Modelling and control of an automotive electromechanical brake

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Abstract

This thesis describes the modelling and control of an electromechanical brake (EMB) for a drive-by-wire vehicle. The investigation comprised two components on the development, identification and assessment of an EMB model, and the development of an improved control algorithm for an EMB.

The first component of the study began with the examination of a simplified model for an electromechanical disk brake without the positive feedback of brake self-energisation. A methodology was proposed for practical identification of the model parameters on an assembled actuator. Experiments were conducted on a prototype EMB, and for the first time the model fidelity was tested in isolation without a feedback controller acting to reject disturbances. Laboratory tests of the model fidelity were complemented with closed-loop simulations against field data from a brake-by-wire test vehicle. It was determined that the EMB model reasonably predicted the key behaviours of the brake apply, force modulations and lockup due to load-dependent stick-slip friction. The limitations of the model were then identified and extensions were considered to describe secondary effects.

The second component of the study utilised the model to develop an improved control algorithm for an EMB, particularly considering the problem of tracking a brake force command from the driver, or from another vehicle control. Existing EMB controllers were seen to have a limited effectiveness; with a suboptimal handling of actuator nonlinearity, they suffered from problems of the load dependent mechanism friction, and they could not maintain performance throughout the operational envelope. These shortcomings were overcome sequentially by the development of a friction compensation algorithm and a modified control architecture to better manage actuator nonlinearity. To address model uncertainty the modified architecture was incorporated within a robust control design, but this gave an overly conservative brake performance. In a more successful approach, the modified architecture was extended with a model predictive control to optimise the EMB performance and a method for updating the control algorithm was proposed to handle uncertainty and adapt to actuator variation. At each stage experimental tests were conducted on a prototype EMB to demonstrate performance and the incremental improvements
achieved. The control improvement was found to be most pronounced for fine manoeuvres whereas large manoeuvres were typically limited by the actuator constraints.

The study outcomes regarding EMB modelling, identification and control are a significant incremental advancement on prior work, and may help to facilitate the development of improved brake-by-wire platforms, anti-lock brake systems and advanced driver assist functions.
Declaration

This is to certify that:

(i) the thesis comprises only my original work towards the PhD,

(ii) due acknowledgement has been made in the text to all other material used,

(iii) the thesis is less than 100,000 words in length, exclusive of tables, maps, bibliographies and appendices

Signed:

Christopher Leonard James Line
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Publications

To date the following publications have resulted from the present work,


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1 Introduction

1.1 Background and motivation

In recent times the potential for electromechanical brakes (EMBs) to succeed hydraulic brakes has stimulated the interest of the brake research community and automotive brake manufacturers. Major producers of foundation brakes such as Advics, Akebono, Bosch, Continental Teves, Delphi, and TRW and the industry partner for this research project, Pacifica Group Technologies (PGT), have developed prototype EMBs or patented designs, a number of which have been deployed on test vehicles. This study dealt with the EMB design that PGT supplied for General Motors’ Sequel: a fuel cell, drive-by-wire concept vehicle (Figure 1-1). An industry consensus has yet to be established regarding EMB technology, but some forecasts predict an eventual introduction on commercial vehicles. Already, related electromechanical park brakes have proven a successful forerunner for the commercialisation of such technology.

Research and development of electromechanical brakes is motivated by a number of potential benefits. Compared with hydraulic brakes they may offer advantages of component reduction, system weight reduction, ‘plug, bolt and play’ modularity and
an improved brake response. EMBs also support the vision of drive-by-wire where a communication network liberates the vehicle control from the constraints of mechanical linkages. Prototype vehicles have showcased increased cabin space with ergonomic and crash compatible controls. Manufacturers may gain product differentiation, new vehicle features and the cost-benefits of a modular vehicle assembly. Moreover, drive-by-wire offers the exciting prospect of coordinating vehicle subsystems electronically such that integrated by-wire throttle, steering and brakes may facilitate a more sophisticated vehicle dynamics control. Foreseeable outcomes in the immediate term include improved electronic stability programs and a system that is suited to the coordination of regenerative braking. Looking further ahead, drive-by-wire systems may ultimately support autonomous vehicles.

Electromechanical disk brakes typically comprise an electric motor driving a mechanism with rotary-to-rectilinear reduction to clamp and release the brake rotor. A cross-section of the EMB used in this study is shown in Figure 1-2. During operation a motor torque may be developed between the stator, ①, and the permanent magnet rotor, ②. The motor rotor drives the planetary gear, ③, and ball screw, ④, to operate the piston, ⑤, and clamp the brake pads, ⑥. The clamp force, \( F_{cl} \), is then reacted over the bridge of the floating calliper, ⑦, by the opposing brake pad.

![Figure 1-2: WO 2005/124180 A1, Actuating Mechanism and Brake Assembly patented by PBR Australia Pty. Ltd. (Wang, Kaganov, Code and Knudtzen, 2005)](image-url)
EMBs are installed on the vehicle with a mechanical, power and communications interface. Brake commands from the driver or vehicle dynamics controller are transmitted via an in-vehicle network using a fault-tolerant, time-triggered communication protocol. Depending on how functionality is arranged between the central vehicle management and the actuator control, brake instructions may command brake torque, brake force, or a particular mode of operation such as standby, off, or anti-lock braking. It is here that the EMB control problem is encountered; a control algorithm is required to respond to the brake commands and operate the brake actuator. To facilitate the development of a control algorithm an EMB model is also sought for simulation and design.

A starting point for investigation may be found in pioneering EMB research that has provided a foundation for modelling and control. However, significant questions remain to be answered and an ensuing literature review will identify avenues for further work. A body of knowledge on mechanical design exists in the form of patents, but there is little on EMB modelling or the development of control algorithms. Numerous studies have been published on vehicle dynamics control, electronic stability programs and the anti-lock brake system (ABS), but these all depend on the performance of the low-layer actuator control. A general lack of information was noted in (Petersen, 2003) with regard to the development of an improved ABS.

*Due to the industrial property rights and very strict proprietary policies within the automotive industry, very little has been published around automotive EMB. There is no literature describing tests and performance results of EMB beside those that have been carried out through the H2C project.* (Petersen, 2003)

EMBs for the H2C project were provided by Continental Teves and further detail in (Schwarz, 1999) and (Lüdemann, 2002) will be discussed in the literature review.
To be useful, an EMB model should describe the key behaviours of the brake apply profile, the frequency response and the lockup that can occur due to stick-slip friction. There should be practical methods to identify the model parameters on an assembled EMB. These should be demonstrated and the model fidelity should be checked against experimental tests without a feedback control system acting to reject disturbances. A literature review will indicate that further work is required on these matters and this motivates an investigation of EMB modelling.

The EMB control problem is differentiated by the somewhat unique requirements on a vehicle brake system. The EMB must be compact, cover a large range of loading beyond 30 kN and provide responsive brake performance. During operation the control must manage the challenges of actuator saturation, nonlinear stiffness and the mechanism friction that can become large at high clamp loads. Given these considerations, it is not yet clear how prior EMB control algorithms perform throughout the operational envelope, what potential exists for improvement, or how this might be achieved. This motivates the further investigation of EMB control.
1.2 Research questions and hypotheses

The following research questions and hypotheses are proposed to help establish the scope of the present study. The investigation is focused on two issues of EMB development:

1) How can an automotive EMB, such as the prototype actuator (Figure 1-2) installed on the GM Sequel (Figure 1-1), be simply modelled for the purpose of brake simulation?

2) How may the prototype EMB control be improved to surpass the performance of standard cascaded proportional-integral control?

For each of these questions a corresponding hypothesis may be proposed.

Hypothesis 1: Key behaviours of the prototype EMB may be described by a lumped parameter, half-calliper model following a method for in-situ parameter identification on the assembled actuator.

Hypothesis 2: The performance of the EMB control can be significantly improved relative to the benchmark established by state-of-the-art cascaded PI control.

While brake operation can depend on many factors, all scenarios are covered by the performance benchmark. Qualitative comparisons may then consider aspects such as the tracking error, rise time, amplitude response, and so on.

The hope is that if the EMB control can be improved then its potential for responsive, high performance vehicle braking may be better realised. Hence, the benefit accrued during demanding operations such as anti-lock braking is also a matter of interest.

In addressing the two research questions and hypotheses, this study aims to build upon prior work on automotive EMB modelling and control. More detail on the state-of-the-art is given in the following literature review.
1.3 Literature review

A literature review on EMB modelling and control was undertaken to establish the state-of-the-art, identify the need for further work and provide a focus for the current study. This review describes a brief history of the EMB, a critique of prior EMB models and the development of EMB control.

A brief history of the automotive EMB

A short overview of automotive EMB development is appropriate to provide a context for this study. The following summary is largely based on the patent review in Appendix 1A. It follows the development of EMBs through the archive held by the United States Patent and Trademark Office (USPTO).

Demonstrations of electric vehicles were first reported between 1834 and 1837 (Westbrook, 2001). In 1900 around 4200 automobiles were sold, 40% steam, 38% electric, and 22% gasoline powered (Husain, 2003). Designs for EMBs existed at this time and there are early patents for both the drum and disk types.

Some of the earliest EMB patents were for railway cars such as the electromagnetic disk brake patented in (Olmsted, 1869). Following some related designs, an electromagnetic brake apparatus was patented for a ‘street-car’ which ran on rails and might now be called a tram or a trolley (Davis, 1895). Around this time the rise of the motor car resulted in a transition to EMB patents for automobiles.

Automotive EMBs may be classified into drum or disk designs and both had emerged by the turn of the 1900s. The first designs were actuated by electromagnets, with early examples being the drum brake described in (Stevens and Penney, 1899) and the disk brake in (Sperry, 1899).
In 1912 electric vehicles outnumbered gas-powered vehicles two to one (Husain, 2003) and various brake systems were in use. Developing sufficient brake force was a challenge and drum brake designs gained favour over those with a disk configuration. Electromechanical band brakes emerged and these were driven by a centralised motor and reduction gearing. An example in Figure 1-3 shows a 1918 patent for an electric band brake system. The controlling switch (3) activated a motor (1) with a reduction gear to drive the cable winding drum (6) and apply the brake band (11). Similar designs persisted until the late 1920s.

In the 1930s electromechanical belt brakes were superseded by internal drum brake designs. These had brake shoes inside the rotating drum and offered improved holding capacity and reliability. A new actuating mechanism became popular where an energised electromagnet would drag upon the rotating assembly to operate a lever and apply the brake shoes. The mechanism exploited a self-energising effect and the design persisted up to the 1960s.
Around the 1970s there was a transition to modern electromechanical drum brake configurations. These have an electric motor driving a mechanism, such as a power screw, to apply the brake shoes. Despite a recent trend towards electromechanical disk brakes, patents have continued on equivalent designs for drum brakes.

Electromechanical disk brake designs were intermittent prior to the 1970s and had difficulty satisfying the large brake force requirement. Various attempts included large electromagnetic clamps and multiple brake rotors. Prospects began to change in the 1970s when electromagnetic levers from self-energising drum brakes were adapted to operate electromechanical disk brakes.

Modern electromechanical disk brake configurations emerged in the late 1980s, with a motor driving a rotary-to-linear reduction to clamp and release the brake rotor. These designs are now pursued more vigorously than electromechanical drum brakes partly because they are less susceptible to thermal effects and brake fade. Variations have explored the possibilities of different drives, transmissions and actuator configurations. Classic modifications have incorporated components such as electric motors, electromagnets, piezoelements, ultrasonic motors, dual drives, dual rotors, power screws, ball screws, planetary roller screws, roller ramps, hydraulic transmission, planetary gears, harmonic drives, eccentric gears, variable gears, locks, couplings, clutches, wedges and other mechanisms for brake ‘self-energisation’. Further details and dates on these innovations are provided in Appendix 1A. A list of USPTO patents is also provided in Appendix 1B.

EMBs are approaching a stage where they have the potential to outperform and succeed current hydraulic brake systems. To provide some guide, a 30kN brake application within 100-150 ms is a reasonable objective for current prototypes. The absolute performance is constrained by both software and hardware. A dominant EMB configuration is yet to be established, but the test actuator used in this study has the successful combination of a motor, planetary gear and ball screw to clamp and release the brake.
**Prior EMB modelling**

An EMB model would provide a valuable tool for brake-by-wire simulation, analysis and design. This has motivated prior work and the following critique is intended to establish the state-of-the-art and identify the need for further work.

**A simplified EMB model**

*Everything should be made as simple as possible, but not simpler.* (Albert Einstein)

The EMB model should be sufficiently complex to describe the brake behaviour, but simple enough that model parameters can be easily identified. An early simulation diagram of a simplified EMB model appeared in (Maron, Dieckmann, Hauck and Prinzler, 1997) and is shown in Figure 1-4 for convenience. It “consists of a concentrated mass encumbered with friction, the gear spindle transmission ratio, and, finally the non-linear characteristic curve of rigidity of the brake calliper and brake pads” (Maron et al., 1997). Little further information was provided, but the concept of a simple, lumped parameter, half-calliper EMB model was established.

![Figure 1-4: Maron et al.’s EMB model (Maron et al., 1997)](image-url)
Maron et al.'s diagram requires interpretation, but the main ideas appear to have been,
1) A simple half-calliper model in which the clamp force, $F_z$, on one brake pad was
   assumed to be reacted over the bridge of the floating calliper by the opposing brake
   pad.
2) A lumped inertia, $\Theta$, a lumped stiffness (from displacement, $x$, to clamp the force,
   $F_z$) and a lumped friction model that was speed and load dependent.
3) A simple torque balance about the motor axle, $\sum M = \Theta \dot{\omega}$, to determine the
   mechanism dynamics.

The EMB model in (Maron et al., 1997) had motor current as an input and the brake
force as an output. Stepping through the simulation diagram from the input, the motor
current first multiplies a flux linkage, $\Psi$, to give the motor torque. A torque balance
is then considered about the motor axle. This accounts for torques from the motor,
clamp load and friction to determine the net torque. The resultant torque is divided by
the equivalent system inertia, $\Theta$, to give the motor acceleration. Integrations
subsequently determine the motor speed, $\omega$, and the position, $\phi$. The motor position,
$\phi$, multiplies a gear ratio to give a nominal spindle position, $x$. A stiffness curve then
links the position, $x$, to a clamp force, $F_z$. A brake pad friction model is used to relate
the clamp force to a braking force.

Such an elegant EMB model as that described by (Maron et al., 1997) would be a
valuable tool for brake-by-wire research and development if it could reasonably
predict the brake behaviour and also the model parameters were practical to identify.
Maron et al. assessed their EMB model against data from a hardware-in-the-loop test
for an ABS brake manoeuvre. A vehicle deceleration from 150 km/h was simulated
and results were compared between the two front wheels, one of which had the EMB
model, while the other included a hardware-in-the-loop EMB on a non-rotating brake
disk. The results from Maron et al. are shown in
Figure 1-5.
Figure 1-5: Maron et al.'s simulated and measured braking in an ABS stop (Maron et al., 1997) showing ABS switching and brake forces (top) and the vehicle and wheel speeds (bottom)

From this experiment one can conclude that the simulation of the actuator works quite good, as the 2 wheels behave almost the same, both qualitatively and quantitatively, even though one actuator exists in reality and the other is only simulated. (Maron et al., 1997)

Maron et al. may have assumed that if the test gave comparable output at the two wheels then the EMB model and physical actuator must be equivalent. However, this assumption is not valid because it neglects the different feedback control that may occur at each wheel. With different input to the simulated and physical EMBs it is difficult to draw a fair comparison between their outputs. For example, it may be that a poor EMB model was compensated by increased controller activity.
Irrespective of the methodology, Maron et al.’s test results exhibit different ABS cycling frequencies, different pressure build patterns and different excursions into wheel slip. Additionally, the moving average of the clamp force is observed to decline in measurements, but not in the simulation. These deviations between the two ABS systems require further explanation.

Maron et al.’s pioneering approach for a simplified EMB model encouraged further work and the progress of the research group was continued by Schwarz et al. The research continued in association with Continental Teves following its purchase of ITT Automotive’s brake division in 1998.

A different version of the simplified EMB model was published in (Schwarz, 1999) and (Schwarz, Isermann, Böhm, Nell and Rieth, 1999). The same model was later published with reference to Schwarz in (Isermann, 2003). In these publications a modular description of the brake lead to the formulation of a reduced model.

A diagram of Schwarz et al.’s simplified EMB model is shown in Figure 1-6, which consolidates Maron et al.’s approach of a half-calliper model with lumped parameters.
Stepping through the block diagram from Schwarz et al., the model input is the motor current, $I_m$, and the output is brake clamp force, $F_{cl}$. The motor current, $I_m$, multiplies a torque constant, $\Psi$, to produce the motor torque, $T_m$. A moment balance is then considered about the motor axle. After accounting for torques due to motor, $T_m$, viscous friction, $T_{Fv}$, Coulomb friction, $T_{Fc}$, and load, $T_{use}$, the net torque is divided by the equivalent system inertia, $\Theta_{tot}$, to determine the motor’s angular acceleration, $\dot{\omega}_m$. The acceleration, $\dot{\omega}_m$, is then integrated twice to give the motor angular velocity, $\omega_m$, and position, $\phi_m$. The motor position, $\phi_m$, multiplies the total gear ratio, $v_{tot}$, to give the spindle position, $x_{sp}$. This determines the output clamp force, $F_{cl}$, via a characteristic stiffness curve.

Schwarz et al. used a 3rd generation EMB from Continental Teves for testing and model validation. The EMB had a brushless DC motor driving a planetary gear coupled in series with a planetary roller screw to apply the floating calliper disk brake. To test the EMB model simulation output was compared with measurements from a test actuator during brake application and release (Schwarz, 1999; Isermann, 2003). The validation test followed a triangular clamp force trajectory that peaked at 20 kN and was run over a period of approximately 8 s. Schwarz’s results are shown in Figure 1-7.
Schwarz’s validation procedure did not isolate the EMB plant from the influence of the feedback control. As a consequence, different current inputs to the physical and simulated actuators are observed in plot (d). With different inputs to the two systems there is little basis for a fair comparison of the outputs or a model assessment.

Schwarz’s test profile of an 8 s, gentle, steady brake application and release was not sufficiently demanding for validating the EMB model. The profile avoided high motor velocity and acceleration, except when the calliper was in the clearance region. As a consequence, the modelling errors for the viscous friction and inertial forces are less exposed. Further, the profile did not adequately test brake behaviours such as the apply time, frequency response and stick-slip friction behaviour. Depending on the actuator and initial clearance, an emergency brake apply will occur in the order of 100 to 200 milliseconds. This is an order of magnitude faster than the 4 s apply shown in

Figure 1-7: Schwarz’s simulation and measurement (messung) showing (a) clamping force and setpoint (sollsignal), (b) motor angular position, (c) motor angular velocity, and (d) motor current (Schwarz, 1999)
Schwarz’s results. Schwarz’s test profile only excites low frequency behaviour, except perhaps for the initial acceleration at 1.3 s where significant discrepancy is observed in the motor velocity, position and clamp force plots.

Schwarz’s results suggest that there may be deficiencies in the EMB stiffness and friction models. In the clamp force plot (a) the friction model failed to capture the measured lockup between 5.3 s and 5.7 s. Also, from 2 to 5 s catching and breakaway can be observed in the measured velocity (c), but is not described by the simulation. The feedback controller attempts to suppress the friction disturbance in the physical plant and motor current spikes can be seen from 2-5 s in plot (d).

Some error in the stiffness model may be observed by comparing the force (a) and position (b), at 7.3 s for example, where different displacements produce similar clamp forces. With similar force profiles in (a) between 2-5 s and 6-9 s, the simulated position in (b) leads the measured position during both the apply and release. This may be related to the unmodelled force hysteresis.

In (Lüdemann, 2002) a version of the simplified EMB model was used for simulation and the development of a brake-by-wire ABS. Lüdemann extended the model from Schwarz et al. with three modifications. Firstly, a model for stiffness hysteresis was included. Secondly, the friction model included curvilinear rather than linear segments and may have been used to describe a Striebeck friction curve. Thirdly, a first order lag and velocity dependent saturation were included to describe the motor current dynamics. This might capture the effect of back EMF (electromotive force) on the limited supply voltage.

(Lüdemann, 2002) compared simulations using the simplified EMB model and a first order approximation (termed nonlinear and linear model in Figure 1-8) with measurements from a test actuator. For convenience, the results from (Lüdemann, 2002) are shown in Figure 1-8. Like earlier tests the system was regulated by a controller, but the input and controller activity were not shown and the degree of controller compensation for model inaccuracy is not apparent.
Irrespective of the active feedback control, the (nonlinear model) prediction showed mixed performance. Discrepancy may be observed during the first step around 0.4 s. Afterwards there is reasonable agreement for the successive steps near 0.7 s and 1.05 s. The first 4.5 kN step at 0.4 s was significantly slower than the 5.5 kN step at 0.7 s and perhaps some unmodelled dynamic was responsible for the discrepancy.

Despite the regulation of feedback control, the simulation showed some error in predicting the steady state response. In addition to steady state error, there was some unmodelled chatter-like behaviour between 0.5-6.5 s, 0.75-1.0 s and 1.1-1.3 s. Whether this was due to friction or some other effect is not clear.

![Figure 1-8: Lüdemann’s results for a simplified EMB model (nonlinear model), a first order approximation (linear model) and measurements from a prototype actuator (Lüdemann, 2002)](image)
In a fresh approach, (Kwak, Yao and Bajaj, 2004) presented a modal analysis of the EMB and simulated a high degree of freedom model against a simplified lumped parameter model. (Kwak et al., 2004) realised the importance of isolating the model performance from controller activity and head-to-head simulations were run on open-loop input. As a result of their analysis, (Kwak et al., 2004) concluded that a rigid body, lumped parameter model was a reasonable approximation to a higher order description of the EMB mechanism.

Prior work on a simplified EMB model has generally been encouraging. Foundational research from Maron et al. and Schwarz et al. established an important first-approach to modelling. Additional work from Lüdemann and Kwak et al. was also positive for model development. Test results have shown potential and do encourage the pursuit of a simplified EMB model. However, some details require further investigation, particularly the actuator stiffness, friction and parameter identification. Further, the reported simulations against controlled actuators and brake-by-wire ABS need to be complemented with tests of the EMB model in isolation. An open-loop test is required to assess the model fidelity without a feedback controller acting to reject disturbances.
Stiffness model

In prior work Schwarz et al. estimated that the EMB stiffness distribution was approximately 60-70% for the calliper body and 20-30% for the brake pads (Schwarz, Isermann, Böhm, Nell and Rieth, 1998; Schwarz et al., 1999). Some materials are known to exhibit viscoelastic stiffness behaviour and this was described for brake pads in (Augsburg and Trutschel, 2003) and (Brecht, Elvenkemper, Betten, Navrath and Multhoff, 2003). When Schwarz et al. measured the effective stiffness of an EMB a similar hysteresis was apparent. For convenience, Schwarz’s results are shown in Figure 1-9.

![Figure 1-9: Schwarz’s plots of force-displacement (kraft-verformung) with calliper (faust), brake pads (beläge) and brake disk (scheibe) (left) and plots of clamp force-motor displacement (spannkraft-motorwinkle) (right) (Schwarz, 1999)](image)

While stiffness hysteresis is known, there is no consensus on how it should be addressed in the simplified EMB model. The simulation diagram from (Maron et al., 1997) in Figure 1-4 showed two curves in the stiffness block from position \( x \) to force \( F_x \), but no detail on a hysteresis model was provided. Schwarz took the approach of approximating a middle line (mittelkennlinie approximierte) to the stiffness curve (Schwarz, 1999). The middle curve was described by a look-up table and the equation, \( F_{x_i}(x_{sp}) = \alpha_b \sqrt{\beta_b} + x_{sp}^2 + \chi_b \), where the clamp force, \( F_{cb} \), was a function of...
the spindle position, $x_{sp}$. This may suggest that stiffness hysteresis was considered a secondary effect that can be neglected without significant loss in model fidelity. However, a model for stiffness hysteresis was added to the EMB model in (Lüdemann, 2002). In this case, the middle force curve was adjusted by the addition or subtraction of a load offset. The offset was accumulated based on the change in force until saturation limits were reached. This hysteresis model differs from the viscoelastic material models used to describe brake pads in (Augsburg and Trutschel, 2003) and (Brecht et al., 2003). These described the stiffness hysteresis using a series spring and spring-damper combination.

To confuse the matter further, a different mechanism of hysteresis was suggested in (Kwak et al., 2004) where it is stated that the “hysteresis curve… is common in brake assembly due to the Coulomb friction losses”. Significantly, this would produce the opposite effect to a viscoelastic material behaviour. Viscoelastic hysteresis introduces lead such that the stiffness is elevated on apply and reduced on release, as suggested by Schwarz’s arrows in Figure 1-9. Conversely, friction opposes motion and would introduce lag, decreasing the output force on apply and leaving it trailing high on release.

To settle the confusion over the EMB stiffness, further work is necessary to confirm Schwarz’s results. Perhaps hysteresis is a secondary effect that can be neglected without significant loss in model fidelity. Otherwise, the standard viscoelastic material model might describe the stiffness behaviour.
Friction model

A significant level of friction may arise from large compressive loads within the EMB mechanism. A suitable model is important because friction affects the actuator’s performance, efficiency and stick-slip behaviour. A consensus is yet to be established on how friction should be described in a simplified EMB model.

Examining Maron et al.’s model of EMB friction in Figure 1-4, the friction torque is seen to be a function of the motor velocity and brake clamp force. In other words, the friction may be written as $T_F(\dot{\theta}, F_{cl})$, where $\dot{\theta}$ is the motor velocity and $F_{cl}$ is the clamp force. Interpreting the block diagram, the model appeared to comprise a friction velocity map with some multiplicative dependence on the clamp load. In fairness, (Maron et al., 1997) presented “first results obtained in an early state of the development process”. However, the friction identification procedure that Maron et al. referenced, (Held and Maron, 1988), was criticised for producing a friction model that was “not causal, that is, the discontinuity at zero speed allows the friction to take on an infinite number of values” (Johnson and Lorenz, 1992).

The EMB friction model from (Schwarz, 1999; Schwarz et al., 1999) was more explicit and may be written in the form, $T_F(\dot{\theta}, F_{cl}) = D\dot{\theta} + (C + GF_{cl})\text{sign}(\dot{\theta})$, where $D$, $C$ and $G$ are friction parameters. Schwarz et al.’s viscous friction, $D\dot{\theta}$, was load independent, unlike Figure 1-4 from (Maron et al., 1997) where the viscous friction was multiplied by a loading factor.

(Lüdemann, 2002) extended the EMB friction model with exponential functions to describe curvilinear friction segments. The friction model from (Lüdemann, 2002) may also be written as a function of the velocity and load, $T_F(\dot{\theta}, F_{cl})$. 
None of the prior EMB friction models from Maron et al., Schwarz et al. and Lüdemann adequately describe the stick-slip friction around zero velocity. The problem may be observed in Figure 1-7 and Figure 1-8. In particular, the use of the sign(\( \dot{\theta} \)) function does not adequately describe static friction behaviour. As noted in (Olsson, Åström, Canudas De Wit, Gäfvert and Lischinsky, 1998), “friction at rest cannot be described as a function of only velocity. Instead it has to be modelled using the external force”. At zero velocity stiction opposes the external force, but the prior friction models do not consider this and are only functions of velocity and clamp force, \( T_f(\dot{\theta},F_{cl}) \). Hence, an amendment is required whereby the friction is also a function of the external, non-friction torque, \( T_f(\dot{\theta},F_{cl},T_E) \).

Another problem of prior EMB friction models arises from the viewpoint of numerical simulation. While the motor velocity, \( \dot{\theta} \), may be small, its numerical representation is unlikely to be precisely zero. Consequently, a zero-velocity lockup state may not be encountered during simulation. Hence further work is required to develop and test a more appropriate EMB friction model.
Identification of model parameters

So that the EMB model may be useful, methods are required to identify the model parameters. Quantities of interest are the gear ratio, effective inertia, motor torque constant, stiffness and friction parameters. Ideally, the procedures would allow relevant EMB parameters to be identified online.

Some of the EMB model parameters are reasonable to estimate. The gear ratio may be calculated from geometry. The effective inertia at the motor may determined from component inertias after accounting for gearing. Also, the motor torque constant can be determined from torque-current measurements. Such methods were mentioned in (Schwarz et al., 1998).

The stiffness curve is also reasonably simple to determine when measurements of motor position and clamp force are available. In a significant contribution on force estimation, (Schwarz et al., 1999) indicated that scaling the stiffness curve was sufficient to account for pad wear and temperature variation. It was demonstrated that the stiffness may be determined online from measurements of motor position and estimates of the clamp force.

Determining the EMB friction parameters is more difficult. The friction identification referenced in (Maron et al., 1997) was originally intended for robotic joints and described the case when friction “just depends on the angular frequency \( \omega \)” (Held and Maron, 1988). The identification did not include measurements of clamp force and there was no extension to identify the load dependency in the EMB friction model.

Schwarz et al. took the approach of separately analysing four EMB sub-components and combining their effects. The components considered were the motor, planetary gear, planetary roller screw and the thrust bearing (Schwarz et al., 1998, 1999). The friction in these components was estimated from a combination of theoretical derivation and measurements (Schwarz et al., 1999). The various friction contributions were lumped together and it was assumed that the behaviour of the individual components could be extrapolated to describe the assembled system.
However, this approach does not capture the complex loading of the mechanism components during a brake apply. For example, uniaxial load was considered, but bending and transverse stresses were neglected. Isermann appears to blame this shortcoming for modelling errors. Describing Schwarz’s modelling results in Figure 1-7, Isermann states,

*Minor differences in clamping force, angle of the motor shaft, angular velocity of the motor shaft, and motor current between the model and brake can be traced back due to the fact that the variation of the friction parameters due to the transverse forces are neglected here.* (Isermann, 2003)

Hence, an improved model identification is desired so that the friction parameters can be determined in-situ on an assembled caliper. This would account for the complex mechanism loading and avoid problems associated with identifying the sub-components in isolation.

Additional methods for online EMB identification would also be desirable. Aside from Schwarz et al.’s online force and stiffness estimation (Schwarz et al., 1999), methods are yet to be developed for online identification of the remaining EMB parameters. Such identification techniques may be valuable for applications such as adaptive control.

*Other categories of EMB design*

This study considers a modern EMB configuration with a single drive and without self-energisation. It might be noted that a different category of EMB with dual motors and a wedge mechanism for positive feedback (or self-energisation) was analysed in (Roberts, Schautt, Hartmann and Gombert, 2003). While the wedge brake is a different class of EMB, the study suggest that with some extensions a simplified EMB model might also describe mechanisms of self-energisation.
**The development of EMB control**

Early electromechanical motorcar brakes were driver-controlled using switches and variable resistors. For example, a patent in 1899 describes an electromagnetic drum brake with a “switch box” and a “suitable switch bar” to regulate the excitation current (Stevens and Penney, 1899). A second patent from the same year describes an electromagnetic disk brake with a switch to adjust a variable resistor and control the brake excitation (Sperry, 1899). Feedback position control was later introduced on some electromechanical brake designs. An early example is the motorised drum brake patented in (Wadsworth, 1914) with a control switch to adjust the commanded brake position. The design is shown in Figure 1-10 and includes an electric circuit with contact rails to drive the motor spindle to positions 0-5.

![Figure 1-10: Feedback position control of a motorised drum brake in (Wadsworth, 1914)](image)

While brake torque control may be preferred to manage the vehicle dynamics, obtaining feedback measurements is challenging. For this reason, EMB designs have mainly deferred to brake clamp force control as the next best, or least removed, alternative.

Force control was pioneered in robotic mechanisms around the 1950s and 1960s (Whitney, 1985). One early application noted in Whitney’s 1985 historical perspective was the use of “electric-servo manipulators with force reflection” for remote radioactive hot lab work (Whitney, 1985). (Eppinger and Seering, 1987) briefly summarised early work on force control and noted the simplicity of explicit force control where “the servo loop is based on force errors”. A “generic scheme” for
force control was proposed by De Schutter in 1987 (Schutter, 1987). It generalised earlier approaches with an architecture that had outer-loop force control and cascaded inner control loops to successively manage position, velocity and acceleration. This general structure persists as a description of modern EMB force control. Later reviews on robot force control were given in (Patarinski and Botev, 1993) and (Schutter, Bruyninckx, Zhu and Spong, 1997), and force control with inner-loop position/velocity control was established as standard.

Around the same period as early robotic force control, a standard motion control architecture was established. Referencing works from the 1950s and 1960s, Leonhard states, “there is general agreement that the most effective control scheme for drives is a cascaded or nested structure with a fast inner control loop” (Leonhard, 2001) (p.81). A structure with control loops for position, velocity and motor current was described by Leonhard and the inclusion of “feed-forward reference signals derived from an external reference generator” was advocated for improved performance (Leonhard, 2001) (p.82).

With ideas from robotic force control and servomotor control as potential influences, force control was introduced to EMBs. One example is Jidosha Kiki’s 1986 patent description of an electromechanical disk brake with feedback force control (Ohta and Kobayashi, 1986a).

In more recent work, clamp force control has been implemented on EMBs with embedded control loops. Air-gap management in the clearance region has been handled with a transition to outer-loop position control across the contact point between the brake pads and rotor. While an outer-loop force control and inner-loop motor current control was used in (Hartmann, Schautt and Pascucci, 2002) and (Underwood, Khalil and Husain, 2004), other research has included a velocity control loop. Cascaded EMB force control with embedded feedback loops for force, motor velocity and current/torque is reported in (Maron et al., 1997; Schwarz et al., 1998; Schwarz, 1999; Isermann, 2003; Line, Manzie and Good, 2004; Krishnamurthy, Lu, Khorrami and Keyhani, 2005; Lu, 2005).
State-of-the-art cascaded EMB control has been demonstrated on test actuators, but has shown mixed performance. For example, Schwarz’s plot (a) shown in Figure 1-7 suggests that a cascaded PI control may suffer a poor handling of static friction. In the measurement (messung), lockup may be observed around 5.5 s and a significant ~2 kN tracking error develops over ~0.3 s before there is any response to the commanded set-point (sollsignal). Meanwhile, Lüdemann’s varying step responses shown in Figure 1-8 might suggest that a cascaded EMB controller does not maintain performance throughout the operational envelope.

Considering the large EMB loading up to 30 kN, actuator saturation, load-dependent friction and nonlinear stiffness, it is not yet clear how well a cascaded PI control may perform throughout the work envelope. Further investigation is required to establish the control limitations and determine the potential for improvement. A method for optimally tuning the control gains would also be desirable. Since the control performance affects brake feel and high level functionality such as ABS and electronic stability control, there is a need for a structured EMB control design and critical assessment of the control performance.
1.4 Contributions

The field of EMB research is still maturing and vehicle technology is undergoing an exciting period of innovation. To help further our understanding in this area the present study contributes to the art of EMB modelling and control.

An EMB model would provide a powerful tool for simulation and system design. Hence this study provides an incremental advance of the existing knowledge of EMB modelling, new practical methods for parameter identification and the first open-loop experimental tests of model fidelity. For completeness, simulations of the closed-loop EMB system are also compared with bench-top tests and field data from a brake-by-wire vehicle.

Main contributions on electromechanical brake modelling:

- New methods for EMB parameter identification
- First open-loop experimental tests of the simplified EMB model fidelity
- Necessary extensions to prior EMB models

An EMB control algorithm should be designed to realise the actuator’s potential as a high performance brake. This study investigates a standard cascaded PI control for an EMB to establish its performance, its limitations and the potential for improvement. Following the discovery of significant deficiencies, a series of control solutions are proposed to address the problems of EMB friction, actuator nonlinearity and performance. These improvements are cumulatively realised by the successive contributions of an EMB friction compensation, a modified control architecture and an EMB model predictive control (MPC). To handle the problem of uncertainty the study contributes a robust control design and contrasts this with a proposed method for an adaptive EMB control. Tests were conducted on a prototype EMB and results are presented to demonstrate the performance achieved with the successive control modifications.
Main contributions on electromechanical brake control:

- Establishing the limitations of prior cascaded PI control on an EMB
- A management of EMB nonlinearity culminating in a modified control architecture
- A practical EMB model predictive control for improved performance
- A management of EMB uncertainty, considering first a robust control design and leading to a framework for an adaptive EMB control

Figure 1-11: Prototype electromechanical brake mounted on brake rotor (PGT)
1.5 *Thesis layout*

This dissertation is structured in two parts to address the related topics of EMB modelling and control. The content chapters are prefaced with this general introduction and Chapter 2 on facilities. EMB modelling is then addressed in Chapter 3 and includes necessary extensions to prior EMB models, system identification and the results from experimental tests.

The EMB control is considered in Chapters 4-9. Chapter 4 first investigates cascaded proportional-integral control as the current state-of-the-art for EMBs. Significant limitations are discovered and this provides motivation for Chapters 5-9 where new solutions are investigated to improve the EMB control. A friction compensation is proposed in Chapter 5 to help alleviate the problems of stick-slip friction. Chapter 6 extends this with a modified control architecture to further alleviate the problems of actuator nonlinearity. Chapter 7 then considers the issue of uncertainty and attempts a robust control design for the EMB. Difficulties encountered in satisfying the design requirement lead to a different approach of optimisation and adaptation. Chapter 8 utilises the modified architecture in Chapter 6 to develop a model predictive control for practical implementation and improved EMB performance. Chapter 9 then revisits the issue of uncertainty and extends the MPC within a framework for an adaptive EMB control.
2 Facilities

Experimental testing of the EMB model and controller performance was undertaken on a test-rig with a prototype actuator. The assessment of the model fidelity was complemented with further tests against field data from a brake-by-wire vehicle. This section describes the experimental test-rig, the prototype actuator and the brake-by-wire test vehicle.

2.1 Experimental test-rig

A bench-top EMB test-rig was set-up for system identification, model validation and performance testing. This section contains a listing of the test rig components and connections. Supplementary detail on the sensors, calibration and data acquisition is provided in Appendix 2A.

The components and connections of the EMB test-rig are shown in Figure 2-1, Figure 2-2 and Figure 2-3. Testing was performed with a version V5 prototype EMB from the industry partner, Pacifica Group Technologies. The test apparatus was equipped with sensors to measure the EMB clamp force, angular motor position, motor torque and the three phase motor currents. Test data was recorded on a laptop computer using LabVIEW and a National Instruments data acquisition card. The laptop was also used to schedule the test routines by issuing set-point commands and operating modes to the EMB over a Controller Area Network (CAN) bus.

The embedded EMB controller was programmable and allowed testing of new control algorithms. Control code was uploaded to the control unit via a Joint Test Action Group (JTAG) interface. The control algorithm was written in the C programming language and high level logic was incorporated in a MathWorks MATLAB/Simulink file. C code was generated from the Simulink file using the Real-Time Workshop build function. The generated code was then combined with a sub-layer of C code and compiled so that it could be written to the target controller.
An auto-test routine was programmed and run on a laptop computer for system identification and performance testing. The auto-test program was written in Python and the component object model interface was used to coordinate data acquisition in LabVIEW and communications in CANape. Once control code was uploaded and the EMB was powered, brake commands could be issued from the laptop over the CAN bus. This was done using Vector’s CANape 5.0.30 and a CANcard inserted in the laptop’s PCMCIA slot. The laptop had National Instruments LabVIEW 7.0 installed for data acquisition. A National Instruments data acquisition (DAQ) card, NI PCI-6221, was inserted in the PCI bay of the laptop’s docking station. The DAQ card interfaced a connector block that was linked to the test rig sensors so that measurements could be recorded using LabVIEW.

Figure 2-2 and Figure 2-3 show the test set-up with the torque sensor connected to the EMB motor. This was the test configuration for identifying the motor torque constant. For all other testing the torque sensor and coupling were removed as shown in Figure 2-1. For convenience, the part numbering is detailed in the component listing that follows.
**Component listing**

Figure 2-2: Test rig components with motor torque sensor

1. Dell precision M60 laptop with a CANcard inserted in the Personal Computer Memory Card International Association (PCMCIA) slot.
2. Docking station and with a National Instruments DAQ card, NI PCI-6221, inserted in the Peripheral Component Interconnect (PCI) slot. The docking station has an external power supply connected to 240 V AC power.
3. National Instruments NI SC-2345 shielded module carrier with external power supply, five NI SCC-FT01 feed-through modules and two NI SCC-SG24 full bridge strain gauge modules.
5. PGT wheel brake control unit (WBCU)
6. Clamp force plate containing three HBM C9B force transducers in a triangular configuration.
7. Steglar Mahilo torque transducer mounted on adjustable stand and connected to the EMB motor cap via a torsionally stiff flexible coupling.
8. Current measurement box with LEM-55 current transducers to measure the 3-Phase motor currents
9. Power supply, 5 V direct current (DC) to the current measurement box and 9V DC to the torque sensor
10. Delta Elektronika SM 45-70 power supply (not shown), 42 V DC to WBCU. A current limit was set on the supply, sufficiently low for protection, but sufficiently high to avoid current limiting and resetting of the WBCU during normal operation.
Connections

A. CAN bus between the CANcard and the WBCU connector.
B. Data cable between the data acquisition card and the shielded module carrier.
C. Connection between the feed-through module in socket CH0/8 of the shielded carrier and the inductive position sensor within WBCU. There are 3 sensor wires that carry the ground, channel A and channel B voltages. These are connected respectively to the AIGND, CH+, and CH– inputs on the feed-through module.
D. Connection between the strain gauge modules located in sockets CH1/9 and CH2/10 of the shielded carrier and the 3 force sensors in the clamp force plate. There are 3 cables, one for each of the force sensors. Within a single cable there are 4 wires that are connected to the +/- excitation and +/- signal terminals of the strain gauge module. The force sensors were connected to the DAQ inputs channels 1, 9 and 2 with channel 10 left unused.
E. Connection between the feed-through module in socket CH3/11 of the shielded carrier and the Steglar Mahilo torque transducer. The +/- signal was connected to the CH+ and CH– inputs on the feed-through module.
F. Connection between the feed-through modules in sockets CH4/12, CH5/13, and CH6/14 of the shielded carrier and the current measurement for motor phases A, B and C.
2.2 Prototype actuator

The EMB has a motor, planetary gear and ball screw to operate the brake calliper. This section contains a component listing and a brief description.

Component listing

1. Motor stator and windings
2. Motor rotor permanent magnets
3. Motor rotor
4. Planetary gear input sun gear
5. Planetary gear planet gears
6. Planetary gear output ring
7. Ball screw shaft
8. Piston
9. Brake pads
10. Thrust bearing
11. Piston sleeve
12. End nut
13. Calliper bridge
14. Control unit and heat sink
15. Power and communications link
16. Motor end cap
17. Journal bearings
18. Flexible seal
19. Support bracket
20. Mount for position sensor and ratchet lock
21. Guide pins
22. Location of brake rotor or clamp plate
23. Location of bracket mounting to vehicle

Figure 2-4: EMB cross section and profile (PGT)
**Brief component description**

The sectioned EMB in Figure 2-5 shows the circuit-board, motor stator, rotor and magnets, planetary gear and ball screw. The following descriptions provide a brief overview of the main actuator components.

**Motor**

The prototype EMB contains a 3-phase brushless DC motor with sinusoidal current commutation, otherwise known as a Permanent Magnet Synchronous Motor (PMSM). The motor has an internal rotor with an external mounting of the permanent magnets. Generally, PMSMs are regarded as good servo drives because of their controllability, high power density and responsive performance.

The mechanical construction of the motor is shown in Figure 2-6. It has 24 stator windings and 16 neodymium-iron-boron, diametral, magnets on the rotor. The magnets are arranged with alternating magnetic poles (north, south, north, etc.). The rare earth magnets have a high flux density and large coercive force such that the permanent magnet motor offers a high power density.
The stator slot pitch may be skewed such that the force harmonics and tendency for cogging are reduced. Torque is developed when the stator’s magnetic field vector is commutated with a component that is perpendicular to the rotor’s magnetic field vector. The rotation and magnitude of the stator field is controlled via the three phase motor currents. These are regulated by switched power transistors in the DC link converter. Operational losses arise due to electrical resistance in the copper windings, eddy currents induced in the stator, magnetic hysteresis losses in the iron and mechanical friction.

It is noted that prior EMB research has suggested the use of switched reluctance motors (SRMs) that provide a rugged construction and an extended torque-speed characteristic (Underwood et al., 2004; Lu, 2005; Klode, Omekanda, Lequesne, Gopalakrishnan, Khalil, Underwood and Husain, 2006). A series of alternate drives were also described by some of the patents listed in Appendix 1B.
**Motor controller**

The EMB has an embedded controller to respond to vehicle brake commands and includes a customised 16-bit hybrid controller, signal conditioning and an integrated power stage. Communication with the central vehicle controller is facilitated via Flexray and CAN bus interfaces. The power stage, also known as the DC link converter, commutates the 42 V DC supply and has 6 metal-oxide silicon field effect transistors (MOSFETs) that are switched to govern the three phase motor currents. A schematic circuit is shown in Figure 2-7.

![Motor drive schematic circuit](image)

**Figure 2-7: Electric circuit for the motor and DC link converter based on that in (Hendershot and Miller, 1994; Salem and Haskew, 1995)**

The stator’s magnetic field vector is rotated based on the angular position of the motor such that it is perpendicular to the rotor’s magnetic field vector. The commutation angle may be reduced if field weakening is applied to extend the motor torque-speed characteristic, but this was not implemented on the prototype EMB.

Rotation of the equivalent stator current vector is managed via the transistor switching pattern. Six voltage vectors may be excited by adjusting the transistor states one at a time. The switching pattern is shown in Figure 2-8. Switching between adjacent vectors, the effective voltage vector may be placed inside the hexagon of possible vectors. Switching occurs at a basic clock frequency and pulse width modulation determines the effective voltage amplitude.
The motor control in Figure 2-9 is simplified with transformations between stator and rotor coordinates. The velocity controller, $K_{\dot{\theta}}$, commands the current, $i_q^*$. PI controllers $K_{iq}$ and $K_{id}$ then regulate the quadrature and direct currents. The quadrature current, $i_q$, develops torque as it produces a stator magnetic field perpendicular to the rotor field. The direct current, $i_d$, produces a parallel field without torque development. Hence, $i_d^*$ is often set to zero, unless field-weakening is used to extend the motor torque-speed characteristic.
Sensors

The EMB responds to commands from the central vehicle controller and the internal sensors indicate the feedback that is available for control. The following section details the prototype EMB sensors that measure the motor current, position and brake clamp force. To avoid confusion, the internal EMB sensors are distinguished from the external sensors in the test-rig.

