Appendices

Appendix 1  Interview questionnaire to ascertain Expert opinion of task complexity.
1a. First section of interview opinion before Williams/Clarke Framework.
1b. Second part of interview: Expert employs Williams/Clarke Framework.

Appendix 2  Task A: Understanding the double derivative.

Appendix 3  Task B: Integration checklist.

Appendix 4  Instrument designed to measure ability.

Appendix 5  Test of mathematical understanding.

Appendix 6  Graduates Project used to scaffold students for Task A.

Appendix 7  One of the four questions on CAT 3 (Analysis Task) Specialist Mathematics 1996
Appendix 1a: Expert Questionnaire

First section of interview (before Williams/Clarke Framework)

1. Place of employment
2. Position held
3. Experience within the area of maths education
4. Experience in the area of education of students of high mathematical ability.
5. You have been presented with Task A and Task B.

Task A: Understanding the meaning of the double derivative
Task B: Developing a flow chart to help in recognition and application of different integral styles.

Do you feel these two tasks:
- i represent the same level of complexity (If yes, go to 7)
- ii differ in their level of complexity

6. Which task do you think is more complex?
7. What aspects of these tasks did you consider when you were evaluating their complexity? (Could you please explain in detail)

Appendix 1b: Expert Questionnaire

Interview sheet 2 (Experts employ the Williams/Clarke Framework).

At different times, people have discussed the complexity of tasks by considering a variety of different variables. Some of these are defined below. Could you please tick to indicate whether you feel these two tasks differ in complexity on each of these variables.

<table>
<thead>
<tr>
<th>Description of Complexity</th>
<th>Task A: Understanding the Double Derivative</th>
<th>Task A: Recognising and Applying Integration Styles</th>
<th>Task A and B</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Linguistic Complexity</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2. Conceptual Complexity</td>
<td></td>
<td></td>
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<tr>
<td>Perception of the relationship between the situation described and the required mathematical procedure.</td>
<td></td>
<td></td>
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<tr>
<td>3. Numerical Complexity</td>
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<td></td>
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<tr>
<td>The types and combinations of operations required to perform the task.</td>
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<td></td>
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<tr>
<td>4. Conceptual Complexity</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>The types and combinations of concepts utilized in developing the task.</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>5. Intellectual Complexity</td>
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<td></td>
<td></td>
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<tr>
<td>William’s Taxonomy: knowledge, comprehension, application, analysis, evaluation, synthesis.</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>6. Representational Complexity</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>The symbols, diagrams, graphs etc which need to be used and interpreted to understand and develop the problem.</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Are there any other variables you have thought of? Could you give detail by adding to the table above. Please indicate which task you feel is more complex with regard to this variable. Do you consider a particular one or two of these variables more important when considering the complexity of an investigation in mathematics? Please circle.
1 2 3 4 5 6 7 8 9
Appendix 3: Task B

Integration Map/Chart

Your aim is to set up a chart which helps order your approach to solving an unfamiliar integration problem. Think about the brainstorming your group generates in finding the integral of each of the following functions. This may help you format a chart. (These are not necessarily ordered from easiest to hardest for your group. It may be best to tackle those which seem more accessible first.)

Work as a group for 15 minutes. One member of the group is then to report to the class for 3-4 minutes on one of the interesting aspects your group has covered so far. This may be:
• an interesting break through in ideas
• part of your map
• one of the integrations you have solved so far

Repeat this process of 15 minutes, reporting, then group work, three times more. A different person is to report each time. Each reporter should comment on any interesting thoughts their group has about what other groups have said. By this time the class should have come to a decision about which maps are useful.

