Direct Numerical Simulation of Turbulent Natural Convection bounded by Differentially Heated Vertical Walls

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Far better to dare mighty things, to win glorious triumphs, even though checkered by failure, than to take rank with those poor spirits who neither enjoy much nor suffer much, for they live in the gray twilight that knows neither victory nor defeat.

THEODORE ROOSEVELT
Abstract

Using new, high-resolution direct numerical simulation (DNS) data, this study appraises the different scaling laws found in literature for turbulent natural convection of air in a differentially heated vertical channel.

The present data is validated using past DNS studies, and covers the Rayleigh number ($Ra$) range between $5.4 \times 10^5$ to $1.0 \times 10^8$. This is followed by an appraisal of various scaling laws proposed by four studies: Versteegh and Nieuwstadt (77), Hölling and Herwig (34), Shiri and George (63) and George and Capp (23). These scaling laws are appraised with the profiles of the mean temperature defect, mean streamwise velocity, normal velocity fluctuations, temperature fluctuations and Reynolds shear stress. Based on the arguments of an inner (near-wall) and outer (channel-centre) region, the DNS data is found to support a $-1/3$ power law for the mean temperature in an overlap region. Using the inner and outer temperature profiles, an implicit heat transfer equation is obtained and a correction term in the equation is shown to be not negligible for the present $Ra$ range when compared with explicit equations found in literature. In addition, I determined that the mean streamwise velocity and normal velocity fluctuations collapse in the inner region when using the outer velocity scale. A similar collapse is noted in the profiles of temperature fluctuations with increasing $Ra$ when normalised with inner temperature and length scale. Lastly, I show evidence of an incipient proportional relationship between friction velocity and the outer velocity scale with increasing $Ra$.

The study is extended to the spectrum of turbulent kinetic energy and temperature fluctuations of the flow. The one-dimensional streamwise spectra collapse onto the $-5/3$ slope, coinciding with the standard Kolmogorov form of the power spectra reported in literature. This collapse is found to occur in the outer region of the flow in the bounds between the peaks of the mean streamwise velocity. In spectrogram form, I find evidence that the spectral peaks correspond to energetic velocity structures in the channel—the structures of streamwise velocity fluctuations appear to stretch half
of the streamwise domain and occur at a quarter intervals in the spanwise direction. From 2-dimensional autocorrelations, the structures of spanwise velocity fluctuations are found to be organised in a hatched pattern in an inner location \((z_i^\times \approx 7)\) and at the channel-centre. The respective pattern angles are \(\theta_i \approx 54^\circ\) and \(\theta_o \approx 48^\circ\), both measured from the horizontal. For the temperature spectrum, the \(-5/3\) collapse is also observed in the same bounds as the velocity spectrum. In pre-multiplied form, the spectral peak is found to occur at the wall-normal location which coincides with the peak temperature fluctuations in the channel. With increasing \(Ra\), the wall-parallel isocontours of temperature are found to show standard features of turbulent pressure-driven boundary layers—streaks with spanwise length of \(100^+\) units.
Declarations

This is to certify that:

i. the thesis comprises only my original work towards the masters except where indicated in the Preface,

ii. due acknowledgement has been made in the text to all other material used,

iii. the thesis is less than 40,000 words in length, exclusive of tables, maps, bibliographies and appendices.
Acknowledgements

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Chapter 1

Introduction and Literature Review

1.1 Introduction

When a body of fluid is subjected to temperature differences, the density of the fluid will be non-uniform and causes motions within the fluid body due to differential gravitational forces. This phenomenon, termed natural convection, occurs regularly in nature and has many applications in design and technologies, for instance, double-paned windows in buildings, wall cavities, double-skin façades, solar chimneys, cooling of electronic components and also of nuclear reactors. Le Corbusier, one of the pioneers of modernist architecture in the 1900s, developed a theory to cool or heat a building by forcing air flow between large glass walls enveloping the building (Corbusier et al. (11)). This idea, although unsuccessfully implemented in the Centrossyus Ministry building in Moscow in October 1928, underpins the general interest of designers and engineers alike to the use of natural convection in the control of heat transfer. One such application is the natural convection of a fluid bounded by two differentially heated, vertical walls.

In such a configuration, the fluid closer to the heated wall experiences an upward buoyant force and vice versa closer to the cold wall, resulting in the continuous shear of the fluid parallel to the direction of gravitational acceleration. Figure 1.1 shows this upward motion of particles—described as streaklines—which are caused by the buoyant forces due to the heated vertical wall (black regions). In the vertical configuration, natural convection has been frequently credited towards the reduction of heat transfer (as has been described by a number of technological applications previously
mentioned). It is in this particular area that studies have been conducted in order to understand, predict and control the physics of this flow. This forms the focus of the present numerical study.

![Chalk particle visualisations of natural convection on a vertical plate for: (a) air driven by constant heat flux from the plate, (b) water driven by constant temperature from the plate, and (c) air also by constant temperature from the plate. Schematic shown on right indicates flow direction. Hot walls are shaded black. Reproduced from Martynenko and Khramtsov (47).](image)

Figure 1.1: Chalk particle visualisations of natural convection on a vertical plate for: (a) air driven by constant heat flux from the plate, (b) water driven by constant temperature from the plate, and (c) air also by constant temperature from the plate. Schematic shown on right indicates flow direction. Hot walls are shaded black. Reproduced from Martynenko and Khramtsov (47).

### 1.2 Natural or free convection in literature

Since the early 1900s, various studies have investigated natural convection in different geometries and through different approaches. One of the early detailed studies on the phenomenon is the theoretical work by Batchelor (2), who made predictions on the heat transfer rate and the Rayleigh number, $Ra$—the dimensionless measure of natural
convection. At the time, Batchelor (2) was focused on the study of the flow in a cavity which was then the typical wall configuration in buildings in Britain. Circa 1950s, there have been various studies of the phenomenon in different configurations. These studies include, for example, the early experimental studies by De Graaf and Van der Held (14), Eckert and Carlson (18) and Elder (19, 20) in cavities, and the more recent studies by Dafa’Alla and Betts (13) and Betts and Bokhari (5); experiments over horizontal plates by Deardorff and Willis (17), Thomas and Townsend (69), Townsend (70); experiments using single vertically-suspended plates by Tsuji and Nagano (71), Lock and Trotter (45), Vliet and Liu (78), Cheesewright (7), Warner and Arpaci (79), Smith (66); and also in cylinders, for example, by Fujii (22) and Niemela et al. (53). In terms of theoretical work, there has also been a number of studies which sought to explain the physics of natural convection, for instance, the works of Priestley (59) and George and Capp (23).

In terms of numerical studies, one of the earliest known work is by Elder (21) who conducted investigations in light of his experimental studies at that time. This was followed by other studies, for instance Korpela et al. (42), Lee and Korpela (43), Phillips (57), Boudjemadi et al. (6), Versteegh and Nieuwstadt (76, 77), Hölling and Herwig (34), Tsujimoto et al. (73). A majority of the earlier studies were focused on natural convection in a vertical slot (i.e. channel with capped ends), while the latter studies—some of which will be referenced hereafter—were numerically simulated in a vertical channel. A review of these past literature reveals significant interest in the research community in the heat transfer phenomena of free convection.

As with the scaling studies of pressure-driven flow problems (and the logarithmic law of the wall arising thereof), the scaling laws for natural convection is of primary interest. These scaling laws can be used to develop equations or wall functions to predict the behaviour of the flow and temperature distribution near the wall, useful in determining the required design parameters for the vertical channel or cavity. George and Capp (23) proposed mean streamwise velocity and temperature equations based on scaling arguments which assume two flow regions: an inner region near the wall and an outer region away from the wall. In the inner region in vicinity of the wall, George and Capp (23) show the linear relationship of the velocity and temperature equations. These relationships are then predicted to take a \(-1/3\) power-law form when the outer portion of the inner region is considered, which George and Capp (23) termed the \textit{buoyant sublayer}. Since then, several studies have validated the \(-1/3\) power-law of the temperature profiles (e.g. Henkes and Hoogendoorn (32), Versteegh and Nieuwstadt (77)), but any form of similarity in the velocity profiles based on the
proposed scaling by George and Capp (23) were elusive (Versteegh and Nieuwstadt (77)).

Using the scaling laws and proposed wall functions, fluid behaviour can be numerically predicted by means of computational fluid dynamics (CFD) simulations. These CFD simulations are typically the Reynolds-averaged Navier–Stokes simulations (RANS) and Large-eddy simulations (LES), both of which use coarser discretised grids in the numerical computation. In contrast, direct numerical simulation (DNS) fully solves the time-dependent Navier–Stokes equations without any models and is accepted to be the proper solution to the turbulent flow being modeled (Wilcox (80)). As such, the computational cost of DNS is generally much higher than LES and RANS, and the Reynolds number ($Re$) range, and thus Rayleigh number, is limited. In RANS and LES, wall models are used to set the boundary conditions for calculations close to the wall. These models help reduce the added computational cost associated with fine grid spacings, which is required to capture rapid changes in flow physics in the near-wall vicinity (Kiš and Herwig (39)).

In a typical RANS or LES, logarithmic wall functions are used, even in the case of a naturally convected flow. Heindel et al. (30) and Barakos et al. (1) have found that these logarithmic wall functions are inaccurate and instead suggest the applicability of the wall functions proposed by George and Capp (23) and Cheesewright (7). Henkes and Hoogendoorn (31) appraised the wall functions arising from the theories proposed by George and Capp (23) and Cheesewright (7) and found that the temperature profile is predicted relatively well, but not the velocity profiles. Tsuji and Nagano (71) and Versteegh and Nieuwstadt (76) also proposed wall functions based on their experimental and DNS data, while on a more theoretical approach, Shiri and George (63) suggested a logarithmic form of the inner wall function based on the outer velocity scale. More recently, Kiš and Herwig (39) proposed wall functions based on their DNS data for the Grashof number, $Gr = Ra/Pr$, up to $4.0 \times 10^6$ and were successful in predicting velocity and temperature profiles, with the former accurate up to the velocity maximum. Interestingly, if one considers a typical double-skin building façade under the climatic influence in Melbourne, Australia, for a 0.5m air gap and assuming a typical internal building temperature of $22^\circ C$, the $Ra$ value ranges approximately between the order of $\sim 10^7 - 10^8$. Compared with the $Ra$ range of DNS in literature, it is only within the recent decade that numerical simulations have been able to achieve the operating conditions in practice.

For textbook heat transfer empirical correlations, it is widely accepted in practice that the correlations adopt a power-law form (cf. Incropera et al. (36)). Examples of
such correlations include,

\[ Nu = 0.105 Ra^{1/3}, \]  

(to leading order) for turbulent natural convection from a isothermal vertical plate (cf. eq. (9), Churchill and Chu (10)), and

\[ Nu = 0.046 Ra^{1/3}, \]

for natural convection in a vertical cavity with comparable aspect ratio and Prandtl number (cf. MacGregor and Emery (46)). A detailed summary of heat transfer correlations for various geometries is available in the text by Incropera et al. (36). However, it is worth noting that the present study focuses on an idealised configuration where the extents of the vertical channel are infinite, as compared to the aforementioned studies. This idealised configuration removes end-effects and may shed light on what is hoped to be a more basic form of the heat transfer correlation. Examples of geometries which give rise to end-effects include capped ends (in the case of an enclosed cavity) and trailing plate edges (for vertically-suspended plates).

Despite the abundance of high quality experimental and DNS data, it has been suggested (Shiri and George (63)) that the development of wall functions should be evaluated with flow data which has achieved a separation of scales to a degree such that the buoyant sublayer can be expected to exist. To measure this scale separation, Shiri and George (63) propose that the ratio of outer to inner length scales, \( h/l_i \), should be greater than 10 (the inner lengthscale, \( l_i \), is later defined in section 4.2). This \( h/l_i \) criterion is used throughout our simulations, and have been calculated to range between 19 to 105.

### 1.3 Aims and outline of research

In the absence of high Ra data mentioned above, the present work in this thesis is focused on obtaining high resolution DNS data for Ra up to \( 1.0 \times 10^8 \). The details of the numerical simulation will be presented in chapter 2.

With the present DNS data, I appraised the scaling laws and wall functions proposed by these studies: George and Capp (23), Versteegh and Nieuwstadt (77), Hölling and Herwig (34) and Shiri and George (63). The results are detailed in chapter 4. For the scaling of velocities, the scaling laws are directly appraised for the mean and turbulent quantities, whereas the temperature scales are evaluated with temperature wall functions. Arising from the temperature scales, a heat transfer relationship is defined
and validated with the results from the present data. The proposed inner lengthscale, \( l_i \), is also appraised by comparing with the relative structure sizes in the temperature isocontours shown in chapter 3.

In addition, a study is conducted on the kinetic energy and temperature spectra of the present flow. Both the spectrum of kinetic energy and temperature exhibit the ‘\(-5/3\)’ Kolmogorov slope at the highest Rayleigh number, \( Ra = 1.0 \times 10^8 \) (for the latter, an analogous ‘\(-5/3\)’ slope is obtained from the equation for temperature spectra derived by Corrsin (12) and Oboukhov (54)). Using the pre-multiplied form of spectra, I further analysed the spectrograms of kinetic energy and temperature, and report the variations of the spectral peaks with increasing \( Ra \) in terms of wavelength and wall-normal locations. The results are reported in chapter 5. These spectral peaks are found to correspond well with energetic structures observed in the isocontours of streamwise and spanwise velocities, shown in chapter 3.
Chapter 2

Background Theory and Numerical Methods

In this chapter, I review the theory behind the flow problem—including a statistical description of energy for the flow in terms of spectra—and the numerical methods employed in the study.

2.1 The flow

The flow problem considered in this study is the turbulent natural convection of air between two differentially heated vertical walls, shown schematically in figure 2.1. Throughout this study, the working fluid has a constant Prandtl number ($Pr$) value of 0.709. In the following sections, I present a basic theory of the flow, however, since the set-up of the simulation parameters are based on the work of Versteegh (75), only the necessary equations and parameters will be defined. For greater detail, the reader is encouraged to refer to the thesis of Versteegh (75).

The streamwise, spanwise and wall-normal directions of the flow are defined in the Cartesian coordinates $x$, $y$ and $z$ respectively, while the origin is set at the base of the hot wall defined as the left channel wall. By doing so, the direction of gravitational force is thus $-x$. The streamwise, spanwise and wall-normal components of the velocity field are defined as $u$, $v$ and $w$ and the temperature field as $T$.

Differing from pressure-driven flows, the driving force of this flow problem is the temperature difference between the two channel walls, which I define as $\Delta T = T_h - T_c$, where $T_h$ and $T_c$ are the temperatures of the hot and cold wall respectively. For a turbulent flow, the mean temperature profile, $\overline{T}$, with respect to a reference tempera-
ture, $T_{ref} = \left( T_h + T_c \right) / 2$, forms a decreasing relationship with increasing wall-normal location. This relationship is shown in blue in figure 2.1.

Arising from the temperature difference, $\Delta T$, the flow experiences an upward buoyant force which causes the fluid to rise (on an average sense) closer to the hot wall and vice versa on the cold wall. The mean streamwise velocity profile resembles a sinusoidal shape with the peak closer to the hot wall and trough closer to the cold (shown in red in figure 2.1).

In subsequent analyses, the Boussinesq approximation is used which assumes that the properties of the fluid are constant throughout the channel—except when considering the buoyancy force term which is a function of temperature difference and fluid density. The relevant measure of the flow, analogous to the Reynolds number, is the Rayleigh number, $Ra$, based on the channel width, $H(=2h)$. The Rayleigh number is defined as

$$Ra = \frac{g \beta \Delta T H^4}{\nu \alpha},$$
2.1. THE FLOW

where,

\[ g = \text{acceleration of gravity} \]
\[ \nu = \text{kinematic viscosity} (= \mu/\rho) \]
\[ \alpha = \text{thermal diffusivity} \]
\[ \beta = \text{thermal expansion coefficient} \]

Taking the Prandtl number \((Pr)\) into consideration, the Grashof number, \(Gr\) can be defined as

\[ Gr = \frac{Ra}{Pr} = \frac{g \beta \Delta T H^3}{\nu^2} \]

and is also commonly used as a measure of naturally convected flows.

2.1.1 Governing equations

The flow is governed by the Navier–Stokes equations with an additional buoyancy term in the momentum equations. In tensor notation, the continuity equation is

\[ \frac{\partial u_i}{\partial x_i} = 0. \] (2.1)

The momentum equation is,

\[ \frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{1}{\rho_o} \frac{\partial p}{\partial x_i} + g_i \beta (T - T_{\text{ref}}) + \nu \frac{\partial^2 u_i}{\partial x_j^2}, \] (2.2)

and the energy equation is,

\[ \frac{\partial T}{\partial t} + u_j \frac{\partial T}{\partial x_j} = \alpha \frac{\partial^2 T}{\partial x_j^2}. \] (2.3)

The definition used is, \(\rho_o\) as the reference density, \(p\) is the pressure of the flow, and the reference temperature, \(T_{\text{ref}}\), as before.

