

# FEOLGIT: A new command for estimating fixed effects ordered logit models

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**Abstract.** This paper describes how to estimate panel data ordered logit models with fixed effects using the new community-contributed Stata command `feologit`. Fixed effects models are increasingly popular for estimating causal effects in the social sciences, since they control for unobserved time invariant heterogeneity in a flexible way. The ordered logit model is the standard model for ordered dependent variables and this command is the first in Stata specifically for this model with fixed effects. The command includes a choice between two estimators, the BUC estimator introduced in Baetschmann, Staub, and Winkelmann [2015] and the BUC- $\tau$  estimator in Baetschmann (2012). Baetschmann et al. (2015) showed that the BUC estimator has good properties and is almost as efficient as more complex estimators such as generalized method-of-moments and empirical likelihood estimators. The command and model interpretations are illustrated with an analysis of the effect of parenthood on life satisfaction using data from the German Socio-Economic Panel.

**Keywords:** `st0001`, `feologit`, panel data, ordered dependent variables, logistic models, fixed effects.

## 1 Introduction

While originating in the biometrics literature, regression models for ordered responses are now ubiquitous in the social sciences (Boes and Winkelmann 2006). One factor contributing to the widespread use of ordered responses is that Likert-type scales are the default way in which individual, household and firm surveys collect information on issues which are otherwise difficult to measure, such as attitudes and beliefs. By far the most common cross-sectional regression models for ordered responses are the ordered logit and ordered probit models. When analysing ordinal panel data, researchers are frequently interested in applying extensions of these models that somehow account for the longitudinal nature of the data. The simplest approach, and the one which we consider in this paper, is specifying an additional unobservable individual-specific error term. Under the assumptions that this error term is normally distributed and

independent of the regressors, the models are known as the random effects ordered logit or random effects ordered probit (see, e.g., Cameron and Trivedi 2005), and implemented in Stata with the commands `xtologit` and `xtoprobit`, respectively.

Often, however, these distributional and independence assumptions on the individual-specific error term are undesirable. Fixed effects models relax them: the distribution of the individual-specific error term and its dependence on the regressors are left completely unrestricted (cf., e.g., Wooldridge 2002). This feature of fixed effects models is useful for the estimation of causal effects since it accounts for any potential endogeneity stemming from time-invariant characteristics. But because no default approach for estimating fixed effects models for ordered responses exist, researchers are often faced with the choice of either estimating linear models, which are often inappropriate for ordinal data, or else estimating random effects ordered logit or probit models, which impose the strong assumptions mentioned above.

Because a fixed effects estimator exists for the *binary* logit model, several different estimators for fixed effects ordered logit models can be obtained using the binary logit model as a building block: the ordinal response variable can be transformed into binary responses, which then can be used for estimation and combined back in different ways to provide a single set of estimates. For the ordered probit model, in contrast, a similar approach is infeasible as no fixed effects estimator for the binary probit model exists. Baetschmann, Staub, and Winkelmann [2015] studied several approaches available for the fixed effects ordered logit model and showed that the so-called BUC (“blow-up and cluster”) estimator has good properties and is almost as efficient as more complex estimators such as generalized method-of-moments (GMM) and empirical likelihood (EL) estimators. In this paper, we discuss the BUC estimator as well as a more restricted version of it that makes it possible to estimate additional model parameters—the BUC- $\tau$  estimator introduced in Baetschmann [2012]—, and show how these estimators can be implemented in Stata using the community-contributed command `feologit`.

An integral part of our discussion focusses on the various potential objects of interest in this model, such as marginal effects, odds ratios, etc., and on whether they can or cannot be estimated, and if so, how. We further introduce a new specification test of the more restrictive assumptions relating to the additional threshold parameters estimated by BUC- $\tau$ . The test is simple to implement and we show in an example using Stata syntax how BUC and BUC- $\tau$  estimates can be used to this end. Finally, we illustrate the use of `feologit` and the interpretation of the estimates in an application of the effect of motherhood on women’s life satisfaction which uses data from the German Socio-Economic Panel (SOEP).

The paper reviews the fixed effects ordered logit model and the BUC and BUC- $\tau$  estimators in the next section. The syntax for `feologit` is presented in Section 3. Next, Section 4 gives a guide on how to interpret estimates, and Section 5 exemplifies the use of `feologit` and possible interpretations of the estimates in the application to life satisfaction, and provides a test of the BUC versus BUC- $\tau$  estimates. Section 6 offers some concluding remarks.

## 2 Fixed effects ordered logit models

The fixed effects ordered logit model uses the latent variable  $y^*$  to relate the observable characteristics  $x$  to the observable ordered dependent variable  $y$ , which can take values  $1, \dots, K$ . The latent variable  $y_{it}^*$  for individual  $i$  at time  $t$  depends linearly on  $x_{it}$  and the two unobservable characteristics  $\alpha_i$  and  $\varepsilon_{it}$ :

$$y_{it}^* = x_{it}'\beta + \alpha_i + \varepsilon_{it}, \quad i = 1, \dots, N \quad t = 1, \dots, T. \quad (1)$$

The vector of covariates  $x_{it}$  does not include an intercept as the  $\alpha_i$  act as individual-specific intercepts. We use a balanced panel for notational simplicity; but extending the model to imbalanced panels ( $t = 1, \dots, T_i$ ) is trivial, and the application in Section 5 uses such an imbalanced panel. The time-invariant, individual-specific part of the unobservables ( $\alpha_i$ ) is called the fixed effect, and can statistically depend on  $x_{it}$ . The following observation rule ties the latent variable  $y_{it}^*$  to the observed ordered variable  $y_{it}$  through the thresholds  $\tau_{ik}$ :

$$y_{it} = k \quad \text{if} \quad \tau_{ik} < y_{it}^* \leq \tau_{ik+1}, \quad k = 1, \dots, K. \quad (2)$$

In the most flexible version of the model, the thresholds can vary between individuals, as indicated by the subscript  $i$  in  $\tau_{ik}$ . Beside the stipulation of the lowest and highest thresholds as plus and minus infinity, the only assumption about the individual-specific thresholds is that they are increasing for each person:

$$\tau_{i1} = -\infty; \quad -\infty < \tau_{ik} < \tau_{ik+1} < \infty, \quad \forall k = 2, \dots, K-1; \quad \tau_{iK+1} = \infty. \quad (3)$$

Moreover, the fixed effects ordered logit model assumes that the time-varying unobservable terms,  $\varepsilon_{it}$ , are i.i.d distributed with standard logistic cumulative density function (cdf), hence the name of the model:

$$F(\varepsilon_{it}|x_{it}, \alpha_i) = F(\varepsilon_{it}) = \frac{1}{1 + \exp(-\varepsilon_{it})} \equiv \Lambda(\varepsilon_{it}). \quad (4)$$

