlated. This technique provided substantial improvements in positioning accuracy when selective availability (SA) was turned on, improving horizontal positioning accuracy from about one hundred metres to around 5 metres (95%) (Parkinson and Enge 1996). With SA now turned off, the improvement in accuracy offered by DGPS is greatly lessened, but is still significant for many applications. The effectiveness of differential positioning decreases as the separation between the reference and roving receiver increases, mainly due to the de-correlation of atmospheric errors.
2.8.3.2 Phase Based Differential Positioning

Phase based differential GPS positioning, referred to here as relative positioning, is similar to DGPS but uses phase measurements to the same satellites from two or more receivers. Code observations are not directly used to estimate parameters of interest, but rather play a supporting role in helping remove some sources of error. If the integer ambiguities are resolved, relative positioning has the potential for much higher precision positioning than DGPS because the measurement noise is much smaller. Accuracies of a few millimetres up to several centimetres may be obtained for a fixed ambiguity solution. Modern techniques allow for the integer ambiguities to be resolved while the roving receiver is static or moving, using so-called OTF ambiguity resolution. In real time applications, relative phase positioning is commonly referred to as RTK. As with DGPS, the effectiveness of the technique is reduced as the baseline length (distance between the reference and roving receivers) increases. Commercial RTK systems quote accuracies of 10mm + 1ppm (horizontal) and 20mm + 1ppm (vertical) for kinematic positioning with baselines up to 30km (Leica Geosystems 2006). For post processing of static data with long observation sessions accuracies of 3mm + 0.5ppm (horizontal) and 6mm + 0.5ppm (vertical) are specified (Leica Geosystems 2006). Over longer baselines ambiguity resolution becomes slower, increasing the time spent “initialising” and thus causing a reduction in productivity. In static applications, the integer ambiguities may be resolved over baselines of almost any length, whilst OTF ambiguity resolution is currently limited to baselines of a few tens of kilometres. Relative positioning is the technique of choice for high accuracy positioning such as surveying and geodesy and is the focus of this thesis.

2.8.4 CORS Networks

Networks of Continuously Operating Reference Stations (CORS) have been established in many regions to support the use of GPS positioning systems. CORS infrastructure, which includes receivers, antennas, monumentation and communication devices, are used to provide the correction information needed by the rover unit for relative positioning. The main benefits of CORS networks are:

- reduced the capital investment required by the rover user
• reduced setup time for the rover user
• rover user does need not to be concerned about security, setup, power, communications and groundmark issues that are important in the establishment of a reference station
• reduced need for groundmarks
• support of scientific applications such as ionospheric studies, meteorology, orbit determination, earthquake studies etc.

If the data from multiple reference stations can be collected at a central processing facility with low latency, error modelling can be used to generate higher quality corrections than are possible with a single reference station (Brown et al. 2005). So-called network corrections can be calculated by interpolating the error at the rover using the known error measured at the reference stations. Real time positioning based on network corrections are often referred to as Network RTK (NRTK) and Network DGPS (NDGPS). The main benefits of network based positioning are the increases in accuracy, reliability and productivity experienced by the rover user when operating over long baselines. Networking the reference stations also reduces the number of reference stations that are required to offer a given level of accuracy and reliability, though it also increases the communications requirements due to the need to reliably get the measurement data to the processing facility with low (preferably sub-second) latency.

2.9 SUMMARY

This Chapter has provided an overview of the basic configuration, observations and error sources of the Global Positioning System. It also outlined the positioning techniques commonly used to determine the positions of GPS antennae. Chapter 3 will build on this by examining in detail relative positioning, with a particular focus on long-baseline static precise positioning. Parameter estimation and ambiguity resolution are also discussed in Chapter 3, which will then form the basis for a detailed assessment of error modelling in Chapter 4.
3. GPS PROCESSING MODELS

3.1 INTRODUCTION

A processing model is required to determine positions from GPS observations. The processing model has two key requirements: 1) to account for the sources of error or bias that contaminate the measurements and 2) to optimally estimate the parameters of interest, typically station coordinates, from the measurements. The mathematical model and the stochastic model, which are the focus of sections 3.2 and 3.3 respectively, are crucial elements of the processing model. Also important is the need to detect and negate gross errors and outliers in the measurements, which is often done as a pre-processing step prior to the estimation of the parameters of interest. The topic of pre-processing and quality control is addressed in section 3.4. The objective of the parameter estimation step is to obtain real-valued estimates for the station coordinates and (possibly) other parameters. The fundamental theory of parameter estimation is dealt with in section 3.5. For phase-based GPS positioning, an additional requirement of optimal estimation of integer-valued ambiguities is added in order to achieve the highest accuracy for the estimates of the station coordinates. Ambiguity resolution is discussed in section 3.6.

Chapter 2 introduces a number of biases resulting from the signal transmission, propagation and measurement processes that affect the GPS code and phase observations. Following from Chapter 2, positions may be determined by GPS either in an absolute sense or relative to a reference receiver. This Chapter focuses on the relative positioning technique, based on GPS phase observations. The fundamental techniques for mitigating each of the errors described in Chapter 2 and obtaining
optimal estimates of station coordinates will be discussed. This Chapter forms the basis for a detailed assessment of error modelling with reference to long baseline positioning in Chapter 4.

3.2 MATHEMATICAL MODELS

3.2.1 Preamble

Mathematical modelling describes the functional relationship between the observations, the parameters of interest and the systematic errors. Systematic errors, as opposed to random errors, are the result of scientifically explainable processes. Random errors, such as measurement noise, are the result of imperfect measurement technique or equipment, and are accommodated by the stochastic model.

In terms of accounting for the influence of systematic errors, a mathematical model may achieve this objective in a number of ways:

- Application of an \textit{a priori} model to correct the measurements.
- Mathematical combinations of multiple measurements to eliminate errors and thus form new “observables” (e.g. measurement differencing).
- Estimation of systematic errors as part of the parameter estimation process.

In GPS relative positioning, mathematical combinations of code and/or phase observations from different satellites, receivers, epochs and frequencies are most commonly used in dealing with systematic errors. In this thesis the term “differencing” is used when referring to mathematical combinations of a single observable from different satellites, receivers or epochs. The term “linear combination” refers to the combination of observables of different types or frequencies. These two broad techniques will be examined below to establish their effectiveness. Estimation of systematic errors is commonly used in circumstances where the first two options above are inadequate.
3.2.2 Differencing

It is often the case that the error in a code or phase measurement from one receiver to a given satellite will be very similar to the error in a simultaneous measurement from a nearby receiver to the same satellite. This is due to spatial correlations in the measurements (Figure 3.1). Consider the case of the error introduced by the ionosphere and troposphere. Since GPS satellites orbit at an altitude of approximately 20,200km, the signals pass through similar parts of the atmosphere when the separation between the receivers is small (a few kilometres or so). In this case there will be strong correlation between the atmospheric errors in the two measurements. Therefore these errors will largely be eliminated by taking between-receiver measurement differences. This type of correlation is known as spatial correlation. Spatial correlation tends to decrease as the separation between receivers increases. A similar approach may be taken with measurements from a single receiver to two satellites, where the receiver clock error will affect both observations and may be eliminated by taking a between-satellite difference. Correlations such as these are the basis for the various differencing techniques. The three types of differences that are commonly used in GPS processing are known as the single, double and triple differences.

Figure 3.1 Correlations in the GPS signals (adapted from Barnes and Cross (1998))
A single difference (SD) may be formed between receivers or between satellites. Usually single differences are formed between receivers, thereby creating a baseline. A baseline is formed when one receiver is positioned relative to another receiver with known coordinates. A typical example is the case of a roving receiver being positioned relative to a receiver from a permanent array of reference stations. A between-receiver single difference for observations taken simultaneously at two receivers (i and j) to a single satellite m is calculated as

$$\Delta \Phi_{ij}^m = \Phi_i^m - \Phi_j^m$$  \hspace{1cm} (3.1)

where $\Phi_i^m$ is the phase pseudorange from equation (2.4) and $\Delta$ is the between-receiver single difference operator. By forming this difference, the satellite clock offset will cancel. Additionally, the spatially correlated ionospheric and tropospheric errors will partially cancel. For baselines of a few kilometres or less in length, the ionospheric and tropospheric errors will cancel almost entirely.

Similarly, the between-satellite single difference for observations taken simultaneously at receiver i to satellites n and m is given by

$$\nabla \Phi_{ij}^{nm} = \Phi_i^{nm} - \Phi_i^n$$  \hspace{1cm} (3.2)

where $\nabla$ is the between-satellite single difference operator. The between-satellite single difference has the effect of cancelling the receiver clock offset (though not satellite specific effects caused clock corrections, see equation (2.16)).

A double difference is formed using two single differences. An equivalent double difference is formed using either two between-receiver or two between-satellite single differences. For observations taken simultaneously at two receivers (i and j) from two satellites (m and n), the double difference is given by

$$\nabla \Delta \Phi_{ij}^{mn} = \Delta \Phi_{ij}^m - \Delta \Phi_{ij}^n$$  \hspace{1cm} (3.3)
\[ = \nabla \Phi_i^m - \nabla \Phi_j^m \]
\[ = (\Phi_i^m - \Phi_j^m) - (\Phi_i^n - \Phi_j^n) \]

where \( \nabla \Delta \) is the double difference operator.

The double differenced (DD) phase observations are generally preferred for high precision GPS applications. Double differencing eliminates the common satellite and receiver clock offsets and can dramatically reduce atmospheric errors over short baselines, thereby giving the observations a simplified structure and making integer ambiguity resolution possible. However, there are some disadvantages to using double differences. For instance, any observations to satellites that are not tracked by both receivers must be discarded and the DD measurement noise is four times higher than for a basic observation. However, the main problem arises from choosing the pairs of satellites used to form the double differences (Goad 1998). The processing algorithm becomes complicated when a satellite is available in one epoch but not the next. A common approach to overcome this problem is to assign a bias to each satellite and to constrain the bias of one satellite (the reference satellite) to zero. However, over a long observation session the reference satellite will likely set, requiring a change in the reference satellite (Figure 3.2). Also, for very long baselines, there may be periods when less than four common satellites are visible. To avoid the problems associated with a change of reference satellite, one approach is to carry an ambiguity parameter for each satellite (SD ambiguities), with one fixed at a reference value. The same reference ambiguity is kept for the entire session. For the example shown in Figure 3.2, the ambiguity \( N_1 \) would be chosen as the reference because it has been observed over the longest period and, therefore, has the most observations (Mervart 1995). For long observation sessions, selection of the reference satellite will influence the precision of the ambiguity estimates; the ambiguities most closely linked (i.e. with a large overlap) to the reference ambiguity will be more precise than those with a weaker link (small or no overlap). In Figure 3.2 the estimated double difference ambiguities \( N_3 - N_1, N_4 - N_1 \) and \( N_5 - N_1 \) will generally be more precise than the ambiguities \( N_2 - N_1, N_6 - N_1, N_7 - N_1 \) and \( N_8 - N_1 \) (Mervart 1995). It is generally
not possible to select the reference parameters so that all ambiguities are likely to be resolved (Mervart 1995).

![Figure 3.2 Satellite visibility plot for a long session and long baseline (adapted from Mervart (1995))](image)

Double differencing can be performed for a single baseline using matrix multiplication such that

\[ \Delta \nabla \Phi = D \Phi \]  \hspace{1cm} (3.4)

where \( D \) is the differencing matrix, \( \Phi \) is a vector of phase (or code) measurements and \( \Delta \nabla \Phi \) is a vector of double difference measurements.

The last difference that is commonly used is the triple difference. A triple difference is the time difference between two double differences

\[ \delta \nabla \Delta \Phi_{ij}^{nm} = \nabla \Delta \Phi_{ij}^{nm} (t + \tau) - \nabla \Delta \Phi_{ij}^{nm} (t) \]  \hspace{1cm} (3.5)

The triple difference has the effect of removing the integer ambiguities. Thus, the triple difference is useful for estimating receiver coordinates and detecting cycle slips, which show up as outliers in the triple difference time series. However, the triple difference solution is less stable and noisier than the double difference and so is generally not used for high precision applications except in a quality control capacity. Due to the influence of changes in the receiver dynamics, the utility of triple
differences in quality control is generally limited to applications involving static receivers.

### 3.2.3 Double Difference (DD) Mathematical Model

After forming the double difference, the common errors will cancel and the spatially correlated error will be reduced. The resulting phase double difference observation equation, derived from equation (2.5) is, e.g. (Teunissen 1998),

$$ \nabla \Delta \Phi_{ij}^{mn} = \rho_{ij}^{mn} - I_{ij}^{mn} + T_{ij}^{mn} + M_{ij}^{mn} + \lambda N_{ij}^{mn} + \varepsilon_{ij}^{mn} $$ (3.6)

where $I_{ij}^{mn}$, $T_{ij}^{mn}$ are the residual ionospheric and tropospheric delay terms, $M_{ij}^{mn}$ is the combined multipath term, $N_{ij}^{mn}$ is the DD phase ambiguity, $\varepsilon_{ij}^{mn}$ is the measurement noise and

$$ \rho_{ij}^{mn} = \rho_i^n(t, t - \tau_i^r) - \rho_j^m(t, t - \tau_j^s) - \rho_i^n(t, t - \tau_i^s) + \rho_j^m(t, t - \tau_j^r) $$ (3.7)

where the $\rho_i^n(t, t - \tau_i^r) = \|x_i^r(t - \tau_i^r) - x_i(t)\|$ is the range between the receiver ($r$) and the satellite ($s$). The term for the antenna phase centre offset and variations from equation (2.5) has been dropped from equation (3.6) on the assumption that it can be removed using an empirically derived a priori model, as explained in section 4.7. Similarly the error introduced by antenna orientation, explained in section 2.7.4.3, has been ignored since it can be removed if the orientation of the antenna is known.

For short baselines the assumption is also often made that the ionospheric and tropospheric errors are eliminated by the formation of the double differences. Multipath is often ignored because it is generally small and difficult to model. The double difference observation equation can then be simplified to:

$$ \nabla \Delta \Phi_{ij}^{mn} = \rho_{ij}^{mn} + \lambda N_{ij}^{mn} + \varepsilon_{ij}^{mn} $$ (3.8)
which is referred to as the ionosphere-fixed model (refer to section 4.4.2). Even with short baselines the residual tropospheric may be significant in the case of large height differences between the two stations. Similarly, the ionospheric error will not cancel effectively with double differencing in the presence of high order ionospheric effects and ionospheric scintillation. For longer baselines residual ionospheric and tropospheric errors cannot be ignored. The double difference observation equation becomes:

\[ \nabla \Delta \Phi_{ij}^{mn} = \rho_{ij}^{mn} - I_{ij}^{mn} + \lambda N_{ij}^{mn} + \epsilon_{ij}^{mn} \] (3.9)

which is referred to as the ionosphere–float model (refer to section 4.4.1). Simplifying the notation by removing the \( \nabla \Delta \) operator and the subscripts for the station and superscripts for the satellites and writing the observation equation for both frequencies gives

\[ \Phi_{L1} = \rho - I + T + \lambda_{L1} N_{L1} + \epsilon_{L1} \] (3.10)
\[ \Phi_{L2} = \rho - \frac{f_{L1}^2}{f_{L2}^2} I + T + \lambda_{L2} N_{L2} + \epsilon_{L2} \] (3.11)

Due to the frequency dependency of the ionospheric delay given in equation (2.11), it is possible and convenient to express the ionospheric error on \( L2 \) as the product of the error on \( L1 \) multiplied by the ratio of the frequencies squared. The equivalent observation equations for the code pseudoranges are

\[ \Phi_{P1} = \rho + I + T + \epsilon_{P1} \] (3.12)
\[ \Phi_{P2} = \rho + \frac{f_{L1}^2}{f_{L2}^2} I + T + \epsilon_{P2} \] (3.13)

Equations (3.10) and (3.11) are the basis for all subsequent processing under the double difference model. The two error sources that require further modelling for ambiguities to be resolved over long baselines are the ionospheric and tropospheric path delays. In practice there will also be residual error due to imperfect orbits. However, since it is difficult to separate the tropospheric and orbit errors the terms are
often bundled together and referred to as geometric error. Two important things to note from equations (3.10) and (3.11) are:

1. The geometric error (due tropospheric delay and orbit error) is the same (in metres) on L1 and L2.
2. The ionospheric delay on L1 and L2 is related to the frequencies of the signals.

Therefore, if a combination of the observations on L1 and L2 is made it is possible to eliminate or further reduce the influence of geometric, ionospheric and tropospheric errors. Such combinations are the second fundamental mathematical modelling technique used in GPS processing.

3.2.4 Linear Combinations

One of the strengths of GPS is that each satellite-receiver range can be measured by both code and phase measurements on two frequencies. For example, if a receiver records L1 and L2 phase measurements plus P1 and P2 code measurements, each satellite-receiver range is measured four times. A linear combination of two or more of these measurements (as distinct from the differences described in section 3.2.2) may be used to estimate or eliminate errors in the GPS signals. For instance, a linear combination of code and phase measurements on a single frequency can be used to estimate the code multipath (Estey and Meertens 1999). Other linear combinations are used in the estimation of a receiver’s position and in ambiguity resolution.

Forming a linear combination from two or more observations can remove some errors and reduce the magnitude of others. However such combinations reduce the number of observations and may also magnify random errors such as noise. Therefore, if distance dependent errors such as the ionospheric delay are adequately cancelled by double differencing it is preferable to work directly with the L1 and L2 measurements rather than with combinations. At some point the distance dependent errors will become a limiting factor in ambiguity resolution. It is at this point that it may be preferable to form a linear combination of the observations. Typically this occurs when baselines are over 10 to 15 kilometres in length (Hugentobler et al. 2001).
A linear combination of two measurements, $\Phi_{LC}$, to the same satellite on different frequencies can be expressed as

$$\Phi_{LC} = a\Phi_{L1} + b\Phi_{L2} = \begin{bmatrix} a \\ b \end{bmatrix} \begin{bmatrix} \Phi_{L1} \\ \Phi_{L2} \end{bmatrix}$$  \hspace{1cm} (3.14)$$

A linear combination may be equivalently formed before or after single, double or triple differencing. For example, a linear combination of double differenced pseudoranges is

$$\Phi_{LC} = a\nabla\Delta\Phi_{ij}^{mn} + b\nabla\Delta\Phi_{ij}^{mn}$$  \hspace{1cm} (3.15)$$

$$= a\left(\Phi_{iL1}^m - \Phi_{jL1}^m - (\Phi_{iL2}^n - \Phi_{jL2}^n)\right) +$$

$$b\left(\Phi_{iL2}^m - \Phi_{jL2}^m - (\Phi_{iL1}^n - \Phi_{jL1}^n)\right)$$

$$= (a\Phi_{iL1}^m + b\Phi_{iL2}^m) - (a\Phi_{jL1}^m + b\Phi_{jL2}^m) - (a\Phi_{iL1}^n + b\Phi_{iL2}^n) -$$

$$\left( a\Phi_{iL1}^n + b\Phi_{iL2}^n \right)$$

For the ambiguities to retain their integer nature, $a$ and $b$ must also be integers. The frequency and wavelength of $\Phi_{LC}$ are given by (Hoffman-Wellenhof et al. 1994)

$$f_{LC} = a f_{L1} + b f_{L2}$$  \hspace{1cm} (3.16)$$

$$\lambda_{LC} = \frac{c}{f_{LC}} = a \lambda_{L1}^2 + b \lambda_{L2}^2$$  \hspace{1cm} (3.17)$$

where $c$ is the speed of light.

Two linear combinations commonly used for parameter estimation are the ionosphere-free (IF) and widelane (WL) linear combinations.
3.2.4.1 Ionosphere-Free Linear Combination

The IF combination is calculated using \( a = 1 \) and \( b = -\frac{f_{l2}}{f_{l1}} \) such that (Hoffman-Wellenhof et al. 1994)

\[
\Phi_{IF} = \Phi_{L1} - \frac{f_{l2}}{f_{l1}} \Phi_{L2}
\]  

(3.18)

where \( \Phi_{L1} \), \( \Phi_{L2} \) and \( \Phi_{IF} \) are the phase measurements in cycles. Equations (3.16) and (3.17) can be used to convert equation (3.18) from cycles to metres as follows,

\[
\Phi_{IF} = \lambda_{IF} \Phi_{IF}
\]

(3.19)

\[
= \frac{c}{f_{IF}} \left( \Phi_{L1} - \frac{f_{l2}}{f_{l1}} \Phi_{L2} \right)
\]

\[
= \frac{c f_{l1}}{f_{l1}^2 - f_{l2}^2} \left( \Phi_{L1} - \frac{f_{l2}}{f_{l1}} \Phi_{L2} \right)
\]

\[
= \frac{1}{f_{l1}^2 - f_{l2}^2} \left( c f_{l1} \Phi_{L1} - c f_{l2} \Phi_{L2} \right)
\]

Simplifying equation (3.19) using \( c = f_{\lambda} \) gives the standard form of the ionosphere-free linear combination,

\[
\Phi_{IF} = \frac{1}{f_{l1}^2 - f_{l2}^2} \left( f_{l1}^2 \Phi_{L1} - f_{l2}^2 \Phi_{L2} \right)
\]  

(3.20)

where \( \Phi_{L1} \), \( \Phi_{L2} \) and \( \Phi_{IF} \) are phase ranges in metres. The ionosphere-free linear combination is very useful for parameter estimation because it eliminates almost all (at least to a first order approximation) of the ionospheric delay. However, the noise on the IF observable is approximately three times greater than that of L1. A further disadvantage of the IF observable is that the ambiguity will be non-integer. This follows from (3.16) and (3.18), since \( b = -\frac{f_{l2}}{f_{l1}} \) is non-integer, the IF ambiguity will be non-integer. Also the L1 and L2 ambiguities become inseparable when the IF
observable is formed and so cannot be estimated directly. As such, the IF observable cannot directly be used to resolve the L1 and L2 integer ambiguities. To facilitate ambiguity resolution, IF may be used in conjunction with another combination, such as the widelane. For the above reasons the IF combination is generally used only for baselines greater than 10km or so where the ionospheric delay between two receivers will not be cancelled effectively by differencing. The IF combination is more typically used for estimation of the receiver’s position, with the integer ambiguities having either been previously fixed by another means or by treating them as real valued parameters. The observation equation for the DD ionospheric-free linear combination may be expressed as

$$\Phi_{IF} = \rho + T + B_{IF}$$ (3.21)

where $B_{IF}$ is often called the ionospheric-bias. The term bias is used to reflect the fact that the ambiguities are no longer integers. The ionospheric-bias is a function of the L1 and L2 integer ambiguities such that

$$B_{IF} = \frac{c}{f_{L1}^2 - f_{L2}^2} (f_{L1}N_{L1} - f_{L2}N_{L2})$$ (3.22)

### 3.2.4.2 Widelane Linear Combination

The widelane linear combination is calculated using $a = 1$ and $b = -1$ (Hoffman-Wellenhof *et al.* 1994)

$$\phi_{WL} = \phi_{L1} - \phi_{L2}$$ (3.23)

where $\phi_{WL}$ is a phase measurement in cycles. The widelane combination can be expressed in terms of phase ranges in metres using equations (3.16) and (3.17) such that

$$\Phi_{WL} = \lambda_{WL} \phi_{WL}$$ (3.24)
\[
\phi_{WL} = \frac{1}{f_{L1} - f_{L2}} \left( f_{L1} (\rho - I + T + \lambda_{L1} N_{L1}) - f_{L2} \left( \rho - \frac{f_{L1}^2}{f_{L2}^2} I + T + \lambda_{L2} N_{L2} \right) \right)
\]

(3.25)

\[
= \frac{1}{f_{L1} - f_{L2}} \left( \rho (f_{L1} - f_{L2}) - \frac{f_{L1}}{f_{L2}} I (f_{L1} - f_{L2}) + T (f_{L1} - f_{L2}) + c (N_{L1} - N_{L2}) \right)
\]

\[
= \rho + \frac{f_{L1}}{f_{L2}} I + T + \lambda_{WL} N_{WL}
\]

where \( N_{WL} = N_{L1} - N_{L2} \). Remembering that L2 is a lower frequency than L1, it is evident from equation (3.25) that the ionospheric delay on WL is actually larger than on L1 by a factor of -1.283 (Table 3.1). However, whilst the ionospheric error is increased, the wavelength of the WL observable is also increased 4.529 times compared to \( \lambda_{L1} \). Thus the ionospheric error is proportionally smaller and has less impact on the estimates of the widelane ambiguities. When the new L5 signal becomes available, a L2-L5 widelane combination with a wavelength of 5.861 metres can be formed. The L2-L5 widelane combination has a relative increase in the ionosphere of \(-1.719\) times L1 and a wavelength increase of 30.799 times L1. Based on these figures, Hatch et al. (2000) predicts that this will enable single-epoch WL ambiguity resolution over baselines of twenty or more kilometres. However, Hatch et al. (2000) and Mowlam et al. (2003) agree that near instantaneous ambiguity resolution will still be difficult for longer baselines.
If the widelane ambiguities are resolved, the determination of the L1 (and hence L2) ambiguities is greatly simplified. Equation (3.22) can be rearranged to give (Mervart 1995)

\[
B_{IF} = c \left( \frac{f_{L2}}{f_{L1}^2 - f_{L2}^2} N_{WL} + \frac{c}{f_{L1} + f_{L2}} N_{L1} \right)
\]

which shows that if the WL ambiguities can be resolved and substituted before the IF combination is formed it is possible to resolve the L1 ambiguities with the ionosphere-free linear combination. Once the L1 and WL ambiguities are known it is then a simple matter to calculate the L2 ambiguities. The term \( \frac{c}{f_{L1} + f_{L2}} \) in equation (3.26) gives the \( N_{L1} \) ambiguities an effective wavelength of approximately 10.7cm. Hence, the \( \frac{c}{f_{L1} + f_{L2}} N_{L1} \) ambiguities are often referred to as narrow-lane ambiguities.

### 3.2.4.3 Melbourne-Wübbena Linear Combination

If high quality dual-frequency code and phase measurements are available, then a linear combination can be formed that will eliminate the geometry (station and satellite coordinates), satellite and receiver clocks, ionosphere and troposphere. The combination is referred to as the Melbourne-Wübbena (MW) combination as it was proposed independently by Melbourne (1985) and Wübbena (1985). The MW combination is formed as

\[
\Phi_{MW} = \frac{1}{f_{L1} - f_{L2}} (f_{L1} \Phi_{L1} - f_{L2} \Phi_{L2}) - \frac{1}{f_{L1} + f_{L2}} (f_{L1} \Phi_{p1} + f_{L2} \Phi_{p2})
\]

where \( \Phi_{p1} \) and \( \Phi_{p2} \) are the P-code pseudoranges on L1 and L2 respectively. The right hand part of the expression is the code equivalent of the widelane combination, which, ignoring the noise term, may be expanded using (3.12) and (3.13) to

\[
\Phi_{WL,\text{code}} = \frac{1}{f_{L1} + f_{L2}} (f_{L1} \Phi_{p1} + f_{L2} \Phi_{p2})
\]
Using (3.25) and (3.28) equation (3.27) simplifies the MW observation equation to

\[ \Phi_{\text{MW}} = \lambda_{\text{WL}} N_{\text{WL}} \]  

(3.29)

Thus the Melbourne-Wübbena combination may be used to directly estimate the widelane ambiguities, which may then be substituted into equation (3.26) to allow the narrow-lane ambiguities to be estimated. The success of this technique is contingent upon having P-code data with a RMS of less than one metre (Hugentobler et al. 2001). Anti-spoofing and code multipath may limit the availability of suitably precise code measurements (Mervart 1995).

