COLLABORATIVE PROBLEM SOLVING IN MATHEMATICS: THE NATURE AND FUNCTION OF TASK COMPLEXITY

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DSME
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Declaration of Originality

This thesis contains no material which has been accepted for any other degree in any university. To the best of my knowledge and belief, this thesis contains no material previously published or written by any other person, except where due reference is given in the text.

Signature

Gaynor Williams
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Collaborative problem solving in Mathematics: the nature and function of task complexity

The nature and function of Task Complexity, in the context of senior secondary mathematics, has been identified through: a search of the research literature; interviews with experts that focused on the nature of task complexity; expert use of the Williams/Clarke Framework of Complexity (1997) as a tool to categorise the complexity of a task, and observation and analysis of the responses of senior secondary mathematics students as they worked in collaborative groups to solve an unfamiliar challenging problem. Although frequently used in the literature to describe tasks, ‘complexity’ has often lacked definition. Expert opinion about the nature of mathematical complexity was ascertained by seeking the opinions of experts in the areas of mathematics, mathematics education, and gifted education. Expert opinion about task complexity was stimulated by questions about the relative complexity of two tasks. The experts then categorised the complexities within each of these tasks using the Williams / Clarke Framework of Complexity. This framework identifies the dimensions of task complexity and was found by experts to be both useful and adequate for this purpose. A theoretical framework was developed to assess student ability to solve challenging problems. This theoretical framework was used to design a test to assess student ability to solve challenging problems. The information this test provided about the problem solving ability of the students in this study informed my analysis of student response to complexity. Case studies of two collaborative groups of final year secondary mathematics students were undertaken and these studies indicated the construct of a Discovered Complexity was a useful tool to analyse student response to complexity. This construct was formulated after preliminary observation of the video data. The task explored by the students was found to contain many potential complexities to discover but the two collaborative groups differed in the number and nature of complexities discovered. The discovery of complexities was found to add a dynamic element to the task as each new complexity altered the students’ perception of the task. The discovery of complexities was found to be associated with increased student engagement with the task and increased conceptual development. The interrelationships between Task Complexity, student engagement and conceptual development suggested by the findings in this study have been explained using a schematic representation I have named ‘Engaged to Learn’. This representation relates the concept of Flow and the concept of the Zone of Proximal Development to the concept of a Discovered Complexity thus relating the cognitive and affective aspects of learning.
CHAPTER 1

INTRODUCTION
1.1 Introduction
Over more than twenty-five years of teaching secondary mathematics, I became dedicated to encouraging students to develop a passion for mathematics by the recognition of its use and beauty. I had noticed there were instances where students experienced a high level of engagement in the learning process and that this appeared to result in the attainment of a higher quality of conceptual development than usual. Upon reflection and experimentation I decided the level and frequency of engagement in the learning process appeared to be dependent upon the structure of the task, the instructional approach, the use of small collaborative groups of students and the composition of these groups. I decided to investigate this process through a study intended to support and extend my continued reflection upon the teaching and learning process. The dual role of teacher and researcher created some additional problems, but these were outweighed by the benefits gained by researching the learning taking place in my own classroom. As I observed and reflected upon the video record of my classes, my understanding of the processes by which students learned and the part I (as teacher) played in creating an atmosphere conducive to learning was extended. I found this reflective process both stimulating and enriching.

This chapter explains: (a) the elaboration of key terms; (b) the impetus for this study; (c) the focus of this study; (d) the research context; (e) the research questions; (f) the purpose of this study; (g) the limitations of the research; and (h) a guide to the format of the thesis.

1.2 Elaboration of Key Terms
The terms ‘unfamiliar challenging problems’, ‘task complexity’, ‘class collaboration’, ‘ability to solve unfamiliar challenging problems’ and ‘group interaction patterns’ are central to my research and have been elaborated to clarify the intended meaning of these terms in this thesis.

1.2.1 Unfamiliar Challenging Problems
The term ‘mathematics problem’ has generally been applied to a broad spectrum of very different activities in mathematics, such as: worded problems that require the student to apply a learned algorithm; and open-ended tasks (with little direction supplied) that require students to connect mathematical ideas in unfamiliar combinations. In this study, the term ‘unfamiliar challenging problems’ refers to tasks: (a) presented before the relevant mathematical concepts have been ‘taught’; (b) that cannot be solved by the application of algorithmic procedures assumed known to the students; and (c) that require students to analyse mathematical representations to connect mathematical ideas and to build concepts new to the students. The types of unfamiliar challenging problems used in this study required students to focus on patterns within the mathematics itself rather than patterns related to a physical context. The mathematical techniques and problem solving strategies utilised by the different
groups of students typically differed in mathematical domain and sophistication. For example, at opposite ends of the solution spectrum of such problems are the solutions that rely upon trial and error through the use of specific numerical examples and the solutions formulated through a clearly justified generalisation.

Different groups would not necessarily reach the same solution to an unfamiliar challenging problem. Amongst the factors that could be responsible for these differences are: the simplifying assumptions students make, the level of accuracy determined by the mathematical techniques the students select, the problem solving strategies students use, and those aspects of the problem the students decide to analyse in depth. As the intention was for students to develop an understanding of new concepts, exploration of a task may at times have provided more benefits than task completion.

1.2.2 Task Complexity
Initially I understood task complexity to depend upon the number of components within a task and the degree of interrelationship between these components. I considered tasks increased in complexity as the number of components increased and/or the degree of interrelationship between these components increased. Although I was unclear about the diversity of form these components and their interrelationships could take, it seemed to me that the intellectual components of the task could add to the complexity and that these intellectual components were a contributing factor to the level of student engagement. My study began with a literature review to determine whether researchers shared a common view of task complexity and if so, whether this view was consistent with mine. As researchers did not share a common view, the elaboration of the term ‘task complexity’, theoretically and empirically, became a major goal of this research.

1.2.3 Class Collaboration
Learning through an instructional technique that relies upon a cycle of small group collaboration followed by whole class collaboration is typified by the use of open-ended tasks undertaken over several lessons by students in small collaborative groups who report their progress to the class at regular intervals. It is the responsibility of each group to evaluate the correctness and usefulness of information provided by their fellow students during these feedback sessions.

This instructional technique has been called ‘Class Collaboration’ for the purpose of this study and is explained in more detail in Chapter 5 when student response to complexity is investigated. Class collaboration, as employed in this study, relies initially on the teacher creating a classroom environment in which: (a) students feel able to take risks; (b) more than one solution pathway is possible for any given problem; (c) students develop and justify ideas for themselves; and (d) collaborative groups value the contributions of all students in the group.
In this approach the teacher does not answer questions, provide hints or indicate whether a solution pathway is correct or not. Instead, the teacher asks questions, which help the students to work these ideas out for themselves. The teacher’s questions help groups: (a) define the problem; (b) recognise when they are ‘off track’; (c) clarify thoughts when an idea is partly formulated; (d) move towards the next breakthrough; (e) analyse their findings; (f) evaluate their progress so far; and (g) generalise their results (Williams 1994). This approach is described in more detail later in this thesis when student response to complexity is analysed (Chapter 5).

1.2.4 Ability to Solve Unfamiliar Challenging Problems

Points of view differ about whether ‘ability’ exists as a general static potential (Binet and Simon, 1980) or whether environmental factors can influence this ability (Krutetskii, 1976; Schoenfeld, 1994; Sweller, 1992). For the purpose of this study, I will use the term ‘ability’ to mean the capacity to solve unfamiliar challenging problems. I conceive ability to solve these problems to be a dynamic rather than static characteristic influenced by both innate and environmental factors. My classroom observation prior to this study led me to believe that students possessed differing capacities to draw upon certain types of thinking when solving unfamiliar challenging problems and that a student’s capacity to solve problems could be altered by environmental factors to which the student was exposed. At any point in time, students differed in their capacity to: define the problem, generate possible solution pathways, evaluate progress, attain closure, provide sufficient justification, generalise results, evaluate conclusions and provide an elegant solution (Victorian Curriculum and Assessment Board (VCAB), 1992a; VCAB, 1992b). These and other characteristics, identified within this study, account for the differences in what I have termed ‘student ability to solve unfamiliar challenging problems’. This ability to solve unfamiliar challenging problems is referred to as ‘Ability’ for the remainder of this study.

1.2.5 Group Interaction Patterns

For the purpose of this study, the term ‘group interaction patterns’ refers to the manner in which the members of a group work together to build an understanding of the mathematics, in particular the extent to which students evaluate, build upon and combine the ideas contributed by individual group members.

1.3 The Impetus for this Study

My decision to undertake formal research about how members of a collaborative group respond to task complexity resulted from feedback I gained prior to this study. This feedback suggested students who learnt mathematics through class collaboration experienced positive changes in how they perceived mathematics as a subject and in how they perceived themselves as mathematicians. I decided to study those factors that appeared to contribute to student engagement in the task and to student development of their own mathematical understanding.
A summation of my transition from teacher to researcher is now provided. It includes discussion of: factors that influenced my development of this teaching approach and the feedback that reassured me this approach was worth pursuing.

1.3.1 My Development of the Class Collaboration Approach
Class collaboration resulted from my refinement (over several years) of classroom practice to cater better for the diversity of learning needs exhibited by secondary school mathematics students. In particular, I had worked to develop an approach that facilitated the development of enriched conceptual understanding for students with high mathematical ability whilst simultaneously providing access to the mathematics curriculum for students who processed mathematical ideas at a slower pace and/or in a less sophisticated manner. I gradually recognised that my former traditional teacher-centred style catered inadequately for students in a mixed-ability classroom. Some students did not have access to the curriculum because they lacked the required background. In my opinion, too many students felt they were failures at mathematics and too few students were challenged or motivated by the mathematics presented. I began to formulate an approach that provided more students with access to the curriculum, success in their mathematical endeavours and challenge within the mathematical tasks they encountered.

1.3.2 Feedback Affirming Class Collaboration Prior to this Research
I received feedback affirming class collaboration from a variety of sources: students studying secondary mathematics through class collaboration, students who had studied secondary mathematics through class collaboration and were now undertaking tertiary mathematics, changes to the proportion of students selecting senior mathematics in the school where instruction through class collaboration had existed for several years and my classroom observations of students undertaking class collaboration.

1.3.2.1 Class Collaboration: feedback from present students
Colleagues from several schools in which I have been mathematics co-ordinator and colleagues from both government and private schools in metropolitan and rural areas have experimented with this instructional approach at all levels in the secondary mathematics curriculum. I began to document feedback from students for several years prior to this study. Students provided many useful comments that assisted my refinement of the class collaboration approach. The positive feedback provided by students encouraged my continued development and use of this approach to teaching. I now provide a selection of students’ comments (made prior to this study) about this approach. Comments such as these encourage my continued interest in learning facilitated through class collaboration. Section 1.4.2 provides more detail about the mathematics subjects these students undertook. A representative selection of student feedback is now included to provide evidence about how students perceive class collaboration as a teaching approach.
On students first encountering class collaboration some uncertainty existed about the expectations and benefits of the approach.

*Have you noticed? She doesn’t answer questions, she just asks another*  
[First month of 1995: Group of four Year 12 Specialist Mathematics students].

*She is never going to tell us, we are just going to have to work it out for ourselves* (and they did)  
[1995: Same group of four Year 12 Specialist Mathematics students a week later].

*At the beginning of year 12 we used to wonder if you (the teacher) really didn’t know or you were just a good actress*  
[1997: Third Year Science/Engineering student (previously 1994 Year 12 Specialist Mathematics and Mathematical Methods student)].

On students developing responsibility for their own learning students became active rather than passive learners.

*I really feel empowered now I realise I can work things out for myself*  
[1996: Year 11 female student belonging to the class of a colleague using class collaboration].

*Are you (the teacher) sure you don’t want to ask us any questions?*  
[Many Mathematical Methods and Specialist Mathematics male and female students during Year 12].

On students becoming excited and engaged in the learning process

*It is so exciting when you can report something you know no one else has found*  
[Second Semester 1995: Year 12 Specialist Mathematics and Mathematical Methods male student].

On appreciation of the process: development of metacognition students began to look beyond the solution to the present problem and consider learning and problem solving from a broader perspective.

*I really enjoy seeing how other people think about a problem, I had always thought we all did it the same way*  
[1994: Year 9 male student]

*When you report (at the board) it really crystallises in your mind. You have to understand it better than when you are just writing a report.*  
[1994: Mathematical Methods male student who had not enjoyed mathematics the previous year but had become interested during Year 12].

*You have made me realise the essential things of maths as well as the essential things of life—how to solve problems*  
[1995: Mathematical Methods female student at the end of the year].

Although a small proportion of the feedback received indicated that certain students would still prefer a more conventional approach, overall student comments were positive and suggested that the group work and the reporting process helped them develop mathematical understanding. In my experience, both males and females became motivated by class collaboration and recognised the benefits of this approach.
1.3.2.2 Class Collaboration: feedback from past students

Thirteen of the sixteen students in my 1994 Specialist Mathematics class undertook tertiary courses containing at least one first year mathematics subject. In their second semester of tertiary mathematics (1995), the opportunity arose to individually ask twelve of these students whether their experiences with class collaboration had helped, hindered or made no difference to their tertiary mathematics studies. All except one student responded in terms of the benefits their class collaboration experience had provided in coping with tertiary mathematics.

The substance of the comments from the majority of the 1994 students resembled these examples:

- *It has helped me at university. My understanding of what the mathematics is about is much better than the understanding of most students.*

- *It has helped me; when I can’t do something for a start, I just persist. I know I am capable of working something out for myself or as part of a group.*

The student with an alternative view stated:

- *It has made university mathematics boring in contrast. . . . you must keep teaching this way though, I would not want to give up my Year 12 experience for anything.*

In summary, students generally believed class collaboration had been of benefit when they continued with tertiary studies in Mathematics. Even the student who found tertiary mathematics boring in comparison to class collaboration believed it was not the class collaboration that needed to be changed.

1.3.2.3 Class Collaboration: influence on subject selection

Prior to this study, my personal observations and other feedback suggested that class collaboration was one of the factors responsible for students’ increased selection of higher-level senior secondary mathematics subjects. In higher level senior mathematics classes (Specialist Mathematics) the number of male and female students increased, and the disproportionate under-representation of girls in these classes became less exaggerated.

1.3.2.4 Proportion of female students increased

In 1994, the new subject ‘Specialist Mathematics’ became a part of the Victorian secondary school curriculum (see Section 1.5.2). The mathematics faculty of the school, at which my research was undertaken in 1996, recognised that many capable mathematicians—particularly girls—were not selecting this subject. It was hoped that the discussion and teamwork facilitated by class collaboration would interest more girls in mathematics.
I just love maths now. It makes such a difference when you can explain what you understand to each other. I get so excited when we solve problems. I now know I can solve them myself.

[1993: Year 11 female student after a semester of class collaboration. This student later successfully undertook tertiary studies in Engineering].

This comment reflects the change in this girl’s perception of herself as a mathematician. She attributes this change to the collaborative group work. Her subsequent enrolment in and success with Engineering demonstrated the long-term effects of these changes in perception.

Increased participation of girls in the more difficult senior secondary mathematics subjects, although not the focus of this study, was one of the motivating factors in the development of this approach. My past experience indicated that girls were often more comfortable and confident in mathematics classes where there was a larger proportion of girls than boys. During 1994 there were only two girls in a class of sixteen students in Specialist Mathematics. In 1996, when there were sufficient students for two classes, girls were all placed in one class (see Table 1.1). Although the proportion of girls in the subject was still less than half, it was hoped by clustering girls together their learning environment would improve. In my experience, girls were more likely to participate in whole class collaboration if their representation in the class was close to or above 50%. At the beginning of the year, girls tended to respond positively to the support of at least one other girl in a small collaborative group. At later stages in the year this was generally not an issue for some of the girls. I hoped the learning environment produced by increasing the proportion of girls in the class would be less threatening to these girls and would thus further encourage girls—in the future—to select senior mathematics. This decision was independent of this research.

1.3.2.5 Increased selection of Specialist Mathematics.

Although the number of students and proportion of girls in the final year of secondary education at this school remained relatively constant, (approximately 140 students), the number of students selecting Specialist Mathematics more than doubled in three years (see Table 1.1).

The number of girls selecting this course during this time had increased seven fold. Provided all other factors were equal, this suggested the teaching approach and/or the clustering of girls together affected both the total number of students selecting this subject and the number of girls selecting this subject (see Table 1.1 ). One other factor that could have made a partial contribution to these changes was student recognition of the high weighting Specialist Mathematics attracted when tertiary entrance scores were calculated.
Table 1.1 Increased Participation of Students and Increased Participation of Girls in Specialist Mathematics (1994 - 1997) in the school where my study was undertaken.

<table>
<thead>
<tr>
<th>Year</th>
<th>Number of Specialist Mathematics classes</th>
<th>Number of students studying Specialist Mathematics</th>
<th>Number of girls studying Specialist Mathematics</th>
<th>Percentage of the Specialist Mathematics students who are girls</th>
</tr>
</thead>
<tbody>
<tr>
<td>1994</td>
<td>1</td>
<td>16</td>
<td>2</td>
<td>13%</td>
</tr>
<tr>
<td>1995</td>
<td>1</td>
<td>22</td>
<td>2</td>
<td>9%</td>
</tr>
<tr>
<td>1996</td>
<td>2</td>
<td>30</td>
<td>5</td>
<td>17%</td>
</tr>
<tr>
<td>1997</td>
<td>2</td>
<td>38</td>
<td>14</td>
<td>37%</td>
</tr>
</tbody>
</table>

As anecdotal evidence suggested other schools were not experiencing similar changes in gender proportions, these differences in weighting do not appear to be the main reason for these changes.

1.3.2.6 My classroom observation

I noticed small collaborative groups frequently became engaged in mathematical discourse—even to the extent that they ignored the bell for the end of class and continued their discussion. On frequent occasions groups of students collaborated in different ways when responding to tasks that I considered differed in complexity. The complexity of a task appeared to alter the manner in which a group interacted when working to solve the task.

Table 1.2 Summary of feedback (prior to this study) related to students studying mathematics by working to solve unfamiliar challenging problems through class collaboration.

<table>
<thead>
<tr>
<th>Source of Feedback</th>
<th>Feedback provided</th>
<th>Significance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Students undertaking secondary studies at the time</td>
<td>Students learning through class collaboration at the time reported they: • were empowered in the learning process • enjoyed mathematics now</td>
<td>Engagement of the students in the learning process.</td>
</tr>
<tr>
<td>Tertiary students</td>
<td>Tertiary students reported Year 12 class collaboration assisted because: • they understood mathematics better than most students • they felt empowered to work out anything they didn’t understand</td>
<td>Teaching through class collaboration increased student ability to cope with subsequent mathematics courses.</td>
</tr>
<tr>
<td>Increased selection of senior mathematics</td>
<td>• increased number of males and females selecting senior secondary mathematics • increased proportion of females in senior secondary mathematics</td>
<td>Teaching through class collaboration increased student selection of senior secondary mathematics.</td>
</tr>
<tr>
<td>My observation</td>
<td>• more students became engaged in the learning of mathematics • group interaction patterns appeared to depend on the task and the group composition</td>
<td>Group composition appeared to affect student engagement and appropriate group composition appeared to vary for different types of tasks.</td>
</tr>
</tbody>
</table>
Table 1.2 summarises the feedback collected, prior to this study, about student response to class collaboration. Comments from present students and past students studying tertiary mathematics, the increased selection of senior mathematics subjects at this school, and my own classroom observations confirmed the benefits students gained from this collaborative approach. Class collaboration appeared to empower students in the learning process, increase the selection of senior secondary mathematics subjects and equip students to cope with tertiary mathematics.

1.3.2.7 My response to these informal findings
It appeared that students improved their perception of themselves as mathematicians when they learnt by class collaboration and as a result selected more senior mathematics courses. As a result of these informal findings (Table 1.2) and because of the contribution I believed the task made to creating this effective learning environment, I decided to investigate both the nature of task complexity and student response to this complexity. Retaining my role as teacher, I made the transition from teacher to teacher/researcher.

1.4 The Focus of this Study
This research investigates the role of task complexity where an instructional approach that includes both small group and whole class collaboration was used with senior secondary mathematics students as they worked to solve unfamiliar challenging problems. The nature of task complexity was explored through the research literature and by a survey of the opinions of appropriate experts (Appendix 1). A measure of student ability to solve unfamiliar challenging problems was developed to inform my subsequent study of student response to complexity. I developed case studies that portrayed students working in collaborative groups to solve unfamiliar challenging problems in mathematics. Student response to task complexity was investigated through analysis of the mathematical discourse in collaborative groups, identification of the complexities encountered and study of student engagement in the task. Inferences were drawn regarding the role of task complexity when solution to such problems was undertaken through small group and whole class collaboration. The implications of these findings for educators and teachers were identified. In particular, the significance of task construction, task selection and instructional approaches utilised was discussed in the context of the facilitation of concept development.

1.5 Research Context
A description of the students, the school, the mathematics subject studied and the unfamiliar challenging problems investigated is now provided.

1.5.1 Students and School
The research subjects were final year secondary students at a Victorian State secondary college with a total enrolment of 1100 students and a reputation for academic
excellence. A large proportion of these students was first generation Australians with English as their second language. Over one-third of the 1996 Specialist Mathematics students began their education in another country. Many of the students had arrived from other countries during mid-secondary education. Approximately a third of the Specialist Mathematics students had come from countries within the former Soviet Union. This was a much larger proportion than these students represented in the general student body within this school (approx 15%). Less than one-third of the students in the Specialist Mathematics classes originated from families who had lived in Australia for more than two generations.

1.5.2 Specialist Mathematics
Specialist Mathematics is a subject available to students in the final year of secondary school. This subject is prerequisite to some tertiary engineering, surveying and physics courses. It is useful to students intending to pursue further studies in calculus at the tertiary level. Amongst the topics studied are complex numbers, vectors, and various methods of integration, applications of integration and mechanics. All students undertaking Specialist Mathematics also undertake a more general subject in calculus (Mathematical Methods).

1.5.3 Tasks Investigated
The senior secondary mathematics (calculus) content upon which these tasks focused related to the second derivative and the different techniques of integration (Appendix 2, Appendix 3). As the students worked with the problems, I expected they would develop an understanding of the significance of the second derivative and how to interpret graphs of the second derivatives of functions (Appendix 2), and formulate a framework to assist their recognition of the algebraic structure associated with standard forms of integration (Appendix 3).

1.6 The Research Questions
To investigate task complexity and student response to task complexity, the following questions were posed:

- What is the nature of task complexity?
- How do students respond to various aspects of complexity when an unfamiliar challenging problem is undertaken through class collaboration?

1.7 The Purpose of this Research Study
This study identifies some of the variables that impinge upon the optimisation of learning in the mathematics classroom. It is hoped that findings about student response to task complexity will promote teacher experimentation with collaborative learning approaches and tasks of varying complexity. This could then provide an opportunity for extension of the teaching repertoires of mathematics teachers.
Chapter 1: Introduction

This study should provide teachers with a framework that could increase their understanding and their application of the concept of task complexity. This should lead to the formulation or selection of unfamiliar challenging problems with the potential to optimise student engagement in the learning process. It should also assist teachers to identify and manipulate some of the key variables that influence the effectiveness of the mathematics classroom.

If the results of this study encourage other researchers to pursue further research in the area of task complexity in mathematics, then the methodological techniques and the measuring instruments I have used could inform the selection of instruments and techniques for any such subsequent research.

1.8 Limitations of the Study

The research within this minor thesis is intended as a starting point only. The key outcomes of this study are: the analysis of task complexity and the inferences drawn about the effects of task selection on student response when students work to solve unfamiliar challenging problems in senior secondary mathematics through class collaboration.

Due to the scope of this minor thesis, further research will be required to test the generality of the conclusions reached. In particular, further studies should sample across a wider range of task types, year levels and student populations.

1.9 Format of Thesis

This thesis relies upon research into three topics that become inter-connected as the research progresses. Findings related to task complexity and student ability to solve unfamiliar challenging problems informed my study of student response to task complexity. To enable the reader to follow the pathway that progressively informed my research, this thesis format allocates the method, results, analysis and discussion for each of these three topics to three successive chapters. The composition of the six chapters contained in this thesis are as follows: Chapter 1: Introduction; Chapter 2: Literature review and development of theoretical constructs; Chapters 3: Task complexity; Chapter 4: Ability to solve unfamiliar challenging problems; Chapter 5: Student response to task complexity; and Chapter 6: General discussion and the formulation of overall conclusions.
CHAPTER 2

LITERATURE REVIEW
2.1 Introduction
This chapter includes a review of literature with subsequent development of theoretical constructs. This thesis draws together several disparate fields (including task complexity, ability studies, learning, and instructional models). As a consequence, consistent with the scale of a minor thesis this literature review contains a representative selection of relevant literature rather than a comprehensive coverage.

2.2 Literature Review
My literature review included research related to several areas of study (student ability to solve unfamiliar challenging problems; the nature of task complexity; the nature and quality of understanding; models of learning; instructional approaches; and affective factors capable of enhancing the learning process). These areas informed various aspects of my study of student response to task complexity.

2.2.1 The Ability to Solve Unfamiliar Challenging Problems
For the purpose of this research the term ‘unfamiliar challenging problems’ relates to the types of open-ended problems in mathematics described in Chapter 1. These problems provide the opportunity for students to formulate mathematical ideas for themselves before a concept is formally taught. To develop a deeper understanding of the nature of unfamiliar challenging problems, I studied literature from a variety of subject domains related to the study of student ability to solve such problems in mathematics. The research studies discussed relate to problem solving in: (a) mathematics; (b) subject areas of which mathematics was a component; and (c) areas other than mathematics. Where a research study related specifically to the area of mathematics, the types of problems solved were sometimes drawn from a broader content domain than the type of unfamiliar challenging problem explored in my study. This literature search informed my formulation of a measuring instrument to assess student ability to solve unfamiliar challenging problems (Krutetskii, 1976; Binet & Simon, 1980; Tannenbaum, 1983; Schoenfeld, 1994).

This following section includes: (a) researchers’ conflicting views about whether the ability to solve problems exists only as the student solves a problem or whether this ability is possessed by the student as a potential to solve problems; (b) the constituent parts of the ability to solve problems—identified as a result of the literature review; (c) the theoretical construct formulated to assess the ability to solve unfamiliar challenging problems; and (d) the assessment of student ability to solve such problems.

2.2.1.1 Ability: potential to solve or a demonstrated skill?
Problem solving ability has been perceived as: (a) an innate characteristic of the individual student (Raven, Court and Raven, 1992); (b) the application of a ‘tool box’
of useful skills a student had acquired as a result of exposure to other problems (Schoenfeld, 1992; Sweller, 1992; Bell 1993); (c) containing an innate component but not limited by this innate component because mathematical ability can be created and developed as problems are solved (Krutetskii, 1976); and (d) sometimes enhanced by social interactions between students working together to solve a problem (Tang, 1993; Schoenfeld, 1992).

Raven, Court and Raven (1992), Schoenfeld (1992), Sweller (1992), Bell (1993) and Krutetskii (1976) recognised a student begins each new problem with their own individual capacity to solve the problem. Some researchers believed innate characteristics influenced this potential (Raven, Court and Raven and Krutetskii) and that this capacity to solve problems could be affected by a student’s previous experience of solving problems (Schoenfeld, Sweller, Krutetskii, Bell). Krutetskii claimed an “innate biological inclination” was “necessary but not sufficient for the subsequent development of an ability” (1976, p.xii). Sweller and Schoenfeld differed in their opinion about environmental influences on a student’s capacity to solve problems. Sweller believed the ability to solve problems was enhanced by student exposure to many specific problem types and Schoenfeld believed student exposure to a generic set of problem solving processes enhanced problem solving ability. The potential for collaborative group work to affect the likelihood that students solved a problem has become a focus of attention over the past ten years (Schoenfeld, 1992; Bell, 1993; Tang, 1993; Leigh-Lancaster, Splitter & Williams, 1997).

The ability to solve unfamiliar challenging problems possessed by the student at the start of the problem solving task is referred to as ‘Ability’ in this study. This study makes a distinction between a student’s capacity to solve problems (Ability) (Krutetskii, 1976) and the demonstrated problem solving achievements of students in a collaborative group as they solve a problem together (Tang, 1993). Whether this Ability is innate, dependant on previous problem solving experiences or a combination of such factors is immaterial to this study. What is of interest is the relative Ability of students in collaborative groups and any effects these differences in Ability may have on student response to task complexity when students solve problems in a collaborative setting.

2.2.1.2 The constituent parts of the ability to solve problems
There are similarities and differences between the opinions of researchers in relation to student ability to solve challenging problems.

Tannenbaum (1983) compares the ‘performers’ (who acquire, comprehend and appreciate) to the ‘creators’ (who synthesise to create new ideas). Krutetskii (1976, p. 86) identifies mathematical ability as ‘creative mastery of the school mathematics course’ and recognised the ability to solve unfamiliar problems as an essential attribute
of mathematical ability. He appears to be describing ability in a manner consistent with Tannenbaum’s ‘creators’ (in the mathematical domain).

Krutetskii (1976, Chapter 6) and his research team analysed the mathematical and psychological literature to develop a hypothesis related to characteristics of mental activity possessed by mathematically gifted school students. These hypothesised characteristics included the ability to: (a) interrelate and connect ideas; (b) analyse for important similarities and differences; (c) operate with numerals and other symbols; (d) follow a logical reasoned argument; (e) curtail mathematical reasoning; (f) switch from a “direct to a reverse train of thought”; (g) think flexibly—“switch from one mental operation to another”; (h) remember in generalisations or overall concepts; and (i) think spatially—connect mathematics to a geometrical or graphical interpretation (Krutetskii, 1976).

Bell (1993) describes student ability to work mathematically as consisting of knowledge of general mathematical strategies including the ability to: (a) learn and develop mathematical concepts; (b) generalise; (c) carry out an investigation successfully; and (d) ask appropriate questions to pursue an investigation. Like Bell, Schoenfeld (1985) identifies the student’s ability to monitor their own progress and to approach the world with a mathematical frame of mind as essential elements of successful problem solving. Balacheff (1982) and Brousseau (1981) stress the importance of the communication of mathematical ideas and the clear development of arguments in effective problem solving.

It appears Bell, Schoenfeld, Balcheff and Brousseau were describing the types of thinking students used as they worked to solve challenging problems and Krutetskii identified the mental activities undertaken as a student employed these types of thinking to solve such problems. I have used Krutetskii’s descriptions of the mental activities he and his researchers identified to form a hierarchical classification of the characteristics of Ability. This classification is based on Bloom’s Taxonomy (1956) which was originally developed “to classify instructional objectives and test items in a hierarchical fashion” (VanTassel-Baska, 1993 p. 303). It has since been adopted as “a hierarchical model for conceptualising high-level thinking skills for gifted learners” (VanTassel-Baska, 1993. p. 303). Assessment of the student responses to the Ability test (Appendix 4) used in this study relied upon the hierarchical order of higher order thinking skills represented in Bloom’s Taxonomy. Additional categories have been added as suggested by Krutetskii’s empirical data and by the terminology (consistent with Bloom’s terminology) that Krutetskii applies.

Different researchers have described metacognition as a higher level thinking skill (Bell, 1993), a different dimension of thinking (Anderson, 1998) and “the processes of developing awareness of one’s own thinking and techniques for controlling and
improving thinking activities” (VanTassel-Baska, 1993, p. 320). For the purpose of this thesis I have used VanTassel-Baska’s definition of metacognition. This is consistent with the framework of Anderson (1998) where metacognition is viewed as a separate dimension of thinking rather than a part of Bloom’s higher level thinking skills. Although metacognition (as defined in this study) is not represented as one of the mental activities assessed by the Ability test, it is expected a student with well developed metacognitive skills (awareness of, monitoring and control of their own thinking, VanTassel-Baska, 1993) would be more likely to employ higher level thinking.