Laminar solution

The laminar equations for this flow problem can be obtained from the governing equations (2.1), (2.2) and (2.3) by assuming the time derivatives of the quantities to be zero and that the flow is homogeneous in the streamwise and spanwise directions. Applying these assumptions and the boundary conditions at the wall: \(u_i(x, y, 0) = u_i(x, y, 2h) = \)
0, \( T(x, y, 0) = \Delta T/2 \) and \( T(x, y, 2h) = -\Delta T/2 \), the mean streamwise velocity and temperature equations are obtained in the following non-dimensional forms:

\[
\frac{\pi}{\alpha/h} = \frac{Ra}{12H^3} (2z^3 - 3z^2 H + zH^2) \tag{2.4}
\]

and

\[
\frac{T - T_{ref}}{\Delta T} = \frac{T_h}{\Delta T} - \frac{z}{H}. \tag{2.5}
\]

Turbulent equations

For a turbulent flow, the fluctuations of the physical quantities are accounted for by applying the standard Reynolds decomposition, where for any turbulent field, \( u_i \), the quantities can be decomposed into the mean and fluctuating components, i.e.,

\[
u_i = \overline{u_i} + u_i'
\]

where \( \overline{u_i} \) is the mean component and \( u' \) the fluctuations. Also, the mean component is assumed to be averaged in time

\[
\overline{u_i} = \lim_{\tau \to \infty} \frac{1}{\tau} \int_{\tau_0}^{\tau_0+\tau} u_i d\tau.
\]

Hence, for a turbulent flow, equations (2.2) and (2.3) can be expressed as

\[
0 = \frac{d}{dz} \left( \nu \frac{d\overline{u}}{dz} - u'w' \right) + g\beta (T - T_{ref}), \tag{2.6}
\]

\[
0 = \frac{d}{dz} \left( \alpha \frac{d\overline{T}}{dz} - w'T' \right). \tag{2.7}
\]

Gravity is defined as acting only in the streamwise direction, \( x \), hence \( g = g_x \) (the subscript \( x \) is omitted for simplicity).

The mean equations for turbulent natural convection, (2.6) and (2.7), are seen to be quite different from that of laminar flow, equation (2.4) and (2.5). The turbulent equations contain non-linear terms which are defined as the Reynolds shear stress, \( -u'w' \) and the turbulent heat flux, \( -w'T' \), and since the quantities are not known a-priori, the equations are thus unclosed. The mean statistics remain dependent on the wall-normal location, \( z \), and are homogeneous in the streamwise and spanwise directions.

In the following chapters, equations (2.6) and (2.7) will be the starting point for
2.2 Spectrum of naturally convected turbulence

The underlying theory of turbulence is that turbulence consists of a range of scales. These scales contain energy which is typically cascaded from the largest scales of motion to the smallest, where finally, the energy is dissipated through viscous interactions. This notion of energy cascade is neatly summarised by Richardson (60):

“...big whirls have little whirls that feed on their velocity, and little whirls have lesser whirls and so on to viscosity—in the molecular sense”

In the following sections, some basic spectral theory for kinetic energy and passive scalar contaminants—temperature, in this study—will be presented, and in chapter 5, I report the results and discussion for the simulations in this study.

2.2.1 The turbulent kinetic energy spectrum

The turbulent kinetic energy (of fluctuating velocity) per unit mass, $k$, can be expressed in wavenumber space, $\kappa$, by the equation:

$$k = \frac{1}{2} (u'^2 + v'^2 + w'^2) = \int_0^\infty E(\kappa) d\kappa,$$

(cf. Pope (58), Wilcox (80)) where $E(\kappa) d\kappa$ is the spectrum of turbulent kinetic energy contained between the wavenumbers, $\kappa$ and $d\kappa$. The wavelength which corresponds to the wavenumber, $\kappa$, is defined by $\lambda = 2\pi/\kappa$.

Kolmogorov’s first similarity hypothesis (Kolmogorov (40)) states that in the energy cascade process, there exists an equilibrium range in wavenumber space, $\kappa$, (e.g., turbulent eddy size) where the energy is governed by the dissipation rate of kinetic energy per units mass, $\varepsilon$, and viscosity, $\nu$. In the wavenumber range where equilibrium exists, the flow is thought of as locally isotropic\(^1\). Here, the rate of dissipation of kinetic energy

\(^1\)Local isotropy is defined as isotropy at small scales, or, small scale structures which are independent of mean shearing motion and exhibit constant averages under rotations or reflections of the coordinate system (c.f. Tennekes and Lumley (68))
energy is defined (approximately) by:

$$\varepsilon = \nu \frac{\partial u_i}{\partial x_j} \frac{\partial u_i}{\partial x_j}$$

(2.9)

Hence, the spectrum function can be expressed in the following form:

$$E(\kappa) = f(\kappa; \varepsilon, \nu).$$

(2.10)

For details of the arguments underlying the functional form of equation (2.10), the reader is referred to the texts of Pope (58, chap 6.5), Wilcox (80) and Tennekes and Lumley (68).

Using an analogous argument to the inertial sublayer (Tennekes and Lumley (68)) for wall-bounded flows, an *inertial subrange* which is within the equilibrium range and independent of viscosity, \(\nu\), can be defined for a sufficiently high Reynolds number. In this subrange, the energy spectrum function is simply:

$$E(\kappa) = C \varepsilon^{2/3} \kappa^{-5/3},$$

(2.11)

where \(C\) is the universal Kolmogorov constant. This equation is the well-known Kolmogorov’s ‘\(-5/3\)’ law. The Kolmogorov constant, \(C\), has been experimentally found (e.g. Saddoughi and Veeravalli (62)) to be approximately 1.5.

The present study focuses on the one-dimensional (1D) spectra in the longitudinal (streamwise) and transverse (spanwise) directions, given by:

$$E_{11}(\kappa_1) = C_1 \varepsilon^{2/3} \kappa_1^{-5/3},$$

(2.12)

$$E_{22}(\kappa_1) = C'_1 \varepsilon^{2/3} \kappa_1^{-5/3},$$

(2.13)

where \(C_1 \approx 0.49\) and \(C'_1 \approx 0.65\) (cf. Pope (58)). This follows from the assumption of isotropic turbulence, that is, the constants, \(C_1\) and \(C'_1\) are related to the Kolmogorov constant through the following equations:

$$C_1 \approx \frac{18}{55} C$$

(2.14)

$$C'_1 \approx \frac{24}{55} C$$

(2.15)

Based on the abundant evidence on \(-5/3\) scaling results (e.g. Saddoughi and Veeravalli (62)) and universality of the Kolmogorov spectrum, Rogallo and Moin (61) suggests that in order to produce reliable datasets, data from numerical simulations should
ideally be able to reproduce the universal trend. In the following sections, the non-dimensional forms of equations (2.12) and (2.13) are used to show that the present study exhibit evidence of this universality, thus indicating the simulations are enough to resolve the scales on the order of the Kolmogorov length.

2.2.1.1 The 1D streamwise energy spectrum

The non-dimensional forms of the 1D energy spectrum functions can be defined with the use of the Kolmogorov scales for:

\[
\begin{align*}
\text{length, } & \eta_K = (\nu^3/\varepsilon)^{1/4}, \\
\text{velocity, } & u_{\eta_K} = (\nu\varepsilon)^{1/4}.
\end{align*}
\]

Equations (2.12) and (2.13) can then be rewritten in non-dimensional form by multiplying with \(\eta_K^{-5/3}\) and rearranging, giving

\[
\begin{align*}
\frac{E_{11}(\kappa_1)}{\varepsilon^{2/3}\eta_K^{5/3}} &= C_1(\kappa_1\eta_K)^{-5/3}, \\
\frac{E_{22}(\kappa_1)}{\varepsilon^{2/3}\eta_K^{5/3}} &= C'_1(\kappa_1\eta_K)^{-5/3}.
\end{align*}
\]

Using the Kolmogorov velocity scale, \(u_{\eta_K}\), and the values of \(C_1 = 0.49\) and \(C'_1 = 0.65\) obtained from literature, the following universal relationships are obtained:

\[
\begin{align*}
\frac{E_{11}(\kappa_1)}{u_{\eta_K}^2} &= 0.49(\kappa_1\eta_K)^{-5/3}, \\
\frac{E_{22}(\kappa_1)}{u_{\eta_K}^2} &= 0.65(\kappa_1\eta_K)^{-5/3}.
\end{align*}
\]

For the spectral analysis in this study, the following conventions are used instead of the canonical forms above,

\[
\begin{align*}
\overline{u'^2} &= 2 \int_0^{\infty} E_{11}(\kappa_1) d\kappa_1, \\
\overline{v'^2} &= 2 \int_0^{\infty} E_{22}(\kappa_1) d\kappa_1.
\end{align*}
\]
Hence, the Kolmogorov equations with modified constants are,

\[
\frac{E_{11}(\kappa_1)}{u_{\eta K}^2 \eta K} = \frac{C_1}{2} (\kappa_1 \eta K)^{5/3} = 0.245 (\kappa_1 \eta K)^{-5/3}, \quad (2.24)
\]

\[
\frac{E_{22}(\kappa_1)}{u_{\eta K}^2 \eta K} = \frac{C_1'}{2} (\kappa_1 \eta K)^{5/3} = 0.325 (\kappa_1 \eta K)^{-5/3}. \quad (2.25)
\]

### 2.2.1.2 The 1D streamwise energy spectrum - pre-multiplied form

The contributions to the spectrum, and thus the energy of the flow, can be analysed by pre-multiplying the 1D energy spectrum with the wavenumber. This is intuitively useful since the area under the pre-multiplied spectrum is equal to the energy contributed by the scale of the flow on the abscissa, defined either in wavenumber of wavelength space. Any peaks in the pre-multiplied spectrum will then indicate the most ‘energetic’ wavenumber or wavelength. In this study, the pre-multiplied spectrum is analysed against the non-dimensional wavelength, \( \lambda_i/h \) where \( \lambda_i = 2\pi/\kappa_i \). Thus, the pre-multiplied equations take the form: \( \kappa_j E_{ii}^\times(\kappa_j) \), plotted against \( \lambda_j/h \), where superscript \( \times \) indicates non-dimensionalisation by the velocity scale which is suitable for the present flow problem\(^1\).

### 2.2.2 The temperature spectrum

Building on the theory of local isotropy in section 2.2.1, an equilibrium range in the spectra of temperature variance should similarly exist if an equilibrium range is present in the spectra of kinetic energy (Tennekes and Lumley (68)). This can be thought of as the mixing of temperature in the fluid by the turbulent motion.

In this equilibrium range, the spectrum for temperature variance, denoted as \( E_\theta(\kappa) \), is dependent only on \( \nu, \varepsilon, \alpha \) and the rate of dissipation of temperature variance, \( \varepsilon_\theta \), defined by:

\[
\varepsilon_\theta = \alpha \frac{\partial T}{\partial x_i} \frac{\partial T}{\partial x_i}.
\]

(2.26)

Using dimensional arguments, \( E_\theta(\kappa) \) can be expressed in the following form:

\[
E_\theta(\kappa) = \varepsilon_\theta e^{-1/3} \kappa^{-5/3} f_\theta(\kappa \eta, Pr),
\]

(2.27)

where \( f_\theta \) represents the non-dimensional temperature spectra and is dependent on the Prandtl number of the fluid.

---

\(^1\)The canonical friction velocity scale, \( u_\tau \), is found not to be a suitable scale for the present \( Ra \) range. However, there is evidence of tendency towards \( u_\tau \) when \( Ra \to \infty \) (see section 4.3.5)
In addition to the Kolmogorov scale, $\eta_\text{K}$, two other relevant lengthscales which are used to characterise the small-scales are the Batchelor scale and Corrsin scale. These are respectively defined as:

\[
\eta_B = \left( \frac{\nu \alpha^2}{\varepsilon} \right)^{1/4} = Pr^{-1/2} \cdot \eta_\text{K},
\]

\[
\eta_C = \left( \frac{\alpha^3}{\varepsilon} \right)^{1/4} = Pr^{-3/4} \cdot \eta_\text{K}.
\]

Batchelor (3) distinguished the use of $\eta_B$ and $\eta_C$ as the relevant scales for the cases where $Pr \gg 1$ and $Pr \ll 1$ respectively (cf. Hill (33)).

Several sub-ranges in the equilibrium range can be described (Tennekes and Lumley (68)) as follows:

1. If $Re$ is sufficiently large and if diffusivity, $\alpha$, is negligible, the sub-range is defined as the inertial-convective sub-range. In this sub-range, temperature fluctuations is simply convected and the spectra should be independent of $\nu$ and $\alpha$. Hence (also from Corrsin (12) and Oboukhov (54)):

\[
E_\theta(\kappa) = B \varepsilon_\theta \kappa^{-1/3} \kappa^{-5/3}.
\]

2. If $Pr \ll 1$, the thermal diffusivity, $\alpha$, becomes important in the inertial sub-range, relative to $\nu$. This sub-range is defined as the inertial-diffusive sub-range and Batchelor et al. (4) predicted the spectra to take the form:

\[
E_\theta(\kappa) = \frac{1}{3} C \varepsilon_\theta \kappa^{-2/3} \alpha^{-3} \kappa^{-17/3},
\]

where $C$ is the Kolmogorov constant.

3. For $Pr \gg 1$, the effects of $\nu$ become more important than $\alpha$ in the inertial sub-range. This defines the viscous-convective sub-range when $\kappa \eta_\text{K} \gtrsim 1$ and $\kappa \eta_B \ll 1$, and the viscous-diffusive sub-range when $\kappa \eta_\text{K} \gg \kappa \eta_B \gtrsim 1$. The theoretical equation proposed by Batchelor (3) for the viscous-convective and viscous-diffusive sub-range is

\[
E_\theta(\kappa) = -\varepsilon_\theta \gamma^{-1} \kappa^{-1} \exp \left( \frac{\alpha \kappa^2}{\gamma} \right) \text{ for } \kappa \gtrsim \frac{1}{\eta_\text{K}}
\]

where $\gamma$ is the single effective least principal rate of strain and is estimated by
Batchelor (3) to be approximately \(-0.5(\varepsilon/\nu)^{1/2}\). As such, the equation above becomes

\[ E_\theta(\kappa) = -\varepsilon_\theta \gamma^{-1} \kappa^{-1} \exp \left[ -2 \left( \frac{\kappa}{\kappa_B} \right)^2 \right], \quad (2.31) \]

where, \( \kappa_B = 1/\eta_B \).

Based on the flow problem of the present study, equations (2.28) and (2.29) appear to be most relevant despite the prerequisite for \( Pr \ll 1 \). The former is applicable at a lower wavenumber range and the latter at a higher wavenumber range.

### 2.2.2.1 The 1D temperature spectrum

As before, the present study focuses on the 1D temperature spectrum which can be used to validate the statistics of the temperature fluctuations. The 1D equation in the inertial sub-range is found (Hill (33)) to be in the similar form as equation (2.28), i.e.:

\[ E_\theta(\kappa_1) = B_1 \varepsilon_\theta \varepsilon^{-1/3} \kappa_1^{-5/3}, \quad (2.32) \]

where the one-dimensional constant, \( B_1 = 5/3B \), from isotropy (Gibson and Schwarz (24), Hill (33)).

Using equation (2.32) and the Kolmogorov lengthscale, \( \eta_K \), the non-dimensional form can be expressed (as before):

\[ \frac{E_\theta(\kappa_1)}{\varepsilon_\theta \varepsilon^{-1/3} \eta_K^{5/3}} = B_1(\kappa_1 \eta_K)^{-5/3}. \]

### 2.2.2.2 The 1D temperature spectrum - pre-multiplied form

The temperature spectrum can also be analysed in the pre-multiplied form to determine the peak wavelength(s) which contribute to the ‘energy’ in the spectrum. Here, the term ‘energy’ is used loosely as an analogy to the kinetic energy spectrum. Pre-multiplying the temperature spectrum by the wavenumber, \( \kappa_i \) and analysing against increasing non-dimensionalised wavelengths, \( \lambda_i/h \) as before, the equation takes the form:

\[ \kappa_i E_\theta^\pi(\kappa_i) \quad (2.33) \]

where, \( E_\theta^\pi(\kappa_i) \) is the temperature spectra, \( E_\theta(\kappa_i) \) non-dimensionalised by the temperature scale from section 4.2. It will be shown later in section 5.2.2 that the spectra at the channel-centre exhibits limited scaling characteristic when scaled with inner and
outer temperature scale.

2.3 Numerical methods

2.3.1 Computational domain and numerical scheme

![Visualization of computational domain](image)

Figure 2.2: Visualization of computational domain. The streamwise ($x$) and spanwise ($y$) spacings are uniform, while the wall-normal spacings ($z$) utilise a cosine-stretching grid.

The governing equations of (2.1), (2.2) and (2.3) are solved numerically over a computational domain size defined by $L_x \times L_y \times L_z$ for two different domain sizes: $24h \times 12h \times 2h$ for Run 1 and $16h \times 8h \times 2h$ for Run 2, where $h$ is the channel half-width. The second set of simulations, Run 2, is numerically simulated using higher resolution. In the present study, the simulations were conducted for five $Ra$ values ranging from $5.4 \times 10^5$ to $1.0 \times 10^8$. The number of nodes used in each direction, defined as $n_x \times n_y \times n_z$, are summarised in table 2.1.