The probability of observing outcome  $k$  for individual  $i$  at time  $t$  is therefore

$$\Pr(y_{it} = k|x_{it}, \alpha_i) = \Lambda(\tau_{ik+1} - x_{it}'\beta - \alpha_i) - \Lambda(\tau_{ik} - x_{it}'\beta - \alpha_i). \quad (5)$$

This probability depends on  $x_{it}$  and  $\beta$ , the parameter of primary interest. However, it also depends on  $\alpha_i$ ,  $\tau_{ik}$  and  $\tau_{ik+1}$ . As can be seen from (5), without further assumptions on the thresholds, only  $\tau_{ik} - \alpha_i \equiv \alpha_{ik}$  is identified as we can always define  $\tilde{\tau}_{ik} = \tau_{ik} + \eta$  and  $\tilde{\alpha}_i = \alpha_i - \eta$  for any  $\eta \in \mathbb{R}$ .<sup>1</sup>

1. On a more fundamental level, the parameters  $\beta$  and  $\alpha_{ik}$  are identified only by the normalizing assumption that  $\varepsilon_{it}$  follows a *standard* (rather than any) logistic distribution, which fixes  $\varepsilon_{it}$ 's variance. Without this normalisation, only  $\beta/\text{var}(\varepsilon_{it})$  and  $\alpha_{ik}/\text{var}(\varepsilon_{it})$  are identified. Thus, identification and consistent estimation in this paper refer to identification and consistency *up to scale*, as is the case for most standard latent variable models, such as logit, probit, and their ordered and multinomial generalizations.

Direct estimation of  $\alpha_{ik}$  is difficult. Generally, estimation of  $\alpha_{ik}$  uses only information of the  $T$  observations of individual  $i$ . Thus, if the time dimension  $T$  is fixed—as is generally assumed in short panels—there are only a finite number of observations to estimate  $\alpha_{ik}$  even when the total number of observations  $NT$  grows to infinity. As a consequence, the fixed effects  $\alpha_{ik}$  cannot be estimated consistently and, in general, their inconsistency spills over to inconsistency of  $\beta$ , the parameters common to all observations. This situation is known as the incidental parameters problem (Neyman and Scott 1948; Lancaster 2000). In short panels, the resulting bias in  $\hat{\beta}$  can be substantial (Abrevaya 1997; Greene 2004). A consistent estimator of  $\beta$  can be obtained by collapsing  $y_{it}$  into a binary variable and then applying the well-known conditional maximum likelihood (CML) estimator (Andersen 1970; Chamberlain 1980).

## 2.1 CML Estimator

The conditional maximum likelihood (CML) estimator is well known. But since the BUC estimator for the fixed effects ordered logit model is based on the CML we present it in some detail to fix notation. In Stata, this estimator is implemented in the command `clogit` and in the panel data command `xtlogit` with option `fe`, which relies on `clogit`. Similarly, `feologit` also relies on `clogit`.

Let  $d_{it}^k$  denote the binary variable that results from dichotomizing the ordered variable at the cutoff point  $k$ :  $d_{it}^k = \mathbb{1}(y_{it} \geq k)$ . This is the dependent variable of the CML estimator. Let

$$g_i^k = \sum_{t=1}^T d_{it}^k$$

be the observed number of ones of the dependent variable for individual  $i$ . Now consider the probability of observing  $d_i^k = (d_{i1}^k, \dots, d_{iT}^k)'$  conditional on observing  $g_i^k$  ones. It can be shown that this probability is

$$\mathcal{P}_i^k(\beta) \equiv \Pr \left( d_i^k \middle| \sum_{t=1}^T d_{it}^k = g_i^k \right) = \frac{\exp(d_i^{k'} x_i \beta)}{\sum_{j \in B_i} \exp(j' x_i \beta)}, \quad (6)$$

where  $j$  denotes a vector of dimension  $T$  with each element  $j_t$  equal to 0 or 1 and with  $\sum_{t=1}^T j_t = g_i^k$ . Further,  $B_i$  denotes the set of all possible  $j$ -vectors with  $g_i^k$  ones and  $T - g_i^k$  zeros. There are  $\binom{T}{g_i^k} = \frac{T!}{g_i^k!(T-g_i^k)!}$  such combinations. Crucially, this probability (6) does not depend on  $\alpha_i$  and the thresholds. Chamberlain (1980) showed that maximizing the conditional log likelihood

$$LL^k(b) = \sum_{i=1}^N \log \mathcal{P}_i^k(b) \quad (7)$$

results in a consistent estimator for  $\beta$ . Therefore,  $\beta$  of the fixed effects ordered logit model can be estimated by first dichotomizing the ordered dependent variable into a binary one and then applying the standard conditional maximum likelihood estimator.

However, different cutoff points  $k$  can be used, and using only one of them leads to loss of information and, therefore, to inefficiency. Further details of the CML estimator such as first order conditions and asymptotic variance can be found, e.g., in Baetschmann et al. (2015).

## 2.2 BUC estimator

Several ideas exist to combine the information of the CML estimators obtained from dichotomizing samples at different cutoff points (see Baetschmann et al. 2015). The BUC estimator presented here combines the log likelihood functions resulting from different cutoff points, leading to a one-step estimator of  $\beta$ . The log likelihood function for this estimator is

$$LL^{BUC}(b) = \sum_{k=2}^K LL^k(b), \quad (8)$$

where  $LL^k(b)$  is defined as in (7), and the BUC estimator is the one which maximizes (8). It can also be regarded as a restricted CML estimator, since it imposes the restriction that  $\hat{\beta}^2 = \dots = \hat{\beta}^K$ . We call this the “blow-up and cluster” (BUC) estimator since this describes how the estimator is implemented: first, every individual’s observations in the sample are replaced with  $K - 1$  copies or clones of itself (“*blow-up*” the sample size); and, then, each clone is dichotomized at a different cutoff point. We then use the entire inflated sample to estimate  $\beta$  by applying the CML estimator. Since the clones of the same individual are not independent of each other, we have to compute standard errors that are *clustered* at the individual level. Baetschmann et al. (2015) have shown that combining the likelihoods leads to a large efficiency gain compared to using only one cutoff. In addition, the BUC estimator has less convergence problems compared to efficient estimators like a two-step generalized method-of-moments (GMM) estimator and the efficiency loss in finite samples is negligible.