### 3.2.4.4 Geometry-Free (GF) Linear Combination

Another useful linear combination is the geometry-free (GF) linear combination, defined as (Blewitt 1989)

\[ \Phi_{\text{GF}} = \lambda_{\text{L1}} \phi_{\text{L1}} - \lambda_{\text{L2}} \phi_{\text{L2}} \]  

(3.30)

or,

\[ \Phi_{\text{GF}} = \Phi_{\text{L1}} - \Phi_{\text{L2}} \]  

(3.31)

The geometry-free linear combination cancels the receiver and satellite clock offsets, tropospheric error and geometry (station coordinates and orbit), leaving only multipath, ionospheric error and the initial phase ambiguities. As such, the geometry-free linear combination is very useful in estimating ionospheric delay. The
observation equation for the DD geometry-free linear combination may be expressed as

$$\Phi_{GF} = \frac{f_{L2}^2 - f_{L1}^2}{f_{L2}^2} I + \lambda_{L1} N_{L1} - \lambda_{L2} N_{L2}$$  \hspace{1cm} (3.32)

Like the IF combination, the ambiguities lose their integer nature when the GF combination is formed. However, the GF combination may be used to resolve the widelane ambiguities. Equation (3.32) can be rearranged to give (Blewitt 1989)

$$\Phi_{GF} = \frac{f_{L2}^2 - f_{L1}^2}{f_{L2}^2} I + \frac{f_{L1}^2 - f_{L2}^2}{f_{L1} f_{L2}} (\lambda_{WL} N_{WL} - B_{IF})$$  \hspace{1cm} (3.33)

where $B_{IF}$ is the ionospheric-bias of equation (3.22). Using equation (3.33) the widelane ambiguities can be expressed as

$$N_{WL} = \frac{1}{\lambda_{WL}} \left( \frac{f_{L1} f_{L2}}{f_{L2}^2 - f_{L1}^2} \left( \Phi_{GF} - \frac{f_{L2}^2 - f_{L1}^2}{f_{L2}^2} I \right) + B_{IF} \right)$$  \hspace{1cm} (3.34)

The ionospheric-bias can be estimated sufficiently well using the ionosphere-free linear combination to enable widelane ambiguity resolution with equation (3.34) (Blewitt 1989). However, as equation (3.34) is still affected by the ionosphere, this approach suffers from the same limitations as the widelane linear combination.

The contributions of geometry, ionospheric delay and noise on the different frequencies are shown in Table 3.1 relative to L1, both in metres and cycles of the respective wavelength. The MW combination has not been included because its noise is determined by the noise of the P-code not that of the phase observations. Note that because the cycles on IF are not integers, IF is shown as not being a wave. However, the formal wavelength of IF is approximately 0.484m. The important things to note from Table 3.1 are:
1. The ionospheric error on the widelane combination, whilst being larger in metres, is smaller in cycles than it is on L1 or L2. Thus ambiguity resolution with the widelane combination is less sensitive to ionospheric error than it is with L1 and/or L2.

2. The noise of all the linear combinations is greater than the noise on either L1 or L2.

The implication of Table 3.1 is that there is no ideal linear combination that eliminates all error sources and makes ambiguity resolution easy. Each linear combination has certain advantages and disadvantages. For this reason, a processing strategy may use a different linear combination for each stage of the processing.

<table>
<thead>
<tr>
<th>LC</th>
<th>λ (m)</th>
<th>a</th>
<th>b</th>
<th>Geometry (m)</th>
<th>Geometry (cycles)</th>
<th>Ionosphere (m)</th>
<th>Ionosphere (cycles)</th>
<th>Noise (m)</th>
<th>Noise (cycles)</th>
</tr>
</thead>
<tbody>
<tr>
<td>L1</td>
<td>0.190</td>
<td>1.000</td>
<td>0.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>L2</td>
<td>0.244</td>
<td>0.000</td>
<td>1.000</td>
<td>1.000</td>
<td>0.779</td>
<td>1.647</td>
<td>1.283</td>
<td>1.000</td>
<td>0.779</td>
</tr>
<tr>
<td>IF</td>
<td>-</td>
<td>1.000</td>
<td>-0.779</td>
<td>1.000</td>
<td>-</td>
<td>0.000</td>
<td>-</td>
<td>2.978</td>
<td>-</td>
</tr>
<tr>
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<td>-0.244</td>
<td>0.000</td>
<td>-</td>
<td>-0.647</td>
<td>-</td>
<td>1.414</td>
<td>-</td>
</tr>
<tr>
<td>WL</td>
<td>0.862</td>
<td>1.000</td>
<td>-1.000</td>
<td>1.000</td>
<td>0.221</td>
<td>-1.283</td>
<td>-0.283</td>
<td>5.742</td>
<td>1.268</td>
</tr>
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</table>

Table 3.1 Sensitivity of the Linear Combinations to Various Error Sources (adapted from Hugentobler et al. (2001))

### 3.3 STOCHASTIC MODELS

#### 3.3.1 Preamble

The stochastic model works in conjunction with the mathematical model and describes the statistical properties of the observations. The mathematical model expresses the expectation (mean or first moment) of the observations and the stochastic model describes the dispersion (variance or second central moment) (Wieser 2001). If a Gauss-Markov model, such as least squares, is used for parameter estimation, the expectation and dispersion are formulated as (Wieser 2001),
\[ E[\ell] = Ax \]  
\[ D[\ell] = E[\ell^T \ell] = C_\ell = \sigma^2 Q_\ell \] (3.35)  
(3.36)

where
- $E[.]$ denotes expectation,
- $D[.]$ denotes dispersion,
- $\ell$ is a vector of reduced observations, or measurements corrected by the mathematical model (observed minus computed observations),
- $Ax$ is the linearised mathematical model based on a design matrix $A$ of rank $u$ and the vector of $u$ functional parameters $x$,
- $C_\ell$ is the variance-covariance matrix of the $m$ reduced observations,
- $Q_\ell$ is a cofactor matrix that, when scaled by the variance factor $\sigma^2$, produces the variance-covariance matrix,
- $u$ is the number of unknowns,
- $m$ is the number of measurements.

The values along the leading diagonal of the variance-covariance (or simply covariance) matrix represent the variance of the corresponding observations. The off-diagonal terms are the covariances.

The sole aim of stochastic modelling is to correctly describe the random deviation of the observations from their expectation $Ax$, and thereby establish the covariance matrix $C_\ell$. If the vector of measurement errors from the true measurements is given by,

\[ v = Ax - \ell \]

then the covariance matrix must satisfy (Wieser 2001)

\[ D[v] = C_v = C_\ell = \sigma^2 Q_\ell \] (3.37)
\[ E[v] = Ax - E[\ell] = 0 \] (3.38)
However, in practical situations some of the observations will have deviations from $E[y] = 0$ that are non-random due to outliers or imperfect modelling of systematic effects. Outliers are individual measurements that vary greatly from the distribution attributed to them. In geodesy, measurements are commonly assumed to have a Normal distribution following the central limit theorem (Wieser 2001). A comprehensive stochastic model should account for both random and systematic deviations (Wieser 2001). Indeed, there is model equivalence between the mathematical and stochastic models. Thus a parameter may be explicitly estimated by the mathematical model or implicitly by the stochastic model (Blewitt 1998). Model equivalence is the basis of the pre-elimination technique described in section 5.3.2. Estimation of additional parameters, through either the mathematical model or the stochastic model, leads to a less precise solution for the parameters due to the lower redundancy.

Often much less emphasis is given to the stochastic model than the mathematical model because noise has less impact on the parameter estimates than most systematic errors. Also the stochastic model is often harder to quantify than the mathematical model. A study by Han and Rizos (1995) found that the parameter estimates differed by only a few millimetres when temporal correlations were alternatively included or ignored. However, ignoring these correlations was found to produce overly optimistic precisions for the estimated parameters. Other researchers have reached similar conclusions (Han 1997; Radovanovic et al. 2000; Radovanovic 2001a; Wang 1998). The impact of the stochastic model on the precision of the parameters was investigated by Barnes et al. (1998) and Barnes and Cross (1998) who reverse-engineered a data set and processed it with its empirically derived ‘true’ stochastic model. The precision of the positions and the fidelity of the quality estimates resulting from using the true stochastic model far exceeded the results obtained using traditional simplistic stochastic models. The influence of the stochastic model on the quality estimates of the estimated parameters has been shown to be critical for efficient and effective ambiguity resolution and validation due to its influence on the size and shape of the ambiguity search space (Euler and Ziegler 2000; Teunissen 1997b; Wang 1999; Wang et al. 2002). In general, an incorrect
A stochastic model will result in unreliable results, incorrect measures of precision and a reduced ability to detect errors.

Stochastic modelling of the GPS observables has been the focus of much research around the world. Before reviewing the existing models it is worth clarifying some terminology used with regard to stochastic modelling.

### 3.3.2 Definitions

#### 3.3.2.1 Stochastic (Random) Process

A random variable $x$ is a measurable characteristic of a real-world phenomenon that may have different realisations if the variable is measured repeatedly. A stochastic process $x(t)$ is a continuous family of random variables indexed to $t$ (usually time). A discrete random process (one which only occurs as the measurement is taken) is called a stochastic function.

A random variable contains both ‘signal’ and noise. The first order moment (expected value or arithmetic mean) of the random variable is assumed to satisfy the mathematical model. The noise characteristics are captured by the second moment (dispersion or variance if the mean is zero).

#### 3.3.2.2 Stationarity

A (strict) stationary process is one in which the statistical properties do not change over time. Therefore the first and higher order moments of the stationary stochastic processes $x(t)$ and $x(t + \tau)$ are independent of $t$. Therefore, $E[x(t)] = \mu$ for all $t \in \mathbb{R}^n$ and $Pr[x(t) \leq z]$ does not depend on $t$ (the distribution of $x$ is invariant) (Cressie 1991).

Stationarity up to order two, known as wide sense stationarity, means that the expectation and dispersion (mean and variance) are stationary. Wide sense stationarity for Normally distributed random variables (Gaussian processes) is identical to strict stationarity (Borre and Tiberius 2000).
3.3.2.3 **Ergodicity**

A random process is ergodic if every realisation of the process is representative of all other realisations of the process. Therefore the ensemble average (the average of all realisations) may be replaced by the time average from a single realisation. A random process must be stationary for it to be ergodic.

3.3.2.4 **Homoskedasticity**

A stochastic process is homoskedastic if its covariance matrix can be represented as $\mathbf{C} = \sigma^2 \mathbf{I}$ where $\mathbf{I}$ is an identity matrix (Cressie 1991). Therefore, homoskedasticity implies stationarity and ergodicity and that all random variables in the process have equal variance.

3.3.3 **Propagation of Variance**

The common and preferred procedure in GPS processing is to use a stochastic model for the one-way (undifferenced) observations to populate a covariance matrix of the observations ($\mathbf{C}_e$). The covariance matrix of the double difference observations ($\mathbf{C}_{\ell\ell}$) can then be calculated using the law of propagation of variance whereby if $\mathbf{x}$ and $\mathbf{y}$ are vectors satisfying the relationship $\mathbf{y} = \mathbf{A} \mathbf{x}$ with covariance matrices $\mathbf{C}_x$ and $\mathbf{C}_y$ respectively, then $\mathbf{C}_y = \mathbf{A} \mathbf{C}_x \mathbf{A}^T$. Therefore the covariance matrix of the DD observations is,

$$\mathbf{C}_{\ell\ell} = \mathbf{D} \mathbf{C}_e \mathbf{D}^T$$

where $\mathbf{D}$ is the differencing matrix from equation (3.4). By calculating $\mathbf{C}_{\ell\ell}$ in this manner the mathematical correlations resulting from the differencing process are rigorously accounted for.

3.3.4 **Stochastic Models Used in GPS Processing**

The stochastic models used in GPS processing software or proposed in the literature may be classified depending on whether they assume the observations to be: uncorrelated; have spatial correlation; or have both spatial and temporal correlations.
Individual models from each of these categories will be examined. A large amount of material has been published on stochastic modelling in GPS, however much of it is similar. The aim of this section is to understand the approaches that have been taken and their limitations. It is not intended to be an exhaustive analysis of all the individual models and variants in the literature.

3.3.4.1 Uncorrelated Measurement Models

Models within this class assume that there are no spatial or temporal correlations in the observations and that the expected value of the measurement error is zero. The model noise is assumed to be stationary and ergodic. The models all have the following characteristics,

\[ \begin{align*}
E[\epsilon_i(t)] &= 0 \quad \Rightarrow \text{the noise is zero mean} \\
E[\epsilon_i(t)\epsilon_j(t)] &= 0 \quad (i \neq j) \quad \Rightarrow \text{no spatial correlation} \\
E[\epsilon_i(t)\epsilon_i(v)] &= 0 \quad (t \neq v) \quad \Rightarrow \text{no temporal correlation}
\end{align*} \]

where \( \epsilon_i(t) \) is the measurement error in phase range \( i \) at epoch \( t \). The consequences of making these assumptions will become apparent as the individual models are reviewed.

3.3.4.1.1 Equal Variance Model

The equal variance model is the most simple as it assumes the observations are homoskedastic. Not surprisingly, the equal variance model is commonly used in GPS processing (Hoffman-Wellenhof et al. 1994; Strang and Borre 1997). Not only is it assumed that there are no correlations between the observations but that observations from all satellites, regardless of circumstances, have equal variance. Such an approach ignores differences in the noise characteristics of signals from different satellite-receiver combinations caused by signal propagation, antenna gain and imprecision of the tracking loops. The equal variance model is then given as,

\[ D[\epsilon_i(t)] = \sigma^2 \]
The covariance matrix for the one-way observations is therefore $C_j = \sigma^2 I$. This model may be acceptable if a high elevation mask is used. Signals from low elevation satellites pass through more of the atmosphere, which leads to a lower SNR, thus reducing the precision of the phase lock loop in equation (2.19). It is often desirable to use the signals from low elevation satellites to improve redundancy and intersection geometry (and hence DOP). Therefore, if a lower elevation mask is to be used a more sophisticated stochastic model should also be employed.

3.3.4.1.2 Elevation Dependent Models
A number of elevation dependant stochastic models have been published. In these models the variance of the measurements is related to the elevation (or zenith) angle of the satellite. The stochastic model is specified as,

$$D[e_i(t)] = \sigma_i(t)^2 = f(\alpha_i)$$

where $\alpha$ is the elevation of the satellite and $f(\ldots)$ is a mathematical function. As discussed above, the precision of GPS observations is related to the elevation of the satellite. Elevation dependence can be demonstrated empirically using data from a zero-baseline. A zero-baseline is achieved by connecting two GPS receivers to a single antenna using an antenna splitter. When the double differences are formed all non-random errors will cancel. Twenty-four hours of 1Hz data were collected using two Leica GRX1200 Pro receivers and an AT504 choke ring antenna. Custom developed software was used to form double differences and to resolve the integer ambiguities. The resulting data, which consists of approximately 830,000 L1, L2 and ionosphere-free DD phase residuals, were partitioned based on satellite elevation. The standard deviations for each partition are shown in Figure 3.3. Clearly, observations from low elevation satellites display larger random errors than those from high elevation satellites, particularly for L2 and IF, confirming the hypothesis of elevation dependent stochastic model.
The elevation dependence is a by-product of the relationship between measurement noise and the SNR as shown in equation (2.19). Lower SNR values occur when the signals propagate through more of the atmosphere. Therefore the elevation dependence is simply an abstract way of using SNR to model the measurement noise. The strong relationship between SNR and satellite elevation can clearly be seen in the zero baseline data (Figure 3.4). The differences between L1 and L2 SNR values are due to the lower L2 transmission power and the codeless measurement technique required to track L2 under AS conditions. Using elevation instead of SNR may be preferable in certain circumstances. For instance, the SNR values are not transmitted in most real time correction formats - SNR is supported in RTCM v3.0 messages 1002, 1004, 1010 and 1012 but not in CMR, CMR+, Leica and RTCM v2.x). For post processing applications the Receiver Independent Exchange Format (RINEX) format is used. The SNR observations S1 and S2 were added to RINEX in v2.10, but are still not very widely used. Most RINEX files contain only mapped SNR values as integers between 0 to 9, making the data less meaningful and also causing unrealistic steps in the computed variances.
A number of elevation dependent weighting functions have been proposed. Han (1997) gives the model,

$$\sigma = s(a_0 + a_1 \exp(-\alpha/\alpha_0))$$ \hspace{1cm} (3.43)

where \(s\), \(a_0\), \(a_1\) and \(\alpha_0\) may be determined empirically. The Bernese GPS processing software v4.2 (Hugentobler et al. 2001) also uses an elevation dependent model based on the cosine of the zenith angle \((z = 90 - \alpha)\),

$$\sigma = \frac{\sigma_0}{\cos^2(z)}$$ \hspace{1cm} (3.44)

to map noise from a zenith value \(\sigma_0\) to a given zenith angle. The zenith value \(\sigma_0\) may be chosen based on the measurement precision of the receiver specified by the manufacturer.
3.3.4.1.3 SNR Models

Equation (2.19) illustrates the relationship between PLL jitter and SNR. As an illustration of this relationship, Figure 3.5 shows the precision of the data from the zero baseline used in the previous section versus the measured SNR. Low SNR values are clearly associated with less precise phase measurements.

![Figure 3.5 DD phase precision versus signal to noise ratio](image)

Based on this relationship, SNR is clearly useful for model measurement noise. Additionally, because multipath and signal diffraction, like signal noise, are usually highest on low elevation satellites, SNR can also be used to model multipath stochastically. Brunner et al. (1999) propose the SIGMA – ε model, which uses SNR to model measurement noise as

\[
\sigma_i^2 = C_i . 10^{-\frac{(C/N_o)}{10}}
\]  (3.45)

where \( C_i \) is a constant related to the bandwidth of the tracking loop of receiver \( i \) and \( C/N_o \) is the carrier to noise ratio from equation (2.18). A similar approach has been taken by Lau and Mok (1999). Brunner et al. (1999) adapts the SIGMA – ε model to
account for signal diffraction by including the difference between the measured carrier-to-noise ratio and the expected (template) value for a clear signal at the same elevation and azimuth. The adapted model, known as \( \text{SIGMA} - \Delta \) is given as,

\[
\sigma^2_i = C_i 10^{(C/N,\text{measured} - \alpha \Delta)/10} \quad (3.46)
\]

with \( \Delta = C/N_{0,\text{template}} - C/N_{0,\text{measured}} \) and the constant \( \alpha \) is included to allow for scaling based on empirical results. Tests by Brunner et al. (1999) show that the \( \text{SIGMA} - \Delta \) model, using \( \alpha = 2 \), produces superior results to the equal variance model. Using smaller values of \( \alpha \) increases the bias (difference between measured and truth value) of the baseline.

Tests conducted by Satirapod and Wang (2000) found that both SNR and satellite elevation are generally suitable as quality indicators, but do not necessarily reflect reality. However, the authors concluded that SNR is a more realistic indicator than satellite elevation.

### 3.3.4.2 Spatially Correlated Measurement Models

Models within this class assume that spatial correlation exists between the observations in a single epoch. They assume that no between-epoch (temporal) correlations are present. As such, the models have the following characteristics

\[
\begin{align*}
E[e_i(t)] &= 0 & \Rightarrow \text{the noise is zero mean} \\
E[e_i(t)e_j(t)] &= \sigma_{ij}(t) & \Rightarrow \text{spatial correlation} \\
E[e_i(t)e_j(v)] &= 0 \quad (t \neq v) & \Rightarrow \text{no temporal correlation}
\end{align*}
\quad (3.47)
\]

#### 3.3.4.2.1 Baseline Length Models

Schaffrin and Bock (1988) derive stochastic models for the L1 and L2 phase observations and the residual ionosphere after double differencing, \( C_{\ell_1}, C_{\ell_2} \) and \( C_{\text{iono}} \) respectively, such that the double difference covariance matrices \( \sigma^2_1 DC_{\ell_1} D^T \), \( \sigma^2_2 DC_{\ell_2} D^T \) and \( \sigma^2_{\text{iono}} DC_{\text{iono}} D^T \) are representative of spatial de-correlation with
increasing baseline length. The terms $\sigma_i^2$, $\sigma_j^2$ and $\sigma_{\text{iono}}^2$ are a priori variances of unit weight for each observable. The principle is that there is baseline dependence to the errors because as the baseline length increases less of the atmospheric error is cancelled (due to lower spatial correlation). The covariance matrices $\sigma_i^2 D_C i_i D^T$, $\sigma_j^2 D_C j_j D^T$ and $\sigma_{\text{iono}}^2 D_C \text{iono} D^T$ are therefore intended to reflect both measurement noise and unmodelled errors by becoming less precise as the baseline length increases. Schaffrin and Bock (1988) represent this baseline dependence by,

$$\sigma_i^2 = 4[a^2 + b^2s_{ij}^2]$$  \hspace{1cm} (3.48)

where $a$ and $b$ are constants, $s$ is the distance in millimetres between stations in $i$ and $j$ and the factor 4 results from having four phase measurements contributing to the double difference. Suggested values for the constants, for baselines longer than approximately 50km, are $a = 10\text{mm}$ and $b = 10^{-8}$.

The general form, $C$, of the covariance matrices for the four phase measurements from receivers A, B and satellites $r$, $s$ contributing to the double difference, as derived by Schaffrin and Bock (1988) is,

$$C = \begin{bmatrix}
\sigma_A^2 & 0 & \sigma_{AB} & 0 \\
0 & \sigma_A^2 & 0 & \sigma_{AB} \\
\sigma_{BA} & 0 & \sigma_B^2 & 0 \\
0 & \sigma_{BA} & 0 & \sigma_{B}^2
\end{bmatrix} = \alpha\beta^2 \begin{bmatrix}
1 & 0 & z & 0 \\
0 & 1 & 0 & z \\
z & 0 & 1 & 0 \\
0 & z & 0 & 1
\end{bmatrix} + \beta^2(1-\alpha) \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}$$  \hspace{1cm} (3.49)

where $\sigma_A^2$ is the variance of the measurement from receiver A to satellite r, $\sigma_{AB}$ is the covariance between the measurements from receivers A and B to satellite r,

$$z = \frac{2}{e^{-c^2\psi} + e^{c^2\psi}} = \text{sech}(c^2\psi), \quad \psi \quad \text{is the arc-length along the earth’s surface between}$$
the two receivers and $\alpha$, $\beta$ and $c$ are constants. The constants $\alpha$ and $\beta$ are selected to ensure $C$ is positive-definite, as such $\beta^2 > 0$ and $0 \leq \alpha \leq 1$. Suggested values for the constants for baselines over 50km, derived during a solar minimum, are (Schaffrin and Bock 1988),

$$\alpha = 1 - \frac{a^2}{\beta^2}$$

$$\beta^2 = a^2 + 0.3(10^4 \text{mm})^2$$

The constant $c$, which is defined by $c^2 = \ln(1 + \sqrt{2})/1.57$, ensures that the point of inflection in the sech function is not reached. With this form the DD covariance matrices are (Schaffrin and Bock 1988),

$$\text{DCD}^T = 4\beta^2(1 - \alpha \text{sech}(c^2\psi))$$

(3.50)

For a zero baseline ($\psi = 0$), the correlations are perfect (i.e. $z = 1$) and equation (3.50) reduces to $4a^2$. The correlation decreases exponentially with baseline length, leading to increasingly higher variances at the double difference level.

The covariance matrix of the observations,

$$C = \begin{bmatrix}
\beta^2 & 0 & \alpha \beta^2 \text{sech}(c^2\psi) & 0 \\
0 & \beta^2 & 0 & \alpha \beta^2 \text{sech}(c^2\psi) \\
\alpha \beta^2 \text{sech}(c^2\psi) & 0 & \beta^2 & 0 \\
0 & \alpha \beta^2 \text{sech}(c^2\psi) & 0 & \beta^2 \\
\end{bmatrix}$$

(3.51)

contains both a constant and distance dependant part following equation (3.48). However, Bock (1998) simplifies equation (3.51) further by assuming that the distance dependent term is negligible with modern precise orbits. Therefore the covariance matrix of the observations is simply,
For the ionosphere, Bock (1998) neglects the constant term of equation (3.48) by selecting $\alpha = 1$ such that,

$$
\begin{bmatrix}
\beta^2 & 0 & \beta^2 \cdot \text{sech}(c^2 \psi) & 0 \\
0 & \beta^2 & 0 & \beta^2 \cdot \text{sech}(c^2 \psi) \\
\beta^2 \cdot \text{sech}(c^2 \psi) & 0 & \beta^2 & 0 \\
0 & \beta^2 \cdot \text{sech}(c^2 \psi) & 0 & \beta^2
\end{bmatrix}
$$

(3.53)

For stations which are close together, where the ionosphere is highly correlated, the ionospheric constraints are tightened (the diagonal values of $C_{\text{iono}}$ are small). For distant stations, the ionospheric model is relaxed (the diagonal values of $C_{\text{iono}}$ are large), allowing the solution to approach the ionosphere-float solution.

Importantly, the baseline length, in conjunction with some empirically derived constants, is relied upon to fully describe the stochastic properties of the double difference observations. All observations are assumed to have equal variance and correlations, regardless of satellite elevation or SNR. It should also be noted that, due to the form of equation (3.49), which was chosen to ensure that the resulting DD covariance matrix would be symmetric-positive-definite for the general case of multiple satellites and stations, it is very difficult to incorporate between-satellite covariances should this be desired.

