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Development of genetic-based models for predicting the resilient modulus of cohesive pavement subgrade soils

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ABSTRACT

Accurate determination of resilient modulus (M_r) of pavement subgrade soils is an important factor for successful design of pavement system. M_r is an important soil property, which is complex in nature as it is dependent on several influential factors, such as soil physical properties, applied stress conditions, and environmental conditions. The aim of this study is to explore the potential of an evolutionary algorithm, i.e., genetic algorithm (GA), and a hybrid intelligent approach combining neural network with GA (ANN-GA), for estimation of M_r of cohesive pavement subgrade soils. To achieve this aim, a reliable database containing results of repeated load triaxial tests (RLT) and other index properties of subgrade soils was utilized. GA was employed to develop a precise equation for predicting M_r of subgrade soils. In addition, GA was used as a tool for determining the optimal values of the weights and the bias of the ANN-GA approach. The developed ANN-GA model was then transferred to a functional relationship for further application and analyses. Several validation and verification phases were conducted to examine the performance of the developed models. The results indicated that both GA and ANN-GA models could accurately predict the M_r of cohesive subgrade soils, and had better performance compared to the available models in the literature. Finally, a sensitivity analysis was conducted to evaluate the effect of utilized parameters on the M_r.

Keywords: Pavement subgrade; Resilient modulus; Genetic algorithm; Optimized neural network; Hybrid.

Introduction

The successful design of a pavement system with various material properties is affected by the stiffness and strength of the pavement layers, i.e. surface, base, subbase, and subgrade. Amongst pavement layers, the subgrade is considered as the foundation of the pavement system that transfers applied loads to the ground. To characterize the subgrade material under different environmental and loading conditions, the resilient modulus (M_r) has been introduced as a fundamental material property that describes the inelastic behavior of material under traffic loading (AASHTO, 2003). As pavement layers undergo repeated traffic loading, the subgrade soil undergoes recoverable as well as permanent strains with each load repetition. When the number of load repetitions is increased, plastic deformation decreases until it becomes all recoverable (Sadrossadat et al., 2016). M_r is defined as the ratio of the applied deviator stress to the recoverable strain (AASHTO, 2003).

M_r of subgrade soils in commonly determined through repeated load triaxial (RLT) test on remoulded or undisturbed samples (AASHTO, 2003). In this test, varying combinations of confining and deviator stresses are applied to the specimen in various sequences to simulate the field conditions. Despite their accuracy and reliability, performing RLT tests requires sophisticated equipment and skilled personnel. In order to avoid performing time consuming and uneconomical RLT tests, several relationships have been proposed that formulate M_r in terms of stress states, physical properties, and strength tests parameters.

M_r mainly depends on stress states and physical properties of the subgrade soils. Generally, models obtained for calculation of resilient modulus of subgrade soils can be classified into two main groups: (I) Correlations with laboratory tests and in situ test results (II) Constitutive equations. Recently, P-wave and S-wave measurements have also been used for estimation of resilient modulus of pavement materials (Schuettpelz et al., 2010). A summary of some of the available equations for estimation of M_r of cohesive soils are summarized in **Table 1**.

Correlations are typically developed by relating the M_r value to the results of laboratory tests such as California bearing ratio (CBR) and unconfined compressive strength (UCS); in-situ tests such as cone penetration test (CPT) and dynamic cone penetrometer (DCP); soil physical properties like moisture content (w) and dry density (γ_d); stress states such as confining pressure (σ_3) and deviator stress (σ_d); or a combination of these parameters. Constitutive models, on the other hand, are obtained by relating the M_r value to various stress invariants, i.e. bulk stress (θ), octahedral shear stress (τ_{oct}), and octahedral normal stress (σ_{oct}). Unknown parameters of the constitutive models can be related to soil physical properties to include both physical properties and stress conditions.