## 2.3 BUC- $\tau$ estimator assuming constant thresholds

The standard ordered logit model for cross sectional data assumes that the thresholds are constant across individuals. The BUC estimator is conformable with a more general class of models, since it is also consistent with models where each individual has different thresholds. If we are willing to make the additional assumption of constant thresholds, Baetschmann (2012) suggested a procedure based on the BUC estimator which allows us to estimate the thresholds, too. We will call this estimator BUC- $\tau$ . The additional assumption for the more restrictive model is that  $\tau_{ik} = \tau_{jk} = \tau_k$  for all individuals  $i$  and  $j$ . Since the rest of the model is unchanged, the probability of observing outcome  $k$  for individual  $i$  at time  $t$  is

$$\Pr(y_{it} = k | x_{it}, \alpha_i) = \Lambda(\tau_{k+1} - x'_{it}\beta - \alpha_i) - \Lambda(\tau_k - x'_{it}\beta - \alpha_i). \quad (9)$$

As in the more flexible model described above, we cannot distinguish between terms which are constant within an individual. So if all thresholds ( $\tau$ ) increase by the same

amount as the individual fixed effect ( $\alpha$ ) the same probability results. We deal with this underidentification by restricting the second threshold to 0. Assumption (3) of the model therefore changes to

$$\tau_1 = -\infty; \quad \tau_2 = 0; \quad 0 < \tau_k < \tau_{k+1} < \infty, \quad k = 3, \dots, K-1; \quad \tau_{K+1} = \infty. \quad (10)$$

Without the restriction  $\tau_2 = 0$  only the differences between the thresholds are identified.

The idea of the BUC- $\tau$  estimator is to dichotomize the observations within a person at different cutoff points and then to apply the standard CML estimator. This allows us to estimate the thresholds, too. Let  $d_i$  denote the resulting vector of the dichotomized dependent variable for individual  $i$  and  $g_i$  the number of ones in  $d_i$ . In addition, define  $\tau_i^{cut}$  as the vector of thresholds used as cutoff points for person  $i$ . The conditional probability of observing  $d_i$  conditional on  $g_i$ , when  $d_i$  results from dichotomizing at different cutoff points is

$$\Pr \left( d_i \middle| \sum_{t=1}^T d_{it} = g_i \right) = \frac{\exp(d_i'(x_i\beta - \tau_i^{cut}))}{\sum_{j \in B_i} \exp(j'(x_i\beta - \tau_i^{cut}))}, \quad (11)$$

where  $j$  denotes again a vector with zeros and ones with  $\sum_{t=1}^T d_{it} = g_i$ , and  $B_i$  the set of all possible  $j$ -vectors with  $g_i$  ones and  $T - g_i$  zeros.

As an example, consider a person who is observed for two time periods. For the BUC estimator, we would produce  $K - 1$  copies of this person's observations—that is,  $K - 1$  clones of the person—and dichotomize each clone at a different cutoff point. Thus, one of these clones, say  $i$ , would for instance be dichotomized at the cutoff point 3, resulting in  $d_i = (\mathbb{1}(y_{i1} \geq 3), \mathbb{1}(y_{i2} \geq 3))'$  with corresponding  $\tau_i^{cut} = (\tau_3, \tau_3)'$ . The next clone,  $j$ , dichotomized at 4, would result in  $d_j = (\mathbb{1}(y_{j1} \geq 4), \mathbb{1}(y_{j2} \geq 4))'$  with corresponding  $\tau_j^{cut} = (\tau_4, \tau_4)'$ .

In contrast, for the BUC- $\tau$  estimator, the first observation of clone  $i$  might be dichotomized at the cutoff point 3 and the second observation at the cutoff point 4, resulting in the vectors  $d_i = (\mathbb{1}(y_{i1} \geq 3), \mathbb{1}(y_{i2} \geq 4))'$  and  $\tau_i^{cut} = (\tau_3, \tau_4)'$ . Thanks to this heterogeneity in the cutoff point within a conditional likelihood contribution (i.e., within a clone), the expression depends also on the thresholds, as can be seen in (11).

In the case of the BUC estimator, we combine the  $(K - 1)$  possible clones of each person to estimate  $\beta$ . However, if different cutoff points within a clone are allowed as for BUC- $\tau$ , the number of possible clones of each person is  $(K - 1)^T$  and the sample size of the inflated data set would be  $N(K - 1)^T$ . In standard applications, this would result in more observations than most of today's computers can handle. Therefore, not all possible clones can be included in the inflated estimation sample and a selection has to be made. We propose to include all clones with no variation in the cutoff point (i.e., the clones corresponding to the sample used by the BUC estimator), and use a limited number of clones with random variation in the cutoff points. The program **feologit**, **threshold** is implemented accordingly, where the default is to include 10 clones of each individual with randomly chosen cutoffs. The user can change the number

of additional clones by using the option `clones(#)`. The process of randomly selecting cutoff points can be influenced by the option `seed(#)`. With replicability of results in mind, the `feologit` command has been programmed in such a way that running the BUC- $\tau$  estimator twice leads to the same results. This should not deflect from the fact that cutoff points are selected randomly.

## 3 The feologit estimation command

### 3.1 Syntax

The command `feologit` is called with the following syntax:

```
feologit depvar indepvars [if] [in] [weight] , [ group(varname) thresholds
      cluster(clustvar) keepsample or otheropts]
```

where *depvar* is an ordered categorical variable. Time series operators are not allowed. Unless a panel identifier has been set with `xtset`, the user must specify `group(varname)`. If a panel identifier has been set with `xtset`, the option `group(varname)` may be omitted; in this case, `feologit` will use the panel identifier and provide a warning.

### 3.2 Description

`feologit` fits fixed effects ordered logit models using the BUC estimator of Baetschmann et al. (2015). It does so by replacing each observation in the dataset by  $K - 1$  copies of the observation (where  $K$  is the number of categories of the ordered dependent variable) and then applying the conditional maximum likelihood estimator `clogit`, clustering the standard errors at the level of the original panel unit. After estimation, the dataset is returned to its original form (unless option `keepsample` is specified). With option `threshold`, `feologit` applies the BUC- $\tau$  estimator of Baetschmann (2012), which assumes that thresholds are constant across panel units.

### 3.3 Options

- `group(varname)` specifies an identifier variable (numeric or string) for the panel units.
- `cluster(clustvar)` sets the identifier variable for clustering standard errors. Standard errors are always clustered; specifying this option overrides the default clustering variable, which is the group identifier.
- `thresholds` calls the BUC- $\tau$  estimator, which includes estimates of the thresholds.
- `clones()` specifies the number of clones used in the estimation when `thresholds` has been specified; default is `clones(10)`. A clone is a copy of all observations of a panel unit.

- `seed()` specifies the pseudo-random-number seed used in the estimation when `thresholds` has been specified; default is `seed(79846512)`.
- or reports the estimated coefficients transformed to odds ratios, that is, `exp(b)` rather than `b`. Standard errors and confidence intervals are similarly transformed.
- `keepsample` specifies that the estimation sample is kept. The estimation sample includes the original data as well as additional observations consisting of copies of the original data. The option `keepsample` generates the following new variables:
  - `dkdepvar`, the dichotomized dependent variable used in the `clogit` estimation step;
  - `dkthreshold`, a variable which indicates at which cutoff point each observation of the ordered dependent variable was dichotomised (to result in `dkdepvar`);
  - `bucsample`, a binary variable which indicates if the observation forms part of the estimation sample of the BUC estimator—this variable exhibits variation only if the option `thresholds` has been specified;
  - `clonegroup`, an integer-valued variable which identifies observations corresponding to each panel unit and clone in the estimation sample;
  - and `clone`, a binary variable which indicates if an observation is part of the original sample (`clone=0`) or a copy (`clone=1`).

