I computed two additional synthetics based on variations upon the density model. Only the coal sequence amplitudes and regions containing noise related to the coal sequence differed appreciably when Gardner's equation was used to replace the density model (Figure 4-7a,b). I then assigned a density of 2.2 g/cc to each layer thus representing a total lack of detail in the density model. This gave a stack response that was almost identical to that obtained from Gardner's equation (Figure 4-7c). These results suggest that density only influences a composite reflection response when there are strong anomalies in the density model (i.e., coal seams). Therefore, accurate and detailed density models are only required in highly active parts of the depth model and at isolated events (i.e., the sea floor reflection).

![Figure 4-7](image)

**Figure 4-7.** The stacked elastic synthetic from the reference depth model a) was quite different within the coal sequence and target zone to the synthetic whose density profile was based on Gardner's equation b). Using only a single density for the entire subsurface gave a synthetic c) that was nearly identical to panel b) suggesting that only large density variations can influence a complex stack response.
4.5 Q ATTENUATION

The quality factors $Q_p$ and $Q_S$ define the amount of inelastic attenuation incurred during P- and S-wave propagation. Previous studies (Hughes and Kennett, 1983) demonstrated that inelastic attenuation only affects the appearance of seismic reflections over a significant depth interval. I found that strong linear noise events can be highly sensitive to $Q$ values in the shallow subsurface, even over relatively short depth intervals. I realised this after setting $Q_p = Q_S = 85$ for the entire depth model during my first attempt at computing the elastic synthetic. This resulted in a poor match to the amplitude of the direct wave and guided waves across the supergather (Figure 4-8a), although this discrepancy is partly obscured in the display by the use of "percentage clipping". In building the reference model, I increased the $Q$ values in the water layer and compaction zone where one might naturally expect to find weaker attenuation.

I initially set $Q_S = Q_p$ based on my assessment of measured $Q$ values for a variety of rock types (Bourbié, et al., 1987), bearing in mind that a lower $Q_S$ value was likely to be more physically reasonable. For the sensitivity analysis, I computed elastic synthetics using $Q_S = 2Q_p$ and $Q_S = 0.5Q_p$ and observed considerable variation in the amplitudes of the S-wave refractions and long traveltime S-wave reflections (Figure 4-8b,c). The stacked synthetics were almost identical to the reference result suggesting that events sensitive to $Q_S$ are well suppressed by P-wave processing.
Figure 4-8. Elastic synthetics revealing the effects of Q attenuation were computed using $Q_p = Q_s = 85$ a), $Q_s = 2Q_p$ b) and $Q_s = 0.5Q_p$ c) for the entire depth model.
4.6 LATERAL STRUCTURE

Structure in highly reflective formations can have a significant impact on a stack response through both the signal and noise events that these formations generate. An example of this was shown in the previous chapter, where I simulated the complex structure of the Miocene carbonates by randomly reordering layers in the reference depth model from 600-1100 m, while maintaining the same overall traveltimes to the deeper reflections (Figure 3-5). The stacked synthetics simulated structural effects in the Miocene carbonate primaries that were consistent with those in the field data while also causing significant alteration of the NZ2 interference in the target zone (Figure 3-7). This result suggested that perturbation of some parts of the 1D elastic depth model can help to fully interpret the noise events in a seismic dataset. This may be far preferable to building a 2D depth model and using a more sophisticated modelling procedure.

4.7 CONCLUSIONS

For seismic data acquired with "typical" recording lengths (2-6 s) and cable lengths (2-5 km), I summarise its sensitivity to the underlying depth model as follows. Highly sensitive scenarios include structure in $V_p$ and near surface variation in $V_p$, $V_S$, $Q_p$ and $Q_S$. For frequencies up to 70 Hz, seismic data are sensitive to the $V_p$ and $V_S$ values of layers whose thicknesses are greater than 1 m. Stacked seismic data appear quite insensitive to the accuracy of $\rho$ (except for some lithologies), $V_S$, $Q_p$ and $Q_S$ and this insensitivity grows with increasing traveltimes.

These guidelines suggest how a detailed elastic depth model might be built from minimal information to achieve an excellent synthetic tie to field data. This is possible because seismic data are insensitive to many parts of the elastic depth model and is a reflection of the nonuniqueness problem that is often encountered in inversion studies. My experience in modelling seismic data from other basins suggests that these guidelines are highly
relevant, despite their qualitative nature and being based upon a highly active depth model. The sensitivity analyses have already played a key role in building depth models and identifying noise in the Java Sea (Indonesia), the Northwest Shelf (Australia) and elsewhere in the Gippsland Basin.

As mentioned earlier, a composite seismic reflection response can be quite insensitive to the Poisson ratio. This suggests that AVO analysis techniques are not as widely applicable as perhaps thought. An interesting addition to my work would be to build an elastic synthetic and perform sensitivity analyses in areas containing suspected AVO anomalies.

Perhaps the most exciting application of this study is in the prestack elastic waveform inversion of seismic data, as the sensitivity analysis outlines a strategy for constraining and parameterising the inversion process. This insight might be combined with the ability to produce an excellent initial guess to enable an accurate and detailed inversion. Monte Carlo inversion techniques could be used to improve the solution using the sensitivity analyses to constrain the depth model and control the adjustment of free parameters during the inversion.
Chapter 5

\( \tau-p \) domain processing

5.1 INTRODUCTION

From a theoretical viewpoint, the \( \tau-p \) transform is a useful processing tool because it provides an increased separation between different seismic wave phenomena and rearranges seismic records into an ideal structure for processing. A simplified interpretation of field records and greater versatility in noise suppression are obtained following a point source \( \tau-p \) transform (Harding, 1985), provided that there is minimal deterioration of the data due to transform artefacts. Successful use of a \( \tau-p \) processing application involves selecting a transform implementation and suppressing artefacts as appropriate to the problem at hand (Dunne and Beresford, 1995). Velocity filtering (Tatham, et al., 1983; Mitchell and Kelamis, 1990) and predictive deconvolution (Brysk, et al., 1987) can then be applied to seismic records in the \( \tau-p \) domain and with significant theoretical advantages. Velocity analysis and stacking can also be applied in the \( \tau-p \) domain and benefit from the use of an elliptical moveout correction (Schultz, 1984).

Previous attempts at \( \tau-p \) domain processing have been unable to determine whether \( \tau-p \) methods hold practical advantages over their \( t-x \) domain counterparts. I believe that this assessment can be made by forming a complete \( \tau-p \) processing sequence where the overall advantage can be judged over several case studies. In achieving this my key innovation is the implementation of hyperbolic velocity filtering (HVF) during a point source \( \tau-p \) transform to eliminate the transform artefacts. HVF also suppresses the S-wave reflections and long period multiples found in seismic data from the Gippsland Basin. In doing so, it further assists the application of gap deconvolution in the \( \tau-p \) domain to suppress the remaining mode converted interbed multiples (NZ1).
In this chapter, I discuss the theory and implementation behind each of the $\tau$-$p$ methods that comprise a $\tau$-$p$ domain processing sequence for the Gippsland Basin. The artefacts peculiar to the implementation of HVF in the proper slant stack were understood using simple test models, the elastic synthetic computed from Well 3 and its neighbouring supergather. This approach led to the development of simple techniques for suppressing the artefacts while ensuring that the noise suppression advantages of the $\tau$-$p$ domain processing sequence would carry over to the field data.

5.2 AN INTRODUCTION TO THE $\tau$-$p$ DOMAIN

5.2.1 The linear $\tau$-$p$ transform

A "$\tau$-$p$ transform" is any discrete transform based upon the Radon transform (Radon, 1917) that can be used to map seismic data to a domain of intercept time ($\tau$) and event slope ($p$). The interpretation of seismic data is simplified in the "linear" $\tau$-$p$ domain following the successful implementation of a "linear" $\tau$-$p$ transform. The simplest $\tau$-$p$ transform is the slant stack, which is based on the linear Radon transform for a single coordinate Cartesian geometry,

$$R_{\varphi}[f(t,x)] = F(\tau, p) = \int_{-\infty}^{\infty} f(\tau + px, x) dx,$$  \hspace{1cm} (5-1)

where $t = \tau + px$,  \hspace{1cm} (5-2)

and $p = \frac{dt}{dx}$,  \hspace{1cm} (5-3)

with $f(t,x)$ the amplitude at time $(t, x)$ and $F(\tau, p)$ the amplitude at $(\tau, p)$.

The implementation of the slant stack can be interpreted as a summation of event amplitudes along sloping lines in the $t$-$x$ domain dataset (Equation 5-4). It is worth noting that for single-sided recordings, the sum over positive ranges neglects small
negative range events that, if recorded, would contribute to the small \( p \) traces during slant stacking.

\[
F(\tau_i, p_j) = \sum_{i=1}^{N} f(\tau_i + p_j x_k, x_k),
\]

where \( N \) is the number of traces,

and \( i, j, k \) are sample locations in the \( \tau-p \) and \( t-x \) domains.

### 5.2.2 The transform variables

The transform variable \( p \) is most simply interpreted as the event slope. In a more physical sense it is the horizontal component of a plane-wave's slowness or the ray parameter,

\[
p = \frac{\sin \theta_n}{v_n},
\]

where \( \theta_n \) is the plane-wave propagation angle within the \( n \)th layer,

and \( v_n \) is the interval velocity of the \( n \)th layer.

Snell's law states that \( p \) is constant in each layer for a given plane-wave. For a seismic wave that is reflected from a dipping interface, Snell's law is complicated by the need to reference \( \theta_n \) to the depth axis rather than to the normal of the dipping interface. In this situation, a \( \tau-p \) transform does not produce a true plane-wave response as reflections from a single plane-wave appear on different \( p \) traces. However, for a \( \tau-p \) transformed CMP gather, \( p \) represents the average horizontal slowness of the upgoing and downgoing waves and, for moderate dips, this averaging property helps to maintain purity in the plane-wave response (Diebold and Stoffa, 1981).

The intercept time \( \tau \) is the traveltime contribution due to the vertical component of a plane-wave's slowness. This can be seen in equation (5-2), where \( px \) represents the
traveltime contribution due solely to the horizontal slowness \( p \) of a plane-wave. For horizontal layers, \( \tau \) is the two-way sum of the vertical transit times through each layer (Diebold and Stoffa, 1981),

\[
\tau_n = 2 \sum_n q_n z_n ,
\]

(5-6)

where \( z_n \) is the thickness of the \( n \)th layer, and \( q_n = \frac{\cos \theta_n}{v_n} \) is the vertical slowness.

For the case of dipping layers on a CMP gather, the layer thicknesses are referenced to the CMP location and \( q_n \) becomes the average of the upgoing and downgoing vertical slownesses for the \( n \)th layer (Diebold and Stoffa, 1981).

