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Robust stability of a class of Networked Control Systems ⋆

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Abstract

Networked Control Systems (NCSs) affected with packet dropouts and scheduling are considered. The undesirable effects of packet dropouts and scheduling, such as instability or deteriorated performance, are addressed by application of a protocol and controller co-design method. The used method exploits Model Predictive Control (MPC) framework and the flexible NCS architecture which allows for distributed computation. Uniform Global Asymptotic Stability (UGAS) is established by assuming a finite bound on the number of consecutive packet dropouts and appropriate modifications to often adopted MPC stability-related assumptions. Two approaches that demonstrate UGAS are provided. The proof of one approach consists of finding an appropriate Lyapunov candidate function, while the other uses a cascade stability idea.

Key words: Uniform Global Asymptotic Stability, Networked Control System, Model Predictive Control, Packet dropouts, Scheduling

1 Introduction

Networked Control Systems (NCSs) use a *communication network* to send and/or receive control-related signals. Communication over a network may reduce price, volume and weight. Moreover, it can lead to a significant simplification of the installation, maintenance, troubleshooting and/or addition of new elements. On the other hand, many challenges arise due to intrinsic network communication phenomena such as packet dropouts, scheduling, delays, quantization, time-varying packet transmission and/or sampling intervals; e.g., see [26] and references therein.

The block diagram of the considered NCS architecture is depicted in Fig. 1. The corresponding network $\Sigma_{\mathfrak{n}}$ is such that it allows access to only one plant input at each time instant. This generates the need for scheduling since for multi-variable plants not all inputs can be simultaneously addressed in one time instant. Furthermore, the network is affected with packet dropouts $w_{\mathfrak{n}}$. Consequently, control values may get lost leading to performance deterioration and possibly instability. In order to mitigate the undesirable effects of packet dropouts and scheduling, a controller and protocol co-design method was proposed in [7]. Succinctly, the con-

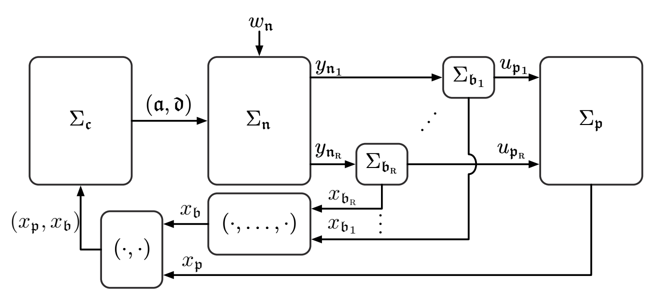

Fig. 1. An NCS architecture considered for stability.

sidered controller and protocol co-design method consists of two main parts. One part consists of employing a *Model Predictive Control (MPC)* framework to generate a sequence of *optimal predicted control values* over a *finite horizon* for an *optimal node (plant input)*. The other part consists of exploiting the *flexible architecture* of NCSs, which allows for *distributed computation*; see also [12, 16, 23, 25]. In the NCS architecture considered, the distributed computation comes from buffers $\Sigma_{\mathfrak{b}_\cdot}$. These are devices which perform very simple computation and have limited memory; more specifically, a parallel-in-serial-out shift register. As shown in [7, 21], the MPC framework enables to define an optimization problem

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which explicitly takes into account packet dropouts and scheduling.

From a theoretical point of view, the considered NCS architecture is important because it provides a platform for analysis of the undesirable effects of packet dropouts and scheduling. Practically, due to technological advancements in electronics and communication in the last decade, the considered NCS architecture is implementable. Namely, potential computational issues, due to on-line optimization, can often be addressed with current generation of microprocessors while the communication networks, wireless or not, have become much faster. It is important to note that even though the networks are much faster, the data frame (used for communication) may still be small, e.g., 8 bytes for a Control Area Network Bus. On the other hand, the data frame might be larger, e.g., 254 bytes for a FlexRay Bus, but only a small fraction of it might be available for control purposes. Thus, the issue of scheduling might be unavoidable. A simple example application captured with the considered NCS is the control of mobile robots in confined space. For instance, the position of robots can be measured with sensors directly connected to the controller while the communication between the controller and robots has to be done through a wireless network.

We consider the same NCS architecture as in [7, 21] and strengthen the stability characterization by establishing Uniform Global Asymptotic Stability (UGAS) of the augmented state of the plant and buffers state. The enabling assumption is the additional lower bound on the stage cost function in terms of control *only*, e.g., see [15]. Using this assumption and standard assumptions for showing stability in MPC setup combined with techniques from [3,5] and [10], was sufficient to establish the UGAS result. In fact, we prove the corresponding result in two ways. One way is more standard, where we find an appropriate Lyapunov function. The other one, perhaps more interesting, uses a cascade idea. The key fact that leads to this is the observation that the *overall* buffer (defined later in the document) as a system is Input-to-State Stable (ISS) with respect to the input being the arriving control sequence. Moreover, the overall buffer is UGAS uniformly in buffer states, e.g., ISS with zero gain. This enables one to consider the corresponding NCS as a cascade and use the corresponding stability analysis. Combining this observation with the fact that the plant trajectories converge (see [7]), our extra assumption on lower bound on the stage cost function in terms of control only and modification and extension of some results from [3,5] and [10] were sufficient to establish the result; please note that explicit consideration of buffer dynamics enabled to establish UGAS of the augmented state. The importance of this result is not only concentrated around strengthening the stability characterization from [7, 21] by establishing UGAS. Namely, UGAS is established via two approaches, both of which provide different insights. Further, both approaches required nontrivial modifications and appropriate application of several results documented in advanced nonlinear analysis literature, such as [3, 5, 10], which technically might be useful to other researchers..

Notation and mathematical preliminaries: Respectively $\mathbb{N}$, $\mathbb{Z}$ and $\mathbb{R}$, denote the set of natural, whole and real numbers. A set $\mathbb{D} \in \{\mathbb{N}, \mathbb{Z}, \mathbb{R}\}$ which is $\diamond$-bounded, $\diamond \in \{\leq, <, >, \geq\}$, by an element c, is denoted with $\mathbb{D}_{\diamond c} := \{\nu \in \mathbb{D} : \nu \diamond c, c \in \mathbb{D}\}$. Symbol $^\top$ is used for transposition while tuple notation denotes a column vector, e.g.: $(\nu_1, \ldots, \nu_i) := \left[\nu_1^\top \ldots \nu_i^\top\right]^\top$, $i \in \mathbb{N}$, where ν_j are vectors for each $j \in \{1, \ldots, i\}$. The origin element of the i-dimensional space of real numbers is denoted by 0^i, e.g.: $0^i := (0, \ldots, 0) \in \mathbb{R}^i$, $i \in \mathbb{N}$. The $i \times i$ identity matrix is denoted via $I_{i\times i} = I_i := \text{diag}(1, \ldots, 1)$ while $i \times i$ zero matrix is denoted via $0_{i\times i} = 0_i := \text{diag}(0, \ldots, 0)$. Sequence of elements is denoted as $\nu_j^k := \{\nu(i)\}_{i=j}^k$ if $j \leq k$ and, $\{\}$ if $j > k$, where $j \in \mathbb{Z}$ and $k \in \mathbb{Z}$. A set of infinite sequences initialized at index 0 whose elements take values from some set $\mathcal{V}$ is denoted as $\mathcal{S}^{\mathcal{V}} := \{\nu_0^\infty : \nu(i) \in \mathcal{V},\ \forall i \in \mathbb{N}_0\}$; $\mathbb{N}_0 := \mathbb{Z}_{\geq 0}$. A function $\rho : \mathbb{R}^n \to \mathbb{R}_{\geq 0}$ is said to be positive definite $\rho \in \mathcal{PD}$ (positive semi-definite $\rho \in \mathcal{PSD}$) with respect to $\nu = c$ if it is continuous, $\rho(c) = 0$ and $\rho(\nu) > 0$ ($\rho(\nu) \geq 0$) for all $\nu \neq c$. A function $\alpha : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ is said to be of class-$\mathcal{K}$ ($\alpha \in \mathcal{K}$) if it is continuous, $\alpha(0) = 0$, and strictly increasing. Further, a function $\alpha : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ is said to be of class-$\mathcal{K}_\infty$ ($\alpha \in \mathcal{K}_\infty$) if $\alpha \in \mathcal{K}$ and, in addition, $\lim_{\nu\to\infty} \alpha(\nu) = \infty$. Furthermore, a function $\sigma : \mathbb{R}_{\geq 0} \to \mathbb{R}_{>0}$ is said to be of class-$\mathcal{L}$ ($\sigma \in \mathcal{L}$), if it is continuous, strictly decreasing, and $\lim_{\nu\to\infty} \sigma(\nu) = 0$. Now, a function $\beta : \mathbb{R}_{\geq 0} \times \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ is said to be of class-$\mathcal{KL}$ ($\beta \in \mathcal{KL}$) if it is class-$\mathcal{K}$ in its first argument and class-$\mathcal{L}$ in its second argument; that is, $\beta \in \mathcal{KL}$ if for each fixed $\omega \in \mathbb{R}_{\geq 0}$, $\beta(\cdot, \omega) \in \mathcal{K}$ and for each fixed $\nu \in \mathbb{R}_{\geq 0}$, $\beta(\nu, \cdot) \in \mathcal{L}$.