-Current sensors
The three phase motor currents, $i_A$, $i_B$ and $i_C$ are evaluated from measurements of the DC link current, $i_{DC}$, in the motor power stage (Balazovic, 2004). A DC link shunt resistor is used to measure the current and sampling is performed on the control board via an analogue to digital converter. PWM timing information is then used to access and evaluate each of the three phase currents.

-Motor position sensor
An inductive position sensor is mounted about a toothed gear wheel on the motor rotor. The sensor has two detection coils that are physically offset to produce two output voltages that vary with the passing of the gear teeth. When running at constant angular velocity the two output signals are sinusoid-like with a constant phase offset. These signals are low pass filtered in hardware and sampled via an analogue to digital converter on the control board to provide high resolution measurement of the angular position. Additionally, the filtered signals pass Schmitt triggers to give a quadrature encoder signal that is sampled to record the passing of gear teeth with accuracy to a quarter tooth. Although the quadrature signal offers less resolution, the use of dedicated hardware allows sampling at a significantly higher frequency so that a lock on the motor position is ensured at high angular velocities.
-Force sensor
A rosette of strain gauge sensors is mounted on the outside of the EMB piston sleeve. While the clamp force is transmitted via the sleeve, evaluations using the strain gauge measurement are complicated by temperature variation, torque transmission and mechanism friction. Standard temperature compensation is employed using a configuration with offset strain gauges.

-Temperature sensor
There is a temperature sensor embedded within the EMB. While this is not used for feedback control, it allows temperature protection and compensation.

**Communications**

Communication between the EMB and the vehicle controller occurs via the Flexray bus. Unlike the event triggered CAN bus, Flexray and its competitor TTP utilise time-triggered communication protocols that are deterministic and tolerant to faults such as non-responsive or ‘babbling’ nodes (Poledna and Kroiss, 1998; Kopetz, 2002). Fault tolerance is also provided via system redundancy.

**Reduction gearing**

The EMB employs significant reduction gearing to develop clamp forces up to 40 kN. The reduction gearing is two-stage with a planetary gear coupled in series with a ball-screw. The motor drives the planetary sun gear such that the output ring gear turns the ball-screw shaft. The piston translates in accordance with the ball-screw pitch to clamp or release the brake rotor.

Standard planetary gears have a sun gear, planet carrier, and ring gear. With one of these held stationary, the remaining two form the input and output. Planetary gears are known for their ability to achieve high reduction ratios in a compact unit. This is desirable on the EMB since compact packaging is a key design consideration.
The planetary gear is shown in Figure 2-10. An additional reduction is achieved at the expense of efficiency by the inclusion of a second ring gear. In this configuration the second ring gear is held stationary. The sun gear is the input and the planet carrier and output ring gear both turn. The number of friction surfaces is increased with the addition of a fourth component.

Ball-screws with ball-bearings between the screw and nut are an efficient alternative to power screws. Friction is drastically reduced as the sliding friction is replaced with an approximate rolling contact. Since the sliding friction has some proportionality with the normal load, a ball-screw is preferred to avoid excessive friction under high brake clamp loads.
**Calliper bridge**

The brake calliper bridge is typically cast in aluminium or steel. The bridge supports the clamp load that is developed during a brake apply. The prototype EMB had an aluminium bridge that underwent visible deformation during a full 40 kN apply.

**Brake pads**

The brake pad material, temperature sensitivity and friction coefficient significantly affect the brake torque developed at the wheel. Brake pad tribology is complex and regularly maintained as proprietary. Generally, the brake pads are a composite material and are maintained as a separate component to allow for replacement after wear. While the pad composition and friction coefficient can vary, the response characteristic of the brake pad and rotor should allow for temperatures of up to 700°C as well as contaminants such as water and grime (Siegert, Glasner, GeiBler, Zanten and Berg, 1999).
2.3 Brake-by-wire vehicle

To complement the EMB bench-top experiments, further test data was available from a brake-by-wire test vehicle. Data from anti-lock braking field tests was provided by the industry partner, Pacifica Group Technologies. Measurements were from a BMW 328i E46 1998 model that was retrofitted with electromechanical brakes (PGT EMB version V4). The steering angle, wheel speeds, estimated vehicle speed, brake demand and brake clamp forces were recorded at 250Hz over a series of emergency brake manoeuvres. The brake-by-wire test vehicle is shown in Figure 2-12.

Figure 2-12: Brake-by-wire test vehicle retrofitted with EMBs
3 Modelling the electromechanical brake

3.1 Introduction

The EMB model should describe the brake apply profile, force modulation and the stick-slip mechanism friction that characterise the brake performance, particularly during an emergency stop and anti-lock brake (ABS) pressure cycling. The model should have parameters that are readily quantified and practical to identify. Additionally, the parameters should have a physical meaning to facilitate interpretation and ‘what if’ analysis. Finally, the model fidelity should be experimentally tested to establish its performance.

Assessing the fidelity of a lumped-parameter EMB model is a primary aim of this study. Firstly, a behavioural description is outlined to facilitate a meaningful interpretation of the model. A preliminary linear system identification indicates significant nonlinearity and leads to a nonlinear model description. Subsequently, new methods for experimental parameter identification are proposed and identification tests are reported. Finally, the first assessment of the EMB model performance including open-loop tests is described, leading to conclusions regarding the fidelity of the EMB model.
3.2 Behavioural description

A behavioural description of the EMB is relevant for understanding the actuator and how it might be described by a mathematical model. The following summary is intended to complement the actuator description in Chapter 2. It provides detail on the mechanism stress paths, motion behaviour on the test bench, mechanism friction and the hardware limitations.

Main stress paths

While it might be challenging to quantify the mechanical stress fields in the EMB without a finite element model, a qualitative description of the main stress paths is helpful to explain the brake’s operation.

Two main stress fields in the EMB are attributable to the motor torque developed between the stator and rotor, and the clamp force reacted from one brake pad, over the calliper bridge, by the opposing brake pad. Other stresses might arise due to the brake rotor dragging and twisting the brake pads. However, the EMB design avoids transmission of these forces through the mechanism via a support bracket that is bolted to the vehicle. Forces in plane with the brake pad are transferred to a sliding channel in the support bracket via tabs on the brake pad. These tabs may be seen in Figure 3-1. Depending on the friction distribution there is a potential for twist and all planar forces are shown for generality, but it is understood that the lateral force is dominant.

![Figure 3-1: Brake pad with generalised force axes](image-url)
Stress path due to motor torque

When motor torque is developed there is an equal and opposite reaction between the stator and the rotor. As shown in Figure 3-2, a stress path arises from the magnets on the rotor, via the planetary gears, to the ball screw. Depending on the reaction forces and component accelerations, the torque is then transmitted from the ball screw, via the sleeve, end nut and calliper body, back to the stator.

A secondary torque stress field develops as a consequence of the planetary gearing. The output torque from the planetary gear is greater than the motor input and the additional input torque is reacted by the stationary ring gear mounted on the sleeve. Depending on component accelerations, the resulting stress path is through the output gear, the ball screw and back along the sleeve.

The ball screw operation is determined by the:
- Input torque from the planetary gear
- Input axial force from the thrust bearing
- Output torque reacted at the keyway between the piston and sleeve
- Output axial force from the near brake pad nearest the motor
- Friction at the relevant locations between sliding surfaces
When applying over the clearance region, a small torque on the calliper body from the net acceleration of rotating components is reacted by the support bracket via the guide pins.

**Stress path due to clamp force**

As shown in Figure 3-3, another stress field arises due to the clamp force between the brake pads and brake rotor. Since the floating calliper may slide along the guide pins, forces on one brake pad are transmitted over the calliper bridge to the opposing brake pad. Tracing the stress path from the pad nearer the motor, a compressive stress is transmitted through the pad, piston, and ball screw, to the thrust bearing. From here, tensile stress acts via the sleeve and over the calliper bridge to compress the far brake pad where the force is reacted.
Motion behaviour on the test bench

The following describes test bench observations of an initial apply, the subsequent release and a successive apply.

Initial apply:
From a released position the EMB piston advanced over the clearance region until the brake pad nearest the motor contacted the clamp plate (replacing the brake rotor). Prior to contact, the reaction was supported by frictional forces without motion of the more massive calliper housing. After contact at the near brake pad, the calliper body translated back over the remaining clearance until the far brake pad also made contact. With no remaining clearance, additional compliance was taken up as strain. In addition to axial deformation, the calliper bridge was subject to bending and tended to yaw open during apply.

Release:
During release, the motor unwound to recover the elastic strain developed during brake apply. Following the recovery of strain, the piston was retracted into clearance. Because the brake pads were separate components, they were not actively withdrawn.

Apply:
For the successive apply the brake pads were not withdrawn from the brake rotor. The clearance mainly comprised the gap between the piston and the near brake pad. From here the piston advanced to the contact between the brake pads and brake rotor. Upon contact there was no significant translation of the floating calliper since a neutral position was established during the initial apply. Clamp force developed as further advancement was accommodated as strain.

It is noted that the one-sided clearance after release on the test bench would be less likely if the EMB were mounted in a vehicle. In practice, vehicle vibration, brake rotor runout and disk thickness variation are expected to return both brake pads into the clearance region.
Mechanism friction

The EMB operates over a large range of force (0-30+ kN) and motor velocity (0-300 rad/s) and significant mechanism friction may oppose brake actuation. The main sources of mechanism friction are shown in Figure 3-4.
1. Friction in the journal bearings and viscous friction
2. Friction between sliding surfaces in the planetary gearbox
3. Friction in the thrust bearing
4. Friction in the ball screw
5. Friction as the piston slides inside the sleeve
6. Friction as the brake pads slide along channels in the support bracket
7. Friction as the calliper body slides along its guide pins

Friction can be difficult to model due to various factors such as surface roughness, work history and lubrication. However, at a given stage in the life-cycle of an EMB, the two main factors affecting mechanism friction are the clamp load and the velocity of moving components. Some secondary friction effects are possible due to bending and transverse forces.

Significant levels of friction may occur in the thrust bearing and ball screw that support the large, 30+ kN clamp loads. Due to the gearing, the rotation is low velocity (~12.5 rad/s peak) and the losses are reduced (~24.2:1) when transferred back to the motor axle.

Calliper bending and transverse forces during braking may increase the friction in the ball screw and sliding friction between the piston and sleeve. There is also some normal force between the piston key and the sleeve keyway due to the motor torque.

The brake pads have tabs that slide along a channel in the support bracket. As the pads drag and twist against the brake rotor, the brake forces are directly reacted by the support bracket. There is negligible pad translation during loading and compliance is mostly accommodated as strain.
To quantify the total friction load on the motor it might be suggested that measurements are taken for each subsystem; the bearings, the gears, the piston etc. This may be useful to determine the component efficiencies, but it fails to capture the interaction when the actuator is assembled. Complex stresses may arise during operation as a result of axial forces, bending, and transverse loading. Hence, a better approach is to measure the net mechanism friction when all the components are installed. With fixed gearing setting the relative velocities of moving components, it is reasonable to superimpose the friction contributions. For example, consider the spur gears in Figure 3-5.

![Figure 3-5: Spur gears with friction](image)

Suppose the gear friction can be represented by friction-velocity maps. The friction torque on Gear 2 can be referred back to Gear 1 via the gear ratio and superimposed to obtain an equivalent friction-velocity map, as depicted in Figure 3-6.
When the gear 2 friction torque is referenced to axle 1 it is adjusted by the gear reduction. The two friction contributions may then be added to determine the net friction.

The losses described so far are mechanical. If the overall system efficiency is of interest, then the electric and magnetic losses should also be considered.
Physical performance limitations

From the viewpoints of modelling, design and control, it is important to establish the physical constraints on the EMB performance. The limitations are generally related to material properties or power capacity and may be broadly classified as either thermal, strength or power restrictions.

Thermal restrictions

The various EMB materials have thermal limits beyond which they may burn, melt or fail. If there is excessive heat exposure then the operation might be regulated to avoid damage.

The components most sensitive to thermal damage are in the power electronics and the electric motor. These are heated by electrical resistance losses and by heat transfer from the brake pads which can reach temperatures up to 700 °C as the vehicle’s kinetic energy is dissipated. The temperature class of the motor insulation is a particular consideration, with cotton, resin, mica, epoxy, and silicon having ratings between 60-125 °C above an ambient temperature of 40 °C (Leonhard, 2001).

Since the embedded controller and power electronics are sensitive to high temperatures, these were distanced and insulated from the brake pads. The electronics had suitably located heat sinks and a temperature sensor was available to regulate protection. Current limits were imposed to avoid burnout of electric components such as the power transistors.

Permanent magnet motors are also liable to demagnetisation at elevated temperatures. While the Curie temperature indicates the upper temperature limit, a more practical operating limit considers the combined influence of temperature, exposure time, loadline and any adverse magnetic field (Trout, 2001).
Atomic thermal motions counteract the coupling forces between the adjacent atomic dipole moments, causing some dipole misalignment, regardless of whether an external field is present... With increasing temperature, the saturation magnetization diminishes gradually and then abruptly drops to zero at what is called the Curie temperature. (Callister, 1997)

Under certain conditions demagnetisation may occur below the Curie temperature. At elevated temperatures there is an increased sensitivity to magnetic reversal within the material. This will occur if the temperature and magnetic field produce an operation condition on the flux density-field intensity curve (B-H curve) that is below the ‘knee’ (Figure 3-7). In this region the B-H curve undergoes a sudden decline as the magnet’s intrinsic limit is approached. The magnetic decay is generally logarithmic and depends upon the exposure time (Trout, 2001). For neodymium-iron-boron magnets that are popular in brushless motors the maximum operating temperatures presently range between 50-200°C depending upon the material grade.

![Figure 3-7: Demagnetisation curves in the 2nd quadrant of the B-H plot (Trout, 2001)](image)
**Strength restrictions**

The strength of the EMB mechanism components limit the brake force that may be developed. While the stiffness may affect the compliance, work and brake apply time, it is not considered a limit in itself. The EMB is designed for clamp loads up to 40 kN and this is sufficient for installation on a large car or sports utility vehicle. Component specifications should consider potential failure due to plastic deformation, fracture or fatigue. Sourced components are typically provided with working limits specified by the manufacturer. Other components should be designed to handle the peak loads and stress cycles that the calliper will endure over its working life.

**Power restrictions**

A quantity of work must be done to apply the brake and clamp the brake rotor. Hence, the apply time is largely determined by the actuator’s power and this is restricted by component limitations. With a 42 V supply the electrical power is bound by current limits that avoid burnout of the power stage and motor. The mechanical power transmission is also restricted by material limits. However, an appropriate design should avoid material extremes.

While rapid actuation requires high power, the brake apply is a low energy event and no mechanical work is done to hold a clamp force. During an apply electrical energy is converted to the kinetic energy of moving components, heat losses, and useful clamp force work. While the EMB develops large forces, surprisingly little energy is required. For example, measurements for a 25 kN apply indicated that the output clamp force work was in the order of tens of joules. This result agrees with an approximate calculation of the potential energy. Given a stiffness of $\sim 25 \times 10^6$ N/m and a nominal travel of $\sim 1$ mm, the strain potential energy is approximately given by,

$$\frac{1}{2} k \Delta x^2 = \frac{1}{2} \times 25 \times 10^6 \times 0.001^2 = 12.5 \text{ J}.$$
**Saturations**

Two additional saturations may arise during motor operation due to the back electromotive force (EMF) and magnetic saturation. The back EMF increases with motor velocity until a maximum motor speed is reached where the applied voltage is counteracted by the back EMF. Approaching the maximum speed the effective voltage and current diminish until, with friction and loading, there is no net accelerating torque. This effect determines the motor torque-speed characteristic and results in a maximum drive speed.

Ferromagnetic materials undergo magnetic saturation when all the magnetic domains are aligned (Callister, 1997). Depending on the properties of the iron stator, magnetic saturation may occur when a high magnetic field intensity is induced by large motor currents. In standard international units the field intensity is quantified in terms of ampere turns per meter. A second effect that is more pronounced at higher currents is the distortion of the air gap magnetic flux density (Upadhyay, Rajagopal and Singh, 2004). As a consequence of the armature reaction and magnetic saturation, the motor flux linkage and inductance may be diminished at higher currents (Ohm, 2000).
3.3 Linear system identification

Before advancing to a nonlinear EMB model it is appropriate to first investigate whether a linear approximation may suffice. In classical system identification a plant transfer function may be obtained from measurements of the input and output during excitation. In the case of the EMB, the motor quadrature current may be regarded as the input while the brake clamp force is the output. To assess the suitability of a linear EMB model a classical system identification was conducted.

As a guide, the input signal for identification should be sufficiently rich to excite the system. The input was chosen as a bias current superimposed with band limited white noise with a discrete period of 4 ms. Due to processor limitations a noise generator could not be run on the embedded controller. Instead, a 2 s sample record was generated using MATLAB, saved and executed from the device memory.

System identification was conducted about 5, 15 and 25 kN using the test apparatus described in Chapter 2. The bias of the input waveform was adjusted to obtain the nominal clamp loads. Each test was run over 200 s and measurements of the input current and output clamp force were recorded. The time records are shown in Figure 3-8 and it may be observed how input currents of similar amplitude produce reduced amplitude excitations at higher clamp loads.

Figure 3-8: Band limited white noise excitation about 5, 15 and 25 kN
To investigate the suitability of a linear EMB model, candidate transfer functions were estimated from the input-output data shown in Figure 3-8. After removal of the average component of the signals, a transfer function estimate was obtained using the function `tfestimate()` in MATLAB. The transfer function is given by $T_{xy}(s) = \frac{P_{xy}(s)}{P_{xx}(s)}$ where $P_{xx}$ is the power spectral density and $P_{xy}$ is the cross power spectral density. The functional evaluation utilised Welch’s averaged periodogram method and involved a fast Fourier transform on windowed data. Since the time records were already periodic due to the 2 s repeating noise input, a rectangular window was chosen to preserve information. The window length was set at 2 s and produced a frequency resolution of 0.5Hz. Given the 200 s record length, the 2 s window resulted in 100 averages. With an excitation period of 4 ms the identification extended up to a Nyquist frequency of 125Hz. The system identification results are shown in Figure 3-9 with and without a fitted model.

Figure 3-9: Identified transfer functions from motor current to brake clamp force, about 5, 15 and 25 kN (left), overlaid in black with a fitted model (right)
In the Bode plots of Figure 3-9 an additional coherence plot indicates how well the linear identification describes the relationship between the input and output data. The coherence, $C_{xy}$, is given by $C_{xy} = \left| \frac{P_{xy}}{P_{xx}P_{yy}} \right|$, where $P_{xx}$ and $P_{yy}$ are the input and output power spectral densities. Coherence values close to 1 would suggest good linear correlation between the input and output signals.

Figure 3-8 and the corresponding transfer functions in Figure 3-9 show how the magnitude of the EMB force response depends on the operating condition. While the coherence subplots may suggest some local linearity about each operating condition, the identified transfer functions exhibit significantly different magnitude gains. It is apparent that due to actuator nonlinearity, no single transfer function can describe the global EMB behaviour.

The investigation may be advanced by fitting a linear model to the data. Such a model can be derived by considering the net torque due to the motor, $T_m$, clamp load, $T_L$, and friction, $T_F$, that determine the acceleration, $\ddot{\theta}$, of the system inertia, $J$.

$$ T_m - T_L - T_F = J \ddot{\theta} $$

such that,

$$ i_q K_t - F_{cl} N - D \dot{\theta} = J \ddot{\theta} \quad (3-1) $$

where $i_q$ is the motor quadrature current, $K_t$ is the motor torque constant, $F_{cl}$ is the clamp force, $N$ is the effective gear ratio, $D$ is the viscous friction coefficient and $\dot{\theta}$ is the motor velocity. Once the linear model is converted from the time domain ($t$) to the frequency domain ($s$), the resulting block diagram is as shown in Figure 3-10.

![Figure 3-10: Linear EMB model from motor current input, $i_q$, to brake clamp force output, $F_{cl}$](image-url)
Assuming ideal current-loop behaviour, the transfer function from the motor current command $i_q^*$ to the clamp force may be written,

$$\frac{F_{cl}(s)}{i_q^*(s)} = \frac{K, KN}{Js^2 + Ds + KN^2}$$  \hspace{1cm} (3-2)

The constant parameters for the linear model were,

\[
\begin{align*}
J &= 2.91\times10^{-4} \quad \text{Effective inertia (kg m}^2) \\
K_t &= 0.0697 \quad \text{Motor torque constant (N m/A)} \\
N &= 2.63\times10^{-5} \quad \text{Gear ratio (m/rad)}
\end{align*}
\]

The identification of these constants is detailed later in the chapter, but for the moment their values are sufficient.

In addition, a measured stiffness curve was linearised about the three operating loads of 5, 15 and 25 kN.

\[
\begin{align*}
K_5 &= 2.3\times10^7 \quad \text{N/m} \\
K_{15} &= 3.6\times10^7 \quad \text{N/m} \\
K_{25} &= 4.3\times10^7 \quad \text{N/m}
\end{align*}
\]

An energy analysis may be used to estimate the viscous friction parameter, $D$, as the motor work was converted to kinetic energy, clamp force work and frictional losses. Denoting the power loss, $P_{loss}$, the friction torque, $T_f$, and the velocity, $\dot{\theta}$,

\[
P_{loss} = T_f \dot{\theta} = D\dot{\theta}^2
\]

\[
\therefore D = P_{loss} \dot{\theta}^2
\]  \hspace{1cm} (3-3)
Given the periodic excitation, an average value for $D$ may be obtained from the time average power loss and the mean squared velocity.

For the three excitations about 5, 15 and 25 kN the analysis gave viscous friction values of,

$D_5 = 0.0528 \text{ N m s/rad}$

$D_{15} = 0.0161 \text{ N m s/rad}$

$D_{25} = 0.0029 \text{ N m s/rad}$

The friction parameter values are specific to each of the manoeuvres and it might be noted that the high frequency noise excitation acted like a dither signal to alleviate the effects of static friction.

From the range of values for $K$ and $D$ it is apparent that both the stiffness and friction are nonlinear and depend on the operating condition. Three transfer functions were obtained for $\frac{F_i(s)}{i_q(s)}$ using the parameter values for operation at 5, 15 and 25 kN.

These transfer functions were overlain on the measured Bode plots in Figure 3-9 for comparison. The agreement between the three linear models and the identified transfer functions suggests that a simple model structure may be appropriate provided the nonlinearity of the stiffness and friction is described globally, throughout the operational envelope.
3.4 Nonlinear EMB model

A simple EMB model may be suitable if the nonlinear stiffness and friction are modelled appropriately. Hence, a nonlinear EMB model is described beginning with a number of simplifying assumptions that reduce the problem complexity. Following this, a table of the model nomenclature is provided as a convenient reference. An equation of motion is then used to describe the mechanism behaviour and a description of the motor is simplified to a single-phase motor model. Finally, friction and stiffness characteristics complete the model description.

General assumptions

A discussion of the model assumptions is relevant to establish its limitations. Broadly, the model describes a floating-calliper, electromechanical disk brake that has a motor driving some rotational-to-rectilinear reduction to clamp and release the brake.

The model is a half-calliper representation that assumes the force at one brake pad is reacted over the calliper bridge by the opposing brake pad. This assumption neglects slight differentials that might occur, such as on initial brake application when one brake pad has a tendency to contact the rotor before the other.

The symmetrical description of the half-calliper model requires interpretation in the clearance region. In reality, the EMB has a tendency to retract the piston while the housing remains stationary. Combined with the effects of vehicle vibrations and brake rotor runout, it is unlikely that the brake pad clearance will be symmetrical on each side of the brake rotor. Hence, the clearance predicted by the model should be interpreted as indicating the total clearance, but not the distribution.
During release the brake pads are assumed to clear the brake rotor and the possibility of residual drag is neglected. A near horizontal mounting is assumed such that the floating calliper does not slide along the guide pins and cause drag. Possible loading due to vehicle roll, cornering and vibration are neglected. The model also neglects sound and does not describe brake squeal.

The model considers lumped parameters for the stiffness, damping and inertia. Accounting for gearing, the inertias of moving components are referred to the motor axle and combined. The resulting system is equivalent provided its energy function remains equivalent in terms of the generalised coordinates. For operation in the clearance region the inertial load depends on which components are accelerated, the calliper body, the piston, or both. However, there is negligible variation in the inertial load at the motor due to an inverse square relationship with the reduction gearing.

The friction model describes a friction-velocity map with clamp load dependence. It captures bending and shear effects that depend on the clamp force, but not the brake torque. It is assumed the transverse loading of the brake pads due to drag on the brake rotor is directly reacted by the support bracket, as intended in the calliper design.

The stiffness is described by a middle curve and variation with pad wear or temperature may be handled by updating the stiffness characteristic. Stiffness hysteresis is initially neglected, but may be included in a later extension.

The model does not describe the brake self-energisation that may occur in some other EMB designs. An extension to include positive feedback based on the brake torque is possible, but may require a wheel model to determine the speed of the brake rotor and the pad friction.

The output of the EMB model is the brake clamp force. This allows the EMB model to be run in isolation or included in a vehicle model as desired.
Model nomenclature

The following parameters are used to describe the model.

- \( C \) = Coulomb friction factor (Nm)
- \( D \) = viscous friction factor (Nms/rad)
- \( \delta \) = electrical angle by which the stator field is offset for field weakening (rad)
- \( e_{a,b,c} \) = back electromotive force induced in each phase (V)
- \( F_{cl} \) = calliper clamp force (N)
- \( G \) = load dependency of friction (N m/N)
- \( i_{a,b,c} \) = motor three phase currents (A)
- \( i_{d,q} \) = motor direct and quadrature currents (A)
- \( i_0 \) = zero current after dq0 transformation of balanced phase currents (A)
- \( i_s \) = magnitude of the stator currents (A)
- \( J \) = effective system inertia lumped at motor axis (kg m^2)
- \( K_b \) = back EMF constant (V s/rad)
- \( K_t \) = motor torque constant (N m/A)
- \( L \) = inductance (H)
- \( \lambda_m \) = motor flux linkage (Wb)
- \( N \) = gear ratio (m/rad)
- \( N_{poles} \) = number of motor poles
- \( \theta \) = motor angular position (rad)
- \( \theta_e \) = motor ‘electrical angular position’ = \( \theta N_{poles}/2 \) (rad)
- \( P_o \) = power output of the motor (J/s)
- \( R \) = resistance (\( \Omega \))
- \( T_E \) = external non-friction torque (N m)
- \( T_F \) = friction torque (N m)
- \( T_L \) = load torque (N m)
- \( T_m \) = motor torque (N m)
- \( T_s \) = static friction torque (N m)
- \( x \) = nominal calliper position (m)
- \( v_{bemf} \) = back electromotive force (V)
- \( v_{d,q} \) = direct and quadrature voltages (V)
**Equation of motion**

The assumptions lead to a greatly simplified, lumped-parameter EMB model. There is one degree of freedom for the motor position, $\theta$, and a torque balance about the motor axis is considered to derive an equation of motion. Applying Newton’s second law of motion, the net torque due to the motor, friction and load determines the acceleration, $\ddot{\theta}$, of the system inertia, $J$.

$$T_m - T_f - T_L = J\ddot{\theta}$$

$$\therefore i_q K_t - T_f - F_{cl} N = J\ddot{\theta}$$ \hspace{1cm} (3.4)

Here the motor torque, $T_m$, is given by the product of a motor quadrature current, $i_q$, and the torque constant, $K_t$. The load torque, $T_L$, is determined by the clamp load, $F_{cl}$, and the gear reduction, $N$.

**Motor model**

While a detailed model of the three-phase, permanent-magnet synchronous motor is possible, with appropriate simplification an equivalent DC motor model is sufficient to describe the mechanical operation of the EMB. The model derivation begins with a three-phase motor description that is transformed to a two-axis (quadrature-direct) reference frame and then simplified to an equivalent DC motor model.

Background on the modelling of permanent magnet synchronous motors may be found in (Fitzgerald, Kingsley and Umans, 1990), (Krause, Waynczuk and Sudhoff, 2002), (Leonhard, 2001), (Hendershot and Miller, 1994), and (Chai, 1998). It is appropriate to first consider an ideal, three-phase machine with a two-pole permanent magnet rotor. The two-pole model may then be extended to describe a machine with $N$ poles. This motor description neglects eddy current and magnetic hysteresis losses, thermal parameter variation and magnetic saturation. In practice the development of eddy currents is reduced by a laminated stator construction, while demagnetisation and magnetic saturation at elevated temperatures are avoided as a design consideration.
An idealised and practical representation of the motor winding is shown in Figure 3-11. In the ideal case it is assumed the windings have a sinusoidal rather than concentrated distribution. “Theory based on a sinusoidal model brings out the essential features of dc machine theory. The results can readily be modified whenever necessary to account for any significant discrepancies” (Fitzgerald et al., 1990). The EMB motor has a near sinusoidal back EMF characteristic.

The flux linkages established in each phase by the permanent magnets are assumed to vary sinusoidally with the rotor electrical position, with magnitude $\lambda_m$:

$$\begin{bmatrix}
\lambda_{am} \\
\lambda_{bm} \\
\lambda_{cm}
\end{bmatrix} = \lambda_m \begin{bmatrix}
\cos \theta_r \\
\cos(\theta_r - 120^\circ) \\
\cos(\theta_r + 120^\circ)
\end{bmatrix} \quad (3-5)$$

The back EMF induced in each phase is the rate of change of the flux linkage:

$$\begin{bmatrix}
e_a \\
e_b \\
e_c
\end{bmatrix} = -\dot{\theta} \lambda_m \begin{bmatrix}
\sin \theta_r \\
\sin(\theta_r - 120^\circ) \\
\sin(\theta_r + 120^\circ)
\end{bmatrix} \quad (3-6)$$
To eliminate variation with angular position it is convenient to use a reference frame that turns with the rotor. Subsequently, the ‘dq0 transformation’ is used to convert quantities such as current, voltage and flux to a rotor reference-frame orientated by the electrical angle, \( \theta_r \). Figure 3-12 shows the rotating d-q coordinate system overlaid on the three-phase abc stator coordinates.

The dq0 transformation and its inverse are given by,

\[
\begin{bmatrix}
S_d \\
S_q \\
S_0
\end{bmatrix}
= \frac{2}{3}
\begin{bmatrix}
\cos \theta_r & \cos(\theta_r - 120^\circ) & \cos(\theta_r + 120^\circ) \\
-\sin \theta_r & -\sin(\theta_r - 120^\circ) & -\sin(\theta_r + 120^\circ) \\
\frac{1}{2} & 1 & 1
\end{bmatrix}
\begin{bmatrix}
S_a \\
S_b \\
S_c
\end{bmatrix}
\]

\[
\begin{bmatrix}
S_a \\
S_b \\
S_c
\end{bmatrix}
= \begin{bmatrix}
\cos \theta_r & -\sin \theta_r & 1 \\
\cos(\theta_r - 120^\circ) & -\sin(\theta_r - 120^\circ) & 1 \\
\cos(\theta_r + 120^\circ) & -\sin(\theta_r + 120^\circ) & 1
\end{bmatrix}
\begin{bmatrix}
S_d \\
S_q \\
S_0
\end{bmatrix}
\]

where the components of the vector \( S \) may represent voltages, currents, flux linkages or charges.

Figure 3-12: Stator abc coordinates and rotor dq0 coordinates
An expression for the motor torque, $T_m$, in direct-quadrature coordinates may be derived from a transformation of the instantaneous power output using $P_o = T_m \dot{\theta}$.

The mechanical power is the product of the motor torque and velocity.

The instantaneous output power due to the current $i$ in each phase is the product of the current with the back EMF, $e$. Hence the total output power is given by,

$$P_o = e_a i_a + e_b i_b + e_c i_c = \begin{bmatrix} e_a^T \\ e_b \\ e_c \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} \tag{3-8}$$

Applying the dq0 transformation to the currents, $i_{abc}$,

$$P_o = -\dot{\theta} \lambda_m \begin{bmatrix} \sin(\theta_r - 120^\circ) \\ \cos(\theta_r - 120^\circ) \\ \sin(\theta_r + 120^\circ) \end{bmatrix} \begin{bmatrix} \cos(\theta_r) & -\sin(\theta_r) & 1 \\ \cos(\theta_r - 120^\circ) & -\sin(\theta_r - 120^\circ) & 1 \\ \cos(\theta_r + 120^\circ) & -\sin(\theta_r + 120^\circ) & 1 \end{bmatrix} \begin{bmatrix} i_d \\ i_q \\ i_0 \end{bmatrix} = \dot{\theta} \lambda_m \begin{bmatrix} 0 & 3/2 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} i_d \\ i_q \\ i_0 \end{bmatrix} \tag{3-9}$$

But, $P_o = T_m \dot{\theta} = T_m (2\dot{\theta} / N_{poles})$, so that the motor torque is

$$T_m = \frac{3}{2} \frac{N_{poles}}{\lambda_m} i_q \tag{3-10}$$

The Kirchhoff equations for the phase circuits are

$$v_{abc} = RI + \frac{d\lambda_{abc}}{dt} \tag{3-11}$$

where the elements of $v_{abc}$ are the phase voltages, $R$ is the resistance of each phase, $I$ is a unit matrix, and $\lambda_{abc}$ is a vector of flux linkages. The latter are due to the phase currents $i_{abc}$ flowing through the circuit inductances, represented by a matrix $L$, and to the permanent magnets: $\lambda_{abc} = Li_{abc} + \lambda_{mabc}$. This relationship is valid in the linear magnetic region. Otherwise, non-linear inductance curves may be used to account for magnetic saturation (Ohm, 2000).
 Applying the dq0 transformation to (3-11) yields the circuit equations

\begin{align*}
    v_d &= R_i d + L \frac{di_d}{dt} - L_i q \dot{\theta}_r \\
    v_q &= R_i q + L \frac{di_q}{dt} + L_i d \dot{\theta}_r + \lambda m \dot{\theta}_r
\end{align*} \tag{3-12}

In a non-salient machine, such as the EMB motor, the d- and q-axis inductances are the same (equal to \( L \) in (3-12)). The flux linkage due to the permanent magnet, \( \lambda m \), is oriented along the direct axis.

The stator phases are supplied with a balanced set of currents:

\[ i_a = i, \cos(\theta + \delta), \quad i_b = i, \cos(\theta + \delta - 120^\circ), \quad i_c = i, \cos(\theta + \delta + 120^\circ) \tag{3-13} \]

Applying the dq0 transformation results in

\[ i_q = i, \cos \delta, \quad i_d = i, \sin \delta, \quad i_0 = 0 \tag{3-14} \]

Now, as shown above, \( T_m = \frac{3}{2} \frac{N_{poles}}{2} \lambda m i_q = K_i i_q \), where \( K_i \) is the motor torque constant. Hence, for a given magnitude of the phase currents \( i_s \), the q-axis current \( i_q \) and the motor torque will be maximised if \( \delta = 0 \). This corresponds to the stator’s magnetic field vector being perpendicular to the rotor’s magnetic field vector.

Motor position feedback is used to commutate the motor currents as in (3-13), with the stator magnetic field turning synchronously with the rotor. The quadrature current \( i_q \) is controlled to produce the required motor torque, while the direct current \( i_d \) is regulated to zero. This motor control scheme is implemented on the prototype EMB and hence the motor model may be simplified under the assumption of zero direct current, \( i_d = 0 \).

The quadrature motor circuit equation in (3-12) simplifies to:

\[ v_q = R_i q + L \frac{di_q}{dt} + \lambda m \dot{\theta}_r = R_i q + L \frac{di_q}{dt} + K_b \dot{\theta} \tag{3-15} \]

where the back EMF constant is \( K_b = \frac{N_{poles} \lambda m}{2} = \frac{2K_i}{3} \).

The resulting equivalent DC motor circuit is depicted in Figure 3-13.
The single-phase motor description captures the maximum current limit and the influence of the back electromotive force. Protection is included in the current controller to avoid excessive motor currents.

From (3-15),
\[ v_q - v_{bemf} = Ri_q + L \frac{di_q}{dt} \]  \hspace{1cm} (3-16)

A transfer function may be determined by Laplace transformation to the frequency domain,
\[ \frac{i_q}{v_q - v_{bemf}} = \frac{1}{R + Ls} \]  \hspace{1cm} (3-17)
In practice, the effective quadrature voltage, $v_q$, is managed by the DC link converter using pulse width modulation and pattern switching of power transistors to control the voltage vector. The drive operation was described in Chapter 2. In simulation the transfer function given in (3-17) may be used to determine the quadrature current from the applied voltage. The back EMF is determined by the motor velocity, $\dot{\theta}$, and the back EMF constant, $K_b = 2/3K_t$, such that $v_{\text{bemf}} = K_b\dot{\theta} = 2/3K_t\dot{\theta}$. Finally, the motor torque, $T_m$, is determined as the product of the motor current, $i_q$, and the torque constant, $K_t$. 

**Stiffness model**

The stiffness curve for the electromechanical brake is shown in Figure 3-14. The plot relates the displacement of the motor to the brake clamp force. The curve shape is nonlinear and has two regions where the stiffness firstly increases with displacement (0-4 rev) before approaching a more linear relationship (4-7 rev). The reduced initial stiffness occurs before there is sufficient compression to overcome a preliminary compliance and may be associated with the compressive alignment of the mechanism components. Thereafter, the stiffness exhibits a more linear elastic material behaviour. During a 30 kN apply the bridge of the aluminium brake calliper visibly deforms and accounts for most of the compliance. Deformation of the brake pads is understood to be the second greatest contribution to the compliance.

A notable characteristic in Figure 3-14 is the stiffness hysteresis. The clamp force was slightly elevated during apply and reduced during the release. There was a slow relaxation in the stiffness and each measurement was allowed to settle for a period of minutes before recording. The entire apply and release was performed over 91 minutes. This ‘static’ hysteresis is consistent with the pad material model of (Augsburg and Trutschel, 2003), which contains a Coulomb friction-like element in parallel with a viscoelastic element.

![Figure 3-14: Stiffness curve for EMB](image)

**Figure 3-14: Stiffness curve for EMB**
Hysteresis in the stiffness is also velocity-dependent and may arise from viscoelastic material behaviour. In isolation, the brake pads are also known to exhibit a viscoelastic stiffness (Brecht et al., 2003). The velocity dependence of the stiffness hysteresis may be observed by running a brake apply and release at a series of velocities. Constant velocity manoeuvres were commanded at 10, 20, 40, 80, 160 and 300 rad/s. The results in Figure 3-15 and Figure 3-16 exhibit the difference between the apply and release force, \( \Delta F(\theta) = F_{up}(\theta) - F_{down}(\theta) \). It may be observed that the stiffness hysteresis is more pronounced at increased velocity.
Figure 3-16: Stiffness hysteresis during 5-25 kN apply and release

Viscoelastic stiffness behaviour may be described by a spring in series with a parallel spring and damper, as shown in Figure 3-17. The transfer function for the viscoelastic stiffness (detailed in Appendix 3A) may be summarised,

\[
F = \frac{K_1K_2 + C_1K_2s}{K_1 + K_2 + C_1s}x
\]

Figure 3-17: Viscoelastic stiffness model
(Augsburg and Trutschel, 2003) used a similar model with nonlinear stiffness and damping to describe viscoelastic brake pad behaviour. However, fitting such models to experimental data can require a large number of tests, especially when the effects of wear and temperature are considered. Instead, it may be convenient to neglect hysteresis in the first instance and approximate the stiffness using a middle curve model (Figure 3-18). The clamp force, $F_{cl}$, may be approximated either as a function of the motor displacement, $\theta$, or the nominal calliper position, $x_{sp} = N\theta$. The stiffness curve may be found from position and force measurements and is readily updated to account for variation with temperature or wear.

Figure 3-18: Nonlinear model to describe middle stiffness curve
Friction model

Since the EMB mechanism friction is affected by the brake clamp force, the friction model should include this dependency. Some common static and dynamic friction models are reviewed in (Olsson et al., 1998) and options such as the Dahl and LuGre models have found recent application in (Kelly, Santibanez and Gonzalez, 2004) and (Canudas De Wit and Kelly, 2007) for example. However, the original Dahl and LuGre friction models require modification to incorporate friction load dependency.

When formulating a suitable EMB friction model, other significant characteristics include static, Coulomb and viscous friction. These friction phenomena particularly affect the mechanism lockup and operational losses.

Another friction behaviour for consideration is the pre-sliding displacement, described by LuGre (Canudas De Wit, Olsson, Åström and Lischinsky, 1995), bristle (Haessig and Friedland, 1991) and other dynamic friction models. The effect of pre-sliding displacement within the EMB mechanism may be less significant if it does not correspond to significant changes in brake force. A static friction model may be sufficient with regard to describing the brake behaviour, but this will be tested experimentally.

In an effort to describe the most significant EMB friction behaviours, a classical friction model from (Olsson et al., 1998) was modified to include the friction load dependency. This resulted in a static EMB friction model that may be described as a friction-velocity map with load dependency. The friction torque model, \( T_F(\dot{\theta}, F_{cl}, T_E) \), is a function of the velocity, clamp load and the external non-friction torque. The friction opposes the mechanism motion and is given by,

\[
T_F = \begin{cases} 
D\dot{\theta} + (C + GF_{cl}) \text{sign}(\dot{\theta}) & \forall |\dot{\theta}| > \varepsilon \\
T_E & \text{if } |\dot{\theta}| < \varepsilon \text{ and } |T_E| < (T_e + GF_{cl}) \\
(T_e + GF_{cl}) \text{sign}(T_e) & \text{otherwise}
\end{cases}
\]  

(3-19)
The expression has three conditions that describe the friction during motion, lockup and the instant when maximum static friction is overcome. $D$ is the viscous friction coefficient, $C$ is the load-independent Coulomb friction torque, $G$ is the friction load-dependency, $T_s$ is the load-independent static friction torque and $\pm \varepsilon$ defines a small zero velocity bound in accordance with the Karnopp remedy for zero velocity detection. The load dependent friction-velocity map is depicted in Figure 3-19.

![Friction model as a friction-velocity map with load dependency](image)

**Figure 3-19: Friction model as a friction-velocity map with load dependency**
Model summary

A block diagram of the nonlinear EMB model is shown in Figure 3-20. The input is the motor current, \( i_q \), and the output is the clamp force, \( F_{cl} \). Stepping through the block diagram, the motor torque, \( T_m \), is given by the motor current and the torque constant, \( K_t \). The net torque about the motor axle determines the acceleration, \( \ddot{\theta} = (T_m - T_F - T_L)/J \), of the effective inertia, \( J \). The load torque, \( T_L = NF_{cl} \), is subtracted from the motor torque, \( T_m \), to give the external, non-friction, torque, \( T_E \). The friction torque, \( T_F \), is then computed from the friction model. The acceleration is integrated to give the motor velocity, \( \dot{\theta} \), and subsequently the position, \( \theta \). Following the first integration, a small bound (\( \pm \varepsilon \)) for zero velocity detection is imposed to match that in the friction model. The motor position, \( \theta \), and gearing, \( N \), then give the nominal calliper displacement, \( x \). Finally the displacement and stiffness curve determine the brake clamp force, \( F_{cl} \).

![Figure 3-20: Block diagram of EMB model](image)

Depending on the application, the EMB model may be augmented with a single-phase motor circuit and current control. For example, \( i_q = \frac{1}{R + L_S} (v_q - v_{bemf}) \) where the voltage, \( v_q \), is governed by a PI control acting on the current tracking error, \( i_q^* - i_q \). This motor approximation is sufficient to describe the mechanical behaviour and it incorporates the limits on the voltage, \( v_{qmin} \leq v_q \leq v_{qmax} \), and current, \( i_{qmin} \leq i_q^* \leq i_{qmax} \).
3.5 Parameter identification

Before the EMB model can be simulated, practical methods are required for parameter identification. The proposed methodology begins with a list of the key model parameters and a summary of the procedure. This is followed by a more detailed description of the EMB parameter identification.

Model parameters to be identified

The key system parameters to be determined are:

- \( C \) = load independent Coulomb friction (N m)
- \( D \) = viscous friction parameter (N m s/rad)
- \( G \) = load dependency of friction (N m/N)
- \( J \) = system inertia lumped at motor axis (kg m\(^2\))
- \( K \) = lumped stiffness (N/m)
- \( K_t \) = motor torque constant (N m/A)
- \( N \) = gear ratio (m/rad)
- \( T_s \) = torque due to stiction (N m)
Identification procedure and results

This section presents the parameter identification for the EMB model. Model parameters were identified on two actuators of the same design, units 9 and 19 of the PGT V5 EMB. In preliminary testing, EMB 9 was used to develop the identification methodology. Once the procedure was refined, some of the repetitive tests were automated so that an increased data set could be measured. Then a comprehensive identification was performed on EMB 19 (since unit 9 had been deployed on a vehicle). While the preliminary data for EMB 9 was less complete, measurements are presented where possible to allow comparison between the two actuators.

The main steps of the parameter identification are outlined in the following summary. The gear ratio is calculated from the transmission geometry, but the remaining parameters are measured.

Summary of the EMB parameter identification

1) From the transmission geometry determine gearing and the reduction, \( N \)
2) Measure the component masses, compute moments of inertia and calculate the effective inertia seen by the motor, \( J \)
3) From measurements of motor torque and current, determine the motor torque constant, \( K_t \)
4) From measurements of motor position and clamp force, determine the stiffness, \( K \)
5) Measure the friction parameters:
   i. From breakaway tests, determine torque required to overcome static friction, \( T_s \)
   ii. From constant-velocity sweeps, determine viscous and Coulomb friction, \( D \) and \( C \)
   iii. From a series of brake applies, determine the friction load dependency, \( G \)
**Gear ratio N**

The EMB transmission comprises a planetary gear coupled to a ball screw. A half-section of the planetary gear is shown in Figure 3-21. There are two ring gears to obtain a greater reduction at the expense of efficiency.