For instance, after BUC- $\tau$  estimation of `feologit` with option `keepsample`, the corresponding BUC estimates can be obtained by issuing the following command:

```
clogit dkdepvar indepvars if bucsample==1, group(clonegroup) cluster(clustvar),
```

where *indepvars* and *clustvar* are the variables which were used in the BUC- $\tau$  estimation.

### 3.4 Stored Results

Many of the results stored in `e()` are similar to `clogit` or `ologit`. Stored results specific to `feologit` are:



## Scalars:

<code>e(N)</code>	number of observations including observations of clones
<code>e(N_true)</code>	number of authentic observations (without observations of clones)
<code>e(N_group)</code>	number of panel units specified by <code>group()</code>
<code>e(cut1)</code>	rescaling factor used for <code>predict</code> and <code>margins</code> postestimation command. This is only stored if the option <code>thresholds</code> has been specified. For use with <code>predict</code> and <code>margins</code> , <code>e(cut1)</code> is added to all thresholds. See Postestimation below and Section 4.4 for details.
<code>e(clones)</code>	number of clones (additional to dichotomised sample)
<code>e(seed)</code>	seed for random-number generator

## Macros:

<code>e(estopt)</code>	estimation type option (basic or thresholds)
<code>e(group)</code>	name of <code>group()</code> variable

### 3.5 Postestimation

The following postestimation commands are available after `feologit`:

- `logitmarg` calculates statistics of marginal effects from sample averages. `logitmarg` uses a routine provided with the `feologit` installation. It uses sample averages to calculate marginal effects (see Section 4.3 for details). It provides standard errors using the Delta method. The following options allow users to modify the reported results.
  - `outcome(outcome)` displays estimated marginal effects only for the category selected by `outcome`, which should be either one value of the dependent variable, or an indicator of the ordered category (`#1`, `#2`, etc.).
  - `dydx(varlist)` displays estimated marginal effects only for the variables listed by `varlist`.
  - `eretstore` store estimates in `e()` instead of `r()`. Existing `e()` results will be lost.
- `predict` creates a new variable containing linear predictions (option `xb`) or predictions of probabilities (called using the same syntax as after `ologit`). Predictions of probabilities are only available after estimation with option `thresholds`. Estimates of probabilities are calculated assuming all fixed effects are equal to `e(cut1)` (see Section 4.4 for details).
- `margins` estimates margins of response for probabilities and linear predictions. Margins for probabilities are only available after estimation with option `thresholds`. Margins of response for probabilities are calculated assuming a value of `e(cut1)` for all fixed effects (see Section 4.4 for details).
- `test` & `testnl` conduct Wald tests of simple and composite linear hypotheses & tests of nonlinear hypotheses. These commands cannot be used on estimates of the second threshold, which is constrained ( $\tau_2 = 0$ ). The second threshold  $\tau_2$  is the first finite threshold and is called `/cut1` in the estimation output.

## 4 Interpretation

In empirical applications, the interest usually lies in the effect of the covariates  $x$  on the dependent variable and the interpretation of  $\beta$  is of primary interest. However, since the

ordered logit model is a nonlinear model, this parameter does not reflect marginal effects of  $x$  on the ordered dependent variable  $y$ . There exist different possibilities of interpreting the regression results, some of which are discussed below.

#### 4.1 Direction and compensating variation

The sign of  $\beta$  indicates the direction in which an increase of  $x$  influences the cumulative distribution of the dependent variable. If  $\beta_l > 0$ , an increase of the regressor  $x_l$  will lead to an unambiguous decrease in the probability of the lowest category  $\Pr(y_{it} \geq 1|x_{it}, \alpha_i)$  and an increase in the probability of the highest category  $\Pr(y_{it} \geq K|x_{it}, \alpha_i)$ . Moreover, the single crossing property of the ordered logit model implies that there will be exactly one change from the probabilities of lower categories, which decrease, to probabilities of higher categories, which increase (see Winkelmann and Boes 2006). Without knowing the thresholds, it is impossible to determine at which category this switch from decrease to increase will take place.

The  $\beta$  can be interpreted as marginal effects of  $x$  on the latent variable  $y^*$ . Since the interest often lies on the ordered dependent variable  $y$  rather than the latent  $y^*$ , this interpretation is rarely used. Another simple interpretation of  $\beta$  with wider application is to compute the compensating variation between variables, e.g. the change in two regressors such that the latent variable, and therefore the ordered dependent variable, remains unchanged. The compensating variation of two variables is given by the ratio of the corresponding  $\beta$ : an increase of  $x_l$  by 1 has the same effect as an increase of  $x_r$  by  $\beta_l/\beta_r$ .

#### 4.2 Odds ratio

The effect size in logit models is often interpreted using odds, which refers to the ratio between the probability of a certain event and the complementary probability. In the case of ordered logit, the odds of individual  $i$  in period  $t$  having a  $y_{it}$  above category  $k$  relative to below or equal to  $k$  is

$$\text{Odds}(k, x_{it}) \equiv \frac{\Pr(y_{it} > k|x_{it})}{\Pr(y_{it} \leq k|x_{it})} = \exp(x'_{it}\beta - \tau_{ik}) . \quad (12)$$

The odds are independent of the fixed effects but still depend on the thresholds. However, the change in the odds if the  $l$ th regressor is modified, solely depends on  $\beta$  and the shift of the regressor:

$$\text{Odds ratio}(k, \Delta x_{itl}) = \frac{\text{Odds}(k, x_{it} + \Delta x_{itl})}{\text{Odds}(k, x_{it})} = \exp(\Delta x'_{itl}\beta) . \quad (13)$$

Therefore an increase of  $x_l$  by 1 increases the odds ratio by  $\exp(\beta_l)$  for all categories except the first one, *ceteris paribus*. Or in other words, a unit increase in  $x_l$  changes the odds by about  $\beta_l \times 100$  per cent (for small  $\beta_l$ ) or by exactly  $(\exp(\beta_l) - 1) \times 100$  per cent. The option `or` displays the results as  $\exp(\beta)$  as in the standard commands for logit models.