2 NCS Architecture

The considered NCS architecture is depicted in Fig. 1. Due to the complexity of the considered system each part is presented separately. However, as the considered NCS architecture is documented in detail in [21], only the essential parts are presented.

2.1 Plant

Discrete-time plants, $\Sigma_\mathfrak{p} : x_\mathfrak{p}(k+1) = f_\mathfrak{p}(x_\mathfrak{p}(k), u_\mathfrak{p}(k))$, $k \in \mathbb{N}_0$, where $x_\mathfrak{p} \in \mathbb{R}^{n_\mathfrak{p}}$ is the state and $u_\mathfrak{p} \in \mathbb{R}^{m_\mathfrak{p}}$ is the input of the plant are considered. The mapping $f_\mathfrak{p} : \mathbb{R}^{n_\mathfrak{p}} \times \mathbb{R}^{m_\mathfrak{p}} \to \mathbb{R}^{n_\mathfrak{p}}$ is general nonlinear. Whenever appropriate, a succinct notation with respect to time is used, e.g., the system $\Sigma_\mathfrak{p}$ is written as

$$\Sigma_\mathfrak{p} : x_\mathfrak{p}^+ = f_\mathfrak{p}(x_\mathfrak{p}, u_\mathfrak{p}). \tag{1}$$

Without loss of generality, it is assumed that the system $\Sigma_{\mathfrak{p}}$ has an equilibrium at the origin, i.e., $x_{\mathfrak{p}}^e = f_{\mathfrak{p}}(x_{\mathfrak{p}}^e, u_{\mathfrak{p}}^e) = f_{\mathfrak{p}}(0^{n_{\mathfrak{p}}}, 0^{m_{\mathfrak{p}}}) = 0^{n_{\mathfrak{p}}}$. Furthermore, the concept of solution mapping for each difference equation is used; see [29]. Thus, in the case of (1), the solution of $\Sigma_{\mathfrak{p}}$, $j-i$ steps into the future, starting at initial condition $x_{\mathfrak{p}}$ at time instant $i \leq j$, under the influence of control input sequence $\{u_{\mathfrak{p}}(k)\}_{k=i}^{j-1}$ is denoted with $\phi_{f_{\mathfrak{p}}}\big(j-i, x_{\mathfrak{p}}, \{u_{\mathfrak{p}}(k)\}_{k=i}^{j-1}\big)$. Note that $\phi_{f_{\mathfrak{p}}}\big(0, x_{\mathfrak{p}}, \{\}\big) = x_{\mathfrak{p}} = x_{\mathfrak{p}}(0)$. Finally, the plant control input is *partitioned* according to (see Fig. 1) $u_{\mathfrak{p}} := (u_{\mathfrak{p}_1}, \ldots, u_{\mathfrak{p}_R})$ where $u_{\mathfrak{p}_r} \in \mathbb{R}^{m_{\mathfrak{p}_r}}$, $m_{\mathfrak{p}_r} \in \mathbb{N}$, $\forall r \in \mathcal{R}$, $\sum_{r=1}^{R} m_{\mathfrak{p}_r} = m_{\mathfrak{p}}$ and $\mathcal{R} := \{1, \ldots, R\}$.

2.2 Network

The considered network is packet based and its channels are modeled as *erasure channels*. The transmission effects are modeled as discrete dropout processes $\{w_{\mathfrak{n}}(k)\}_{k \in \mathbb{N}_0}$ where

$$w_{\mathfrak{n}}(k) := \begin{cases} 0, & \text{if dropout occurs at time } k, \\ 1, & \text{if dropout \textit{does not} occur at time } k. \end{cases}$$

The set of dropout outcomes is denoted with $\mathcal{D} := \{0, 1\}$ while the set consisting of all successful transmission time instants is defined as $\mathbb{K} := \{k \in \mathbb{N}_0 : w_{\mathfrak{n}}(k) = 1\}$. The elements of this set are denoted as k_i and they are numbered in ascending order. The number of consecutive packet dropouts between successful transmission instants k_i and k_{i+1} is defined as

$$\Delta k_i := k_{i+1} - k_i - 1. \tag{2}$$

At each time instant the network receives a *packet* π (sent by the controller). This packet is a tuple consisting of *address* and *data* field. In particular, $\pi := (\mathfrak{a}, \mathfrak{d}) \in \mathcal{R} \times \mathbb{R}^{L \cdot \bar{m}_{\mathfrak{p}}}$ where L is the length [1] of the buffer and $\bar{m}_{\mathfrak{p}} := \max\{m_{\mathfrak{p}_1}, \ldots, m_{\mathfrak{p}_R}\}$. The number of network outputs corresponds to the number of plant inputs. However, at each time instant only one of these outputs will be "active" with respect to the plant. Namely, if $r = \mathfrak{a}(k)$, then $y_{\mathfrak{n}_r}(k) = w_{\mathfrak{n}}(k)\mathfrak{d}(k)$. However, if $r \neq \mathfrak{a}(k)$, then $y_{\mathfrak{n}_r}(k)$ is "inactive" with respect to the plant.

2.3 Buffer

As depicted in Fig. 1 there is one buffer for each plant input, adding up to R buffers. These buffers are devices consisting of a memory unit and a simple processing unit. The dynamics of each buffer is captured by the dynamics of a linear switched system, namely $\Sigma_{x_{\mathfrak{b}_r}}$:

$$\begin{cases} x_{\mathfrak{b}_r}^+ := \begin{cases} S_{m_{\mathfrak{p}_r}} x_{\mathfrak{b}_r}, & r \neq \mathfrak{a} \wedge w_{\mathfrak{n}} = 0, \\ S_{m_{\mathfrak{p}_r}} \mathfrak{d}, & r = \mathfrak{a} \vee w_{\mathfrak{n}} = 1, \end{cases} \\ y_{\mathfrak{b}_r} := \begin{cases} [I^{m_{\mathfrak{p}_r}} \; 0^{m_{\mathfrak{p}_r}} \cdots 0^{m_{\mathfrak{p}_r}}] x_{\mathfrak{b}_r}, & r \neq \mathfrak{a} \wedge w_{\mathfrak{n}} = 0, \\ [I^{m_{\mathfrak{p}_r}} \; 0^{m_{\mathfrak{p}_r}} \cdots 0^{m_{\mathfrak{p}_r}}] \mathfrak{d}, & r = \mathfrak{a} \vee w_{\mathfrak{n}} = 1, \end{cases} \end{cases}$$

for each $r \in \mathcal{R}$, where

$$S_{m_{\mathfrak{p}_r}} := \begin{bmatrix} 0_{m_{\mathfrak{p}_r}} & I_{m_{\mathfrak{p}_r}} & 0_{m_{\mathfrak{p}_r}} & \cdots & \cdots & 0_{m_{\mathfrak{p}_r}} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0_{m_{\mathfrak{p}_r}} & \cdots & 0_{m_{\mathfrak{p}_r}} & I_{m_{\mathfrak{p}_r}} & \cdots & 0_{m_{\mathfrak{p}_r}} \\ 0_{m_{\mathfrak{p}_r}} & \cdots & \cdots & 0_{m_{\mathfrak{p}_r}} & \cdots & I_{m_{\mathfrak{p}_r}} \\ 0_{m_{\mathfrak{p}_r}} & \cdots & \cdots & \cdots & \cdots & 0_{m_{\mathfrak{p}_r}} \end{bmatrix}$$

is a *shift* matrix which belongs to $\mathbb{R}^{L \cdot m_{\mathfrak{p}_r} \times L \cdot m_{\mathfrak{p}_r}}$. (Note, $\forall r \in \mathcal{R}$, $y_{\mathfrak{b}_r} = u_{\mathfrak{p}_r}$, and thus, $y_{\mathfrak{b}} = u_{\mathfrak{p}}$) To simplify the presentation, the overall buffer is defined as $x_{\mathfrak{b}} := (x_{\mathfrak{b}_1}, \ldots, x_{\mathfrak{b}_R}) \in \mathbb{R}^{L \cdot m_{\mathfrak{p}}}$. (Note, $n_{\mathfrak{b}} := L \cdot m_{\mathfrak{p}}$.) Then, $\Sigma_{\mathfrak{b}}$ is described via

$$\left.\begin{aligned} x_{\mathfrak{b}}^+ &:= f_{\mathfrak{b}}(x_{\mathfrak{b}}, \mathfrak{a}, \mathfrak{d}, w_{\mathfrak{n}}) := \Xi(\mathfrak{a}, w_{\mathfrak{n}}) S x_{\mathfrak{b}} + S \Pi(\mathfrak{d}, \mathfrak{a}, w_{\mathfrak{n}}) \\ y_{\mathfrak{b}} &:= h_{\mathfrak{b}}(x_{\mathfrak{b}}, \mathfrak{a}, \mathfrak{d}, w_{\mathfrak{n}}) := \Gamma\Big(\Xi(\mathfrak{a}, w_{\mathfrak{n}}) x_{\mathfrak{b}} + \Pi(\mathfrak{d}, \mathfrak{a}, w_{\mathfrak{n}})\Big) \end{aligned}\right\} \tag{3}$$

