CHAPTER 5

CONFINED SWIRLING FLOW - INTERACTION BETWEEN INERTIA AND ELASTICITY

5.1 INTRODUCTION

The behaviour of both Newtonian and non-Newtonian fluids in swirling flows was reviewed in chapter 2. In the torsionally driven cavity, the interaction between inertial and viscous forces governs the flow kinematics for Newtonian fluids. Rotation of the bottom lid generates variations in the centrifugal force in the boundary layer, which propels the fluid near the rotating lid radially outwards. This results in a secondary flow for Newtonian fluids where the fluid moves outwards along the rotating disk, up the side stationary walls, inwards along the stationary lid and then down the central axis. At high Reynolds numbers \((Re > 1000)\), vortex breakdown occurs for a Newtonian fluid where flow stagnation points form on the central axis with weak recirculation zones, as pictorially shown in figure 2.1. The existence domain of vortex breakdown with respect to two governing dimensionless groups, the cylinder aspect ratio \((H/R)\) and a rotational Reynolds number \((Re)\), was developed by Escudier (1984) and featured one, two and three recirculation bubbles as shown in figure 2.4.

The introduction of non-Newtonian fluids into the torsionally driven cavity can introduce two more sets of governing parameters associated with shear thinning viscosity and/or fluid elasticity. The phenomena of vortex breakdown with fluids which are considered highly shear thinning but inelastic was studied by Böhme et al. (1992). They observed a shift in the domain of breakdown to higher aspect ratios and associated this shift with shear thinning, as shown in figure 2.7. The flow was also predicted using numerical methods with a generalised Newtonian model used to describe the shear thinning rheology of the fluids while elasticity was ignored. They
used a shear thinning parameter $\beta$ to evaluate the degree of shear thinning for their two fluids which had values of $\beta = 0.13$ and $\beta = 1$, where $\beta = 0$ corresponds to a fluid of constant viscosity.

The effects of slight fluid elasticity on vortex breakdown have not been considered previously. However, it is well known that polymer concentrations of the order of parts per million can have an extraordinary effect on such systems as turbulent pipe flows by causing drag reduction. While the flows to be examined in the torsionally driven cavity are not turbulent, vortex breakdown has often been considered as the prelude, or transition stage, for turbulence in swirling flows. It is of interest here to examine the interaction of fluid elasticity with inertia for the phenomena of vortex breakdown. As well as developing an understanding of how elasticity interacts with inertia, the research is also conducted to provide experimental observations suitable for numerical prediction as a test of non-Newtonian constitutive equations.

The following chapter investigates the interaction between inertia and elasticity by examining the effect of fluid elasticity on the phenomena of vortex breakdown. Low-viscosity elastic fluids are used for situations where inertial forces dominate the flow kinematics. All the fluids used have essentially a constant viscosity with the largest value of the shear thinning parameter being $\beta = 0.000136$ for the 75 ppm xanthan gum Boger fluid. The fluids are slightly elastic ($EI < 0.01$) and are therefore referred to as low-viscosity Boger fluids. Details on their rheology, including constitutive model parameters, was summarised in sections 4.3.6 and 4.5. The main two sets of test fluids examined contain either dilute concentrations of polyacrylamide or semi-dilute concentrations of xanthan gum. Polyacrylamide and xanthan gum are polyelectrolyte polymers with flexible and semi-rigid conformations in solution respectively. Solutions containing dilute and semi-dilute concentrations of polyethyleneoxide, which is a flexible non-ionic polymer, are also examined. Initially, the observations from flow visualisation and the results from particle image velocimetry (PIV) will be presented. This will then be followed by a discussion of the phenomena observed.
5.2 SWIRLING FLOW OF LOW-VISCOSITY BOGER FLUIDS

Flow visualisation images using fluorescent dye will initially be shown for the 45 ppm polyacrylamide solution for three sets of cylinder aspect ratios. The second part of this section will examine the effect of polymer, and hence elasticity, on the existence domain of vortex breakdown. Velocity measurements are shown in the third part of this section to assist in the establishment of possible reasons for a change in the flow behaviour which results from fluid elasticity, and to provide quantitative data suitable for comparison to numerical studies.

5.2.1 Flow Visualisation

The flow visualisation images of the secondary flow patterns for 45 ppm polyacrylamide are illustrated in figures 5.1, 5.2, and 5.3 for cylinder aspect ratios \((H/R)\) of 2.5, 2, and 1.5 respectively. Only the central portion of the secondary flow cell is shown, with corresponding dimensions of 83.9 \(\times\) 175 mm, 69.2 \(\times\) 140 mm, and 50.6 \(\times\) 105 mm for a \(H/R\) of 2.5, 2 and 1.5 respectively, to illustrate the changing structure of the vortex core as breakdown occurs. The flow structures presented are similar to those observed and described by Escudier (1984) for a Newtonian liquid, and by Böhme et al. (1992) for shear thinning fluids. The fluid is generally flowing downwards along the central axis in the images, which is the same direction as for a Newtonian fluid.