The Theoretical Construct for the ability to solve problems is now explained in more detail. The criteria used to measure student ability to solve unfamiliar challenging problems are explained and the justification for their hierarchical order is given. This hierarchy was based on the work of Bloom (1956) and Krutetskii (1976).

In descending hierarchical order Bloom’s six cognitive processes are evaluation, synthesis, analysis, application, comprehension and knowledge (Bloom, 1956). Explanations of each cognitive process in Bloom’s Taxonomy are described in Table 2.1.

<table>
<thead>
<tr>
<th>Term from Bloom’s Taxonomy</th>
<th>Description of cognitive activity</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Evaluation</strong></td>
<td>Proof or justification of conclusions reached;</td>
</tr>
<tr>
<td><strong>Synthesis</strong></td>
<td>Recombination of concepts to create something new;</td>
</tr>
<tr>
<td><strong>Analysis</strong></td>
<td>A search for patterns, similarities and differences or dissection of the whole into its composite parts;</td>
</tr>
<tr>
<td><strong>Application</strong></td>
<td>The use of knowledge in a real life situation;</td>
</tr>
<tr>
<td><strong>Comprehension</strong></td>
<td>Demonstration of understanding of knowledge;</td>
</tr>
<tr>
<td><strong>Knowledge</strong></td>
<td>Information that can be recalled but is not necessarily understood.</td>
</tr>
</tbody>
</table>

Krutetskii uses the terms ‘Analytic-synthesis’ and ‘Evaluative-synthesis’ to describe categories shown by his empirical data to lie below and above synthesis on the hierarchy. While Evaluative-synthesis is consistent with Bloom’s evaluation, analytic synthesis appears to be consistent with the extra category between analysis and synthesis as suggested by the work of Anderson (1998) and Victorian Curriculum and Assessment Board (1992a).
Table 2.2 Hierarchical (descending order) Theoretical Construct of Ability to Solve Unfamiliar Challenging Problems. Validation through comparison of research from Krutetskii (1976) and Bloom (1956).

<table>
<thead>
<tr>
<th>Characteristic of Ability to enable the inference of certain mental activities</th>
<th>Type of student Krutetskii found displayed this characteristic</th>
<th>Terminology employed by Krutetskii.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Recognise inconsistent information</td>
<td>Capable students</td>
<td>Progressive evaluation during synthesis</td>
</tr>
<tr>
<td>Combine concepts to create an original concept</td>
<td>Capable students</td>
<td>Synthesis</td>
</tr>
<tr>
<td>Explain the need for extra information when insufficient information is provided to solve a problem</td>
<td>Capable students, Average students (few)</td>
<td>Analytic-synthetic</td>
</tr>
<tr>
<td>Use more than one pathway</td>
<td>Capable students, Average students (some)</td>
<td></td>
</tr>
<tr>
<td>Recognise the need for extra information</td>
<td>Capable students, Average students (some)</td>
<td>Analysis</td>
</tr>
<tr>
<td>Build on a taught concept</td>
<td>Capable students, Average students</td>
<td>Analysis</td>
</tr>
<tr>
<td>Understand learnt concepts</td>
<td>Capable students, Average students, Relatively incapable students (some)</td>
<td>Comprehension</td>
</tr>
<tr>
<td>Repeat taught information</td>
<td>Capable, Average, Relatively incapable students (most)</td>
<td>Knowledge</td>
</tr>
</tbody>
</table>

Krutetskii categorised the mental activity to ‘build on a taught idea’ as both Application and Analysis. Application has not been included as a category in my theoretical construct (Table 2.2) because the types of unfamiliar challenging problems used in this study were related to abstract mathematical ideas rather than mathematics in a physical context. For this reason, to ‘build on a taught idea’ is only included as part of ‘Analysis’.

2.2.1.3 Assessment of the ability to solve problems
Krutetskii (1976) and Tannenbaum (1983) both observed students undertaking tasks in order to study the students’ problem solving ability (Krutetskii) or capacity to work with the task (Tannenbaum). The instrument designed to measure Ability in my study utilised this approach. The Ability test required students to solve unfamiliar
challenging problems (of limited magnitude without the assistance of a collaborative group). The mental processes students displayed as they worked with these activities were used as a measure of student capacity to solve such problems (Ability).

Krutetskii and his co-workers (Krutetskii, 1976, Chapter 8) designed tests to study the differences in mental activities undertaken by capable, average and relatively incapable students in mathematics. Mental activity was inferred by analysis of student responses as they ‘thought out loud’ to solve problems. Krutetskii believed the mental activities undertaken by students in mathematics fell into three categories: gathering, processing, and retaining information. In Krutetskii’s terms, and for the purpose of this study, the assessment of Ability has been limited to the processing of information because this study focuses on students undertaking a problem solving task. Criteria have been further limited to include only those criteria that can be assessed by a pen and paper test administered during a 40 minute mathematics class.

Krutetskii’s researchers (1976) observed and questioned students solving problems individually to explore the mental activities exposed as the students solved the problems. Krutetskii used two indices in assessing student ability to solve each problem. These two indices related to the solution a student could attain working alone and the solution a student could attain working with a more expert other (Vygotsky, 1978). It appeared Krutetskii did not consider the impact of the researcher’s questions on the individual student’s solution processes. Recent research indicates that the oral communication of ideas contributes to the understanding a student constructs (Cobb, Wood, Yackel & McNeal, 1992; Cohen, 1994). Krutetskii’s criteria that identify student ability to solve challenging problems are consistent with the more recent descriptions provided by other researchers. It would be useful to consider whether the criteria identified by Krutetskii are sufficient to describe problem solving in a collaborative group or whether other student attributes are of assistance. My study, which focused on student interactions with the task and with each other, is informed by knowledge of which attributes of the ability to problem solve (Krutetskii, 1976) students bring with them to the collaborative group. Other student attributes may also add to student ability to solve problems in collaborative groups. Such attributes are considered when student response to complexity is analysed.

2.2.2 The Nature of Task Complexity
Task complexity refers to complexities within the task and is distinct from Complexity Theory (Lewin, 1993; Casti, 1994) which considers how a small change in one variable can create a marked change in outcome. I explored the research literature to ascertain whether the research community expressed a common view about the nature of task complexity. I found the term ‘complex’ was frequently used to describe mathematics tasks (Schoenfeld, 1992; Clarke, Wallbridge & Fraser, 1992; Sweller, 1992; Tang,
1993; Landvogt, 1998) without explicit explanation of the meaning of ‘complex’. Where explanations or partial explanations of task complexity—explicit or implicit—existed, there were differences between the nature of task complexity as described by different researchers.

Where an implicit or explicit explanation of the nature of task complexity existed, task complexity most frequently appeared to be equated with intellectual complexity (Bloom, 1956; Tannenbaum, 1983; Cohen, 1994; Maker & Nielson, 1996; Smith and Stein, 1998). These descriptions of the intellectual complexity of a task included requirements such as use of trial and error; use of analysis; evaluation of progress and resynthesis of ideas (creation through interconnection of ideas); and the use of logic, abstraction, proof and generalisation. These intellectual requirements of the task were either identified as equivalent to the complexity of the task (Krutetskii, 1976) or as one aspect of task complexity (Bell, 1993).

Other aspects of task complexity identified included: (a) cognitive complexity defined by the number of primitive operations required for a computer to complete the task (Ohlsson, Ernst & Rees, 1992); (b) socio-cognitive complexity, defined as the number of interactions required between students (Bouvier, 1987); (c) procedural complexity (Anthony, 1996); (d) metacognitive complexity (Anderson, 1998; Bell 1993); or (e) the extent or complexity of the implicit links between the intellectual demands of the task and the other task features (understanding of context, transformation of representations, manipulation of symbols, realisation that an idea can be considered another way, development of new knowledge) (Bell 1993).

The literature demonstrated that task complexity was generally not clearly defined and that those definitions that did exist differed in detail and content domain. Intellectual complexity generally formed at least part of each definition. As a consequence of the lack of common understanding of task complexity within the education community, I surveyed appropriate experts in the initial stages of my research. This literature review informed the framing of questions used when seeking the expert’s opinions about the nature of task complexity.

2.2.3 The Nature and Quality of Understanding
Development of mathematical understanding, student personal construction of mathematical concepts and student learning have been used interchangeably in this study. The literature demonstrates there is considerable overlap between the definitions of these terms and differentiation between these terms is outside the scope of this thesis.

This section describes a model of conceptual development that focused on developmental stages attained by the individual (Piaget, Inhelder & Szeminska, 1960).
A brief explanation is then provided of the two frameworks (Skemp, 1976; Biggs and Collis, 1982) to which I referred when assessing the quality of mathematical understanding indicated by student responses to the test of mathematical understanding (Appendix 5). My use of Skemp’s framework of mathematical understanding is similar to the use made of this framework by another teacher/researcher (Cavanagh, 1996). Cavanagh was studying the development of mathematical understanding in senior secondary students introduced to calculus through investigation in an Australian context.

2.2.3.1 Concept development: focus on the individual
Constructivist theory gained popularity as a result of Piaget’s studies of his own children constructing an understanding of the world around them (Bouvier, 1987; Howe, 1996). Piaget used the terms ‘assimilation’ and ‘accommodation’ to describe two types of changes to an individual’s cognitive schema theorised to explain the acquisition of an increased understanding of a concept. When changes in understanding of a concept relied on extending the concept already developed, Piaget named the process assimilation and explained this as the individual’s process of attaching new parts to the present cognitive schema. When the changes in understanding indicated the presently held concept was inconsistent with the new information and a different conceptual understanding was required, Piaget named this process accommodation. He described accommodation as a re-organisation of the present cognitive schema. Piaget believed the level of cognitive processing a child could use was dependent upon the attainment of developmental levels (Howe, 1996). This theory suggested the need for a sequential progression through the curriculum, withholding exposure to opportunities for higher level thinking until the developmental stage was considered to have been attained by most students. This theory is inconsistent with recent findings about how students learn in socio-cognitive settings (Brown, 1994; Howe, 1996). In this study, I use the terms ‘Assimilation’ and ‘Accommodation’ to refer to the extension or re-organisation of cognitive schema whether this change is attributed to the individual working alone or to concept development in a socio-cognitive setting. Using these terms in this manner synthesises relevant concepts developed by Piaget with later findings on learning in a socio-cognitive setting.

2.2.3.2 Frameworks used to infer mathematical understanding
Skemp’s (1976) framework of types of mathematical understanding was my primary frame of reference for the interpretation of student understanding when analysing responses to the test of mathematical understanding (Appendix 5). Using Skemp’s framework of mathematical understanding, a student has achieved a low level of mathematical understanding (instrumental understanding) when he or she is able to
repeat a mathematical procedure without knowledge of why the procedure works. A higher level of mathematical understanding has been achieved where a student understands how a procedure works, why it works that way and how that procedure is related to other mathematical concepts and procedures (relational understanding).

It was sometimes more appropriate to interpret a question or a student response through the framework of the SOLO Taxonomy (Biggs and Collis, 1982). The SOLO Taxonomy (Structure Of Learning Outcomes; Biggs and Collis, 1982) uses a different but complementary frame of reference because it focuses on describing the understanding in terms of the outcome. A deeper understanding of the mathematics was deemed to have been achieved when a student moved from considering specific pieces of information in isolation to simultaneously considering more than one piece of information and finally considering the interconnections between information and formulating a generalisation (Biggs and Collis, 1982).

A general understanding (formal or logical understanding) was the highest level of understanding described by both Skemp (1979) and the SOLO Taxonomy. This level of understanding was achieved when a student developed a generalisation that interconnected mathematical ideas and the student was able to communicate a logical argument (or explanation) of this generalisation to others.

If students attain at least relational and preferably formal (or logical) understanding the fragmented nature of a student’s understanding will be decreased. The fragmented nature of instrumental understanding with its lack of interconnections between ideas is more likely to lead to the development and retention of misconceptions. Formal or logical understanding signifies the interlinking of concepts by the student and an ability to understand and explain these links. This results in a reduced likelihood that misconceptions will develop and a greater likelihood that any misconception developed will be recognised by the student. With formal or logical understanding there is less likelihood of misapplied and overgeneralised rules (Ginsberg, 1986; Bouvier, 1987).

The test of mathematical understanding (Appendix 5) developed and administered during this study was used as a support to my video analysis rather than as a primary source for determining the level of understanding developed. I considered a simple qualitative analysis of student responses to these tests, through reference to Skemp’s framework and the SOLO Taxonomy, sufficient to support the inferences I drew through analysis of the video data.

### 2.2.3.3 Introduction to senior mathematics through discovery

Cavanagh (1996) used the work of Barnes (1990) to develop an instructional approach to introduce calculus through investigation. Students investigated calculus concepts through numerical and graphical representations through the use of technology. The
investigation in my study used student collaboration rather than technology to promote investigation although students could use technology to test a finding. Cavanagh assessed the student’s mathematical understanding through the use of Skemp’s (1976) mathematical understanding that was the primary framework I used. Cavanagh used a pen and paper test to assess mathematical understanding and I used such a test to support my video analysis.

Forster (1999) is another teacher/researcher who has studied the learning gains in Australian senior secondary mathematics students. Forster worked with investigations in a variety of senior secondary mathematics topics where students utilised technology.

2.2.4 Development of Understanding: socio-cognitive models

Through the 20th Century the predominant learning theory changed from behaviourist (focused upon stimuli and response) to cognitive (focused upon students constructing their own learning) and finally to socio-cognitive learning (construction of learning within a social setting) (Brown, 1994; Howe, 1996). Focus on socio-cognitive learning also highlighted possible gender differences in learning styles. Research claiming that learning for girls is enhanced where learning is undertaken in small groups within co-operative settings (Peterson & Fennema, 1985) added to the interest in instruction through models that include the use of learning in small groups.

Socio-cognitive explanations of concept development became the focus of interest towards the end of the 20th Century. This section discusses two socio-cognitive models of learning including:-(a) the work of Vygotsky (1978) developed early in the 20th Century but not disseminated outside Russia until the second half of the 20th Century (Howe, 1996); and (b) learning as part of a community of learners.

2.2.4.1 Development of understanding: individual and expert other

In contrast to Piaget (Peterson, 1996), Vygotsky (1978) described an individual’s opportunities for construction of new understanding as taking place in a social setting where an adult or a more able peer interacted with a student to facilitate the conceptual development of the student. It was claimed that the potential level of conceptual development that could be attained by a student at a given time was influenced by the ability of the student to understand the concepts and by the capabilities of the adult or more able peer. The difference between the student’s present level of conceptual understanding and potential level of conceptual development was referred to as the ‘Zone of Proximal Development’ (Vygotsky, 1978).

2.2.4.2 Development of understanding: community of learners

Schoenfeld (1992) recognised the importance of the enculturation of the individual and the effects that the degree and the quality of student interaction with others can have on student ability to solve problems. Tang’s (1993) study of spontaneously formed
groups of tertiary Physiotherapy students illustrated the improved quality of response to problems where groups collaborated in an initial exploration of the problem. Tang attributed these quality gains to the autonomy of the groups. Characteristics of autonomy identified by Tang included: (a) the spontaneous decision of students to form a group outside class time; (b) groups selecting their own members; and (c) the group’s decision about how to focus their investigation and/or the problem solving pathways they would follow (Schoenfeld, 1992; Tang, 1993). The effective problem solving environment created through student communication in collaborative groups (Schoenfeld, 1994; Tang, 1993) was one of my reasons, as a teacher, for developing the Class Collaboration instructional approach. The spontaneity of group formation and self-selection of groups in Tang’s study was not a feature of Class Collaboration but Tang’s discussion of spontaneity focused my analysis on the spontaneous behaviours demonstrated as collaborative groups of students responded to the complexities within an unfamiliar challenging problem.

Where students collaborate, evidence suggests student discourse within these socio-cognitive settings can lead to an increased conceptual development for the students (Cobb et al., 1992; Bell, 1993; Brown 1994; Williams, 1997a). Cobb et al., (1992) described the dynamic nature of a ‘Taken as Shared’ understanding. Brown referred to increasingly higher levels of the multiple Zones of Proximal Development within which Grade 2 students negotiated a joint understanding of concepts within the domain of science in a socio-cognitive setting. Brown’s (1994) belief that the learning community can be limited or empowered by the combined knowledge of its members parallels Vygotsky’s belief that the potential Zone of Proximal Development for a student is a function of both the student’s ability and the capacities of the more able other. In both cases, the combined capacity of those taking part in the social interaction is seen to set the boundaries for possible outcomes.

Bell (1993) emphasised the importance of students building on their present knowledge in order to develop new concepts. He advocated that the teacher provide scaffolding to increase the commonality of mathematical background possessed by the students. Without the teacher’s scaffolding to assist students to construct more sophisticated understanding, there is an increased likelihood students will attain only instrumental understanding and may hold informal concepts and learned algorithms simultaneously without any recognition of the relationship between them.

Vygotsky’s (1978) requirement of the presence of a more capable other as part of learning interactions conflicts with Bell’s suggestion that conceptual development is effectively facilitated by social interactions between students with common backgrounds.
My study of student response to complexity adds to our knowledge about how conceptual development is facilitated when complex tasks are used in a collaborative setting. It also has the potential to increase our understanding of the conditions that facilitate learning in collaborative groups in the absence of an expert other.

2.2.5 Group Interaction Patterns and Group Composition

My research focused on student learning through collaboration at two levels within the classroom. Each small group collaborated as they worked with the problem but the class as a whole also collaborated to share information at regular intervals. For this reason, literature related to small group discourse and classroom discourse has been explored with an emphasis on group interaction patterns and group composition. This section discusses: the differences between co-operative and collaborative groups, the effects of varying group composition with regard to Ability and group interactions patterns that have been found useful in other studies of learning.

2.2.5.1 Co-operative groups

The designation ‘co-operative group’ is not perceived to possess the same characteristics by all researchers, educators and teachers (Robinson, 1991). Although many assume heterogeneous groups—mixed ability groups—are an essential requirement (Slavin, 1990a; Slavin, 1990b; Robinson, 1991; Gallagher, 1993), homogeneous ability groups are an acceptable option in the co-operative group instructional model for others (Kulik & Kulik, 1993).

In co-operative (as opposed to collaborative) groups, the group work together to assist each other but the members of the group do not share a common goal (Tang, 1993). Co-operative rather than collaborative group interactions generally exist when individual students worked alone and group members checked each other’s work at regular intervals; or a peer tutoring model exists and a more expert student explains a concept to one or more other students. The motivating factor in co-operative group work is often an extrinsic reward (for example, a certificate for groups that met certain criteria) (Slavin & Karweit, 1985).

2.2.5.2 Collaborative groups

This terminology is used to describe groups that share a common goal (Tang, 1993). The collaborative groups in my research study belonged to a subset of groups collectively named collaborative groups. To provide a portrait of the type of collaboration I strive, as teacher, to generate amongst students in my classroom, I will describe my ideals about how Class Collaboration should function. I will relate these ideals to research on effective learning undertaken with collaborative groups that functioned in a similar manner. Students in this subset of collaborative groups (groups learning through class collaboration) are encouraged to work together to gain new
insights. These insights appear to result in an intrinsic (rather than extrinsic) reward derived from the satisfaction of the intellectual achievement (Damon & Phelps, 1989; Tang, 1993). By the use of logic, analysis, trial and error, evaluation of progress and resynthesis of ideas students strive for a solution they find acceptable (Bloom, 1956; Krutetskii, 1976; Maker & Nielson, 1996; Tannenbaum, 1983; Williams, 1996). Learning is promoted by a classroom of inquiry (Brown, 1994; Cobb, 1995; Leigh-Lancaster et al., 1997) where students work together to build their understanding. Logical argument is developed and shared by students (Schoenfeld, 1992). Opportunity is provided to either accept an argument, consider it further, or provide justification for perceived flaws (Bouvier, 1987; Brown, 1994; Cobb et al., (1992); Barnes, 2000; Schoenfeld, 1992; Williams, 1994). This develops richly-connected bodies of knowledge that Bell (1993) claimed are well-retained.

Brown’s (1994) description of learning through a ‘Community of Learners’ in the Grade 2 science classroom contains elements of the characteristics that Tang (1993) identifies as responsible for the higher level thinking skills developed by the tertiary level Physiotherapy students who spontaneously formed and participated in study groups outside class time. These characteristics are also present in Class Collaboration (Williams, 1997a). In all these learning situations students work together on a broad topic within which the group select the focus area of exploration and the pathways to pursue. At some stage, in the collaborative process undertaken by the students described by Brown (1994), Tang (1993) and Williams (1997a), oral presentations of material were required to enable the findings to be shared by a group of students (collaborative group or whole class). This community of learners (Brown, 1994) resembles Robinson’s (1991) description of the Group Investigation method of group learning. As higher level thinking skills are evident in the students studying Grade 2 Science, senior secondary Mathematics and tertiary Physiotherapy, this adds to the evidence that students can use higher level thinking without having to wait until they reach a certain developmental stage (Howe, 1996).

2.2.5.3 Differences between collaborative and co-operative groups
In the past ten to twelve years there has been an increased focus on student discourse and its role in concept development (Damon & Phelps, 1989; Schoenfeld, 1992; Webb, 1991; Cobb et al., 1992; Bell, 1993; Brown, 1994; Cobb & Bauersfeld, 1995; Barnes, 2000). Few authors (Damon & Phelps, 1989; Robinson 1991; Tang, 1993) have drawn the distinction between co-operation and collaboration. Co-operative learning is more directed by the teacher and collaborative learning provides more student autonomy (Robinson, 1991; Tang, 1993; Brown 1994; Barnes, 2000). Robinson (1991) noted that the nature of the task altered the group interactions and the
task could lead to the group functioning with a more co-operative or collaborative nature.

Tang (1993) identified differences between co-operative and collaborative learning. These differences included the type of accountability, the reward structure and whether the teacher or students initiated the groups and/or directed what work was to be undertaken. Where co-operative groups generally operated with individual accountability and group extrinsic reward, collaborative groups operated with group accountability and individual intrinsic reward. The lasting effects of such intrinsic reward were also noted in Brown’s (1994) observation that the excitement and learning generally continued long term, well after the accountability of the group for the topic.

For the purpose of this study, collaborative groups are groups that worked together to achieve a shared goal. This ‘working together’ is characterised by student discourse that allowed ideas to build as a result of contributions from each individual group member.

2.2.5.4 Composition of groups with regard to Ability
Research has been undertaken to identify effective methods of grouping students to maximise learning opportunities (Gallagher, 1993; Oakes, 1985; Slavin, 1993; Tang, 1993; Kulik & Kulik, 1991; Robinson, 1991). Slavin and Karweit (1985) identified learning gains in the co-operative learning model (Team Assisted Individualisation) where a mixed ability class is divided into small heterogeneous groups to undertake individual work. Slavin (1990b) advocated heterogeneous grouping drawing on evidence that homogeneous grouping frequently did not provide increased learning gain for high ability students but reduced the learning gains and self-esteem of low ability students. Slavin believed this lower learning gain was due to the reduced pace of learning teachers often delivered in these instances, the lack of other students to stimulate their thinking and the stigma attached to these groupings (Oakes, 1985). On the basis of a large scale meta-analysis of research into ability grouping, Kulik & Kulik (1991) claimed learning gains for high ability students were greater in within-class homogeneous groups, if an appropriately differentiated curriculum was provided. Slavin (1993) conceded the top 3% of students may benefit from homogeneous grouping.

As the Abilities of the students in my study were assessed, this study could contribute to our knowledge of the effects of the relative Ability of group members on the collaborative group’s response to complexity.
2.2.5.5 Identifying group interactions associated with learning

When considering group dynamics, researchers use different perspectives to study the significance of group interactions. The factors involved in group interaction include: (a) giving versus receiving information (Webb, 1991; Gooding & Stacey, 1993); (b) readiness to receive information (Webb, 1991; Cohen, 1994; Barnes & Williams, 1997) (c) topic-related versus non-topic-related interactions (Johnson & Johnson, 1979; Webb, 1991; Csikszentmihalyi, M. & Csikszentmihalyi, I., 1992a; Gooding & Stacey, 1993); (d) the value placed on the information provided by the teacher in comparison to information provided by the group or the rest of the class (Cohen, 1994; Cobb & Bauersfeld, 1995); (e) the interaction differences in co-operative and competitive groups (Robinson, 1991); and (f) harmony versus conflict in group interactions (Johnson & Johnson, 1979; Bell, 1993; Cobb & Bauersfeld, 1995).

Webb (1991) found that the giver of information generally learned more than the receiver. If the receiver requested the information though, a readiness to learn was more likely to be present. Barnes and Williams (1997) cited a lack of this readiness as a possible reason for senior secondary calculus students missing the significance of an idea the first time another group member presented it. Webb uses the frequency of topic-related versus non-topic-related information as an indicator of the quantity of learning taking place.

For the purpose of this study of student response to complexity, relevant indicators included: (a) ‘on-task and off-task talk’ as a contributing indicator to student engagement; and (b) ‘readiness and lack of readiness to receive information’. These factors were investigated through analysis of student discourse and inferences about the level of conceptual development.

2.2.6 Affective factors that may enhance the learning environment

In this section, reference is made to opportunities for individuals to work at the edge of their present understanding (Vygotsky, 1978; Bell, 1993, Csikszentmihalyi, M. & Csikszentmihalyi, I., 1992; Williams, 1997a) and to other conditions claimed necessary to create an enhanced learning environment (Csikszentmihalyi, M. & Csikszentmihalyi, I., 1992). M. Csikszentmihalyi (1992b) describes an optimal condition, ‘Flow’, and acknowledges that group flow can also exist. Groups in flow experience an optimal level of engagement; they are so involved in the task that they lose all consciousness of time, self and the world around them. All energies are being directed towards the process at hand. Students are not in a state of flow if they can be easily distracted from the task at hand. The students’ response or lack of response to non-topic related interactions from others, or the production of non-topic related interactions themselves, provides an indicator of whether flow could be present. The level of engagement of students in this study is analysed through observation of body
language to investigate the level of student engagement displayed when students respond to task complexities.

The optimal conditions that create flow are present when an individual is working just above their present skills level with a challenge almost out of reach (Csikszentmihalyi, M. & Csikszentmihalyi, I., 1992). Smith and Stein (1998) believe the challenge in a mathematics task is the intellectual complexity. Flow results from optimal learning conditions and the anticipation of the intrinsic reward—‘jolt of thrills’—received when a challenge is overcome (Sato, 1992 p.92). People in many different fields of endeavour have described similar pleasurable experiences across a variety of cultures. These fields of endeavour include: sport, work, leisure, writing, research and mathematics (Csikszentmihalyi, M. & Csikszentmihalyi, I., 1992).

M. Csikszentmihalyi found ‘No activity can sustain it [flow] for long unless both the challenges and skills become more complex’ (Csikszentmihalyi, M. & Csikszentmihalyi, I., 1992, p. 30). Bell (1993) and Williams (1994) identified challenge as being the quest for an elegant solution, with the attainment of that elegant solution as the intrinsic reward. Bell recognised that interaction in the classroom could be exploited to help provide a climate which engages (Bell, 1993). Henningsen and Stein (1997) recognised a variety of classroom factors that affected the length of time students remained engaged in a task during a lesson. These factors included whether: (a) tasks built on student prior knowledge; (b) scaffolding was provided; (c) an appropriate amount of time was allocated; (d) high level performance was modelled by the teacher and/or by other students; (e) a sustained pressure for explanation and meaning existed; (f) self monitoring was undertaken by the student; and (g) conceptual connections were drawn by the teacher.

Henningsen and Stein’s (1997) criteria include many features that promote the conditions necessary but not sufficient for ‘flow’. If the task builds on prior knowledge and the students have the required background, the skills in the task are less likely to be too far out of reach. If students are required to justify ideas and the process of inquiry is role modelled, the intellectual challenge should not be too far out of reach.

Class Collaboration encourages most of the features Henningsen and Stein identify as contributing to sustained student engagement in the task. One difference between the features identified by Henningsen and Stein and the factors I believe operate during Class Collaboration is related to who makes the conceptual connections within the classroom. Conceptual connections were identified as a feature of both Henningsen and Stein’s classrooms and classrooms working through Class Collaboration but
Henningsen and Stein identified the teacher as making the conceptual connections whereas the connections are frequently drawn by the students in Class Collaboration.

The present study considers student response to complexity by analysing student dialogue during Class Collaboration to identify times when students are working with new mathematical ideas where the complexity of the task appears to challenge the students. As these are the necessary conditions for flow, it is expected this study could contribute to our understanding of student engagement with the task.

*The ‘Aha Experience’ may be equivalent to the ‘jolt of thrills’ described by Sato (1992, pp. 100). Davis and Hersch (1980) claim the ‘Aha’ experience (or Barnes’ ‘Magical Moment’; 2000) provided an external cue to the break-through to a new insight or development of a new concept. This breakthrough was accompanied by feelings of pleasure. The ‘jolt of thrills’ experienced by Japanese motor cycle gangs (as they mastered new techniques and thus overcame the challenge of eluding the police in their next ride through town) is similar to the feelings of pleasure experienced when challenges are overcome in mathematics. In both cases new skills and/or concepts are developed as a challenge is overcome. People have been found to work to replicate the experience (Csikszentmihalyi, M. & Csikszentmihalyi, I., 1992).* 

Barnes (2000) discussed a student, Naidra, who described breakthroughs or insights in mathematics as ‘magical’. He claimed that such moments often occurred for him as a result of his participation in the mathematics class to which he belonged and that when they did it was ‘great’. As Naidra was undertaking mathematics through Class Collaboration (during Barnes’ study), if Naidra is representative of students learning mathematics through Class Collaboration, this present study should further our understanding of ‘magical moments’ in mathematics.

**2.3 Theoretical Constructs**

This study made use of theoretical frameworks and operational constructs related to task complexity, student ability to solve unfamiliar challenging problems, and student engagement. In this section I discuss the frameworks for task complexity and student Ability. The operational construct for student engagement is outlined in Chapter 5 where student response to task complexity is considered.

**2.3.1 Williams/Clarke Framework of Task Complexity**

A theoretical framework for task complexity was proposed for use in the initial stages of this study. Williams and Clarke (1997) developed a framework of dimensions of complexity to provide a common framework to which expert opinion (Appendix 1) and student behaviours could both be related.
This facilitated the comparison of the opinions of different experts. In Table 2.3 the Williams/Clarke Framework of Complexity (1997) is compared with the features of complexity described by Bell (1993). The similarities and differences between Bell’s descriptions and the Williams/Clarke Framework of Complexity are identified.