To ensure that the statistics are sampled over a sufficiently long interval, data from Run 1 were initially collected over a time interval where the ratio of simulation timescale to eddy turnover time

$$\frac{t_{\text{sim}}}{t_{\text{eddy}}} = \frac{t_{\text{sim}}}{\text{length scale/velocity scale}} = \frac{t_{\text{sim}}}{h/(g\beta f_w h)^{1/3}}$$

is greater than 100, with the exception of $Ra = 2.0 \times 10^7$. However, it was discovered in the later course of study that higher resolutions were required in order to adequately resolve the smallest scales. These smallest scales are measured by the Kolmogorov lengthscale, $\eta_K$, and will be discussed in detail in section 2.3.5.
Table 2.1: Simulation parameters for this study. Run 2 uses a 4th-order staggered differencing scheme—similar to Run 1—but with the QUICK scheme, higher resolution and reduced computational domain size.

In the subsequent refined simulation, Run 2, the statistics were collected for approximately 50 washout cycles (comparable with the DNS simulation of Kisser and Herwig (39)), defined based on the maximum mean streamwise velocity:

$$\text{washout cycles} = \frac{\tau_{\max}}{L_x/t_{\text{sim}}}.$$  

However, due to limited computational hours for $Ra = 1.0 \times 10^8$, only 12 washout cycles of data was collected. For all simulations, a constant zero mass flux algorithm is implemented to facilitate convergence of the simulations.

The governing equations are spatially discretised using the fully conservative fourth-order staggered scheme of Morinishi et al. (49) and marched in time using the low-storage third-order Runge–Kutta scheme of Spalart et al. (67). The velocity field is projected onto a divergence-free field after each Runge–Kutta stage via the fractional-step method (e.g. Kim and Moin (37)). For Run 2, the QUICK scheme (Leonard (44)) is implemented to overcome the artifacts arising from artificial numerical diffusion due to the implementation of the central differencing scheme. The artifacts were observed as wiggle-like isocontours in the temperature field from the data in Run 1. Grid spacings in the streamwise and spanwise directions are uniform (see figure 2.2), while the wall-
normal spacings utilise a cosine-stretching function, defined by

$$\Delta z(i_z) = \frac{L_z}{2} \left\{ \cos \left( \pi \frac{i_z - 1}{n_z} \right) - \cos \left( \frac{\pi i_z}{n_z} \right) \right\}$$  \hspace{1cm} (2.34)

where $\Delta z(i_z)$ is the wall-normal spacing and $i_z \in [1, n_z]$. This code is a variant of an in-house developed code written in C and has been used in past studies, for instance, in Matheou and Chung (48). A comparison of the numerical schemes used in the DNS studies for the present flow is summarised in table 2.2. The legend is used in the validation plots in the following section.
<table>
<thead>
<tr>
<th>Studies</th>
<th>Legend</th>
<th>$L_x \times L_y \times L_z$</th>
<th>Resolution</th>
<th>Numerical setup</th>
</tr>
</thead>
<tbody>
<tr>
<td>VN (77)</td>
<td>○</td>
<td>$24h \times 12h \times 2h$</td>
<td>$432 \times 216 \times 96$</td>
<td>2nd-order FV, with Richardson extrapolation</td>
</tr>
<tr>
<td>Pallares et al. (55)</td>
<td>x</td>
<td>$8\pi h \times 4\pi h \times 2h$</td>
<td>$121 \times 121 \times 100$</td>
<td>2nd-order FV, with staggered grid</td>
</tr>
<tr>
<td>Kiš and Herwig (39)</td>
<td>□</td>
<td>$24h \times 12h \times 2h$</td>
<td>$640 \times 320 \times 145$ (maximum)</td>
<td>Pseudo-spectral Chebychev-tau</td>
</tr>
<tr>
<td></td>
<td></td>
<td>($Ra \le 1.1 \times 10^7$)</td>
<td>$12h \times 6h \times 2h$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>($Ra = 1.7 \times 10^7$)</td>
<td>$384 \times 192 \times 151$</td>
<td></td>
</tr>
<tr>
<td>Second-order</td>
<td>-</td>
<td>$24h \times 12h \times 2h$</td>
<td>$432 \times 216 \times 96$</td>
<td>2nd-order FD, with staggered grid</td>
</tr>
<tr>
<td>Run 1</td>
<td>-</td>
<td>$24h \times 12h \times 2h$</td>
<td>$432 \times 216 \times 96$</td>
<td>4th-order FD, with staggered grid</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$768 \times 384 \times 192$ (R1.4)</td>
<td></td>
</tr>
<tr>
<td>Run 2</td>
<td>-</td>
<td>$16h \times 8h \times 2h$</td>
<td>$384 \times 192 \times 96$</td>
<td>4th-order FD, with staggered grid and QUICK scheme</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$512 \times 256 \times 96$ (R2.3)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$832 \times 416 \times 192$ (R2.4)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$1536 \times 768 \times 384$ (R2.5)</td>
<td></td>
</tr>
</tbody>
</table>

Table 2.2: Comparison of domain size, resolution and numerical setup of DNS studies, past and present.
2.3. NUMERICAL METHODS

2.3.2 Validation with published data

The present simulations were validated with the mean and turbulent statistics from Versteegh and Nieuwstadt (77), Pallares et al. (55) and Kiš and Herwig (39) for $Ra = 5.4 \times 10^5$. The comparisons are shown below in figure 2.3. Also shown in the figure is the preliminary DNS data which was conducted on a second-order staggered scheme, shown as ‘---’ (where available). It is found that this second-order-accurate statistics exhibit a higher magnitude of the streamwise velocity fluctuations, $u'u'$, at the channel-centre (almost matching the second-order DNS results of Pallares et al. (55)) when compared to the fourth-order-accurate results of Versteegh and Nieuwstadt (77). This motivated the present use of the fourth-order code which were found to closely overlap with the DNS results of Versteegh and Nieuwstadt (77). The fourth-order-accurate results are shown in the figures as ‘---’ for Run 1 and ‘---’ for Run 2.

2.3.3 Mean statistics

For the mean statistics, I find that the profiles of the present data is consistent with published DNS results. In particular, the fourth-order-accurate mean streamwise velocity profile, $\overline{u}$, in figure 2.3a matches the data from Versteegh and Nieuwstadt (77) and Pallares et al. (55) and exhibits the same velocity gradient at the wall and channel-centre. However, the peak mean streamwise velocity is marginally lower for the preliminary second-order and fourth-order (Run 1) simulations, and is almost coincident with the data from Kiš and Herwig (39). The locations of the velocity maximum for the datasets are found to be relatively constant at $z/h \approx 0.16$. Despite the minor discrepancies in mean streamwise velocity profile, the mean temperature profile for both second and fourth-order simulations matches well with the published data (see figure 2.3b).

2.3.4 Turbulent statistics

For the normal velocity and temperature fluctuations, the quantities match quite closely with the exception of the profiles of streamwise velocity fluctuations, $u'u'$, at the channel-centre. The results of preliminary second-order and fourth-order (Run 1) simulations appear to match more closely to the results of Pallares et al. (55) and Kiš and Herwig (39). These results are noticeably higher than the results from the fourth-order-accurate data of Versteegh and Nieuwstadt (77) and the present fourth-order (Run 2) simulation, suggesting that the turbulent statistics are sensitive to domain-size (Run 2 has a smaller domain-size compared to Run 1). At present, I am unable to account for the higher magnitude in the streamwise velocity fluctuations of the preliminary
second-order run at the channel-centre. Nevertheless, the profiles of normal velocity fluctuations generally agree and subsequent investigations in this study will present data from both Run 1 and 2, where available, for a complete view of the flow. A comparison between the Reynolds shear stress, $u'w'$, and heat flux fluctuations, $w'T'$, (see figures 2.3e and 2.3f) show good agreement with the DNS data in literature.
2.3. NUMERICAL METHODS

2.3.5 Effects of grid resolution

In the course of analysing the spectra of the flow in chapter 5, there was a noticeable distortion of the high-wavenumber viscous roll-off in the 1D velocity spectra at $\kappa_1 \eta_K \approx 1$ and $\kappa_2 \eta_K \approx 1$, where $\kappa_1$ and $\kappa_2$ are the streamwise and spanwise wavenumbers, and $\eta_K$ is the Kolmogorov lengthscale. This distortion was evident during the analysis of the data from Run 1 (see dashed spectra profiles in figure 5.1) and is speculated to be caused by the usage of a grid resolution which accounts for a maximum wave number less than $1/\eta_K$. That is to say, if the Kolmogorov lengthscale is used as the measure of the smallest scales of motion (e.g. Grötzbach (29) and Shishkina et al. (64)), the resolution for Run 1 appears insufficient to fully resolve the energy at the high wavenumber range. Based on this finding, there was an inherent need to refine the grid resolution for the present set of simulations.

I start by defining the Kolmogorov lengthscale

$$\eta_K = \left( \frac{\nu^3}{\langle \varepsilon \rangle_{t,V}} \right)^{1/4}. \tag{2.35}$$

Here, $\langle \varepsilon \rangle_{t,V}$ is the global time and spatially-averaged rate of kinetic energy dissipation.
CHAPTER 2. BACKGROUND THEORY AND NUMERICAL METHODS

per mass:

\[ \langle \varepsilon \rangle_{t,V} = \frac{1}{\Delta t \Delta V} \int_t^T \int_V \alpha \left( \frac{\partial u_i}{\partial x_j} \right)^2 dV dt. \]  

(2.36)

Using the Kolmogorov lengthscale, Grötzbach (29) defines a maximum wave number, \( \kappa_{\text{max}} = \frac{\pi}{\Delta} \), which should be exceeded by the global mean mesh resolution, \( \Delta = (\Delta_x \Delta_y \Delta_z)^{1/3} \), through the inequality

\[ \kappa_{\text{max}} \geq \frac{1}{\eta_K}, \]  

(2.37)

which can be rewritten as

\[ \frac{\Delta}{\eta_K} \leq \pi. \]  

(2.38)

In the present study, since there is sufficient data to analyse the resolutions in each directions, equation (2.38) is rewritten as a function of wall-normal distance. Hence, I introduce the \( xy \) plane-average dissipation rate (due to homogeneity in \( x \) and \( y \))

\[ \langle \varepsilon(z) \rangle_{\text{local}} = \frac{1}{\Delta t L_x L_y} \int_t^T \int_{A_{x,y}} \alpha \left( \frac{\partial u_i}{\partial x_j} \right)^2 dA_{x,y} dt. \]  

(2.39)

The one-dimensional form of equation (2.38) based on the local dissipation rate is then

\[ \frac{\Delta_i}{(\nu^3/\varepsilon_{\text{local}})^{1/4}} = \frac{\Delta_i}{\eta_{K,\text{local}}} \leq 3, \]  

(2.40)

where, for simplicity, the threshold value, \( \pi \), is rounded down to 3.

Thus, the threshold equation for computing minimum resolution in each direction is

\[ \frac{\Delta_i}{\eta_{K,\text{local}}} \leq 3. \]  

(2.41)

It is highlighted here that the Kolmogorov lengthscale is selected as the smallest measure of the scales, since \( Pr = 0.709 \leq 1 \). The relevant lengthscales for different \( Pr \) values have been previously discussed in section 2.2.2. The computed ratios \( \Delta_i/\eta_{K,\text{local}} \) for Runs 1 and 2 are shown in figure 2.4.
2.3. NUMERICAL METHODS

Figure 2.4: Plot of spacing ratio, $\Delta_i/\eta_K$, local, against wall-normal distance, $z/h$, for (a) Run 1, and (b) Run 2 for increasing $Ra$ (notation from table 2.1). Streamwise, spanwise and wall-normal ratios are shown as $\circ$, $\cdot\cdot\cdot$ and $\cdot\cdot\cdot$. In contrast to Run 1, the spacing ratio for Run 2 is less than the threshold value of 3 (shown as $\cdot\cdot\cdot$) throughout the channel, except in the near-wall region for the streamwise and spanwise resolutions.
Generally, it is found that the streamwise and spanwise spacings are more restrictive in a computational sense since the selected resolution in the respective directions result in magnitudes of ratios greater than that of the wall-normal resolution. I also find that the magnitude of $\Delta_x/\eta_{K,\text{local}}$ and $\Delta_y/\eta_{K,\text{local}}$ is maximum at the wall and, for the lowest $Ra$, $Ra = 5.4 \times 10^5$, drops off to a constant value at the location $z/h \approx 0.05$. This location of the drop-off moves closer to the wall with increasing $Ra$. Since viscosity is constant, this indicates that the local dissipation rate is higher in the vicinity of the wall, which is consistent with the dissipation profiles of Deardorff and Willis (17).

For the wall-normal ratios, $\Delta_z/\eta_{K,\text{local}}$, the values are lower than the streamwise and spanwise profiles, and peaks in the channel-centre. Another noticeable feature is a small region for $0.01 \lesssim z/h \lesssim 0.05$ where the gradient of the wall-normal profile is approximately 0.

Based on the threshold inequality of equation (2.41), the $\Delta_i/\eta_{K,\text{local}}$ ratios for $Ra \geq 2.0 \times 10^6$ indicate the resolution in the streamwise and spanwise directions are inadequate to resolve the Kolmogorov lengthscales since the $\Delta_x/\eta_{K,\text{local}}$ and $\Delta_y/\eta_{K,\text{local}}$ profiles are greater than 3 (shown as -- in figure 2.4). The wall-normal spacings, however, appear to be favourable as the ratio of $\Delta_z/\eta_{K,\text{local}}$ are less than 2 in general. Hence, I focused on the grid refinement in the streamwise and spanwise directions.

To satisfy the threshold requirement, the streamwise and spanwise resolutions for Run 1 are increased by first assuming an average $\Delta/\eta$ profile throughout the channel width. This assumption is reasonable since the dissipation, and hence Kolmogorov scale ($\eta_K$), is found to be relatively uniform throughout the channel except in the vicinity of the wall (Grötzbach (29)). In addition, the assumption serves as a trade-off in terms of computational resources since a fully-resolved simulation (i.e. $\Delta/\eta$ in vicinity of wall is less than 3) requires approximately double the number of nodes in the streamwise and spanwise directions, at the very least (an estimate from figure 2.4b). The average profile is then similar to the inequality of equation (2.41):

$$\frac{\Delta}{\eta_{K,\text{global}}} \leq 2.5$$

(2.42)

where the value, 2.5, is a conservative estimate of an empirical average value.

The required resolution, $n = L/\Delta$, is then

$$n \geq \frac{L}{2.5\eta}.$$  

(2.43)

Applying the inequality to the data from Run 1, I obtain the new resolution listed...
in table 2.1 for Run 2 with a trade-off in the computational domain, an attempt to optimise resource allocation. The new distribution profiles of \( \Delta/\eta_K \) are shown in figure 2.4b. From the figure, I obtained a more desirable result of the profiles which have average values of below 3, except in the regions close to the wall. For the purposes of the present study, I conclude that the simulation data is adequately resolved. Where applicable, I will present and interpret flow statistics by superimposing results from both Run 1 (coarser resolution, larger domain) and Run 2 (finer resolution, smaller domain) in order to have a complete picture of the flow.

In light of the works of Shishkina et al. (64) and Grossmann and Lohse (25), there are considerable areas for optimising the grid specifications for the present flow problem, especially in the determination of the number of grid points in the thermal and kinetic boundary layer. For more accurate statistics, particular attention should be paid to the mesh grading in the streamwise and spanwise directions close to the wall to account for the high dissipation regions. If the current meshing strategy is continued, a four-fold increase in resolution—arising from an estimated doubling of the streamwise and spanwise resolution (see figure 2.4b)—is required to fully resolve the near-wall regions.

### 2.3.6 Resolution based on global kinetic energy (KE) and thermal dissipation rates

For naturally convected flow between horizontal walls, i.e., Rayleigh-Bénard convection, the requirements for DNS resolution can be determined a priori (cf. Shishkina et al. (64)) by making use of the exact closed-form analytical relations for the global kinetic energy and thermal dissipation rates:

\[
\langle \varepsilon_u \rangle_{t,V,RB} = \frac{\nu^3}{16\beta h^4} (Nu - 1) Ra Pr^{-2}, \tag{2.44}
\]

\[
\langle \varepsilon_T \rangle_{t,V,RB} = \alpha \frac{\Delta T}{4h^2} Nu. \tag{2.45}
\]

Here, the channel half-width is defined as \( h \), for consistency with the present flow problem. The following steps illustrate the difference for the present flow.