Figure 5.1 illustrates the secondary flow patterns for 45 ppm polyacrylamide at a aspect ratio of \(H/R = 2.5\). At low Reynolds number, a straight line of dye flows down the centreline into the Ekman boundary layer and recirculates outwards along the rotating disk, up the side walls, along the stationary lid, and down the central vortex core. However, as the Reynolds number is increased to \(Re = 2015\) (figure 5.1a), wavy dye streak lines are observed around the centreline and what appears like
Figure 5.1 - Flow visualisation images for the 45 ppm polyacrylamide Boger fluid at $H/R = 2.5$ for (a) $Re = 2015$, $We = 0.41$, (b) $Re = 2297$, $We = 0.47$, (c) $Re = 2373$, $We = 0.48$, (d) $Re = 2443$, $We = 0.5$, (e) $Re = 2636$, $We = 0.54$, and (f) $Re = 2922$, $We = 0.6$. 
a spiral on the centreline. A spiral of dye is usually observed at Reynolds numbers just below those required to cause breakdown and is an artefact of flow visualisation, and therefore, it should not be confused as an asymmetry in the flow (Hourigan, Graham & Thompson 1996). A stagnation point is produced upon increasing the Reynolds number to $Re = 2300$ (figure 5.1b), which is an indicator of the incipient breakdown state. The stagnation point moves upstream once breakdown takes place, and an adverse pressure gradient causes the production of a weakly flowing recirculation zone between two stagnation points. A single vortex breakdown bubble is clearly visible at a Reynolds number of $Re = 2375$ (figure 5.1c). The flow is also observed to decelerate just downstream from the bubble, where a build up of dye is shown. On subsequent increases in Reynolds number, another stagnation point occurs with the production of a second breakdown bubble, which is shown for a Reynolds number of $Re = 2445$ (figure 5.1d). Further increases in Reynolds number leads to increases in bubble size and a shift of the second bubble upstream towards the stationary lid, as shown in figure 5.1(e) and 5.1(f).

Figures 5.2 and 5.3 show the development of the secondary flow for the occurrence of a single vortex breakdown bubble at $H/R = 2$ and $H/R = 1.5$ respectively, for 45 ppm polyacrylamide. At $H/R = 2$, no second vortex breakdown bubble is observed using 45 ppm polyacrylamide, although the second vortex breakdown bubble is observed for a Newtonian fluid. In both figures 5.2 and 5.3, the single breakdown bubble is observed to initially grow in size and then change shape as the Reynolds number is increased. Figure 5.2(e) and figure 5.3(c) show a flattening out of the bubble downstream near the stagnation point. As the Reynolds number is increased further, the stagnation point rises and ultimately vortex breakdown disappears, as shown in figure 5.2(g).
Figure 5.2 - Flow visualisation images for the 45 ppm polyacrylamide Boger fluid at $H/R = 2.0$ for (a) $Re = 1059$, $We = 0.22$, (b) $Re = 1686$, $We = 0.34$, (c) $Re = 1760$, $We = 0.36$, (d) $Re = 1974$, $We = 0.40$, (e) $Re = 3245$, $We = 0.66$, (f) $Re = 3442$, $We = 0.70$, and (g) $Re = 3625$, $We = 0.74$. 
Figure 5.3 - Flow visualisation images for the 45 ppm polyacrylamide Boger fluid at $H/R = 1.5$ for (a) $Re = 1403$, $We = 0.29$, (b) $Re = 1574$, $We = 0.32$, (c) $Re = 1808$, $We = 0.37$, and (d) $Re = 1959$, $We = 0.40$. 
5.2.2 Existence Domain of Vortex Breakdown

The steady state existence domain of vortex breakdown, in terms of Reynolds number and aspect ratio, for the polyacrylamide and xanthan gum Boger fluids are shown in figures 5.4 and 5.5 respectively for polymer concentrations of up to 45 ppm. The existence domain for two polyethyleneoxide solutions is shown in figure 5.6. The solid lines in the diagrams represent the existence domain for the single upstream vortex breakdown bubble while the dashed lines represent the existence domain of the second vortex breakdown bubble. Upon increasing the rotation speed of the disk, the flow can become unsteady initially but then return to a stable flow field after several minutes. Therefore, it is difficult ascertaining the unsteady regime observed by Escudier (1984) for Newtonian fluids (see figure 2.3). Also, when a high disk rotation rate is maintained, severe viscous heating occurs where the temperature of the fluid can rise by more than 1 °C after only a couple of minutes, which subsequently causes the material properties to be altered. Therefore, the unsteady regime is not examined due to the difficulties in ascertaining its existence domain combined with the adverse effects associated with viscous heating. The flow field beyond the upper limit of vortex breakdown is also not examined.