where:

$$\begin{cases} \Xi(\mathfrak{a}, w_{\mathfrak{n}}) := \begin{cases} \operatorname{diag}(I_{L \cdot m_{\mathfrak{p}_1}}, \ldots, I_{L \cdot m_{\mathfrak{p}_{\mathfrak{a}-1}}}, \\ \qquad 0_{L \cdot m_{\mathfrak{p}_{\mathfrak{a}}}}, I_{L \cdot m_{\mathfrak{p}_{\mathfrak{a}+1}}}, \ldots, I_{L \cdot m_{\mathfrak{p}_R}}), & w_{\mathfrak{n}} = 1, \\ I_{L \cdot m_{\mathfrak{p}}}, & w_{\mathfrak{n}} = 0, \end{cases} \\ S := \operatorname{diag}(S_{m_{\mathfrak{p}_1}}, \ldots, S_{m_{\mathfrak{p}_R}}), \\ \Pi(\mathfrak{d}, \mathfrak{a}, w_{\mathfrak{n}}) := \begin{cases} (0^{L \cdot m_{\mathfrak{p}_1}}, \ldots, 0^{L \cdot m_{\mathfrak{p}_{\mathfrak{a}-1}}}, \\ \qquad \mathfrak{d}, 0^{L \cdot m_{\mathfrak{p}_{\mathfrak{a}+1}}}, \ldots, 0^{L \cdot m_{\mathfrak{p}_R}}) & w_{\mathfrak{n}} = 1, \\ 0^{L \cdot m_{\mathfrak{p}}}, & w_{\mathfrak{n}} = 0, \end{cases} \\ \Gamma := [\varsigma_1 \; \cdots \; \varsigma_R], \text{ where } \forall r \in \mathcal{R}, \end{cases}$$

$$\varsigma_r := \begin{bmatrix} 0_{m_{\mathfrak{p}_1}} & 0_{m_{\mathfrak{p}_1}} & \cdots & 0_{m_{\mathfrak{p}_1}} \\ \vdots & \vdots & \cdots & \vdots \\ 0_{m_{\mathfrak{p}_{r-1}}} & 0_{m_{\mathfrak{p}_{r-1}}} & \cdots & 0_{m_{\mathfrak{p}_{r-1}}} \\ I_{m_{\mathfrak{p}_r}} & 0_{m_{\mathfrak{p}_r}} & \cdots & 0_{m_{\mathfrak{p}_r}} \\ 0_{m_{\mathfrak{p}_{r+1}}} & 0_{m_{\mathfrak{p}_{r+1}}} & \cdots & 0_{m_{\mathfrak{p}_{r+1}}} \\ \vdots & \vdots & \cdots & \vdots \\ 0_{m_{\mathfrak{p}_R}} & 0_{m_{\mathfrak{p}_R}} & \cdots & 0_{m_{\mathfrak{p}_R}} \end{bmatrix} \in \mathbb{R}^{m_{\mathfrak{p}} \times L \cdot m_{\mathfrak{p}_r}}.$$

2.4 Controller

An essential part of an MPC controller is the model it uses to generate optimal control predictions; e.g., see [18]. For the ease of presentation, all plant inputs are

[1] The number of memory spaces.

assumed to have the same dimension. This enables to consider only a node $r \in \mathcal{R}$ instead of each node independently. The corresponding nominal prediction model becomes

$$\Sigma_{\mathfrak{p}}^{\mathfrak{m}}: \begin{cases} \tilde{x}_{\mathfrak{p}}(k+i+1) := f_{\mathfrak{p}}(\tilde{x}_{\mathfrak{p}}(k+i), \tilde{u}_{\mathfrak{p}}^{r}(k+i)), \\ \tilde{x}_{\mathfrak{p}}(k) = x_{\mathfrak{p}}(k),\ k \in \mathbb{N}_0,\ i \in \{0, \ldots, \mathfrak{h}-1\} \end{cases} \tag{4}$$

where $\mathfrak{h} \in \mathbb{N}$ is a *finite horizon.* Note that at each time instant k, model (4) is initialized with the corresponding measurement of the plant state $\tilde{x}_{\mathfrak{p}}(k) = x_{\mathfrak{p}}(k)$ (and the overall buffer state $\tilde{x}_{\mathfrak{b}}(k) = x_{\mathfrak{b}}(k)$, which is explained in the sequel). Moreover, note that in each of these optimization problems we can only minimize over the control values of the corresponding node while control values of other nodes are fixed. This is denoted by $\tilde{u}_{\mathfrak{p}}^{r} \in \mathbb{R}^{m_{\mathfrak{p}}}$ which is defined as

$$\tilde{u}_{\mathfrak{p}}^{r} := \begin{bmatrix} \Gamma_{m_1}\left(\Xi(r, \tilde{w}_{\mathfrak{n}})\tilde{x}_{\mathfrak{b}} + \Pi(\{\tilde{u}_{\mathfrak{p}_r}\}_k^{k+\mathfrak{h}-1}, r, \tilde{w}_{\mathfrak{n}})\right) \\ \vdots \\ \Gamma_{m_{r-1}}\left(\Xi(r, \tilde{w}_{\mathfrak{n}})\tilde{x}_{\mathfrak{b}} + \Pi(\{\tilde{u}_{\mathfrak{p}_r}\}_k^{k+\mathfrak{h}-1}, r, \tilde{w}_{\mathfrak{n}})\right) \\ \tilde{u}_{\mathfrak{p}_r} \\ \Gamma_{m_{r+1}}\left(\Xi(r, \tilde{w}_{\mathfrak{n}})\tilde{x}_{\mathfrak{b}} + \Pi(\{\tilde{u}_{\mathfrak{p}_r}\}_k^{k+\mathfrak{h}-1}, r, \tilde{w}_{\mathfrak{n}})\right) \\ \vdots \\ \Gamma_{m_R}\left(\Xi(r, \tilde{w}_{\mathfrak{n}})\tilde{x}_{\mathfrak{b}} + \Pi(\{\tilde{u}_{\mathfrak{p}_r}\}_k^{k+\mathfrak{h}-1}, r, \tilde{w}_{\mathfrak{n}})\right) \end{bmatrix}$$

where:

$$\begin{cases} \Sigma_{\mathfrak{b}}^{\mathfrak{m}}: \tilde{x}_{\mathfrak{b}}^{+} := f_{\mathfrak{b}}(\tilde{x}_{\mathfrak{b}}, r, \{\tilde{u}_{\mathfrak{p}_r}\}_k^{k+\mathfrak{h}-1}, \tilde{w}_{\mathfrak{n}}),\ \tilde{x}_{\mathfrak{b}} = x_{\mathfrak{b}}, \\ \tilde{u}_{\mathfrak{p}_r} \in \{\tilde{u}_{\mathfrak{p}_r}\}_k^{k+\mathfrak{h}-1}, \\ \tilde{w}_{\mathfrak{n}} \in \{\tilde{w}_{\mathfrak{n}}\}_k^{k+\mathfrak{h}-1} := \{1, 0, \ldots, 0\}, \\ \Gamma = \left[\Gamma_{m_{\mathfrak{p}_1}}^{\top} \cdots \Gamma_{m_{\mathfrak{p}_R}}^{\top}\right]^{\top}, \Gamma_{m_{\mathfrak{p}_r}} \in \mathbb{R}^{m_{\mathfrak{p}_r} \times \mathfrak{h} \cdot m_{\mathfrak{p}}}, \forall r \in \mathcal{R}. \end{cases}$$

Remark 1 (Overall buffer state) *Control predictions depend on the content of the overall buffer, which are assumed directly available. Instead of this approach one might use acknowledgments of receipt. Another approach is to adopt a stochastic control framework as in [1, 17], or use a control which accounts for all transmission scenarios as in [6].* □

For presentation purposes it is convenient to introduce the "NCS state" as $x := (x_{\mathfrak{p}}, x_{\mathfrak{b}}) \in \mathbb{R}^{n_{\mathfrak{p}} \times \mathfrak{h} \cdot m_{\mathfrak{p}}}$. The cost function is then defined as

$$J(x, \{\tilde{u}_{\mathfrak{p}_r}\}_k^{k+\mathfrak{h}-1}) := g(\phi_{f_{\mathfrak{p}}}(\mathfrak{h}, x_{\mathfrak{p}}, \{\tilde{u}_{\mathfrak{p}}^{r}\}_k^{k+\mathfrak{h}-1})) + \sum_{i=k}^{k+\mathfrak{h}-1} l(\phi_{f_{\mathfrak{p}}}(i-k, x_{\mathfrak{p}}, \{\tilde{u}_{\mathfrak{p}}^{r}\}_k^{k+\mathfrak{h}-1}), \tilde{u}_{\mathfrak{p}}^{r}(i)) \tag{5}$$

where $l : \mathbb{R}^{n_{\mathfrak{p}}} \times \mathbb{R}^{m_{\mathfrak{p}}} \to \mathbb{R}_{\geq 0}$ is a stage cost function and $g : \mathbb{R}^{n_{\mathfrak{p}}} \to \mathbb{R}_{\geq 0}$ is a terminal cost function. (Recall that there are R nodes, resulting in R cost functions.)