### 4.3 Marginal effects

In empirical applications, the interest often lies in the marginal probability effects, that is, the change in the probabilities of observing  $y_{it} = k$  if a covariate  $l$  is changed by a small amount:

$$ME_{itkl} \equiv \frac{\partial \Pr(y_{it} = k | x_{it}, \alpha_i)}{\partial x_{itl}} = \frac{\partial \Pr(y_{it} \leq k | x_{it}, \alpha_i)}{\partial x_{itl}} - \frac{\partial \Pr(y_{it} \leq k-1 | x_{it}, \alpha_i)}{\partial x_{itl}}. \quad (14)$$

Since the probabilities depend on the thresholds and the individual fixed effects, any marginal probability effects will also depend on these parameters, in general. And since they are not estimated, estimates for marginal effects cannot be obtained, in general, either. For the ordered logit model, the marginal effect has the specific form

$$\begin{aligned} ME_{itkl} &= \{ \Lambda(\tau_{ik+1} - x_{it} - \alpha_i) [1 - \Lambda(\tau_{ik+1} - x_{it} - \alpha_i)] \\ &\quad - \Lambda(\tau_{ik} - x_{it} - \alpha_i) [1 - \Lambda(\tau_{ik} - x_{it} - \alpha_i)] \} \beta_l \\ &= \{ \Pr(y_{it} \leq k | x_{it}, \alpha_i) [1 - \Pr(y_{it} \leq k | x_{it}, \alpha_i)] \\ &\quad - \Pr(y_{it} \leq k-1 | x_{it}, \alpha_i) [1 - \Pr(y_{it} \leq k-1 | x_{it}, \alpha_i)] \} \beta_l. \end{aligned} \quad (15)$$

Thus, we see immediately that the relative size of the marginal effects of two covariates  $l$  and  $r$  is equal to the relative size of their  $\beta$  coefficients,

$$ME_{itkl} / ME_{itkr} = \beta_l / \beta_r, \quad (16)$$

a quantity which can readily be estimated from  $\hat{\beta}_l / \hat{\beta}_r$ .

Moreover, from the second equality in (15) it is clear that for given probabilities of the dependent variable  $y_{it}$ , the marginal effect is just a function of  $\beta_l$ , and straightforward to calculate with an estimate  $\hat{\beta}_l$ . One can therefore calculate marginal effects for any interesting probabilities of the dependent variable. An obvious choice for such probabilities are the sample proportions. We called this the marginal effect at the average and it can be computed by the feologit postestimation command `logitmarg` for each regressor and each possible outcome category. Then an estimate of the marginal effect of regressor  $l$  for category  $k$  is

$$\overline{ME}_{kl} = \{ \bar{d}^{k+1} (1 - \bar{d}^{k+1}) - \bar{d}^k (1 - \bar{d}^k) \} \hat{\beta}_l, \quad (17)$$

where  $\bar{d}^k$  is the sample average of  $d_{it}^k$ . Standard errors for  $\overline{ME}_{kl}$  can be obtained via the Delta method. Depending on whether one is interested on  $\overline{ME}_{kl}$  as an estimate of the marginal effect at the sample average or as an estimate of the marginal effect at the population mean, standard errors need to account for sampling variation from estimation of only  $\beta$  or, in addition, from estimation in  $\bar{d}^{k+1}$  and  $\bar{d}^k$ . The command `logitmarg` provides standard errors for the marginal effect at the sample average.

This is a simple and arguably useful object of interest. It is, however, different from the average marginal effect,  $E(ME_{itkl})$ , which is infeasible. It is also different from the marginal effect at the average of the regressors, defined as (15) evaluated at  $\bar{x} = \sum_i \sum_t x_{it} / (NT)$ , which is also infeasible. Both these more widely used objects of interest depend on the unavailable individual thresholds and fixed effects (see expression after the first equality of (15)), while the marginal effect at the average of the dependent variable that we propose circumvents this problem by focussing instead on the expression after the second equality of (15).

Finally, another potentially useful quantity which is identified and easily estimable is the average semi-elasticity of the continuation probability at category  $k$  with respect to regressor

*l.* The identification and estimation of the average semi-elasticity in *binary* fixed effects logit models was demonstrated by Kitazawa (2012) (see also Santos Silva and Kemp 2016), and here we generalize it to the ordered case:

$$E \left[ \frac{\partial \ln P(y_{it} \geq k | x_{it}, \alpha_i)}{\partial x_{itl}} \right] = \beta_l (1 - \bar{d}^k).$$

#### 4.4 Thresholds

The assumption of constant thresholds for all individuals used by the BUC- $\tau$  estimator allows for additional interpretations. A model with constant thresholds can be estimated with **feologit** by using the option **threshold**. For the interpretations that follow, we change our conceptual perspective: up to this point, all interpretations were cast in terms of probabilities. The  $\varepsilon_{it}$  was treated as a random variable. Now we keep  $\varepsilon_{it}$  fixed, and a change of a regressor leads to a deterministic effect of either pushing the ordered variable to a different category or staying in the same category.

If the spaces between adjacent thresholds is almost equal, a change of the regressors has similar effects independently of the specific category. This does, however, not apply for the tails of the distribution, i.e., the lowest and the highest category. If the spaces are unequal, a change of a regressor has a smaller effect for categories where the thresholds are far apart.

Regarding the effects of covariates, even a marginal change can lead to a switch of category since we do not know the exact value of the latent variable. However, since an estimate of the differences between the thresholds is available, we can also compute the change in the regressors which surely leads to a switch of category. For example, a person in the third category will surely rise up to the next higher category if  $x_{itl}$  increases by  $(\tau_3 - \tau_4)/\beta_l$ , everything else being equal.

When both the thresholds and  $\beta$  are known, the only unknown parameter in the formula for the marginal effect (15) is  $\alpha_i$ . For such situations in fixed effects models, Stata's **margins** postestimation command assumes  $\alpha_i = 0$  for all  $i$ . This is the case, for instance, when using **margins** after **xtlogit** with option **fe** (the binary fixed effects logit).

If  $\alpha_i \neq 0$  for all  $i$ , the object estimated assuming  $\alpha_i = 0$  for all  $i$  will, in general, not be consistent for the average marginal effect. Nevertheless, we have equipped **feologit** with a similar postestimation capability. When **margins** is called after **feologit** with option **thresholds**, an average marginal effect is computed assuming that  $\alpha_i$  is constant across individuals. Because of the underidentification of  $\tau_{ik}$  and  $\alpha_i$ , and the normalisation  $\tau_2 = 0$ , calculating probabilities at  $\alpha_i = 0$  for all  $i$  is often a particularly poor choice. Therefore, we use  $\alpha_i = \tilde{\alpha}$  for all  $i$  instead, which is defined as the estimate of the constant in a binary (cross-sectional) logit of the dichotomised dependent variable at the first cutoff ( $d_{it}^2$ ) with a single regressor  $z_{it} \equiv x_{it}\hat{\beta}$  whose coefficient is restricted to 1:

$$\tilde{\alpha} = \max_a \sum_i \sum_t d_{it}^2 \log \Lambda(a + z_{it}) + (1 - d_{it}^2)(1 - \Lambda(a + z_{it})), \quad (18)$$

with first order condition

$$\bar{d}^2 = (NT)^{-1} \sum_i \sum_t \Lambda(\tilde{\alpha} + z_{it}). \quad (19)$$

The estimate  $\tilde{\alpha}$  is stored in **e(cut1)** after **feologit** with option **thresholds**. A potentially better estimate for  $\tilde{\alpha}$  could be obtained by using all  $d^k$  instead of basing it only on  $d^2$ . However,

our intention here is only to provide such an estimate as a suggestive result and we caution against relying on these marginal effects (see also, e.g., Santos Silva and Kemp 2016, who argue persuasively against using marginal effects based on  $\alpha_i = 0$ ).