There is a shift in the existence domain of vortex breakdown for the polyacrylamide solutions to higher Reynolds number and higher aspect ratios, due to the influence on elasticity that the addition of flexible polymer to the Newtonian solvent has. Figure 5.4 indicates that the degree of shift in the vortex breakdown existence domain increases with polyacrylamide concentration. A greater Reynolds number is required for the appearance and disappearance of the first and second recirculation bubbles, at a constant aspect ratio, for the polyacrylamide solutions than for a Newtonian fluid. In the case of the 45 ppm polyacrylamide Boger fluid, the critical Reynolds number required for breakdown increases by about 20%. Also, the minimum critical aspect ratio, below which vortex breakdown is not observed, increases with polyacrylamide concentration. In the case of the 45 ppm polyacrylamide Boger fluid, the minimum critical aspect ratio for the first breakdown bubble increases by approximately 20%,
Figure 5.4 - Existence domain for vortex breakdown of low-viscosity polyacrylamide Boger fluids and the Newtonian solvent.
while the minimum critical aspect ratio for the second bubble increases by approximately the same absolute magnitude as that for the first breakdown bubble. The upper domain curves for each polyacrylamide concentration, where the single vortex breakdown bubble disappears, merge to a common curve and are offset at greater Reynolds numbers than for the Newtonian fluid. Vortex breakdown is not observed at all for 75 ppm polyacrylamide for the aspect ratios examined ($H/R < 2.8$).

The vortex breakdown existence domain is also altered by the presence of semi-rigid xanthan gum polymer, as shown for polymer concentrations of 25 and 45 ppm in figure 5.5. The area bounded by the vortex breakdown domain curves decreases with increasing xanthan concentration, and no second breakdown bubble is observed at aspect ratios below $H/R = 2.5$ for the 45 ppm xanthan gum. There is only a slight increase in the Reynolds numbers corresponding to the formation of the first breakdown bubble, however, there is a substantial decrease in the Reynolds numbers corresponding to the disappearance of the first and second breakdown bubbles with increasing xanthan concentration. This corresponds to about a 25% decrease in Reynolds number required for the disappearance of the first breakdown bubble for the 45 ppm xanthan gum Boger fluid. There is also a small shift ($\approx 5\%$) to higher Reynolds numbers for the occurrence of the second breakdown bubble for the 25 ppm xanthan gum Boger fluid. The xanthan gum also causes the minimum critical aspect ratio required for breakdown to increase with polymer concentration. Vortex breakdown is not observed for aspect ratios of less than $H/R = 2.8$ for 75 ppm xanthan gum.

It should be noted that the xanthan gum solutions are slightly shear thinning above shear rates of $\dot{\gamma} \approx 1-10$ s$^{-1}$, while the Reynolds number is based on the zero-shear rate viscosity. For the overall flow kinematics, the degree of shear thinning for the xanthan solutions is considered negligible when it is compared to the highly shear thinning fluids of Böhme et al. (1992). The shear thinning parameter for the 75 ppm xanthan gum Boger fluid is $\beta = 0.000136$, while for the fluids of Böhme et al.
Key:

- Newtonian
- 25 ppm XG
- 45 ppm XG

Solid line & filled symbols: 1 breakdown
Dashed line, empty symbols & shaded area: 2 breakdowns.

Figure 5.5 - Existence domain for vortex breakdown of low-viscosity xanthan gum Boger fluids and the Newtonian solvent.
(1992), $\beta \geq 0.13$. Therefore, with values for the shear thinning parameters of close to zero, the slight shear thinning behaviour of the xanthan gum Boger fluids will have no effect on the minimum critical aspect ratio required for breakdown, which Böhme et al. (1992) found to be the major effect associated with shear thinning. Hence, the only effect of the slight shear thinning behaviour of the xanthan gum solutions is the difficulty associated with defining a viscosity to use in the Reynolds number. By defining the viscosity as the zero-shear rate value, the Reynolds number will be underestimated considering that the flow kinematics are governed by shear rates above $\dot{\gamma} \approx 1 \text{ s}^{-1}$ since the rotation rate of the disk is generally greater than $\Omega \approx 1 \text{ s}^{-1}$. If a lower value of viscosity is used to define the Reynolds number for the xanthan solutions, the Reynolds number will be larger than those reported here. This causes the vortex breakdown domain curves for the xanthan gum solutions to be shifted to higher Reynolds numbers and therefore be similar to the domain curves for the polyacrylamide Boger fluids.