<table>
<thead>
<tr>
<th>Gear</th>
<th>Teeth</th>
</tr>
</thead>
<tbody>
<tr>
<td>Input sun</td>
<td>(N_1)</td>
</tr>
<tr>
<td>Planets</td>
<td>(N_2)</td>
</tr>
<tr>
<td>Fixed ring</td>
<td>(N_3)</td>
</tr>
<tr>
<td>Output ring</td>
<td>(N_4)</td>
</tr>
</tbody>
</table>

**Figure 3-21: Planetary gear half-section and number of teeth**

The gear ratios in Table 3-1 were calculated using the standard tabular method (Townsend, 1991). This involved rotating planetary train under two imaginary conditions and summing the results to find the overall gear ratio. The procedure may be summarised,

1. Rotate entire system clockwise one revolution and tabulate rotations
2. Fix planet carrier, rotate grounded member anti-clockwise and tabulate rotations
3. Add the first two rows and tabulate results in the third row
4. Finally, calculate the gear ratio between any two components

<table>
<thead>
<tr>
<th>Rotate forward</th>
<th>Input sun</th>
<th>Planet carrier</th>
<th>Planets</th>
<th>Fixed ring</th>
<th>Output ring</th>
</tr>
</thead>
<tbody>
<tr>
<td>+1</td>
<td>+1</td>
<td>+1</td>
<td>+1</td>
<td>+1</td>
<td>+1</td>
</tr>
<tr>
<td>Fix carrier and rotate fixed ring back</td>
<td>(N_1/N_2) (N_2/N_1)</td>
<td>0</td>
<td>(N_1/N_2)</td>
<td>-1</td>
<td>(-N_1/N_2) (N_2/N_1)</td>
</tr>
<tr>
<td>Net rotation</td>
<td>1+(N_2/N_1)</td>
<td>+1</td>
<td>1-(N_2/N_1)</td>
<td>0</td>
<td>1-(N_2/N_1)</td>
</tr>
</tbody>
</table>

**Table 3-1: Calculation of planetary gear ratios**

The planetary gear reduction (11/266) was coupled to a ball screw with a pitch of 4.0 mm/rev. Hence, the total reduction was \(N = 26.3 \times 10^{-6} \text{ m/rad.}\)
**Effective Inertia J**

The inertial load on the motor may be calculated from component masses and moments of inertia after accounting for gearing. The effective inertia is determined by the square of the gear ratio. This square of proportionality may be found by considering a simple gear train as in Figure 3-22 and its equivalent system,

![Figure 3-22: Original and equivalent system for a simple gear train](image)

Expressing the kinetic energy, \( T_{KE}(q, \dot{q}) \), as a function of the generalised coordinates, \( q \), Lagrange’s equation of motion is,

\[
\frac{d}{dt} \frac{\partial T_{KE}}{\partial \dot{q}} - \frac{\partial T_{KE}}{\partial q} = Q \tag{3-20}
\]

where, \( Q \) is the generalized force.

The two systems are equivalent if their kinetic energy is equivalent in terms of the generalised coordinates. This constraint may be used to determine the effective inertia of the equivalent system. Let the motion be described by the generalized coordinate \( \theta_1 \). With the moments of inertia \( I_1 \), \( I_2 \) and \( I_3 \) for gears 1, 2 and 3 and a 1: \( N \) reduction from Gear 1 to Gear 2, the kinetic energy of the original system is given by,

\[
T_{KE} = T_{KE_{gear1}} + T_{KE_{gear2}}
\]

\[
\therefore T_{KE} = \frac{1}{2} I_1 \dot{\theta}_1^2 + \frac{1}{2} I_2 (N \dot{\theta}_1)^2 \tag{3-21}
\]

\[
\therefore T_{KE} = \frac{1}{2} (I_1 + I_2 N^2) \dot{\theta}_1^2
\]
Similarly, the kinetic energy of the equivalent system may be written:

\[
T_{KEq_{equiv}} = T_{KEq_{gear}}
\]

\[
\therefore T_{KEq_{equiv}} = \frac{1}{2} I_s \dot{\theta}_1^2
\]  

Equating the two kinetic energy functions, the inertia of the equivalent system is,

\[
I_s = I_1 + I_2 N^2
\]  

Hence, the effective component inertias are determined by the square of the gear ratio, \(N^2\). The various contributions are summarised in Table 3-2. Summing the inertial contributions about the motor axis, the effective system inertia was found as

\[
J = 0.291 \times 10^{-3} \text{ kg m}^2.
\]

<table>
<thead>
<tr>
<th>Rotating components</th>
<th>Inertia (kg (\text{m}^2))</th>
<th>Effective inertia (kg (\text{m}^2))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Motor rotor</td>
<td>2.05\times10^{-4}</td>
<td>2.05\times10^{-4}</td>
</tr>
<tr>
<td>Motor magnets and glue</td>
<td>5.43\times10^{-5}</td>
<td>5.43\times10^{-5}</td>
</tr>
<tr>
<td>Motor bearing (\times 2)</td>
<td>1.65\times10^{-5}</td>
<td>1.65\times10^{-5}</td>
</tr>
<tr>
<td>Motor rotor cap</td>
<td>1.25\times10^{-5}</td>
<td>1.25\times10^{-5}</td>
</tr>
<tr>
<td>Epicyclic sun gear</td>
<td>1.47\times10^{-6}</td>
<td>1.47\times10^{-6}</td>
</tr>
<tr>
<td>Epicyclic planet gears:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-about central axis of sun gear</td>
<td>5.08\times10^{-6}</td>
<td>7.84\times10^{-7}</td>
</tr>
<tr>
<td>-about own axis</td>
<td>2.03\times10^{-7}</td>
<td>4.15\times10^{-7}</td>
</tr>
<tr>
<td>Ball screw</td>
<td>1.37\times10^{-5}</td>
<td>2.34\times10^{-5}</td>
</tr>
<tr>
<td>Epicyclic output ring gear</td>
<td>1.14\times10^{-5}</td>
<td>1.95\times10^{-5}</td>
</tr>
<tr>
<td>Ball screw thrust bearing</td>
<td>5.51\times10^{-6}</td>
<td>9.42\times10^{-7}</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Translating components</th>
<th>Inertia (kg)</th>
<th>Effective inertia (kg (\text{m}^2))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Motor stator</td>
<td>0.9810</td>
<td>6.80\times10^{-10}</td>
</tr>
<tr>
<td>Brake pad</td>
<td>0.4707</td>
<td>3.26\times10^{-10}</td>
</tr>
<tr>
<td>Brake pad load spreader</td>
<td>0.1491</td>
<td>1.03\times10^{-10}</td>
</tr>
<tr>
<td>Motor end cover</td>
<td>0.0861</td>
<td>5.97\times10^{-11}</td>
</tr>
<tr>
<td>Ball screw shaft</td>
<td>0.1630</td>
<td>2.13\times10^{-12}</td>
</tr>
</tbody>
</table>

| Total (kg \(\text{m}^2\)):  | \textbf{0.291\times10^{-3}} |

Table 3-2: Effective component inertias
**Motor torque constant \( K_t \)**

The motor torque constant, \( K_t \), may be determined from measurements of the motor torque, \( T_m \), and current, \( i_q \). Some background is relevant to describe the torque and the computation of quadrature current, \( i_q \), from the three-phase motor currents, \( i_{abc} \).

As a first test, the quadrature current was commanded to \(~10\) A and the motor torque was measured through one complete motor revolution. The results shown in Figure 3-23 exhibit the motor torque pulsations. These may arise from variation in magnetic reluctance or distortion of the magnetic field. The 15° degree oscillation period corresponds to the motor geometry. During each revolution the rotor magnets pass 24 stator windings that are distributed at intervals of 15° degrees. The average torque constant, \( K_t \), neglects the ripple and only describes the mean relationship between the motor current and torque.

![Figure 3-23: Torque (Nm) versus motor angle (degrees) at a constant quadrature current of 10.73 A](image)

All bench tests undertaken in this investigation were conducted at room temperature. While high temperatures can affect magnetic saturation and demagnetisation, the EMB motor should not approach these conditions during its operation. Thermal variation of the stator winding resistance was managed by feedback current control.
The quadrature motor current, $i_q$, may be determined from measurements of the three-phase currents, $i_{abc}$. As shown in Figure 3-24 (a), the three-phase currents appear sinusoidal when commutated near constant velocity. Applying the Clark transformation to the three-phase currents, $i_{abc}$, results in the two-phase representation, $i_{αβ}$. The currents $i_{dq}$ may then be found by applying the Park transformation which was applied with the simplifying assumption that the direct current was regulated to zero ($i_d \approx 0$). In other words, $i_q$ is approximated by the instantaneous vector magnitude as portrayed in Figure 3-24 (b). The assumption of zero direct current is a reasonable approximation except at high velocities beyond the commutation capacity of the discrete-time current control. For completeness, the Clark and Park transformations are given in Figure 3-25 and Figure 3-26.

![Figure 3-24: Sinusoidal three-phase motor currents, $i_{abc}$, during motion (a) and after transformation to two-phase currents, $i_{αβ}$ (b)]](image)

\[
\begin{bmatrix}
  i_{α} \\
  i_{β} \\
  i_{o}
\end{bmatrix} = \frac{2}{3} \begin{bmatrix}
  1 & -\frac{1}{2} & -\frac{1}{2} \\
  -\frac{1}{2} \sqrt{3} & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \\
  \frac{1}{2} & \frac{1}{2} & \frac{1}{2}
\end{bmatrix} \begin{bmatrix}
  i_a \\
  i_b \\
  i_c
\end{bmatrix}
\]

![Figure 3-25: Clark transformation from three-phase $i_{abc}$ coordinates to two-phase $i_{αβ}$ coordinates](image)
It may be noted that the combined Clark-Park transformation is equivalent to the previous dq0 transformation.

Having mathematical tools to analyse the three-phase currents, the motor torque constant, $K_t$, may be identified as the slope of the torque-current ($T_m$-$i_q$) plot. To this end, the motor torque was measured at standstill for a series of quadrature currents. The torque transducer was coupled to the motor output as described in Chapter 2. Measurements were recorded with brake pad clearance to avoid clamp load. The results are shown in Figure 3-27 and are summarised in Table 3-3.
<table>
<thead>
<tr>
<th>Actuator</th>
<th>Torque constant, $K_t$ (N m/A)</th>
<th>Regression line $R^2$ value</th>
</tr>
</thead>
<tbody>
<tr>
<td>EMB 19</td>
<td>0.0697</td>
<td>0.9998</td>
</tr>
<tr>
<td>EMB 9</td>
<td>0.0701</td>
<td>0.9999</td>
</tr>
</tbody>
</table>

Table 3-3: Torque constant, $K_t$

As expected, a linear relationship was observed between the motor torque and quadrature current. After linear regression to find the motor torque constant, the coefficient of determination, $R^2$, was greater than 0.99. When extrapolated to the origin, the trend lines had a small offset that may be due to the positive transformation of noise in the current measurement. This did not affect the torque constant that was indicated by the slope.
Stiffness curve

A stiffness curve was determined from concurrent measurements of the clamp force and motor displacement. During tests a force measurement plate was substituted for the brake rotor as shown in the experimental setup of Chapter 2. The identification was a slow test to determine the middle stiffness curve and measurements were recorded at room temperature using unworn brake pads. The results are shown in Figure 3-28 with curves fitted to the measured data. While the two actuators had mechanisms of the same design, EMB 19 had a rear calliper bridge that was less stiff than the front calliper bridge of EMB 9.

The zero position was defined as the contact between the brake pads and rotor. The motor position, \( \theta \), was reduced by, \( N \), to determine the nominal piston translation, \( x \).

With the clamp force, \( F_{cl} \), in kilonewtons and the nominal piston translation, \( x \), in millimeters, the measured stiffness for EMB 19 may be approximated from 0 to 30 kN by,

\[
F_{cl} = \begin{cases} 
-7.23x^3 + 33.7x^2 - 3.97x & x > 0.125 \\
0.1295x & \text{otherwise}
\end{cases}
\] (3-24)

Figure 3-28: Stiffness curve
Static friction $T_s$

The maximum static friction may be determined from a series of breakaway tests. To obtain each measurement the motor torque (indicated by $T_m = K_I i_q$) was slowly increased until static friction was overcome and motion occurred. The tests were conducted in the clearance to avoid clamp load. Figure 3-29 depicts the breakaway events as the torque was increased to exceed the maximum static friction.

![Figure 3-29: Static friction breakaway tests on EMB 9 (N m)](image)

At standstill with zero load and acceleration, the maximum static friction, $T_s$, was indicated by the breakaway friction torque ($T_s = T_F$ at breakaway),

$$T_p = T_m - T_L - J\ddot{\theta} \quad (T_L = 0 \text{ and } \dot{\theta} = 0)$$

$$\therefore T_s = T_{mBreakaway} = K_I i_{qBreakaway} \quad (3-25)$$

Preliminary measurements were recorded using EMB 9 and the procedure was later automated so that a greater number of measurements were possible on EMB 19. The accuracy of the automated test was refined by applying fine torque increments slowly until breakaway motion was detected. Following each breakaway the motor torque was reset and the procedure was iterated. The tests were maintained in the brake pad clearance region.
The static friction measurements on EMB 19 had a normal distribution and are shown in Figure 3-30. The results for both actuators are summarised in Table 3-4. Some variability may be expected between the units and measurements indicate that the static friction on EMB 9 was almost double that on EMB 19. The friction variation may arise from differences in lubrication and the efficiency of sub-components in the transmission.

![Figure 3-30: Static friction measurements on EMB 19](image)

<table>
<thead>
<tr>
<th>Actuator</th>
<th>Mean static friction, $T_s$ (N m)</th>
<th>Observations</th>
<th>Standard deviation (N m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>EMB 19</td>
<td>0.0379</td>
<td>1000</td>
<td>0.0055</td>
</tr>
<tr>
<td>EMB 9</td>
<td>0.0558</td>
<td>28</td>
<td>0.0071</td>
</tr>
</tbody>
</table>

Table 3-4: Static friction, $T_s$
Viscous and Coulomb friction parameters, $D$ and $C$

The viscous and Coulomb friction were determined from constant-velocity sweeps over an extended brake pad clearance. With zero average acceleration ($\dot{\theta} \approx \text{constant}$) and no clamp load, the friction was indicated by the average motor torque necessary to maintain constant velocity. With this approach, the friction was determined over a series of velocities to develop a friction-velocity map.

The friction was found at a series of velocities from the average drive torque,

$$T_p = T_m - T_L - J\ddot{\theta}$$  \hspace{1cm} (\text{for } T_L = 0 \text{ and } \ddot{\theta} \approx 0)

$$\therefore T_p \approx T_{\text{mSweepAverage}}$$  \hspace{1cm} (3-26)

Subsequently, the viscous and Coulomb parameters $D$ and $C$ were fitted to the friction-velocity map. Under the condition of motion and with no clamp load, the friction may be written,

$$T_p = D\dot{\theta} + (C + GF_{cl}) \text{sign}(\dot{\theta})$$  \hspace{1cm} (\text{for } F_{cl} = 0 \text{ and } \dot{\theta} \geq 0)

$$\therefore T_p = D\dot{\theta} + C$$  \hspace{1cm} (3-27)

A number of preliminary measurements (85) were available from preliminary test development on EMB 9. After test automation, a larger set of measurements (1488) was recorded using EMB 19. The results are shown in Figure 3-31. The static friction, $T_s$, was added as a data point at zero velocity.
The results obtained for EMB 9 exhibit a nonlinear characteristic below 20 rad/s that was not apparent in the later tests on EMB 19. This effect on EMB 9 may be due to the low-speed control becoming irregular and under-driven towards zero-velocity. Otherwise, the velocity control was found to accurately regulate the velocity command. EMB 19 had an improved velocity controller at a later stage of development. The corresponding friction-velocity data showed an increased spread only at speeds under 2 rad/s. To avoid the low-velocity irregularity, particularly on EMB 9, the trend-lines were fitted to the linear section of the data.

The viscous and Coulomb friction parameters for the two actuators are summarised in Table 3-5. These values correspond to the fitted trendlines seen in Figure 3-31. As previously observed with the static friction, the friction parameters on EMB 9 were approximately double those on EMB 19.

<table>
<thead>
<tr>
<th>Actuator</th>
<th>Coulomb friction, $C$ (N m)</th>
<th>Viscous friction, $D$ (N m s/rad)</th>
</tr>
</thead>
<tbody>
<tr>
<td>EMB 19</td>
<td>0.0304</td>
<td>$3.95 \times 10^{-3}$</td>
</tr>
<tr>
<td>EMB 9</td>
<td>0.0600</td>
<td>$8.20 \times 10^{-4}$</td>
</tr>
</tbody>
</table>

Table 3-5: Viscous and Coulomb friction parameters, $D$ and $C$
**Friction load dependency, \( G \)**

The friction load dependency, \( G \), was determined from a series of brake applies. The identification was two-stage, firstly calculating the friction torque and then fitting the friction parameters by linear regression.

The instantaneous friction was estimated using \( T_f = T_m - T_L - J\ddot{\theta} \) over a series of brake applies. The calculation was based on measurements of the motor current \( (T_m=K_e i_d) \), brake clamp force \( (T_L=NF_{cl}) \) and motor position. The high-resolution position measurement was differentiated to obtain signals for the velocity, \( \dot{\theta} \), and acceleration, \( \ddot{\theta} \). The instantaneous friction estimates are shown in Figure 3-32 and appear to be strongly correlated with the clamp load.

![Figure 3-32: Friction during brake apply on EMB 19 (top) and EMB 9 (bottom)](image-url)
Numerical differentiation of the motor position was performed using a centred finite difference equation,

\[ f'(x_i) \approx \frac{f(x_{i+1}) - f(x_{i-1})}{2\Delta x} \quad (3-28) \]

Prior to differentiation the noise was filtered using a zero-phase, low-pass digital filter.

On EMB 19 the motor position was recorded at 25 kHz to differentiate accurate velocity and acceleration signals. Because the analysis was not developed when 11.9 kHz data was recorded from EMB 9, only limited back-testing was possible. At the reduced sampling rate the velocity tracking was lost during the fast-and-large brake applies. Consequently, the friction load-dependency was only measured up to 15 kN on EMB 9.

After the instantaneous friction was calculated, the friction parameters could then be estimated by linear regression. Regression matrices were constructed using

\[ T_{F} = D\dot{\theta} + (C + GF_{cl})\text{sign(}\dot{\theta}\text{)} \]

to obtain a least-mean-squares estimate of the friction parameters,

\[
\begin{bmatrix}
T_{F1} \\
T_{F2} \\
\vdots
\end{bmatrix} =
\begin{bmatrix}
\dot{\theta}_1 & \text{sign}(\dot{\theta}_1) & F_{cl1}\text{sign(}\dot{\theta}_1\text{)} \\
\dot{\theta}_2 & \text{sign}(\dot{\theta}_2) & F_{cl2}\text{sign(}\dot{\theta}_2\text{)} \\
\vdots & \vdots & \vdots
\end{bmatrix}
\begin{bmatrix}
D \\
C \\
G
\end{bmatrix}
\]

\[ \therefore Y = X\hat{\Theta} \]

\[ \therefore \hat{\Theta} = (X^TX)^{-1}X^TY \quad (3-29) \]

where \[ \hat{\Theta} = \begin{bmatrix} D \\ C \\ G \end{bmatrix} \]

\[ Y = \begin{bmatrix} T_{F1} \\ T_{F2} \\ \vdots \end{bmatrix} \]

and \[ X = \begin{bmatrix} \dot{\theta}_1 & \text{sign}(\dot{\theta}_1) & F_{cl1}\text{sign(}\dot{\theta}_1\text{)} \\ \dot{\theta}_2 & \text{sign}(\dot{\theta}_2) & F_{cl2}\text{sign(}\dot{\theta}_2\text{)} \\ \vdots & \vdots & \vdots \end{bmatrix} \]

While this also yields estimates for the viscous and Coulomb friction parameters, \( D \) and \( C \), the approach is most suited to identifying the dominant friction load-dependency, \( G \) (shown in Table 3-6). Constant-velocity sweeps in the clearance region are preferred for a more sensitive identification of \( D \) and \( C \) without the influence of the load-dependent friction.
The final estimates of the friction load-dependency are summarised in Table 3-6. As before, the friction parameter for EMB 9 was around double that of EMB 19. The elevated friction on EMB 9 may also be seen in Figure 3-32. While identification was only performed on two actuators, the results might suggest a large range in the mechanism friction.

<table>
<thead>
<tr>
<th>Actuator</th>
<th>Friction load-dependency, $G$ (N m/N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>EMB 19</td>
<td>$1.17 \times 10^{-5}$</td>
</tr>
<tr>
<td>EMB 9</td>
<td>$2.62 \times 10^{-5}$</td>
</tr>
</tbody>
</table>

Table 3-6: Friction load dependency, $G$
**Parameter summary**

The model parameters for the two EMBs used in this investigation are summarised in Table 3-7.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Units</th>
<th>EMB 19</th>
<th>EMB 9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gear ratio, $N$</td>
<td>m/rad</td>
<td>$26.3 \times 10^{-6}$</td>
<td>$26.3 \times 10^{-6}$</td>
</tr>
<tr>
<td>Inertia, $J$</td>
<td>kg m$^2$</td>
<td>$0.291 \times 10^{-3}$</td>
<td>$0.291 \times 10^{-3}$</td>
</tr>
<tr>
<td>Torque constant, $K_t$</td>
<td>N m/A</td>
<td>0.0697</td>
<td>0.0701</td>
</tr>
<tr>
<td>Static friction, $T_s$</td>
<td>N m</td>
<td>0.0379</td>
<td>0.0558</td>
</tr>
<tr>
<td>Coulomb friction, $C$</td>
<td>N m</td>
<td>0.0304</td>
<td>0.0600</td>
</tr>
<tr>
<td>Viscous friction, $D$</td>
<td>N m s/rad</td>
<td>$3.95 \times 10^{-4}$</td>
<td>$8.20 \times 10^{-4}$</td>
</tr>
<tr>
<td>Friction load-dependency, $G$</td>
<td>N m/N</td>
<td>$1.17 \times 10^{-5}$</td>
<td>$2.62 \times 10^{-3}$</td>
</tr>
</tbody>
</table>

Table 3-7: EMB model parameters

The stiffness curve is approximated from 0-30 kN with $F_{cl}$ in kN and $x$ in mm by,

EMB 19, \[ F_{cl} = \begin{cases} 
-7.23x^3 + 33.7x^2 - 3.97x & x > 0.125 \\
0.1295x & \text{otherwise} 
\end{cases} \]

EMB 9, \[ F_{cl} = \begin{cases} 
-16.18x^3 + 51.04x^2 - 6.73x & x > 0 \\
0 & \text{otherwise} 
\end{cases} \]
3.6 Model performance

The best one can do is to test the model by trying to falsify (invalidate) it, looking for defects – (Walter and Pronzato, 1997) explain the misnomer of ‘model validation’.

Since the fidelity of the EMB model was a primary concern, a series of tests were conducted to investigate its performance. The testing considered the model’s ability to describe the brake apply profile, modulations and the stick-slip friction behaviour. These characteristics are relevant to operations such as emergency and anti-lock braking.

The model performance was assessed against the measured EMB response and its fidelity was evaluated firstly in open-loop tests, then with feedback control active and finally against field data from ABS braking on the test vehicle. The open-loop tests are important to assess the model in isolation, without feedback control rejecting disturbances. If the plant model is found to be reasonable, then it may later be used for controller development and simulation.

The closed-loop tests were intended to provide a complementary model assessment when feedback control was included. Similarly, the model assessment against vehicle brake data acts as a verification test for the bench-top results. However, a limitation was that the test vehicle had PGT V4 EMBs while bench-top tests were performed on the later version, PGT V5 EMB. To clarify, the V5 actuator was simulated against bench-top V5 measurements and the V4 simulation was compared with V4 vehicle data. The V5 parameters were taken from the prior identification and estimates of the V4 parameters were provided by the industry partner. All the model parameters are summarised in Appendix 3B.

The results of the model-fidelity tests are presented in two parts. Firstly, the open-loop simulation is compared with measurements from the test-rig described in Chapter 2. Later, the closed-loop simulation is compared with measurements obtained from the bench-top apparatus and also from brake data from the test vehicle. Since some behaviours were consistent throughout the results, the discussion is largely withheld until the end, together with conclusions regarding the model fidelity.
Open-loop tests of model fidelity

During a series of controlled brake manoeuvres, measurements were recorded for the motor current, position and clamp force. The recorded current was then used as an open-loop input to the simulation model. The fidelity of the EMB model could then be assessed by comparing the simulated and measured responses.

Open-loop simulation of step-response

The model’s description of the brake apply profile was tested over a series of step-responses. All comparisons were performed on different data to that used for the parameter identification. The results for EMB 19 and EMB 9 are shown in Figure 3-33. For repeatability the tests were started from a nominal 100 N clamp load as this could be determined more precisely than the zero-load contact point. While the EMB 9 parameters were determined from preliminary measurements before the identification procedure was refined, some back-testing was possible and the results are presented for completeness.

Figure 3-33: Measured and simulated step-response to 2.90, 4.65 and 6.11 A on EMB 19 (a) and 5.80 and 7.78 A on EMB 9 (b)
Further results shown in Figure 3-34 compare the simulation output with measurements of the clamp force and motor velocity. These two signals describe the state of the EMB. A third subplot shows the input motor current and there is only one current trace as the measurement was used as an open-loop input to the simulation.

Figure 3-34: Measured and simulated step-responses to 6, 13, 21 (a, b, c) and 31 kN (d) on EMB 19
In addition to comparing the states, the equivalence of two systems may be assessed by comparing the internal system energy. Figure 3-35 shows a measured and simulated apply (a), with the corresponding losses (b), power (c) and energy (d).

The internal system energy was determined by,

Input motor work, \[ W_{IN} = \int T_{m}(t)\dot{\theta}(t)dt \]

Kinetic energy of moving components, \[ K_E = \frac{1}{2} J\dot{\theta}(t)^2 \]

Frictional losses, \[ F_L = \int T_F(t)\dot{\theta}(t)dt \]

Output clamp force work, \[ W_{OUT} = \int F_c(t)\dot{x}(t)dt \]
Open-loop simulation of modulations

The simulation of brake modulations was assessed about a series of clamp loads. The results given in Figure 3-36 show 10% modulations about 5, 15 and 25 kN at 4Hz and 15 kN at 8Hz.

Figure 3-36: 10% modulations on EMB 19 about 5, 15 and 25 kN at 4Hz (a, b, c) and 15 kN at 8Hz (d)
Each plot in Figure 3-36 comprises three subplots showing the clamp force, motor velocity and current. As previously, the manoeuvres were all started from 100 N for consistency. Measurements were recorded during closed-loop force control and the current record was used as an open-loop input to the simulation. The 4Hz modulations were chosen as relevant to some ABS operation, but it is understood that anti-lock braking may exhibit significant variation depending on the conditions and control. To broaden the results, a more demanding 8Hz modulation is shown in Figure 3-36 (d). Since there is some fine detail in the results given in Figure 3-36, enlarged versions are provided in Appendix 3C.

In addition to the brake apply and modulations, it is important that the model can describe the stick-slip friction behaviour. In some instances the static friction can significantly affect the brake response by causing the mechanism to lock up. Figure 3-37 shows enlargements of some stick-slip friction events in Figure 3-36 (c).

Figure 3-37: Stick-slip motion during 10% modulation on EMB 19 about 25 kN at 4Hz
Closed-loop tests of model fidelity

The closed-loop tests assess the EMB simulation when the plant model is extended with a single-phase motor circuit (Figure 3-13) and brake control. Regulation was performed by a cascaded PI control with feedback loops to manage clamp force, motor velocity and current. Further detail on the controller is given in Chapter 4. Presently it is sufficient to note that the closed-loop tests assess the regulated model from input force commands to the brake force output \((F_c \rightarrow F_{cl})\). The results complement prior open-loop tests of the plant model from input current to output clamp force \((i_q \rightarrow F_{cl})\).

Model performance against bench-top test results

A number of brake manoeuvres are simulated and measured in subsequent chapters on EMB control. To avoid repetition, the data shown in Figure 3-38 was chosen as being representative of later results.

Figure 3-38: Simulated and measured 15 kN brake apply on EMB 19 with cascaded PI control
**Model performance against vehicle brake data**

The prior tests of model fidelity were all conducted on the bench-top apparatus described in Chapter 2. While the test-rig provides a controlled test environment, it is important that the results are complemented with field data from the test vehicle. Hence, ABS brake data from the industry partner was used for an additional model assessment. The response of the controlled model, from the force command to the output \( F_{cl} \rightarrow F_{cl} \), was compared with the vehicle brake data. The results are shown in Figure 3-39 and enlargements are provided in Appendix 3D.

![Graphs comparing force command to output](image)

**Figure 3-39:** Braking during ABS stop with PGT EMB V4 at vehicle front-left (a), front-right (b), rear-left (c) and rear-right (d)
Assessment of model fidelity

The fidelity of the EMB model may be assessed on its ability to predict the measured brake response and vehicle brake data. The behaviours that should be described are the brake apply profile, modulations and the stick-slip friction. These behaviours are featured in the open-loop simulations of the brake apply and modulation. The open-loop tests are particularly demanding since there is no regulation and simulation errors may accumulate. The closed-loop tests provide a further assessment when feedback control is included.

In all results for the brake apply profile, the simulated response consistently lead the measurement. The lead in the open-loop simulation translated to a faster apply in the predicted closed-loop response. The simulation lead might arise from underestimates of the friction and inertia parameters or an overestimate of the stiffness. These properties are more susceptible to identification error than the motor torque constant or the gear ratio. For example, a comparison of the force and velocity subplots in Figure 3-34 might suggest a slight error in stiffness model. Further, some error in the friction estimate is suggested by the result in Figure 3-35 (b). Also, a small underestimate of the inertia parameter was suspected at a later stage in the investigation.

Features that help quantify the model evaluation are the brake-apply peak force, peak time and apply rate. These properties are depicted in Figure 3-40 and the test results in Figure 3-33 are quantified in Table 3-8.

Figure 3-40: Step response features
The largest discrepancy occurred for the open-loop EMB 9 results in Figure 3-33. The model produced errors of 18.7% and 13.5% for the measured peak forces of 12.3 and 17.8 kN. However, errors in the corresponding peak-time were less than 2%. The simulation error may be partly due to the limited parameter identification on EMB 9 data before measurement procedures had been refined.

Following refinement of the parameter identification, the open-loop EMB 19 results showed an improved agreement between the simulated and measured brake-apply profile. It is seen in Table 3-8 that the simulation produced errors of 5.1%, 2.3% and 1.2% for the measured peak forces of 9.3, 17.1 and 24.7 kN. The corresponding peak-time errors were 5.7%, 3.6% and 1.7%. Generally, the characteristics of the larger brake applies were predicted with less relative error.

When compared with the measured EMB response, the model should predict the system states and the internal energy. Results from the open-loop simulation demonstrated reasonable predictions in both cases. To verify the energy analysis given in Figure 3-35, the strain potential energy can be checked using an approximate stiffness of $25 \times 10^6$ N/m and nominal travel of $0.9 \times 10^{-3}$ m, in which case the clamp force work in a 20 kN apply may be estimated as $\frac{1}{2} k x^2 = \frac{1}{2} \times 25 \times 10^6 \times (0.9 \times 10^{-3})^2 = 10.1$ J. This coarse approximation is in the same order as the 7 J of measured clamp force work.

<table>
<thead>
<tr>
<th>EMB 19</th>
<th>Peak force (kN)</th>
<th>Peak time (s)</th>
<th>Peak apply rate (kN/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Simulated</td>
<td>Measured</td>
<td>Simulated</td>
</tr>
<tr>
<td>2.90A</td>
<td>9.3</td>
<td>9.8</td>
<td>0.425</td>
</tr>
<tr>
<td>4.65A</td>
<td>17.1</td>
<td>17.5</td>
<td>0.373</td>
</tr>
<tr>
<td>6.11A</td>
<td>24.7</td>
<td>25.0</td>
<td>0.350</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>EMB 9</th>
<th>Peak force (kN)</th>
<th>Peak time (s)</th>
<th>Peak apply rate (kN/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Simulated</td>
<td>Measured</td>
<td>Simulated</td>
</tr>
<tr>
<td>5.80A</td>
<td>14.6</td>
<td>12.3</td>
<td>0.296</td>
</tr>
<tr>
<td>7.78A</td>
<td>20.2</td>
<td>17.8</td>
<td>0.316</td>
</tr>
</tbody>
</table>

Table 3-8: Peak force, time and apply rate for step responses in Figure 3-33
One feature not described by the EMB model is the force relaxation following the peak response. Without motor motion, there is a significant relaxation in the measured clamp force. Some decay may be observed in Figure 3-33 and for clarity the EMB 19 result was plotted with an extended time axis in Figure 3-41. For the 25 kN apply, the force relaxation was 0.9 kN at 1.44 s after the peak. This relaxation might be attributed to the viscoelastic stiffness that was neglected in the model.

![Figure 3-41: Measured and simulated step-response to 2.90, 4.65 and 6.11 A on EMB 19](image)

In addition to predicting the step brake apply, the open-loop simulation described force modulations with reasonable accuracy. For example, the oscillation results in Figure 3-36 show satisfactory agreement between the predicted and measured responses. The simulation tended to lead during the initial apply in a manner that was consistent with other results. Also, the model output tended to have a decreased relative error for the larger brake manoeuvres.

The agreement in the open-loop modulation results was encouraging, particularly since the simulation was run without feedback regulation. However, it may be noted that the brake tends to modulate about an equilibrium level where the motor torque is largely balanced by the clamp load. In other words, there is a negative feedback against error accumulation as the actuator tends towards the equilibrium load. For example, this might explain the behaviour in Figure 3-36 (a) where the simulated response converges towards the measured force level following some initial overshoot.
Enlargements of the zero-velocity crossings in Figure 3-37 show the stick-slip friction and demonstrate the model’s ability to describe the behaviour. The result is significant because for some operations the static friction can drastically attenuate the response and may even cause mechanism lockup.

During general testing, some slight motor motions were observed without a corresponding change in clamp force. However, the slight reversal compliance was negligible in terms of modelling the macro-motion brake behaviour.

Since numerous closed-loop brake applies are presented in later chapters, one representative result was shown in Figure 3-38. It might be anticipated that the controlled model response would show greater agreement with measurements than during open-loop tests. Interestingly, the lead of the open-loop simulation translated to a faster apply rate during the controlled brake apply. Another consistent discrepancy was the small force relaxation after the peak measurement. There are more sources of error when the mechanism model is extended with feedback control and a single-phase motor model. For example, sensor models were not included and ideal feedback was assumed in simulation. Despite the simplifying assumptions, Figure 3-38 shows reasonable agreement between the simulated and measured brake apply. There was an approximate error of 5% for both the 15.9 kN peak force and the 0.143 s peak time. After peaking, the force relaxation settled towards 0.35 s and the steady-state simulation error was reduced to approximately 1% of the measured 15.5 kN.
As further confirmation, the simulation predicted the controlled EMB response when compared with test-vehicle ABS brake data in Figure 3-39. Again, the simulation initially predicted a slightly increased apply rate, but overall the root-mean-square (RMS) error of the prediction was 0.36, 0.38, 0.36 and 0.38 kN for the front-left, front-right, rear-left and rear-right brakes. This accuracy was achieved despite a 0.1 kN discretisation in the vehicle brake force data.

Finally, the EMB model was developed with clamp force as the output so that it may be incorporated with standard tyre and vehicle models. In a common approach, the brake torque, $T_B$, may be determined from the clamp force, $F_{cl}$, brake pad friction coefficient, $\mu$, and the effective brake rotor radius, $R$, using $T_B = \mu F_{cl} R$. It might be noted that the brake pad force distribution and centre-of-pressure will vary during the apply. The variation in the force centre-of-pressure for EMB 19 is shown in Figure 3-42 and may approximated by, $\Delta Y_{COP}(\text{mm}) = 83 \times 10^{-5} F_{cl}^3 - 0.063 F_{cl}^2 + 1.8 F_{cl}$ with $F_{cl}$ in kN.

![Figure 3-42: Force measurements from 3 clamp-plate sensors (left) and the corresponding centre-of-pressure (middle). Equilateral triangle configuration of the sensors (right)](image-url)
3.7 Conclusions

The test results generally indicate that a simplified EMB model is suitable for simulation and following a practical parameter identification, it was possible to predict the brake apply profile, modulations and stick-slip behaviour with satisfactory accuracy. The model fidelity was demonstrated in open-loop simulations and with active brake control. During closed-loop simulations, the prediction accuracy was demonstrated against bench-top measurements and vehicle brake data, with for example, the force response during ABS braking being predicted with an RMS error of 0.38 kN. Based on the test results, the EMB model appears to be suitable for the subsequent work on controller design. It may also be incorporated in a vehicle model and used to simulate brake manoeuvres such as ABS emergency braking.
4 Cascaded proportional-integral control

4.1 Introduction

A control algorithm is required to operate the EMB and track commands from either the driver or a vehicle control system. Ideally, the controller should realise the EMB’s potential for responsive braking. A suitable EMB model was described in Chapter 3 and subsequent chapters present an investigation of the EMB control design.

EMBs have previously been managed using cascaded proportional-integral (PI) control. As described in Chapter 1, a typical state-of-the-art EMB force control has embedded control-loops to regulate clamp force, motor velocity and current. However, it was not clear how the PI gains should be tuned or how the controller would perform throughout the work envelope. Some challenges might be anticipated given the large range of loading, actuator saturation, load-dependent friction and nonlinear stiffness. Given the importance of brake performance, there is a need for further investigation and a structured EMB control design.

This chapter presents an investigation of EMB cascaded PI control in three parts. Additional background on the EMB control problem and cascaded PI control is provided to establish a starting point for analysis. The problem of tuning the PI control gains is then explored by an optimisation approach, and the resulting controller was implemented on an EMB to allow testing. Experimental and simulation results are presented leading to discussion and conclusions regarding the performance, problems and opportunities for the EMB control.
4.2 The brake force control problem

The EMB control is required to execute brake commands from a higher level control, such as the driver. While the brake response is limited by the actuator constraints, important characteristics are a quick brake apply and a capacity for brake force modulation. Control of brake torque \( T_b=\mu RF_c \) might be preferred for managing vehicle dynamics, but obtaining feedback is difficult. For this reason, practical applications typically defer to clamp force control as the next best, or least removed, alternative. Hence, a control algorithm is required to regulate the brake clamp force, \( F_{cl} \), to the set-point command, \( F^*_c \), by adjusting the motor current, \( i_q \), (or motor torque, \( T_m=K_i i_q \)) as a control variable. The control structure is shown in Figure 4-1.

The EMB plant may be described as a nonlinear, single-input single-output (SISO) system that is slowly time-varying. In particular, the friction and stiffness are subject to variation with wear or temperature.

Actuator nonlinearity can present a significant control challenge. To demonstrate how the EMB response was affected, three related excitations are shown in Figure 4-2. These measurements were recorded during open-loop operation with input current oscillations about 2 A. Significant variation may be observed when the input amplitude was reduced from 195% to 155%. If the system were linear, the response characteristic would be independent of the input amplitude. Instead, the nonlinearity is evident as the output sinusoids are distorted, ‘capped’ and attenuated, particularly by static friction.
When motor current (or torque) is applied, the brake tends to be driven towards an equilibrium load. Hence, maintaining closed-loop stability is not challenging since the brake apply is open-loop stable. A more difficult issue may be the closed-loop brake performance.

The EMB control algorithm is executed in discrete time on an embedded controller. Measurements are available for feedback control of the brake clamp force, motor position, velocity and current. During standby the brake pad clearance is maintained with a transition to position control. For other operation, a clamp force controller is required to manage the brake application.
4.3 Cascaded proportional-integral control

Cascaded PI control presents a candidate solution for managing the EMB clamp force. It has been used in prior work, has a simple structure, establishes a baseline performance standard and has been successful in classic motion control problems. Cascaded PI control is a sensible first approach and provides a starting point for the control investigation.

Cascaded control structure

Cascaded PI control for the brake clamp force may be structured with embedded feedback loops to regulate force, motor velocity and current. Each control-loop is implemented with proportional-integral action on the tracking error. The control architecture is shown in Figure 4-3.

\[
\begin{align*}
F_{cl}^* & \quad \text{Force Controller} \\
\dot{\theta}^* & \quad \text{Velocity Controller} \\
i_q^* & \quad \text{Current Controller} \\
v_q & \quad \text{EMB}
\end{align*}
\]

\[
\begin{align*}
F_{cl} (N) & \quad \text{Force (N)} \\
\dot{\theta} (\text{rad/s}) & \quad \text{Velocity (rad/s)} \\
i_q (A) & \quad \text{Current (A)}
\end{align*}
\]

\[F_{cl}\] denotes the brake clamp force, \[\dot{\theta}\] is the motor velocity, \[i_q\] is the quadrature current and \[v_q\] is the effective quadrature voltage. The set-point commands are indicated by the superscript ‘*’. The voltage is pulse-width modulated (PWM) by power transistors in the dc-link converter as described in Chapter 2. On the prototype EMB, the controller was operated to regulate the force at 4 ms, velocity at 0.8 ms and current at 200 \(\mu\)s.
The cascaded architecture allows multi-rate operation such that the electronic and mechanical dynamics may be controlled at appropriate frequencies. Further, the structure allows state feedback control for explicit regulation of the velocity and force.

Omitting the velocity loop to obtain a cascaded force-current controller would be possible. However, the three-loop, force-velocity-current control is preferred for improved performance. In particular, the inclusion of a velocity control-loop allows active deceleration approaching the force set-point. In contrast, the force-current control drives towards the force set-point and relies on passive deceleration, until overshoot causes a change in the sign of the tracking error, $e_{Fcl}$.

**The cascaded control problem**

With the cascaded control structure in Figure 4-3, the control problem may be described as a series of sub-problems. The force controller should maintain the clamp force, $F_{cl}$, at the reference set-point, $F_{cl}^*$, by adjusting the motor velocity command, $\dot{\theta}^*$. In turn, the velocity controller should track the set-point, $\dot{\theta}^*$, by commanding an appropriate motor current, $i_q^*$. Finally, the current controller should track the reference by adjusting the PWM voltage at the power stage.

**Proportional-integral control**

Within the cascaded structure, the action of each proportional-integral controller is based on the tracking error, $e$, and the PI control gains,

$$u(t) = Pe(t) + \int e(t)dt$$  \hspace{1cm} (4-1)$$

such that the force, velocity and current control is given by,

$$\dot{\theta}^*(t) = P_f e_{Fcl}(t) + I_f \int e_{Fcl}(t)dt$$

where, $e_{Fcl} = (F_{cl}^* - F_{cl})$

$$i_q^*(t) = P_v e_{\dot{\theta}}(t) + I_v \int e_{\dot{\theta}}(t)dt$$

$$v_q(t) = P_v e_{iq}(t) + I_v \int e_{iq}(t)dt$$

$$e_{\dot{\theta}} = (\dot{\theta}^* - \dot{\theta})$$

$$e_{iq} = (i_q^* - i_q)$$  \hspace{1cm} (4-2)$$
**Integral anti-windup**

A variation to the cascaded PI control was the addition of integral anti-windup. This limited the accumulation, $\int e(t)dt$, during saturation to avoid windup and possible overshoot. Two common anti-windup schemes are limited and conditional integration (Bohn and Atherton, 1995; Visioli, 2003; Åström and Hägglund, 2006). These particularly avoid the introduction of additional tuning variables. In the case of limited integration, bounds are imposed on the magnitude of the integral term. The approach is computationally inexpensive and suitable for the high frequency motor current control. For the slower velocity and force controllers, a more sophisticated conditional integration was employed for integral anti-windup. With this approach, the integration was suspended during saturation (when $u=u_{\text{min}}$ or $u=u_{\text{max}}$) if the tracking error and control output had the same sign ($\text{sign}(e)=\text{sign}(u)$).

**Clearance management**

A further variation was the addition of clearance control. Clearance was regulated during standby, maintenance, or pad replacement with a transition to position control across the contact point between the brake pad and rotor. The transition may be performed with a switch as shown in Figure 4-4 (a) or alternatively, a single controller may operate on merged force-position feedback where clearance is represented by negative force values as shown in Figure 4-4 (b). The latter was preferred as it avoids the complexity of a switch and supports simple initialisation and diagnostic routines.

![Diagram](image)

*Figure 4-4: Air-gap management by outer-loop switch (a) or merged force-position feedback (b)*
**Inner-loop current control**

**Motor control architecture**

A diagram of the motor control architecture is shown in Figure 4-5. The permanent magnet synchronous motor and drive were described in Chapters 2 and 3.

The current controllers $K_{iq}$ and $K_{id}$ had PI action with limited integration anti-windup to regulate the quadrature and direct currents. The controllers operated in the d-q rotating reference frame and transformations were performed between the two-phase $dq$ rotor coordinates and three-phase $abc$ stator coordinates. The Park and Clark transformations were described in Chapter 3.

Motor torque is developed by the quadrature current, $i_q$, as it induces a magnetic field perpendicular to the rotor field. The direct current, $i_d$, induces a parallel field without torque and was commanded to zero ($i_d^* = 0$).

**Field-weakening**

Future work might consider field-weakening at high motor velocity. Maximum velocity may be reached when the supply voltage is saturated by induced back-EMF such that the motor current, torque and acceleration approach zero. As a countermeasure, the stator magnetic field can be rotated away from the perpendicular q-axis by adjusting the direct current. Field-weakening reduces the back-EMF and torque constants to extend the motor torque-speed characteristic.
Compensation

As the motor accelerates during a brake apply the back-EMF \((\dot{\theta}, \lambda_m)\) can act like a 'ramp' voltage disturbance and this may be alleviated using a feed-forward compensation. In addition, the quadrature and direct circuits are coupled by the induced voltages \(-\dot{\theta}_r Li_q\) and \(\dot{\theta}_r Li_d\). Each of these terms appear in the d-q voltage equations in Chapter 3,

\[
v_d = R_i_d + L \frac{di_d}{dt} - \dot{\theta}_r Li_q
\]

\[
v_q = R_i_q + L \frac{di_q}{dt} + \dot{\theta}_r \lambda_m + \dot{\theta}_r Li_d
\]

(4-3)

A feed-forward decoupling is recommended and a block for this is included in Figure 4-5.