Qn. 1) \[
\frac{x^3 + x^2 + 2}{x^2 + 1}
\]
Qn. 2) \[
\frac{x(4-x)^{\frac{3}{2}}}{\sqrt{4-x^2}} + (2x + 3)^{\frac{3}{2}}
\]
Qn. 3) \[
\sin^2 x + \sin x \cos x + \sec^2 x \tan x
\]
Qn. 4) \[
\frac{(x^2 + 2)^3}{x}
\]
Qn. 5) \[
(e^{2x} + e^{-x})
\]
Qn. 6) \[
\frac{\cos x}{\sqrt{1 - \cos^2 x}}
\]
Qn. 7) \[
\frac{x^3 + 3x + 1}{(x+2)(x^2 + 1)}
\]
Qn. 8) \[
\cos x
\]
Qn. 9) \[
\cos x
\]
Qn. 10) \[
\frac{x^3 + 3x + 1}{(x+2)(x^2 + 1)}
\]
Qn. 11) \[
\cos x
\]
Appendix 4a: Instrument used to measure Ability

Graphs: Inverse Functions and Gradients
For each part of each question, quickly go through for a start and put a tick on any sections where you have already been taught this work, so, know the answer without having to do any thinking for yourself.

Put a cross on work you have not been taught, so are working out for the first time in this text.

Question 1.
a) Which of these graphs could have a gradient function equivalent to the original function.

b) Explain how you decided this.

c) Can you evaluate your decision by thinking about this another way?

Question 2.
You are told the series expansion of
\[ e^x = 1 + x + x^2/2! + x^3/3! + x^4/4! \ldots \]

a) Find a way to decide on whether this is reasonable or not.

Appendix 4b: Instrument used to measure Ability (cont)

Question 3.
You are given this gradient function.

\[ y = f(x) \]

a) What does it tell you about the sketch of the original graph?

b) Can you think of another way to look at your result and decide whether it is reasonable or not?

c) Do you need more information before you can sketch the original function, showing key features?


Question 4.
What is an inverse function?

Question 5.
a) Sketch the inverse of \[ y = e^{x-3} \].
Appendix 4c: Instrument used to measure Ability (cont)

Graphs: Inverse Functions and Gradients 3
b) Explain how you did this.

b) Can you check your answer by thinking about this problem another way? Explain.

Question 6.
a) Sketch the inverse of \( y = f(x) \)

\[\begin{align*}
&y = f(x) \\
&(-1, 1) \\
&x
\end{align*}\]

b) If you used a different method to that in 5b, explain what you did, and why the method works.

c) Does anything special happen with the sketch of the inverse in this case?

d) If you have not already done so, explain why this is so.

Appendix 4d: Instrument used to measure Ability (cont)

Graphs: Inverse Functions and Gradients 4

Question 7.
y = Tan\(^{-1}\)x is the inverse function of \( y = Tanx \) (\( \pi \) indicates the principal domain for \( y = tanx \). This domain includes \( x = 0 \) and is as wide an interval as possible for the inverse to still exist.)
a) What could the principal domain be? Explain your reasoning carefully.

b) Sketch \( y = Tan^{-1}x \) explaining your method.

d) Sketch the graph of the gradient of \( y = Tan^{-1}x \).

e) Can you use another method to check this gradient function is reasonable.
Appendix 4e: Instrument used to measure Ability

Question 8.

(a) Comment on the feasibility of finding a quadratic if you are supplied with the following information. (Explain your reasoning carefully)

Two points on the graph are (2,1) and (0,9). The gradient is -6 at the point (0,9) and the turning point of the graph occurs when x = 4.

b) If you were allowed to change a piece of information, explain carefully whether this could be helpful, and which piece of information, if any, would be preferable.

Appendix 5a: Test of mathematical understanding

Derivatives.

You have thirty minutes to answer as much of this test as you can. Don’t be concerned if there are some ideas you have not seen before. I just want to know who knows what before we start some group work in this area.

Question 1. Below is the graph of y = f(x)

(a) Sketch the graph of the derivative of y = f(x) i.e. y = f'(x)

(b) Explain how you did this.