A large range of polyethyleneoxide Boger fluids have also been examined in the torsionally driven cavity. However, the vortex breakdown existence domain is altered from the Newtonian case for only a few of these solutions. This is at least in part due to the low elasticity number of the solutions, as tabulated in section 4.3.6, which are more than two orders of magnitude lower than those for the polyacrylamide and xanthan gum Boger fluids. Mechanical degradation is also considered as a reason in some instances for only small departures from Newtonian behaviour, particularly for the solutions consisting polyethyleneoxide with a molecular weight of 4 million. Mechanical degradation, as well as other difficulties which arise when using the polyethyleneoxide solutions, have been discussed in section 4.3.5.

The existence domain of vortex breakdown for the two highest concentrations used for the 1 million molecular weight (1000 ppm PEO-1M) and 2 million molecular weight polyethyleneoxide solutions (1000 ppm PEO-2M) are shown in figure 5.6. The existence domain for the 1000 ppm PEO-1M, which is a dilute polymer solution, is similar to the Newtonian vortex breakdown domain. Vortex breakdown of the
Figure 5.6 - Existence domain of vortex breakdown for the low-viscosity polyethyleneoxide Boger fluids.
semi-dilute 1000 ppm PEO-2M Boger fluid occurs at critical Reynolds numbers which are approximately 20% larger than those for Newtonian fluids, while there is a negligible increase in the critical aspect ratio below which vortex breakdown does not occur. The shift in the existence domain, in this case, is similar to that for the 45 ppm polyacrylamide Boger fluid. However, for the 1000 ppm PEO-2M the area in the domain curve for the aspect ratios tested is essentially equivalent to the Newtonian case for both the first and second breakdown bubbles, which is contrary to the decrease in domain area observed for the polyacrylamide and xanthan gum Boger fluids. In other words, the domain curves for the 1000 ppm PEO-2M Boger fluid is almost an exact shift of the domain curve to 20% higher Reynolds numbers.

5.2.3 Velocity Measurements

Particle image velocimetry has been used to determine the axial and radial velocity field in the secondary flow plane for the Newtonian solvent, and the polyacrylamide and xanthan gum Boger fluids with polymer concentrations of 45 ppm and 75 ppm. The following section will initially analyse the full vector field for the 75 ppm polyacrylamide Boger fluid at a single Reynolds number. The axial velocity profiles down the central axis for all the fluids is then compared for various Reynolds numbers.

Vector Field Diagrams

The vector field and sectional streamline plot for the 75 ppm polyacrylamide Boger fluid is shown in figure 5.7 for the case of a Reynolds number of $Re = 2100$ and aspect ratio of $H/R = 2$. At these conditions, vortex breakdown is observed for a Newtonian fluid but it is suppressed for the 75 ppm polyacrylamide Boger fluid. The vector map and streamline plot shows the direction of the flow to be downwards in the central region as the fluid moves towards the rotating disk. The streamline plot shows a slight divergence of the streamlines from the central axis such that they have a slight degree of 'waviness'. The size of the arrows in the vector plot are proportional to the velocity, with a reference vector indicated in the accompanying legend. The values for the axial and radial components of the velocity are also shown.
Figure 5.7 - Secondary flow field for the 75 ppm polyacrylamide Boger fluid at $Re = 2100$, $We = 0.7$, and $H/R = 2.0$ showing the (a) vector field, and (b) sectional streamline patterns.
Figure 5.8 - Velocity contour diagrams for the secondary flow field of the 75 ppm polyacrylamide Boger fluid at $Re = 2100$, $We = 0.7$, and $H/R = 2.0$ showing the (a) axial velocity, and (b) radial velocity.
in figure 5.8 as contour plots. In the contour plots, solid lines indicate a positive velocity while dashed lines indicate a negative velocity noting that the origin is located at the centre of the rotating disk. The velocity is constant along a contour line with the absolute velocity given in the legend.

By examining the vector plot as well as the velocity contour diagrams in figure 5.7 and 5.8 respectively, the secondary flow is examined. Starting from the origin in the middle of the rotating disk, fluid is quickly pumped outwards along the disk to the stationary side walls. The radial velocity (figure 5.8b) shown in the region closest to the disk is extremely difficult to visualise, and therefore, the velocity in this region is high in error, as discussed in section 3.6. Fluid then flows upwards along the stationary side wall, with a maximum axial velocity of around 32 mm/s (figure 5.8a), before turning inwards as it encounters the top stationary lid. The fluid then flows inwards along the stationary lid with a similar radial velocity to that present along the rotating disk (figure 5.8b). The fluid then flows down the central region where it passes through a peak in the axial velocity of more than 32 mm/s at a height of $z = 120$ mm for a radius of $r = \pm 15$ mm (figure 5.8a). The fluid then slows down as it moves down the central axis to a region of almost constant axial velocity for $z < 100$ mm with $-12 < V_z < -16$ mm/s, before passing through another small peak at $z \approx 15$ mm. The flow along the central core region is dominated by the axial velocity with only small radial fluctuations indicated at $z \approx 100$ mm, which results from the divergence of the streamlines (figure 5.7 and 5.8b).

