IN WHAT WAYS ARE YEAR ONE STUDENTS ABLE TO REPRESENT THEIR MATHEMATICAL UNDERSTANDING?

Bronwyn Deagan
Bachelor of Education (Primary) Honours

Submitted in (partial) fulfilment of the requirements of the degree of Master of Education in the Faculty of Education at The University of Melbourne
March, 2006
Thesis Supervisor:

Professor David Clarke
Faculty of Education,
The University of Melbourne

Author:

Bronwyn Deagan
Student Number: 205818
# TABLE OF CONTENTS

*Chapter One: Introduction*

<table>
<thead>
<tr>
<th>Section</th>
<th>Page Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1 Background</td>
<td>1</td>
</tr>
<tr>
<td>1.2 Statement of problem</td>
<td>4</td>
</tr>
<tr>
<td>1.3 The context of the study</td>
<td>5</td>
</tr>
</tbody>
</table>

*Chapter Two: Literature Review*

<table>
<thead>
<tr>
<th>Section</th>
<th>Page Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1 Introduction</td>
<td>7</td>
</tr>
<tr>
<td>2.2 Background</td>
<td>8</td>
</tr>
<tr>
<td>2.3 What is an understanding?</td>
<td>9</td>
</tr>
<tr>
<td>2.4 Concrete material representation (Physical/Kinaesthetic)</td>
<td>11</td>
</tr>
<tr>
<td>2.5 Written representation (Visual)</td>
<td>17</td>
</tr>
<tr>
<td>2.6 Oral representation (Linguistic)</td>
<td>21</td>
</tr>
<tr>
<td>2.7 Children’s thinking and mathematical instruction</td>
<td>23</td>
</tr>
<tr>
<td>2.8 Key points</td>
<td>25</td>
</tr>
</tbody>
</table>

*Chapter Three: Methodology. The Conduct of Investigation*

<table>
<thead>
<tr>
<th>Section</th>
<th>Page Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.1 Introduction</td>
<td>28</td>
</tr>
<tr>
<td>3.2 Data collection through focused learning sessions</td>
<td>29</td>
</tr>
<tr>
<td>3.3 Lesson one: written addition task</td>
<td>32</td>
</tr>
<tr>
<td>3.4 Lesson two: written addition task</td>
<td>33</td>
</tr>
<tr>
<td>3.5 Lesson three: physical addition task</td>
<td>34</td>
</tr>
<tr>
<td>3.6 Lesson four: physical addition task</td>
<td>36</td>
</tr>
<tr>
<td>3.7 Lesson five: oral addition task</td>
<td>38</td>
</tr>
<tr>
<td>3.8 Lesson six: oral addition task</td>
<td>41</td>
</tr>
<tr>
<td>3.9 Lesson seven: individual student selection task</td>
<td>42</td>
</tr>
<tr>
<td>3.10 Research methods</td>
<td>45</td>
</tr>
<tr>
<td>3.11 In summary</td>
<td>47</td>
</tr>
</tbody>
</table>
Chapter Four: Results and Discussion

4.1 Introduction ............................................ 48
4.2 Physical representational mode .................. 51
4.3 Visual representational mode .................. 57
4.4 Linguistic representational mode ............ 65
4.5 Analysis of students task selection ............ 70
4.6 Summary of findings ............................... 78

Chapter Five: Conclusion

5.1 Introduction ............................................ 80
5.2 Enjoyment of the actual activity and valuing of product .... 80
5.3 Comfort, confidence and peer influence .......... 83
5.4 Problematic connection and too many options ........ 86
5.5 Willingness to change ............................... 88
5.6 Understanding to represent .......................... 89
5.7 Recommendations arising from this investigation .... 92

References

References .................................................. 95

Appendix

Appendix One (Consent Forms)
Appendix Two (Interview Questions)
Appendix Three (Interview Transcripts)
Appendix Four (Work Samples)
Appendix Five (Plain Language Statement)
<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>Table 3.1</td>
<td>Six sub-strands and three representational modes</td>
<td>....30</td>
</tr>
<tr>
<td>Table 3.2</td>
<td>Six focused teaching session framework</td>
<td>....31</td>
</tr>
<tr>
<td>Table 4.1</td>
<td>Gender Distribution</td>
<td>....49</td>
</tr>
<tr>
<td>Table 4.2</td>
<td>Final task representational mode break down</td>
<td>....49</td>
</tr>
<tr>
<td>Table 4.3</td>
<td>Task selection</td>
<td>....50</td>
</tr>
<tr>
<td>Table 4.4</td>
<td>Would you share your understanding with the Class or Partner?</td>
<td>....68</td>
</tr>
<tr>
<td>Table 4.5</td>
<td>Student behaviour during six focused group sessions</td>
<td>....70</td>
</tr>
<tr>
<td>FIGURE</td>
<td>Description</td>
<td>PAGE NUMBER</td>
</tr>
<tr>
<td>----------</td>
<td>-----------------------------------</td>
<td>-------------</td>
</tr>
<tr>
<td>Figure 3.1</td>
<td>Hands on materials/equipment</td>
<td>35</td>
</tr>
<tr>
<td>Figure 3.2</td>
<td>Mode description posters</td>
<td>43</td>
</tr>
<tr>
<td>Figure 4.1</td>
<td>Student physical representations</td>
<td>53</td>
</tr>
<tr>
<td>Figure 4.2</td>
<td>Student written representations</td>
<td>58</td>
</tr>
<tr>
<td>Figure 4.3</td>
<td>Representing $12+7=$</td>
<td>61</td>
</tr>
<tr>
<td>Figure 4.4</td>
<td>Unsuccessful attempt</td>
<td>61</td>
</tr>
<tr>
<td>Figure 4.5</td>
<td>Addition poster</td>
<td>66</td>
</tr>
</tbody>
</table>
Declaration

1. Bronwyn Deagan, as a student of the University of Melbourne, Faculty of Education, declare that this research is original, and can be supported upon request by documents, transcripts and video recordings that will attest to this. I further declare that this document has never been presented for academic accreditation in any forum for a Master of Education Degree, or any other degree, and to which end it is both original and unpublished.

[Signature]

Bronwyn Deagan

March, 2006
Acknowledgments

This work would never have been possible without the support and contributions of many people that I would like to sincerely thank. Their encouragement, assistance and advice was invaluable to the completion of this thesis.

Professor David Clarke, my supervisor, for providing valuable insights into the topic of mathematics and guiding me to ensure that I maintained a focussed approach during this research.

To the members of our research group, for their continued support, words of wisdom and for making me believe in myself.

To the Year One students of 2005 who participated in this project, readily providing me with material and participating in the one-on-one interviews, I am very much indebted to them for enabling me to complete this minor thesis. Their love of learning and enthusiasm towards mathematics is inspiring.

Grateful thanks to my loved ones for their continued guidance and support.
ABSTRACT

The Early Years of schooling are a crucial part of a student’s education. Recent years have seen the implementation of new Literacy and Numeracy programs in primary school classrooms. The key area of Mathematics (Numeracy) has been closely monitored and funded by political and educational bodies (Clarke, Cheeseman, Gervasoni, Gronn, Horne, McDonough, Montgomery, Roche, Sullivan, Clarke, & Rowley, 2002; Association of Independent Schools of South Australia, 2004). The new Numeracy programs have been introduced into the school curriculum to ensure that all students’ needs are catered for in the classroom program. However, standardised testing using pencil and paper is still being used as the accepted form of assessment. The Victorian State Government uses the Achievement Improvement Monitor (AIM) to assess students’ mathematical achievement levels. This pencil and paper test is conducted for students in Years Three, Five, Seven and Nine and is used to sort the students into a percentile group. Other than the ‘Early Numeracy in the Classroom’ program (2002) used by Victorian schools as a Prep to Three program, where a one-on-one interview is used as a form of assessment, there is currently no program that offers students the opportunity to choose how best to represent their own mathematical understanding. Although, the learning needs of students are being better catered for within the classroom, students are being disadvantaged by the way in which they are assessed.

This research investigates whether, given the correct opportunities, students are able to represent their own mathematical learning. It has been my observation that students are capable of representing their mathematical learning, if given some control over the mode of representation. This investigation gave Year One students exposure
to the three representational modes: Physical, Written and Oral, through six structured mathematics lessons dealing with the concept and procedures of two-digit addition. The lessons exposed students to a variety of ways of using these representational modes in relation to addition sums. The lessons were taught over a four week period during the students’ regular numeracy classes. At the completion of the six lessons, the students were given a two-digit addition sum in a seventh lesson and asked to each select their own way to represent their understanding by using one of the three representational modes.

The qualitative investigation of ten students’ final task selections indicated that students as young as Year One were able to choose a medium they best felt could represent what they understood about an addition sum. Although only a small sample was used in this study, they were all able to produce an appropriate representation: An important aspect of the study was that the students needed to be taught how to manipulate equipment, use the correct language and how to present their work for them to be able to make an informed decision as to their preferred representational mode. The findings from this investigation indicate that if the classroom numeracy program supported this inclusive approach to student representation of learning, then a more flexible approach to individual assessment could be used by all classroom teachers. Student selection of representational mode would not be used as the only form of assessment, but would supplement the current assessment practises used by each teacher. It would provide the teacher with an additional piece of assessment that both indicates the child’s understanding and their preferred representational modes. This would assist in the development of a more accurate individualised student mathematical profile by ensuring that students had the best possible opportunity to
display the results of their learning. As a result of improving the quality of
information about students’ understanding teachers would be better equipped to plan
their instructional program.

It is a recommendation of this study that a larger project, employing a similar
research design, be undertaken with students in the same year levels as those taking
the AIM test. The students’ performance could then be compared with their AIM
results. Such a project would give a clearer indication of the extent to which pencil
and paper tests constrain students’ ability to show what they know. This thesis reports
the initial investigation of this question, and reveals differences in preferred
representational mode between Year One students. Classroom teachers should
broaden the representational options available to their students. Both teachers and
students should also be encouraged to develop expertise and confidence using a
variety of representational modes to communicate their understanding of
mathematics.
CHAPTER ONE

Introduction

Background

Mathematics is a subject of study that proves to be a challenge for primary school teachers as it contains abstract ideas, symbols, patterns and relationships (Lewis, 1996), all of which represent a significant conceptual hurdle for young learners. The challenge is, therefore, how do you teach this in a way that caters for all learners in your classroom?

Research in the area of Mathematics education is continuing to inform teaching practices. The last decade has seen a far more complete picture of the process whereby students acquire skills and understandings and how they relate their learning to the real world. The focus in mathematics teaching has shifted from group outcomes to implementing a teaching program that focuses on the individual learner. Therefore, the classroom program now is designed to accommodate the student, rather than pressure the student to fit the program (Cockcroft, 1982; Grouws, 1992).

The use of individualised programs in mathematics and the goal of catering for the needs of individual students have been central elements in the research and curriculum development agenda within the last two decades. Yet, mathematics assessment strategies and prescribed learning outcomes are still being designed to address the group of students as a whole and do not allow for individual preferences. The classroom program caters for diversity but the assessment is often pen and paper tests that only allow students to represent their answers in a particular representational mode.
The Victorian State Government’s implementation of the Early Years Numeracy Program into Prep to Year Two classrooms has seen children being encouraged to display their mathematical understandings in a variety of ways. These include using counters to make ‘groups of’, students verbalising what they have learned through share time at the end of a lesson, or by completing a written activity or a learning log. The students’ progress in Numeracy is monitored by a one-on-one interview with their classroom teacher and activities, where observational or anecdotal notes are recorded in a teacher’s assessment folder. However, when the children enter Year Three, they are confronted with ‘formal’ testing, which requires them to sit a pencil and paper test and where no concrete materials are available.

*When such formal tests are approaching, most teachers will modify their normal teaching strategies to prepare children for ‘test-taking’ processes. While we might rightly question the educational value of such learning, formal testing is still a reality and we would be disadvantaging students if we did not prepare them for aspects such as working to a time limit, answering multiple-choice questions and dealing with questions they do not know how to answer (Bobis, Mulligan and Lowrie, 2004: 283).*

This leads to questions such as: Are students disadvantaged by pencil and paper testing, when they understand what is required but are unable to represent their understanding? Are classroom teachers providing students with enough opportunities
and representational modes to enable students to demonstrate their mathematical learning and understandings?

Currently, an evolving pedagogy of mathematics is taking into account the styles or patterns in which individual students learn, and the teaching methodologies which are created to match student learning styles. Keefe (1987, 1989), Midkiff and Thomasson (1993) accept that the promotion of a single learning style is unlikely to maximise learning for every student and, therefore, programs must be designed to suit particular student characteristics and aptitudes. Seeing, hearing, touching and feeling enable learning to occur, with learners demonstrating preference for receiving information by visual, auditory or kinaesthetic means or a combination of them (Lewis, 1996).

The Early Numeracy Research Project (Clarke et al, 2002), which resulted in the implementation of the Early Numeracy in the Classroom program into Victorian Primary Schools, endorses and puts into practice the theory of teaching and learning in which students are introduced to mathematical concepts through the use of concrete materials. The students are encouraged to explore mathematics through the manipulation of materials. Through teacher guidance and support the students are then helped to make the link to the written stage and verbalise their learning. Learning through the use of concrete materials in mathematics and identifying that students learn best through physical and mental activity that was related to their experiences is nothing new in education. Froebel (Bruce, 1992), Montessori (Mooney, 2000) and Pestalozzi (Sowell, 1989) were all advocates of concrete material usage.
The advantages of teaching according to a student's preferred way of learning is something that many teachers have been aware of and have been catering for in their classroom programs. The teacher should be able to identify through a variety of forms of assessment where a student requires additional assistance. However, the point needs to be addressed as to whether it is not that the student does not understand the assessment task, but rather that they are unable to express their understanding through the representational mode chosen as the form of assessment. Mismatches of instruction and individual need include such things as the teaching of mathematics by visual methods to learners who prefer to receive information kinaesthetically. However, even if teachers are catering for a particular student in the classroom program, then this student's preferred learning style should also be catered for in relation to assessment.

**Statement of the Problem**

Reflecting upon the theories on catering for the needs of individual students in a mathematics classroom and the issues of different learning styles led to the conclusion that further research was required into assessment and student preferences for representing mathematical learning.

This investigation, therefore, was intended to contribute new and potentially useful information to existing or emerging bodies of knowledge about the possible alignment of assessment with students' preferred representational modes. It was devised to determine Year One students' preferences regarding the representation of their mathematical understanding. The main focus of the study was on how the use of
particular representational modes could constrain or afford students’ ability to represent their understanding.

The key question “In what ways are Year One Students able to represent their mathematical learning?” addresses the need not just to provide a classroom program that caters for the individual students but also ensure that assessment techniques allow for individual preferences and styles. It was intended to ensure that students are all given enough opportunities for them to demonstrate their understanding.

**The context of the study**

The participants of this investigation were a group of ten students in Year One at an Independent school in the Western Suburbs of Melbourne. This school is a Prep to Year Three campus that has 143 students. The class sizes are capped at 20 students. The Year One classroom teacher whose students were involved in the study is the Numeracy Coordinator and teaches using the ‘thinking’ curriculum and thrives on catering for the individual needs of the students in her class. The students were halfway through their second year of schooling.

The school uses the ‘Early Numeracy in the Classroom program’. This is the numeracy classroom program that came about from the Early Numeracy Research Project conducted by Clarke, Cheeseman, Gervasoni, Gronn, Horne, McDonough, Montgomery, Roche, Sullivan, Clarke, & Rowley, 2002. Therefore, the students had taken part in the Early Years one-on-one numeracy interview at the beginning of each school year. The students were all familiar with using a variety of different representational modes and their classroom activities had allowed them to explore
learning through the use of concrete materials, the verbalisation of their learning, open-ended activities and written tasks. The ten students were selected for this study not on the basis of their mathematical ability, but because of the interesting and diverse ways they displayed their learning in class. That is, the ten students were chosen in order to maximise the range of learning styles and preferred representational modes available for analysis in the study.

Throughout this thesis, ‘learning’ refers to the process of knowledge construction, while ‘understanding’ is the outcome of that process. These and other key constructs are identified and discussed in greater detail in the next chapter. It must be emphasised that the understanding being represented is the student’s own understanding, which may or may not correspond to the teacher’s understanding. But it is only through identifying the student’s understanding, that the teacher can judge how best to act instructionally. Pencil and paper tests may deny the teacher this essential information.

The following chapters of this thesis contain an examination of the research literature relating to the problem being investigated, information relating to the design of the research, an analysis of the data collected and a final interpretation of results.
CHAPTER TWO
Literature Review

2.1 Introduction

A number of key issues in mathematics education have emerged in recent years and are presently continuing to evolve. The last ten years has seen the significant identification that the Early Years of schooling are a crucial part of a student’s education. The focus on Literacy and Numeracy has been closely monitored and funded by political and educational bodies (Clarke et al, 2002; Association of Independent Schools of South Australia, 2004). In Australia, studies on the importance of providing students with solid mathematical foundations have resulted in an overhaul of mathematics teaching in many primary school classrooms. The focus has moved away from the ‘chalk and talk’ mode where the work was written on the board and the students were to copy down and complete, to a more one-on-one approach ensuring that the individual needs of students are catered for and that students are provided with different opportunities to learn mathematics.

This chapter contains an examination of research literature in the field of mathematical representations. The review is presented in the following sections:

2.2 Background

2.3 What is an Understanding?

2.4 Concrete Materials (Physical Mode)

2.5 Written Representation (Visual Mode)

2.6 Oral Representation (Linguistic Mode)

2.7 Children’s thinking and Mathematical Instruction

2.8 Key Points.
2.2 Background

Young children identifying and selecting a preferred method of representing their learning is an area that is still relatively new. For many years students have been told how to solve a problem and what steps need to be taken for the correct answer. Recent reforms such as the Early Numeracy Research Project (2002) have encouraged teachers to look at other ways of identifying what children know. This can be done at the beginning and/or the end of each lesson, before the start of the school year or the introduction of a new topic.

“Drawing a mathematics lesson to a close is important with current approaches to mathematics education. This is difficult to do well because it involves much more than simple restating the mathematics addressed. Children are often encouraged to reflect on their learning and to explain or describe strategic thinking” (Cheeseman, 2003: 17).

Booker, Briggs, Davey and Nisbet (1992) identified that:

“Just as there is a wide range of levels of cognitive, physical and social development among the children in any class group, so too there is diversity in the children’s preferred modes of working, thinking, and learning different aspects of Mathematics. In any situation there may be some who would prefer to operate chiefly in a visual mode (for example, where their understanding and learning is enhanced by appropriate visual representations and responses) whilst others prefer to operate chiefly in a verbal mode involving considerable oral discussions and writing. Others may prefer to work in a concrete mode
where the process is developed through manipulation of concrete materials or through dramatic participation or 'acting out' as in trading games and shopping activities” (page 366).

Booker, Briggs, Davey and Nisbet’s (1992) reference to identifying a teacher’s classroom mathematical diversity, addressed the wide range of students' preferred modes in relation to the working, thinking and learning processes of mathematics. This leads to other research into the preferred modes that young students use to represent their mathematical learning and how the use of particular representational modes can constrain or afford student learning and their ability to represent their learning. However, this links to the idea as to whether the students understand a topic enough to be able to choose a preferred mode to represent their understanding.