5.2.3 The appearance of seismic events in the linear \( \tau-p \) domain

The linear \( \tau-p \) transform performs a sum along lines in the data at sampled values of \( \tau \) and \( p \) so that a linear event in the \( t-x \) domain maps to a point in the \( \tau-p \) domain. Conversely, a point in the \( t-x \) domain maps to a line in the \( \tau-p \) domain demonstrating that information is not lost during transformation as all points in the \( t-x \) domain are completely described by lines in the \( \tau-p \) domain. Using the two-term reflection traveltime approximation (Equation 5-7) and the relation between the transform variables one can easily show that a linear \( \tau-p \) transform maps hyperbolic reflections to ellipses (Equation 5-8), as depicted in Figure 5-1,

\[
t^2(x) = t^2(0) + \frac{x^2}{V_{RMS}^2}
\]

(5-7)

\[
\tau(p) = \tau(0) \sqrt{1 - p^2 v_{RMS}^2}
\]

(5-8)
Figure 5-1. A reflection hyperbola a) maps to an ellipse b) in the $\tau$-$p$ domain.

The $\tau$-$p$ domain is well suited to the interpretation of prestack seismic data because the linear $\tau$-$p$ transform provides an excellent separation between different seismic events (Dietrich, 1988). For example, primary reflection ellipses never cross each other in the $\tau$-$p$ domain even if their corresponding reflection hyperbolae cross in the $t$-$x$ domain. The ray parameter ensures that any two events on a $p$ trace have shared the same raypath up to the point where the shallow horizon was reached. Therefore a reflection from a deeper horizon can not arrive earlier because it must travel further.

However, multiples and S-wave reflections can cross deeper P-wave primaries in the $\tau$-$p$ domain. Fortunately, predictive deconvolution is ideal for suppressing such multiples due to their exact periodicity and consistent amplitude decay along $p$ traces. S-wave reflections are generally well separated from P-wave events due to the decomposition of data into traces of equal angle of incidence at the surface (Figure 5-2). As a result their reflection amplitudes are typically quite low at the small angles where they cross deeper P-wave reflections ($X_1$ and $X_2$). The reverse situation occurs in the $t$-$x$ domain where the amplitudes of both P- and S-wave events are usually significant at the crossover points.
Figure 5-2. S-wave events generated in a marine environment a) are well separated from P-wave events in the \( \tau-p \) domain b).

Figure 5-3 indicates the appearance of other seismic wave types in the \( \tau-p \) domain. The direct wave (D) maps to a small region near \( \tau = 0 \), illustrating the ability of a linear \( \tau-p \) transform to separate strong linear events from the primary signal. The first head-wave refraction (H₁) is also a linear event and is tangential to the first reflection ellipse (R₁) so it maps to a small region on R₁. The deeper reflection ellipse (R₂) terminates at \( p_{\text{crit}} \) corresponding to the critical angle (\( \theta_{\text{crit}} \)) for propagation into the second layer. For \( p \geq p_{\text{crit}} \) the head-wave (H₁) then propagates parallel to the interface with the apparent velocity of the lower layer. Therefore the termination point of R₂ has the same \( \tau \) as H₁ thus accounting for the termination of R₂ at H₁ in the \( \tau-p \) domain. Supercritical reflections such as R₁ exist for \( p \geq p_{\text{crit}} \) and their strong multiples (guided waves) often dominate seismic records obtained near hard sea floors. Once again, the \( \tau-p \) transform places these events well away from the deep primaries.
5.3 PLANE-WAVE DECOMPOSITION VIA THE PROPER SLANT STACK

5.3.1 Theory and implementation

A plane-wave decomposition (PWD) is a set of plane-wave responses to the subsurface, in which each $p$ trace represents a single angle of incidence (at the surface) and contains event amplitudes that are directly related to the reflectivity series (Wang and McCowan, 1989). A PWD can be constructed from a shot record or CMP gather using a $\tau$-$p$ transform that allows for the nature of the source excitation. The slant stack merely
suns events along sloping lines in the $t$-$x$ domain and produces a PWD for a line source only. However, the marine seismic source is usually approximated better by a point source that generates seismic waves having a cylindrical geometry. The Radon transform must be recast into cylindrical geometry to compute a PWD for point source data (Brysk, et al., 1986),

$$R_p[f(t,r)] = F(\tau, p) = \int_{-\infty}^{\infty} f(\tau + p \cdot r, r) dr,$$

(5-9)

where $p$ is a propagation vector with azimuth $\alpha$,

and $r$ is a position vector with azimuth $\phi$.

An axisymmetric source array and horizontal layering are required to ensure energy propagation from the point source in terms of cylindrical rings. Given these assumptions, seismic data recorded along a line can be substituted for data missing from the implied areal coverage of Equation 5-9 and used to form the PWD. Point source $\tau$-$p$ transforms use this assumption in implementing Equation 5-9 and an example of this is the "proper slant stack" (Harding, 1985),

$$F(\tau, p) = \int_{-\infty}^{\infty} d\omega \exp(-i\omega\tau) \int_{0}^{\infty} r dr J_0(\omega \rho r) \int_{-\infty}^{\infty} dt \exp(i\omega t) f(t, r)$$

(5-10)

Equation 5-10 is easily interpreted as the cascaded application of a Fourier transform, a Hankel transform and an inverse Fourier transform. For single-sided recordings, it incorporates the small negative range contributions through the incoming wave part of the zero order Bessel function ($J_0$). It is worth noting that I have borrowed the name "proper slant stack" from a closely related implementation, which is formulated in terms of the angle of incidence rather than the ray parameter (Treitel, et al., 1982).

One of the main advantages in using the proper slant stack is the simplicity of its implementation. As shown in Figure 5-4, an FFT is applied to the input data prior to the
numerical evaluation of the Hankel transform using the trapezoidal rule. After performing the Hankel transform for each $\omega$ and $p$, an inverse FFT is the applied to produce the $\tau$-$p$ dataset. The FFT introduces wraparound effects that are most evident when large amplitudes fall near the edges of the time window. A larger time window reduces this effect but at greater computational expense.

![Flowchart](image)

**Figure 5-4.** A flowchart describing my implementation of the proper slant stack (PWS). The runtime is given by the number of times each loop is performed multiplied by the time each loop takes to run.

Several other point source $\tau$-$p$ transforms exist, for example the cylindrical slant stack (Brys and McCowan, 1986a) and the corrected slant stack (Chapman, 1981). Each $\tau$-$p$ transform has its own advantages and limitations and these are usually related to accuracy and cost (Kappus, et al., 1990). Selection of the most appropriate transform
depends on the intended application (e.g., velocity filtering, deconvolution), upon the source geometry (e.g., line source, point source) and on the type of seismic experiment (e.g., marine, wide aperture marine, land) (Dunne and Beresford, 1995). I expected the proper slant stack to give the highest quality PWD for marine seismic data from the Gippsland Basin, although it may prove slightly slower than other methods.

Aliasing during the forward $\tau-p$ transform can arise from undersampling in both time and space. Given that the input dataset is adequately sampled in time, $\tau-p$ aliasing occurs when $\Delta x$ is large enough so that a number of constructive summation directions exist about an event being transformed. In this situation the range of $p$ values that are transformed includes more than one cycle from neighbouring offset traces. Aliasing lowers the frequency content of the $\tau-p$ dataset thereby reducing the resolution of events and, in severe cases, it produces artificial events in the $\tau-p$ domain. The nature of $\tau-p$ aliasing is easily understood using a graphical method (Figure 5-5).

![Figure 5-5](image)

**Figure 5-5.** A graphical interpretation of aliasing in the forward $\tau-p$ transform. If slopes approaching $\phi$ are included in the transform, aliasing results from constructive summation across separate events (redrawn from Turner, 1990).

An approximate guideline for the prevention of aliasing can be derived by assuming a sinusoidal input, with period $T$ and negligible moveout between the input traces (Turner,
1990). Aliasing then occurs for \( \Delta x \tan \theta > \frac{1}{2} T \) and is strongest at \( \Delta x \tan \theta = T \).

Substituting \( T = 1/f_{\text{max}} \) and \( \tan \theta = p_r \) gives the requirement to avoid aliasing,

\[
\Delta x < \frac{1}{2 p_r f_{\text{max}}},
\]

(5-11)

where \( p_r \) is the range of \( p \) values "scanned".

For typical group intervals, an unaliased response is usually obtained by limiting the resolution in \( \tau \) due to a reduced \( f_{\text{max}} \). Alternatively it is possible to maintain resolution by permitting higher frequencies in the construction of smaller \( p \) range \( \tau-p \) panels (Fokkema, et al., 1992). Coherency measures can also be used to reduce aliasing by isolating signal in the \( \tau-p \) domain. Examples include use of the \( \tau-p \) semblance (Stoffa, et al., 1981), an amplitude-ratio testing filter (Moon, et al., 1986) and a local slope calculation (Singh, et al., 1989; Yilmaz and Taner, 1994). However, these methods tend to make unrealistic assumptions about the amplitude of reflections and should be avoided if a true PWD is required.

Aliasing in the proper slant stack is complicated by the need to evaluate the Bessel function in the Hankel transform. Harding (1985) approximated the Bessel function using an asymptotic approximation valid for \( \omega pr > 1 \),

\[
J_0(\omega pr) = \frac{1}{(2\pi \omega pr)^{1/2}} \left\{ \exp[i(\omega pr - \pi/4)] - \exp[-i(\omega pr - \pi/4)] \right\}
\]

(5-12)

This approximation is adequate for most marine seismic data except for a deep sea floor where low \( p \) events are recorded at the near offsets (Kappus, et al., 1990). Equation 5-12 consists of two zero order Hankel functions representing the incoming and outgoing wave contributions. As \( p \) increases, the incoming wave contributions cut steeply across the sloping seismic events thus lowering the aliasing frequency. Harding reduced aliasing by only including the incoming wave contribution for small \( p \) values. This is quite
acceptable as only the outgoing waves will contribute significantly to the amplitude of large $p$ events. Despite including Harding's innovation in my implementation of the proper slant stack, it was not used in the subsequent tests as the small $p$ values were muted when HVF was applied.

In implementing a point source $\tau$-$p$ transform, the finite amount of data actually transformed means that fewer terms contribute near the data edges. This prevents the complete destructive interference of non-events during transformation and is the source of coherent linear artefacts known as edge effects. These artefacts originate at the near and far offset traces, where the destructive interference of non-events is weakest during transformation. Although the offset weighting applied during a point source $\tau$-$p$ transform suppresses near offset edge effects, it has the undesirable effect of boosting those from the far offset traces (Brysk and McCowan, 1986a). These edge effects can be reduced by tapering amplitudes on the large offset traces (Brysk and McCowan, 1986b) although this further degrades the purity of the PWD (Dobbs, et al., 1990).