A contour diagram of the azimuthal component of the vorticity is shown in figure 5.9 with a positive vorticity being directed into the page. In examining the left half of figure 5.9, there is an area of negative azimuthal vorticity near the central axis which, according to Lopez (1990), is a necessary condition for vortex breakdown to occur. The streamlines and vector plot in figure 5.7 indicate that the streamlines have a concave form, which is also considered a necessary condition before breakdown occurs. Yet, vortex breakdown is not observed at any Reynolds number at this aspect ratio for the 75 ppm polyacrylamide Boger fluid, which suggests that although
Figure 5.9 - Azimuthal component of vorticity for the 75 ppm polyacrylamide Boger fluid at $Re = 2100$, $We = 0.7$, and $H/R = 2.0$. 

Key:
Level $\theta_0 (\text{s}^2)$
G 3
F 2.6
E 2.2
D 1.8
C 1.4
B 1.0
A 0.6
9 0.2
8 -0.2
7 -0.6
6 -1.0
5 -1.4
4 -1.8
3 -2.2
2 -2.6
1 -3.0
the criteria of Lopez (1990) may be a necessary condition required for breakdown to occur, a negative azimuthal vorticity in the central core region does not necessarily indicate that breakdown will occur. Although large errors can be generated when experimental results are differentiated, figure 5.9 shows very good anti-symmetry which suggests that, in this case, the errors are minimal.

Contour diagrams for four components of the rate of strain tensor are shown in figure 5.10. The rate-of-strain components shown are \( \gamma_r, \gamma_\theta, \gamma_\phi \), and \( \gamma_a \), and the contour plots are all relatively symmetric (or anti-symmetric) despite differentiation being performed on experimental measurements. Figure 5.10(a) shows \( \gamma_r \), which corresponds to the component of shear rate in the \( z \)-plane acting in the \( r \)-direction, or vice-versa for its equivalent symmetrical term \( \gamma_\theta \). This quantity reflects the change in axial velocity with the radial position, and the change in the radial velocity with axial position. Figure 5.10(b) shows \( \gamma_\theta \), with the negative contours indicating compression (dashed lines) and the positive contours indicating extension (solid lines) in the \( r \)-direction. Areas of fluid extension in the \( r \)-direction are situated at the origin, at the corners between the stationary side walls and the stationary lid, and in the centre of the flow cell at around \( z \approx 100 \text{ mm} \) (\( \gamma_\theta \approx 0.8 \text{ s}^{-1} \)), which corresponds to the divergence of the streamlines in figure 5.7. Figure 5.10(c) shows \( \gamma_\phi \) with the contours indicating compression and extension in the \( \theta \)-direction (into or out of the page). This component of the rate-of-strain tensor is determined using the continuity equation. Finally, the \( \gamma_a \) component of the rate-of-strain is shown in figure 5.10(d) and indicates the extension and compression in the axial direction. There is a large peak in the extension rate to \( \gamma_a \approx 3.4 \text{ s}^{-1} \) just below the stationary lid corresponding to \( z \approx 130 \text{mm} \) and \( r \approx \pm 15 \text{ mm} \). This is then followed by a region of compression as the fluid flows down the centre of the flow cell before a region of near constant velocity for \( z < 90 \text{ mm} \), such that the compression and/or extension is negligible.
Figure 5.10 - Components of the rate-of-strain tensor for the secondary flow field for the 75 ppm polyacrylamide Boger fluid at $Re = 2100$, $We = 0.7$, and $H/R = 2.0$ showing the $(a) \dot{\gamma}_r$, $(b) \dot{\gamma}_z$, $(c) \dot{\gamma}_{zz}$ and $(d) \dot{\gamma}_{zz}$. 