Five of the aspects of complexity implicitly identified by Bell (1993) and one additional aspect of complexity—linguistic complexity—were included as dimensions on the Williams/Clarke Framework (1997). Unlike Bell, Williams and Clarke did not include metacognition, (‘thinking about thinking’). The difference may be in the application of the term, metacognition, as Bell’s descriptions of metacognition appeared to lie within the cognitive processes described in Bloom’s Taxonomy (1956) and located in the Williams/Clarke Framework of complexity within the dimension of Intellectual Complexity.
Table 2.3 Similarities and differences between the Williams/Clarke Framework of Complexity and Bell’s (1993) categories of Complexity

<table>
<thead>
<tr>
<th>Williams/Clarke Dimensions (Appendix 1b: short elaboration of terms)</th>
<th>Bell’s categories of complexity</th>
<th>Similarities and Differences</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Linguistic</td>
<td>Context</td>
<td>Williams/Clarke model only</td>
</tr>
<tr>
<td>2. Contextual</td>
<td>Manipulation of the symbolic expressions</td>
<td>√</td>
</tr>
<tr>
<td>3. Numerical</td>
<td>Revelations leading to the development of a new aspect of mathematical knowledge or a new mathematical procedure</td>
<td>√</td>
</tr>
<tr>
<td>4. Conceptual</td>
<td>Intellectual — abstracting, representing, symbolising, generalising, proving and formulating new questions, degree of intensity (focussing in detail on a small aspect and developing interconnections and understanding through discussion, reflection and review — metacognition)</td>
<td>Except for metacognition, these cognitive processes were consistent with Bloom’s Taxonomy categories (see Table 2.1, 2.2). Thus Williams/Clarke and Bell provided the same description of intellectual complexity except for Bell’s inclusion of metacognition.</td>
</tr>
<tr>
<td>5. Intellectual</td>
<td>Transformation of mathematical representations</td>
<td>Williams/Clarke included use and interpretation of representations. It appears Bell focused on the transformation of representations as part of representational complexity. Williams and Clarke considered each different representation to be part of representational complexity and the transformation from one representation to another to be intellectual complexity.</td>
</tr>
</tbody>
</table>

Key: √ Williams and Clarke articulated a similar meaning to Bell

Table 2.3 indicates Bell had implicitly recognised (but not explicitly categorised) many of the complexities identified as dimensions within the Williams/Clarke Framework. Table 2.3 also highlights differences in the interpretation of terms such as metacognition.

2.3.2 Ability to solve problems: evaluating the construct

A theoretical framework for student ability to solve challenging problems included the mental activities in Table 2.1 and 2.2 (Bloom, 1956; Krutetskii, 1976; VanTassel-Baska, 1993). The theoretical construct of Ability was developed earlier in this chapter (Table 2.2). The sufficiency of this theoretical construct as a measure of Ability is now discussed.

The mental activity of progressively evaluating a problem during task completion was emphasised by Krutetskii, as a characteristic possessed only by capable students. As Krutetskii did not include a way to measure this attribute by a pen and paper test, I
added an additional theoretical characteristic ‘the ability to recognise contradictory information’ to elaborate Krutetskii’s characteristics of “evaluation of the solution method during solution” and “not so much direct attempts at solving a problem as a means of thoroughly investigating it” (1976, p 292) (Table 2.2). The relevant question in the instrument I developed required the student to detect contradictory information. This was intended to reveal a student’s ability to investigate, synthesise and progressively evaluate information during the solution process.

Krutetskii’s data related to students solving problems was collected as the researchers listened to students ‘thinking out loud’. The Ability test used in my research required students to provide a written response. As the students in my study were used to providing both written and oral responses about their mathematical thinking as part of Class Collaboration, a written rather than oral response from students was not expected to limit the data collection.

The inclusion (in my theoretical construct of ability) of theoretical characteristics that related to each level of the constructed hierarchy indicates the mental characteristics assessed are representative of the full spectrum of ability to process information while solving the unfamiliar challenging problem. I therefore contend that the data collected from the Ability test (prior to students undertaking Task A) provided sufficient information to infer the students’ mental capacity and analyse the effects of the relative Ability of students in a collaborative group on the group’s response to task complexity.
CHAPTER 3

THE NATURE OF TASK COMPLEXITY
3.1 Introduction
My study investigated student response to the complexities within an unfamiliar challenging problem—Task A (Appendix 2)—where students worked for periods of time in small collaborative groups and periods of time undertaking whole class collaboration (explained in detail in Chapter 5). In order to investigate student response to complexity, I required an understanding of the nature of task complexity in general and an understanding of the particular complexities that existed within Task A. My search of the literature (Chapter 2) indicated that the research community had not established a common understanding of the nature of task complexity. To clarify this concept I interviewed experts from the fields of Mathematics, Mathematics Education and Gifted Education (referred to as Experts for the remainder of this thesis). This chapter focuses on the collection and analysis of these Experts’ opinions about the nature of task complexity and their opinions about the relative complexity of two unfamiliar challenging problems, Task A and Task B (Appendix 3). The purpose of this comparison was not to draw conclusions as to which task was more complex in any absolute sense, but rather to use both tasks as a mechanism for eliciting Expert opinion about task complexity.

The interviews with Experts consisted of two stages (Appendix 1); the initial stage relied on open-ended questions that elicited each Expert’s opinion about the relative complexity of Task A and Task B. The second stage—which followed immediately after the first stage—required each Expert to compare the complexity of Task A and Task B with respect to each of the dimensions of the Williams/Clarke Framework of Complexity (Williams & Clarke, 1997). Expert opinion was sought about the existence of other dimensions of complexity not present in the Williams/Clarke Framework. The Williams/Clarke Framework was developed to ensure Experts provided opinions using a common set of dimensions of complexity, so comparison of Expert opinion was possible.

This chapter focuses on the nature of task complexity in general and the complexities within Task A in particular. Included in this chapter are the: (a) rationale for my study of task complexity; (b) method and data collection techniques utilised; (c) analysis of results related to Expert opinion; and (d) my formulation of the nature of Task Complexity as a result of my analysis of Expert opinion.

3.2 Rationale
A theoretical framework for task complexity was prerequisite to the study of student response to complexity. Discussions with mathematics teachers, prior to the research period, demonstrated a diversity of opinions among the teaching community about the nature of task complexity. Often their ideas had not been previously articulated or were partially formulated. The concept of task complexity appeared to be ill defined within
the community of secondary mathematics educators in Australia and so required further clarification.

The question of whether the consistency of application of the term ‘complexity’ matters is central to this research study. The scarcity of definition in the literature and the lack of clarity of definition amongst teachers raises questions about the usefulness of the term ‘complexity’ as utilised in the elaboration of criteria for External Assessment of students’ attempts to solve unfamiliar challenging problems (Challenging Problems: Victorian Curriculum and Assessment Board, 1992a and 1992b) and in the documentation related to VCE Mathematics 2000 (Victorian Board of Studies Mathematics Study Design 2000; 1999). Clarity about the nature of task complexity and consistency in its application will assist teachers in the interpretation of curriculum documents and the consistency of assessment. Just as importantly, those advocates of the use of complex mathematical tasks for either instruction or assessment presently lack a structure (and a vocabulary) by which to identify and evaluate the different characteristics of such tasks and their relation to the sort of student activity they promote.

3.3 Collection of Data
This section describes the Experts from whom data was collected and the data collection techniques employed.

3.3.1 Selection of Experts
The selection of Experts was not random. Individuals were selected because in combination they possessed various relevant combinations of foci in academic study and represented a variety of Victorian tertiary academic institutions. Three experts were selected from each of the areas of (a) Mathematics; (b) Mathematics Education; and (c) Gifted Education. All Experts with a major area of expertise in Mathematics or Mathematics education reported working extensively or for concentrated periods of time with students of high intellectual potential. One member of the Expert panel (Respondent No. 7) was not working as a University academic but was a secondary mathematics teacher undertaking university higher degree studies at the time. Even though this Expert was unavailable for interview (and so provided a written response to the interview questions) I decided it was important to include these responses in the analysis to retain the equal distribution of responses from each field of expertise. In combination, I expected data collected from Experts in the areas of Mathematics, Mathematics Education and Gifted Education to provide a broad perspective of the nature of task complexity. I expected the Experts from different areas of expertise would focus to varying degrees on the mathematics encountered, effective conditions for learning and the aspects of complexity that may engage gifted students. I considered that each of these perspectives would inform my study of student response to complexity in a mixed-ability mathematics classroom where learning takes place through Class Collaboration.
3.2 Method and Data Collection Techniques
Data was collected through interview. The stimulus material provided for the Experts before and during the interview and the structure of the interview are detailed below.

3.3 Stimulus Material Provided for the Interview
Expert response to the nature of complexity was stimulated by the provision of: (a) two unfamiliar challenging problems, Tasks A and B (Appendix 2 and 3), compared during the interview process; (b) my rationale as teacher for the use of these problems; (c) a brief description of the pre-task student background required and the intended purpose of each task; (d) excerpts from transcripts of video tapes of other students undertaking these two problems (prior to this research study); (e) the initial questions to be used in the interviews with Experts; and (f) the Williams/Clarke Framework of Complexity (1997). All materials except the Williams/Clarke Framework of Complexity were provided to each Expert at least a day prior to the interview to allow the Expert to consider the material and the questions to which they would be required to respond.

3.4 Structure of the Interview
The interview (Appendix 1) was designed to determine each Expert’s opinion in relation to: (a) the nature of complexity; (b) the relative complexity of Tasks A and B; (c) the dimensions most important when considering the overall complexity of a task; and (d) the strength with which the Experts held their opinions.

By asking Experts to discuss the relative complexity of Tasks A and B (Appendix 2 and 3), I was able to draw inferences about Expert opinion of the nature of complexity. Experts were required to ‘think out loud’ as they considered the tasks and I transcribed their spoken thoughts and asked clarifying questions where appropriate.

The Williams/Clarke Framework of Complexity provided focus for the second part of the interview. The structured questions in the second part of the interview required Experts to decide which of Tasks A and B was more complex along each of the dimensions within the Williams/Clarke Framework. Experts could add further dimensions if, in their opinion, this list of dimensions was not exhaustive.

After the Experts had ticked boxes to indicate which task they considered more complex on each dimension and ‘thought out loud’ as their decisions were made, they were asked which one or two dimensions of complexity they considered most important in determining the overall complexity of a mathematics task. The Experts were also asked whether they wanted to re-evaluate their original decision regarding the relative complexity of the two tasks. This enabled me to gauge the strength with which Experts held their initial concept of task complexity.

3.4.1 Task A and Task B: nature of tasks and rationale for use
The information supplied to Experts is described in Figures 3.1, 3.2, and 3.3. This includes: (a) the rationale for use of these types of problem solving tasks (Figure 3.1);
(b) a description of Task A: ‘Understanding the Double Derivative’ (Figure 3.2); and
(c) a description of Task B: ‘The Integration Chart/Map’ (Figure 3.3).

**Figure 3.1 Teacher’s rationale for use of these tasks**

These tasks form part of the delivery of higher level mathematics to students in their final year of secondary school. There is a certain open-endedness to the sample tasks. At the stage at which these tasks were given, students were being encouraged to think for themselves and verbalise their mathematical ideas in language with which they felt comfortable. Correct language and notation were supplied as the course developed. This generally occurred as students searched for words to describe a particular mathematical idea they had developed. The tasks were deliberately phrased in common language terms for two main reasons: (a) to render the task more accessible to students no matter what their previous mathematical background; and (b) to eliminate the expectation that there is a single correct precise answer. It has been my experience (teacher/designer of these tasks) that precise mathematical language actually inhibits the sort of exploratory mathematical investigation characterised in this type of instruction. Both tasks are designed for students working in small groups. The tasks elicit rich student discussion of conceptual ideas. They are student-oriented tasks where the students must engage with the task and help each other to develop and demonstrate concepts. Feedback from groups to the class as a whole occurs at regular intervals throughout the lessons.

**Figure 3.2 Task A. Understanding the Double Derivative.**

This task (Appendix 2) was devised to allow students to develop their own concept of why the double derivative in combination with the derivative gives us information about the shape of the curve to the function. By the end of two lessons, students as a class have generally analysed graphs of different functions and developed their own ‘table of double derivative rules’. This task assumed the prerequisite knowledge that the derivative of a function is the gradient to the graph of that function. It also assumed that no double derivative work had been developed previously.

**Figure 3.3 Task B. The Integration Map/Chart.**

By the time students attempted this task (Appendix 3), integration techniques had already been taught as part of the Specialist Mathematics course. Integration techniques were taught during term 2 and applications of integration were developed during term 3. The teacher introduced this task at the beginning of term 4 to revise and improve the understanding of integration techniques. Students were required to design a chart that could be used to identify the type of integration necessary. Recognition of different types of integration was expected to improve as students analysed ways in which the mathematical formulation differed in expressions requiring different integration techniques. I (teacher/task designer), like Hewitt (1996), recognised that a skill improved if students were set a task that required more than just demonstration of competence with the skill. This task required the student to undertake some analysis and synthesis reliant upon frequent utilisation of the skill to be developed. For this reason, I expected the integration skills and the understanding of the interrelationships between the expressions to be integrated to improve for these students.

### 3.3.4.2 Williams/Clarke Framework of Complexity

In the second part of the interview process the Experts responded in terms of the Williams/Clarke Framework. Williams and Clarke (1997) designed their theoretical Framework of Complexity during the interview design phase for this study. It was intended to: (a) provide responses along the same dimensions of complexity to facilitate comparison of Expert opinion; (b) investigate the degree of consistency between Experts in their perception of complexity; and (c) to assist researchers and teachers to develop a common understanding of the term ‘complexity’ in relation to
unfamiliar challenging problems. The Williams/Clarke Framework consisted of the following six dimensions of complexity—linguistic, contextual, numerical, conceptual, intellectual and representational (see Table 3.1).

### Table 3.1 Dimensions of Complexity (Williams & Clarke, 1997)

<table>
<thead>
<tr>
<th>Dimension of Complexity</th>
<th>Short elaboration of each dimension of Complexity</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Linguistic</td>
<td>Vocabulary and/or sentence structure.</td>
</tr>
<tr>
<td>2. Contextual</td>
<td>The relationship between the situation described and the required mathematical procedure/s.</td>
</tr>
<tr>
<td>3. Numerical</td>
<td>The types and combinations of operations required to perform the task.</td>
</tr>
<tr>
<td>4. Conceptual</td>
<td>The types of concepts students encounter as they worked with the task and diversity of concepts students combine to create new concepts.</td>
</tr>
<tr>
<td>5. Intellectual</td>
<td>Broadly conceived in the same terms as Bloom’s Taxonomy (1956) as commonly used to describe higher level thinking (Victorian Curriculum and Assessment Board, 1992a): knowledge; comprehension; application; analysis; evaluation; synthesis).</td>
</tr>
<tr>
<td>6. Representational</td>
<td>The symbols, diagrams, graphs etc. which need to be used and interpreted to understand and develop the problem</td>
</tr>
</tbody>
</table>

Table 3.1 provides a brief description of the intended meaning of each dimension of complexity within this framework. Experts were provided with a similar elaboration of each dimension (Appendix 1). Distinction is now made between Task Complexity and Student Difficulty. ‘Task Complexity’ relates to the attributes of the task (Williams & Clarke, 1997) and ‘Student Difficulty’ relates to the student’s interaction with the task. If a student lacks the background required to undertake a certain task, the student will find the task difficult. For example, a student will experience difficulty if they have not previously encountered: (a) the mathematics required to begin the task; (b) the symbolic notation in which the task is expressed; or (c) the combination of concepts required to create new concepts. A student could also find a task difficult due to their prior experiences or lack of prior experiences with that particular context (Clarke and Helme, 1995). This could lead to a misconstrual of the situation. This is different to the contextual complexity of the task that refers to the relationships between the situation and the mathematical procedures required.

As Williams and Clarke developed this framework to clarify the nature of task complexity, each dimension was accompanied by a short elaboration only (Appendix 1, Table 3.1). The extended and enriched elaboration of Task Complexity informed by Expert opinion (Table 3.12, p 55) appears later in this chapter.
3.4 Data Analysis

The goals of the data analysis were to ascertain: (a) the nature of task complexity; (b) the sufficiency of the Williams/Clarke Framework to describe the nature of task complexity; (c) the complexities within Task A, ‘Understanding the Double Derivative’, as opposed to Task B, ‘Integration Map/Chart’; (d) the dimensions of complexity considered most important in assessing the complexity of a mathematics task; (e) changes in Expert opinion as a result of the interview process; and (f) whether other facets of complexity existed and appeared to be relevant to the study of student response to Task Complexity.

3.4.1 The Nature of Task Complexity

For the purpose of this research, ‘task complexity’ is taken to be a characteristic of the task alone, ‘difficulty’ is a consequence of the interaction of the student with the task and ‘mediating factors’ are those factors which facilitate student interaction with the task and affect the likelihood of task completion (Williams & Clarke, 1997).

Expert opinion about the nature of task complexity differed and often included attributes not directly related to the task. Experts’ descriptions of task complexity included all or some of the following attributes: (a) task attributes (Task Complexity); (b) student attributes related to task completion (Difficulty); and (c) environmental factors which affect the student’s interaction with the task (Mediating factors). Table 3.2 shows my allocation of characteristics Experts identified as task complexity to either Task Complexity, Difficulty or Mediating Factors.

All Experts mentioned some task attributes, seven of the nine Experts mentioned at least one student attribute, and three of the nine Experts mentioned at least one attribute of the environment. I subdivided each reported attribute into its component characteristics and these characteristics were elaborated (Table 3.2).

For example, the Task Complexity dimension of ‘Concepts’ included a variety of subcategories. A task reported to contain a high level of complexity in relation to the characteristic ‘concepts’ could contain some or all of the following: (a) a large number of concepts involved; (b) a diversity in the nature of the concepts involved; (c) the necessity for the student to develop a new concept by the unfamiliar combination of other concepts and/or (d) the lack of task guidance through the development of intermediate steps when the task requires a new concept to be developed (large conceptual leap). Alternatively a task described as possessing a low level of complexity in relation to the characteristic ‘concepts’ may require the utilisation of a single simple concept or two concepts which are similar in nature and the student may be taught the concept rather than be expected to develop the concept for themselves in the course of completing the task.
Table 3.2: Characteristics Experts identified as “task complexity” allocated to Task Complexity, Difficulty or Mediating Factors

<table>
<thead>
<tr>
<th>Task Characteristics</th>
<th>Student Attributes</th>
<th>Environmental Factors</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Task Complexity</strong></td>
<td><strong>Difficulty</strong></td>
<td><strong>Mediating Factors</strong></td>
</tr>
<tr>
<td>Clarity of problem definition (2)</td>
<td></td>
<td>Teacher assistance (1)</td>
</tr>
<tr>
<td>• Intricate analysis of language or language structure and interpretation of subtleties of language</td>
<td>Background (5)</td>
<td>• Whether the student is required to develop the solution pathway or the teacher tells the student what to do</td>
</tr>
<tr>
<td>Mathematical operations (4)</td>
<td></td>
<td>• Whether the teacher responds to a student question with a question (compared with whether the teacher provides the student with task specific information)</td>
</tr>
<tr>
<td>• Many high level operations</td>
<td></td>
<td>Quality of understanding expected/sought (1)</td>
</tr>
<tr>
<td>• Task designed to encourage students to use unfamiliar (to the student) combinations of mathematical operations</td>
<td>Perceptions (3)</td>
<td>• Formal/logical; relational or instrumental (Skemp; 1976)</td>
</tr>
<tr>
<td>Concepts (8)</td>
<td></td>
<td>Classroom management (1)</td>
</tr>
<tr>
<td>• Many concepts</td>
<td>• Style of learning preferred</td>
<td>• Classroom: physical set up</td>
</tr>
<tr>
<td>• Diverse concepts</td>
<td>• Whether the student considers this type of task ‘doing mathematics’</td>
<td>• Opportunity for interaction with others</td>
</tr>
<tr>
<td>• Student required to generate concepts as the task is undertaken</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• Size of unscaffolded jump required to develop the new concept</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Thought processes (9)</td>
<td>Preference (3)</td>
<td></td>
</tr>
<tr>
<td>• Analyse, synthesise and evaluate not just know and comprehend</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Directedness of task (4)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• Approach to be taken is not specified</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Closure not clearly defined (4)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• Lack of clear specification of what constitutes completion</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Representations (1)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• Examples: algebraic, graphical, numerical, tabular, geometric</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• Interrelate, construct or interpret</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• Task involves many representations</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Key (x): number of the nine Experts who identified this characteristic

The number of Experts (e.g. 4 is four out of nine Experts) have been included under the different characteristics of Task Complexity, Difficulty and Mediating Factors in Table 3.2 to indicate the proportion of Experts who identified a particular characteristic. Further discussion of Difficulty and Mediating Factors occurs when student response to complexity is explored later in this research study.

3.4.2 The Sufficiency of the Williams/Clarke Framework

The task attributes identified by Experts (Task Complexity) were described and categorised and then mapped against the Williams/Clarke Framework to investigate the sufficiency of the six dimensions within this framework to encompass the attributes identified by the Expert group.
Table 3.3 Mathematics Education Experts: categorisation of key terms and phrases employed during initial opinion of task complexity

<table>
<thead>
<tr>
<th>Expert</th>
<th>Complexity</th>
<th>Difficulty</th>
<th>Mediating Factors</th>
</tr>
</thead>
</table>
| 1      | Mathematical operations  
• “mathematical manipulation”  
Concepts  
• “conceptual”  
Thought processes  
• “level of requirement; understanding, sufficiency”  
• “conjecturing they generate” | Student background  
• “mathematics involved”  
Student preferences  
• “student orientation to engage and demonstrate” | Quality of understanding expected  
• “task alone doesn’t tell what is actually expected in class” |
| 2      | Concepts  
• “conceptual”  
Thought processes  
Recognition of Closure  
• “optimisation/refinement or ‘once it is done it is done’” |  |  |
| 3      | Mathematical operations  
• “the number of possibilities”  
Concepts  
• “development of understanding”  
Thought processes  
• “development of understanding”  
• “the number of possibilities”  
Directedness of task  
• “whether there is a starting point”  
Recognition of Closure  
• “how open ended the task is”  
• “amount of refinement possible”  
• “do you know when you have reached the end” | Student background  
• “context and background of the student” | Teacher assistance  
• “some teacher, rather than student, development of the task” |
### Table 3.4 Mathematics Experts: categorisation of key terms and phrases employed during initial opinion of task complexity

<table>
<thead>
<tr>
<th>Expert</th>
<th>Complexity</th>
<th>Difficulty</th>
<th>Mediating Factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>Concepts</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
|        | • “variety of different techniques or same type of task throughout”  
|        | • “possibility of a quick logical determination”  |
|        | Thought processes |            |                  |
|        | • “interpret through graphical representations”  
|        | • “requirement to draw conclusions”  |
|        | Directedness of Task |            |                  |
|        | • “recognising what has to be done”  
|        | • “many possible constructions or well defined rules”  
|        | • “amount of guessing and lateral thinking”  
|        | • “assumptions necessary”  |
|        | Representations |            |                  |
|        | • “opportunity to interpret through graphical representations”  |
|        | Student background |            |                  |
|        | • “Student background”  |
|        | Classroom management |            |                  |
|        | • “the opportunity to work in groups”  |
| 5      | Concepts   |            |                  |
|        | • “how searching the things they are asked to consider are”  |
|        | Thought processes |            |                  |
|        | Directedness of Task |            |                  |
|        | • “selection they are asked to consider is haphazard or organised”  
|        | • “is there a focus”  
|        | • “predictions about what?/ are students told critical points?”  |
|        | Recognition of Closure |            |                  |
|        | • “open ended”  |
| 6      | Clarity of problem definition |            |                  |
|        | • “lack of definition of terms like prediction”  |
|        | Concepts   |            |                  |
|        | • “conceptualisation”  |
|        | Thought processes |            |                  |
|        | Recognition of Closure |            |                  |
|        | • “whether there is a feeling of end or solution”  |
|        | Student perception |            |                  |
|        | • “student perception of progressive achievement”  |
Table 3.5 Gifted Education Experts: categorisation of key terms and phrases employed during initial opinion of task complexity

<table>
<thead>
<tr>
<th>Expert</th>
<th>Complexity</th>
<th>Difficulty</th>
<th>Mediating Factors</th>
</tr>
</thead>
</table>
| 7      | Mathematical operations  
  - “number of steps students need to combine”  
  - “whether required to combine in the order learnt or the reverse”  
  Concepts  
  - “number of different concepts to the rule they are trying to find”  
  - “understanding mathematical concepts required to use”  
  Thought processes  
  - “requirement to generate and test” | | |
| 8      | Thought processes  
  Directedness of task  
  - “very directed, or immediately required to think”  
  Recognition of Closure  
  - “whether finite or not” | | |
| 9      | Clarity of task definition  
  - “understanding the requirements”  
  - “presentation: spacing, size of writing, language”  
  - “organisational group work instructions and requirements”  
  - “need for student to identify the important parts”  
  Mathematical operations  
  - “mathematical operations utilised”  
  - “diversity of algebraic manipulation”  
  Concepts  
  - “create, design an actual framework or just solve”  
  - “conceptual development”  
  - “differences in algebraic ideas needed”  
  Thought processes  
  - “extrapolating back”  
  - “create, design an actual framework or just solve” | | |

Student perceptions  
- “first impressions”  
Student preferences  
- “prefer not to brainstorm”  
- “do not want to have to write about their mathematical thoughts”  
Student background  
- “student background”  
Student perception  
- “whether feels within reach”  
- “framing of questions to indicate a solution or an exploration”
3.4.2.1 Initial Expert opinion: description and categorisation
Table 3.3 (Mathematics Education Experts), Table 3.4 (Mathematics Experts) and Table 3.5 (Gifted Education Experts) contain the categorisation of key terms and phrases that each group of Experts employed as they ‘thought out loud’ about task complexity. I have retained the Experts’ own words to describe the subcategories developed. Although the intended meaning of some terms and phrases employed by Experts is not always readily apparent from the quote, I considered it important to provide readers with the opportunity to gain some understanding of the individual viewpoint of each Expert. My sub-categorisation and elaboration of the key characteristics of Expert opinion was informed by the key terms and phrases employed by Experts as they provided their opinion about the nature of task complexity and, where appropriate, by the answers to the additional clarifying questions I posed to facilitate accurate interpretation of Expert opinion.

3.4.2.2 Comparison of Expert opinion
‘Directedness of Task’ and ‘Recognition of Closure’ have been categorised within ‘Thought Processes’ but listed as subcategories to demonstrate the high number of responses specific to these particular subcategories. Experts were of the opinion that tasks that provided little direction and/or had no clearly defined end required decision making by the student. Where a task lacked direction, there was a necessity for students to analyse the wording to determine the meaning within the structure of the language and then decide on the direction to take (evaluation). Where closure was difficult to recognise Experts considered there was a necessity for the student to continually evaluate their progress against the intended goal of the task.

All Experts included ‘Thought Processes’ as an attribute of task complexity. Experts tended to report attributes of complexity by the use of verbs such as create, test, combine, develop and recognise (Thought Processes in Table 3.1) in conjunction with one or more of the other task attributes. e.g. combine concepts to create; develop understanding of concepts; recognise critical points, analyse representations to evaluate the relative benefits of each representation.

Only one Expert (E8, Gifted Education) identified ‘Thought Processes’ as the sole attribute of task complexity when considering Tasks A and B. The placement of such emphasis on ‘Thought Processes’ by this Expert is in accord with the prominence placed on the development of higher level thinking skills in Gifted Education (Tannenbaum, 1983). The inclusion of ‘Thought Processes’ as the sole attribute of complexity (by this Expert) might also be attributed to a view that other attributes of complexity cannot exist in isolation from ‘Thought Processes’.

In summary, Thought Processes and Concepts were the dimensions of complexity given the greatest ‘collective weight’ by Experts. Each other attribute of task complexity was mentioned by less than half of the nine Experts.
3.4.2.3 Expert opinion and the Williams/Clarke Framework compared

Correspondence was established between the Williams/Clarke framework and initial Expert opinion of Task Complexity (Table 3.2) along the five dimensions: Linguistic, Numerical, Conceptual, Intellectual and Representational Complexity. This correspondence is displayed in Table 3.6. It can be assumed that Contextual complexity was not elicited as part of initial Expert opinion because of the nature of the tasks (Task A and Task B do not involve mathematics set in a real life context).

During the second stage of the interview, several other dimensions of complexity were suggested by Experts. These suggestions were either identified with one of the six dimensions of complexity in the Williams/Clarke Framework or with student difficulty or mediating factors. For example, ‘Operational Complexity’ (suggested as an additional dimension by a Mathematics and a Mathematics Education Expert) was intended (by Williams and Clarke) to be part of Numerical Complexity. It appeared that Numerical Complexity was sometimes interpreted by the Experts in a limited domain that included numbers only and not algebra and operations. Upon consideration of this feedback, Clarke and I decided ‘Numerical Complexity’ was a misnomer for this particular dimension and renamed it ‘Operational Complexity’

Table 3.6 Reconciliation: Expert opinion (Table 3.2) and the Williams/Clarke Framework of Complexity.

<table>
<thead>
<tr>
<th>Williams/Clarke Dimension</th>
<th>Initial Expert Opinion</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Linguistic</strong></td>
<td>Clarity of problem definition</td>
</tr>
<tr>
<td><strong>Contextual</strong></td>
<td>*</td>
</tr>
<tr>
<td><strong>Numerical</strong></td>
<td>Mathematical operations</td>
</tr>
<tr>
<td><strong>Conceptual</strong></td>
<td>Concepts</td>
</tr>
<tr>
<td><strong>Intellectual</strong></td>
<td>Thought processes</td>
</tr>
<tr>
<td></td>
<td>Directedness of task</td>
</tr>
<tr>
<td></td>
<td>Recognition of Closure</td>
</tr>
<tr>
<td><strong>Representational</strong></td>
<td>Representations</td>
</tr>
</tbody>
</table>

* Experts did not refer to contextual complexity when they discussed their initial opinions

All characteristics of task complexity identified by Experts were consistent with the dimensions of complexity within the Williams/Clarke Framework illustrating the sufficiency of this framework as a tool to describe the dimensions of complexity for these types of mathematical tasks.

3.4.3 The Relative Complexities within Task A and Task B
This section reports the analysis of Expert opinion of the relative complexity of Tasks A and B. Experts did not specifically mention Task A as more or less complex along the dimensions of contextual or representational complexity.

Comments from Experts were drawn from both the initial stage of the interview (Table 3.7) and from each Expert’s application of the Williams/Clarke Framework of Complexity (Williams & Clarke, 1997) (Table 3.8).

The two purposes of this analysis were to: (a) gauge the degree of consistency between Experts in their comparison of the complexity of these two tasks; and (b) increase my understanding of the complexities within the task undertaken by students at a later stage in this research (Task A).

**Table 3.7 Summary of initial Experts opinion that portrayed Task A as More Complex than Task B or Task A as Less Complex than Task B**

<table>
<thead>
<tr>
<th>Dimension of complexity</th>
<th>Task A: Double Derivative More Complex than Task B</th>
<th>Task A: Double Derivative Less Complex than Task B</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Linguistic</strong></td>
<td>Problem definition is more limited for Task A. Task A does not tell students as much about what to do.</td>
<td>It was easier to see what to do with Task A because it had well defined rules (the Expert considered the task clearly indicated to students that they were to work with gradients). The instructions were expressed in simple language with well-spaced writing.</td>
</tr>
<tr>
<td><strong>Numerical</strong></td>
<td>In Task A there is insufficient definition about what mathematics is required. Students will need to make assumptions about scale.</td>
<td>Task A is less complex because of the limited number of possible constructions (ways to think about the problem). Task A is less complex because it is just a matter of looking at where the gradients are positive and negative.</td>
</tr>
<tr>
<td><strong>Conceptual</strong></td>
<td>The focus of the investigation in Task A is not given. Students are not told what types of patterns to look for. Task A is conceptually difficult, a quick logical determination of the overall concept is unlikely due to the size of the single unscaffolded conceptual leap required. In comparison Task B relies upon small changes in conceptual understanding.</td>
<td>Task A is less complex because the concept of developing f’’(x) from f’(x) is not entirely repetition, but the same sort of task. It is not really the development of a new concept but just a matter of extrapolating back.</td>
</tr>
</tbody>
</table>
| **Intellectual**        | Task A is more complex. Students required to:  
- demonstrate understanding of why the patterns occur  
- discuss the sufficiency of information about a function f’’(x) in generation of f(x)  
- draw general conclusions  
Task A is a constructive task where students are required to synthesise to generate new concepts  
The analysis to be undertaken by the student is not defined because the key features are not indicated and the critical points are not given. A distinct end to Task A is not initially evident so the student is required to continually evaluate their progress against their intended goal. | Task A is less complex because there is a natural way of doing the process. It is more closed than Task B. Once Task A is done, it is done (finite) but Task B requires continual optimisation. |
When Experts were asked to discuss the relative complexity of Tasks A and B, two Experts initially considered that the tasks were too different to compare. One Expert was of the opinion that the tasks were ‘like chalk and cheese’ and another Expert was of the opinion that Task A was a constructivist task but Task B was revision of work already covered. Upon further reflection, these Experts were able to compare task characteristics.