## 5 Application: Effect of motherhood on life satisfaction

To illustrate the empirical application of the `feologit` command, we analyze the effect of the birth of the first child on his or her mother's life satisfaction using the German Socioeconomic Panel, a large representative household survey (Wagner et al. 2007). The data were collected yearly between 1984 and 2009 and it is therefore possible to follow a person up to 26 years. The sample includes women between the ages of 20 and 60 who either were mothers or became mothers during the observation period. The application is based on data from Baetschmann, Staub, and Studer [2016] where additional information about data and estimation of causal effects can be found. The dependent variable is life satisfaction, an ordinal variable that ranges from 0 (completely dissatisfied) to 10 (completely satisfied). Below we tabulate its distribution.

```
. tab lifesat
```

life satisfaction		Freq.	Percent	Cum.
0. Completely dissatisfied	0	650	0.52	0.52
	1	498	0.40	0.92
	2	1,485	1.19	2.11
	3	3,332	2.67	4.79
	4	4,657	3.74	8.53
	5	16,464	13.22	21.74
	6	13,979	11.22	32.97
	7	26,793	21.51	54.48
	8	36,127	29.00	83.48
	9	13,669	10.97	94.45
10. Completely satisfied	10	6,908	5.55	100.00
Total		124,562	100.00	

About two third of the responses lie in the upper part of the distribution (seven or higher). The modal answer is category eight with a proportion of around 29%. We want to analyze the effect of the first child on mother's life satisfaction and are especially interested in the evolution of a women's general life satisfaction in the first years after the birth of her first child. Our specification also includes a small set of additional regressors: age, logarithm of household income and a dummy indicating if the respondent is working. While controlling for potentially endogenous factors such as income and labor force participation can lead to biases in the estimates of the effects of interest, we include these variables here only with the intention of illustrating the regression output of the `feologit` command.

```
. describe lifesat kidage01 age lhinc work
```

variable name	storage type	display format	value label	variable label
lifesat	byte	%31.0g	p1110109	life satisfaction
kidage01	byte	%9.0g		age of first child
age	byte	%9.0g		age
lhinc	float	%9.0g		log of monthly household income
work	byte	%8.0g		worked last week

To estimate the effect of having the first child on life satisfaction and the following dynamics, we include five dummy variables, representing the age of the first child in years. For example, the variable `kidage01_2` is equal to 1 when the first child's age is two and 0 otherwise. Such a flexible form approach is common in the literature on adaptation to life events (e.g. Clark et al. 2008). Women who are happier might be more likely to get married and have children (Stutzer and Frey 2006). To control for this selection into motherhood, we want to control for time-invariant characteristics like genetic happiness disposition. And since the dependent variable is ordered we use a fixed effects ordered logit model estimated with the BUC estimator. First, we declare the panel variable with the `xtset` command

```
. xtset idpers
    panel variable: idpers (unbalanced)
```

and then estimate the model with the BUC estimator using `feologit`. We issued the following command

```
. feologit lifesat kidage01_0-kidage01_4 age lhinc work
```

which resulted in the following output

```
note: group() not specified; assuming group(idpers) from panel identifier
note: multiple positive outcomes within groups encountered.
Iteration 0: log conditional likelihood = -172931.52
Iteration 1: log conditional likelihood = -170560.88
Iteration 2: log conditional likelihood = -170558.47
Iteration 3: log conditional likelihood = -170558.47
Fixed-effects ordered logistic regression
```

				N. of obs. (inc. copies) =	469318
				N. of observations =	115257
				N. of panel units =	11247
				Wald chi2(8) =	1356.60
				Prob > chi2 =	0.0000
Log conditional likelihood = -170558.47			Pseudo R2 =	0.0222	
			(Std. Err. adjusted for 11,247 clusters in idpers)		

lifesat	Coef.	Robust Std. Err.	z	P> z	[95% Conf. Interval]	
kidage01_0	.7795942	.0526895	14.80	0.000	.6763246	.8828637
kidage01_1	.6458431	.0514869	12.54	0.000	.5449306	.7467555
kidage01_2	.2123843	.0488413	4.35	0.000	.1166571	.3081115
kidage01_3	.1570779	.0457228	3.44	0.001	.0674628	.246693
kidage01_4	.0118047	.0426905	0.28	0.782	-.0718671	.0954765
age	-.059859	.0022636	-26.44	0.000	-.0642956	-.0554224
lhinc	.5426902	.0262043	20.71	0.000	.4913307	.5940497
work	.2002279	.0212617	9.42	0.000	.1585558	.2419

The output shows that the algorithm for maximizing the log conditional likelihood converged after three steps. For estimating the model parameters, only individuals (panel units), who have variation in their dependent variable, are informative. Individuals who are observed only once or have always the same life satisfaction score over time are excluded by the program

(as their log-likelihood contribution is zero). This condition is met by 11,247 individuals which results in 115,257 observations. On average, people in the estimation sample are therefore observed about 10 times. The ordered dependent variable has 11 categories, so ten different dichotomizations are possible. However, since not all dichotomizations lead to copies with variation in the binary dependent variable, we end up with 469,318 copies which contribute to the estimation procedure. Since the copies are not independent of each other, **feologit** calculates cluster-adjusted standard errors at the individual level (11,247 individuals).

The Wald test indicates that all eight included variables are jointly statistically significant. Regarding the effect of having the first child, the effect is highest in the year of birth (coefficient on *kidage01\_0*). Thereafter, the effect decreases and reaches a nonsignificant level after four years. Age has a negative effect after controlling for time invariant characteristics and the other variables in the model. As expected, household income and working have a positive effect on life satisfaction. The compensating variation between work and log household income is about 0.37, meaning that log household income has to increase by 0.37 to compensate for not working. This is equivalent to saying that household income has to increase by 45% ( $\exp(0.37) - 1$ ) to offset a nonworking status.

Odds ratios for interpreting the effect sizes can be obtained by using the option **or**. Below the code and an excerpt of the output:

```
.               feologit, or
...

```

lifesat	Odds Ratio	Robust Std. Err.	z	P> z	[95% Conf. Interval]	
kidage01_0	2.180587	.1148941	14.80	0.000	1.966636	2.417814
kidage01_1	1.907595	.0982161	12.54	0.000	1.724489	2.110143
kidage01_2	1.236623	.0603983	4.35	0.000	1.123734	1.360853
kidage01_3	1.170087	.0534997	3.44	0.001	1.06979	1.279786
kidage01_4	1.011875	.0431974	0.28	0.782	.9306546	1.100183
age	.9418973	.0021321	-26.44	0.000	.9377277	.9460854
lhinc	1.720629	.0450879	20.71	0.000	1.63449	1.811309
work	1.221681	.025975	9.42	0.000	1.171817	1.273667

Having the first child increases the odds ratio by about 118% in the year of birth, about 91% in the first, about 24% in the second, and about 17% in the third year thereafter. In the fourth year, the effect is essentially no longer present.