(c) Sketch the graph of the derivative of f'(x) i.e. y = \frac{d[f'(x)]}{dx}

also called y = f''(x)

(d) Explain how you did this.
Appendix 5b: Test of mathematical understanding (cont)

Question 2.
(a) What does the graph of \( y = f'(x) \) tell you about the graph of \( y = f(x) \)? Explain your thought process clearly. Make sure you have clearly explained why this is so.

(b) The graph of \( y = g(x) \) is shown below. From this graph, draw the graph of \( y = g''(x) \). Try to choose the most elegant method. Clearly explain what you did.

Question 3.
(a) The graph of \( y = h''(x) \) is given below. What can you tell about the graph of \( y = h(x) \)?

(b) If you feel more information would allow you to find more about the graph of \( y = h(x) \), explain what information would be helpful and why.

Appendix 5c: Test of mathematical understanding (cont)

Question 4.
The graph below is the double derivative of \( f(x) \) and you are told \( f'(x) = 0 \) when \( x = 1 \), and \( f(3) = 0 \).

\[ f''(x) = \frac{d^2 f}{dx^2} \]

(a) If you have appropriate information, sketch the graph of \( y = f(x) \).

(b) If the information is not appropriate, explain why.

If a sketch is possible, explain how you worked out what the sketch should look like.
Appendix 6: Gradient Project

The Gradient of a Function

Work in groups of four. Discuss what you think you have to do. Explain your thought process to each other. It will help you understand.

For each of the following, sketch the graph of the gradient of \( y = f(x) \) on the set of axes underneath.

Clue: Think about the size of the gradient for each value of \( x \).

Are there some points on the graph where it is really easy to see the size of the gradient?

\[ f(x) \]  
\[ f'(x) \]

Compare the findings of the different groups.

For each graph, a member of your group will come out to the board and draw the graph your group has decided on. Other students will then ask questions so they can understand how your group is thinking. If your findings are the same as another group you can just say this. If they differ, the reporter needs to explain how they differ and why your group is thinking the way it is.

Appendix 7: Question from CAT 3

This ninety-minute analysis task consisted of 4 questions. The other three questions covered areas including the area of enclosed spaces, integrations, etc. on knowledge about inverse trigonometric functions and the integral of the cube of a sine function, complex numbers, vectors, dynamics, and inclined planes.

Question 3: Analysis Task Specialist Mathematics 1996

Michelle, who is speeding in her car along a straight road at a constant speed of 20 m/s, passes a stationary police motorcycle. John.

Three seconds later John starts in pursuit. He accelerates for 6 seconds, at which time he has reached a speed of 18 m/s per second which he maintains until he overtakes Michelle.

Let \( t \) seconds be the time elapsed since Michelle passed John.

John's speed \( v(t) \) m/s until he reaches the constant speed of 18 m/s, is given by

\[ v(t) = \frac{12}{t} - 18t^2 + 180 \]  
\( 0 \leq t \leq 9 \)

as shown in the velocity-time graph.

a. Find the value of \( V \).

b. Find an expression \( a(t) \), \( 0 \leq t \leq 9 \), for John's acceleration in m/s² and hence find the time when he achieves maximum acceleration.

c. i. Sketch the velocity-time graph for Michelle on the above axes.

ii. Write down an expression for the number of metres travelled by Michelle in terms of \( t \) for \( 0 \leq t \leq 9 \).

d. i. How far has John travelled when he reaches his maximum speed \( V \) at \( t = 9 \)?

ii. If \( d(t) \) is the distance in metres John has travelled after \( t \) seconds, write down an expression for \( d(t) \), for \( t \geq 0 \).

e. i. For what value of \( t \) does John draw level with Michelle?

ii. How far does John travel before he draws level with Michelle?

f. From the moment John draws level with Michelle, she starts to reduce speed. Find the time and distance required for her to stop if, while reducing speed, her acceleration is given by

\[ \frac{dv}{dt} = -\frac{1}{3}t^2 \]

where \( t \) is the time measured in seconds from when Michelle starts to reduce speed and \( v = 0 \) is her speed at time \( t \).
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Appendices

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