Considering Kim (2004) equation in **Table 1**, regression coefficients for cohesive subgrade soils (A-6 soils) can be obtained using following equations (Hanittinan, 2007):

$$\begin{aligned} k_1 &= a_1 a_3^{a_2} + a_3\left(\frac{S_r}{100}\right)^{a_4} + a_5 q_u + a_6 PI + a_7 (LL - w) \\ &+ a_8 (w_{opt} - w) + a_9 (P_{200} - a_{10}) \end{aligned} \tag{1}$$

$$k_2 = b_1 \sigma_3^{b_2} + b_3\left(\frac{S_r}{100}\right)^{b_4} + b_5 q_u^{b_6} + b_7 PI + b_8 LL \tag{2}$$

$$a_1 = a_{11} + a_{12}\left(\frac{w_{opt} - w}{w_{opt}}\right) \tag{3}$$

$$b_1 = b_{11} + b_{12}(w - w_{opt}) \tag{4}$$

where a_1 to a_{12} and b_1 to b_{12} are constants of Kim's model. Detailed description of the Kim's regression coefficients can be found in Hanittinan (2007). As can be seen in **Table 1**, constitutive models such as those provided by Seed et al. (1967), Witczak and Uzan (1988), and Puppala et al. (1996) have a pre-defined structure, and unknown coefficients of these equations are typically obtained by performing regression analysis. Considering Kim (2004) equation in **Table 1**, a second set of regression analysis is required to relate the obtained coefficients, i.e., k_1 and k_2, to soil parameters as summarized in **Eq.** (1-4). As can be seen, the procedure for finding the final model is complicated and time consuming. On the other hand, while regression based equations may perform well on the utilized datasets, they are not typically tested and validated on new datasets (Gandomi et al., 2013, Ghorbani et al., 2018). The complexity of the M_r factor as well as the importance of considering the nonlinear interaction between variables necessitates the use of more advanced techniques for prediction of M_r of subgrade soils.

To overcome the limitations of the traditional modeling techniques, artificial intelligence (AI) methods have been employed by various researchers for solving complicated engineering problems (Ghorbani et al., 2018, Chen et al., 2019, Jayawardana et al., 2019, Ghorbani and Hasanzadehshooiili, 2018). Unlike traditional modeling techniques, artificial intelligence approaches are capable of determining the nonlinear relationship between variables in a model effectively, without considering any prior assumptions about the problem. Kim et al. (2014) and Hanittinan (2007) used ANNs, and Khoury and Maalouf (2018) employed support vector machine method for prediction of resilient modulus of subgrade soils. Sadrossadat et al. (2018b) used a variant of genetic programming, namely linear genetic programming (LGP) for indirect estimation of M_r of cohesive subgrade soils. Amongst computation intelligence methods, artificial neural networks (ANNs) are more widely used than other methods due to their inherent features, which include managing complex problems with large datasets, handling problems with multiple outputs, and predicting the unseen data effectively. However, ANNs have disadvantages such as slow learning rate and getting stuck in local minima. Furthermore, ANNs are known as black-box systems as they do not provide a distinct relationship between inputs and the output (Ziaee et al., 2015).

To improve the prediction capability of ANNs, evolutionary algorithms such as genetic algorithm (GA) and particle swarm optimization (PSO) have been applied to find the optimal values of the weights for the ANN. Evolutionary algorithms can aid ANNs in converging to the global minima, and hence improving the prediction performance of the network. In this regard, Mousavi et al. (2017) proposed a hybrid neural network and simulated annealing for prediction of the daily solar radiation. Alsarraf et al. (2019) applied the PSO-ANN technique for prediction of exergetic performance of a building integrated photovoltaic/thermal system. Mosallanezhad and Moayedi (2017) investigated the potential of an integrated imperialist competitive algorithm ANN to estimate the pull-out resistance of screw piles. While evolutionary algorithms have been found efficient for solving engineering problem, application of these methods in the field of pavement geotechnics is limited to date. To date, there has been limited studies on the application of evolutionary algorithms and hybrid methods for providing formulations of resilient modulus of subgrade soils.

This paper proposes the application of GA as well as a hybrid ANN-GA approach for predicting the M_r of cohesive subgrade soils. GA was employed to establish a precise equation for prediction of M_r of subgrade soils. The hybrid ANN-GA model was developed by using GA to determine the optimal values of weights and bias of the ANN-GA approach, which can result in a more robust model. To achieve this aim, a comprehensive and reliable set of data, including the results of RLT tests on cohesive subgrade soils was utilized to develop models. To evaluate the performance of the proposed models, several validation and verification study phases were considered. Furthermore, results were compared with available equations in the literature, to verify the superiority of the proposed models. The developed ANN-GA model was also converted to a tractable formula for hand calculation and pre-design purposes to reduce the time and cost associated with performing RLT tests.