The kinetic and thermal energy equations are first obtained from the conservative form of the momentum equation, equation (2.2), and the energy equation, equation...
(2.3), by multiplying with \( u_i \) and \( T \) respectively:

\[
\begin{align*}
  u_i \frac{\partial u_i}{\partial t} + u_i u_j \frac{\partial u_i}{\partial x_j} &= -\frac{u_i}{\rho_o} \frac{\partial p}{\partial x_i} + g_i \beta u_i (T - T_0) + \nu u_i \frac{\partial^2 u_i}{\partial x_j^2}, \\
  T \frac{\partial T}{\partial t} + u_j T \frac{\partial T}{\partial x_j} &= \alpha T \frac{\partial^2 T}{\partial x_j^2}.
\end{align*}
\]

(2.46) (2.47)

Now, if the time and volume-integral average of equations (2.46) and (2.47) are applied together with the boundary conditions of the flow problem, i.e., periodicity in the streamwise and spanwise directions and no-slip condition at the walls, the expressions for KE and thermal dissipation rates are obtained as follows:

\[
\begin{align*}
  \langle \varepsilon_u \rangle_{t,V} &= \frac{g \beta}{\Delta t \Delta V} \int_{t,V} u(T - T_{\text{ref}}) dV \, dt, \\
  \langle \varepsilon_T \rangle_{t,V} &= \frac{\Delta T}{4h^2} \frac{T}{\kappa}.
\end{align*}
\]

(2.48) (2.49)

The unclosed form of the global KE dissipation rate, \( \langle \varepsilon_u \rangle_{t,V} \), for this flow problem cannot be determined a priori as in equation (2.44). For the present study, the resolution used in the DNS for Run 1 was initially based on the study by Versteegh and Nieuwstadt (77), however, as have been shown in section 2.3.5, the simulation was not able to satisfactorily resolve the smallest scales on the order of Kolmogorov lengthscale. This resulted in a trial-and-error approach to obtain high-resolution mesh-sizes which are used in Run 2, which are subsequently confirmed by the analysis of \( \Delta \)-spacings to Kolmogorov scale, \( \eta_K \) which are less than 3 (figure 2.50). To facilitate benchmarking for required resolutions of future DNS studies for this flow problem, I present the global KE dissipation rate, \( \langle \varepsilon_u \rangle_{t,V} \), defined as

\[
\langle \varepsilon_u \rangle_{t,V} = \nu \left( \frac{\partial u_i}{\partial x_j} \frac{\partial u_i}{\partial x_j} \right)_{t,V}
\]

(2.50)

and also the computed values of the buoyancy integral, equation (2.48) for increasing \( Ra \) from Run 2\(^1\). The dissipation rates can be used for future DNS studies to derive the baseline resolution based on the Kolmogorov lengthscale, \( \eta \), from equation (2.43).

\(^1\)Dissipation rates from Run 1 are not reported as they have been found to be almost two times the order of magnitude compared to Run 2, and it has been previously shown that the resolutions used were not able to resolve the Kolmogorov lengthscale (see figure 2.4a)
These values are presented in figure 2.5, with the equation line

$$\frac{\langle \varepsilon_u \rangle_{t,V}}{(\nu^{3/4}/h)^4} = 0.03377 \, Ra^{1.362}$$

(2.51)

Figure 2.5: Trend of normalised global KE dissipation rates, $\langle \varepsilon_u \rangle_{t,V}$, with increasing $Ra$ for the highly-resolved DNS data, Run 2. Data shown are computed for the buoyancy-integral expression, equation (2.48) and the definition from equation (2.50).
Chapter 3

Visualisations

In this chapter, the structures in the flow which are visually observed using isocontour plots of temperature and velocity.

3.1 Temperature contour plots

The isocontours of temperature, $T$, are shown in figures 3.2 to 3.3 and are coloured from red (hottest at $T = \Delta T/2$) to blue (coldest, at $T = -\Delta T/2$). As a reference, the length scales for 100 buoyancy length scales and 100 viscous length scales, i.e., 100 $z^-$ units (blue bar) and 100 $z^+$ units (green bar), are included in the visualisations. For the current $Ra$ values, the buoyancy length scale is greater than the viscous length scale, and both scales reduce with increasing $Ra$.

In figure 3.1, the wall-normal ($x$-$z$) slices depict the variation of temperature across the channel-width. Closer to the hot wall on the left, the flow moves upwards and results in structures in the temperature isocontours which appear downward-tilted. Similar upward-tilted structures are likewise visible closer to the cold wall, where the flow moves downwards in the direction of gravity.

In figure 3.2, wall-parallel plots ($x$-$y$ slices) of temperature isocontours are taken at the locations of peak temperature fluctuation for the respective $Ra$. These locations have been determined to be relatively constant at $z_i \approx 3$ (see section 4.4) for all $Ra$ values. From the slices, it is observed that the cooler regions of the fluid, seen as yellow/white ‘puffs’ in the thermal field, decrease in size relative to the channel with increasing $Ra$ and are interstitially filled with hotter fluid (in red). The interstitial regions also appear to occasionally form streaks, which is evident in plot for $Ra \geq 2.0 \times 10^7$. This suggests that as $Ra$ increases, the temperature boundary layer starts
to show standard features of turbulent pressure driven boundary layers, i.e., streaks with spanwise length of $100^+$ units. These streaks appear to dominate the 'puffs' of cooler regions in the fluid at higher $Ra$. In general, the size of the 'puffs' appears to qualitatively scale much better with $l_i$ (‘×’ units) than with the viscous length scale, $\delta_v$ (‘+’ units). Thus, it can be concluded that $l_i = \left[ \frac{\alpha^3}{(g\beta|f_w|)^{1/4}} \right]$ is a suitable length scale for the present flow problem.

In the spanwise plots ($y$-$z$ slices) shown in figure 3.3, the structures forming in the near-wall vicinity appear to have no preferred orientation. Again, these structures are seen to decrease with increasing $Ra$. 
Figure 3.1: Isocontours of temperature in the $x$-$z$ plane for Run 2, at the channel mid-span, $L_y/2$. Upward flow on the left walls appear to result in downward tilted angles of the temperature field, and vice versa on the cold wall. In the near-wall vicinity, the thermal boundary layers—regions of contiguous red and blue—appear to decrease in thickness with increasing $Ra$. 
Figure 3.2: Isocontours of temperature in the $x$-$y$ plane for Run 2 (cont.), at the location of peak temperature fluctuations, $z^x \approx 3$. Cooler regions of the fluid are seen as yellow/white ‘puffs’, and decrease in size with increasing $Ra$. The interstitial regions are filled with regions of hotter fluid, and appear contiguous, seen in (d) the slice for $Ra = 2.0 \times 10^7$. Colour legend as per figure 3.1.
Figure 3.2: Isocontours of temperature in the $x$-$y$ plane for Run 2, at the location of peak temperature fluctuation, $z^* \approx 3$. Cooler regions of the fluid are seen as yellow/white 'puffs', and decrease in size with increasing $Ra$. The interstitial regions are filled with regions of hotter fluid, and appear contiguous, seen in (d) the slice for $Ra = 2.0 \times 10^7$. Colour legend as per figure 3.1.
Figure 3.3: Isocontours of temperature in the $y$-$z$ plane for Run 2, taken at mid-streamwise location, $L_x/2$. Structures are seen to form in the near-wall vicinity, but appear to have no preferred orientation. These structures appear decreasing in size with increasing $Ra$. Colour legend as per figure 3.1.
3.2 VELOCITY CONTOUR PLOTS

3.2.1 Streamwise velocity, $u$

The isocontours of streamwise velocity, $u$, are shown in figures 3.4 to 3.6 for increasing $Ra$ from Run 2. To investigate the possible existence of flow structures, I qualitatively define contiguous regions of positive (blue) and negative fluctuations (red) as the bounds of these structures.

In section 5.1.2.1, the dominant wavelength of $u'u'$ energy spectrogram was found to be relatively constant for all $Ra$ in both low and high resolution runs. These wavelengths—interpreted as the most energetic wavelengths in $u$—occur at the channel-centre and are found as a ratio of the respective domain sizes as:

$$\frac{\lambda_1}{L_x} \approx 0.5, \quad \text{and} \quad \frac{\lambda_2}{L_y} \approx 0.25.$$  

From these wavelengths, it can be inferred that, at the channel-centre, the streamwise velocity structures stretch half of the streamwise domain, $L_x$, and occur at quarter-spacings of the spanwise domain, $L_y$. This scaling result is rather surprising in the sense that these structures scale with the domain size, however, it is also possible that this scaling with $L_x$ and $L_y$ occurs because of the smaller domain-size used in Run 2. As such, this remains an open question. To assess these size estimates, the streamwise and spanwise wavelength values are overlayed as a scale on the wall-parallel velocity isocontours in figure 3.5.

When compared with the scales (shown in black lines of length $L_x/2$, spanning $L_y/4$), the large streamwise velocity structures appear to extend to approximately half of $L_x$, and on occasions, seemingly extend throughout the entire streamwise domain. The discrepancy of the latter is postulated to be influenced by domain-size effects which will be shown later (in section 5.1.2.1) to be non-trivial. The spanwise spacings of the structures are found to be approximately spaced at quarter intervals, agreeing with the predicted spacings above. Also, with increasing $Ra$, the sizes of the large structures (which stretch half of the streamwise domain) are found to remain approximately constant in size. Despite the discrepancy in the size of the large scale structures, there appears to be significant evidence pointing to a relatively fixed size of the large coherent structures of streamwise velocity fluctuation, at the channel-centre. Interestingly, these large streamwise structures have been similarly observed in the studies for turbulent Couette flow, for example Komminaho et al. (41), Papavassiliou and Hanratty (56), Tsukahara et al. (74) and Kitoh and Umeki (38). In these studies, the streaky
structures are reported as regions of high- and low-speed events and are reported to be sensitive to domain-size—similar to the findings of the present study.

In addition to the large coherent structures observed, there is a noticeable increase in smaller scales with increasing $Ra$. The small scales appear to cumulatively make up a large structure in the flow. In the wall-normal and spanwise slices shown in figures 3.4 and 3.6, it is clearly seen that the flow is separated into an upward-moving flow (in blue) in the left half of the channel closer to the hot wall, and vice versa (in red) in the right half. In figure 3.6, plume-like structures are observed to traverse from the hot to cold wall, and vice versa.
Figure 3.4: Isocontours of $u'u'$ for Run 2, at the channel mid-span $L_y/2$. $Ra$ values is increasing from left to right.
Figure 3.5: Isocontours of $u'u'$ in the $x$-$y$ plane for Run 2, at the channel-centre location. This location corresponds to peak $u'u'$ in the channel. Colour legend as per figure 3.4.
Figure 3.5: Isocontours of $u'u'$ in the $x$-$y$ plane for Run 2 (cont.), at the channel-centre location. This location corresponds to peak $u'u'$ in the channel. Colour legend as per figure 3.4.
Figure 3.6: Isocontours of $u'u'$ for Run 2, taken at mid-streamwise location, $L_x/2$. Ra values is increasing from left to right. Colour legend as per figure 3.4.
3.2. VELOCITY CONTOUR PLOTS

3.2.2 Spanwise velocity, $v$

The wall-parallel isocontours of spanwise velocity, $v$, are shown in figure 3.7. Using the same qualitative identification of the coherent structures, the isocontours are colour-graded from blue to red, indicating regions of positive and negative $v$, and are plotted at the locations of the inner and outer spectrogram peaks (see section 5.1.2.2).

From the velocity plots, large scale spanwise velocity structures are observed for the $Ra$ range. These structures appear to be oriented in a hatched pattern, where the hatching angle for the velocity plots at the inner peak are qualitatively greater than the angle at the outer peak. From the 2-dimensional (2D) autocorrelation plot (see figure 5.6), the inner and outer angles are determined to be relatively invariant for the present $Ra$ range and are estimated to be,

$$\theta_i \approx 54^\circ \quad (3.1)$$
$$\theta_o \approx 48^\circ. \quad (3.2)$$

The associated hatched patterns based on the inner and outer angles are drawn in the lower left quadrant of the isocontours, and appears to align well with the angles of the large scale velocity structures observed. Interestingly, similar tilted-angle observations have been previously reported by Pallares et al. (55) in their study of conditionally-averaged flow structures. In their study, the angles were obtained from the evolution of vortex structures and are reported to influence the local heat transfer rate at the hot wall. Coupled with the present observations of hatched spanwise velocity structures, this presents an interesting opportunity for future studies to investigate the influence of large scale structures on the heat transfer for the present flow problem.
Figure 3.7: Isocontours of $v'v'$ in the $x$-$y$ plane for Run 2, in the inner peak (IP, on left) and outer peaks (OP, on right). Corresponding inner and outer angles are $54^\circ$ and $48^\circ$. Regions of positive and negative fluctuations are coloured blue and red respectively.
Figure 3.7: Isocontours of $v'v'$ in the $x$-$y$ plane for Run 2 (cont.), in the inner peak (IP, on left) and outer peaks (OP, on right). Corresponding inner and outer angles are $54^\circ$ and $48^\circ$. Regions of positive and negative fluctuations are coloured blue and red respectively.
Figure 3.7: Isocontours of $v'v'$ in the $x$-$y$ plane for Run 2 (cont.), in the inner peak (IP, on left) and outer peaks (OP, on right). Corresponding inner and outer angles are $54^\circ$ and $48^\circ$. Regions of positive and negative fluctuations are coloured blue and red respectively.
Chapter 4

Scaling Laws and Wall Functions

The content of this chapter has been accepted for publication in the International Journal of Heat and Fluid Flow under the title “Turbulent natural convection scaling in vertical channel” (Ng et al. (52)), and has previously been presented at the 18th Australasian Fluid Mechanics Conference 2012 (Ng et al. (51)). Here, I include the analysis for the highest, recently-obtained data at $Ra = 1.0 \times 10^8$.

4.1 Introduction

Previous studies in the past, such as the study by George and Capp (23), have arrived to different conclusions on the choice of scales for velocity, temperature and length for the present flow problem. These studies generally begin from scaling arguments analogous to that used for pressure-drive flows. From these arguments, the studies propose different forms of wall functions for the flow parameters.

In this study, we compare the proposed scaling and wall functions by George and Capp (23), Hölling and Herwig (34), Shiri and George (63), Versteegh and Nieuwstadt (77) with the statistics of the present DNS data. The proposed forms of the wall functions from these studies are summarised in table 4.1.

4.2 Comparison of scaling analyses

We begin by defining the flow parameters used in the scaling arguments in literature (George and Capp (23)). Starting from the integration of equation (2.7), giving

$$\alpha \frac{dT}{dz} - \overline{w' T'} = -\frac{q_w}{\rho C_p} \equiv f_w,$$

(4.1)
which describes a characteristic heat flux constant, \( f_w \), equivalent to the wall heat flux, \( q_w \), flowing from left to right divided by density, \( \rho \), and specific heat, \( C_p \). From equation (4.1), it follows that \( f_w \) is independent of location and can be deduced as a characteristic parameter for describing the flow in the channel. We now describe the inner-outer scaling approach—adopted by the four studies—which defines an inner layer of the flow close to the wall and an outer layer at the core of the channel. From equations (2.6) and (2.7), the parameters \( \alpha, \nu, g\beta \) and \( h \), are used to form the necessary scales (cf. George and Capp (23)).

The respective scales proposed by the four studies are thus summarised in tables 4.2 and 4.3. From the comparison, we find that all three studies seemingly agree on the choice of length and inner temperature scales, however, the choices for velocity and outer temperature scales differ. These differences potentially arose, at the time, from limited availability of high \( Ra \) data to validate the scaling laws at the asymptotic limit.

In the following sections, we will first appraise the temperature scales which extend to a heat transfer law, and subsequently perform a straightforward comparison of the velocity scales.

### 4.3 Mean statistics

#### 4.3.1 Temperature scale

By virtue of adopting the two layer approach described in section 4.2, we argue that the outer temperature scale, \( T_o \), depends on the channel half-width, \( h \). Hence, we do not appraise the outer temperature scale proposed by Hölling and Herwig (34), and,
4.3. MEAN STATISTICS

**Inner scaling**

<table>
<thead>
<tr>
<th>Source</th>
<th>Scaling Equation</th>
<th>Inner Length Scale</th>
<th>Outer Length Scale</th>
</tr>
</thead>
<tbody>
<tr>
<td>Versteegh and Nieuwstadt (77)</td>
<td>$(g\beta</td>
<td>f_w</td>
<td>\alpha)^{1/4}$</td>
</tr>
<tr>
<td>Hölting and Herwig (34)</td>
<td>$(g\beta</td>
<td>f_w</td>
<td>\alpha)^{1/4} Pr^{-1}$</td>
</tr>
<tr>
<td>Shiri and George (63)</td>
<td>$(g\beta</td>
<td>f_w</td>
<td>h)^{1/3}$</td>
</tr>
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<td>f_w</td>
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</tr>
</tbody>
</table>

Table 4.2: Comparison of inner layer scales, differences highlighted in red.

**Outer scaling**

<table>
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</tr>
</tbody>
</table>

Table 4.3: Comparison of outer layer scales, differences highlighted in red.

From tables 4.2 and 4.3, the inner and outer temperature scales of interest are thus:

$$T_i = \left( \frac{|f_w|^3}{g\beta \alpha} \right)^{1/4}, \quad T_o = \left( \frac{|f_w|^2}{g\beta h} \right)^{1/3},$$

with the respective inner and outer length scales:

$$l_i = \left( \frac{\alpha^3}{g\beta |f_w|} \right)^{1/4}, \quad l_o = h.$$

Using the scalings above and the gradient-matching approach, the temperature wall
functions take the following power-law forms:

\[
\frac{T_h - T}{T_i} = -c_1 \left( \frac{z}{l_i} \right)^{-1/3} - c_2(Pr),
\]

(4.2)

\[
\frac{T - T_{ref}}{T_o} = c_1 \left( \frac{z}{l_o} \right)^{-1/3} + c_3,
\]

(4.3)

(cf. George and Capp (23), Shiri and George (63), Versteegh and Nieuwstadt (77)) where \(c_1, c_2(Pr)\) and \(c_3\) are constants to be empirically determined. As first argued by George and Capp (23), \(c_2(Pr)\) is a Prandtl number-dependent constant, however, since \(Pr\) is constant in this study, \(c_2(Pr)\) is assumed to be a constant. With the present data for higher \(Ra\) from Run 1 and 2, we attempt to determine the constants for the temperature wall functions based on the assumption that the \(-1/3\) form of the temperature wall functions are valid.