### 3.3.4.2.2 Angular Separation Models

Radovanovic (2001b) developed a stochastic model for the residual troposphere error in which correlation is related to the angular separation between the satellites. A tropospheric zenith variance is mapped to an arbitrary elevation using the same mapping function as used for mapping the tropospheric zenith path delay (see section 4.5). The zenith variance is calculated as,
\[ \sigma_t^2 = m(\alpha_t^r)^2 \sigma_t'^2 \] (3.54)

where \( \sigma_t^2 \) is the tropospheric error variance for receiver \( r \) and satellite \( s \). \( m \) is the mapping function, \( \alpha_t \) is the elevation angle and \( \sigma_t'^2 \) is the tropospheric zenith variance which is estimated or determined empirically. Radovanovic (2001b) also considered three types of correlation in the stochastic model. The first, between-satellite covariance for satellites \( s \) and \( t \) is expressed as,

\[ \sigma_{st}^2 = m(\alpha_t^s)\sigma_t'^2 \exp(-\theta/\Omega) \] (3.55)

where \( \theta \) is the angle of separation calculated by,

\[ \cos\theta = \sin \alpha_t^s \sin \alpha_t^t + \cos \alpha_t^s \cos \alpha_t^t \cos(A_t^s - A_t^t) \] (3.56)

with \( A_t^s \) and \( A_t^t \) representing the azimuth of each satellite. The correlation angle \( \Omega \) is determined empirically. A similar approach is taken with between-receiver covariances between receivers \( q \) and \( r \), except that the baseline length \( D \) and a correlation distance \( d \) are used such that,

\[ \sigma_{qr}^2 = m(\alpha_q^r)\sigma_t'^2 \exp(-d/D) \] (3.57)

The third type of correlation considered by Radovanovic (2001b) was between measurements with no common receivers or satellites (which may be relevant for multi-baseline positioning). For this case a combination of the above two models was used, resulting in the covariance,

\[ \sigma_{qr}^{st} = m(\alpha_q^s).m(\alpha_t^r).\exp(-\theta/\Omega).\exp(-d/D)\sigma_t'^2 \] (3.58)

Test results from Radovanovic (2001b) using a network of nine reference stations, show an improvement in both positioning accuracy and reliability of the quality estimates when the above stochastic model is used, compared to the equal variance model.
3.3.4.2.3 Empirical Models

Wang et al. (1998a) used the procedure of Minimum Norm Quadratic Unbiased Estimation (MINQUE) to generate an empirical DD covariance matrix using residuals from an adjustment based on a standard (equal variance) stochastic model. The MINQUE estimation technique works on the basis that the DD covariance matrix is linear in terms of its parameters and can therefore be written as (Rao 1971),

\[ C = \theta_1 T_1 + \theta_2 T_2 + \ldots + \theta_k T_k = \sum_{i=1}^{k} \theta_i T_i \]  \hspace{1cm} (3.59)

where \( k \) is the number of variance-covariance components \( \theta_i \) that are to be estimated and \( T_i \) are the so-called accompanying matrices. For example, in the equal variance model (section 3.3.4.1.1) where \( C = \sigma^2 \mathbf{I} \) and \( C_{\ell\ell} = D C_i D^T = \sigma^2 \mathbf{D}^T \mathbf{D} \) then \( \theta_i = \sigma^2 \) and \( T_i = \text{diag}(\mathbf{D}^T) \) and \( k = 1 \).

Wang et al. (1998a) uses this approach to produce three different stochastic models A, B and C. Model A uses the equal variance approach and is the most simple. Model B uses an elevation dependant model similar to equation (3.43) to give a different variance to each satellite, but still ignores any spatial correlations between measurements. Model C is the most comprehensive, calculating every element in the DD covariance matrix for each epoch (no temporal correlations were considered). Model C addresses both mathematical and spatial correlations in the data. Tests conducted by Wang et al. (1998a) show that Model C offers significant improvements in ambiguity resolution over the equal variance model or models that assume only mathematical correlation. The down side of this approach is that generation of the empirical covariance matrices using MINQUE requires significant computational overhead. Wieser (2001) warns that using a technique such as MINQUE to estimate a large part of the covariance matrix without imposing physically meaningful constraints may be inappropriate. The same comment may be made regarding all approaches that use filtering of residuals to extract the unmodelled systematic error, such as the wavelet technique of Satirapod et al. (2001).
3.3.4.3 *Spatially and Temporally Correlated Measurement Models*

Models within this class allow for physical correlation between measurements within a single epoch and between measurements from different epochs. As such, the models have the following characteristics,

\[
E[\varepsilon_i(t)] = 0 \quad \Rightarrow \text{the noise is zero mean} \quad (3.60)
\]

\[
E[\varepsilon_i(t)\varepsilon_j(t)] = \sigma_{ij}(t) \quad \Rightarrow \text{spatial correlation exists}
\]

\[
E[\varepsilon_i(t)\varepsilon_i(t + \tau)] = E[\varepsilon_i(t)\varepsilon_i(t + \tau)] = \sigma_i(t, t + \tau)
\]

\[
\Rightarrow \text{temporal correlation exists}
\]

3.3.4.3.1 *Elevation Dependant Models*

Radovanovic *et al.* (2000) and Radovanovic (2001a) apply the elevation dependent model to multipath error. Short baselines (<10m) were used in a test scenario similar to a zero baseline. Over a short baseline atmospheric and antenna offset errors cancel leaving only multipath and noise. The behavior of measurement noise is well understood, so multipath error may be analysed by removing the measurement noise profile from the overall noise profile of the residuals. Using this approach Radovanovic *et al.* (2000) found an apparent link between satellite elevation and multipath variance at both the DD and the undifferenced phase observation level.

For each site a multipath zenith variance is calculated empirically using a very short baseline over which atmospheric errors will almost entirely cancel, leaving only multipath and noise. If the noise characteristics of the receiver are known, then the stochastic behaviour of the multipath can be determined by removing the noise profile. Working off the basis that multipath is higher for low elevation satellites, a mapping function is used to map zenith variance to an arbitrary elevation, such that,

\[
\sigma_m^2 = \sigma_{m,0}^2 m(\alpha) \quad (3.61)
\]

where \( \sigma_m^2 \) is the multipath variance for a single phase observation, \( \sigma_{m,0}^2 \) is the zenith multipath variance, \( m \) is a mapping function and \( \alpha \) is the satellite elevation. The
mapping function used by Radovanovic (2001a) was derived from the antenna gain pattern and the expected temporal correlation of the multipath was modelled as an exponential delay. Radovanovic (2001a) found no link between correlation length and satellite elevation. Tests results by Radovanovic (2001a) found that modelling of multipath and noise as a function of satellite elevation had little effect on the estimated parameters but greatly affected the accuracy of the associated quality estimates. The conclusion from the tests was that ignoring temporal correlations had the biggest influence on the reliability of the quality estimates.

The notion of mapping multipath error or variance from a zenith value is conceptually questionable. In the typical situation where a receiver is in the open, i.e. with no trees or other objects above it, the multipath error and variance at zenith should be zero. No reflected or attenuated signals should arrive at the zenith of the antenna in such a situation. Also the mapping function used by Radovanovic (2001a) assumes the multipath variance depends only on the elevation angle. As such the reflector geometry, which is one defining characteristic of the multipath at a site, does not influence the multipath variance. That is to say, the stochastic model assumes that the multipath variance is azimuthally isotropic, which it obviously is not.

3.3.4.3.2 Empirical Models

Wang et al. (2002) model the temporal and spatial correlation between DD observations using a first-order autoregressive model,

\[
\begin{bmatrix}
\varepsilon_1(t) \\
\varepsilon_2(t) \\
\vdots \\
\varepsilon_n(t)
\end{bmatrix} =
\begin{bmatrix}
\rho_{11} & \rho_{12} & \cdots & \rho_{1n} \\
\rho_{21} & \rho_{22} & \cdots & \rho_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
\rho_{n1} & \rho_{n2} & \cdots & \rho_{nn}
\end{bmatrix}
\begin{bmatrix}
\varepsilon_1(t-1) \\
\varepsilon_2(t-1) \\
\vdots \\
\varepsilon_n(t-1)
\end{bmatrix} +
\begin{bmatrix}
u_1(t) \\
u_2(t) \\
\vdots \\
u_n(t)
\end{bmatrix} \tag{3.62}
\]

or

\[
\varepsilon(t) = R\varepsilon(t-1) + u(t) \tag{3.63}
\]
where $\epsilon_i(t)$ is the error in a DD observation, $\rho_a$ represents the temporal correlation and $\rho_i$ represents inter-temporal correlation between different DD receiver-satellite pairs. A random variable $u_i(t)$ has been added to model the random noise component and is assumed to be uncorrelated in space or time such that,

$$E[u_i(t)u_j(t)] = 0 \quad (i \neq j)$$
$$E[u_i(t)u_j(t)] = 0 \quad (t \neq v)$$

and therefore its covariance matrix $C_u$ has a simplified structure. Wang et al. (2002) transforms the error vector $\epsilon$ into the new vector $u$ using a transformation matrix $G$ such that,

$$Ge = u$$

(3.65)

The basic least squares model (refer to section 3.5.2) of equation (3.69) is then re-written as,

$$\tilde{1} = \tilde{A}x + u$$

(3.66)

where $\tilde{1} = GI$ and $\tilde{A} = GA$. An iterative approach is used to estimate the correlation matrix $R$ and the transformation matrix $G$ beginning with residuals from an ambiguity float adjustment based on a standard (equal variance) stochastic model. The MINQUE procedure is then used to estimate the variance and covariance elements for each epoch as discussed previously.

Using three test baselines, Wang et al. (2002) compared two variants (C and D) of this model with the standard equal variance model (A) and a model that derived temporal correlation from an exponential function (B). The first variant (C) assumed the inter-temporal correlations were zero, the second variant (D) used the first-order autoregression model of equation (3.63). The results obtained indicated that both models C and D offered improvements in integer ambiguity resolution and validation.
However, the results were not conclusive as the reliability of the ambiguity resolution decreased for two of the baselines. Since the residuals were more random using models C and D, Wang et al. (2002) argues that the test statistics from these models were more realistic. The validity of the transformation was verified because the temporal correlation coefficients of the transformed measurements were much closer to zero.

### 3.4 Pre-processing and Quality Control

#### 3.4.1 Preamble

A critical step in high precision positioning is data screening and quality control. Outliers and undetected cycle slips will bias the estimation process, leading to non-optimal results. Mis-specifications in the mathematical or stochastic models will also cause erroneous results. Thus some model validation technique is required. In real-time kinematic positioning, a sophisticated testing procedure such as the detection, identification and adaptation algorithm (DIA), see e.g. Salzmann (1993), is required. In long baseline processing, which is the focus of this thesis, a static receiver must be used, at least for high precision applications. Thus a simpler testing procedure than DIA may be used. This section briefly outlines the basic data cleaning steps required for geodetic positioning using GPS. In geodetic GPS processing, selection of the appropriate mathematical and stochastic models is often the responsibility of the person doing the processing. No single processing strategy is suitable for all situations. Processing strategies applicable to long baseline positions are reviewed in Chapter 4.

Quality control can also be used for site/receiver performance monitoring and plays an important role in geodetic or other high accuracy uses of permanent GPS reference stations (Brown et al. 2003b). Monitoring the receiver’s tracking and measurement performance helps to identify hardware, site location and environmental issues (such as RF interference) which may impact on the quality of the collected data. Quality indicators such as data completeness (including satellite acquisition and tracking on L1 and L2, obstructions, data outages), number of cycle slips, code
multipath RMS (MP1, MP2), receiver clock stability and signal to noise ratios are important for such monitoring.

3.4.2 Cycle Slip Detection and Repair

Discontinuities in the phase observations, referred to as cycle slips, must be detected if high precision results are sought. Often breaks in the tracking of a satellite will result in cycle slips of hundreds or thousands of cycles, which are quite easy to detect. However, slips of only a few cycles also occur, and these are more difficult to find. Also, in areas of high ionospheric activity, such as equatorial and polar regions, rapid fluctuations in the TEC may be interpreted by the receiver as small cycle slips (Hugentobler et al. 2001). Under such conditions, processing software that relies on the flagging of cycle slips by the receiver may unnecessarily introduce a new ambiguity parameter. Alternatively, a receiver may fail to detect a small cycle slip. Thus the processing software should perform its own cycle slip detection to ensure cycle slips are correctly identified and dealt with.

If possible, cycle slips should be repaired by correcting all phase measurements after the slip by the appropriate number of lost integer cycles. If cycle slip repair is not possible, another ambiguity parameter must be created adding to the number of unknowns to be estimated. Cycle slips may be detected and repaired using undifferenced, double differenced or triple differenced observations. The geometry-free and widelane linear combinations are also commonly used in cycle slip detection. Often a combination of approaches is used to ensure the detected cycles slips are valid and have been correctly repaired.

3.5 PARAMETER ESTIMATION

3.5.1 Preamble

Least squares estimation and Kalman filtering (for kinematic data) are the techniques commonly used to estimate coordinates and other parameters from GPS measurements. This is true for both absolute and relative positioning techniques. Least squares estimation is used because it is able to provide optimal, unbiased estimates
from an over-determined system of observations and is computationally simple. However, least squares and Kalman filtering require correct weighting of the measurements to provide unbiased estimates of the parameters. Residual and unmodelled systematic errors in the mathematical model and correlation between the GPS observables make the determination of the covariance matrix (and hence the weight matrix) difficult (Barnes et al. 1998; Keenan and Cross 2001; Wang 1999). Thus careful selection of the mathematical and stochastic models is critical to achieving optimal parameter estimates. In the previous sections some basic mathematical and stochastic models used in GPS positioning were presented. This section will briefly cover the basics of least squares estimation. Least squares is the fundamental basis for the modified algorithms used in Chapter 4 that have been developed to overcome practical difficulties and improve efficiency in long baseline GPS processing.

### 3.5.2 Least Squares Estimation

The least squares (LS) algorithm is commonly used to estimate coordinates and other parameters along with their precision from GPS measurements. An observation equation of the form,

$$\ell = f(x)$$  \hspace{1cm} (3.67)

is constructed where,

- $\ell$ is an observation,
- $x$ is a vector of u parameters, and
- $f(\ldots)$ is a non-linear mathematical model.

The vector of parameters may contain a mixture of parameters of interest and so-called nuisance parameters. Nuisance parameters are parameters that are of no direct interest but which must be solved in order to obtain good estimates for the parameters of interest. Examples of possible nuisance parameters include integer ambiguities, receiver clock offsets and tropospheric delay estimates.
Typically, geodetic measurements are non-linear in terms of the parameters and must be linearised to estimate the parameters using least squares. A first order Taylor’s series is the method of choice for linearisation. After linearisation, the linearised observation equation,

\[
\ell - f(x'_1, x'_2, ..., x'_u) = \frac{\partial f}{\partial x_1} \Delta x_1 + \frac{\partial f}{\partial x_2} \Delta x_2 + ... + \frac{\partial f}{\partial x_u} \Delta x_u
\]  

(3.68)

results where \( x'_1, x'_2, ..., x'_u \) are approximate values (\textit{a priori} estimates) of the receiver coordinates and other parameters. The terms \( \Delta x_1, \Delta x_2, ..., \Delta x_u \) are increments to the approximate values and are the unknowns being sought in the LS solution. Therefore, \( x_1 = x'_1 + \Delta x_1, \ x_2 = x'_2 + \Delta x_2 \), and so on, or in vector form \( \mathbf{x} = \mathbf{x}' + \Delta \mathbf{x} \).

The \( m \) measurements are then represented in linear form using,

\[
\ell = A\mathbf{x} + \mathbf{e}
\]  

(3.69)

or

\[
\mathbf{v} = A\mathbf{x} - \ell'
\]  

(3.70)

where,

\( \ell' \) is a \((m \times 1)\) vector of reduced (observed minus computed) observations \( \ell' = \ell - f(x_0) \),

\( A \) is a \((m \times u)\) design matrix containing the partial derivatives \( \frac{\partial f}{\partial x_{x=x_0}} \) of \( f(x) \) with respect to the parameters,

\( \mathbf{x} \) is a \((u \times 1)\) vector of parameters (increments to the \textit{a priori} estimates), and

\( \mathbf{e} \) is a \((m \times 1)\) vector of measurement errors,

\( \mathbf{v} \) is a \((m \times 1)\) vector of residuals.
A \((m \times m)\) covariance matrix \(C_r\) is constructed based on the stochastic model to describe the statistical behaviour of the observations in the context of the mathematical model. The diagonal elements of \(C_r\) are the variances \((\sigma_r^2)\) of the measurements.

The least squares optimised solution to a redundant (over determined) system of linear equations is obtained when (Mikhail 1976),

\[
y^T P y = \text{minimum} \tag{3.71}
\]

where \(P\) is the weight matrix, defined as the inverse of the variance-covariance matrix \((P = C_r^{-1})\).

The least squares optimised solution for the parameters is given by solving the set of linear equations,

\[
Nx = b \tag{3.72}
\]

where \(N = A^T PA\) is known as the normal matrix and \(b = A^T P\ell\) is referred to as the right hand side (RHS) of the normal equations. If the normal matrix is non-singular, there will exist a matrix such that,

\[
NN^{-1} = I
\]

If the solution is under-determined, such that \(m < u\), or the normal matrix \((N)\) is not of full rank, then no solution for the parameters will exist. If the normal matrix is not singular, the solution for the parameters is,

\[
\hat{x} = N^{-1} b = (A^T PA)^{-1} A^T P\ell \tag{3.73}
\]
and has the properties,

\[ E[\hat{x}] = E[\hat{x}] = \hat{x} \]  
(3.74)

\[ D[\hat{x}] = \sigma_0^2 (A^T PA)^{-1} = \sigma_0^2 Q = C \]  
(3.75)

where \( \sigma_0^2 \) is the variance factor, \( \hat{v} = A\hat{x} - \ell \) are the least squares residuals and \( (m - u) \) is the degrees of freedom (DOF). Thus the expected values of the parameters are the actual values of the parameters themselves, meaning the solution is unbiased. The factor \( \sigma_0^2 \) is often known in advance because the dispersion of the observations is known, if only approximately and is referred to as the a priori variance factor. An \textit{a posteriori} variance factor,

\[ \hat{\sigma}_0^2 = \frac{\hat{v}^T P \hat{v}}{(m - u)} \]  
(3.76)

can be calculated and used as a test value to detect model errors. If the mathematical and stochastic models are correct and the observations are normally distributed, the test statistic

\[ f \frac{\hat{\sigma}_0^2}{\sigma_0^2} \approx \chi_i^2 \]  
(3.77)

has a \( \chi^2 \) distribution with \( f = m - u \) degrees of freedom.

In some circumstances it may be necessary to constrain a particular parameter \( x \). Constraints may be used if \textit{a priori} information is available on the parameter or if the solution is under-determined. The constraint may be applied by creating an artificial observation known as a pseudo-observation with a given variance (Leahy 1999). The constraint observation equation takes the form,

\[ \hat{x} = x \]  
(3.78)
or in reduced form,

\[ \hat{x} - x' = x \]  

(3.79)

The constraint observation is appended to the system of observation equations (3.70), increasing the number of observations and the degrees of freedom by one. Similarly it is possible to constrain one parameter relative to another. This may be necessary, for example, when estimating tropospheric error. The tropospheric delay is expected to change slowly. Thus a more stable solution may result if subsequent tropospheric parameters are constrained at least to some extent to the preceding estimate.

If the observations can be partitioned into a number of independent sets, each set may be processed separately. The full solution, equivalent to processing all observations together, can be obtained by adding the normal equations. The sets are only independent if \( C_\ell \) has a block diagonal structure. If the observations are separated into two sets,

\[
\begin{align*}
\mathbf{v} = \mathbf{Ax} - \ell &= \begin{pmatrix} \mathbf{v}_1 \\ \mathbf{v}_2 \end{pmatrix} = \begin{pmatrix} \mathbf{A}_1 \\ \mathbf{A}_2 \end{pmatrix} \mathbf{x} - \begin{pmatrix} \ell_1 \\ \ell_2 \end{pmatrix}, \quad C_\ell = \begin{pmatrix} C_1 & 0 \\ 0 & C_2 \end{pmatrix}
\end{align*}
\]

(3.80)

the solution, equation (3.73), can be re-written as,

\[
\hat{x} = (\mathbf{A}_1^T \mathbf{P}_1 \mathbf{A}_1 + \mathbf{A}_2^T \mathbf{P}_2 \mathbf{A}_2)^{-1} (\mathbf{A}_1^T \mathbf{P}_1 \ell_1 + \mathbf{A}_2^T \mathbf{P}_2 \ell_2)
\]

(3.81)

\[
= (\mathbf{N}_1 + \mathbf{N}_2)^{-1} (\mathbf{A}_1^T \mathbf{P}_1 \ell_1 + \mathbf{A}_2^T \mathbf{P}_2 \ell_2)
\]

Using the addition of normal equations approach it is possible to reduce the dimensions of the normal equation matrix that must be stored and inverted to obtain the parameter estimates. This results in a very efficient computation. Addition of normal equations is an important part of the pre-elimination technique described in section 5.3.2.
3.6 AMBIGUITY RESOLUTION

3.6.1 Preamble

In section 2.3.2 it was shown that the GPS carrier phase observations contain a real valued component comprising satellite-receiver geometry and observation errors plus the integer ambiguity. When the integer ambiguities are fixed the phase ranges become more precise and allow determination of positions at the millimetre level. However, fixing the ambiguities to their correct integer values is non-trivial. This section reviews the general problem of ambiguity resolution.

The overall parameter estimation process may be broken into three distinct phases. The first phase, called the float solution, is used to obtain real-valued estimates of the coordinates and ambiguities along with their quality estimates. Additionally the float solution may be used to estimate nuisance parameters, such as tropospheric zenith delays, that will be used in the following step. After the float solution is the ambiguity resolution and validation stage in which an attempt is made to resolve the ambiguities to their correct integer values. The final step is the fixed solution in which the least squares adjustment is performed with the ambiguities constrained to their integer values. The fixed solution provides the most precise estimates of the station coordinates (Teunissen 1998).

3.6.2 Float Solution

Consider the least squares model of equation (3.70),

\[ \mathbf{v} = \mathbf{A} \mathbf{x} - \ell \]

where \( \mathbf{x} \) contains a number of real-valued parameters (coordinates etc.) plus the integer ambiguities and \( \mathbf{P} = \mathbf{C}^{-1} \) is the weight matrix of the double difference observations. The observation system can be re-written as,

\[ \mathbf{v} = \mathbf{Aa} + \mathbf{Bb} - \ell \]

(3.82)
where $\hat{a}$ contains the $n$ integer ambiguities and $\hat{b}$ contains $m$ other parameters. In the float solution all parameters are estimated as real numbers using an ordinary unconstrained least squares adjustment. The real-valued estimates of the parameters are denoted as $\hat{a}, \hat{b}$ and have a covariance matrix of

$$C = C_{\hat{a},\hat{b}} = \hat{\sigma}_0^2 \begin{pmatrix} Q_{\hat{a}\hat{a}} & Q_{\hat{a}\hat{b}} \\ Q_{\hat{b}\hat{a}} & Q_{\hat{b}\hat{b}} \end{pmatrix}$$  \hspace{1cm} (3.83)

where $Q_{\hat{a}}$ is the ambiguity co-factor matrix. Once the ambiguity estimates and covariance matrix have been calculated, the next step is to estimate the ambiguities as integers.

### 3.6.3 Ambiguity Resolution and Validation

The integer-valued solution for the ambiguities is denoted $\tilde{a}$ and is generally based on the integer least squares minimisation problem (Teunissen 1998)

$$(\tilde{a} - \hat{a})^T Q_{\hat{a}}^{-1}(\tilde{a} - \hat{a}) = \text{minimum with } \tilde{a} \in Z^n.$$  \hspace{1cm} (3.84)

In order to make this minimisation problem feasible, the space of all integers ($Z^n$) must be replaced with a smaller subspace that includes the integer least squares solution. The subspace is chosen to match a region bounded by,

$$(\tilde{a} - \hat{a})^T Q_{\hat{a}}^{-1}(\tilde{a} - \hat{a}) \leq \chi^2$$  \hspace{1cm} (3.85)

and is known as the ambiguity search space (Teunissen 1998). The size of the search space is determined by the positive constant $\chi^2$ and its shape and orientation are governed by $Q_{\hat{a}}^{-1}$ . Han (1997) suggests that a suitable value for the constant $\chi^2$ is,

$$\chi^2 = t \hat{\sigma}_0^2 \xi_{t, \alpha, \alpha - 1}$$  \hspace{1cm} (3.86)
where \( t \) is the number of real valued parameters, \( m \) is the number of integer parameters, \( n \) is the number of observations and \( \xi_{F_{n-t-m,1-\alpha}} \) is the one-tailed boundary of the \( 1 - \alpha \) confidence interval for the Fisher's distribution statistic with \( t \) and \( n - t - m \) degrees of freedom.

There are a number of techniques that can be used to apply the minimisation criterion of equation (3.84) to select the best set of integer ambiguities, which must then be validated. Many different ambiguity resolution algorithms have been developed and are optimised for certain data depending on the baseline length and frequency used (L1/L2, widelane/narrowlane etc.). An algorithm called Quasi-Ionosphere Free (QIF), which is applicable for long baselines, is described in section 5.2.4.

It has been said previously that the stochastic model influences the ambiguity search space and, hence, the efficiency and effectiveness of ambiguity resolution and validation. As such, information about the ambiguity search space is useful in assessing a stochastic model. Teunissen (1997a) has proposed an ambiguity DOP (ADOP) that captures the main intrinsic characteristics of the ambiguity covariance matrix and is representative of the ambiguity search space. The ADOP is given as

\[
\text{ADOP} = \sqrt{\frac{1}{n}}
\]

The trace of the ambiguity variance matrix would be much simpler to calculate, however it ignores the significant correlations between the ambiguities (Teunissen 1997a). The trace also lacks some important properties of invariance, with transformations (e.g. change of reference satellite) resulting in changes in the trace (Teunissen 1997a). As with the position based DOPs, smaller values indicate a more precise solution.
The F-ratio test is used in GPS to discriminate between two possible sets of ambiguities. The F-ratio is a ratio of variances and, in the context of ambiguity resolution, may be written as (Wang 1999)

\[ F = \frac{\Omega_i}{\Omega_j} \quad (3.88) \]

where \( \Omega_i \) and \( \Omega_j \) are the quadratic form of the residuals. \( \bar{v}_i \) and \( \bar{v}_j \) are the residuals from two least squares adjustments based on two sets of candidate integer ambiguities such that

\[
\begin{align*}
\Omega_i &= \bar{v}_i^T P \bar{v}_i + (\bar{v}_i - \bar{v}_j)^T Q_{\tilde{i}}^{-1} (\bar{v}_i - \bar{v}_j) \\
\Omega_j &= \bar{v}_j^T P \bar{v}_j + (\bar{v}_i - \bar{v}_j)^T Q_{\tilde{j}}^{-1} (\bar{v}_i - \bar{v}_j)
\end{align*}
\quad (3.89)\]

The assumption is made that the errors are normally distributed and that the estimates of the variance are independent. The set of ambiguities that produces the maximum value of \( F \) is the best estimate. The F-ratio of the best set of integers with the second best set can be used to validate the solution. The larger the F-ratio is between the best and second best solutions, the more confident one can be that the correct set was chosen as the best estimate. Therefore, the F-ratio may be used to compare the effectiveness of stochastic models in terms of ambiguity resolution and validation.

One problem with the F-ratio is that its distribution is generally not known. A more rigorous ambiguity discrimination test is the W-ratio proposed by Wang et al. (1998b). The W-ratio is given by

\[ W = \frac{d}{\sigma_d^2} \quad (3.90) \]

where \( d = \Omega_i - \Omega_j \) and \( \sigma_d^2 = \tilde{\sigma}_d^2 (\bar{v}_i - \bar{v}_j)^T Q_{\tilde{d}}^{-1} (\bar{v}_i - \bar{v}_j) \) is the estimated variance of \( d \) and \( \tilde{\sigma}_d^2 \) is the \textit{a posteriori} variance factor from the float solution. The W-ratio then
has a Student’s $t$ distribution. As with the F-ratio, the larger the value the more likely it is that the ambiguity set is the correct one.

### 3.6.4 Fixed Solution

For efficiency, the residual $(\hat{\alpha} - \bar{\alpha})$ is used to adjust the parameter estimates from the float solution to obtain the fixed solution (Teunissen 1998)

$$\bar{b} = \hat{b} - Q_{\alpha \alpha}^{-1}(\hat{\alpha} - \bar{\alpha})$$

(3.91)

and, treating the integer ambiguities as constants, the law of propagation of variance gives the co-factor matrix of the fixed solution as (Teunissen 1998)

$$Q_b = Q_b - Q_{\alpha \alpha} Q_{\alpha \alpha}^{-1} Q_{\alpha b}$$

(3.92)

Obviously the covariance matrix of the fixed solution is more precise than that of the float solution. Additionally, when the ambiguities are fixed, the number of unknowns decreases reducing the computational overhead of estimating other parameters such as tropospheric delays, orbits and earth orientation parameters.

