SWIRLING FLOW OF VISCOELASTIC FLUIDS

A thesis submitted in fulfilment of the requirements of the degree of

Doctor of Philosophy

by

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PREFACE

I hereby declare that, to the best of my knowledge, this thesis is original in its entirety and contains no material which has been previously published by any other person, except where due reference is stated. In addition, no part of this work has been submitted for the award of any other degree or diploma in any university.

I certify that the work has been undertaken solely by the candidate and the text of this thesis, exclusive of tables, figures, bibliography and appendices, does not exceed 100,000 words.

Jason R. Stokes
October 1998
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SUMMARY

The ability to understand and predict the flow behaviour of non-Newtonian fluids in swirling flow is industrially important for the efficient design and performance of processes which utilise fluids with complex rheological properties. In particular, fluids with elastic properties are not well described by non-Newtonian constitutive models, such that predictions using such models must be carefully validated. A benchmark problem is proposed here which provides a well defined geometry to study the swirling flow of non-Newtonian fluids as a test case for the validation of constitutive models. The confined swirling flow utilised is a torsionally driven cavity where the test fluid is confined in a cylinder with a rotating bottom lid, and stationary side walls and top lid. The flow field is three-dimensional and consists of both a primary motion, which is directed azimuthally, and a secondary motion, which is located in the radial and axial plane of the cylinder and driven by inertial and/or elastic forces.

Constant viscosity elastic liquids, ie. Boger fluids, are used to examine the effect of elasticity, in the absence of shear thinning effects, in confined swirling flow. The Boger fluids range from low to high viscosity, and consist of either dilute concentrations of flexible polyacrylamide or semi-dilute concentrations of semi-rigid xanthan gum. Using flow visualisation and the optical technique of particle image velocimetry (PIV), the effect of elasticity is examined for situations ranging from where inertial forces dominate the secondary flow to where elasticity dominates the secondary flow in the torsionally driven cavity.

Low-viscosity Boger fluids are used to examine the effect of dilute polymer concentrations on the phenomena of vortex breakdown which is observed using Newtonian fluids. The introduction of elasticity results in a 20% and 40% increase of the minimum critical aspect ratio required for vortex breakdown to occur, using polyacrylamide and xanthan gum respectively at concentrations of 45 ppm. When the concentrations of either polyacrylamide or xanthan gum is raised to 75 ppm, vortex
breakdown is entirely suppressed for the aspect ratios examined. Radial and axial velocity measurements along the axial centre line show that the alteration in existence domain is linked to a decrease in the magnitude of the peak in axial velocity along the central axis. The minimum peak axial velocities along the central axis for the 75 ppm polyacrylamide and 75 ppm xanthan gum Boger fluids are 67% and 86% lower in magnitude respectively than for the Newtonian fluid at Reynolds number of $Re \approx 1500 - 1600$. This decrease in axial velocity is associated with the interaction of elasticity in the governing boundary on the rotating base lid and/or the interaction of extensional viscosity in areas with high velocity gradients.

A wealth of phenomena is observed as the degree of inertia, elasticity and viscous forces are varied by using a range of medium to high-viscosity flexible polyacrylamide Boger fluids and a semi-rigid xanthan gum Boger fluid. As the inertia is decreased and elasticity increased by using polyacrylamide Boger fluids, the circulation rates for the Newtonian-like secondary flow decrease until flow reversal occurs due to the increasing magnitude of the primary normal stress difference. For each polyacrylamide fluid, the flow became highly unstable at a critical combination of Reynolds number and Weissenberg number. Each fluid is characterised by a dimensionless elasticity number and a correlation with Reynolds number is found for the occurrence of the instability. In the elasticity dominated flow of the polyacrylamide Boger fluids, the instability disrupted the flow dramatically and causes an increase in the peak axial velocity along the central axis by as much as 400%. In this case, the core vortex spirals with the primary motion of fluid and is observed in some cases at Reynolds numbers much less than unity. The instability across the whole flow domain is therefore considered an elastic flow instability. Elastic 'reverse' flow is observed for the xanthan gum Boger fluid at high Weissenberg number. As the Weissenberg number is decreased, and Reynolds number increased, counter-rotating vortices flowing in the inertial direction form on the rotating lid. The peak axial velocity decreases for the xanthan gum Boger fluid with decreasing Weissenberg number.
The rheological properties of the test fluids are examined in detail and comparisons are made with predictions of these properties using several single mode and multi-mode constitutive models, such as the Oldroyd-B, Giesekus, FENE-P, and KBKZ for the flexible polyacrylamide Boger fluids, and the rigid dumbbell model for the semi-rigid xanthan gum Boger fluids. In particular, the primary normal stress difference of the medium and high-viscosity polyacrylamide Boger fluids varies quadratically with shear rate, at low shear rates, and this behaviour is predicted using the aforementioned models. Therefore, the polyacrylamide Boger fluid are characterised using a single relaxation time and a constant viscosity at moderate to low shear rates.

This experimental investigation of a complex three-dimensional flow, using a large range of well characterised fluids, provides the information necessary for the validation of non-Newtonian constitutive models through numerical analysis of the torsionally driven cavity flow.
PUBLICATIONS


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