Methodology

Genetic algorithm (GA)

GA is a heuristic search and optimization algorithm introduced by Holland (1975). GA was inspired by Darwin's theory of evolution and imitates the process of natural evolution and selection. In addition, unlike conventional optimization methods, GA requires less information about the problem and is well suited for more complex problems. In GA, a population of individuals are randomly generated to solve a problem. Each solution is encoded as a fixed-length binary string of 0s and 1s, known a chromosome. A chromosome has several genes and total number of chromosomes indicate the population size (Holland, 1975). The superiority of an individual over other solutions is evaluated using a fitness function. Solutions are evolved in successive iterations (or generations) until a satisfactory criterion, i.e. the maximum number of generations or a predefined fitness value is met. In the process of evolution, several operators, i.e., mutation and crossover are used to modify solutions transferred to next generations (Muduli and Das, 2015). **Fig. 1** summarizes steps taken in the GA for reaching the optimal solution.

Hybrid ANN-GA approach

ANNs are a branch of artificial intelligence techniques that aim to mimic the behavior of human brain and nervous system for solving complex problems. Amongst different variants of ANN, multi-layer perceptron (MLP) neural networks is the most widely used (Cybenko, 1989). A MLP neural networks consists of three distinct layers, i.e. input layer, hidden layer, output layer. Each layer is connected to the subsequent layer through computing elements known as nodes. Except for the input nodes which are fixed nodes, each node is a neuron or processing element with a nonlinear activation function. In a neuron, an input from previous layer is multiplied by a weight coefficient which connects two layers. The output of each neuron is then calculated by passing through a nonlinear activation function (Ziaee et al., 2015). Training of the network is done by adjusting the weights of the network so that the network's predicted values matches the target values in the datasets. To do so, back propagation algorithm is typically used in which the calculated error is back propagated and the network weights are altered accordingly to minimize the prediction error (Ghorbani and Hasanzadehshooiili, 2018).

One of the advantages of GA over classic search methods is its ability to perform a global search and hence avoiding the risk of trapping into a local minimum (Gandomi et al., 2013). Thus, a hybrid ANN-GA model uses GA as a powerful search algorithm to find the best parameters for the ANN. In the ANN-GA method, GA is employed to adjust the weights of the ANN in a way that the error of the GA-ANN model (i.e., fitness value) is minimized. A fitness function (i.e., RMSE) is considered to measure the fitness of each solution vector. In case of reaching the defined termination criterion, the simulation procedure stops and the results are represented. The termination condition is typically set to the maximum value until which the algorithm is iterated to find better solutions.

Database and model variables

The database used for development of ANN-GA model was collected from a study conducted by Hanittinan (2007), which reported on data collected from earlier studies by Kim (2004), Huang (2001), and Rodgers (2006). The database was composed of the physical soil properties, unconfined compressive strengths, and the results of 283 RLT tests performed on cohesive subgrade soils. Soils were mainly silt, clay, and silty clay, classified as A-6 based on the AASHTO soil classification code. Samples were collected from seven different locations and tests were performed in several US universities (Hanittinan, 2007). RLT tests were performed at a range of moisture contents from 4% below optimum moisture content to 3% above optimum moisture content, based on AASHTO designation T294-94 (Hanittinan, 2007).