The sequence of optimally predicted control values for the corresponding node $\{u_{\mathfrak{p}_r}^{*}\}_k^{k+\mathfrak{h}-1}$ is obtained via

$$V(x, \{u_{\mathfrak{p}_r}^{*}\}_k^{k+\mathfrak{h}-1}) := \min_{\{\tilde{u}_{\mathfrak{p}_r}\}_k^{k+\mathfrak{h}-1}} J(x, \{\tilde{u}_{\mathfrak{p}_r}\}_k^{k+\mathfrak{h}-1}).$$

The sequence of optimally predicted control values that produces the minimal cost value is obtained via

$$V(x) = V(x, \{u_{\mathfrak{p}_{r^*}}^{*}\}_k^{k+\mathfrak{h}-1}) := \min_{r} V(x, \{u_{\mathfrak{p}_r}^{*}\}_k^{k+\mathfrak{h}-1}). \tag{6}$$

Extracting the values for the optimal node and its corresponding sequence of optimal control values reduces to

$$\begin{cases} r^{*} := \operatorname{argmin}_r V(x, \{u_{\mathfrak{p}_r}^{*}\}_k^{k+\mathfrak{h}-1}), \\ \{u_{\mathfrak{p}_{r^*}}^{*}\}_k^{k+\mathfrak{h}-1} := \operatorname{argmin}_{\{\tilde{u}_{\mathfrak{p}_{r^*}}\}_k^{k+\mathfrak{h}-1}} J(x, \{\tilde{u}_{\mathfrak{p}_{r^*}}\}_k^{k+\mathfrak{h}-1}). \end{cases}$$

The packet that the controller sends is then formed as

$$\pi = (\mathfrak{a}, \mathfrak{d}) := (r^{*}, (u_{\mathfrak{p}_{r^*}}^{*}(k), \ldots, u_{\mathfrak{p}_{r^*}}^{*}(k+\mathfrak{h}-1)).$$

Finally, the closed-loop system Σ_x is given as

$$\begin{aligned} x^{+} &= \begin{bmatrix} f_{\mathfrak{p}}(x_{\mathfrak{p}}, h_{\mathfrak{b}}(x_{\mathfrak{b}}, r^{*}, \{u_{r^*}^{*}\}_k^{k+\mathfrak{h}-1}, w_{\mathfrak{n}})) \\ f_{\mathfrak{b}}(x_{\mathfrak{b}}, r^{*}, \{u_{r^*}^{*}\}_k^{k+\mathfrak{h}-1}, w_{\mathfrak{n}}) \end{bmatrix} \\ &=: f(x, w_{\mathfrak{n}}). \end{aligned} \tag{7}$$

Remark 2 (Computational issues) *Note that R optimization problems that the controller needs to solve can be carried out in parallel. Thus, the computational burden scales linearly with the number of plant inputs (nodes) leading to computation times comparable to those of regular MPC.* □

Remark 3 (Network in feedback) *Within the used scheme, one could use state observers with intermittent observations [4, 8, 9, 20]. Alternatively, one could also restrict the controller to only calculate control sequences at instances of successful sensor-data receptions.* □

3 Uniform Global Asymptotic Stability

First, assumptions necessary for further presentation due to notation are presented.

3.1 Assumptions

Reasons for packet dropouts may include excessive (infinite) delays, packet collisions, traffic congestion and/or failed transmissions. Moreover, they are inevitable, see [2,11,13,19], and, they occur *randomly.* Hence, imposing any deterministic finite bound on the number of *consecutive* packet dropouts may seem unrealistic. However, the networks are designed to have a high throughput, thus, from a practical point of view (and experience), dropouts often occur with low probability. In these cases it makes sense to assume a deterministic finite bound on the number of *consecutive* packet dropouts.

Assumption 1 (Packet dropouts) *There exists*[2] $B \in \mathbb{N}$ *such that* $B \leq L$ *and*[3] $\Delta k_i \leq B-1$ *for each* $k_i \in \mathbb{N}_0$. □

Due to the former assumption, the definition of a set consisting of *successful transmission instants* such that the number of consecutive packet dropouts is at most $B-1$, changes to $\mathbb{K}_B := \{k_i \in \mathbb{K} : \Delta k_i \leq B-1\}$. Next, a set consisting of all infinite sequences of dropout outcomes satisfying Assumption 1 becomes $\mathcal{S}^{\mathcal{D}}_{\mathbb{K}_B} := \{\{w_\mathfrak{n}\}_0^\infty \in \mathcal{S}^{\mathcal{D}} : k_i \in \mathbb{K}_B\}$. Furthermore, note the following equality

$$\begin{aligned}\{w_\mathfrak{n}\}_{k_i}^{k_{i+1}-1} &= \{w_\mathfrak{n}\}_{k_i}^{k_i+B} \\ &= \{w_\mathfrak{n}(k_i), w_\mathfrak{n}(k_i+1), \ldots, w_\mathfrak{n}(k_{i+1}-1)\} \\ &= \{1, 0, \ldots, 0\},\end{aligned} \tag{8}$$

which enables to write $\forall \{w_\mathfrak{n}\}_0^k \subset \{w_\mathfrak{n}\}_0^\infty \in \mathcal{S}^{\mathcal{D}}_{\mathbb{K}_B}$, $\{w_\mathfrak{n}\}_0^k = \{\{w_\mathfrak{n}\}_0^{k_0-1}, \{w_\mathfrak{n}\}_{k_0}^{k_1-1}, \ldots, \{w_\mathfrak{n}\}_{k_i}^{k_{i+1}-1}, \{w_\mathfrak{n}\}_{k_{i+1}}^{k_{i+2}-1}, \ldots, \{w_\mathfrak{n}\}_{k_{j-1}}^{k_j-1}, \{w_\mathfrak{n}\}_{k_j}^{k}\}$. Before proceeding with MPC related assumptions, a remark regarding the prediction *horizon* is stated.

Remark 4 (Prediction horizon) *Prediction horizon in MPC is assumed greater or equal than the* bound *on the number of consecutive packet dropouts* B. *However, for the presentation purposes it is set equal to it,* $\mathfrak{h} := B$, $\forall r \in \mathcal{R}$. *Practically, one would set the prediction horizon to be sufficiently long, and then would claim the stability for a situation where the actual bound on the number of consecutive packet dropouts is smaller than this horizon. Indeed, increasing the horizon does require larger buffers and more computation but that is something one has control over.* □

The first MPC assumption is concerned with a stage and a terminal cost function, see (5).

Assumption 2 (Cost functions) *There exist class-*$\mathcal{K}_\infty$ *functions* $\alpha_{x_\mathfrak{p}}$ *and* $\alpha_{u_\mathfrak{p}}$ *such that*

$$\begin{cases} l(x_\mathfrak{p}, u_\mathfrak{p}) \geq \alpha_{x_\mathfrak{p}}(|x_\mathfrak{p}|), \\ l(x_\mathfrak{p}, u_\mathfrak{p}) \geq \alpha_{u_\mathfrak{p}}(|u_\mathfrak{p}|), \\ l(0^{n_\mathfrak{p}}, 0^{m_\mathfrak{p}}) = 0, \end{cases} \quad \text{and} \quad \begin{cases} g(x_\mathfrak{p}) \geq 0, \\ g(0^{n_\mathfrak{p}}) = 0, \end{cases}$$

for each $x_\mathfrak{p} \in \mathbb{R}^{n_\mathfrak{p}}$ *and each* $u_\mathfrak{p} \in \mathbb{R}^{m_\mathfrak{p}}$. □

[2] Recall that $L \in \mathbb{N}$ is the length of a buffer.
[3] Definition of Δk_i is provided in (2).

Remark 5 ("Extra" lower bound) *Additional lower bound on the stage cost function strengthens a standard stability assumption; for instance, see [15].* □

The following assumption is a modified "standard" stability-related assumption.