### 3.7 SUMMARY

In this Chapter the mathematical and stochastic models used in precise GPS positioning have been reviewed. The use of these models in the parameter estimation and ambiguity resolution processes has been explained. The double differencing approach is effective at eliminating the clock errors. However, the effectiveness of DD in removing atmospheric error is related to the baseline length. Thus for long baselines other modelling techniques may be required.

It was shown that the ionospheric error can be eliminated using a combination of dual-frequency phase measurements known as the ionosphere-free linear combination. Unfortunately, in forming the IF combination the ambiguities lose their integer nature, thereby precluding integer ambiguity resolution, unless the widelane ambiguities have already been solved. The widelane linear combination is less
sensitive to ionospheric error than L1 or L2 and therefore can be more effective over longer baselines. Tropospheric error can be eliminated using the geometry-free linear combination. However, like IF the GF combination cannot be used to resolve integer ambiguities. Therefore, there is no linear combination of phase only measurements that can eliminate both the ionospheric and the tropospheric error. If precise dual-frequency code pseudoranges are available, then the Melbourne-Wübbena combination can be formed to eliminate both the ionospheric and tropospheric errors, thereby greatly simplifying the ambiguity resolution process. Due to multipath and measurement noise, which is amplified by reconstruction of the P-codes by unauthorised receivers under anti-spoofing conditions, sufficiently precise dual-frequency code observations may not always be available. Also, the Melbourne-Wübbena combination only helps in fixing the widelane ambiguities and as a further step the narrowlane ambiguities must be resolved, which is more difficult due to the short 10.7cm wavelength. As such, alternative techniques are required to model the residual ionospheric and tropospheric errors for long baselines where the effectiveness of double differencing is limited.

Also in this Chapter, a number of stochastic models used in GPS or proposed in the literature have been reviewed. They show a great variety of approaches and attempt to model one or more of measurement noise, multipath, ionospheric and tropospheric errors. None of the models have attempted to model all four error sources. Test results have indicated that many of the models show significant improvements in positioning fidelity and ambiguity resolution over the equal variance and elevation dependent models. However, the testing of most models has been done with short baselines (less than 10km) with small height changes where residual ionospheric and tropospheric errors will generally be negligible.

Elevation and SNR have been seen to be good indicators of measurement precision. SNR can also be useful for modelling the dispersion due to signal diffraction and multipath. Some models, such as the elevation dependent and baseline dependent models, heavily abstract the physical processes that are causing the correlations. Empirically derived stochastic models are questionable because they do not necessarily have any physical meaning. Additionally, these models operate at the DD level, whereas the errors would ideally be propagated from the undifferenced level
where they originate. Most of the models do not consider all the possible forms that the correlations may take, e.g. between-receiver, between-satellite and temporal.
4.

4. MODEL PERFORMANCE IN LONG BASELINE POSITIONING

4.1 INTRODUCTION

In this chapter the specific techniques for modelling each of the errors described in Chapter 2 will be reviewed. The aims of this review are to:

1. Highlight deficiencies in current techniques.
2. Identify the most appropriate technique for long baselines.

This will lead to some specific problems to be addressed by this research, following the overall research problem introduced in Chapter 1.

Using the double difference and linear combination techniques outlined in Chapter 3, most of the systematic error can be eliminated from the observations. This results in greatly simplified observation equations and therefore a simplified mathematical model. The mathematical models used in GPS positioning are well defined and documented and are generally accepted (Bock et al. 1986; Hoffman-Wellenhof et al. 1994; Strang and Borre 1997). However, some systematic errors (such as multipath) are not removed by these methods and residual errors remain in the observations. The magnitude of the error remaining after differencing depends on the:

- mathematical model (i.e. choice of linear combination, estimated parameters)
• use of *a priori* models (e.g. antenna phase centre offsets and variations, antenna orientation, troposphere, ionosphere)
• presence of multipath
• ionospheric activity
• baseline length
• height difference
• ephemeris type (broadcast or precise)
• session length

In this section each of the error sources identified in section 2.4 will be examined in order to identify the modelling techniques that are most effective and to understand the limitations of those techniques.

### 4.2 Satellite and Receiver Clocks

As GPS is based on time, it is essential to know the time of transmission and time of reception of the signals precisely and in the same time system. The satellite clock offset is cancelled when between-receiver single-differences are formed. Similarly, the receiver clock offset is cancelled by differencing between satellites. However, residual error remains because the receiver clock offset must be estimated in order to calculate the GPS time of signal transmission and reception. These times are needed to calculate the coordinates of the satellites and, hence, the geometric distance between the receiver and the satellite. As long as the receiver clock is estimated to within 1µs the error in the computed geometric distance will be less than 1mm (Hugentobler *et al.* 2001). This is simple to achieve because code measurements need only to be measured to better than 300m for this requirement to be satisfied. Assuming clock corrections, equation (2.16), have been correctly applied and the receiver clock offset has been taken into consideration in the calculation of the satellite coordinates, it is reasonable to conclude that the formation of DD remove the influence of all clock errors.
4.3 ORBIT ERRORS

The influence of errors in the satellite orbits on the position estimate is proportional to the baseline length and can be approximated by (Leick 1990)

\[
\frac{\delta b}{b} = \frac{\delta r}{r}
\]  

(4.1)

where $\delta b$ is the error in the baseline of length $b$ caused by an error in the orbit of $\delta r$ by a satellite at a topocentric distance of $r$. Using the Final Orbits product of the IGS, which provides satellite positions accurate to better than 5cm (IGS 2006a), the error in a baseline of up to 400 kilometres in length will be less than a millimetre (Figure 4.1). However, for ultra long baselines (>4000km) the error in the baseline approaches one centimetre. Long observation sessions and good satellite geometry can help mitigate this error. In general, residual orbit errors are insignificant and can be ignored in the mathematical model. Instead their influence can be modelled by adding a term to the stochastic model to account for this baseline-dependent degradation in precision. One such model by Schaffrin and Bock (1988) was discussed in section 3.3.4.2.1.

![Figure 4.1 Error in Baseline with IGS Final Orbit](image)
4.4 IONOSPHERE

The best ionosphere models that are currently available are only able to model the monthly mean TEC to approximately 10% and there is a 20-25% day to day variation from the monthly mean (Klobuchar 1996). Therefore, ionospheric models are generally of most benefit when using single frequency receivers, which are not able to make use of the frequency dependent nature of the ionosphere.

The first-order effect of the ionosphere is eliminated when the ionospheric-free (IF) linear combination is formed. The second and third order effects on the IF combination, which may be removed with triple frequency measurements, are commonly in the range of 1-2cm (Wang et al. 2005). As such the ionospheric-free combination is very useful in the ambiguity float and fixed solutions. However, because the ambiguities on IF are not integers, another approach must be taken to model the ionosphere during the ambiguity resolution process. If precise dual-frequency code and phase observations are available, the Melbourne-Wübbena linear combination can be used to eliminate the ionospheric and tropospheric error. However, sufficiently precise code observations may not be available because of multipath and noise introduced by the code reconstruction under AS. Multipath on the P-code pseudoranges of around 1.3 metres in a benign environment and 4 to 5 metres in a highly reflective environment may be expected (Evans and Hermann 1990). An investigation of code multipath for a typical reference station network, in which multipath is a consideration in the site and antenna selection, is given in Appendix A. The results of the investigation show that the requirement of a code RMS of better than one metre can generally be satisfied. However, for long baselines, orbit error is a significant problem in resolution of the narrow lane ambiguities and therefore limits the application of the Melbourne-Wübbena linear combination. Assuming the precise orbits are accurate to 5cm, for a baseline of 4000km, the orbit error will be in the order of one narrow lane cycle. In order to reliably resolve the integer ambiguities, the float estimates must be accurate to much less than one cycle. As such, the two-step widelane/narrowlane ambiguity resolution procedure required by the Melbourne-Wübbena linear combination is not practical. Based on this assumption, this discussion and subsequent research investigation will proceed on the
assumption that the mixed code and phase linear combinations are not suitable for long baseline ambiguity resolution.

For dual-frequency receivers there are three general modelling techniques that may be used with phase only data (Odijk 2000); ionosphere-fixed model, ionosphere-weighted model and the ionosphere-float model.

4.4.1 Ionosphere-float Model

In the ionosphere-float model the DD residual ionospheric error is estimated simultaneously with the baseline components and ambiguities using the L1 and L2 observations. As explained previously, the ionosphere can change very rapidly so it is necessary to estimate the ionospheric delay for each satellite every epoch. Thus a very large number of parameters must be estimated.

As no a priori information on the state of the ionosphere is required, the ionosphere-float model is particularly suited to long baselines. For short to medium baselines the ionosphere-float technique is not appropriate because it introduces a high number of parameters and, for short baselines, the estimated parameters will be close to the noise level. For long observation sessions, which are generally required for long baselines, the additional number of parameters that must be estimated becomes a problem due to practical limitations regarding the computational overhead. One ionospheric parameter is estimated per satellite per epoch. Thus if n satellites are observed for p epochs, the observations and unknowns will be,

\[2 \times (n-1) \times p \text{ observations (one per double difference, per frequency, per epoch),}
3 \text{ receiver coordinates,}
(n-1) \times 2 \text{ ambiguities (one per double difference, per frequency),}
n \times p \text{ ionosphere parameters (one per satellite, per epoch).}\]

Consider the case where a baseline is observed for three hours with a sampling interval of 30 seconds. If there is an average of seven satellites above the elevation mask, there will be \(3 + 12 + 2,520 = 2,535\) unknown parameters and 4,320 DD
observations. Even with this moderate session length there is a large number of parameters that need to be solved. To do this by ordinary least squares would require the inversion of a (2,535 x 2,535) matrix, a computationally intensive task even for modern computers. For a twenty-four hour data set commonly used in geodetic applications, this becomes 34,560 observations and at least 20,187 parameters, remembering that satellites will rise and set during the period. To make this approach feasible in practical terms, the ionospheric parameters must be estimated by alternative means, such as pre-elimination (refer to section 5.3.2). (Brown 2003) and (Brown et al. 2003a) have shown that pre-elimination is not ideal for long baselines because it relies on being able to treat the residual ionospheric delays as white noise. Due to this short-coming, the ionosphere-float model suffers from a similar limitation to that of the ionosphere-weighting model discussed below.

4.4.2 Ionosphere-fixed Model

For short baselines of about 10km or less where the residual ionospheric error is expected to be sufficiently small, it is possible to use a simple model that neglects the contribution of the ionosphere. The L1, L1/L2 or widelane observations may be used in the ionosphere-fixed model. As explained in section 3.2.4, the widelane linear combination is less sensitive to ionospheric error than L1 and can therefore make the ionosphere-fixed model a viable approach for baselines well beyond 10km. Further improvements may be made if a priori information on the ionosphere is available, e.g. from a deterministic model. However, deterministic models, which are discussed in section 4.4.4, do not always correct for the entire ionospheric delay and so the ionospheric-fixed model is not well suited to long baselines.

4.4.3 Ionosphere-weighted Model

In the ionosphere-weighted model, the residual ionospheric error after differencing and the corrections from a deterministic model are treated stochastically. Bock et al. (1986) introduce the idea of using ionospheric constraint observations to model the residual ionospheric error. A suitable covariance model for this approach developed by Schaffrin and Bock (1988) was discussed in section 3.3.4.2.1.
The ionosphere-weighted model is flexible because, depending on the variance that is given to the ionosphere, the model can be made to be equivalent to the ionosphere-float model or the ionosphere-fixed model. A variance of zero for the ionosphere will give the same solution as the ionosphere-fixed model, and a variance of infinity would produce the same estimates as the ionosphere-float model. Test results by Odijk (2000) have shown this approach to be very effective and flexible for medium baselines. The ionosphere-weighted approach is only suitable for long baselines if the residual ionospheric error is sufficiently small to be treated as noise. It will be shown in the next section that the deterministic models do not necessarily model the systematic part of the ionospheric delay accurately. For long baselines the residual error after differencing can be ten or more metres. Increasing the variance of the ionospheric error to compensate is not an effective approach and will reduce the precision of the estimates making ambiguity resolution more difficult. Thus the ionospheric-float solution is better suited for long baselines because it is not reliant on having good \textit{a priori} information on the ionosphere.

4.4.4 Deterministic Ionosphere Models

As alluded to in the previous sections, it is possible to divide ionospheric error into two parts: a deterministic component and a stochastic component. The deterministic and/or the stochastic component may be modelled, depending on the situation. One technique is to model the deterministic part using a single layer model (SLM) (Schaer 1999). The SLM assumes that all free electrons are concentrated on an infinitesimally thin shell at a specified height (H) above the mean surface of the earth (R) (Figure 4.2). A typical value for H is 450km.
The total electron content $E$ is calculated as the integral of the electron density along the signal path (Schaer 1999)

$$E = \int N_e(s)ds$$

(4.2)

and can be mapped to the vertical by

$$E_v = F_I(z').E$$

(4.3)

where $z'$ is the zenith angle of the satellite at the ionospheric pierce point. Based on equation (4.3) the ionospheric mapping function

$$F_I(z') = \frac{1}{\cos(z')}$$

(4.4)

where $z' = \arcsin\left(\frac{R}{R + H}\sin(z)\right)$ is used to map the path delay at the receiver to a vertical TEC at the ionospheric pierce point. The coordinates of the ionospheric pierce point

---

**Figure 4.2 Single layer ionosphere model (adapted from Schaer (1999))**
are expressed as geographic or geomagnetic latitude and sun-fixed longitude. The geometry-free linear combination, formed using undifferenced code or phase measurements, may be used to determine the ionospheric model (or map) from a network of reference stations, following the equations (Schaer 1999)

\[
\Phi_{L4} = -\alpha \left( \frac{1}{f_1^2} - \frac{1}{f_2^2} \right) F_1(z') \zeta (\beta, s) + B_4
\]  

(4.5)

\[
\Phi_{P4} = -\alpha \left( \frac{1}{f_1^2} - \frac{1}{f_2^2} \right) F_1(z') \zeta (\beta, s) + b_4
\]  

(4.6)

where \( \Phi_{L4} \) is the phase geometry-free linear combination, \( \Phi_{P4} \) is the code geometry-free linear combination, \( \beta \) is the geographic or geomagnetic latitude and \( s \) is the sun-fixed longitude. The term \( B_4 \) is the bias from the L1 and L2 phase ambiguities. For undifferenced phase the term \( B_4 \) also includes the initial (fractional) phase and the differential L1-L2 bias (refer to section 2.7.4.4). If equation (4.5) is formed using DD phase, the initial phase and the differential L1-L2 bias will be cancelled. If the ambiguities are not known the bias \( B_4 \) must be estimated. The term \( b_4 \) is the differential code bias. If equation (4.6) is formed using DD code measurements the term \( b_4 \) disappears. There are two approaches commonly used to estimate a deterministic ionosphere model. The first uses DD phase measurements based on equation (4.5). As the ionospheric delay is partially cancelled by differencing, a sparse reference network must be used. The second approach uses zero-difference code measurements based on equation (4.6). To improve the results, the code measurements are smoothed using the carrier phase to reduce the measurement noise (Schaer 1999).

The ionosphere is modelled either as a two-dimensional Taylor’s series expansion or as a spherical harmonic expansion, the degree and order of which depends on the size of the geographic region to be modelled (Hugentobler et al. 2001). Due to the sparse amount of data available, one map is typically generated for each two-hour period. Thus the models are not able to capture the short-term variations of the ionosphere.
For example, Global Ionosphere Maps (GIM) are calculated by the Centre for Orbit Determination in Europe (CODE) as a spherical harmonic expansion to degree and order fifteen using a subset of the IGS network. The maps are available from [http://www.aiub.unibe.ch/ionosphere.html](http://www.aiub.unibe.ch/ionosphere.html) in both the ionosphere exchange format (IONEX) and the Bernese GPS Software ION format.

If such an ionosphere model is used to provide a priori corrections to the measurements, it should theoretically be possible to solve ambiguities over long baselines using the widelane or ionosphere-weighted approach. However, these deterministic models do not necessarily model the ionosphere to a sufficiently high degree of accuracy. To illustrate this limitation, data from a 4000km baseline (designated ALCO) between Alice Springs in central Australia and Cocos Island in the Indian Ocean was processed using the Bernese GPS Software v4.2. The L1 and L2 integer ambiguities were resolved using the QIF strategy (refer to Chapter 5) before estimating the residual ionospheric error every two minutes using the geometry-free linear combination. Two days data are used in the following examples corresponding to Julian days (JD) 189, during winter in the southern hemisphere, and 265, near the vernal equinox, of 2002. On each day, the ionospheric errors were estimated twice; once using no a priori corrections, and once using the global ionospheric map from CODE. The results are presented in Figure 4.3 and Figure 4.4. Each of the linear features (arcs) shown in Figure 4.3 and Figure 4.4 corresponds to a particular satellite pair. Clearly the ionospheric error has a strong time correlation, even with an observation interval of two minutes. In fact the temporal correlation between consecutive epochs is as high as 0.9992 for some satellites and is generally between 0.85 and 0.98.

Note that the residual ionospheric error, even after the deterministic model has been applied still reaches a maximum of 12m. Also note that whilst the deterministic model generally reduces the residual ionospheric error, it sometimes causes it to increase. An example can be seen in Figure 4.4 at approximately 10 hours where the ionospheric error increases in magnitude from –4m to –12m after applying the deterministic model. Whilst only these two examples are shown here, similar results can be seen in many other long baseline datasets. In such conditions, the ionosphere-float model is most appropriate because no *a priori* information on the ionosphere is
required and the magnitude of the residual ionosphere error will not limit the effectiveness of the model. Hence, for long baselines the ionosphere-float model offers the most flexible solution despite the disadvantage that it requires many more parameters to be estimated.

Figure 4.3 ALCO on JD 189 2002 using (top) no a priori model and (bottom) using a global ionosphere map
4.5 TROPOSPHERE

For precise positioning over long baselines, tropospheric models based on a standard atmosphere, such as the (Hopfield 1969) model, are inadequate (Hugentobler et al. 2001). It is, however, possible to model the troposphere by mapping a reference zenith (vertical) delay to a signal path as e.g. (Langley 1998b)

\[
T_r^s = m_{\text{dry}}(z)T_{\text{dry}}^0 + m_{\text{wet}}(z)T_{\text{wet}}^0
\]  

(4.7)

where \(T_r^s\) is the tropospheric path delay between receiver \(r\) and satellite \(s\), \(T_{\text{dry}}^0\) is the zenith hydrostatic (dry) delay (ZHD), \(T_{\text{wet}}^0\) is the zenith wet delay (ZWD), and \(m_{\text{dry}}\) and \(m_{\text{wet}}\) are mapping functions expressed as a function of the satellite zenith angle \(z\).

The ZHD can be predicted to better than 1 mm if surface pressure measurements accurate to 0.3 millibars or better are available (Bock 1998). The ZWD

Figure 4.4 ALCO on JD 265 2002 using (top) no a priori model and (bottom) using a global ionosphere map
can be predicted to no better than 1-2 cm using surface measurements (Langley 1998b). The reason for this is that water vapour has wide variations in concentration, unlike the dry gases which are uniform (Spilker Jr 1996). The ZWD can be estimated using a relatively expensive ground-based, high precision water vapour radiometer, barometer and thermometer measurements. However, such measurements are often not available or not desirable because an error of one percent in the relative humidity will cause a height bias of up more than one centimetre (Hugentobler et al. 2001).

Another option is to use the GPS observations themselves to estimate tropospheric zenith delays. The disadvantage of estimating the tropospheric zenith delay from the GPS measurements is that a single (combined dry and wet) zenith delay must be estimated, which precludes the use of separate mapping functions for the dry and wet components. The difference between the dry and wet mapping functions is small for data above 15° elevation. If surface measurements are available the ZHD can be predicted and only the ZWD is estimated, thus enabling the use of separate mapping functions. In either case, equation (4.7) simplifies to

\[ T_r^e = m(z)T^0 \]  

(4.8)

Additionally, tropospheric gradients, implemented as a tilt to the direction of tropospheric zenith, may also be estimated to account for azimuthal asymmetries in the tropospheric delay. A generalised form of the gradient functions given by MacMillan (1995) and Davis et al. (1993) in Rothacher et al. (1997)

\[ T_r^e = m_r(z)T^0 + \frac{dm_r(z)}{dz} T_n^0 \cos \alpha + \frac{dm_r(z)}{dz} T_e^0 \sin \alpha \]  

(4.9)

where \( \alpha \) is the azimuth of the satellite, \( m_r \) is the mapping function, and \( T_n^0 \) and \( T_e^0 \) are the gradient parameters for the north-south and east-west directions respectively. Results obtained by CODE in processing IGS data have shown that estimating tropospheric gradients significantly improves the horizontal accuracy of the estimated station coordinates (Rothacher et al. 1997). This improvement may partially result
from the absorption of multipath and antenna phase centre variations by the gradient parameters (Rothacher et al. 1997).

The main problem with estimating tropospheric parameters from the GPS measurements is that they are highly correlated with the vertical component of the station coordinates. To overcome this problem, observation sessions of several hours or more are used and the zenith delays are estimated as arc parameters (i.e. the tropospheric error is treated as being constant for a given interval). Typically, a new set of zenith delays will be estimated every one to two hours (Hugentobler et al. 2001). Higher resolution estimates of the tropospheric zenith delays are possible, but require the use of relative constraints on the parameters (Hugentobler et al. 2001). A low elevation mask (0, 5 or 10 degrees) also helps improve the tropospheric estimates by improving the satellite geometry, at the expense of introducing noisier data (Niell et al. 2001; Rothacher et al. 1997). The impact of the noisier data from the low elevation satellites can be overcome by using an appropriate stochastic model. The use of low elevation data and an appropriate stochastic model can significantly improve the accuracy of the station height (Rothacher et al. 1997). It is critical to use appropriate mapping functions, as many are inaccurate when using low elevation data (Mendes and Langley 1994). The mapping function of Niell (1996), referred to as NMF, is widely used in GPS and VLBI processing due to its accuracy for low elevations and independence from surface meteorological data. The Niell (1996) mapping function has the form,

\[
m(e) = \frac{1 + \frac{a}{b + c + 1}}{\sin(e) + \frac{a}{b + c + 1}}
\]  

\[
(4.10)
\]

where \(e\) is the geometric elevation of the satellite and \(a\), \(b\) and \(c\) may be constants or functions of other variables such as isobaric height, latitude and day of year. The NMF was computed using data only from the northern hemisphere. The inversion of the seasons was accounted for by adding six months to the phase for southern latitudes (Niell 1996). Studies by Boehm et al. 2007 show that the NMF has temporal
deficiencies that reach a maximum around January. These deficiencies are most pronounced at high southern latitudes, in Japan, the northern part of Europe, the western part of Canada, and Alaska.

If a suitable mapping function is used, the troposphere can be modelled to sub-centimetre accuracy using zenith delays (Mendes and Langley 1994; Niell et al. 2001). Thus, only a relatively small number of tropospheric parameters must be estimated to achieve high accuracy. Whilst the tropospheric delay can be estimated well enough for ambiguity resolution, it is still a limiting factor in the accuracy of the fixed solution. Mis-modelling of the tropospheric error can have two effects: a station height bias, and a scale bias in the baseline length.

A tropospheric bias common to both stations in a baseline causes a scale bias that may be approximated as (Beutler et al. 1988)

$$\frac{\Delta \ell}{\ell} = \frac{\Delta T_{a,z}}{R \cos(z_{max})}$$  \hspace{1cm} (4.11)

where

- $\Delta \ell$ is the baseline bias,
- $\ell$ is the baseline length,
- $\Delta T_{a,z}$ is the absolute tropospheric error in the zenith direction,
- $R$ is the radius of the earth, and
- $z_{max}$ is maximum zenith angle of the observation session.

A height bias is caused when one station has a tropospheric error relative to another. The effect of such differential troposphere biases may be approximated as (Beutler et al. 1988)

$$\Delta h = \frac{\Delta T_{a,z}}{\cos(z_{max})}$$  \hspace{1cm} (4.12)
where
\[ \Delta h \] is the station height bias,
\[ \Delta T_{r,z} \] is the relative tropospheric error in the zenith direction.

### 4.6 Multipath

When signals from the GPS satellites are reflected by a surface and arrive at the antenna via an indirect path, multipath error results. Multipath is a function of the reflective properties of the receiver’s environment and the satellite constellation and is unique for each site. As such multipath is not cancelled by measurement differencing. Signal-to-noise ratios in conjunction with antenna gain patterns have been shown to be effective in identifying reflected signals, though difficult to apply (e.g. Barnes et al. (1998)). Due to the difficulty in modelling multipath it is best avoided (where possible) by careful site selection. Appropriate antenna design and signal processing by the receiver can help reduce multipath. The best approach, though not always practical, is to place the antenna in a low multipath environment. In geodetic applications, the assumption is made that over long periods (several hours) that multipath will behave more like noise than a systematic error (Schaer 1999). Since long observation sessions are required to estimate tropospheric delays, it is convenient to simply assume that the net effect of multipath error will be insignificant. This assumption can be validated by estimating the multipath error and examining its effect on the station coordinates.

Multipath may be estimated using a number of techniques. One technique is to use ray-tracing to directly calculate the multipath error. Calculation of the multipath error requires that the position, orientation and reflectance of all reflective surfaces in the vicinity of the antenna be known. For many applications the development of the site model required for the ray-tracing approach is impractical. Alternatively the fact that the GPS constellation repeats every sidereal day (approximately every 23h 56m) can be used to estimate multipath error. The multipath error at a site will be similar under the same satellite geometry if the reflectors have not changed. However, perturbations to the satellites orbits and changes to the reflectance properties of the reflectors (because of rain, snow, dust etc.) mean that multipath will not be identical from day to day. Radovanovic (2000) shows that multipath correlation between
consecutive days is highest within 12 seconds of 23hr 56min (two times the orbital period of the GPS satellites). Modelling of multipath by its day to day repeatability can show improvements in accuracy of 25% to 40% but requires a fixed antenna and stable reflectors and reflectance’s (Radovanovic 2000). To validate the assumption that multipath can be ignored for long observation sessions, multipath error was estimated using an adaptive filter (AF). The estimation process is described in Appendix B. The task of the adaptive filter is to separate the noise and multipath error from two sets of residuals collected at a site on two consecutive days. If the antenna position, satellite geometry and reflectors are the same then the filter should be able to isolate the multipath error. The multipath estimates can then be used to correct the measurements and provide a “multipath-free” solution. Figure 4.5 shows filter results for satellite pair G7-G8 from a 275km baseline between Melbourne and Horsham measured on two consecutive days, Julian days 210 and 211 of 2002. The four series within Figure 4.5 correspond to the JD 210 residuals, the JD 211 residuals, the estimated measurement noise and the estimated multipath. The residuals were derived from a float solution using the ionosphere-free linear combination with data collected at a five second recording interval.