2.3 What is an Understanding?

When teaching, teachers cater for the learning needs and understandings of students. In relation to assessment, teachers tend to rely on the standard tests or work samples. This poses the questions, “What is an understanding?” and “How do we determine the nature of a child’s understanding?” Many theorists and researchers have their own definition of understanding. Booker, Briggs, Davey and Nisbet (1992) see understanding in these terms: “It is understanding that allows ideas and techniques to be adapted to new ends” (1992: 3). When relating ‘understanding’ to a mathematical context, Booker, Briggs, Davey and Nisbet continue to explain that

“Understanding

- Allows knowledge to be transformed to match new problem situations.
• Provides more efficient and effective learning.
• Assists retrieval of facts and processes’ (Page 4)

Jean Piaget’s ‘Cognitive Theory’ (Myers, 1986: 88) that studied children’s thinking processes was “focused on the development of Mathematical and logical concepts. This theory has been applied extensively to teaching practice and curriculum design in elementary education” (Bybee and Sund, 1982). This cognitive theory along with Bruner (1966) and Vygostky (1978) looks at the stages of cognitive development. Piaget identifies four stages of cognitive or intellectual development (developmental stages) “Sensorimotor, preoperations, concrete operations, and formal operations.” (Copeland, 1970: 24-31) Each developmental stage was linked to an age group.

“He (Piaget) considered that every individual passed through these four stages in the same order although there would be variations in the ages associated with each stage. It should be noted that an individual does not end one stage suddenly and commence another. Rather, there is a gradual merging from one to another” (Marsh, 1997: 18.)

Piaget’s theory provides teachers with an insight into how students think and their learning behaviours. In relation to the concrete operations stage and the link to mathematics, Gage and Berliner (1992: 124) believe that “Children who have not played with beads, rods and lumps of clay may have difficulty understanding addition, subtraction, multiplication and division”. Bruner identified three stages of growth as the “enactive, iconic and symbolic stages” (Marsh, 1997: 22). The enactive mode is
based on learning by doing, iconic involves using imagery but not language and symbolic is where the children obtain understanding through the use of symbol systems.

Although these theories guide our teaching, there is not one specific theory that pinpoints what a child understands and what form this level of understanding takes. Piaget and Bruner have provided us with research-based developmental stages, however, it is important to remember that children can develop at different levels and time spans. The theorists’ knowledge of how children think, the use of concrete materials and the sequencing of instructions can help teachers to develop their classroom program. But do our current forms of assessment allow us to relate these theories to a child’s mathematical understanding and how we ask students to represent their understanding? The use of the word understanding is frequently linked to developmental stages but the Macquarie Dictionary defines understanding as “to perceive the meaning of; grasp the idea of; comprehend” (1982: 1849). The big question still is, “How do we measure or document a child’s mathematical understanding?” In this study, we interpret this question as, “By what representational modes might the child demonstrate or display their understanding most effectively?”

2.4 Concrete Material Representation (Physical/Kinaesthetic)

Thompson’s (1994) paper on ‘Concrete materials and teaching for Mathematical Understanding’ reflected on the role of concrete materials in teaching for mathematical understanding. He identified that in the early nineteen hundreds the use of concrete materials were encouraged. In the early nineteen sixties, Zolton Dienes (1960) and Jerome Bruner (1961) provided theoretical justification for the use of
concrete materials, and as a result their use in mathematical classrooms accelerated. However, since then theorists' views on the effectiveness of concrete materials have varied. According to Thompson (1994) many researchers have concluded that the use of concrete materials for younger students has proven to be successful especially in relation to base ten models and yet the use of concrete materials by older students has had little effect. Thompson felt that this may be a result of the context in which the concrete materials had been used, he believed that activities needed to engage students and address the total instructional environment. The use of Multi-Base Arithmetic Blocks (MAB) to understand place value has always created great discussions when linked to concrete material use, as students have trouble visualising hundreds as small wooden cubes. Young students in the Early Years are being encouraged to use icy pole sticks to assist their understandings of place value. Thompson stresses in his article that concrete materials do not always automatically carry mathematical meaning for students. Teachers have to help with that development.

"Concrete materials are used appropriately for two purposes. First, they enable you and your students to have grounded conversations. Their use provides something 'concrete' about which you and they can talk.......... Second, concrete materials provide something on which students can act” (1994: 556-558).

In support of Thompson’s article, it has been my observation that students are only able to represent their mathematical learning when an understanding is apparent. Not just an understanding of the topic but also the representational mode, and in this case
concrete materials. Students being able to manipulate and discuss their findings confidently is a reflection of the classroom learning environment that the students have been provided with. Have the students' individual needs been met and have they been given the opportunity to thoroughly explore the representational mode?

**Understanding place value:** A case study of the Base Ten Game came about from the Australian Government Numeracy Research and Development Initiative in 2001. One of the critical factors in developing students' understanding of the number system identified by this research project were that

"Concrete materials, such as those used in Base Ten Game, can make a significant contribution to the development of students' conceptual and procedural knowledge about the number system across all year levels" (Association of Independent Schools of South Australia, 2004: v).

The Base Ten Game concept is in relation to place value and building from the base ten concept to have a greater understanding of the number system and place value. The project team's research found that Kamii (1986) cited results of research in the United States, Canada and Switzerland which found that most students in their first and second year of schooling do not understand that the 1 in 16 indicates that there is 1 ten. Therefore, the use of concrete materials such as icy pole sticks bundled in ten and then six sticks by themselves would visually assist students to understand what the number 16 looked like. This report indicates that students should be taught using concrete materials and then be allowed to show their understanding. In my
research process, students could use the icy pole sticks to represent the numbers and to aid their understanding of two digit number addition. Thompson’s (1994) opinion that students need to understand the context of using the concrete materials would therefore, become apparent as would their understanding of base ten numbers.

The Base Ten Game research project also tried to create the link and bridge the gap between the use of concrete materials and the formal written mode (algorithmic procedures) (Hart, 1989). Students were asked to make and manipulate a base ten number and then present their answer in a table format. The students were not only to write the findings but also to draw what they had made. The MAB concrete materials also provided difficulties for students and their mathematical understandings in this research. The concept of looking at a wooden cube and being told there were a thousand cubes was a lot more difficult for students to understand than creating bundles of icy poles sticks that they could move and manipulate.

The findings of this research concluded that there were many classroom implications for teachers. Again the need for catering for the needs of individuals was addressed, establishing the current constructs of each learner was required to provide a learning environment that would challenge and support the students (Association of Independent Schools of South Australia, 2004). Concrete materials were also seen as an essential feature of the Base Ten Game as they provide a visual model of the number system.

Building on what the students already know and using concrete materials was a key feature of the Early Years Numeracy Research Project that was established in
1999. The research team spent three years implementing and establishing a numeracy program with a developmental framework, assessment, professional development and appropriate school support to enhance young children’s mathematical learning. The outcomes from this research resulted in the implementation of the ‘Early Numeracy in the Classroom’ program into the majority of Victorian Primary Schools. One form of assessment that has resulted through this research project is the Early Years Interview.

"The Prep Interview (Detour) within the Early Years Numeracy Interview enabled a very clear picture of the mathematical knowledge and understanding that young children bring to school, and the development of these aspects during the first year of school. Most Prep children arrive with considerable skills and understandings in areas that have traditional content for this grade level" (Clarke, Cheeseman, Gervasoni, Gronn, Horne, McDonough, Montgomery, Roche, Sullivan, Clarke, & Rowley, 2002: 25).

The findings of this report indicate how important it is for teachers to identify where students are at before they introduce a new topic. The traditional Prep programs appear to be no longer suitable for all children entering the Prep Year. This would also indicate that the traditional Year One program may also no longer be appropriate to all students. What is important to note about the interview process is that different representational modes are used for students to display their understanding.

"The interviews are an opportunity for the teacher to develop deeper understanding of the students’ thinking processes through seeing how
they respond to a task." (Department of Education, Employment and Training, 2001: 17).

The tasks that the students are required to respond to are based on counting, place value, addition and subtraction strategies, multiplication and division strategies, time, length and measurement, space (properties and visualisation). The interview is supported by an interview kit that is filled with teddies for counting, scales for measuring, calculators for number awareness and so on. The use of concrete materials is a key component of this program and the students use the materials to represent their understanding. The Early Years Numeracy Interview was not the only aspect of the program where concrete materials were used. The teachers of trial schools found that one of the changes with the implementation of the project was that they were now providing more practical and hands on activities in their daily numeracy teaching period.

The development of 'growth points' in young children's mathematical understanding was a key feature of this Research Project. The 'growth points' are able to be supported by the use of concrete materials and activities assisting students to achieve particular growth points. These activities are aimed to be taught either as a whole class or in a small group where students are immersed in plastic teddies for counting, icy poles sticks, tens frames and counters for place value and so on. Like the Base Ten Games research, students have also been encouraged not just to use concrete materials but are also being taught the skills of how to transfer their understanding to the written mode. Current Australian research has provided a strong relationship between mathematical representational modes, stressing the importance of not just
being strong in one aspect. However, when students are asked to select their own preferred way to represent their understanding, do they display a preference for one particular mode? My research will aim to answer this question.

When looking at research into the use of concrete materials in Primary School mathematics classrooms, it mainly focuses on the manipulation of materials. However, the physical mode of representation could also include Kinaesthetics. Students can use their bodies to assist them in classroom mathematics. It also poses the questions as to whether students show a preference for individual or group work. Thompson (1994) saw concrete materials being used for both individual and group activities. The Early Numeracy Research Project (2002) growth point activities ranged from class activities to individual work and covered the transition from concrete to abstract thinking. Of course there are other types of concrete materials in mathematics especially in the area of measurement. However, as my research is based on numbers and two digit addition, this is the concrete materials representational mode this study is focussing on.

The types of concrete materials being used is a major factor as to how effective their use is in a mathematics classroom. Students must also be able to understand why and how the concrete materials are being used, for them to truly and successfully represent their mathematical learning.

2.5 Written Representation (Visual)

organised and represented data and also the relationship between their organization and representation of data". Recent reforms in mathematics education and a call for an earlier introduction of data handling into elementary schools lead to the development of this study. With a vast majority of research on data handling and representation being focused on older students it is refreshing to note that Mathematical organizations have identified a lack of research and knowledge of younger students representing their understanding.

Nisbet et al. (2003) studied students from Grades One to Five from a mid-west American school. In order for students to display their data the researchers conducted data exploration classes where the students were introduced to the construction of different graphs and how to use data that had already been provided for them. Jones et al. (2000) Statistical Thinking Framework was what this study was based on, as it provided expectations of the levels and the kinds of 'thinking children engage in when representing and organising sets of data'. The research team identified from the results of their study that

"Clearly further research is needed on modes of presentation and context in data exploration with a view to identifying and classifying tasks and contexts that are suitable for young children. Young children in kindergarten and in primary grades regularly engage in sorting activities with blocks and other manipulatives. However, there is obviously a need for research to build a more pervasive understanding of children’s sorting schemata not only in context of manipulative
blocks but also in the context of data exploration” (Nisbet, Jones, Thornton, Langrall and Mooney, 2003: 54).

Atkinson and Clarke (1991) researched children’s mathematics and found that the language and notations of mathematics can make understanding for students difficult. They focused on making the link between home and school understandings and when it is the appropriate time to introduce mathematical symbols. The structure of the standard notion perhaps has been introduced too early into schooling and students are still working on how to represent their own understanding. Although, students may not write the standard notation for a sum in the conventional methods, they may be able to represent their understanding in visual representations. Scribble, drawings, copied symbols and shorthand could be a variety of children’s own methods of representing their own learning. It is important to remember that students’ own recordings can be helpful in telling us what the students really understand.

Nisbet, Jones, Thornton, Langrall and Mooney (2003) believe data handling should be introduced earlier into the primary program. This would be a structured way to set out a student’s findings, yet, some students in the early years of numeracy are still in the ‘drawing’ visual representation stage of their understanding. If they were then directed to complete a table format or graph would this confuse the student and change or interfere with their understanding? Atkinson and Clarke (1991) found that children’s own symbols held enormous meaning for them and supported their understanding.
“Many teachers report that when children see a need to record for themselves, the recordings usually help the children’s thinking processes. However, they report some confusion in thinking when recording is for another (usually adult) reason” (Atkinson and Clarke, 1991: 42).

The Early Years one-on-one interview which came about through the Early Numeracy Research Project (2002) has a written component in its assessment. The students are asked to draw what a clock looks like. The student’s written representations would give a clear indication as to their level of understanding. The teacher is able to identify, does this clock have hands? Does it have numbers? How many numbers does it have? Did the student draw an analogue or digital clock? The students are then asked to give an oral recount of their drawing. Again, a cross over of representational modes has taken place. The visual mode is supported by the Oral mode.

Reflective journals or learning logs have recently been introduced and used as another form of students representing what they have learnt during their mathematics lesson in the written mode.

“Journal entries may be structured, focusing on a particular aspect of mathematics, such as place value or using a ruler; or they may be more general, for example a review of a mathematics session”

These journals can either be completed as a class or individually. Some of these entries have also been supported with the use of photographs. This is a visual form of representation where the children have manipulated or created a model to complete a mathematics task. It is essential that teachers know what mathematical understanding their children have.

Research is showing that students are able to represent their understanding through the visual mode but the mode selection works better when it is supported with the Oral representations. Studies have also supported the need for students to represent their understanding in a way that makes meaning to them whether this is through diagrams or through writing. However, assessment and state benchmark testing in Victoria relies on the use of multiple choice answers where the students indicate their understanding though colouring in a letter or a number that will be processed through a computer. When students enter a school environment, teachers are required to teach them the standard notion of how to set out a sum and what is required. Of course students need to be shown a variety of methods but they should also be exposed to different ways to represent their understanding in the visual form.

2.6 Oral Representation (Linguistic)

"Conceptually-orientated" thinking or asking a student "how did you work that out?" is one way that students can verbalise their learning (Thompson, Philipp, Thompson, Boyd, 1994). Hiebert and Wearne (1992) called it the connecting phase of mathematical learning, creating the link and the natural talk in mathematics. The hardest part of the Oral mode of representation is keeping on track, ensuring that what is being discussed is relevant and suitable. Thompson and Thompson (1994)
found that the type of language (discourse) used in a classroom setting had been thoroughly researched as a result of recent reforms in mathematics education. When the teacher is confidently using ‘Maths Talk’ whilst in the mathematics classroom, this will in turn encourage the use of the mathematical language by the students. The next phase is whether the students understand this language, are able relate to it, use it confidently and in the correct context. Therefore, as suggested by Thompson and Thompson (1994) being sensitive to children’s thinking whilst delivering mathematical instruction is essential when encouraging students to speak mathematically.

The Early Years one-on-one interview is structured enabling the students to explain their thought processes and mathematical understandings. How and why questions are used, building children’s relational understanding. Watching the students manipulate the concrete materials and justify how they derived an answer can help a teacher discover how a student arrives at an answer. For example: When adding 9+6 Did the student add to the nearest ten by taking one away from the six and adding it to the 9 to now make the sum 10+5 =? When a student writes the answer on a page and uses the visual mode then the process of how they solved the problem becomes lost. The use of the written learning log at the end of the lesson is a visual way to represent learning, however, an oral reflective chat is also becoming a part of a numeracy classroom and has been incorporated into the daily numeracy Early Years block. Share time provides students with the opportunity to discuss and reflect on their learning during the class. Students are encouraged to verbalise their understandings and share their findings. Teachers are encouraged to make notes and record details ensuring every student is given the opportunity to verbalise their understanding.
Classroom projects, problem solving and team work activities often require an oral presentation at the conclusion. When students are working in a group or completing an activity the use of informal language can assist a teacher to tap into the level of a student’s understanding. Informal language or maths talk are often the key when identifying how much a student understands.

Gould (2004) has found that creating a classroom in which communication and reflection are valued is the first step in the learning of language in a mathematics classroom. The language used by teachers needs to be relevant and thought should be given to the introduction and usage of language. When expressing and interpreting ideas in mathematics, the order of the words is very important. Language in mathematics can become confusing. In order for students to be able to represent their mathematical learning through the Oral mode they need to be continually exposed to the correct language. Again the trouble may be that students do understand the task they just have difficulty explaining it. English as a Second Language (E.S.L.) students are often facing this hurdle and at times traditional beliefs may prevent students from representing their understanding in the Oral mode. Teachers should encourage the use of ‘maths’ talk in their classroom to allow students to participate and gain confidence in this area. Gould (2004) also suggested that older students should learn how to argue mathematically. That way students are encouraged and forced to justify their answers. In a year one classroom, however, this may be a little too difficult.

2.7 Children’s Thinking and Mathematics Instruction
A four-year study on a teacher developmental program that focused on helping the teachers understand the development of children’s mathematical thinking allowed
teachers to reflect on their own teaching styles. The study placed the teachers into a position similar to ones that their students are faced with on a daily basis. This procedure allowed the teachers to identify the processes children go through to complete a task.

“They based their work on a socioconstructivist epistemology of children’s learning and wrote instructional activities designed to provide opportunities for children to construct their own mathematical knowledge. Teachers engaged in workshop activities that guided their reflection about their instruction and students’ thinking” (Fennema, Carpenter, Franke, Levi, Jacobs, Empson 1996: 404).

The study of the teaching and learning of addition and subtraction in a Year One classroom was based on a model and research that “had identified regularities in children’s solutions to different types of problem situations through detailed analyses of content domains” (Carpenter, 1985, Fuson, 1992; Greer, 1992, cited in Fennema Carpenter, Franke, Levi, Jacobs, Empson 1996: 404). The findings resulted in the teachers seeing the importance of allowing the children to verbalise their mathematical understanding. A major concern of the teachers was that although this study had proven effective they were concerned with the crowded curriculum and completing set units with the school mathematics text. However, as stated by a teacher in that study:

“We have learned to gear our instruction towards what the children know and what they are thinking rather than trying to push a certain
method down the children's throats as far as how they are to learn to do math. They come up with their own ideas and you just branch off from that" (Fennema, Carpenter, Franke, Levi, Jacobs, Empson 1996: 419).

This framework allows teachers to view their teaching procedures from a child's perspective. As Gould (2004) suggested teachers need to be aware of the language they use in order for the students to become proficient at using it. Correct modelling of procedures, language use and mathematical concepts assists students with their understanding. Once the students have an understanding it is also the teacher's role to help the students express their understanding. Asking students to choose their own representational mode to indicate their understanding can assist the teacher to identify to what extent and what level the student understands particular concepts in mathematics.

2.8 Key Points
Both Qualitative and Quantitative methods were used in the research reviewed. Interviews, surveys, sample groups, work samples, video taping, questionnaires, case studies and classroom observations were included. The data was then presented in a variety of tables, graphs and transcripts. The researchers themselves clearly exploited the advantages of a variety of representational modes, in both data collection and in reporting their findings. Obviously, all the research involved young children. It can be seen from this extended literature review that there are both advantages and disadvantages of using a variety of different representational modes. Interestingly, the advantages seem evident from the students' perspective, while it is in the challenge to
the teacher's existing repertoire and the additional load of an already crowded curriculum that multiple representations might be seen as a disadvantage. However, none of the research reviewed offered the students the opportunity to choose their own way to complete a task.

