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Book Review. Mathematics & Mathematics Education: Searching for Common Ground

Fried, M. N., & Dreyfus, T. (Eds.). (2014). *Mathematics & mathematics education: Searching for common ground*. Dordrecht: Springer. 402 pp. Hard Cover: ISBN 978-94-007-7472-8 \$129.00 [US]. eBook: ISBN 978-94-007-7473-5 \$99.00 [US].

The publicity material for the book states the following:

“This book originated in a symposium held in May 2012, at the Ben-Gurion University of the Negev, Israel, on the occasion of the retirement of Ted Eisenberg. It begins by addressing issues of dialogue and mutual expectations between the two communities of Mathematics & Mathematics Education. The main section of the book continues with a series of themes of common interest, including the history of mathematics and mathematics education, problem-solving, mathematical literacy, visualisation, justification and proof, policy, collaboration, and a post-script. It is structured in the form of a position paper followed by a chapter containing a series of short reflections and responses. The final section of the book contains about 50 pages of appendices, including a homage to Eisenberg and reprints of previous dialogues.”

The stated aim of this book was, and remains, to encourage dialogue between university mathematicians and members of the mathematics education community in an attempt to overcome what was perceived as a growing division between the two communities. From this perspective of further developing their mutual willingness to do so through their attempts to understand and to respect the diversity of opinion, it is clear that a great deal of effort has been made on the part of all symposium participants. The importance of communication is stressed throughout the chapters; both from the formal, technical side in relation to the important aspects of the discipline of mathematics, as well as from the interpersonal, social, and cultural aspects of communication between people, where meaning is far more open to interpretation, contextually situated, and where ambiguity, intended or otherwise, can occur. Looking back over the book as a whole, communication, in its many forms, is a central feature. The discussions of the history of mathematics, problem solving, mathematical literacy, visualisation, justification and proof, show the *act of communicating* to be a crucial aspect of thinking mathematically, of doing mathematics with and for other people, and in helping oneself and others to understand and to be able to act in the world — even if this sometimes means deciding not to follow the “best” mathematical solution on the basis of critical contextual information. Communicating *about* curriculum, teaching, and learning mathematics in terms of mutual expectations and collaboration, as well as about policy — its planning, enactment, and implementation, also contestation, within and beyond the institution of education — is also vital and a major theme. It is especially important in the ongoing struggle for greater sophistication and renewal of mathematics as an academic discipline, and of mathematics education as a region of knowledge, an interface between disciplines (Bernstein, 2000).

This book offers a wealth of informed opinions and considerable experience and reflections for the reader to draw upon, and I now offer a selection of personal observations — based upon my own positioning as a teacher in adult and vocational mathematics, including some undergraduate mathematics, and workplace mathematics. I am also a researcher involved in workplace and vocational mathematics. The richness of this book means that any review will necessarily be partial, and I hope that other readers with different experiences and perceptions

will also find it of value. They are also encouraged to submit their own reviews to this and other journals. I address both the mathematical themes of the book and the powerful underlying theme of communication.

Mathematical Themes

Mathematical themes form the kernel of the book. The introduction to Chapter 8, *Reflections on Problem-Solving*, states that: "... mathematical problem solving (including problem posing, conjecturing and proving) is *the* central activity in mathematics as a living science, and thus it has been hoped that it would also become *the* central activity in mathematics education" (p. 114). However, although there have been many studies and an accumulation of knowledge on how problem solving occurs, there remain many unresolved fundamental issues concerning its role in mathematics education, as compared to its clear role in mathematics. In his reflective summary, Boris Koichu notes that the continuing gap between the promise of the use of problem solving and its actual realisation often becomes a source of frustration; as does the sporadic and complicated developmental process, impacted and influenced by politics (including standardised testing regimes) and the evolution of technology, for example. The chapter includes calls for the study the tacit cognitive and affective processes underlying expertise in mathematics; and for problem-based task design to enhance conceptual understanding rather than the current (seemingly universal) focus on the acquisition of techniques needed in preparation for tests and examinations. For me, the question is this: If mathematical problem solving, including problem posing, conjecturing, and proving, is central to the discipline of mathematics, and also outside of the academy in vastly different contexts, as an important part of what people might do on a daily basis at work and elsewhere, why do these frustrations of mathematics educators continue?