Clearly, there is a strong multipath signal that is common to the residuals from both days. The maximum correlation between the two series was 0.803 at a lag of 4.2 minutes. The filter is able isolate the signal common to both days. The noise series, which is the JD 211 residuals minus the estimated multipath, is essentially the multipath corrected residuals. The noise series has less systematic trend than the uncorrected residuals, but is noisier because it contains the measurement noise from both days plus any discrepancy between the multipath errors on the two days (multipath noise). As explained in Appendix B, the effectiveness of the filter can be assessed by checking the correlation between the signals. The correlations between the signals for the Melbourne-Horsham baseline are given in Table 4.1 using the notation:

| X | Input Signal (Day 210 residuals) |
| D | Desired Signal (Day 211 residuals) |
| Y | Predicted (multipath) Signal |
| E | Error (noise) Signal |
As expected, the noise signal has low correlation with the residual and multipath series, suggesting that it is more random. Conversely, the multipath series is highly correlated with the two sets of residuals indicating that the multipath signal is the dominant structure in both series.
Satellites | XD | XY | XE | DY | DE | YE
---|---|---|---|---|---|---
7-8 | 0.803 | 0.977 | -0.269 | 0.821 | 0.321 | -0.277
8-11 | 0.703 | 0.940 | -0.076 | 0.812 | 0.607 | 0.028
11-26 | 0.654 | 0.921 | -0.069 | 0.777 | 0.653 | 0.030
26-27 | 0.555 | 0.884 | -0.328 | 0.548 | 0.495 | -0.455
27-28 | 0.594 | 0.868 | -0.307 | 0.577 | 0.451 | -0.468
28-29 | 0.708 | 0.988 | -0.288 | 0.721 | 0.458 | -0.286
26-28 | 0.689 | 0.974 | -0.258 | 0.699 | 0.493 | -0.277
29-4 | 0.709 | 0.939 | -0.224 | 0.768 | 0.457 | -0.219
4-9 | 0.728 | 0.944 | -0.195 | 0.786 | 0.464 | -0.183
8-26 | 0.781 | 0.975 | -0.291 | 0.800 | 0.332 | -0.300
7-26 | 0.773 | 0.981 | -0.276 | 0.780 | 0.369 | -0.294
24-5 | 0.584 | 0.896 | -0.335 | 0.649 | 0.461 | -0.376

Table 4.1 Correlations between input, desired, multipath and noise signals

Using this approach, multipath was estimated with a test data set of five consecutive days data from three baselines: Melbourne-Ballarat (101km), Melbourne-Hamilton (259km) and Melbourne-Horsham (275km). The multipath corrections were applied to the measurements and the solutions were recomputed. The data was processed in three batches using two, four and eight hours of data respectively. The mean differences in position between the normal and multipath corrected solutions are shown in Table 4.2 along with their RMS. The mean difference and RMS values are all within a few millimetres indicating that the multipath error has very little influence on the solution. Note that one of the days of data for the MEBA baseline produced an outlier in the 8 hour uncorrected dataset (due to undetected outliers in the data), causing an high RMS. After removal of this outlier, the RMS drops to 0.001m. Importantly, there are limitations in the practical use of this approach, since it requires that the same DD observations are available from day to day, which is often not the case due to the changing satellite constellation. As such, multipath error will be disregarded in this investigation.
<table>
<thead>
<tr>
<th>Baseline</th>
<th>Mean / RMS</th>
<th>Observation Period</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>2 Hours</td>
</tr>
<tr>
<td>MEBA</td>
<td>Mean</td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td>RMS</td>
<td>0.002</td>
</tr>
<tr>
<td>MEHA</td>
<td>Mean</td>
<td>0.001</td>
</tr>
<tr>
<td></td>
<td>RMS</td>
<td>0.001</td>
</tr>
<tr>
<td>MEHO</td>
<td>Mean</td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td>RMS</td>
<td>0.002</td>
</tr>
</tbody>
</table>

Table 4.2 Position difference in metres between normal and multipath corrected solutions

### 4.7 Phase Centre Variations and Antenna Orientation

For short baselines, the antenna phase centre offset and variation will, assuming manufacturing tolerances are negligible, cancel if the antennas are the same model and have the same orientation. For longer baselines, a phase centre model should be used with the antennas oriented to grid north. As mentioned in section 2.7.4.2, relative calibrations introduce small biases in long baselines due to the differences in the satellite elevations. Hence, it is preferable to use absolute calibrations when available.

Bock et al. (1998) report relative phase-center accuracies, based on a sample of 34 choke ring antennas, as about 0.5 mm in the horizontal and 1.7 mm in the vertical, with a precision of about 0.1-0.2 mm in the horizontal and 0.5 mm in the vertical. Testing by Krantz et al. (2001) has shown that the phase centre variations of the Zephyr antenna are consistent with the manufacturer’s model to 0.2mm. Based on these tests it is reasonable to assume that antenna phase centre offsets and variations can be modelled to the millimetre level or better using empirically derived models and so will not be considered any further in this investigation.

For long baselines the signals will arrive at the two antennas from different directions causing an effect know as phase wind-up (refer to section 2.7.4.3). On very long baselines the error due to antenna orientation can reach several centimetres and should be corrected using the formulae given by Wu et al. (1993).
4.8 Measurement Noise

The stochastic model is used to account for measurement noise by giving the observations appropriate weight in the parameter estimation process. Since measurement noise is typically small, its effect on the parameter estimates is also small. The stochastic models presented in section 3.3.4 are able to adequately model measurement noise by modelling its relationship with the SNR or the satellite’s elevation.

4.9 Summary

If the most appropriate modelling techniques are employed with a long session and high quality dual-frequency data sets, ambiguity resolution is theoretically possible over baselines of almost any length (only limited by the requirement that both sites must track common satellites). Some sources of error, such as phase centre offsets and variations and the satellite and receiver clock offsets, can be modelled to such an extent that they will have negligible effect on the solution. With modern precise ephemerides, orbit error can be disregarded in most circumstances. For ultra long baselines an additional distance-dependent term can be added to the stochastic model to account for residual orbit errors, though with long observation sessions this is not generally necessary. Measurement noise is well understood and is easily modelled by quite simple stochastic models. It was shown in section 4.6 that phase multipath has little influence on the coordinate estimates for observations sessions of two or more hours, assuming that well-sited, choke ring or similar antennas are used. The remaining error sources that may interfere with ambiguity resolution or limit the precision of the parameter estimates are the ionosphere and troposphere.

The tropospheric delay can be estimated or modelled with sufficiently high accuracy to permit ambiguity resolution over any baseline length given sufficient data. However, the troposphere is still a limiting factor in the precision that can be obtained using long baseline GPS.

The ionosphere can be effectively eliminated in the ambiguity float and fixed solutions using the ionosphere-free linear combination. Unfortunately the ionosphere-
free combination can not be used in the ambiguity resolution process or only in conjunction with the widelane linear combination. In the absence of precise dual-frequency P-code observations, deterministic ionosphere models can help reduce the bias due to the ionosphere. However, current deterministic models for the ionosphere were shown to be less than ideal for long baseline positioning, with residual errors of some metres remaining after the models have been applied. As such, the ionosphere-fixed and ionosphere-weighted models are not appropriate, requiring the use of the computationally more demanding ionosphere-float model. The ionosphere-float model can be impractical for long sessions because of the large number of parameters that must be estimated. A more rigorous estimation technique than pre-elimination, which is used by high-end software such as Bernese, is required to make full use of the ionosphere-float model. Pre-elimination relies on being able to treat the, clearly systematic, residual ionospheric delay as noise. Additionally, incorporation of knowledge regarding the strong temporal correlation in the residual ionospheric delay over long baselines, may lead to more efficient estimation algorithms.
5. THE QUASI-IONOSPHERE-FREE PROCESSING STRATEGY

5.1 INTRODUCTION

Having reviewed current modelling techniques and processing strategies in the earlier Chapters, it is now appropriate to bring this knowledge together as a processing strategy. The term processing strategy is used because different mathematical and stochastic models are used in different stages of the process.

5.2 MATHEMATICAL MODEL

The need to resolve the phase ambiguities to integers imposes the requirement of a multi-stage parameter estimation process. The result is that different mathematical models are used for each of the three main stages of parameter estimation. These three stages, described in section 3.6.1, are the float solution, ambiguity resolution and validation, and fixed solution. The mathematical model for each of these stages to be used in this research will be described. The fundamental mathematical model that has been chosen is the one recommended for baselines of 100km to 2000km when using the Bernese GPS Software version 4.2 (Hugentobler et al. 2001). The ambiguity resolution strategy for this model is the Quasi-Ionosphere-Free (QIF) strategy, whereby L1 and L2 ambiguities may be resolved directly for baselines of almost arbitrary length (Mervart 1995). The requirements of this strategy are to have long sessions (several hours) of static dual-frequency data. The QIF processing strategy is shown in Figure 5.1 and the various steps are outlined in sections 5.2.1 through 5.2.5.
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<th>Amb. Resolution</th>
<th>AIMS</th>
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<tbody>
<tr>
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<td>• Fixed L1/L2 integer ambiguities (where possible)</td>
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<tr>
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<td>• Ionosphere-free phase data</td>
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<td></td>
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</tbody>
</table>

**Figure 5.1 QIF processing strategy**
5.2.1 Pre-Processing

In the pre-processing stage data checking and cleaning is performed. The main tasks of the pre-processing are to:

- Remove outliers
- Remove unpaired observations (typically L1 observations with no corresponding L2 observations)
- Detect and repair cycle slips
- Estimate approximate station coordinates (if not already known)

5.2.2 Float Solution 1

The main task of the first float solution is to provide good real-valued estimates of the coordinates and the tropospheric delays. Estimation of the integer ambiguities is done in a second float solution. Two float solutions are used because the ionosphere-free linear combination provides the most reliable estimates of the troposphere but is unsuited to ambiguity resolution as discussed previously. The mathematical model of the first float solution is based on:

- DD ionosphere-free phase observations, equation (3.19)
- Ionosphere-free ambiguities estimated as real numbers
- Estimation of tropospheric zenith delays and gradients, equation (4.9)
- Precise orbits, antenna phase centre variation and orientation models, ocean tide loading model applied

If processing a network of reference stations, it is standard to process all baselines together in order correctly model the mathematical correlations and to obtain more reliable estimation of the troposphere parameters.

5.2.3 Float Solution 2

A second float solution is performed to estimate the L1 and L2 ambiguities and their covariance matrix for the ambiguity resolution step. The second float solution is an
ionosphere-float model, with the troposphere estimates introduced as known values from the first float solution. The main characteristics of the second float solution are:

- DD L1 and L2 phase observations, equations (3.10) and (3.11)
- L1/L2 ambiguities estimated as real numbers
- Estimation of ionospheric delay for each satellite/epoch
- (Optional) Deterministic ionosphere model applied
- Troposphere estimates introduced from previous float solution
- Precise orbits, antenna phase centre variation and orientation models, ocean tide loading model applied

The second float solution and subsequent ambiguity resolution steps are typically performed on a baseline by baseline case due to the very high number of parameters that are estimated.

### 5.2.4 Ambiguity Resolution and Validation

The float solution provides real-valued estimates for the double difference ambiguities $\hat{N}_{L1}$ and $\hat{N}_{L2}$. To find the optimal set of integers based on these estimates using the Quasi-Ionosphere-Free (QIF) ambiguity resolution strategy (Mervart 1995), the following steps are performed.

Using the float estimates, the ionospheric-bias $B_{IF}$ of equation (3.22) is calculated for each set of ambiguities as,

$$
\hat{B}_{IF} = \frac{c}{f_{L2}^2 - f_{L1}^2} (f_{L1} \hat{N}_{L1} - f_{L2} \hat{N}_{L2})
$$

(5.1)

Converting to narrow lane cycles equation (5.1) becomes,

$$
\hat{b}_{IF} = \frac{f_{L1} + f_{L2}}{c} \hat{B}_{IF}
$$

(5.2)
where \( \beta_1 = \frac{f_{L1}}{f_{L1} - f_{L2}} \) and \( \beta_2 = \frac{-f_{L2}}{f_{L1} - f_{L2}} \). When the ambiguities have been resolved as integers \( \hat{N}_{L1} \) and \( \hat{N}_{L2} \), the difference between the real-valued and integer ionospheric-bias

\[
\xi = \left| \hat{b}_{IF} - \bar{b}_{IF} \right|
\]  

(5.3)

may be used to select the best set of integers (Mervart 1995), provided the float estimates are accurate and precise. For static baselines with sufficient data, this criterion is easily achieved for most ambiguities (remember from section 3.2.2 that with long observation sessions it is not likely that all ambiguities will be resolved). However, many possible sets of integers \( \hat{N}_{L1}, \hat{N}_{L2} \) will give similar values of \( \xi \). The RMS of the ionospheric bias,

\[
\sigma = \sigma_0 \sqrt{\beta_1^2 Q_{11} + 2 \beta_1 \beta_2 Q_{12} + \beta_2^2 Q_{22}}
\]  

(5.4)

where \( Q_a \) is derived from the ambiguity variance-covariance matrix, is used to sort the ambiguity pairs in ascending order of their precision (Mervart 1995). For those ambiguity pairs for which \( \sigma < \sigma_{\text{max}} \), search ranges are defined and the pair of integers \( \hat{N}_{L1} \) and \( \hat{N}_{L2} \) that has the smallest test value

\[
\xi = \left| \beta_1 (\hat{N}_{L1} - \bar{N}_{L1}) + \beta_2 (\hat{N}_{L2} - \bar{N}_{L2}) \right|
\]  

(5.5)

is selected as a solution unless \( \xi \geq \xi_{\text{max}} \) (Mervart 1995). Recommended values for the constants are \( \sigma_{\text{max}} = 0.03 \) and \( \xi_{\text{max}} = 0.1 \) (Hugentobler et al. 2001). An iterative process is then used whereby each ambiguity pair, in ascending order of their RMS is tested. Should a set of integers be accepted, the least squares adjustment is
recomputed with the ambiguities held fixed at their integer values and the process repeated until all ambiguities have been tested. By conducting the search in \((N_{L1}, N_{wl})\) search space, the number of ambiguity pairs that has to be tested may be reduced due to the small gradient difference between a line of constant \(N_{wl}\) and a line of constant \(\xi\). The shape of the QIF search space is illustrated in Figure 5.2.

![Figure 5.2 QIF Search Space (adapted from (Mervart 1995))](image)

In Figure 5.2 the float estimate \((\hat{N}_{L1}, \hat{N}_{L2})\) is indicated by the symbol \(o\). The solid line passing through the float estimate is the line of constant \(\xi\), which also passed through the “true” ambiguity pair \((0, 0)\). The second black line represents the constant widelane ambiguity that is selected as “true”. The bounds of the \((N_{L1}, N_{L2})\)
search space are shown as a dashed rectangle. The $\left(N_{L1}, N_{WL}\right)$ search space is indicated by the dash-dot trapezoid. A search range of 0.5 widelane cycles is recommended (Hugentobler et al. 2001).

5.2.5 Fixed Solution

The fixed solution is very similar to the first float solution, the only difference is that the ambiguities that were resolved as integers in the previous step are treated as constants and removed from the observation equations. Ambiguities that were not resolved are carried as real-valued parameters. The ionosphere-free linear combination is employed and the full tropospheric zenith delays are re-estimated for each station as before. The main characteristics of the fixed solution are:

- DD ionospheric-free phase observations
- Resolved L1/L2 ambiguities are treated as constants
- Unresolved ionosphere-free ambiguities estimated as real numbers
- Estimation of tropospheric zenith delays
- Precise orbits, antenna phase centre variation and orientation models, ocean tide loading model applied

As with the first float solutions, multiple baselines are normally processed together in this step.

5.3 Parameter Estimation in the Ionosphere-Float Model

5.3.1 Preamble

The second float solution, which is required to obtain real valued estimates for the L1 and L2 ambiguities, is based on the ionosphere-float model. As outlined in section 4.4.1, estimation of the ionospheric delays in every measurement introduces a large number of unknowns. For long sessions ordinary least squares becomes impractical because of the computational requirements of inverting a very large matrix.
5.3.2 Pre-Elimination

It was mentioned in section 3.3.1 that there is model equivalence between the mathematical model and the stochastic model. Therefore, a parameter may be estimated explicitly within the mathematical model, or it can be estimated implicitly within the stochastic model. This model equivalence is the basis of the pre-elimination technique. Consider the case where the parameters $\mathbf{x}$ contain two subsets; parameters of interest $\mathbf{a}$; and nuisance parameters $\mathbf{k}$. Equation (3.70) can then be re-formulated as,

$$
\ell = \mathbf{A}\mathbf{x} - \ell
= \mathbf{A}\mathbf{a} + \mathbf{B}\mathbf{k} - \ell
=[\mathbf{A} \quad \mathbf{B}]
\begin{bmatrix}
\mathbf{a} \\
\mathbf{k}
\end{bmatrix}
- \ell
$$

The equivalent implicit solution to equation (3.73), referred to as pre-elimination, is given by (Blewitt 1998),

$$
\hat{\mathbf{a}} = (\mathbf{A}^T\mathbf{V}\mathbf{A})^{-1}\mathbf{A}^T\mathbf{V}\ell
$$

where $\mathbf{V}$ is the reduced weight matrix,

$$
\mathbf{V} = \mathbf{P} - \mathbf{PB}(\mathbf{B}^T\mathbf{P})^{-1}\mathbf{B}^T\mathbf{P}
$$

However, $\mathbf{V}$ is singular and, hence, $\mathbf{A}^T\mathbf{V}\mathbf{A}$ is also singular. Therefore it is not possible to estimate the parameters using the usual matrix inversion approach. This singularity can be overcome by treating the nuisance parameters as stochastic variables with their own covariance matrix $\mathbf{C}_k$. An augmented stochastic model is thus constructed,

$$
\mathbf{C}' = \mathbf{C} + \mathbf{BC}_k\mathbf{B}^T
$$
where the matrix $C_k = \sigma_k^2 I$ is a source of process noise and $\sigma_k$ is chosen to be sufficiently large that the nuisance parameters are not over constrained and, therefore, the data has a strong influence on the values in $k$. Process noise means that the parameters themselves are a random process. The parameters $k$ are essentially modelled as white noise with an expectation $E[k] = 0$.

The reduced weight matrix can be written as (Blewitt 1998),

$$V = C'^{-1}$$

$$= P - PB(B^T PB + C_k^{-1})^{-1}B^T P$$

(5.10)

The solution for the parameters of interest, equation (5.7), will be equivalent to the traditional least squares solution if (Blewitt 1998):

1. $\sigma_k^2$ is sufficiently large;
2. The augmentation to the stochastic model, expressed by the term $BC_k B^T$ in equation (5.9), accounts for correlations resulting from the observation’s functional dependence on the nuisance parameters (as defined by matrix $B$);
3. The augmentation to the stochastic model does not include a priori information on the actual variance of the process noise.

Writing the equation (5.7) in normal equation form,

$$Nx = b = \begin{bmatrix} A^T VA & A^T VB \\ B^T VA & B^T VB \end{bmatrix} \begin{bmatrix} a \\ k \end{bmatrix} = \begin{bmatrix} A^T V \ell \\ B^T V \ell \end{bmatrix}$$

(5.11)

or more simply

$$Nx = b = \begin{bmatrix} N_{11} & N_{12} \\ N_{21} & N_{22} \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

(5.12)
the solution, equation (5.7), may be expressed as,

\[
\tilde{N}_{i1} = \tilde{b}_1
\]  

(5.13)

with

\[
\begin{align*}
\tilde{N}_{i1} &= N_{i1} - N_{i2} N_{21}^+ N_{21} \\
\tilde{b}_1 &= b_1 - N_{i2} N_{21}^+ b_2
\end{align*}
\]  

(5.14)

(5.15)

In this notation it becomes clear that it is possible to estimate the parameters of interest without having to explicitly estimate the nuisance parameters. Using this technique, the full solution can be obtained by processing each epoch, pre-eliminating the nuisance parameters (in this application to ionospheric path delays) and combining the epochs using addition of normal equations (equation (3.81)). The pre-elimination technique will work even if in any given epoch there are more unknowns than observations, which will be the case with the ionosphere-float model. For example, if six satellites are observed on L1 and L2 for a single epoch, there will be 10 double difference observations and a total of 19 parameters (3 coordinate parameters, 5 L1 ambiguities, 5 L2 ambiguities and 6 ionospheric path delays).

Pre-elimination assumes that the ionosphere parameters \( k = k_{\text{meas}} - k_{\text{apr}} \) have an expectation of \( E[k] = 0 \) and a dispersion of \( \text{D}[k] = C_k = \sigma_k^2 I \). The a priori ionosphere estimates \( k_{\text{apr}} \) may come from an ionosphere model or, for baselines up to 500km, \( k_{\text{apr}} = 0 \) may be used (Hugentobler et al. 2001). The value recommended for \( \sigma_k^2 \) in the Bernese GPS Software is 0.25 metres. A smaller value may cause the ionosphere parameters to be too tightly constrained, forcing error into the coordinate and ambiguity estimates. A larger value will result in decreased precision of the estimated parameters, which can interfere with ambiguity resolution. Thus in order to estimate ambiguities over long baselines using the QIF approach with pre-elimination of the ionosphere parameters also requires a good deterministic model of the ionosphere. Therefore the advantages of the ionosphere-float model are lost. Also, it was shown in section 4.4.4 that current ionosphere models are not necessarily accurate and, in that
example, residual error of up to 12m remained after the deterministic model was applied.

5.3.3 Sequential Phased Adjustment

The phased adjustment algorithm offers a rigorous method for estimating parameters in batches. The measurements are segmented into blocks, with each block containing only a subset of the overall parameters. Each block will contain parameters that are unique to that block (inner parameters), plus parameters that are common to more than one block (junction parameters). The application of sequential phased adjustment to ionospheric estimation for long baseline GPS positioning has been published in Brown (2003) and Brown et al. (2003a).

An example of the segmenting process, for a linear series of parameters, is shown in Figure 5.3. Such a linear relationship between the parameters may arise because the parameters relate to a linear process. Examples are: coordinates along a railway line (linear in space) or ionospheric parameters through time (linear in time). The parameters in Figure 5.3 have been segmented into four blocks. The circles representing inner parameters have been filled, whilst the circles representing junction parameters have not.

![Figure 5.3 Example of a segmented linear network](adapted from Leahy and Collier (1998))

The solution is obtained by sequentially processing the blocks in what is known as a pass. Both forward pass and a reverse pass are required to obtain rigorous estimates for all parameters. The forward pass is shown diagrammatically in Figure 5.4, using the following notation (Leahy and Collier 1998):

- $I_1, I_2, I_3, I_4$ inner parameters for each of the four blocks,
- $J_{12}, J_{23}, J_{34}$ junction parameters between neighbouring blocks,
partial estimates of the junction parameters resulting from the forward pass,
covariance matrices of the partial estimates of the junction parameters from the forward pass,
rigorous estimates and covariance matrix of all parameters of the last block.

The linear systems of equations for each block are,

\[ \ell_1 = \begin{bmatrix} \Delta_{11} & \Delta_{12} \end{bmatrix} \begin{bmatrix} I_1 \\ J_{12} \end{bmatrix} \]

\[ \ell_2 = \begin{bmatrix} \Delta_{21} & \Delta_{22} & \Delta_{23} \end{bmatrix} \begin{bmatrix} J_{12} \\ I_2 \\ J_{23} \end{bmatrix} \]

\[ \ell_3 = \begin{bmatrix} \Delta_{31} & \Delta_{32} & \Delta_{33} \end{bmatrix} \begin{bmatrix} J_{23} \\ I_3 \\ J_{34} \end{bmatrix} \]

\[ \ell_4 = \begin{bmatrix} \Delta_{41} & \Delta_{42} \end{bmatrix} \begin{bmatrix} J_{34} \\ I_4 \end{bmatrix} \]

where \( \ell_i \) are vectors of observations and \( \Delta_{ij} \) are design matrices.
Each block is processed in sequence, producing partial solutions for the junction parameters, which are then fed into the next block. The partial estimates of the inner parameters are stored for use in the reverse pass. The partial estimate is calculated using the inverse of a partitioned symmetric matrix. The inverse of a partitioned matrix $E$ (of full rank) is (Leahy 1999),

$$E^{-1} = \begin{bmatrix} E_{11} & E_{12} \\ E_{21} & E_{22} \end{bmatrix}^{-1} = \begin{bmatrix} (E_{11} - E_{12}E_{22}^{-1}E_{21})^{-1} & 0 \\ 0 & (E_{22} - E_{21}E_{11}^{-1}E_{12})^{-1} \end{bmatrix} \begin{bmatrix} I & -E_{12}E_{22}^{-1} \\ -E_{21}E_{11}^{-1} & I \end{bmatrix}$$

(5.16)

Adopting the notation of equation (3.72), the normal equations of Block 1 take the form,

$$\begin{bmatrix} N_{11} & N_{12} \\ N_{21} & N_{22} \end{bmatrix} \begin{bmatrix} \hat{I}_1 \\ \hat{I}_{12} \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

(5.17)

and the full solution for Block 1, using equation (5.16), is then,

$$\begin{bmatrix} \hat{I}_1 \\ \hat{I}_{12} \end{bmatrix} = \begin{bmatrix} N_{11} & N_{12} \\ N_{21} & N_{22} \end{bmatrix}^{-1} \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

(5.18)

$$\begin{bmatrix} \hat{I}_1 \\ \hat{I}_{12} \end{bmatrix} = \begin{bmatrix} (N_{11} - N_{12}N_{22}^{-1}N_{21})^{-1} & 0 \\ 0 & (N_{22} - N_{21}N_{11}^{-1}N_{12})^{-1} \end{bmatrix} \begin{bmatrix} I & -N_{12}N_{22}^{-1} \\ -N_{21}N_{11}^{-1} & I \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

(5.19)

As only the measurements from this block are used in the forward pass, the solution will not be rigorous. The partial solution for the junction parameters $\hat{J}_{12}$ is (Leahy 1999),

$$\hat{J}_{12} = \begin{bmatrix} 0 \\ (N_{22} - N_{21}N_{11}^{-1}N_{12})^{-1} \end{bmatrix} \begin{bmatrix} I & -N_{12}N_{22}^{-1} \\ -N_{21}N_{11}^{-1} & I \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

(5.19)

with a covariance matrix of,
The partial estimates of the junction parameters and their covariance matrix from equations (5.19) and (5.20) are passed into the next block where they are treated as pseudo-measurements. The pseudo-measurements are uncorrelated with the observations in the next block, such that,

\[
\begin{bmatrix}
\tilde{J}_{12} \\
\ell_{2}
\end{bmatrix} = \begin{bmatrix}
1 & 0 & 0 \\
A_{21} & A_{22} & A_{23}
\end{bmatrix}
\begin{bmatrix}
J_{12} \\
J_{22} \\
J_{23}
\end{bmatrix}
\] (5.21)

and,

\[
C = \begin{bmatrix}
C_{112} & 0 \\
0 & C_{22}
\end{bmatrix}
\] (5.22)

The partial solution of the next set of junction parameters, \(J_{23}\), is solved for and the process repeats until the last block has been processed, thereby completing the forward pass. In summary, the computational sequence for the forward pass is (Leahy and Collier 1998),

1. Adjust Block 1.
2. Pass junction parameter estimates and their covariance matrix from Block 1 \((\hat{x}_{12}, \hat{C}_{12})\) to Block 2.
3. Adjust Block 2 treating \((\hat{x}_{12}, \hat{C}_{12})\) as pseudo-measurements.
4. Pass junction parameter estimates and their covariance matrix from Block 2 \((\hat{x}_{23}, \hat{C}_{23})\) to Block 3.
5. Adjust Block 3 treating \((\hat{x}_{23}, \hat{C}_{23})\) as pseudo-measurements.
6. Pass junction parameter estimates and their covariance matrix from Block 3 \((\hat{x}_{34}, \hat{C}_{34})\) to Block 4.
7. Adjust Block 4 treating \((F \hat{x}_{34}, F C_{34})\) as pseudo-measurements. The solution for all parameters in Block 4 is rigorous as it contains the influence of all measurements.