An important consideration in using a $\tau$-$p$ transform is the low pass filtering action associated with the slant stacking process (Phinney, et al., 1981). This effect is often overlooked because it does not appear in the forward transform expression. It is only when the inverse transform is sought that a frequency dependent term is introduced to compensate for the frequency loss incurred during both the forward and inverse transforms. This term is known as a "rho filter" (Claerbout, 1985) and for the proper slant stack its effect is to introduce a uniform phase shift (-90°) and a linear frequency weighting given by,

$$\rho_{_{\text{fps}}} (\omega) = -i\omega,$$

where the subscript FPS denotes a forward point source $\tau$-$p$ transform.
The rho filtering effect upon the inverse transform (below) indicates an overall $\omega^2$ reduction in the frequency content of an uncorrected forward/inverse $\tau$-$p$ transform,

$$\rho_{\text{IPS}}(\omega) = i\omega,$$

(5-14)

where the subscript $\text{IPS}$ denotes an inverse point source $\tau$-$p$ transform.

Using these simple relations it is possible to correct both the forward and inverse $\tau$-$p$ transforms for their frequency filtering effects, thereby ensuring data consistency throughout a $\tau$-$p$ domain processing sequence. However, later in this chapter I show that the effect of HVF upon the frequency content is not so easily predicted. In any case, other processing techniques cause additional frequency filtering effects that obscure those of the $\tau$-$p$ transform. After some testing, I elected to correct only for the phase shift during the proper slant stack to maintain consistency in the timing of events. Subsequent spectral analyses vindicated this decision by revealing only minor differences in the frequency content of stacked seismic data processed in the $t$-$x$ and $\tau$-$p$ domains.

### 5.3.2 Synthetic testing

In this section I test the proper slant stack using several synthetics with the same recording specifications (e.g., $\Delta T$, no. of channels) as the supergathers from the 1992 regional seismic survey. A key feature of these tests is the way in which I progress from simple to more complex synthetics. The simple synthetics helped to reveal the inner workings of each processing method, while the complex synthetics provided an invaluable link to the field data experiments. This progression proved essential to understanding the complexities of the subsequent field data processing.
The first test model (Figure 5-6a) consisted of a single hyperbolic reflection and was computed using the "Synthetic Trace Generator" on ProMAX\textsuperscript{TM} with the specifications tabulated below:

<table>
<thead>
<tr>
<th>T(_0) (ms)</th>
<th>V(_{RMS}) (km/s)</th>
<th>Amplitude</th>
<th>Wavelet Length (ms)</th>
<th>Dip (°)</th>
<th>f(_{dom}) (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000</td>
<td>2.00</td>
<td>1</td>
<td>300</td>
<td>0</td>
<td>35</td>
</tr>
</tbody>
</table>

Table 5-1. Specifications of the single hyperbolic reflection that comprised test model 1.

The second test model (Figure 5-6b) consisted of several isolated P- and S-wave reflections and refractions computed without surface-related multiples using the reflection matrix method. The modelling parameters were identical to those used in the previous chapters (see Table 2-1) and the depth model is tabulated below:

<table>
<thead>
<tr>
<th>(V_p) (km/s)</th>
<th>(\rho) (g/cc)</th>
<th>(h) (km)</th>
<th>(V_S) (km/s)</th>
<th>(Q_p)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.50</td>
<td>1.00</td>
<td>0.050</td>
<td>0.00</td>
<td>100</td>
</tr>
<tr>
<td>1.80</td>
<td>1.50</td>
<td>0.200</td>
<td>0.40</td>
<td>100</td>
</tr>
<tr>
<td>2.00</td>
<td>2.00</td>
<td>0.500</td>
<td>0.80</td>
<td>100</td>
</tr>
<tr>
<td>2.50</td>
<td>2.25</td>
<td>0.750</td>
<td>1.10</td>
<td>100</td>
</tr>
<tr>
<td>3.00</td>
<td>2.50</td>
<td>1.000</td>
<td>1.50</td>
<td>100</td>
</tr>
<tr>
<td>4.00</td>
<td>3.00</td>
<td>0.500</td>
<td>2.20</td>
<td>100</td>
</tr>
</tbody>
</table>

Table 5-2. The elastic depth model used to construct test model 2.

The third test model was the full elastic synthetic computed from the depth model for Well 3, although the \(\omega^2\) (rho filtering) frequency reduction was allowed for during the inverse \(\tau-p\) transform to enable a direct comparison with the exact \(\tau-p\) response (Figure 3-8). The fourth test model was the nearby supergather 842 from Line A (Figure 2-10a).
These four test models allowed the main features of the $\tau$-$p$ domain processes to be studied, although several other test models were actually used during the research. I have also omitted tests involving variations on the parameter selections. The optimal parameter selections are tabulated as they appeared on the ProMAX™ user interface (Tables 5-3, 5-4). In each test I used a dB colour display, which spanned a range of 50 dB to reveal even the weakest artefacts. This enabled an unprecedented assessment of the behaviour of the proper slant stack and played a key role in the adaptation of
hyperbolic velocity filtering to a point source $\tau$-$p$ transform. However, the screen interpolation sometimes produced spurious events in regions containing null data values.

<table>
<thead>
<tr>
<th>Plane Wave Decomposition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of ray parameters</td>
</tr>
<tr>
<td>Minimum slowness</td>
</tr>
<tr>
<td>Maximum slowness</td>
</tr>
<tr>
<td>Swapped slowness value in Hankel transform</td>
</tr>
<tr>
<td>Minimum frequency</td>
</tr>
<tr>
<td>Maximum frequency</td>
</tr>
</tbody>
</table>

Table 5-3. The parameters used in the proper slant stack (PWD) experiments.

The proper slant stack experiments were performed beyond the aliasing frequency, as described by Equation 5-11, in order to test the alias suppression properties of HVF. This aliasing is only apparent in the PWD of the first test model (Figure 5-7a) where there are several regions of mild to weak aliasing. The strongest and most extensive of these regions lies below the reflection ellipse and at small $p$ values and would interfere with any deeper reflections. The edge effect associated with the near offset (upper arrow) is quite weak due to the $p$ weighting term in the Hankel transform. In contrast, the $p$ weighting term boosted the far offset edge effect, which interfered with the deep response (lower arrow) and also produced a wraparound event near $(\tau_{\text{max}}, p_{\text{max}})$.

The PWD of the second test model (Figure 5-7b) contained several far offset edge effects, the strongest of which was associated with the direct wave (upper arrow). Weaker edge effects were associated with the P-wave refractions, although one example originated from the far offsets of a pre-critical reflection (lower arrow). Aliased events and the near offset edge effects were over 50 dB weaker than the direct wave. This example illustrated the excellent separation achieved between P- and S-wave reflections in the $\tau$-$p$ domain, as the S-wave reflections have low amplitudes at the crossover points.
Figure 5-7. The proper slant stack (PWS) applied to the first test model a) showed distinctive edge effects from the near (upper arrow) and far offsets (lower arrow). Its application to the second test model b) only indicated edge effects from the far offsets.
Figure 5-8. The proper slant stack (PWS) applied to the elastic synthetic a) and supergather 842 b) revealed strong edge effect artefacts in the field data due to the stronger refractions and guided waves.

Weak edge effects associated with the guided waves persisted in the PWS of the elastic synthetic (Figure 5-8a). It was only when this result was compared to the exact $\tau$-$p$ response (Figure 3-8) that other deleterious effects were revealed. Firstly, $\tau$-$p$ aliasing caused a significant reduction in the frequency content of the large $p$ events (e.g., the
guided waves). Secondly, the limited aperture of the $t$-$x$ domain input synthetic meant that some wide-angle events were not fully represented in the PWD, although this effect was far less important than I had expected. This observation suggests that an increased cable length may offer only minor improvements to the imaging of wide-angle events in this part of the Gippsland Basin.

A large number of strong edge effects appeared in the PWD of supergather 842 (Figure 5-8b) and these originate from the strong refractions and guided waves at the far offsets. Ignoring the edge effects, the elastic synthetic PWD provided a good match to the main reflections in the field data PWD. An exception occurred at the S-wave reflections just below and to the right of the coal sequence, which were stronger in the field data. The excellent separation between P- and S-wave reflections, and also the linear noise events, suggests that the $\tau$-$p$ domain is well suited to improving images from the Gippsland Basin. In the next section, I show how hyperbolic velocity filtering provides a effective means for suppressing both the noise events and the artefacts without adverse effects upon the plane-wave response.

5.4 HYPERBOLIC VELOCITY FILTERING DURING THE PROPER SLANT STACK

5.4.1 Theory and implementation

Hyperbolic velocity filtering (HVF) is a form of time and offset variant filtering that only allows each point in the $t$-$x$ domain to contribute to a small number of $p$ traces during a $\tau$-$p$ transform (Schultz and Claerbout, 1978). Until now, it has only been applied during the slant stack to suppress coherent noise events and transform artefacts with a significantly different moveout to the expected reflection response. HVF requires an approximate stacking velocity function to define a range of "geophysically possible" stacking velocities $\left( V_{\text{min}}, V_{\text{max}} \right)$ that specify a range of $p$ values about the true ray
parameter of an event (Equation 5-15). At each point in the input gather, the $p$ values outside this range are omitted from the transform thereby preventing the transformation of non-events (Figure 5-9). As a result, the HVF velocities must be defined at each point in the input gather.

$$\frac{x}{tV_{\text{max}}^2} < p < \frac{x}{tV_{\text{min}}^2}, \quad (5-15)$$

where $p_{\text{ave}}(t,x) = \frac{x}{tV_{\text{RMS}}^2}$ and $V_{\text{max, min}} = (1 \pm k / 100)V_{\text{RMS}}$,

where $k$ is the percentage velocity corridor halfwidth,

and the $t-x$ dependence of the velocities has been omitted to simplify the notation.

**Figure 5-9.** Hyperbolic velocity filtering in action, indicating how the stacking velocity range restricts the summation to a small range of $p$ values a) and thus suppresses aliasing and edge effects b).

HVF was first used to attenuate ground roll (Tatham, et al., 1983) and also to obtain an S-wave stacked section using the PSSP reflections resulting from mode conversion at the sea floor (Tatham and Goolsbee, 1984). Using a conventional velocity analysis, a rough estimate of the S-wave velocity function and velocity limits of $\pm 25\%$, Tatham and
Goolsbee performed two HVF/slant stacks and inverse slant stacks to obtain separate P- and S-wave stacked sections. Their S-wave section contained more noise but provided an independent structural confirmation of the P-wave section. HVF has also been used to suppress diffraction events associated with small, near surface anomalies in a hard sea floor (Noponen and Keeney, 1986).