The PWM voltage driver may include a compensation to account for the dead-time during power transistor switching. To allow for slight differences between the on and off switch times, a programmed dead-time avoids short circuit across transistors at the high and low side of the DC supply. However, the dead-time reduces the effective voltage and its contribution depends on the rate of PWM switching. Dead-time compensation pre-adjusts the PWM rate to obtain the desired voltage.

Controller frequency

The discrete-time control frequency should be set relative to the system dynamics. The electronic transients are determined by the winding resistance, inductance and voltage disturbances. The 200 \(\mu\)s current control period was an order of magnitude faster than the current step response. It would be preferable if the controller could be run a couple of orders faster than the system dynamics.

Implementation

The control algorithm was executed on a 16 bit embedded controller. A code library was available to support low-level motor commutation, diagnostics and control.
4.4 Tuning the cascaded PI control

Cascaded controllers are normally tuned progressively outwards from the innermost control-loop. Tuning the EMB cascaded PI control may be approached in two parts. Firstly, the inner-loop current control may be tuned for an appropriate electronic response. Secondly, the outer-loop velocity and force controllers may be tuned for an appropriate mechanical response. Given a suitable inner-loop current control, the fast electronic transients are not significant to the slower mechanical dynamics. It is the velocity and force controllers, in addition to hardware constraints, that largely determine the mechanical response and brake performance.

Tuning the motor current controller

The PI current controllers \(K_{iq}\) and \(K_{id}\) in Figure 4-5) were tuned manually by the electronics group of the industry partner, Pacifica Group Technologies. The manual tuning provided a satisfactory current control response, regulated the fast electronics and supported control of the slower mechanical dynamics. For consistency, the same motor current controller was utilised throughout the investigation.
Tuning the velocity and brake force controllers

The tuning of the PI velocity and force controllers is significant in determining the brake’s mechanical response. Reviews of methods to tune PID controllers are given by (Åström, Hägglund, Hang and Ho, 1993), (Gorez and Calcev, 1997), (Tan, Wang, Hang and Tore, 1999), (Cominos and Munro, 2002), and (Moradi, 2003). Various approaches include manual, Ziegler-Nichols, Cohen-Coon, pole placement, gain and phase margin, and optimal tuning.

A plant model is generally helpful for control design and analysis. For example, a classical control design might be attempted using a linear plant approximation. However, there is no appropriate global linearisation to describe the EMB and a better approach might exploit the information contained in the nonlinear EMB model.

Optimal PI tuning

An improved control design is generally possible when more detailed information is available on the plant behaviour. Hence, an attractive approach for tuning the control gains is a nonlinear optimisation using the EMB model described in Chapter 3. The optimisation may be constructed to seek the PI gains that minimise a penalty on the tracking error and controller effort.

Since the tracking error and power demand are functions of the force and velocity control gains \( (P_f, I_f, P_v, \text{ and } I_v) \), an optimal tuning may be found for a given manoeuvre by minimising an appropriate quadratic cost function,

\[
\min_{P_f, I_f, P_v, I_v} \sum_{k=1}^{n} e_k Q e_k + u_k R u_k
\]  

(4-4)

where, \( e_k = F_{cl}^* - F_{cl} (P_f, I_f, P_v, I_v) \) is the clamp force tracking error, \( u_k^* = J_I (P_f, I_f, P_v, I_v) K_f \omega_m^* (P_f, I_f, P_v, I_v) \) is the power demand and \( n \) is the number of periods to the end of the manoeuvre.
The weights $Q$ and $R$ were selected such that the cost of the tracking error was dominant. A small penalty on the power demand was mainly included to avoid a flat cost function during actuator saturation.

The optimal tuning was performed using the nonlinear EMB model. A numerical solution was obtained using the MATLAB routine `lsqnonlin()` that was a trust-region approach for nonlinear minimisation subject to bounds. At each iteration, the nonlinear cost function was approximated over a neighbourhood trust-region and used to find a sub-optimal solution with a lower cost. The procedure was repeated until termination conditions, such as the rate of convergence, were satisfied.

The search algorithm has the potential to identify local minima. For example, with an inappropriate initial guess near zero, the optimisation can identify a solution where the control gains are zeroed to minimise the term $\mu_p^* R \mu_p^*$. The optimised response profiles were reviewed to avoid erroneous solutions.

**PI tuning results**

The force and velocity control gains ($P_f$, $I_f$, $P_v$, and $I_v$) were optimally tuned over a series of step manoeuvres. Figure 4-6 shows the optimal gains for brake application from a nominal 100 N. Figure 4-7 and Figure 4-8 present the optimal gains for additional step manoeuvres throughout the work envelope. The steps are defined by an array of ‘start’ and ‘end’ force levels. In each case the initial position was determined from the start force via the stiffness curve. Meanwhile, the initial velocity was set finitely small at $0^\circ$ and with the right heading to avoid the zero velocity state.
An appropriate proportional force gain, $P_f$, was most significant in reducing the optimisation cost, followed by the velocity gain, $P_v$, and the two integral gains $I_f$ and $I_v$. The spread of the numerically optimised gains was inversely proportional to their significance on the cost. For example, Figure 4-6 shows a large variation in the integral velocity gain, $I_v$, as the cost function was relatively flat with respect to this less significant parameter. Conversely, there was a stronger trend for the proportional force gain, $P_f$. This gain was significant because it affected the velocity command and thus the brake force rate. In other words, it scheduled the brake force response that was the slowest and dominant actuator dynamic. While the proportional gains and especially $P_f$ are dominant, the integral gains are also presented for completeness.

![Figure 4-6: Optimal gains $P_f$, $I_f$, $P_v$, and $I_v$ for step applies to the ‘end force’ from a nominal 0.1 kN initial load.](image)
Figure 4-7: Optimal proportional force and velocity gains, $P_f$ (a) and $P_v$ (b), for step manoeuvres defined by the ‘start’ and ‘end’ forces.

Figure 4-8: Optimal integral force and velocity gains, $I_f$ (a) and $I_v$ (b), for step manoeuvres defined by the ‘start’ and ‘end’ forces.

Figure 4-6 and Figure 4-7 indicate the optimal proportional gains are elevated for small magnitude manoeuvres. This is also shown by the high gain ridge in the contour plot. The proportional gains are elevated at higher clamp forces where the system is driven under load. Some similar features may be observed in the integral gain plots of Figure 4-8.
The trends in the optimally tuned gains arise due to actuator nonlinearity. The solution noise is due to the numerical optimisation, particularly for the less influential integral gains seen in Figure 4-8. The high gain ridge for fine manoeuvres may be partly attributed to friction having greater significance during fine operation. Fine manoeuvres tend to have smaller tracking errors and so elevated gains may drive a faster response. When actuator saturation occurs during large manoeuvres the reduced gains may decrease the cost on power demand. The proportional force gain also acts to schedule actuator deceleration based on the tracking error. The motor torque is limited and greater momentum is developed during large manoeuvres. Hence, to avoid excessive overshoot, the deceleration period may be extended for larger manoeuvres with a reduced proportional force gain.

Due to the variation in the optimal PI gains, it is clear that no one set of fixed gains can be close to optimal for all possible steps. Furthermore, the optimal gains depend on the details of the brake manoeuvre, precluding a simple gain schedule based on the clamp force.

Having to select a sub-optimal set of fixed gains, a conservative choice is prudent. Based on the results in Figure 4-6, the gains were set \( (P_f=0.034, I_f=0.15, P_v=0.51, I_v=4.2) \) and were most appropriate for large manoeuvres.
4.5 Results and discussion

The EMB cascaded PI control was tested to investigate its performance, limitations and the potential for improvement. As no fixed set of control gains can provide global optimality, the performance degradation associated with suboptimal tuning is of interest. The following results were obtained with the same fixed tuning ($P_f=0.034$, $I_f=0.15$, $P_v=0.51$, $I_v=4.2$), unless otherwise stated. The cascaded PI control was implemented on EMB 19 and a set of experimental tests complement a more comprehensive suite of simulation results. The results establish a baseline performance standard and facilitate critical assessment.

Maintaining stable closed-loop control on an EMB should not be difficult given the brake apply is open-loop stable. The optimal gain tuning was conducted with long time horizons (0.35 s) to help ensure a convergent response. The simulated phase-plane portraits in Figure 4-9 demonstrate convergence throughout the work envelope. The three points of attraction correspond to brake force commands of 10, 20 and 30 kN from different initial conditions.

![Figure 4-9: Phase-plane portraits for 10, 20 and 30kN commands from different initial conditions](image)

Since no fixed set of gains can maintain global optimality, the consequence of suboptimal tuning was investigated in simulation. In Figure 4-10, a set of gains near optimal for the full apply ($P_f=0.034$, $I_f=0.15$, $P_v=0.51$, $I_v=4.2$) is overly conservative for a light apply. Conversely, Figure 4-11 shows that gains appropriate for a light apply ($P_f=0.17$, $I_f=0.15$, $P_v=0.51$, $I_v=4.2$) resulted in overshoot and increased settling time during a larger apply. The performance degradation due to suboptimal tuning is significant. A single set of fixed gains cannot cover the EMB work envelope without a substantial degradation in performance relative to the optimal set.
It is noted that only the proportional force gain, $P_f$, was adjusted between the examples in Figure 4-10 and Figure 4-11. This dominant control parameter has a significant influence on the EMB performance.

In addition to the EMB step response, brake force modulations are important from the viewpoint of anti-lock brake pressure cycling. Figure 4-12 presents large 80% and small 3% modulations simulated about 15 kN at 1, 4 and 8 Hz. In each figure, an input-output diagram omits the time dimension to compare the normalised results. It shows the normalised output plotted against the command over a series of frequencies. The frequency is indicated by a colour bar and the ideal 1:1 response is shown by a diagonal reference line. Further detail on the interpretation of input-output diagrams is provided in Appendix 4 A.

Preliminary plots displayed in Figure 4-12 show that the cascaded PI control is reasonably capable of tracking large and slow manoeuvres, such as the 80% amplitude oscillation at 1 Hz. As might be expected, there is increased attenuation and lag at higher frequency. The fine oscillations are also seen to be particularly affected by static friction. Further results will help to characterise the control performance.
Figure 4-12: Large 80% (top) and small 3% modulations (bottom) about 15 kN at 1, 4 and 8Hz

For a more complete perspective, simulated input-output results in Figure 4-13 show a progression from small 2% to large 80% oscillations about 15 kN at 1-10Hz. The results indicate that performance is degraded in two regions, during particularly fine or large modulations, at amplitudes of <5% or >30%. The fine control suffers a poor handling of static friction and flat-top lockup events may be observed. Otherwise, the large amplitude response is ultimately constrained by actuator saturation.

Figure 4-13: Normalised input-output diagrams for small 2-9% modulations (top) and large 10-80% modulations (bottom) about 15 kN at 1-10Hz
Another depiction in Figure 4-14 indicates the oscillation magnitude. The x-axis shows the commanded amplitude from 80-20% (left) and 10-2% (right) as a percentage of the 15 kN load bias. The y-axis shows the commanded frequency from 0-10Hz. The z-axis and colour indicate the range of the sinusoidal output as a percentage of the commanded amplitude such that 100% would be ideal.

Again the EMB response is seen to be attenuated in two regions, at very small and large oscillation amplitudes of <5% or >30%. A poor handling of friction during fine manoeuvres is aggravated by the fixed control gains being more suited to large manoeuvres. Meanwhile, the large amplitude response is ultimately constrained by the actuator limits.

**Figure 4-14: Output range as a percentage of the commanded sinusoidal oscillation**
**Experimental results**

To facilitate experimental testing, the cascaded PI control was implemented on the embedded controller of EMB 19. All controller performance tests in this and subsequent chapters were performed on the same actuator. The brake force controller was programmed in MATLAB/Simulink. The faster velocity control was coded in a C sub-layer. Real Time Workshop was used to generate C code from Simulink and this was combined with the lower level code. After the compiled code was loaded to the embedded controller, it was ready for testing.

Figure 4-15 shows measurements and simulations for a 5, 15 and 25 kN brake apply. The manoeuvres were started from 100 N. Generally, there is good agreement between the simulated and experimental results. The simulation slightly leads the measured response and is consistent with the model behaviour reported in Chapter 3.

![Figure 4-15: Measured and simulated brake applies to 5, 15 and 25 kN with cascaded PI control](image-url)
The performance of large manoeuvres, such as the 15 and 25 kN brake applies in Figure 4-15, is largely constrained by actuator saturation. In this case, the current is saturated at 40 A until approximately 0.03 s after which the velocity is limited to 300 rad/s. Cumulatively, the actuator is saturated for approximately 2/3 the duration of these large manoeuvres. Hence, there is little potential for improvement by adjusting the controller. A slightly faster brake response might be controlled by decelerating later and harder.

Small manoeuvres, such as the 5 kN brake apply in Figure 4-15, are performed conservatively by the cascaded PI control. The measured peak time for the 5 kN apply (0.126 s) is similar to that for the 15 kN apply (0.144 s). The motor does not reach the velocity limit and there is a protracted deceleration. This occurs because the suboptimal fixed gains are most suited to large manoeuvres and are conservative for fine operation.

The series of large oscillations seen in Figure 4-16 show 75% modulations about 15 kN at 1, 4 and 8Hz. Generally there is good agreement between the measured and simulated responses. In terms of the control performance, there is reasonable tracking during the slow 1Hz oscillation. Some phase lag (~38°) was observed at 4Hz. At higher frequency the response was ultimately limited by actuator saturation. The modulation was significantly attenuated at 8Hz due to the persistent 40 A current limiting.

![Figure 4-16: Measured and simulated 75% modulations about 15 kN at 1, 4 and 8Hz with cascaded PI control](image)
The series of small oscillations seen in Figure 4-17 show 5% modulations about 15 kN at 1, 4 and 8Hz. During these fine manoeuvres, the measured response is degraded by static friction and lockup. The problem of friction is slightly worse than anticipated by the simulation and the disturbance is pronounced at low frequency due to extended periods of lockup. For example, the velocity measurement at 1Hz shows significant perturbation and catching. The corresponding measured current is elevated as the control struggles to overcome the friction disturbance. The deficiency of the fine control is compounded by suboptimal fixed gains that are more suited to large manoeuvres. Lag and attenuation arise as the conservative controller only utilises a fraction of the available motor current (40 A).

Figure 4-17: Measured and simulated 5% modulations about 15 kN at 1, 4 and 8Hz with cascaded PI control

The fine EMB control is particularly degraded at large clamp forces due to the load dependent friction. The cascaded PI control is also liable to suffer a perpetual hunting. The example shown in Figure 4-18 shows measurements of a 1 kN modulation about ~25 kN at 4Hz. Prior to the oscillation there is a constant force command.
Hunting results from an interaction between the integral control action and static friction. Figure 4-18 shows an initial slight steady-state force error and slow integral wind-down of the motor current until 0.15 s. At this time the friction is overcome, breakaway occurs and the system ‘twitches’. A new steady-state error is established and the process continues with integral wind-up of the motor current between 0.2 s and 0.3 s. The extrapolation of this behaviour produces a hunting cycle.

The cascaded PI control fails to execute the fine oscillation observed in Figure 4-18 due to a poor handling of the static friction. There is some response after 0.55 s due to the initial offset bias and integral accumulation. After the tracking is centred, the response is largely attenuated by frictional lockup.

![Figure 4-18: 0.5kN amplitude oscillation about ~25 kN at 4Hz with cascaded PI control](image)

The degraded fine brake response in Figure 4-18 is due to controller deficiency and not a consequence of actuator saturation. Sufficient motor torque is available to overcome the static friction and actuate a response.
To help quantify the friction at various clamp loads, Figure 4-19 shows the simulated friction during a 30 kN apply. The plot includes aggregate contributions of the viscous and load dependent Coulomb friction. The peak torque values are summarised in Table 4-1.

After gearing, 30 kN of brake force corresponded to a 0.79 Nm load on the motor. The friction contributed a significant additional load, peaking around 0.41 Nm at high clamp force. However, with appropriate control, the 3 Nm of available motor torque is sufficient to overcome static friction and actuate a response.

![Figure 4-19: Estimated friction contributions during a 30 kN brake apply](image)

<table>
<thead>
<tr>
<th>Peak value</th>
<th>Symbol</th>
<th>(Nm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Motor torque</td>
<td>$T_m$</td>
<td>3.0</td>
</tr>
<tr>
<td>Load torque</td>
<td>$T_L$</td>
<td>0.79</td>
</tr>
<tr>
<td>Total friction</td>
<td>$T_F$</td>
<td>0.41</td>
</tr>
<tr>
<td>Coulomb friction</td>
<td>$(C + GF_{cl})\text{sign} \dot{\theta}$</td>
<td>0.38</td>
</tr>
<tr>
<td>Viscous friction</td>
<td>$D\dot{\theta}$</td>
<td>0.12</td>
</tr>
</tbody>
</table>

Table 4-1: Estimated peak torque values during a 30 kN brake apply
4.6 Conclusions

Cascaded PI control presents a simple first-approach for EMB clamp force control, but its performance is limited and there is potential for improved fine control.

Large-magnitude brake response

The performance of large EMB manoeuvres is largely constrained by actuator saturation, not the cascaded PI control. For example, a 15 kN brake apply was restricted by motor current and velocity saturation for 2/3 of the apply duration. Hence there is little room for faster or improved operation by adjusting the control. Future work should seek to improve the large-magnitude brake response by improving the hardware design.

Small-magnitude brake response

The small-magnitude EMB response (<5 kN) was limited by the cascaded PI control performance. Fine manoeuvres (<1 kN) were particularly degraded as a consequence of the control.

The performance of small-amplitude EMB operations may be reduced by a suboptimal tuning of the cascaded PI control. Due to actuator nonlinearity, no single set of fixed control gains can maintain optimality throughout the work envelope. The controller may be tuned for a particular brake operation, but it is then suboptimal for other manoeuvres. When the gains are set conservatively, the tuning is more suitable for large manoeuvres while the fine control is degraded.

Degradation in the fine brake control is compounded by significant mechanism friction. The load dependent friction can produce a considerable disturbance at high brake loads. The problem manifests as a tendency to suffer frictional lockup. As a consequence, the fine EMB control may be greatly attenuated.
Limitations in the small-amplitude EMB control present the greatest opportunity for improvement beyond the first-approach cascaded PI control. Seeking an improved management of friction and actuator nonlinearity, subsequent chapters consider techniques of friction compensation (Chapter 5), feedback linearisation and gain scheduling (Chapter 6). The investigations described in these chapters is aimed at improving the EMB response.
5 Friction compensation

5.1 Introduction

In Chapter 4 it was reported that friction in the EMB mechanism can significantly degrade the fine brake control. The friction disturbance may be pronounced at high clamp forces due to its load dependency and in the worst case, the brake control may fail to execute some small adjustments. The static friction can also couple with integral control action to produce an undesirable hunting. To address these deficiencies, an improved handling of the EMB friction is sought for a more responsive brake control.

EMB designs usually include efficient transmissions to help reduce the severity of mechanism friction. Components with rolling contacts, such as ball screws, are commonly preferred to those with sliding contacts, such as power screws (Appendix 1B). However, a significant degree of friction is inherently due to the large compressive brake loads, necessitating a suitable control management.

A model-based friction compensation is proposed in this chapter to better manage the EMB friction disturbance. The compensation accounts for the load dependency of the mechanism friction. It is also compatible with an adaptive implementation by online estimation of the friction parameters. Extension to an adaptive routine is considered later in Chapter 9.

After examining prior work on controlling machines with friction, a new EMB friction compensation is proposed based on the friction model described in Chapter 3. The results of testing on the prototype EMB are presented, followed by discussion and conclusions regarding the performance of the EMB friction compensation.
Preliminary considerations

The challenge of controlling machines with friction is encountered in fields such as robotics, precision servomechanisms, machining, aerospace and telescopy. A brief review provides background for the formulation of a new EMB friction compensation. The brake control problem is differentiated by the large range of loading, fast operation and restrictions on the actuator size and cost. Hence, an oversized drive is not a feasible solution.

Seminal papers on friction modelling and control management are (Armstrong-Hélouvry, Dupont and Canudas De Wit, 1994) and (Olsson et al., 1998). It is documented that friction may cause hunting, catching at zero velocity, stick-slip motion and a disturbance to velocity control. Of these issues, a considerable static friction disturbance during clamping is perhaps the greatest challenge to fine EMB control. The actuator control can also suffer hunting.

Integral dead-band

A simple technique to avoid hunting is the inclusion of a small dead-band on the input to the integral control (Armstrong-Hélouvry et al., 1994). By introducing a precision threshold some steady-state error is sacrificed to avoid hunting. An alternative implementation is possible by appropriate integral resetting.

Dither, knocking and impulse control

Early friction management devices such as a mechanical governor for the control of steam turbines (Bennett, 1979) used a high-frequency dither perturbation to overcome static friction.

Because the turbines produced substantially less vibration than older reciprocating machines sticking was observed in the governors and a mechanism was introduced solely to produce vibration. (Armstrong-Hélouvry, 1991)
Mechanical vibrators and dither were used in autopilot gyroscopes in the 1940s (Olsson et al., 1998). Such vibrators, sometimes called dipplers, are still used for gun mounts and other large pointing systems (Armstrong-Hélouvry et al., 1994).

With an external vibrator there may be more freedom to target the dither perturbation. Alternatively, high-frequency dither may be superimposed on the control input. A recent example is known as the ‘knocker’ and has a pulse sequence added to the control input (Hägglund, 1997). A related concept is impulse control whereby the control action is translated to an equivalent sequence of high magnitude impulses. However, in some applications the methods of high frequency excitation may be attenuated by contact compliance (Armstrong-Hélouvry et al., 1994). Moreover, a superimposed perturbation is undesirable for the brake actuator.

**Joint torque control**

In the field of robotics, a high-bandwidth feedback control on transmission output has been included for improved rejection of frictional disturbances (Armstrong-Hélouvry et al., 1994). However, feedback measurements of the output transmission torque would require an additional sensor and hence the approach is not suitable for the EMB.

**Learning control**

Learning control has also been used in robotics to develop a table of feed-forward corrections for specific trajectories. However, when a friction model is available it might be utilised for an improved compensation. Variation in the friction may then be managed by adaptation rather than learning.

**Model-based friction compensation**

An EMB friction compensation may be based on the friction model identified in Chapter 3. In this chapter an estimate of the nonlinear friction is used to apply a compensating motor torque. Extension to an adaptive implementation is considered in Chapter 9.
5.2 EMB friction compensation

Friction in the EMB mechanism may be considered as a nonlinear load disturbance and counteracted with an appropriate compensation. A suitable compensation can be estimated from the EMB friction model and then superimposed on the control action.

The friction compensation scheme is shown in Figure 5-1. The friction estimate, $\hat{T}_{sj/c}$, is used to determine an appropriate compensation for the Coulomb and static friction. An appropriate current compensation, $i_{q1}^* = \frac{\hat{T}_{sj/c}}{K_i}$, is then superimposed on the control action to counter the nonlinear component of the mechanism friction, $T_F = D\dot{\theta} + T_{cl}$.

![Figure 5-1: Friction in the plant (red) and compensation in the controller (blue)](image)

In Figure 5-1, an estimate of the static and Coulomb friction, $\hat{T}_{sj/c}$, is based on measurements of the brake clamp force, $F_{cl}$, motor velocity, $\dot{\theta}$, and the orientation of the command, $\dot{\theta}^*$. The friction compensation is superimposed on the current command, $i_{q1}^*$, from the controller $K_c$.

A ‘mechanism’ block shown in Figure 5-1 allows further detail, and the friction is depicted as a function of the velocity, clamp force and motor torque, $T_F = T_F(\dot{\theta}, F_{cl}, T_M)$. This is consistent with the EMB model description in Chapter 3. During motion the friction torque, $T_F$, is a function of the brake clamp load, $F_{cl}$, and the mechanism velocity, $\dot{\theta}$. At standstill the friction is a reaction to the external torque from the motor and the load, $T_E = T_M - T_L = T_M - NF_{cl}$. 
The concept of friction compensation is depicted in Figure 5-2 using friction-velocity maps with a term for the load dependency, $GF_{cl}$. To negate the nonlinear friction torque (left) a compensating motor torque is applied (middle) so that the remaining friction is mainly linear (right).

\[ T_c = \frac{C + GF_{cl}}{G} \text{sign}(\hat{\theta}) \]

\[ \hat{T}_{s/c} = \begin{cases} (C + GF_{cl})\text{sign}(\hat{\theta}) & \forall|\hat{\theta}| > \varepsilon_1 \\ (T_s + GF_{cl})\text{sign}(\hat{\theta}^*) & \text{if}|\hat{\theta}| < \varepsilon_1 \text{ and } |\hat{\theta}^*| > \varepsilon_2 \\ 0 & \text{otherwise} \end{cases} \] (5-1)

where $C$ is the Coulomb friction, $G$ is the friction load dependency and $T_s$ is the static friction. The compensation described by equation (5-1) is based on the friction model identified in Chapter 3 so that the friction parameters are available from Table 3-7.

The small interval $\pm\varepsilon_1$ is used to implement the Karnopp remedy for zero velocity detection (Olsson et al., 1998). A small velocity dead-band, $\pm\varepsilon_2$, might also be considered to avoid switching if measurement noise is an issue. Setting $\varepsilon_2$ to zero was sufficient in this investigation.

The friction-compensating current is determined from the friction model as $i_{qc}^* = \hat{T}_{s/c} / K_t$. To counter friction, the compensation torque is applied in the direction of motion, or the desired direction. If the velocity command is not available then the desired heading at standstill may be determined from the force tracking error. For the estimate of $\hat{T}_{s/c}$ in (5-1), the term $\hat{\theta}^*$ may readily be replaced by $(F_{cl}^* - F_{cl})$. 

Figure 5-2: Mechanism friction (left), compensation (middle) and the remaining uncompensated viscous friction (right)
5.3 Results and discussion

A series of tests was conducted to assess the EMB friction compensation. Since disturbances may be perfectly rejected in simulation, experimental tests were undertaken to assess the compensation effectiveness in practice. However, the simulation was useful to investigate various degrees of over- and under-compensation.

Experimental performance

Friction compensation was programmed on the controller of EMB 19 to support the operation of the cascaded PI control from Chapter 4.

A series of brake modulations was run to measure the performance with and without friction compensation. Tests manoeuvres were run back-to-back with the compensation enabled and disabled. Fine modulations at elevated clamp load provided suitably challenging test profiles. Force oscillations were commanded about 25 kN at 4Hz and 8Hz. The command amplitudes were 2, 4 and 6% with the corresponding results shown in Figure 5-3, Figure 5-4 and Figure 5-5. Enlargements are provided in Appendix 5 A. Each figure compares the brake response with and without friction compensation. Subplots indicate the brake clamp force, motor velocity and quadrature current.

Figure 5-3: 2% modulations about 25 kN at 4Hz (left) and 8Hz (right), with and without friction compensation
The operation and benefit of friction compensation is perhaps most apparent in Figure 5-3 for the fine 2% modulation about 25 kN at 4Hz. For this manoeuvre there is almost complete frictional lockup without friction compensation. Only slight adjustments in the motor velocity and clamp force are evident as the control struggles to manage the static friction. Conversely, when the friction compensation is enabled the static friction is overcome and the tracking is significantly improved.
The function of the friction compensation is particularly evident in the velocity and current plots of Figure 5-3. At each zero-velocity crossing, a step change in the compensation current counteracts the static and Coulomb friction. This alleviates catching about zero velocity as the nonlinear friction disturbance is rejected. Improved zero-velocity crossings may also be observed in Figure 5-4 and Figure 5-5. The reduced tendency for catching helps to alleviate ‘flat-topped’ force tracking and lag.

The benefit of friction compensation appears to be more pronounced at higher frequency. For example, Figure 5-4 shows a greater relief in attenuation at 8Hz than at 4Hz. At the faster rate there is less opportunity for integral control action to overcome the static friction.

The benefit of friction compensation is also more pronounced for small amplitude oscillations. For example, there is a significant improvement during the fine 2% (1 kN) modulations, but less improvement at amplitudes of 4% and 6%. During catching at increased amplitudes, the tracking error and PI control action develop more rapidly to overcome the static friction.

While the friction compensation can significantly improve the fine brake modulations, it is not perfect. For example, there is still some catching in the 4Hz plot of Figure 5-3 around 0.75 s, 1.0 s, and 1.25 s. Interestingly, the catching is only during release. Conversely, there is slight velocity overdrive on the apply zero-crossing. This velocity behaviour might suggest a slight under-compensation on release and over-compensation on apply.

Generally, the friction compensation significantly improved the fine brake control. The nonlinear friction disturbance was reduced, but not eliminated. As the friction estimate is imperfect, there appeared to be slight under or over-compensation. The performance sensitivity to varying degrees of imperfect compensation may be investigated further in simulation.
**Over and under-compensation**

The consequence of over and under-compensation is readily investigated by simulation. Figure 5-6 shows fine 4Hz modulations simulated about 25 kN with 0, 50, 100 and 150% friction compensation.

![Figure 5-6: 4Hz modulation about 25 kN with friction compensation at 0, 50, 100 and 150%](image)

As might be expected, with friction under-compensation there is catching about zero velocity, frictional lockup and an attenuated response due to static friction. With over-compensation the actuator is ‘knocked’ at each reversal. In this case, the over-excitation adds lead and overshoot to the response. Meanwhile, the 100% ideal compensation shows the simulated response with perfect compensation.

A significant feature seen in Figure 5-6 is the large tolerance to imperfect compensation. Even with ±50% imperfect compensation, the brake response is improved relative to that without compensation.
**Integral dead-band**

It was noted in the preliminary considerations that hunting may result from an interaction between the integral control and static friction. This may also be affected by the introduction of imperfect friction compensation. The standard remedy of a dead-band on input to the integral control was tested with 0% and 150% friction compensation. Figure 5-7 presents the simulated response with and without a 25 N dead-band on the integral action. A degree of steady-state precision is sacrificed with the dead-band to eliminate the hunting behaviour. The technique remains effective over a range of friction (a) under and (b) over-compensation.

![Figure 5-7: Response about steady-state with and without integral dead-band and with friction compensation at (a) 0% and (b) 150%](image)

The double breakaway is an interesting feature in Figure 5-7 (b) without the dead-band. This behaviour is caused by the force and velocity integral controllers accumulating at different rates. However, the hunting behaviour is generally prevented with a small dead-band on the input to the integral controller.
5.4 Conclusions

Since friction in the EMB mechanism can significantly degrade the fine brake control, a model-based friction compensation was proposed. By estimating the nonlinear friction disturbance it was possible to apply a compensating motor torque. In experimental tests, the compensation was effective and significantly improved the operation of fine brake manoeuvres (<1 kN). The benefit of the friction compensation was more pronounced for small brake adjustments. In a dramatic example, friction compensation was the difference between a failure to respond and executing a 0.5 kN modulation about a 25 kN clamp load. During other manoeuvres, the friction compensation alleviated an undesirable catching around zero velocity.

The results in this chapter demonstrated the effectiveness of fixed friction compensation at one point in the life-cycle of an EMB. However, to maintain accurate compensation and account for friction variation, an extension to an adaptive implementation is considered in Chapter 9.
6 Modified control architecture

6.1 Introduction

As described in Chapter 4, a cascaded PI controller was unable to maintain performance throughout the operational envelope of an EMB due to actuator nonlinearity. The fine brake control was particularly degraded as a consequence of mechanism friction and a suboptimal set of fixed control gains. Chapter 5 describes how the friction disturbance was effectively counteracted with an EMB friction compensation. This chapter considers further modifications to address some of the remaining actuator nonlinearity with the aim of maintaining control performance for both large and fine brake operation.

A modified control architecture is proposed using techniques of feedback linearisation and gain scheduling to manage the EMB actuator nonlinearity. The new EMB control architecture is introduced after other approaches are first considered. Simulations and bench testing on the prototype EMB are presented to show its performance, followed by discussion and conclusions.

EMB control design progression:

Baseline cascaded PI control

arrow

Modified control architecture
- Feedback linearisation
- Gain scheduling (Management of nonlinearity)
**Preliminary considerations**

Some possible modifications to the baseline cascaded PI control described in Chapter 4 include PID control, feed-forward action, feedback-linearisation (including friction compensation) and gain scheduling.

**PID control**

The inclusion of derivative control action (proportional to the derivative of the tracking error, \( u_D = D \times \text{de/dt} \)) can add lead to the control response (Åström and Hägglund, 2006). PID control may offer a small improvement for a subset of EMB manoeuvres, but the benefit is not maintained through the operational envelope with a set of fixed gains. Further, the derivative action can amplify the effect of high frequency noise.

**Feed-forward reference generator**

The baseline cascaded control may be extended with a feed-forward reference generator. As seen in Figure 6-1, the reference commands drive and augment the feedback control action.

The detail of the reference generator may be different to that shown in Figure 6-1. In this example, the current reference is the computed load, \( i_{qref} = F_c(N)/K_t \), where \( N \) is the gear reduction and \( K_t \) is the motor torque constant. To obtain the velocity reference, \( \dot{\theta}_{ref} \), a second order filter is firstly applied to the force command to describe a suitable actuator response. A corresponding position, \( x_{ref} \), is then
determined by an inverse stiffness mapping. The velocity reference is finally obtained following differentiation.

The addition of feed-forward control action may significantly improve the range of EMB operation over which performance is maintained. However, it is challenging to develop a reference generator that is appropriate for all manoeuvres. For example, the described profile generator could maintain the large-amplitude performance and concurrently improve the small-amplitude brake response (<5 kN). However, the fine control (<1 kN) would then have a tendency to be overexcited and suffer overshoot.

It is possible that a more sophisticated reference generator may offer a further improvement. For example, the reference profiles might be generated using rate limits, a more detailed plant model, scheduling or some system inverse. Before pursuing a more complex implementation however, other approaches should first be considered. For example, it may be possible to address the actuator nonlinearity more directly by techniques of gain scheduling and feedback linearisation.

**Feedback linearisation**

Feedback linearisation aims to counter nonlinear disturbances with a suitable control action based on feedback measurements and a system model. One example is the EMB friction compensation described in Chapter 5. In this case, a friction estimate was determined from feedback measurements so that the nonlinear disturbance could be effectively compensated. Similarly, feedback linearisation might be considered for the nonlinear EMB stiffness load.

**Gain scheduling**

Since an appropriate control depends on the EMB operation, gain scheduling may present a solution to the problem of actuator nonlinearity. The classic method of gain scheduling involves linearising the plant about operating points, designing a set of linear controllers, and interpolating the control based on a scheduling variable. The control structure is shown in Figure 6-2.
The popularity of gain scheduling since the 1990s may be linked to the availability of digital controllers that avoid costly and complex analogue implementations (Rugh and Shamma, 2000). Gain scheduling may be considered as “a divide and conquer approach whereby the nonlinear design task is decomposed into a number of linear sub-tasks” (Leith and Leithead, 2000). As a guideline, the scheduling variable should capture plant nonlinearity and be slowly varying relative to the system dynamics (Shamma, 1988).

While the brake clamp force might be considered as a scheduling variable, it does not vary slowly relative to the mechanical dynamics. Hence, the classic approach of scheduling over a set of linear control designs may not be suitable.

In Chapter 4, it was found that the optimal PI control gains depend on the details of the brake manoeuvre. In another approach, the optimal gains could be stored and scheduled from a 3D lookup table based on the states \( \dot{\theta} \) and \( F_{cl} \) and the brake force command \( F_{\text{cmd}} \). However, the memory requirement might be demanding without suitable compression. Further, the preliminary optimisation is computationally demanding and may prohibit online recomputation to adjust for actuator variation.

A more elegant gain scheduling for the EMB control might be achieved by the application of inverse functions to counter the actuator nonlinearity. The concept is to compensate a nonlinear gain with an inverse gain function. The approach may be interpreted as a form of gain scheduling (Rugh and Shamma, 2000).
6.2 A modified control architecture

An improved management of the EMB actuator nonlinearity is proposed using techniques of gain scheduling and feedback linearisation. A modified control architecture may be constructed with three components to address the nonlinear stiffness, loading and friction.

**Inverse gain function to address nonlinear stiffness**

The first control modification is the addition of an inverse gain schedule to address the nonlinear stiffness. The inverse function may be applied within the controller such that the compound gain from the position, \( x \), to the control variable, \( \nu \), is linear. Figure 6-3 shows the inverse function applied to both the force measurement, \( F_{cl} \), and the command, \( F_{cl}^* \). The formation of a linear compound gain is shown in Figure 6-4. Subsequently, the controller, \( K_c \) in Figure 6-3, is partly isolated from the stiffness nonlinearity.

![Figure 6-3: Inclusion of an inverse function for a linear compound stiffness gain](image)

![Figure 6-4: Inverse function for linear compound gain from position, ‘\( x \)’, to control variable, ‘\( \nu \)’](image)
**Feedback linearisation of nonlinear stiffness load**

Secondly, feedback linearisation for the nonlinear stiffness load is proposed. In Figure 6-5, the load torque, $T_L$, has a nonlinear dependence on the mechanism position, $x$, due to the stiffness curve. The nonlinear load disturbance may be feedback linearised with a suitable control action. The concept is to apply a compensation torque, $T_{mc2}$, so that the compensated load ($T_{Le} = T_L + T_{mc2}$) appears linear to the controller $K_c$. The approach may be interpreted as shaping the stiffness load as depicted in Figure 6-6. Within actuator limits, the compensation torque, $T_{Mc2}$, may be applied to shape the apparent load, $T_{Le}$, as desired.

![Figure 6-5: Inclusions of feedback linearisation for the nonlinear stiffness load, $T_L$](image)

![Figure 6-6: Compensation torque, $T_{Mc2}$, such that the compensated load, $T_{Le}$, is feedback linearised](image)

The plots in Figure 6-6 are oriented to reflect the shape of the stiffness curve. It may be noted that the y-axis of the middle plot ($T_{Mc2}$) increases downwards. To clarify, positive motor torque (red) reduces the apparent load, $T_{Le}$, while the negative motor torque (blue) increases the compensated load. The net effect is a feedback linearised stiffness load.
It is seen from Figure 6-5 that signals for both the position, $x$, and force, $F_{cl}$, enter the compensation torque block for $T_{Mc2}$. Given a desired load characteristic ($T_{Lc} - x$ in Figure 6-6), the position and force signals are sufficient to determine the stiffness load characteristic ($T_{L} - x$) and the corresponding compensation, $T_{Mc2}$. Meanwhile, if the compensation is predetermined ($T_{Mc2} - x$ or $T_{Mc2} - F_{cl}$), then only one signal is required, either $x$ or $F_{cl}$.

While Figure 6-5 shows the general configuration, special case simplifications are possible. For example, a simple feedback linearisation would be to negate the load, $T_{Mc2} = T_{Lc} = NF_{cl}$. The compensating current command, $i_{qc}^{*} = NF_{cl} / K_r$, would then be advantageously based on the measured clamp force, $F_{cl}$. However, this would require greater compensation than a more subtle shaping like that seen in Figure 6-6. Hence, the compensation magnitude should be compared with the available motor torque to assess suitability. In this case, direct compensation would utilise a fraction of the 3 Nm motor torque (13% at 15 kN).

**Feedback linearisation of friction**

The third component of the modified EMB control architecture comes from Chapter 5 where an effective friction compensation was given by $i_{qc1}^{*} = \hat{T}_{sfc} / K_r$, and $\hat{T}_{sfc}$ was estimated from a friction model.

**Net compensation action**

The total compensating motor torque, or compensating current, may be determined by superimposing the stiffness and the friction feedback linearisation. The net action is given by the compensating current, $i_{qc}^{*} = i_{qc1}^{*} + i_{qc2}^{*}$.
**Modified EMB control architecture**

A practical version of the modified EMB control architecture is shown in Figure 6-7. The nonlinearities associated with the stiffness and friction (coloured red) are managed by an inverse gain schedule, load compensation and friction compensation (coloured blue). For example, $\hat{T}_{s/c}$ and $T_{mc2}$ are compensation torques for the nonlinear friction and stiffness load. The controller, $K_c$, is isolated from the system nonlinearity, except for the unavoidable case of actuator saturation. In other words, the apparent plant is shaped by the modified control architecture to alleviate the challenges of actuator nonlinearity.

Appendix 6A shows a version of Figure 6-7 that retains the generalised stiffness compensation block from Figure 6-5.

When the modified control architecture (Figure 6-7) was simulated and tested on the prototype EMB the friction compensation was the same as that described in the Chapter 5. Cascaded PI control was retained for $K_c$, but it was necessary to adjust the control gains. The control gains were retuned following the optimisation method described in Chapter 4. The optimal control gains were approximately constant for various step brake applies due to the management of nonlinearity with the modified control architecture.
6.3 Results and discussion

The benefit of the modified EMB control architecture may be assessed against the baseline performance with cascaded PI control (Chapter 4). It may be recalled that due to actuator nonlinearity, the fixed gains of the baseline control were sub-optimal for some brake operations. As shown in Figure 6-8 and Figure 6-9, the gains could be optimally tuned for a small or large brake apply, but not concurrently. By comparison, the modified EMB control architecture provides a greater level of performance throughout the operational envelope due to an improved management of the actuator nonlinearity.

![Figure 6-8: Modified control simulated against the cascaded PI control with fixed gains suitable for the large 30 kN apply](image)

Comparing the 1 kN and 30 kN applies in Figure 6-8, it may be noted that the system behaviour is still nonlinear with the modified control. For example, the shape of the brake response profile depends on the amplitude of the force command. The remaining nonlinearity is due to unavoidable actuator saturation. With limitations on the motor torque and speed, the power and rate of clamp force work are constrained.

![Figure 6-9: Modified control simulated against the cascaded PI control with fixed gains suitable for the small 1 kN apply](image)
The slight overshoot in the 1 kN step responses of Figure 6-9 correspond to the optimal gain tuning from Chapter 4. Tuning the PI gains to minimise a weighted quadratic cost on the integral error and power demand does not translate to zero overshoot.

Figure 6-10 shows measurements and simulations for a 5, 15 and 25 kN brake apply with the modified control. Generally, the model predicts the brake force profile and apply time with reasonable accuracy. The simulation slightly leads the measured response and this is consistent with the model behaviour in Chapter 3.

A consistent discrepancy seen in Figure 6-10 is the early decrease in the simulated velocity. This occurs because the modelled brake apply is slightly fast. Interestingly, the simulated deceleration also begins at a larger force error. Due to the simulation lead, less integral control action develops to maintain a saturated velocity command.
A series of large oscillations shown in Figure 6-11 and Figure 6-12 depict 75% and 5% modulations about 15 kN at 1, 4 and 8Hz. As previously, the performance of the large modulations is constrained at 8Hz due to the current saturation. The control is almost ‘bang-bang’ as the controller drives the actuator to its limit. Meanwhile, the fine brake response with the modified control is significantly improved relative to the cascaded PI control as described in Chapter 4.

The greatest simulation discrepancy in the large oscillations seen in Figure 6-11 occurs at 8Hz. The simulation slightly leads the measured response in a manner that is consistent with the slightly fast model behaviour described in Chapter 3. For the small oscillations seen in Figure 6-12, the most significant discrepancy may be observed at the low frequency of 1Hz. The measured velocity exhibits more catching than is predicted by the simulation. The irregular disturbance might suggest that the mechanism friction has a stochastic nature which is not captured by the friction model. In response, the embedded control utilises greater motor current to execute the sinusoidal oscillation.

Figure 6-11: Measured and simulated 75% modulations about 15kN at 1, 4 and 8Hz with modified control
To assess its benefit, the modified control architecture may be further compared with the baseline cascaded PI control described in Chapter 4. Measurements of large and fine brake applies with the two controllers are shown in Figure 6-13. Two fine modulations are also presented in Figure 6-14.

The brake applies in Figure 6-13 have an initial 7.5 kN load to avoid any influence of the clearance management. As noted in Chapter 4, the force and position control are merged with a single feedback variable. Hence the initial clamp force avoids any influence of the force-position handover region at low loads.

The modified control architecture only provides a slight improvement for the large brake apply from 7.5 kN to 25 kN seen in Figure 6-13. The brake response is largely constrained by the current up to 0.04 s, whereafter the velocity is limited to approximately 0.06 s. In this case, there is little margin for improvement.

The benefit of the modified EMB control architecture is more pronounced during fine brake operation. For example, there is a significant improvement for the small apply from 7.5 kN to 8.5 kN in Figure 6-13. The modified control drives the actuator harder, develops a higher peak velocity and decelerates at an increased rate. Consequently, the brake force rise time (10-90%) is almost halved from 0.066 s to 0.035 s.
The fine brake response is significantly improved by the modified control architecture. For the 5% modulation about 15 kN, the phase lag is decreased from 83° to 76° and the amplitude is increased from 43% to 77%. The benefit is even more pronounced for the 2% modulation about 25 kN. In this case, there is no brake response until the modified control executes the oscillation at 58% of the desired amplitude.

The action of friction compensation in the modified control is particularly apparent in the 2% modulation about 25 kN. At each velocity reversal there is a step change in the compensating motor current (and torque) to reject the nonlinear friction disturbance.
Some slight displacements may be observed without a change in the clamp force. For example, the 2% modulation shows a flat-topped brake force lagging the velocity reversals. The motor turns approximately 8º before there is a response in the force, which would correspond to an axial displacement of 3.5 microns with rigid gearing. Backlash is not an explanation since the mechanism was persistently loaded with a positive motor torque ($i_q > 0$). There is some other directional mechanism compliance which might arise from a directional load distribution within the mechanism. Such an effect could possibly be caused by the distributed mechanism friction.

![Graphs showing force, velocity, and current over time]

Figure 6-14: Modulations of 5% about 15kN and 2% about 25kN at 8Hz with the modified control architecture and the baseline cascaded PI control

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<th>Phase lag (º)</th>
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</tr>
</tbody>
</table>

Table 6-1: Amplitude and phase lag with cascaded PI control and the modified control architecture for the modulations of 5% about 15 kN and 2% about 25 kN
6.4 Conclusions

The modified EMB control significantly improved upon the performance of the baseline cascaded PI control described in Chapter 4, offering comparable performance for large manoeuvres while improving the small-amplitude (<5 kN) and fine (<1 kN) brake control. For example, the rise time for a 1kN adjustment at 7.5kN was almost halved from 0.066 s to 0.035 s.