Marginal effects at the average can be obtained by using the postestimation command **logitmarg** after estimating the model. To compute them, the relative frequencies of the corresponding categories in the estimation sample are used. Below the code and the output. We specified the option **dydx(kidage01\_0)** to limit the output to the marginal effects of the year of birth.

```
.               logitmarg, dydx(kidage01_0)
Marginal effects at the average          N. of observations=    115257
                                         N. of panel units =    11247

```

	Margin	Std. Err.	z	P> z	[95% Conf. Interval]	

1	-.0039435	.0002665	-14.80	0.000	-.0044659	-.0034211
2	-.0031271	.0002113	-14.80	0.000	-.0035414	-.0027129
3	-.0090451	.0006113	-14.80	0.000	-.0102433	-.007847
4	-.0196532	.0013283	-14.80	0.000	-.0222566	-.0170498
5	-.0253752	.001715	-14.80	0.000	-.0287365	-.0220139
6	-.0718074	.0048532	-14.80	0.000	-.0813194	-.0622954
7	-.0401357	.0027126	-14.80	0.000	-.0454523	-.0348191
8	-.0197687	.0013361	-14.80	0.000	-.0223874	-.0171501
9	.0881895	.0059604	14.80	0.000	.0765074	.0998716
10	.0661833	.0044731	14.80	0.000	.0574163	.0749503
11	.0384832	.0026009	14.80	0.000	.0333855	.0435809

Note that `logitmarg` enumerates categories starting at 1 and ignores the actual (arbitrary) labels of the dependent variable, which in our case starts at 0. Since having a child has a positive effect on life satisfaction in the first year, the marginal probability effects at the average are negative for the lower categories and positive for the ninth and higher categories. For example, in the first year having a child decreases the probability of falling into the sixth category by 7.2%-points and increases the probability of having the highest rating by 3.8%-points for this average person, *ceteris paribus*. The effects for the lowest categories are very small since only few people have such a low life satisfaction status.

Estimating the thresholds is possible by using the option `thresholds`, which calls the BUC- $\tau$  estimator. This requires the assumption that the spacing between the thresholds is the same for all individuals. Below the output:

```
. feologit lifesat kidage01_0-kidage01_4 age lhinc work, group(idpers) threshold
note: multiple positive outcomes within groups encountered.
Iteration 0: log conditional likelihood = -335358.34
Iteration 1: log conditional likelihood = -317080.65
Iteration 2: log conditional likelihood = -316611.97
Iteration 3: log conditional likelihood = -316611.28
Iteration 4: log conditional likelihood = -316611.28
Fixed-effects ordered logistic regression
                                N. of obs. (inc. copies) = 1544471
                                N. of observations      = 116665
                                N. of panel units       = 11725
                                Wald chi2(17)          = 33921.94
                                Prob > chi2            = 0.0000
Log conditional likelihood = -316611.28 Pseudo R2       = 0.5250
                                (Std. Err. adjusted for 11,725 clusters in idpers)
```

lifesat	Coef.	Robust Std. Err.	z	P> z	[95% Conf. Interval]	
kidage01_0	.7835162	.0537423	14.58	0.000	.6781832	.8888491
kidage01_1	.6396636	.052942	12.08	0.000	.5358992	.743428
kidage01_2	.2046826	.0501304	4.08	0.000	.1064288	.3029365
kidage01_3	.1624746	.047537	3.42	0.001	.0693037	.2556455
kidage01_4	.0107345	.0432988	0.25	0.804	-.0741297	.0955986
age	-.0578395	.002264	-25.55	0.000	-.0622769	-.0534021
lhinc	.5471535	.0269294	20.32	0.000	.4943729	.5999342
work	.200715	.021935	9.15	0.000	.1577232	.2437068
/cut1	0	(constrained)				
/cut2	.7284609	.0615201			.6078836	.8490381



/cut3	1.637104	.0677637	1.504289	1.769918
/cut4	2.666144	.0709232	2.527137	2.805151
/cut5	3.424063	.0726216	3.281727	3.566399
/cut6	4.872104	.0734364	4.728172	5.016037
/cut7	5.696417	.0749376	5.549542	5.843292
/cut8	7.036473	.0766951	6.886153	7.186792
/cut9	9.097256	.0807096	8.939068	9.255444
/cut10	10.65767	.0872651	10.48663	10.8287

In contrast to the procedure without thresholds, the cutoff point within a likelihood contribution (clone) can change. Therefore, even when the ordered dependent variable is constant, the resulting dichotomized dependent variable with different cutoff points can vary. This increases the number of individuals in the estimation sample slightly to 11,725 and the number of observations to 116,665. The estimator includes all contributions of the BUC estimator plus ten copies of each individual with random variation in the cutoff point. Therefore, the number of included copies increases noticeably to over 1.5 million.

The regression coefficient  $\beta$  has the same interpretation as before. It is also possible to display the odds ratios. From the output, we see that the exact estimates changed only slightly. Since life satisfaction can take eleven different values and the first finite threshold /cut1 (corresponding to  $\tau_2$ ) is normalized to zero, the output shows estimates for the second to the tenth threshold. Careful inspection of the spaces show that there is a moderate tendency that differences increase towards the top. Where the difference between the first and second, and second and third, is 0.728 and 0.909, respectively, the two spaces at the upper end are 2.06 and 1.56. This implies that changes of regressors have a larger effect on the observed ordered dependent variable for unhappy individuals compared to happy individuals.

The spaces range from 0.73 to 2.06 where the second smallest difference is 0.76. The size of the marginal effects of the different regressors are rather small compared to these spaces. Except for the unhappiest people (lowest category), having a child can never increase life satisfaction by more than one point, everything else being equal. The same is true for doubling the income ( $\exp(0.547) = 1.73$ ). Regarding age, for a person with a life satisfaction score of nine, 26 years need to pass for her to surely change to the next lower category  $((10.35 - 8.91)/0.057)$ .

The BUC and BUC- $\tau$  estimator are both consistent under the additional assumption of constant thresholds. Therefore, we can test this assumption in the form of a generalized Hausman specification test by comparing the two estimates. If they are statistically different, we can reject the null hypothesis of constant thresholds.<sup>2</sup> There are different possibilities to implement such a test in Stata. The most obvious way would be to use the command `suest`. Using this command requires estimating the model without clustered standard errors and, in a second step, computing a joint covariance matrix of both estimators adjusting for clusters. However, since computing the BUC estimator without clusters can be misleading and adjusting them after using `clogit` can lead to error messages if the group variable is not properly nested within the cluster variable, we decided to work with interaction terms. In a first step, the command `feologit, threshold` with the additional option `keepsample` is used:

```
.qui: feologit lifesat kidage01_0-kidage01_4 age lhinc work, group(idpers) threshold keep
```

The option `keepsample`, here shortened to `keep`, keeps the inflated dataset of clones which

2. The precision of the test might be improved by increasing the number of clones used in estimation.

is used to compute the BUC- $\tau$  estimator after the estimation. In addition, the dichotomized dependent variable is stored under “dkdepvar”, the corresponding cutoff-point under “dkthreshold” and the variable “bucsample” indicates if the cutoff varies within the clone (bucsample=0) or not (bucsample=1). Observations with bucsample=1 constitute the sample of the BUC estimator. The rest of the observations are primarily used to estimate the thresholds, but contribute to the estimates of  $\beta$ , too. Under the null hypothesis of constant thresholds, both samples (bucsample=0 and bucsample=1) can be used to estimate  $\beta$ . Statistical differences between the two estimates indicate that the thresholds are not constant. Thus, after having used `feologit` with the options `thresholds` and `keepsample`, we can implement this test by interacting the variable “bucsample” with all regressors and estimating the model with `clogit` while grouping on the clone variable and clustering on the individual level. Finally, we test if the differences of the interactions terms are jointly equal to zero.