4.3.1.1 Inner temperature scale

By differentiating equation (4.2) and writing in non-dimensional notation (indicated by superscript \(\times\)), we obtain the mean temperature gradient

\[
\frac{dT_i^\times}{dz_i^\times} = \left( \frac{c_1}{3} \right) \left( z_i^\times \right)^{-4/3},
\]

(4.4)

where

\[
T_i^\times = \frac{T_h - T}{T_i} = \frac{T_h - T}{|f_w^3/(g\beta\alpha)|^{1/4}},
\]

\[
z_i^\times = \frac{z}{l_i} = \left[ \frac{\alpha^3/(g\beta|f_w|)}{1/4} \right].
\]

Using equations (4.2) and (4.4), we can define diagnostic quantities for the constants \(c_1\) and \(c_2\), similar to the approach by Moser et al. (50):

\[
c_1 = 3z_i^\times(4/3) \frac{dT_i^\times}{dz_i^\times},
\]

(4.5)

\[
c_2 = -3z_i^\times \frac{dT_i^\times}{dz_i^\times} - T_i^\times,
\]

(4.6)

Assuming the power-law form in section 4.3.1 is valid, equations (4.5) and (4.6) should converge to constant values—inherently, \(c_1\) and \(c_2\)—in some region of \(z_i^\times\). This diagnostic approach is further emphasised by plotting the mean quantities against
4.3. MEAN STATISTICS

$z_i^{x^{-1/3}}$, shown for example in figure 4.1. In this text, we refer to equations (4.5) and (4.6) as compensated profiles.

On inspection of the compensated inner profiles in figure 4.1a, we find that the profiles collapse between $0.6 \lesssim z_i^{x^{-1/3}} \lesssim 0.9$ (equivalently $1 \lesssim z_i^x \lesssim 5$), with the exception of two highest $Ra$, $Ra = 2.0 \times 10^7$ and $Ra = 1.0 \times 10^8$ which increases in magnitude, whilst the other profiles are decreasing. Despite this discrepancy, the collapse suggests that the inner temperature function, equation (4.2), can model the mean temperature profile of the flow in this $z_i^x$ region up to a certain extent. The uncertainty comes in from the fact that the compensated profiles consistently deviates in some apparent trend, which suggests a change in the mean profile with increasing $Ra$. In the following section, we find also that this region overlaps with the outer compensated temperature gradients, hence suggesting some similarity in the solution of the mean temperature profile.

As such, we determine the constants for the inner mean temperature equation as $c_1 \approx 4.2$ and $c_2 \approx -5.0$, which are the same constants obtained in the study by Versteegh and Nieuwstadt (77). The new inner temperature wall function based on the present DNS data is

$$\frac{T_h - T}{T_i} = -4.2 \left( \frac{z}{y_i} \right)^{-1/3} + 5.0.$$  \hspace{1cm} (4.7)

Since the constant, $c_1$, also appears in the outer mean temperature equation, equation (4.3), we extend the analysis of the compensated form of the mean temperature profile to the outer temperature wall function.
Figure 4.1: Compensated profiles from Runs 1 (-----) and 2 (------) to determine constants, $c_1$, $c_2$ and $c_3$. The estimated values of the constants are respectively $c_1$: '---' (4.2), $c_2$: '---' (-5.0), and $c_3$: '---' (-4.5). The collapsing region is shaded in grey.
4.3. MEAN STATISTICS

4.3.1.2 Outer temperature scale

Using the similar approach as described in section 4.3.1.1, the compensated equations for the outer region are

\[ c_1 = -3z_o^{(4/3)} \frac{dT_o^x}{dz_o^x}, \quad (4.8) \]
\[ c_3 = T_o^x + 3z_o^{(4/3)} \frac{dT_o^x}{dz_o^x}, \quad (4.9) \]

and plotting against the outer length scale, \( z_o^{x-1/3} = (z/h)^{-1/3} \), we obtain the compensated profiles shown in figure 4.1b.

Based on similarity of the solution, the inner overlap region is mapped to outer coordinates. The mapping resulted in a ‘converging’ region which coincides with the intermediate inflection points of the compensated profiles (peaks for \( c_1 \) and troughs for \( c_3 \)), and we find that the points move closer to the wall with increasing \( Ra \). Again, due to the diverging trend of the \( c_3 \) curve for \( Ra = 2.0 \times 10^7 \) and \( 1.0 \times 10^8 \), we omit the inflection point for this dataset and extrapolate the trend of the troughs, giving \( c_3 \approx -4.5 \). This value is lower than the value obtained by Versteegh and Nieuwstadt (77). At this stage, the diverging trends of the two highest \( Ra \) values is apparent in the \( c_3 \) curves, and it may be prudent to conservatively accept the estimation of the new constants.

The outer temperature wall function with new constants is then

\[ \frac{T - T_{\text{ref}}}{T_o} = 4.2 \left( \frac{z}{l_o} \right)^{-1/3} - 4.5. \quad (4.10) \]

Table 4.4 summarises the wall functions proposed to date, including this study. To appraise these wall functions, we qualitatively assess the respective fits to the present DNS data in both inner and outer scales, shown in figures 4.2a and 4.2b.

In the region near the wall, the inner temperature scale, \( T_i^x \) varies linearly with the inner length scale \( z_i^x \), observed in figure 4.2a for \( z_i^{x-1/3} \gtrsim 1.0 \) (\( z_i^x \lesssim 1.0 \)). The wall function from the present study, similar to that by Versteegh and Nieuwstadt (77), appears to model the temperature profile for lower \( Ra \) more accurately, while the wall function proposed by Hölling and Herwig (34) fits the temperature profile in outer region of the channel, between \( 0.4 \lesssim z_i^{x-1/3} \lesssim 0.5 \) (\( 8 \lesssim z_i^x \lesssim 16 \)). As \( Ra \) increases, it is apparent that the present wall function is unable to model the temperature profile for \( z_i^{x-1/3} \lesssim 0.6 \) (or \( z_i^x \gtrsim 4.6 \)), suggesting a limited applicability of the power-law form for
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<table>
<thead>
<tr>
<th></th>
<th>Inner scaling</th>
<th>Outer scaling</th>
</tr>
</thead>
<tbody>
<tr>
<td>Versteegh and Nieuwstadt (77)</td>
<td>$-4.2(z_i^x)^{-1/3} + 5.0$</td>
<td>$4.2(z_o^x)^{-1/3} - 5.0$</td>
</tr>
<tr>
<td>Hölling and Herwig (34)</td>
<td>$0.4 \log(z_i^x) + 1.9$</td>
<td>-</td>
</tr>
<tr>
<td>Shiri and George (63)</td>
<td>no constants proposed</td>
<td></td>
</tr>
<tr>
<td>George and Capp (23)</td>
<td>$-5.6(z_i^x)^{-1/3} + 4.2$</td>
<td>-</td>
</tr>
<tr>
<td>Present study</td>
<td>$-4.2(z_i^x)^{-1/3} + 5.0$</td>
<td>$4.2(z_o^x)^{-1/3} - 4.5$</td>
</tr>
</tbody>
</table>

Table 4.4: Summary of inner and outer temperature wall functions.

the mean temperature profile. Similarly, in the outer temperature plot in figure 4.2b, equation 4.3 from the present study and by Versteegh and Nieuwstadt (77) qualitatively matches the temperature profile at the channel-centre up to $Ra \approx 2.0 \times 10^7$. This model deviates considerably from the highest $Ra$, $Ra = 1.0 \times 10^8$. However, at present, there is no other findings which may explain the deviation.
Figure 4.2: DNS temperature data plotted in inner and outer scaling with evidence of adherence to power-law behaviour for $0.6 \lesssim z_i^{x-1/3} \lesssim 0.9$ (region shaded in grey) and $1.0 \lesssim z_o^{x-1/3} \lesssim 3.5$. Data from Run 1 is shown as ‘--’ and Run 2 as ‘- - - -’.
To further appraise the constants, we now attempt to validate the heat-transfer and \( Ra \) relationship derived from the equations (4.2) and (4.3) with the present simulation data.

### 4.3.2 The heat transfer law

The typical non-dimensional measure of heat transfer used in literature, for example Versteegh and Nieuwstadt (77), is the Nusselt number, \( Nu \), defined as

\[
Nu = \frac{|f_w| (2h)}{\Delta T \alpha},
\]

where \( f_w \) is the characteristic heat flux constant from equation (4.1), and \( \Delta T, \alpha \) are the same flow parameters defined earlier.

By matching equations (4.2) and (4.3), and introducing \( Pr \) and \( Ra \) into the expression, the heat transfer relationship with Rayleigh number can be expressed in the following implicit form

\[
Nu^{3/4} \left[ c_3 (Nu \cdot Ra \cdot Pr)^{-1/12} - c_2 \right] = \frac{1}{2} (Ra \cdot Pr)^{1/4}.
\]  
(4.11)

Equation (4.11) can be further simplified to an explicit form if we assume the term which is multiplied by \( c_3 \) to be small for the present \( Ra \) range,

\[
Nu = \left( \frac{-1}{2c_2} \right)^{4/3} (Ra \cdot Pr)^{1/3}.
\]  
(4.12)

This equation is in the similar form as the common one-third power expression for the \( Nu-Ra \) relationship found in literature, for example in the experimental studies of Tsuji and Nagano (72), Cheesewright (7), and Siebers et al. (65), and the numerical studies of Henkes and Hoogendoorn (31, 32) and Versteegh and Nieuwstadt (77). More recently, Kiš and Herwig (39) proposed a variation to equation (4.12) based on their DNS data: \( Nu \sim (Ra \cdot Pr)^{1/3.2} \).

Equivalent forms of equations (4.11) and (4.12) have also been proposed by George and Capp (23), Shiri and George (63) using the dimensionless group \( Ra^* = \frac{\beta |f_w| H^4}{\nu \alpha^2} \),

but they are not considered in this study.
4.3. MEAN STATISTICS

The proposed heat transfer laws and respective empirical constants are compared with the DNS data from the present study, as well as from Kiš and Herwig (39), Versteegh and Nieuwstadt (77) in figure 4.3. Of the six equations shown in figure 4.3, two underpredicted the \( Nu \) values from the data: the explicit \(-1/3\) equation (equation (4.12)) using the constants from the present study (—), and the same equation with constants from George and Capp (23) (—). From this observation, we venture that for the present \( Ra \) range, the \( c_3 (Nu \cdot Ra \cdot Pr)^{-1/12} \) term in equation (4.11) is non-negligible, as been previously noted by George and Capp (23). However, and as previous studies have emphasised, there is ample evidence of a \( Nu \sim (Ra \cdot Pr)^{1/3} \) relation and the constant can easily be deduced from curve-fitting approaches. As such, the conclusions here are two-fold: firstly, for the present \( Ra \) range, the \( Nu–Ra \) relationship may potentially be modelled ‘precisely’ by equation (4.11) with constants determined through the approach described in sections 4.3.1.1 and 4.3.1.2; secondly, a relatively more simple empirical one-third expression can also be obtained independently from the \( Nu–Ra \) dataset and is equivalently valid.
4.3.3 Contributions to the heat transfer law

It would be interesting to analyse the assumption for the simplified form of the heat transfer law, equation (4.12). Starting from the heat transfer equation:

\[
\frac{c_3 (Ra \cdot Pr)^{-1/12} Nu^{2/3}_I}{c_2 \cdot Nu^{3/4}_II} - \frac{c_3 (Ra \cdot Pr)^{-1/12} Nu^{2/3}_I}{c_2 \cdot Nu^{3/4}_II} = 0.5 (Ra \cdot Pr)^{1/4},
\]

the assumption, restated here, is that at sufficiently high \( Ra \), Term I can be neglected and thus simplifying equation (4.12) to the familiar one-third form of the heat transfer law. By analysing the individual contributions of the terms in equation (4.11), the relative importance of the terms can be investigated. The individual terms, normalised by the highest contributing term, Term II, is shown in figure 4.4.

![Figure 4.4: Normalised plot of the individual terms in the heat transfer law from Run 1 (×) and Run 2 (△). The contribution of Term I appears to decrease with increasing Ra.](image)

From figure 4.4, we note that the contribution of Term I appears to be converging slowly to 0. However, when compared with Term III, both contributions are still of \( O(-1) \) which again asserts the importance of Term I in equation (4.11) for the present \( Ra \) range. This suggest that Term I plays the role of a correction term for the heat transfer equation.
4.3.4 Velocity scale

Here, we directly appraise the two different inner velocity scales proposed (see table 4.2):

\[
\begin{align*}
    u_{i,h} &= (g\beta f_{w|h})^{1/3}, \\
    u_{i,\alpha} &= (g\beta f_{w|\alpha})^{1/4},
\end{align*}
\]

by scaling the present data with both velocity scales and plotting against the inner length scale, \( l_i \). The comparison of scales are shown in figure 4.5.

The key difference between the two velocity scales is the choice of the parameter \( h \) over \( \alpha \), the latter of which is proposed by Versteegh and Nieuwstadt (77) and Hölting and Herwig (34) based on the classical two-layer approach. Conversely, Shiri and George (63) argue that the buoyancy-induced flow away from the wall drives the velocity in the vicinity of the wall, hence the outer velocity scale, \( u_o = (g\beta f_{w|h})^{1/3} \), is applicable in the inner region and thus, \( u_{i,h} = u_o \). The velocity profiles are shown in figure 4.5a and figure 4.5b respectively.

The velocity profiles scaled by \( u_{i,h} \) (figure 4.5a) show a much better collapse for \( z^x \lesssim 1 \) compared to the profiles scaled by \( u_{i,\alpha} \) (figure 4.5b) where we observe systematic departure with increasing \( Ra \). This evidence of collapse with \( u_{i,h} \) supports the hypotheses of Shiri and George (63), and thus we propose the inner velocity scale, \( u_{i,h} = (g\beta f_{w|h})^{1/3} \) as a candidate scale for obtaining universality of velocity profiles for the present flow problem.
Figure 4.5: DNS streamwise velocity data showing: (a) collapse for \( z_i^{x-1/3} \gtrsim 1 \), when scaled with outer velocity scale, \( u_o = (g\beta|f_w|h)^{1/3} \), and: (b) systematic departure with increasing \( Ra \), when scaled with inner velocity scale, \( u_{i,\alpha} = (g\beta|f_w|\alpha)^{1/4} \). Region with power-law behaviour for inner temperature function is shaded in grey. Data from Run 1 is shown as ‘--’ and Run 2 as ‘- - - -’. 
4.3. MEAN STATISTICS

4.3.5 The relationship with shear velocity, $u_\tau$

In the asymptotic analysis of the inner velocity scale, Shiri and George (63) and George and Capp (23) hypothesised that the ratio of the wall shear velocity, $u_\tau$, to the velocity scale, $u_{i,h}$, approaches a constant as $Ra \to \infty$. With the present data and the data from Versteegh and Nieuwstadt (77) and Kiš and Herwig (39), the ratio of $u_\tau/u_{i,h}$ against $\log(Ra)$ is shown in figure 4.6.

From the figure, we note that $u_\tau/u_{i,h}$ appears to slowly exhibit a proportional relationship with increasing $Ra$, which suggests perhaps that the present $Ra$ is not sufficiently high—in this case, to test the hypothesis proposed by Shiri and George (63) and George and Capp (23).

![Figure 4.6: A plot of $u_\tau/u_{i,h}$ versus $\log(Ra)$ showing an incipient proportional relationship with increasing $Ra$. DNS data shown here are: ×, Run 1; △, Run 2; +, Versteegh and Nieuwstadt (77); and ○, Kiš and Herwig (39).](image-url)
CHAPTER 4. SCALING LAWS AND WALL FUNCTIONS

4.4 Turbulent statistics

In this section, we appraise the scaling for the Reynolds stresses, temperature fluctuations and the temperature flux for the present flow problem. The time and spatially-averaged Reynolds stresses, $u_i' u_i'$, are found to be maximum in the channel centre for all $Ra$, as shown for instance in figure 2.3c. The magnitude of streamwise velocity fluctuations, $u' u'$, is higher than the velocity fluctuations in the spanwise direction, $v' v'$, followed by wall-normal fluctuations, $w' w'$. The profile of Reynolds shear stress, $u' w'$, begins with a zero gradient at the wall, and drops slightly below zero before going to a positive maximum at the channel centre. For the temperature fluctuations, $T' T'$, the profiles consistently show a common peak at $z_i^* \cong 2$ for the present $Ra$ range. Finally, the temperature flux, $w' T'$ profile is appraised, and is shown to collapse when scaled with the characteristic heat flux constant, $f_w$.