Over the past decades, many studies have examined the factors affecting the M_r of subgrade soil. Results of laboratory studies indicate that the M_r of pavement materials is highly influenced by the stress state parameters, i.e., σ_3 and σ_d. For cohesive subgrade soils, increasing the σ_d at a constant σ_3 decreases the M_r value (Seed et al., 1967, Huang, 2001, Kim, 2004). It is known that increasing the σ_3 increases the M_r of soils (Kim et al., 2014, Sadrossadat et al., 2016). In addition to stress state parameters, several studies have been performed to study the influence of soil physical properties, compaction characteristics, and environmental factors on the M_r. These factors should be considered in the model development for providing more comprehensive and robust models. Moisture content and degree of saturation (S_r) describe the changes in the soil environment and seasonal variations. Soil physical properties such as percentage of soil particles passing through #200 sieve (P_{200}), liquid limit (LL), and plasticity index (PI) are commonly used for classification and identifying the soil type. Besides, UCS has been identified as a static strength parameter which is positively correlated with the M_r (Kim, 2004, Hanittinan, 2007). There are several equations in the literature which relate the M_r to strength parameters such as UCS and CBR. On the other hand, compaction characteristics of soils is important factor in design of pavement layers. Herein, soil optimum moisture content (w_{opt}) which is defined as the soil moisture content at its maximum dry density is incorporated in the model development procedure.

In addition to aforementioned factors, viscous nature of cohesive soils can cause creep deformations under the applied loads. The developed time dependent shear strains under sustained or repeated loads can reduce the M_r of cohesive soils (Viyanant et al., 2007). In saturated soil conditions, application of load cycles generates excess pore water pressure in cohesive soils which can cause creep deformations (Holzer et al., 1973).

With an aim to incorporate most of the influential parameters and considering the available database, the model for prediction of M_r of cohesive subgrade soils is expressed as a function of following parameters:

$$M_r = f(LL, PI, P_{200}, w_{opt}, S_r, w, q_u, \sigma_3, \sigma_d) \quad (5)$$

To have a better understanding of the model variables, key descriptive statistics of the variables are summarized in **Table 2**.

Data pre-processing

Artificial intelligence techniques utilize data to determine the optimum model that best describes the relationship between input and output parameters. One key issue in finding this relationship is known as overfitting, which needs to be avoided to have a model with better generalization. Overfitting occurs when the model has relatively small error on the trained dataset, while when a new set of data is introduced to the model, error value becomes very high. To avoid overfitting, the dataset needs to be divided into two subsets: 1. Training data and 2. Testing data. The training is used to construct the models and testing data is used to evaluate its performance on unseen data. Accordingly, the testing dataset is used to find a model with better generalization. It is suggested by many researchers that a range of 15-30 % of the data should be used for testing the model performance (Gandomi et al., 2013). It is worth noting that while the results of the model on training data represents the ability of the model to learn the behavior of variables in the database, the test performance indices show the performance on the model on unseen data which indicates its generalization capability. In this study, 80% and 20% of the datasets are used for training and testing the model, respectively.

The number of datasets in database have a significant effect on the model performance in training stage of the network. The models with larger number of datasets are more reliable and safer for further analysis. It is suggested that the minimum ratio of datasets in a database to the variables of the model should be more than five (Gandomi et al., 2013). In this study, this ratio is equal to 31, which is much higher than the required value.

Model performance assessment

In general, several important criteria need to be checked to evaluate the accuracy and generalization of the developed ANN-GA model. Correlation coefficient (R), mean absolute error (MAE), round mean squared error (RMSE) are among essential statistical criteria that indicate the overall performance of the developed model (Ghorbani et al., 2018):

$$R^2 = \frac{\left(\sum_{i=1}^{n}(M_{rm}-\overline{M}_{rm})(M_{rp}-\overline{M}_{rp})\right)^2}{\sum_{i=1}^{n}(M_{rm}-\overline{M}_{rm})^2\sum_{i=1}^{n}(M_{rp}-\overline{M}_{rp})^2} \tag{6}$$

$$RMSE=\sqrt{\frac{\sum_{i=1}^{n}(M_{rp}-M_{rm})^2}{n}} \tag{7}$$

$$MAE=\frac{1}{n}\sum_{i=1}^{n}\left|M_{rp}-M_{rm}\right| \tag{8}$$

in which M_{rm} and M_{rp} are the measured and predicted values of the i^{th} output, $\overline{M}_{rm}$ and $\overline{M}_{rp}$ are the average values of the measured and predicted results, and n is the number of samples.

Smith (1986) suggests that there exists a strong correlation between the predicted and measured values if the correlation coefficient $R^2 \geq 0.64$. Additionally, error values, i.e., MAE and RMSE, should be minimum for both training and testing data.