Because the Experts were asked which task was more complex, they tended to focus on the differences in the complexity of Task A and Task B rather than similarities between the tasks. Inconsistencies existed between the opinions of the Experts about the relative complexity of the two tasks. Experts differed about whether Task A (relative to Task B) was (a) more directed; (b) more clearly defined; and/or (c) conceptually more complex (Table 3.7). Experts also differed in their perception of the intellectual demand required by Task A.

Table 3.8  Expert opinion on the relative complexity of each task using the Williams/Clarke (1997) framework of dimensions of complexity.

<table>
<thead>
<tr>
<th>Dimension of Complexity</th>
<th>Task A more complex</th>
<th>Task B more complex</th>
<th>Equivalent complexity or unable to select the more complex</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>[respondent code no. ; area of expertise]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1. Linguistic</td>
<td>[E4 ; M] [E6 ; M]</td>
<td>[E2 ; Me] [E9 ; G]</td>
<td>[E1 ; Me] [E3 ; Me] [E5 ; M] [E7 ; G] [E8 ; G]</td>
</tr>
<tr>
<td>2. Contextual</td>
<td>[E3 ; Me] [E7 ; G]</td>
<td>[E4 ; M] [E9 ; G]</td>
<td>[E1 ; Me] [E2 ; Me] [E5 ; M] [E6 ; M]</td>
</tr>
<tr>
<td>3. Numerical</td>
<td>[E8 ; G] [E7 ; G]</td>
<td>[E1 ; Me] [E3 ; Me] [E5 ; M] [E6 ; M] [E9 ; G]</td>
<td>[E2 ; Me] [E4 ; M]</td>
</tr>
<tr>
<td>4. Conceptual</td>
<td>[E1 ; Me] [E2 ; Me] [E5 ; M] [E6 ; M] [E8 ; G]</td>
<td>[E3 ; Me] [E4 ; M] [E7 ; G]</td>
<td>[E9 ; G]</td>
</tr>
<tr>
<td>5. Intellectual</td>
<td>[E1 ; Me] [E6 ; M] [E7 ; G] [E9 ; G]</td>
<td>[E2 ; Me] [E3 ; Me] [E4 ; M] [E5 ; M] [E8 ; G]</td>
<td></td>
</tr>
<tr>
<td>6. Representational</td>
<td>[E2 ; Me] [E6 ; M] [E7 ; G]</td>
<td>[E3 ; Me] [E9 ; G]</td>
<td>[E1 ; Me] [E4 ; M] [E5 ; M] [E8 ; G]</td>
</tr>
</tbody>
</table>

Key: Tasks. A; Understanding the double derivative. Experts. Me; Mathematics, teacher education B; Integration map or chart M; Mathematics, tertiary mathematics G; Gifted Education Expert
The variety of opinions expressed by Experts about the complexities or lack of complexities within Task A (Table 3.7) increased my understanding of the number and diversity of concepts to consider when studying task complexity. I found the interviews with Experts assisted my recognition of the various subcategories within each dimension of complexity and the varying degree of complexity that could exist within each of these subcategories for a particular task. The interviews with Experts also heightened my awareness of the possibility that students may simultaneously work with different dimensions of complexity within a task.

Experts also provided opinions about the relative complexity of Task A and B along each of the dimensions of complexity in the Williams/Clarke Framework (Table 3.8). When applying the provided framework for dimensions of complexity, Expert opinion differed on almost all categories of complexity. All Experts considered Tasks A and B differed in Intellectual complexity and all Experts except Expert 9 considered Tasks A and B differed in Conceptual complexity but consensus was lacking amongst Experts as to which of Tasks A and B was more conceptually or intellectually complex.

Even within the same field of expertise, Mathematics or Mathematics Education for instance, Experts held different opinions about which task was more intellectually or conceptually complex. Experts who thought Task A was conceptually more complex did not necessarily think this same task was intellectually more complex. When Experts considered the tasks in relation to their Linguistic, Contextual and Representational complexity, there was a greater tendency to consider the tasks to be of equivalent complexity (Table 3.8).

Although two Experts considered Task A more Numerically Complex and two Experts were of the opinion that Tasks A and B were equivalent in Numerical complexity, Task B was considered more numerically complex by five experts. Differences in Expert opinion with regard to Numerical Complexity could relate to the particular Expert’s interpretation of this dimension of complexity and whether they included mathematical operations as part of the definition of Numerical Complexity.

3.4.3.1 Further analysis of inconsistencies: eg. Conceptual Complexity

Although Experts reported finding the Williams/Clarke Framework useful for focusing their thoughts on the various dimensions of complexity, differences existed between the Experts’ opinions about the relative complexity of the two tasks. To investigate this further, I have analysed one dimension of complexity to a greater depth to look further at the reasons for the differences in Expert opinion. As Conceptual Complexity was reported more frequently than any other dimension of complexity as an important contributor to an Expert’s opinion of the overall complexity of a mathematics task (Table 3.10), and because Experts provided detailed information about their understanding of Conceptual Complexity, I selected this dimension of complexity for further analysis.
Table 3.9 explains the apparent inconsistencies between Experts’ opinions of the relative conceptual complexity of Tasks A and B. Initially, it appeared Experts differed in their opinion about the relative conceptual complexity of the tasks (Table 3.8). Further analysis of Table 3.7 revealed that the Experts agreed on the relative Conceptual Complexity of the two tasks within a given subcategory of conceptual complexity but differed in the subcategory of conceptual complexity they considered most important (Table 3.9). The only subcategory of Conceptual Complexity for which Task A was considered more complex was the ‘size of unscaffolded conceptual leap’. The importance placed on this subcategory by more than half the Experts was evidenced by their opinion that Task A was more complex conceptually (Table 3.8). If Experts based their decision about the relative Conceptual Complexity of the tasks on the ‘diversity of concepts’ then Task A was considered less complex conceptually than Task B because ‘f’(x) is not entirely a repetition but the same type of process’.

Table 3.9 Subcategories of Conceptual Complexity and Expert opinion related to the relative complexity of Task A and B.

<table>
<thead>
<tr>
<th>Subcategory</th>
<th>More Complex</th>
<th>Comments from Experts</th>
</tr>
</thead>
<tbody>
<tr>
<td>Size of Unscaffolded Conceptual Leap</td>
<td>Task A</td>
<td>Students were required to develop a new concept in Task A without task support in the development of several intermediate concepts first.</td>
</tr>
<tr>
<td>Number of Concepts</td>
<td>Task B</td>
<td>A larger number of concepts are required in Task B. In Task A there are a finite number of ‘key features’ and ‘critical points’ to consider.</td>
</tr>
<tr>
<td>Diversity of Concepts</td>
<td>Task B</td>
<td>A diversity of concepts is involved for Task B. The concepts required in Task A are related to a more limited domain than those for Task B.</td>
</tr>
<tr>
<td>Conceptualising the task</td>
<td>Task B</td>
<td>To start Task B, the ‘original conceptual procedure’ (understanding the concept of formulating a chart or map) must be understood before the student can begin to develop the chart.</td>
</tr>
<tr>
<td>Refine Concept</td>
<td>Task B</td>
<td>In Task B, the students can improve their chart as they recognise interrelationships between different parts of the chart. ‘continual optimisation and refinement’</td>
</tr>
</tbody>
</table>

In summary, although Experts differed in their determination of whether a task was more complex conceptually, there was general consensus amongst Experts as to which Task was more complex with regard to each of five identified subcategories of conceptual complexity. These subcategories are: (a) size of the unscaffolded conceptual leap; (b) number of concepts; (c) diversity of concepts; (d) conceptualisation of the task; and (e) refinement of concepts. This suggests Experts
differed, not so much in their perception of the relative conceptual complexity of the tasks but in the subcategories they considered more important.

In summary, when Experts considered the complexity of a mathematics task they: (a) differed as to which dimensions of complexity they considered more important in the decision making process; (b) held different and often opposing opinions as to which task was more complex along a particular dimension; and (c) did not consider numerical complexity as one of the most important dimensions. Most Experts considered two rather than one dimension as more important than others when the complexity of a mathematics task was considered. In cases where the Expert selected one dimension as more important than all other dimensions, the Expert selected either Conceptual or Intellectual complexity (Table 3.10). As Experts generally selected one or the other of these dimensions but not both when considering the important dimensions to consider the overall complexity of a task, I suggest a study of the interrelationship between Intellectual and Conceptual complexity may further our knowledge about these dimensions. Such a study is beyond the scope of my minor thesis.

3.4.4 The Relative Importance of Each Dimension of Complexity
To gain a more comprehensive understanding of the nature of task complexity, Expert opinion was sought not only about the constituent parts of task complexity but also the relative importance of the constituent parts in determining the overall complexity of a mathematics task (in general).

<table>
<thead>
<tr>
<th>Expert No. and Area</th>
<th>Linguistic</th>
<th>Contextual</th>
<th>Numerical (Operational)</th>
<th>Conceptual</th>
<th>Intellectual</th>
<th>Representational</th>
</tr>
</thead>
<tbody>
<tr>
<td>1Me</td>
<td>A</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2Me</td>
<td>A</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3Me</td>
<td></td>
<td>B</td>
<td>B</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4M</td>
<td></td>
<td>B</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5M</td>
<td></td>
<td></td>
<td>A</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6M</td>
<td>A</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7G</td>
<td></td>
<td></td>
<td></td>
<td>A</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8G</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>B</td>
<td></td>
</tr>
<tr>
<td>9G</td>
<td>B</td>
<td>B</td>
<td>B</td>
<td>A</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Key: A Task A more complex along this dimension  Me Mathematics Education Expert
      B Task B more complex along this dimension  M Mathematics Expert
      = Tasks of equivalent complexity along this dimension  G Gifted Education Expert

Conceptual or Intellectual complexity were among the most important dimensions of complexity selected by all Experts. Only one Expert selected both Intellectual and Conceptual complexity among the dimensions they considered most important (Table 3.10). The exception was Expert 9 who considered all dimensions equally important at
different stages in task performance. Conceptual complexity was selected as one of the most important dimensions for considering the complexity of a task in mathematics more frequently than any other dimension.

Expert 9 believed that complexities could hinder the student’s progress at varying stages of task development. Certain types of complexities were perceived by this Expert to occur at varying stages of task completion. For example: (a) initially representational: “the presentation”; then (b) linguistic: “they may be stopped by the first sentence with three words they don’t understand”; next (c) the conceptual complexity: “have they come across this idea before?” and then (d) intellectual: “words like sufficient, test”.

3.4.5 Changes in Expert opinion on Task Complexity
At the end of the interview the nine Experts were consulted about their final opinion with regard to the complexity of the two tasks.

Table 3.11 Expert comparison of complexity of Task A and Task B before and after Framework sighted.

<table>
<thead>
<tr>
<th>[Respondent No; field of expertise]</th>
<th>[1;Me]</th>
<th>[2;Me]</th>
<th>[3;Me]</th>
<th>[4;M]</th>
<th>[5;M]</th>
<th>[6;M]</th>
<th>[7;G]</th>
<th>[8;G]</th>
<th>[9;G]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Considered more complex pre-framework</td>
<td>unsure</td>
<td>maybe B</td>
<td>maybe B</td>
<td>just B</td>
<td>A</td>
<td>A</td>
<td>A</td>
<td>B</td>
<td>B</td>
</tr>
</tbody>
</table>

* Changed opinion about which task was more complex

Results indicated: (a) four Experts had altered their original decision (Table 3.11); (b) two Experts who initially expressed tentative decisions reported they were now more certain about their initial decisions; and (c) the other three Experts maintained their original decision.

These results (supported by the comments made by Experts during the interview process) indicated that over half the Experts may not have held a clearly defined construct of the nature of task complexity at the start of the interview process.

3.4.5.1 The usefulness of the Williams/Clarke Framework
Experts reported that the Williams/Clarke Framework focused their thinking on the multi-dimensional nature of task complexity and assisted them as they analysed the tasks and identified the complexities. This framework encouraged Experts to consider dimensions of complexity they may not have previously thought about. In some cases, the use of the framework led to a change in opinion about the more complex task (Table 3.11) and in other cases it led to the consolidation of a previously held opinion. Many Experts commented about the contribution of this interview and the Williams/Clarke Framework to the clarification of their own concept of the complexity
of mathematics tasks. None of the Experts challenged the legitimacy of the
dimensions of complexity identified in the Williams/Clarke Framework.

These findings affirm the usefulness and adequacy of the Williams/Clarke Framework
of Complexity to characterise the dimensions of Task Complexity (once Numerical
Complexity was renamed Operational Complexity). These findings also raise the
question about whether further classification of dimensions of complexity into
subcategories would be useful and/or would lead to more consistency between Experts
in their classification of the complexity of a task. Such subcategories were not
employed in the analysis of dimensions of complexity other than conceptual
complexity in this study and a thorough investigation of their use is beyond the scope
of this research.

3.4.6 Another Contributing Factor to the Nature of Task Complexity
As a result of my interviews with Experts, I began to realise there was another
contributing element to the nature of Task Complexity. This element of task
complexity emerged as Expert 4 considered how students might respond to Task B.

3.4.6.1 Expert Number 4 discovers a complexity
This Mathematics Expert considered Task B (the Integration Map/Chart) from the
perspective of a student in order to judge the complexities. Initially, the Expert’s
opinion of Task B was encapsulated by this comment:

“less exciting, need to chew on their pencil more”

The implication of this quote, and of subsequent comments of the Expert, was the
opinion that students would experience a lack of engagement with the task. The Expert
believed the task would just be a matter of the student sitting and thinking the bits out
and mapping it.

The Expert then started to categorise and work with the task and consider ‘out loud’
some of the interrelationships. As the Expert started to consider the process of
producing a map—from the perspective of a student—there was a change in the
behaviour of the Expert. There was an increase in the speed and diversity of thought
articulated and a change in the inflection of the Expert’s voice. It became apparent that
the Expert had become more intense. The task—when viewed from the students’
perspective—had engaged the Expert. The Expert then paused and appeared to reflect
before commenting:

“... anyone who tries to make up a flow chart has to be very brave”

In this example, the complexities of the interrelationships between different
expressions requiring integration were considered by the Expert. The interconnections
prerequisite to student formation of an elegant chart were initially not evident to the
Expert. When the Expert considered further and verbalised some of the intricacies
involved in producing a clear, simple chart or map, the Expert became more aware of the interconnections a student may recognise when attempting the task.

This excerpt of the interview with the Expert in Mathematics illustrates that the discovery of a complexity can transform an ‘unexciting task’ into a challenging task. (This interview also raises the question of possible differences in the opinions of Experts if they had been required to work with each task—rather than just consider each task—prior to the interview). Since the interviews proved extremely fruitful in identifying and clarifying the nature of Task Complexity, this lack of in depth interaction by each Expert with each task does not call the conclusions into question. It does however suggest that future studies might provide greater opportunity for respondents to actually attempt the interview tasks.

3.4.6.2 Relationships between the discovery of complexities and Flow 

Prior to this research study I had frequently observed collaborative groups of students maintain a high level of engagement for at least the period of time allocated to task completion. I had often reflected about the possible reasons for this sustained engagement that sometimes resulted in students continuing to discuss the problem long after the class had concluded. When I considered Csikszentmihalyi’s (1992a, 1992b) model of optimal learning conditions in conjunction with the responses from Experts 4 and 9, a possible explanation of this process began to crystallise and this influenced the focus of my study of student response to complexity (Chapter 5).

M. Csikszentmihalyi (1992a, 1992b) claims optimal learning conditions exist when a person or a group is in a state of Flow. Flow is characterised by an intense engagement in the task where a person loses all sense of time, self and the external environment. Csikszentmihalyi claims these conditions are created when a person works just above their present skill level on a challenge almost out of reach. Csikszentmihalyi found ‘Flow’ could not be maintained for long without an increase in the skill level expected and an increase in challenge.

Expert 4 demonstrated that the discovery of a complexity during task performance could provide a challenge with the potential to engage the person undertaking the task. Expert 9’s opinion that different complexities become the focus of attention at differing stages during task completion suggested to me that tasks could be structured so complexities were progressively discovered as groups of students undertook the task. In this way, an appropriate amount of challenge could be evident to the students at each stage of the solution process and students would not be overwhelmed by challenges ‘too far out of reach’. This type of task structure could provide one of the two conditions Csikszentmihalyi has found necessary for the maintenance of optimal learning conditions—the students would be exposed to a challenge ‘almost out of reach’ over a sustained period of time by the successive discovery of new challenges.

The second condition Csikszentmihalyi identified as necessary for the creation of
optimal learning conditions was the application of skills just above the present skill level of the students. Hence my analysis of student response to complexity (Chapter 5) investigated whether the mathematics undertaken by the students was new to them or involved the use of mathematics to which the students had previously been exposed.

I formulated the construct of a Discovered Complexity to study student response to complexity by investigating the interrelationships between instances when students were: (a) working with new mathematical skills; (b) working to resolve a challenge created by a newly discovered complexity; (c) engaged in the task and/or (d) increasing their understanding of the mathematics. This construct is described in more detail in section 3.5.

By investigation of the level of student engagement when a complexity was discovered and any conceptual development evident in student discourse, I would be able to draw inferences about whether Csikszentmihalyi’s conditions for optimal learning were sufficient to create student engagement in the task and an increased mathematical understanding.

3.5 Elaboration of Task Complexity
For the remainder of this study, the nature of Task Complexity relates to: (a) the dimensions of complexity (Williams/Clarke Framework, 1997) (Table 3.12); and (b) the potential for Discovered Complexities within a task.

3.5.1 The Williams/Clarke Framework
The Williams/Clarke Framework (Table 3.12) has been elaborated as a result of Expert opinion. Many interrelationships between the dimensions of complexity are apparent in this elaboration of the Williams/Clarke Framework. Many components requiring simultaneous consideration are displayed. For example, as a student employs higher level cognitive processes, they simultaneously employ lower level cognitive processes and in addition these cognitive processes are used to understand complexities related to at least one other dimension of complexity.

This detailed elaboration of the Williams/Clarke Framework of Complexity (Table 3.12) should add to both the commonality of understanding of ‘task complexity’ within the education community and provide a common language that could assist educators as they consider different aspects of complexity and as they share ideas about task structure. This elaboration of the nature of Task Complexity should also assist teachers and researchers as they ‘unpack’ a task to determine the complexities a student will encounter or design a task with specific outcomes in mind.
Table 3.12 Elaboration of Dimensions of Complexity in the Williams/Clarke Framework (informed by Expert Opinion).

<table>
<thead>
<tr>
<th>Dimension of Complexity</th>
<th>Elaboration of dimensions of complexity</th>
</tr>
</thead>
</table>
| **Linguistic**          | • The use of many sophisticated words sometimes including subtle differences in meanings (both technical and non-technical) or a small amount of simple language.  
                          | • Complicated sentence structure with many ideas within a sentence or simple structure with one idea to a sentence. |
| **Contextual**          | • The relationship between the mathematics and the situation is not explicit and involves many different links between the two or there is one simple link between the mathematics and the situation. |
| **Operational**         | • Many sophisticated mathematical operations or one simple mathematical operation.  
                          | • Many diverse and unfamiliar combinations of mathematical procedures or similar simple mathematical procedures.  
                          | • The mathematics is expressed in generalities or numerical examples. |
| **Conceptual**          | • The number of concepts involved for successful task completion.  
                          | • The diversity of the domains from which the concepts were drawn.  
                          | • The concept is taught or students discover the concept.  
                          | • The degree of assimilation or accommodation necessary to build a framework which explains a collection of mathematical ideas. |
| **Intellectual**        | Intellectual complexity is described through Bloom’s Taxonomy (1956) and includes the ability to make sense of, utilise, extend or look for reasons behind information arising from one or more of the other dimensions of complexity. Higher level thinking skills are more complex because they require the simultaneous use of lower level thinking skills. In the examples that follow the more complex examples have been included in the earlier dot-points. The intellectual aspects of each complexity have been italicised:  
                          | • The analysis and synthesis of what underpins a set of similarities and differences in an attempt to find out why they occur (evaluation).  
                          | • The original—to those undertaking the task—combination of recalled and understood concepts to find an appropriate mathematical application to solve a problem (synthesis).  
                          | • Analysis of information to decide whether it fits within an understood context.  
                          | • The application of an understood idea to a slightly different context.  
                          | • The understanding of a recalled representation.  
                          | • The recall of linguistic information. |
| **Representational**    | • The medium in which the information is presented (symbolic, graphical, verbal, numeric, algebraic, diagrammatic, geometric, tabular, etc).  
                          | • The number of different representations involved in framing and/or solving the task. |

3.5.2 Discovered Complexity: a new construct
I have defined a Discovered Complexity as a complexity that becomes apparent during task completion and requires each member of the collaborative group to work with unfamiliar mathematical ideas to understand the complexity discovered. By defining a Discovered Complexity in this way, optimal learning conditions would exist each time a complexity was discovered.

Reflecting upon student experiences with Tasks A and B prior to this study and the opinions of Experts, I believed Tasks A and B would provide opportunities to study
dynamic tasks where students would discover complexities. Dynamic Tasks—for the purpose of this research—are tasks that contain complexities not apparent at the commencement of the task that become the focus of student attention at some time during task performance. A task providing the opportunity for Discovered Complexities may be consistent with Bell’s (1993) dynamic task.

The focus of my investigation of student response to complexity (Chapter 5) was clarified by this study of Expert opinion of task complexity (Chapter 3). It informed my understanding of the complexities within Task A and assisted me to recognise the possible causes of sustained student engagement.
CHAPTER 4
STUDENT ABILITY TO SOLVE PROBLEMS
4.1 Introduction

In Chapter 2, I formulated a theoretical construct for the ability to solve unfamiliar challenging problems (Ability) (Table 2.2, p.18). This construct consists of a hierarchy of cognitive activities based on Bloom’s Taxonomy (1956) and Krutetskii’s (1976) empirical data. These cognitive activities in descending order include the ability to: recognise inconsistent information, combine concepts to create an original concept, explain the need for extra information, use more than one pathway, recognise the need for extra information, build on a learned idea, and repeat taught information. This construct has been used to design a test to measure Ability. Explanation is now provided about how this Ability test was designed, administered and interpreted. These assessments of student Ability assisted my selection of two collaborative groups for a case study (Chapter 5).

To support the construct validity of the test I designed as a measure of Ability, comparisons were made between the Ability test, the Standard Ravens Progressive Matrices (Raven, Court & Raven, 1992) and the Common Assessment Task 3 (Victorian Board of Studies, 1996). The Standard Ravens Progressive Matrices (referred to as the ‘Ravens’ for the remainder of this study) measures the ‘ability to reason’ and the Common Assessment Task 3 (referred to as ‘CAT 3’ for the remainder of this study) is claimed to measure the ‘ability to solve unfamiliar analysis tasks in mathematics under examination conditions’. An example of a task question can be found in Appendix 7. The cognitive processes identified in my theoretical construct of Ability were compared with the cognitive processes required by the Ravens and CAT 3 to determine the characteristics that distinguish Ability from these other two measures. Student responses to the three tests were then compared to confirm whether a similar profile of differences in performance was reflected in student responses.

This chapter describes: (a) the Ability test; and (b) the use of the Ravens, and (c) the CAT 3 to support the validity of the Ability test. The use of the Ravens as an indicator of student Ability is discussed.

4.1.1 The Ability Test

This section discusses the test design in relation to the theoretical and operational constructs of Ability (Table 2.1, 2.2, Appendix 4) and the process by which inferences were drawn about student Ability (Table 4.1, p.61).

4.1.1.1 Design of the Ability test

The thirty-five minute Ability test designed for this research study consisted of eight questions (see Appendix 4). The test questions required students to explain their thoughts as they worked to solve short unfamiliar challenging problems. As the subject students were often required to demonstrate their ability to understand a problem from more than one perspective and to provide an argument as to why their solution was
Chapter 4: Student Ability to Solve Problems

reasonably as they collaborated in the classroom, I expected students would not have difficulty expressing their own ideas. For this reason, I expected the Ability test to provide a valid indicator of the cognitive processes employed by each student.

Although the initial stages of this test utilised mathematical concepts to which the subject students had previously been exposed, the students were expected to progressively build upon and interrelate familiar ideas and to develop and explain unfamiliar concepts. The particular cognitive processes employed by students in response to a question often depended on whether the student had previously encountered the same or a similar type of problem. Higher level cognitive processes were more likely to be activated if the problem type was unfamiliar to the student (Krutetskii, 1976; Smith and Stein, 1998). For valid inferences to be drawn from the Ability test and to produce test results from which comparisons between students could be drawn, students were supposed to have a common mathematical background within the mathematical domain addressed in the Ability test. The Gradients Project Investigation (Appendix 6) was undertaken in Mathematical Methods 3/4 in the month prior to the administration of the Ability test in an attempt to provide students with a common understanding of gradients to curves. Mathematical Methods 3/4 is another Year 12 mathematics subject undertaken by all Specialist Mathematics students. In Mathematical Methods 3/4, all students participated in student discussions related to the gradient function and attempted to sketch the gradient function using information from a sketch of the original function. All students were thus exposed to the common base of mathematical experience upon which the Ability test relied. As a further check of student background, students were asked to put a cross beside any familiar mathematics presented in each item on the Ability Test.

To stimulate the use of higher level cognitive processes as students worked to solve unfamiliar challenging problems, test items required students to use familiar concepts either in: (a) an unfamiliar situation; (b) an unfamiliar sequence; or even (c) an original combination. From the test responses, I was able to draw inferences about the nature of each student’s cognitive activity and classify students according to the hierarchical categories of Ability (see Table 4.2, p.64).

The conditions imposed on the administration of the Ability test attempted to control for comparability of prior knowledge before administration of the Ability test. Instances where student mathematical knowledge prior to the Ability test was not consistent with the expected level of knowledge were generally detected by the students’ indication of mathematical ideas they had seen before.

4.1.1.2 Interpreting the Ability test

Test items (Appendix 4) included questions that allowed students to demonstrate the characteristics of Ability discussed in Chapter 2. Table 4.1 includes a selection of
student responses to test items and my interpretations of the relationships between these responses and the characteristics of Ability are discussed below.

**Recognition that some information contradicts other information.** Table 4.1 displays the equation William found for the quadratic in Question 8 (Appendix 4) and his statement that the quadratic he had found did not fit one of the other pieces of information he was given in relation to this question. This example illustrated William’s finding that information he developed through calculation was inconsistent with other information provided in the question. William demonstrated his argument by using substitution to support his claim.

**Combination of two or more taught ideas to create own design to solve a problem.** In Question 2, William attempted to demonstrate the equivalence of the derived series and the series for e^x. He developed his argument by applying his knowledge of derivatives to a series unknown to him before the administration of this test (the series for e^x). When the Ability test was administered, students from this school had not been exposed to the exponential series in class. William confirmed that the series was unknown to him as part of his test response. It can be inferred that William developed the idea to use a derivative to support his argument. He therefore demonstrated the ability to combine ideas in unfamiliar ways to create his own design to solve a problem. Even though William’s argument was not quite complete, he demonstrated the ability to synthesise mathematical information to generate new concepts.

**Use of more than one pathway to solve a problem.** In Question 1, Lida used more than one pathway to solve a problem. She provided both an algebraic and a graphical (Lida stated ‘geometrical’) argument and demonstrated Graph A does not have a gradient equivalent to the original graph. Lida has demonstrated analytic-synthesis (Table 4.2).

**Build on a taught idea to answer a question with a slight change.** During a lesson in the week prior to administration of the Ability test (May 1996), I guided the class as they applied a restriction to the domain of the sine function so an inverse sine function existed. No other work related to restricted domains of inverse circular functions had been undertaken by these students prior to the Ability test. It can be inferred that Gerard used the same thought process with a new example (finding the appropriate restricted domain of the tan function).

Gerard demonstrated the ability to analyse mathematical information and recognise the similarities to a previous problem. This recognition resulted in his application of an understood procedure in a new situation. ‘Analysis’ was the highest level of cognitive activity demonstrated by Gerard when he determined the restriction to be placed on the domain of the tan function to ensure the existence of the inverse tan function. Even
though further consideration of the endpoints of the domain were required, Gerard has demonstrated the ability to apply his understanding to a new situation.

Table 4.1 Examples of the cognitive activities elicited when students responded to Ability Test questions (Appendix 4, Table 2.2)

<table>
<thead>
<tr>
<th>Cognitive activity</th>
<th>[Question number; student]</th>
<th>Student response (italics) : researcher comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>• recognise some information contradicts other information</td>
<td>[8b; William]</td>
<td>The information about the point (2,1) seems to be invalid since $y = \frac{3}{4}(2)^2 - 2(2) + 9 \neq 1$</td>
</tr>
<tr>
<td>• combine two or more taught ideas to create own design to solve a problem</td>
<td>[2; William]</td>
<td>Take the derivative of both sides. LHS $\frac{dy}{dx} = e^x$ RHS $\frac{dy}{dx} = 1 + 2x + \frac{3x^2}{2!} + \frac{3x^3}{3x^2!}$ this will continue, it shows that both are the same. (William had shown the cancelling)</td>
</tr>
<tr>
<td>• be able to explain why extra information is needed</td>
<td>[Talei; 3d]</td>
<td>Looking at the above graph, the function can be determined knowing the y intercept. Once the function is known we can anti-differentiate to work out the equation (however it is important to know a set of coordinates to determine the value of c) You can solve it algebraically and geometrically. eg. A; Its equation is $f(x) = ax^3 + bx^2 + cx + d$ $f'(x) = 3ax^2 + 2bx + c$ so $f(x) = f'(x)$ Geometrically (then Lida has drawn the original graph and underneath it is the gradient of that graph with lines indicating the turning points on the graph have become the x intercepts on the gradient graph)</td>
</tr>
<tr>
<td>• use more than one pathway</td>
<td>[Lida; 1c]</td>
<td></td>
</tr>
<tr>
<td>• recognise extra information is needed</td>
<td>[3c; Jeff]</td>
<td>Yes (no response to 3d) The principal domain would be $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$ (the end points of the domain are incorrect but the main idea is evident)</td>
</tr>
<tr>
<td>• build on a taught idea to answer a question with a slight change</td>
<td>[7a; Gerard]</td>
<td></td>
</tr>
<tr>
<td>• understand concepts behind a taught idea</td>
<td>[1b; Arthur]</td>
<td>B is a function of $e^x$ The derivative is also $e^x$</td>
</tr>
<tr>
<td>• reproduce a taught idea</td>
<td>[3a; Andrew]</td>
<td>We can find the equation of this function and then anti-differentiate</td>
</tr>
</tbody>
</table>

Table 4.1 Examples of the cognitive activities elicited when students responded to Ability Test questions (Appendix 4, Table 2.2)
Gerard’s test response highlighted the necessity for (researcher) knowledge of student background in the interpretation of responses to the Ability test. Had Gerard been exposed to teaching about the required domain restriction for a tan function prior to the administration of this test, his response would have been interpreted as ‘the ability to repeat a taught idea’.