4.4.1 Reynolds stresses

Beginning from the mean momentum equation, equation (2.6), a cursory analysis does not appear to immediately yield any basis for scaling arguments for the velocity variances. However, from the relatively successful collapse of the velocity profiles in section 4.3.4, we hypothesise that the velocity variances can also be scaled with the velocity scale, $u_{i,h}^2 = (g \beta |f_w| h)^{2/3}$. The scaled profiles of normal velocity variances from this study are shown in figure 4.7a, and compared with the DNS data from Versteegh and Nieuwstadt (77) and Kiš and Herwig (39).

Interestingly, all three profiles exhibit satisfactory collapse for $z_i^* \cong 0.8$ ($z_i^* \cong 2.0$) (see figure 4.7a), whereas the scaling with $u_i^2 = (g \beta |f_w| h)^{1/2}$ shows departing trends. This observation again gives some assurance that the velocity scale $u_{i,h} = (g \beta |f_w| h)^{1/3}$ is a suitable scale for this flow problem.
4.4. TURBULENT STATISTICS

Figure 4.7: Plot of normal velocity variances showing: (a) collapse of profiles when normalised by $u_{i,h}^2 = (g|\beta| f_w |h|)^{2/3}$, and; (b) systematic departure when normalised by $u_{i,\alpha}^2 = (g|\beta| f_w |\alpha|)^{1/2}$. DNS data shown are: , Run 1; , Run 2; Versteegh and Nieuwstadt (77); and , Kiš and Herwig (39). Region with power-law behaviour for inner temperature function is shaded in grey.
4.4.2 Reynolds shear stress

The scaling analysis is extended to the Reynolds shear stress profiles in a similar fashion as above and the results are shown in figure 4.8. Both plots show consistently diverging profiles, unlike the trends observed in the prior analyses for mean and normal velocity variances. Based on this finding, we conclude that at present, there appears to be no evidence that the velocity scale, \( u_{i,h} = (g\beta|f_w|h)^{1/3} \), can provide a universal expression for the Reynolds shear stress profiles.

4.4.3 Temperature variance

The temperature variances are scaled with the inner and outer temperature scales, \( T_i = \left( \frac{|f_w|^3}{g\beta\alpha} \right)^{1/2} \quad T_o = \left( \frac{|f_w|^2}{g\beta h} \right)^{2/3} \)

and the plots are shown in figure 4.9. When scaled with the inner scale, \( T_i \), (see figure 4.9b), the profiles collapse in the inner region for \( z_i^{x-1/3} \gtrsim 0.8 \) (or \( z_i^x \lesssim 2.0 \)) with increasing \( Ra \). This collapse is in contrast with the scaling with \( T_o \), shown in figure 4.9a, and supports the proposed inner temperature variance scale, \( T_i \) for \( z_i^x \lesssim 2.0 \). It is worth noting also that the present dataset does not show a collapse in the outer region, for which Versteegh and Nieuwstadt (77) reported scaling relationships at \( z_i^x \gtrsim 7.0 \).

4.4.4 Wall temperature flux

As has been argued by Versteegh and Nieuwstadt (77), the characteristic heat flux, \( f_w \) is constant in the channel and hence should be a choice candidate to scale the wall-normal heat flux, \( w'T' \). This is shown in figure 4.10, which shows a clear collapse of the profiles up to the channel centre and furthermore, validates the numerical solution of the flow in the region.
4.4. TURBULENT STATISTICS

Figure 4.8: Plots of Reynolds shear stress, $\overline{u'w'}$, both showing departing trends when normalised by (a) $u_i^2 = (g\beta|f_w|h)^{2/3}$, and; (b) $u_i^2 = (g\beta|f_w|\alpha)^{1/2}$. DNS data shown are: Run 1; Run 2; Versteegh and Nieuwstadt (77); and Kiš and Herwig (39). Region with power-law behaviour for inner temperature function is shaded in grey.
Increasing $Ra$

Figure 4.9: Plots of temperature fluctuations, $T'/T'$, showing: (a) departing trends when normalised by $T_o^2$, and; (b) collapse for $z_i^{-1/3} \geq 0.8$ when normalised by $T_i^2$. DNS data shown are: -- -- , Run 1; - - - , Run 2; - - - - , Versteegh and Nieuwstadt (77); and - - - - - , Ks and Herwig (39). Region with power-law behaviour for inner temperature function is shaded in grey.
Figure 4.10: Plot of temperature flux, \( w'T' \) showing collapse when scaled with the characteristic heat flux, \( f_w \). DNS data shown are: Run 1 (---) and Run 2 (—).
4.5 Chapter summary

Past studies have proposed ‘$-1/3$’ power-laws for wall functions, for instance, George and Capp (23) and Versteegh and Nieuwstadt (77). But, other studies, e.g. Hölling and Herwig (34), have instead argued for the existence of a logarithmic law. Of particular interest is the mean temperature equation, of which a matching of the ‘$-1/3$’ power-law form will lead to ‘$-1/3$’ heat transfer law. In this study, we show that the inner ‘$-1/3$’ temperature wall function (with constants obtained from the present high-Ra DNS data) predicts the mean temperature profile accurately between $1.3 \lesssim z_i^* \lesssim 4.6$. However, it is noted that the constants obtained are the same as those from Versteegh and Nieuwstadt (77), and there is evidence that these constants are not universal at $Ra > 2.0 \times 10^7$. The outer ‘$-1/3$’ temperature equation is found only to qualitatively predict the outer temperature profile for $0.02 \lesssim z_o^* \lesssim 1.0$. The associated heat transfer law derived from the present constants is able to predict the $Nu-Ra$ relationship up to $Ra = 1.0 \times 10^8$, which indicates that the implicit form of the heat transfer equation is equivalently valid as the prevalent explicit ‘$-1/3$’ form of the $Nu-Ra$ relationship used in industry. The explicit ‘$-1/3$’ $Nu-Ra$ equation based on the present constants is found to underpredict the relationship.

The outer velocity scale is shown to scale the mean streamwise velocity profile and the normal velocity variances in the inner region (figures 4.5a and 4.7a) However, scaling of the Reynolds shear stress using the inner and outer velocity scale was unsuccessful. This suggests a somewhat limited universality of the outer velocity scale.

With regards to the friction velocity $u_\tau$, we present evidence of an incipient proportionality relationship with the outer velocity scale, $u_{i,h}$, at high $Ra$ as suggested by Shiri and George (63) and George and Capp (23) (see figure 4.6). Interestingly, this implies a logarithmic relationship at sufficiently high $Ra$ (cf. Shiri and George (63)). However, the apparent gradual trend of the $u_\tau/u_{i,h}$ ratio indicates that the present $Ra$ range can be considered moderate at best, illustrating the need for data at higher $Ra$ to truly validate the scaling theories appraised in this chapter.

For temperature variance, there is evidence of collapse in the inner region for $z_i^* \lesssim 2.0$ at increasing $Ra$, when the inner temperature scale is used (see figure 4.9b). This is in contrast to the collapse in outer region ($z_i^* \gtrsim 7.0$) reported by Versteegh and Nieuwstadt (77). Thus, it is proposed that the temperature variance scales more reasonably with $T_i = [f_{i3}^2/(g\beta\alpha)]^{1/2}$ closer to the wall.
Chapter 5

Spectrum of the Naturally Convected Turbulence

This chapter describes the study of the velocity and temperature spectra for the naturally convected flow in a vertical channel. In particular, the 1D spectra is analysed in pre-multiplied form using the equations defined in section 2.2. The pre-multiplied spectra are further analysed as spectrograms to determine the variations of the energy peaks with \( Ra \), described as a function of wall-normal locations and wavenumber (ergo, wavelength).

5.1 Streamwise Energy Spectra

The 1D, streamwise spectra equations for the streamwise and spanwise velocity fluctuations are

\[
\frac{E_{11}(\kappa_1)}{u_{\eta K}^2 \eta K} = 0.245(\kappa_1 \eta K)^{-5/3},
\]

\[
\frac{E_{22}(\kappa_1)}{u_{\eta K}^2 \eta K} = 0.325(\kappa_1 \eta K)^{-5/3},
\]

(cf. section 2.2.1), with the following convention

\[
\overline{u'u'} = 2 \int_{0}^{\infty} E_{11}(\kappa_1) d\kappa_1.
\]  

At the channel-centre, the velocity fluctuations are found to be maximum (see section 4.4.1) and it is hypothesised that the 1D spectra at the channel-centre may contain
the universal equilibrium range—more specifically, the inertial subrange—where the energy exhibits the characteristic ‘$-5/3$’ decay, as described in the 1D spectra equations.

In addition, an analysis is carried out to study the changes in the spectra, before and after the location of the peak mean streamwise velocity, $\overline{u}_{\text{peak}}$, which were found to be $z_{i,\text{peak}}^x \approx 2.1, 2.4, 3.3, \text{ and } 4.9$ with increasing $Ra$. The non-dimensional 1D spectra are plotted at associated wall-normal locations of $z_i^x = 0.01, 0.5$ and at the channel-centre, shown in figure 5.1.

In this figure, the lighter curves refer to spectra plotted at $z_i^x = 0.01$ and 0.5 and the darker curves are associated with spectra at the channel-centre. The spectra for Run 1 (lower resolution, larger domain) and 2 (higher resolution, smaller domain) are shown as dashed and solid lines respectively. The ‘$-5/3$’ slope from the Kolmogorov equations (2.24) and (2.25) are also traced in both plots for reference. In both plots of spectra for $u'u'$ and $v'v'$, the spectra at wall-normal locations of $z_i^x = 0.01$ and 0.5 (i.e., before $z_{i,\text{peak}}^x$) are found to be in the lower energy range. However, when the spectra is plotted for increasing wall-normal locations, the curves collapse onto the ‘$-5/3$’ slope for the spectra of $u'u'$, whereas for $v'v'$, the slope appears shallower than ‘$-5/3$’. Both collapsing of curves are shown by the darker plots in figure 5.1. This collapse after $\overline{u}_{\text{peak}}$ suggests that the transition from low energy range to universal collapse appears to occur at the location of $\overline{u}_{\text{peak}}$. 
5.1. STREAMWISE ENERGY SPECTRA

Figure 5.1: Plots of 1D streamwise spectra for (a) $u'u'$, and (b) $v'v'$. The spectra at the channel-centre (dark curves) which are greater than $z_i^{\times}$ peak collapse onto the $'−5/3'$ slope. The colours associated with increasing $Ra$ are blue, green, red, dark red, and black. Run 1 (lower resolution, larger domain) is shown as dashed lines and Run 2 (higher resolution, smaller domain) are shown as solid lines.
5.1.1 Pre-multiplied streamwise and spanwise energy spectra

The non-dimensional 1D streamwise, pre-multiplied energy spectra for $u'u'$ and $v'v'$ takes the form

$$\kappa_1 E_{11}^{\times}(\kappa_1), \quad \text{and}$$

$$\kappa_1 E_{22}^{\times}(\kappa_1) \quad (5.2)$$

(cf. section 2.2.1.2) and are plotted respectively in figures 5.2 and 5.3 against normalised streamwise and spanwise wavelength coordinates, defined as $\lambda_1/h = 2\pi/(\kappa_1 h)$ and $\lambda_2/h = 2\pi/(\kappa_2 h)$. In the pre-multiplied form, the proportion of energy contributed by the wavelengths is readily obtained by computing the area under the spectra curve. Typically, it is desirable for the energy at very high and low wavenumbers to be small, which is indicative of a sufficiently fine resolution (low wavelengths) and large domain (large wavelengths). This mitigates any possible artifacts which cause a redistribution of the energy across the wavelengths and have been previously noted in past studies, for example, in the DNS pipe flow study by Chin (9). For the present study, the scaling behaviour of the energy spectra is also analysed by non-dimensionalising the spectra with inner velocity scale, $u_{i,\alpha} = (g\beta |f_w|\alpha)^{1/4}$, (figure 5.2a and 5.3a) and outer velocity scale, $u_o = (g\beta |f_w|h)^{1/3}$, (figure 5.2b and 5.3b). The inner and outer scales were previously appraised in section 4.2.

From the 1D streamwise, pre-multiplied energy spectra for $u'u'$ (see figure 5.2), it is found that for all $Ra$, the spectra of Run 2 (solid lines) at the highest wavelengths do not drop-off to a sufficiently small value. For Run 1 (dashed lines), the spectra at highest wavelengths drop-off considerably more as compared to Run 2, but the energy captured is still of comparable magnitude to the peaks in spectra. This lack of drop-off at the highest wavelengths appear to be attributed to the choice of streamwise domain length, and based on the present data, suggests that a streamwise domain size, $L_x > 24h$ is necessary to resolve the energy at the longest wavelengths. Due to the smaller computational domain in Run 2, the spectral peaks in Run 1 appear to shift to a lower wavelength range in Run 2 by approximately 20%. This again highlights the necessity to use a larger streamwise domain size to mitigate the redistribution of energy across wavelengths and in so doing, provide greater accuracy for higher order statistics.

In terms of scaling, the streamwise spectra of $u'u'$ scaled with the inner velocity scale, $u_{i,\alpha}$, are found to fan out with increasing $Ra$ (see figure 5.2a). In contrast, when scaled with the outer velocity scale, $u_o$, the spectra appear to cluster between $\lambda_1/h \gtrsim 2$
and \( \lambda_1/h \lesssim 4 \). Although the clustering indicates some form of scaling behaviour for the streamwise spectra of \( u'u' \), it is noted that the spectra is plotted for the channel-centre, which is in the outer region (as defined in chapter 4), whereas previously in section 4.4.1, there was evidence of successful scaling of the streamwise velocity fluctuations in the inner region (see figure 4.7a\(^1\)). As such, there appears to be no conclusive evidence that the outer velocity scale, \( u_o \), is a suitable scale for streamwise velocity fluctuation in the outer region.

In contrast, the streamwise pre-multiplied spectra for \( v'v' \), (figure 5.3) exhibit a significant energy drop-off from the spectral peaks to values close to zero—seen at the highest wavelengths and for both runs. Additionally, the spectral peaks appear to be relatively constant at \( \lambda_1/h \approx 3.5 \). Both findings for the drop-off and invarying spectral peaks indicate that the large-scale spanwise energy is captured by runs 1 and 2. This also suggests that the 1D spectra for \( v'v' \) is insensitive to the reduction in the streamwise domain-size, \( L_x \), from 24\( h \) in Run 1 to 16\( h \) in Run 2. However, due to the need to obtain resolved statistics in the 1D streamwise spectra for \( u'u' \), it is crucial to ensure that \( L_x > 24h \).

The scaling behaviour of the 1D pre-multiplied spectra of \( v'v' \) is now considered. When scaled with the inner velocity scale, \( u_{i,\alpha} = (g\beta|f_w|\alpha)^{1/4} \), the spectra consistently departs with increasing \( Ra \) (see figure 5.3a). However, when scaled with the outer velocity scale, \( u_o = (g\beta|f_w|h)^{1/3} \), the spectra exhibit collapse in the region between the spectral peak and highest wavelength range (see figure 5.3b). This scaling behaviour suggests that the outer velocity scale, \( u_o \), is a viable scale for 1D spectra of \( v'v' \) and appears to be consistent with the collapse of the profiles of \( v'v' \) observed in figure 4.7a in the inner and outer regions.

\(^1\)In figure 4.7a, the profiles of \( u'u' \) scaled with the outer velocity scale, \( u_o \), exhibit departures in the outer region.
Figure 5.2: Plots of 1D streamwise, pre-multiplied energy spectra for $u'u'$ at channel-centre, scaled with (a) inner velocity scale, $u_{i,\alpha} = (g\beta|f_w|\alpha)^{1/4}$ and (b) outer velocity scale, $u_o = (g\beta|f_w|h)^{1/3}$. Run 1 (lower resolution, larger domain) is shown as dashed lines and Run 2 (higher resolution, smaller domain) are shown as solid lines.
5.1. STREAMWISE ENERGY SPECTRA

Figure 5.3: Plots of 1D streamwise, pre-multiplied energy spectra for $v'v'$ at channel-centre, scaled with (a) inner velocity scale, $u_{i,\alpha} = (g\beta|f_w|\alpha)^{1/4}$ and (b) outer velocity scale, $u_o = (g\beta|f_w|h)^{1/3}$. Run 1 (low resolution, larger domain) is shown as dashed lines and Run 2 (higher resolution, smaller domain) are shown as solid lines.
CHAPTER 5. SPECTRUM OF THE NATURALLY CONVECTED TURBULENCE

5.1.2 Spectrograms of Velocity Fluctuations

The pre-multiplied spectra is plotted against inner wall-normal coordinates, \( z^* = z/\left[\alpha^3/(g\beta|f_w|)\right]^{1/4} \) (see section 4.2), giving isocontours with a definitive peak, for instance in figure 5.4. These so-called “energetic” peaks correspond to certain wavelengths which are interpreted as the most energetic wavelengths. In the subsequent sections, I report characteristic wavelengths in the velocity spectrogram of \( u'u' \), \( v'v' \) and \( w'w' \)—found to generally depend on the outer length scale, \( h \)—and relate the wavelengths to the structures observed in the isocontours of velocity fluctuations at the associated locations (chapter 3).