With the focus of government and educational bodies being in the Early Years of Schooling, my research in numeracy education is relevant and supports the research reviewed. My research would also ask many questions and lead to further research into benchmark and scholarship testing and whether these give a true indication of a student's mathematical understanding. The recent research projects such as the Early Numeracy Research Project (2002), Understanding Place Value: The Base Ten Game (2004) and other research reviewed looked at the roles of classroom teachers and how students best learn mathematics using a variety of modes. Only the Early Years Numeracy Research Project also looked at the role of assessment and the implementation of growth points. According to Gould (2004), Thompson and Thompson (1994), Fennema, Carpenter, Franke, Levi, Jacobs, and Empson (1996), the role of the mathematics teacher is essential in assisting a student with their mathematical understanding. The teacher should model the mathematical language, ensure students are using concrete materials correctly and know why they are using them. It is the major premise of this thesis that teachers must provide the opportunity for students to use their own mathematical recordings, if they are to obtain useful insights into what the students really understand. This research addresses the questions: Do students understand a mathematical task but have difficulty expressing their understanding through the directed representational mode? Are students being
disadvantaged by not being able to represent their mathematical understanding in their preferred mode?
CHAPTER THREE
Methodology
Conduct of Investigation

3.1 Introduction

The Methodology of this investigation involved qualitative research into the way
students choose to represent their mathematical understanding. After an extended
Literature Review of previous research in the area of students learning and using
representational modes it became apparent that classroom observations, follow-up
audio-taped one-on-one interviews, the use of classroom video footage, participant
observer journals and a collection of work samples would obtain worthwhile data
from students of this age. It would also maximise the ways the students use and
manipulate materials provided to them. This Chapter is presented in the following
sections:

3.2 Data Collection through focused learning sessions
3.3 Lesson One
3.4 Lesson Two
3.5 Lesson Three
3.6 Lesson Four
3.7 Lesson Five
3.8 Lesson Six
3.9 Lesson Seven
3.10 Research Methods
3.11 In Summary

Data collection took place within the mathematics classroom and during the
allocated Numeracy periods during a four week period. The students were involved in
a normal classroom session. The only obvious difference was the video camera placed
in the corner of the classroom recording the mathematics sessions. The video camera was placed in the classroom for two weeks prior to data collection in order for the students’ to become familiar with its presence.

In order to maintain the desired contextual focus, my methodology was chosen using a teaching experiment to capture first hand the students’ mathematical learning and understanding.

“*A primary purpose for using teaching experiment methodology is for researchers to experience, firsthand, student's mathematical learning and reasoning. Without the experiences afforded by teaching, there would be no basis for coming to understand the powerful mathematical concepts and operations students construct or even for suspecting that these concepts and operations may be distinctly different from those of researchers*” (Steffe & Thompson, 2000: 267).

Videoing the students’ as they completed their tasks allowed the researcher to view the students’ body language, behaviour and the interaction with their peers. The students were given the opportunity to explore their own preferred styles and not be told how to complete their task.

### 3.2 Data Collection through focused learning sessions.

The data collection took place whilst the participants were engaged in six focused addition sessions and a follow up one-on-one interview with the participants after they had completed their own representational selection task. When I taught the six
focussed sessions, I ensured that two sessions focused on each of the three chosen representational modes of 'physical (kinaesthetic), oral (linguistic) and written (visual)'. The sessions were taught linking the six sub-strands and the three representational modes as indicated on table 3.1.

Table 3.1: Six sub-strands and three representational modes

<table>
<thead>
<tr>
<th>Written (Visual)</th>
<th>Physical (Kinaesthetic)</th>
<th>Oral (Linguistic)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Session 1 A: Drawings/diagrams</td>
<td>Session 3 C: Individual using concrete materials</td>
<td>Session 5 E: Public by talking to the whole class</td>
</tr>
<tr>
<td>Session 2 B: Words/Statements</td>
<td>Session 4 D: Group work using our bodies</td>
<td>Session 6 F: Private by sharing and talking to a partner</td>
</tr>
</tbody>
</table>

As the research was conducted during classroom teaching periods, the data collection focused specifically on ten students chosen from those who had consented. The selection of students was intended to maximise diversity of student characteristics such as ability and gender. Selection of students was informed by observations of prior participation in classroom activities. Notes about each student's strengths and weaknesses had previously been recorded on an annotated class list. This method was chosen to ensure that the students that were selected and granted permission from parents and guardians were from a mixed ability group that showed strengths and weaknesses in particular areas. In order to ensure that no student was disadvantaged by selection or participation, and that my research was valid, all students were required to have had previous experience in each of the six activities. In the case of one of the girls, even though she was absent from the two sessions focussed on the
Visual mode, she had been involved in previous class lessons where the same activities were conducted.

The six focused sessions were taught as individual lessons consisting of 45 minute blocks. However, due to the class timetable these lessons were grouped and taught in a double period. The two stand-alone lessons were taught grouped by the representational mode. The final individual representation task was given a single lesson allocation of 45 minute block. The lesson directly after the seventh session was used to conduct the one-on-one student interviews. The six focused teaching session framework is indicated in table 3.2.

*Table 3.2: Six focused teaching session framework*

<table>
<thead>
<tr>
<th>Session</th>
<th>Activity</th>
<th>Time allocation</th>
</tr>
</thead>
<tbody>
<tr>
<td>One</td>
<td>Written addition task where the students write down worded addition sums to express their understanding.</td>
<td>45 minutes</td>
</tr>
<tr>
<td>Two</td>
<td>Written addition task where students represent their learning through diagrams and pictures.</td>
<td>45 minutes</td>
</tr>
<tr>
<td>Three</td>
<td>Physical addition task working individually using concrete materials.</td>
<td>45 minutes</td>
</tr>
<tr>
<td>Four</td>
<td>Physical addition task working in a group and using the body.</td>
<td>45 minutes</td>
</tr>
<tr>
<td>Five</td>
<td>Oral addition task focusing on sharing learning by sitting in the chair at the front of the class.</td>
<td>45 minutes approx 2 minutes per student</td>
</tr>
<tr>
<td>Six</td>
<td>Oral addition task where the students share their learning with a partner.</td>
<td>45 minutes approx 2 minutes per student</td>
</tr>
<tr>
<td>Individual student</td>
<td>Students select their own way to complete a simple addition sum. The students were told if they wanted to represent their understanding through the Oral mode then they were to notify the teacher and that would be organised for them.</td>
<td>15 minutes</td>
</tr>
<tr>
<td>Selection Task (Clinical Interview)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
3.3 Lesson One: Written addition task where the students write down worded addition sums to express their understanding

The lesson commenced with the students playing a quick game of dice addition. They rolled two dice and added them together (example 6+3=). This was used as a tuning in activity to get the students ready for addition. The students were then read the story ‘Add and Subtract at Sea’ written by Susan Frame (2003). This big book uses worded addition examples with supporting pictures. The students were asked to focus on the words. After reading each page a student was asked to use the information that was provided to them through the story to write the addition sum on the whiteboard. After the completion of the big book the students were given an example of how to write a worded addition sum on the board. The students were then asked to give an example of a written addition sum orally.

The students were given two sums that were written up on the board.

- 12+5=
- 9+5=

They were then required to write two separate worded addition sums. The language of addition was discussed (example: plus, more, equals, altogether). The students were not required to draw a supporting picture. They were only required to write the two worded addition sums, one on each side of the page. At the conclusion of the lesson, students were selected to read their stories out aloud. This was so that we could discuss the use of correct language and real life examples.
3.4 Lesson Two: Written addition task where students represent their learning through diagrams and pictures.

The lesson was introduced by reflecting on the worded addition sums that we had completed in the previous numeracy lesson. The Big Book 'Add and Subtract at Sea' was referred to, this time we just used the pictures to work out the addition sum (Example: There were 6 green yachts and 5 red yachts). The words were covered at the top of the page so therefore the students were only using the pictures to identify the addition sum. Individual students were selected to use the pictures to write the addition sum on the board. Continued reference was made to the big book and how clear the pictures were. As a class the students were invited to complete a few examples on the whiteboard. The students were then given two sums in which they were required to draw or represent their answer through pictures. The same two sums from our previous lesson were written up on the board.

- 12 + 5 =
- 9 + 5 =

The children were encouraged to use pencils, textas and crayons to complete their drawings. Six students asked what pictures they had to draw, but they were instructed to draw any pictures they liked to represent the addition sums. At the conclusion of the lesson a few students were selected to share their drawings/diagrams with the class. Through this sharing further discussion were held as to representing what we knew in addition through drawing and writing. Many of the students took a lot of time and put a lot of detail into their drawings. It was interesting to note that these were the students who enjoyed drawing. When checking these students’ work, some of them
spent more time focussing on their drawings so that the intended outcome of the lesson was not met.

3.5 Lesson Three: Physical addition task working individually with concrete materials.

The students were asked to reflect back on our previous two lessons: "What were we required to do? How did we solve our addition sums?" After a brief reflection the students were asked to participate in a class brainstorm. The focus question being ‘Other than using pencil and paper, writing words, numbers and drawing pictures, what other way can I solve an addition sum?’ The students’ answers are listed below:

- Use icy pole sticks
- Use counters
- Use a dice
- Count using pencils
- Use my fingers and toes
- Use blocks
- Count on
- Count all
- Use leaves from outside
- Use ourselves (This required a little prompting. Once a student identified this, an example using the students was required).

After the brainstorm, a variety of hands-on materials and equipment were introduced (Figure 3.1)
Individual students were selected from the class to show their peers how they would use this equipment to solve the addition sum 12+3=. The students modelled this sum using the equipment which supported the physical mode. The students were then divided into groups with a specific piece of equipment and asked to make the sum 15+11=. A rotation took place allowing all students to represent the same sum using
the selected concrete materials. At the completion of the lesson, individual students were asked to share their findings and understanding with the class. An oral retell of using the physical mode was undertaken for each of the five pieces of equipment. The lesson concluded with a ‘What did you learn today?’ session. Interestingly enough one student said that in today’s lesson she learned “That you don’t always have to write it down”. Another student said “that she could see the two groups that you have to make before you add them altogether”. The last child said that he learned that “15+11 made 26”. The students were reminded just before the conclusion of the lesson that today’s class was based on using the Physical mode.

3.6 Lesson Four: Physical addition task working in a group and using our bodies

This lesson was a part of a double period that linked to lesson three. That decision was made so that the students could see a variety of ways of using Physical representation and also to provide a link between the different ways. After reflecting on what we had completed in the previous session and using a variety of concrete materials, we referred back to our brainstorm that the class participated in at the beginning of lesson three. This was still visible on the whiteboard and it remained on the board until both lessons were completed. After reading through the list, I asked the students to discuss using our bodies to solve an addition sum. For example: Using our fingers and toes and using ourselves. Students were asked to provide a demonstration of what that meant. As a class we decided that using our fingers and toes could be linked to the previous lesson on using concrete materials but adding another category: using counters, blocks, pegboards, fingers/toes, plastic links and icy pole sticks. We then decided we could have completed the same 15+11 = sum as a group. The students were then asked to work back in their groups from the previous lesson and
instead of completing this task individually they were asked to complete this task as a group. The students were used to working as a group and throughout all key learning areas students work together. It was noted that students must be able to work productively in a group before this type of lesson could take place.

The students were only asked to use one type of concrete material (example: counters) and were then asked to explain to their peers through share time how they worked together to solve the problem. After this, the students were then introduced to using themselves to solve an addition sum. For example: 7+9= Seven boys and nine girls makes how many students altogether? The students appeared to enjoy being chosen to stand up to be included in the sum. Chairs and tables in the classroom were moved to make enough space for this physical mode to take place. The students were then shown the sum 8+10 = on a piece of paper. They were then required to make this without teacher assistance. After a few ‘group’ discussions which included that we didn’t have enough boys to make 10, and another student asking her friends “do we have to be broken down into groups of girls and boys?” the class completed the task. The last sum they had to solve in a physical mode was 3+1= . Many students made it gender specific; three boys and one girl or three girls and one boy. There were a few groups that were mixed genders. The students were then asked to explain how they came about their groupings. One student answered, “It didn’t have written in the sum what there was three of so it could have been anything. So we just made it three groups of children and one more child.” This lead to a discussion on what the question was actually asking. The lesson concluded again with students sharing orally what they had learned today. Again, students’ comments were centred on having fun
making groups and numbers. This lesson also concluded with the students being told that today they learned how to solve addition sums using the physical mode.

3.7 Lesson Five: Oral addition task focussing on sharing learning with peers in a public setting

After a brainstorm of the different ways in which we have been completing our addition sums in class, the students were asked if there another way in which we represent or share our understandings. After a lot of prompting the students were introduced to the concept of sharing what they understood through explaining it to the class. To assist the students with understanding through the Oral mode the students were encouraged to sit on their hands and answer a few simple addition sums. These sums were focused on doubles and near doubles. These number facts were something the students have been working on in their numeracy block. Students were then asked to explain how they came about their answer.

Example: 7+7= 14 therefore 7+8= 15

Students were asked to explain to the class how they got the answer. In order to ensure that the students were comfortable with the situation, they were encouraged to smile when they got the answer to the sum. That way, as the teacher, I ensured that all students had come up with the answer before I asked them. The students were then asked to give the answer to the sum and then explain how they came about that answer.

Example: 11+3 =

Some examples of answers given include:
* I took away 1 to make it 10 and then added 1 to the 3 to make 4. Then I knew that 10+4 = 14

* I used the tens and ones. 3 and 1 made 4 and then I added the one to the front to make 14.

Through this type of discussion the students were encouraged to think of the many different ways in which we could have completed the sum. This oral exercise of sharing how a particular student came about the answer encouraged other students to say things like “I got the same answer but I did it differently”. As each student shared their strategy I did a written example on the board to help the other students in the class visualise how some students solved the sum. Again the link was made between the different representational modes. A class discussion took place on adding to the nearest 10. The exposure to this representational mode at first seemed challenging for many of the students, but after a few students had been involved and explained their understanding it encouraged others to get involved. The use of language did prove to be a challenge as many children had trouble expressing how they understood it. This led to a revision of terms that are used in our mathematics class.

Numeracy Terms discussed:

- Count all: to count all of the numbers to get the answer.
- Count on: To count on from one number to the other. Example: 7+3 (7,8,9,10).
- Known Fact: Something that I just know.
- Add to the nearest 10: The student add to the nearest 10 and use this as their base number.
After a revision of mathematical terms, the students became more confident in explaining their answers. It proved to be essential that the students all have a similar understanding of mathematical terms and language in order for them to understand each others’ explanation. However, some of the students were quite articulate when giving their description. This session proved that in order for the students to use the language, they must be familiar with it.

The next part of the lesson required the students to orally explain a ‘real life’ story and then provide the answer. The students were asked to sit on the chair at the front of the classroom to explain their stories. The sum was written on the board and the students read the sum and then made their story.

Example: 15+2 =

One student’s story was: ‘when he went to the shops to get strawberries and chocolate. He then asked his Mum if he could get them all’. In this example, the child forgot to put in the numbers. When this was explained and modelled by the teacher, he was then able to complete this task confidently. These tasks were all modelled before the students were encouraged to participate.

At the conclusion of the lesson the students shared what they had learned that session. One of the children couldn’t believe that was a maths lesson because “we didn’t do any work”. It is slightly amusing to think that unless students’ hands are busy they don’t think they are learning anything. This session also identified the students’ different learning and thinking styles.
3.8 Lesson Six: Oral addition task where the students share their learning in a one-on-one interview with their peers.

As a result of the school timetabling, this lesson was included in a double period with lesson five. However, a clear distinction was made between the two lessons. To start this lesson, the students were paired up and given three dice. The students rolled the three dice and then had to explain out aloud the answer.

Example: 3+3 makes 6 and 4 more makes 10.

This was seen as an easy way to get the students talking with a partner without them realising what they were doing. As a result all students were involved in orally explaining how they came about an answer. Whilst still in their pairs the students were then given a sum and they had to tell a story using the sum. This was identical to the task that was modelled and completed in lesson five. However, this setting gave every student in the classroom the opportunity to be involved and share their understanding. It was again stressed that these stories needed to be ‘real life’ situations. The teacher roamed around the room listening to students’ stories. Each student shared three stories. When they were not sharing a story, they were listening to their partners. The last task was that the students had to give their partner an addition sum for them to solve. Not only were the students explaining their understanding through the use of stories, they were also showing and displaying their understanding of the make up of addition sums when orally giving their partner a sum to solve.
This non-threatening setting allowed all students to participate including the English as a Second Language (ESL) students who at times are a little hesitant to be involved in group discussions. The teacher was able to monitor the level of a student’s ‘mathematical language’ and understanding by listening and sitting in on a group. As with lesson five, in my teaching, I have observed that in order for students to speak mathematically they need to be provided with a model of ‘maths’ talk and be given the opportunity to talk maths and explain how they came about their answer.

3.9 Lesson Seven: Individual Student Selection Task

This forty-five minute Numeracy lesson commenced with the students being re-read the story ‘Add and Subtract at Sea’ (Pye, 2003). This big book uses worded addition examples with supporting pictures. As the students had heard the story a few times they were keen to read along and count the pictures. The students were asked to think back to the last few addition lessons when they were given the opportunity to explore a variety of ways to complete a simple addition sum. Three posters were displayed on the white board with an example of the different modes the students had used (See Figure 3.2).
Figure 3.2: Mode description posters
We discussed the posters and worked through an example of each. The students were then told that in a few minutes they were going to be given an addition sum in which they were required to work out the answer. They were asked to decide which way they felt they could show they understood addition sums. It was discussed that if the students would like to to they could complete the activity using the oral mode, they were just required to come and speak to me. Due to the age of the children, they were encouraged not to choose the same activity just because their friend did it. They were also told that at the end of the lesson there would be a ‘share’ time and that they would be required to explain why they chose that mode. The students were then given 15 minutes to complete the addition sum 12+7=.

During the fifteen minutes the room remained relatively quiet. The students were all engaged in their activity and they didn’t feel the need to interact with their peers. When the students had completed their activity and were happy with their work they were required to bring it to the carpet and sit in a circle. Each student explained what mode they used and many of them referred to the posters to ensure they were using the correct words. To conclude the lesson the students were asked how they felt about being able to select their own way to complete an activity.

Interview

After the completion of the individual selection task, ten students were interviewed in relation to their task selection. The one-on-one interviews took place in the students’ classroom and the work that they completed was placed on the table in front of them. The video camera was sitting on a slight angle behind the student and it was used to record answers to the interview as well as body language. The students were told that there was a camera situated in the room before their interview commenced. During the interview the
students were encouraged to use the equipment and to explain how and why they chose that particular representational mode. The interview data was then transcribed.

3.10 Research Methods

One-on-One Interview

One-on-one interviews with students have most typically been promoted for helping teachers understand how children think about mathematics (Buschman, 2001; Ginsburg, Jacobs, & Lopez, 1998; Long & Ben-Hur, 1991). The Early Years Numeracy Research Project (2002: 27) found that “the interview was particularly powerful in the respect that it provided detailed, reliable information on what children knew and could do, collected in an enjoyable way for young children.” Through the use of hands-on materials and mathematics games the children have found this interview to be enjoyable. Specifically, interviews provide researchers and teachers with opportunities to practice eliciting and building on children’s thinking by engaging them in discussions of problems. This also assists researchers and teachers to identify the words the students use and what level of understanding they have of a chosen topic. After the completion of the student selection task the students were interviewed and asked why they chose that particular method. Students were also asked to reflect on their feelings about completing the task using the Oral mode. Student interviews were recorded on a video recorder and transcribed.

Observations

As the classroom teacher and the researcher, observations were used as a natural part of my teaching activity. The ten participants were observed during the focused sessions and the data was recorded by annotations on a class list as I made observations. The observations that were made were based on how the students participated in classroom activities. The students were identified according to who showed a particular strength and
weakness for a representational mode and how they responded to the sub-strands. A video camera was also placed in the corner of the classroom. This allowed for the student’s body language and facial expressions in group activities to be recorded.