Reproduction of a taught idea was inferred from Andrew’s response to Question 3a. The response indicated that he remembered a procedure and was not able to demonstrate an understanding of how to apply this procedure to another example. Andrew demonstrated the ability to recall taught information but did not demonstrate an understanding of this information.

4.1.2 Ravens Progressive Matrices and CAT 3

This section includes a description of the Standard Ravens Progressive Matrices (1958 Edition), and the Common Assessment Task 3 for Specialist Mathematics November 1996 (Victorian Board of Studies, Victorian Certificate of Education, 1996) (see Appendix 7).

4.1.2.1 Standard Ravens Progressive Matrices

This task was designed to measure the eductive component of general ability which is the ability to generate new, largely non-verbal, concepts that make it possible to think clearly (Raven, Court and Raven 1992). This test consisted of 5 sets of 12 diagrammatic pattern recognition tasks. Each of the 60 tasks required recognition of the final pattern in a 3 x 3 grid. Students were required to select the correct alternative from eight patterns supplied.

Two of the main benefits of the Ravens test were its non-verbal nature and the short administration time. The non-verbal nature of the test reduced language bias against the ESL (English as a Second Language) students. As class time was a precious resource, the preferred alternative was the twenty minute timed rather than un-timed Standard Ravens Progressive Matrices (1958 edition). Raven et al. (1992) have demonstrated comparable measures are achieved using the timed and un-timed version of the test with different conversion tables. A percentile ranking for eductive ability was generated using the score from the Ravens and the appropriate conversion table (de Lemos, 1995). I used this ranking to allocate students to six groups ranked according to ‘student ability to reason’.

Although the Ravens has a ceiling and therefore does not distinguish clearly between students of extremely high ability (Raven et al., 1992), the results were sufficient to classify the twenty-eight students into six hierarchical categories because all students who scored above the ninety-fifth percentile were in the same group.
4.1.2.2 Common Assessment Task 3

This ninety-minute test formed part of the external assessment for Specialist Mathematics. The test consisted of four analysis tasks each of which was sequenced so successive sections of a question became progressively more complex. An example of one of these analysis tasks is in Appendix 7. To answer the questions, students were required to use skills developed in Specialist Mathematics in unfamiliar combinations (Victorian Board of Studies, 1994). As students were expected to combine mathematical ideas in unfamiliar ways, it was expected that student performance on CAT 3 would reflect some aspects of Ability; in particular analysis and synthesis. CAT 3 does not require extensive evaluation to determine whether closure has been attained or whether optimisation is possible because, unlike the unfamiliar challenging problems, explicit information is given about the type of solution required.

Where a student had not become familiar with the mathematics within the Specialist Mathematics course, or alternatively where a student had prepared thoroughly by working with an extensive variety of analysis tasks, for the purpose of this study, CAT 3 was not expected to provide a valid assessment. The assessments for such students would reflect student ‘ability to prepare for an examination’ and student ‘ability to apply Specialist Mathematics skills’ rather than student ‘ability to solve unfamiliar analysis tasks’. In the first case, the test may present Difficulty due to the students’ lack of the required mathematical background and in the second case some or all of the analysis tasks may not be ‘unfamiliar’.

4.2 Results and Analysis

In this section, student responses to the Ability test are analysed and each student has been allocated to one of the hierarchical categories of Ability (Table 4.2). The cognitive activities elicited by the Ravens and CAT 3 are compared with the cognitive activities that characterise Ability (Table 4.2) and these comparisons are used to predict comparative student performance on the three tests. This provides information about the construct validity of the Ability test.

4.2.1 Ability Test: Analysis of Results

Where student absences occurred, the Ability test was not administered because the two students concerned returned to their group on return to class. The collaborative group work related to Task A would then have familiarised students with mathematical techniques required to be unfamiliar for valid administration of the Ability test. Although this limited the number of student results available for comparison, withholding the commencement of Task A (to allow the administration of the Ability test) was not in the best educational interest of the students. The method of allocation of the twenty-eight students to Ability categories is now described.
Table 4.2 Theoretical Construct of Ability formulated using Bloom’s Taxonomy (1956), Krutetskii (1976) (see Table 2.2).

<table>
<thead>
<tr>
<th>Categories of Ability</th>
<th>Hierarchy (descending): Mental activity activated at this level and each level below</th>
</tr>
</thead>
<tbody>
<tr>
<td>Evaluative-synthesis</td>
<td>• recognise inconsistent information</td>
</tr>
<tr>
<td>Synthesis</td>
<td>• combine two or more concepts to create an original concept</td>
</tr>
<tr>
<td>Analytic-synthesis</td>
<td>• explain the need for extra information</td>
</tr>
<tr>
<td></td>
<td>• use more than one pathway</td>
</tr>
<tr>
<td>Analysis</td>
<td>• recognise the need for extra information</td>
</tr>
<tr>
<td></td>
<td>• build on a concept to answer a question with a slight change</td>
</tr>
<tr>
<td>Comprehension</td>
<td>• understand a concept</td>
</tr>
<tr>
<td>Knowledge</td>
<td>• repeat a taught idea</td>
</tr>
</tbody>
</table>

4.1.1.1 Ability test: method of allocation to Ability categories

Students were categorised according to the highest level of cognitive activity displayed (see Tables 4.3 and Table 4.4) with reference to the six different Ability categories identified using the theoretical construct of Ability (Table 4.2).

For example, a student who demonstrated Evaluative-synthesis had ‘recognised inconsistent information’ and at least one operation from each of the lower level in the Ability hierarchy whereas a student whose highest demonstrated cognitive activity was Analytic-synthesis had demonstrated competent use of at least one of the operations: ‘explained the need for extra information’, or ‘used more than one pathway’. Such a student was unable to ‘recognise inconsistent information’ or ‘combine concepts to create an original concept’ to solve a problem. A student who was allocated to the category 'Knowledge' could ‘repeat taught information’ but could not carry out any of the other operations listed.

Table 4.3 and Table 4.4 summarise the inferred cognitive activity elicited by the Ability test for each student. Whatever the highest level of cognitive activity demonstrated by any student other than Lanie, this student also demonstrated the use of at least one cognitive activity at each level lower than this. Lanie on the other hand appeared to activate Evaluative-synthesis without activating mental activities representative of the intervening levels.

4.1.1.2 Ability test: consideration of Lanie’s discrepant results

In an attempt to find why Lanie’s performance did not fit the theoretical hierarchy, background information was sought about Lanie and essential prerequisite for valid application of the Ability test is further emphasised.
In Term 1 1996, Lanie’s attention was directed primarily towards achieving excellence in extra curricula activities and this appeared to be responsible for her limited progress in mathematics at that time. My teacher’s diary recorded Lanie tended to flit quickly from one thought to another (in first term of 1996) rather than persevering with an idea. The Ability test (Appendix 4) was administered towards the end of Term 1 and the first test of Mathematical Understanding (Appendix 5) was administered in the first week of Term 2. Lanie’s behaviour and demonstrated knowledge in Term 1 (teacher’s diary) coupled with her responses on the test of mathematical understanding provided evidence to demonstrate Lanie did not possess an understanding of the mathematical prerequisite for valid interpretation of her Ability test results.

Lanie’s lack of background does not explain her apparently successful employment of the highest level cognitive activity though. Even if the test was not valid for Lanie, her performance at the highest cognitive level was inconsistent with my classroom observations of Lanie’s Ability (teacher’s diary). My diary entries recorded observations of Lanie demonstrating analysis and analytic-synthesis but not synthesis or evaluative synthesis. The test item designed to illustrate the ability to ‘recognise some information contradicts other information’ was at the end of the test (Qn. 8, Appendix 4) and even though many students attempted the question, inconsistencies were only recognised by William and Lanie (Table 4.3). A ‘solution’ to Question 8 could be obtained using algorithms taught early in Year 11 Mathematics but a student obtaining this ‘solution’ would not necessarily ‘recognise the inconsistencies’. They would have obtained an equation but unless they checked further they would not realise this equation did not fit all the information. As this was one of the few test items Lanie could access with her limited mathematical background, she probably spent a considerable amount of time on this question. With this extra time Lanie may have found the inconsistencies by obtaining the solution using one method then checking the solution using another method. If Lanie found the inconsistencies using this process, she has demonstrated ‘analysis’ and ‘analytic-synthesis’ rather than ‘evaluative-synthesis’. Evaluative-synthesis involves progressive evaluation of progress rather than simply evaluating at the end of the problem. If Lanie has used ‘analysis’ and ‘analytic-synthesis’ rather than ‘evaluative-synthesis’, her results are consistent with the hierarchy in the Ability framework.

4.1.1.3 Ability test: validation of hierarchy

The constraints within which this Ability test is valid were highlighted by Lanie’s result. This test measures the ability of a student to solve unfamiliar challenging problems where the student had acquired the level of mathematical background developed in the Gradient Function Investigation (Appendix 6). Lanie could not validly be assessed by the Ability test if she had not developed the required background.
As Lanie was the only student whose test performance was not initially consistent with the proposed hierarchical order of cognitive activity and Lanie’s results have been explained, the results in general support the hierarchical order of these cognitive activities and my proposed theoretical model and operationalised model formulated to assess the ability of a student to solve unfamiliar challenging problems.

### 4.1.1.4 The Ability of each student

Table 4.5 summarises the Ability categories (in order of descending Ability) to which each student was allocated using their Ability test responses (Table 4.3, Table 4.4).

<table>
<thead>
<tr>
<th>Category of Ability</th>
<th>Subcategory</th>
<th>Class 1</th>
<th>Class 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Evaluative-synthesis</td>
<td>Synthesis</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>a</td>
<td>Talei</td>
<td>Alistair</td>
</tr>
<tr>
<td></td>
<td>b</td>
<td>Gerard</td>
<td>Dean</td>
</tr>
<tr>
<td></td>
<td>c</td>
<td></td>
<td>Tony</td>
</tr>
<tr>
<td>Analytic-synthesis</td>
<td></td>
<td>Damien</td>
<td>Angelos</td>
</tr>
<tr>
<td></td>
<td>a</td>
<td>Brandon</td>
<td>Rez,</td>
</tr>
<tr>
<td></td>
<td>b</td>
<td>Lida</td>
<td>Jarrod</td>
</tr>
<tr>
<td></td>
<td>c</td>
<td>Lanie?</td>
<td>Jacob,</td>
</tr>
<tr>
<td>Analysis</td>
<td></td>
<td>Jeff,</td>
<td>Harry</td>
</tr>
<tr>
<td></td>
<td>a</td>
<td>Jack</td>
<td>Alec</td>
</tr>
<tr>
<td></td>
<td>b</td>
<td>Sergiy, Arthur</td>
<td></td>
</tr>
<tr>
<td></td>
<td>c</td>
<td>Nina</td>
<td></td>
</tr>
<tr>
<td>Comprehension</td>
<td>Knowledge</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>a</td>
<td>Amy</td>
<td>Paulos, Nick</td>
</tr>
<tr>
<td></td>
<td>b</td>
<td>Edward, Aliek</td>
<td></td>
</tr>
<tr>
<td></td>
<td>c</td>
<td></td>
<td>Gene</td>
</tr>
</tbody>
</table>

Absent: Andy, Bazza.

Where students within a particular category differed in the frequency of demonstration of an operation or where students differed in the number of operations demonstrated at the highest level of cognitive processing they achieved on the Ability test, they were listed on separate lines in the table.

For example, Alistair, Dean and Tony were all assessed at Ability Level 2 but Alistair demonstrated the ability to combine ideas to create his own solution most frequently and Tony demonstrated this ability least frequently.

### 4.1.2 Comparison: Ravens, CAT 3 and Ability Construct

This section compares the cognitive activities required for successful performance on the Ravens and CAT 3 with the cognitive activities within the Ability construct (Table 4.2).

By analysis of the Ravens and CAT 3 I was able to identify differences in the cognitive activities required by the Ravens and CAT 3. Although the three measures are not the same, consistent with the analysis in Table 4.6 some of the characteristics required by the Ravens and CAT 3 could be considered as a subset of the characteristics measured by the Ability test (Figure 4.1). Figure 4.1 has been used to represent the cognitive activities within the construct of Ability and illustrate that both CAT 3 and the Ravens
require some of the cognitive activities represented in the Ability construct but not all of these cognitive activities. For example, the analysis and synthesis required by CAT 3 and the Ravens is not as extensive as the analysis and synthesis represented in the Ability construct.

Table 4.6 Summary of differences between the characteristics of Ability (Table 2.2), Standard Ravens Progressive Matrices and CAT 3.

<table>
<thead>
<tr>
<th>Type of Cognitive Activity Required</th>
<th>Differences between the Ability Construct, Ravens and CAT 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Analysis</td>
<td>Standard Ravens Progressive Matrices</td>
</tr>
<tr>
<td></td>
<td>• analysis of geometric patterns.</td>
</tr>
<tr>
<td></td>
<td>• number of possibilities comparatively limited.</td>
</tr>
<tr>
<td></td>
<td><strong>Common Assessment Task 3</strong></td>
</tr>
<tr>
<td></td>
<td>• analysis of English language and mathematical language</td>
</tr>
<tr>
<td></td>
<td>• tests from previous years could familiarise students with</td>
</tr>
<tr>
<td></td>
<td>the style of analysis.</td>
</tr>
<tr>
<td></td>
<td>• analysis of Specialist Mathematics topics.</td>
</tr>
<tr>
<td></td>
<td>• domain of mathematics specified; many possibilities exist.</td>
</tr>
<tr>
<td></td>
<td><strong>Ability Construct</strong></td>
</tr>
<tr>
<td></td>
<td>• type of analysis unfamiliar and varied.</td>
</tr>
<tr>
<td></td>
<td>• some analysis of English and mathematical language.</td>
</tr>
<tr>
<td></td>
<td>• no limit to the domain of familiar mathematics required.</td>
</tr>
<tr>
<td>Evaluation</td>
<td>Standard Ravens Progressive Matrices</td>
</tr>
<tr>
<td></td>
<td>• check pattern across page with pattern down the page.</td>
</tr>
<tr>
<td></td>
<td>With accuracy, a high score could be achieved without</td>
</tr>
<tr>
<td></td>
<td>evaluation.</td>
</tr>
<tr>
<td></td>
<td><strong>Common Assessment Task 3</strong></td>
</tr>
<tr>
<td></td>
<td>• possible for accurate students to achieve a high score</td>
</tr>
<tr>
<td></td>
<td>with minimal evaluation.</td>
</tr>
<tr>
<td></td>
<td>• informal evaluation of progress not generally required</td>
</tr>
<tr>
<td></td>
<td>but possible, useful and varies in nature.</td>
</tr>
<tr>
<td></td>
<td>• not open tasks—a solution exists so little evaluation of</td>
</tr>
<tr>
<td></td>
<td>progress against intended goal or optimisation required.</td>
</tr>
<tr>
<td></td>
<td><strong>Ability Construct</strong></td>
</tr>
<tr>
<td></td>
<td>• request to evaluate not generally explicit but without</td>
</tr>
<tr>
<td></td>
<td>evaluation inconsistencies may not be recognised.</td>
</tr>
<tr>
<td></td>
<td>• the ability to recognise inconsistencies is an element of</td>
</tr>
<tr>
<td></td>
<td>Ability.</td>
</tr>
<tr>
<td></td>
<td>• open task; requires optimisation and evaluation of whether</td>
</tr>
<tr>
<td></td>
<td>closure has been attained.</td>
</tr>
<tr>
<td>Synthesis</td>
<td>Standard Ravens Progressive Matrices</td>
</tr>
<tr>
<td></td>
<td>• combine patterns to produce an overall pattern. Limited</td>
</tr>
<tr>
<td></td>
<td>number of possibilities.</td>
</tr>
<tr>
<td></td>
<td><strong>Common Assessment Task 3</strong></td>
</tr>
<tr>
<td></td>
<td>• use familiar mathematics from the restricted domain of</td>
</tr>
<tr>
<td></td>
<td>the Specialist Mathematics (and Mathematical Methods) in</td>
</tr>
<tr>
<td></td>
<td>unfamiliar combinations.</td>
</tr>
<tr>
<td></td>
<td><strong>Ability Construct</strong></td>
</tr>
<tr>
<td></td>
<td>• required to create new concepts by combining concepts in</td>
</tr>
<tr>
<td></td>
<td>unfamiliar ways.</td>
</tr>
<tr>
<td></td>
<td>• expected to combine mathematical ideas from diverse</td>
</tr>
<tr>
<td></td>
<td>content domains.</td>
</tr>
<tr>
<td></td>
<td>• no indication of the mathematical domain from which the</td>
</tr>
<tr>
<td></td>
<td>mathematics is drawn.</td>
</tr>
</tbody>
</table>

Even though evaluation can increase accuracy on CAT 3 and the Ravens, and is required for some sections of the analysis tasks in CAT 3, the spontaneous use of progressive evaluation during synthesis, integral to the ability to solve unfamiliar challenging problems, is not a requirement for CAT 3 or the Ravens.
For each test, students were required to interconnect ideas and develop concepts or techniques to solve a problem (Table 4.6). Where the Ability test and CAT 3 required the interconnection of mathematical concepts, the Ravens required the interconnection of patterns across a set of diagrams. Each of the three tests (Ability test, Standard Ravens Progressive Matrices and CAT 3) differed with regard to: (a) the breadth of ideas from which students must select to analyse and synthesise; (b) the types of evaluation elicited by the items in the test; (c) the open or closed nature of the task; and (d) the importance placed on evaluation as a cognitive process in the assessment (Table 4.6).

![Venn diagram representing the relationship between the characteristics of Ability, and the higher level cognitive activities required by CAT 3 and Standard Ravens Progressive Matrices.](image)

Several differences between the Ability and the cognitive activities elicited by CAT 3 and the Ravens include: (a) evaluative-synthesis is an integral part of the assessment on the Ability test where the use of this mental activity is not given the same weight in assessment on the other two tests; (b) CAT 3 and the Ability test rely on background knowledge in mathematics and the English language but the Ravens (which relies on diagrammatic representations) does not require such a knowledge; and (c) the analysis tasks on CAT 3 may not be ‘unfamiliar’ to students who prepare extensively.

### Expected differences in student performance on tests

The analysis in Table 4.6 was then used to identify those groups of students who would be expected to demonstrate different levels of performance on the three tests. The types of students were described and justification provided for the predicted differences in performance. Comparison of the Ability Construct (Table 2.2) with Ravens and CAT 3 highlights those characteristics of Ability that differ from the cognitive processes required by the other two tests. Student Ability is characterised by the ability to select, analyse and synthesise mathematical information from many disparate and broad mathematical domains and the ability to synthesise and progressively evaluate during task performance.
The Ability test required students to undertake extensive and progressive evaluation as part of task completion. Evaluative-synthesis is considered an integral aspect of successful task performance rather than only a technique to reduce the number and size of errors (see Table 4.6). Where a fast accurate student lacking evaluative-synthesis could achieve a high level of success on the Ravens and on CAT 3, this is not the case for the Ability test. I expected to find a group of students who achieved their best result for the Ravens Progressive Matrices and did not perform as well on the Ability test. It was expected these students would be fast and accurate with demonstrated ability to analyse and synthesise but not the ability to progressively evaluate their solution.

CAT 3 and the Ability test: rely on background knowledge in Mathematics and English. English as a Second Language (ESL) students may be denied the opportunity to demonstrate their ‘ability to solve unfamiliar challenging problems’ and or demonstrate their ‘ability to solve analysis tasks’. ESL students may be missing mathematical background due to language problems. They may also experience language difficulties with the task. ESL students with a high level Ability who found the mathematics and/or English Difficult would be expected to achieve their highest score on the Raven Standard Progressive Matrices.

Analysis tasks in CAT 3 are not unfamiliar to some students so I expected to find a group of students who achieved their best performance on CAT 3 because their extensive preparation for CAT 3 included work with analysis tasks of a very similar nature. For these students, their performance on CAT 3 could demonstrate ‘the ability to prepare for examinations’ and the ‘ability to apply Specialist Mathematics skills but not the ‘ability to solve unfamiliar analysis tasks under test conditions’.

<table>
<thead>
<tr>
<th>Name of Test</th>
<th>Method of allocation to one of six levels</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ability Test</td>
<td>• cognitive activities elicited were inferred&lt;br&gt;• subsequent allocation to one of six categories (Table 4.2)</td>
</tr>
<tr>
<td>Standard Ravens Progressive Matrices</td>
<td>• percentile ranking in the wider community (standardised test)</td>
</tr>
<tr>
<td>Specialist Mathematics CAT 3</td>
<td>• according to the grade from A to E within the Specialist Mathematics population&lt;br&gt;• percentile ranking in 1996 Specialist Mathematics population</td>
</tr>
</tbody>
</table>

As expected, comparison of student performance on the three tests highlighted three types of students with markedly different performances on the three tests. Markedly Different Performance for the purpose of this study includes students whose assessments on the three tests differed by more than two categories.

Small differences in performance were not considered discrepant because different criteria were used to allocate students to categories for each of the three tests. Ability
test categories were formed according to the cognitive activities a student displayed, Ravens categories related to the percentile rank of the student in the whole population and CAT 3 categories related to the student’s percentile ranking in the Specialist Mathematics population (see Table 4.7).

Approximately two thirds of the students were assessed at similar levels in their hierarchical placement on each of the three tests indicating each of these students demonstrated a similar level of ability to ‘solve challenging problems’, ‘perform unfamiliar analysis tasks’ under test conditions and ‘reason’ (Table 4.8).

**Table 4.8 Comparison of Ability Test, Ravens Standard Progressive Matrices and CAT 3. The number of levels spanned by each of the 20 students measured on the three different tests**

<table>
<thead>
<tr>
<th>Number of levels Spanned</th>
<th>Number of students</th>
<th>Percentage of the students</th>
<th>Markedly different results</th>
<th>Student (highest measure)</th>
</tr>
</thead>
</table>
| All within 1 level or across 2 levels | 13 | 65% | | Damien (Ravens)  
Brandon (Ravens)  
Jack (Ravens) |
| 3 | 5 | 15% | X | Jeff (CAT 3)  
Alistair (CAT 3) |
| more than 3 | 2 | 10% | X | Zaroff (Ravens)  
Paulos (Ravens) |

4.1.1.2 Students with markedly different performances

As expected, there were marked differences in the student performance on the three tests for three types of students: (a) students who were fast and accurate but did not analyse problems in depth nor progressively evaluate their results (Damien, Brandon and Jack); (b) highly motivated students who gained familiarity with some of the types of analysis tasks in CAT 3 (Jeff and Alistair); and (c) ESL students (Zaroff and Paulos). Table 4.9 provides the sources of the evidence upon which these categorisations were based.

Fast, accurate analysis but evaluative-synthesis not demonstrated (Table 4.3). Damien, Brandon and Jack each gained scores that spanned three hierarchical levels on the three tests and gained their highest score on the Ravens.

Evidence of the superficial approach of these students was recorded in my teachers diary and is now summarised. In July 1996, I recognised the tendency for Damien,
Brandon and Jack to reach rapid task closure due to their lack of depth of analysis and lack of progressive evaluation of their findings. This had become evident as they prepared for CAT 1 (the unfamiliar challenging problem in Specialist Mathematics undertaken as a school based task in July/August 1996). My diary record indicated I alerted these three students to their operating style in preparation for CAT 1. I drew each student’s attention to the superficial nature of their approach to challenging problems. I confirmed their work was fast and accurate but emphasised they focused too much on rushing to a solution and did not deliberate about: (a) why patterns occurred; (b) whether findings were reasonable; and (c) whether there were exceptions to the patterns found. It can be concluded from the performances of Jack, Brandon and Damien that the Ability test measures characteristics that are not a constituent part of the characteristics measured by the Ravens. These characteristics include ‘ability to recognise inconsistent information’ by progressive evaluation during synthesis.

Table 4.9 Characterisation of the discrepant students and evidence for this characterisation

<table>
<thead>
<tr>
<th>Student Names, (highest score achieved)</th>
<th>Characteristic</th>
<th>Evidence for classification</th>
</tr>
</thead>
</table>
| Damien; Brandon; Jack (Ravens)          | • Quick, Accurate  
• Lack depth of analysis  
• Rapid and inappropriate closure  
• Lack evaluation        | Teacher’s diary (July 1996) |
| Jeff; Alistair (CAT 3)                  | • Motivation  
• Persistence                                      | Teacher’s diary for past two years  
Further confirmation in the November Exam revision undertaken by these students. |
| Zaroff; Paulos (Ravens)                 | • English language hindered acquisition of mathematics content  
• English language hindered test interpretation for CAT 3 and possibly Ability test  
• Ability to problem solve demonstrated orally in class | Language other than English at home  
Ravens relied on geometric patterns not English language  
My diary observations identify oral problem solving ability demonstrated by both students  
Zaroff arrived Australia from Russia during mid-secondary schooling  
Paulos’ Primary mathematics acquisition was hindered by language barriers (diary) |

Students for whom some analysis task questions were not as unfamiliar. Jeff and Alistair’s highest score on CAT 3 reflected their familiarity with the types of analysis questions asked in CAT 3.

Alistair produced his highest score on CAT 3 and his lowest score on the Ravens. My opinion of Alistair expressed in my teacher’s diary in Term 1 1996 is supported by the findings in Chapter 5, Alistair was a student with a very high Ability who thought ideas
through carefully, slowly and effectively and often recognised ideas other students had not yet considered. His lower assessment on the Ravens would have been a function of his slow, deliberate and thoughtful approach to problems and the test’s time constraints. His highest performance on CAT 3 indicated the amount of time he spent preparing for this test and the familiarity he gained with the types of analysis tasks used. Alistair was able to achieve an extremely high grade despite his slow deliberate approach. This is an indication of Alistair’s familiarity with the types of analysis tasks in CAT 3.

Notes in my teacher’s diary indicated the large and diverse number analysis tasks undertaken by Alistair and Jeff in preparation for CAT 3. This evidence confirms these student’s familiarity with the type of analysis tasks on CAT 3 and suggests the Ability test included a component that assessed student ability to work with unfamiliar problems.

*Standard Ravens Progressive Matrices: indicator of Ability in ESL students.*  Zaroff and Paulos like Brandon, Damien and Jack gained their highest score on the Ravens but Zaroff and Paulos’s scores were discrepant by more levels than any other students (see Table 4.8). Both Zaroff and Paulos speak a language other than English at home and each reported to me that their acquisition of mathematics had been hindered by lack of understanding of the English language (see Table 4.9). Evidence in my teacher’s diary confirms each student lacked a sound mathematical background and that Zaroff still experienced language difficulties. Zaroff and Paulos’s results suggest the Ravens Progressive Matrices could be used as a preliminary test to detect Ability for some ESL students.

**4.1.1.3 Summary**

Comparison of the Ability Construct (Table 2.2), Ravens and CAT 3 for Specialist Mathematics and analysis of the differences in performance of individual students on the three tests supports the validity of the Ability test as a measure of student ability to solve unfamiliar challenging problems. This analysis suggests the Ability test items elicit evaluative-synthesis (progressive evaluation) and synthesis (combine mathematics in unfamiliar ways). This analysis also highlights the need for alternative ways to detect Ability in students lacking the mathematical background required by the Ability test.

When language barriers experienced by ESL students were considered, the non-verbal nature of the Ravens increased the students’ ability to interpret the test. The Ability test could present Difficulties for ESL students where language difficulties had hindered their acquisition of mathematical concepts at some stage in their schooling or where present difficulties with English denied the student the opportunity to interpret and respond to the Ability test. As the Ravens has been demonstrated to detect student
ability to analyse and synthesise this test could be used as a preliminary tool in the identification of student Ability in ESL students.

4.3 Conclusions

The hierarchical nature of the Ability construct has been empirically validated during this study (Table 4.3, Table 4.4) and the validity of the Ability test as a measure of student ability to solve unfamiliar challenging problems has been supported (Section 4.2.2). This study highlights the importance of considering prerequisite conditions when using the Ability test. As predicted and demonstrated during this study, the Ability test does not provide a valid measure of Ability when a student experiences Difficulty with the mathematics or the English language required by the task. Although it would be unrealistic to consider students shared a background common in all respects, the common investigation undertaken by students prior to the Ability test, the limited topic domain of the test and the cross-checking procedures used (as students undertook the test) would have assisted in reducing and detecting the differences in student background. Even though the Standard Ravens Progressive Matrices did not test the depth of cognitive activity characterised by Ability, it possesses several advantages in certain instances. It identifies ESL students who may possess higher Ability than indicated by the Ability test. The Ravens was also fast to administer and did not require a specific mathematical background.

There were several disadvantages to this test (Ravens) that would require consideration if this test were selected as an indicator of Ability. As the timed version of the Ravens relies on fast accurate reasoning and a slow, deliberate approach to problem solving is characteristic of some students with high Ability (Krutetskii, 1976), the un-timed version of the Ravens may be a more appropriate indicator of Ability. The Ravens does not detect the presence or absence of evaluative-synthesis. If the Ravens Standard Progressive Matrices were selected for use as an indicator of Ability further analysis would be required to complete the assessment of student Ability.

The relative Ability of students in this research study (as assessed by the Ability test) informed my selection of two collaborative groups to use as case studies when investigating student response to complexity. The characterisation of Ability as elaborated through this research enriched my study of student response to complexity (Chapter 5); in particular it informed my study of the influences the relative Ability of a collaborative group may have on the student response to complexity.
CHAPTER 5

STUDENT RESPONSE TO TASK COMPLEXITY
5.1 Introduction

‘Effectiveness’, for the purpose of this study, is used to indicate the usefulness of a particular task or other contributing factor in the facilitation of conceptual development in students. This chapter provides an account of my exploration of the relationship between task effectiveness and task complexity by study of student response to the complexities within an unfamiliar challenging problem (Task A).

The analysis was undertaken through case studies of two small collaborative groups of senior secondary mathematics students who worked to solve unfamiliar challenging problems through Class Collaboration. Through these case studies I examined the role complexity played in these student responses by analysis of data from several sources including videotapes, tests, interviews, and diary entries. Table 5.1 provides further information about the types of data collected.

<table>
<thead>
<tr>
<th>Type of Data</th>
<th>Description of Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Video tapes</td>
<td>Video tapes of students undertaking unfamiliar challenging problems</td>
</tr>
<tr>
<td>Tests</td>
<td>Test of Mathematical Understanding administered before and after individual instruction and Class Collaboration as students worked to solve Task A (see Appendix 2)</td>
</tr>
<tr>
<td>Interviews</td>
<td>Several students were interviewed after Task A had been undertaken. This provided support for interpretations of the video data (Clarke 1996)</td>
</tr>
<tr>
<td>Diary entries</td>
<td>Entries in my diary record informal discussions with students throughout the year in which the study was undertaken.</td>
</tr>
</tbody>
</table>

Initial observation of video data suggested possible interrelationships between: (a) student discovery of complexities; (b) student engagement in the task; and (c) student development of mathematical concepts. My analysis focused on associations between these three types of student responses and enabled me to draw inferences about the role of Task Complexity where Class Collaboration was used as students worked to solve unfamiliar challenging problems.

It should be noted that I was the ‘researcher’, the ‘classroom teacher’ and the ‘mathematics coordinator’. My primary role at different stages in the research process has been included in brackets where this information is relevant to the interpretation of the research findings.