My implementation of HVF within the proper slant stack endeavours to preserve plane-wave amplitudes while removing transform artefacts and some plane-wave events. However, HVF can not be applied in the $t$-$x$ domain because the proper slant stack is computed in the frequency domain. To get around this I use a reformulation of Equation 5-15 that applies HVF using an equivalent set of mutes in the $\tau$-$p$ domain (Mitchell and Kelamis, 1990),

$$\frac{-\tau}{2x} + \sqrt{\left(\frac{\tau}{2x}\right)^2 + \left(\frac{1}{V_{\text{max}}}\right)^2} < p < \frac{-\tau}{2x} + \sqrt{\left(\frac{\tau}{2x}\right)^2 + \left(\frac{1}{V_{\text{min}}}\right)^2}$$

(5-16)

The flowchart for my implementation indicates that each offset must be transformed separately and this increases the computation time according to the product of the number of input traces and the time it takes to compute an inverse FFT (Figure 5-10). On the positive side, a far simpler velocity interpolation is achieved because primary reflections never cross in the $\tau$-$p$ domain.

In the early $t$-$x$ domain implementations it appears that the stacking velocities were not interpolated across the $t$-$x$ domain due to the difficulties associated with crossing reflections. In their $\tau$-$p$ domain implementation, Mitchell and Kelamis also used the normal incidence velocities by restricting the range of $p$ values and appealing to the small moveout of reflection ellipses at traveltimes where the stacking velocity gradient is usually steepest. I have taken the more general approach of interpolating the stacking velocities defined at normal incidence across the $\tau$-$p$ domain. I achieve this using the
two-term traveltime approximation (Equation 5-8) as indicated in an example computed
from the Well 3 depth model (Figure 5-11).

Figure 5-10. A flowchart describing my implementation of HVF during the proper slant stack.

Until now, HVF has only been applied during the slant stack. In applying HVF during
the proper slant stack, the subsequent τ-p domain processing must allow for the
incomplete nature of its PWD. For example, an inversion requiring a full PWD would
need to simulate HVF during the forward modelling phase of its operation, or instead
model only the plane-wave phenomena expected after HVF.
Figure 5.11. Interpolated RMS velocities for the Well 3 depth model as used in my implementation of HVF during the proper slant stack.

After experiencing some difficulties in applying HVF during a point source τ-p transform, due to the offset weighting of the far offset edge effects, I introduced a number of new features into the implementation. Firstly, the accurate interpolation of stacking velocities across the τ-p domain allowed smaller velocity corridors to be used at the far offsets. I also introduced a triangular weighting (Bartlett window) to the HVF mute specification to reduce the effects of the offset weighting upon the edge of the mute. In the next section I show how this innovation helped to avoid troublesome artefacts that are otherwise obtained at the low p mute end of supercritical reflections. Finally, I incorporated a postmute to zero any events formed beyond the critical angle (see Appendix E).
5.4.2 Synthetic testing

In testing my implementation of HVF during the proper slant stack (HVFPWD) I aimed to determine whether it suppresses or compounds the problems associated with the τ-p transform artefacts. The main parameter selections (Table 5-4) were identical to those used in the PWD experiments. An appropriate velocity function was used for the simple test models and the RMS velocity function for Well 3 was used in the elastic synthetic and supergather tests. I used a velocity corridor of ±15% with the intention of suppressing the NZ2b events. I applied a fixed reduction in the velocity corridor at the far offset traces, commencing at 15% at 0.75x_{max} and decreasing in a linear fashion to 3.75% at x_{max}.

<table>
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<tr>
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</tr>
<tr>
<td>Swap slowness value in Hankel transform</td>
</tr>
<tr>
<td>Minimum frequency</td>
</tr>
<tr>
<td>Maximum frequency</td>
</tr>
<tr>
<td>Velocity corridor halfwidth %</td>
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<tr>
<td>Reduce velocity corridor near Xmax?</td>
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<tr>
<td>Low p mute tau restriction (ms)</td>
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</tr>
<tr>
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<tr>
<td>SELECT Velocity parameter file</td>
</tr>
</tbody>
</table>

Table 5-4. The parameters used in the hyperbolic velocity filtering (HVFPWD) experiments.

The artefacts that persist after HVFPWD were interpreted by comparing the results of tests run with and without Bartlett windowing on the mute specification. Without Bartlett windowing, test model 1 contained three artefacts that detract from the positive features of the result (Figure 5-12a). When compared to Figure 5-7a, it was clear that the aliased events, the near offset edge effect and most of the far offset edge effect were suppressed by HVFPWD. However, a large section of the far offset edge effect remained (white arrows) because it fell within the pass zone. A mild elliptical shaped artefact (upper black arrow), with its curvature related to V_{min}, was generated at the high
$p$ edge of the HVF mute. The low $p$ edge of the HVF mute generated a much stronger artefact (lower black arrow) due to the offset weighting term in the proper slant stack. This artefact originated from the large $p$ part of the reflection ellipse so that supercritical effects could render it even stronger.

Without Bartlett windowing, test model 2 contained further examples of these artefacts including a far offset edge effect from the direct wave (lower white arrow) that fell within the pass zone alongside the deep primary reflection (Figure 5-12b). The vertical shaped artefacts were related to the shape of the interpolated velocity field and were again caused by the low $p$ edge of the HVF mute. A particularly strong event caused by the low $p$ edge of the HVF mute (black arrow) showed the reverse curvature that is characteristic of this type of artefact.

These unique artefacts can also be seen in the papers by Mitchell and Kelamis but are less obvious due to the lack of offset weighting in the slant stack. The offset weighting in the proper slant stack meant that the amplitudes passed by each HVF mute were boosted towards the low $p$ edge. These artefacts can be reduced by applying a Bartlett window to the mute specification. For test model 1, this reduced the HVF artefacts by over 20 dB while maintaining consistent amplitudes near the centre of the event (Figure 5-13a). Similar improvements were obtained with test model 2, where the vertical shaped artefacts are no longer visible (Figure 5-13b). In addition to the excellent artefact suppression, almost all of the S-wave reflections were suppressed and only the short period P-wave interbed multiples remain.
Figure 5-12. HVFPWD applied to the first test model a) and the second test model b) without the Bartlett weighting applied to the mute specification. The white arrows indicate the far offset edge effects that fell within the pass zone, while the black arrows indicate the artefacts caused by the sharp edges of the HVF mute.
Figure 5-13. HVFPWD applied to the first test model a) and the second test model b) with the Bartlett weighting applied to themute specification.
A similar level of improvement was observed in the tests performed using the elastic synthetic and the supergather. Without Bartlett windowing, these tests contained strong artefacts associated with the low $p$ edge of the HVF mute, which can be seen from their interference through the centre of the main reflections (Figure 5-14). These events are strongest near the Miocene carbonates where they are boosted by the high amplitude supercritical reflections. Although not apparent in the dB colour display, the reverse curvature of this type of artefact was observed in raw amplitude "wiggle" displays. Strong far offset edge effects from the guided waves were identified below the coal sequence and to the right of the target zone. The "wiggle" displays also helped to confirm the linear character of this type of artefact.

The use of Bartlett windowing brought about a dramatic improvement in the elastic synthetic and supergather results (Figure 5-15). Only a small portion of the guided wave edge effects remained, more so in the supergather due to the greater strength of the recorded guided waves. In comparison with the PWD result (Figure 5-8b), HVF with Bartlett windowing has gone a long way towards eliminating the $\tau$-$p$ transform artefacts. It is possible that other windowing functions might give slightly better results by passing more signal while still reducing the sharp mute edges.

The guided wave edge effects are the only $\tau$-$p$ transform artefacts that now remain within 50 dB of the strongest events. It is worth recognising the role of the velocity interpolation in keeping these artefacts as small as possible. With accurate velocities specified at the wide-angle events, it became possible to apply smaller velocity corridors at the far offsets. Without this procedure, the guided wave edge effects appeared considerably wider in the HVFPWD of the elastic synthetic and the supergather (Figure 5-16). Furthermore, the wide-angle events at the Miocene carbonates have quite a different appearance when the normal incidence velocities are used to apply the HVF mutes.
Figure 5-14. HVFPWD applied to the elastic synthetic a) and the supergather b) without the Bartlett weighting applied to the mute specification.
Figure 5.15. HVFPWD applied to the elastic synthetic a) and the supergather b) with the Bartlett weighting applied to the mute specification.
Figure 5-16. HVFPWD applied to the elastic synthetic a) and the supergather b) with Bartlett weighting but without velocity interpolation and the velocity corridor reduction at the far offsets.

From Figure 5-15 it is clear that many of the main noise contributions to seismic data from the Gippsland Basin were suppressed by HVF. HVF is an excellent method for suppressing guided waves and S-wave reflections because it discriminates against these events on the basis of both velocity and angle of incidence. The P-wave surface
multiples appear partially attenuated and only the primaries and short period multiples remain almost fully intact. Deconvolution in the $\tau$-$p$ domain is now well placed to attack these multiples because many of the possible distractions have been removed.

Changes in noise content due to HVFPWD also appeared in spectral analyses of the elastic synthetic. The spectrum of the rho filtered version of the elastic synthetic (Figure 5-17a) indicated the low frequency enhancement of the $\tau$-$p$ transform when compared to the spectral analysis obtained from its PWD (Figure 5-17b). However, the $\tau$-$p$ reorganisation of the dataset provided differences in the averaged amplitude spectra that the rho filter ($-i\omega$) could not account for. Further reductions in the frequency content resulted from the use of HVF (Figure 5-17c) and were probably due to the suppression of the high frequency guided waves. Finally, the spectral analysis of the PWD closely approximated that of the exact $\tau$-$p$ reflection response (Figure 5-17d) and this confirmed the ability of the proper slant stack to provide a high quality PWD.

**Figure 5-17.** Averaged amplitude spectra of the rho filtered, $\tau$-$x$ domain elastic synthetic a), its PWD b), its HVFPWD c) and its exact $\tau$-$p$ reflection response d), shown from 0-70 Hz.
5.5 OTHER \( \tau-p \) DOMAIN PROCESSING APPLICATIONS

In this section I provide a brief outline of the theory behind the other \( \tau-p \) domain processing applications used in this thesis. Some common processing applications, such as DMO and prestack migration, were not investigated as they were not expected to prove important to the subcoal imaging problem.