Key:

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Key:

(c) Level $\hat{y}_{00}(s')$

- G: 3
- F: 2.6
- E: 2.2
- D: 1.8
- C: 1.4
- B: 1.0
- A: 0.6
- 9: 0.2
- 8: -0.2
- 7: -0.6
- 6: -1.0
- 5: -1.4
- 4: -1.8
- 3: -2.2
- 2: -2.6
- 1: -3.0

Key:

(d) Level $\hat{y}_{00}(s')$

- I: 3.4
- H: 3
- G: 2.6
- F: 2.2
- E: 1.8
- D: 1.4
- C: 1.0
- B: 0.6
- A: 0.2
- 9: -0.2
- 8: -0.6
- 7: -1.0
- 6: -1.4
- 5: -1.8
- 4: -2.2
- 3: -2.6
- 2: -3.0
- 1: -3.4
A local Deborah number ($De_{ij}$) may also be determined from any of the components of the strain rate tensor, and is defined here as the ratio of the characteristic time of the fluid ($\lambda_M$) and the characteristic time of the local deformation process ($1/\dot{\gamma}_{ij}$) such that $De_{ij} = \lambda_M \dot{\gamma}_{ij}$. The Deborah number therefore measures the importance of local elastic effects in the flow field. The components of the rate of strain tensor are therefore directly proportional to the local Deborah number and can then be used to examine the relative importance of elasticity in the secondary flow plane. The maximum magnitude for any of the measured individual components of the strain rate tensor is about 3.4 s$^{-1}$. This corresponds to a maximum local Deborah number of $De \approx 0.13$, which is low compared to the effects associated with inertia since $Re = 2100$. For all of the normal-components of the strain rate tensor, areas of highest compression or extension are where elasticity can have the largest effect on the flow behaviour. As determined from figure 5.10, the areas of highest extension or compression, and of highest local Deborah number, are mainly located along the two lids, in the corners of the cylinder, and just below the centre of the stationary lid.

**AXIAL VELOCITY MEASUREMENTS**

The axial velocity close to the central axis, where -5 mm $< r < 5$ mm, has been determined for the Newtonian fluid, 45 ppm and 75 ppm polyacrylamide Boger fluids, and the 45 ppm and 75 ppm xanthan gum Boger fluids, at various Reynolds numbers before and after breakdown. The results are shown for an aspect ratio of $H/R = 2$ in figures 5.11 to 5.15, while the results for aspect ratios of $H/R = 1.5$ and $H/R = 2.5$ are shown in appendix B. The axial velocity is shown in dimensionless form by dividing by the maximum azimuthal velocity, which corresponds to that produced at the edge of the rotating disk ($2\pi R \Omega$). Lines shown on the diagrams represent lines which best fit the data. The rotating disk corresponds to $z = 0$, such that a negative axial velocity indicates that the fluid is flowing towards the rotating disk.
Figure 5.11 - Dimensionless axial velocity measurements along centreline for the Newtonian solvent at $H/R = 2.0$
The axial velocity distribution for the Newtonian fluid at an aspect ratio of $H/R = 2$ in figure 5.11 is used to describe the axial flow profiles. Note that the first and second breakdown bubbles occur for a Newtonian fluid with an aspect ratio of $H/R = 2$ at $Re \approx 1450$ and $Re \approx 1800$ respectively with the second bubble disappearing at $Re \approx 2300$. No bubble is observed at all above $Re \approx 3000$. The axial velocity distribution initially has a minimum velocity just above the rotating lid at low Reynolds number with a narrow axial velocity distribution. As the Reynolds number is raised to $Re = 380$, the axial velocity distribution becomes broader and the minimum moves to a position further away from the rotating disk. At Reynolds numbers of $Re > 990$, the minimum in axial velocity moves closer to the stationary disk and the magnitude in velocity just above the rotating disk is low. The axial velocity distribution becomes a very narrow peak ($1.1 < z/R < 2$) with the minimum close to the stationary lid as the Reynolds number is raised to $Re = 1290$, corresponding to the stage just prior to breakdown. The velocity near the rotating lid is small in magnitude with a flat distribution governing a majority of the axial length ($z/R < 1.1$). This low velocity, or almost stagnant region, continues to migrate to a higher axial length until a stagnation point forms and a small region of flow reversal occurs. This vortex breakdown is shown for $Re = 1510$, where a positive velocity is indicated at $z/R = 1.3$. Above $z/R = 1.3$, the axial velocity peak becomes more narrow and the magnitude of the minimum axial velocity decreases slightly from its value prior to breakdown. Two breakdown bubbles are observed at higher Reynolds numbers, as shown for $Re = 2144$ at $z/R \approx 1$, where the axial velocity is slightly above zero and the magnitude of the minimum axial velocity is much lower than that prior to breakdown.

The progression of the axial velocity distribution for 45 ppm polyacrylamide and 45 ppm xanthan gum Boger fluids are very similar to those of the Newtonian solvent, with the measurements shown in figures 5.12 and 5.13 respectively. As previously mentioned, vortex breakdown occurs at larger Reynolds numbers for the 45 ppm polyacrylamide Boger fluids than for the Newtonian solvent, and this is reflected in the velocity profiles. At the same Reynolds number, the velocity profile for the polyacrylamide solution is less developed than that for the Newtonian fluid, such that
Figure 5.12 - Dimensionless axial velocity measurements along centreline for the 45 ppm polyacrylamide Boger fluid at $H/R = 2.0$ where $El = 49 \times 10^4$. 