5.2 Study Design: Analysis of Student Response

The groups selected for case study, the task undertaken, learning through Class Collaboration and the data collection techniques are now described and the methods of analysis are explained.
5.2.1 Case Studies: Individual students and groups selected

Two collaborative groups with similar Ability distributions (see Table 4.5), Group 1 (William, Talei and Gerard) and Group 2 (Dean, Tony, Alistair and Rez) from Class 1 and Class 2 respectively, were chosen for analysis. Group 1 had originally contained four students but Damien was absent for the Class Collaboration session and consequently was not included in the Group 1 case study. All three students in Group 1 had demonstrated the Ability to ‘combine concepts to create an original concept’ and William had also demonstrated the ability to ‘recognise inconsistent information’ (Table 4.3). Three students in Group 2 had demonstrated the ability to ‘combine concepts to create an original concept’ and the highest cognitive activity demonstrated by Rez was ‘analytic-synthesis’ (Table 4.4). Each group consisted of students representative of two adjacent Ability levels and the majority of the members in each group were members of the Synthesis category of Ability (Table 4.5).

5.2.2 Selection of Case Studies: rationale

I taught both Specialist Mathematics classes—Class 1 and Class 2—at this school (see Chapter 1 for information about the school and students). Class Collaboration was used in conjunction with other teaching approaches for both classes during the year. By collecting data related to student response to complexity from collaborative groups from both classes I obtained a broader data base from which to select two collaborative groups for study. I selected two groups with similar distributions of student ability to solve unfamiliar challenging problems (referred to as Ability).

One of the main reasons for selecting Group 1 (Talei, William and Gerard) and Group 2 (Dean, Alistair, Tony and Rez) was the unexpected differences in learning outcomes for Gerard (Group 1) and Dean (Group 2). These two students possessed comparable ability to solve unfamiliar challenging problems as measured by the Ability test (Chapter 4) but demonstrated different conceptual development as they worked within the task (see Section 5.4.1, p.85). I undertook analysis to discover whether these differences in learning outcomes could be attributed to differences in response to complexity by the two groups.

I have chosen to use the terminology ‘worked within the task’ (in the previous paragraph and throughout this thesis) to convey my prior experiences with students undertaking these types of tasks. They do not tend to focus immediately on producing a solution but rather explore ideas within the mathematical context set up by the task.

5.2.3 The Unfamiliar Challenging Problem: Task A

Task A, ‘Understanding the Double Derivative’ (see Appendix 2) was the task used to investigate student response to complexity. See Chapter 3 for more information about this task, my rationale for task design and use, and Expert opinion about the complexity of Task A.
5.2.4 Instruction: Class Collaboration and individual instruction

Although this study focused on student response to complexity in a situation where Class Collaboration was used as students worked to solve Task A, students spent an equal amount of time working individually with this task.

5.2.5 Class Collaboration

The introduction to this thesis provided an overview of Class Collaboration and its evolution as a result of my observations and reflections as a classroom teacher.

<table>
<thead>
<tr>
<th>Feature</th>
<th>Elaboration of this Feature</th>
</tr>
</thead>
</table>
| The learning culture | • Build from each student’s present understanding  
|                   | • Any justified solution pathway is acceptable  
|                   | • Reasonable justification to be the authority upon which student acceptance of a solution pathway is based  
|                   | • Value all contributions                                                                 |
| Type of Task     | • Open ended task  
|                   | • A maximum of ten minutes formal teaching prior to the task  
|                   | • Undertaken over several lessons                                                           |
| Classroom management | • Small collaborative groups  
|                 | • Feedback from each group to the whole class at regular intervals                          |
| Questioning technique | As groups brainstorm and develop ideas, the teacher:  
|                    | • visits each group regularly for 30 seconds to 2 minutes  
|                    | • listens to and sometimes participates in discussions  
|                    | • demonstrates an interest in the students’ thoughts  
|                    | • shares the excitement when students have found an idea new to them  
|                    | The teacher does not:  
|                    | • provide hints  
|                    | • indicate whether a pathway is correct  
|                    | • answer questions.  
|                    | The teacher asks questions to assist groups to:  
|                    | • clarify an idea  
|                    | • refocus upon a productive solution pathway  
|                    | • experience a breakthrough or an enlightenment  
|                    | • analyse the mathematics generated  
|                    | • evaluate progress  
| Reporting process | • The pace and nature of the reporting session is determined by students  
|                   | • Value of information presented is decided by each group  
|                   | • A member of the class may ask the student, who is reporting at the board, for further clarification. This may be to clarify the questioner’s or the reporter’s understanding of the justification provided or to provide clarification for the questioner about a possible flaw in the justification  
|                   | A group may decide to:  
|                   | • discard ideas presented  
|                   | • use ideas presented  
|                   | • build further in the direction suggested.  
|                   | When students are reporting, two classroom ‘rules’ are:  
|                   | • questions are constrained to information the reporter has presented  
|                   | • a student is not to be contradicted while presenting a report  

I continually modified my approach always seeking to create a classroom where students developed ideas for themselves and extended their learning by sharing ideas with others. I believe Class Collaboration facilitates student learning and that certain
key features are integral to its effectiveness. These key features include the learning culture of the classroom, the structure of the task, the classroom management, the questioning techniques and the reporting process. Table 5.2 provides a brief description of the features of Class Collaboration including information about what the approach aims to achieve. The questioning technique and the reporting process are then described in more detail.

The learning culture or ‘Didactic Contract’ (Brousseau, 1986, pp 51, trans. Clarke) for this classroom places the mathematical authority in the hands of the students. The questioning process and the reporting process are now explained including the contribution each makes to the learning environment.

5.2.5.1 The questioning process
The teacher’s questions are intended to model effective problem solving strategies. Table 5.3 contains examples of these questions.

Table 5.3 Examples of types of questions asked by the teacher during Class Collaboration.

<table>
<thead>
<tr>
<th>Aim of Question</th>
<th>Example of Question</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clarify the problem or an idea</td>
<td>Well, what do you know so far?</td>
</tr>
<tr>
<td></td>
<td>What do you think the problem is about?</td>
</tr>
<tr>
<td></td>
<td>Does it say you have to do that in the question or is it something you get to decide?</td>
</tr>
<tr>
<td>Refocus on a productive solution pathway</td>
<td>Can you explain that to me again?</td>
</tr>
<tr>
<td></td>
<td>How does it differ from what you were saying before?</td>
</tr>
<tr>
<td></td>
<td>Can both ideas hold true at the same time?</td>
</tr>
<tr>
<td></td>
<td>How can you work out which one is true…or if either is true?</td>
</tr>
<tr>
<td></td>
<td>Is there any way you can test your theories further…I don’t know…with some other examples maybe…or by working out why it is so….I don’t know…… [Students confirm I do not give a direction nor indicate anything I say will necessarily be productive. See student quotes in Chapter 1].</td>
</tr>
<tr>
<td>Experience a breakthrough</td>
<td>What have you looked at?</td>
</tr>
<tr>
<td></td>
<td>Can you tell me more?</td>
</tr>
<tr>
<td></td>
<td>Oh! I wonder why that happened?</td>
</tr>
<tr>
<td></td>
<td>Does it always happen?</td>
</tr>
<tr>
<td>Analyse the mathematics generated</td>
<td>Have you found any patterns?</td>
</tr>
<tr>
<td></td>
<td>Does it work every time?</td>
</tr>
<tr>
<td>Evaluate progress</td>
<td>How broadly can you hold this conclusion?</td>
</tr>
<tr>
<td></td>
<td>Do you know whether it works for all graphs?</td>
</tr>
<tr>
<td></td>
<td>Why does it happen?</td>
</tr>
</tbody>
</table>

These questions place the authority in the hands of the students rather the mathematical authority being situated with the teacher and/or the text book (Schoenfeld, 1994).
5.2.5.2 The reporting process

By requiring collaborative groups to report to the class at regular intervals it was intended to infuse ideas related to the development of concepts throughout the whole class. Table 5.4 details the features of the reporting process I developed to increase the participation of students in the learning process. By experimentation, I realised that certain constraints stopped the reporting session from degenerating into a series of student challenges and that this provided an environment in which a reporter could discuss the partially formulated ideas of the group without fear of confrontation. Table 5.4 lists the desired features and the constraints I imposed. Some of these features are described in more detail below and are also further elaborated in Williams (1996).

Table 5.4 Description of the reporting process identifying aspects related to student autonomy, student motivation and imposed constraints.

<table>
<thead>
<tr>
<th>Impact on students</th>
<th>Aspect of reporting process</th>
<th>Purpose of this aspect</th>
</tr>
</thead>
<tbody>
<tr>
<td>Participation in the learning process</td>
<td>• Order in which groups report</td>
<td>• Opportunity to build from present understanding • Opportunity for all groups to present a new idea</td>
</tr>
<tr>
<td></td>
<td>• Prime the reporter (Williams, 1996)</td>
<td>• All group members are accountable for the report</td>
</tr>
<tr>
<td>Autonomy</td>
<td>• Pace</td>
<td>• Determined by the students</td>
</tr>
<tr>
<td></td>
<td>• Nature</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Questions asked</td>
<td>• Clarify • Locate flaw in justification</td>
</tr>
<tr>
<td></td>
<td>• Use of information</td>
<td>• Accept • Reject • Use to build further</td>
</tr>
<tr>
<td>Constraints</td>
<td>• Type of questions</td>
<td>• Must lie within the ideas and content presented</td>
</tr>
<tr>
<td></td>
<td>• No contradictions during the report</td>
<td>• Leave contradictions until subsequent reports or summary at the end</td>
</tr>
</tbody>
</table>

The order of reporting was determined as a result of my visits to each group during the collaborative group work. The groups reported in an order that was intended to enable each group to report at least one new idea and to facilitate the construction of understanding by enabling each report to build on ideas already reported (Williams, 1996)

Priming the reporter was a process I developed to help create cohesive groups where all members were accountable for the reporting process. Groups ‘Primed’ their reporter by initially listening to what the reporter intended to report then working together as a group to modify this report until group consensus was reached.
The pace and nature of the reporting sessions was determined by the students in an attempt to increase student autonomy. Members of the class were encouraged to ask the reporter for further clarification. Each group evaluated the information presented and decided whether to discard ideas presented, use them, or build further in the direction suggested.

Imposed constraints resulted from my classroom observation over many years. I believe the reporter is more likely to discuss a hypothesis developed by the group if the reporter is not challenged during the articulation of this hypothesis. To create an environment in which student discourse was promoted but the reporter was not ‘challenged’ while away from the support of the collaborative group, two classroom ‘rules’ evolved. Students could: (a) only ask questions within the domain of mathematics presented by the reporter; and (b) not contradict the reporter during their report. Contradictions were postponed until the report of a later group or the summary time at the end of the reporting session. In this way, the ideas of the collaborative group as a whole were challenged rather than the reporter threatened by the perception of challenges made to themselves as an individual rather than to the group as a whole. Schoenfeld (1994, pp 63) is addressing the same issue when he comments that it is “the mathematics at stake in the conversation, not the students!”.

5.2.5.3 Group Composition

I have not included ‘group composition’ in Table 5.2 because this is an area in which I am still experimenting. My classroom experience has led me to believe groups are more effective where the Ability distribution within the group does not cover the full spectrum of Ability in the classroom and where at least one group member possesses personality characteristics that enable them to promote a supportive group environment within the collaborative group. Both these factors were considered when groups were composed for this study. Based on observations prior to this study, I believed Talei (Group 1) and Dean (Group 2) would each promote a supportive group environment.

5.3 Data Collection Techniques

Findings from the preliminary investigations in this research increased my ability to recognise and categorise complexities in general and Discovered Complexities in particular (see Chapter 3). This informed my analysis of student response to Task Complexity. My analysis was facilitated by use of the Williams/Clarke Framework of Complexity (see Chapter 2) and informed by Expert opinion regarding the complexity of Task A (see Chapter 3).

5.3.1 Areas of Analysis Undertaken

Collaborative groups were video taped as they worked to solve Task A, ‘Understanding the Double Derivative’ (Appendix 2). By observation of the video, I was able to infer the complexities discovered, the progressive conceptual development of students, and
the level of student engagement in the task. The analysis that led to these inferences is explained in the following sections. This data was supplemented and supported by student interviews (researcher/teacher), my diary record of informal discussions with students, and a test to monitor the development of mathematical understanding in students (see Appendix 5).

Table 5.5 Information sought and data sources analysed to draw inferences about Discovered Complexities, student engagement and progressive conceptual development in students.

<table>
<thead>
<tr>
<th>Area about which inferences were drawn</th>
<th>Process questions requiring answer for inferences to be drawn.</th>
<th>Data source</th>
<th>Parallel analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discovered Complexity</td>
<td>• Has the group discovered an idea they want to investigate? • Are the mathematical concepts new to all group members? • To which dimensions of complexity does this idea relate?</td>
<td>• Video data • Test of mathematical understanding • Video data • Video data • Task A</td>
<td>Student discourse: focus of the investigation Student discourse and Test of Mathematical Understanding Student discourse: analysis of student discourse informed by nature of complexity (Chapter 3)</td>
</tr>
<tr>
<td>Student development of Mathematical Concepts</td>
<td>• Has the Taken as Shared understanding of the group changed? • In what way? • What gains in conceptual development have the students undergone during the task? (Table 5.6)</td>
<td>• Test of Mathematical Understanding • Video data</td>
<td>Test of Mathematical Understanding and Student discourse: explanations indicating more than procedural understanding.</td>
</tr>
<tr>
<td>Level of student engagement in the learning process</td>
<td>• Is the level of engagement of the students high? (Table 5.7)</td>
<td>• Video data</td>
<td>Body language</td>
</tr>
</tbody>
</table>

Associations between the Discovered Complexities encountered by students, the level of student engagement in the learning process, and changes to the articulated mathematical understanding of the students were drawn by parallel analysis of the information (Clarke, 1996). Table 5.5 lists the information I sought and the data sources used.

5.3.1.1 Discovered Complexities

Video data was analysed for Group 1 and Group 2 undertaking Task A. The mathematical focus of each discussion was identified and the time devoted to this topic
recorded (see Tables 5.8, p.86 and 5.10, p.91). Some complexities were explicit and visible to all students from the outset; others were ‘discovered’ in the course of attempting the task. As students generated mathematics and mathematical ideas, previously unrecognised complexities were discovered and explored by the students. Consistent with my definition of a Discovered Complexity in Section 3.5, but elaborating this concept as a result of preliminary observation of the video data, a Discovered Complexity, as formulated for the purpose of this study, possesses two key features: (a) students focused on a search to answer a question implicitly or explicitly formulated by the group; and (b) this search encompassed mathematical ideas and concepts new to all group members.

5.3.1.2 Mathematical Understanding/Conceptual Development
For the purpose of this research, development of mathematical understanding and the development of a deeper understanding of mathematical concepts are used interchangeably. The video data and test of mathematical understanding were used to determine the progressive development of mathematical understanding as students worked within the task (Table 5.5).

<table>
<thead>
<tr>
<th>Conceptual Understanding of Individual or Group</th>
<th>Inferred from</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Individual Student</strong></td>
<td>Student:</td>
</tr>
<tr>
<td></td>
<td>• Explanation of a concept to others</td>
</tr>
<tr>
<td></td>
<td>• Indication of an understanding of a concept explained by another group member</td>
</tr>
<tr>
<td></td>
<td>• Displayed a deeper understanding of the concept in their responses to the post-test of mathematical understanding</td>
</tr>
<tr>
<td><strong>Taken as Shared Understanding of group</strong></td>
<td>Group:</td>
</tr>
<tr>
<td></td>
<td>• members accepted explanations of a mathematical idea</td>
</tr>
<tr>
<td></td>
<td>• accepted a point of view without challenge</td>
</tr>
<tr>
<td></td>
<td>• asked for further clarification of a new idea.</td>
</tr>
<tr>
<td></td>
<td>• built upon each other’s ideas.</td>
</tr>
</tbody>
</table>

Table 5.6 explains how inferences were drawn from student discourse about conceptual development attained within the group. For example, one way an individual student might have demonstrated understanding was to explain a concept to others. Where student discourse indicated a change in the ‘taken as shared’ conceptual understanding of the group, each group member had achieved at least this level of understanding (Cobb, Wood, Yackel, and McNeal, 1992).

Detailed analysis of mathematical understanding is outside the scope of this minor thesis but inferences about conceptual development were also drawn from a simple qualitative analysis of student responses to the Test of Mathematical Understanding (Appendix 5). It was concluded that the students had improved their development of a
concept where the understanding displayed in the video and post collaborative group work test of mathematical understanding was deeper than the understanding displayed on the pre-test of mathematical understanding (Skemp 1976, 1979).

5.3.1.3 Student Engagement: individual and group
To draw inferences about student engagement, I selected relevant aspects of body language by reference to body language literature (Quilliam, 1995), individual reflection, and comparison of my interpretations with the interpretations of four colleagues undertaking their PhD in Education at the University of Melbourne. Body language as an indicator of engagement was separated into indicators of individual engagement and group engagement (Quilliam, 1995). Table 5.7 summarises the features of body language used to infer student engagement.

### Table 5.7 Indicators used to infer the level of engagement in the task of individuals and groups.

<table>
<thead>
<tr>
<th>Engagement of the Individual or Group</th>
<th>Inferred from</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Individual Engagement</strong></td>
<td>Student:</td>
</tr>
<tr>
<td></td>
<td>• Leaned forward</td>
</tr>
<tr>
<td></td>
<td>• Eyes on the task material as it developed</td>
</tr>
<tr>
<td></td>
<td>• Watched the group member speak about or work with the task</td>
</tr>
<tr>
<td></td>
<td>• Disregarded the camera</td>
</tr>
<tr>
<td></td>
<td>• Listened to explanations</td>
</tr>
<tr>
<td></td>
<td>• Asked questions</td>
</tr>
<tr>
<td></td>
<td>• Explained to others</td>
</tr>
<tr>
<td></td>
<td>• Wrote information about the task (individually or for the group)</td>
</tr>
<tr>
<td><strong>Group Engagement</strong></td>
<td>Group:</td>
</tr>
<tr>
<td></td>
<td>• Brainstormed ideas together</td>
</tr>
<tr>
<td></td>
<td>• Pointed to the same task sheet in discussion</td>
</tr>
<tr>
<td></td>
<td>• Continued to discuss the mathematics when teacher called for attention</td>
</tr>
<tr>
<td></td>
<td>• Team worked to develop a concept by students ‘Latching’ to the comments of others</td>
</tr>
<tr>
<td></td>
<td>• Group members together displayed those characteristics that indicate individual engagement</td>
</tr>
</tbody>
</table>

In the context of this research study, ‘Latching’ describes the process of students connecting their comments to the comments of others by completing a sentence, or adding an extension to an idea. ‘Brainstorming’ is the process of Latching comments to build and extend group understanding. Brainstorming is perceived as different to ‘confronting’ which is characterised by students ‘cutting across’ each other’s comments with the demand for a justification in a manner that indicates ideas are individually owned rather than shared by the group.

It was possible for individual members of a group to be engaged in a task alone, some members of the group to be engaged together, or for the whole group to be engaged.

An individual was taken to display a high level of individual engagement where their behaviour included a variety of the representative features of body language listed in Table 5.7. The engagement of the group as a whole was inferred where each group...
member was observed to display body language that indicated individual engagement in the task, and all group members demonstrated a combination of body language from which group engagement could be inferred (Table 5.7). For example, group engagement was inferred if students brainstormed ideas together, continued mathematical discourse with no apparent awareness the teacher had called the class to attention, and Latched comments to the comments of others group members thus jointly developing a concept.

5.3.1.4 Tape Recorded Interviews and Diary Entries
Where data relevant to my research study arose in informal discussion with a student, this was recorded in my teacher’s diary. Several students were interviewed in the week following each research period. These interviews were used to check the validity of interpretations made and conclusions drawn from the video data. They also provided data related to affective factors in student learning of mathematics and the composition of groups.

5.4 Results
This section includes: (a) the rationale for group selection; (b) analysis of Group 1; (c) analysis of Group 2; and (d) a comparison of the response to complexity of Group 1 and Group 2.

5.4.1 Results for Group 1 and 2: Similarities/Differences
Analysis of video data for each group was undertaken with emphasis on the complexities discovered, student engagement and student conceptual development. Associations were then drawn about the interrelationships between these variables. The similarities and differences in the findings for each group informed the conclusions I drew.

5.4.2 Analysis: Group 1 (Gerard, Talei and William)
The excerpt of video data analysed for Task A includes the first 14 minutes of collaborative group work. The buzzer intended to end the group work sounded 13 minutes after the session commenced but Group 1 continued to work on the task.

This collaborative group work—Group 1’s first exposure to Task A—was followed by a feedback session with the rest of the class. This cycle was then repeated. The first fourteen minutes of collaborative group work on Task A have been studied to ascertain the number and nature of complexities discovered and the last of these fourteen minutes has been studied in detail as students were engaged and did not respond to the buzzer. The interval of time analysed was representative of the group work undertaken by Group 1 (William, Talei and Gerard) whose general interaction patterns (as evidenced by the video record) are described later in Figure 5.1 (p. 89).

My analysis included identification of the mathematical focus of group discussion and
identification of the Discovered Complexities (Table 5.8). By identification of times when students Discovered Complexities, I was able to draw associations between the occasions when students Discovered Complexities, the level of student engagement and the concepts students developed.

Table 5.8 Group 1, Task A: Mathematical Content of First Collaborative Session (Talei, William, and Gerard)

<table>
<thead>
<tr>
<th>Amount of Time</th>
<th>Mathematical Content emergent in student dialogue</th>
<th>Complexities and * Discovered Complexities</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 mins</td>
<td>How to draw the gradient graph.</td>
<td>• Conceptual.</td>
</tr>
<tr>
<td></td>
<td>What a sign diagram is and the construction in this case.</td>
<td>• Conceptual.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Conceptual: the concept of each representation.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Representational.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Intellectual (analysis).</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Intellectual (analyse to represent sign diagram from information in graphical form. Simple synthesis to interconnect the representations).</td>
</tr>
<tr>
<td>2 mins</td>
<td>Return to problem to clarify the next requirement (to analyse).</td>
<td>• Linguistic.</td>
</tr>
<tr>
<td></td>
<td>Analyse graphs and diagrams searching for patterns.</td>
<td>• Intellectual (understand the requirement, analyse).</td>
</tr>
<tr>
<td>1 min</td>
<td>Analysis of differences and relative benefits of sign diagram and graphical representation.</td>
<td>*Discovered Complexity</td>
</tr>
<tr>
<td></td>
<td>Recognition of insufficiency of information from ( f''(x) ) if required to generate ( f'(x) ) and ( f(x) ).</td>
<td>• Intellectual (Analysis, simple synthesis and evaluation).</td>
</tr>
<tr>
<td></td>
<td>Analysis (through algebra) of extra information required.</td>
<td>*Discovered Complexity</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Intellectual (Analysis, simple synthesis and evaluation).</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Conceptual.</td>
</tr>
<tr>
<td>2 mins</td>
<td>Recognition that shape is known but not position.</td>
<td>*Discovered Complexity</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Intellectual (analysis, evaluation, synthesis of different representations to comprehend a concept).</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Conceptual.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Representational.</td>
</tr>
<tr>
<td>1 min</td>
<td>Continue searching for patterns. ( f''(x) ) shows inflection but not turning points. Why?</td>
<td>*Discovered Complexity</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Intellectual (Analysis).</td>
</tr>
<tr>
<td></td>
<td></td>
<td>*Discovered Complexity</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Intellectual (analysis, evaluation).</td>
</tr>
<tr>
<td>2 mins</td>
<td>Further analysis of differences and relative benefits of sign diagram and graphical representation.</td>
<td>*Discovered Complexity</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Intellectual.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Representational.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Conceptual.</td>
</tr>
<tr>
<td>1 minute</td>
<td>Amount of information required.</td>
<td>*Discovered Complexity</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Intellectual (Analysis).</td>
</tr>
</tbody>
</table>

In addition I analysed the body language of the group and juxtaposed descriptions of body language against a transcription of the student discourse for the final minute of collaborative work (Table 5.9) when an understanding of the overall relationship between \( f''(x) \) and \( f(x) \) became clear.
Discovered Complexities existed when the focus of discussion included the quest to answer a group-instigated question (not evident to the group at the start of the task) and required the use of new mathematical ideas (see Table 5.8). It is a major premise of this study that an analysis in terms of Discovered Complexities advances our understanding of both student collaborative activities and the function of mathematical tasks in promoting this activity. Students in Group 1 discovered a variety of complexities during the fourteen minutes of group collaboration analysed. During this time, the group worked for short time intervals to develop each of many mathematical concepts new to the students.

Each of the Discovered Complexities (see Table 5.8) can be related to a question implicitly asked by the group. By analysis of the video data I have inferred the implicit student-generated question related to each Discovered Complexity. A summary of these questions includes: (a) what are the relative benefits of a sign diagram and a graphical representation? (b) what characteristics of \( f(x) \) are evident (shape) and are not evident (position) through analysis of \( f''(x) \)? (c) what do the critical features of \( f''(x) \) indicate (inflections) and not indicate (turning points) about \( f(x) \)? (d) how much information is required to generate \( f(x) \) from \( f''(x) \) and (e) why isn’t the turning point of \( f(x) \) represented by a critical feature of the graph of \( f''(x) \)? These questions were spontaneously generated by the students rather than being explicit in the task. Nakamura (1992), discussing ‘flow’ as experienced by mathematics students, believes the challenge and use of new skills promote the engagement. Although further research is required, observation of the video data indicates the students leaned forward to explore the idea once the focus question became apparent. This suggests the Discovered Complexity precede the engagement.

Each Discovered Complexity (see Table 5.8) involved Intellectual Complexities (Williams and Clarke, 1997) such as analysis, synthesis and evaluation. In each case, another Complexity—generally Conceptual and/or Representational—created the vehicle through which this Intellectual Complexity was manifested. For example, analysis of the relative benefits of the sign diagram and graph as a means to represent information included the intellectual demands of analysis and evaluation of two different representations of information. In order to undertake this analysis, the students first worked to comprehend the concepts involved in each representation.

Student Engagement could be inferred by analysis of the body language displayed by Group 1 as they undertook collaborative group work. The body language for Group 1 remained relatively consistent for the majority of the time. The description of body language in Table 5.9 (before Gerard becomes aware of the whole class again) is representative of the body language of Group 1 during the first thirteen minutes of collaborative group work. All three students leaned forward with their eyes focused on the work sheet. There were frequent occasions where two or three of the students pointed to diagrams with their pens. It was common for students to Latch their
comment to a comment of another student. ‘Latching’ is where a student connects a comment to the comment of another by completing the sentence or adding an extension to an idea. All students contributed ideas of value (to the group) during this time as could be inferred from the Latches students made to each other’s comments and the reflective expressions as students considered the comments of others. The body language became more pronounced as complexities were discovered. For example, voices became faster, students leant further over, Latching increased and more students had their pens on the page at the same time. In summary, the level of engagement for this group remained high throughout this collaborative group work session and increased even further when complexities were discovered.

*Conceptual Development* for members of Group 1 can be traced in Table 5.8 in the instances where Discovered Complexities correspond to Conceptual complexities. As some of the original concepts developed were prerequisite to an understanding of later concepts, and because all students contributed progressively to the development of ideas in the first thirteen minutes, it can be assumed all students developed a further understanding of these concepts. For example, students initially shared their understandings of the concept of a sign diagram. Subsequently students considered their first Discovered Complexity: the relative benefits of a sign diagram and a graph. As the development of concepts related to the relative benefit of these two representations relied upon the understanding of each representation, this Discovered Complexity could not have arisen without initial familiarity with the meaning of a sign diagram. The video record demonstrated all three students had understood the concept of a sign diagram as they all contributed to the discussion about the relative benefits of these two representations.

### 5.4.2.1 Student characteristics: Group 1

The Abilities (see Table 4.3) and personalities of Gerard, William and Talei appeared to contribute to the interaction patterns that facilitated and maintained group cohesion. Attributes of each group member appeared to be integral to the creation of this cohesion (see Figure 5.1).

It appeared Gerard was unable to combine concepts to create new concepts at the same pace as Talei and William, so once the formulation of a new concept had been completed by William and Talei, Gerard questioned the other two students. Gerard then appeared to process their explanations quickly. Talei provided most of the explanations for Gerard; she explained quickly and succinctly and was also able to immediately begin to consider any new statement or question William proposed as a precursor to the discovery of the next complexity.
Chapter 5: Student response to complexity

Figure 5.1 Usual interaction pattern in Group 1 during collaborative group work.

Key: ………………… members of the group working towards the same goal but sub-groups exist

START

William considered the task as he simultaneously listened and contributed to the explanations Talei provided for Gerard. Talei and William’s ability to quickly assess new ideas and decide upon the next complexity contributed to the rapid development of concepts within this group. The cycle (Figure 5.1) would then begin again with all three students contributing to the initial investigation of the next complexity.

5.4.3 Analysis: Group 2 (Rez, Alistair, Dean and Tony)

The first 13 minutes of collaborative group work was also analysed for Group 2 and student responses to complexity were again determined by identifying Discovered Complexities (Table 5.10), student engagement (Table 5.11) and student conceptual development (Tables 5.10 and 5.11). The response to complexity in Group 1 and Group 2 was very different. In an attempt to explain these differences, I explored a difference in the instructional sequence for the two groups. Whereas Group 1 had begun the task as a collaborative group, Group 2 had spent time working on the task individually before the collaborative session. My analysis of Group 2 included: (a) observations as students undertook individual work; and (b) analysis of video data of collaborative work. I used my diary record and personal reflections to identify relevant student responses to Task A during the individual work session prior to Class Collaboration for Group 2.
5.4.3.1 Individual Instruction: Group 2

Individual Instruction for Group 2 was undertaken prior to Class Collaboration for Task A. As the classroom teacher, I moved from student to student asking each student about what they were thinking and asking each student for explanations about any graphs, diagrams or mathematics they had generated. The questions were similar to the questions asked during collaborative group work.

I found students undertaking individual work appeared to develop and retain more misconceptions than students working in collaborative groups. This was evident from student responses to my questions and student explanations of their diagrams. For example, student responses on the tests of mathematical understanding and evidence from the video data in the subsequent collaborative session confirmed the presence of these misconceptions.

5.4.3.2 Collaborative group work: Group 2

Collaborative group work was undertaken after individual instruction. The mathematical discourse during the 13 minutes of collaborative group work is described in four time intervals (see Table 5.10). The group began by sharing ideas generated during individual work. Rez presented his findings and other group members challenged many ideas. Alistair then used Rez’s sheet to justify his disagreement with Rez’s point of view. There was then one brief mention of differing gradients at different points before the group discussed hybrid functions and rejected them as not the most appropriate way to explore the task. The final two minutes were devoted to discussion of $f''(x)$ and analysis of whether an increasing negative gradient could still be negative. Each of these time intervals is now discussed in more detail to illuminate the connections between mathematical discourse, Discovered Complexities and body language.