Assumption 3 (Terminal control law) *For some node* $r \in \mathcal{R}$ *there exists a* terminal control law $\kappa_r : \mathbb{R}^{n_\mathfrak{p} \times \mathfrak{h} \cdot m_\mathfrak{p}} \to \bar{\mathbb{U}}_{\mathfrak{p}_r}$ *such that*

$$\begin{cases} g(f_\mathfrak{p}(x_\mathfrak{p}, \kappa_r(x_\mathfrak{p}, x_\mathfrak{b}))) - g(x_\mathfrak{p}) + l(x_\mathfrak{p}, \kappa_r(x_\mathfrak{p}, x_\mathfrak{b})) \leq 0, \\ f_\mathfrak{p}(x_\mathfrak{p}, \kappa_r(x_\mathfrak{p}, x_\mathfrak{b})) \in \mathbb{R}^{n_\mathfrak{p}}, \\ \kappa_r(x_\mathfrak{p}, x_\mathfrak{b}) \in \bar{\mathbb{U}}_{\mathfrak{p}_r}, \end{cases}$$

holds for each $(x_\mathfrak{p}, x_\mathfrak{b}) \in \mathbb{R}^{n_\mathfrak{p} \times \mathfrak{h} \cdot m_\mathfrak{p}}$ *where* $\bar{\mathbb{U}}_{\mathfrak{p}_r} := 0^{m_{\mathfrak{p}_1}} \times \cdots \times 0^{m_{\mathfrak{p}_{r-1}}} \times \mathbb{R}^{m_{\mathfrak{p}_r}} \times 0^{m_{\mathfrak{p}_{r+1}}} \times \cdots \times 0^{m_{\mathfrak{p}_R}}$. □

Please note that the latter assumption is conservative as it requires knowledge of a global Control Lyapunov Function. This requirement is justified by seeking to establish a global result. Relaxation of the latter assumption is left for future work. The final assumption is related to the optimal value function.

Assumption 4 (Optimal value function) *There exists a class-*$\mathcal{K}$ *function* γ_V *such that* $V(x) \leq \gamma_V(|x|)$ *holds for each* $x = (x_\mathfrak{p}, x_\mathfrak{b}) \in \mathbb{R}^{n_\mathfrak{p} \times \mathfrak{h} \cdot m_\mathfrak{p}}$. □

Notice, that the latter bound can be ensured by assuming asymptotic controllability as in Section III of [14].

3.2 Results

First, UGAS of the system Σ_x (see (7)) is defined.

Definition 1 (UGAS) *Consider the system* Σ_x *(see (7)). We say that* Σ_x *is UGAS if there exists a class-*$\mathcal{KL}$ *function* β *such that for each* $x(\bar{k}_0) = x$, *each* $\bar{k}_0$ *and each* $\{w_\mathfrak{n}\}_{\bar{k}_0}^{k-1} \subset \{w_\mathfrak{n}\}_0^\infty \in \mathcal{S}^{\mathcal{D}}_{\mathbb{K}_B}$ *the following holds* $|\phi_f(k-\bar{k}_0, x, \{w_\mathfrak{n}\}_{\bar{k}_0}^{k-1})| \leq \beta(|x|, k-\bar{k}_0)$ *for each* $k \geq \bar{k}_0 \geq 0$. □

To establish UGAS of the system Σ_x according to Definition 1, we first establish that the corresponding system is *UGAS at successful transmission instants from set* $\mathbb{K}_B$ (defined in the sequel). Then, we address the "inter-sample" behavior (see Fig. 2). This idea originates from [10, Theorem 2]. First, we define *UGAS at successful transmission instants from the set* $\mathbb{K}_B$ (UGAS-$\mathbb{K}_B$) of the system Σ_x (see (7)).

Definition 2 (UGAS-$\mathbb{K}_B$) *Consider the system* Σ_x *(see (7)). We say that* Σ_x *is UGAS at successful transmission instances from the set* $\mathbb{K}_B$ *if there exist a class-*$\mathcal{KL}$ *function* β *such that for any initial condition*

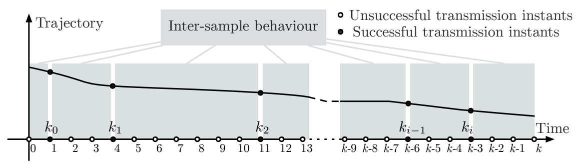

Fig. 2. Illustration of what we refer to as inter-sample behavior; k denotes current discrete time.

$x(k_0) = x$, any k_0 and any $\{w_{\mathfrak{n}}\}_{k_0}^{k_i-1} \subset \{w_{\mathfrak{n}}\}_0^{\infty} \in \mathcal{S}_{\mathbb{K}_B}^{\mathcal{D}}$ we have $|\phi_f(k_i - k_0, x, \{w_{\mathfrak{n}}\}_{k_0}^{k_i-1})| \leq \beta(|x|, k_i - k_0)$, for all $k_i \in \mathbb{K}_B$. □

Next, we define *Uniform Global Boundedness over B* (UGB-B) of the system Σ_x (see (7)), which can be seen as a counterpart to a discrete-time system being UGBT in [10]. Sufficient conditions for UGB-B can be obtained by following the steps provided in Lemma 3 and 4 in [10].

Definition 3 (UGB-B) *Consider the system Σ_x (see (7)). We say the solutions of the system Σ_x are Uniformly Globally Bounded over B if there exist a class-$\mathcal{K}_\infty$ function γ_B such that $|\phi_f(k - \bar{k}_0, x, \{w_{\mathfrak{n}}\}_{\bar{k}_0}^{k-1})| \leq \gamma_B(|x|)$ for all $\bar{k}_0 \in \{0, \ldots, k_i + \Delta k_i\}$, $k_i \in \mathbb{K}_B$ and $\{w_{\mathfrak{n}}\}_{\bar{k}_0}^{k-1} \subset \{w_{\mathfrak{n}}\}_0^{\infty} \in \mathcal{S}_{\mathbb{K}_B}^{\mathcal{D}}$ where $k \in \{\bar{k}_0, \ldots, \bar{k}_0 + B\}$.* □

The last two definitions enable us to state the main technical result which we use to establish UGAS of the NCS.

Lemma 1 *The system Σ_x (see (7)) is UGAS, if the following conditions hold:*

(1) The system Σ_x is UGAS-$\mathbb{K}_B$,
(2) The solutions of the system Σ_x are Uniformly Globally Bounded over B (UGB-B). □

Proof: follows the proof of Theorem 2 in [10]. □

Establishing UGAS-$\mathbb{K}_B$ is essential. In what follows we will establish this property via two approaches because of the different insight they provide. We will first present an approach which uses the results on stability of cascaded systems. This approach reveals how one can exploit the fact that the overall buffer as a system is ISS with respect to the input being the arriving control sequence. Moreover, the application and modification of techniques used in the proofs are interesting and useful since they provide an alternative approach for addressing packet dropouts and scheduling. This approach will be followed by a "standard" approach in which one constructs an appropriate Lyapunov function [4]. The merit of this approach is the corresponding construction.

[4] Notice that this approach is also documented in [22] and we include it here for completeness of the presentation.

3.2.1 *Cascade approach*

In order to establish UGAS-$\mathbb{K}_B$, we use and/or modify several results. First, we exploit the result that establishes the equivalence between "$\mathcal{KL}$ stability with respect to two measures" on one hand, and "uniform stability and global boundedness" and "uniform global attractivity" on the other hand; for more details, please see [3] for the continuous-time case and [5] for the discrete-time case. The mentioned result will be modified and used for plant state and control trajectories. Then, by using the fact that the overall buffer is ISS with respect to the input being the arriving control sequence, we establish an appropriate bound on the buffer trajectories. Moreover, the fact that the overall buffer is UGAS uniformly in buffer states (i.e., ISS with zero gain), allows us to regard the corresponding system as a cascade (although in principle it is a feedback connection). This further enables us to adopt stability proofs for cascade systems, such as the one provided in [27], to establish UGAS-$\mathbb{K}_B$ of the system Σ_x (see (7)). We begin with a modification of the definition of $\mathcal{KL}$ stability with respect to two measures from [5].

Definition 4 (Modified Definition 2.1 from [5]) *Let $\mathcal{PSD} \ni \rho_i : \mathbb{R}^{n_{\mathfrak{p}}} \times \mathbb{R}^{\mathfrak{h} \cdot m_{\mathfrak{p}}} \to \mathbb{R}_{\geq 0}$, $i \in \{1, 2\}$. Consider the system Σ_x (see (7)). We say that Σ_x is $\mathcal{KL}$-stable with respect to (ρ_1, ρ_2) if there exist a class-$\mathcal{KL}$ function β such that for each initial condition $x(k_0) = x$, each k_0 and any $\{w_{\mathfrak{n}}\}_{k_0}^{k_i-1} \subset \{w_{\mathfrak{n}}\}_0^{\infty} \in \mathcal{S}_{\mathbb{K}_B}^{\mathcal{D}}$ the following inequality holds $\rho_1(\phi_f(k_i - k_0, x, \{w_{\mathfrak{n}}\}_{k_0}^{k_i-1})) \leq \beta(\rho_2(x), k_i - k_0)$ for all $k_i \in \mathbb{K}_B$.* □

The following result establishes the equivalence.