Chapters 13 and 14 address *Justification and Proof*. David Tall wrote the position chapter and opens with a quotation, stressing the human origins of mathematics:

The cognitive development of mathematical thinking and proof is based on fundamental human aspects that we all share: human perception, action and the use of language and symbolism that enables us to develop increasingly sophisticated knowledge structures. It is based on ... blending together perception, operation and reason. (p. 224)

In his framework, *practical* mathematics based on experiences with shape and space lead to their conceptual embodiment, and experiences with number lead to operational symbolism in arithmetic and algebra. Focusing on their properties leads to *theoretical* mathematics of Euclidean proof in geometry, symbolic proof in algebra, and a blending of embodied and symbolic reasoning using language. Properties of these, in turn, lead to axiomatic formalism and formal proof at the highest level of mathematical thinking. In other words, practical, theoretical, and formal mathematics develop with maturation, but, once developed, all forms continue to interact and lead to different forms of proof at ever more sophisticated levels. However, Tall recognises that not all forms of mathematical knowledge build logically on previous experience, and that some early generalisations may become problematic in more sophisticated contexts. This observation helps to explain difficulties experienced by individual learners of all ages, as well by different cultures throughout history.

In Chapter 14, I have chosen to focus on the section by Gila Hanna, *Proof in the Curriculum: Reflecting Modern Mathematical Practice*, due to her focus on epistemological issues. She observes that over the last 30 years there has been a shift in interest of philosophers towards

“the ways in which today’s mathematicians actually devise and judge proof, on the reasoning styles they use, and on the ways in which they present and weigh evidence” (p. 239). Hanna notes that although there is a standard definition of mathematical proof which is universally accepted as the ideal, in actual practice the vast majority of proofs do not conform to the syntactical derivations due to the high cost of such demands, and the fact that informal proofs may provide more insight and understanding. Accordingly, she recommends that mathematics educators should pay more attention to the explanatory role of proof than its justificatory role. This could include the use of computer graphics and other forms of experimentation and visualisation in mathematics teaching. Hanna concludes that providing and understanding proofs depend on a good understanding and familiarity with the mathematical concepts involved, and hence they form a valuable part of mathematics education.

Chapters 9 and 10 address *Mathematical Literacy*. Paul Goldenberg asserts that mathematical literacy is an inadequate metaphor, and that even the OECD definition offers no clear meaning with respect to phrases such as “well founded judgements,” “use and engage with mathematics,” and “meet the needs of that individual’s life” (see <https://stats.oecd.org/glossary/detail.asp?ID=5388>). With respect to the last, he asks whether it refers to an individual’s life as it is? As it might be? Or does it concern opening up new options? He continues:

Mathematics absolutely *requires* precise communication so much so that it has elaborate and extensive formal languages and a vast technical vocabulary. But its essence is in its logic, manner of thinking, investigative style, and unique approach to establishing truth and causality: proof. No natural extension of the meaning of “literacy” captures this essence of mathematics. (p. 141)

Instead, Goldenberg asserts that reasoning — what he terms *habits of mind* — should be at the core of curricula, and gives many examples including mathematical reasoning, explaining, justifying, visualising, critically interpreting diagrams, and so forth. In the Reflections chapter, Anna Sfard supports Goldenberg’s emphasis on teaching mathematical ways of thinking as the most important outcome of compulsory mathematics education. However, she suggests instead the phrase *habits of communication*, to also include habits that regulate communicating with other people, thus bringing in an important interpersonal dimension which, she asserts, may help to overcome the situatedness of learning. In her summary of the chapter, Sfard puts forward the view that mathematical literacy is more demanding than formal mathematical competency, it is more than the addition of applications to formal mathematics, and actually requires knowing mathematics in a new, more complex way due to the need to be more varied and flexibly adaptable to the specificities of each situation where mathematical routines and thinking are required. In Bernstein’s (2000) terms, this means developing a greater repertoire. My question is this: How might complex problems from actual (not imaginary) work and life situations which are rich in contexts supporting and/or constraining possible solutions, and quite unlike the brief textual form typical of school mathematics problems, find a place in the mathematics curriculum?

Chapters 11 and 12, on *Visualization in Mathematics and Mathematics Education*, begin with a historical overview of the research by Ken Clements, who pleads for greater precision in terminology due to the many meanings attributed to the term *visualization* across disciplines and fields. In the Reflections chapter, Elena Nardi observes that mathematics education research is far from reaching a consensus on its possible roles and benefits. Nardi’s experience of working among mathematicians at university level, shows that they use

diagrams as a third language to complement words and symbols. These mathematicians perceive visualization as fostering “a fluent interplay between analytical rigour and (often visually based) intuitive insight” (p. 209). A new didactical contract at university allows students to use facts not formally established, but which they are later expected to formalise. They are encouraged to make use of visualization but in a sophisticated way. In her contribution, Norma Presmeg raises several difficulties with visualization, each implicitly related to compartmentalisation, and she listed thirteen significant questions related to implementation of visualization in mathematics education. In her summary of contributions made to the chapter, the issues raised by Nardi include:

1. Not teaching visualization as an end in itself but allowing it to be explored across the curriculum (clearly supported in several other chapters of this book).
2. Acknowledging that visualization has a logic of its own.
3. Acknowledging that radical change is needed in the didactic contract between secondary and tertiary mathematics so that visual thinking is regarded as legitimate in mathematics and is part of the explicit intended curriculum.
4. Fostering the learning of the grammar and syntax of diagrams, as for written mathematics, and not assuming that visuality means transparency and automatic understanding.
5. Encouraging students to engage with the linguistic and kinaesthetic registers of mathematics as well as the symbolic and the visual.
6. Acknowledging that visualization utilising technology needs a reorganisation of the curriculum: its content, order of difficulty, structure.

Nardi concludes that although visualization is time-consuming, this is “compensated by its power to give insight and to support our power for abduction” (p. 217). Visualization clearly plays an important role in peoples’ lives at work and elsewhere in our three-dimensional world of space and shape, real and imagined, so how might it gain greater recognition and acceptance in mathematics education at all levels?

Chapters 6 and 7 address *The History of Mathematics*. Although, as Michael Fried noted in Chapter 1, neither the discipline of mathematics nor mathematics education place the history of mathematics at their centres, it may end up being common ground through its contribution to mathematical literacy and to “the meaning of being mathematically educated” (p. 17). In Chapter 6, Hans Niels Jahnke adopts a hermeneutic approach to history in mathematics education, in which students are asked to examine in detail an example of historic source material related to their current studies and with which they are already familiar, to explore its various historical, cultural, and scientific contexts. He observes that there are two distinct but equally important outcomes of this approach. First, students deepen both their understanding of the history of mathematics as well as of their own personal conceptualisations. Second, students are faced with the task of being able to take on the perspective of people living long ago, in order to be able to argue from their assumptions, and to use their symbols and methods, thus posing completely new demands on students’ abilities to argue, to justify, and to prove mathematically. Students are both doing mathematics and reflecting on mathematics. In concluding, he considers the idea of mutual respect as a necessary condition for communication between people, but adds the connotation of *epistemological* to respect, because no single perspective has the right to claim exclusive access to truth.

In Chapter 7, Luis Radford reflects on the history of mathematics from a cultural-historical perspective. Considering the *nature of mathematics* he notes the dangers of reducing

mathematics to its technical aspect, because when historical-cultural actions are devalued, it removes the individual from the discipline, leading to student disengagement. He argues that if we consider mathematics as an “historically constituted social practice, a cultural form of reflection and action ..., something practiced not in a vacuum but *with* and *for* others” (p. 105), mathematics would not be something to acquire but a practice “where we step into the public space” (p. 105). It would be “a place where we come to hear others’ voices and perspectives and to speak out” (pp. 105-106). In his reflective summary, Radford states that the history of mathematics cannot simply be imported into the classroom. It “needs to appear as a problematic field, where one can interrogate notions and ideas that we usually leave unthematized, such as mathematics, its development, and its relationship to culture” (p. 107). He concludes with the hope that coming to see these things in concrete human activity and finding new forms of thinking mathematically beneath the technicist approaches that are currently dominant, it might be possible to extract “the aesthetic, intersubjective and embodied dimensions of mathematics and mathematical thinking” (p. 107). Both chapters reinforce the value and the contribution that communicating *in* and *about* mathematics respectfully *with* and *for* others. To me, this includes peoples’ ways of using and doing mathematics in their roles as citizens, and as workers in all vocations and professions, in their capacities to learn about and respect contextual information such as taking account of the needs and perspectives of others and the environment, previous histories of activities, considering alternatives, and so on.

Communication About Mathematics Education and Mathematics, and Collaboration

From consideration of mathematical themes closely related to the content and pedagogy of teaching and learning, I now turn to sections of the book which address significant issues for both communities at what I consider to be at a different level: policy, mutual expectations, and practical collaboration. Each of these have a bearing on the possibilities for finding common ground.