5.5.1 The inverse transform

The ability to stack seismic data in the \( \tau-p \) domain meant that the inverse transform was not required in my \( \tau-p \) domain processing sequence. However, it is used in the reflectivity method to construct a \( t-x \) domain version of the elastic synthetic. The inverse transform for the proper slant stack is almost identical in implementation to the forward transform. The main difference is the rho filter term \((\omega^2)\) in Equation 5-17, which I included as an option to be used when constructing \( t-x \) domain synthetic to test the \( \tau-p \) transform methods.

\[
f(t, r) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \omega^2 d\omega \exp(-i\omega\tau) \int_{0}^{\infty} p dp J_0(\omega pr) \int_{-\infty}^{\infty} dt \exp(i\omega t) F(\tau, p) \tag{5-17}
\]

The inverse transform suffers from aliasing and edge effects in the same manner as the forward transform, with the requirement to avoid aliasing given by Equation 5-18. Although \( \Delta p \) can be set to avoid aliasing, events at the large \( p \) values can give rise to particularly strong edge effects due to the \( p \) weighting term in the Hankel transform.

\[
\Delta p < \frac{1}{2x_r f_{max}}, \tag{5-18}
\]

where \( x_r \) is the range of \( x \) values "scanned".

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5.5.2 The elliptical moveout correction and stack

The velocity analysis and stacking of \( \tau-p \) gathers is achieved with significant theoretical advantages using an elliptical moveout correction (EMO) to flatten reflection ellipses (Cutler and Love, 1980). For example, NMO experiences difficulties with crossing reflections and this is avoided in the \( \tau-p \) domain where primary P-wave reflection ellipses never cross. The EMO correction also minimises wavelet stretch since shallow reflections have a smaller moveout in the \( \tau-p \) domain (Mitchell and Kelamis, 1990). Despite having a larger moveout, deeper reflections experience a natural reduction in wavelet stretch as a result of Q attenuation in the shallow subsurface (Figure 5-18).

![Diagram](image)

**Figure 5-18.** The elliptical moveout correction (EMO) reduces the wavelet stretch of shallow, small moveout reflections in the \( \tau-p \) domain without adverse affects upon deep, large moveout reflections.

Perhaps the most important advantage of stacking in the \( \tau-p \) domain is the wider applicability of the two-term EMO approximation to the exact reflection traveltime for a horizontally layered medium (Appendix C). A simple explanation derives from the observation that equal intervals in \( p \) correspond to increasing intervals in \( x \), so that the
best stacking ellipse is naturally weighted towards zero offset where Dix's approximation is most valid (Stoffa, et al., 1982).

By applying the $\tau$-$p$ transform to CMP gathers it is possible to stack seismic data in the $\tau$-$p$ domain thus eliminating the need for the inverse transform. This reduces the $\tau$-$p$ processing time by half and avoids the artefacts associated with the inverse transform. Another advantage is the exact spherical divergence correction that is obtained from the PWD (Wang and McCowan, 1989). However, previous studies have suggested that the advantage of stacking in the $\tau$-$p$ domain is largely held in the knowledge that the amplitudes should be more representative of the reflectivity response (Goodrum, et al., 1986; Brysk, et al., 1987).

My implementation of the EMO correction includes a stretch factor that mutes the corrected gather beyond a specified percentage of $p_{\text{crit}}$. For each $\tau(0)$ and each $p$, the correction is performed by interpolating the amplitude in the location described by,

$$\tau_{\text{EMO}} = \tau(0)\sqrt{1 - p^2 V_{\text{EMO}}^2}$$

(5-19)

I tested the EMO correction upon the HVFPWD of the elastic synthetic (Figure 5-15a) using the RMS velocity function from Well 3 and a fixed stretch mute of 90% of $p_{\text{crit}}$ (Table 5-5). The result appeared quite reasonable and suggested that some intermediate period multiples persist in the target zone (Figure 5-19).

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</thead>
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<td>SELECT Velocity parameter file</td>
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</table>

Table 5-5. The parameters used to test the elliptical moveout (EMO) correction.
Figure 5.19. The elliptical moveout correction (EMO) applied to the HVFPWD of the elastic synthetic (Figure 5.15a).

5.5.3 S-wave elliptical moveout correction

Substantial mode conversion can occur at hard sea floors (White and Stephen, 1980) enabling the S-wave arrivals from deeper reflectors to be recorded. The S-wave reflections are often referred to as PPSP/PSPP and PSSP to denote their propagation through the water as P-waves. S-waves might assist a seismic interpretation in a number of ways. A stacked S-wave section would provide an independent structural confirmation of events in the P-wave stack. Under favourable conditions it may even be possible to exploit the greater resolution (smaller λ) of S-wave events. Velocity and amplitude analyses could produce direct estimates of the S-wave interval velocities for use in AVA studies, modelling and inversion. Furthermore, the ratio $V_p/V_S$ is a direct hydrocarbon indicator due to the S-wave velocity's insensitivity to fluid content (Tatham and Stoffa, 1976).
As shown in Figure 5-2, mode converted events have a clearer expression in the $\tau$-$p$ domain due to their smaller moveout and their angular separation in the $\tau$-$p$ domain. Using a rough estimate of the PSSP stacking velocity, HVF could be applied to provide gathers containing enhanced PSSP reflections. An EMO correction that decomposes the event into its P- and S-wave contributions would then enable a velocity analysis of the PSSP arrivals (Carrion and Hassanzadeh, 1985). For each $\tau_{\text{PSSP}}(0)$ below $\tau_{W}(0)$ and each $p$, this correction is performed by interpolating the amplitude in the location described by,

$$\tau_{\text{EMO}} = \tau_{W}(0)\sqrt{1 - p^2 V_w^2} + (\tau_{\text{PSSP}}(0) - \tau_{W}(0))\sqrt{1 - p^2 S_{\text{EMO}}^2} \quad (5-20)$$

where $V_w$ is the P-wave velocity in water, $\tau_{W}(0)$ is the two-way travelt ime to the water bottom, and $S_{\text{EMO}}$ is the stacking velocity for PSSP events.

### 5.5.4 $\tau$-$p$ domain semblance analysis

As shown in Chapter 7, the theoretical advantages of the EMO correction also help to improve the resolution of semblance peaks computed from $\tau$-$p$ gathers. I obtained further improvements in resolution from the use of supercritical reflections. My implementation of the $\tau$-$p$ domain semblance analysis simply applies the EMO correction over a range of trial stacking velocities and calculates the semblance envelope over a short time window. I also programmed an equivalent $t$-$x$ domain semblance analysis in order to provide meaningful comparisons.

### 5.5.5 Deconvolution

In the $t$-$x$ domain, the periodicity of multiples varies with offset over the gather and also between successive multiple reflections on each trace (Figure 5-20). In the $\tau$-$p$ domain, successive multiples are exactly periodic on each $p$ trace because they share the same
angle of incidence within the multiple-generating layer (Taner, 1980). Furthermore, the amplitude decay of the multiples is consistent as they also share a common reflection coefficient. A horizontally layered subsurface is not essential to ensure periodic multiples in the $\tau$-$p$ domain as only the main multiple-generating layers require a uniform thickness.

![Diagram](image)

**Figure 5-20.** The exact periodicity of multiple reflections in the $\tau$-$p$ domain b) for the case where first-order water bottom multiples are recorded from a flat sea floor a).

The periodicity of multiples still varies across the $\tau$-$p$ gather, with each $p$ trace requiring different prediction lengths as described by Equation 5-21 (Alam and Austin, 1982). This correction could be easily applied during a $\tau$-$p$ domain deconvolution and would probably improve the suppression of long period multiples. At this stage, I have not implemented this procedure because I only expect to apply deconvolution at the small $p$ traces following HVFPWD.

$$\Delta \tau_m(p) = \Delta \tau_m(0) \sqrt{1 - p^2 v^2}$$  \hspace{1cm} (5-21)

where $v$ is the interval velocity of the multiple-generating layer.
5.6 CONCLUSIONS

The backbone of my $\tau$-$p$ domain processing sequence is the application of HVF during the proper slant stack. This step ensures that the potentially troublesome $\tau$-$p$ transform artefacts are kept to a minimum and provides an excellent suppression of guided waves and S-wave reflections. It delivers a predominantly P-wave dataset that can then be deconvolved and stacked in the $\tau$-$p$ domain with further advantages over $t$-$x$ domain techniques. It achieves all of this without prior muting or filtering of the strong linear events so that wide-angle reflections can be used to improve the deep stack response.

It is worth reiterating that many of the innovations that I introduced into the $\tau$-$p$ processes originated directly from the tests that were run. In using a testing procedure that encompassed both simple and complex models, I was able to develop a detailed understanding of the artefacts peculiar to my implementation. The solutions to these problems were then tested on field data with support from the equivalent tests performed upon the elastic synthetic.

Irrespective of the likely advances in computing over the next decade, $\tau$-$p$ processing techniques will only be adopted by the oil industry if the increase in cost is offset by the reduction in exploration risk. My ongoing research has shown that the cylindrical slant stack formulation (Brysk and McCowan, 1986a) might provide a basis for an accurate and economical implementation of HVF during a point source $\tau$-$p$ transform (Appendix D). With such an implementation the full range of $\tau$-$p$ applications might then become amenable, such as 3D seismic data processing, AVA analysis, full waveform inversion and S-wave data processing from conventional and ocean bottom seismic acquisition.
Chapter 6

Processing through to stack in the \( t-x \) and \( \tau-p \) domains

6.1 INTRODUCTION

Previous investigations into \( \tau-p \) domain processing have been unable to claim significant improvements over their \( t-x \) domain counterparts (Goodrum, et al., 1986; Brysk, et al., 1987; Wang and McCowan, 1989; Vanić, 1989). As a result, seismic data continue to be processed in the \( t-x \) domain, with only temporary excursions into other domains for filtering purposes. It is likely that the \( \tau-p \) transform artefacts hindered these early attempts at \( \tau-p \) domain processing, in which point source transforms were applied without artefact suppression measures. Mitchell and Kelamis (1990) made stronger claims about the success of their \( \tau-p \) processing stream, in which they applied HVF during the slant stack. Their field data example showed a considerable improvement in continuity although short period multiples appeared to dominate the result.

The insight gained from my elastic modelling study indicated that several noise types deteriorate the subcoal image. In the previous chapter, I reported that HVF during the proper slant stack suppresses guided waves and S-wave reflections and is therefore likely to further improve the \( \tau-p \) domain deconvolution of the remaining multiples. An elastic synthetic computed with greater Q attenuation in the near surface region (Figure 4-8a) provided a glimpse of wide-angle reflections in the target zone. These events are muted in the "production" processing, therefore further improvements in the subcoal image might be obtained by stacking these in the \( \tau-p \) domain.

Using a 4 km segment of Line A, I sought to determine whether the advantages of \( \tau-p \) domain processing could be achieved through to stack. From the previous chapter, I expected to find: 1) that the \( \tau-p \) transform artefacts have very little influence upon the
stack response; 2) significant improvements due to the stacking of wide-angle reflections, and 3) a better deconvolution in the $\tau$-$p$ domain. I confirmed these expectations by studying the performance of my $\tau$-$p$ domain processing stream relative to the "production" processing used in the 1992 regional survey. Equivalent tests were again performed upon the elastic synthetic computed from Well 3 to assist my interpretation of the field data results. In taking this approach, I observed gradual improvements in the subcoal image due to each stage of the $\tau$-$p$ domain processing stream.