Another form of observation that was utilised was the Participant Observer Journal. It is a data gathering device that commenced at the beginning of the study and continued throughout. Observations about individual participants in learning sessions, the last task and during the interviews enhanced the richness of the accompanying data. The entries were dated and pseudonyms were used to ensure confidentiality. The journal was added to as frequently as possible and when it was required.

Observation was chosen to supplement the other methods of data collection to add more depth to the concepts and themes that became apparent. As Robert Burns states, “In addition to its independence of a subject’s ability to report, observation is also independent of the subjects’ willingness to report. There are occasions when research meets with resistance from the person or group being studied” (Burns, 2000: 23). Therefore, observing the students during the six sessions and using a Participant Observer Journal, other issues that may arise that would not necessarily be specified in an interview format. As the children were of a young age, the information gathered through observations may assist with why children chose a specific representational mode.

“Observations play an important role as the researcher notes the interviewee’s body language” (Marshall and Rossman, 1999: 107).
3.11 In Summary

The three modes of Physical, Visual and Oral representations were focused on in a Year One mathematics classroom. Each of the six research lessons that were taught involved the whole class in all the processes associated with the different representational modes. In a seventh lesson, students were offered the opportunity to select a representational mode by which to display their understanding of a particular addition problem. From the students' responses, ten students were chosen for detailed analysis and to participate in the one-on-one interview. Observations and notes were made on all students throughout the six focussed lessons. The ten students that are the focus of the next chapter were selected on the basis of their task selection process. The data collection and student selection for interviews aimed to maximise diversity of student characteristics such as ability and gender. They were also chosen for the interesting ways in which they chose to represent their mathematical learning.

A detailed analysis of the results and a discussion of the findings of the investigation including representational selection and student learning styles follows in the next chapter.
CHAPTER FOUR

Results and Discussion

4.1 Introduction

This Chapter reports the results of a qualitative investigation of the way Year One students represent their Mathematical learning. The results and interesting ways in which students chose to represent their learning in relation to the representational modes discussed in previous chapters will be presented and evaluated. This chapter is presented in the following sections:

4.2 Physical Representational Mode
4.3 Visual Representational Mode
4.4 Oral (Linguistic) Representational Mode
4.5 Analysis of students’ task selection
4.6 Summary of findings

After the completion of the six focussed learning sessions the children were presented with the addition sum of 12+7=. They were then provided with the option of completing the addition task by one of the three representational modes of Visual, Physical or Oral. The students were given fifteen minutes to complete the task in their chosen preferred mode. At the conclusion of the lesson, the students were involved in a one-on-one interview, where they were asked to explain why they chose to represent their understanding in a particular representation mode.

Ten students were involved in the investigation and the gender balance was even. The students’ ages were either six or seven years old. It is the second year of schooling for all the children involved in the study.
Table 4.1: Gender Distribution

<table>
<thead>
<tr>
<th>Gender</th>
<th>Number of Students</th>
</tr>
</thead>
<tbody>
<tr>
<td>Male</td>
<td>Five (5)</td>
</tr>
<tr>
<td>Female</td>
<td>Five (5)</td>
</tr>
</tbody>
</table>

During the seventh lesson which required the students to select their own preferred representational mode they chose to work individually, although the option was available if the students wanted to work in a small group. The break-up of how the students completed the final activity can be located in table 4.2:

Table 4.2: Final task representational mode break down

<table>
<thead>
<tr>
<th>Representational Mode</th>
<th>Number of students who selected this mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>Visual</td>
<td>Five students</td>
</tr>
<tr>
<td>Physical</td>
<td>Five students</td>
</tr>
<tr>
<td>Oral</td>
<td>No students</td>
</tr>
</tbody>
</table>

This table indicates that the students either chose to represent their understanding through the Visual or Physical mode. No student chose to represent their mathematical understanding of addition through the Oral mode. There were a variety of ways in which the students could represent their understandings in these modes. The task selections are identified in table 4.3.
Table 4.3: Task Selection

<table>
<thead>
<tr>
<th>Representational Mode</th>
<th>Specific Task Representations</th>
<th>Number of students who selected this mode</th>
<th>Gender</th>
</tr>
</thead>
<tbody>
<tr>
<td>Visual</td>
<td>Drawing a Picture and Writing a Story</td>
<td>One Student</td>
<td>Female</td>
</tr>
<tr>
<td>Visual</td>
<td>Writing a Story</td>
<td>One Student</td>
<td>Female</td>
</tr>
<tr>
<td>Visual</td>
<td>Drawing a Picture</td>
<td>Three Students</td>
<td>Male (2) Female</td>
</tr>
<tr>
<td>Physical</td>
<td>Icy Pole Sticks</td>
<td>One student</td>
<td>Male</td>
</tr>
<tr>
<td>Physical</td>
<td>Peg Boards</td>
<td>Two Students</td>
<td>Female (2)</td>
</tr>
<tr>
<td>Physical</td>
<td>Plastic Links</td>
<td>Two Students</td>
<td>Male (2)</td>
</tr>
<tr>
<td>Oral</td>
<td>Presenting to the Whole Class</td>
<td>No Students</td>
<td>N/A</td>
</tr>
<tr>
<td>Oral</td>
<td>Presenting to a Small Group</td>
<td>No Students</td>
<td>N/A</td>
</tr>
</tbody>
</table>

The students’ final task selection, as indicated in the previous chapter, was conducted in a stand-alone lesson. The students had been asked to reflect on the way they felt more comfortable in representing their addition sum. They were also asked in their one-on-one interview the factors that influenced their selection. These will be discussed later on in this Chapter. A comparison will also be made with student actions during the six focussed sessions and each student’s final task selection.
4.2 Physical Representational Mode

With half of the sample choosing to represent their understanding through the Physical mode, it is interesting to note that, other than in the Early Years Numeracy Project (Clarke, Cheeseman, Gervasoni, Gronn, Horne, McDonough, Montgomery, Roche, Sullivan, Clarke, & Rowley, 2002) and the Base Ten Program (Association of Independent Schools of South Australia, 2004), students are seldom offered the opportunity to represent their understanding through the Physical mode. There was a variety of different concrete materials that were used by the students and no student chose to use their body or work in groups. The students used materials such as the icy pole sticks, plastic links and peg boards. Other equipment such as the counters and unifix blocks were also placed on the floor but no child chose to use this equipment for their preferred mode. So what led the students to make these decisions? Did they copy a friend? Did they have a preferred piece of equipment? Did they find this task easy during the focussed group session and feel comfortable using this mode? Did the students choose what they preferred or what they thought they were good at?

Through the one-on-one interviews with the students many of these questions were answered. The ability to manipulate the equipment appealed to three of the five students. “Like I said it would also be better to actually join the links together.” Student Two felt that it was easy to add the two numbers together when you were able to join the plastic links together and to count them. Student Five liked the ability to put on and take off the pegs on the peg board and that is why she chose to use the peg boards. “Why did you decide to do peg boards?” ..... “Um, because they can take off and you can put them on easily”. When this student was asked whether it was
important to be able to take things off and put them back on in mathematics she replied "To me it is". The students liked the ability to be able to manipulate the equipment and to make changes. Student Six experimented through completing the task in two ways and by ending up with two answers. The first way he didn’t connect the plastic links and the second way he connected the plastic links together. He wanted to identify which was easier:

**1. So why have you got two?**

_S6_. Um cos. Um I felt like doing two.

**1. So did you find it easier to connect it in one row to count them altogether?**

_S6_. Yeah, and then I was seeing if that was easier (pointing to the rows of links joined together) or that one was easier (pointing to the links that he had made into a circle).

In the end this student kept both pieces to present them to the class. It appeared through these few transcripts that the ability to manipulate the equipment made the task easier for them to complete and understand. The student who selected icy pole sticks was not bothered that he was the only student to use icy pole sticks. He decided straight away that he wanted to represent his answer using this equipment. When he was asked why “I think it is easier because I like using things…. It felt comfortable and easy to um do.”

The ‘fun’ factor and using equipment that the students had not previously used before, other than the six focussed sessions, also appeared to appeal to the five students who chose the Physical representational mode. An interesting fact to note was that the students have used counters on a regular basis in the Year One mathematics program and using the unifix blocks is a part of the Preparatory mathematics program. Yet, these were the two pieces of equipment that no student
chose to use. The icy pole sticks are often used during class time and they are used by the students independently. The links and peg boards are used in class for other purposes than addition. Therefore, this could be why the peg boards and plastic links were the preferred methods for some students to use in relation to representing their mathematical learning.

(Student Six chose to represent his understanding in two ways; by connecting the plastic links together and by not connecting the plastic links)

*Figure 4.1: Student physical representations*
All five students who chose the Physical representational mode said that, if they were required to do the same activity tomorrow but with a different sum, they would choose another way to represent their understanding. One student would have chosen the Oral mode of telling a story to the class but the remaining four students would have chosen another type of equipment and still remained in the Physical representation stream. This suggests that the children do have a preferred representational mode and although they would use another type of equipment they would prefer to stay in the category of the Physical Representation. The students’ recent experiences and the fact that they would stay in this mode representation suggests that their preferred mode preference is well-established. They were asked if they would consider using the Visual form of representation and four of the five students said they would not want to write an addition story:

S5. Well, because... because it’s a bit, because you have to write you have to draw and write stories and sometimes you can’t think of a story quickly as you need to.

I. So you were worried that you wouldn’t be able to think of a good story?

S5. Yes

I. You didn’t want to draw a picture?

S5. No.

I. Why not?

S5. Because, it is like the writing where you gotta do the writing except you gotta draw the picture.

Student Five was very concerned that she would not be able to think of a good story to write and that is why she chose to use pegs to complete her answer. She liked the ability to be able to put on and take off the pegs. The fact that she couldn’t think of a good story may also lead the fact that to achieve success and complete the task to
the best of her ability, she needed to make it good. To her, a bad story would not be seen as successful. Student Two had a similar opinion about completing the task by writing.

**I. So why did you decide today to complete your activity using the links?**

**S2.** Well, you see I said in my mind it would be better than writing, and um like I said it would also be better to actually join the links together.

**I. Can I ask why it is better than writing?**

**S2.** Well, because you see writing is some..... You know how I told the story and you know how I did that writing (The student is referring to Lesson Number one) I actually did a takeaway sum instead of a plus.

**I. Oh so, when you do your writing you get confused and do a take away?**

**S2.** Yes (student replied very enthusiastically and agreeing)

**I. So you knew that, is that why you chose to do the links today because you knew you would be able to do plus?**

**S2.** Yes

Was it again that perhaps the fear of not completing the sum correctly that led this student to select a mode that perhaps was not preferred, but in fact it was chosen because it would guarantee the correct answer? Student One supports this view:

**I. Great. Now why did you decide to use pegs and not do some writing?**

**S1.** Because, Pegs is a little bit fun and I’m not really good at writing that much.

**I. Really? I think you are very good at writing.**

**S1.** Even though I try to do my best.

**I. You try at home do you?**

**S1.** Yes

**I. Do you try and do Maths writing at home?**

**S1.** Yes
Student One obviously practises her writing at home and believes that she is not good at writing. Therefore, she chose the Physical representational mode as she knew she would be good at this. Student Six just had one of those days where he felt he just didn’t want to write:

S6. Um, well probably……. pause for 5 seconds…. um

I. Well why didn’t you do the writing?

S6. Because um um, I just didn’t feel like it

I. You didn’t feel like it. When I get you to write in maths do you mind doing writing? Or don’t you like writing?

S6. I like writing a bit ……but not really.

I. So you like writing a bit?

S6. Yeah

This student doesn’t mind writing but he just felt like using equipment. Student Six is also very good at completing mathematical sums in his head. In previous mathematical tasks his head was working faster than his hand. He usually struggles with tasks that involve writing and therefore, completing the task took too long for him to write his ideas and he managed to get confused. Student Four who chose icy pole sticks to complete his task had no comments to make about writing. He said he would complete the task using something else on another day, he likes to do things differently. When questioned about using the Oral mode he said “he would consider it but he was more comfortable using this” (student points to the icy pole sticks lying on the table in front of him).
From the transcripts, it is obvious that the comfort and fun factor was important to all the students as they decided to complete the tasks. They wanted to use something that was different, that they were comfortable with and something they knew they would be able to use correctly. Therefore, is it the preferred mode that leads students to represent their understanding or is it the feeling that they know they will complete the task correctly? All five students completed the addition sum successfully and correctly. They were also able to verbally explain how they set out their equipment and completed the task. The ability to verbalise their selection and the steps involved in completing the task could also be considered as using the Oral mode to express their understandings. When reviewing the Literature in Chapter Two, it was apparent that a key point was that for children to represent their understanding using concrete materials, they must be able to understand how and why the materials are being used for them to truly and successfully represent their mathematical learning. The five students who chose this form of representation have definitely fulfilled that criterion. Through completing their tasks they have also supported the view that representational modes can overlap. Although the students completed the task using concrete materials, they also explained their understanding using the Oral mode.

4.3 Visual Representational Mode

Similar to the Physical mode, the Visual Representational Mode was selected by half of the sample group. Of the five students who chose to represent their understanding of addition through this mode, three were female and two were male. The task selection types varied with one student electing to include both a sentence and a
supporting picture, one student chose to write a story and three students elected to use a picture as their form of representation. The work samples can be found in figure 4.2.

| 12 + 7 = 19 |
| 1 went swimming i found 12 jellys and 7 more. I have 19 jellys. |
| Student Three |

| Student Seven |

| Student Eight |

| Student Nine |

| Student Ten |

*Figure 4.2: Student written representations*

The two students who chose to complete their task through writing were both females. One of the factors that influenced their decision to write was that they both enjoyed writing. Student Three who just used writing to represent her understanding saw this as a good opportunity to practise her writing:
I. I would like you to tell me about your activity that you completed today.

S3. By writing

I. Now why did you choose to do writing?

S3. Because, I thought it would be a good idea if I practise writing at some places.

I. Now do you do a lot of writing at home?

S3. Yep

I. So you like to do lots of writing. Do you prefer to do writing or drawing?

S3. Writing

I. Ok, so you prefer to do writing? Now, why did you decide to do writing? Just because you wanted to practice?

S3. Um

I. Or did you like the story that I read that was all about writing?

S3. I like doing writing in lots of places and also that I like to see beautiful writing.

I. Oh me too! Now, if it was possible for you to complete it, the activity doing the oral way by talking on the chair or by talking to a friend about a story would you have chosen that today. Or would you still have chosen to do writing?

S3. I would still have chosen to do writing.

When Student Three was asked how she came about the answer of 19, she said she used her fingers and toes. Again, the overlapping of representational modes is evident. But yet, this student was not comfortable or confident enough to complete the task using the Physical Representational mode or simply preferred written representations to the alternative. Student Eight who chose both writing and drawing saw this task as an opportunity to show off what she can do. She drew the picture so she could count how many there were and she decided to write a story so she could tell people about something that she has done. This student through her mode selection has indicated a very good understanding of the concept of addition.
1. I would like you to tell me about your activity that you completed today.

S8. Um, I . . . done this because I like drawing and I like writing things and I like writing about that.

I. Ok you chose to do the writing and the picture. Why did you choose to do both?

S8. So you can see the picture, what it is. So you

I. So you needed?

S8. (Student speaks over the teacher) So you can count.

I. Oh so you put the picture there so you can count what you have written?

S8. Student nods head up and down to indicate yes.

I. Ok, so how did you decide that this was the way you were going to do it today?

S8. What do you mean?

I. Well with all these things on the floor (Interviewer points to equipment still placed at the front of the room) why did you decide to do it this way?

S8. It is a bit easier to count that.

I. Why didn’t you decide to use perhaps the hands on mode?

S8. Cos, I think it’s a bit fun to do that (student picks up the piece of paper on which she had drawn and written her answer sitting on the desk in front of her)

Similar to the students who chose the Physical representational mode, these two students chose to write as they enjoy doing it. The remaining three students that chose to draw a picture also enjoy drawing. Student Seven said that he chose to draw a picture because it was "comfy" with him. Again, the comfort factor appealed to students when they selected their form of representation. Student Ten thought that drawing a picture would look nice and Student Nine really liked drawing and that is why he chose it.
Students Nine and Ten both drew the standard twelve of one item and seven of another item. Example: 12 love hearts and 7 suns. Then they counted the pictures altogether to get the correct answer. This was a very straight forward way to answer the question and mirrored the way the students were shown how to draw a picture to represent their understanding in focussed lesson number two.

![Figure 4.3: Representing 12 + 7 =](image)

Student Nine

Student Ten

Student Seven was very interesting and somewhat abstract in the way he chose to represent his understanding. He enjoys drawing and is often distracted in class by decorating or drawing a picture that not necessarily related to the task. In most situations, he genuinely has an idea in his head and he will get carried away with it.

![Figure 4.4: Unsuccessful attempt](image)

Student Seven Drawing

61
Student Seven drew a picture of two cars, with three cats inside one car and a person driving the other. He also drew one bird in the air. He was asked to explain how he drew this addition representation.

I. Tell me what the first thing it was that you did? Did you draw the picture first or write the sum up the top?

S7. Well first thing is I drewed, I drewed the sum then I drew the picture.

I. I actually did notice that. Did you write the sum to remind you of what the picture should be about?

S7. The student nods his head up and down to indicate yes.

The student was very confident that he read the sum correctly and drew the matching picture as a form of representation. During the one-on-one interview the student was asked to explain where 12+7 was shown in his picture:

I. Ok. Now if I show you this card which is the same as you have on the top of your sheet. Can you show me 12+7 in your picture?

S7. 12 plus 7..... I’m not sure what you really mean.

I. Ok, well remember we’ve got twelve and you said kittens this time. So twelve kittens and seven kittens. Can you show me where they all are in your picture?

S7. Well I didn’t really have enough, so I tried to do as much as I could. This is one kitten that well there is a sun roof up here so it’s enjoying the sun. (The student is pointing to the picture of his car in his drawing) This kitten is thinking that one of his toys has gone down the street.

Clearly his picture does not show how he worked out that 12+7 =. At the beginning of his interview the student was asked to tell the interviewer about the activity he had completed on that day. When it came to explain his activity Student Seven had forgotten about the cars and the bird and he discussed kittens.
S7. Well this is a picture about how much kittens I bought. And on Monday I bought twelve kittens and three days later um on Thursday I bought seven more. And in this picture you can see me driving back to home.

1. Ok where is your twelve where are all your, where is your twelve? And where is your seven?

S7. My twelve is here (pointing to the number 12 at the top of his page) and here is my seven (again pointing to the number 7 written at the top of his page)

The student was actually referring to the numbers in the sum he had written at the top as a way of justifying his understanding. Clearly, this student’s mental imagery of the twelve kittens and seven kittens is substantial but his picture does not indicate that he understands the concept of drawing a representation of addition. Therefore, technically, he is able to represent his understanding through the Oral mode better than the Visual mode that he selected. So, why did Student Seven choose the visual mode? Was it because it was ‘comfy’?