As an auxiliary step, we first use a small loop to create the interaction variables (prefixed “tau.”) and store them in the local macro `interact`:

```
.      local interact=""
.      foreach i of var kidge01_0-kidge01_4 age lhinc work {
2.          qui: gen tau_`i' = `i'*(bucsample==0)
3.          local interact="`interact' tau_`i'"
4.      }
```

We then estimate the model with interactions using the `clogit` command, grouping on the clone variable and clustering on the individual level:

```
.      clogit dkdepvar kidge01_0-kidge01_4 age lhinc work `interact' i.dkthreshold, group(clo
> negroup) cluster(idpers)
note: multiple positive outcomes within groups encountered.
note: 132,242 groups (827,909 obs) dropped because of all positive or
      all negative outcomes.

Iteration 0:  log pseudolikelihood = -335337.34
Iteration 1:  log pseudolikelihood = -317228.3
Iteration 2:  log pseudolikelihood = -316599
Iteration 3:  log pseudolikelihood = -316594.58
Iteration 4:  log pseudolikelihood = -316594.58
Conditional (fixed-effects) logistic regression

                                     Number of obs   = 1,544,470
                                     Wald chi2(25)     = 34242.88
                                     Prob > chi2       = 0.0000
Log pseudolikelihood = -316594.58      Pseudo R2     = 0.5251
                                     (Std. Err. adjusted for 11,725 clusters in idpers)
```

dkdepvar	Coef.	Robust Std. Err.	z	P> z	[95% Conf. Interval]	
kidge01_0	.7795942	.0526894	14.80	0.000	.6763248	.8828635
kidge01_1	.6458431	.0514868	12.54	0.000	.5449308	.7467553
kidge01_2	.2123843	.0488412	4.35	0.000	.1166573	.3081113
kidge01_3	.1570779	.0457228	3.44	0.001	.0674629	.2466928
kidge01_4	.0118047	.0426904	0.28	0.782	-.0718669	.0954764
age	-.059859	.0022636	-26.44	0.000	-.0642956	-.0554225
lhinc	.5426902	.0262043	20.71	0.000	.4913307	.5940496
work	.2002279	.0212616	9.42	0.000	.1585559	.2418999
tau_kidge01_0	.008167	.0306995	0.27	0.790	-.052003	.068337
tau_kidge01_1	-.013622	.029474	-0.46	0.644	-.07139	.0441459
tau_kidge01_2	-.0168844	.0268302	-0.63	0.529	-.0694707	.0357018

tau_kidage01_3	.0119503	.0251759	0.47	0.635	-.0373935	.0612941
tau_kidage01_4	-.0023741	.024214	-0.10	0.922	-.0498327	.0450844
tau_age	.0040727	.0008054	5.06	0.000	.0024942	.0056513
tau_lhinc	.0093463	.0121449	0.77	0.442	-.0144573	.0331498
tau_work	.0009298	.0102628	0.09	0.928	-.0191849	.0210444
dkthreshold						
2	-.7305085	.061547	-11.87	0.000	-.8511384	-.6098786
3	-1.638868	.0678167	-24.17	0.000	-1.771786	-1.505949
4	-2.667425	.0709832	-37.58	0.000	-2.80655	-2.5283
5	-3.424848	.072681	-47.12	0.000	-3.567301	-3.282396
6	-4.871923	.0734845	-66.30	0.000	-5.01595	-4.727896
7	-5.695513	.0749785	-75.96	0.000	-5.842468	-5.548557
8	-7.034464	.0767388	-91.67	0.000	-7.184869	-6.884058
9	-9.093563	.0807186	-112.66	0.000	-9.251769	-8.935358
10	-10.65302	.0873042	-122.02	0.000	-10.82414	-10.48191

As can be seen, the estimates of the non-interacted regressors are numerically equivalent to the previous BUC estimates. The estimates of the interaction terms mirror the differences between the BUC and the BUC- $\tau$  estimates, but are not identical since the thresholds adjust to the additional regressors. The differences are small in size and most of them are not individually significant. To formally test whether they are jointly different, we can use Stata's `test` command:

```
.      test `interact`
( 1)  [dkdepvar]tau_kidage01_0 = 0
( 2)  [dkdepvar]tau_kidage01_1 = 0
( 3)  [dkdepvar]tau_kidage01_2 = 0
( 4)  [dkdepvar]tau_kidage01_3 = 0
( 5)  [dkdepvar]tau_kidage01_4 = 0
( 6)  [dkdepvar]tau_age = 0
( 7)  [dkdepvar]tau_lhinc = 0
( 8)  [dkdepvar]tau_work = 0
      chi2( 8) =   39.13
      Prob > chi2 =   0.0000
```

Despite the differences being small in size, the joint hypothesis that all these terms are equal to zero can be rejected at the 0.1% level ( $\chi^2(8) = 39.13$ ) due to the large sample. Therefore, formally we reject the null hypothesis of constant thresholds and should use the BUC instead of the BUC- $\tau$  estimator to interpret the results. However, given that differences in the estimates in this case are so small, the thresholds might still shed some light on the data generating process, but should not be over-interpreted.

## 6 Conclusion

This paper presented and discussed the BUC and BUC- $\tau$  estimators of the fixed effects ordered logit model and introduced a new community-contributed Stata command that implements these estimators in Stata, `feologit`.

BUC and BUC- $\tau$  both offer consistent estimates of the slope parameters  $\beta$ . In addition, BUC- $\tau$  obtains consistent estimates of the thresholds  $\tau$ , under the slightly more restrictive assumption that they do not vary between individuals. While the calculation of average marginal

effects is not possible with these estimators, useful identified objects of interest include odds ratios, compensating variation and other quantities. Of particular interest is a particular marginal effect at the average, which we proposed in this paper and for which a dedicated postestimation command is available with `feologit`. Finally, we also presented a new specification test that can be used to evaluate the assumption of constant thresholds by comparing BUC and BUC- $\tau$  estimates, and which can be easily implemented with a few lines of code in Stata.

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