At the end of the forward pass, all parameters in the final block will have been solved for rigorously; the solution is equivalent to that obtained by processing all measurements and parameters together in a traditional least squares adjustment. All the other parameters will have been solved for non-rigorously. To obtain rigorous estimates for the other parameters, a reverse pass is performed. The computational procedure for the reverse pass, shown in Figure 5.5, is (Leahy and Collier 1998),

1. Adjust Block 4.
2. Pass junction parameter estimates and their covariance matrix from Block 4 \((r \hat{x}_{43}, r C_{43})\) to Block 3.
3. Adjust Block 3 treating \((r \hat{x}_{43}, r C_{43})\) as pseudo-measurements.
4. Pass junction parameter estimates and their covariance matrix from Block 3 \((r \hat{x}_{32}, r C_{32})\) to Block 2.
5. Readjust Block 3 including \((r \hat{x}_{32}, r C_{32})\) and \((F \hat{x}_{23}, F C_{23})\) to obtain rigorous solution for Block 3.
6. Adjust Block 2 treating \((r \hat{x}_{32}, r C_{32})\) as pseudo-measurements.
7. Pass junction parameter estimates and their covariance matrix from Block 3 \((r \hat{x}_{21}, r C_{21})\) to Block 1.
8. Readjust Block 2 including \((r \hat{x}_{32}, r C_{32})\) and \((F \hat{x}_{12}, F C_{12})\) to obtain rigorous solution for Block 2.
9. Adjust Block 1 treating \((r \hat{x}_{21}, r C_{21})\) as pseudo-measurements to obtain rigorous solution for Block 1.
Figure 5.5 Computational procedure for the reverse pass (adapted from Leahy and Collier (1998))

To ensure that the normal equations are always non-singular, constraint observations are used. If the unconstrained least squares model is,

\[ y = Ax - \ell \]

then the constrained solution will be,

\[ y = \begin{bmatrix} A \\ I \end{bmatrix} x - \begin{bmatrix} \ell \\ \hat{x} \end{bmatrix} \]  \hspace{1cm} (5.23)

where,
is the vector of reduced constraints based on the latest estimates of the parameters $\chi_i'$, $i=1,n$ with a covariance matrix of $\sigma^2 I$. The covariance matrix of the measurements will have the form,

$$C = \begin{bmatrix} C & 0 \\ 0 & \sigma^2 I \end{bmatrix}$$

(5.24)

Using equation (3.73) the solution of the parameters is,

$$\hat{x} = (A^T C^{-1} A + \sigma^2 I)^{-1} A^T P m$$

(5.25)

where $m = \begin{bmatrix} \ell \\ \hat{x} \end{bmatrix}$. Thus if the constraint variance is made large, the constrained solution will be very close to the unconstrained solution since the term $\sigma^{-2} I$ in equation (5.25) will diminish. Leahy (1999) shows that if $\sigma_c$ is 100 times larger than the expected precision of the parameters the constraints will have an insignificant effect on the parameter estimates. The variances of the constraints will be approximately 10000 times the magnitude of the observations. This imbalance can cause rounding problems within the matrix inversion algorithm. To overcome this problem, the normal matrix can be scaled by the transformation (Leahy 1999),

$$M = T N T$$

(5.26)

where $T$ is a diagonal matrix of the form,
and \( n_{ii} \) is the element at the leading diagonal of the \( i^{th} \) row of \( \mathbf{N} \). The transformation of equation (5.26) sets the values along the leading diagonal of \( \mathbf{M} \) to one whilst maintaining the relative magnitude of the off-diagonal terms. The reverse transformation is performed after the normal matrix has been inverted as follows (Leahy 1999),

\[
\mathbf{N}^{-1} = \mathbf{T} (\mathbf{T} \mathbf{N} \mathbf{T})^{-1} \mathbf{T}
\]

Thus no loss of accuracy is incurred because of the vastly different magnitudes of the measurement and constraint variances. The forward and reverse passes are iterated until the solution converges. After each iteration, the \textit{a priori} estimates and constraint observations are updated to the most recent estimates of the parameters.

As the entire (deterministic plus stochastic) ionospheric delay can be estimated using phased adjustment, there is no need to apply a deterministic ionosphere model even for very long baselines, a weakness of the widelane and pre-elimination approaches. Table 5.1 highlights the main differences between the pre-elimination and phased adjustment approaches in the estimation of the ionospheric delay. Phased-adjustment offers a more generic and flexible option than pre-elimination. The disadvantage of phased adjustment is that it is much less efficient than pre-elimination. The reasons for this are that two passes must be made on the data and the results from each block must be stored. Also, the phased adjustment requires more iterations for the solution to converge than ordinary least squares or pre-elimination. However, since rigorous estimates of the ionospheric parameters are not required, the phased adjustment algorithm can be reduced to just the first pass to make it more efficient, see Brown (2003).
<table>
<thead>
<tr>
<th>Criteria</th>
<th>Pre-Elimination</th>
<th>Sequential Phase Adjustment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rigorous</td>
<td>Yes, depending on whether it is reasonable to model nuisance parameters as noise and if an appropriate variance is used</td>
<td>Yes, providing that appropriate variances are used</td>
</tr>
<tr>
<td>Precise</td>
<td>Depends on variance given to nuisance parameters</td>
<td>Yes</td>
</tr>
<tr>
<td>Temporal Correlation</td>
<td>Not modelled</td>
<td>Can be modelled. Partial estimates from the current epoch can be used as a priori values for the next epoch</td>
</tr>
<tr>
<td>Estimation of Ionosphere</td>
<td>Not required (implicit)</td>
<td>Required</td>
</tr>
<tr>
<td>Parameters</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Deterministic Model</td>
<td>Required</td>
<td>Not required</td>
</tr>
<tr>
<td>Relative Efficiency</td>
<td>High</td>
<td>Low</td>
</tr>
<tr>
<td>Efficiency Relative to Least Squares</td>
<td>Very High</td>
<td>High</td>
</tr>
</tbody>
</table>

Table 5.1 Pre-elimination versus phased adjustment

The application of phased adjustment to estimating the ionospheric path delay closely follows the example given above. In this application, the blocks follow a linear pattern where one block represents one epoch or several consecutive epochs of observations. The inner parameters, which are unique to a particular block, are the ionospheric delays. The junction parameters are the station coordinates and ambiguity parameters, which are common to multiple blocks. The station coordinates will be common to every epoch and the ambiguities will be common to a series of epochs. At any given epoch, the junction parameters will be the station coordinates plus all ambiguities that have had observations in the current or previous epoch.

The parameters are separated in the same fashion as was used for the pre-elimination technique, shown in equation (5.6),

\[
\textbf{v} = \textbf{Ax} - \ell \\
= \textbf{A}a + \textbf{B}k - \ell \\
= \begin{bmatrix} \textbf{A} \\ \textbf{B} \end{bmatrix} \begin{bmatrix} a \\ k \end{bmatrix} - \ell
\]

where \(a\) contains the station coordinates and the integer ambiguities and \(k\) contains the ionospheric path delays. Expanding equation (5.29) gives,
where \( t_i \) indicates the epoch. In fact the vector of station coordinates and ambiguities \( \mathbf{a} \) is also not static as it grows whenever ambiguity parameters are first processed.

The inner and junction parameters of each block are:

<table>
<thead>
<tr>
<th>Block</th>
<th>Inner Parameters</th>
<th>Junction Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>( k(t_0) )</td>
<td>( \mathbf{a}_0 )</td>
</tr>
<tr>
<td>1</td>
<td>( k(t_1) )</td>
<td>( \mathbf{a}_1 )</td>
</tr>
<tr>
<td>\vdots</td>
<td>\vdots</td>
<td>\vdots</td>
</tr>
<tr>
<td>p</td>
<td>( k(t_p) )</td>
<td>( \mathbf{a}_p )</td>
</tr>
</tbody>
</table>

The solution of the junction parameters follows the example given above. If the user has no interest in the ionospheric path delays, then a forward pass will be sufficient because the junction parameters are carried through to the last epoch and will therefore be rigorously estimated.

Applying the constraints of equation (5.23) to the partitioned least squares model for the ionospheric delays, equation (5.6), gives,

\[
\begin{align*}
\begin{bmatrix}
\ell(t_0) \\
\ell(t_1) \\
\vdots \\
\ell(t_p)
\end{bmatrix} &= \begin{bmatrix}
\mathbf{A}(t_0) & \mathbf{B}(t_0) & 0 & \cdots & 0 \\
\mathbf{A}(t_1) & 0 & \mathbf{B}(t_1) & \cdots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
\mathbf{A}(t_p) & 0 & 0 & 0 & \mathbf{B}(t_p)
\end{bmatrix} \begin{bmatrix}
\mathbf{a} \\
k(t_0) \\
k(t_1) \\
\vdots \\
k(t_p)
\end{bmatrix} \\
\begin{bmatrix}
\ell_a \\
\ell_k
\end{bmatrix} &= \begin{bmatrix}
\mathbf{A} & \mathbf{B} \\
\mathbf{I} & 0
\end{bmatrix} \begin{bmatrix}
\mathbf{a} \\
k
\end{bmatrix}
\end{align*}
\]

(5.30)
constraints will be $\sigma_s = 0.1\,\text{m}^2$ for the coordinates and ambiguities and $\sigma_i = 1.0\,\text{m}^2$ for the ionospheric delays. The covariance matrix will then be,

$$
C = \begin{bmatrix}
\sigma^2 & 0 & 0 \\
0 & \sigma^2_i & 0 \\
0 & 0 & \sigma^2_i
\end{bmatrix}
$$

(5.32)

It was demonstrated earlier in the paper that, whilst the ionospheric delays between epochs are different, they are often highly correlated. Thus the ionospheric delay at epoch $t_i$ will be similar to that of epoch $t_{i-1}$. This *a priori* knowledge can be incorporated into the phased adjustment to strengthen the solution. The implementation is achieved by using the (non-rigorous) estimates of the ionospheric delay from epoch $t_{i-1}$ as the constraints of the ionospheric delay for epoch $t_i$. The expected value of the ionospheric delay will be,

$$
E[I_i(t_p)] = I'_i(t_{p-1})
$$

(5.33)

with a dispersion of

$$
D[I_i(t_p)] = \sigma_{\text{ion}}^2
$$

(5.34)

where $I_i$ is the ionospheric delay for the $i$th double difference. The dispersion of the ionosphere will depend on such things as TEC activity, baseline length and the amount of atmosphere the signals pass through, which is related to the satellite elevation. Using this approach it is possible to incorporate knowledge regarding the stochastic behaviour of the ionosphere into the adjustment.

### 5.4 STOCHASTIC MODEL

Typically only a basic stochastic model is used to model the noise characteristics of the measurements. Due to the fact that different makes of GPS receiver do not use a common method for measuring SNR, the various models of antenna have different antenna gain patterns and often only mapped SNR values are contained in RINEX
files, SNR is rarely used as a indicator of measurement noise. Rather elevation is preferred because it is readily available and provides a good approximation of the general measurement noise behaviour as explained in section 3.3.4.1.2. For example, the Bernese GPS Software v4.2 (Hugentobler et al. 2001) uses a stochastic model based on the zenith angles given in equation (3.44). Even with the use of precise orbits, with long baselines there will be correlation between the reduced observations due to the residual orbit error. However with long observation sessions, neglecting this type of correlation has little impact on the estimates or ambiguity resolution.

5.5 SUMMARY

This Chapter discussed the quasi-ionosphere-free processing strategy that is used in GPS positioning to be able to resolve ambiguities over hundreds and even thousands of kilometres. The aim of the QIF approach is to be able to estimate the individual satellite-receiver path delays caused by the ionosphere, the most problematic error source for long baseline ambiguity resolution. Due to the vast number of parameters that must be estimated, a more efficient estimation technique than least squares is required to make the procedure more practical. The method currently in use today, pre-elimination, requires that the ionospheric path delays are random in nature. However, it was shown in section 4.4.4 that with current deterministic models of the ionosphere, the residual ionospheric path delays are far from random. An alternative approach, namely sequential phased adjustment, was suggested as a more robust estimation technique for use in the ionosphere-float QIF processing strategy. Whilst it is not as computationally efficient as pre-elimination, sequential phased adjustment does not require that the ionospheric parameters are white noise parameters with an expectation of zero. As such, sequential phased adjustment makes it possible to model correlations and estimate the remaining systematic error.
6. **TEST RESULTS AND ANALYSIS**

6.1 **INTRODUCTION**

In this Chapter empirical testing is used to evaluate the performance improvement gained using sequential phased adjustment to estimate the ionosphere over the accepted pre-elimination technique. Tests are conducted with and without the use of deterministic ionosphere models in order to measure the reliability of the approach on \textit{a priori} ionosphere data. Section 6.2 describes the test setup including the GPS raw data, orbit and other input data, software, computation procedure and data flow. Section 6.2 also introduces some analysis techniques that can be used to measure the randomness of residuals. In section 6.3 the processing results are analysed in terms of the accuracy of the float solution, the success of the QIF ambiguity resolution, accuracy of the fixed solution and the randomness of the resulting residuals. A summary of the results is given in section 6.4.

6.2 **TEST SETUP**

6.2.1 **Test Environment**

The Bernese GPS Software Version 4.2 has been chosen as the basic processing engine. The Bernese software has undergone over 15 years of development and testing and is regarded in the geodesy community as one of the premier GPS processing packages. The number of the IGS analysis centres that use Bernese for processing reflects its acceptance by the wider geodetic community. As such, Bernese also provides the ideal basis with which to compare the approach developed in this
research against industry standard algorithms. Note that during the research Bernese GPS Software v5.0 was released. Bernese v5.0 includes a lot of user interface, data flow changes and automisation changes, support for new formats, improved zero difference processing and many other changes (Fridez 2004; Hugentobler et al. 2006). For long single baseline DD processing, the important changes are the addition of a new a priori troposphere model based on the work of Niell (1996), the piece-wise linear estimation of tropospheric parameters and a constant added to the mapping function used with the SLM. In this testing no a priori troposphere model was used and the tropospheric parameters were estimated as piece-wise constants. The impact of these changes should be that the tropospheric estimates are slightly improved in Bernese v5.0. However, these changes in Bernese do not influence the ionospheric estimation tests performed in this research. Since the IONEX maps used in this research were calculated based on the older mapping function, consistency is maintained by using Bernese v4.2.

The implementation and testing of the new sequential phased adjustment based approach was achieved through a mixture of modifying and recompiling the Bernese Fortran 77 source code and writing an independent test application in C++ that interfaces to Bernese via some custom log files written by the specially adapted version of Bernese. The structure of the Bernese modules meant that implementation directly in Bernese would be very time consuming and unnecessarily complicated. Hence, Bernese was used for the computations using pre-elimination and, during the processing, custom output files were written so that the processing could be exactly reproduced in another software program. Using this approach meant that the precise ephemeris and earth rotation parameter handling, ocean tide loading, troposphere modelling and other corrections that Bernese applies did not have to be reprogrammed or tested. The output extracted from Bernese included:

1. The components of the design, observed minus computed and covariance matrices for each epoch.
2. Information on the frequency of each observation and the type of each parameter.
3. Auxiliary information such as the station coordinates and internal identifiers and values of the reference ambiguities.
It was necessary to extract parts of the necessary data from a number of Bernese modules including GPSEST, AMBREF and BLDDBL. Some data management issues regarding the indexing of parameters, removal of unused parameters and handling of reference ambiguities arose as a result. The consequence of this was that the processing as implemented in the test application is not as efficient as Bernese. The pre-elimination technique was implemented in the test software and the results compared with Bernese to make sure the data was correctly extracted and interpreted. The test application was able to produce identical estimates for the float ambiguities and coordinates and to resolve the same integer ambiguities as Bernese.

### 6.2.2 Test Procedure

The first step was to collect the necessary data for the comparison of pre-elimination and sequential phased adjustment. Various types of data from a range of sources were needed for the test as shown in Table 6.1. Note that Bernese v4.2 does not correct for phase wind-up.

<table>
<thead>
<tr>
<th>Data Type</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Raw GPS Data</td>
<td>Data from the ARGN or Geoscience Australia,</td>
</tr>
<tr>
<td>Precise orbits and earth rotation</td>
<td>IGS Final orbits and earth rotation parameters,</td>
</tr>
<tr>
<td>parameters</td>
<td><a href="http://igscb.jpl.nasa.gov/components/prods.html">http://igscb.jpl.nasa.gov/components/prods.html</a>,</td>
</tr>
<tr>
<td></td>
<td>ftp://cddis.gsfc.nasa.gov/gps/products/</td>
</tr>
<tr>
<td>IONEX maps</td>
<td>The global ionosphere maps from Centre for Orbit Determination Europe,</td>
</tr>
<tr>
<td></td>
<td><a href="http://igscb.jpl.nasa.gov/components/prods.html">http://igscb.jpl.nasa.gov/components/prods.html</a>,</td>
</tr>
<tr>
<td></td>
<td>ftp://cddis.gsfc.nasa.gov/gps/products/ionex/</td>
</tr>
<tr>
<td>Antenna PCV models</td>
<td>The IGS relative calibration models used with the Bernese software,</td>
</tr>
<tr>
<td></td>
<td>ftp://ftp.unibe.ch/aiub/BSWUSER42/GEN</td>
</tr>
<tr>
<td>Satellite specific data and</td>
<td>The IGS models used with the Bernese software,</td>
</tr>
<tr>
<td>manoeuvre files</td>
<td>ftp://ftp.unibe.ch/aiub/BSWUSER42/GEN</td>
</tr>
<tr>
<td>Ocean tide loading models</td>
<td>Provided on request by the Astronomical Institute, University of Berne</td>
</tr>
<tr>
<td>Site coordinates</td>
<td>SINEX files of the IGS and Geoscience Australia weekly solutions,</td>
</tr>
<tr>
<td></td>
<td><a href="http://igscb.jpl.nasa.gov/components/prods.html">http://igscb.jpl.nasa.gov/components/prods.html</a>,</td>
</tr>
<tr>
<td></td>
<td>ftp://cddis.gsfc.nasa.gov/gps/products/</td>
</tr>
</tbody>
</table>

**Table 6.1 Data types and sources**
The GPS raw data used for the testing is described in detail in section 6.2.3. After compiling and configuring the Bernese GPS Software v4.2 and importing the data, the processing steps shown in Figure 6.1 were performed. As discussed in section 5.2, the processing consists of pre-processing, ionosphere-free float solution, L1/L2 float solution, ambiguity resolution and ambiguity fixed solution steps. Figure 6.1 also illustrates the flow of data between the processing steps and the two software applications. The pre-processing steps are common to all subsequent estimation steps using pre-elimination and sequential phased adjustment. The Bernese software was used for all pre-processing steps. Detailed information on the individual steps within the pre-processing and the Bernese modules that were used is given in Table 6.3.

The computation procedure for estimation and ambiguity resolution steps is listed in Table 6.4 together with the input and output data flow. Bernese does not allow for output of residuals or ionospheric path delay estimates when using pre-elimination due to the restrictions of the computation procedure, so this information was not available for testing. Ionospheric path delays and residuals were calculated and output for the sequential phased adjustments. Normally a further step in the processing would be performed, an IF float solution that constrains the fixed ambiguities to their integer values and re-estimates non-fixed float ambiguities and the tropospheric zenith delay and gradient parameters. This final estimation step provides the highest accuracy solution. However, this step was not needed for this test and was not practical due to the problem of importing data from the test application back into Bernese. For this reason the final fixed solution is shown using dashed lines in Figure 6.1.

For the sake of brevity, steps 5 through 8 in Table 6.4 will be referred to in the following sections PE, PE+I, SPA and SPA+I as described in Table 6.2.
<table>
<thead>
<tr>
<th>Step</th>
<th>Name</th>
<th>Description</th>
<th>Software Used</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>PE</td>
<td>Pre-elimination with no <em>a priori</em> ionosphere information</td>
<td>Bernese</td>
</tr>
<tr>
<td>6</td>
<td>PE+I</td>
<td>Pre-elimination with <em>a priori</em> ionosphere information</td>
<td>Bernese</td>
</tr>
<tr>
<td>7</td>
<td>SPA</td>
<td>Sequential phased adjustment with no <em>a priori</em> ionosphere information</td>
<td>Test application</td>
</tr>
<tr>
<td>8</td>
<td>SPA+I</td>
<td>Sequential phased adjustment with <em>a priori</em> ionosphere information</td>
<td>Test application</td>
</tr>
</tbody>
</table>

Table 6.2 Abbreviations used to describe the test procedure
<table>
<thead>
<tr>
<th>Bernese</th>
<th>Test Application</th>
</tr>
</thead>
</table>
| **Pre-Processing** | **INPUT**  
- Raw code and phase  
- Orbits, coordinates |
| **OUTPUT**  
- SD phase  
- Outliers and unpaired observations removed  
- Cycle slips repaired  
- Approximate station coordinates |
| **INPUT**  
- SD ionosphere-free phase  
- Orbits, coordinates, ocean tide loading model |
| **OUTPUT**  
- Station coordinates  
- IF float ambiguities  
- Tropospheric zenith delays and gradients  
- Residuals |
| **Least Squares** |
| **INPUT**  
- L1/L2 float ambiguities and covariances |
| **OUTPUT**  
- Fixed L1/L2 integer ambiguities (where possible)  
- Updated L1/L2 float ambiguities, covariances and station coordinates |
| **Fixed Solution** |
| **INPUT**  
- SD ionosphere-free phase  
- Fixed L1/L2 ambiguities  
- Orbits, coordinates, ocean tide loading |
| **OUTPUT**  
- Station coordinates  
- IF float ambiguities  
- Tropospheric zenith delays and gradients  
- Residuals |
| **Least Squares** |
| **INPUT**  
- Station coordinates  
- L1/L2 float ambiguities and covariances  
- Custom output for test application |
| **OUTPUT**  
- Station coordinates  
- IF float ambiguities  
- Tropospheric zenith delays and gradients  
- Residuals |
| **Fixed Solution** |

**Figure 6.1 Processing flow used for testing of ionospheric estimation**
<table>
<thead>
<tr>
<th>Step</th>
<th>Description</th>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>The CODSPP module was used to estimate receiver clocks and detect outliers in the code data.</td>
<td>Undifferenced GPS raw data, orbits</td>
<td>Receiver clock estimates</td>
</tr>
<tr>
<td>2</td>
<td>Single differenced phase observations were formed using the SNGDIF module. Predefined baselines were used as described in section 6.2.3.</td>
<td>Undifferenced GPS raw data</td>
<td>Single differenced GPS phase data</td>
</tr>
<tr>
<td>3</td>
<td>Cycle slip detection and repair and data screening was performed using the module MAUPRP.</td>
<td>Single differenced GPS phase data, orbits, coordinates</td>
<td>Single differenced GPS phase data</td>
</tr>
<tr>
<td>4</td>
<td>An ambiguity float solution based on the IF combination was used to estimate tropospheric zenith delays and gradient parameters using the module GPSEST. The Niell (1996) mapping function was used and no \textit{a priori} troposphere model was applied.</td>
<td>Single differenced GPS phase data, orbits, coordinates, ocean tide loading models</td>
<td>IF float ambiguities, Tropospheric zenith delays and gradients, residuals</td>
</tr>
</tbody>
</table>

**Table 6.3 Pre-processing and IF float solution procedure used for testing of ionospheric estimation**
<table>
<thead>
<tr>
<th>Step</th>
<th>Description</th>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>A L1+L2 float solution was performed to estimate real valued ambiguities and</td>
<td>Single differenced GPS phase data, orbits, coordinates, ocean tide</td>
<td>L1+ L2 float ambiguities and covariance information, fixed ambiguities,</td>
</tr>
<tr>
<td></td>
<td>ionospheric path delays using the module GPSEST. The ionospheric path delays</td>
<td>loading models, tropospheric zenith delays and gradients</td>
<td>coordinates and quality information from the float and fixed solutions,</td>
</tr>
<tr>
<td></td>
<td>were pre-eliminated each epoch and the normal equations added. The</td>
<td></td>
<td>intermediate data outlined in section 6.2.1</td>
</tr>
<tr>
<td></td>
<td>tropospheric estimates from the pre-processing step were introduced as</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>known values. No <em>a priori</em> information on the ionosphere was used. The</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>QIF technique was used to resolve the integer ambiguities. The final</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>coordinates and quality estimates were calculated after ambiguity resolution</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>was completed.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>A L1+L2 similar to step 5 above was performed with the difference that <em>a</em></td>
<td>As above plus global IONEX maps</td>
<td>As above</td>
</tr>
<tr>
<td></td>
<td>priori information on the ionosphere was introduced.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>A L1+L2 float solution was performed to estimate real valued ambiguities</td>
<td>Single differenced GPS phase data, orbits, coordinates, ocean tide</td>
<td>L1+ L2 float ambiguities and covariance information, fixed ambiguities,</td>
</tr>
<tr>
<td></td>
<td>and ionospheric path delays using sequential phased adjustment as</td>
<td>loading models, tropospheric zenith delays and gradients via the</td>
<td>coordinates and quality information from the float and fixed solutions,</td>
</tr>
<tr>
<td></td>
<td>implemented in the test application. The tropospheric estimates from the</td>
<td>intermediate Bernese output from step 5</td>
<td>ionospheric path delay estimates, residuals</td>
</tr>
<tr>
<td></td>
<td>pre-processing step were introduced as known values. No <em>a priori</em></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>information on the ionosphere was used. The QIF technique was used to</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>resolve the integer ambiguities. The final coordinates and quality</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>estimates were calculated after ambiguity resolution was completed.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>A L1+L2 similar to step 7 above was performed with the difference that <em>a</em></td>
<td>As above plus global IONEX maps</td>
<td>As above</td>
</tr>
<tr>
<td></td>
<td>priori information on the ionosphere was introduced.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 6.4 Ambiguity estimation and resolution procedure used for testing of ionospheric estimation
6.2.3 Test Data

A set of stations from the ARGN was chosen for this testing, most of which also contribute to the IGS network. The chosen sites are listed in Table 6.5 and are shown in Figure 6.2. The sites cover a range of latitudes and longitudes, from COCO in the Indian Ocean at 12° 11’ S, 96° 50’ E to DAV1 in Antarctica at 68° 34’ S, 77° 58’ E and MAC1 in the Pacific Ocean at 54° 29’ S, 158° 56’ E. All stations are permanent reference stations with IGS standard choke ring antennas.