*During the first four minutes*, Rez began by sharing his findings while the other group members looked at the sheet and listened. They did not lean forward and the hands of all group members were never on the page at the same time. Students did not Latch—build upon each other’s ideas. The discussion was not at a rapid pace. Alistair remained a greater distance from Rez’s page than the other group members did. Alistair occasionally looked at his own sheet during this time and listened to the discussion. Students challenged some of Rez’s findings. Analysis of student responses to tests of mathematical understanding and observation of the video record, indicated ideas that arose in the first four minutes were familiar to all group members (and therefore did not include any Discovered Complexities). Group 2’s body language indicated students were generally focused on the task but the level of engagement was not high.
Table 5.10 Group 2, Task A: Mathematical Content of First Collaborative Session (Rez, Alistair, Dean and Tony)

<table>
<thead>
<tr>
<th>Amount of Time</th>
<th>Mathematical Content emergent in student discourse</th>
<th>Complexities and *Discovered Complexities</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 minutes</td>
<td>Derivatives considered as algebraic manipulation. Investigation and generalisation of the derivative of a derivatives considered algebraically</td>
<td>Numerical (Comprehension and Analysis)</td>
</tr>
<tr>
<td>5 minutes</td>
<td>Comparison of ( f(x) ) and ( f'(x) ) through interpretation of graphs with focus on gradient. Explanation of the change in gradient, disagreement with Rez’s ( f'(x) ) graph</td>
<td>Representational (Comprehension and Analysis)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 minutes</td>
<td>Exploration of the possibility of a hybrid graph. A comment about the relative magnitude of the gradient at two points.</td>
<td>Representational</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Intellectual (analysis, synthesis)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Intellectual (Analysis) Conceptual, Representational (a precursor to the Discovered Complexity)</td>
</tr>
<tr>
<td>2 minutes</td>
<td>Articulation of need to explore links between ( f(x) ) and ( f''(x) ).</td>
<td>Intellectual (Comprehension of task requirement)</td>
</tr>
<tr>
<td></td>
<td>Recognition of the gradient of the gradient.</td>
<td>Linguistic</td>
</tr>
<tr>
<td></td>
<td>Discussion that a negative gradient can be increasing.</td>
<td>Intellectual (Analysis)</td>
</tr>
<tr>
<td></td>
<td>*Discovered Complexity arising from comment at the start of the 10th minute.</td>
<td>Intellectual (Synthesis) Conceptual</td>
</tr>
<tr>
<td></td>
<td>Articulation by Tony: “double derivative gives the curve of the graph”.</td>
<td></td>
</tr>
</tbody>
</table>

During the fifth, sixth, seventh, eighth and ninth minute, Alistair introduced the concept of change in gradient and justified his disagreement with Rez’s \( f'(x) \) graph. Video evidence indicated these ideas were not new to Alistair and had been used to generate his \( f'(x) \) graph during individual work. As Alistair justified his disagreement with the \( f'(x) \) graph Rez had generated during the session when students had worked individually, the body language of the group members changed (see Table 5.11). The students leaned forward, frequently all students pointed to the page at once, and the pace of the discourse increased. Unlike Group 1, these students did not Latch—build upon each other’s comments—but generally cut across (interrupted) each other’s comments as they disputed the ideas developed and asked or demanded the
clarification of ideas. Although the level of engagement increased during this time interval, it was not as high as the engagement displayed by Group 1 as they Discovered Complexities. Two examples of the lower level of engagement of Group 2 are evident in Table 5.11. Both examples indicated students had not ‘lost all sense of self’ which is an indication of group flow (Csikszentmihalyi & Csikszentmihalyi, 1992 pp.86). For Group 2, student discourse was characterised by ‘cutting’ rather than ‘Latching’ and when Alistair discussed Rez’s sheet, Rez responded as though this had been a personal attack on his work.

During the tenth and eleventh minutes. At the beginning of the 10th minute, Rez commented on the relative gradient of the graph at two points and all students leaned forward and looked at the sheet indicating the level of engagement had increased. The group then discussed hybrid functions for about a minute and a half and looked at whether this would help them solve the problem. Students had previously been exposed to hybrid functions so no new ideas were discussed and no additional complexities were discovered. The idea of a ‘negative gradient increasing’ did not become the focus of attention until the last two minutes.

During the last two minutes there was a decrease in the frequency of students’ ‘cutting across each other’s comments but still ‘Latching’ did not occur. The relationship between the rate at which the gradient changes and the $f''(x)$ graph became the mathematical focus. This mathematical content appeared to be outside the present understanding of Tony, Rez and Dean when Alistair introduced it in the fifth minute. The video data (just before the tenth minute) showed Tony pointing to these ideas on the sheet Alistair generated during the individual work time thus indicating Alistair was aware of the initial concept of ‘gradient of gradient’ before this collaborative session began. The concept of whether ‘an increasing negative gradient can still be a negative amount’ became the focus of student attention next and considering ‘change in gradient’ from this perspective appeared to be unfamiliar to all group members as the discussion began. It appears Alistair, like the members of Group 1, had generated graphs where a negative gradient increased but had not considered that these graphs represented a negative gradient increasing. The members of Group 1 demonstrated it was possible to reach a solution to Task A without specifically considering the concept of a ‘negative gradient increasing’ (Table 5.8).

In the first half of the 13 minutes, students frequently cut across each other’s comments but this did not occur as frequently in the last few minutes. Latching did not occur between members of Group 2 during this collaborative session.
**Table 5.11 Group 2, Task A: Excerpt of Collaborative Group (Dean, Rez, Tony, and Alistair).**

<table>
<thead>
<tr>
<th>Student</th>
<th>Student utterances</th>
<th>Time</th>
<th>Body Language</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alistair</td>
<td>(Alistair is discussing the graph of f’(x) Rez generated during individual time). When you look at this point here (on f(x)), you see that the gradient is zero, but then it suddenly it increases.</td>
<td>5, 6th, 7th, and 8th and 9th minute</td>
<td>Alistair points to the positive gradient of f(x) to the right of the stationary point of inflection. Other students watch, lean over and listen and point to the page.</td>
</tr>
<tr>
<td>Rez</td>
<td>Well not sudden</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Alistair</td>
<td>Oh, well it increases all the time, it doesn’t actually decrease which is um/ (Alistair points from the f(x) graph to the f’(x) graph)</td>
<td></td>
<td>All are now leaning in together across the page much more than at any time in the last five minutes.</td>
</tr>
<tr>
<td>Rez</td>
<td>/ It decreases as you get towards that point</td>
<td></td>
<td>Dean, Rez and Alistair are leaning over the page pointing and talking. Tony is also leaning in, he is silently watching.</td>
</tr>
<tr>
<td>Dean</td>
<td>How did you get the um the sort of stationary part there? (points to the f’(x) graph of Rez’s)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rez</td>
<td>Because at this part here, it is basically not quite/</td>
<td></td>
<td>Students are pointing to the set of graphs Rez has generated.</td>
</tr>
<tr>
<td>Alistair</td>
<td>When you look at this section here, you see that the gradient is decreasing (f’(x)) but the gradient is increasing (f(x))/</td>
<td></td>
<td>Alistair points from f’(x) to f(x). Other students lean forward and watch.</td>
</tr>
<tr>
<td>Rez</td>
<td>/Where is it decreasing?</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Alistair</td>
<td>Take the derivative of this section and you see this curve implies that the gradient is decreasing/</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rez</td>
<td>/It is decreasing/</td>
<td></td>
<td>Sharp voice</td>
</tr>
<tr>
<td>Alistair</td>
<td>I am talking about the change in gradient, um/</td>
<td></td>
<td>Quiet voice</td>
</tr>
<tr>
<td>Rez</td>
<td>/What do you mean? Look,/</td>
<td></td>
<td>Sharp voice</td>
</tr>
<tr>
<td>Alistair</td>
<td>/the gradient is decreasing because it is more at that value than it is at that</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rez</td>
<td>if the gradient was decreasing, the graph would go like that, that is decreasing gradient, if it was increasing gradient, it would be coming up.</td>
<td></td>
<td>Quiet voice, clarity through</td>
</tr>
<tr>
<td>Alistair</td>
<td>I don’t understand what you are saying...... Oh, don’t worry</td>
<td></td>
<td>Quiet voice, not as clear at the end of the sentence</td>
</tr>
</tbody>
</table>

Key:  (*italics*)

/student cuts across the comment of another (interrupts)

[ ] [student Latches to the comment of another.]

Table 5.11 continued over the page
<table>
<thead>
<tr>
<th><strong>Student</strong></th>
<th><strong>Student utterances</strong></th>
<th><strong>Time</strong></th>
<th><strong>Body Language</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Rez</td>
<td>No...can I have a look at your graph? perhaps we could.......</td>
<td>5, 6th, 7th, 8th and 9th minute (cont)</td>
<td>Loud voice</td>
</tr>
<tr>
<td>Dean</td>
<td>Rez, you are coming on a bit too strong there/ /no why/</td>
<td></td>
<td>Abrupt</td>
</tr>
<tr>
<td>Rez</td>
<td>/Calm down/</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tony</td>
<td>The gradient keeps on increasing <em>(points to Alistair’s sheet, unclear which graph he points to)</em> until about that point there when it is about maximum then it comes back down again..... OK. First of all we are saying that the maximum is about there right?</td>
<td>10th and 11th min</td>
<td>Tony takes Alistair’s sheet and quietly starts to talk pointing to the sheet as he does so.</td>
</tr>
<tr>
<td>Rez</td>
<td>Maximum gradient yes</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tony</td>
<td>Yes, right, now looking at that./</td>
<td>12th and 13th min</td>
<td>All students are now leaning in pointing</td>
</tr>
<tr>
<td>Rez</td>
<td>/The gradient at that point is less than the gradient at that point isn’t it? <em>(discussion of whether a hybrid function could be useful)</em></td>
<td></td>
<td>Student are looking at the sheet but not leaning in and pointing.</td>
</tr>
<tr>
<td>Tony</td>
<td>We haven’t actually discussed any links between f(x) and f”(x)</td>
<td></td>
<td>Students are still all leaning in and pointing to the sheet.</td>
</tr>
<tr>
<td>Dean</td>
<td>Here it is decreasing? You are saying here it is decreasing?</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rez</td>
<td>Here you are looking at the gradient of that. You are looking at the rate of gradient at that point versus the rate of gradient at that point and it is increasing so in other words, say this is the gradient of -3, this is the gradient of -2. <em>(The buzzer goes to indicate collaborative group time is almost finished)</em></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dean</td>
<td>Who is speaking?</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Teacher</td>
<td>You have about a minute</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tony</td>
<td>Excuse me, what are we going to report? <em>(Some discussion of who will report. And what the report will entail. Buzzer to indicate report time)</em></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tony</td>
<td>By the way, the double derivative gives you the curve of f’(x). Its the curve, I mean.....that is what you are saying.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rez</td>
<td>No/</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tony</td>
<td>/the curve/</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rez</td>
<td>It shows you the rate of gradient</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tony</td>
<td>Yes, it is still the same sort of thing.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Tony’s final comments about ‘the curve’ appeared to be a concept he had developed and clarified through the collaborative discussions. The group was not receptive to this concept so no Discovered Complexity emerged.

**Conceptual development** was minimal for members of Group 2. The number of concepts developed by Group 2 throughout the whole 13 minutes was far less than the number of concepts developed by Group 1 (see Table 5.8 and 5.10). Two contributing factors to the slower conceptual development for Group 2 appeared to be: (a) the lack of responsiveness of the group to some suggestions from Alistair; and (b) the tendency to hold strongly to misconceptions developed individually.

The interaction patterns in Group 1 differed from the interaction patterns in Group 2. Where Group 1 Latched to each other’s comments to generate new ideas, Group 2 cut across each other’s comments for the first two thirds of the session. In the last third of the session, the students in Group 2 no longer cut across each other’s comments but ‘Latching’ still did not occur. Where Group 1 remained focused on the task, there was an interval in the middle of the fifth to ninth minute time interval when Group 2 focused on the behaviour of the Group members rather than the task.

### 5.4.4 Discovered Complexity, engagement, concepts developed

Where a Discovered Complexity became evident, student exploration of this complexity resulted in increased student engagement and the development of conceptual understanding (Tables 5.8, 5.9, 5.10, and 5.11). It appeared engagement resulted from the challenge associated with the Intellectual Complexity of the question upon which students decided to focus.

**Discovered Complexities create engagement** as demonstrated by Group 1 who discovered more complexities and were engaged for the majority of the fourteen minutes. Group 2 experienced a high level of engagement for a little over two of their thirteen minutes. Examples are now provided to illustrate a time when each group experienced a high level of engagement.

**Group 1**, William, Gerard and Talei, in the fourteen minutes of the collaborative session (see Table 5.9) worked with the graphs of \( f(x) \), \( f'(x) \) and \( f''(x) \) in Task A trying to make sense of their findings regarding the apparent importance of inflections and lack of importance of turning points from the \( f(x) \) graph (when considering the \( f''(x) \) graph). Table 5.9 illustrates Talei and William displaying a high level of engagement as they investigated this Discovered Complexity precipitated by William’s statement that there ‘must be something more’. William, Talei (and initially Gerard) did not respond to the buzzer but instead continued to lean forward over the page and Latch comments. The three students did not appear to hear the teacher providing instructions about the specific requirements for the reporting process. William and Talei did not appear to hear Gerard who had turned to listen to the teacher’s instructions and then turned to ask Talei and William a question about his forthcoming
report for the group (Table 5.9). After further consideration of the patterns they had found, William exclaimed—with the accompanying hand movement tracing a minimum point in the air—‘It is always turning the same way’. The video recording (Table 5.9) captured the instant where William suddenly appeared to grasp the overall concept. His utterance “That’s it!” and the accompanying hand movement together with the apparent expression of enlightenment on his face indicated something had suddenly become clear. Talei’s hand movement at almost the same instant indicated she had also grasped the concept.

Group 2 discovered only one complexity. This was discussed for less than four of the thirteen minutes of collaborative group work. Whereas Group 1 brainstormed together and built a new understanding, Group 2 listened and questioned as Alistair—with the occasional assistance of Tony—explained a new idea. Each member of the group was focused on the same goal: to develop—in the group—a higher level of conceptual understanding of the rate of change of gradient and it’s relationship to \( f(x), f'(x) \) and \( f''(x) \).

<table>
<thead>
<tr>
<th>Table 5.12 Differences in functioning of Group 1 and Group 2.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Characteristic</td>
</tr>
<tr>
<td>Engagement in the task</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Interconnection between student discourse</td>
</tr>
<tr>
<td>Number of Discovered Complexities</td>
</tr>
<tr>
<td>The relationship between the content of the mathematical discourse and the present conceptual development of the students.</td>
</tr>
<tr>
<td>The primary mode of group operation</td>
</tr>
<tr>
<td>Number of areas of observable conceptual development of the group</td>
</tr>
</tbody>
</table>

The body language of all group members indicated a higher level of engagement (when they encountered the Discovered Complexity) than previously but not as high a level of
engagement as for Group 1. Group 2 did not brainstorm and Latch comments. However students leaned forward, looked at the sheet, looked at the others and spoke with a greater intensity than in the first twelve minutes of group work.

*Differences between the groups* were evident in the number of complexities they discovered and the conceptual development that resulted. Group 1 discovered more complexities, maintained a higher level of engagement in the task, and developed an understanding of a greater number of concepts during the time interval. This group continually worked just above their present mathematical understanding and brainstormed to create new mathematical ideas together. This differed from Group 2 who mainly worked at a level below the present mathematical understanding of one or more group members and tended to challenge and demand further explanations rather than create new ideas and extend the concept development of all group members. The majority of the collaborative session, for Group 2, focused on peer tutoring and challenges to the ideas of others. Table 5.12 summarises the differences between the two groups. Group 1 displayed the higher level of engagement (indicated by Latching and brainstorming), discovered more complexities and worked for the majority of the time on ideas just beyond their present level of understanding to develop a greater gain in mathematical understanding than Group 2.

### 5.5 Discussion: Student Response to Complexity

Student response to complexity in Task A contributed to my understanding of task features that create an effective task. Higher levels of engagement occurred for both groups of students at the times when they Discovered Complexities. In Group 1, successive discovery of complexities during task completion led to maintenance of a high level of student engagement in the task (Table 5.8, 5.9). Based on the work of M. and I. Csikszentmihalyi (1992), I contend the Discovered Complexity creates the challenge that is a pre-requisite condition for Flow so the complexity is discovered first and the engagement follows (see Figure 5.2).

**Figure 5.2 Prerequisite factors for the enhanced learning environment experienced by Group 1 and not Group 2.**

The provision of tasks that provide the opportunity for students to Discover Complexities is not sufficient to ensure complexities will be discovered. It appears other factors like student Difficulty and Mediating Factors can alter the likelihood that complexities are discovered. In summary, where students with common mathematical
background work in appropriately composed collaborative groups to solve unfamiliar challenging problems within a learning environment that encourages student autonomy (and relies upon students justifying their mathematical ideas), complexities are more likely to be discovered.

5.5.1 Discovery of Complexities

Task A provided opportunities for collaborative groups to discover a variety of complexities that varied in nature and size of the associated conceptual leap (see Table 5.8 and 5.10). Each Discovered Complexity involved an intellectual complexity and each time a complexity was discovered the level of student engagement in the task increased. Where more complexities were discovered (Group 1) gains in conceptual development were greater.

The discovery of complexities is idiosyncratic to the particular group and students do not necessarily discover the same complexities as they solve a problem. For example, Group 1 solved the problem without focusing on the Discovered Complexity Group 2 focused upon—‘whether a negative gradient can be increasing’.

5.5.2 Common Background

Each class undertook individual instruction and Class Collaboration in a different sequence. Class 1 undertook Class Collaboration first and Group 2 undertook individual work first. This sequencing decision was made independently to this research study. Prior to this study students generally undertook Class Collaboration before extended individual work but when the need for change occurred, I had not expected the sequencing of instruction to affect student response to complexity.

The evidence provided in this research study supports the use of Class Collaboration prior to individual instruction where students work to solve unfamiliar challenging problems. Where students begin unfamiliar challenging problems individually (Group 2), more misconceptions appear to develop and there appears to be an inertia when attempts are made to dislodge these misconceptions (Table 5.11). Student misconceptions appear to contribute to the reduced discovery of complexities for Group 2 compared to Group 1 (Table 5.8 and 5.10). The progressive evaluation, by the group, of new ideas as they are suggested (Group 1, Table 5.8) appeared to decrease the number of misconceptions developed.

5.5.3 Key Factors in Class Collaboration

This study demonstrates the potential for learning where a pedagogical approach such as Class Collaboration is utilised but the study also highlights the variety of variables that require consideration if use of such an approach is to be effective. Where student autonomy has been developed other researchers have also found learning gains. With further exploration of the relationships between group composition and group dynamics our understanding of student learning could be increased.
5.5.3.1 Student Autonomy
Tang (1993) identified factors that increased student autonomy in his spontaneously formed groups. The majority of these factors identified by Tang are present in Class Collaboration and include: (a) group selection of the focus for their exploration; (b) any justifiable pathway was accepted; (c) a reporting process existed that was student directed and enabled students to question and pursue the ideas of interest to individual students and collaborative groups of students.

In Group 1, student autonomy was evident as the group progressively selected a series of foci that challenged and engaged them.

5.5.3.2 Group composition
Group 1 worked together to generate new ideas and Group 2 operated primarily within a peer tutoring model, it appeared that greater conceptual development resulted when students spent their collaborative session generating new ideas.

Group 2 spent the majority of their time working below Alistair’s Zone of Proximal Development. Alistair spent much of the time involved in peer tutoring to raise the Zone of Proximal Development of the other students. There was only one time when Alistair appeared to work within his Zone of Proximal Development and developed a new concept (Table 5.10, Table 5.11; in the last two minutes of group work). The lack of overlap between the zones of proximal development of the students seems to account at least partially for the group’s lack of responsiveness to Discovered Complexities that could have resulted from Alistair’s perceptive comments early in the collaborative session. Alistair’s quiet manner and his decision not to continue immediately with an idea when the rest of the group were not receptive may have slowed the pace at which complexities could have been discovered. Although, if students did not have the required mathematical background, Alistair may not have been able to alter the pace. These ideas require further investigation in another study. Alistair’s persistence in presenting the idea again helped the group to discover at least one complexity. Little opportunity was available for Alistair to work within his Zone of Proximal Development in this session. Possibly, if the group had collaborated for longer, more opportunities would have arisen as it was apparent Alistair had begun to raise the Zone of Proximal Development for other students in Group 2 (see Table 5.11).

Evidence now provided (from other sources within this study and external to this study) suggests that there are factors associated with group composition that affect the learning opportunities that arise within a group. Evidence from several sources is provided below:

1) Lida (see Chapter 4) when grouped with William and Talei early in 1996 commented “I know you don’t usually do this but I really need to change groups.
William and Talei are really helpful and explain what they have found but I find as I struggle to think about and take in one idea, they have already jumped two ideas further ahead”. This supports my interpretation that part of the cohesiveness of Group 1 resulted from the pace at which Gerard could process new ideas if they were explained to him.

2) Dean was part of a study the previous year (Barnes and Williams, 1997). In this Year 11 class of mine, Dean (with pseudonym Simon in the Year 11 study) was a valuable contributor to the discovery of many complexities in the group. Some of the complexities discovered can be ascertained by study of Barnes’ recent paper (2000). This demonstrates that Dean’s lack of contribution in my present study was a function of something other than the personality of the student.

3) Rez and Bazza in a follow up interview late in my present study commented that it depended on whether you ‘got a good group or not’. When asked what a good group was, they explained it was a group where everyone contributed.

4) A conscientious 1994 Mathematical Methods student with higher than average ability commented on his final feedback sheet that he preferred to be grouped with students who thought at the same speed as he did. He said he found he could not work as well with students who thought a great deal faster than he did.

5) Gerard, in October 1996 (after undertaking Task B with Talei, William and Damien) commented in an informal interview that he had found it intimidating to work with these students on this task because they thought so fast. Gerard was a lot quieter in the video record of Task B (not studied in detail for this present study) than in Task A. It appeared that a different task and an additional student in the group altered the learning environment.

These examples suggest there are factors other than the task that affect the learning environment but the process by which this occurs is an area for further study. These examples suggest further exploration of group composition could be productive.

5.6 Research subjects: generalisation

Even though the two collaborative groups are from the same school, Year level and subject, there are differences that should be considered when comparisons are made. The groups belong to different Specialist Mathematics classes (taught by the same teacher) and the gender composition of these classes differs as do the sizes of the collaborative groups selected for case study. Although the small number and specificity of these groups would suggest the results should not be generalised without further investigation, the findings of Brown (1994) with grade 2 science students and Tang (1993) with first year tertiary physiotherapy students support the finding that student autonomy appears to be associated with student engagement and the resultant learning gains.
5.6.1.1 Support for my interpretations (triangulation)

To guard against possible areas of bias that may have arisen with the researcher as the classroom teacher, I continually reflected on the sources of evidence I used and how my own opinions and observations could be cross-checked. When you know the type of response that would help demonstrate the research results, it is a challenge to keep an unbiased perspective. For this reason, the triangulation of results (Clarke, 1996) becomes even more important in this instance. Wherever possible I supported my perspective with data from other sources, including tests of mathematical understanding, interviews with students, interpretations from colleagues and reference to my diary notes (written before the implications of the responses from a research perspective were evident).
### Table 5.9

<table>
<thead>
<tr>
<th>Speaker</th>
<th>Utterances (mathematical discourse)</th>
<th>Body language</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>William</strong></td>
<td>The shape</td>
<td>All students look at the page, lean forward and have their pen pointing to the graph.</td>
</tr>
<tr>
<td><strong>Talei</strong></td>
<td>Uh hu</td>
<td></td>
</tr>
<tr>
<td><strong>William</strong></td>
<td>To there</td>
<td>William moves his pen along the graph. All three students are looking at the page.</td>
</tr>
<tr>
<td></td>
<td><em>(buzzer, teacher starts talking)</em></td>
<td>All three in the group appear to be unaware of buzzer. The teacher talks to the class throughout the rest of Group 1’s conversation. None of the group appear to be aware of this at this stage.</td>
</tr>
<tr>
<td><strong>William</strong></td>
<td>Well if you look between there and there it is curving</td>
<td>William points to the graph</td>
</tr>
<tr>
<td><strong>Talei</strong></td>
<td>yes</td>
<td>Talei also points to the graph</td>
</tr>
<tr>
<td><strong>William</strong></td>
<td>the curve is getting less and less</td>
<td>Gerard leans over and watches</td>
</tr>
<tr>
<td><strong>Talei</strong></td>
<td>yes, it’s, the gradient yes it’s</td>
<td></td>
</tr>
<tr>
<td><strong>William</strong></td>
<td>till there which means it’s negative,.....it’s steep</td>
<td></td>
</tr>
<tr>
<td><strong>Talei</strong></td>
<td>yes</td>
<td></td>
</tr>
<tr>
<td><strong>William</strong></td>
<td>till there when it starts to get more</td>
<td></td>
</tr>
<tr>
<td><strong>Talei</strong></td>
<td>so when the curvature is</td>
<td></td>
</tr>
<tr>
<td><strong>William</strong></td>
<td>and more</td>
<td></td>
</tr>
<tr>
<td><strong>Talei</strong></td>
<td>smaller but that’s</td>
<td></td>
</tr>
</tbody>
</table>

Group 1 (Talei, William and Gerard)

Last of the fourteen minutes of group collaboration.

Mathematical discourse and body language juxtaposed.
### Table 5.9 cont

<table>
<thead>
<tr>
<th>Speaker</th>
<th>Utterances (mathematical discourse)</th>
<th>Body language</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>William</strong></td>
<td>but what about between there and there, what happens? Why is that?/</td>
<td>All three have been looking intensely at the page and William and Talei have had their pens on the page moving them around the different parts of the curve as they think and talk. Gerard now becomes aware of what is happening in the class around him and realises they—probably he—will soon be required to report. William and Talei generally prime him for this report.</td>
</tr>
<tr>
<td><strong>Gerard</strong></td>
<td>What are we going to talk about? Do you want/</td>
<td>The other two do not hear him, they continue to think and move their pens on the graph and continue their conversation</td>
</tr>
<tr>
<td><strong>William</strong></td>
<td>Why is that....... a positive?/</td>
<td></td>
</tr>
<tr>
<td><strong>Gerard</strong></td>
<td>to talk?/</td>
<td>Gerard is looking around at the preparation for reporting in the rest of the class. His body is tense.</td>
</tr>
<tr>
<td><strong>Talei</strong></td>
<td>It’s sort of.......that its.......that’s true..../</td>
<td></td>
</tr>
<tr>
<td><strong>Gerard</strong></td>
<td>Do you want to talk?/</td>
<td>No one hears</td>
</tr>
<tr>
<td><strong>William</strong></td>
<td>It turns there but it doesn’t turn and go the other way.....it always turns the same way. THAT’S IT!</td>
<td>William and Talei are pointing backwards and forwards between the three graphs and appear to be oblivious to anything which is happening around them. Gerard looks around at the activity in the rest of the class and his body is still tense. William snaps his fingers, a smile of enlightenment comes across his face, he becomes less intense, sits back a bit in his seat and moves his hand in the shapes of a curve with a minimum turning point. This action appears to be part of William’s thought process, he watches his hand move. Gerard looks back at the group with apparent interest for an instant but the imminence of the report appears to have taken precedence.</td>
</tr>
<tr>
<td><strong>Talei</strong></td>
<td>Yes</td>
<td>Talei makes a small hand movement of her own, apparently as part of her thought process.</td>
</tr>
<tr>
<td><strong>William</strong></td>
<td>It’s positively turning that way.</td>
<td>Gerard is still looking around the room, his body tense.</td>
</tr>
</tbody>
</table>
CHAPTER 6

DISCUSSION AND CONCLUSIONS


6.1 Introduction

Within the discussion, and conclusions I have included my new understanding of the nature of task complexity and the interconnections I have found between task complexity, student ability to solve unfamiliar challenging problems, and student response to complexity. These interconnections informed the theoretical model, the Engaged to Learn model, I propose to explain my findings. The results of my analysis help to explain the engagement or lack of engagement of students as they work to solve unfamiliar challenging problems. The limitations of the study and the potential for generalisation are discussed and the implications of its findings are identified. This chapter concludes with a summary of the questions arising from this study.

6.2 Task Complexity: extending my initial understanding

I initially understood task complexity to depend upon the number of components within a task and the degree of interrelationship between these components (Section 1.2.2). One way of analysing task complexity that seems to provide useful insights into student engagement with unfamiliar challenging problems is to consider the constituent ‘components’ of complexity as the dimensions of complexity in the refined Framework of Complexity (Table 3.12, p.55) developed by Williams and Clarke (1997). Each of these dimensions has its own structure (Table 3.2, 3.3, 3.4, 3.5, p.40-43; Table 3.9, p.49). On the basis of this study it appears that tasks structured to provide students with the opportunity to discover complexities are also those with a high degree of interrelationship between the dimensions of complexity (Table 5.8, p.86; Table 5.10, p.91). The following section focuses on the features of task structure that enable the identification of the complexities of a task and, depending on the teacher’s purpose, the modification of that task.

6.2.1.1 The nature of task complexity

The complexity of a mathematics task can be determined and/or altered by considering the degree of complexity of each relevant ‘component’ of the task structure. When considering the complexity of a task, the contributing factors are: (a) the degree of complexity within each dimension of complexity; (b) the number of dimensions that exhibit a high degree of complexity; and (c) the ‘degree of constructed interrelationship’ between the dimensions of complexity.

Where a task is structured to provide the opportunity for students to discover complexities, potential exists for a high ‘degree of constructed interrelationship’ between the dimensions of complexity. The consequence of such a high degree of interrelationship was evident as students responded to Discovered Complexities (each containing an intellectual component) within the task they explored and as a result developed an increased conceptual understanding (Conceptual Complexity). This
exploration required students to consider other dimensions of complexity at the same time. For example: students analysed the differences between various representations to determine the usefulness of each representation (See Table 5.8, Representational Complexity, Conceptual Complexity and Intellectual Complexity). It is a major proposition to emerge from this research that the process of recognising and exploring ‘Discovered Complexities’ in a task is a fundamentally constructive and connective activity that inevitably promotes student awareness of the interrelationship of task components and maximises a form of conceptual connectedness.

The constituent characteristics within the Ability construct (Table 2.2) developed and used in this study to determine student ability to solve unfamiliar challenging problems, were the characteristics students displayed when they explored Discovered Complexities. This demonstrates the usefulness of the concept of a Discovered Complexity as a tool to analyse student responses to unfamiliar challenging problems.

### 6.3 Student Response to Complexity

The two case studies undertaken highlighted the different responses of two collaborative groups to the same task. Group 1 discovered many more complexities than Group 2 and more conceptual development was evident for students in Group 1 than students in Group 2 (see Section 5.4.4, p.97, Differences between the groups). Similarities and differences between the groups are now summarised and reference is made to these similarities and differences as the Engaged to Learn model is developed to explain these findings.

Both groups consisted of students who demonstrated a high level of ability to solve unfamiliar challenging problems; all members of each group demonstrated an ability to solve challenging problems employing cognitive activities of a higher level than analysis (see Table 4.5, p.66). All except one student in each group demonstrated ‘the ability to synthesise’ as the highest cognitive activity used to solving unfamiliar challenging problems (see Table 4.3 and 4.4, p.64ab). William (Group 1) also demonstrated the higher level ability to employ analytic-synthesis and Rez (Group 2) who did not demonstrate the ‘ability to synthesise’ did demonstrate the ability to use ‘analytic-synthesis’ (the next level of cognitive activity in the descending hierarchy). These findings indicate both groups contained students with the capacity to ‘combine concepts to form new concepts’ yet this activity occurred more frequently in Group 1.

Students in Group 1 and Group 2 differed in the background knowledge they brought to the task. Group 2 worked individually on the task before the group work session and had developed misconceptions as a result (Section 5.4.3.1, p.90, Individual instruction). Group 1 began the task in their collaborative group and appear to have shared a common background as a result of the Gradient of a Function Investigation.
This research facilitated my own conceptual leap when I realised that tasks providing the opportunity for Discovered Complexities contained a dynamic aspect that could explain the sustained engagement of members of a collaborative group. I now discuss the interconnections between Task Complexity, student engagement, student development of new concepts, student Ability, student difficulty and mediating factors as found in this research then propose the Engaged to Learn Model (Figure 6.1, p.107) to explain these interconnections.