Prior to the modified EMB control, the baseline control suffered a poor handling of the nonlinear friction and stiffness. By applying techniques of gain scheduling and feedback linearisation, the controller was largely isolated from nonlinearity except for the unavoidable saturation. The modified EMB control architecture improved the management of nonlinearity, maintained a higher level of performance and particularly improved the fine brake control.
7 Robust $H_\infty$ optimal control

7.1 Introduction

A modified EMB control architecture was proposed in Chapter 6 to improve the management of actuator nonlinearity. With scheduling and compensation, brake performance was achieved over a greater range of large and fine operation. However, due to an imperfect model for control design and compensation, some further consideration of the model uncertainty is required. An allowance for uncertainty is important, particularly as the brake parameters may vary with wear and temperature. This chapter extends the modified architecture outlined in Chapter 6, beginning with a parameter sensitivity analysis and assessment of uncertainty, leading to a description of a robust control design for the EMB.

A robust control design is proposed for the EMB to guarantee stability and a level of performance over a specified model uncertainty. The design includes consideration for parameter uncertainty and imperfect compensation. Significant factors are identified in a parameter sensitivity analysis and assessment of the uncertainty and this leads to the design of the robust $H_\infty$ optimal control for the EMB. Experimental and simulation results are followed by discussion and conclusions regarding the design success and whether a useful degree of uncertainty can be robustly tolerated without an overly conservative EMB control.


**Preliminary considerations**

*Consideration of prior work on robust EMB control*

In (Lu, 2005) and (Krishnamurthy et al., 2005) a cascaded proportional control was derived for an EMB by a method of robust backstepping. Proportional control gains \( k_1, k_2, \) and \( k_3 \) were applied within embedded feedback loops to regulate force, velocity and torque. Under assumptions of bounded uncertainty, it was shown that Lyapunov functions might be written when certain inequalities are satisfied. Lu et al. conclude that, “the force tracking error can be regulated to an arbitrarily small compact set by picking \( k_1, k_2, \) and \( k_3 \) large enough.” A stable response was simulated on a perturbed system with cascaded *PID-P-P* control, but little guidance was provided on how to tune the control gains for performance.

Since the EMB apply is open-loop stable from motor torque to clamp force, it is not difficult to maintain stability under closed loop control. A more difficult problem is managing robust performance. This latter challenge might be approached with the application of modern robust control design.

*Robust control design*

The approach to robust control design has progressed through periods of classical sensitivity (1927-1960), state-variable (1960-1975) and modern robust control (1975-present) (Dorato, 1986). Robust control gained in popularity in the late 1970s when it was realised that other methods of control lacked assurances of stability or performance under uncertainty (Bhattacharyya, Chapellat and Keel, 1995). In response, modern robust \( H_{\infty} \) control was developed to address issues of uncertainty.

Modern robust control theory may be found in texts such as (Mackenroth, 2004), (Zhou and Doyle, 1998), (Zhou, Doyle and Glover, 1996), (Green and Limebeer, 1995), and (Bhattacharyya et al., 1995). Further reading may be found in (Dorato, Tempo and Muscato, 1993) and surveys on robust control in robotics such as (Abdallah, Dawson, Dorato and Jamshidi, 1991) and (Sage, Mathelin and Ostertag, 1999).
**Robust $H_\infty$ optimal control**

The concept of $H_\infty$ optimal control is to treat the worst case scenario (Green and Limebeer, 1995). A robust design may assure stability and a limit on the worst case performance across a set of model uncertainty.

Designing a robust $H_\infty$ optimal control for the EMB largely consists of suitably expressing the control problem within the general framework of Figure 7-1. Once this is done, standard robust control theory may be applied to synthesise a controller. For convenience, the relevant theory is summarised in Appendix 7A. The present discussion is only intended as an introductory overview.

![Figure 7-1: General framework for robust control design](image)

In Figure 7-1, the controller, $K$, regulates an extended plant, $P$, that is perturbed by a set of bounded uncertainty, $\Delta$. The signal $u$ is the controlled input to the plant, $P$, and $y$ is the feedback to the controller, $K$. $z_p$ is the input to the model uncertainty, $\Delta$, and $w_p$ is the uncertain feedback. $w$ is an input vector that may include noise, disturbances or a reference set point and $z$ is the set of controlled output, such as tracking errors.

Without uncertainty ($\Delta=0$), the framework shown in Figure 7-1 reduces to an optimal $H_\infty$ control problem. For example, if $w$ and $z$ were the set-point command and tracking error then the optimisation would seek the controller, $K$, that minimised the $H_\infty$ norm of the transfer function from the reference, $w$, to the tracking error, $z$. 
The $H_\infty$ norm of the transfer matrix $G$ is the supremum of the maximum singular value, $\sigma$, over all frequencies, $\omega$, $\|G\|_\infty = \sup_{\omega} \|\sigma(G(i\omega))\|$. The $H_\infty$ norm describes the peak matrix gain across all frequencies and all input directions.

The peak matrix gain is central to the robust control theory due to the nature of the uncertainty. The model uncertainty is assumed to be stable and bounded, but without specified phase behaviour the system stability can only be deduced from magnitude information. Hence, a stability criterion is conservatively based on the small gain theorem (Appendix 7A). Generally, if the loop gain is less than unity the Nyquist plot cannot encircle the minus one point and the system will be internally stable.

Following an elegant problem arrangement, the “robust performance problem is equivalent to a robust stability problem with augmented uncertainty” (Zhou and Doyle, 1998). With further explanation in Appendix 7A, the problem arrangement conforms with the structure shown in Figure 7-2. $M=F_L(P,K)$ is the lower fractional transformation of the extended plant, $P$, and controller, $K$. The uncertainty, $\Delta$, is augmented to incorporate the robust performance objective.

![Figure 7-2: Uncertain feedback system](image)

Let the nominal feedback system $M=F_L(P,K)$ be internally stable and let $\gamma>0$. Then the following assertions are equivalent.

(i) For every $\Delta \in M(\Delta)$ with $\|\Delta\|_\infty < 1/\gamma$ the feedback system depicted in Figure 7-2 is well-posed, internally stable and fulfils the inequality $\|F_u(M,\Delta)\|_\infty < \gamma$.

(ii) The following inequality holds: $\sup_{\omega \in \mathbb{R}} \mu_{\Delta}(M(i\omega)) \leq \gamma$. (Mackenroth, 2004)
Where \( \Delta_p = \left\{ \begin{bmatrix} \Delta & 0 \\ 0 & \Delta_1 \end{bmatrix} \right| \Delta \in \Delta, \Delta_1 \in C^{q_1 \times p_2} \right\} \)

In other words, for every uncertainty, \( \Delta \), in the set of structured uncertainty, \( \mathcal{M}(\Delta) \), with a peak uncertain gain, \( \| \Delta \|_\infty < 1/\gamma \), the uncertain feedback system is well-posed, internally stable and satisfies \( \| F_U(M, \Delta) \|_\infty < \gamma \). This assertion is equivalent to a bound on the supreme structured singular value, \( \sup_{\omega \in R} \mu_{\Delta_p}(M(i\omega)) \leq \gamma \), for the augmented performance uncertainty \( \Delta_p \). Further definitions, such as the structured singular value, are provided in Appendix 7A.

The significance of the theorem is that, “the peak of the \( \mu \)-plot of \( M \) indicates the magnitude of the perturbations for which the feedback loop is robustly stable and has additionally robust performance” (Mackenroth, 2004). A controller may then be synthesised by an algorithm known as the \( D-K \) iteration. With further detail given in Appendix 7A, the algorithm seeks an approximate solution to minimise the supremum, \( \sup_{\omega \in R} \mu_{\Delta_p}(F_L(P, K)(i\omega)) \leq \gamma \).

**Further considerations for a robust EMB control design**

A robust \( H_\infty \) control design for the EMB would ensure stability and a limit to the worst case performance over a set of model uncertainty. Before presenting the control design, significant factors are identified in a sensitivity analysis and assessment of the parametric uncertainty. Since there tends to be a trade-off between the level of uncertainty and performance, a careful structuring of the uncertainty is important. When formulating the problem in the framework of Figure 7-1, the design should avoid unnecessarily conservative perturbations whilst providing a meaningful description of the uncertainty. Once an appropriate formulation is developed, standard tools of \( \mu \) synthesis may be applied to obtain a robust control law.
7.2 Sensitivity

Since the significance of parameter uncertainty depends on the system sensitivity, an analysis can help to guide a suitable robust control design. If the system is sensitive then a small parameter uncertainty may produce significant variation. Conversely, if the system is insensitive then a large uncertainty may have little effect.

To assess the EMB sensitivity, the model described in Chapter 3 was used to simulate the brake apply with perturbed parameters. Figure 7-3 shows a set of open-loop brake applies with the variables increased by 25%, one at a time. The input was a step 6.5 A quadrature current. Table 7-1 summarises the peak apply rates and steady-state forces relative to the nominal response with no parameter adjustment.

Figure 7-3: Simulated effect of increasing each variable by 25% for a 6.5A step current input

<table>
<thead>
<tr>
<th>+25% increase</th>
<th>Peak apply rate (%)</th>
<th>Steady-state force level (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>No parameter adjustment</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>Motor torque constant, $K_t$</td>
<td>133</td>
<td>127</td>
</tr>
<tr>
<td>Stiffness</td>
<td>109</td>
<td>101</td>
</tr>
<tr>
<td>Inertia, $J$</td>
<td>91</td>
<td>101</td>
</tr>
<tr>
<td>Gear ratio, $N$</td>
<td>102</td>
<td>86</td>
</tr>
<tr>
<td>Friction parameters, $C$, $G$ and $D$</td>
<td>89</td>
<td>88</td>
</tr>
</tbody>
</table>

Table 7-1: Effect of 25% parameter increase on apply rate and steady-state force for 6.5A input

Table 7-1 indicates the brake apply rate was increased by a higher torque constant, stiffness and gear ratio. In order of sensitivity, the apply rate was affected by variation in the torque constant, friction, stiffness, inertia and gear ratio. The steady-state force level increased with the torque constant, but decreased with a higher gear ratio and friction. The steady-state force was most sensitive to these parameters.
7.3 Uncertainty

When uncertainty is introduced, the uncertain EMB model describes a set of possible actuators. The model uncertainty may allow for parameter uncertainty or unmodelled dynamics. For example, the EMB parameters may contain identification inaccuracy or there may be some uncertain variation between actuators or over time.

The description of model uncertainty may be structured or unstructured. For example, Figure 7-4 (a) shows an unstructured input-multiplicative uncertainty ($|\Delta|<1$) that could allow for various unmodelled dynamics, while (b) shows uncertainty ($|\delta|<1$) structured within the model. This describes uncertainty in the parameter $a = a_0 + \delta_m \delta$, where $a_0$ is the nominal value and $\delta_m$ is the magnitude applied to the uncertainty $|\delta|<1$. The benefit of structured uncertainty is that it can describe a more specific set of uncertain systems. Otherwise, the uncertainty must be covered with a larger and more conservative perturbation (Zhou and Doyle, 1998).

Once uncertainty is appropriately described in a model, the problem formulation involves a rearrangement known as ‘pulling out the deltas’. The bounded uncertainties may then be isolated to obtain the general problem framework of Figure 7-1. The examples in Figure 7-5 show rearrangements for the systems seen in Figure 7-4. After the problem is expressed within the general framework, standard robust control theory and controller synthesis may be applied.

![Figure 7-4: Systems with multiplicative input uncertainty (a) and parametric uncertainty (b)](image1)

![Figure 7-5: Arrangement into the general problem framework by ‘pulling out the deltas’](image2)
Ideally, the parameter uncertainty would be determined from a large survey of EMB actuators throughout the brake lifecycle. However, only two actuators were available for testing in this study. With little information, the bounds on parameter uncertainty were specified to provide a reasonable allowance.

Table 7-2 categorises the parameter uncertainty into approximate categories of low (<15%), medium and high (>100%). Excluding damage, the torque constant, $K_t$, and gear ratio, $N$, might vary in the order of 1%. The inertia, $J$, could vary a few percent between actuators. The stiffness might vary by scores of percent with temperature and pad wear. Finally, the friction parameters may double or halve depending on the actuator, lubrication and wear.

<table>
<thead>
<tr>
<th>Uncertainty:</th>
<th>Between new units of the same design</th>
<th>Throughout lifecycle</th>
<th>Following system identification</th>
</tr>
</thead>
<tbody>
<tr>
<td>Motor torque constant, $K_t$</td>
<td>Low</td>
<td>Low</td>
<td>Low</td>
</tr>
<tr>
<td>Stiffness</td>
<td>Low</td>
<td>Medium</td>
<td>Low</td>
</tr>
<tr>
<td>Inertia, $J$</td>
<td>Low</td>
<td>Low</td>
<td>Low</td>
</tr>
<tr>
<td>Gear ratio, $N$</td>
<td>Low</td>
<td>Low</td>
<td>Low</td>
</tr>
<tr>
<td>Friction parameters, $C, G$ and $D$</td>
<td>High</td>
<td>High</td>
<td>Low</td>
</tr>
</tbody>
</table>

Table 7-2: Parametric uncertainty classified as low, medium or high, between units, over a lifecycle and following system identification.

It may be anticipated that the friction and stiffness will have the greatest uncertainty given the possible variation with wear and temperature. Table 7-1 indicates that the simulated brake response is moderately sensitive to these parameters. Hence, tolerance of the friction and stiffness uncertainty will be an important consideration in the robust control design. Meanwhile, the uncertainty associated with the gear ratio, torque constant and system inertia is less significant.
7.4 Robust control design for an EMB

A robust EMB control design is proposed as an extension to the modified control architecture described in Chapter 6. Previously, the modified architecture isolated the controller, $K_c$, from actuator nonlinearity except for the unavoidable saturation. In this chapter, the inner controller $K_c$ is synthesised by a robust control design to allow for parameter uncertainty and imperfect compensation. The control design is presented in three parts that describe the problem formulation, controller synthesis and controller reduction.

**Problem formulation**

The problem formulation involves crafting the model uncertainty and performance objective within the general framework of Figure 7-1 so that standard robust control theory and tools may be applied.

**Weighted plant with two-degree-of-freedom control**

Given the desire for a high level of robust brake performance, a two-degree-of-freedom control structure is utilised. With this structure the controller receives separate reference and measurement signals, rather than an aggregate tracking error. Figure 7-6 shows the general structure of the extended plant with mixed sensitivity performance weights, $W_{1,2,3}$.

![Figure 7-6: Extended plant, $P$, with two-degree-of-freedom control configuration and mixed sensitivity performance weights, $W_{1,2,3}$](image)
Figure 7-6 shows the extended plant, $P$, and controller, $K$, without uncertainty, $\Delta$. The structure may be compared with the general framework of Figure 7-1. The control feedback, $y=[y_m, r]=[v, v^*]$, includes the measurement and set-point reference. $u=i_q^*$ is the control action entering the nominal plant, $G$. The input, $w=[n, r]=[n, v^*]$, includes noise and the reference signal. The controlled output, $z=[z_1, z_2, z_3] = [W_1 e, W_2 u, W_3 y]$, comprises a weighted tracking error, input and output. The weight $W_n$ is included to satisfy the rank condition for $H_\infty$ synthesis (Appendix 7B). It is chosen as a small constant and has little effect on the control design.

Since uncertainty is not yet included, the $H_\infty$ optimal control problem would be to minimise the $H_\infty$ norm of the transfer matrix between the inputs, $w$, and outputs, $z$. During optimisation, the performance weights, $W_{1,2,3}$, specify the desired mixed-sensitivity transfer functions, $S_{r \rightarrow e}$, $KS_{r \rightarrow u}$, and $T_{r \rightarrow y}$ (Appendix 7C).

**Mixed sensitivity performance weights**

A $S$-$KS$ mixed-sensitivity weighting is suitable for the robust EMB control design. It allows the desired tracking performance to be specified in the frequency domain and may account for the actuator saturation. The performance weight $W_1$ may be used to shape the desired sensitivity, $e=S(s)r$. Meanwhile, $W_2$ may be used to avoid saturation by bounding the transfer function from the reference to the control action, $u=K(s)S(s)r$. Since $S+T=1$, the weight $W_3$ and output $z_3$ are excluded to avoid over-specification.

A suitable form for the performance weights is given by,

$$W_1 = \frac{1}{M_s} \frac{s+\omega_{11}}{s+\omega_{21}}$$

where $M_s$ = peak sensitivity

$$W_2 = \frac{\omega_{22}}{M_u \omega_{12}} \frac{s+\omega_{12}}{s+\omega_{22}}$$

where $M_u$ = peak transfer magnitude from reference to input
As shown in Figure 7-7, the inverse magnitudes of $W_1$ and $W_2$ shape the specification of $S_{r \rightarrow e}$ and $KS_{r \rightarrow u}$.

![Figure 7-7: Sensitivity, S, with inverse weighting, 1/W1 (a) and KS with inverse weighting, 1/W2 (b)](image)

The sensitivity magnitude, $|S|$, may be reduced at low frequency for a small tracking error. At higher frequencies, the sensitivity magnitude approaches unity as the magnitude of the tracking error approaches the reference amplitude.

Bode’s sensitivity integral is relevant when specifying $W_1$ since when the loop transfer function is stable, $\int_0^\infty \ln |S(j\omega)| d\omega = 0$ (Zhou and Doyle, 1998). Hence, a ‘waterbed’ effect occurs where a sensitivity below unity causes elevation above unity across other frequencies. The weight $W_2$ may shape the transfer function $KS_{r \rightarrow u}$ to reflect the actuator saturation and it may also be rolled-off to attenuate high frequency noise.

**Inclusion of model uncertainty**

An initial step towards the general problem framework described in Figure 7-1 is the inclusion of model uncertainty. An appropriate structuring of model uncertainty is important for a meaningful description and to avoid unnecessarily conservative perturbations. The robust control design considers a linear plant model with bounded uncertainty. This might seem mismatched to a description of the nonlinear EMB actuator, but the modified control architecture described in Chapter 6 reduces the exposure of the controller, $K_c$, to system nonlinearity and avoids a large uncertainty that would otherwise be associated with unmodelled nonlinear dynamics. The robust design may also account for the saturation using $W_2$ to bound the transfer function $KS_{r \rightarrow u}$.
With the robust controller incorporated in the modified EMB control architecture, some degree of uncertainty corresponds to imperfect compensation and parameter uncertainty.

**Allowance for imperfect compensation**

A starting point for including model uncertainty is the nonlinear EMB model described in Chapter 3, following the progression depicted in Figure 7-8. Diagram (a) of this figure shows the friction model expanded under a condition of motion (\(\forall|\dot{\theta}| > \varepsilon\)). Diagram (b) includes estimation errors for imperfect compensation of the stiffness load, as well as static and Coulomb friction. Diagram (c) is rearranged with the error for Coulomb friction compensation, \(T_{c} - \hat{T}_{c}\), as an input disturbance. In diagram (d), the compensation errors are bounded by uncertainties, \(W_{AC,F,G}\). The stiffness curve in the feedback path is conservatively over-bounded across the work range with a gain that will later be denoted, \(K_{w}\). Diagram (e) shows the controller, \(K_{c}\), in relation to the imperfectly compensated plant and it regulates the linear control variable, \(u\), to the set-point command, \(u^{*}\). \(K_{c}\) will be synthesised as a robust controller to allow for the model uncertainty. In diagram (f) the compound linear stiffness is denoted by \(K_{p}\). The fast current transients are neglected, but the robust control design does consider the current saturation.
Figure 7-8: Inclusion of model uncertainty for imperfect compensation

The diagrams in Figure 7-8 show the:

a) Nonlinear EMB plant

b) Errors for imperfect compensation of stiffness load, Coulomb and static friction

c) Rearrangement with imperfect compensation of $T_c$ as an input disturbance

d) Allowance for imperfect compensation with bounded uncertainties, $W_{ΔC,G,F}$

e) Uncertain plant and the robust controller, $K_c$, within the modified architecture

f) Stiffness and inverse combined to give the linear compound gain, $K_v$
The uncertainties $W_{AC,G,F}$ for imperfect feedback linearisation of the friction and stiffness load are given in Table 7-3. Specific sets of uncertainty will be defined prior to the controller synthesis. For each specification, the success of the robust control synthesis will be indicated by the $\gamma$-value achieved during optimisation.

<table>
<thead>
<tr>
<th>Uncertainty from imperfect compensation of…</th>
<th>Specification</th>
<th>Magnitude</th>
<th>Uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coulomb friction</td>
<td>$W_{AC} = W_C A_C$</td>
<td>$W_C = constant$</td>
<td>$|\Delta_C|_{\infty} \leq 1$</td>
</tr>
<tr>
<td>Load dependent friction</td>
<td>$W_{AG} = W_G A_G$</td>
<td>$W_G = constant \times G_o$</td>
<td>$|\Delta_G|_{\infty} \leq 1$</td>
</tr>
<tr>
<td>Stiffness load</td>
<td>$W_{AF} = W_F A_F + W_{Foffset}$</td>
<td>$W_F = constant \times N_o$</td>
<td>$|\Delta_F|_{\infty} \leq 1$</td>
</tr>
</tbody>
</table>

Table 7-3: Model uncertainty associated with imperfect feedback linearisation

The uncertainties, $\Delta_{C,G,F}$, are assumed stable with bounded magnitude $\|\Delta_{C,G,F}\|_{\infty} \leq 1$. The weights, $W_{C,G,F}$, specify the magnitude of the uncertainty. A finitely small offset, $W_{Foffset}$, is included to ensure the rank condition is fulfilled for $H_\infty$ synthesis.

$W_{AC}$ is an uncertainty to allow for imperfect compensation of the load independent, static and Coulomb friction. These friction contributions are generally small (0.03-0.06 Nm) relative to the load dependent friction (~0.35 Nm at 30 kN) or the maximum motor torque (3 Nm). After compensation, the uncertain disturbance should be smaller again. If the multiplicative input uncertainty was specified with $W_C = 0.1$, it would allow for uncertain loads up to 10% of the instantaneous motor torque. Meanwhile, a component of the $W_{AG}$ uncertainty may be interpreted as providing some cover for uncertain loads if the motor torque approaches zero. Except when the motor torque and clamp force both approach zero, there is allowance for imperfect friction compensation during most brake operation.

$W_{AG}$ is an uncertainty to allow for imperfect compensation of the load dependent friction. For example, a weighting of $W_G = 1.15 \times G$ would allow for a doubling of the nominal load dependent friction and an additional force measurement error of 15%.
$W_{AF}$ is an uncertainty to allow for imperfect compensation of the stiffness load. For example, setting $W_F = 0.15$ would allow a 15% error in the force measurement such that the compensation would be 85% effective.

The stiffness curve is over-bounded by $K_w = \text{(maximum design force, 30 kN) / (corresponding displacement at low temperature and high pad wear)}$. $K_w$ is a stiff linearisation that overestimates the true clamp force so that the subsequent uncertainties $W_{AG}$ and $W_{AF}$ remain conservative.

**Allowance for parameter uncertainty**

Having allowed for imperfect compensation, the next consideration is the description of parameter uncertainty. In Table 7-4 the model parameters are specified with some uncertain perturbation.

<table>
<thead>
<tr>
<th>Parametric uncertainty for...</th>
<th>Specification</th>
<th>Magnitude</th>
<th>Uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>Viscous friction</td>
<td>$D = D_o + D_{offset} + \delta_{mD}\delta_D$</td>
<td>$\delta_{mD} = \text{constant} \times D_o$, $D_{offset} = \text{constant} \times D_o$</td>
<td>$|\delta_D|_\infty \leq 1$</td>
</tr>
<tr>
<td>Compound stiffness</td>
<td>$K_i = K_i + K_{offset} + \delta_{mKv}\delta_{Kv}$</td>
<td>$\delta_{mKv} = \text{constant} \times K_{i0}$, $K_{offset} = \text{constant} \times K_{i0}$</td>
<td>$|\delta_{Kv}|_\infty \leq 1$</td>
</tr>
<tr>
<td>Inertia</td>
<td>$1/J = 1/J_o + \delta_{mJ}\delta_J$</td>
<td>$\delta_{mJ} = \text{constant} / J_o$</td>
<td>$|\delta_J|_\infty \leq 1$</td>
</tr>
<tr>
<td>Torque constant</td>
<td>$K_t = K_t + \delta_{mKt}\delta_{Kt}$</td>
<td>$\delta_{mKt} = \text{constant} \times K_{t0}$</td>
<td>$|\delta_{Kt}|_\infty \leq 1$</td>
</tr>
<tr>
<td>Gear reduction</td>
<td>$N = N_o + \delta_{mN}\delta_N$</td>
<td>$\delta_{mN} = \text{constant} \times N_o$</td>
<td>$|\delta_N|_\infty \leq 1$</td>
</tr>
</tbody>
</table>

Table 7-4: Parametric model uncertainty

In Table 7-4 the weights, $\delta_{mx}$, specify the magnitude of the uncertainty. The uncertainty, $\delta_x$, is assumed to be stable and to have a bounded magnitude $\|\delta_x\|_\infty \leq 1$. The subscript ‘o’ is used to denote nominal values. The parameter $D_{offset}$ is used to shift the uncertain bound when the nominal value is not centred. $K_v$ represents the composite stiffness gain and has some uncertainty due to the imperfect linearisation of the inverse function.
Adding parametric uncertainty to the imperfectly compensated plant in Figure 7-8, the system may be redrawn as shown Figure 7-9. The bounded uncertainties, $\delta$ and $\Delta$, correspond to imperfect compensation and parameter uncertainty. The extended plant in Figure 7-10 is finally obtained by including the performance weights, $W_1$, $W_2$ and $W_n$, shown in Figure 7-6. As mentioned, $W_1$ and $W_2$ will be used to specify the desired mixed sensitivity transfer functions, $S$ and $KS$.

Figure 7-9: Uncertain feedback system

Figure 7-10: Extended plant, $P$, with model uncertainty and performance weights
**Rearrangement to general problem framework**

The extended plant, $P$, in Figure 7-10 may be rearranged to the general problem framework of Figure 7-1 by separating the uncertainty and ‘pulling out the deltas’. The machinery of standard robust control theory may then be applied to synthesise the controller, $K_c$.

![Figure 7-11: General problem framework for robust electromechanical brake control](image)

In Figure 7-11, the input, $w=[n,r]=[n,v^r]$, comprises the noise and reference, while the output, $z=[z_2,z_1]=[W_2u,W_1e]$, is the weighted input and tracking error. The controlled input is $u=i_q$ and the control feedback, $y=[r,y_m]=[v^r,v]$, is the set-point reference and measurement. The signals $z_{px}$ and $w_{px}$ for the uncertain feedback are the same in Figure 7-10 and Figure 7-11.

Having expressed the problem in the general framework of Figure 7-11, the controller, $K_c$, may be obtained by $\mu$ synthesis. The success of the design will be indicated by the magnitude of the $\gamma$-value achieved during optimisation.
Controller synthesis

After defining the uncertainty and performance weights, the Matlab function \textit{dksyn()} was used to synthesise a robust EMB controller by standard \textit{D-K} iteration (Appendix 7A). Some adjustment of the uncertainty and performance specification was necessary to achieve a successful design, as described in the following.

The success of a design is indicated by the $\gamma$-value achieved during $\mu$ synthesis. The parameter $\gamma=1$ is ideal and would indicate robust stability and performance across the entire specified uncertainty. Larger $\gamma$-values indicate robustness over a reduced magnitude of uncertainty, $\|\Delta\|_\infty < 1/\gamma$. The specification of uncertainty and performance may be adjusted until a successful design is achieved or large $\gamma$-values indicate there is no appropriate solution.

Design specification 1 – reasonable performance and allowance for uncertainty

The design specification defines the level of uncertainty and performance weights. A first design specification in Table 7-5 describes a reasonable level of uncertainty and satisfactory performance.

<table>
<thead>
<tr>
<th>Imperfect feedback compensation of:</th>
<th>Uncertainty</th>
<th>Lower bound</th>
<th>Upper bound</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coulomb friction</td>
<td>$W_C$</td>
<td>-0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>Load dependent friction</td>
<td>$W_{x3}$</td>
<td>-1.15$G_o$</td>
<td>1.15$G_o$</td>
</tr>
<tr>
<td>Stiffness load</td>
<td>$W_F$</td>
<td>-0.15$N_o$</td>
<td>0.15$N_o$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parametric uncertainty for:</th>
<th>Uncertain parameter</th>
<th>Nominal value - %</th>
<th>Nominal value + %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Viscous friction</td>
<td>$D$</td>
<td>-50</td>
<td>100</td>
</tr>
<tr>
<td>Compound stiffness</td>
<td>$K_c$</td>
<td>-20</td>
<td>10</td>
</tr>
<tr>
<td>Inertia</td>
<td>$J$</td>
<td>-2</td>
<td>2</td>
</tr>
<tr>
<td>Torque constant</td>
<td>$K_t$</td>
<td>-1</td>
<td>1</td>
</tr>
<tr>
<td>Gear reduction</td>
<td>$N$</td>
<td>-1</td>
<td>1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>S-KS performance weights:</th>
<th>Peak value</th>
<th>1$\text{st}$ corner frequency</th>
<th>2$\text{nd}$ corner frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tracking error weight, $W_1$</td>
<td>$M_1 = 1.2$</td>
<td>$\omega_1 = 25$</td>
<td>$\omega_2 = 0.01 \times \omega_1$</td>
</tr>
<tr>
<td>Control input weight, $W_2$</td>
<td>$M_u = 40/300000$</td>
<td>$\omega_2 = 500$</td>
<td>$\omega_2 = 100 \times \omega_1$</td>
</tr>
</tbody>
</table>

Table 7-5: First robust control design specification
For convenience, it may be recalled that \( W_1 = \frac{1}{M_1 s + \omega_{11}} \) and \( W_2 = \frac{\omega_{22}}{M_u \omega_{12}} \frac{s + \omega_{22}}{s + \omega_{22}} \).

The design specification indicates the range of uncertainty, \( x \in [x_{\text{low}}, x_{\text{high}}] \). The uncertainties may be rewritten in the form shown in Table 7-3 and Table 7-4, \( x = x_o + x_{\text{offset}} + \delta_x \delta x \) where \( x_o \) is the nominal value and \( \| \delta_x \| \leq 1 \).

Table 7-5 specifies a reasonable allowance for uncertainty and satisfactory performance. However, the controller synthesis outputs a large value of \( \gamma=2.07 \) indicating that robust performance cannot be guaranteed over the specified uncertainty. For the purpose of continuing the investigation, the design specification was adjusted.

**Design specification 2 – low uncertainty and moderate performance**

Over a number of design iterations, the level of uncertainty was decreased to find a more achievable specification. Table 7-6 specifies a low level of uncertainty and moderate performance. This was a more successful design with \( \gamma=1.39 \).

<table>
<thead>
<tr>
<th>Imperfect feedback compensation of:</th>
<th>Uncertainty</th>
<th>Lower bound</th>
<th>Upper bound</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coulomb friction</td>
<td>( W_{\Delta e} )</td>
<td>-0.01</td>
<td>0.01</td>
</tr>
<tr>
<td>Load dependent friction</td>
<td>( W_{\Delta f} )</td>
<td>-0.1 ( G_o )</td>
<td>0.1 ( G_o )</td>
</tr>
<tr>
<td>Stiffness load</td>
<td>( W_{\Delta F} )</td>
<td>-0.05 ( N_o )</td>
<td>0.05 ( N_o )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parametric uncertainty for:</th>
<th>Uncertain parameter</th>
<th>Nominal value - ( % )</th>
<th>Nominal value + ( % )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Viscous friction</td>
<td>( D )</td>
<td>-5</td>
<td>5</td>
</tr>
<tr>
<td>Compound stiffness</td>
<td>( K_c )</td>
<td>-5</td>
<td>5</td>
</tr>
<tr>
<td>Inertia</td>
<td>( I/J )</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Torque constant</td>
<td>( K_t )</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Gear reduction</td>
<td>( N )</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>S-KS performance weights:</th>
<th>Peak value</th>
<th>1st corner frequency</th>
<th>2nd corner frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tracking error weight, ( W_1 )</td>
<td>( M_1 = 1.2 )</td>
<td>( \omega_{b1} = 15 )</td>
<td>( \omega_{b2} = 0.01 \times \omega_{b1} )</td>
</tr>
<tr>
<td>Control input weight, ( W_2 )</td>
<td>( M_u = 1.5 \times 40/30000 )</td>
<td>( \omega_{b2} = 500 )</td>
<td>( \omega_{b2} = 100 \times \omega_{b1} )</td>
</tr>
</tbody>
</table>

Table 7-6: Second robust control design specification – low uncertainty and moderate performance
**Design specification 3 – low performance and moderate uncertainty**

For the purpose of comparison, another design was formulated with reduced performance. Table 7-7 specifies a low performance and moderate level of uncertainty. A controller was successfully synthesised for this design with $\gamma \approx 1.36$.

<table>
<thead>
<tr>
<th>Imperfect feedback compensation of:</th>
<th>Uncertainty</th>
<th>Lower bound</th>
<th>Upper bound</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coulomb friction</td>
<td>$W_{w}$</td>
<td>-0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>Load dependent friction</td>
<td>$W_{k}$</td>
<td>-0.75$G_o$</td>
<td>0.75$G_o$</td>
</tr>
<tr>
<td>Stiffness load</td>
<td>$W_{st}$</td>
<td>-0.1$N_o$</td>
<td>0.1$N_o$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parametric uncertainty for:</th>
<th>Uncertain parameter</th>
<th>Nominal value</th>
<th>Nominal value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Viscous friction</td>
<td>$D$</td>
<td>-30</td>
<td>30</td>
</tr>
<tr>
<td>Compound stiffness</td>
<td>$K_s$</td>
<td>-20</td>
<td>10</td>
</tr>
<tr>
<td>Inertia</td>
<td>$l/J$</td>
<td>-2</td>
<td>2</td>
</tr>
<tr>
<td>Torque constant</td>
<td>$K_t$</td>
<td>-1</td>
<td>1</td>
</tr>
<tr>
<td>Gear reduction</td>
<td>$N$</td>
<td>-1</td>
<td>1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>S-KS performance weights:</th>
<th>Peak value</th>
<th>1st corner frequency</th>
<th>2nd corner frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tracking error weight, $W_1$</td>
<td>$M_1 = 1.2$</td>
<td>$\omega_{11} = 10$</td>
<td>$\omega_{21} = 0.01 \times \omega_{11}$</td>
</tr>
<tr>
<td>Control input weight, $W_2$</td>
<td>$M_2 = 1.5 \times 40/30000$</td>
<td>$\omega_{22} = 500$</td>
<td>$\omega_{22} = 100 \times \omega_{11}$</td>
</tr>
</tbody>
</table>

Table 7-7: Third robust control design specification – low performance and moderate uncertainty
Controller reduction

Since $\mu$ synthesis may produce controllers of high order, a reduction may be desired for practical realisation. The controller from the second design specification in Table 7-6 was reduced from 8 states to 5 by the MATLAB function `reduce()` that used Hankel singular values to preserve the most energetic states. After discretisation, the balanced controller, $K_c$, was described by,

$$
x_{k+1} = K_{ca} x_k + K_{cb} y_k
$$

(7-1)

$$
u_k = K_{cc} x_k + K_{cd} y_k
$$

where $K_c = \begin{bmatrix} K_{ca} & K_{cb} \\ K_{cc} & K_{cd} \end{bmatrix}$ has a period of $T = 4$ ms and,

$$
K_{ca} = \begin{bmatrix} 9.995 & 0.014 & -0.135 & -0.003 & 0.0005 \\ 0.701 & -2.859 & -34.531 & -3.127 & -0.615 \end{bmatrix} \times 10^{-1} \\
K_{cb} = \begin{bmatrix} -4.414 & -773.872 \\ -0.911 & -108.345 \times 10^6 \\ -3.200 & -16.726 \end{bmatrix} \\
K_{cc} = \begin{bmatrix} 0.952 & 0.392 & 1.904 & 0.605 & -0.103 \\ 0.037 & 0.457 & 7.775 & 0.406 & -0.532 \end{bmatrix} \\
K_{cd} = \begin{bmatrix} 0.047 & -0.111 & 1.753 & 0.646 & 8.292 \end{bmatrix} \\
K_{cd} = \begin{bmatrix} -48.19 & 141.28 & 256.87 & -353.14 & -12.16 \end{bmatrix}
$$

The inputs to the controller are the reference signal and the measurement, $y=[r, y_m]^T = [v^*, u]^T$. The controller output is $u=i_q$. A Bode plot for the controller is shown in Figure 7-12 with good agreement between the reduced and high-order representation.

![Bode Diagram](image-url)

Figure 7-12: Controller before and after reduction from 8 states to 5
7.5 Results and discussion

Robust EMB controllers were synthesised for the three design specifications given in Table 7-5, Table 7-6 and Table 7-7, and the results are summarised in Table 7-8. When a reasonable level of uncertainty and performance was detailed in the first specification, a large $\gamma$-value (2.07) indicated the design could not be satisfied. It was possible to relax the specification of uncertainty and performance to obtain a more successful design (with a low $\gamma$-value), but this outcome was less than satisfactory. For example, a moderate performance could only be achieved with a low level of uncertainty. Conversely, a moderate level of uncertainty could be tolerated only with conservative performance.

Conservatively avoiding the actuator saturation is a significant limitation. To achieve more successful designs, the second and third specifications in Table 7-7 and Table 7-8 have increased magnitudes for the performance weight $M_u$. In these cases, the control no longer avoids the 40 A current saturation for force commands approaching a magnitude of 30 kN, thereby reducing the operations for which robustness is guaranteed.

<table>
<thead>
<tr>
<th>Design</th>
<th>Uncertainty</th>
<th>Performance</th>
<th>$\gamma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st specification</td>
<td>Satisfactory</td>
<td>Satisfactory</td>
<td>2.07</td>
</tr>
<tr>
<td>2nd specification</td>
<td>Low</td>
<td>Moderate</td>
<td>1.39</td>
</tr>
<tr>
<td>3rd specification</td>
<td>Moderate</td>
<td>Low</td>
<td>1.36</td>
</tr>
</tbody>
</table>

Table 7-8: Results of robust controller synthesis

While the robust EMB control design has some limitations, it is important to confirm the anticipated performance by experiment. To facilitate testing, the robust controller from the second specification in Table 7-6 was implemented on EMB 19, with a safety modification of overdrive protection to limit the motor velocity. By forecasting the velocity ahead two control periods, the acceleration and motor current could be appropriately limited to avoid exceeding the maximum velocity. Figure 7-13 compares measurements and simulations for a 5, 15 and 25 kN brake apply with the robust EMB control.
The predicted brake apply with the robust controller shows a greater discrepancy than earlier results reported in Chapters 4 and 6 with the modified and cascaded PI controllers. The difference is particularly evident in the velocity subplot of Figure 7-13. For convenience, enlarged versions of the results are provided in Appendix 7D. Because the robust controller is higher order, its response may be more sensitive to the initial conditions. While the simulated controller could be initialised, the experimental controller had an operational history and had to be settled at an initial state. The discrepancy may be exaggerated by dissimilar periods of actuator saturation during which the 5th order state-space controller may windup to a different extent. While there are some discrepancies, the simulated brake force profiles were generally seen to be reasonable.
Figure 7-14 shows large 75% brake force modulations about 15 kN at 1, 4 and 8Hz. There is reasonable agreement between the simulated and measured brake force response. However, the simulation exhibits more controller activity, particularly at 4Hz for example. The excitation corresponds to deviation in the mechanism velocity and may be related to the slightly faster response of the simulated actuator.

Figure 7-15 presents fine 5% modulations about 15 kN at 1, 4 and 8Hz. The results show further discrepancy between the simulation and measurement, particularly at 1Hz and 8Hz due to static friction. The friction appears to be under-compensated and there is no brake force response at 8Hz. The controller fails to provide robust performance because the design specification in Table 7-8 has an insufficient allowance for imperfect compensation. The design was formulated with a low level of uncertainty to satisfy the moderate performance specification.
The robust EMB control performance may be benchmarked against the baseline cascaded PI control described in Chapter 4. As an example, Figure 7-16 compares measurements for a light 5 kN brake apply. As might be expected, the robust control performance is conservative because the design allows for the specified uncertainty.
In a further comparison, the robust and baseline controllers were simulated on a perturbed actuator. Model parameters for two perturbed actuators were constructed by arranging the uncertainty in Table 7-5 for a best and worst case scenario. For example, a degraded actuator included characteristics such as increased friction, greater inertia, decreased stiffness and a decreased motor torque constant.

Figure 7-17 shows the open-loop response to a 5 A motor current, simulated on the nominal and perturbed actuators. It may be observed that the uncertainty shown in Table 7-5 produces reasonable variation in the open-loop EMB response. With a high rate of apply, the enhanced actuator approaches 25 kN before undergoing reversal. Meanwhile, the nominal and degraded actuators lock-up without reversal due to static friction.

Brake applies were simulated on the perturbed actuators to compare the robust and cascaded PI control. While the uncertainty seen in Table 7-5 exceeds the robust design specification in Table 7-6, the large perturbations may better expose the control behaviour. Figure 7-18 presents a 10 kN brake apply with two curves for each controller showing the best and worst case scenarios. While both controllers reject the disturbances in Figure 7-17 by the mechanism of feedback, the brake apply profile with the conservative robust control is less sensitive to perturbation. It would appear that some level of performance is sacrificed for a greater degree of certainty.
The robust EMB control appears be less sensitive to actuator perturbations, but it has the disadvantage of being more conservative. As seen in Figure 7-18, the brake apply time with robust control (~0.2 s) is significantly longer than with cascaded PI control (~0.15 s) on both of the perturbed actuators. The robust design could not allow for a useful degree of uncertainty without a significant reduction in performance. With the moderate performance specification described in Table 7-6, the allowance for uncertainty was less than adequate. The insufficient tolerance of uncertainty was particularly apparent during the failed 8Hz modulation of Figure 7-15.

Finally, the robust control design attempts to guarantee stability and performance, but it does not consider the discrete nature of the fixed-point embedded controller. The expectations of the robust design might be questioned on this basis.
7.6 Conclusions

In this chapter a robust control design was proposed for the EMB. The design described a meaningful set of model uncertainty, including parameter uncertainty and imperfect compensation, while avoiding unnecessarily conservative perturbations. However, despite a two-degree of freedom control, a careful structuring of uncertainty, and the use of the modified control architecture described in Chapter 6, the robust control performance was overly conservative when compared with the benchmark established by cascaded PI control.

The need for brake performance conflicted with a conservative handling of uncertainty. In allowing for uncertainty, the performance of the robust control was slow relative to that of the cascaded PI control. The robust controller could manage moderate performance with low uncertainty, or moderate uncertainty with low performance, but not both concurrently. Neither of these outcomes satisfies the original intent of the robust EMB control design.

When benchmarked against the cascaded PI control described in Chapter 4, the robust EMB control was less sensitive to perturbations, but its performance was overly conservative. For example, the time for a 5kN brake application was ~50% longer when a small tolerance for uncertainty was specified. Given the desire for responsive brake performance, the robust control design may not be the most suitable approach for handling uncertainty. Further work in Chapters 8 and 9 will instead concentrate on improving the brake performance and a different approach for the management of actuator uncertainty.
8 Model predictive control

8.1 Introduction

Chapter 7 presented a robust control design for the EMB as an extension to the modified control architecture from Chapter 6. However, the robust control produced an overly conservative brake performance. Instead, this chapter concentrates on optimising the brake performance by extending the modified control architecture with a model predictive control (MPC) design.

A model predictive control is proposed for the EMB to optimise the brake response. The concept of the control design is to use a predictive model to ‘look-ahead’ and optimise the control trajectory. However, a drawback is that the optimisation can be computationally expensive. Hence, a practical implementation is sought to allow real-time solution on the embedded controller of the current EMB.

After some preliminary considerations, the chapter considers the constrained MPC problem for the EMB. A series of simplifications are applied to obtain a practical realisation that is computationally efficient. Experimental and simulation results are presented to demonstrate the performance of the MPC on the test actuator. An ensuing discussion leads to conclusions regarding the benefit of the proposed MPC for the EMB.
Preliminary considerations

Model predictive control may also be known as ‘model based predictive control’ or simply ‘predictive control’. It originated in the late seventies and has been greatly developed since then (Camacho and Bordons, 2004). Its application is becoming more widespread, as indicated by surveys such as (Qin and Badgwell, 2003) and the past reviews referenced therein.

Theory of MPC may be found in texts such as (Maciejowski, 2002) and (Camacho and Bordons, 2004). The concept of MPC is to use a predictive model to look-ahead and plan the control trajectory to minimise an objective cost function, such as a penalty on the tracking error. The structure and operation of a MPC is shown in Figure 8-1. In a common approach, the control trajectory is optimised over a receding horizon. The trajectory is recomputed at each time step using measurements to update the model states. This results in closed-loop control with the mechanism of feedback to handle disturbances.

![Figure 8-1: Structure of a model predictive control](image)
The approach of using a predictive model and appropriate cost function to optimise the control trajectory is intuitive and appealing. The control objective is well defined and information on the plant may be incorporated in the predictive model. MPC also has the unique advantage that it can directly account for actuator saturation. Unlike other control designs, it does not depend on fixes such as integral anti-windup. Hence, it has been particularly successful in industry because it allows operation closer to constraints (Maciejowski, 2002).

One challenge of MPC is that optimisation can be difficult or numerically intensive. Solving the optimal control problem can be complicated for nonlinear systems with constraints, such as the EMB. While methods for optimisation exist, intensive computation can be prohibitive, particularly for online implementations where computational resources are limited.

As computer processors have improved it has become feasible to perform MPC optimisation on systems with faster dynamics. However, with a limited processor and fast brake dynamics, real-time solution may prove challenging on the embedded controller of the present EMB. A full brake apply occurs in the order of 0.15 s and the optimal control trajectory must be computed at 4 ms intervals. Realising a practical implementation is a significant consideration in the MPC design.