The five students who chose the Visual Representation Mode were all asked if they were going to complete the same activity tomorrow with a different sum, would they choose the same representational mode to display their understanding. Two of these five students would have used the peg boards from the Physical Mode. The reasons behind these selections were again because they are fun and also because of their confidence in their ability to put on and take off the pegs. One student would have chosen the Oral mode of telling a story to the whole class had that been available and the remaining two students would have selected the same way of either drawing a picture or by writing a story. One student did state that she would write a different story though. Other than Student Seven the remaining four of the five students who chose the Visual Representational Mode were able to count their pictures or read their story correctly.
The students who chose the Visual Representational Mode were also asked what influenced their decision to use the Visual mode to represent their understanding. Student Eight felt there were too many options in relation to the concrete materials. She felt if there were only two pieces of equipment on the floor then it would make her decision easier. That is then why she chose to write and draw her understanding, ironically, giving herself an almost infinite variety of possible choice. Student Ten waited for a while and looked at what her fellow class members had selected. She saw that a few people had selected drawings so she thought that she would also like to do that. Student Three felt that using the materials maybe a little tricky and that you can confuse them, although, she did feel very grown up about being able to make her own decision. Student Nine considered using the counters to complete the task, but then he really felt like drawing, so he thought he would do that. Student Seven just enjoys drawing and that is why he chose that mode on that day.

The five students that selected to complete their task through the Visual mode did not specifically choose this mode because they knew they would get the answer correct. Many of them chose this mode as it was comfortable for them and something they enjoyed doing. The selection of Visual representation was not one that was fixed in concrete, three of the five students said that if they were to complete the task again they would choose another representational mode. As with the Physical mode, there has again been an overlap of modes with the Oral mode supporting the Visual mode. As found in the literature on the Written or Visual mode, research is showing that students are able to represent their understanding through the Visual mode but the mode selection works better when it is supported with the Oral representation (Clarke, Cheeseman, Gervasoni, Gronn, Horne, McDonough, Montgomery, Roche,
Sullivan, Clarke, & Rowley, 2002). It is very important to note that students’ writing their understanding through stories, drawings or symbols is a lot different from completing a multiple choice task, where the students are just asked to colour in a letter which indicates the correct answer. Unfortunately, a lot of standardised testing which incorporates the Visual mode does not allow students to write their own representation but to just fill in the missing sections or colour in the appropriate letter. Students are not always given the opportunity to explain their answer to a task using the Oral mode as the students in this investigation were. Four out of the Five students who selected this mode to represent their understanding completed the task successfully.

4.4 Oral (Linguistic) Representational Mode

It was interesting to think that out of a sample of ten not one student directly chose to use the Oral mode as a form of representing their mathematical understanding. The students had been given two forty-five minute lessons in which the specific focus was on delivering addition sum through the oral mode. Although, these two sessions did start a little slowly, the students appeared to catch on and enjoy the associated activities. There are many factors to consider as to why the students may not have chosen this form of representation in the final lesson.

Through reading literature on the Oral (Linguistic) mode it became apparent that for students to express their understanding through the oral mode they needed to be comfortable expressing themselves orally. The students studied in this investigation were either six or seven years of age and their oral language skills are still developing. Other than show and tell, answering questions in class or by telling some news to their
peers they are not really given time or exposure to sitting and talking in front of a group of children. The age and maturity of the students may have therefore been a key factor. One of the students was also an English as a Second Language (ESL) student and she is not confident with speaking in front of the class in any situation. Another point raised in the literature about the Oral (Linguistic) mode is the use of a common language, the students being comfortable and familiar with the words they are using. During focussed lesson number five it did become apparent that the use of common language was a problem. During that lesson we discussed and made a list of words we use in mathematics (Figure 4.5).

![Addition poster](image)

**Figure 4.5: Addition poster**

That poster had been on display since lesson five and was still hanging up and displayed during the final task selection. The students' lack of confidence in using this language perhaps influenced their decision to choose a mode that suited them and that felt comfortable for them. This may have discouraged students to choose the Oral mode.
At the beginning of the seventh lesson the students were given the option of completing the task through the Oral mode. They were told that if they were considering using the Oral form of representation they were to let the teacher know and it would be organised for them. This additional requirement may have discouraged students from attempting this approach. Obviously, no child approached the teacher. Indirectly the Oral mode was used by all ten students involved in this investigation. In the follow up one-on-one interview the students were all asked to explain how they completed the task, this required them to explain orally the processes involved. As previously mentioned, Student Seven completed his task through drawing a picture, yet his picture was actually incorrect. His understanding of the task and his concept of addition was evident through the oral retell that he made during his one-on-one interview. When this student was asked if he would consider sitting in the chair and telling a story he said he would not have chosen that.

I. Ok, now if it was possible for you to sit on the chair and tell the class a story using those numbers would you have chosen to do that?

S7. Not quite.

I. Why?

S7. Because Well I don’t really think I would, I don’t really think it is my type.

I. Your Type? (the interviewer smiles as she repeats this statement) What do you mean type?

S7. My thing. I don’t really like doing it.

I. Ok, what about if it just had of been you and another person and if you were just working in pairs?

S7. That would be fine.

The student was able to represent his understanding through the oral mode which became evident in the interview, yet, he was not comfortable or confident enough to
share his understanding with the class; although, he did say that he would be fine to share his understanding with a partner.

The ten students studied in this investigation had mixed opinions when asked if they would choose to share their understanding with the class or with a partner. The results are shown in the tables below:

*Table 4.4: Would you share your understanding with the Class or Partner?*

<table>
<thead>
<tr>
<th>Student</th>
<th>Whole Class</th>
<th>Partner</th>
</tr>
</thead>
<tbody>
<tr>
<td>One</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Two</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Three</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Four</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Five</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Six</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Seven</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Eight</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Nine</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Ten</td>
<td>No</td>
<td>Yes</td>
</tr>
</tbody>
</table>

**Share understanding with the whole class**

<table>
<thead>
<tr>
<th>Yes</th>
<th>No</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 Students</td>
<td>6 Students</td>
</tr>
</tbody>
</table>

**Share understanding with a partner**

<table>
<thead>
<tr>
<th>Yes</th>
<th>No</th>
</tr>
</thead>
<tbody>
<tr>
<td>6 Students</td>
<td>4 Students</td>
</tr>
</tbody>
</table>
The results indicate that over half the students did not want to speak to the whole class and the same number of students were happy to speak to a partner. However, there were three students who did not want to share their understanding through speaking to the class or a partner. They were not comfortable and did not prefer the Oral mode as a form of representation. There were five students who varied with some recording results of yes to partner and no to class or no to partner and yes to class. The remaining two students were comfortable in presenting their understanding to either the whole class or a partner.

Sharing what is understood or learned is not unusual for the students involved in this study. They were often required by the teacher to share what they had learned at the end of their numeracy lessons, as a part of their share time. The five minute allocation of time at the end of each lesson allowed the children to orally share with the class what they found interesting, different or any other information they wanted to share about their mathematical lesson. At completion of the individual selection task in this research the students sat in a circle and shared with their peers what representational mode they chose, why they chose that task and they also presented their finished product. During this informal Oral (Linguistic) session all students participated.

The Oral mode did not appeal to any of the students as the sole representational mode, but obviously they were keen to use it to support what they had completed or in a one-on-one interview. The need for a common mathematical language was a key point that arose during this investigation and had previously been identified by other researchers (Thompson and Thompson, 1994). The students will become more
comfortable and confident in using words they are familiar with. The fact that no student chose the Oral mode to represent their mathematical understanding does not mean they do not have an understanding to express, it just means that this mode was not one that the students felt comfortable with or preferred.

4.5 Analysis of students’ task selection

What led the students to select a particular mode? It was evident through their one-on-one interviews that they did not choose a mode because their friend chose it. They chose what they felt comfortable with and what they would enjoy. It is interesting to see whether the students were already comfortable with a particular mode and had developed a well-established preference prior to this study or whether the six focused group sessions influenced their decision. The table 4.5 gives a brief overview of each student’s behaviour during the six focused group sessions and what their final task selection was. The shaded cells indicate the lessons corresponding to the students’ final choice of representational mode.

Table 4.5: Student behaviour during six focused group sessions

<table>
<thead>
<tr>
<th>Student Number</th>
<th>Lesson 1 Written Stories or statements</th>
<th>Lesson 2 Drawings or diagrams</th>
<th>Lesson 3 Concrete materials</th>
<th>Lesson 4 Group Activity using our bodies</th>
<th>Lesson 5 Public speaking sharing with a group</th>
<th>Lesson 6 Private speaking sharing with a partner</th>
<th>Lesson 7 Individual Task Selection</th>
</tr>
</thead>
<tbody>
<tr>
<td>One</td>
<td>Absent</td>
<td>Absent</td>
<td></td>
<td>Student used pegboards during the class session. Did not try to use other equipment. Student involved in oral sharing at the end of the task.</td>
<td>Student was an active participant who tried to organise the group to get the correct answer.</td>
<td>Not engaged at all. Did not participate in the class discussion.</td>
<td>Interacted well with her partner. Enjoyed the activity.</td>
</tr>
<tr>
<td>Two</td>
<td>Wrote how many less instead of how many altogether. Scored when working alone. Last to complete the task.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>--------</td>
<td>------------------------------------------------------------------------------------------------------------</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Started task away, simple pictures that matched the sum he had written up the top.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Participated in the class discussions about using concrete materials. He offered suggestions on materials we could use. Used links during the hands on session.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Student was a non-participant in maths talk or organisation. He did what he was told.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Not engaged at all. Did not participate in the class discussion.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Interacted well with partner. Articulate in describing process and addition stories. He did forget to include the numbers in addition story.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Plastic Links</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Three</th>
<th>Very confident, worked well.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Started task quickly, very neat and orderly in presentation of her work.</td>
</tr>
<tr>
<td></td>
<td>Student enjoyed using counters during the hands on session. Keen to start early. Helped others and showed them how to set the counters out.</td>
</tr>
<tr>
<td></td>
<td>Enjoyed counting the students involved in the group work. Although at times she counted the children incorrectly.</td>
</tr>
<tr>
<td></td>
<td>Not engaged at all. Did not participate in the class discussion.</td>
</tr>
<tr>
<td></td>
<td>Interacted well with her partner. Enjoyed the activity.</td>
</tr>
<tr>
<td></td>
<td>Writing a Story</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Four</th>
<th>Needed help at his table to start writing a sentence.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Very detailed picture for one sum, simple for second sum.</td>
</tr>
<tr>
<td></td>
<td>Student provided a class example of using icy pole sticks for addition when the materials were introduced.</td>
</tr>
<tr>
<td></td>
<td>Student was a non-participant in maths talk or organisation. He did what he was told.</td>
</tr>
<tr>
<td></td>
<td>Student involved in sharing how he worked out the answer.</td>
</tr>
<tr>
<td></td>
<td>Interacted well with his partner. Enjoyed the activity</td>
</tr>
<tr>
<td></td>
<td>Icy pole sticks</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Five</th>
<th>Completed task quickly and confidently. Did not speak to other students.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Counted her pictures a number of times to make sure she had drawn enough.</td>
</tr>
<tr>
<td></td>
<td>Student enjoyed using counters during the hands on session. Happy to use the two different colours to</td>
</tr>
<tr>
<td></td>
<td>Student was not engaged. Did not appear to be enjoying this activity.</td>
</tr>
<tr>
<td></td>
<td>Student involved in sharing how she worked out the answer.</td>
</tr>
<tr>
<td></td>
<td>Interacted well with her partner. Enjoyed the activity.</td>
</tr>
<tr>
<td></td>
<td>Peg boards</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>Six</td>
<td>Knew in his head what he wanted to write but when he reread it, it didn't make sense.</td>
</tr>
<tr>
<td>Seven</td>
<td>Looked at how the other students at his table were completing the task before he started. Answered correctly.</td>
</tr>
<tr>
<td>Eight</td>
<td>Started task straight away. Helped others at her table. Very detailed picture that was completed correctly.</td>
</tr>
<tr>
<td>Nine</td>
<td>Worked well, wrote two very good stories. Took a lot of time to complete the task. Very structured in the setting out of his work.</td>
</tr>
<tr>
<td>Ten (ESL)</td>
<td>Engaged in reading of story. Confused about writing story. Spoke to people at her table asking for help.</td>
</tr>
</tbody>
</table>

Shading indicates the cell that corresponds to the students’ final selection.

When comparing what had happened in the six focussed sessions and the overall task selection many different patterns emerged. Interestingly enough, there were factors that occurred during the six focussed lessons that may have influenced the students’ task selection. Student One was absent from school when the first two sessions on the Visual mode were taught. Although, she had been involved in classroom lessons prior to this study that required her to complete addition tasks through the visual representation mode. During lesson three she enjoyed using peg boards and this is what she selected to complete her final task. In her interview she also mentioned that her Grandpa has peg boards and she was used to playing with them at his place.

Student Two mentioned in his interview that he knew he couldn’t write the stories or statements and when referring back to the data that had been recorded on that focussed session the student did require assistance from the teacher to complete the
task. However, during the concrete materials session he was an active participant and he spent his time manipulating the plastic links. He wasn’t too keen to participate in the whole class sharing during the focussed group sessions but, when asked, he said that he “could of” chosen that for the task selection activity:

Student Three showed a real interest in the Visual/Written mode during the focussed group sessions and she also enjoyed the Physical mode, where she enjoyed showing others how to complete the task correctly. In contrast, she did not enjoy the Oral focussed sessions. Student Three was one of the students who, if given the opportunity, would not choose any of the Oral modes to represent her understanding. Based on my observations of the student during the focussed sessions this student could have perhaps chosen either the Physical or Visual modes to answer her question as she appeared comfortable and confident with either. It could be assumed that because she enjoyed writing and seeing beautiful writing that is what influenced her decision to use her preferred mode of writing a story.

During the written lesson involving writing a story, Student Four required teacher assistance as he became very confused. This was the only session in which he struggled. It was interesting to reflect back to lesson three when he used icy pole sticks to provide an example of addition to show the class. Perhaps, it was the fact that the student had already presented this mode to the class and got it correct that influenced him to select it for his final task or was it that he had already developed a comfort factor with using that specific piece of equipment? His confidence with icy pole sticks was such that without any direction from the teacher he grouped the ten icy pole sticks with an elastic band to represent ten.
I. Ok, so you thought it was easier because you like using things. Ok, good boy. Now why did you decide to use icy pole sticks?

S4. Because, it felt comfortable and it’s easy to um do.

I. So you felt comfortable?

S4. Yes

Student Five did not show any preference for a particular mode during the six focussed group sessions so it was interesting to see what her final task selection was. She chose to represent her understanding using the peg boards from the Physical Mode. This student had not had any prior experience with the peg boards and perhaps that is why it was chosen as it was something new and a different experience. In contrast, most of the other students chose a representational mode and equipment in which they were comfortable. So what led this student to use pegboards and is this a preferred mode? As Student Five did not show a preference during the six sessions is she yet to have developed a preferred mode? In her one-on-one interview the ability to manipulate and remove the pegs was important to her and her mode selection. She was also concerned in doing something that other people may or may not be doing. If given the opportunity to complete the task again she said she would have a go at something else. When asked if she would consider speaking to the class she was not keen but was more enthusiastic at the thought of sharing her understanding with a partner. From the data, observations and interview transcripts this student does not appear to have found a preferred Representational mode.

Student Six struggled during the Visual representational mode sessions and he verbalised this when he re-read the story that he had written. “Oh I wrote it down wrong.” His drawings were also very unstructured. In contrast, the four remaining
focused sessions were handled extremely well. This student chose to represent his understanding through the plastic links, but he indicated that he would also consider using the Oral mode in the future. This student has a history of struggling when it comes to writing.

As it has previously been discussed Student Seven elected to draw his understanding in the final task. In the second focused group session of drawing, he did exactly the same as he did in his final task. He wrote the sum up the top and his picture did not match his sum. During the other five sessions, he completed the tasks correctly and interacted in the class discussions. He showed a preference for the plastic links during the concrete materials and he enjoyed sharing his story with the class. Interestingly enough, he said he would not have selected to have spoken to the class. Therefore, did the love of drawing influence this student’s task selection? He was spoken to in session two and with the teacher they worked through how he could have perhaps drawn his addition representation. Obviously, this child was still having difficulties understanding the concept. The preferred mode in this case was drawing, but this is not the best way for Student Seven to represent his understanding. He was able to verbalise his understanding during the one-on-one interview and he does have an understanding to represent. Unfortunately, this student selected the mode he was comfortable but not the one that would best represent his understanding.

Student Eight was a very confident student. As with Student Five, she did not show a preference to a particular mode throughout the six focused sessions. She completed each session task correctly and was involved in classroom discussions. So what led her to choose writing and drawing? This student was confident during the
class literacy activities and she enjoyed writing stories. It seems likely that this influenced her decision. Writing appears to be her preferred mode to represent her understanding.

Student Nine has a slight stutter and is not usually confident in presenting to the class. During session five he shared his addition process with the class, although, he got confused and was unable to come up with the correct answer. Therefore, it came as a complete surprise when he mentioned that he would be happy to share his understanding with the whole class or partner. He is an avid drawer so his task selection was what had been expected. He appeared more engaged in the drawing focussed session than he was in any other session. Perhaps, the comfort factor and getting the answer correct was very important to this student when he made his task selection.

Student Ten also chose drawing. Throughout all sessions this student would seek clarification from her peers before completing a task. During the oral sessions she did not appear to be engaged, but as with Student Nine she was happy to share her understanding with a partner, if she was given the opportunity. When comparing her picture in session two and her final task selection her pictures were identical.

I. Oh ok, good girl. Now if we were to do this tomorrow but I had the sum 8+5 on the board. Would you have chosen to do a picture or would you have chosen to use or do something else?

S10. Um, I would do a picture.

I. So you would draw a picture again.

S10. Yeah

I. Why would you do a picture again?
S10. Because I think it looks nice.

Student Ten appears to be very comfortable with drawing pictures and it is important to her that they look nice. This student has been involved in an ESL programme run by the school where the research has taken place. This has helped her with her oral language and speaking in front of a class. However, during this research she has been very hesitant to participate in classroom discussions or sharing her mathematical ideas. Although, she was prepared to share her understanding with a partner if the opportunity had arisen.

4.6 Summary of Findings

When analysing the data many interesting patterns emerged. These included:

- Students chose what they were comfortable with to represent their understanding. But the meaning of “comfortable” varied between students. Students identified comfort as confidence, enjoyment or the ability to participate without being directed what to choose.

- Each student had an understanding to represent. Every student understood the concept of addition. Nine out of the ten students were able to represent their understanding correctly by completing the task. One student could not show his understanding through the mode he selected. He was still able to explain his understanding of addition in the follow up one-on-one interview.

- None of the sample chose to represent their understanding through the Oral (Linguistic) mode. Although, if they were to complete the same activity tomorrow, some of the students indicated they would consider using that mode.
• Some of the representational modes overlapped. For example: Students explained orally what their representation was about.

• Students chose the representational mode that they felt they would achieve success with.

Further discussions of findings and the major conclusions of the investigation are reported in Chapter Five.
CHAPTER FIVE

Conclusion

5.1 Introduction

This investigation was devised primarily to find out, ‘In what ways are Year One students able to represent their mathematical learning?’ As a result of the data collected and analysed in the investigation, a number of major conclusions can be drawn. The findings of this study have important implications for further research. The conclusions are presented in this chapter in the following sections.

5.2 Enjoyment of actual activity and valuing of product

5.3 Comfort, confidence and peer influence

5.4 Problematic connection and too many options

5.5 Willingness to change

5.6 Understanding to represent

5.7 Recommendations arising from this investigation

Each section represents a distinct set of findings arising from this research.