Chapters 15 and 16, by Mogens Niss and Nitsa Movshovitz-Hadar, concern *Policy*. Niss identifies a comprehensive list of people and bodies who are involved in policy decision making and implementation concerning mathematics education, as well as a second list of policy agents pursuing philosophical, ideological, or commercial interests. In this regard he observes that there are likely to be differing, sometimes conflicting, priorities and agendas. He offers three interrelated explanations in relation to the existence of differing and sometimes incompatible views of policy protagonists regarding (a) the nature and essence of mathematics as a discipline, (b) conceptions of mathematics as a discipline and its role in society and culture, and (c) mathematical pedagogy in relation to particular groups of students. Movshovitz-Hadar argues that coherent policy relies upon mathematicians and mathematics educators working together to improve outcomes for mathematics education. A number of provocative questions were raised by her and by Jonas Emanuelsson concerning teacher education. Recommendations for mutual collaboration were made by Davida Fischman, including university policies supporting collaboration through funding support for joint teaching, research, joint meeting and conference attendance, mentoring of junior mathematics staff by both groups, and, most importantly, recognition through the promotion and tenure process.

Fischman also raised what I consider to be two related crucial issues with respect to policy and its outcomes: student assessment and teacher accountability. In many countries around the world, actual practice in mathematics classrooms (or mathematics lessons in primary school classrooms) is governed by state-mandated assessment and reporting policies which

control to a large extent, explicitly and implicitly, what teachers teach and how they teach it. Julian Williams (2012) captures this in an example drawn from a larger research project in the UK, in the examination-oriented teaching of an “advanced” mathematics class, where mathematics teachers “teach to the test” to the exclusion of encouraging students’ curiosity-driven questions. This then leads to the kinds of problems outlined by Uri Onn in Chapter 5, where students who have succeeded in gaining entry to prestigious university mathematics courses through high-stakes examination systems which are “*product oriented and not process oriented*” (p. 60), end up with “damaged perception[s] of mathematics objects as if they were discrete sets of points in space with no connections” (pp. 58-59).

At this point I am wondering why the crucial topic of assessment which clearly has major implications for mathematicians, mathematics educators, mathematics teachers, students and their families, among others, was not included in this symposium. I am also concerned that the related issue of the increasing control over the work of mathematics (and other) teachers, and, more recently, in many countries, the work of academics, driven by ideologies of governments usually elected for relatively short terms, has the potential to override much of the goodwill and productive efforts at fostering collaboration and mutual respect.

This brings us back to Chapter 5, concerning *Mutual Expectations Between Mathematicians and Mathematics Educators*, by Tommy Dreyfus. In this chapter, Tommy is also joined by Joanna Mamona-Downs and Steve Lerman, and a central theme is transition for students and for teachers. From the student perspective, and following from the discussion above, the transition from school to university can be very difficult. Especially for many young people, the post-school social and cultural transition to independence may be difficult, in addition to the very different styles of teaching at university and expectations placed on students, notwithstanding the serious efforts made by many university lecturers to overcome such problems. This is in addition to the difficulties raised by Tall (2013) concerning the transition from theoretical mathematics to formal mathematics. From the teacher education perspective, Mamona-Downs highlights the problems faced by pre-service teachers as they attempt to bridge the worlds of academia, and their strong disciplinary knowledge of mathematics, with the less scholarly approaches to mathematics that they might find in school mathematics texts and practices. Over the years, many mathematics teacher educators have made similar complaints about inconsistencies between the pedagogical approaches pre-service and beginning mathematics teachers have developed at university and what they experience as powerful norms in school practice.

For Lerman, this chapter involves people, and hence implies a sociological perspective concerning boundaries, status, and control — control of both curricular content as well as pre- and in-service mathematics teacher education — thus bringing us back to policy issues. In closing he raises the perpetual question of the location and responsibility for the preparation of future mathematics teachers. Lerman observes that curricular and pedagogic choices “are driven by ideology, by beliefs and values” (p. 66), and reminds us that such decisions can involve not only academic mathematicians and mathematics educators, but also political interests such as governments who have the power to make such decisions *with or without* consultation of academics, to undertake selective or token consultation to serve their own interests. Lerman also offers a valuable discussion on the theoretical differences between mathematics and mathematics education (pp. 65-66). In his summary, Dreyfus raises many important issues around the expectations of one another’s cohort by mathematicians and by mathematics educators. He outlines a wide range of possible misconceptions or confusions about respective roles and responsibilities, agency and structural impediments in policy and

practice, knowledge structures and research outcomes, epistemologies, roles of technology, and even the definition of “applied mathematics” — to which I shall return below. In closing he raises the perpetual question of the location and responsibility for the preparation of future mathematics teachers. (To this I would add continuing education and professional development.)

Chapter 17, *Mathematics and Education: Collaboration in Practice*, offers a detailed account of collaboration between a research mathematician, Hyman Bass, and a practising teacher and mathematics educator, Deborah Loewenberg Ball, both regarded as eminent in their respective fields. From their work spanning over 20 years, they explain what they did and how they did it, as well as reflect on the affordances of their complementary perspectives, along with the challenges and problems in pursuing the question: “What is mathematical about mathematics teaching, and what are the mathematical demands of that work?” For anyone interested in considering such a partnership this chapter is well worth reading.