6.2 PROCESSING STREAMS FOR THE GIPPSLAND BASIN

I designed my $\tau$-$p$ domain processing sequence to provide meaningful comparisons to a $t$-$x$ domain processing sequence that resembled the "production" flow used in the initial processing of the 1992 regional survey (Figure 6-1). The sequences differ most in the pre-processing stage because trace muting, velocity filtering and the spherical divergence correction are addressed during the $\tau$-$p$ transform itself. Several optional processes, such as FK demultiple and gap deconvolution, were included to provide a full assessment of the advantages of each processing stream. In the $\tau$-$p$ domain, it is sometimes necessary to follow a deconvolution with an inside $p$ mute in order to suppress the multiples not predicted due to the shape of the low $p$ HVF mute.

Some standard seismic processes, such as DMO and prestack migration, were omitted from the test processing streams as they were not required for my field data examples. It is possible that these also carry theoretical advantages when implemented in the $\tau$-$p$ domain. Other processes that may be important in some exploration permits, such as statics, trace equalisation and spectral whitening, can be applied either before or after the test processing streams. For the examples presented in this chapter, spike and noise burst editing was applied to the input data and I later examine the effect of a poststack migration upon the data output from each sequence.
The CMP sorting stage is crucial to both processing streams as it can provide considerable suppression of linear noise events according to the stack array principle (Anstey, 1986). For both processing sequences there appears to be considerable merit in enforcing the stack array principle from the start by processing supergathers rather than decimated and/or preconditioned CMP gathers. Although increasing the runtime, the finer spatial sampling of the supergathers helps to eliminate the transform artefacts generated during the $\tau-p$ transform and the 2D FFT used in the FK filtering processes.

Figure 6-1 A $t-x$ domain processing stream for marine seismic data a) and its $\tau-p$ domain "equivalent" b). The key differences occur in the pre-processing and CMP sorting stages, with options (dashed boxes) to perform FK demultiple, gap deconvolution and velocity analysis prior to stacking.
To be consistent between the experiments, I used the same mutes, filters, gain functions and a single stacking velocity computed from the depth model for Well 3 (Table 6-1). This ensured that the differences between each result were solely due to the inclusion of a new process. I have arranged the figures in a way that allows the reader to simulate the screen swapping used to analyse the differences between each result. I have elected not to present my attempts at improving upon the "production" processing. These experiments resulted in only minor, and sometimes dubious, changes to the subcoal image and would only serve to confuse the conclusions drawn from this chapter.

The parameter selections were based upon the "production" processing stream and from the tests performed in the previous chapter. An important exception is the number of ray parameters (NP) used in the τ-p transforms, which should be just large enough to provide a stacked trace that is independent of NP. Stacking in the τ-p domain will then have an effective fold equal to the number of traces in the input t-x gather. I selected NP = 90 based on the extent to which the field data stack response had stabilised following the application of HVFPWD for different NP values.

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<td>PWD</td>
<td>NP=90, νr=0.01-0.45 s/km, f=0-70 Hz</td>
</tr>
<tr>
<td>processing</td>
<td>HVFPWD</td>
<td>NP=90, νr=0.01-0.45 s/km, f=0-70 Hz, k=15%, V™1m model</td>
</tr>
<tr>
<td>t-x stacking</td>
<td>NMO</td>
<td>V™1m model, 50% stretch mute</td>
</tr>
<tr>
<td></td>
<td>Stack</td>
<td>Mean normalisation</td>
</tr>
<tr>
<td></td>
<td>Form stack array from CDPs</td>
<td>Add traces and divide by 2</td>
</tr>
<tr>
<td>τ-p stacking</td>
<td>EMO</td>
<td>V™1m model, 90% p™crit mute</td>
</tr>
<tr>
<td>Display</td>
<td>BP filter (Ormsby)</td>
<td>5-10-60-70 Hz</td>
</tr>
<tr>
<td></td>
<td>True amplitude recovery</td>
<td>5 dB/s</td>
</tr>
<tr>
<td></td>
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</tr>
<tr>
<td></td>
<td>Postscript plotting</td>
<td>97.5% not clipped</td>
</tr>
</tbody>
</table>

Table 6-1. Processing parameters used in each flow segment during this chapter.
6.3 PRE-PROCESSING

The pre-processing stage of the "production" flow attempts to reduce the influence of the strong refractions and guided waves that dominate the far offset recordings. A large part of their suppression occurs during CMP stacking provided that the field data acquisition satisfies the stack array criterion (Anstey, 1986). For marine seismic data, acquired with the group interval equal to the group length, the source interval must be half the group interval to meet this criterion. In less specific terms, the linear noise events must not become aliased in the CMP gathers after sorting from the shot records. The stack array concept can also be viewed in terms of the wavenumber response of the hydrophone array and the CMP array response. With the elements of each array equally weighted, the stack array is obtained when the notches in the hydrophone array response coincide with, and thus cancel, alias peaks in the CMP array response (Duncan and Beresford, 1993).

In the "production" processing stream, a "2 on 1" trace decimation is performed in the shot domain prior to CMP sorting. This is achieved using a rough NMO correction, which is reversed after adding together each set of adjacent traces. For Line A, this halves the number of CMP gathers and also halves the effective offset sampling interval when the CMP gather is moveout corrected and stacked. This provides a substantial reduction in the aliasing of coherent linear events in the CMP domain, although a further "2 on 1" decimation is required to satisfy the stack array criterion. If necessary, this can be achieved after stacking by adding adjacent stacked traces. An alternative approach is to interleave the traces of adjacent CMP gathers to form a supergather. For Line A, the stack array criterion can be satisfied by forming one supergather for every 4 CMP gathers. This option is particularly attractive given the planned application of a point source $\tau$-$p$ transform, in which edge effects will be reduced when more traces are transformed. In this section, I examine the effects upon the $t$-$x$ and $\tau$-$p$ domain processing due to several pre-processing options.
6.3.1 \( t-x \) domain

The influence of pre-processing in the \( t-x \) domain was assessed relative to processing stream 1, in which I did not attempt to specifically suppress the linear noise events. For consistency with the "production" flow, I included a single "2 on 1" decimation prior to FK demultiple and stacking. It was not surprising to find that FK demultiple had very little effect upon the final result (Beresford, et al., 1993). The stacked section was dominated by the linear noise events and only the strongest reflections were visible (Figure 6-2). There was a change in the character of the noise interference below 3.0 s where the guided waves reach the end of the cable. From this result it is clear that the guided waves must be suppressed before tackling the other noise contributions.

To suppress the linear noise events I applied muting and then FK filtering in the shot domain. The results of these tests were quite similar to each other and to the result obtained when both were used. Processing stream 2 resembled the "production" flow in applying a top mute to remove the P-wave refractions and then FK filtering using a fan filter with an upper limit of 2.20 km/s. The "2 on 1" decimation and FK demultiple were again applied to produce a stacked section in which the reflections were visible at all times (Figure 6-3).

Side-scattered noise appeared below 2.0 s with its characteristic linear moveout across the stacked section. In Chapter 3, I observed both forward and backscattered events in shot records from Line A. By comparing this result with that obtained using processing stream 1 (Figure 6-2), I realised that the FK filter had suppressed the forward scattered events, which dip upwards in the shooting direction (right side of figure). If necessary, the remaining backscattered noise could be suppressed using an appropriate FK filter, either before or after stacking.
Figure 6-2. Line A, t-x domain processing stream 1: No pre-processing, decimated CMP gathers and FK demultiple. Only the strongest reflections are visible amongst the guided wave interference, until 3.0 s where the guided waves reach the end of the cable.
Figure 6-3. Line A, t-x domain processing stream 2: Mute/FK filter pre-processing, decimated CMP gathers and FK demultiple. Signal and noise reflections are now visible at all times although backscattered noise appears in the deep response.
Figure 6-4. Line A, $t$-$x$ domain processing stream 3: Mute/FK filter pre-processing, decimated CMP gathers and FK demultiple with full stack array formed after stacking. The target zone response was unchanged apart from the reduction in the backscattered noise.
To determine whether the linear noise events still persist in the stacked section I enforced the stack array criterion by performing a second trace decimation after stacking. Processing stream 3 was equivalent to processing stream 2, except that the full stack array was formed prior to display. The stacked section showed a significant reduction in the side-scattered noise, although several of the stronger events remained (Figure 6-4). Allowing for the reduced number of traces, it appeared that an interpretation of the target zone response would remain unchanged. This suggested that muting and FK filtering had suppressed the linear noise events.

An elastic synthetic computed with greater Q attenuation in the near surface region (Figure 4-8a) indicated that the target zone reflections continue out to the far offsets. I found that the muting had removed a large portion of these wide-angle reflections, which might otherwise be used to improve the subcoal image. I made very little progress in establishing a trade-off between suppressing the linear events and using the wide-angle reflections. There are several plausible explanations for this: 1) the linear noise events are simply too strong (and variable) to rely solely upon the stack array for suppression; 2) the wide-angle reflections suffer from NMO stretch and are automatically muted prior to stacking, and 3) FK demultiple generates artefacts that interfere with the weak target zone reflections.

It is likely that each of these effects played a part in reducing the quality of the t-x domain sections. I found considerable support for the third explanation by forming the stack array prior to stacking. By applying FK demultiple to supergathers rather than decimated CMP gathers, I observed a considerable improvement in the continuity of signal and noise reflections. It is likely that the guided waves produce artefacts during FK demultiple in a similar manner to those generated during HVFPWD. In contrast, the equivalent tests performed upon the elastic synthetic were insensitive to the CMP sort method because the weaker (isotropic) linear events did not produce strong artefacts.
6.3.2 \( \tau-p \) domain

Pre-processing was not applied in the \( \tau-p \) domain processing streams, as the linear noise events were more conveniently muted following or during the \( \tau-p \) transform. Although not indicated in the flowchart (Figure 6-1b), the \( \tau-p \) transform can be applied to either decimated CMP gathers or supergathers. As suggested in the previous section, stronger edge effects are produced when fewer traces are transformed, and these could easily contaminate the weak target zone reflections. In applying HVFPWD to the decimated CMP gather 842, I observed strong edge effects throughout much of the \( \tau-p \) transformed gather (Figure 6-5a). However, when HVFPWD was applied to supergather 842, these artefacts were almost entirely eliminated and appear unlikely to influence the stack response (Figure 6-5b).