5.2 Enjoyment of the actual activity and valuing of product

This study has demonstrated that Year One students are able to represent their mathematical understanding in a variety of ways. When selecting a mode of representation, students took a variety of factors into account. From the findings of this investigation, it was interesting to see that students based their selection of a representational mode primarily on what they enjoyed doing. Year One students are still of a young age, developing new skills and are continually being exposed to new learning situations. Through the one-on-one interviews in this investigation, the
students indicated that they selected representational modes based on whether they enjoyed using that equipment or those particular representational or communicative skills (e.g. "I like drawing"). Another factor that influenced students’ selection of representational mode was the value that they attached to the product of their representational activity. Student Three selected writing to represent her understanding as she likes to see beautiful writing, "I thought it would look nice." Student Nine really liked drawing and that was why he chose that mode of representation. The students' choice of representational mode did not only represent their understanding of mathematical concepts and procedures, their choices can also be taken as an indication of what was enjoyable to them and what they valued.

The young age of the students may have influenced both their choices and the way they chose to justify them. For example, they may have felt that the "nice/ neat" type of response was more likely to get the teacher's approval. They were aware that the tasks they were involved in and the data they provided would be used to assist in a study. They were unaware of the manner in which the information they had supplied would be used. Most commonly, the students said that they chose as representational mode those actions they enjoyed doing. When reading the Literature on students' mathematical representations and their understandings, it was hard to find research that connected student enjoyment to its influence on student's choice of representational mode. For all of the reasons cited earlier in this thesis, this aspect of mathematical behaviour should be researched further. When students are enjoying themselves and are comfortable in their environment then research suggests students would find it easier to be engaged and motivated to learn (Csikszentmihalyi & Csikszentmihalyi, 1988, Csikszentmihalyi & Rathunde, 1992). In particular, the use
of concrete materials in Primary School mathematics classrooms can be justified on cognitive as well as affective grounds:

"Children who have not played with beads, rocks and lumps of clay may have difficulty understanding addition, subtraction, multiplication and division" (Gage and Berliner, 1992: 124).

The interviews conducted in this study suggest that through the manipulation of materials the students not only learned effectively, but also enjoyed their exposure to mathematical concepts.

The length of time which the students took to complete their final task indicates that the students valued the product of the process of representation. They were given an allocated time of fifteen minutes, yet, four students required longer to ensure that they had completed their representational mode successfully. Student Six independently chose two ways of using the plastic links to ensure that his mathematical understanding was represented. This student valued the product of the process.

Based on the findings of this research, teachers should try to ensure that their classroom mathematics program provides students with an opportunity not only to learn, discover and invent but also to have fun and to value what they are doing. In other words, teachers should consider the affective as well as the cognitive consequences of their instructional decisions.
5.3 Comfort, confidence and peer influence

The students not only based their task selection on what they enjoyed and valued, but also what they felt comfortable and confident with. Two of the students in this sample chose to use concrete materials that they had not used before, but a majority of the sample used what they were familiar or comfortable with.

Student Seven chose to represent his findings through drawing. "Because it's comfy with me, it suits me." When this student was asked if he would like to share his understanding by the oral presentation mode he declined because "I don't think I would, I don't really think it is my type". The language used by this student indicates that he has already identified a preferred mode of representation. Interestingly enough, interview data made it clear that Student Seven did have some understanding of the relevant content but did not correctly represent this through his preferred mode of drawing. Although, he felt comfortable using this representational mode, he was not actually able to produce an adequate drawing to represent what he understood. In the interview, however, he was quite capable of demonstrating his understanding orally. There were other students in the sample that chose the representational mode that they were comfortable with but these students were able to correctly represent their understanding using their preferred mode. So what does this mean for practising teachers? If the student is using their preferred mode successfully then this success must be interpreted as a true indication of a students' mathematical understanding. However, if as identified in this study a student is using their preferred mode unsuccessfully, in cases of lack of success, it is the teacher's responsibility to ensure that the student is offered another representational mode by which to attempt to express their understanding. In the case of this study the student was able to represent
his understanding using an oral retell because of the teacher’s capacity to probe the student’s statement. After an unsuccessful attempt, the classroom teacher should conduct an interview with the student.

Classroom teachers need to ensure that all students have experience using different representational modes and have the skills to construct different representational types successfully. Accommodating students’ preferred representational modes in assessment settings is not intended to encourage student specialization in a particular mode, only to get the best possible insight into the student’s understanding. Ideally, all students should become experts in all representational modes. Although, it may not be the preferred mode by a student they may need to use these skills in different situations. Providing students with a variety of experiences does not restrict the use of their preferred mode. Instead, it offers them a range of new opportunities to express their understandings. That way, the teacher is able to identify in a much more accurate and detailed fashion, the nature of a student’s understanding, however incomplete or unsophisticated this understanding might be.

Through the one-on-one interviews another factor that influenced the students’ representational selections was the students’ reported perceptions of what they were good at. The students wanted to achieve the correct answer, so they mostly chose a mode that allowed them to be successful. Student Four chose to represent his understanding through a mode which he had correctly used in the focussed addition session. Student Two chose to use concrete materials for his final task selection.
"Well, because you see writing is some ... you know how I told the story and you know how I did that writing" (The student is referring to Lesson Number one) "I actually did a takeaway sum instead of a plus".

This student thought that if he chose writing he would not complete the task correctly. These examples show that the students' confidence and the ability to answer the question correctly influenced the way they chose to represent their mathematical understanding.

With only ten students being a part of this investigation, the results and data were limited in certain respects. With exactly half the students selecting either the Physical or Visual representational modes it is interesting to note that the factor of peer influence appeared to have little impact on the students' selection of representational mode. The majority of the students reported that they had already decided which representational mode they would use and the equipment they would require when they were sitting on the floor listening to instructions. However, Student Ten waited to see what the other students had selected before she chose her representational mode:

"Well, I was waiting for the children to choose things and then I saw a few kids doing some pictures, and then I thought a picture would be nicer."
This student provides an important example of the possibilities of peer influence, but it is noteworthy that the majority of the students did not allow their friends' choices to influence their selection of representational mode.

5.4 Problematic connection and too many options

In this investigation, of the ten students, only one student had difficulties correctly representing his understanding of the addition sum 12+7=. Student Seven really liked drawing and believed that this was the best way for him to represent his understanding. However, when analysing the drawing he submitted, there was no indication of how he had worked out the sum and no real representation of 12+7=.

The problem here is the connection between his representation and the actual mathematics (See Section 5.3). When the student was asked to discuss his drawing, his oral explanation indicated that he understood the concept of addition.

**S7.** Well this is a picture about how much kittens I bought. And on Monday I bought twelve kittens and three days later um on Thursday I bought seven more. And in this picture you can see me driving back to home.

**I.** Ok where is your twelve where are all your..., where is your twelve? And where is your seven?

**S7.** My twelve is here (pointing to the number 12 at the top of his page) and here is my seven (again pointing to the number 7 written at the top of his page)

**I.** You tell me what the first thing it was that you did? Did you draw the picture first or write the sum up the top?

**S7.** Well first thing is I drawed. I drewed the sum then I drew the picture.

**I.** I actually did notice that. Did you write the sum to remind you of what the picture should be about?

**S7.** (The student nods his head up and down to indicate yes)
This study was limited by the size of its sample group and it would be interesting to see if such problematic connections between mathematical concept, mathematical procedure and mode of representation were a common problem in a bigger study. A larger study could also address questions such as whether there is a gender-specific aspect to that tension between preferred representational mode and the students' successes in using that mode to demonstrate understanding.

Nonetheless, in this study there were a number of students who were able to correctly demonstrate their understanding through at least two of the representational modes. Although, none of the students in this sample chose to represent their understanding orally, through the one-on-one interview the students were able to explain their understanding. The use of multiple representational modes was a point that has been discussed in the research literature (Thompson, 1994; Hart, 1989; Clarke, Cheeseman, Gervasoni, Gronn, Horne, McDonough, Montgomery, Roche, Sullivan, Clarke, & Rowley, 2002). This was also supported through the implementation of the Early Years Numeracy Interview, which was one aspect of the Early Numeracy Research Project (ENRP) (Clarke, Cheeseman, Gervasoni, Gronn, Horne, McDonough, Montgomery, Roche, Sullivan, Clarke, & Rowley, 2002). The students completed tasks using the Physical representational mode and they explained their thought processes orally. The ENRP also allowed the students to use multiple representational modes.

The availability of equipment and the opportunity for the students to independently select the way they wanted to represent their understanding was enjoyed by nine of the ten students in this investigation. One student had difficulties
deciding what she wanted to use as her form of representation. “If we choose, we get, we have a long time to think, and it is a bit hard to choose with all that stuff”. When this student was asked if she would be happier if there were only two things on the floor, she replied “Yes”. From this statement, it could be concluded that this student did not have a strong preference regarding how to represent her understanding. She was confident in all representational modes and would have preferred the teacher to limit her selection. Ironically, this student chose both writing and drawing to complete her task selection, and from her subsequent interview statements it appeared that she did enjoy the opportunity to decide on her own representational mode.

5.5 Willingness to change

This investigation has shown that students are able to represent their mathematical understandings in a variety of ways within the broad categories of Physical and Visual modes. Within this demonstrated general capacity, students may have a preferred way to complete addition tasks. The students in this investigation gave varying responses to the question of what they would choose if they were required to do the same task again. Two students would like to do exactly the same thing as they did on this task selection “yes, but I would write a different story.” This student did not want to change to concrete materials as “it’s a bit tricky.” Seven students would choose a different type of representation and do something else “Because, I don’t like playing with the, the same stuff every single day”. Interestingly enough, three said they would tell a story: which no student chose in this study. This suggests that despite the students’ appreciation of the opportunity to use their preferred mode of representation, they are also sufficiently confident to anticipate using a different mode next time. Student Two was keen to either tell a story or stay within the physical representational
mode. He was sure that he did not want to choose anything from the Written Represen
tational mode.

1. If I was to do this same activity again tomorrow but I didn’t give you 
12 + 7 =. Let’s say tomorrow I gave you 8 + 5 = would you choose to do
that again? Would you choose to use links again tomorrow? Or would
you choose something else?

S2. Maybe I would tell a story or I would use the peg boards.... Yeah.

1. So you wouldn’t choose to do writing?

S2. No

Student Seven, who did not represent his understanding correctly through drawing,
indicated that he would choose to use pegboards if he was to complete the task again.
“I would choose peg boards because you can take them off”. This response suggests a
higher level of trust in the Physical manipulation of concrete objects, than in written
or drawn representations. One student would choose a different type of concrete
manipulation but stay within the physical representational mode.

It might be expected that the students’ young age and their willingness to learn
and discover could have inclined them to use a variety of different representational
modes. Yet, three of the students in this study were keen to stay under the umbrella of
the same representational mode. Therefore, it could be concluded that students as
young as six and seven years old already had developed a preferred way to represent
their understanding.

5.6 Understanding to represent.

The focus of the students’ representations in this study was their understanding of the
concept and procedures of ‘two digit addition’. It does not matter what
representational mode they ultimately chose, the central concern is the understanding being represented. In this investigation, one student did not correctly represent his understanding through the Visual representational mode. However, through the one-on-one interview he was able to establish that he understood the concept of addition. This child had an understanding, which he just could not represent correctly. Of equal interest, are students with an incomplete understanding. It is hoped that the choice of representational mode leads to a clearer indication of the nature of students' difficulties. This study was more concerned with maximising student opportunity to demonstrate existing understanding that might not be displayed in the more conventional pencil and paper tests.

In the Literature review (Chapter 2) of this thesis many meanings of the term 'understanding' were identified and discussed. Davey and Nisbet believed that "understanding allows knowledge to be transformed to match new problem situations" (1992: 4). Through this investigation that definition of understanding has become crucial. That is, the capacity to employ multiple representations has emerged as a key indicator of understanding. Our goal as teachers must be for students to be able to understand the concept and then be able to transfer their knowledge to another form or a different representational mode and then be able to express their knowledge clearly in their chosen representational mode.

Allowing students to select their own representational mode can prove a challenge for classroom implementation; however, offering students a choice of representational mode seems likely to prove a lot more informative as a form of assessment. The teacher is more likely to be able to identify whether or not the student
does understand the concept. The current exam format does not allow students to represent what they understand in a mode that is preferred by them. It is entirely possible that the student does understand the question, but they are just not given adequate opportunity to show what they understand because their representational options are restricted by the existing traditions of assessment.

Regardless of what representational mode a student selects, the goal is the best possible representation of their understanding. As demonstrated in this study, a student may "get the answer wrong," not because they do not understand the task or relevant concept, but because the chosen mode of delivery did not clearly represent what they understood. In the event of a student's chosen representation suggesting incomplete understanding, the student must be offered the opportunity to employ a different representational mode to display their understanding.

These findings also emphasise the need for classroom teachers to provide students with continual exposure to different modes of representations. A student may not always be given a question that they are able to answer using their preferred representational mode, therefore, they should have another mode to fall back onto. Classroom teaching should not channel students to use their preferred mode, but provide students with skills in all areas. Instruction should promote non-preferred learning styles. The teacher would then be able to identify two things, the level of the student's understanding and whether or not an appropriate representation was used. These two elements: understanding and appropriate representation, are both required for the successful choice and implementation by a student of the mode by which to represent their understanding. Both teachers and students should be encouraged to
develop expertise and confidence using a variety of representational modes to communicate their understanding of mathematics.

5.7 Recommendations arising from this investigation

This small scale study has certainly shown that there is value in allowing students to select which way they would like to represent their understanding within the mathematics curriculum. Students as young as six were able to successfully identify in what way they could represent their understanding of addition. The study has also highlighted the fact that some students do have different preferred learning and representational styles and that all styles should be catered for and promoted in both instruction and in assessment. Classroom mathematics programs are designed to accommodate the individual needs of students in classroom activities, but in most cases assessment takes a predictably narrow and restrictive form throughout the class, year level or even in the case of state-mandated assessment.

This study was based on students in a Year One classroom. A similar study should be conducted in Years Three, Five, Seven and Nine. These are the year levels where State Benchmarking in Victoria takes place. Using results from the Achievement Improvement Monitor (AIM) testing and comparing the outcomes with a task when the students are given a choice as to how they represent their understanding would definitely help to identify the level and form of a student's understanding and whether pencil and paper testing allowed this understanding to be correctly and successfully represented.
This study provided students with the opportunity to complete their task selection in either the Visual, Physical or Oral modes and in two forms within each of these modes. This selection was limited due to the age of the students and that it was only their second year of formal schooling. Students in the older year levels should have options in their mode selection due to their more sophisticated learning experiences. For example: Visual would include graphical; Physical would include model construction; and, Oral would include algebraic or any other symbol system with established semantic and syntactical conventions. One thing that has proven to be particularly successful in this investigation was giving students the opportunity to be involved in a one-on-one interview to allow them to explain their thought processes and their understandings, supported by their chosen representation.

A key finding from this research is that many students may be disadvantaged by restriction to a prescribed way of representing what they understand. Pencil and paper tests, and multiple choice formats disadvantage students for whom this is not their preferred way to present what they know. If schools or teachers prefer to use the visual mode as the dominant form of assessment then there are other options to consider in addition to just multiple choice and short answer questions. The Visual mode as a form of representation in assessment should be broadly interpreted and should incorporate reflective writing as well, supporting the emphasis on the processes by which the student gets the answer. Every student should be given as many opportunities as possible for them to show what they know and they should not be limited to a standard method. It is also possible that when working in their preferred mode, students are able to develop new knowledge of a concept through
their confidence in manipulating or utilising their preferred medium (example: Student Four spontaneously grouping ten icy pole sticks).

The Year One students in this investigation all had some form of understanding to represent and they chose which way they felt truly was the best way to represent their understanding. These selections fell under the two umbrellas of the Visual and Physical representational modes. However, the nature of this research was such that all students also had the opportunity to also demonstrate their understanding orally in the one-on-one interviews.

Continued research is warranted as a result of the findings and recommendations of this investigation, to assist in the development of an assessment protocol that allows the learning styles and preferred representational modes of students to be accommodated with the school mathematics curriculum. Ultimately, our goal must be to develop student expertise in the widest possible range of representational forms. More flexible and inclusive assessment is one step towards this goal.
References


APPENDIX ONE

(Consent Forms)
How can Year One Students Represent their Mathematical Learning?
Consent Form for Students, and Parents or Guardians

I, __________________________________________, consent to the participation of my child,
(Name of Parent or Guardian)

__________________________________________, in the production of video material,
(Name of Student)

used to research How Year One student’s represent mathematical learning.

- I consent to the videotaping of my child consistent with the information provided to me.
  Parent or Guardian’s Initial: ______________________

- I consent to my child being interviewed and for the interview to be videotaped or audio taped. I am aware that my child’s name will not be used.
  Parent or Guardian’s Initial: ______________________

I acknowledge that:

(a) The project is for the purpose of research and the improvement of teaching and learning in mathematics;
(b) Participation in this study will not involve any student in any type of activity different from what would normally occur in class;
(c) I understand that my child may withdraw from this research at any time and that any unprocessed data will also be withdrawn should this occur;
(d) Data collected during this study will be stored securely by Bronwyn Deagan for approximately five years and then be destroyed. Confidentiality of the data collected will be maintained subject to legal requirements.

Signature: __________________________________________ Date: ______________________

(Signature of Parent or Guardian)
Tuesday, 14th June 2005

Dear (Student name),

As you know I am also going to school. I go to Melbourne University and I am doing a project to find out how you learn mathematics. When I finish my project it will be part of my degree, called a "Masters". My teacher, Professor David Clarke, helps me with my project. He is called my "supervisor". We both work in "the Department of Science and Mathematics Education".

Mr Ryan, Mr Barklamb and Miss Lentini have given me permission to send you this letter to tell you a bit about my project. Once you have read the letter you can decide if you would like to take part. You should talk to your parents about the project too. They have also been given a letter.

If you want to be part of the project, it is no different to what we do in the classroom during our maths lessons. The only difference you will notice is a video camera in the corner videoing how you complete your work. Just like in our normal lessons I will be there to explain about the questions and collect the answers at the end. If you want to stop doing the questions, you can tell me and we can stop videoing. Some students will be interviewed at the end of the session and what they say will be taped on a tape recorder.

Only my supervisor and I will see your answers, so please don’t worry that anyone else will see them. After the project is over, I will lock all the information away safely in a filing cabinet and I have to do this because it is a University rule.

Remember, you don’t have to take part unless you want to. If you have any questions you should talk to your parents or me. If they don’t know the answer to your question, they can contact my supervisor, or the Research Ethics Office at the University for you.

If you want to be part of my project, and your parent/s agree, please sign your name below where it says "student".

Thank you
From
Miss Deagan

Student’s Endorsement
I understand the purpose of the proposed videotaping and research. I am happy to participate in it.

Student: ___________________________ Date: ______________
APPENDIX TWO

(Interview Questions)
Interview questions for one-on-one interview
after the completion of the 'Individual student selection task'

The interviews are designed to take fifteen minutes. However, due to the age of the students this time may vary. A lot of the interview questions will be based on the observations made during the task. Therefore the questions below are non specific and will change according to how the student represented their mathematical learning.

Interview Process:

Thank you .................. for chatting to me today about your activity that you completed. As we have our chat I may write down some things. You will also notice that we have a tape recorder in the room. That is there for me to listen to our chat again later. Is there anything you would like to know about our meeting before we start our chat?

1. I would like you to tell me about your activity that you completed today.

2. Why did you decide to complete your activity in that way?

3. If it was possible to complete the activity by working with a group or by telling the class what you have found, would you have chosen that?