Some recent work on MPC has attempted to reduce the computational burden for practical implementation. Approaches such as explicit MPC tend to shift the intensive optimisation offline to obtain a simpler online solution. A survey on explicit approaches for constrained MPC may be found in (Grancharova and Johansen, 2004).

Explicit MPC for linear systems with state and input constraints may be achieved following the approach given in (Bemporad, Morari, Dua and Pistikopoulos, 2000) and (Bemporad, Morari, Dua and Pistikopoulos, 2002). It is shown that because the problem is a piecewise affine function of the states, the same properties are inherited by the controller. As a result, the MPC optimisation may be solved offline and explicitly implemented as a piecewise linear and continuous function of the states.
Explicit MPC for nonlinear systems is feasible by performing computationally intensive optimisation offline and implementing the stored solution online. For example, (Grancharova and Johansen, 2004) suggests partitioning the state-space, solving the optimisation at each node, and saving the optimal control in a search tree that acts like a multi-resolution lookup table.

While explicit MPC solutions can provide efficient implementation, the offline component of the solution is not readily recalculated to re-calibrate the predictive model for slowly-time varying systems. This should be considered since the EMB is subject to parameter variation with wear and temperature. As an extension, parameter variation may be incorporated in an offline solution by performing optimisation over a set of uncertain plants. The optimal control would then be stored in a lookup table with extra dimensions that adjust for the uncertain parameters. However, incorporating parameter variation may exponentially increase the demand on numerical optimisation and require lookup tables with high dimensions.

Another approach to obtain an efficient MPC may be with suitable problem simplification. Simplification for the EMB MPC is discussed in the following section, beginning with the nonlinear control problem and working towards a practical implementation.
8.2 Constrained MPC

Optimisation using the nonlinear EMB model

A MPC may be designed for the EMB using an appropriate predictive model and cost function. A direct optimisation could utilise the EMB model from Chapter 3 and some cost penalty on the tracking error. However, the optimisation would then be nonlinear and challenging to solve.

Solving the nonlinear optimisation for the EMB may be possible using dynamic programming. Various methods are based on Bellman’s principal of optimality which recognises that a section of an optimal policy is itself optimal (Bertsekas, 2005). Hence, a piecemeal construction of the optimal control trajectory (Figure 8-1) is possible by successively optimising an increased end section, starting from the endpoint and working backwards to the initial state. Such approaches are known as backwards dynamic programming. The concept is powerful and suited for general application. Future dynamic programming may provide an ultimate control approach, but presently its computational demand prohibits practical real-time solution for an EMB.

Problem simplification

Since the optimisation must be solved quickly (4 ms) on a limited embedded processor, a simplification to the EMB MPC problem is required for practical computation. In a special case, the optimal solution might reduce to a bang-bang control such as that in Appendix 8A. However, this lacks generality and suffers the problem of control chatter.

In a more general approach, approximate dynamic programming may simplify the optimisation using a value function or ‘cost-to-go’ that is computed offline. A further step would be to solve the entire optimisation offline and save the control solution in a multidimensional lookup table based on the system states and set-point command. All the intensive computation would then be performed offline.
Solving part of the MPC optimisation offline may facilitate a more practical implementation, but it does have disadvantages. Depending on the method, storing part of the control solution can be demanding of memory. Moreover, the offline component of the solution may not be readily updated to handle online plant variation. For example, the EMB friction and stiffness may vary with wear and temperature.

**Problem simplification using the modified EMB control architecture**

A significant simplification to the EMB MPC is possible using the modified control architecture from Chapter 6. Within this control structure, the MPC would be largely isolated from actuator nonlinearity except for the unavoidable case of actuator saturation. Hence, the apparent plant may be reasonably approximated as a linear system with an appropriate saturation on the control input. With this approach, the corresponding MPC problem is greatly simplified.

When the MPC is incorporated within the modified control architecture, the remaining nonlinearity is due to limits on the motor current and velocity. These may be incorporated by constructing the MPC problem with a hard constraint on the motor current and a soft constraint on the motor velocity. System outputs, such as the velocity, are usually relaxed to soft constraints to ensure that a feasible solution exists.

**Constrained MPC problem**

Within the modified EMB control architecture, a constrained MPC may be formulated as the solution to a quadratic programming problem. At each time step, an optimal adjustment of the control trajectory, \( \Delta i_q^* \), is sought to minimise a weighted quadratic cost on the tracking error, \( e_f \), changes in the command, \( \Delta i_q^* \), and the violation, \( \varepsilon \), of the soft velocity constraint,

\[
\min_{\Delta i_q^*} \left( e_f^T Q e_f + \Delta i_q^*^T R \Delta i_q^* + \rho \| e_f \|^2 \right) \\
\text{subject to } i_{q \min} < i_q^* < i_{q \max}
\]

(8-1)

where, \( \Delta i_q^* = \begin{bmatrix} \Delta i_q^* (k) \\ \vdots \\ \Delta i_q^* (k + H_u - 1) \end{bmatrix} \) and \( e_f = \begin{bmatrix} e_f (k) \\ \vdots \\ e_f (k + H_p) \end{bmatrix} \)
The cost function penalises the force tracking error, \( e_f(k) \), up to the prediction horizon, \( H_p \), and changes in control trajectory, \( \Delta i_q^*(k) \), up to the control horizon, \( H_u \). The change in the current control at the sample instant \( k \) is \( \Delta i_q^*(k) = i_q^*(k) - i_q^*(k-1) \).

Meanwhile, the tracking error is \( e_f(k) = v^*(k) - v(k) \), where \( v \) is the linearised force variable within the modified control architecture from Chapter 6. \( i_{q_{\min}}^* \) and \( i_{q_{\max}}^* \) define a hard constraint on the motor current command. Meanwhile, the slack variable, \( \varepsilon \), describes the violation of the soft velocity constraint.

The matrices \( Q = \text{diag}(Q,\ldots,Q) \) and \( R = \text{diag}(R,\ldots,R) \) weight the penalty on the tracking error and changes in control input. Values for \( Q \) and \( R \) were selected such that the penalty on the tracking error was dominant. A small cost on changes in control was only included to avoid chatter. Meanwhile, \( \rho \) weighted a penalty on the violation, \( \varepsilon \), of the soft velocity constraint.

An extended prediction horizon, \( H_p \), may be chosen for improved trajectory planning and stability. The prediction horizon was set to \( H_p=40 \) or \( 0.16 \) s with the control period at \( 0.004 \) s. This was sufficient to look-ahead beyond the duration of a full brake apply. A reduced number of control moves may be chosen to decrease the complexity of an iterative solution. \( H_u=20 \) was set as a sufficient number of control moves.

When the MPC is embedded within the modified control architecture, the apparent plant may be approximated as a linear system with input constraint. The predictive state-space model is given by,

\[
\dot{x} = \begin{pmatrix} -D/J & 0 \\ NK_q & 0 \end{pmatrix} x + \begin{pmatrix} K_v/J \\ 0 \end{pmatrix} i_q^*
\]

\[
v = (0 \ 1)x
\]

where \( D \), \( J \) and \( K_v \) are parameters for the viscous friction, inertia and motor torque constant from Chapter 3. The states are \( x = \begin{bmatrix} \dot{\theta} \\ v \end{bmatrix} \), where \( \dot{\theta} \) is the motor velocity and \( v \) is the linearised force output (Chapter 6). \( K_v=25.6 \) kN/mm was the compound gain of
the stiffness and the chosen inverse function. For implementation, the predictive state-space model in (8.2) was converted to a discrete-time equivalent,

\[ x_{k+1} = Ax_k + Bu_k \]
\[ y_k = Cx_k \]  \hspace{1cm} (8-3)

The MPC formulation neglects that background compensation in the modified control architecture requires a fraction of the available motor torque. However, this is small for most operation and since the predictive model cannot be perfect, some error will be managed by the mechanism of feedback.

**Solving the constrained MPC problem**

While the quadratic programming problem in (8-1) does not have a closed-form algebraic solution, there are methods for numerical solution. These solutions may be obtained from interior-point or active-set methods and algorithms can be found in commercial software such as MATLAB. In offline simulation, the constrained optimisation was solved using Dantzig-Wolfe’s active-set method in MATLAB’s model predictive control toolbox.
8.3 Unconstrained MPC with post-constraint

Unconstrained MPC for online implementation

A further simplification of the MPC was considered necessary to reduce the computational demand for implementation on the current EMB. This was possible by relaxing the constraints so that an analytic closed-form solution was available for the unconstrained optimisation problem,

\[
\min_{\Delta i_q} \left( e^T Q e + \Delta i_q^T R \Delta i_q \right)
\]  \hspace{1cm} (8-4)

With this approach the analytic MPC solution may be computed entirely online.

Application of post-constraint

Limits on the motor current and velocity are important to avoid burnout or overdrive. A post-constraint was applied to the MPC solution to restrict the current command, \( i_q^* \), to the range \([i_{q_{\min}}^*, i_{q_{\max}}^*]\). In addition, the velocity limit was imposed with a further bound on the current command, \([i_{q\theta_{\min}}^*(\dot{\theta}), i_{q\theta_{\max}}^*(\dot{\theta})]\). This was adjusted dynamically to restrict the motor torque and avoid exceeding the velocity limit. When the bounds were combined, the current command was limited by,

\[
\max(i_{q_{\min}}^*, i_{q\theta_{\min}}^*(\dot{\theta})) \leq i_q^* \leq \min(i_{q_{\max}}^*, i_{q\theta_{\max}}^*(\dot{\theta}))
\]  \hspace{1cm} (8-5)

The dynamic constraint, \([i_{q\theta_{\min}}^*(\dot{\theta}), i_{q\theta_{\max}}^*(\dot{\theta})]\), was calculated to prevent acceleration beyond the velocity limits over \( n \) future control periods of duration \( T \). The acceleration bounds were determined by,

\[
\dot{\theta}_{\text{min}} = \frac{(\dot{\theta}_{\text{min}} - \dot{\theta})}{nT} \quad \text{and} \quad \dot{\theta}_{\text{max}} = \frac{(\dot{\theta}_{\text{max}} - \dot{\theta})}{nT}.
\]

The motor torque was then constrained by,

\[
J \ddot{\theta}_{\text{min}} \leq T_m - T_L - T_F \leq J \ddot{\theta}_{\text{max}}
\]

such that,

\[
J \ddot{\theta}_{\text{min}} + T_L + T_F \leq T_m \leq J \ddot{\theta}_{\text{max}} + T_L + T_F
\]  \hspace{1cm} (8-6)
The friction torque, $T_F$, may be neglected conservatively since it opposes motion and always acts to reduce the velocity,

$$ J\dot{\theta}_{\text{min}} + T_L \leq T_m \leq J\dot{\theta}_{\text{max}} + T_L $$

or in terms of the velocity limits,

$$ \frac{J(\dot{\theta}_{\text{min}} - \dot{\theta})}{nT} + F_{cl}N \leq i_{q\theta}^* K_f \leq \frac{J(\dot{\theta}_{\text{max}} - \dot{\theta})}{nT} + F_{cl}N $$

(8-7)

Limits on the current command are then given by,

$$ i_{q\theta \text{min}}^*(\dot{\theta}, F_{cl}) = \frac{J(\dot{\theta}_{\text{min}} - \dot{\theta})}{K_i nT} + \frac{F_{cl}N}{K_f} $$

$$ i_{q\theta \text{max}}^*(\dot{\theta}, F_{cl}) = \frac{J(\dot{\theta}_{\text{max}} - \dot{\theta})}{K_i nT} + \frac{F_{cl}N}{K_f} $$

(8-8)

where $n$ is a small number of control intervals, two in this case, over which the acceleration is restricted to avoid velocity overshoot.

To maintain feasibility with (8.5), the dynamic limits are restricted by,

$$ i_{q\theta \text{min}}^*(\dot{\theta}) < i_{q\theta \text{max}}^*(\dot{\theta}) $$

(8-9)

**Solving the unconstrained MPC problem**

The unconstrained optimisation problem has a known analytic solution and for convenience a derivation is provided in Appendix 8B. The result is that optimal changes in the input are given by,

$$ \Delta i_q^* = -(\Gamma^T Q\Gamma + R)^{-1} \Gamma^T Q(\Psi - \psi^*) $$

(8-10)

where $\psi^*$ is the projected command and the matrices $\Gamma$ and $\Psi$ are determined from the state-space model, the measured states and the prediction horizons as follows.
With the prediction model of (8-2) expressed in the discrete form of (8-3),
\[ x_{k+1} = Ax_k + Bu_k \]
\[ y_k = Cx_k \]

and re-writing the quadratic cost function from (8-4) as,
\[ J = (v - v^*)^T Q (v - v^*) + \Delta i_q^T R \Delta i_q^* \]  \hspace{1cm} (8-11)

the target trajectory, \( v^* \), input changes, \( \Delta i_q^* \), projected output, \( v \), and the penalty weights, \( Q \) and \( R \), are given by,
\[
\begin{bmatrix}
 v_{k+1}^* \\
 \vdots \\
 v_{k+H_p}^*
\end{bmatrix}
\begin{bmatrix}
 \Delta i_{q_k}^* \\
 \vdots \\
 \Delta i_{q_k+H_{p-1}}^*
\end{bmatrix}
\begin{bmatrix}
 v_k \\
 \vdots \\
 v_{k+H_p}
\end{bmatrix}
= \begin{bmatrix}
 C \\
 \vdots \\
 0
\end{bmatrix}
\begin{bmatrix}
 x_{k+1} \\
 \vdots \\
 x_{k+H_p}
\end{bmatrix}
= \begin{bmatrix}
 \Omega \\
 \vdots \\
 \Omega
\end{bmatrix}
\]
\[
Q = \begin{bmatrix}
 Q_1 & 0 & 0 \\
 0 & \ddots & 0 \\
 0 & 0 & Q_{H_p}
\end{bmatrix}
\text{and } R = \begin{bmatrix}
 R_0 & 0 & 0 \\
 0 & \ddots & 0 \\
 0 & 0 & R_{H_{p-1}}
\end{bmatrix}
\hspace{1cm} (8-12)
\]

and the matrices \( \Psi \) and \( \Gamma \) are,
\[
\Psi = \begin{bmatrix}
 A \\
 \vdots \\
 A^{H_p}
\end{bmatrix} x_k + \begin{bmatrix}
 B \\
 \vdots \\
 \sum_{i=0}^{H_{p-1}} A'B
\end{bmatrix} u_{k-1} \text{ and } \Gamma = \begin{bmatrix}
 B & \cdots & 0 \\
 AB + B & \cdots & 0 \\
 \vdots & \ddots & \vdots \\
 \sum_{i=0}^{H_{p-1}} A'B & \cdots & A + B \\
 \sum_{i=0}^{H_{p-1}} A'B & \cdots & \sum_{i=0}^{H_{p-2}} A'B
\end{bmatrix}
\hspace{1cm} (8-13)
\]

(Maciejowski, 2002) suggests the solution (8-10) may be computed efficiently as,
\[
\Delta i_q^*(k)_{opt} = \begin{bmatrix} K_1 & K_2 & K_3 & K_4 \end{bmatrix}
\begin{bmatrix}
 v^*(k) \\
 \dot{\theta}(k) \\
 v(k) \\
 i_q^*(k-1)
\end{bmatrix}
\hspace{1cm} (8-14)
\]

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Solving the unconstrained optimisation (8-4) with a post constraint on the input (8-5) is different to, and less desirable than, solving the constrained optimisation problem (8-1). However, this simplification was necessary for practical implementation given the computational limits of the embedded controller.

A limitation of the unconstrained MPC is that the optimised control trajectory does not explicitly account for the actuator limits. For example, the unconstrained MPC could over-anticipate its access to decelerating torque and suffer overshoot. To help avoid this scenario, the cost-function weight $R$ may be re-tuned for a greater penalty on input changes. This tends to produce smaller control adjustments towards a steady-state command so that excursions into saturation may be reduced to a manageable level. The trade-off is that a gentler control trajectory leads to a more conservative brake response. While unconstrained MPC has an efficient implementation, it will drive a more conservative brake response than the constrained MPC.

When the weight $R$ is tuned to limit excursions into saturation, an appropriate value depends on the number of control moves, $H_u$. The control action can be tempered with $R$ and $H_u$ set to produce many small moves or a few large moves. Taking the latter approach for a more rapid response, the control horizon, $H_u$, was reduced. The prediction horizon for the unconstrained MPC was set to $H_p=38$ and the number of control moves was $H_u=3$. This gave a prediction horizon of 0.152 s with the discrete control period at 0.004 s. The prediction horizon was sufficient to look-ahead over a full brake apply.
8.4 Results and discussion

The results are presented beginning with simulations of the constrained MPC for the EMB. The constrained and unconstrained MPC are then compared in simulation, the latter being a less sophisticated version for real-time computation on the present EMB. Following implementation, experimental results are presented for the unconstrained MPC with post-constraint. Its performance is benchmarked against the cascaded PI control and the modified control from Chapters 4 and 6. Further, its benefit is assessed during simulated ABS braking.

Constrained MPC with trajectory planning

Ideally, the EMB command would include not only the set-point reference, but also the intended brake trajectory. The planned trajectory could then be used to improve the tracking performance. Anticipating the brake demand may be difficult when the driver's intent is unknown, but there may be some possibilities. For example, a high pedal apply rate during panic braking might be used to anticipate an emergency brake manoeuvre. Otherwise, some advantage may be possible with antilock brake systems that command brake trajectories rather than a set-point. Alternatively, a future collision-avoidance program may schedule a brake force trajectory to coordinate an evasive manoeuvre.

To test the benefit of commanding a brake trajectory rather than a set-point, the MPC was simulated with and without knowledge of the intended trajectory. Without the trajectory plan, the set-point was projected to the prediction horizon with a zero-order hold,

\[ u^*_i = u^*_k \quad \text{for} \quad i = k+1, \ldots, k+H_p \]  

(8-15)

Other projections such as a first-order hold are possible in place of (8-15), but the alternatives also have limitations in terms of anticipating driver intent.

Figure 8-2 shows a demanding 5kN brake modulation about 20kN at 8Hz, simulated using the constrained MPC with and without knowledge of the intended trajectory.
The result suggests that a significant improvement is possible when a trajectory plan is available. It is apparent that the lag in the brake response largely results from a limited knowledge of the intended brake trajectory.

![Figure 8-2: Simulated 5 kN modulation about 20 kN at 8Hz using constrained MPC with and without knowledge of the intended brake trajectory](image)

**Constrained MPC versus unconstrained MPC with post-constraint**

The less sophisticated unconstrained MPC with post-constraint reduced the computational burden to facilitate implementation on the present EMB. However, it had an increased penalty weight on input changes, $R$, to avoid excessive excursions into saturation. Hence, the trade-off was a more conservative response as shown in Figure 8-3. In this example, the brake response with the unconstrained MPC shows increased lag and attenuation as the control drives the brake more conservatively.

![Figure 8-3: Simulated 5 kN modulation about 20 kN at 8Hz with the constrained MPC and unconstrained MPC with post-constraint](image)
Unconstrained MPC with post-constraint in experimentation

Comparing Figure 8-2 and Figure 8-3, there was a reduction in performance without knowledge of the intended brake trajectory and a further reduction with the less sophisticated MPC. While some performance was sacrificed, the unconstrained MPC with post-constraint reduced the computational burden to allow implementation on the present EMB. Once the control was programmed, experiments were conducted to test its performance.

Figure 8-4 compares measurements and simulations for a 5, 15 and 25 kN brake apply using the unconstrained MPC with post-constraint. A marginally faster simulated response was consistent with the EMB model behaviour in Chapter 3. Other than the slight lead, the simulation produced reasonable predictions for the control action, mechanism velocity and brake force profile.

![Figure 8-4: Measured and simulated brake applies to 5, 15 and 25 kN using unconstrained MPC with post-constraint](image-url)
In addition to the step brake applies, the simulation fidelity was also tested for large
and fine modulations. Figure 8-5 shows the results for a large 75% amplitude
modulation about 15 kN. Meanwhile, Figure 8-6 shows a fine 5% modulation.
Generally, there is good agreement between the simulation and measured results.

As previously in Chapters 4 and 6, the brake performance was limited during large
modulations at 8Hz due to the actuator saturation. In this case, the saturated control is
almost bang-bang as it struggles to execute the commanded manoeuvre. Conversely,
the performance of the fine modulation at 8Hz is the result of the control software and
not limited by the actuator hardware.

![Figure 8-5: Measured and simulated 75% modulations about 15kN at 1, 4 and 8Hz using
unconstrained MPC with post-constraint](image-url)
Comparison with the baseline and modified control

To assess its benefit, the unconstrained MPC with post-constraint was tested against the cascaded and modified controllers from Chapters 4 and 6. Experimental tests were run for a large brake apply (Figure 8-7), a small apply (Figure 8-8), and a fine modulation (Figure 8-9) to determine the improvement. The brake applies were started from a light 7.5kN load to avoid the clearance control zone near zero load.

For large manoeuvres, such as the brake apply in Figure 8-7, the unconstrained MPC has little benefit because the response is mainly limited by actuator constraints. In this example, the current is limited up to $t\approx0.04$ s, whereafter the velocity is limited to approximately $t\approx0.06$ s. Only a slight improvement is managed with the unconstrained MPC by maintaining the maximum velocity for longer and decelerating harder.
Figure 8-7: A large magnitude brake manoeuvre comparing the responses of the cascaded PI control, the modified control and the unconstrained MPC with post-constraint

The unconstrained MPC has a greater benefit during small manoeuvres, such as the brake apply in Figure 8-8. Here a significantly reduced rise time is achieved with the unconstrained MPC (0.019 s) when compared with the modified control (0.035 s), and the baseline cascaded PI control (0.066 s). The improvement is achieved through a better utilisation of the available motor torque.

<table>
<thead>
<tr>
<th>EMB controller</th>
<th>Rise time $t_{10-90%}$ (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unconstrained MPC with post-constraint</td>
<td>0.019</td>
</tr>
<tr>
<td>Modified control (Chapter 6)</td>
<td>0.035</td>
</tr>
<tr>
<td>Cascaded PI control (Chapter 4)</td>
<td>0.066</td>
</tr>
</tbody>
</table>

Table 8-1: 10-90% rise times for the 1kN apply in Figure 8-8
It might be noted that Figure 8-8 shows slight initial motor motions without a significant corresponding change in the measured clamp force. This also occurs during the velocity reversals, between 0.04-0.06 s and 0.07-0.13 s, where there is negligible change in the clamp force. The same phenomenon was noted in Chapter 6 and may be further observed in Figure 8-9 for the cascaded control at 2% amplitude where there are slight motions without variation in the measured force. Hence, there appears to be some internal mechanism compliance. To avoid the effect, the 10-90% rise time was chosen as a suitable performance measure and was compared in Table 8-1.
Figure 8-9 shows 8Hz modulations of 2% about 25 kN and 5% about 15 kN. These demanding manoeuvres were chosen to test the control performance. The cascaded PI control tended to suffer lockup with a poor handling of the static friction. The modified control improved the management of the actuator nonlinearity using compensation and an inverse gain schedule. A further improvement was then achieved by including the unconstrained MPC with post-constraint. Improvements in the amplitude and phase response are summarised in Table 8-2 and the benefits are more pronounced during fine manoeuvres.

<table>
<thead>
<tr>
<th>5% modulation about 15 kN</th>
<th>Amplitude (%)</th>
<th>Phase lag (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cascaded PI control</td>
<td>43</td>
<td>83</td>
</tr>
<tr>
<td>Modified control</td>
<td>77</td>
<td>76</td>
</tr>
<tr>
<td>Unconstrained MPC with post-constraint</td>
<td>88</td>
<td>61</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>2% modulation about 25 kN</th>
<th>Amplitude (%)</th>
<th>Phase lag (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cascaded PI control</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td>Modified control</td>
<td>58</td>
<td>106</td>
</tr>
<tr>
<td>Unconstrained MPC with post-constraint</td>
<td>84</td>
<td>85</td>
</tr>
</tbody>
</table>

Table 8-2: Results summary for 8Hz modulations with cascaded PI control, the modified control and the unconstrained MPC with post-constraint.
**Anti-lock brake performance**

The benefit of the unconstrained MPC with post-constraint was investigated during simulated ABS braking. The ABS maintains vehicle steering by avoiding wheel lockup and may also act to reduce the stopping distance. A CarSim model of the BMW test vehicle and its ABS was available from (Oliver, 2007). This provided a test environment for incorporating the EMB model and simulating the brake-by-wire vehicle.

Straight line ABS stops were simulated to compare the EMB performance with the unconstrained MPC and the baseline control from Chapter 4. The example in Figure 8-10 shows the speed of the vehicle and front right wheel during a simulated ABS stop on a dry road. The plot also shows the corresponding wheel slip, \((v_{\text{vehicle}} - v_{\text{wheel}})/v_{\text{vehicle}}\). Excursions into slip are influenced by the tyre dynamics, suspension and the body dynamics of the vehicle. However, the simulation predicts significantly reduced excursions into wheel slip with the unconstrained MPC control and this is desirable for maintaining steerability.

The stopping distances were similar in Figure 8-10 for the two braking scenarios. Actually, the unconstrained MPC produced a slight increase as the corresponding ABS regulation was further below the peak friction at 8% slip. Since the ABS was not re-tuned for the MPC, future work might consider an improved ABS algorithm to exploit the enhanced EMB control.

Further ABS stops were simulated on a wet and tiled (ice-like) surface. The braking distance was measured from the point of brake application \((F_{cl} > 20 \text{ N})\) until the vehicle velocity decreased below 0.5 m/s. The results are summarised in Table 8-3 for the cascaded PI control and unconstrained MPC. While the results are mixed, future work might reduce the vehicle stopping distance by designing the ABS to utilise the improved EMB control.
Figure 8-10: Simulated ABS stop on a dry surface with PGT V5 EMBs using the baseline cascaded PI control and the unconstrained MPC with post-constraint

<table>
<thead>
<tr>
<th>Surface – Dry from 98 km/h</th>
<th>Braking distance (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>V5EMB - Cascaded PI control</td>
<td>47.0</td>
</tr>
<tr>
<td>V5EMB - Model predictive control</td>
<td>47.5</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Surface – Wet from 103 km/h</th>
<th>Braking distance (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>V5EMB - Cascaded PI control</td>
<td>82.1</td>
</tr>
<tr>
<td>V5EMB - Model predictive control</td>
<td>81.4</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Surface – Tiles (simulated ice) from 48 km/h</th>
<th>Braking distance (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>V5EMB - Cascaded PI control</td>
<td>41.9</td>
</tr>
<tr>
<td>V5EMB - Model predictive control</td>
<td>40.9</td>
</tr>
</tbody>
</table>

Table 8-3: Simulated ABS braking distance on a dry, wet and tiled surface, comparing the cascaded PI control and the unconstrained MPC with post-constraint
8.5 Conclusions

Extending the modified control architecture from Chapter 6 with a MPC design to optimise performance produced an incremental improvement in the EMB control.

To reduce the computational burden of the optimisation, a simplification was devised for the EMB MPC problem. The modified control architecture provided a framework whereby the apparent actuator seen by MPC could be approximated using a linear system with an input constraint. While this would be the preferred design, a further simplification was necessary for real-time implementation on the present EMB. Hence, a less sophisticated unconstrained MPC with post-constraint was proposed, sacrificing some performance for computational efficiency.

Since the large EMB manoeuvres were constrained by hardware, the benefit of the MPC control was most pronounced during fine brake manoeuvres. In comparison with the modified control from Chapter 6, the rise time for a 1 kN adjustment at 7.5 kN was improved from 35 ms to 19 ms. Meanwhile, the amplitude response for a fine 2% modulation about 25 kN at 8Hz was improved from 58% to 84%. The improvement was even more dramatic against the baseline cascaded PI control from Chapter 4. During simulated ABS manoeuvres, the improved EMB response translated to reduced excursions into wheel slip such that the vehicle steerability was maintained. Future work might manage a further improvement in ABS and other driver assist functions with algorithms designed to utilise the enhanced EMB control.
9 Online estimation and adaptive control

Given that the EMB parameters, particularly those for friction and stiffness, are uncertain and may vary with wear and temperature, some tolerance or management should be considered in the control design. To allow for model uncertainty, the modified control architecture described in Chapter 6 was extended with a robust control design in Chapter 7, but this produced an overly conservative brake performance. Taking another approach, Chapter 8 incorporated a model predictive control within the modified architecture to optimise the brake performance and this chapter considers an adaptive implementation to manage parameter variation.

An adaptive implementation is proposed for the EMB control, using online parameter estimation to update the prediction model of the MPC (Chapter 8), maintain effective friction compensation and to adjust the inverse stiffness function in the modified control architecture. In the first part of this chapter, new methods are proposed for online estimation of the EMB parameters. In the second part, the parameter estimates are used to update the control and maintain its performance. The estimation routines are demonstrated on measured data and the control is simulated on a perturbed actuator. The results indicate the feasibility of the adaptive control and its potential for maintaining optimal EMB performance.
9.1 Methods for online parameter estimation

Three estimation routines are proposed to identify parameters for the EMB inertia ($J$), friction ($D, C, G$) and stiffness ($\alpha$). Each routine is demonstrated on measurements from the EMB. The proposed inertia estimate is aimed at improving on the convenience and accuracy of the identification model described in Chapter 3. However, it is not necessary for adaptive control since the inertia should remain relatively constant. Conversely, the EMB friction and stiffness estimates may be particularly useful to update the control and account for variation with wear and temperature.

Measured signals

The measurements available for online estimation were the brake clamp force, $F_{cl}$, motor current, $i_q$, and position, $\theta$. Signals for the motor velocity, $\dot{\theta}$, and acceleration, $\ddot{\theta}$, were derived from the position measurement.

Signal processing commonly involves low-pass filtering to reduce high frequency noise. Because a number of measurements may be used for each estimate, some consideration is important to avoid relative phase lag between the signals. The relative phase shifting of data signals can be minimised with careful filter design. Alternatively, zero-phase filters may be applied at the expense of buffering a data window, as shown in Figure 9-1. It is sufficient for the parameter estimation to be performed on recent historical data since the parameters are slowly time varying.

![Figure 9-1: Time buffer to filter historical data with zero phase shift](image)
Signals for the velocity, $\dot{\theta}$, and acceleration, $\ddot{\theta}$, were differentiated from measurements of the motor position, $\theta$. After high frequency noise was attenuated with a zero-phase low-pass digital filter, numerical differentiation was performed by a centred finite-divided-difference (Chapra and Canale, 1989),

$$f'(x_i) = \frac{f(x_{i+1}) - f(x_{i-1})}{2\Delta x}$$  \hspace{1cm} (9-1)

Digital signal processing was performed at a high rate, $f_1=25$ kHz, and sampled at lower frequency, $f_2=1$ kHz, for the estimation routines in this section. The parameters for the inertia ($J$), friction ($D,C,G$) and stiffness ($\alpha$) are either constant or slowly-time varying such that estimation may be performed at a reduced rate and with gains set for slow convergence.
Effective inertia ($J$)

Signals and parameters

An observer for the effective inertia, $J$, is proposed using measurements of the EMB motor current, $i_q$, acceleration, $\ddot{\theta}$, direction, $\text{sign}(\dot{\theta})$, and the clamp force, $F_{cl}$. The estimation assumes knowledge of the gear ratio, $N$, and torque constant, $K_t$, and these parameters are often easier to identify accurately.

Estimation routine

An inertia estimate, $\hat{J}$, is based on knowledge that the friction always opposes the direction of motion. The approach does not require knowledge of friction parameters or the friction magnitude, $|T_F|$, only its direction as indicated by the velocity. With the positive directions shown in Figure 9-2, $\text{sign}(T_F) = \text{sign}(\dot{\theta})$.

Figure 9-2: True friction torque opposing the direction of motion

To determine its direction, the friction torque is calculated using,

$$\hat{T}_F(k) = i_q K_t - F_{cl} N - \hat{J} \ddot{\theta}$$  \hspace{1cm} (9-2)

Equation (9-2) may be re-formulated with $T_F$ as the true friction and $e_{TF}$ as the estimation error,

$$(T_F + e_{TF}) = i_q K_t - F_{cl} N - \hat{J} \ddot{\theta}$$  \hspace{1cm} (9-3)

Error in the inertia estimate, $\hat{J}$, is apparent when it produces a friction value, $\hat{T}_F$, that erroneously acts in the same direction as the velocity, $\text{sign}(\hat{T}_F) \neq \text{sign}(\dot{\theta})$. The precise error, $e_{TF}$, is unknown, but may be conservatively estimated as the distance to
the origin, \( e_{TF0} \), since the estimate, \( \hat{T}_F \), and the true value, \( T_F \), are known to act in opposite directions,
\[
|e_{TF}| > |\hat{T}_F| \quad \text{if sign}(\hat{T}_F(k)) \neq \text{sign}(\dot{\theta}(k))
\]
\[
|e_{TF}| \geq 0 \quad \text{otherwise}
\]

(9-4)

An example of the first condition is shown on the number line in Figure 9-3.

Figure 9-3: Number line showing friction, \( T_F \), and the estimate, \( \hat{T}_F \), when they erroneously act in opposite directions such that \( \text{sign}(\hat{T}_F) \neq \text{sign}(\dot{\theta}) \)

When the error \( e_{TF0}(k) = -\hat{T}_F(k) \) is detected the inertia estimate may be updated,
\[
\hat{J}(k) = \hat{J}(k - 1) - L_1 \frac{e_{TF0}(k)}{\dot{\theta}(k)} \quad \text{when sign}(\hat{T}_F(k)) \neq \text{sign}(\dot{\theta}(k)) \text{ and } |\dot{\theta}(k)| > \varepsilon_{\dot{\theta}}
\]

(9-5)

otherwise, \( \hat{J}(k) = \hat{J}(k - 1) \).

The first condition in (9-5) enables the update law when the friction estimate and velocity erroneously act in the same direction. The second condition on the velocity magnitude avoids a finitely small band, \( \pm \varepsilon_{\dot{\theta}} \), around zero velocity. An additional slew rate limit, \( |L_1 e_{TF0}/\dot{\theta}| \leq \Delta_{\text{max}} \), was applied to limit any peak noise effects.

The observer may be designed for slow convergence (Appendix 9A) of the estimate. If the estimator gain is set less than unity, \( L_1 < 1 \), the inertia estimate is updated with less than the true error, \( |e_{J0}(k)| = \left| \frac{e_{TF0}(k)}{\dot{\theta}} \right| \ll |J - \hat{J}(k)| \). Once the inertia estimate converges it is possible to hold the result.
**Simulated results**

Using the EMB model described in Chapter 3, the inertia observer was tested in simulation where the true value was known. Figure 9-4 shows the outcome over a sinusoidal 4Hz brake excitation with the estimate initialised at zero, \( \dot{J}(0) = 0 \). After 20 s the inertia estimate, \( \dot{J} \), has converged to 97.1% of the true value. Some steady-state error may remain due to the conditions imposed on the update law (9-5), such as the velocity bound \( \pm \varepsilon_\theta \). The conditions are also the cause of the event-driven adjustments in the estimate.

![Figure 9-4: Simulated inertia estimate, \( \dot{J} \), normalised against the true value](image-url)
Experimental results

The inertia estimate was tested on measurements taken from EMB 19 and compared with the value from system identification in Chapter 3. The results given in Figure 9-5 show the inertia estimate converging to within 107% of the previously identified value.

![Figure 9-5: Experimental inertia estimate, \( \dot{J} \), normalised against the value identified in Chapter 3](image)

Further tests were undertaken to investigate the 7% discrepancy between the inertia estimate and the identification reported in Chapter 3. Figure 9-6 shows a 4Hz modulation and the friction estimate from (9-2) using the previously identified inertia. Error in the friction estimate may be observed when it acts in the same direction as the velocity. These periods are highlighted red, and occur when the acceleration and motor torque are high during deceleration on release.

The friction was re-computed on the same data using the 7% increased inertia estimate. Figure 9-7 shows the adjusted friction estimate changes sign nearer to each reversal than in Figure 9-6. This might suggest the inertia estimate is more accurate than the identification in Chapter 3, or it could indicate the estimate is better adjusted to some error in the parameters \( N \) and \( K_t \) or the measurements in (9-2).
The EMB inertia estimate converged towards the known value in simulation and agreed with the previous identification to within 7%. The new estimate provides a simple alternative to the lengthy calculation of component masses and moments of inertia described in Chapter 3.
Friction parameters (D, C, G)

Approaches for identifying friction in other systems have included regression analysis, model reference estimation and Lyapunov-based methods (Armstrong-Hélouvry et al., 1994). The latter two approaches particularly avoid calculations with acceleration signals. For example, a parameter estimate may be updated with PI action on the error between the measured and modelled velocity. Alternatively, an update law may be obtained from a Lyapunov-like argument involving the position error. When compared with regression, “the other algorithms may be seen as implementing filtered versions of the normal equations” (Armstrong-Hélouvry et al., 1994). While a reasonable acceleration signal may be obtained from EMB measurements, the friction estimate must identify parameters that include load dependency.

Signals and parameters

An estimate of the viscous, Coulomb and load-dependent friction parameters, D, C and G, is proposed using measurements of the EMB motor current, \( i_q \), acceleration, \( \ddot{\theta} \), and the clamp force, \( F_{cl} \). The estimation assumes knowledge of the gear ratio, \( N \), the torque constant, \( K_t \), and the inertia, \( J \), which is reasonable given these parameters are not expected to vary significantly once identified.

Estimation routine

The procedure has two steps that involve calculating the friction, \( \hat{T}_r \), and then estimating the friction parameters, D, C, and G, by a least-squares regression.

The friction torque is estimated by,

\[
\hat{T}_r = i_q K_t - F_{cl} N - J \ddot{\theta} \tag{9-6}
\]

During motion, a linear regression for the friction parameters can be constructed from,

\[
\hat{T}_r = D \dot{\theta} + (C + GF_{cl}) \text{sign}(\dot{\theta}) \quad \text{when } |\dot{\theta}| > \varepsilon \tag{9-7}
\]
For example, regression matrices may be written,

\[
\begin{bmatrix}
\hat{T}_F^1 \\
\hat{T}_F^2 \\
\vdots \\
\hat{T}_F^{e1} \\
\end{bmatrix} =
\begin{bmatrix}
\hat{\theta}_i \\
\hat{\theta}_j \\
\vdots \\
\hat{\theta}_e \\
\end{bmatrix}
\begin{bmatrix}
F_{e1} \text{sign}(\hat{\theta}_i) \\
F_{e2} \text{sign}(\hat{\theta}_j) \\
\vdots \\
F_{e} \text{sign}(\hat{\theta}_e) \\
\end{bmatrix}
\begin{bmatrix}
\hat{D} \\
\hat{G} \\
\hat{C} \\
\end{bmatrix}
\text{ when } |\hat{\theta}| > \varepsilon
\tag{9-8}
\]

A recursive least-squares solution is appropriate for online implementation, in which case, the regression matrices at each sample instant, \(k\), are,

\[
\hat{T}_F(k) =
\begin{bmatrix}
\hat{\theta} \\
F_e \text{sign}(\hat{\theta}) \\
\vdots \\
F_e \text{sign}(\hat{\theta}) \\
\end{bmatrix}
\begin{bmatrix}
\hat{D} \\
\hat{G} \\
\hat{C} \\
\end{bmatrix}
\text{ when } |\hat{\theta}| > \varepsilon \text{ and } |\hat{T}_F| < \varepsilon_2
\]

and may be written as,

\[
y(k) = \mathbf{x}^T(k) \hat{\Theta}
\tag{9-9}
\]

The second condition, \( |\hat{T}_F| < \varepsilon_2 \), provided a feasibility check that the friction magnitude was less than a large 3 Nm bound.

Recursive least-squares estimates are well known and one solution in (Ljung, 1999) is given by,

\[
\hat{\Theta}_k = \hat{\Theta}_{k-1} + L(k)[y(k) - \mathbf{x}^T(k) \hat{\Theta}_{k-1}]
\tag{9-10}
\]

where,

\[
L(k) = \frac{P(k-1)\mathbf{x}(k)}{\bar{\lambda}(k) + \mathbf{x}^T(k)P(k-1)\mathbf{x}(k)}
\]

\[
P(k) = \frac{1}{\bar{\lambda}(k)} \left[ P(k-1) - \frac{P(k-1)\mathbf{x}(k)\mathbf{x}^T(k)P(k-1)}{\bar{\lambda}(k) + \mathbf{x}^T(k)P(k-1)\mathbf{x}(k)} \right]
\]

and \(0 < \bar{\lambda} \leq 1\) is the forgetting factor.

The friction parameters may be slowly time-varying and a slow forgetting factor, \( \bar{\lambda} \), was specified close to unity.
Simulated results

The friction estimation was first tested in simulation where the true parameter values were known, using the EMB model described in Chapter 3. The results given in Figure 9-8 show the estimated parameters during a 4Hz brake modulation following zero initialisation. After 1s the parameter estimates for $D$, $G$, and $C$ have all converged to within 1% of the true value. As mentioned previously, a slow convergence of the estimate is suitable since the friction parameters are slowly time-varying.

![Figure 9-8: Simulated estimate of the friction parameters $D$, $G$ and $C$, normalised against the true values](image)

Here the root-mean-square-error is given by $\sqrt{[(D - \hat{D})^2 + (G - \hat{G})^2 + (C - \hat{C})^2]/3}$
Experimental results

The friction estimate was further tested on measurements taken from EMB 19. The inertia observer from (9-4) was activated to improve the friction torque estimate, $\hat{T}_F$. Figure 9-9 shows convergence of the estimated friction parameters over the same 100 s data record shown earlier in Figure 9-5. The parameters are plotted against those identified in Chapter 3 and the steady-state values are given in Table 9-1.

![Figure 9-9: Estimated friction parameters $D$, $C$ and $G$, and the values identified in Chapter 3](image)

<table>
<thead>
<tr>
<th></th>
<th>$D$ (Nm/rad/s)</th>
<th>$C$ (Nm)</th>
<th>$G$ (Nm/N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>System identification</td>
<td>$3.95 \times 10^{-4}$</td>
<td>$3.04 \times 10^{-2}$</td>
<td>$1.17 \times 10^{-5}$</td>
</tr>
<tr>
<td>Parameter estimate</td>
<td>$1.73 \times 10^{-4}$</td>
<td>$5.31 \times 10^{-2}$</td>
<td>$1.38 \times 10^{-5}$</td>
</tr>
</tbody>
</table>

Table 9-1: Friction parameters from the system identification and adaptive estimate
Comparing the estimated friction parameters with those identified in Chapter 3, there was reasonable agreement for the friction load dependency, \( G \), the most significant friction parameter, with an 18% difference in the steady-state value. Figure 9-10 shows the dominant correlation between the estimated friction and the clamp load. The data scatter is partly due to viscous friction, static friction at reversals and measurement noise.

The significance of the differing parameter values is apparent when they are used to model friction. Figure 9-11 shows the friction and calculated values using the adaptive and static friction parameters. Reasonable estimates are obtained from both parameter sets, but there is an improvement with the adaptive friction estimate. It is possible that the mechanism friction changed since the earlier parameter identification, in which case, an adaptive estimate for the friction parameters would be preferred.

Figure 9-10: Friction, \( \hat{\tau}_f \), from 50-100s in Figure 9-9 plotted against the clamp force, \( F_{cl} \)
Generally, the friction parameter estimate was successful and converged to the known values in simulation. Moreover, it produced an adaptive friction estimate that suitably described the EMB mechanism friction.
Extension to bi-directional friction estimation

With the improved inertia estimate, the friction apparent in Figure 9-10 appeared slightly elevated on apply. A directional friction bias might arise from the friction distribution within the EMB mechanism. For example, consider the two scenarios in Figure 9-12. In one case, \( F_b = F_{cl} + F \) and in the other, \( F_b = F_{cl} - F \), with the frictional bias depending on the direction of motion. As shown in Figure 9-13, the scenario is comparable to that within the EMB mechanism. Hence, the load, \( F_b \), on the thrust bearing depends on the direction of motion. The proportional friction loss within the thrust bearing, \( c \), will also depend on the direction of motion. With a similar effect in other parts of the transmission, it is apparent how a directional friction bias may be produced.

\[
\begin{align*}
F_b &= F_{cl} + F \\
F_b &= F_{cl} - F
\end{align*}
\]

Figure 9-12: Forces on a sliding element

Figure 9-13: Frictional losses (a,b,c...) between the components (A,B,C...) of the mechanism
Considering the possibility of a friction bias, a bi-directional estimation was investigated. The estimation is the same as before, except two sets of parameters, $(\hat{D}, \hat{C}, \hat{G})_{\text{apply}}$ and $(\hat{D}, \hat{C}, \hat{G})_{\text{release}}$, are updated depending on the direction of motion. The approach was tested over the same data as that provided in Figure 9-9. The convergence of the friction parameters is shown in Figure 9-14 and the steady-state values are summarised in Table 9-2. The friction on release appeared to be approximately 80% of that on apply.

![Figure 9-14: Estimated bi-directional friction parameters against values identified in Chapter 3](image)

<table>
<thead>
<tr>
<th></th>
<th>D (Nm/rads$^{-1}$)</th>
<th>C (Nm)</th>
<th>G (Nm/N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>System identification</td>
<td>$3.95 \times 10^{-4}$</td>
<td>$3.04 \times 10^{-2}$</td>
<td>$1.17 \times 10^{-5}$</td>
</tr>
<tr>
<td>Parameter estimate – apply</td>
<td>$1.83 \times 10^{-4}$</td>
<td>$6.76 \times 10^{-2}$</td>
<td>$1.48 \times 10^{-5}$</td>
</tr>
<tr>
<td>Parameter estimate - release</td>
<td>$1.60 \times 10^{-4}$</td>
<td>$4.41 \times 10^{-2}$</td>
<td>$1.25 \times 10^{-5}$</td>
</tr>
</tbody>
</table>

Table 9-2: Friction parameters from the system identification and directional adaptive estimate
The benefit of the bi-directional friction estimate is shown in Figure 9-15. Where the symmetric friction model slightly underestimates on apply and overestimates on release, the bi-directional estimate provides a more agreeable fit. Both the adaptive estimates improved upon the static friction model described in Chapter 3, perhaps because the friction had changed since the earlier identification tests.