Development of GA model

In this section, GA is employed to find a precise equation for prediction of M_r of subgrade soils. In this regard, a code is written in MATLAB environment (Mathworks, 2017). The input and output values are normalized before model development to increase the capability of algorithm in finding the relationship between input variables and the output. The database is normalized to lie between 0 and 1 using following equation:

$$X_n = \frac{X - X_{\min}}{X_{\max} - X_{\min}} \tag{9}$$

where X_{min}, X_{max}, and X_n are the minimum, maximum, and normalized values of the variable X, respectively**.** The de**-**normalized value of the output can be calculated as:

$$X = X_n(X_{\max} - X_{\min}) + X_{\min} \tag{10}$$

General form of the considered equation for prediction of M_r of subgrade soils is given in **Eq. (11)**:

$$\begin{aligned} M_r &= w_1 LL^{a_1} + w_2 PI^{a_2} + w_3 P_{\#200}{}^{a_3} + w_4 W_{opt}{}^{a_4} + w_5 W_c{}^{a_5} + \\ & w_6 S_r{}^{a_6} + w_7 q_u{}^{a_7} + w_8 \sigma_3{}^{a_8} + w_9 \sigma_d{}^{a_9} + b \end{aligned} \tag{11}$$

where w_1 to w_9 and a_1 to a_9 are the coefficients of the equation and b is the bias value. GA is employed as a tool for finding the optimal values of unknown coefficients. RMSE function is considered as the fines function to evaluate solutions in each iteration.

Several parameters such as population size (N_{pop}), crossover probability, mutation rate, and maximum number of iterations affect the prediction capability of the GA approach. These values are typically obtained by trial and error method or using recommended values of other researchers. The combination of parameters considered to find the optimal model is summarized in **Table 3**. Several preliminary runs were performed to come up with a parameter setting to provide a robust model with high generalization capability. Crossover and mutation rates were selected based on some previously suggested values (Momeni et al., 2014, Khandelwal and Armaghani, 2016, Rostami et al., 2018). Proper determination of population size is dependent on the size and the complexity of the investigated problem. To investigate the effect of population size on the performance of the GA model, a parametric study was done as shown in **Fig. 2**. It was observed that model with N_{pop} = 400 had the lowest RMSE value. Considering different values for parameters resulted in 2 × 5 × 2 × 2 = 40 combinations of parameters. Also, 5 replications of each parameter combination was tested and evaluated. In total, 200 runs with different combinations of parameters were conducted. The best values of population size, mutation rate, and crossover probability for the developed GA model were 400, 30%, and 70%, respectively. The optimal coefficients of the developed equation based on GA optimization are summarized in **Table 4**.

Considering one of the testing samples (i.e. LL = 31, PI = 12, P_{200} = 56 %, w_{opt} = 13.4 %, w = 11.4 %, S_r = 66.44 %, q_u = 696.13 kPa, σ_3 = 41.37 kPa, σ_d = 40.76 kPa, M_r = 112.81 MPa), the normalized values of LL, PI, P_{200}, w_{opt}, w, S_r, q_u, σ_3, and σ_d are equal to 0.617, 0.82, 0, 0.476, 0.337, 0.412, 1, 0.999, 0.494, and 0.602, respectively. By substituting the obtained coefficient values in **Eq. (11)**, M_r is calculated equal to 0.6076. Using **Eq. (10)**, the de-normalized value of M_r is calculated equal to 113.7 (MPa).

Hybrid ANN-GA model development

In order to develop the ANN-GA model for prediction of resilient modulus of subgrade soils, a code is written in MATLAB environment (Mathworks, 2017). The procedure for modeling of M_r using ANN-GA is illustrated in **Fig. 3**. The model includes nine input variables and M_r is the only output of the model.