4. Why would/ wouldn’t you?

5. When you completed your activity I noticed that ................................. Why did you / didn’t you do that?

6. If we were do to this activity again tomorrow would you choose the same way to answer your question? Why / why not

7. When I teach in class I usually tell you which way to answer the questions. Did you find it hard to decide which way to complete your activity today? Why?

8. Would you prefer me to tell you how to complete the answer? Why?

9. How did you decide which way to complete the activity?
10. [Using a card prompt with a 12 + 9 = ] How could you show me you understand this?

11. Is there another way you could have shown this?

12. Thank you for chatting to me I really appreciate it. I just have one last question; Is there any thing else you would like to tell me about today’s activity?

Thank you so much ............. You have been fantastic today!
APPENDIX THREE

(Interview Transcripts)
Interview Transcript
Student One

Thank you ................. for chatting to me today about your activity that you completed. As we have our chat I may write down some things. You will also notice that we have a video recorder in the room. That is there for me to listen to our chat again later. Is there anything you would like to know about our meeting before we start our chat?

The student made reference to the setting out of her sum throughout her interview.

I. I would like you to tell me about your activity that you completed today.

S. Well... I (3 second pause) I used the peg boards and I um and I used pegs and I already knew to add up. I already knew that all of that was 12. Then I didn’t count the first one so then I took that one away and then I said one. And then I kept on taking them all off.

I. Oh, Ok

S. And I used and cos I used, I did the longer one cos there was still a little bit more pegs left and I wanted to use them and I did it because just so you know what answer I knew it was. (referring to the numeral 19 she made at the bottom of her board)

I. Ok. So you did that for me?

S. Yes

I. Great. Now why did you decide to use pegs and not do some writing?

S. Because, Pegs is a little bit fun and I’m not really good at writing that much.

I. Really? I think you are very good at writing.

S. Even though I try to do my best.

I. You try at home do you?
S. Yes

I. Do you try and do Maths writing at home?

S. Yes

I. Ok, alright. Now if it was possible for us today for you to sit on the chair and you tell a story to the class or if you had of sat and told a story to a friend. Would you have chosen to do that? Or would you have still chosen to do pegs?

S. Um, chosen to do pegs.

I. Why?

S. Because, sometimes I don’t like telling stories, but um.. I had a really scary story and I didn’t want to tell the class because it might have scared the class.

I. Ok., Now when I normally teach the class, I tell you which way to answer the question don't I?

S. Yes

I. I normally tell you that you have to do it in your book, or you have to do that. Did you find it hard to make your own decision today? Or did you like making your own decision?

S. Um, I liked making my own decision.

I. Why?

S. Well, because I don’t know why I guess I really still like playing with peg boards. Because my Grandpa used to have peg boards but now he doesn’t.

I. Oh ok, so is it something you are used to playing with?

S. Yes

I. If I was to do this activity again tomorrow but write a different sum on the board and put out all the equipment would you choose to use pegs again tomorrow? Or would you choose something else?

S. Something else?

I. Why?

S. Because, I don’t like playing with the, the stuff every single day. The same stuff every single day.

I. Oh, Ok. So you would have liked to do something different?
S. Yes

I. Alright, I would just like to ask, how did you decide to set them out in rows? (pointing to the peg board)

S. Um, I just thought id do it like that because if I left a big gap like that bit there, then they would think that, all of that is, that all of that is um I wasted so much space. Then I wasted so much space and um....

I. That's ok. Now if I was to give you this (12+7= on a piece of card placed on the table in front of her) can you show me how you understood it? So show me how you did it.

S. Um, First, what I did (student takes out all of her pegs from the board) Um, I used it in colours so first I used blue and then a lot of red, of the reds and I used, used it by. Well, I knew the answer was 19 already. But I um.. (pause)

I. How did you know that?

S. Because. When we were on the floor I already said 12 and 7 and I took the one away and then it equalled 19.

I. Oh ok, so you knew that 7 plus 2 made 9 and one more made 19?

S. Yes.

I. Oh ok, thank you for telling me all about this today. I just have one last question? Is there anything else you want me to know about what you've done today?

S. Um... No.

I. So you are happy with everything?

I. Oh well thank you for everything today. You were fantastic and you are a very good writer. Thank you once again.
Interview Transcript
Student Two

Thank you for chatting to me today about your activity that you completed. As we have our chat I may write down some things. You will also notice that we have a video recorder in the room. That is there for me to listen to our chat again later. Is there anything you would like to know about our meeting before we start our chat?

The student made reference to the setting out of his sum throughout his interview.

I. I would like you to tell me about your activity that you completed today.

S. Well, I wanted to make a pattern but I actually wrecked it because I realised it made 19. So I took 1 away so it made 19.

I. So you had 20 to start off with did you?

S. MMM.

I. So why did you decide today to complete your activity using the links?

S. Well, you see I said in my mind it would be better than writing, and um like I said it would also be better to actually join the links together.

I. Can I ask why it is better than writing?

S. Well, because you see writing is some.... You know how I told the story and you know how I did that writing (The student is referring to Lesson Number one) I actually did a takeaway sum instead of a plus.

I. Oh so, when you do your writing you get confused and do a takeaway?

S. Yes (student replied very enthusiastically and agreeing)

I. So you knew that, is that why you chose to do the links today because you knew you would be able to do plus?
S. Yes

I. That is a really interesting thing that you have told me. Now, if it was possible for you to sit on the chair and you tell a story to the class or if you had sat and told a story to a friend. Would you have chosen to do that? Or would you have still done the links?

S. Well, I could have chosen to do that but I would have still chosen to do the links instead.

I. Why?

S. Just because of the same reason?

I. Oh ok, just because of the same reason? Now, when you completed your activity I have actually noticed that you have joined them altogether. How did you work out that you had 12 and 7?

S. Well, see, because am I actually (pause 3 seconds, as student counts his links to 7 and separates them making two rows) I made a row of 7 and then I made a row of 12.

I. Ok, can you just count this for me so we can check if there are 12?

S. (Student counts) 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12

I. Ok, and how many are here? (Pointing to the second row of links)

S. 1, 2, 3, 4, 5, 6, 7

I. Ok, so altogether? So did you then clip them together? (student clips the links together to make one row of links)

S. So then I clip them so I can .... I knew it would be easier, because I did this first (student takes three links off the end) 1, 2, 3 taking them off and then I actually went to the floor and then I got back up to my table because I realised it didn’t make 2.

I. If I was to do this same activity again tomorrow but I didn’t give you 12 \& 7 , Let’s say tomorrow I gave you 8 \& 5 — would you choose to do that again? Would you choose to use links again tomorrow? Or would you choose something else?

S. Maybe I would tell a story or I would use the peg boards.... Yeah.

I. So you wouldn’t choose to do writing?

S. No

I. So you wouldn’t do writing

S. Well, I guess I could do writing (student laughs)
I. You certainly could. Now in class I normally tell you what to do. I normally tell you that you have to do writing, or you have to do dice games. Did you find it hard to make your own decision today? Or did you like that?

S. Well, you see, I liked it the way I thought, because I have never used the links before.

I. Oh ok! Did you watch and see what the other people were doing?

S. Yeah!

I. Oh ok, would you prefer me as the teacher to tell you what to do or would you like to decide?

S. Um. well I like to decide and I like people telling me what to do too.

I. Ok, Now show me how you understand that. I’ve given you 12 + 7. You show me how you understand it.

S. Well I understand it. well I ‘understood’ it by um I don’t know how I. I forgot how I understood it.

I. Was that when you put it in a group of 12 and a group of 7?

S. Yes.

I. Now thank you for chatting to me today, is there anything else you wanted to tell me about your activity today?

S. Well, not really, no.

I. Well thank you so much for today.
Interview Transcript
Student Three

Thank you for chatting to me today about your activity that you completed. As we have our chat I may write down some things. You will also notice that we have a video recorder in the room. That is there for me to listen to our chat again later. Is there anything you would like to know about our meeting before we start our chat?

\[12 + 7 = 19\]
I went searching and found 12 belbies and 7 more. I have 19 belbies.

I. I would like you to tell me about your activity that you completed today.

S. By writing.

I. Now why did you choose to do writing?

S. Because, I thought it would be a good idea if I practise writing at some places.

I. Now, do you do a lot of writing at home?

S. Yep.

I. So you like to do lots of writing? Do you prefer to do writing or drawing?

S. Writing.

I. Ok, so you prefer to do writing? Now, why did you decide to do writing? Just because you wanted to practice?

S. Um.

I. Or did you like the story that I read that was all about writing?

S. I like doing writing in lots of places and also that I like to see beautiful writing.

I. Oh me too! Now, if it was possible for you to complete it, the activity doing the oral way by talking on the chair or by talking to a friend about a story would you have chosen that today? Or would you still have chosen to do writing?

S. I would still have chosen to do writing.
I. Oh ok so that wouldn't have changed your mind? Ok, when you were completing your activity today. I actually heard you say "Miss Deagan, I haven't worked out the answer" How did you work out the answer?

S. Well I used my fingers and toes.

I. Ok, so what other mode did you use? You just didn't do the writing today. You also used the...? (Student directed to look at the 3 posters on the board displaying the three types of representational modes)

S. Counting.

I. Yes you also used the hands on mode as you used your fingers. So you are very clever as you chose to do it two ways.

S. Then I thought that I shouldn't do it that way. I felt it wouldn't be that good. So then I started doing it in my head.

I. So you did it in your head. Wow! Can you read me your story as it is actually very good?

S. I went walking I found 12 Barbies and 7 more. I have 19 Barbies.

I. You certainly do. Good Girl. Now, if we were to do this same activity tomorrow but Miss Deagan wrote on the board $8 \div 5$, would you have chosen to write or would you have chosen another way to do it?

S. Umm to write

I. You would have done writing again? Would you have made up a different story though?

S. Yes.

I. Oh ok. Now, when I teach in class I normally tell you what to do. Did you find it hard deciding what to do today by yourself?

S. Nope

I. Why not?

S. I just thought that doing those things (student pointed to materials that were still set up on the floor) may be a bit tricky, when you can confuse them.

I. So you thought that using that would be a bit tricky?

S. Yes, you would confuse bits so I just thought I should just do some writing.
I. Oh, interesting. Would you prefer me to tell you what to do or do you like to decide by yourself?

S. Um, I like to decide by myself.

I. Ok, that’s very grown up that you like to do it by yourself. Now, how did you decide which way to complete it? Did you just decide that writing was the best way?

S. Yes.

I. Now I’m going to show you the card that we used 12 + 7. How can you show me that you understand this?

S. Um, it’s just that, I kept looking up at the board and when I realised that I should write the answer down here. I just did it.

I. Oh alright. Thank you for chatting to me today. Is there anything else you wanted to tell me about our activity?

S. Nope.

I. Did you enjoy it?

S. Yes.

I. Well thank you for talking to me today.
Thank you ................ for chatting to me today about your activity that you completed. As we have our chat I may write down some things. You will also notice that we have a video recorder in the room. That is there for me to listen to our chat again later. Is there anything you would like to know about our meeting before we start our chat?

I. I would like you to tell me about your activity that you completed today.

S. I completed the hands on mode.

I. Why did you choose that way?

S. Um, I think it's easier. (pause 4 seconds) than, because I like using things.

I. Ok, so you thought it was easier because you like using things? Ok, good boy. Now why did you decide to use icy pole sticks?

S. Because, it felt comfortable and it's easy to um do.

I. So you felt comfortable?

S. Yes

I. Wonderful. Now if it was possible for you to complete the oral way today by sitting on a chair and talking to the class or telling a friend. Would you have chosen to do that today? Or would you have still used your icy pole stick?

S. Pause. I think I'd use these (pointing to the icy pole sticks)

I. Why?

S. Um, the same reason as before.

I. Ok, so you are not comfortable talking in front of people?
S. Um, yes but I feel more comfortable doing this (student again points to icy pole sticks lying on the table in front of him)

I. Ok, Now when you completed your activity I noticed that you used a bundle. Can you actually explain to me how you set it out today?

S. What do you mean?

I. Well why have you got a group here and a group there? Can you explain that for me?

S. I have got it here like that

I. (Interviewer points to the bundle of ten) Ok, but why have you used that?

S. This? (Student points to the bundle of ten icy pole sticks)

I. Yes.

S. Because twelve has one ten.

I. So twelve has one ten and you have the two ones that makes twelve. What about down there (Interviewer points to the second row of icy pole sticks)

S. That’s seven

I. Ok, can you count it for me altogether?

S. 1, 2, 3, 4, 5, 6, 7, 8, 9, (student pauses when he approaches the bundle of ten, he now recounts) 10, 11, 12, 13, 14, 15, 16, 17, 18, 19

I. Ok good boy. Now if tomorrow I was to do the same activity and introduce it the same and have the posters and put on the board 8 x 5. Would you still have chosen to use the icy pole sticks? Or would you have chosen something else?

S. I think I would have chosen something else, because I would have liked to do something different.

I. You would have liked to do something different? Now, normally in class I tell you what to do like today we are going to answer this question using icy pole sticks or today you are going to complete a worksheet and stick it in your book. Did you find it hard to decide what to do on your own today?

S. Um, no

I. No, why not?

S. Um I don’t know

I. Did you feel grown up making a decision?
S. Um, Yes.

I. Would you prefer me to tell you what to do or would you prefer to decide by yourself?

S. Decide by myself.

I. Oh, that's really interesting! How did you decide to use icy pole sticks and the hands on mode? Did you watch the other children and see what they grabbed or you did it straight away?

S. I did it straight away.

I. Oh good. I was going to ask you to explain to me how you could understand this and show me how you understand it (Interviewer places card with 12 + 7 in front of student)

S. (Student gives a puzzled look)

I. But by looking at what you have done, I think you have already shown me by putting 12 in a pile and 7 in a pile. So I think you have already shown me that you understand that. Am I right?

S. Yes

I. Is there another way you could have done this today with icy pole sticks?

S. Um, I don't think so

I. The only way I thought of is perhaps take the rubber band off and have the ten sticks lying out. That could have been another way, couldn't it?

S. Yes

I. But that is a very grown up way. Thank you for chatting to me today. Is there anything else you wanted to tell me?

S. Um No

I. Ok well thank you very much.
Thank you ................ for chatting to me today about your activity that you completed. As we have our chat I may write down some things. You will also notice that we have a video recorder in the room. That is there for me to listen to our chat again later. Is there anything you would like to know about our meeting before we start our chat?

I. I would like you to tell me about your activity that you completed today.

S. I decided to do the peg boards

I. Oh ok, why did you decide to do peg boards?

S. Um, because they can take off easily and you can put them on easily.

I. Is that important when you do maths that you can take things off and put them back together.

S. To me it is.

I. To you it is? Why did you decide to complete your activity like this today? Out of all the things on the floor you decided to do the hands on mode. You didn’t want to do the written or the oral mode. What made you decide to do this?

S. Well, because... because it’s a bit, because you have to write you have to draw and write stories and sometimes you can’t think of a story quickly as you need to.

I. So you were worried that you wouldn’t be able to think of a good story?

S. Yes

I. You didn’t want to draw a picture?

S. No.

I. Why not?
S. Because, it's like the writing where you gotta do the writing except you gotta draw the picture.

I. Ok. If it was possible for you today to sit on the chair and tell a story to the children on the floor, would you have chosen to do that?

S. Um ...Um

I. Or would you have still done the peg boards?

S. Well, I would, cos I was thinking of doing the blocks... but um

I. You wouldn't have told a story on the chair?

S. Um, no.

I. Do you like talking in front of people?

S. No

I. Not really, so you wouldn't have done that.

I. What if you could have sat with perhaps Megan and told Megan a story would you have done that?

S. Yes

I. So you would have done it with Megan? Tomorrow if I was to write 8 + 5 on the board and have all the same equipment out, would you choose to do the same thing tomorrow or would you choose to do something different?

S. I would do something different.

I. Why?

S. Because you get to have a turn at something.

I. Because you get to have a turn at something else?

I. In class I normally say, today we are all going to get a turn at using blocks or today we are all going to do a worksheet. Did you find it hard to decide which way you wanted to do it and what you were going to use or was that easy?

S. That was a bit hard because I was thinking about maybe getting the blocks. I mean the peg boards because I was thinking that maybe no-one else would do it. Then I was looking at the blocks and then I thought more people would get the blocks. I didn't know why, but I said that probably more people would get the blocks. But more people got the peg boards.
I. Now, would you prefer me to tell you how to do the answer or do you like to decide?

S. I like to decide.

I. Why?

S. Because, um, because then I can use my fingers and something to work it out.

I. So you like to do it the way you like to do it.

S. Yes

I. (directing the student to her pegboard. The student also has the flash card with 12÷7 in front of her) I want you to show me how you this. I can actually see...what have you got across the top row?

S. um. I’ve got 12

I. Ok, so you have got the 12, and what have you got down the second row?

S. I’ve got that 7.

I. Ok, so why did you decide to do a row of 12 and a row of 7?

S. Because then I don’t have to get them. I just do that. (Pointing at the pegs one at a time)

I. Oh so is it easier to count?

S. (the student nods her head indicating yes)

I. So can you count that for me and show me how you got the answer?

S. 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19

I. Ok, and the answer is 19?

S. Yes

I. Is there any other way you could have done that?

S. (The student points to the next row between the two rows of pegs she made) I could have put them there.

I. Yes, you could have. But also is there another way you could have shown that on the peg board?

S. What do you mean?
I. 12+7?

S. Um... I'm not that sure.

I. Not sure... Is there anything else you would like to tell me about your activity today?

S. (The student nods to indicate no)

I. Thank you so much for all your help today. I really appreciate it.
Interview Transcript
Student Six

Thank you for chatting to me today about your activity that you completed. As we have our chat I may write down some things. You will also notice we have a video recorder in the room. That is there for me to listen to our chat again later. Is there anything you would like to know about our meeting before we start our chat?

*** The student started off with two lots of links. The first group had 18 links in a circle. The second group of links as displayed above were broken into the two rows of ten and seven. The student refers to this throughout the interview ***

I. I would like you to tell me about your activity that you completed today.

S. um... well, actually at first I didn’t have all that I need, I really had just the seven and the twelve. Then I joined them.

I. (Seeking clarification) so you had a row of seven and a row of twelve?

S. Yeah, and then I joined them.

I. And then you joined them?

S. Yeah.

I. So why have you got two?

S. Um cos. Um I felt like doing two.

I. So did you find it easier to connect it in one row to count them altogether?

S. Yeah, and then I was seeing if that was easier (pointing to the rows of links joined together) or that one was easier (pointing to the links that he had made into a circle).

I. Ok. Now why did you decide to complete your activity by using links today?
S. Um, well probably... pause for 5 seconds.... um

I. Well why didn't you do the writing?

S. Because um, um, I just didn't feel like it

I. You didn't feel like it. When I get you to write in maths do you mind doing writing? Or don't you like writing?

S. I like writing a bit... but not really.

I. So you like writing a bit?

S. Yeah

I. Now if it was possible for you to sit on the chair today and tell a story to the class would you have chosen to do it that way?

S. Yes.

I. Why?

S. Because... (3 second pause) I just um... like.... Um I'm good at them.

I. You are good at them.

I. Do you do a lot of maths in your head?

S. Yeah

I. So you don't usually use blocks all the time?

S. No

I. So if you had sat down with another child today and told him the story would you have liked to do that?

S. No, I still would have preferred to do it to the class.

I. When you completed your activity I noticed you completed it very quickly. You were one of the first done. How did you count it? How did you know to do it?