<table>
<thead>
<tr>
<th>Name</th>
<th>Code</th>
<th>Location</th>
<th>Receiver</th>
<th>Antenna</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alice Springs</td>
<td>ALIC</td>
<td>Northern Territory</td>
<td>AOA ICS-4000Z ACT</td>
<td>AOAD/M_T DOME</td>
</tr>
<tr>
<td>Casey</td>
<td>CAS1</td>
<td>Antarctica</td>
<td>AOA ICS-4000Z ACT</td>
<td>AOAD/M_T DOME</td>
</tr>
<tr>
<td>Ceduna</td>
<td>CEDU</td>
<td>South Australia</td>
<td>AOA ICS-4000Z ACT</td>
<td>AOAD/M_T DOME</td>
</tr>
<tr>
<td>Cocos Island</td>
<td>COCO</td>
<td>Indian Ocean</td>
<td>AOA SNR-12 ACT</td>
<td>AOAD/M_T DOME</td>
</tr>
<tr>
<td>Davis</td>
<td>DAV1</td>
<td>Antarctica</td>
<td>AOA ICS-4000Z ACT</td>
<td>AOAD/M_T DOME</td>
</tr>
<tr>
<td>Hobart</td>
<td>HOB2</td>
<td>Tasmania</td>
<td>AOA ICS-4000Z ACT</td>
<td>AOAD/M_T NONE</td>
</tr>
<tr>
<td>Macquarie Island</td>
<td>MAC1</td>
<td>Pacific Ocean</td>
<td>AOA ICS-4000Z ACT</td>
<td>AOAD/M_T DOME</td>
</tr>
<tr>
<td>Mawson</td>
<td>MAW1</td>
<td>Antarctica</td>
<td>AOA ICS-4000Z ACT</td>
<td>AOAD/M_T DOME</td>
</tr>
<tr>
<td>Tidbinbilla</td>
<td>TIDB</td>
<td>Australian Capital Territory</td>
<td>AOA ICS-4000Z ACT</td>
<td>AOAD/M_T JPLA</td>
</tr>
<tr>
<td>Yaragadee</td>
<td>YAR2</td>
<td>Western Australia</td>
<td>AOA ICS-4000Z ACT</td>
<td>AOAD/M_T DOME</td>
</tr>
</tbody>
</table>

Table 6.5 Sites used for testing of ionospheric estimation

![Figure 6.2 Sites and baselines used for testing of ionospheric estimation](image-url)
For this research a selection of eight baselines listed in Table 6.6 and shown in Figure 6.2 in bold were used. The baselines range between 636 and 4,080 kilometres in length. These combinations give two independent baselines for four categories of length: medium (approx. 500 to 1000km), medium-long (approx 1500km), long (approx. 2500km) and very-long (approx. 4000km). With each baseline processed four times, a total of 480 solutions were computed.

<table>
<thead>
<tr>
<th>Baseline</th>
<th>Station 1</th>
<th>Station 2</th>
<th>Length (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ALCE</td>
<td>ALIC</td>
<td>CEDU</td>
<td>907</td>
</tr>
<tr>
<td>ALCO</td>
<td>ALIC</td>
<td>COCO</td>
<td>4,040</td>
</tr>
<tr>
<td>CADA</td>
<td>CAS1</td>
<td>DAV1</td>
<td>1,405</td>
</tr>
<tr>
<td>DAMW</td>
<td>DAV1</td>
<td>MAW1</td>
<td>639</td>
</tr>
<tr>
<td>HOMA</td>
<td>HOB2</td>
<td>MAC1</td>
<td>1,553</td>
</tr>
<tr>
<td>TIMA</td>
<td>TIDB</td>
<td>MAC1</td>
<td>2,246</td>
</tr>
<tr>
<td>YACA</td>
<td>YAR2</td>
<td>CAS1</td>
<td>4,080</td>
</tr>
<tr>
<td>YACO</td>
<td>YAR2</td>
<td>COCO</td>
<td>2,686</td>
</tr>
</tbody>
</table>

Table 6.6 Baselines used for testing of ionospheric estimation

Five days of data from each of summer, winter and spring were selected for the testing. The dates and times are listed in Table 6.7. The data are from 2002, near the peak of a solar maximum and, as such, provide challenging conditions for ionospheric estimation. To illustrate the activity of the ionosphere during the test period, Figure 6.3 shows the vertical TECU values for the three Julian days 6, 189 and 265 at about noon Australian Eastern Standard Time (AEST) for the years 2002 and 2006. The same colour gradients are used for all TEC maps with deep blue representing 0 VTECU and deep red representing greater than 90 VTECU. Clearly the test data encompasses a period of elevated ionospheric activity. The use of data from near the solstices and vernal equinox provides a range of solar input in addition to the overall high level of ionospheric activity during 2002.

<table>
<thead>
<tr>
<th>Time Period</th>
<th>Calendar Date</th>
<th>Julian Day</th>
<th>GPS Week</th>
</tr>
</thead>
<tbody>
<tr>
<td>Summer</td>
<td>6-10 January 2002</td>
<td>006-010</td>
<td>1148, day 0-4</td>
</tr>
<tr>
<td>Winter</td>
<td>8-12 July 2002</td>
<td>189-193</td>
<td>1174, day 1-5</td>
</tr>
<tr>
<td>Spring</td>
<td>22-26 September 2002</td>
<td>265-269</td>
<td>1185, day 0-4</td>
</tr>
</tbody>
</table>

Table 6.7 Dates used for testing of ionospheric estimation
One station, YAR2, is missing data for the whole of JD9. On JD10 MAC1 has only 67% data completeness due to data gaps. The average number of L1+L2 observations, ionospheric path delay parameters and ambiguity parameters for the various baselines are shown in Figure 6.4. The two baselines CADA and DAMW both have a high number of ambiguities due partly to cycle slips detected by the MAUPRP module and partly due to the short observation periods of each satellite due to the low latitude and east-west orientation of the baselines. Figure 6.5 shows the dependence of the degrees of freedom on the baseline length. Long baselines typically have fewer common satellites and, hence, less redundancy.

![Figure 6.3 Vertical TECU for Julian days 6, 189 and 265 for 2002 (left) and 2006 (right) at 12 noon AEST](image)
Figure 6.4 Number of observations and parameters per baseline

Figure 6.5 Average degrees of freedom against baseline length
6.2.4 Tools for Evaluation of Model Performance

6.2.4.1 Randomness of the Residuals

The more successful the modelling process is in removing or modelling the systematic and noise components of the measurements, the closer the residuals will reflect the noise characteristics of the measurements. As such, it is expected that the residuals from the adjustment will be more random when more effective mathematical and/or stochastic models are used. Most geodetic measurements are assumed to be Gaussian white-noise random variables. The closer the characteristics of the residuals resemble those of the zero-baseline the better the mathematical/stochastic model combination is.

6.2.4.1.1 Zero Baseline Analysis

Baselines of zero length (two receivers connected to a single antenna via an antenna splitter) are useful for assessing receiver performance such as noise and inter-channel biases. Clock, atmospheric, antenna, orbit and multipath errors will be fully correlated in a GPS zero baseline and will cancel almost perfectly leaving only the measurement noise. As such the pure noise characteristics of the double difference observations can be seen in zero-baseline data. This then provides a benchmark against which the residuals from non-zero baseline results can be compared. One approach is to plot the covariance function of the residuals from both adjustments. Another is to plot sequential variance estimates on a logarithmic scale and examine the gradient.

6.2.4.1.2 Covariance Functions

A covariance function (also called a variogram) is used to model the variance and covariance between two sequences for a specified lag (separation or interval) operator. Typical lags that are used in geodesy are time and distance. The covariance function of a single sequence with itself is called the auto-covariance function. The covariance function of residuals from a more effective modelling technique will tend to be closer to that of the zero-baseline than those from a less effective processing model.
Consider a random variable $x$ that is sampled across some dimension $t$. The auto-covariance function of $x$ is given as:

$$C(t_1 - t_2) = \text{cov}(x(t_1), x(t_2))$$  \hspace{1cm} (6.1)

Empirical covariance can be calculated from sample data for a given lag. For GPS measurements, which are taken at discrete times, the intervals are simply a fixed time separation or number of epochs. The sample covariance for random variables $x$ and $y$ with $m$ distinct pairs and a mean of $\bar{x}$ and $\bar{y}$ for a given interval $j$ is:

$$\hat{C}(j) = \sigma_{xy,j} = \sigma_{x,i} \sigma_{y,i+j} = \frac{1}{m} \sum_{i=0}^{m} (x_i - \bar{x})(y_{i+j} - \bar{y})$$  \hspace{1cm} (6.2)

The correlation coefficient may be calculated by

$$\rho_{xy} = \frac{\sigma_{xy}}{\sigma_x \sigma_y}$$  \hspace{1cm} (6.3)

If the process is truly random (white noise) then the random variables will be uncorrelated (but not necessarily independent) random variables. The correlation will then be:

$$\rho_{xy} = \begin{cases} 1 & x = y \\ 0 & x \neq y \end{cases}$$  \hspace{1cm} (6.4)

The sample variance and covariances form an empirical covariance function for the data which can be plotted against the lag variable. Plotting the empirical covariance function provides an effective way of checking if two random variables are correlated. The covariance function assumes stationarity and is therefore not useful for identifying changes in the covariance behaviour through time.

If the receiver noise were truly Gaussian white noise then one would expect zero correlation for all lags except zero and the covariance function of the double-
difference residuals to drop immediately and oscillate around zero. Figure 6.6 shows the empirical covariance function of DD residuals from a zero baseline. It can be seen that the variance (time lag equals zero) varies between the double differences but that the covariance is effectively zero for all non-zero lags.

![Figure 6.6 Covariance function of DD residuals from a zero baseline](image)

Such an approach to the analysis of zero-baseline data can be misleading because the noise in the measurements may also be correlated between the two receivers. The reason for this is twofold. Firstly most of the noise in the measurements comes from the preamplifier in the antenna (Gourevitch 1996). Secondly, recall that the noise is a function of the $C/N_0$, which is determined by the signal strength, signal loss during propagation and the antenna gain pattern. Therefore the measurement noise will be similar for both receivers and is thus correlated. The result of this correlation is that the measurement noise will be partially cancelled in a zero baseline through differencing.

6.2.4.1.3 Sequential Variance Plots

Gourevitch (1996) outlines an alternative method that can be used to test whether the residuals are truly white noise. This method is based on the premise that for statistically independent data the variance is inversely proportional to the averaging time. Sequential variance estimates are calculated by averaging the residuals from
zero to a maximum averaging time. Consider the case where there are \( m \) residuals in a series of residuals \( v \), then \( k \) residual sequences would be formed as

\[
s_1 = \begin{bmatrix} v_1 & v_2 & v_3 & \ldots & v_m \end{bmatrix}
\]

\[
s_2 = \begin{bmatrix} (v_1 + v_2)/2 & (v_3 + v_4)/2 & \ldots & (v_{m-1} + v_m)/2 \end{bmatrix}
\]

\[ \vdots \]

\[
s_k = \begin{bmatrix} \frac{1}{k} \sum_{i=0}^{k} v_i & \frac{1}{k} \sum_{i=k}^{2k} v_i & \ldots & \frac{1}{k} \sum_{i=m-k}^{m} v_i \end{bmatrix}
\]

The variance \( \sigma_i^2 \) of each sequence \( S_i \) is then calculated and plotted on a log scale graph with the x-axis the averaging time (\( k \)) and the y-axis as (\( \sigma_i^2 * k \)). If the residuals are white noise then \( \sigma_i^2 \) should be half \( \sigma_1^2 \); \( \sigma_3^2 \) should be one third of \( \sigma_1^2 \) and so on. Therefore we would expect the plot to show a straight horizontal line if the residuals are uncorrelated. This technique allows for numerical comparison of results through the use of the line gradients.

Figure 6.7 shows Gourevitch’s white noise test computed using data from a zero baseline. Each series in the plot represents a sequence of DD residuals. As the averaging time increases the variance calculation becomes unstable due to the low number of samples. However, it is clear looking at the averaging times less than 10 epochs that the gradient is almost zero (the average gradient is \(-0.041\)) indicating (nearly) random data.
6.3 RESULTS AND ANALYSIS

6.3.1 Preamble

The key performance indicators resulting from the computations are the following:

- *a posteriori* sigma of unit weight from the L1+L2 ambiguity float solution
- accuracy of the ambiguity float solution
- *a posteriori* sigma of unit weight from the L1+L2 ambiguity fixed solution
- accuracy of the ambiguity fixed solution
- the percentage of fixed ambiguities
- the randomness of the residuals

The *a posteriori* sigma of unit weight, $\hat{\sigma}_0$ from equation (3.76), is an overall indicator of modelling errors and, for this kind of processing, is expected to be in the range of 1.0 to 1.5 (Hugentobler et al. 2001). Accuracy has been calculated as the difference between the estimated and known coordinates and is useful for detecting any biases in the solution. The percentage of fixed ambiguities is an indicator of both
precision and accuracy since the RMS of the ionospheric bias (equation (5.1)) must be small and the ambiguities close to integers (equation (5.5)) for the ambiguities to be fixed. The accuracy value provides a check on the ambiguity resolution since the position estimate will be biased if an ambiguity is fixed incorrectly. The values of the key performance indicators resulting from each baseline and day are tabulated in Appendix C.

As a general note, the computation time required for the phased adjustment can be substantially higher than for pre-elimination. Compared with pre-elimination, sequential phased adjustment requires more matrix operations (multiplications and inversions). Also, in this testing it was seen that two iterations (each consisting of a forward and reverse pass) were needed for sequential phased adjustment, whereas one sufficed with pre-elimination. The final reverse pass could be skipped since the ionospheric estimates and residuals are not normally of interest. However, the main difference compared to pre-elimination is that the size of the matrices grows due to the increasing number of inner parameters (i.e. ambiguities) that must be carried forward into the next block. The time needed to invert the $N_{i1}^{-1}$ matrix increases exponentially with the number of inner parameters. Since this matrix must be inverted for every block, the processing time can become considerable (minutes or even tens of minutes) if there are hundreds of ambiguity parameters. Pre-elimination on the other hand is much more linear in the way that processing time relates to the number of parameters. Increasing the number of epochs in each block can help to reduce the number of inversions. However, due to the number of observations, pseudo measurements and ionospheric constraints, processing more than a few epochs at once increases the time to perform the adjustment.

Note that not all graphs and figures are presented in the results. Rather an illustrative subset of the data has been chosen to present the key results. The chosen subset is representative of the overall data.

6.3.2 Ambiguity Float Solution

The effectiveness of the estimation techniques can be evaluated simply by comparing the accuracy of the estimated parameters. Since no truth values are known for the
ambiguities, the station coordinates must be used. For this comparison, the station coordinates from the IGS weekly solution for the same period as the test data were used as the truth values. The IGS weekly solution estimates combine the processing results of ten IGS Analysis Centres and are the best available estimates of the station coordinates. The estimated accuracy of the weekly solutions is 3mm horizontal and 6mm vertical. For the site COCO, which is not part of the IGS network, the weekly solution of the Space Geodesy Analysis Centre, Geoscience Australia was used instead. The absolute difference between the solutions from the IGS and Geoscience Australia for the other sites is on average 7mm and is consistent for each of the weekly solutions.

Figure 6.8 shows the average improvement in 3D accuracy of the station coordinates from the L1+L2 float solutions for each baseline relative to the results of the pre-elimination strategy with no a priori ionosphere information. On average, all other approaches showed improved accuracy compared with PE, though the improvement is only small. On average the improvements in accuracy of the float solution for PE+I, SPA and SPA+I are 2.6%, 2.3% and 2.2% respectively. This result reflects the problem associated with the assumption made in the pre-elimination technique that the expectation of the ionosphere parameters must be zero. Whilst a difference of 2% may not seem significant, the importance of this difference is clearly seen in the results shown in section 6.3.3. However, when the IONEX models are applied pre-elimination is able to perform at the same level as sequential phased adjustment. It can be seen from Figure 6.9 that the improvement in accuracy relative to the PE approach is higher in summer and spring when the ionospheric is more active.

As is often the case, the results are somewhat dataset dependent. The baseline DAMW shows a decline in accuracy, though at only 0.04mm, the difference is insignificant. For the two baselines ALCO and YACA the improvement with sequential phased adjustment is noticeably more than with PE+I, but otherwise there are no major differences between methods. Interestingly these cases are the two longest (4000km) baselines and those which have the least observations. In contrast, the average a posteriori sigma of unit weight $\hat{\sigma}_0$ (Figure 6.10) indicates that
sequential phased adjustment is better at handling the ionospheric path delays than PE and PE+I. Since the raw data, pre-processing and parametisation is the same in all cases, the difference in $\hat{\sigma}_0$ cannot be due to data problems such as undetected cycle slips.

Figure 6.8 Average improvement of 3D accuracy by baseline for the L1+L2 float solutions relative to pre-elimination

Figure 6.9 Average improvement of 3D accuracy by season for the L1+L2 float solutions relative to pre-elimination
The quality of the SPA float solution can also be verified by comparing the residuals to those of the ionospheric-free float solution. Figure 6.11 shows the histogram of IF residuals from the IF float solution and L1 residuals from the SPA solution for the 900km baseline ALCE on JD6. Similarly, Figure 6.12 shows the histogram of residuals from the 4000km baseline ALCO for the same day. The histograms all have a shape similar to the Normal distribution. The standard deviation of the IF residuals is 3.4 times higher than the L1 residuals as expected from the noise characteristics of the two observables given in Table 3.1. Similar histograms were produced for the other baselines and for the SPA+I adjustment, indicating that the sequential phased adjustment is able to effectively estimate the ionospheric path delays. This is confirmed by the sequential white noise plots, Figure 6.13 and Figure 6.14 for the baseline CADA and Figure 6.15 and Figure 6.16 for the baseline ALCO. The y-scale is different due to the different magnitudes of the residuals, though the patterns are identical. The average gradients of 0.733 and 0.722 for the CADA and ALCO baselines indicate that the residuals are not very random, probably due to residual orbit, troposphere and multipath error. Unfortunately, no residuals are available for the PE and PE+I as discussed in section 6.2.2.
Figure 6.11 Histogram of residuals from the IF and SPA float solutions for CADA on JD6

Figure 6.12 Histogram of residuals from the IF and SPA float solutions for ALCO on JD6
Figure 6.13 Sequential variance white noise test for the residuals from the CADA IF float solution on JD6

Figure 6.14 Sequential variance white noise test for the residuals from the CADA SPA float solution on JD6
6.3.3 Ambiguity Resolution

Figure 6.17 shows the average percentage of ambiguities that were fixed per baseline for each of the four tested strategies. On average, the improvements in the number of ambiguities fixed relative to PE are 16.2%, 24.9% and 25.8% for PE+I, SPA and SPA+I respectively. The PE approach clearly has trouble fixing ambiguities, particularly with the ALCO baseline. In fact, it is due to the baselines ALCO and YACO that the improvement in fixed ambiguities by season, Figure 6.18, shows a
remarkable difference between summer and the other seasons. For the summer data, the ALCO baseline showed improvements in fixed ambiguities for PE+I, SPA and SPA+I of 378%, 442% and 453% respectively. It can be seen in Figure 6.19 that the PE approach has extreme difficulties in fixing ambiguities for four of the five days in January, during which time the other approaches were much more consistent. Similarly, the YACO baseline showed improvements in fixed ambiguities for PE+I, SPA and SPA+I of 195%, 217% and 217% respectively. If the ALCO and YACO baselines are excluded from the calculation, the improvement is approximately the same for each season. In comparison, the results for the baseline HOMA (Figure 6.20) are similar for all datasets and for all approaches.

In order to explain the differences between the results of the two baselines ALCO and HOMA, the DD ionospheric delays were computed using the geometry free linear combination (equation (3.31)). Two representative days were chosen for this analysis, JD6 for the problematic data and JD265 for the normal data. On examination of the histograms of DD ionospheric delays for JD6 (Figure 6.21), it can be seen that the curve for ALCO is much flatter with more data in the tails and less around zero. However, on JD265, when all approaches were similarly successful for both baselines, the histograms have basically the same shape (Figure 6.22). Aside from the additional data in the tails for ALCO/JD6, particularly above 12m, the overall the magnitudes of the DD ionospheric delays are not substantially different for the two days. In order to determine if the absolute error or the randomness of the error is the cause of the problems with the PE approach, the sequential variance test of section 6.2.4.1.3 was calculated and the results are shown on the secondary y-axis of Figure 6.19 and Figure 6.20. Whilst the average gradient for HOMA is lower than for ALCO (0.61 versus 0.80), there does not appear to be a correlation between the gradient and the amount of ambiguities that are resolved. This result supports the idea that treating the ionosphere as noise, which is the basis of pre-elimination, causes problems if a significant number of observations have residual ionosphere error exceeding about 12m. Sequential phased adjustment on the other hand is still able to perform effectively in this situation.
Figure 6.17 Average percentage of fixed ambiguities by baseline

Figure 6.18 Average relative improvement in fixed ambiguities by season
Figure 6.19 Fixed ambiguities by day for the ALCO baseline

Figure 6.20 Fixed ambiguities by day for the HOMA baseline
The RMS of the estimated ambiguity parameters for the baselines ALCO and HOMA for the JD6 dataset are shown in Figure 6.23 and Figure 6.24 respectively. The differences in the RMS values between the pre-elimination and sequential phased adjustment approaches are much more pronounced in the ALCO than in HOMA. The average RMS values for ALCO are 3.57, 1.48, 0.65 and 0.65 cycles for PE, PE+I,
SPA, SPA+I respectively. This corresponds to a reduction of RMS by 142%, 448% and 450% relative to PE. For the HOMA baseline the relative decrease in average RMS are a more modest 45%, 89% and 89% respectively. The pattern in the RMS is the same for all approaches, though for sequential phased adjustment the peaks are flattened out.

Figure 6.23 RMS of float ambiguities for the ALCO baseline on JD6

Figure 6.24 RMS of float ambiguities for the HOMA baseline on JD6
6.3.4 Ambiguity Fixed Solution

Figure 6.25 shows the average improvement in coordinate accuracy between the ambiguity float and fixed solutions. The average improvements in accuracy for the fixed solution relative to the float solution are -11.6%, 4.4%, -10.0% and 6.0% for PE, PE+I, SPA and SPA+I respectively. A drop in accuracy between the float and fixed solutions could indicate that one or more ambiguities have been resolved incorrectly. Not surprisingly, PE shows a significant drop in accuracy. More surprising is the fact that SPA also shows a drop in accuracy between the float and fixed solutions. In apparent conflict with this result, the \( a \text{ posteriori sigma} \) of unit weight clearly indicates that both SPA and SPA+I are effectively modelling the data (Figure 6.26). In order to further investigate this discrepancy, the residuals for the datasets where a drop in accuracy occurred were examined. Two such cases are the CADA baseline on JD6 and the ALCO baseline on JD268. The CADA/JD6 baseline showed a drop in accuracy from 6.9mm to 10.0mm for SPA but an improvement in accuracy from 6.9mm to 1.9mm for SPA+I. Similarly, the ALCO/JD268 baseline showed a drop in accuracy from 20.2mm to 29.2mm for SPA but an improvement in accuracy from 20.2mm to 14.2mm for SPA+I. In both cases, the quality of the float adjustment is verified by the histogram of residuals, which can be seen for the two datasets in Figure 6.27 and Figure 6.29 respectively. The SPA and SPA+I adjustments produce nearly identical residuals that closely resemble the Normal distribution, corroborating the \( a \text{ posteriori sigma} \) of unit weight values near one. The sequential variance white noise test also produces nearly identical results. For the baseline CADA/JD6 the average gradient was 0.733 for the SPA adjustment and 0.734 for the SPA+I adjustment, see Figure 6.14 and Figure 6.28 respectively. Similarly for the baseline ALCO/JD268 the average gradient was 0.824 for the SPA adjustment and 0.825 for the SPA+I adjustment, see Figure 6.30 and Figure 6.31 respectively. However, whilst the accuracy of the float solutions, \( a \text{ posteriori sigma} \) of unit weight and distribution of residuals are nearly identical for the SPA and SPA+I adjustments, the estimated values for the ambiguities are not.

Figure 6.32 and Figure 6.33 show the values of the estimated ambiguities by the SPA and SPA+I approaches for the CADA/JD6 and ALCO/JD268 datasets respectively. The ambiguity estimates from the SPA adjustment are clearly different
from those estimated using SPA+I for the 1,400km baseline CADA. For the 4,000km baseline ALCO the differences are even more pronounced. It is important to note that the ambiguities estimated by SPA are generally larger than those estimated by SPA+I. A reasonable explanation for this behaviour is that the ambiguity parameters are absorbing part of the ionospheric path delays. Recall that the IONEX maps, which are applied in SPA+I but not SPA, have a temporal resolution of only 2 hours. As such, they only remove the large, slow changing part of the ionosphere. Over short periods of time there is a strong correlation between the ionospheric path delays and the ambiguities. Each block in the phased adjustment, for reasons of efficiency, contains only one or at most several epochs of data. With only a snapshot of data it is not possible for the estimation routine to properly separate the two parameters. This is essentially the same problem as with pre-elimination: the expectation of the ionosphere parameters $k = k_{\text{mean}} - k_{\text{spr}}$ must be $E[k] = 0$ for the estimation to function correctly. So despite the fact that sequential phased adjustment is capable of estimating the full path delays, irrespective of their magnitude, the practical use of this is limited by the difficulty in separating the path delays from the ambiguities. This problem cannot be overcome by tightening the constraint values that are used for the ambiguities or the ionospheric path delays since the magnitudes of both parameters can vary substantially.

![Graph](attachment:image.png)

**Figure 6.25** Average relative improvement of 3D accuracy between float and fixed solutions by baseline
Figure 6.26 The average \textit{a posteriori} sigma of unit weight for the L1+L2 fixed solutions.

Figure 6.27 Histogram of residuals from the SPA and SPA+I solutions for the baseline CADA on JD6.
Figure 6.28 Sequential variance white noise test for the residuals from the CADA SPA+I float solution on JD6

Figure 6.29 Histogram of residuals from the SPA and SPA+I solutions for the baseline ALCO on JD268
Figure 6.30 Sequential variance white noise test for the residuals from the ALCO SPA float solution on JD268

Figure 6.31 Sequential variance white noise test for the residuals from the ALCO SPA+I float solution on JD268
In this Chapter empirical testing was undertaken to evaluate sequential phased adjustment in comparison to pre-elimination for the estimation of coordinates, ambiguities and ionospheric path delays to aid ambiguity resolution over long GPS baselines. A modified version of the Bernese GPS Software v4.2 was used for the testing together with an independent test application that could interface with Bernese via custom log files. Fifteen days of data from ten stations of the ARGN were used for
the testing. The data included five days in each of the (southern hemisphere) summer, winter and spring of the year 2002 providing a range in solar input and an overall high level of ionospheric activity. The key statistics resulting from the tests are shown in Table 6.8.