The spectrograms generally exhibit a peak in the channel-centre, which are consistent with the profiles of averaged velocity fluctuations shown previously in figure 4.7a. For the spectrograms of \( v'v' \), a second inner peak (close to the wall) is present at a relative constant wall-normal location of \( z^*_i \approx 7 \).

For brevity, only the spectrograms of \( Ra = 2 \times 10^7 \) for both runs will be shown in the following analyses—spectral peaks are drawn as thick, green (Run 1) and red (Run 2) markers, whereas the remaining \( Ra \) cases are indicated with thin markers.

5.1.2.1 Spectrograms for \( u'u' \)

The streamwise and spanwise spectrogram of \( u'u' \) for \( Ra = 2 \times 10^7 \) are shown in figure 5.4 where a single peak is evident at the channel-centre for both plots.

For the streamwise spectrogram, the corresponding peak wavelength is constant for Run 1 at \( \lambda_1/h \approx 12 \), however for Run 2, the peak wavelength appears to vary between \( \lambda_1/h \approx 8 \) and \( \lambda_1/h \approx 16 \) with increasing \( Ra \). This variation can be explained from the previous plots of 1D pre-multiplied spectra in figure 5.2 which did not sufficiently drop-off from the spectral peak at the highest wavelengths. The variation also appears to be an artifact of the reduced streamwise domain size, \( L_x \), for Run 2 and indicates that energy was not captured at longer wavelengths in the simulations. In turn, it causes a redistribution of energy across the wavelengths, which is reflected in the downward shift of the spectral peak to a lower wavelength value. In the spanwise spectrogram, the peak wavelengths for Run 1 occur at \( \lambda_2/h \approx 3 \) whereas the peaks for Run 2 occurs at \( \lambda_2/h \approx 2.7 \). This drop in spanwise wavelength value appears to again be an artifact of the reduced spanwise size in the computational domain, which is noticeable in the spanwise spectrogram as contractions of the isocontours from Run 1 to Run 2 (seen as ‘early’ closing of the isocontours at the highest wavelengths). The contraction resulted in a downward shift of the peak wavelength values.

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5.1. STREAMWISE ENERGY SPECTRA

Figure 5.4: Spectrograms of $u'u'$ for $Ra = 2.0 \times 10^7$ in (a) streamwise, and (b) spanwise direction, normalised by the outer velocity scale, $u_o = (g\beta|f_w|h)^{1/3}$. The dot-dash contours represent data from Run 1, and the solid contours are from Run 2. Contour values start from 0.15 at the outermost, with increments of 0.15. Spectral peaks are shown in green (Run 1) and red (Run 2), thick markers represent spectral peaks for $Ra = 2.0 \times 10^7$. 

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If the contraction of isocontours is ignored and the peak wavelength values of Run 1 is considered, the relatively constant values of $\lambda_1/h$ and $\lambda_2/h$ suggest some sort of scaling behaviour of the large-scale structures—inferred from these “energetic” wavelengths. To test this hypothesis, the peak wavelength values of Run 1 are scaled with the streamwise and spanwise domain sizes of $L_x = 24h$ and $L_y = 12h$ respectively, giving the ratios

$$\frac{\lambda_1}{L_x} = 0.5,$$

$$\frac{\lambda_2}{L_y} = 0.25.$$

The streamwise ratio of $\lambda_1/L_x = 0.5$, suggests that the most energetic structures of $u'u'$ will stretch across half of the streamwise channel domain, whereas the spanwise ratio of $\lambda_2/L_y = 0.25$ suggests that the structures occur at quarterly intervals in the spanwise direction. To visualise this, wall-parallel isocontours of the streamwise velocity fluctuations at the channel-centre are plotted in figure 3.5, p. 40. In the figure, positive streamwise fluctuations are coloured as blue, whereas negative fluctuations are coloured red. From the figure, I find regions of contiguous positive and negative fluctuations which stretch in the streamwise direction for approximately half of $L_x$, whereas the spanwise spacings of these regions are ordered approximately quarter of $L_y$. Both observations agree with the ratios found for the energetic streamwise and spanwise wavelengths, and suggest the existence of long structures in the channel-centre. At present, the physics of these structures have not been explored, but are noted to be strikingly similar to the streaky streamwise velocity structures reported in literature for plane Couette flows, for example, by Komminaho et al. (41), Papavassiliou and Hanratty (56), Tsukahara et al. (74) and Kitoh and Umeki (38).
5.1. STREAMWISE ENERGY SPECTRA

5.1.2.2 Spectrograms for $v'v'$

In a similar manner to the analysis for the spectrograms of $u'u'$, the streamwise and spanwise spectrograms of $v'v'$ (see figure 5.5) are found to contain a second inner peak close to the wall, while the outer peak is found to occur at the channel-centre. The occurrence of both peaks agree with the averaged profiles of $v'v'$ (see figure 4.7) where 2 inflection points are evident with increasing $Ra$. In the spectrograms, the inner and outer peaks are labeled as $\times$ and $+$ respectively.

From both streamwise and spanwise spectrograms, I find that the inner peak occurs at $z_i \approx 7$ and at wavelengths $\lambda_1/h \approx 4$ and $\lambda_2/h \approx 3$, whereas the outer peak occurs at the channel-centre for the same wavelengths. Only one spectral peak is visible in both streamwise and spanwise spectrogram for the lowest $Ra$ of Run 2 although 2 peaks are present for the same $Ra$ in Run 1. It is speculated that for the lowest $Ra$, the effects of periodicity of the streamwise and spanwise wavelengths of $v'v'$ become significant and result in artificial saturation in the simulation by the energetic wavelengths. This is somewhat similar to the “contamination” reported by Chin et al. (8) in the DNS study of pipe flow for a short streamwise pipe-length. A simulation using a larger domain size for the lowest $Ra$ may shed light on the discrepancy noted. Despite the distortions in the spectrograms for same contour levels, the inner and outer peak wavelengths as well as the wall-normal location of the inner peak appear to be relatively constant with increasing $Ra$. These findings suggest that, in the inner region, the peak wavelength can be ascribed to a characteristic structure in the spanwise velocity field at a fixed wall-normal location and is invariant for the present domain-size as well as $Ra$ range.
Figure 5.5: Spectrograms of $v'v'$ for $Ra = 2.0 \times 10^7$ for (a) streamwise direction, and (b) spanwise direction, normalised by the outer velocity scale, $u_o = (g\beta f_w |h|)^{1/3}$. The dot-dash contours represent data from Run 1, and the solid contours are from Run 2. Contour values start from 0.1 at the outermost, with 0.1 increments. For both spectrograms, an inner peak ($\times$) is present $z_i^* \approx 7$ at wavelengths, $\lambda_1/h \approx 4$ and $\lambda_2/h \approx 3$. 

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5.1. STREAMWISE ENERGY SPECTRA

To characterize the structures in the spanwise velocity field, I define the aspect ratio of the structures for the respective inner and outer locations as:

\[
\frac{\lambda_{1,i}}{\lambda_{2,i}}, \quad \text{and} \quad \frac{\lambda_{1,o}}{\lambda_{2,o}}
\]  

(5.4)

The inner and outer aspect ratios are computed for both runs to determine the variation with increasing \( Ra \), and are summarised in tables 5.1 and 5.2.

<table>
<thead>
<tr>
<th></th>
<th>R1.1</th>
<th>R1.2</th>
<th>R1.3</th>
<th>R1.4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \lambda_{1,i}/\lambda_{2,i} )</td>
<td>1.200</td>
<td>1.200</td>
<td>1.600</td>
<td>1.600</td>
</tr>
<tr>
<td>( \lambda_{1,o}/\lambda_{2,o} )</td>
<td>1.143</td>
<td>1.143</td>
<td>1.000</td>
<td>1.000</td>
</tr>
</tbody>
</table>

Table 5.1: Run 1 – aspect ratio for structures in \( v'v' \) at the inner and outer spectrogram peaks. For notation of simulation runs, refer to table 2.1.

<table>
<thead>
<tr>
<th></th>
<th>R2.1</th>
<th>R2.2</th>
<th>R2.3</th>
<th>R2.4</th>
<th>R2.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \lambda_{1,i}/\lambda_{2,i} )</td>
<td>n.a.</td>
<td>1.500</td>
<td>1.200</td>
<td>1.500</td>
<td>1.500</td>
</tr>
<tr>
<td>( \lambda_{1,o}/\lambda_{2,o} )</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.200</td>
<td>1.500</td>
</tr>
</tbody>
</table>

Table 5.2: Run 2 – aspect ratio for structures in \( v'v' \) at the inner and outer spectrogram peaks. For notation of simulation runs, refer to table 2.1.

From both tables, the inner aspect ratios are found to be greater than the outer aspect ratios for the current \( Ra \) range on a qualitative sense. The average of the aspect ratios are found to be,

\[
\frac{\lambda_{1,i}}{\lambda_{2,i}} \approx 1.413
\]  

(5.5)

\[
\frac{\lambda_{1,o}}{\lambda_{2,o}} \approx 1.110.
\]  

(5.6)

The data from equations (5.5) and (5.6) suggest that the aspect ratio of the structures is closer to 1 in the outer peak. In the inner peak, the number suggests that the structures are longer in the streamwise than the spanwise direction. Both numbers agree qualitatively with the visual analysis of the spanwise velocity fluctuations in figure 3.7, p. 44. Interestingly, the visual data in figure 3.7 exhibits large inclined structures at both inner and outer locations. Since these structures appear organised, a 2D autocorrelation is performed to investigate the orientation of the spanwise velocity structures.
2D autocorrelations of $v'v'$

The autocorrelation results for $Ra = 2.0 \times 10^7$ from Run 2 is shown below in figure 5.6.

![2D autocorrelation of $v'v'$ (R2.4) at the inner peak (a) and outer peak (b).](image)

Figure 5.6: 2D autocorrelation of $v'v'$ (R2.4) at the inner peak (a) and outer peak (b). The plots show rectangular isocontours with inclination angles of approximately $54^\circ$ and $48^\circ$ respectively, corresponding to the inclined large-scale structures in the isocontour plots of $v'v'$ (figure 3.7).

From the autocorrelation plot, the shape of the highly-correlated contour levels appear rectangular, and at the lowest contour levels, the isocontours are tapered at a diagonal. These observations suggest that the spanwise velocity structures are weakly correlated at a diagonal and can be interpreted as oriented at an incline in the inner and outer peaks. On a qualitative basis, the inner inclination angles appear to be greater than the outer inclination angles. The average for both runs in the present $Ra$ range are approximately,

$$\theta_i \approx 54^\circ$$
$$\theta_o \approx 48^\circ$$
From these results, there appears to be some evidence of hatched orientation in the spanwise velocity fluctuations at the inner peak, $z_i^c \approx 7$, and at the channel-centre. These hatched orientations appear to be relatively constant for the present $Ra$ range: at the inner peak, the hatching angle, $\theta_i$ is approximately $54^\circ$ and in the channel-centre, $\theta_o$ is approximately $48^\circ$. In addition, the aspect ratios of the structures—computed from the spectral peaks—are found to be higher in the inner peak ($\lambda_{1,i}/\lambda_{2,i} \approx 1.413$) than at the channel-centre ($\lambda_{1,o}/\lambda_{2,o} \approx 1.110$).
5.1.2.3 Spectrograms for $w'w'$

Lastly, the streamwise and spanwise spectrograms of $w'w'$ for $Ra = 2.0 \times 10^7$ are shown in figure 5.7. Both spectrograms exhibit a single peak which occurs at the channel-centre and at $\lambda_1/h \approx \lambda_2/h \approx 2$. Again, the location of the spectral peak is found to agree with the averaged profiles of the wall-normal velocity fluctuations as shown previously in figure 4.7. Compared to the spectrograms of $u'u'$ and $v'v'$, the spectrogram contours of $w'w'$ appear to be less affected by the smaller computational domain used in Run 2. In general, the shapes of spectrograms for $w'w'$ from both runs are found to be very similar, which suggest that the contribution from $w'w'$ to the turbulent kinetic energy is less affected by the reduction in computational domain-size. Also, it becomes apparent that future simulations should place greater emphasis on resolving the spectra for $u'u'$ and $v'v'$ in order to sufficiently resolve the large scale energy of the flow.

The wavelength ratios are calculated again, as before, and the results are shown in tables 5.3 and 5.4. From the tables, I find that the streamwise ratios in Run 2 are generally higher than that of Run 1 by approximately 34%, while the spanwise ratios in Run 2 are higher than that of Run 1 by approximately 64%. At present, I am unable to relate the peak wavelength values to any observed structures of the wall-normal fluctuations.

<table>
<thead>
<tr>
<th>$Ra$</th>
<th>Runs 1</th>
<th>Runs 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$5.4 \times 10^5$</td>
<td>0.1250</td>
<td>0.2000</td>
</tr>
<tr>
<td>$2.0 \times 10^6$</td>
<td>0.1250</td>
<td>0.2000</td>
</tr>
<tr>
<td>$5.0 \times 10^6$</td>
<td>0.0833</td>
<td>0.1000</td>
</tr>
<tr>
<td>$2.0 \times 10^7$</td>
<td>0.1111</td>
<td>0.1000</td>
</tr>
<tr>
<td>$1.0 \times 10^8$</td>
<td>n.a.</td>
<td>0.1429</td>
</tr>
<tr>
<td>average</td>
<td>0.1111</td>
<td>0.1486</td>
</tr>
</tbody>
</table>

Table 5.3: Streamwise wavelength ratios $\lambda_{i,x}^h/L_x^h$ for R1 and R2.

<table>
<thead>
<tr>
<th>$Ra$</th>
<th>Runs 1</th>
<th>Runs 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$5.4 \times 10^5$</td>
<td>0.1429</td>
<td>0.2000</td>
</tr>
<tr>
<td>$2.0 \times 10^6$</td>
<td>0.1250</td>
<td>0.2000</td>
</tr>
<tr>
<td>$5.0 \times 10^6$</td>
<td>0.1429</td>
<td>0.2500</td>
</tr>
<tr>
<td>$2.0 \times 10^7$</td>
<td>0.1250</td>
<td>0.2000</td>
</tr>
<tr>
<td>$1.0 \times 10^8$</td>
<td>n.a.</td>
<td>0.2500</td>
</tr>
<tr>
<td>average</td>
<td>0.1340</td>
<td>0.2200</td>
</tr>
</tbody>
</table>

Table 5.4: Spanwise wavelength ratios $\lambda_{i,y}^h/L_y^h$ for R1 and R2.
Figure 5.7: Spectrograms of $w'w'$ for $Ra = 2.0 \times 10^7$ for (a) streamwise direction, and (b) spanwise direction, normalised by $u_o = (g\beta|f_w|h)^{1/3}$. The dot-dash contours represent data from Run 1, and the solid contours are from Run 2. Contour values start from 0.05 at the outermost, with 0.05 increments.
CHAPTER 5. SPECTRUM OF THE NATURALLY CONVECTED TURBULENCE

5.2 Spectra of Temperature Variance

5.2.1 1D temperature spectra

From section 2.2.2, the non-dimensional temperature spectra in the inertial-convective range can be expressed as:

$$E_\theta(\kappa) = B_1 (\kappa \eta_K)^{-5/3}.$$

I now repeat the analysis of the trend in the spectra curves at the wall-normal locations $z/l_i = z_i^\kappa = 0.01, 0.5$ and at the channel-centre. The result is shown in figure 5.8.

From the plot, both streamwise and spanwise spectra of temperature are found to collapse when plotted at wall-normal locations which are greater than the location of $\bar{u}_{\text{peak}}$—similar to the analysis of the 1D streamwise and spanwise velocity spectra. The curves also exhibit a $-5/3$ region which extends to approximately one decade at the highest $Ra$. There are several papers which propose values for the one-dimensional constant $B_1$ for the streamwise temperature spectra equation (e.g. Hill (33)), however, it is not the intention of this study to estimate this constant. At present, it is sufficient to say that at the channel-centre, the temperature spectra exhibits an inertial-convective region in the streamwise and spanwise spectra which spans approximately one decade at the mid-range wavenumbers.
Figure 5.8: 1D spectra of temperature fluctuations at $z_i^x = 0.01, 0.5$ and the channel-centre, for (a) streamwise, and (b) spanwise directions. The spectra at channel-centre (dark curves) which are greater than $z_i^x$ peak collapse onto the $-5/3$ slope of the inertial-convective subrange, and extend approximately one decade. The colours associated with increasing $Ra$ are blue, green, red, dark red, and black. Run 1 (lower resolution, larger domain) is shown as dashed lines and Run 2 (higher resolution, smaller domain) is shown as solid lines.
5.2.2 1D pre-multiplied temperature spectra

The peaks of the temperature fluctuations, $T'T'$, have been determined in section 4.4 to occur at $z_i^* \approx 2.0$. At this location, the intensity of temperature fluctuations is expected to be the greatest so it is natural to analyse the pre-multiplied temperature spectra at this location. Using equation (2.33) and the normalised wavelength coordinates, $\lambda_1/h = 2\pi/(\kappa_1 h)$ and $\lambda_2/h = 2\pi/(\kappa_2 h)$, the spectra is plotted and shown in figure 5.9.