S. Because when I was on the floor, I, I just in my head I already knew I just worked it out.

I. So when you were on the floor you just worked it in your head? Then you just did it straight away.

S. Yeah
I. Ok, then tomorrow if I was to write on the board $8 + 5 = $ would you choose to use links or would you choose to use something else tomorrow?

S. I would probably choose telling a story.

I. You'd tell a story tomorrow?

S. Yeah.

I. Ok, ok fantastic. Now, when I teach the class I normally tell you what to do. Like today you are going to do writing or tomorrow we are going to do a worksheet. Did you like deciding what to do today?

S. Yes

I. How did you decide which was the best to use?

S. Well, um because really I've never used links before.

I. Ok. Now I'm going to show you this (Flash card with $12 + 7 =$ placed on table) you show me how you understood how to do that.

S. Easy, I already know what $2 + 7$ makes.

I. Ok, but how did you set it out? Did you set out $12 + 7$?

S. Yes.

I. Can you do that for me now just here please?

S. Student pulls apart the links he had already made together in a chain. He counts out six and then realises he only had six. He then adds another link to that row. He then just pulls the other links tight and places them underneath his row of seven. He does not count these to check the answer.

I. Wonderful, is there any other way in which you could have done this today?

S. The students shakes his head from side to side to indicate no.

I. No?

I. Ok, now is there anything else you would like to tell me about today's activity?

S. Not really.

I. Alright, well thank you for helping me out today.
Thank you for chatting to me today about your activity that you completed. As we have our chat I may write down some things. You will also notice that we have a video recorder in the room. That is there for me to listen to our chat again later. Is there anything you would like to know about our meeting before we start our chat?

I. I would like you to tell me about your activity that you completed today.

S. Yes I could.

I. Fantastic. Tell me about it.

S. Well this is a picture about how much kittens I bought. And on Monday I bought twelve kittens and three days later um on Thursday I bought seven more. And in this picture you can see me driving back to home.

I. Ok where is your twelve where are all your, where is your twelve? And where is your seven?

S. My twelve is here (pointing to the number 12 at the top of his page) and here is my seven (again pointing to the number 7 written at the top of his page)

I. Oh ok, well then how did you decide or why did you decide to draw a picture today and not use anything else?

S. Because it’s comfy with me.

I. It’s.
S. (Speaks over teacher), it suits me.

I. So it suits you? And you like drawing?

S. Yes.

I. Ok, now if it was possible for you to sit on the chair and tell the class a story using those numbers would you have chosen to do that?

S. Not quite.

I. Why?

S. Because, well I don’t really think I would, I don’t really think it is my type.

I. Your Type? (The interviewer smiles as she repeats this statement) What do you mean type?

S. My thing. I don’t really like doing it.

I. Ok, what about if it just had of been you and another person and if you were just working in pairs?

S. That would be fine.

I. So that would be fine?

I. Oh ok, I just like the way you used the word type. I thought that was really cute. Now, when you were completing your activity I actually noticed the first thing that you did. No you tell me what the first thing it was that you did? Did you draw the picture first or write the sum up the top?

S. Well first thing is I drewed, I drewed the sum then I drew the picture.

I. I actually did notice that. Did you write the sum to remind you of what the picture should be about?

S. (The student nods his head up and down to indicate yes)

I. Ok, now if tomorrow I was to write 8+5 on the board and put all the equipment down the front again would you choose to draw a story or picture again? Or would you choose something else?

S. I would choose something else.

I. What would you choose and why?

S. I would choose peg boards because you can pick them off.

I. And you like being able to take them off? I did notice that you had to use a rubber.
S. Yes.

I. Now normally I tell you what to do. Like ... (student's name)... It's time to use the blocks or it's time to do a worksheet. Today you had to decide, was that hard?

S. Nope

I. No, then how did you know what to choose?

S. Well, because I really like drawing.

I. Oh ok then.

S. So, I chose drawing.

I. Would you prefer me to tell you what to do all the time?

S. Not really.

I. But you'd like me to tell you sometimes?

S. (Student nods head to indicate yes)

I. Ok. Now if I show you this card which is the same as you have on the top of your sheet. Can you show me 12 + 7 in your picture?

S. 12 plus 7... I'm not sure what you really mean.

I. Ok well remember we've got twelve and you said kittens this time. So twelve kittens and seven kittens. Can you show me where they all are in your picture?

S. Well I didn't really have enough, so I tried to do as much as I could. This is one kitten that well there is a sun roof up here so it's enjoying the sun. (The student is pointing to the picture of his cat in his drawing) This kitten is thinking that one of his toys has gone down the street.

I. Oh ok. Thanks. Is there anything else you wanted to tell me about today's activity?

S. Not really.

I. Is there any other way you could shown 12 + 7 = ?

S. No.

I. Could you have perhaps drawn the twelve kittens in a row? Or?

S. I could...

I. Alright, well thank you for your help today.
Thank you for chatting to me today about your activity that you completed. As we have our chat I may write down some things. You will also notice that we have a video recorder in the room. That is there for me to listen to our chat again later. Is there anything you would like to know about our meeting before we start our chat?

I. I would like you to tell me about your activity that you completed today.

S. Um. I... done this because I like drawing and I like writing things and I like writing about that.

I. Ok you chose to do the writing and the picture. Why did you choose to do both?

S. So you can see the picture, what it is. So you

I. So you needed?

S. (Student speaks over the teacher) so you can count.

I. Oh, so you put the picture there so you can count with your words?

S. (Student nods head up and down to indicate yes)

I. Ok, so how did you decide that this was the way you were going to do it today?

S. What do you mean?
I. Well with all these things on the floor (Interviewer points to equipment still placed at the front of the room) why did you decide to do it this way?

S. It is a bit easier to count that.

I. Why didn’t you decide to use perhaps the hands on mode?

S. Cos, I think it's a bit fun to do that (student picks up the piece of paper sitting on the desk in front of her)

I. So you thought it would be more fun to do the writing? Now, if it was possible for you to sit on the teacher chair today and tell a story to the class. Would you have chosen to do it that way?

S. I guess, yeah!

I. Yes, why?

S. Because I can tell them something that I’ve done and stuff.

I. To tell them what you have done?

S. (Student nods head to indicate yes)

I. So would you, if you were sitting here with a partner would you have perhaps done partner work as well?

S. Yes (5 second pause)

I. Ok, fantastic. Now can you tell me when you completed your activity did you write this up the top first? What was the first thing you did?

S. I wrote this there (student points to the top of her paper where the sum had been written)

I. Ok, so why did you write that up the top?

S. So there is more room up the top and room for the picture down there.

I. Is there any other reason why you wrote the sum up the top?

S. (Student shakes head to indicate no)

I. Now, if we were going to do this activity tomorrow but I wrote 8+5= on the board would you write and draw a picture tomorrow or would you do it another way?

S. Another Way

I. What way would you do it tomorrow?
S. Umm...a fun way.

I. You'd do a fun way. What is a fun way?

S. Peg boards.

I. The pegboards?

S. (Student shakes head to indicate yes)

I. You'd choose the peg boards. Normally in class I tell you what to do, like I would say we are going to do a worksheet, we are going to use counters. Did you find it hard today deciding what to do by yourself or did you find it easy?

S. Hard

I. Why?

S. Because, um if we choose we get, we have a long time to think, and it is a bit hard to choose with all that stuff.

I. Did it help that I had posters up on the board?

S. Yes.

I. So the posters helped? Ok, so you prefer me to say, today we are all going to use counters.

S. Yes

I. Ok, Now ......

S. (student cuts teacher off) or if there were only two things on the floor to choose.

I. So you'd be happy if there were only two things on the floor?

S. (student nods head to indicate yes)

I. Were there too many things?

S. Yes.

I. Oh No!! Now, I want to show you this card 12+7. (Interviewer places flash card with 12+7 on the table) Can you show me how you understand this? Well let's get you to read your story to me first just to show me that you understand 12+7

S. One day I went out to catch some fish. I saw 12 hammerheads and 7 birds. How many things did I see?
I. Ok good girl. Now why did you circle the numbers?

S. Just so you know it’s the numbers in the story.

I. Ok, is there another way you could have done your pictures or set this out today?

S. (4 seconds pause) No I can’t think

I. No, you can’t think? Is there anything else you want to tell me about your activity today?

S. Um, that it was fun to choose.

I. It was fun to choose! Oh good, thank you for telling me that. Well, thank you so much for helping me out today.
Interview Transcript
Student Nine

Thank you ................ for chatting to me today about your activity that you completed. As we have our chat I may write down some things. You will also notice that we have a video recorder in the room. That is there for me to listen to our chat again later. Is there anything you would like to know about our meeting before we start our chat?

I. I would like you to tell me about your activity that you completed today.

S. Um, well I did, well I did a picture and I did um 12 cars and 7 dinosaurs. And um I really like drawing so that’s why I did that. But I won’t always um pick drawing.

I. Ok, well I was just going to say if it was possible for you to sit on the chair today and tell a story to the class. Would you have chosen to do that?

S. Um, I would of but I just wanted to choose something different.

I. Ok, if you were sitting at a table talking to a friend would you have chosen to do that? Could you have chosen to do that? Or are you not happy with that?

S. Not happy. cos then they might copy my work.

I. Oh so you are worried that they might copy your work?

S. Yes

I. Can you tell me what was the first thing you did on your paper today?

S. Um the cars,
I. So you did the cars? When did you write the sum?

S. Um I did the sum; I did like the sum and then the cars. (Student points to the picture as he answers the question)

I. So you did the sum first? Now, tomorrow if I was to write 8+5 on the board and I had the same equipment out. What would you choose tomorrow? Would you choose to do a drawing again? or would you choose something else?

S. I would choose to go up on the chair and tell a story.

I. So you would go on the chair and tell a story? Why would you choose that one?

S. Because, um it is fun more when writing about something about Maths, but not like saying what you did on your holidays that's why I would have chosen that next time, tomorrow.

I. When I teach I usually tell you how to answer a question like I would normally say today we are going to use calculator, counters or a book. Did you find it hard deciding what to do because you got to choose?

S. (Student nods head up and down to indicate yes) yes.

I. Ok, then what made you choose to do it in this way?

S. Well I thought that, well I was actually going to do drawing, no I mean writing. But I wanted to do some pictures so that's why I chose pictures. So that's why.

I. Now, I'm going to show you this card (interviewer places flash card with 12/7 on the table) and I want you to show me how you did 12/7. You show me that.

S. What do you mean by that?

I. Show me where 12/7 are?

S. It's there (student points to the sum 12/7= on the top of the page)

I. Yes, What have you got 12 of?

S. Cars

I. What have you got 7 of?

S. Dinosaurs.

I. Can you count them altogether for me please?

S. 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19
I. Wonderful. Now is there anything else you wanted to tell me about today’s activity?

S. Yes a little bit more. I would like to tell you well um the thing I wanted to do before the picture was I wanted to use the counters. But then I decided that I wanted to do some drawing. Then I wanted to do some drawing.

I. So what just because you wanted to draw you changed from counters?

S. Yeah

I. Alright. Well thank you very much for talking to me today. I have learnt so much about how you learn Maths.
Thank you for chatting to me today about your activity that you completed. As we have our chat I may write down some things. You will also notice that we have a video recorder in the room. That is there for me to listen to our chat again later. Is there anything you would like to know about our meeting before we start our chat?

I. I would like you to tell me about your activity that you completed today.

S. Well, I was going to do the love hearts red then I thought to do this bit red and that bit pink (Student points to the pictures on her page). For the suns I thought I should do it orange.

I. Ok, why did you decide to do the picture way today and not write a story or make something using the equipment?

S. Well, cos I thought it would look nice.

I. You thought it would look nice. Is that why you decided to do it this way today?

S. Yeah

I. Is there any other reason why?

S. No

I. Ok, now if it was possible for you to complete your activity today by sitting in the chair and telling a story would you have wanted to do that today?

S. No

I. Why not?
S. Um, (4 second pause) I don't really know why.

I. You don't really know why? Is it that you don't like talking to a big group of children?

S. No

I. No. Um ok, what if you were just sitting at a table and Aisha was sitting here with you. Would you have been able, would you have liked to have told Aisha a story?

S. Yeah

I. Ok, so a small group was ok but a bigger group was too hard?

S. Yeah

I. Not a problem. Now, when you did your activity today I actually noticed that you did all the pictures first and then you wrote the sum down the bottom. Why did you leave it to the end to write your sum?

S. Because, if I done that first it would take me more longer to do that. (Student was pointing to the sum and then the pictures)

I. Ok, but how did you know how many hearts and suns to draw?

S. I looked there (student pointed to the white board which had the sum displayed on it)

I. Ok, so you looked at the board?

S. Yeah

I. Oh ok, good girl. Now if we were to do this tomorrow but I had the sum 8+5 on the board. Would you have chosen to do a picture or would you have chosen to use or do something else?

S. Um, I would do a picture.

I. So you would draw a picture again?

S. Yeah

I. Why would you do a picture again?

S. Because I think it looks nice.

I. So you wouldn't want to use the pegs or the links?

S. No
I. So do you like drawing?

S. Yes.

I. Now, when I teach the class I normally tell you what to do. I normally say now this is a worksheet or I just want you to use the counters today. Did you find it hard deciding what to do today?

S. Yes

I. Why?

S. Well, Um, I didn't really know if I should do a picture or something different. And then I thought I should do a picture.

I. Is that because you are good at them?

S. Yes

I. So would you prefer Miss Deagan to say everybody use the pegs today or everyone use the links?

S. No

I. So you would want to choose?

S. Yes

I. Ok, Now how did you decide to do a picture? Did you wait for the other children to grab things first to see what the other children were doing? Or did you think about it when you were on the carpet?

S. Well, I was waiting for the children to choose things and then I saw a few kids doing some pictures. And then I thought a picture would be nicer.

I. Fantastic. Ok. I'm going to show you this card now (Interviewer places card 12 + 7 on the table) let's read it out together.

S. 12 + 7 =

I. Can you show me where you have got your 12?

S. There (the student points to the number 12 written on her page)

I. Yes, but where is your picture showing me your 12?

S. There (student points to the hearts)

I. Great, can you count them and just check them for me please?
S. 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12

I. Good Girl. And where is your 7?

S. There (student points to the pictures of the suns)

I. Can you just check them for me please?

S. 1, 2, 3, 4, 5, 6, 7

I. And how did you know how many you had altogether? How did you know that you had 19?

S. I counted them altogether.

I. Can you do that now for me please?

S. 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19

I. Ok, fantastic. Is there another way you could have shown 12+7 = today?

S. Yes, by 2's.

I. By 2's. How would you have done that?

S. 2, 4, 6, 8, 10, 12..., pause..., 15..., pause..., 17..., pause..., 18..., pause, 19. (Student places finger on two of each picture as she counts)

I. Ok, well done. Is there anything else you wanted to tell me about your activity today?

S. Not really.

I. Not really? So you didn’t want to write your sentence?

S. No

I. Ok, well thank you for helping me out today and completing your activity.
APPENDIX FOUR

(Work Samples from Lesson Seven)
I have 19 Begles.

I went looking! Found 7 more.

17 + 7 = 24
did I see. And I saw 12 hammer heads. I caught some fish. One day I went out.
APPENDIX FIVE

(Plain Language Statement)
Tuesday, 14th June 2005

Dear Parents,

As you may be aware I am currently completing my Masters degree at Melbourne University. As a part of this degree I wish to complete my research based on 'How Year One students represent their mathematical learning?'. Mr Ryan has given me permission to collect data during class time and I am now asking for volunteers who would be interested in allowing their child to participate in this research. I have attached a Plain Language Statement for you to read and if you feel you need more information I will be happy to answer any questions you may have.

If you are happy for your child to participate then please sign and return the consent form at your earliest convenience. The research will take place during the first few weeks of Term Three.

With every good wish,

Kind Regards,

Bronwyn Deagan
Plain Language Statement for Parents/Guardians

Research Topic: How can Year One students represent their mathematical learning?

Principal Investigator: Professor David Clarke  (Phone: 8344 1140)

Other Investigator: Bronwyn Deagan  (Phone: 03 9731 9555)

The Aim of This Research:

The aim of this research is to determine ‘How Year One students represent their mathematical learning”. The main focus will be on the different ways in which a student might show what they have learned. There are three main types of representation that will guide my research and data collection:

- Physical Representation: Do the students show a preference for individual or group work?
- Oral representation: Do students prefer to share their understandings publicly or privately?
- Written Representation: Do students prefer to use drawings or written statements?

This research will take place during mathematic classes at Williamstown Grammar School and will be based on the topic of two digit addition. Example: 12 + 9 =. After six learning sessions where the children practice each of the six representational modes, the students will be given a set of addition sums where they are able to choose the way in which they represent their understanding and learning. The student’s material; example drawings and written statements will be collected for analysis. The students will also be interviewed about their chosen form of representation.

What does it mean to take part in this study?

The students taking part in this study will all come from Year 1.D. Students will only take part if their parents/guardians give written permission. The School Principal has given permission to use the school premises and conduct the research during school time. The Ethics Committee at Melbourne University have also approved this research project. The students involved in this study have been taught Mathematics using the Early Years Numeracy Framework and have all been involved in previous one-on-one numeracy interviews.

Department of Science and Mathematics Education
(Incorporating IT in Education and Health & Physical Education)
Doug McDonell Building
The University of Melbourne Victoria 3010 Australia
T: +61 3 8344 8443/8419  F: +61 3 8344 8739
The lessons on addition will be no different to those that would normally be taught. A video camera will be used to record class activities. Individual students will be interviewed in the same way as for the Early Years Numeracy interviews and interviews are not anticipated to go for any longer than fifteen minutes. Material from ten students will be selected to be analysed in this study. Participation in this study will not involve any student in any type of activity different from what would normally occur in class.

Will participation prejudice my child in any way?

Please be advised that your child's participation in this study is completely voluntary. Should you or your child wish to withdraw at any stage, or to withdraw any unprocessed data you have supplied, you are free to do so without prejudice. This research is not a part of Westbourne Grammar School therefore it will have no effect on the students classroom or school based assessment.

Confidentiality

I intend to protect your child's anonymity and the confidentiality of their responses to the fullest possible extent, within the limits of the law. Although video footage will be analysed the student's names will not be used. In the final report, your child will be referred to by a pseudonym. The data collected for this research will be stored securely for five years after completion of analysis before being destroyed.

Contact us with any questions:

If there are any questions or concerns regarding this research you may wish to contact any of the investigators on the above contact details. If you have any concerns regarding the conduct of the research project you can contact the Executive Officer, Human Research Ethics, the University of Melbourne, Vic 3010, ph: (03) 8344 2073; fax: (03) 9347 6739.

Thank you for your assistance in this research.
Minerva Access is the Institutional Repository of The University of Melbourne

Author/s:
Deagan, Bronwyn

Title:
In what ways are year one students able to represent their mathematical understanding?

Date:
2006

Citation:

Publication Status:
Unpublished

Persistent Link:
http://hdl.handle.net/11343/39519

File Description:
In what ways are year one students able to represent their mathematical understanding?

Terms and Conditions:
Terms and Conditions: Copyright in works deposited in Minerva Access is retained by the copyright owner. The work may not be altered without permission from the copyright owner. Readers may only download, print and save electronic copies of whole works for their own personal non-commercial use. Any use that exceeds these limits requires permission from the copyright owner. Attribution is essential when quoting or paraphrasing from these works.