![Figure 9-15: Friction estimates between 53.5-55s using the adaptive and static parameters](image)

Extending the friction estimate to a bi-directional model produces a marginal improvement to the description of the EMB mechanism friction. Whether or not this justifies the small increase in complexity may depend on the application as Figure 9-15 indicates the simpler symmetric friction model may be sufficient.
**Stiffness (α)**

*Signals and parameters*

An observer for the EMB stiffness scaling, $\alpha$, is proposed using measurements of the motor position, $\theta$, and the clamp force, $F_{cl}$. This version of the estimate assumes knowledge of the gear ratio, $N$.

*Estimation routine*

Identifying the EMB stiffness is straightforward given measurements of position and force. (Schwarz et al., 1999) noted that the stiffness variation with wear and temperature could be described by scaling a stored characteristic curve. A similar approach is taken here.

An observer for the stiffness scaling, $\alpha$, is shown in Figure 9-16. The force is estimated from the scaled stiffness curve and the error, $e_F = F_{cl} - \hat{F}_{cl}(\hat{\alpha})$, is used to update $\hat{\alpha}$. Updating was enabled during brake application, at clamp forces above a small threshold, $F_{cl} > \varepsilon$. The gain, $L_2$, may be set for slow convergence (Appendix 9B) and the stored stiffness curve was taken from the identification given in Chapter 3. Finally, the observer was converted to a discrete-time equivalent operating at 1 kHz for testing.

![Figure 9-16: $\alpha$ observer to scale the stored stiffness curve](image-url)
Experimental results

The stiffness observer was tested on measurements from EMB 19. As expected, Figure 9-17 shows convergence of $\hat{\alpha}$ towards unity. It may be seen that updating is intermittently suspended in the zero-force clearance region, as in the first 3s for example. Once the stiffness estimate converged it showed good agreement with the measured clamp force. Figure 9-18 indicates the agreement between the measured and estimated stiffness. It may be seen that the middle-stiffness curve does not describe the slight hysteresis. However, the estimate provides a sufficient description of the EMB stiffness.

Figure 9-17: Convergence of $\alpha$ stiffness scale estimate

Figure 9-18: Stiffness measured and estimated with the steady-state $\alpha$ value
9.2 Adaptive control

Since the stiffness and friction may be identified online, the EMB control may be updated to account for variation with wear and temperature. A structure for an adaptive control is shown in Figure 9-19 and includes the MPC (Chapter 8) within the modified control architecture (Chapter 6). The controller receives the brake command, \( F_{cl} \), and feedback measurements. As an extension, it also receives parameter estimates for the stiffness, \( \alpha \), and friction, \( D, C, \) and \( G \). These may be used to update the predictive model of the MPC, adjust the friction compensation and maintain the inverse stiffness linearisation. As a result, the control performance may be better maintained.

\[
\begin{align*}
\hat{\alpha}, \hat{D}, \hat{C}, \hat{G} & \quad \text{Parameter Observer} \\
F_{cl}^o (N) & \quad \text{MPC} \\
i_q (A) & \quad \text{EMB} \\
i_q, \Theta, F_{cl} & \quad F_{cl}^o (N)
\end{align*}
\]

Figure 9-19: Adaptive control structure

Updating the modified control architecture

The modified control architecture may be maintained by updating the friction compensation and inverse stiffness function to reflect changes in the actuator. The adjustment is straightforward given the parameter estimates. Since the identified stiffness is scaled by \( \hat{\alpha} \), the operation of the inverse function may be adjusted proportionally. Meanwhile, the model based friction compensation may be updated with the estimated parameters \( \hat{C} \) and \( \hat{G} \). As an approximation, the static friction parameter may be maintained in proportion to the Coulomb friction (\( \hat{T}_s = 1.25 \hat{C} \) in this case).
Updating the model predictive control

The predictive model of the MPC may be updated using the estimated stiffness and friction parameters. The predictive state-space model may be re-formed with the estimated viscous friction parameter $\hat{D}$ and the scaled linear stiffness $\alpha \hat{K}$,

$$\dot{x} = \begin{pmatrix} -\hat{D}/J & 0 \\ N\alpha \hat{K} & 0 \end{pmatrix} x + \begin{pmatrix} K_i/J \\ 0 \end{pmatrix} u$$

$$\nu = (0 \quad 1)x \quad (9-11)$$

The optimal control may then be determined using the solution provided in Chapter 8. In the simulations that follow, the parameter observers were run at $f_s=250\,\text{Hz}$ and the unconstrained MPC matrix $[K_1 \quad K_2 \quad K_3 \quad K_4]$ was recomputed at $10\,\text{Hz}$ using the updated model.

Management of uncertainty

The adaptive EMB control was compared with the fixed version given in Chapter 8 (unconstrained MPC with post-constraint) by simulation on a perturbed actuator. As a preliminary test, both controllers were simulated on the nominal actuator. In this case, Figure 9-20 shows that the responses with the two controllers are identical.

![Figure 9-20: Simulated 1kN steps comparing the fixed and adaptive MPC on the nominal EMB using a perfect model of the stiffness and friction](image)

The operation of the adaptive MPC is only apparent when the parameters of the EMB model are perturbed. To expose the difference in performance, the stiffness was halved and the friction was doubled. The first plot in Figure 9-21 shows the initial adjustment of the adaptive MPC control as the new parameters are identified. While
the fixed MPC manages the parameter disturbance to some extent by the mechanism of feedback, the performance is better maintained by the adaptive control. The second plot in Figure 9-21 shows the performance after 9 s when the parameter estimates have converged. At 10 s the simulated performance is compared with MPC measurements on the prototype EMB described in Chapter 8. Even though the simulation has a perturbed actuator with the stiffness halved and the friction doubled, it may be seen that the adaptive control maintains the brake performance.

![Figure 9-21: Simulated 1kN steps comparing the fixed and adaptive MPC on a perturbed EMB with halved stiffness and doubled friction](image_url)

The adaptive control performance is similar on the nominal (Figure 9-20) and perturbed (Figure 9-21) actuators, but not identical because the adjusted control trajectory, $\Delta i_q^*$, attracts a marginally different cost penalty. The effect is small since the MPC cost function was largely weighted towards minimising the tracking error.

The parameters for the stiffness, $\alpha$, and friction, $D$, $C$, and $G$, are independent of the control excitation, $i_q$, and the observer gains may be chosen such that the estimates converge to the true parameter values. Subsequently, the MPC control performance may be maintained with the updated EMB parameters.
9.3 Conclusions

An adaptive implementation is feasible for the EMB model predictive control described in Chapter 8. The control may be updated using estimates of the stiffness and friction parameters to account for variation with wear or temperature. The parameters may be estimated during normal brake operation and observers were demonstrated in simulation and on measurements from the EMB. Using the estimated stiffness and friction parameters, it was possible to update the prediction model of the MPC, maintain the friction compensation and adjust the inverse stiffness gain-schedule. Subsequently, the adaptive EMB control had the benefit of a more consistent brake performance. For example, the fine brake control was maintained in simulation on an actuator with halved stiffness and doubled friction. The adaptive EMB control manages uncertain parameter variation by adjusting to the stiffness and friction parameters identified during normal brake operation.
10 Conclusions

This investigation complements the development of automotive EMBs in the context of a next generation of brake-by-wire system, and reaches significant conclusions regarding issues of EMB modelling, identification and control. A first aim of the study was to determine whether EMB behaviour could reasonably be described by a lumped parameter, half-calliper model following parameter identification on an assembled actuator. A second aim was to determine whether a baseline cascaded PI control may be significantly improved to better realise the potential of the EMB for responsive brake performance. The main outcomes and conclusions are summarised here,

**EMB calliper modelling**

A lumped parameter, half-calliper model of the EMB would be ideal for simulation and design if it described the brake behaviour with reasonable fidelity, and if methods were available to identify the model parameters. After reviewing prior EMB models it was apparent that the standard lumped-parameter model required extension to describe the load-dependent stick-slip friction in the mechanism. Using measurements of brake clamp force, motor current and motor position, new and practical methods were demonstrated to better identify the EMB parameters on an assembled calliper. Following system identification, the fidelity of the simplified EMB model was assessed for the first time in open-loop tests without a feedback controller rejecting disturbances. Based on the results it was concluded that the simplified lumped-parameter EMB model and parameter identification could provide a valid description of the brake apply profile, modulations, and lockup behaviour due to the load dependent friction in the brake mechanism.

Experimentation revealed some secondary behaviour that was not described by the simplified EMB model. Hysteresis in the stiffness curve had been noted in prior work and was confirmed in this study. The EMB stiffness hysteresis was found to depend on the rate of strain (or mechanism velocity) in a similar manner to the viscoelastic material behaviour known in brake pads. Two new behaviours were also discovered
in the EMB mechanism. Firstly, the EMB friction was found to have a directional bias. Secondly, an internal mechanism compliance other than backlash was observed at velocity reversal. The compliance manifested as slight motions in the motor position without a corresponding change in the brake clamp force. While identifying these effects is important for understanding the behavior of the EMB mechanism, the results indicated that it was reasonable to neglect these secondary effects without a significant loss in the EMB model fidelity for normal brake operation.

**EMB clamp force control**

A brake control algorithm is required to operate the EMB and realise its potential for responsive braking. A cascaded PI control had been used in prior work with embedded feedback loops to manage brake clamp force, motor velocity and motor current (or torque). After attempting to tune the control and characterise its performance it was realised that although it might be suited to a particular EMB manoeuvre, it suffered degraded performance for other operations due to a deficient handling of the actuator nonlinearity. This degradation in performance was pronounced during fine operation and in some instances complete lockup was observed due to the load-dependent mechanism friction. Since the cascaded PI control could be tuned for specific operations, but not throughout the operational envelope of the EMB, a significant potential for improvement was apparent.

A series of modifications was proposed to improve the EMB control. A model-based friction compensation was suggested and back-to-back tests were conducted on a prototype EMB with and without the compensation active. Experimental results indicated that the EMB friction compensation was effective for alleviating the problem of the load-dependent friction. The benefit was most pronounced during fine operation and for some brake manoeuvres where complete frictional lockup was observed until the friction compensation was activated.
Further control modifications were proposed to better manage the nonlinear stiffness characteristic of the EMB. Incorporating feedback linearisation and an inverse gain schedule with the friction compensation led to a modified control architecture that mainly isolated the controller from actuator nonlinearity except for the unavoidable case of actuator saturation. Experimental and simulation tests were conducted to benchmark the modified control against the baseline cascaded PI control. Test results indicated that the modified control architecture better maintained performance throughout the operational envelope and improved fine operation. As an example, the measured performance of a large brake apply from 7.5 kN to 25 kN was similar between the two EMB controllers due to actuator constraints, but the rise time for a small apply from 7.5 kN to 8.5 kN was almost halved (down from 66 ms to 35 ms) with the modified control.

**Dealing with EMB uncertainty**

Since the EMB characteristics may vary with wear and temperature it was necessary to consider the issue of model uncertainty. To this end the modified control was extended with a robust $H_\infty$ control design for the EMB. The intention was to ensure a level of performance across a useful set of uncertain actuators. However it was found that the robust control design was unable to satisfy the competing demands for brake performance and tolerance to a reasonable level of actuator uncertainty. To achieve a feasible design it was necessary to significantly reduce the specification of actuator uncertainty. When benchmarked against the cascaded PI control the robust control appeared less sensitive to simulated perturbations, but its performance was overly conservative.

**Optimise and adapt**

A different strategy was formulated to optimise the EMB control performance on the nominal actuator and then consider the handling of uncertainty. To improve EMB performance, the modified control architecture was extended with a model predictive control. The modified architecture simplified the optimisation with its management of nonlinearity such that the ‘apparent plant’ could be approximated using a linear, state-space prediction model with input saturation. In simulation the constrained
MPC was found to offer an improved performance, particularly when the brake trajectory could be planned. A further simplification was necessary for real-time computation on the embedded controller of a current EMB. In a trade-off to reduce the computational burden a practical implementation was found by designing an unconstrained MPC and post-applying actuator constraints. Experimental tests were conducted to compare the MPC performance with that of the modified control and the baseline cascaded PI control. Only a slight improvement was achieved with the MPC for large manoeuvres, such as a brake apply from 7.5 kN to 25 kN, since the performance was mainly limited by actuator constraints. However, a significant improvement was achieved for fine operation. For a brake apply from 7.5 kN to 8.5 kN, the EMB rise time with the MPC was less than a third of that with the benchmark cascaded PI control (down from 66 ms to 19 ms).

Returning to the issue of EMB uncertainty, an adaptive control was proposed to adjust for actuator variation. Methods for online EMB parameter identification were developed to maintain effective compensation and to update the predictive model of the MPC. It was confirmed that on-line stiffness identification was feasible using measurements of the EMB clamp force and motor position. New methods for identification of the EMB friction and inertia parameters were demonstrated on measurements from the prototype actuator, and found to be effective. Subsequently, it was concluded that the control and its predictive model could be updated to adjust for actuator variation. The adaptive MPC was demonstrated in simulation on a perturbed EMB and its performance was compared with that of the fixed MPC. Simulation results indicated that the adaptive EMB control may provide more consistent performance when subject to actuator variation.
**Significant outcomes of this thesis**

The results of this study suggest that a lumped-parameter, half-calliper model is adequate to describe electromechanical brake behaviour for control design and system performance simulations. Further, a more responsive EMB control may be designed by incorporating model predictive control within a modified architecture to handle actuator nonlinearity. In addition, online EMB parameter identification may be feasible to update the control and account for actuator variation. The outcomes described in this thesis are relevant to future brake-by-wire development and may facilitate the design of improved anti-lock brake systems and vehicle dynamics control in the immediate future.
10.1 Future work possibilities

EMB modelling

The simplified EMB model investigated in this work may provide a useful tool for further brake research. Analysis and simulation may be conducted with greater confidence in the capacity of a lumped parameter EMB model. Applications range from actuator optimisation to the simulation of brake-by-wire vehicles. This may facilitate the simulation and design of other vehicle technologies such as ABS or electronic stability control.

The simplified EMB model may provide a useful analytical tool, but it does have limitations. While it describes the ‘input-output’ brake behaviour, the simplifying assumptions mean that it provides minimal information on the internal mechanism operation. This is where a more detailed model may allow insight that could be relevant for improving the actuator design.

For a greater depth of understanding, further investigation might explain the EMB operation, mechanism friction, stiffness and the stress-strain distribution in more detail. For example, better explanations may be sought for the bi-directional friction, the small compliance between motor displacement and brake force, and the effects of loading during brake application. A detailed understanding of the EMB behaviour is particularly important to appreciate the consequences of model assumptions and simplification.

Since internal measurements of the EMB mechanism are difficult during operation, a finite element model could be used to estimate the dynamic stress-strain distribution that arises from clamping, bending and transverse loads. For example, an analysis of how loading relates to the mechanism friction may be valuable since this has a significant effect on EMB performance. A detailed model of the mechanism loading may also facilitate improvements to actuator design.
**Parameter identification**

Methods of estimation and parameter identification are important for EMB modelling and control. Estimation routines that have a practical online implementation are particularly valuable for diagnostics, calibration and adaptive control. Further work may look to develop better EMB estimation routines. The calculation of error bounds with consideration for sensor inaccuracy would also be useful.

Since parameters were only identified for two actuators in this study, it would be desirable to identify a larger sample in further work. Measurements on a larger sample would allow statistical analysis and a better estimate of parameter variability. For example, in this study the friction parameters were twice as large on one test actuator. Whether this is a typical or if the friction parameters remain in proportion for a given EMB is yet to be established. Once this is established the control design may consider a more representative specification of actuator variability.

**EMB design optimisation**

While a large body of information exists in the form of EMB patents, a significant opportunity remains for innovative EMB designs. Even when an actuator configuration is decided there is a need for design optimisation. Although hardware development tends to be iterative, the EMB model may greatly assist the design process.

**Model extension**

The EMB model may be readily included in brake-by-wire vehicle simulations. Some variation may be necessary in cases such as self-energising brakes, but otherwise the model may be used as presented. EMB vehicle simulation may facilitate drive-by-wire research and the development of advanced vehicle dynamics control. For example, the industry partner found brake-by-wire vehicle simulation to be a useful tool for ABS tuning. Once new vehicle control algorithms have been developed offline, the performance may be confirmed by running vehicle manoeuvres at the test track.
When the EMB model is included in a vehicle simulation a brake-pad friction model is also required. Basic friction models use a friction coefficient to calculate the brake force from the clamp load. Methods to estimate the friction coefficient and the brake torque would be particularly useful for vehicle dynamics control.

**Online constrained MPC**

Future work may improve the EMB control by developing practical methods for online computation of the constrained MPC rather than using an unconstrained MPC with post-application of constraints. In this case, a stability constrained MPC might be considered within the framework for adaptive control. The MPC might also consider a variable saturation in the predictive model to explicitly allocate motor current (or torque) for the parallel operation of compensation and feedback linearisation.

Another approach to implement constrained MPC on an EMB would be to solve the control problem offline and save the solution to a look-up table. The optimal control action could then be retrieved based on the actuator states and the command. To handle changes in EMB stiffness and friction, variation in these parameters could be added as dimensions to the optimisation problem. Since these parameters may be identified online, the optimal control may be maintained.

**EMB diagnostics and failure modes**

This study only considered control design for a functional EMB. However, further analysis may consider diagnostics and modes of brake failure. In these cases estimation routines can provide useful diagnostics checks. A more robust brake system may be realised if sensor information were used to identify and manage faults.

**Large scale testing**

Further testing of the adaptive MPC should be conducted on a number of EMBs to confirm the anticipated performance. Extensive testing is appropriate given the safety critical nature of the vehicle brake system.
References


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Appendices

Appendix 1A – Review of US EMB patents

The United States Patent and Trademark Office (USPTO) holds a valuable database on automotive technology, having attracted patents from both domestic and foreign companies. This review presents the development of EMB designs held by the USPTO.

EMB patents date to the period of early electric vehicles and an interest in electromagnetic brakes for railway cars that offered a means for remote control along a train’s length. An early example was Olmsted’s electromagnetic disk brake for a railway car (Olmsted, 1869). Following a series of related designs for railway cars, an electromagnetic brake apparatus for a ‘street-car’ was patented (Davis, 1895). This also ran on rails and might now be called a tram or a trolley. From rail to road, the development of the motor car resulted in a transition to EMB patents for automobiles.

Demonstrations of electric vehicles were first reported between 1834 and 1837 (Westbrook, 2001). In 1900 around 4200 automobiles were sold, of which 40% were steam, 38% electric and 22% gasoline-powered (Husain, 2003).
**Electromechanical drum brake patents**

An electromagnetic drum brake for a motor car was patented as early as 1899 by Stevens and Penny (Stevens and Penney, 1899). They were experienced boat builders from Britain and their design, shown in Figure 1, had brake blocks, $h$, on pivoted levers, $i$, that were applied to the drum brake by solenoids.

![Figure 1: Stevens and Penney’s electric brake (Stevens and Penney, 1899)](image)

In 1912 electric vehicles outnumbered gas-powered vehicles two to one (Husain, 2003) and various types of brakes were in use. Two illustrative examples of early EMBs are patent designs by Wadsworth (Wadsworth, 1914) and Adler (Adler, 1919). Both designs had electric motors and gearing to apply a drum brake. Wadsworth’s design (Figure 2) has a motor (15) that drives a spindle thread (16) which is linked to a bell crank (12). The crank parts a split ring (11) and brakes the drum (10).

Adler’s “electrically-operated vehicle-brake” (Figure 3) has an electric control switch, motor (7), gear transmission (10, 15, 17) and levers (33-37) to operate a band type drum brake (39). Reliability was a concern, as it is today, and Adler’s patent offers an early example of redundancy. He states, “I prefer to maintain the foot brake mechanism so the brake can be operated by foot power.” An additional connection to the foot brake (63) is shown in his diagram.
Other patent designs of band type electromechanical drum brakes from this time are described by Hartford (Hartford, 1918, 1921, 1924), Davis (Davis, 1923) and Parsons (Parsons, 1926). Andres’ brake design was similar, but the rear brake construction was not prescribed (Andres, 1927). Hartford’s designs had a central 6V electric motor and a 2500:1 worm reduction to winch a cable and apply the drum bands. Hartford states, “a 3000 lb. pull (=13.3kN total or 6.7kN to each rear wheel) is capable of being developed within approximately 2 seconds… so tightly as to lock the wheels” (Hartford, 1918). Parson’s design also had a central motor and worm, but rod linkages were applied to all four wheels and an additional foot brake actuation was added for redundancy. Sullivan modified this design with the addition of a power screw transmission and dual internal and external drum brake actuation at each wheel (Sullivan, 1925).
During the 1930s electromechanical belt drum brakes were superseded by internal drum brakes. These had brake shoes inside the rotating drum and offered improved holding capacity and reliability. A new actuation mechanism became popular whereby an energised electromagnet would drag upon the rotating assembly to operate a brake lever. The brake force was then amplified via the lever to engage the brake shoes inside the drum. The solenoid actuator was in direct competition with electric motor and hydraulic designs. To cover these alternatives a patent by Apple and Rauen described the same drum brake with each of the three actuators, a motor, solenoid, and hydraulic piston (Figure 4) (Apple and Rauen, 1933).

![Figure 4: Apple’s brake with motor, solenoid or hydraulic actuation (Apple and Rauen, 1933)](image)

The electromagnetic solenoid mechanism was simple and captured the self-energising effect. It persisted in drum brake patents well into the 1960s and was later adapted for electromechanical disk brakes. Some early examples of electromagnetic actuation include a series of patent designs by (Warner, A. P. and Cadman, 1931; Whyte, 1931; Warner, 1932; Penrose, 1933; Whyte, 1934; Penrose, 1942). Later examples that illustrate the design are described in patents by Penrose (Penrose, 1944) and Stocker (Stocker, 1951) (Figure 5 and Figure 6).
Modern electromechanical drum brakes have an electric motor driving a gear mechanism, often a power screw, to part the brake shoes and engage the inside lining of the drum. Despite the recent trend towards electromechanical disk brakes, equivalent designs for the drum brakes continue to be patented. Some illustrative examples include those illustrated in Figure 7 and Figure 8, as well as designs by (Hickey, Green Jr. and Kozak, 1992) and (Shaw, Schenk, Hallinan and Newton, 1994).
**Electromechanical disk brake patents**

Patents on electromechanical disk brakes may be found as far back as those for drum brakes. A patent for an electric spot type disk brake (Sperry, 1899) (Figure 9) has electromagnets \((L)\) to actuate the brake on the surface of a disk \((k^{11})\). Following his experience with the Sperry Electric Railway Company, Sperry was well placed to transfer his railway brake knowledge to an application in motor cars. In a 1942 patent, Penrose credits Sperry as a founder of the automotive EMB in the USA. He writes, “development may be conveniently traced by reference first to the early U.S. patents to Sperry and to Williams” (Penrose, 1942). Williams patented a “magnetic friction clutch” building on Sperry’s earlier work (Williams, 1906).

![Figure 9: Sperry’s “electric brake” (Sperry, 1899).](image)

Early electromagnetic disk brakes directly clamped the brake rotor and were challenged when it came to developing large brake forces, leading to the subsequent incorporation of large electromagnet banks to meet the force requirement (Apple, 1934). The front wheel brakes are shown in Figure 10. A power cable leads to a bank of electromagnet coils. When these are energised the friction linings are clamped to arrest the brake rotor. Additional drum bands were included on the rear wheels to provide a park brake.
While electromechanical drum brakes were significantly advanced from the 1930s to 1960s, equivalent disk brakes were only patented intermittently. The large brake force requirement persisted as a challenging problem. A solution from the Westinghouse Electric & Manufacturing Company was to incorporate a series of brake rotors within the electromagnetic clamp (Arnold, 1940). This reduced the necessary clamp force for an equivalent brake torque. The patent design is shown in Figure 11 and an annular electromagnet (15) may be seen to clamp a ‘plurality’ of friction rings, (9).

Electromechanical disk brakes remained unpopular and lacked the ‘self-energisation’ that allowed smaller and more practical actuators in equivalent drum brakes. Self-energising brakes capture the brake drag force to assist further brake application. Eventually, a means for incorporating this effect in disk brakes was patented (Kershner and Hahn, 1966). This design had a single brake rotor, but innovatively reacted the brake torque via a ball ramp support. As the ball elements tended to ride up the cam surfaces, these were separated allowing further brake application. To compensate for brake wear a self-adjusting screw mechanism was used to limit back-off.
Renewed interest in electromechanical disk brake patents began in the 1970s, particularly with the use of levered electromagnets adapted from equivalent drum brake designs. This innovation provided a simple mechanism for effective ‘self-energisation’ and was a breakthrough for practical disk brakes. In his 1972 patent, Wolf describes how he combined electromagnetic drum brake technology with that of mechanical disk brakes to arrive at his new electromagnetic disk brake (Wolf, 1972) (Figure 12). The design has an electromagnet mounted across the diameter of the brake disk for mechanical leverage. The energised electromagnet would drag on the brake rotor to tilt the lever. This operated a ball-ramp mechanism to spread the brake pads and engage the rotor flanges. Further self-energisation would then be realised as the brake pad force was reacted via the ball-ramp to engage the brake pads further. Despite the large mechanical advantage and self-energisation, Wolf implies the design might be suitable for a vehicle “such as a trailer” (Wolf, 1972).

Electromagnetic lever actuation for electromechanical disk brakes became the standard during the 1970s. Kelsey-Hayes Company patented two similar brake designs with this configuration (Hoffman and Jansen, 1974; Hoffman, 1976), the most recent of which, an “electrically operated disk brake”, is shown in Figure 13. It has a levered electromagnet (52) that drags on the rotor when energised to circumferentially translate the brake pads (24,44). As a result of tilting supports (29) the circumferential shift is translated to an axial movement of the brake pads. In a similar manner, brake drag then causes further interference and self-energisation. A comparable electromagnetic brake was patented by ITT Industries Inc. (Dixon, 1975). In this design however, the levered electromagnet operated a hydraulic piston to impart axial pressure on the brake pads.
The emergence of electromechanical disk brakes with electric motors and or piezoelectric devices in the late 1980s saw the demise of levered electromagnetic actuation. Innovation by Japanese manufacturers such as Jidosha Kiki and Honda lead the debut of piezoelectric actuation in United States patents. Later, other companies such as Toyota and Nissan would also champion this technology.

The piezoelectric effect describes the mechanical distortion of some materials when a voltage is applied. The effect is exhibited by crystals such as quartz and also some ceramic materials. When compared with other actuators, piezoelectric elements are generally compact, light, responsive, and capable of developing very large forces. Realising these advantages, Jidosha Kiki patented an electromechanical disk brake with a piezoceramic element in 1986 (Ohta and Kobayashi, 1986a) (Figure 14). Although the piezoelement (13) used to generate the brake force had a limited extension, it was configured in series with a motor (11) and power screw (18) for macro-adjustment and to compensate wear.

A second patent by Jidosha Kiki had a ‘walking’ piezoelectric core to overcome the problem of macro-adjustment (Ohta and Kobayashi, 1986b). A central piezoelement (13) was used to develop axial displacement. Meanwhile, ‘walking’ was achieved through the coordinated operation of four stabilising piezoceramics elements (16,17,18,19). Each of these had a length of 20mm and area of 300mm$^2$ to generate around 5kN of restraining thrust. Using positive and negative voltages the force range
could be doubled to around 10kN. However, the piezoelectric actuation required
200V and this might pose a hazard. Currently, 42V automotive systems are being
adopted as the safe vehicle standard.

Since piezoelectric elements only offer small displacements there is little opportunity
for mechanical gearing or leverage to amplify the brake force. Instead, Honda
stacked many piezoelectric units and gained leverage via a seesaw rocker (Yamatoh,
Ogura, Kanbe and Isogai, 1989) (Figure 15). To compensate for wear, a separate
means for adjustment (20) was employed to vary the location of rocker support (21).

Figure 14: Jidosha Kiki’s piezoelectric brakes patented in 1986 with a motor (a) and ‘walking’
core (b) to allow macro-adjustment (Ohta and Kobayashi, 1986a, b)

Figure 15: Honda’s brakes patented in 1989 with a piezoelement (a) and ultrasonic motor (b)
(Yamatoh and Ogura, 1989; Yamatoh et al., 1989)
A promising EMB configuration by Honda incorporated a piezoelectric ultrasonic motor (Yamatoh and Ogura, 1989). As shown in Figure 15, the ultrasonic motor (5) was used to drive a ball-screw (82) and generate clamp force.

Generally, ultrasonic motors have an annular stator and rotor in frictional contact. The stator has a ring of piezoelements with coordinated excitation to convert an ultrasonic travelling wave into rotational motion (Figure 16). Ultrasonic motors typically output high torque at low speed. They have a simple construction, low inertia, high frequency response, quiet operation, large temperature range, and holding torque when not energised. Disadvantages include wear between the motor surfaces and cost. Among other prominent research groups, the National Aeronautics and Space Administration (NASA) has pursued ultrasonic motors for use in robotic arms and space rovers (Bar-Cohen, Bao and Grandia, 1988; Das, Bao, Bar-Cohen, Bonitz, Lindemann, Maimone, Nesnas and Voorhees, 1999; Bar-Cohen, Sherrit, Bao, Chang and Lih, 2003). The European Space Agency has also been involved with ultrasonic motor development for space applications (Six, Letty, Seiler and Claeyssen, 2004). The motors have also found commercial applications such as auto-focus drives in camera lenses.

![Figure 16: Piezoelectric ultrasonic motor core (Das et al., 1999)](image)

EMB designs with ultrasonic motors were advanced by Toyota and the company was granted two patents in 2000 (Shirai, Yokoyama and Oota, 2000a; Shirai, Yoshino, Takemura and Yokoyama, 2000b). These designs, one of which is shown in Figure 17, had dual ultrasonic motors (30, 32) driving a planetary gear (48) and ball-screw (66, 72) to actuate the brake calliper.
A patent by SKF Engineering and Research Centre B.V. combined piezoelectric elements (19) with an integrated motor (4) and roller screw (3) (Vries, Olschewski, Kapaan, Winden, Druet and Muller, 2002). The series motor-piezoelement configuration, shown in Figure 18, was in the same spirit as Jidosha Kiki’s 1986 design. With coordinated control, it allowed macro-adjustment via the motor and high frequency, accurate, micro-adjustment using the piezoelement.

Concurrently to the advancement of piezoelectric brakes, in the late 1980s United States patents emerged on electromechanical disk brakes with electric motors. From this time forward, the electric motor became the most popular drive and has been included in many designs with various means of gearing or self-energisation. Secondary features such as locks, clutches, mechanical redundancy, lubrication, and couplings have also been incorporated. Beyond these variations however, the basic electric motor, gearing, piston arrangement is now established as the most popular patent configuration.

As with prior designs, the first electric motor disk brakes were challenged by the large clamp force requirement. As feasible solutions emerged there was a series of transitional patent designs in the late 1980s. One of the first from Bendix France had dual rotors to reduce the brake force requirement (Carre and Thioux, 1988). A novel wedge and roller transmission was employed to achieve a high reduction and avoid the inefficiency of a power screw. As shown in Figure 19, the ‘L’ shaped actuator had an opposed motor (8) driving a plunger against a wedge (17) to translate the
piston (15). A second patent was granted to Bendix France (Fargier, 1989). In the latter version, the plunger’s screw drive was replaced by a centrifugal fly-weight mechanism that extended according to the motor speed and load.

Self-energisation was utilised in some early electric motor disk brakes to achieve a magnified clamp force. The Aisin Seiki patented a design (Figure 20) with a tilting pad support for self-energisation (Karnopp and Yasui, 1989) that was conceptually similar to the tilting pad support described by Kelsey-Hayes in 1974 (Hoffman and Jansen, 1974). The tilting pin support (35) is mounted behind one brake pad (11) such that brake drag during an apply causes additional interference and brake force. A related, but more elaborate self-energisation scheme with dual callipers and a ball-ramp pad support was patented by General Motor’s Corporation (Shaw and Schenk, 1993).

Dual electromechanical and hydraulic operation was included in some late 1980s designs for redundancy and improved reliability. While the hydraulic backup is understandable as a transitional stage, these actuators were large and somewhat defeated the main objective of by-wire actuators to eliminate the need for mechanical linkages between the vehicle’s controls and its actuators. Examples of such designs appear in patents by Allied-Signal Inc. (Taig, Grabil and Jackson, 1988; Taig, 1989b). The former design is shown in Figure 21. The piston (330) may be displaced by either pressurised hydraulic fluid or a screw (388) that is driven via a planetary gear and motor (340).
Towards the end of the 1980s the configuration of an electric motor, gear and piston began to emerge as the standard for electromechanical brake actuators. Allied-Signal’s designs by Allistair Taig in 1988 and 1989 are commonly referenced by later United States EMB patents. Figure 22 shows his “electrically operated disc brakes” (Taig, 1989c). The design has a motor (40), planetary gear (50), and screw configuration. A significant development by Allied-Signal at this time was the introduction of a high-reduction planetary gearbox. In both the 1988 and 1989 patents for an “electrically operated disc brake” (Taig et al., 1988) a large reduction was achieved using a planetary gear that had dual rings. The rotatable ring had fewer teeth than the fixed ring such that each rotation of the planet carrier caused the rotatable ring to advance a number of teeth.

Although large gear reductions magnified the drive force, efficiency and friction remained a common issue in patent designs. Friction may pose a control problem and the matter of ‘back-drivability’ is regularly mentioned. A significant reduction in friction was achieved with the inclusion of ball bearing screws in EMB designs. These are considerably more efficient than the power-screws of other early designs. Patents assigned to Honda (Yamatoh and Ogura, 1989), Allied-Signal (Taig, 1989a) and Bendix France (Fargier and Pressaco, 1989) provide some early examples of ball-screw designs. Even with this addition, Fargier and Pressaco note that with inferior ball-screw configurations, “the release of the brakes is found to be ‘lazy’ as a result of difficulties in getting the screw to rotate under the force of reaction” (Fargier and Pressaco, 1989).
During a brake manoeuvre the drag on the brake pads is usually reacted mostly via pad abutments to the support bracket. Inadvertently however, there is some level of calliper deformation. Under heavy braking axial loading and bending can increase the sliding interference between moving components, and particularly along the piston. Consequently, Honda patented a piston slide bearing (Fujita, Aral and Ogura, 1992). The design shown in Figure 23 has a motor (30) and worm gear (35) that are perpendicular to the drive axle of a ball screw (37). The worm gear is non backdrivable and offers a hold function.

![Figure 23: Honda’s “motor disc brake system” (Fujita et al., 1992)](image)

Although straight ball-ramp mechanisms had appeared in previous EMB patents, an electromagnet disk brake patent by Robert Bosch was one of the first to introduce an annular ball ramp (Holl, Keller, Kaehler, Kramer and Winner, 1994). Such ball-ramps or roller-ramps convert rotational motion to translation with a compact reduction. Motor driven designs with roller-ramps were later patented by Robert Bosch (Blosch and Keller, 2000), by DaimlerChrysler (Reimann and Roess, 2000a), by INA Walzlager Schaeffler OHG (Zernickel, 2001), by Akebono Brake Industry (Takahashi and Kawase, 2001), by Tokico (Usui and Ohtani, 2002a; Usui, 2005), by Continental Teves (Weiler, Balz, Denhard and Heiderich, 2004) and by Advics (Yokoyama, Takeshita and Arakawa, 2005).
An EMB configuration proposed in 1994 by Akebono Brake Industry (Takahashi, Miyake, Kunimi, Ogawahara, Kobayashi and Fukaya, 1994) (Figure 24) had a hydraulic transmission between the piston and drive. This design achieved a hydraulic boost effect and was credited by later patents with providing a means to avoid the issues of back-drivability, loading disturbance, and excessive inertia. The motor (13) drives spur gears (4), a screw (5), and a hydraulic transmission (10), to clamp the brake pad (11) against the brake rotor (12). A calliper bridge was added to the drawing for clarity. The hydraulic boost was achieved by having a smaller piston area on input to the fluid chamber. Continental Aktiengesellschaft subsequently utilised the boost of a hydraulic transmission to eliminate the need for less efficient gearing such as a planetary gear (Hauck, Dieckmann, Maron and Bergmann, 1998). EMB designs with a hydraulic transmission were patented by DaimlerChrysler (Reimann and Roess, 2000c) and Lucas Industries (Mohr and Wagner, 2001). The latter design had a hydraulic pump of the positive displacement type. The pump comprised an inclined disk with axial pistons.

A significant advancement towards more compact EMBs was the development of integrated motors. Brakes with integrated motors were patented by Continental Aktiengesellschaft (Hauck et al., 1998; Schoner, Bergmann, Dieckmann and Prinzler, 1998) and ITT Automotive Europe (Halasy-Wimmer, Bill, Balz, Kunze and Schmitt, 1998). Rather than having a separate motor unit inline, opposed, or parallel to the piston axis, the integrated motors have a coaxial arrangement and typically wrap around the gear transmission. This allows a considerable size reduction as the working envelope and actuator volume are better utilised. Another benefit is the increased motor torque that results from a larger diameter rotor. While this reduces the reliance on gearing to achieve large clamp forces, a downside is the increased rotational inertia. ITT Automotive’s patent design is shown in Figure 25. The integrated motor (6) is coupled to a planetary roller screw (30) to drive the calliper piston.
While the integrated motor became standard in EMB designs, a significant advancement by Robert Bosch was a new construction and winding arrangement (Hilzinger, Schumann, Blosch and Kastinger, 2004) (Figure 26). It has a transverse flux motor with a separate yoke for each phase. In other words, there are three separate annular phase windings. When current is passed through a phase winding (22), the corresponding yoke (24) is magnetised and attracts the rotor poles (30) to apply torque. The motor configuration offers improved packaging and a volumetric size reduction. The need for permanent magnets is eliminated and without complex motor windings the manufacturing effort and cost is reduced. More motor poles can be accommodated than is possible with standard windings. Also, the patent claims improved motor efficiency and higher power density.
As EMBs remained challenged by the issues of large forces, dynamic performance, efficiency, and size, there was a strong motivation for innovative gear transmissions. United States EMB patents have intermittently incorporated specialised drives such as harmonic gears or eccentric gears.

Harmonic drives consist of a wave-generator, a flexspline, and a circular spline (Tuttle, 1992). Rotation of the inner wave generator distorts the flexible spline to engage the rigid circular ring. The gear advancement is illustrated by Tuttle (Tuttle, 1992) and shown in Figure 27.

![Figure 27: Harmonic drive advancement as illustrated in (Tuttle, 1992)](image)

Eccentric gears typically have a rigid, externally toothed gear inside an internally toothed ring. The internal gear then undergoes some orbital motion such that the ring is advanced at a reduced rate.

Two EMB designs with harmonic drives were patented by DaimlerChrysler in 2000 (Reimann and Roess, 2000b, a), while Lucas Industries patented an EMB with a harmonic drive as an alternative to a wobble swash-plate transmission in 2002 (Poertzgen, Worsdorfer, Erben and Zenzen, 2002). EMB designs with eccentric gears were subsequently patented by SKF Engineering and Research Centre B.V. (Kapaan, Zwarts and Broersen, 2004c), by Advics (Yokoyama et al., 2005) and by Tokico (Usui, 2005).
Since advancements of EMB disk brake designs in the early 1990s, self-energisation lost favour for meeting clamp force requirements, possibly because of difficulties of control and of positive feedback. The level of reinforcement changes with the brake torque and is affected by factors other than clamp force such as the wheel speed, temperature, wear, and contamination. If the control problem were managed however, then a significant reduction in actuator size and power is possible with self-energising brakes.

Self-energisation for electromechanical disk brakes re-emerged in a patent assigned to Deutsches Zentrum für Luft- und Raumfahrt e.V. in 2001 (Dietrich, Gombert and Grebenstein, 2001). Shown in Figure 28 (a), the patent suggested an arrangement in which the brake drag force was reacted via wedges (18) so that an additional normal load was developed. Complementary to this self-energisation, a large ring gear across the diameter of the brake disc allowed an efficient direct drive motor (32). The patent claims that with this configuration the energy requirement can be reduced to between 2% and 17% of the previous customary value.

With potential to drastically reduce the necessary drive force, Ford Global Technologies LLC Co advanced the design of the self-energising EMB by adding a second motor such that the wedge mechanism and level of self-energisation was controllable (Hartsock, 2004) (Figure 28 (b)).

![Figure 28: Self-energising brakes patented by (a) Deutsches Zentrum für Luft- und Raumfahrt (Dietrich et al., 2001) and (b) Ford Global Technologies (Hartsock, 2004)](image-url)
A possible benefit of EMBs is the potential for active pad return to avoid residual brake drag, energy loss, and higher vehicle fuel consumption. This is in contrast to current hydraulic systems that have passive return and can suffer continued brake drag after release. The inclusion of an active return mechanism is not trivial. The withdrawal must accurately maintain a small clearance, compensate for wear, and actively return brake pads on both the piston and claw side of the calliper. An innovative solution by Tokico (Usui and Ohtani, 2002b) incorporated dual ball ramps to control the extension of both the piston and claw. Return springs were included to retract the brake pads from the disk rotor.

To avoid excess actuator friction during an apply, a number of EMB patents describe an attempt to isolate moving components from the bending and transverse stress fields. Friction or interference at high loads may cause wear, damage, or inefficiency in the spindle, motor, gears, and piston.

To help prevent transverse and bending loads in the gear train, a 2001 patent of SKF Engineering and Research Centre included a ‘rod-socket’ coupling between the pad support and piston mechanism (Vries, Olschewski, Kapaan and Haas, 2001). Ball-socket couplings at the same location were included in two patents by Robert Bosch in 2002 (Schaffer, Wolfram, 2002a, b). A Continental Teves patent later in 2002 described a push rod, modular construction, and remote positioning of rotating components to enhance isolation from the stress field (Rieth, Schwarz, Kranlich, Jungbecker, Schmitt, Hoffmann and Nell, 2002). SKF Engineering subsequently incorporated a pressure pad support (Rinsma and Zwarts, 2003).

Attention then returned to dual ball-socket couplings at both ends of the piston mechanism (Kapaan, Fucks, Gurka and Dubas, 2004a). Also in 2004, a Continental Teves patent described a universal joint coupling on the planetary gear output (Backes, Zernickel, Hartmann, Grau, Dorsch, Rieth, Jungbecker, Schmitt, Schwarz, Hoffmann and Nell, 2004).
A number of EMB patents focused on more efficient gearing and designs that were less sensitive to load disturbances. There is concern that the efficiency of planetary roller screws is particularly susceptible to transverse and bending loads, and the approximately spherical rolling contacts within ball-screws might better handle non-axial loads. To help reduce friction, oil film lubrication for the ball-screw was proposed in patents by Nissan Motor (Tamasho, Kubota and Tsukamoto, 2002) and SKF Engineering and Research Group B.V. (Kapaan, Fucks, Gurka, Dubus, Boch, Druet, Bundgart and Visconti, 2004b). A patent by SKF Engineering and Research Group B.V. described a coarser ball-screw thread to allow larger balls. The overall gear reduction was maintained with a high-ratio eccentrically positioned gear (Kapaan et al., 2004c).

Brake callipers with dual brake rotors are an attractive configuration as they effectively halve the necessary clamp force. Disadvantages of a second rotor include increased wheel inertia and unsprung mass, more components, and a calliper bridge that is elongated and less stiff. The dual rotor disk design did not feature in USA EMB patents for nearly two decades after the Bendix France patent of 1988 (Carre and Thioux, 1988) when Delphi Technologies reintroduced it in 2005 (Osterday, Fiste and Hill, 2005).

Less pertinent features of EMB patent designs include locking mechanisms, clutches, variable gear ratios, provision for failure modes, and back-off protectors. Brake hold and park brake functionality are typically provided using either a lock or a non-back driving gear. Non-back driving gears are included in patents by Allied-Signal (Taig, 1989c) and Honda (Fujita et al., 1992). Some examples of lock mechanisms appear in patents from Continental Aktiengesellschaft (Dieckmann, Leitemann and Henken, 1999), Continental Teves (Bill, Balz and Dusil, 2000; Schwarz, 2001) and Robert Bosch (Schaffer, Walfram, 2001c).
A concern related to the ‘back-drivability’ of the EMB is the brake’s failure mode, which can be either hold or release. For example, a series of patents of Robert Bosch describe arrangements for back-drivability and brake release in the event of malfunction (Schaffer, Walfram, 2001c; Schaffer, Wolfram, 2001a; Schumann, 2001a, b). Other patents include features to avoid failure such as can occur if the brake is excessively unwound. A safe-guard against this was provided by Allied-Signal with a “back-off protector” (Taig, 1989b). Protection is also explicitly included in patents held by Toyota (Otomo and Shirai, 2001) and Robert Bosch (Schaffer, Wolfram, 2001b).

In an attempt to improve the EMB reversal time a number of patents have incorporated electromagnetic clutches. An early example is a design by Bendix France (Fargier and Pressaco, 1989). Without this decoupling the reversal time can be adversely affected by the apply momentum. In particular, the motor can develop high speeds traversing the large angular displacement that results from the heavy reduction gearing. Some later examples of EMB designs with clutches may be found in patents by Allied-Signal (Taig, 1989c), DaimlerChrysler (Reimann and Roess, 2000c), and Robert Bosch (Schaffer, Wolfram, 2001b, 2002a, b).