<table>
<thead>
<tr>
<th>Result</th>
<th>PE</th>
<th>PE+I</th>
<th>SPA</th>
<th>SPA+I</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fixed ambiguities [%]</td>
<td>64.7</td>
<td>75.2</td>
<td>80.9</td>
<td>81.4</td>
</tr>
<tr>
<td>Improvement in fixed ambiguities [%]</td>
<td>-</td>
<td>16.2</td>
<td>24.9</td>
<td>25.8</td>
</tr>
<tr>
<td>Accuracy of the float solution [mm]</td>
<td>13.6</td>
<td>13.3</td>
<td>13.3</td>
<td>13.3</td>
</tr>
<tr>
<td>Improvement in accuracy of the float solution [%]</td>
<td>-</td>
<td>2.6</td>
<td>2.3</td>
<td>2.2</td>
</tr>
<tr>
<td>Accuracy of the fixed solution [mm]</td>
<td>15.4</td>
<td>12.7</td>
<td>14.8</td>
<td>12.6</td>
</tr>
<tr>
<td>Improvement in accuracy of fixed solution compared to float solution [%]</td>
<td>-11.7</td>
<td>4.7</td>
<td>-10.1</td>
<td>5.6</td>
</tr>
</tbody>
</table>

Table 6.8 Key results of ionospheric estimation testing

The results of the ambiguity float solution indicate that sequential phased adjustment is better able to estimate the ionospheric path delays than pre-elimination, producing an *a posteriori* sigma of unit weight closer to one and an overall improvement in accuracy of the station coordinates of 2.2%. The QIF ambiguity resolution strategy was able to fix more ambiguities when based on the results of the float solution using sequential phased adjustment with or without *a priori* information on the ionosphere. Sequential phased adjustment provides on average a 25% improvement in fixed ambiguities compared to pre-elimination and a 9% improvement compared to pre-elimination with a deterministic ionosphere model. However, the results of the ambiguity fixed solution indicate that *a priori* information on the ionosphere is still important for ambiguity resolution as otherwise the ambiguity parameters will absorb some of the ionospheric error leading to incorrect ambiguity fixes. This requirement applies to the ionosphere float model in general and is not specific to when using sequential phased adjustment. When using global IONEX models, the QIF strategy combined with the sequential phased adjustment gave better results in terms of accuracy of the float and fixed solutions and number of ambiguities fixed than the pre-elimination technique used in Bernese. Sequential
phased adjustment with a deterministic ionosphere model shows an average improvement in accuracy between the float and fixed solutions of 5.6% compared to only 4.7% for pre-elimination. The downside of the sequential phased adjustment approach is that it is computationally more demanding than pre-elimination, requiring more matrix manipulation. Also the computation time is linked to the number of inner parameters (coordinates and ambiguities) that must be estimated in each block.
7. CONCLUSIONS AND RECOMMENDATIONS

7.1 CONCLUSIONS

Satellite-based navigation systems make it possible to determine the relative positions of points on the earth with very high accuracy. With suitable user equipment and appropriate data processing techniques, accuracies at the centimetre level can readily be achieved over baselines up to several thousand kilometres in length. The applications of long baseline precise positioning include the measurement of the motion of the earth’s tectonic plates, isostatic rebound, absolute sea level monitoring and datum definition. In order to achieve the highest possible accuracy it is necessary to resolve the carrier phase integer ambiguities. The problem of ambiguity resolution lies in the contamination of the measurements by a wide range of systematic affects including ionospheric delay, tropospheric delay, orbit errors and multipath. For the applications listed above, long observation sessions of hours or even days can be used to help in the estimation and mitigation of some error sources. Observations on two or more frequencies are essential for long baseline processing because the atmospheric errors do not effectively cancel when forming double differences. It was shown that the two most problematic error sources are the ionosphere for ambiguity resolution and the troposphere for accuracy of the final coordinates.

The problem addressed in this thesis is the modelling of the ionosphere during ambiguity resolution process. The preferred approach for handling the ionosphere is the ionospheric float model in which dual frequency phase observations are used to
estimate path delays for each satellite and epoch in addition to the station coordinates and ambiguities. For the 24-hour, 30 second datasets commonly used in geodetic applications, the number of unknowns to be estimated is typically in the order of 20,000. Estimation of such a high number of parameters is not practical using ordinary least squares and standard workstation type computing resources. The problem becomes more significant when large networks, such as the 333 active stations of the IGS network (IGS 2006b), and higher rate data are processed daily. One solution for this problem, and the one that is used in the Bernese GPS Software, is to use the technique of pre-elimination. Pre-elimination divides the unknowns into parameters of interest (the coordinates and ambiguities) and nuisance parameters (the ionospheric path delays). The nuisance parameters are treated as stochastic variables and modelled as process noise. Using matrix operations, the nuisance parameters can be removed from the system of linear equations. If pre-elimination is done for each epoch and the normal equations are stacked, it is only necessary to invert small matrices and the final solution will be equivalent to the rigorous least squares solution. The main limitation of the pre-elimination approach stems from treating the residual ionosphere, which is clearly a systematic error, as noise with an expectation of zero. It was shown in section 4.4.4 that, at least during periods of high ionospheric activity, current ionosphere models are not sufficiently accurate for the assumption of zero expectation to be valid. Simply giving a higher variance to the ionosphere does not help since it leads to less precise estimates of the ambiguities and thereby interferes with ambiguity resolution. Hence, the aim of this research was to identify an alternative estimation technique that provides more effective estimation of the ionospheric path delays but is still sufficiently computationally efficient to be of practical use.

The proposed solution is to use sequential phased adjustment, a technique for rigorously estimating parameters in batches. In sequential phased adjustment, the measurements are segmented into blocks based on the measurements. Any given block contains measurements involving only a subset of the overall parameters. Each block will contain parameters that are unique to that block, the so-called inner parameters, which in this application are the ionospheric path delays. Parameters that are common to more than one block are known as junction parameters and include the station coordinates and ambiguities. For each block a partial solution, so-called
because it uses only a subset of the observations, can be computed separately for the junction and inner parameters. The blocks are adjusted sequentially in a two-step process involving a forward and reverse pass. In this process, the partial estimates from each block are introduced as pseudo-observations into the next block. At the end of the forward pass, when all blocks have been processed, the inner parameters will have been estimated rigorously. After the reverse pass all parameters will have been rigorously estimated. The main advantage of this approach over pre-elimination is that it is not necessary to assume that the expected value of the ionospheric path delays is zero. Also, since the ionospheric path delays are explicitly estimated the residuals can be calculated enabling further analysis of the results. The disadvantage of phased adjustment is that it is computationally less efficient than pre-elimination. The reasons for this are that two passes must be made on the data, the results from each block must be stored during the processing and it is necessary to invert larger matrices because the inner parameters accumulate as each block is processed. However, compared to ordinary least squares sequential phased adjustment requires substantially less computational power and is efficient enough to be practical for this application.

In order to compare and contrast the sequential phased adjustment approach with pre-elimination, a set of medium to very long baselines were processed and evaluated in terms of the accuracy of the float solution, the number of ambiguities that could be resolved and the reliability of the ambiguity fixing. For the empirical testing, ten stations from the ARGN were chosen and used to form eight baselines of between 636 and 4,080 kilometres in length. In total fifteen days of data from the summer, winter and spring of 2002 were used for the testing. The data represents a period of elevated ionospheric activity since the solar cycle was near its maximum during the period when the data were collected. The Bernese GPS Software v4.2 was used as the basis for the computations. All pre-processing steps, which include cycle slip detection and repair, formation of baselines, orbit modelling and estimation of tropospheric zenith and gradient parameters were done using Bernese. Bernese was also used for the ambiguity float solutions estimated using pre-elimination and the subsequent QIF ambiguity resolution and fixed solutions. The ambiguity float solutions using sequential phased adjustment and the associated ambiguity resolution and fixed solutions were computed using an external test application written in C++. To enable a common pre-processing strategy, orbit handling and to maintain general
consistency, the Bernese code was modified to output custom log files of the data necessary to form the system of linear equations, compute the estimates and perform ambiguity resolution. Each baseline was processed four times; using pre-elimination and sequential phased adjustment both with and without \textit{a priori} ionospheric information, in the form of global IONEX maps produced by CODE.

As expected, the results show that unlike pre-elimination sequential phased adjustment does not need \textit{a priori} information on the ionosphere to achieve an accurate float solution. Compared to the most accurate reference values available, the IGS weekly solution, sequential phased adjustment gives an improvement in accuracy of approximately 2\% compared to pre-elimination. If a deterministic ionosphere model is used, the results of pre-elimination and sequential phased adjustment are similar. The \textit{a posteriori} sigmas of unit weight calculated from the adjustments clearly indicate that sequential phased adjustment is better at estimating the parameters. This is also reflected in the higher precision of the parameter estimates.

In terms of fixed ambiguities, sequential phased adjustment provides on average a 25\% improvement compared to pre-elimination and a 9\% improvement compared to pre-elimination with a deterministic ionosphere model. Sequential phased adjustment provides consistent results regardless of whether a deterministic ionosphere model is used or not. Improvements were seen for all times of the year and all tested baseline lengths. However, the results of the ambiguity fixed solution indicate that \textit{a priori} information on the ionosphere is still important for ambiguity resolution as otherwise the ambiguity parameters will absorb some of the ionospheric error leading to incorrect ambiguity fixes. This is a general problem of the ionosphere float model due to the strong correlation between the parameters and is not specific to sequential phased adjustment. Sequential phased adjustment with a deterministic ionosphere model shows an average improvement in accuracy between the float and fixed solutions of 5.6\% compared to only 4.7\% for pre-elimination.

When using global IONEX maps, the QIF ambiguity resolution strategy combined with the sequential phased adjustment gave better results in terms of accuracy of the float and fixed solutions and number of ambiguities fixed than the pre-elimination technique used in Bernese. Thus the conclusion can be made that the
hypothesis given in section 1.3, that estimation of the ionospheric path delays can be improved thereby enhancing long baseline ambiguity resolution, has been proven.

A list of publications, software and awards derived from this research is given in Appendix D.

7.2 RECOMMENDATIONS

Given the small but significant increase in accuracy and fixed ambiguities provided by the sequential phased adjustment approach, it is recommended that this approach be used for long baseline applications demanding the highest possible accuracy. Further testing and validation of the approach is important given the diverse range of datasets that it could be used with and which could not be tested in this research. In particular, further tests should be conducted using data from different periods of the solar cycle and with stations closer to the geomagnetic equator and stations in the northern hemisphere. To facilitate further testing, sequential phased adjustment should be integrated into a processing software such as Bernese so that it is convenient to process more datasets. Such an implementation would also facilitate the computation of final step in the processing strategy, the ionosphere-free fixed solution in which the resolved ambiguities are introduced as known values and the unresolved ambiguities are re-estimated together with the tropospheric zenith and gradient parameters.

As the new satellites of modernised GPS and Galileo come into service, long baseline GNSS processing will be improved by:

- an additional measurement of the ionosphere using the third frequency,
- an ultra wide L2-L5 widelane observable with a wavelength of 5.861 metres, and
- improved availability of common satellites.

The QIF ambiguity resolution algorithm will need to be adapted to support a third frequency. Also, the third frequency and additional satellites will greatly strengthen the solution and enhance redundancy. However, the basic problem of modelling the ionosphere during ambiguity resolution will remain. Sequential phased
adjustment is a useful method for attaining high accuracy parameter estimates for long baseline processing. The additional ambiguity parameters that must be estimated with the triple frequency modernised GPS and Galileo will further increase the processing load. Hence, further investigation into the efficiency of the algorithm should be made, for example by finding the optimum number of epochs to be included in each block and through the use of matrix inversion routines designed for sparse matrices.
REFERENCES


APPENDIX A

Code Multipath Estimates for a Reference Station Network

The code multipath RMS may be estimated using the linear combination of code and phase measurements (Estey and Meertens 1999),

\[
\Phi_{MP1} = \Phi_{p1} - \left(1 + \frac{2}{\alpha - 1}\right)\Phi_{L1} + \left(\frac{2}{\alpha - 1}\right)\Phi_{L2}
\]

\[
= M_{p1} + B_{L1} - \left(1 + \frac{2}{\alpha - 1}\right)M_{L1} + \left(\frac{2}{\alpha - 1}\right)M_{L2}
\]

\[
\Phi_{MP2} = \Phi_{p2} - \left(\frac{2\alpha}{\alpha - 1}\right)\Phi_{L1} + \left(\frac{2\alpha}{\alpha - 1} - 1\right)\Phi_{L2}
\]

\[
= M_{p2} + B_{L2} - \left(\frac{2\alpha}{\alpha - 1}\right)M_{L1} + \left(\frac{2\alpha}{\alpha - 1} - 1\right)M_{L2}
\]

where \(\Phi_{MP1}, \Phi_{MP2}\) are the multipath linear combinations, \(\alpha = \frac{f_{L1}^2}{f_{L2}^2}\) and the bias terms are,

\[
B_{L1} = -\left(1 + \frac{2}{\alpha - 1}\right)\lambda_{L1}N_{L1} + \left(\frac{2}{\alpha - 1}\right)\lambda_{L2}N_{L2}
\]

\[
B_{L2} = -\left(\frac{2\alpha}{\alpha - 1}\right)\lambda_{L1}N_{L1} + \left(\frac{2\alpha}{\alpha - 1} - 1\right)\lambda_{L2}N_{L2}
\]

The bias terms \(B_{L1}\) and \(B_{L2}\) will be constant unless there is a cycle slip. The phase multipath terms \(M_{L1}\) and \(M_{L2}\) will be much smaller than the code multipath terms \(M_{p1}\) and \(M_{p2}\), so in the absence of any cycle slips the variation of equations (A1) and (A2) will mainly result from the code multipath. Assuming the average multipath error is zero, the bias terms can be removed using a time average. Thus code multipath RMS can be estimated. A moving average of 50 observations is used
by software such as Leica GNSS QC or UNAVCO TEQC to estimate the multipath RMS. Thus only short-term variations in the multipath are estimated because any low frequency multipath bias gets absorbed by the \( B_{L1} \) and \( B_{L2} \) terms. Thus the absolute bias does not have a significant impact on the RMS estimates.

The average code multipath RMS estimates for a set of stations from the Victorian GPSnet reference station network, located in south east Australia, based on data for the month of April 2003, are shown in Table A1.

<table>
<thead>
<tr>
<th>Reference Station</th>
<th>Antenna</th>
<th>P1 Multipath RMS (m)</th>
<th>P2 Multipath RMS (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>GPSnet Ballarat</td>
<td>TRM29659.00</td>
<td>0.115</td>
<td>1.335</td>
</tr>
<tr>
<td>GPSnet Benalla</td>
<td>TRM22020.00+GP</td>
<td>0.113</td>
<td>0.826</td>
</tr>
<tr>
<td>GPSnet Cann River</td>
<td>TRM29659.00</td>
<td>0.081</td>
<td>0.137</td>
</tr>
<tr>
<td>GPSnet Clayton</td>
<td>TRM22020.00+GP</td>
<td>0.618</td>
<td>0.569</td>
</tr>
<tr>
<td>GPSnet Colac</td>
<td>TRM29659.00</td>
<td>0.174</td>
<td>0.262</td>
</tr>
<tr>
<td>GPSnet Epsom</td>
<td>TRM23903.00</td>
<td>0.162</td>
<td>1.168</td>
</tr>
<tr>
<td>GPSnet Geelong</td>
<td>LEIAT302+GP</td>
<td>0.272</td>
<td>0.195</td>
</tr>
<tr>
<td>GPSnet Hamilton</td>
<td>TRM29659.00</td>
<td>0.174</td>
<td>0.223</td>
</tr>
<tr>
<td>GPSnet Horsham</td>
<td>TRM29659.00</td>
<td>0.130</td>
<td>0.219</td>
</tr>
<tr>
<td>GPSnet Irymple</td>
<td>TRM29659.00</td>
<td>0.132</td>
<td>0.275</td>
</tr>
<tr>
<td>GPSnet Melbourne Observatory</td>
<td>TRM29659.00</td>
<td>0.763</td>
<td>0.787</td>
</tr>
<tr>
<td>GPSnet Melbourne RMIT</td>
<td>TRM41249.00</td>
<td>0.104</td>
<td>0.214</td>
</tr>
<tr>
<td>GPSnet Mt Buller</td>
<td>TRM29659.00</td>
<td>0.105</td>
<td>0.146</td>
</tr>
<tr>
<td>GPSnet Shepparton</td>
<td>TRM29659.00</td>
<td>0.088</td>
<td>0.140</td>
</tr>
<tr>
<td>GPSnet Swan Hill</td>
<td>TRM29659.00</td>
<td>0.103</td>
<td>0.178</td>
</tr>
<tr>
<td>GPSnet Walpeup</td>
<td>TRM23903.00</td>
<td>0.090</td>
<td>1.033</td>
</tr>
<tr>
<td>GPSnet Yallourn</td>
<td>TRM29659.00</td>
<td>0.035</td>
<td>0.063</td>
</tr>
<tr>
<td>Mean</td>
<td></td>
<td>0.192</td>
<td>0.457</td>
</tr>
<tr>
<td>Median</td>
<td></td>
<td>0.115</td>
<td>0.223</td>
</tr>
</tbody>
</table>

Table A1 Average Code Multipath RMS for GPSnet during April 2003

The stations have a median RMS of 0.115 metres and 0.223 metres for P1 and P2 respectively. The average values are substantially higher than the median because of the high RMS values for Clayton and Melbourne Observatory. Note that some stations, which have older receivers that use a less sophisticated code reconstruction technique (cross correlation), the P2 RMS is considerably higher than the P1 RMS. Most of the GPSnet base stations use a Dorne-Margolin model T specification choke ring antenna to help reduce multipath. Additionally, careful site selection was used to ensure that the base stations are in low multipath environments. Recalling that a code RMS of better than one metre is required for the Melbourne-Wübbena linear
combination, it is apparent that the requirement for “good” P-code observations can generally be satisfied. This conclusion is substantiated by Krantz et al. (2001) who give average multipath RMS values of approximately 0.3m for both the choke ring and Zephyr antennas. However, the multipath RMS estimates given by equations A1 and A2 are likely to be optimistic because they force the multipath error to have a mean of zero. Therefore the RMS estimates do not reflect the dominant systematic multipath bias.
APPENDIX B

Estimation of Multipath Using an Adaptive Filter

Dodson et al. (2001) and Ge et al. (2000) show how an adaptive filter can be used to separate the noise and multipath contributions from GPS data. Two data sets are collected on consecutive days with the same satellite and reflector geometry. The assumption is made that the error that is common to both data sets will be multipath and the rest will be noise and other error signals.

Consider two digital signals \( y(n) \) and \( x(n) \) whose relationship can be expressed as,

\[
y(n) = D\{x(n)\} \quad \text{(B1)}
\]

An equivalent expression for equation (B1) is,

\[
y(n) = \sum_{q=0}^{Q-1} b_q x(n-q) + \sum_{p=0}^{P-1} a_p y(n-p) \quad \text{(B2)}
\]

where \( \{ b_q \} \) and \( \{ a_p \} \) are the sequences of parameters (\( b \) and \( a \) in vector form) of the digital filter \( D\{\} \) and \( Q \) is the filter length. Such a filter is called an Infinite Impulse Response Filter or IIR. IIR filters can be thought of as feedback filters because the previous value of the filter affects the current value. If \( a_p = 0 \) for all \( p \) then the digital filter can be simplified to,

\[
y(n) = \sum_{q=0}^{Q-1} b_q x(n-q) \quad \text{(B3)}
\]

and is simply a moving average (MA) filter. This type of digital filter is called a Finite-duration Impulse Response (FIR) filter and is a feedforward filter. FIR filters have the advantage of being both simple and stable. (Dodson et al. 2001) and (Ge et
al. 2000) use this type of filter to estimate multipath. The main reasons for using a FIR to estimate multipath over an IIR are:

1. The previous value of the multipath error is not expected to influence the current value. Therefore, no further information is gained by using an IIR filter.
2. FIR filters are more stable than IIR filters. If the feedback coefficients are not chosen well an IIR filter may oscillate.
3. Ease of computation.

The tasks of the filter and the adaptive algorithm are to produce an output and to determine optimised estimates for the filter parameters \{b_q\}. The initial values of \{b_q\} may be chosen arbitrarily.

Consider two digital signals \(x(n)\) and \(d(n)\) with sample size \(N\). The signal \(x(n)\) is the input signal and \(d(n)\) is the desired or reference signal. The desired signal could be the real measurement time series or another realisation. The input signal \(x(n)\) is run through the filter and the filter output (or prediction) signal \(y(n)\) results following equation (B3). The error signal is given by,

\[
e(n) = d(n) - y(n)
\]  

and is used to update the filter coefficients via an adaptive algorithm. The updated filter coefficients are then used to process the next input sample and the process repeats. A schematic representation of this process applied to modelling an unknown or difficult system can be seen in Figure B1.
The simplest and most often used adaptive algorithm for updating the filter coefficient is the steepest descent algorithm (Embree 1995). The aim of any such algorithm is to update the filter coefficients in such a way that they approach the optimal minimum mean squared error (MMSE). To solve for the optimal filter coefficients the mean squared error (MSE) $E = \sum_{n=0}^{N-1} e^2(n)$ is minimised. The partial derivatives of MSE with respect to the filter coefficients yield a set of linear equations that generate the optimal estimates. However, the process may be simplified by applying a first-order approximation of the error performance surface, known as the method of steepest descent. Bozic (1994) gives the method of steepest descent as,

$$b_{k,n+1} = b_{k,n} + \mu e_{n} x_{n-k} \quad (B5)$$

where $\mu$ controls the rate of change of the update.

Selection of the two filter parameters, the filter length $Q$ and the convergence $\mu$ are critical to the stability and effectiveness of the filter. In order to ensure the filter is not too sensitive to noise but still adapts to the input, $\mu$ should be in the range (Bozic 1994),
\[ 0 < \mu < \frac{2}{Q \cdot P_x} \]  \hspace{1cm} (B6)

where \( P_x = \frac{1}{Q} \sum_{n=0}^{N-1} d^2(n) \).

The filter length, or the number of coefficients in the adaptive filter should be selected such that it the minimum length required to model the regular features or constraints that give the data its special structure. One optimal and robust technique for this is the minimum description length (MDL) criterion. The MDL model is given as (Haykin 2002),

\[ \text{MDL}(m) = -L(\hat{\theta}_m) + \frac{1}{2} m \cdot \ln(N) \]  \hspace{1cm} (B7)

where \( m \) is the number of independently adjusted parameters in the model (i.e. the number of filter coefficients). \( L(\hat{\theta}_m) \) is the natural logarithm of the maximum likelihood estimates of the estimated filter coefficients, and is defined as (Haykin 2002),

\[ L(\hat{\theta}_m) = \max_{\hat{\theta}_m} \sum_{i=0}^{N-1} \ln f_{u_i}(u_i | \hat{\theta}_m) \]  \hspace{1cm} (B8)

where \( f_{u_i}(u_i | \hat{\theta}_m) \) is the conditional probability density function of the data \( u_i \) given \( \hat{\theta}_m \), the estimated vector of parameters that model the process, and \( m \) is the model order or filter length.

Consider the case where the input and desired signals are both made up of a systematic and a random component,

\[ x(n) = S + R \]
\[ d(n) = S' + R' \]
Simulations by Dodson et al. (2001) have confirmed the relationships between the R, S and the error and predicted sequences shown in Table B1.

<table>
<thead>
<tr>
<th>Desired Signal d(n)</th>
<th>Input Signal x(n)</th>
<th>R, R’ uncorrelated S, S’ correlated (Case 1)</th>
<th>R, R’ correlated S, S’ uncorrelated (Case 2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Error Sequence e(n)</td>
<td>R</td>
<td>S</td>
<td></td>
</tr>
<tr>
<td>Predicted Sequence y(n)</td>
<td>S</td>
<td>R</td>
<td></td>
</tr>
</tbody>
</table>

**Table B1 Relationship between FIR filter output and systematic and random components of the signal**

If the random parts of the signals are uncorrelated but the systematic components are correlated then the adaptive filter is able to separate the random and common systematic components. The error sequence contains the random error and the predicted sequence contains the error common to both the input and desired signals. In the application of multipath estimation, the input and desired signal are the residuals from a baseline measured on two consecutive days. The task of the filter is then the separate the measurement noise and multipath from the residuals. The breakdown of the sequences is:

- **Input Sequence** x(n): DD phase residuals from Day 1
- **Desired Sequence** d(n): DD phase residuals from Day 2
- **Error Sequence** e(n): The combined Day 1 and Day 2 measurement noise.
- **Predicted Sequence** y(n): The multipath error common to both Day 1 and Day 2.

The input (or desired) signal is offset by approximately four minutes so that the satellite constellation is the same for both series of residuals. The exact offset is found by calculating the lag that produces the maximum correlation between the two series of residuals. Both the Day 1 and Day 2 residuals will contain measurement noise and multipath error. The measurement noise should be uncorrelated between days. If the two datasets are collected under the same conditions (static receivers, satellite geometry and reflectors), the multipath error will be strongly correlated. The filter should then be able to separate the measurement noise from the multipath error.
The effectiveness of the adaptive filter, and the choice of the filter parameters $\mu$ and $Q$, may be assessed by comparing the cross-correlation between the input and design signals and the error and predicted signals. Following Case 1 in Table B1, it is expected that if $R$ and $R'$ are white noise with zero mean then the following relationships would be expected hold:

$$E[S_n S_{n-j}] = P_j \quad \forall j, n$$
$$E[S_n R_{n-j}] = 0 \quad \forall j, n$$
$$E[S_n R'_{n-j}] = 0 \quad \forall j, n$$
$$E[R_n R'_{n-j}] = 0 \quad \forall n, j \neq 0$$

where $P_j$ is the cross correlation function of the signals. If the filter has been effective then,

1. The error signal should have low correlation with the input, desired and predicted signals.
2. The predicted signal should have high correlation with the input and desired signals.
APPENDIX C

Test Data Summary and Results
<table>
<thead>
<tr>
<th>Day of Year</th>
<th>Observations</th>
<th>Ambiguities</th>
<th>Iono Path Delays</th>
<th>DOF</th>
</tr>
</thead>
<tbody>
<tr>
<td>006</td>
<td>36486</td>
<td>90</td>
<td>21123</td>
<td>16710</td>
</tr>
<tr>
<td>007</td>
<td>36046</td>
<td>106</td>
<td>18150</td>
<td>168</td>
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<td>008</td>
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<td>88</td>
<td>80</td>
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<td>269</td>
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**Baseline Parameter Summary**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>ALCE</th>
<th>CADA</th>
<th>DAMW</th>
<th>HOMA</th>
<th>YACO</th>
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<tr>
<td>Observations</td>
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<td>92</td>
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<td>Iono Path Delays</td>
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<td>23187</td>
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</tr>
<tr>
<td>DOF</td>
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Table C8 Results for baseline YACA
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APPENDIX D

List of Publications, Softwares and Awards Derived from this Research

Publications


Software

The Leica GNSS QC software is derived from software developed during research into reference station data quality done as part of this PhD candidature. The software can be downloaded from www.leica-geosystems.com/corporate/en/ndef/lgs_29436.htm.

Awards

• Student Award, ION GPS/GNSS 2003, Portland, Oregon
Minerva Access is the Institutional Repository of The University of Melbourne

**Author/s:**
Brown, Neil E

**Title:**
Sequential phased estimation of ionospheric path delays for improved ambiguity resolution over long GPS baselines

**Date:**
2006-12

**Citation:**

**Publication Status:**
Unpublished

**Persistent Link:**
http://hdl.handle.net/11343/39223

**File Description:**
Sequential phased estimation of ionospheric path delays for improved ambiguity resolution over long GPS baselines

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