From the figure, the spectra curves are observed to drop-off at both low and high wavelengths, with a more rapid drop-off rate in the lower wavelengths. As compared to the pre-multiplied energy spectra curves in section 5.1.1, the drop-offs at the higher wavelengths are more significant: the spectra for $Ra = 1.0 \times 10^8$ (black curve) at the highest wavelength is approximately 60% to 70% (in the streamwise and spanwise spectra) from the peak in the spectra compared to the absence of a drop-off in the 1D streamwise velocity spectra in figure 5.2b. These findings indicate that both simulation runs are able to resolve the ‘energy’ in the temperature fluctuations at both low and high wavelengths, which is desirable. At higher wavelength values and after the peak in the streamwise spectra, there is a noticeable collapse of the profiles with the exception of the highest $Ra$. In the spanwise spectra, the profiles appear to consistently fan out after the peak location. The deviation of the spectra at the highest $Ra$ may be attributed as an artifact of the reduced computational domain size, albeit not as significant as the spectra for the streamwise velocity fluctuations (figure 5.2). In the next section, the deviation is again observed as contractions of the isocontours in the spectrograms of temperature, at the high and low wavelength range.
5.2. SPECTRA OF TEMPERATURE VARIANCE

Figure 5.9: Plots of 1D pre-multiplied temperature spectra along (a) streamwise, and (b) spanwise direction, at the location of peak temperature fluctuation, $z_i^* \approx 2.0$. Spectra for Run 1 is shown as ---, and Run 2 as ===. Except for the highest $Ra$, the spectra appear to collapse after the peak wavelengths.
5.2.3 1D spectrograms of temperature fluctuations

The streamwise and spanwise spectrogram of temperature for $Ra = 2.0 \times 10^7$ is shown in figure 5.10 for runs 1 and 2. To highlight the spectral peaks, the same marker notation from section 5.1.2 is used.

Both runs show similar distributions in the spectrograms despite the cut-off at higher wavelengths for Run 2, attributed to the smaller computational domain. There is also a slight contraction of the isocontours for Run 2 at the low and high wavelength similar to the velocity spectrograms. Despite the influence of reduced domain-size in Run 2, the temperature spectrograms appear to be adequately resolved at both lowest and highest wavelengths for the present $Ra$ range. This suggests that the computational parameters of the present flow problem (for air at $Pr = 0.709$) are relatively more dependent on the resolution and domain-size requirements of the velocity flow field and less on the temperature field.

For all $Ra$ values, the spectrograms contain a single peak at $z^* \approx 2.0$ coinciding with the location of the peak temperature fluctuations (see figure 4.9). At the peak, the normalised wavelengths for the streamwise and spanwise spectrograms, $\lambda_1/h$ and $\lambda_2/h$, are of the order of approximately 2. On a qualitative basis, the peak wavelengths in the streamwise spectrogram appear to move to the low wavelength range with increasing $Ra$. However, it is inconclusive whether this trend is caused by the smaller computational domain size used. The peak wavelength in the spanwise spectrogram appears relatively constant.

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5.2. SPECTRA OF TEMPERATURE VARIANCE

Figure 5.10: Temperature spectrogram for $Ra = 2.0 \times 10^7$ in (a) streamwise, and (b) spanwise directions. Isocontours for Run 1 are shown as '---' and for Run 2, as '-----'. The contourlines begin from the outermost at 0.2 with increments of 0.2.
5.3 Chapter summary

The velocity spectra for natural convection in a vertical, differentially heated channel differs to that of typical pressure-driven flow problems: the peaks in the velocity spectrograms occur at the channel-centre, coinciding with the peak in velocity fluctuations. In the region between the peaks of mean streamwise velocity, \( \pm \tau_{\text{peak}} \), the 1D velocity spectra collapses onto the ‘\(-5/3\)’ slope in an inertial subrange governed by the Kolmogorov scale, \( \eta_K \). From the analysis of 1D pre-multiplied spectra, I find that that the small scales, or small wavelengths, are adequately resolved. However, the energy at the highest wavelengths for the streamwise spectra of \( u'u' \) were not adequately captured by the simulation since there remains significant energy at the longest possible wavelengths captured by the simulation domain (see figure 5.2). The streamwise spectra of \( v'v' \) were found to be adequately resolved at high wavelengths. These findings suggest that the streamwise domain-size, \( L_x \), should ideally be greater than \( 24h \) in order to capture the large-scale energy in the flow, particularly for the streamwise velocity. Due to the smaller domain-size used for Run 2, the 1D spectra are found to be contracted and appear to redistribute energy across the wavelengths. This is especially evident in the velocity spectrograms, figures 5.4, 5.5 and 5.7, where contractions in the isocontours are visible and appear to cause the spectral peaks to shift to a lower wavelength range.

The peak wavelengths in the spectrograms of \( u'u' \) are found to be constant at \( \lambda_1/h \approx 12 \) and \( \lambda_2/h \approx 3 \) (for Run 1). When scaled with the domain size, the energetic wavelengths are found to stretch across half of \( L_x \) and are spaced at quartered intervals of \( L_y \). These are found to correspond to structures in the wall-parallel isocontour plots of streamwise velocity fluctuations at the channel-centre (see figure 3.5). From the spectrograms of \( v'v' \), a second inner peak (close to the wall) is present for all \( Ra \) at a relatively constant location of \( z_i^+ \approx 7 \). Using the peak wavelength values of the inner and outer peaks, I obtain two average aspect ratios, \( \lambda_{1,i}/\lambda_{2,i} \approx 1.413 \) and \( \lambda_{1,o}/\lambda_{2,o} \approx 1.110 \) which indicate that the structures at the location of the inner peak are slightly elongated in the streamwise direction compared to the spanwise velocity structures at the channel-centre. From 2D autocorrelation, the spanwise velocity structures were found to be organised in a hatched configuration, with the inner hatching angle, \( \theta_i \approx 54^\circ \) and the outer, \( \theta_o \approx 48^\circ \). These angles correspond well with the inclined structures observed in figure 3.2.2.

For temperature, the 1D spectra also collapse onto the ‘\(-5/3\)’ slope for approximately one decade, when analysed at the wall-normal locations between \( \pm \tau_{\text{peak}} \). The streamwise and spanwise temperature spectrograms exhibit one peak at \( z_i^+ \approx 2.0 \), which
coincide with the location of peak temperature fluctuations. From both spectrograms and 1D pre-multiplied temperature spectra, I find that the energy at the lowest and highest wavelengths are resolved by the simulation, evidenced by the peak drop-off of approximately 60% to 70% (see figure 5.9). At present, no relation was found between the spectrogram peaks for $w'w'$ and $T'T'$ and any observed structures.
CHAPTER 5. SPECTRUM OF THE NATURALLY CONVECTED TURBULENCE
Chapter 6

Conclusions

The present study reports DNS results for the naturally convected flow in a vertical channel for $Ra$ between $5.4 \times 10^6$ to $1.0 \times 10^8$, driven by a fixed temperature difference between the channel walls. This $Ra$ range have been determined to be adequate to achieve a large scale separation, measured by the criterion $h/l_i > 10$, which is higher than in past studies. This criterion is the ratio of the channel half-width to inner length scale (see table 6.1). Using the present $Ra$ data, I investigated two general areas of interest for the flow problem: first, the scaling laws and wall functions for the flow, and second, the establishment of a robust relationship between heat transfer, $Nu$, and $Ra$. The first investigation appraises the scaling laws in lieu of the differing opinions of the form of the wall-functions (power-law or logarithmic law) from past literature, while the second appraises the $Nu-Ra$ relationship arising from the derived wall-functions. In addition to the two areas, the turbulent statistics from the present DNS study were also analysed in the form of spectra and correlations. From the spectra, the most energetic wavelengths were identified and corresponded to large-scale structures found in the associated regions of the flow. This suggests the possibility of new physics in the very high $Ra$ regime. Throughout the study, two sets of DNS data—Run 1 and 2—are used, the first with a larger domain size of $L_x \times L_y \times L_z = 24h \times 12h \times 2h$ and the second with $16h \times 8h \times 2h$. For Run 2, higher resolutions are used to ensure the flow is resolved of the order of the Kolmogorov scales (see table 2.1 for summary of the simulation parameters).
6.1 Scaling laws and wall functions

In the first investigation, I appraised the scaling laws from four past literatures: Versteegh and Nieuwstadt (77), Hølling and Herwig (34), Shiri and George (63), and George and Capp (23). This investigation found evidence that provides some support for the ‘$-1/3$’ power law for the mean inner and outer temperature wall function in an overlap region (see figure 4.2, p. 55) based on the temperature and length scale proposed by George and Capp (23). The constants for the temperature wall functions are determined using the compensated temperature gradient profiles (figure 4.1, p. 52) for both the inner and outer scaling. However, it was found that the compensated profiles start to diverge in the overlap region from $Ra \geq 2.0 \times 10^7$, which suggests a limited similarity solution for the mean temperature profile. In contrast to the power-law, the logarithmic form is found only to be applicable in the outer region of the flow.

In the analysis of the temperature variances, I find evidence of scaling with the inner length and temperature scale: the maximum temperature variances occurs at the same location of $z_i^\infty \approx 2$ for the $Ra$ range studied, and the profiles collapse with increasing $Ra$ (see figure 4.9, p. 66). This is in contrast to the findings reported by Versteegh and Nieuwstadt (77) of a collapse only in the outer region.

6.2 Heat transfer relation

A matching of the inner and outer temperature wall functions produces the heat transfer equation which is found to contain a non-negligible correction term (see equation (4.11), p. 56) for the present $Ra$ range\(^1\). This correction term gives rise to an implicit form for the heat transfer relation which slightly underpredicts the heat transfer relationship between $Ra = 5.4 \times 10^5 \approx 4\%$ to $Ra = 1.0 \times 10^8 \approx 13\%$. Despite the inaccuracy at high $Ra$, the scaling arguments underlying the implicit equation appear to provide some theoretical support for obtaining a robust $Nu-Ra$ equation. If the asymptotic limit is considered (also from Versteegh and Nieuwstadt (77)), the implicit equation simplifies to the prevalent explicit power-law expression,

$$Nu = \gamma (Ra Pr)^p,$$

\(^1\)This has been previously highlighted by George and Capp (23)
where $p = 1/3$ from the present scaling analysis and $\gamma$ is a function of the inner wall function constant. This explicit equation is also shown to remain accurate when the constants in literature (e.g. Versteegh and Nieuwstadt (77)) are used, whereas if the constants from this study are applied, the equation underpredicts quite significantly albeit reducing with increasing $Ra$: at $Ra = 5.4 \times 10^5$ by approximately 37%, and at $Ra = 1.0 \times 10^8$ by approximately 30%

### 6.3 Outer velocity scale

Using the outer velocity scale, the mean streamwise velocity, $\overline{u}$, and normal velocity fluctuations, $u'_i u'_i$, are found to collapse in the near-wall region for $z^+ \lesssim 1$ (see figure 4.5a, p. 60 and figure 4.7a, p. 63). This collapse is in support of the scaling theory\(^1\) proposed by Shiri and George (63), who postulated that the buoyancy-induced flow away from the wall drives the velocity in the near-wall region. In contrast, the Reynolds shear stress profiles show departing trends when scaled with the same outer velocity scale. This leads to a conclusion that the velocity scale has limited universality and implicates a more detailed study of the velocity scaling for the naturally convected flow, crucial especially when deriving wall functions to be used in CFD simulations.

In summary, the appraised scales with evidence of scaling behaviour are:

<table>
<thead>
<tr>
<th>Inner scaling</th>
<th>Outer scaling</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u_i$</td>
<td>$u_o$</td>
</tr>
<tr>
<td>$(g\beta</td>
<td>f_w</td>
</tr>
<tr>
<td>$T_i$</td>
<td>$T_o$</td>
</tr>
<tr>
<td>$(</td>
<td>f_w</td>
</tr>
<tr>
<td>$l_i$</td>
<td>$l_o$</td>
</tr>
<tr>
<td>$(\alpha^3</td>
<td>g\beta f_w</td>
</tr>
</tbody>
</table>

Table 6.1: Summary of appraised scales with evidence of scaling behaviour.

In the study of friction velocity ($u_r$) trend with Rayleigh number, I find evidence of an incipient proportional relationship with the outer velocity scale at high $Ra$ values (see figure 4.6, p. 61). This relationship has been suggested by Shiri and George (63) and George and Capp (23), and implies a logarithmic relationship at very high $Ra$ (cf. Shiri and George (63)). However, the apparent gradual trend of the $u_r/u_{i,h}$ ratio indicates that the present $Ra$ range can be considered moderate at best, perhaps illustrating the

\(^1\)Interestingly, the use of this outer velocity scale has been suggested previously in past studies on planetary boundary layers (e.g. Deardorff (15, 16)).
need for data at even higher $Ra$ to truly validate the scaling theories considered in this thesis.

6.4 Spectra, domain-size effects and flow structures

With the available DNS data, the spectral content of the flow field has also been investigated. Both the streamwise 1D spectra for streamwise and spanwise velocity fluctuations, $u'u'$ and $v'v'$, as well as temperature fluctuations, $T'T'$, are found to collapse onto the $-5/3$ slope in the region enveloped between the maximum and minimum mean streamwise velocity, $\pm \overline{u}_{\text{peak}}$, (see figure 5.1, p. 71 and figure 5.8, p. 87). To determine the peak contributing wavelengths, the pre-multiplied spectra is analysed for both velocity and temperature. The streamwise 1D energy spectra is found to be susceptible to domain-size effects as the energy at large wavelengths are not captured in the simulations (seen as truncation of the spectra in figure 5.2). The 1D pre-multiplied spectra for $v'v'$ and $T'T'$ appear less affected by domain-size effects (see figures 5.3 and 5.9), although when comparing the larger-domain, lower-resolution simulation of Run 1 to the smaller-domain, higher-resolution simulation of Run 2, the peaks increase in magnitude and are shifted to the lower wavelengths. These streamwise and spanwise domain-size effects are more evident in the spectrogram analyses (see figures 5.4, 5.5, 5.7 and 5.10). As such, it is concluded that in order to resolve the large-scales of the flow, a proposed minimum domain size of the following aspect ratio, $L_x \times L_y = 24h \times 12h$ would be necessary for high $Ra$ simulations.

Despite the domain-size effects, the energetic wavelengths are found to be relatively constant, in the same order of magnitude, and correspond well with the large-scale structures observed in isocontour plots of velocities at the associated wall-normal locations (see chapter 3). For streamwise velocity, the energetic wavelengths stretch across half of the streamwise domain and are spaced at quarterly intervals in the spanwise direction (see figure 3.5, p. 40). In the spanwise velocity, hatching patterns are identified at two locations—termed the inner (near-wall) and outer (channel-centre) locations—and correspond to the angles determined in the 2D autocorrelation diagrams of the spanwise velocities at these locations (see figure 5.6, p. 82). These inner and outer angles are found respectively to be $\theta_i \approx 54^\circ$ and $\theta_o \approx 48^\circ$ and appear to be relatively constant for the present $Ra$ range.
6.5 Future work

6.5.1 Scaling theories

Based on the varying degrees of success in scaling of the quantities, there are opportunities for exploring other scaling theories. Of significant interest is the Grossmann-Lohse theory, termed the GL theory in literature (Grossmann and Lohse (25, 26, 27, 28)), which considers the global-average thermal and kinetic energy dissipation rates, the kinetic and thermal boundary layer thicknesses, and that the boundary layer flow is “...considered to be scalingwise laminar Prandtl-Blasius flow...” (cf. Shishkina et al. (64). It may be useful to apply an analogous theory to the present flow problem, and possibly shed light on the $Nu-Ra$ heat transfer relationship for vertical natural convection. However, a difference worth noting is that the kinetic energy dissipation rate for this present flow cannot be determined a priori (see chapter 2.3.6), as compared to the Rayleigh-Bénard convection problem for which the GL theory is centred upon. In addition, Prandtl number effects can also be studied for the flow problem and may be of particular interest in engineering applications where a different working fluid is used, for instance in the cooling of nuclear reactors.

6.5.2 Numerical

The domain-size effects have been proven significant to resolve the physics of large-scale structures at the high $Ra$ regime. This presents a two-fold need for a sufficiently high $Ra$ data, and for a domain-size which is at the minimum aspect ratio of $L_x \times L_y = 24h \times 12h$.

6.5.3 Experimental

To verify the DNS results, which are more often complementary in nature, experiments may also carried out. This is also in light of the advancement of new experimental techniques, for instance in Particle Image Velocimetry (PIV), in order to accurately capture higher order statistics, and would give better insight into the large-scale physics at the channel-centre.

6.5.4 Passive flow control

With the available higher order statistics and understanding of the spectra and correlation, there are opportunities to implement flow control to obtain the desired heat
transfer across the channel. For instance, the introduction of spanwise baffles to limit
the streamwise flow development may change the large scale physics of the flow and
perhaps lead to desirable heat transfer conditions. In essence, these baffles function as
large wavelength filters based on the energetic wavelengths determined from the flow
spectra. This may be numerically studied in the same notion as have been done in
the artificial removal of turbulent motions in the outer region via the use of minimal
spanwise computational domain, e.g., by Hwang (35). This can be an interesting area
to explore especially for the building industry where façade insulation is important to
regulate heat gain and losses in the building.
Bibliography


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