Key:
- $Re$:
  - ● 365
  - ○ 920
  - ▲ 1300
  - △ 1890

Lines: best fit of data
FIGURE 5.13 - Dimensionless axial velocity measurements along centreline for the 45 ppm xanthan gum Boger fluid at $H/R = 2.0$ where $EI = 3.33 \times 10^3$. 

Key:

\( Re: \)
- • 390
- ○ 712
- ▲ 1067
- △ 1322
- ▼ 1780

Lines: best fit of data
to obtain a similar velocity distribution, a higher Reynolds number is required for the polyacrylamide Boger fluid. A similar Reynolds number is required for vortex breakdown of the xanthan gum solution compared with the Newtonian solvent which is also reflected by similar velocity profiles at similar Reynolds numbers in the two cases. However, the most notable difference in the axial velocity profiles is the minimum velocity for both the 45 ppm polymer Boger fluids which is lower in magnitude than for the Newtonian solvent. Prior to breakdown, the minimum velocity for the Newtonian fluids is approximately a constant value of $V_\phi/(2\pi R \omega) \approx -0.11$ for $380 < Re < 1290$, and seemingly independent of aspect ratio for $1.5 \leq H/R \leq 2.5$ (see appendix B). However, the minimum velocity prior to breakdown occurring for the Boger fluids was 13% and 27% lower, with values of $V_\phi/(2\pi R \omega) \approx -0.096$ and $V_\phi/(2\pi R \omega) \approx -0.08$ for the 45 ppm polyacrylamide and 45 ppm xanthan gum Boger fluids respectively. This decrease in magnitude of the minimum velocity correlates with the 20% and 40% increase in the minimum aspect ratio required for vortex breakdown to occur for the 45 ppm polyacrylamide and 45 ppm xanthan gum Boger fluids when compared to that for a Newtonian fluid. As displayed in the existence domain plots in figure 5.4 and 5.5, the minimum aspect ratios for the Newtonian, 45 ppm polyacrylamide and 45 ppm xanthan gum Boger fluids was about 1.25, 1.5 and 1.75 respectively. Therefore, these results indicate that there is a connection between the magnitude of the minimum velocity and the minimum aspect ratio required for breakdown to occur.

Vortex breakdown does not occur for the 75 ppm polyacrylamide or 75 ppm xanthan gum Boger fluids for aspect ratios equal to or less than $H/R = 2.8$. The velocity distributions for these two Boger fluids are shown in figures 5.14 and 5.15 respectively. While showing a similar progression to the velocity profiles for the Newtonian solvent, the Boger fluids in this case do not reach very low minimum axial velocities and the velocity distributions are flatter. There is no longer a sharp peak in the axial velocity at Reynolds numbers where vortex breakdown would occur for the Newtonian fluid. At Reynolds numbers of $Re \approx 1500-1600$, the velocity distribution for the Newtonian fluid (figure 5.11) passes through a sharp minimum at $z/R \approx 1.7$ to
Key:

$Re$:  
- 380  
- 1050  
- 1570  
- 2100  

Lines: best fit of data

Figure 5.14 - Dimensionless axial velocity measurements along centreline for the 75 ppm polyacrylamide Boger fluid at $H/R = 2.0$ where $El = 332 \times 10^6$. 

231
FIGURE 5.15 - Dimensionless axial velocity measurements along centreline for the 75 ppm xanthan gum Boger fluid at $H/R = 2.0$ where $El = 9.4 \times 10^3$. 
\( V_z/(2\pi R \Omega) \approx -0.084 \), before increasing to a positive value in the location of the breakdown bubble at \( z/R \approx 1.3 \). However, the velocity distribution for the polyacrylamide (figure 5.14) and xanthan gum (figure 5.15) Boger fluids do not have a sharp minimum peak and the minimum velocities are 67\% and 86\% lower in magnitude respectively than for the Newtonian case. For \( Re \approx 1500 - 1600 \), the minimum axial velocity for the polyacrylamide Boger fluid is only \( V_z/(2\pi R \Omega) \approx -0.028 \) at an aspect ratio of \( z/R \approx 1.3 \), while the minimum velocity for the xanthan gum Boger fluid is only \( V_z/(2\pi R \Omega) \approx -0.012 \) at an aspect ratio of \( z/R \approx 1.975 \). Therefore, the results suggest that the total suppression of breakdown for these two Boger fluids is due to the reduced magnitude of the peak axial velocity, which has been caused by the action of elasticity.