Proposition 1 *Let $\mathcal{PSD} \ni \rho_i : \mathbb{R}^{n_{\mathfrak{p}}} \times \mathbb{R}^{\mathfrak{h} \cdot m_{\mathfrak{p}}} \to \mathbb{R}_{\geq 0}$, $i \in \{1, 2\}$. Consider the system Σ_x (see (7)). The following are equivalent:*

- *The system Σ_x is $\mathcal{KL}$-stable with respect to (ρ_1, ρ_2),*
- *The following two properties hold:*
  1. Uniform stability and global boundedness: *There exists a class-$\mathcal{K}_\infty$ function γ such that for each initial condition $x(k_0) = x$, each k_0 and any $\{w_{\mathfrak{n}}\}_{k_0}^{k_i-1} \subset \{w_{\mathfrak{n}}\}_0^{\infty} \in \mathcal{S}_{\mathbb{K}_B}^{\mathcal{D}}$ it holds*

$$\rho_1(\phi_f(k_i - k_0, x, \{w_{\mathfrak{n}}\}_{k_0}^{k_i-1})) \leq \gamma(\rho_2(x)), \quad (9)$$

  for each $k_i \in \mathbb{K}_B$,
  2. Uniform global attractivity: *For each $\delta > 0$ and $\epsilon > 0$, there exists $K(\delta, \epsilon) \geq 0$ such that that for each initial condition $x(k_0) = x$, each k_0 and any $\{w_{\mathfrak{n}}\}_{k_0}^{k_i-1} \subset \{w_{\mathfrak{n}}\}_0^{\infty} \in \mathcal{S}_{\mathbb{K}_B}^{\mathcal{D}}$ it holds*

$$\left.\begin{array}{r}\rho_2(x) \leq \delta \\ k_i - k_0 \geq K(\delta, \epsilon)\end{array}\right\} \Rightarrow \rho_1(\phi_f(k_i - k_0, x, \{w_{\mathfrak{n}}\}_{k_0}^{k_i-1})) \leq \epsilon$$

  for each $k_i \in \mathbb{K}_B$. □

Proof: follows the same lines as the proof of [5]. □

Remark 6 *The latter proposition originates from [5] and [3]. Note that in these references, functions ρ_1 and ρ_2 (denoted, respectively, as σ_1 and σ_2) are required to be continuous but we require only positive semi-definiteness. Additionally, note that Proposition 2.2 in [5] (and our Proposition 1), in its proof do not use the continuity of ρ_1 and ρ_2. Moreover, note that results in [5] concentrate on difference inclusion and indeed, even in our case, due to scheduling, we have a difference inclusion. However, due to the MPC controller, which at each time instant chooses only* one *optimal node and the corresponding control sequence, our corresponding difference inclusion reduces to a difference equation, see (7). Hence, the reason why we use it in Proposition 1.* □

Having stated Proposition 1, next, we study "uniform stability and global boundedness" and "uniform global attractivity". First, we establish two lemmas that are needed for showing the mentioned properties.

Lemma 2 *Let Assumptions 1, 2 and 3 be satisfied. Then*[5]

$$\Delta V(x(k_i)) := V(\phi_f(\Delta k_i + 1, x, \{w_\mathfrak{n}\}_{k_i}^{k_i+\Delta k_i})) - V(x) \leq - \sum_{j=k_i}^{k_i+\Delta k_i} l(\phi_{f_\mathfrak{p}}(j - k_i, x_\mathfrak{p}, \{u_\mathfrak{p}^{r^*}\}_{k_i}^{k_{i+1}}), u_\mathfrak{p}^{r^*}(j)), \quad (10)$$

holds for each $x(k_i) = x$ and each k_i. □

Proof: follows very similar steps as the proof of Theorem 3 in [7] and some parts of Lemma 2 in [21]. □

Lemma 3 *Let Assumptions 1, 2 and 3 be satisfied. Then $\Delta V(x(k_i)) := V(\phi_f(\Delta k_i + 1, x, \{w_\mathfrak{n}\}_{k_i}^{k_i+\Delta k_i})) - V(x) \leq -\sum_{j=k_i}^{k_i+\Delta k_i} \alpha_{u_\mathfrak{p}}(|u_\mathfrak{p}^{r^*}(i)|)$ holds for each $x(k_i) = x$ and each k_i.* □

Proof: follows almost identical lines[6] as the proof of Lemma 2. □

Finally, we have all the necessary ingredients to state lemmas that establish an appropriate $\mathcal{KL}$ bound on the plant state and control trajectories, respectively.

Lemma 4 *Let conditions of Lemma 2 and Assumption 4 be satisfied. Then, the system Σ_x (see (7)) is $\mathcal{KL}$-stable with respect to (ρ_1, ρ_2) where $\rho_1(x(\cdot)) := \alpha_{x_\mathfrak{p}}(|x_\mathfrak{p}(\cdot)|)$ and $\rho_2(\bullet) := |\bullet|$.* □

Proof: see Appendix A. □

[5] Note that we write $u_\mathfrak{p}^{r^*}$ instead of $u_\mathfrak{p}^{*r^*}$ to simplify notation.

[6] Namely, when applying Assumption 2, we employ the lower bound in terms of the plant control input

Lemma 5 *Let conditions of Lemma 3 and Assumption 4 be satisfied. Then, the system Σ_x (see (7)) is $\mathcal{KL}$-stable with respect to (ρ_1, ρ_2) where $\rho_1(x(\cdot)) := \alpha_{u_\mathfrak{p}}(|u_\mathfrak{p}(\cdot)|)$ and $\rho_2(\bullet) := |\bullet|$.* □

Proof: follows the same lines as the proof of Lemma 4, see Appendix A. □

Before stating the result that establishes UGAS-$\mathbb{K}_B$, we need a lemma that establishes Input-to-State Stable (ISS) of the overall buffer (see (3)) with respect to the input being the data $\mathfrak{d}$ sent by the controller. This lemma is stated next.

Lemma 6 *Consider the overall buffer given by (3) and consider any sequence of dropout outcomes $\{w_\mathfrak{n}\}_{k_0}^{k_i} \subset \{w_\mathfrak{n}\}_0^\infty \in \mathcal{S}_{\mathbb{K}_B}^{\mathcal{D}}$. Then, the overall buffer is ISS with respect to the input being the data $\mathfrak{d}$. Namely, it follows that $|x_\mathfrak{b}(k_i)| \leq \beta(|x_\mathfrak{b}(k_j)|, k_i - k_j) + \gamma(\sup_{k_j \leq \tau \leq k_i} |\mathfrak{d}(\tau)|)$ where $\beta \in \mathcal{KL}$, $\gamma \in \mathcal{K}$ and $k_i \geq k_j \geq k_0$; also, recall that $\mathfrak{d}(\tau) = (u_{\mathfrak{p}_{r^*}}^*(\tau), \ldots, u_{\mathfrak{p}_{r^*}}^*(\tau + B - 1))$.* □

Proof: see Appendix B. □

Finally, we are ready to state the theorem that establishes UGAS-$\mathbb{K}_B$ using this cascade-based approach.

Theorem 1 (UGAS-$\mathbb{K}_B$ – Cascade approach) *Consider the system Σ_x given by (7). Let the following conditions be satisfied:*

- **(Lemma 4)** *The system Σ_x is $\mathcal{KL}$-stable with respect to $(\rho_1(x(\cdot)) := \alpha_{x_\mathfrak{p}}(|x_\mathfrak{p}(\cdot)|), \rho_2(\bullet) := |\bullet|)$,*
- **(Lemma 5)** *The system Σ_x is $\mathcal{KL}$-stable with respect to $(\rho_1(x(\cdot)) := \alpha_{u_\mathfrak{p}}(|u_\mathfrak{p}(\cdot)|), \rho_2(\bullet) := |\bullet|)$,*
- **(Lemma 6)** *The overall buffer (3) is ISS with respect to the input being the data $\mathfrak{d}$.*

Then the system Σ_x is UGAS-$\mathbb{K}_B$. □

Proof: see Appendix C. □

3.2.2 Lyapunov approach

In the previous approach we established UGAS-$\mathbb{K}_B$ of the system Σ_x (see (7)) by focusing on trajectories of the controller, the overall buffer and the plant. This led to an approach that is not so common in the NCS literature. In contrast, in this section, we use a more standard approach of showing stability, namely, by constructing an appropriate Lyapunov function; this function is provided in the proof of the following theorem.

Theorem 2 (UGAS-$\mathbb{K}_B$ – Lyapunov approach) *Consider the system Σ_x given by (7). Consider any sequence of dropout outcomes $\{w_\mathfrak{n}\}_{k_0}^{k_i-1} \subset \{w_\mathfrak{n}\}_0^\infty \in \mathcal{S}_{\mathbb{K}_B}^{\mathcal{D}}$. Let the conditions of Lemma 2 and 5 be satisfied. Further, suppose that*

- **(Lemma 3)** *Dissipation inequality,* $\Delta V(x(k_i)) := V(\phi_f(\Delta k_i+1, x, \{w_{\mathfrak{n}}\}_{k_i}^{k_i+\Delta k_i})) - V(x) \leq -\sum_{j=k_i}^{k_i+\Delta k_i} \alpha_{x_{\mathfrak{p}}}(|\phi_{f_{\mathfrak{p}}}(j-k_i, x_{\mathfrak{p}}, \{u_{\mathfrak{p}}^{r^*}\}_{k_i}^{k_i+\Delta k_i})|)$ *holds for each* $x(k_i) = x$ *and each* k_i,
- **(Lemma 5)** *The system* Σ_x *is* $\mathcal{KL}$*-stable with respect to* $(\rho_1(x(\cdot)) := \alpha_{u_{\mathfrak{p}}}(|u_{\mathfrak{p}}(\cdot)|), \rho_2(\bullet) := |\bullet|)$.