For both the \( t-x \) and \( \tau-p \) processing streams, there appears to be substantial motivation for meeting the stack array criterion prior to stacking. Therefore, supergathers should be used whenever a transform processing method is applied in the CMP domain. For the moderate slopes encountered in the much of the Gippsland Basin it is likely that the reduction in lateral resolution is tolerable and should enable milder muting to remove the linear noise events.
Figure 6-5. Strong edge effects were generated when HVFPWD was applied to the decimated CMP gather 842 a), but these were well suppressed when HVFPWD was applied to supergather 842 b).
6.4 HYPERBOLIC VELOCITY FILTERING

In this section, I demonstrate that the application of HVF during the proper slant stack (HVFPWD) fulfills its theoretical expectations regarding noise and artefact suppression. I achieved this by computing sections using the proper slant stack (PWD) from both decimated CMP gathers and supergathers. This approach revealed gradual improvements in the target zone response resulting from the suppression of the guided wave edge effects. By allowing for the edge effects that persist in the PWD results, I observed an improvement over the "production" processing due to the stacking of wide-angle reflections in the target zone.

I first applied the proper slant stack (PWD) to decimated CMP gathers with the full stack array formed after stacking (Figure 6-6). When compared to its equivalent $t$-$x$ domain result (Figure 6-4), the greatest difference occurred at the Miocene carbonates where the supercritical reflections were included in the stack. Their different phase and frequency content reduced the time resolution of the stack response. Several new events appeared in the target zone (1.7-2.0 s) possessing the low frequency character expected after transmission through the coal sequence. The frequency character of the coal sequence reflections was unaffected suggesting that this observation can not be explained by the frequency reduction associated with the $\tau$-$p$ transform. Without the FK filter pre-processing, both forward and backscattered noise appeared in the deep response.

In applying the proper slant stack to supergathers the guided wave edge effects were significantly reduced in the $\tau$-$p$ gathers and I observed several subtle improvements in the target zone response (Figure 6-7). Examples of improved continuity at 1.55 s and between 1.8-2.0 s suggested that edge effects were affecting the weak target zone reflections in the previous result. The greater continuity of the shallow events also indicated that the edge effects were reduced, although it is not clear whether they have been eliminated.
There are several indications that HVFPWD brought further improvements in the target zone response when it was applied to the supergathers from Line A (Figure 6-8). Firstly, the gathers contained only weak edge effects at the large $p$ values in the target zone (Figure 6-6b). Secondly, the greater strength, continuity and low frequency character of reflections in the target zone (1.5-2.0 s) suggested that the NZ2 events were also suppressed to some extent. The NZ3 surface multiples remained intact but this could be due to the lack of deep targets as indicated in semblance analyses from this part of Line A (see Chapter 7). Finally, the same target zone events appeared when HVF was applied during the slant stack and cylindrical slant stack (Appendix D), thereby providing an independent confirmation. The slant stack result provided greater support as its edge effects were significantly weaker in the $\tau$-$p$ gathers.

The computation time can be reduced by only including the ray parameters that are used to form the target zone stack response. This reduction acted as a large $p$ mute upon the Miocene carbonate supercritical reflections, giving the shallow section a similar appearance to the $t$-$x$ domain results (Figure 6-9). The changes in this section reaffirmed the notion that $\tau$-$p$ domain processing is able to exploit a greater portion of reflection events.

Each of the $\tau$-$p$ domain sections showed further improvements in the target zone events due to the greater suppression of the guided wave edge effects. I observed the same events by applying HVF during the slant stack (Appendix D) and this suggests that the transform artefacts were under control in the HVFPWD result. Despite interference from the guided wave edge effects, it was possible to identify the new target zone events in the PWD result obtained from decimated CMP gathers. By comparing this result to its $t$-$x$ domain equivalent it was clear that the wide-angle reflections had been stacked in the $\tau$-$p$ domain. The co-interpretation of these $\tau$-$p$ domain brute stacks suggested that the stack array, wide-angle reflections, the EMO correction and HVF each contribute to the improved target zone image.
Figure 6-6. Line A, τ-p domain processing stream 4: Decimated CMP gathers and PWD with full stack array formed after stacking. When compared to its equivalent t-x domain result (Figure 6-4), the greatest difference occurred at the Miocene carbonates where the supercritical reflections were included in the stack. Several new events appeared in the target zone (1.7-2.0 s) possessing the low frequency character expected after transmission through the coal sequence.
Figure 6.7. Line A, $\tau$-$p$ domain processing stream 5: Supergathers and PWD. Further improvements in the target zone (1.55 s, 1.8-2.0 s) resulted from the reduction in guided wave edge effects.
Figure 6-8. Line A, $\tau$-$p$ domain processing stream 6: Supergathers and HVPPWD. The new target zone events appeared with greater continuity and strength resulting from the suppression of the guided wave edge effects and the NZ2 reflections, although the NZ3 surface multiples were only slightly reduced.
**Figure 6-9.** Line A, $\tau$-$p$ domain processing stream 6, stacked using only the first half of the $\tau$-$p$ gather. The changes in this section reaffirm the notion that $\tau$-$p$ domain processing is able to exploit a greater portion of reflection events.
6.5 SHORT PERIOD DECONVOLUTION

A significant theoretical advantage of $\tau$-$p$ domain processing is that multiple reflections have a constant period and uniform amplitude decay along each $p$ trace after a point source $\tau$-$p$ transform. Therefore multiples, particularly those with long periods, might be better suppressed by applying gap deconvolution in the $\tau$-$p$ domain. For seismic data from the Gippsland Basin, the coal sequence interbed multiples (NZ1) persist in the early part of the target zone and also act as a transmission effect upon deeper signal and noise events. The consistency of S-wave contributions expected along each $p$ trace might also help to improve the deconvolution of these events in the $\tau$-$p$ domain. It is the uniform amplitude decay aspect, due to the angular decomposition and the spherical divergence correction, which offers significant hope for an improved short period deconvolution.

I obtained strong support for these arguments from spectral analyses of the $t$-$x$ and $\tau$-$p$ gathers obtained before and after a short period gap deconvolution. Figure 6-10 contains the amplitude spectra computed at each trace of these gathers, with each analysis scaled as a percentage of the maximum power in that analysis. The $f$-$x$ spectral analysis of the pre-processed, decimated CMP 842 indicated several large peaks associated with the water bottom and NZ1 multiples (Figure 6-10a). The same peaks were present in the $f$-$p$ spectral analysis of supergather 842 obtained after HVFPWD (Figure 6-10b). In each case, the position of the spectral peaks varied across the gather according to the moveout of the multiples. However, the relative amplitude of the peaks was only preserved across the entire analysis for the $\tau$-$p$ domain result. This is a clear indication of the deconvolution advantages that can be gained in the $\tau$-$p$ domain.

After some trial and error testing using the elastic synthetic, I applied gap deconvolution to the $t$-$x$ and $\tau$-$p$ gathers using a 40 ms gap length and a 200 ms operator length. In both cases, this removed the main spectral peaks to give a bell-shaped spectrum, which I found preferable to a fully whitened spectrum. The $f$-$x$ spectral analysis showed a strong
offset dependence, presumably caused by the aperiodicity of multiples in the \( t-x \) domain (Figure 6-10c). The \( f-p \) spectral analysis was more uniform across the \( \tau-p \) gather, except at the small \( p \) traces where the purity of the PWD was compromised by the HVF mute (Figure 6-10d). The \( \tau-p \) spectrum fell away more steeply at the higher frequencies and this might be related to the frequency reduction suffered during the \( \tau-p \) transform.

**Figure 6-10.** Normalised \( f-x \) and \( f-p \) spectral analyses computed from decimated CMP gather 842 processed in the \( t-x \) domain a) and supergather 842 after HVFPWD in the \( \tau-p \) domain b). The spectral analyses computed after short period gap deconvolution applied in the \( t-x \) domain c) and the \( \tau-p \) domain d) indicated a much better result in the \( \tau-p \) domain.

I applied the same deconvolution during the "production" processing flow and found that some of the more conspicuous multiples were well suppressed (Figure 6-11). This was probably due to the dependence of the \( t-x \) domain section upon the near offset traces,
which the spectral analyses indicated were deconvolved adequately. Despite this, the
target zone events did not possess the low frequency character expected following
transmission through a coal sequence. The t-x domain section can not be trusted because
it appears that the NZ2 events are still stronger than the target zone primaries.

The short period deconvolution brought about several significant changes in the target
zone when applied during the τ-p processing stream (Figure 6-12). There appeared to be
considerable suppression of the NZ1 multiples within the coal sequence and the early part
of the target zone, especially near the centre of the section between 1.5-1.7 s. Spectral
analyses averaged over the results presented in this section indicated a significant
increase in the higher frequencies, which accounts for the slightly noisier appearance of
the deconvolved sections. Further work is required to determine whether the τ-p domain
deconvolution delivered a stacked section that can be tied to nearby well-logs.

6.6 POSTSTACK PROCESSING

Although most of the events in the 4 km section from Line A are flat-lying, it is
important to consider the effects of image correction processes that might be required to
interpret the results from other lines. A poststack migration was applied in the
"production" flow, so I applied a similar migration (selected from those included with
ProMAX™) to the t-x and τ-p results shown in the previous section. For Line A, the
primary effect of the finite difference time migration was the dip filtering of the side-
scattered energy. For the t-x domain stack, this provided greater continuity in the
reflections, although the deep response now appears slightly "wormy" (Figure 6-13). In
the τ-p domain stack, the reflections in the middle part of the target zone (1.8-2.0 s) can
be traced more easily across the section (Figure 6-14). In some circumstances it may
also be necessary to apply a mild spectral balancing to the τ-p domain section so that it
can be co-interpreted with the "production" section.
Figure 6-11. Line A, t-x domain processing stream 3, with a short period gap deconvolution. The "noise contaminated" character of the target zone response remained despite indications that some of the short period multiples were suppressed.
Figure 6-13. Line A, τ-p domain processing stream 6, with a short period gap deconvolution and stacked using only the first half of the τ-p gather. There appeared to be considerable suppression of the NZ1 multiples within the coal sequence and the early part of the target zone, especially near the centre of the section between 1.5-1.7 s.
Figure 6-13. Line A, t-x domain processing stream 3, with short period gap deconvolution and a poststack migration applied. The side-scattered noise appeared partly suppressed by the dip filtering action of the migration.
Figure 6-14. Line A, $\tau_p$ domain processing stream 6, with a short period deconvolution and stacked using only the first half of the $\tau_p$ gather. The subsequent poststack migration reduced the side-scattered noise and thereby improved the continuity of events in the middle part of the target zone (1.8-2.0 s).
6.7 CONCLUSIONS

In this chapter, I studied the effect of the main processing steps on \( \tau-x \) and \( \tau-p \) domain processing sequences appropriate for the Gippsland Basin. In doing so, I found that the combination of the stack array principle and HVF eliminates the artefacts associated with the \( \tau-p \) transform. HVF also exploits the excellent separation of linear events and S-wave reflections in the \( \tau-p \) domain to allow a greater portion of the P-wave reflections to be stacked. The theoretical advantages associated with the EMO correction and the deconvolution of plane-wave events help to further suppress the remaining noise events. In presenting a gradual progression of processing improvements it became clear that the \( \tau-p \) domain processing scheme had provided a target zone response with the low frequency character expected following transmission through a coal sequence.