Chapter 18, *Reflections on Collaboration Between Mathematics and Mathematics Education*, by Pat Thompson with Michèle Artigue, Günter Törner, and Ehud de Shalit, focuses on aspects of collaboration between disciplines and organisations that might offer support, such as professional societies or, on a more personal level, engaging in a shared task. Once again, this is a valuable chapter for its insights into the personal experiences of the authors. One conclusion is that both individuals and institutions matter, and that challenges as well as affordances may involve structural and/or cultural issues. Artigue (pp. 316-317) identifies seven forms of collaboration that she has experienced at various levels, from the personal to international projects such as those of ICMI (the International Commission on Mathematical Instruction) (see <http://www.mathunion.org/icmi/icmi/overview-of-icmi/>). The sections by Thompson and de Shalit describe powerfully their efforts to build the bridges in their own workplaces, and de Shalit’s second example of working outside the academy in organising a multicultural mathematics exhibition is highly commended. Törner raises an issue that I had been expecting to see much earlier in the book: co-operation between pure mathematics and applied mathematics departments. For me it has seemed strange that the definition of *mathematician* is never made clear as to who exactly is included and excluded by this term; also that the terms *applied mathematics* and *applied mathematicians* are scarcely mentioned.

Conclusion

In terms of content, the main section of the book represents a snapshot of the views of invitees to this symposium on mathematical themes considered to be of significance to both communities. From this perspective, the book could be fruitfully used as a resource by university mathematics teaching staff, along with mathematics education lecturers, researchers, and graduate students wishing to gain a broad overview of many of the significant curricular and pedagogical issues for mathematics education, in school and at university. I could imagine it supporting a productive series of colloquia for doctoral students, or professional development sessions for school or university teaching staff, including graduate students employed as tutors.

In terms of communication about mathematics and mathematics education, between members of these two communities, this book has raised a wealth of substantive issues and demonstrated evidence of fruitful collaboration even in the face of philosophical and pedagogical disagreement. In all workplaces, not least education workplaces, mutual respect for the formal knowledge structures, as well as the lived experience of other professionals (among other stakeholders), need to be acknowledged, understood, and respected in order for

productive collaboration of any kind, at any level, to begin. The book, *Educational Interfaces Between Mathematics and Industry: Report on an ICMI-ICIAM-Study* (Damianian, Rodrigues, & Sträßer, 2013), is the result of a collaboration between applied mathematicians and mathematics educators, and with industrial partners. The chapter by Nakagawa and Yamamoto (2013) spells out in detail the process of mathematicians and engineers arriving at a mutual understanding and eventual collaborative solution to a significant problem in a major industrial setting. The mathematicians' innovative solution actually led to the creation of new mathematical knowledge! At the end of Chapter 13, Tall echoed these sentiments in relation to the pivotal issue of mathematicians (pure & applied), mathematics educators, and teachers coming together as different communities of practice for mutual benefit. He recognises

the need for an openness and willingness to listen to other points of view and to see the relevance of various viewpoints in different contexts. It should be possible for a community to realise that viewpoints that may be essential in their own context may not be appropriate in others. (p. 233)

Not only is it important for mathematicians and mathematics educators to communicate with their peers, in person or online, at meetings and conferences, or via refereed publications, it is also important to communicate with other stakeholders such as potential students, their teachers and their families (cf. de Shalit, Chapter 18); also members of other faculties, and members of the business community who can be very supportive educationally and powerful allies in the political domain. The collaboration of our two communities will strengthen their influence with such stakeholders.

Given that respectful understanding of others' viewpoints and mutual cooperation are essential to successful collaboration, it appears that students of mathematics gain very little, if any, such experience in their formal education beyond their classrooms. Explaining, justifying, and convincing others in one's own mathematics classroom is an important first step, and should be commended. However, further collaborative work is needed to help students operate in an inter-disciplinary way within the school or university, not only in the sciences but also in the humanities. Vocational mathematics education, in theory at least, should be the ideal setting for such collaboration (see, e.g., Bakker & Akkerman, 2014; Coben & Weeks, 2014), but it seems that, even here, long traditions of compartmentalisation are difficult to overcome.

Searching for common ground is just the beginning, and congratulations to Michael Fried and Tommy Dreyfus for their monumental efforts! Now, more than ever before, for the sake of the discipline and its survival supported by mathematics education at all levels, respectful interactions across boundaries are needed to continue the work already begun by the people who attended the symposium in 2012.

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