Then the system Σ_x *is UGAS-*$\mathbb{K}_B$. □

Proof: see Appendix D. □

Remark 7 (Extensions to Economic MPC) *It is possible to extend some results from [24] so that the presented UGAS result for standard MPC can be used "off the shelf" to establish UGAS of a NCS governed by Economic MPC; for details see [28].*

4 Conclusion

We considered an NCS architecture in which control signals are sent over a network. The issues of packet dropout and scheduling are induced by the network and are mitigated through a protocol and controller co-design method. UGAS is established by making appropriate assumptions on the number of consecutive packet dropouts and bounds on stage and terminal cost functions. This property is demonstrated via two approaches, namely, by constructing an appropriate Lyapunov function and by exploiting the structure of NCS and using cascade stability idea. Future challenges include addressing more network-induced communication issues (e.g., delays and quantization) and extension of the scheme to include state and input constraints.

Appendices

A Proof of Lemma 4

Let the conditions of Lemma 2 and Assumption 4 be satisfied. From Assumption (2) it follows $l(x_{\mathfrak{p}}, u_{\mathfrak{p}}) \geq \frac{\alpha_{x_{\mathfrak{p}}}(|x_{\mathfrak{p}}|)+\alpha_{u_{\mathfrak{p}}}(|u_{\mathfrak{p}}|)}{2} := \alpha(x)$.Using latter inequality in inequality (10) it follows that $V(x(k_{i+1})) - V(x(k_i)) \leq -\alpha(x(k_i))$. Now, consider time instances for the set $\{k_0, \ldots, k_i\} \in \mathbb{K}_B$ and the corresponding inequalities of the form given in the previous inequality. Namely, consider $V(x(k_1)) - V(x(k_0)) \leq -\alpha(x(k_0))$, $\ldots, V(x(k_i)) - V(x(k_{i-1})) \leq -\alpha(x(k_{i-1}))$. Adding all previous inequalities results in

$$V(x(k_i)) \leq V(x(k_0)) - \sum_{j=k_0}^{k_{i-1}} \alpha(x(j)). \tag{A.1}$$

Note that, due to Assumption 2 and the definition of cost function given in (5), it holds[7] that $V(x(k_i)) \geq \alpha(x(k_i))$. This inequality, combined with the inequality from Assumption 4 and inequality given in (A.1) yields

$$\alpha(x(k_i)) \leq \gamma_V(|x(k_0)|) - \sum_{j=k_0}^{k_{i-1}} \alpha(x(j)). \tag{A.2}$$

To show *uniform stability and global boundedness*, $-\sum_{j=0}^{i-1} \alpha(x(k_j)) \leq 0$ is used, resulting in $\alpha(x(k_i)) \leq \gamma_V(|x(k_0)|)$. Obtaining inequality (9) amounts to

$$\rho_1(\cdot) := \alpha(\cdot), \tag{A.3a}$$
$$\gamma(\cdot) := \gamma_V(\cdot), \tag{A.3b}$$
$$\rho_2(\cdot) := |\cdot|. \tag{A.3c}$$

To demonstrate *uniform global attractivity*, let $\epsilon > 0$ and $\delta > 0$ be given. Define $K = K(\delta, \epsilon) := \frac{\gamma(\delta)-2\cdot\epsilon}{\epsilon}$. Let $\rho_2(x) \leq \delta$. From $V(x(k_i)) - V(x(k_{i-1})) \leq -\alpha(x(k_{i-1}))$, established above, it follows $\alpha(x(k_i)) \leq \gamma_V(|x(k_{i-1})|)$. Using (A.3), the latter inequality becomes $\rho_1(x(k_i)) \leq \gamma(\rho_2(x(k_{i-1})))$, which demonstrates a decreasing behavior. Using this in (A.2), together with (A.3), yields $\rho_1(x(k_i)) \leq \gamma(\rho_2(x)) - \sum_{j=k_0}^{k_{i-1}} \rho_1(x(j)) \leq \gamma(\delta) - \sum_{j=k_0}^{k_0+K} \rho_1(x(j)) - \sum_{j=k_0+K+1}^{k_{i-1}} \rho_1(x(j)) \leq \gamma(\delta) - \sum_{j=k_0}^{k_0+K} \rho_1(x(j))$. Now, because of the decreasing behavior and the properties of $\mathcal{K}$ and $\mathcal{K}_\infty$ functions[8], $\sum_{j=k_0}^{k_0+K} \rho_1(x(j)) \geq (K+1)\mu(x(k_0+K)) \geq (K+1)\epsilon$, where $\mu := \min_{i\in\{0,\ldots,K\}} \bar{\mu}_i(x(k_0+K))$; where $\bar{\mu}_i$ are functions obtained from the decreasing behavior and the properties of $\mathcal{K}$ and $\mathcal{K}_\infty$ functions. With this, it follows $\rho_1(x(k_i)) \leq \gamma(\delta) - (K+1)\epsilon = \epsilon$ as desired.

B Proof of Lemma 6

Let the Lyapunov candidate function be of the following form

$$W(x_{\mathfrak{b}}) := \sum_{r=1}^{R} x_{\mathfrak{b}_r}^\top \Pi_r x_{\mathfrak{b}_r}, \tag{B.1}$$

where $\mathbb{R}^{\mathfrak{h}\cdot m_{\mathfrak{p}_r}\times \mathfrak{h}\cdot m_{\mathfrak{p}_r}} \ni \Pi_r^\top = \Pi_r > 0$ for each $r \in R$. Next, let us consider the difference of the previous Lyapunov function between successful time instances k_i and k_{i+1}, namely $\Delta W(x_{\mathfrak{b}}) := W\big(\phi_{f_{\mathfrak{b}}}(\Delta k_i + 1, x_{\mathfrak{b}}, \{\mathfrak{a}\}_{k_i}^{k_i+\Delta k_i}, \{\mathfrak{d}\}_{k_i}^{k_i+\Delta k_i}, \{w_{\mathfrak{n}}\}_{k_i}^{k_i+\Delta k_i})\big) - W(x_{\mathfrak{b}})$. Now, recall that $\{w_{\mathfrak{n}}\}_{k_i}^{k_i+\Delta k_i} = \{1, 0, \ldots, 0\}$; see (8). Thus, $\{\mathfrak{a}\}_{k_i}^{k_i+\Delta k_i} = \{r^*(k_i), \ldots, r^*(k_i)\}$, $\{\mathfrak{d}\}_{k_i}^{k_i+\Delta k_i} = \{\mathfrak{d}(k_i), S_L\mathfrak{d}(k_i), \ldots, S_L^{\Delta k_i}\mathfrak{d}(k_i)\}$ where $\mathfrak{d}(k_i) = (u_{\mathfrak{p}_{r^*}}^*(k_i), \ldots, u_{\mathfrak{p}_{r^*}}^*(k_i + B - 1))$; recall that L is the length of each

[7] Note that it also holds $V(x(k_i)) \geq \alpha_{x_{\mathfrak{p}}}(|x_{\mathfrak{p}}(k_i)|)$

[8] The inverse and composition properties of $\mathcal{K}$ and $\mathcal{K}_\infty$ functions, cf. [27, Lemma 4.2].

individual buffer. It follows that

$$\begin{aligned}\Delta W(x_{\mathfrak{b}}) =& \sum_{\substack{r=1\\ r\neq r^*}}^{R} x_{\mathfrak{b}_r}^\top \big((S^{\Delta k_i+1})^\top \Pi_r S^{\Delta k_i+1} - \Pi_r\big) x_{\mathfrak{b}_r} \\ &- x_{\mathfrak{b}_{r^*}}^\top \Pi_{r^*} x_{\mathfrak{b}_{r^*}} + \mathfrak{d}^\top \Pi_{r^*} \mathfrak{d} \\ \leq& - \sum_{\substack{r=1\\ r\neq r^*}}^{R} x_{\mathfrak{b}_r}^\top \Lambda_{(r,\Delta k_i)} x_{\mathfrak{b}_r} - x_{\mathfrak{b}_{r^*}}^\top \Pi_{r^*} x_{\mathfrak{b}_{r^*}} + \mathfrak{d}^\top \Pi_{r^*} \mathfrak{d} \\ \leq& - \underbrace{\min_r \{\min_{\Delta k_i}\{\Lambda_{(r,\Delta k_i)}\}, |\Pi_r|\}}_{c_1} \sum_{r=1}^{R} |x_{\mathfrak{b}_r}|^2 + \underbrace{\max_r\{|\Pi_r|\}}_{c_2} |\mathfrak{d}|^2 \\ \leq& -c_1 |x_{\mathfrak{b}}|^2 + c_2 |\mathfrak{d}|^2 \\ \leq& -\frac{c_1}{2} |x_{\mathfrak{b}}|^2, \ \forall |x_{\mathfrak{b}}| \geq \sqrt{\frac{2c_2}{c_1}} |\mathfrak{d}|,\end{aligned} \tag{B.2}$$

where $(S^{\Delta k_i+1})^\top \Pi_r S^{\Delta k_i+1} - \Pi_r = -\Lambda_{(r,\Delta k_i)}$, for each $r \in R$ and where $\mathbb{R}^{\mathfrak{h}\cdot m_{\mathfrak{p}_r} \times \mathfrak{h}\cdot m_{\mathfrak{p}_r}} \ni \Lambda_{(r,\Delta k_i)}^\top = \Lambda_{(r,\Delta k_i)} > 0$. The latter is due to the fact that for any Δk_i, satisfying $1 \leq \Delta k_i \leq B-1$, the resulting matrix $S^{\Delta k_i+1}$ is nilpotent, and thus, has all of its eigenvalues located strictly inside the unit circle. Thus, as desired, inequality (B.2) captures the corresponding ISS property.