Clearly there is a lot more work that can be done to improve upon these results and to further investigate the \( \tau-p \) approach. In the next chapter, I show how further improvements might be obtained in the Line A section through the application of velocity analysis in the \( \tau-p \) domain. Using the full capabilities of the \( \tau-p \) processing scheme, I then complete the study by processing two regional seismic lines that were expected to contain greater structure in the target zone.
Chapter 7

Model-based velocity analysis

7.1 INTRODUCTION

Velocity analysis is possibly the most important stage of a seismic processing sequence because even errors as small as 5% in the stacking velocities can cause significant changes in the stack response. Semblance analyses indicate a range of velocities that can be used to flatten reflections in prestack seismic data and are the primary tool used to perform a velocity analysis. However semblance analyses may prove quite misleading, even for high quality seismic data, so "mini-stacks" and interval velocity estimates are often used to constrain velocity picking. When the noise contributions are severe, there is a danger of picking noise events that are either misinterpreted or unresolved from primary reflections in the semblance analysis.

In this chapter, I show how a velocity analysis can be improved using elastic modelling to interpret the noise contributions to the semblance analysis of an elastic synthetic seismogram. I used the elastic synthetics computed from Well 3 to guide the velocity analysis of Line A and, in doing so, I found that interbed multiples had often been picked in the original velocity analyses.

I also demonstrate the improved resolution of semblance peaks obtained by computing semblance in the $\tau$-$p$ domain using the elliptical moveout correction. I gained even further improvements in resolution by using wide-angle reflections from the Miocene carbonates that were well separated from linear noise events in the $\tau$-$p$ domain. The field data examples reveal how a model-based velocity analysis can produce subtle changes in the stack amplitudes that may have a significant impact upon an interpretation.
7.2 EVENT INTERPRETATION IN SEMBLANCE ANALYSES

7.2.1 Field data comparison

An accurate synthetic description of prestack seismic data from the region of interest is required to perform a model-based velocity analysis. In Chapter 2, I found that realistic synthetics and accurate ties can be obtained using the reflectivity method. This requires a detailed elastic depth model, which must often be built from little more than a sonic log. In Chapter 4, I performed a sensitivity analysis of elastic synthetics to the underlying depth model and this resulted in a set of guidelines for building the depth model. I applied these techniques to construct an elastic synthetic from Well 3 that allowed a detailed noise interpretation of the supergathers from Line A.

An accurate match was also obtained between the semblance analysis of the supergather (Figure 7-1a) and the elastic synthetic (Figure 7-1b) in terms of the semblance values and the positioning of both signal and noise events. The main differences occurred in the deep response and were due to the lack of primaries below 2.0 s in the synthetic and the effect that this had upon the "time slice" scaling method. Despite this, each of the noise zones identified in the synthetic (Table 2-2) was represented in the supergather semblance and remained identifiable in semblance analyses computed away from the well. An impressive example was the weak S-wave reflection zone (NZ2b) that could be seen to the left of the main semblance trend between 1.5-2.0 s. For this region, the importance of the model-based approach can be judged in terms of the large discrepancy between the main semblance trend and the RMS P-wave velocity function ($PP$) for the depth model.
Figure 7-1. A detailed and accurate match between the semblance analysis of supergather 842 a) and the elastic synthetic from Well 3 b) allowed the noise events to be interpreted and avoided in the velocity analysis of Line A.
7.2.2 Using additional synthetics

As in Chapter 3, the noise events in the semblance analysis of the elastic synthetic were interpreted using additional synthetics that were computed from variations on the depth model and by restricting the wave types modelled. Perhaps the most informative of these was the synthetic that contained only P-wave primary reflections as it revealed the full extent of noise contributions to the semblance display. Other synthetics, such as those computed without surface multiples and S-wave contributions, also helped to identify the noise contributions to the elastic synthetic.

The semblance analysis computed from the "P-wave primaries" synthetic can also be used to judge the success of prestack processing techniques and to assess the remaining difficulties for velocity analysis. I applied a P-wave refraction mute, FK demultiple and AGC prior to computing semblance analyses for CMP 842 (Figure 7-2a) and the "P-wave primaries" synthetic (Figure 7-2b). In the latter case, the RMS P-wave velocity function for the 16 m average-resampled depth model (PP) passed through the centre of the main semblance trend and was well removed from several spurious semblance peaks. However, the "processed CMP gather" semblance was a poor match to the "P-wave primaries" semblance and this suggested that conventional processing did little to improve the target zone semblance. This result suggests that a model-based velocity analysis is essential in this part of the Gippsland Basin.
Figure 7.2. The full extent of the difficulties in performing a velocity analysis for complex areas such as the Gippsland Basin can be seen in the semblance comparison between a processed CMP gather a) and the "P-wave primaries" synthetic b).
7.2.3 Using RMS velocity functions

A less computationally intensive interpretation of the events in a semblance analysis can be obtained using RMS velocity functions for both signal and noise events. As used in the previous figures, the location of "P-wave primary" semblance peaks can be predicted from the RMS value of the P-wave velocity model for a given traveltime. RMS velocity calculations can also be used to predict the location of noise events by specifying the noise type and traveltime at which the noise is generated. For example, the RMS velocity for first-bounce P-wave surface multiples (PPPP) is calculated using the P-wave velocity model with the transit time through each layer doubled. The RMS velocity for mode converted reflections is calculated using the P-wave velocity model until the traveltime where conversion occurs and then using the S-wave velocity model.

RMS velocity functions can be easily calculated for the main noise reflections found in marine seismic data and the results used as an overlay for the semblance analysis of the elastic synthetic. However, it is important to remember that velocity functions only predict the position of signal and noise events, not their semblance value. As shown in Figure 7-3a, the RMS velocity for first-bounce P-wave surface multiples (PPPP) corresponds to semblance peaks at traveltimes where these events are strong in the synthetic (0.95, 2.1-2.7 s). Similarly, semblance peaks corresponding to the interbed multiples and S-wave reflections generated between the Miocene carbonates and the coal sequence can be found at traveltimes that are related to the timing difference between these formations. For example in Figure 7-3b, PPSP/PSPP reflections were found at 1.7 s (NZ2b) and PSSP reflections were found at 2.8 s (NZ2c) and these times correspond to the timing difference between the noise-generating formations given that \( V_p/V_s = 2.5 \) in the Lakes Entrance formation. The velocity functions also revealed the role of the low velocity Lakes Entrance formation in providing interbed multiples (NZ2a) with insufficient differential moveout to the P-wave primary trend.
Figure 7-3. The RMS velocity function for first-bounce P-wave surface multiples corresponds to semblance peaks at traveltimes where these events are strongest in the synthetic a). Similarly, the interbed multiples and S-wave reflections generated between the Miocene carbonates and deeper horizons can be found along the trajectories indicated by their respective RMS velocity functions b).
7.3 SEMBLANCE ANALYSIS IN THE $\tau$-$p$ DOMAIN

Considerable effort is required to prepare a detailed elastic depth model and to compute and interpret the elastic synthetics used in a model-based velocity analysis. This effort may seem pointless given that semblance peaks are often poorly resolved. However, it is only via such an approach that one can decide where the primary picks lie in a complex and unresolved semblance analysis. If the semblance peaks were better resolved then a model-based approach would enable a more detailed interpretation of semblance analyses. I found that substantial resolution improvements can be obtained by computing semblance analyses in the $\tau$-$p$ domain.

Semblance analyses in the $\tau$-$p$ domain are computed using an elliptical moveout correction (EMO) to flatten reflection ellipses (Cutler and Love, 1980). The EMO correction holds several theoretical advantages over the normal moveout correction (NMO) performed in the $t$-$x$ domain (see Chapter 5). The resolution of semblance peaks can also be improved using high amplitude, wide-angle reflections, which are usually muted along with the first-break refractions in the $t$-$x$ domain (Schultz, et al., 1983). In the $\tau$-$p$ domain, refractions map to points at their respective critical angles, while supercritical reflections extend reflection ellipses up to the point of critical reflection in the layer above. In Chapter 3, I identified supercritical reflections associated with the Miocene carbonates in a $\tau$-$p$ domain version of the elastic synthetic. Although these events reduce the resolution of the shallow stack response, they can be used to further improve the resolution of semblance analyses computed in the $\tau$-$p$ domain.

An identical semblance calculation and display method were applied in both the $t$-$x$ and $\tau$-$p$ domains to give a true indication of the benefits of $\tau$-$p$ domain processing and the EMO correction. The $t$-$x$ domain semblance (Figure 7-4a) was computed from a CMP gather in which the supercritical reflections were suppressed (along with the direct wave and P-wave refractions) by FK filtering, muting and a 350 ms AGC. FK demultiple and
an inside trace mute were also applied but did little to suppress the strong surface-related multiples that dominate the deeper response.

The $\tau$-$p$ domain semblance (Figure 7-4b) was computed from a supergather following the application of hyperbolic velocity filtering (HVF) during the proper slant stack. A 5x resolution improvement was obtained at the shallow carbonates from the combined advantages of the EMO correction and the use of supercritical reflections. A 2x resolution improvement was obtained at the coal sequence and in the target zone and this probably represented the improvement due the EMO correction alone. The deep response appeared unresolved probably due to the lack of deep targets near Well 3. Despite this, it appears that HVF has suppressed some of the surface-related multiples since these are now less focussed in the semblance analysis. In Chapter 8, I show a more convincing example of improved semblance in the deep response using Line C, which contained stronger indications of deep structure.

7.4 FIELD DATA EXAMPLES

The model-based velocity analysis for Line A revealed that interbed multiples had often been mistaken for primaries in the original velocity analyses. Strong interbed multiples are generated between the Miocene carbonates and the coal sequence and, as Figure 7-3b suggests, their lack of differential moveout to the true primaries is due to the low velocity Lakes Entrance formation. This resulted in velocity picks that were typically 5-10% lower than the primary stacking velocities in the target zone. I investigated the potential for improvement in stacked data from the Gippsland Basin, using a 5% reduction in the model-based velocities to represent the case where interbed multiples had been picked.
Figure 7-4. Supergather semblance analyses computed in the t-x a) and τ-p domains b) indicated a 5x resolution improvement at the Miocene carbonates through the use of supercritical reflections, while elsewhere the EMO correction provided a 2x resolution improvement.