In the first phase of constructing the model, the dataset should be normalized between -1 and 1 to facilitate model development using following equation (Mathworks, 2017):

$$X_n = \frac{X - X_{\min}}{X_{\max} - X_{\min}} - \frac{X_{\max} - X}{X_{\max} - X_{\min}} \tag{12}$$

Considering **Eq. (13)**, the de-normalized value of the output can be calculated as**:**

$$X = 0.5(X_n + 1)(X_{\max} - X_{\min}) + X_{\min} \tag{13}$$

In the hybrid ANN-GA model development, there are several parameters in both the ANN and GA that need to set. The precision of the ANN models increases with increasing the number of nodes in the hidden layer. However, increasing the nodes would result in a more complicated model with many different parameters. Cybenko (1989) indicated that a single hidden layer neural network would provide satisfactory results for approximation of nonlinear problems. Thus, in this study, only one hidden layer consisting of three nodes was considered to develop a simple model that can be represented as a tractable formulation rather than a complicated black-box. The activation function for the hidden layer nodes was Tangent-sigmoid as follows (Mathworks, 2017):

$$f(x) = \frac{2}{1+e^{-2x}} - 1 \qquad \textbf{(14)}$$

The crossover and mutation rates were chosen equal to the values adopted for developing the GA model. In addition, a parametric study was performed by developing several ANN-GA models to find the optimal value of N_{pop}. Increasing population size usually increases the chance of obtaining better results, however, it decreases the speed of the model development. The population size was varied from 50 to 400 and maximum number of generations was set to 500. **Fig. 4** shows the effect of population size on the model performance based on the RMSE criteria. As evident, the model with N_{pop} = 400 had the lowest RMSE value. It is also evident that after about 300 generations, there is no significant change in the RMSE value. The hybrid ANN-GA model was run several times with 3 × 5 × 2 × 2 = 60 different combinations of the parameters and 5 replications for each combination which resulted in a total of 300 runs. The parameter settings during the ANN-GA model development are summarized in Table 5.

The best ANN-GA model for prediction of resilient modulus of cohesive subgrade soils has the following parameters: Population size = 400; Mutation rate = 10%; Crossover rate = 70%. In order to transform the optimal ANN-GA model into a tractable formulation for further analysis, following function is used (Ziaee et al., 2015):

$$h = f_{HO}\left(bias_h + \sum_{k=1}^{h} V_k\, f_{IH}\left(bias_{hk} + \sum_{i=1}^{m} w_{ik} x_i\right)\right) \qquad (15)$$

where $bias_h$ is the hidden layer bias; V_k is the weight connection between neuron k of the hidden layer and the single output neuron; $bias_{hk}$ would be the bias at neuron k of the hidden layer **(**k =1, h**)**; w_{ik} denotes the weight connection between the input variable **(**i=1, m**)** and neuron k of the hidden layer; x_i would be the i^{th} input parameter; f_{HO} is the transfer function between the hidden layer and the output layer; and f_{IH} is the transfer function between the input and hidden layer**.**

After de-normalization of the output, the optimal ANN-GA model for prediction of M_r of cohesive subgrade with nine inputs (LL, PI, P_{200}, w_{opt}, w, S_r, q_u, σ_3, σ_d) can be expressed as:

$$\left(M_{r_i}\right)_{ANN-GA} = 83.77\left(M_{r_i}\right)_n + 11.9 \qquad (16)$$

where,

$$\left(M_{r_i}\right)_n = \sum_{k=1}^{3} V_k \tanh(A_k) + bias_h \tag{17}$$

$$\begin{aligned} A_k = {} & w_{1k}LL_n + w_{2k}PI_n + w_{3k}P_{200_n} + w_{4k}W_{opt_n} + w_{5k}W_{c_n} + \\ & w_{6k}S_{r_n} + w_{7k}q_{u_n} + w_{8k}\sigma_{3_n} + w_{9k}\sigma_{d_n} + bias_k \end{aligned} \tag{18}$$

where LL_n, PI_n, P_{200n}, w_{optn}, w_n, S_{rn}, q_{un}, σ_{3n}, and σ_{dn} are the normalized input values obtained from **Eq. (12)**, and k is the number of hidden layer nodes (i.e., 3). The obtained values of bias and weights of the optimal model for input-hidden and hidden-output layers are summarized in **Table 6** and **Table 7**, respectively.