While a high gear reduction allowed a large clamp force to be obtained, the gearing was unnecessary at low loads and slowed the apply rate. To address the matter a few patents proposed variable gear ratios. For example, a patent design by Bendix France had a first gear ratio for traversing the air-gap and then a lower gear ratio was engaged during the apply (Fargier and Pressaco, 1989). In some EMB designs with ball ramps the reduction mechanism was only engaged after the air-gap was traversed, for example Tokico Ltd’s design in (Usui and Ohtani, 2002a). A later ball-ramp design by Advics included a variable gear ratio mechanism (Yokoyama et al., 2005). Additionally, some ramps might have been shaped such that the reduction increased throughout brake apply.
As EMB configurations have matured, patent designs have become more specialised in describing a particular component or part. New proposals are still emerging however, and a preferred arrangement is not yet established. There is certainly a great deal of interest and patent activity from major automotive manufacturers. This includes companies such as Advics, Akebono Brake Industry, Continental Teves, DaimlerChrysler, Delphi, Deutsches Zentrum für Luft- und Raumfahrt, Ford, General Motors, Honda, Kelsey-Hayes, Lucas, Nissan, Robert Bosch, Siemens, SKF, Tokico, and Toyota. From popular designs it would appear that the modern EMB will be a compact unit with an integrated electric motor, planetary gear, and either a ball-screw, planetary roller gear, or roller-ramp.
## Appendix 1B – Listing of US disk EMB patents

Patents held by the United States Patent and Trademark Office (except first entry):

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<th>Year</th>
<th>Patent Number</th>
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<td>EM</td>
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Table 1: Electromechanical disk brake patents held by the USPTO
Companies

Advics Advics Co. Ltd.
Aisin Seiki Aisin Seiki Kabushiki Kaisha
Akebono Akebono Brake Industry Co., Ltd.
Allied-Signal Allied-Signal Inc.
ArvinMeritor ArvinMeritor Technology LLC
ArvinMeritor HV ArvinMeritor Heavy Vehicle Braking Systems Ltd.
Bendix Bendix Brake Company
Bosch Robert Bosch GmbH
Continental Continental Aktiengesellschaft
Continental Teves Continental Teves AG & Co. OHG
DaimlerChrysler DaimlerChrysler AG
Delphi Delphi Technologies Inc.
DLR Deutsches Zentrum fur Luft- und Raumfahrt e.V.
eStop eStop GmbH
Ford Ford Global Technologies
GM General Motors Corporation
Hitachi Kabushiki Kaisha Hitachi Seisakusho
Honda Honda Giken Kogyo Kabushiki Kaisha
INA INA Walzlager Schaeffler oHG
ITT ITT Industries Inc.
Jidosha Kiki Jidosha Kiki Co. Ltd.
Kelsey-Hayes Kelsey-Hayes Company
Lambert Brakes Lambert Brake Corporation
Lucas Lucas Industries public limited company
Lucas Automotive Lucas Automotive GmbH
Nissan Nissan Motor Co. Ltd.
NTN NTN Corporation
Siemens Siemens Aktiengesellschaft
SKF SKF Engineering and Research Centre B.V.
SKF Industrie SKF Industrie S.p.A
Tokico Tokico Ltd.
Toyota Toyota Jidosha Kabushiki Kaisha
Westinghouse Westinghouse Electric & Manufacturing Company

Transmission

G Gear
Gb Bevel gear
Gbt Belt drive
Ge Eccentric gear
GG Dual gears
Gh Harmonic gear
Gm Multi ratio gear
Gp Planetary gear
Gp Pinion gear
Gs Spur gear
Gw Worm gear

S Screw
SB Ball screw
Sd Differential screw
Spr Planetary roller screw
SS Duel screws
St Threaded spindle
R Ramp
Rb Ball ramp
Rr Roller ramp
RR Duel ramps
CF Centrifugal mechanism

Features

H Hydraulic chamber
Hp Hydraulic pump
M Wedge
SE Self-energisation
C Coupling
CL Clutch
CLR Release
LK Lock
DD Duel disk
DC Duel callipers
STi Internal support
STd Duel support
MC Modular components
BOP Back-off protection
B Band brake

The patent listing only includes electromechanical disk brake patents intended for automobiles, excluding designs for other vehicles such as trains, trams or aircraft. Park brakes and remotely motored hydraulic callipers are also excluded, as are patents where an EMB was mentioned, but was not a main design focus.
Since some patents contain vague language and various embodiments to maintain
generality, the table description was based on the first, preferred or main embodiment.
Sometimes details were interpreted from accompanying diagrams. The list does not
fully capture the patent scope, but is intended to identify the main players in EMB
development.

It is noted that although the large American market attracts foreign patents, the list of
US patents is biased towards domestic innovation and does not give a balanced
representation of the significant EMB developments in European and Asian countries
such as Germany and Japan.
Appendix 2A – Measurement and calibration

Motor current measurement

A current measurement unit was constructed to measure the three phase motor currents. The unit had four channels with one for each of the three current phases and one spare. An isolated regulated supply, NMXD0515SO, with smoothing capacitors was used to power four LEM 55-P, hall effect, current transducers. The measurement signal was transmitted via current to drop-down resistors where the measurement could be recorded as a voltage. The output was nominally 100 mV/A and calibration factors were measured for each channel.

![Figure 29: Current measurement unit (a), internal circuit board (b) and inside cover (c)](image)

Each channel of the current measurement unit was calibrated with a DC power supply (GW Instek GPC-303DQ) and two Finest 701 Multimeters from Fine Instruments Corp. The first multimeter (S/N 031001500) was used to measure the current input while the second (S/N 031001491) recorded the voltage output of the current measurement unit. The calibration indicated good linearity and provided calibration factors for each channel. From a linear regression of the data, the calibration factors were 99.9, 99.6, 99.7, and 99.8 mV/A for the black, red, yellow and blue channels that measure the DC bridge, phase A, phase B, and phase C currents. Channel independence was also tested with a maximum interdependence of 0.14 mV/A measured for physically adjacent channels.
Figure 30: Calibration results for current sensor box
**Clamp force measurement**

A clamp plate was assembled with a triangular configuration of three HBM C9B, 20 kN, force transducers to carry the clamp load (S/N 82410069, 82410088, and 82410086). Calibration data is provided by the manufacturer for the sensitivity, linearity deviation, and hysteresis for each transducer over a full load and release cycle. The transducer sensitivity was nominally 1 mV/V of excitation. The excitation was at 10 V and following signal amplification, x100, using National Instruments’ SCC-SG24 strain gauge module, the full 0-20 kN force range was nominally represented by a 0-1 V signal. With three sensors in parallel the clamp force plate had a nominal capacity of 0-60 kN to satisfy all test procedures.

![Figure 31: Clamp force plate and EMB mounting stand](image)
**Motor position measurement**

Motor position measurements were recorded from an inductive sensor. This was mounted over a toothed gear wheel on the motor’s rotor. The sensor has two detection coils that are physically offset to produce two output voltages that indicate the passing of the gear teeth. The angular rotation of the motor was reconstructed from the two channel inductive measurements. As an indication of how this was done, measurements from the two channels are plotted in Figure 32 as the rotor is turned. Each trace around the ellipse represents the passing of a single gear tooth. If the channel outputs were precisely sinusoidal with a 90° phase offset then a circle would be traced. Instead, the phase angle was determined from the ellipse shape and used to develop a direct mapping.

![Figure 32: Inductive sensor measured output of channel A vs. B](image)

**Figure 32: Inductive sensor measured output of channel A vs. B**


**Motor torque measurement**

A Steglar Mahilo torque transducer with a nominal 1 V/Nm output was coupled to the EMB rotor and used to measure the motor torque. The torque sensor was calibrated against a known force and lever arm length. The lever arm length was measured using an Easson ES-8 digital readout. The applied force was determined using an EK4000H AND scale. The torque sensor output voltage was measured with a Finest 701 multimeter. The transducer exhibited good linearity and the calibrated output was determined to be 1.03 V/Nm.

![Figure 33: Torque sensor calibration set up](image)

![Figure 34: Torque sensor calibration results](image)
Data acquisition

Sensor signals for the motor position, clamp force, motor torque, and motor current were connected to National Instruments modules, NI SCC-FT01 and SCC-SG24. These were mounted in a shielded module carrier, NI SC-2345. Measurements were recorded using a 16 bit, 250 kS/s, National Instruments data acquisition card, NI PCI-6221.

![National Instruments SCC modules in shielded carrier SC-2345](image)

Data acquisition channels:

<table>
<thead>
<tr>
<th>Channel</th>
<th>Description</th>
<th>Voltage range</th>
<th>Nominal physical range</th>
</tr>
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<td>0/8</td>
<td>Motor position signals A and B</td>
<td>0 to 5 V</td>
<td>0 to 5 V</td>
</tr>
<tr>
<td>1/9</td>
<td>Force 1, Force 2</td>
<td>0 to 1 V</td>
<td>0 to 20 kN</td>
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<td>Force 3</td>
<td>0 to 1 V</td>
<td>0 to 20 kN</td>
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<tr>
<td>3/11</td>
<td>Torque</td>
<td>-3 to 3 V</td>
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<td>4/12</td>
<td>Current PHA</td>
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<td>7/15</td>
<td>-</td>
<td>-</td>
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Appendix 3A – Viscoelastic stiffness model

![Figure 36: Stiffness model](image)

With no inertia in the system, the net force of the parallel spring and damper pair, $F_1$, is instantly and equally reacted by the second spring force, $F_2$, such that $F = F_1 = F_2 = F_{\text{support}}$. This relation may be used to derive a transfer function between the force, $F$, and displacement, $x$.

$F_1 = F_2$

$\therefore K_1 x_1 + C_1 \dot{x}_1 = K_2 x_2$

Following a Laplace transformation from the time domain to the frequency domain,

$\therefore K_1 x + C_1 x s = K_2 x_2$

$\therefore x = \frac{K_2}{K_1 + C_1 s} x_2$

Since the total compression, $x$, is the sum of compressions $x_1$ and $x_2$, it is seen that,

$x = x_1 + x_2$

$\therefore x = \frac{K_2}{K_1 + C_1 s} x_2 + x_2$

$x = \frac{K_1 + C_1 s}{K_1 + K_2 + C_1 s} x_2$

Returning now to the forces that are developed over the displacement $x$,

$F = F_2$

$\therefore F = K_2 x_2$

$\therefore F = K_2 \frac{x_2}{x} x$

$\therefore F = \frac{K_1 K_2 + C_1 K_2 s}{K_1 + K_2 + C_1 s} x$
Appendix 3B – EMB model parameters

The parameters for the PGT V5 EMB were identified in this study. Meanwhile, the parameters for the PGT V4 EMB were estimated by the industry partner. The V5 EMB was designed to have a more compact packaging than the earlier V4 actuator. Also, the V5 model had an aluminium calliper bridge while the V4 bridge was steel.

<table>
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<th>Parameter</th>
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<th>PGT V5 EMB 9</th>
<th>PGT V4 EMB</th>
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<td>Gear ratio, $N$</td>
<td>m/rad</td>
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<td>$26.3 \times 10^{-6}$</td>
<td>$26.5 \times 10^{-6}$</td>
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<td>Inertia, $J$</td>
<td>kgm$^2$</td>
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<td>$0.291 \times 10^{-3}$</td>
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<td>Static friction, $T_s$</td>
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<td>Coulomb friction, $C$</td>
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<td>Viscous friction, $D$</td>
<td>Nm s/rad</td>
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<td>Friction load-dependency, $G$</td>
<td>Nm/N</td>
<td>$1.17 \times 10^{-5}$</td>
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</table>

Table 2: EMB parameter summary

Stiffness curve, approximated from 0-30kN with $F_{cl}$ in kN and $x$ in mm by,

**PGT V5 EMB 19,**

$$ F_{cl} = \begin{cases} 
-7.23x^3 + 33.7x^2 - 3.97x & x > 0.125 \\
0.1295x & \text{otherwise}
\end{cases} $$

**PGT V5 EMB 9,**

$$ F_{cl} = \begin{cases} 
-16.18x^3 + 51.04x^2 - 6.73x & x > 0 \\
0 & \text{otherwise}
\end{cases} $$

**PGT V4 EMB,**

$$ F_{cl} = \begin{cases} 
45x - 5 & x > 0.22 \\
-0.0541x^3 + 101.3x^2 - 0.00145x & \text{otherwise}
\end{cases} $$
Appendix 3C – Open-loop modulation results (enlarged)

The following measurements were taken on PGT V5 EMB 19 and the measured current record was used as an open-loop input to the model simulation.

Figure 37: Measured and simulated 10% modulation about 5kN at 4Hz
Figure 38: Measured and simulated 10% modulation about 15kN at 4Hz
Figure 39: Measured and simulated 10% modulation about 25kN at 4Hz
Figure 40: Measured and simulated 10% modulation about 15kN at 8Hz
Appendix 3D – Simulation results against vehicle brake data (enlarged)

PGT V4 EMB simulations were compared with ABS vehicle data provided by the industry partner.

Figure 41: Braking during ABS stop with PGT EMB V4 at vehicle front-left
<table>
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<th>Time (sec)</th>
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<td>8</td>
<td>10</td>
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<tr>
<td>8.5</td>
<td>5</td>
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</table>

Figure 42: Braking during ABS stop with PGT EMB V4 at vehicle front-right
Figure 43: Braking during ABS stop with PGT EMB V4 at vehicle rear-left
Figure 44: Braking during ABS stop with PGT EMB V4 at vehicle rear-right
Appendix 4A – Input-output diagrams

The input-output plots show the instantaneous output against the instantaneous input and the magnitudes may be normalised. In terms of tracking control, the ideal response would be represented by a line with unity slope. Colour may be used to show how the response varies with input frequency.

As an example, the plots in Figure 45 show the influence of phase lag relative to the ideal response.

As a second example, an input-output diagram from 1-5Hz was constructed for the transfer function $G(s) = \frac{10}{s+10}$ and is shown alongside the Bode plot in Figure 46.
Appendix 5A – Friction compensation results (enlarged)

Figure 47: 2% modulations about 25kN at 4Hz with and without friction compensation

Figure 48: 2% modulations about 25kN at 8Hz with and without friction compensation
Figure 49: 4% modulations about 25kN at 4Hz with and without friction compensation

Figure 50: 4% modulations about 25kN at 8Hz with and without friction compensation
Figure 51: 6% modulations about 25kN at 4Hz with and without friction compensation

Figure 52: 6% modulations about 25kN at 8Hz with and without friction compensation
Appendix 6A – Modified EMB control architecture

The modified EMB control architecture in Figure 53 isolates the controller, $K_c$, except for the unavoidable case of actuator saturation. It includes an inverse function to obtain a linear compound stiffness from the position, $x$, to the control variable, $u$. $\hat{T}_{s/c}$ is a compensation torque to reject the nonlinear component of the EMB friction. $T_{mc2}$ is an arbitrary compensation for the nonlinear stiffness load, $T_L$.

Figure 53: Modified EMB control architecture
Appendix 7A – Summary of robust control theory

$H_\infty$ norm

Robust control theory deals with bounded uncertainties where phase information is unknown. Hence, the magnitude of matrix gains is significant and analysed in terms of the $H_\infty$ norm.

The $H_\infty$ norm of the transfer matrix $G$ is the supreme maximum singular value, $\bar{\sigma}$, over all frequencies, $\omega$,

$$\|G\|_\infty = \sup_{\omega} \bar{\sigma}(G(i\omega))$$

The $H_\infty$ norm describes the peak matrix gain across all frequencies and all input directions. For a single-input-single-output transfer function it is the peak value on the Bode magnitude plot of $|G(j\omega)|$.

Maximum singular value, $\bar{\sigma}$

The singular values, $\sigma_i$, indicate the ‘size’ of a matrix and are given by the singular value decomposition. For a $m \times n$ matrix, $A$, there exist unitary matrices $U$ and $V$ of dimension $m \times m$ and $n \times n$ such that, $A = U \Sigma V^*$

$$\Sigma = \begin{bmatrix} \Sigma_1 & 0 \\ 0 & 0 \end{bmatrix} \quad \Sigma_1 = \begin{bmatrix} \sigma_1 & 0 & \cdots & 0 \\ 0 & \sigma_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \sigma_p \end{bmatrix} \quad (p = \min\{m,n\})$$

The singular values are ordered $\sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_p \geq 0$ such that the maximum singular value is given by $\bar{\sigma}(A) = \sigma_1$. The maximum singular value indicates the maximum ‘gain’ of the transfer matrix.
Robust control overview

The framework for robust control design is shown in Figure 54. The block $K$ indicates the controller, $P$ the extended plant and $\Delta$ the uncertainty. The design may provide a controller with robust stability and performance over a degree of uncertainty.

![Figure 54: General framework for an uncertain feedback system](image)

The extended plant, $P$, describes the matrix transformation,

$$
\begin{bmatrix}
  z_p \\
  z \\
  y
\end{bmatrix} = P(s) \begin{bmatrix} w_p \\ w \\ u \end{bmatrix} = \begin{bmatrix}
  P_{11}(s) & P_{12}(s) & P_{13}(s) \\
  P_{21}(s) & P_{22}(s) & P_{23}(s) \\
  P_{31}(s) & P_{32}(s) & P_{33}(s)
\end{bmatrix} \begin{bmatrix} w_p \\ w \\ u \end{bmatrix}
$$

Without uncertainty ($\Delta=0$) the general framework reduces to an optimal $H_\infty$ control problem. For example, if $w$ was selected as the reference and $z$ was the tracking error then the optimisation would seek the controller, $K$, that minimised the $H_\infty$ norm of the transfer matrix from the reference, $w$, to the tracking error, $z$.

Robust stability

When model uncertainty, $\Delta$, is included it is assumed to be stable and bounded. Without information on phase behaviour the internal stability can only be deduced from the system magnitudes. Hence, a criteria for robust stability is conservatively based on the small gain theorem that considers the loop transfer function, $L_o=-P_{22}K$, shown in Figure 55.
Let $P_{22} \in \mathbb{R}H_{\infty}$, $K \in \mathbb{R}H_{\infty}$, and additionally $\|L_o\|_{\infty} < 1$. Then the feedback system is well-posed and internally stable. (Mackenroth, 2004)

It might be noted that the small gain theorem ensures satisfaction of the generalized Nyquist criteria for internal stability. If the loop transfer function $L_o$ has a peak magnitude less than unity then its Nyquist plot cannot encircle the minus one point and the feedback system will be stable.

![Figure 55: System with input and output disturbances $d_1$ and $d_2$](image)

To relate Figure 54 to the form of Figure 55, it may be rearranged as in Figure 56 with $M$ as the lower linear fractional transformation, $F_L(P, K) = F_{zw} = M = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}$.

The feedback system comprising $M_{11}$ and $\Delta$ has the form of Figure 55 and may be tested for stability. In terms of $\overline{\sigma}(\Delta)$, the smallest perturbation beyond which stability is no longer assured by the small gain theorem occurs when $\det(I-M\Delta)=0$.

![Figure 56: Uncertain feedback system](image)

To avoid conservative perturbations a set of structured uncertainty may be defined, $\Delta = \{\text{diag}(\delta_1 I_{i_1}, \ldots , \delta_r I_{r}, \Delta_1 , \ldots , \Delta_f) \mid \delta_i \in \mathbb{C}, \Delta_j \in \mathbb{C}^{n_j \times m_j} \}$.
Dimensioned such that, \( \sum_{i=1}^{r} r_i + \sum_{j=1}^{F} n_j = n, \quad \sum_{i=1}^{r} r_i + \sum_{j=1}^{F} m_j = m \)

A bound, \( 1/\gamma \) may be placed on the size of the perturbation, \( \sigma(\Delta) \), such that \( \det(I-M\Delta) = 0 \),
\[
\min\{\sigma(\Delta) \mid \Delta \in \Delta, \det(I-M\Delta) = 0\} \geq 1/\gamma \quad (\gamma > 0)
\]

This may be rearranged as,
\[
\gamma \geq \frac{1}{\min\{\sigma(\Delta) \mid \Delta \in \Delta, \det(I-M\Delta) = 0\}}
\]

For an arbitrary real or complex \( m \times n \) matrix \( M \) the structured singular value is defined by \( \mu_{\Delta} = \frac{1}{\min\{\sigma(\Delta) \mid \Delta \in \Delta, \det(I-M\Delta) = 0\}} \), if there is a perturbation \( \Delta \in \Delta \) such that \( \det(I-M\Delta) = 0 \). Otherwise let \( \mu_{\Delta} = 0 \). (Mackenroth, 2004)

The structured singular value, \( \mu_{\Delta} (M_{11}) \), measures the size of the system \( M_{11} \) and is a generalisation of the ordinary singular value (Mackenroth, 2004). The reciprocal of the largest singular value is a measure of the smallest ‘destabilising’ perturbation matrix (Zhou and Doyle, 1998).

With \( \mathcal{M}_{\Delta} \) denoting the set of model perturbations and \( \mathbb{C}_+ \) as the closed right half-plane,

Let the nominal feedback system \( M = F_L(P,K) \) be internally stable and let \( \gamma > 0 \). Then the following assertions are equivalent:

(i) \( \mu_{\Delta} (M_{11}(s)) \leq \gamma \) for every \( s \in \mathbb{C}_+ \).

(ii) The perturbed feedback system in Figure 56 is well-posed and internally stable for all \( \Delta \in \mathcal{M}_{\Delta} \) with \( \|\Delta\|_{\infty} < 1/\gamma \).

If \( \gamma \) denotes the peak of the \( \mu \)-plot of \( M_{11} \), then \( 1/\gamma \) is the largest magnitude of the perturbations for which the feedback loop is robustly stable. (Mackenroth, 2004).
**Robust performance**

A robust performance limit indicates the worst-case performance associated with a given level of perturbations. In a problem rearrangement, the performance may be treated as a form of uncertainty such that the robust performance problem is equivalent to a robust stability problem with augmented uncertainty (Zhou and Doyle, 1998). The inclusion of the performance uncertainty, $\Delta_I$, is shown in Figure 57.

![Figure 57: Introduction (a) and inclusion (b) of performance uncertainty $\Delta_I$ within the feedback system](image)

Let the nominal feedback system $M = F_L(P,K)$ be internally stable and let $\gamma > 0$. Then the following assertions are equivalent.

(i) For every $\Delta \in M(\Delta)$ with $\|\Delta\|_\infty < 1/\gamma$ the feedback system depicted in Figure 57 is well-posed, internally stable and fulfils the inequality $\|F_L(M,\Delta)\|_\infty < \gamma$.

(ii) The following inequality holds: $\sup_{\omega \in R} \mu(\Delta_p(M(i\omega))) \leq \gamma$.

Where $\Delta_p = \left\{ \begin{bmatrix} \Delta & 0 \\ 0 & \Delta_1 \end{bmatrix} \mid \Delta \in \Delta, \Delta_1 \in C^{p_2 \times p_2} \right\}$

Hence the peak of the $\mu$-plot of $M$ indicates the magnitude of the perturbations for which the feedback loop is robustly stable and has additionally robust performance (Mackenroth, 2004).
Robust control synthesis by D-K iteration

The largest perturbation, $\|\Delta\|_\infty$, for which the system has robust stability and performance is $1/\gamma$. The objective is to minimise $\gamma$.

Determine a controller $K$ such that the supremum $\sup_{\omega \in \mathbb{R}} \mu_{\Delta_\omega}(F_\omega(P, K)(i\omega)) \leq \gamma$ is as small as possible.

The D-K iteration seeks an approximate solution to this optimisation problem. A scaling matrix $D$ is introduced as shown in Figure 58. The algorithm then comprises two steps,

1) Given a scaling matrix $D$, calculate the $H_\infty$ controller $K$ for the plant $\hat{P}$ which has been scaled by $D$ and $D^{-1}$.

2) Minimise $\sigma(D_\omega F_\omega(P, K)(i\omega)D_\omega^{-1})$ for each frequency $\omega$ (here $P$ is the original unscaled plant). This leads to the scaling matrix $D$. (Mackenroth, 2004)

The first step is a standard $H_\infty$ optimisation that may be solved by a Ricatti, linear matrix inequality (LMI) or maximum entropy approach. A summary of $H_\infty$ controller synthesis by the Ricatti approach is provided in Appendix 7B and further detail may be found in (Mackenroth, 2004) or (Zhou and Doyle, 1998). The second step is a standard convex optimisation that can be solved at finitely many points in the frequency domain.

Figure 58: Feedback system (left) and scaled plant (right)
**Appendix 7B – \( H_\infty \) control problem and synthesis**

Without uncertainty (\( \Delta=0 \)), the framework for robust control shown in Figure 54 reduces to the system in Figure 59 and may be associated with the \( H_\infty \) control problem,

\[
\text{Find all admissible controllers } K(s) \text{ which minimize the } H_\infty \text{ norm of the feedback system, i.e. all admissible controllers that minimize } \|F_{cz}\|_\infty. \quad (\text{Mackenroth, 2004})
\]

Since finding the optimal controller is difficult and the solution is not unique, it is more common to solve the suboptimal \( H_\infty \) control problem,

\[
\text{For a given } \gamma > 0, \text{ find all admissible controllers } K(s) \text{ with } \|F_{cz}\|_\infty < \gamma. \text{ Such a controller is denoted as suboptimal.} \quad (\text{Mackenroth, 2004})
\]

![Figure 59: Standard system configuration](image)

The \( H_\infty \) control optimisation minimises the maximum singular value of the transfer matrix from the set of inputs \( w \) to the set of outputs \( z \). The inputs might include the reference set-point, disturbances, or noise while the outputs are signals to be minimised, such as a tracking error.

For example, the extended plant \( P \) may be described by,

\[
\begin{align*}
\dot{x} &= Ax + B_1w + B_2u \\
z &= C_1x + D_{11}w + D_{12}u \\
y &= C_2x + D_{21}w + D_{22}u
\end{align*}
\]
A suboptimal $H_{\infty}$ controller may then be characterised by means of Riccati equations (Zhou et al., 1996; Mackenroth, 2004). The result is that an $H_{\infty}$ suboptimal controller with $\|F_{zw}\|_{\infty} < \gamma$ is given by,

$$K_{\text{sub}}(s) = \begin{bmatrix} \hat{A}_{\infty} & -Z_{\infty}L_{\infty} \\ F_{\infty} & 0 \end{bmatrix}$$

with,

$$\hat{A}_{\infty} = A + \gamma^2 B_2 B_1^T X_{\infty} + B_2 F_{\infty} + Z_{\infty} L_{\infty} C_2$$

$$F_{\infty} = -B_2^T X_{\infty}, \quad L_{\infty} = -Y_{\infty} C_2^T, \quad Z_{\infty} = (I - \gamma^2 Y_{\infty} X_{\infty})^{-1}$$

If and only if the following conditions are fulfilled,

i) $H_{\infty} \in \text{dom}(\text{Ric})$ and $X_{\infty} = \text{Ric}(H_{\infty}) \geq 0$

ii) $J_{\infty} \in \text{dom}(\text{Ric})$ and $Y_{\infty} = \text{Ric}(J_{\infty}) \geq 0$

iii) $\rho(X_{\infty} Y_{\infty}) < \gamma^2$

Where $\rho(A)$ is the spectral radius of $A$.

Here $H_{\infty}$ and $J_{\infty}$ are the Hamilton matrices,

$$H_{\infty} = \begin{bmatrix} A & \gamma^2 B_1 B_1^T - B_2 B_2^T \\ -C_1^T C_1 & -A^T \end{bmatrix}$$

$$J_{\infty} = \begin{bmatrix} A^T & \gamma^2 C_1^T C_1 - C_2^T C_2 \\ -B_1 B_1^T & -A \end{bmatrix}$$

that correspond to the Riccati equations for $X_{\infty}$ and $Y_{\infty}$,

$$X_{\infty} A + A^T X_{\infty} - X_{\infty} (B_2 B_2^T - \gamma^2 B_1 B_1^T) X_{\infty} + C_1^T C_1 = 0$$

$$AY_{\infty} + Y_{\infty} A^T - Y_{\infty} (C_2^T C_2 - \gamma^2 C_1^T C_1) Y_{\infty} + B_1 B_1^T = 0$$
The solution is subject to assumptions that,

i) $(A,B_1)$ is stabilisable and $(C_1, A)$ is detectable

ii) $(A,B_2)$ is stabilisable and $(C_2, A)$ is detectable

iii) $D_{12}^T C_1 = 0$ and $D_{12}^T D_{12} = I$

iv) $B_1 D_{21}^T = 0$ and $D_{21} D_{21}^T = I$

v) $D_{11} = 0$ and $D_{22} = 0$

These assumptions may be restrictive and are commonly relaxed to a weakened set that can be satisfied by scaling and loop shifting (Mackenroth, 2004). The relaxed assumptions are,

i) $(A,B_2)$ is stabilisable and $(C_2, A)$ is detectable

ii) $\begin{bmatrix} A - i\omega I & B_2 \\ C_1 & D_{12} \end{bmatrix}$ has full column rank for every $\omega$

iii) $\begin{bmatrix} A - i\omega I & B_1 \\ C_2 & D_{21} \end{bmatrix}$ has full row rank for every $\omega$

iv) $D_{12}$ has full column rank and $D_{21}$ has full row rank

When these assumptions are satisfied a 2-Ricatti solution with loop shifting may be used to solve the $H_\infty$ sub-optimal control problem. As an example, Matlab’s `hinfsyn()` Ricatti approach is based on (Glover and Doyle, 1988; Doyle, Glover, Khargonekar and Francis, 1989) and (Saifonov, Limebeer and Chiang, 1989).
Appendix 7C – Mixed sensitivity design

A mixed sensitivity design shapes the system transfer functions to determine its response.

![Figure 60: Standard feedback configuration with input disturbance](image)

Relating the reference, \( r \), and input disturbance, \( d_i \), to the tracking error, \( e \), the input, \( u \), and the output, \( y \), it may be shown that,

\[
\begin{align*}
e &= S(s)r - S(s)G(s)d_i \\
u &= K(s)S(s)r - T(s)d_i \\
y &= T(s)r + S(s)G(s)d_i
\end{align*}
\]

Where the sensitivity, \( S \), and complementary sensitivity, \( T \), are,

\[
S = \frac{1}{1+L(s)} \quad \text{and} \quad T = \frac{L(s)}{1+L(s)}
\]

For this configuration the complete input-output behaviour is described by the four transfer functions \( S, SG, KS, T \). A mixed sensitivity design attempts to shape these functions using the performance weights \( W_{1,2,3} \) shown in Figure 61. The weights are associated with the error \( e \), the control \( u \), and the plant output \( y \).

![Figure 61: Mixed sensitivity configuration](image)
The transfer matrix from inputs \( w = [r] \) to outputs \( z = [z_1, z_2, z_3] \) is given by the lower linear fractional transformation,

\[
F_L(P, K) = F_{zw} = \begin{bmatrix}
W_1 S \\
W_2 KS \\
W_j T
\end{bmatrix}
\]

The weights \( W_{1,2,3} \) may be used to shape the mixed sensitivity functions, \( S, KS, \) or \( T \) in the frequency domain when the \( H_\infty \) design is used to minimise \( \|F_{zw}\|_\infty \). The functions tend to be shaped inversely to the corresponding weights. Also, since \( S+T=1 \), mixed sensitivity designs often consider a sub-selection of the outputs \( z_1, z_2 \) and \( z_3 \).
Appendix 7D – Robust control results (enlarged)

Figure 62: Measured and simulated brake apply to 5kN with robust control

Figure 63: Measured and simulated brake apply to 15kN with robust control
Figure 64: Measured and simulated brake apply to 25kN with robust control
Appendix 8A – An EMB bang-bang control

An optimal bang-bang EMB control may be constructed to drive the actuator hard towards the set-point and decelerate at the latest possible moment. The deceleration can be timed for zero overshoot based on an energy analysis.

Let $F_2^* = F_{cl}$ be the reference force setpoint and $F_1 = F_{cl}$ be the measured force. Let $x_1$, $x_2$, $\theta_1$ and $\theta_2$ be the corresponding nominal piston, and motor displacements. Also, let $\dot{\theta}_1$ and $\dot{\theta}_2$ be the corresponding motor velocity at the two conditions.

Control towards the force command is limited by the maximum motor velocity, $|\dot{\theta}| < \dot{\theta}_{\text{max}}$. Hence, maximum velocity will be the drive command. Scheduling the deceleration for zero overshoot can be performed by calculating the minimum stopping distance from an energy analysis. For operation between points 1 and 2 the following internal energies may be defined:

Motor work, \[ W_{\text{IN}} = \int_{\theta_1}^{\theta_2} T_m(t) d\theta \]

Kinetic energy of moving components, \[ K_E = \frac{1}{2} J\dot{\theta}(t)^2 \]
Frictional losses, \[ F_L = \int_{\theta_1}^{\theta_2} T_f(t) d\theta \]

Output clamp force work, \[ W_{OUT} = \int_{x_1}^{x_2} F_{cl}(x) dx \]

The maximum motor work between position 1 and 2 is given by, \[ W_{max} = \int_{\theta_1}^{\theta_2} T_{max} d\theta \]

The motor will not be able to dissipate the kinetic energy before reaching point 2 if, \[ K_E > F_L + W_{OUT} + W_{max} \]

Frictional dissipation is difficult to estimate with accuracy, but it always opposes motion and may be neglected conservatively.

\[ \therefore K_E > F_L + W_{OUT} + W_{IN} > W_{OUT} + W_{max} \]

Now the system may be decelerated when,

\[ K_E > W_{OUT} + W_{max} \]

\[ \therefore \frac{1}{2} J\dot{\theta}(t)^2 > \int_{x_1}^{x_2} F_{cl}(x) dx + \int_{\theta_1}^{\theta_2} T_{max} d\theta \]

\[ \therefore \frac{1}{2} J\dot{\theta}(t)^2 - \int_{\theta_1}^{\theta_2} F_{cl}(N\theta)N d\theta > T_{max} (\theta_2 - \theta_1) \]

\[ \therefore \theta_2 > \frac{1}{T_{max}} \left( \frac{1}{2} J\dot{\theta}(t)^2 - \int_{\theta_1}^{\theta_2} F_{cl}(N\theta)N d\theta \right) > \theta_1 \]

Position \( \theta_2 \) is known from the target force \( F_2 \) and the stiffness curve. The integral may be found by approximating the stiffness curve with a polynomial. Hence, the inequality may readily be computed to schedule the deceleration for zero overshoot.
Appendix 8B – Unconstrained MPC solution

The unconstrained model predictive control problem is to minimise the quadratic cost function,

\[ J = (Y - Y_{des})^T Q (Y - Y_{des}) + \Delta U^T R \Delta U \]

for a system represented by the state-space model,

\[ x_{k+1} = Ax_k + Bu_k \]
\[ y_k = Cx_k \]

where the output, target trajectory, input changes are,

\[
Y = \begin{bmatrix} y_{k+1} \\ \vdots \\ y_{k+H_p} \end{bmatrix}, \quad Y_{des} = \begin{bmatrix} y^*_{k+1} \\ \vdots \\ y^*_{k+H_p} \end{bmatrix} \quad \text{and} \quad \Delta U = \begin{bmatrix} \Delta u_k \\ \vdots \\ \Delta u_{k+H_s-1} \end{bmatrix}
\]

and the penalty weights are given by,

\[
Q = \begin{bmatrix} Q_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & Q_{H_p} \end{bmatrix} \quad \text{and} \quad R = \begin{bmatrix} R_0 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & R_{H_s-1} \end{bmatrix}
\]

The optimal solution is known (Maciejowski, 2002) and a derivation is only provided here for convenience.

The future states may be predicted by iteratively applying the state space model. This leads to the following result,

\[
\begin{bmatrix} x_{k+1} \\ \vdots \\ x_{k+H_p} \end{bmatrix} = \begin{bmatrix} A \\ \vdots \\ A^{H-p} \end{bmatrix} x_k + \begin{bmatrix} B \\ \vdots \\ \sum_{i=0}^{H_s-1} A^i B \end{bmatrix} u_{k-1} + \begin{bmatrix} B & \cdots & 0 \\ AB + B & \cdots & 0 \\ \vdots & \ddots & \vdots \\ \sum_{i=0}^{H_s-1} A^i B & \cdots & B \\ \sum_{i=0}^{H_s-1} A^i B & \cdots & A + B \\ \vdots & \ddots & \vdots \\ \sum_{i=0}^{H_s-1} A^i B & \cdots & \sum_{i=0}^{H_s-H_s} A^i B \end{bmatrix} \begin{bmatrix} \Delta u_k \\ \vdots \\ \Delta u_{k+H_s-1} \end{bmatrix}
\]
The future outputs are given by the predicted states,

\[
Y = \begin{bmatrix}
    y_{k+1} \\
    \vdots \\
    y_{k+H_p}
\end{bmatrix} = \begin{bmatrix}
    C & 0 & x_{k+1} \\
    \vdots & \ddots & \vdots \\
    0 & \ddots & C
\end{bmatrix} \begin{bmatrix}
    x_{k+1} \\
    \vdots \\
    x_{k+H_p}
\end{bmatrix} = \Omega \begin{bmatrix}
    x_{k+1} \\
    \vdots \\
    x_{k+H_p}
\end{bmatrix}
\]

This may also be written in the form,

\[
Y = \Psi + \Gamma \Delta U
\]

where \( \Psi = \Omega \begin{bmatrix}
    A \\
    \vdots \\
    A^T_{H_p}
\end{bmatrix} + \Omega \sum_{i=0}^{H_u-1} A^i B \]

and \( \Gamma = \Omega \begin{bmatrix}
    B & \cdots & 0 \\
    AB + B & \cdots & 0 \\
    \vdots & \ddots & \vdots \\
    \sum_{i=0}^{H_u-1} A^i B & \cdots & A + B \\
    \sum_{i=0}^{H_u-1} A^i B & \cdots & \sum_{i=0}^{H_u-1} A^i B
\end{bmatrix} \)

Now substituting \( Y \) into the cost function and expanding,

\[
J = (\Delta U^T \Gamma^T + (\Psi - Y_{des})^T)Q(\Gamma \Delta U + (\Psi - Y_{des})) + \Delta U^T R \Delta U
\]

\[
J = (\Psi - Y_{des})^T Q(\Psi - Y_{des}) + \Delta U^T \Gamma^T Q(\Psi - Y_{des}) + (\Psi - Y_{des})^T Q \Gamma \Delta U + \Delta U^T (\Gamma^T Q \Gamma + R) \Delta U
\]

The stationary point may be found by taking the derivative with respect to the control trajectory, \( \Delta U \), and setting the result to zero,

\[
\frac{dJ}{d\Delta U} = 2\Gamma^T Q(\Psi - Y_{des}) + 2(\Gamma^T Q \Gamma + R) \Delta U = 0
\]

The stationary point is a minima with a positive-definite double derivative when positive weight values are chosen for \( Q \) and \( R \). Hence the optimal control trajectory adjustments are given by,

\[
\Delta U_{opt} = -(\Gamma^T Q \Gamma + R)^{-1} \Gamma^T Q(\Psi - Y_{des})
\]
Appendix 9A – Stability of the inertia observer

An inertia estimate, \( \hat{J}(t) \), may be adjusted when it produces an erroneous friction estimate, \( \hat{T}_F(\hat{J}(t),t) \), that acts in the wrong direction (Figure 66). An erroneous estimate may be detected when \( \text{sign}(\hat{T}_F(t)) \neq \text{sign}(\dot{\theta}(t)) \) since the friction always opposes the direction of motion. The positive directions are defined in Figure 67.

\[
\begin{align*}
\hat{T}_F &\quad 0 \quad T_F \\
\text{Figure 66: Number line showing friction, } T_F, \text{ and the estimate, } \hat{T}_F, \text{ when they erroneously act in opposite directions such that } \text{sign}(\hat{T}_F) \neq \text{sign}(\dot{\theta})
\end{align*}
\]

With \( J \) as the true inertia, the error in the estimate is given by,
\[
e_j(t) = J - \hat{J}(t)
\]

A friction estimate is calculated by,
\[
\hat{T}_F(t) = i_q(t)K_I - F_{cl}(t)N - \dot{\hat{J}}(t)
\]

While the true friction may be written as,
\[
T_F(t) = i_q(t)K_I - F_{cl}(t)N - \dot{\hat{J}}(t) + \varepsilon(t) \quad \text{with } |\varepsilon(t)| < h_1 \quad \text{and } |T_F(t)| < h_2
\]

where \( \varepsilon(t) \) is an error associated with an imperfect parameters \( K_I \) and \( N \) and error in the measurements \( i_q(t) \), \( F_{cl}(t) \) and \( \dot{\hat{J}}(t) \). The magnitude of the error and the friction may be anticipated within reasonable bounds \( h_1 \) and \( h_2 \).

Hence the error in the friction estimate is given by,
\[
e_{TF}(t) = T_F(t) - \hat{T}_F(t)
\]
Assumption A1: There exists a value of \( \hat{J} \) that minimizes \( |T^*_F(t) - T_F(t)| \)

**Theorem:** Given assumption A1, an update law given by,

\[
\frac{d\hat{J}}{dt} = \begin{cases} 
-L \frac{e_{TF0}(t)}{\theta(t)} & \text{if } \text{sign}(\hat{T}_F(t)) \neq \text{sign}(\dot{\theta}(t)) \\
0 & \text{otherwise}
\end{cases}
\]

with \( e_{TF0}(t) = -\hat{T}_F(t) \), results in a stable convergence of \( \hat{J} \) towards \( J \) such that the magnitude of the estimation error is bounded.

**Proof:** Defining a candidate Lyapunov function as,

\[
V_i = \frac{1}{2} e_j^2
\]

and taking the derivative of \( V_i \) with respect to time,

\[
\dot{V}_i = \frac{dV_i}{de_j} \frac{de_j}{dt}
\]

\[
\dot{V}_i = e_j \frac{dV_i}{de_j} \frac{de_j}{dt}
\]

\[
\dot{V}_i = e_j \frac{d(J - \hat{J}(t))}{dt}
\]

\[
\dot{V}_i = -e_j \frac{d\hat{J}(t)}{dt}
\]

\[
\dot{V}_i = e_j L \frac{e_{TF0}(t)}{\theta(t)} \quad \text{when the observer is active, } \text{sign}(\hat{T}_F(t)) \neq \text{sign}(\dot{\theta}(t))
\]

\[
\dot{V}_i = e_j L \frac{-\hat{T}_F(t)}{\theta(t)}
\]

\[
\dot{V}_i = e_j L \frac{T_F - \hat{T}_F(t) - T_F}{\theta(t)}
\]
Substitution of $T_F(t)$ and $\hat{T}_F(t)$ leads to,

$$\dot{V}_1 = -e_j L \frac{\bar{\theta}(t)(J - \bar{J}) + \varepsilon(t)}{\bar{\theta}(t)} - e_j \frac{L T_F(t)}{\bar{\theta}(t)}$$

$$\therefore \dot{V}_1 = -e_j^2 L - e_j \frac{L}{\bar{\theta}(t)}(\varepsilon(t) + T_F(t))$$

This is negative definite, $\dot{V}_1 < 0$, when,

$$|e_j^2 L| > \left| e_j \frac{L}{\bar{\theta}(t)}(\varepsilon(t) + T_F(t)) \right|$$

$$|e_j| > \left| \frac{\varepsilon(t) + T_F(t)}{\bar{\theta}(t)} \right|$$

In which case the inertia estimate will converge to within a bounded error depending on the acceleration $\bar{\theta}(t)$, the friction, $|T_F| < h_2$, and the parameter and measurement error, $|\varepsilon(t)| < h_1$. It might be noted that the acceleration tends to be large during estimator updating about changes in direction.
Appendix 9B – Stability of the α observer

A stiffness scale factor, $\alpha$, may be estimated with the observer shown in Figure 68.

Here the clamp force estimation error is given by,

$$e_c(t) = F_{cl}(t) - \hat{F}_{cl}(t)$$

Denoting the stiffness characteristic as $K(x)$, the clamp force estimate is given by,

$$\hat{F}_{cl}(x(t)) = K(x)\hat{\alpha}$$

With $\alpha_{opt}$ being the optimal scaling, the true clamp force is given by,

$$F_{cl}(x) = K(x)\alpha_{opt} + \epsilon(x) \quad \text{with} \quad |\epsilon(x)| < h_1 \quad \text{for} \quad x \in X.$$  

where $\epsilon(x)$ is a continuous and smooth error associated with an imperfect stiffness curve shape $K(x)$ and $X$ is the set of possible displacements $x=N\theta$.

**Assumption A1:** There exists a value of $\hat{\alpha}$ that minimizes $|F_{cl}(x) - \hat{F}_{cl}(x)|$ over the domain $x \in X$.

**Theorem:** Given assumption A1, an update law given by,

$$\frac{d\hat{\alpha}}{dt} = L_2 e_c$$

results in a stable convergence of $\hat{\alpha}$ towards $\alpha_{opt}$ such that the magnitude of the force error is bounded.
**Proof:** Defining a candidate Lyapunov function as,

\[
V_1 = \frac{1}{2} e_F^2
\]

and taking the derivative of \(V_1\) with respect to time,

\[
\dot{V}_1 = \frac{dV_1}{de_F} \frac{de_F}{dt} = e_F \frac{de_F}{dt}
\]

Now the force error may be written,

\[
\alpha(t) = F_{c}(t) - \hat{F}_{c}(t)
\]

\[
\therefore e_F(t) = K(x)(\alpha_{\text{opt}} - \hat{\alpha}(t)) + \epsilon(x)
\]

Therefore,

\[
\dot{V}_1 = e_F K(x) \frac{d(\alpha_{\text{opt}} - \hat{\alpha}(t))}{dt} + e_F \frac{d\epsilon(x)}{dt}
\]

\[
\therefore \dot{V}_1 = e_F (-K(x) \frac{d\hat{\alpha}(t)}{dt} + \frac{d\epsilon(x)}{dt})
\]

Substituting the update law for \(\hat{\alpha}(t)\),

\[
\dot{V}_1 = -K(x) L_2 E_F^2 + e_F \frac{d\epsilon(x)}{dx} \frac{dx}{dt}
\]

This is negative definite, \(\dot{V}_1 < 0\), when,

\[
-K(x) L_2 E_F^2 + e_F \frac{d\epsilon(x)}{dx} \frac{dx}{dt} < 0
\]

\[
e_F \frac{1}{K(x) L_2} \frac{d\epsilon(x)}{dx} \frac{dx}{dt} < E_F^2
\]

This is satisfied when,

\[
e_F \frac{1}{K(x) L_2} \frac{d\epsilon(x)}{dx} \frac{dx}{dt} < E_F^2
\]
\[ \left| \frac{1}{K(x) L_2} \frac{d\varepsilon(x)}{dx} \frac{dx}{dt} \right| < |e_p| \]

In which case the force estimate will converge to within a bounded error. It may be observed that the error bound is related to error in the stiffness characteristic, \( \varepsilon(x) \), and depends on the velocity. The velocity is limited, \( |dx/dt| < l_t u_d \), by the maximum EMB motor speed.
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Modelling and control of an automotive electromechanical brake

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