6.3.1 Identification of factors contributing to sustained engagement

The differences in learning outcomes experienced by Group 1 and Group 2 demonstrate the effect Discovered Complexities might have on the quality of student interactions. The successive discovery of complexities by Group 1 (Table 5.8) was accompanied by sustained engagement and conceptual development for each group member. The majority of the student discourse in Group 1 focused on exploratory talk with small intervals of cumulative talk (Mercer, 1995; Barnes, 1999) and/or peer tutoring (see Figure 5.1, p.89). The members of Group 2, who discovered only one complexity, demonstrated little conceptual development and a lower degree of engagement in the task than Group 1 (Table 5.11, p. 93-94; Section 5.4.4, p. 95-96). Student discourse between the members of Group 2 was predominantly peer tutoring. As both groups of students were undertaking the same task, these findings demonstrate that although the task provided opportunities for Discovered Complexities and that the discovery of these complexities led to increased engagement and conceptual development for the students, the task contributed to, but was not sufficient to ensure, the high quality of the learning environment for Group 1.

Differences in the areas of focus investigated by each group demonstrated each group’s idiosyncratic exploration of the problem (Table 5.8, p.86; Table 5.10, p.91). The use of a task that provides the opportunity for students to discover complexities in a learning environment like class collaboration (Table 5.2, p.78) supports and encourages student autonomy and promotes mathematical exploration. This autonomy makes the actual direction of the groups’ exploratory activities more difficult to predict (see Section 5.5.1, p.98). The uncertainty in the direction a group might explore and the differences in the mathematical experiences of each collaborative group could create a new set of challenges for some teachers. This issue requires further exploration in another study. The types of issues that may need to be addressed include: concerns about how to plan a curriculum and deliver this curriculum within the time constraints of the school; and teacher concern about their capacity to understand and work with the ideas developed by the students.

The complete involvement of William and Talei as they synthesised their findings to create a new concept captures the essence of student engagement and illustrates the
affective responses that can accompany problem resolution:

William focused the attention of Group 1 on a Discovered Complexity with his comment ‘there must be something more’. This precipitated the intellectual challenge to synthesise an overall understanding of f’’(x) (Table 5.8, 5.10). Group engagement and insensitivity to distraction were apparent when the three students (Gerard, Talei and William) did not respond to the buzzer (the end of collaboration). Lack of response to external stimuli and a loss of sense of time were evident when Williams and Talei—atypically—did not respond to Gerard’s question about the imminent report (Table 5.9). Feelings of pleasure could be inferred from William’s exclamation and facial expression when the meaning of f’’(x) suddenly became clear. The behaviour of Talei and William during this interval of time indicates they were experiencing ‘flow’ (Csikszentmihalyi, M. & Csikszentmihalyi, I., 1992). These student responses to the Discovered Complexity within the task can be explained using the Engaged to Learn model.

6.3.2 The Engaged to Learn Model

The Engaged to Learn model (Figure 6.1) explains the sustained engagement of Group 1 and the lower and fluctuating level of engagement of Group 2. This model interrelates affective and cognitive factors associated with learning gains; it interrelates the theoretical model of M. and I. Csikszentmihalyi’s optimal learning conditions (Csikszentmihalyi, M. & Csikszentmihalyi, I., 1992) with some aspects of Vygotsky’s (1978) zone of proximal development and my Discovered Complexities.

The construct of a Discovered Complexity was developed as a tool to explore student response to complexity in instances where optimal leaning conditions were present (Csikszentmihalyi, M. & Csikszentmihalyi, I., 1992). M. Csikszentmihalyi demonstrated the presence of two conditions (necessary but not sufficient) for the existence of flow. These conditions—working above the person’s perceived skill level on a challenge the person perceived was almost out of reach—translated to a mathematical context become ‘working with mathematics unfamiliar to all members of the group to resolve the intellectual challenge created by the focus question associated with the Discovered Complexity’ (Sections 2.2.6, p.28; 3.5.2, p.55; and 5.3.1.1, p.82). The link between the challenges in a task and the Intellectual Complexity of the task are supported by the work of Smith and Stein (1998). A Discovered Complexity exists where students spontaneously work to solve a question formulated by group members and the resolution of this question requires all students to work with unfamiliar mathematics. Every complexity discovered by either Group 1 or Group 2 contained a high level of Intellectual Complexity.
Figure 6.1 Engaged to Learn model, to explain sustained engagement, interrelates M and I Csikszentmihalyi’s (1992, p 259) optimal learning conditions, Vygotsky’s zone of proximal development (1978) and my Discovered Complexities.
The Engaged to Learn schematic representation illustrates the interrelationships between the concept of flow, the zone of proximal development and Discovered Complexity. This schematic representation is an extension of M. and I. Csikszentmihalyi’s graphical representation of flow with Vygotsky’s ‘Zone of Proximal Development’ integrated into this representation to explain the links between: students working with new skills and concepts, students developing new concepts and the affective factors that affect learning including student engagement and flow. M. and I. Csikszentmihalyi represent the challenge ‘just’ out of reach through the use of diagonal lines on the graphical representation (See Section 6.3.4.2, p.110). As Discovered Complexities were developed as a tool to analyse task dynamics and explore optimal conditions for learning, the two conditions that identify Discovered Complexities have been represented on the Engaged to Learn model as equivalent to Csikszentmihalyi’s ‘increase in skills and increase in challenges’. The zone of proximal development as used in this representation relates to student development of new concepts but does not rely upon students working with an expert other.

It has been demonstrated that students do not engage with the focus question in instances where the present skill and concept level is not sufficient for all students in the group to recognise the complexity and so ‘discover’ it as a group (See Table 5.11). In Group 2 early in the fifth to ninth minute time interval Alistair talked about a gradient that is increasing and at this stage Rez did not know what Alistair meant. It was the start of the 10th minute before students in Group 2 formulated a focus question around a new Conceptual Discovered Complexity. They explored whether it is possible ‘for a negative gradient to be increasing’. This complexity could have been discussed several minutes earlier if this concept had been ‘just’ out of reach for all students. Instead there was an intervening period when Rez and Dean clarified their understanding about gradients increasing and decreasing. Students then discovered the complexity indicating all students had now developed skills and concepts to the level where the mathematics related to this complexity was not too far out of reach.

6.3.3 Explanation of the Axes System

I now explain the concepts represented on the horizontal and vertical axes, of this schematic representation of the Engaged to Learn Model (Figure 6.1), and explain how a plotted co-ordinate can indicate the presence or absence of flow. At any instant in time, each student is represented by a co-ordinate on this axis system. If the student feels their level of skills and conceptual understanding has increased or the magnitude of the challenge has changed, this alters the student’s position on the co-ordinate axes system. It is the student’s perception of their skill and concept development and the student’s perception of the challenge rather than the actual level of each that are important (Csikszentmihalyi, M. & Csikszentmihalyi, I., 1992)
6.3.3.1 The horizontal axis (Skills and Concepts)

The horizontal axis represents mathematical skills and concepts the student presently perceives himself or herself to possess (the tool box of knowledge, strategies and understandings relevant to the task at hand) (Schoenfeld, 1993). Vygotsky’s (1978) zone of proximal development for each student can also be represented on the Skills and Concepts axis (as an interval adjacent to and beyond the perceived present level of skill and concept acquisition of the student). This zone of proximal development represents the student’s present potential to acquire new skills and concepts and is likely to differ in magnitude for each student.

6.3.4 The vertical axis (Intellectual Challenge)

The vertical axis represents the intellectual challenge of the task as perceived by the student at the present time. The level of perceived intellectual challenge is raised each time a new complexity is discovered so a dynamic task such as Task A for Group 1 was composed of successive discrete challenges with each new challenge arising with the discovery of each new complexity. As the students in Group 1 demonstrated an understanding of the previous Discovered Complexity before the discovery of the next complexity (Table 5.8) these students were not overwhelmed by too great a challenge. For each student, the next Discovered Complexity opened a new zone of proximal development and provided a new challenge. Each challenge was overcome before the next challenge was discovered as evidenced in Table 5.8. The graph shows the incremental increases in challenge created by each Discovered Complexity (Figure 6.1, for example the length FH).

6.3.4.1 How a Discovered Complexity alters the graph

A simple way to think about the process whereby a group engages with a task is by using the concept of a complexity that is discovered. Through the discovery of a complexity, students: (a) work to resolve a focus question spontaneously formulated by group members (Section 5.3.1.1, p.82, Discovered Complexities); and (b) are required to use mathematics that is unfamiliar to all group members to resolve the focus question. As all students in Group 1 developed new concepts as they Discovered Complexities, each student was working within their own zone of proximal development. As the students were working with the same unfamiliar mathematics to resolve the same focus question, the zones of proximal development for these students overlapped. As the resolution of each focus question for both Group 1 and Group 2 involved students working with Intellectual Complexity, it appears the presence of the Intellectual Complexity and the resulting intellectual demand provided challenge for the students. This is consistent with the work of Smith and Stein (1998) who claim the Intellectual Complexity of a mathematics task provides the challenge.
6.3.4.2 Representation of flow on the axis system.

A student is in flow if the co-ordinate for a student lies between the diagonal lines on the Engaged to Learn model (Figure 6.1). This student is working just above their present perceived skill level with a challenge almost out of reach (Csikszentmihalyi, M. & Csikszentmihalyi, I., 1992). These diagonal lines were used in M. and I. Csikszentmihalyi’s (Csikszentmihalyi, M. & Csikszentmihalyi, I., 1992) graphical illustration of the conditions that create flow. For example, a student at H is required to use skills above their perceived level of skill acquisition (A) to overcome a challenge greater than they can easily achieve (D). The student’s present zone of proximal development represents the increase in skill and concept development that could be expected for that student at that time. M. and I. Csikszentmihalyi’s model includes diagonal parallel lines as a schematic representation of the presence or absence of flow for a person. Where the increase in challenge is not commensurate with the increase in skill and concept development, the co-ordinate will lie outside the diagonal parallel lines (as explained above). In such a case, the person is not experiencing flow so the learning conditions are not optimised.

For group flow to exist, each person within the group must be individually represented on the set of axes between the parallel lines. Although their respective positions relative to the two axes may differ slightly, provided that all students are working on the same new mathematics experiencing the same challenge, differences in position on the graph will be minimal because student positions on the horizontal axis will be almost equivalent if the same mathematics is new to all students.

6.3.4.3 Sustained engagement—sustained flow

Sustained engagement results from the discovery of another complexity each time a complexity has been investigated and the challenge overcome. The level of skill and concept development for each student increases each time a focus question associated with a Discovered Complexity is resolved. This is evidenced in Group 1 where the conceptual understanding required for discovery of a complexity was often the focus of an earlier Discovered Complexity (Table 5.8, Table 5.10, Section 5.4.2, p.85, conceptual development). Without a commensurate increase in challenge, the student will ‘fall out of flow’ (see Figure 6.1). For example, if a student’s skills and understanding of concepts increased so their position moved from E to G (with no subsequent rise in challenge to move the student’s position to I) the student would be positioned below the parallel lines and the student would not be experiencing optimal affective conditions for learning (‘flow’).

If students in a collaborative group are positioned too far apart on the skills and concepts axis, the members of the collaborative group can not remain in group flow. This explains student response in Group 2 in the first nine minutes. Due to the differences in student background that resulted from the misconceptions developed
during individual work, students initially did not find a focus question related to mathematics unfamiliar to all group members. The lack of a high level of engagement for Group 2 in the first nine minutes indicates this group was not experiencing flow at that stage. If the students’ zones of proximal development no longer overlap, the amount of challenge appropriate for individual group members will differ sufficiently for some members of the group to ‘fall out of flow’. For example, two students whose skill levels were represented by the skills and concepts ordinate of E may possess zones of proximal development of different magnitudes due to their ability to solve unfamiliar challenging problems. One student may progress to F and another to G as their skills and concepts develop. Talei and Gerard provide an example of students progressing at different rates through their zones of proximal development in the next section (Section 6.3.5). Using the Engaged to Learn model to consider the affective state of each student at this time, each student would require a challenge of a different magnitude (HF as opposed to GI) to remain in flow; the same complexity level being inappropriate for both students. In some instances the students, due to their relative abilities and inclinations, may mediate to facilitate a continual overlap in zones of proximal development for group members. For example, Group 1, Figure 5.1 p.89 demonstrates the small time intervals of cumulative talk and peer tutoring interspersed between the exploratory talk as Talei assists Gerard to move through the new zone of proximal development opened to him by the change in student interactions. Talei and Gerard’s capacity and inclination to work in this way assisted the group to remain in flow.

### 6.3.5 Sustained Engagement: the effects of group interactions

Comparisons were made between the Abilities of Gerard, Talei and William. These differences in Ability were used to support the inferences I made from the video evidence about the effects of group interactions on sustaining the level of student engagement. The horizontal axis of the Engaged to Learn model is now used to hypothesise the behaviour of Group 1 and Group 2 as they worked within Task A.

Table 4.3 indicates that Gerard, Talei and William all demonstrated the Ability to synthesise, but Gerard did not demonstrate the Ability to work out ‘why extra information was needed’ and he demonstrated less Ability than Talei and William to ‘build on a concept to answer a question with a slight change’. These differences in cognitive processing could account for differences in the zone of proximal development open to each of these students by the task under investigation. The remainder of this comparison refers only to Talei and Gerard because these two students possessed more similarities in Ability than William and Gerard and also because Talei and Gerard spent short intervals of time working together (Figure 5.1). It appears Talei was able to employ cognitive processes more effectively and to draw on a wider variety of cognitive processes to assist the acquisition of a new mathematical understanding. This is
represented in Figure 6.1 where the magnitude of Talei’s zone of proximal development (AC is greater than the magnitude of Gerard’s zone of proximal development AB). Analysis of the video data supports this supposition; all three students began the generation of a new idea together, then William and Talei completed the idea, then Talei and Gerard worked together as Gerard completed his understanding of the new idea (Figure 5.1). The cohesiveness of Group 1 appears to rely upon Talei’s ability to operate in different and appropriate ways at different stages of the collaborative group work. It is unclear whether the key role played by Talei is a result of her Ability lying between the Ability of the other two students, a function of her personality, a combination of these two factors, or whether other factors are operating. Talei’s zone of proximal development was probably more likely to overlap with the zones of proximal development of Gerard and William even when the zones of proximal development of William and Gerard did not overlap, Talei’s inclination to play a mediating role could have been a key factor in group cohesion.

Talei, William and Gerard each played complementary roles in maintaining an overlap between the zones of proximal development of the group. All three students were generally able to commence the resolution of the focus question associated with each Discovered Complexity. When Gerard had moved through the zone of proximal development opened to him by the Discovered Complexity under exploratory talk (Figure 6.1, AB) (Mercer, 1995; Barnes, 1999), he listened to, then questioned Talei and William when they had completed the generation of a new idea (Figure 6.1, Talei at C). Talei and Gerard then generally engaged in cumulative talk with some peer tutoring from Talei for a short period of time (Figure 5.1) (Vygotsky, 1978). This created a new zone of proximal development for Gerard (BC). Talei assisted Gerard to move rapidly through this zone of proximal development because Gerard was able to understand new ideas quickly (see Figure 5.1 and Figure 6.1).

The overlap in zones of proximal development for Talei and Gerard is represented in Figure 6.1 along the horizontal axis. At the point when William and Talei have just completed the development of a new idea, Gerard is at B and Talei is at C on the horizontal axis and their zones of proximal development no longer overlap. For simplicity of explanation and because the student interactions discussed in this diagram are related to Talei and Gerard, William has not been included on the diagram. At that instant in time, Gerard has moved through the zone of proximal development opened for him by the discovery of a complexity and the group working to resolve the focus question through exploratory talk (AB). Talei has moved further under the same conditions (AC). Talei’s acquisition of more skills and concepts than Gerard is evidenced in the cumulative talk and peer tutoring that followed each interval of exploratory talk (Figure 5.1). The new zones of proximal development are unlikely to overlap unless Talei assists Gerard to move through BC first.
The Engaged to Learn model can be used to explain the lack of sustained engagement and the lack of group flow that would result if the zones of proximal development of William, Talei and Gerard do not continue to overlap. For another complexity to be discovered, all three students need to be able to work together on the same new material. Talei’s cumulative talk and peer tutoring with Gerard is integral to sustained student engagement because this opens out a new zone of proximal development for Gerard under cumulative talk and peer tutoring rather than exploratory talk. Unless Gerard progresses along the horizontal axis to C, Talei will require a greater challenge than Gerard (GI and opposed to FH). If this progression is too slow, Talei may spend too long working without a challenge and she might ‘fall out of flow’.

With Group 2, it appears the varying misconceptions developed during individual work reduced the common background between the students (Table 5.11, p.93; 5.12, p.96) and led to a disparity in the positions of each group member on the skills and concepts axis. One consequence appears to be the absence of overlap between the zones of proximal development for this group. As group members lacked a shared understanding about concepts such as gradient, their level of shared understanding which became a starting point for their discussions was below the level of understanding of at least one group member (Alistair). This level of shared understanding from which discussion can begin is called intersubjectivity (Lerman, 1996). Students in Group 2 focused initially on developing a shared understanding of the meaning of gradient. A Discovered Complexity did not arise at this stage and the prerequisite conditions for flow were not met because this mathematics was not unfamiliar for all group members. The lack of engagement in the absence of Discovered Complexities was evidenced in Group 2 and these observations support the finding that when students discovered complexities engagement resulted.

Figure 6.2. represents my hypothesis about the lack of overlap of the zones of proximal development for the students in Group 2. The horizontal axis of the Engaged to Learn model has been used to explain the responses and lower level of conceptual development for Group 2.

For Group 2, the discovery of a complexity towards the end of the collaborative session indicated an emerging area of commonality. This area of commonality was related to the exploration of positive and negative gradients and appears to have developed during the group work session. It is possible that Group 2 might have discovered more complexities if the collaborative session had continued. Rez did not demonstrate the ‘ability to synthesise’ on the Ability test (Table 4.4). This is consistent with his responses during the collaborative session (Table 5.10, Table 5.11). Other students spent considerable time explaining ideas to Rez in an attempt to dislodge Rez’s misconceptions.

During these times, intersubjectivity developed around mathematics which was
generally not new to Alistair and sometimes not new to Tony. Dean’s level of understanding was not evident through most of the student discourse although he did make comments during the 5th and 12th minute that indicated he was experiencing trouble understanding the ideas. Dean’s test of mathematical understanding also indicated he had retained misconceptions about how to find the ‘gradient of the gradient’ throughout the session. During the fifth minute Dean indicated he did not understand (or possibly did not agree with) how Alistair found a stationary point. His comment in the 12th minute indicates it was probably a lack of understanding. In the 12th minute Dean questioned Rez about where he was saying the gradient was decreasing.

Alistair and Tony spent long periods of time working at a level below their zone of proximal development due to the time spent explaining to Rez (Table 5.11). The time Rez took to alter his point of view was probably a function of the strength with which he developed his misconceptions during individual work and his lack of ‘ability to synthesise’ without the support of more expert others (Table 4.4). With the support of others, Rez demonstrated a limited ‘ability to synthesise’ when he explained the increase in the negative gradient in terms of a specific example rather than in generalities (Table 5.11). Dean’s test of mathematical understanding indicated he had not clarified the concept of gradient of gradient during the group work session.

**Figure 6.2 Hypothesised position on the Skills and Concept axis (Figure 6.1) for members of Group 2 at the start of the collaborative work.**

Start of collaborative session.

Key:

- Zone of proximal development (ZPD)

The zones of proximal development open to Alistair, Rez, Dean and Tony later in the session (as evidenced by student discussion/dialogue) were related to a slightly different domain of mathematical understanding (whether a negative gradient could be increasing rather than broader mathematical domain of the relationship between f(x), f’(x) and f’’(x)).

Consideration of the positions of the members of Group 2 on the skills and concepts axis of the Engaged to Learn model explains why these students did not discover complexities early in the session. Although Table 5.11 indicates intersubjectivity was
achieved when different ideas were discussed, a high level of engagement was not generally present because students were involved in cumulative talk and peer tutoring at a lower level than the zone of proximal development of some group members. Exploratory talk was evident when a complexity was discovered (Mercer, 1995).

Evidence from this study challenges the need for an expert other (Vygotsky, 1978) to assist students to move through their individual zones of proximal development (Group 1 in Table 5.8) and in fact suggests learning opportunities for the group are not maximised in a predominantly peer tutoring model (Group 2 in Table 5.10, 5.12). The prerequisite conditions for flow are not present when peer tutoring occurs because the tutor is working within their present mathematical understanding and when students are in a state of flow, the learning environment is enhanced. The peer tutoring in Group 1 was in short intervals and the group members then returned quickly each time to exploratory talk where the mathematics was unfamiliar to all group members. It appears that the lack of exploratory talk was a function of the reduced likelihood for the group to discover complexities. The likelihood for the group to discover complexities was reduced because of the disparity between the skills and concepts levels possessed by group members and the probable lack of overlapping zones of proximal development—a condition that appears to be necessary for group exploration of new mathematical ideas.

6.3.6 Maximising Opportunities to Learn

This study illustrates the need for a common mathematical background in members of a collaborative group in order to promote exploration of new ideas (Group 1). Without this common background students may not possess overlapping zones of proximal development in relation to the task and this could reduce the amount of exploratory activity undertaken by the group (Group 2). This study also illustrates the potential for exploration where tasks provide opportunities for Discovered Complexities and where the composition of the group facilitates the continual overlap of the zones of proximal development amongst its members.

6.4 Limitations / Potential for Generalisation

This research is based on two case studies from one school after analysis of two unfamiliar challenging problems by a small number of Experts. There were differences in the sizes and gender composition of the collaborative groups but the groups did have the same teacher and attempted the same task. All students in these two collaborative groups studied higher level senior secondary mathematics and possessed higher than average ability to solve unfamiliar challenging problems.

Discovered Complexity, although a useful tool, relies upon recognition of whether mathematics is new to each group member. Although the tests of mathematical understanding assisted in determining each student’s level of conceptual development
in relation to the task, it would be appropriate to consider other ways to ascertain student conceptual development in future studies. The methods of analysis used in this study would provide a starting point for such consideration.

The conclusions I have drawn are tentative. In particular, I suggest generalisation without further investigation would be inadvisable. The marked differences in learning outcomes achieved by these two groups (even when so many characteristics of the groups were similar) should encourage others to investigate the conditions necessary for effective learning where students work collaboratively with unfamiliar challenging problems that provide opportunities for Discovered Complexities.

If we can isolate the factors that create such differences in group effectiveness, we will be better informed to advise teachers about the effective use of collaborative learning in their classrooms.

6.5 Implications of this Study

Evidence from this research could inform teachers and researchers about the environments that optimise learning potential for students. In particular, the following implications are discussed below: (a) the usefulness of the concept of task complexity; (b) factors that may affect the learning potential of students in a collaborative learning environment; (c) the benefits of sequencing collaborative group work before individual work; (d) the potential of tasks to promote learning where the task is undertaken before the mathematics is taught; (e) the implications of the findings of this study for VCE 2000; and (f) the use of complex tasks to develop the characteristics of a mathematician.

6.5.1 Task Complexity—a useful concept

My results support recent research and confirm the conceptual development that results when students are engaged with a task (Bell, 1993; Brown, 1994; Cobb, 1995b; Smith and Stein, 1998; Henningsen and Stein, 1997). My results also extend our understanding of some of the factors affecting student engagement in mathematics through the identification of the nature of task complexity and the relationships between task complexity, student engagement and conceptual development.

This study identified the effects that task complexity can have on the learning environment and it also identified the lack of clarity and commonality in the opinions of Experts and teachers about the nature of task complexity. This suggests the need for a common framework and an associated vocabulary with which to discuss task complexity within the mathematics education community. There is also a need for professional development of mathematics teachers and training of student teachers to heighten awareness of the nature of task complexity and the effects this complexity can have on the student’s interaction with mathematics tasks. The Williams/Clarke
Framework (as employed in this study) is a useful and adequate tool to facilitate analysis and discussion using the six dimensions of complexity (Williams and Clarke, 1997). The construct of a Discovered Complexity has been demonstrated to be a useful tool to relate task dynamics, student engagement and conceptual development. By using the Williams/Clarke Framework and the construct of Discovered Complexity, teachers and researchers could be assisted as they construct and select appropriate tasks. This is particularly so when the goal is effective use of collaborative groups to promote student engagement.

6.5.2 Factors Affecting the Opportunities to Learn

As this study illustrated, marked differences in learning outcomes resulted from apparently small changes in the learning environment. Once this is recognised, Complexity Theory (Landvogt, 1998)—as distinct from task complexity—becomes an essential overlay to any analysis of the teaching and learning environment. This study identifies general factors that could alter the level of engagement of students and thus alter their potential to learn. Some conventional classroom practices should be reconsidered in the light of the Engaged to Learn model. These current teaching practices include: whole class instruction as the primary mode of teaching, using investigations for revision rather than generation of knowledge, and formation of groups mainly for the purpose of peer tutoring. As Class Collaboration promotes and encourages student autonomy and the spontaneous and idiosyncratic exploration of mathematics, this instructional approach provides insight into the Mediating Factors (Chapter 3) operating between the student and the task. Another contributing factor was the composition of the group. Intersubjectivity within the group is not a sufficient condition for student engagement as related to flow in this study because intersubjectivity can exist when the students are not working on new material and there is no challenge almost beyond reach. If greater conceptual development occurs when zones of proximal development continue to overlap, the peer tutoring model may not be appropriate as the primary form of interaction in group work. Instead, within-class grouping of students to facilitate the continual overlap of zones of proximal development of the students in a group should assist in the optimisation of learning conditions. This recommendation suggests that activity in the Zone of Proximal Development can occur without direct interaction with a more expert other. I suggest both the relative Ability of group members and the inclination of the students to undertake roles that facilitate the overlap in the students’ zones of proximal development (Group 1: Figure 5.1) contributed significantly to maintained student engagement for the members of the collaborative group.

6.5.3 The Benefits of Collaborative Work Before Individual Work

The findings from this study support the findings of Hewitt (1996) and call into question the use of individual work before group work. It appears the misconceptions
developed and the strength with which these misconceptions were retained when initial individual work was undertaken altered the type of student discussion in subsequent collaborative sessions. After individual work, the collaborative session consisted predominantly of confrontational statements, cumulative talk and peer tutoring with a small amount of exploratory talk. One explanation is that extended individual work exacerbates existing differences in understanding and minimises the opportunities for students to spontaneously focus on a question generated by group members that results in members of the collaborative group focusing on unfamiliar mathematics.

6.5.4 Teachers Teach and/or Students Learn?
At present, many unfamiliar challenging problems used in the classroom are presented at the end of the topic for consolidation rather than at the beginning of the topic to generate new ideas. This study demonstrates that students can produce new mathematical ideas in a supportive learning environment. It also demonstrates students can become engaged in the mathematics in such classroom situations and that conceptual development can result. Research in recent years has begun to focus in more depth upon the particular role a teacher can take to maximise the effective selection and use of tasks (Hewitt, 1996; Chazan & Ball, 1999).

6.5.5 Implications for VCE 2000
School-based assessment tasks to be introduced as part of the Victorian Certificate of Education 2000 (BOS, 1999) return the role of task selection, implementation and assessment to the schools. Schools may choose to allocate more time to learning by selecting assessment tasks that simultaneously develop mathematical concepts. This was the initial intention of the Victorian Certificate of Education Mathematics as expressed to teachers in pilot schools (VCAB, 1989). The use of assessment tasks deliberately designed to change student understanding through the process of task completion is a novel and challenging notion (Clarke et al., 1992; Williams, 1999). In addition to gains in the time allocated to learning, the selection and use of these tasks in conjunction with initial collaborative sessions may limit the misconceptions students develop and engage students in the mathematics (Williams, 1999).

6.5.6 Developing the Characteristics of a Mathematician
Landvogt (1998) suggests if teachers are able to recognise the characteristics possessed by students who demonstrate talent in an area of learning, educators may be able to refine teaching methods to develop these characteristics in students in general and gifted students in particular. Focusing teacher attention on the profile of student ability to solve unfamiliar challenging problems developed in this study (Tables 2.2) could accelerate teacher recognition of this particular capability.
6.6 Questions arising from this study

Many unanswered questions have arisen:

1) Do these conclusions apply more generally; for example for other tasks in the Specialist Mathematics curriculum; for students possessing average and/or below average ability to solve unfamiliar challenging problems; for other mathematics subjects at Year 12; or for other levels in the school system?

2) Is conceptual development generally an outcome of student recognition and use of Discovered Complexities?

3) How task dependent is the functioning of a particular collaborative group?

Of particular interest to me is: What characteristics of the learning environment assisted students to move through a zone of proximal development in the absence of a more expert other (Table 5.9)? It is of interest to consider how Vygotsky himself might have refined his theory in the light of the socio-cognitive learning theories (Brown, 1994) that have gained recognition in recent years.

These questions identify some of the directions our research could follow in the quest to optimise the learning environment for students of mathematics. We then need to consider the next and possibly even greater challenge: If we, as researchers, develop an understanding of the factors that contribute to learning in the classroom and the interrelationship between these factors, how do we heighten the awareness of teachers and assist them to implement these changes in their mathematics classrooms?

6.7 Summary of this research

It this study I investigated the nature of task complexity associated with unfamiliar challenging problems in secondary mathematics. Although the concept of task complexity was not clearly defined by the research community, this study illustrates the usefulness of Task Complexity in assisting teachers and researchers to optimise the effectiveness of the task through appropriate selection and design of tasks. The Williams/Clarke Framework was a useful and sufficient tool to enable researchers and teachers to identify the dimensions of complexity and could be effectively used to facilitate discussion about Task Complexity.

Discovery of complexities by collaborative groups was idiosyncratic and associated with engagement and conceptual development. The graphical representation of ‘flow’ (M. and I. Csikszentmihalyi’s, 1992) has been extended to include the concept of Vygotsky’s Zone of Proximal Development and my Discovered Complexities to explain the sustained engagement of a collaborative group as they undertake mathematical problem solving (Figure 6.1) (Engaged to Learn Model).

The findings of this research support the research of Hewitt (1996), Bell (1993),
Brown (1994) and M. and I. Csikszentmihalyi (1992). The development of new skills should be embedded in tasks that challenge students to explore new ideas. These challenges can alter the affective state of the students and lead to engagement and increased conceptual development. Unlike Vygotsky’s findings, the present study demonstrates students can move through their zone of proximal development without direct interaction with an expert other. This was found to occur when the task presented opportunities for students in a collaborative group to discover complexities. This study demonstrated the provision of such tasks was a necessary but not sufficient condition to optimise learning conditions. Future research should focus on other necessary factors to facilitate student engagement and conceptual development. These factors appear to include group composition, student Difficulty with the task, and mediating factors such as the instructional approach.

Student response to Task Complexity can be affected by a variety of factors. If we are to develop in our students a love of learning and an enjoyment of mathematics, it is essential we support teachers as they work to provide instruction based upon student exploration rather than student revision and regurgitation. It is a challenging task but as M. Csikszentmihalyi explains, students will want to replicate the pleasurable affective state.
Figure 6.1 Engaged to Learn model, to explain sustained engagement, interrelates M. and I. Csikszentmihalyi’s (1992, p 259) optimal conditions, Vygotsky’s zone of proximal development (1978) and my discovered complexities.

The interactions between Gerard and Talei are used as examples when explaining the model. To simplify the diagram, William has not been included (William’s ZPD appeared to be of similar magnitude to Talei’s ZPD).

Key: Zone of Proximal Development (ZPD). The ZPD are intervals along the skills and concepts axis. They have been placed just above the axis in the diagram so each interval can be seen clearly.


**Bibliography**


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