Considering the same testing sample used for calculating the output of the GA model (LL = 31, PI = 12, P_{200} = 56 %, , w_{opt} = 13.4 %, w = 11.4 %, S_r = 66.44 %, q_u = 696.13 kPa, σ_3 = 41.37 kPa, σ_d = 40.76 kPa, M_r = 112.81 MPa), the procedure for calculating the M_r is described as:

Step 1: Normalization of the input dataset. Using **Eq. (12)**, the normalized values of LL, PI, P_{200}, w_{opt}, w, S_r, q_u, σ_3, and σ_d are 0.235, 0.64, -1, -0.048, -0.325, -0.176, 1, 0.99, -0.011, respectively.

Step 2: Calculation of hidden nodes parameters. Using the values summarized in **Table 6**, the values of A_1 to A_3 are calculated as: A_1 = 1.522; A_2 = -1.73; A_3 = -2.89.

Step 3: Prediction of the M_{rn}. The output of each hidden layer neuron is calculated by passing through an activation function (i.e., tansig). M_{rn} can then be calculated as the summation of output of each neuron which is multiplied to the hidden-output weights summarized in **Table 7**.

Step 4: De-normalization of the M_r. Using **Eq. (13)**, M_r is calculated in the range of datasets. The output of the ANN-GA model is obtained equal to 110.46 MPa.

Performance analysis of ANN-GA and GA models

In order to represent the capability of the obtained ANN-GA and GA models, the predicted versus measured values of M_r for training and testing datasets are depicted in **Fig. 5 (a)** and **(c)**, respectively. In addition, histograms of the errors obtained by each method for training and testing datasets is illustrated in **Fig. 5 (b)** and **(d)** to have a general view of the frequency of the errors in different intervals.

As evident in these figures, the ANN-GA model has R^2 values of 0.97 for both training and testing datasets, and RMSE values are 5.5 and 5.2 for training and testing datasets, respectively. On the other hand, the R^2 of GA model for both training and testing datasets is equal to 0.87. The RMSE value for training and testing datasets are 11.1 and 11.3 for training and testing data, respectively. Considering the high R^2 values and low RMSE values, it can be concluded that the developed models are capable of predicting the M_r of subgrade soils with a high degree of accuracy. In addition, close values of R^2 and RMSE values for training and testing datasets indicate that overfitting is avoided (Ghorbani et al., 2018, Sadrossadat et al., 2018a). This means that the proposed models would have a satisfactory performance on unseen data and thus has suitable generalization performance. It should be noted that while the functional structure of the GA model is simpler, the ANN-GA model outperforms GA model with a high degree of accuracy.

Comparative study

To further examine the performance of the proposed models, their performance is compared with the LGP model (Sadrossadat et al., 2018b) and model proposed by Kim (2004). The Kim model was chosen for the purpose of comparison as it outperformed other existing regression based models in the literature (Kim, 2004). In this regard, testing datasets are used to evaluate the prediction capability of all methods. Results of comparative study is represented in **Fig. 6**. Residual error (RE) is calculated as the difference between the measured and predicted values of each model. As evident in **Fig. 6**, the ANN-GA approach outperforms other three methods in terms of all indicators. The R^2 values for ANN-GA, GA, LGP, and Kim's model are 0.97, 0.87, 0.83, and 0.56, respectively. The max |RE| of the ANN-GA model is about 11, while this value for GA, LGP, and Kim's model are around 29, 33, and 62, respectively. Furthermore, it is evident that AI-based methods perform notably better than the regression based model proposed by Kim (2004).

While R, MAE and RMSE together give an overall perspective of the performance of each model, other criteria need to be satisfied on the testing datasets to ensure the external validation of a prediction model. In this regard, a ranking index (RI) is used in this study that incorporates some other statistical criteria to compare the performance of the models (Ghorbani et al., 2018). This statistical procedure is based on the following four criteria:

1. The equation of best fine line of predicted (M_{rp}) versus measured (M_{rm}) resilient modulus ($M_{r\text{-}fit}/M_{rm}$), along with corresponding coefficient of determination (R^2). It is worth noting that model with ($M_{r\text{-}fit}/M_{rm}$) and R^2 closer to 1 has the best performance.