C Proof of Theorem 1

Let the conditions of Lemmas 4, 5 and 6 be satisfied. Consider the first successful transmission instant $k_0 \in \mathbb{K}_B$. The conclusions of Lemmas 6 and 4 provide

$$\left.\begin{aligned}&|x_{\mathfrak{b}}(k_i)| \leq \beta_1(|x_{\mathfrak{b}}(k_j)|, k_i - k_j) + \gamma_1(\sup_{k_j \leq \tau \leq k_i} |\mathfrak{d}(\tau)|)\\ &|x_{\mathfrak{p}}(k_i)| \leq \beta_2(|x_{\mathfrak{p}}(k_j)|, k_i - k_j)\end{aligned}\right\} \tag{C.1}$$

where $k_i \geq k_j \geq k_0$, β_1 and β_2 are class-$\mathcal{KL}$ and γ_1 is a class-$\mathcal{K}$ function. Now, recall that $\mathfrak{d}(\tau) = (u^*_{\mathfrak{p}_{r^*}}(\tau), \ldots, u^*_{\mathfrak{p}_{r^*}}(\tau + B - 1))$ and note that the conclusion of Lemma 5 results in $|u_{\mathfrak{p}}(\tau)| \leq \beta_3(|x(k_j)|, \tau - k_j)$ where β_3 is a class-$\mathcal{KL}$ function. Now, due to the properties of class-$\mathcal{KL}$ function it follows $\sum_{j=\tau}^{\tau+B-1} |u_{\mathfrak{p}}(j)| \leq (B-1)\beta_3(|x(k_j)|, \tau - k_j)$. Further, due to properties of Euclidean norm, $|\mathfrak{d}(\tau)| \leq \sum_{j=\tau}^{\tau+B-1} |u_{\mathfrak{p}}(j)|$, thus we have

$$|\mathfrak{d}(\tau)| \leq (B-1)\beta_3(|x(k_j)|, \tau - k_j). \tag{C.2}$$

Further, let $k_j = \lceil \frac{k_i + k_0}{2} \rceil$. Then

$$|x_{\mathfrak{b}}(k_i)| \leq \beta_1\Big(\Big|x_{\mathfrak{b}}\Big(\Big\lceil \frac{k_i + k_0}{2}\Big\rceil\Big)\Big|, \Big\lceil \frac{k_i - k_0}{2}\Big\rceil\Big) + \gamma\Big(\sup_{k_j \leq \tau \leq k_i} |\mathfrak{d}(\tau)|\Big). \tag{C.3}$$

To estimate $|x_{\mathfrak{b}}(k_j)|$, we apply the first inequality of (C.1) with $k_j = k_0$ and k_i replaced with k_j, which yields

$$|x_{\mathfrak{b}}(k_j)| \leq \beta_1\Big(|x_{\mathfrak{b}}(k_0)|, \Big\lceil \frac{k_i - k_0}{2}\Big\rceil\Big) + \gamma\Big(\sup_{k_0 \leq \tau \leq k_j} |\mathfrak{d}(\tau)|\Big). \tag{C.4}$$

Using the inequality (C.2), results in

$$\left.\begin{aligned}\sup\nolimits_{k_0 \leq \tau \leq k_j} |\mathfrak{d}(\tau)| &\leq (B-1)\beta_3(|x(k_0)|, 0),\\ \sup\nolimits_{k_j \leq \tau \leq k_i} |\mathfrak{d}(\tau)| &\leq (B-1)\beta_3\Big(|x(k_0)|, \Big\lceil \tfrac{k_i - k_0}{2}\Big\rceil\Big).\end{aligned}\right\} \tag{C.5}$$

Substituting the inequality (C.4) in the inequality (C.3) and using the inequality (C.5) together with the inequalities $|x_{\mathfrak{p}}(k_0)| \leq |x(k_0)|$, $|x_{\mathfrak{b}}(k_0)| \leq |x(k_0)|$ and $|x(k_i)| \leq |x_{\mathfrak{p}}(k_i)| + |x_{\mathfrak{b}}(k_i)|$, yields $|x(k_i)| \leq \beta_4(|x(k_0)|, k_i - k_0)$ where $\beta_4(t,s) = \beta_1\Big(\beta_1\Big(t, \frac{s}{2}\Big) + \gamma_1((B-1)\beta_3(t,0)), \frac{s}{2}\Big) + \gamma_1\Big((B-1)\beta_3\Big(t, \frac{s}{2}\Big)\Big) + \beta_2(t,s)$ as desired.

D Proof of Theorem 2

Let the conditions of Lemmas 2 and 5 be satisfied. Let the Lyapunov function be of the following form $L(x) := cU(x) + V(x) + W(x_{\mathfrak{b}})$, $c > 0$ where $V(x)$ is given in equation (6). Furthermore, $W(x)$ is given in the equation (B.1), while $U(x)$ is defined as follows $U(x) := \sum_{j=k}^{\infty} \upsilon_{\mathfrak{p}}^\top(\phi_f(j-k, x, \{w_{\mathfrak{n}}\}_k^{k+j}))\Omega\upsilon_{\mathfrak{p}}(\phi_f(j-k, x, \{w_{\mathfrak{n}}\}_k^{k+j}))$ for each $j \geq k \geq 0$ where $\mathbb{R}^{m_{\mathfrak{p}} \times m_{\mathfrak{p}}} \ni \Omega^\top = \Omega > 0$, $\upsilon_{\mathfrak{p}}(\phi_f(j-k, x, \{w_{\mathfrak{n}}\}_k^{k+j})) := u_{\mathfrak{p}}(j)$ and $\upsilon_{\mathfrak{p}} : \mathbb{R}^{n_{\mathfrak{p}} \times \mathfrak{h}\cdot m_{\mathfrak{p}}} \to \mathbb{R}^{m_{\mathfrak{p}}}$ is possibly a *discontinuous* function. One can easily show that both, V and W are radially unbounded. Namely, for V one would use Assumption 2 and 4, while for W one would use the fact that the function is quadratic. Moreover, since V and U have the same argument and V is radially unbounded there is no need to show that U is radially unbounded. Hence, we conclude that L is radially unbounded. We proceed with analyzing the difference of L between time instances k_i and k_{i+1}, namely, $\Delta L(x(k_i)) := L(\phi_f(\Delta k_i + 1, x, \{w_{\mathfrak{n}}\}_{k_i}^{k_i+\Delta k_i})) - L(x(k_i)) = c\Delta U(x(k_i)) + \Delta V(x(k_i)) + \Delta W(x_{\mathfrak{b}}(k_i))$. Note that $\Delta V(x(k_i))$ is considered in the inequality (10) while $\Delta W(x_{\mathfrak{b}}(k_i))$ is considered in the inequality (B.2). Thus, we concentrate on $\Delta U(x(k_i)) := U(\phi_f(\Delta k_i + 1, x, \{w_{\mathfrak{n}}\}_{k_i}^{k_i+\Delta k_i})) - U(x(k_i)) \leq -\sum_{j=k_i}^{k_i+\Delta k_i} \upsilon_{\mathfrak{p}}^\top(\phi_f(j-k, x, \{w_{\mathfrak{n}}\}_k^{k+j}))\Omega\upsilon_{\mathfrak{p}}(\phi_f(j-k, x, \{w_{\mathfrak{n}}\}_k^{k+j})) \leq -\lambda_{\min}(\Omega) \cdot \sum_{j=k_i}^{k_i+\Delta k_i} |u_{\mathfrak{p}}(j)|^2 \leq -\lambda_{\min}(\Omega) \cdot |\mathfrak{d}(k_i)|^2$. Finally, using the latter inequality together with inequalities provided in (10) and (B.2) and choosing c so that $(c\lambda_{\min}(\Omega) - c_2) \geq 0$, allows us to bound $\Delta L(x(k_i)) \leq -\alpha_{x_{\mathfrak{p}}}(x_{\mathfrak{p}}(k_i)) - c_1|x_{\mathfrak{b}}(k_i)|^2 - (c\lambda_{\min}(\Omega) - c_2)|\mathfrak{d}(k_i)|^2 \leq -\alpha_{x_{\mathfrak{p}}}(x_{\mathfrak{p}}(k_i)) - c_1|x_{\mathfrak{b}}(k_i)|^2$, as desired.

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