5.3 DISCUSSION

The confined swirling flow has been dramatically affected due to the addition of dilute and semi-dilute concentrations of high molecular weight polymers, of both flexible and semi-rigid conformation respectively, to a Newtonian solvent. The critical aspect ratio for the occurrence of vortex breakdown is shifted to greater values as the concentration of polymer is increased. Vortex breakdown is not observed using 75 ppm polyacrylamide or 75 ppm xanthan gum for the aspect ratio’s examined (\( H/R < 2.8 \)). Velocity measurements indicate that the shift in the critical aspect ratio and ultimate suppression of breakdown is due to the lowering of the maximum magnitude of the dimensionless axial velocity along the centreline (\( r \approx 0 \)). The mechanisms by which elasticity affects the axial velocity and the suppression of vortex breakdown is discussed in the following.

Normal stresses induced in flow arise as a result of fluid elasticity. In swirling flow, normal stresses cause a tension along curvilinear streamlines, with a resultant force acting inwards, against the outward normal of the curved streamlines, and in the opposite direction to inertial forces. Although the primary normal stress difference is not measurable for the low-viscosity Boger fluids, it may be estimated from the
storage modulus using simple fluid theory with the relation:
\[ \lim_{\gamma \to 0} \frac{N_x}{2\gamma^2} = \lim_{\omega \to 0} \frac{G'}{\omega^2} \] (see Bird et al. 1987a). Therefore, the radial velocity in the
governing boundary layer positioned on the rotating disk, which drives the whole
secondary flow, is reduced by the action of normal stresses. A slight reduction in the
radial velocity out of the governing boundary layer will result in a lower axial
velocity down the centreline out of the boundary layer located on the stationary top
lid. The waviness observed in the sectional streamlines prior to breakdown (figure
2b) result from inertial fluctuations, and are referred to as inertial waves. These
inertial waves are considered critical for the occurrence of breakdown and their
degree of waviness is controlled by the upstream axial velocity. A decrease in the
axial velocity magnitude from the stationary lid, will therefore lead to a reduction and
suppression of the degree of the waviness in the sectional streamlines and
subsequently hinder the occurrence of vortex breakdown.

Extensional effects often play a role in phenomena associated with viscoelastic fluids,
such as in drag reduction, where extensional viscosity is associated as the cause in
turbulence reduction through the suppression of eddy formation. It is therefore
expected that extensional viscosity will play a role in the suppression of vortex
breakdown. The apparent extensional viscosity, as given in section 4.3, shows that, in
comparison to Newtonian fluids, the low-viscosity Boger fluids exhibit very different
extensional behaviour, with the Trouton ratios well above the Newtonian value of \( Tr = 3 \). The polyacrylamide Boger fluids are strain rate thickening while the extensional
viscosity of the xanthan gum Boger fluids are constant with strain rate. Therefore, at
even small extension rates, the resistance to extension from the xanthan gum Boger
fluids is higher than for Newtonian fluids and this is likely to have an effect on the
flow kinematics. For the polyacrylamide Boger fluid, reasonable extension rates are
required for the extensional viscosity to be above the Newtonian value, due to its
strain rate thickening behaviour. Figure 5.10 shows the components of the rate-of-
strain tensor in the secondary flow plane for the 75 ppm polyacrylamide Boger fluid
at \( Re = 2100 \). The areas of highest extension are situated just below the stationary lid
with \( \dot{\gamma}_z \approx 3.4 \text{ s}^{-1} (De_{zz} \approx 0.13) \), and along the stationary walls from the corner of the

234
rotating disk, where $z < 50 \text{ mm}$ with $\dot{\gamma}_m = 2.2 \text{s}^{-1}$. Therefore, these areas have the highest resistance to extension and are the areas where elasticity will have the greatest effect on the secondary flow field for the polyacrylamide Boger fluid. A greater resistance to extension for the Boger fluids, when compared to Newtonian fluids, will result in a lowering of the local velocity. In particular, the minimum velocity along the centreline will be suppressed. This is observed in the velocity profiles for the polyacrylamide and xanthan gum Boger fluids throughout figures 5.12 to 5.15.

It is clear that even a small degree of elasticity can change the character of a swirling flow field dramatically, even when it is dominated by inertial forces. Predictions of the flow of non-Newtonian fluids are limited, both for when inertia is negligible or when inertial effects are high, such as in turbulent or swirling flows. The experiments in this chapter were carried out to provide experimental observations and quantitative data in a well defined flow field, using well characterised fluids, in order to examine the effect of elasticity on an established steady flow phenomena observed for Newtonian fluids. Inertia is the dominant force governing the flow, and hence the confined swirling flow experiment provides an excellent test case for prediction as a prelude to predicting other inertia dominated flows such as those encountered in mixing processes and in drag reduction. Chapter 6 will examine the secondary flow field as inertia is gradually removed from the problem, such that the flow becomes totally dominated by elastic forces.
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