2. The arithmetic mean (μ) and standard deviation (σ) of M_{rp}/M_{rm}. It is suggested that a model with μ closer to 1 and σ closer to 0 has a better performance in prediction of the M_r.

3. The 50% cumulative probability ($P_{50\%}$) of M_{rp}/M_{rm}. To calculate the $P_{50\%}$, the values of M_{rp}/M_{rm} are arranged in an ascending order, and the cumulative probability is calculated using following equation:

$$P = \frac{i}{n+1} \tag{19}$$

The value of the $P_{50\%}$ for the optimal model should be close to one.

4. The coefficient of efficiency (E). This parameter evaluates how well each model describes the variance of the datasets. E can be calculated using following equations:

$$E = \frac{E_1 - E_2}{E_1} \tag{20}$$

$$E_1 = \sum_{i=1}^{n} (M_{rm} - \overline{M}_{rp})^2 \tag{21}$$

$$E_2 = \sum_{i=1}^{n} (M_{rp} - M_{rm})^2 \tag{22}$$

The overall performance of the each model model can be measured in terms of rank index (RI) which is the sum of the ranks of each sub criteria:

$$RI = R_1 + R_2 + R_3 + R_4 \tag{23}$$

where R_1, R_2, R_3, and R_4 are the ranks from each of the explained sub criteria. The model with the lowest RI has the best performance for prediction of M_r of cohesive subgrade soils. The best fit line of M_{rp}/M_{rm} for each investigated model is illustrated in **Fig. 7**. As evident, the ANN-GA model has the best performance among all models, followed by GA, LGP and Kim model. Therefore, ANN-GA model is ranked 1 (i.e. R_1 = 1) based on the first criterion. While AI-based models are highly capable of predicting the M_r, Kim's model tends to underestimate the M_r values. The results of the first criterion are summarized in the column corresponding to R_1 in **Table 8**.

Considering the cumulative probability criteria, the plots of M_{rp}/M_{rm} versus cumulative probability (%) are illustrated in **Fig. 8**. As evident from this figure, more than 95% of the predicted values by the ANN-GA model have the M_{rp}/M_{rm} values in a range of 0.8-1.2. The $P_{50\%}$ for the ANN-GA, ANN, LGP, and Kim's model are 1, 1, 1, and 0.8. A similar trend can be found in all other criteria as summarized in **Table 8**. It can be concluded that ANN-GA approach is highly capable of predicting the M_r of subgrade soils considering several validation criteria, followed by GA model and the LGP model. Generally, AI-based methods perform notably better compared to the regression based method by Kim (2004).

Sensitivity analysis

To measure the relative importance of each input variable and its contribution to the final model, a sensitivity analysis (SA) is performed. SA aims to measure the strength of the relationship between model inputs and the output variable. In this study, Cosine amplitude method (CAM) is used as an indicator of significance of each input variable (Majdi and Rezaei, 2013). Considering a set of n data samples in the common X-space, a data array X can be defined as:

$$X = \{X_1, X_2, ..., X_m\} \tag{24}$$

Each of the elements, x_i, in the data array X is a vector of lengths of m, that is:

$$X_i = \{X_{i1}, X_{i2}, ..., X_{im}\}_i \tag{25}$$

Each data sample can therefore be regarded as a point in m-dimensional space, where each point requires m coordinates for full description. Each element of relation, r_{ij} is the result of a pairwise comparison of two data samples, x_i and x_j. The strength of the relation between these two data pairs is in a 0 to 1 scale and is expressed by the following equation (Sadrossadat et al., 2016):

$$r_{ij} = \sum_{k=1}^{m} x_{ik} . x_{jk} \Big/ \sqrt{\sum_{k=1}^{m} x_{ik}{}^2 . \sum_{k=1}^{m} x_{jk}{}^2} \tag{26}$$

Fig. 9 describes the relative importance of each input variables in the developed models. The closer the r_{ij} to 1, the more impact corresponding variable has on the M_r value. As evident, the r_{ij} values for all input variables of both methods are between 0.75 and 0.92, which indicates the significance of all input variables for model development. In other words, all input variables are approximately equally important for prediction of M_r of subgrade soils, and their importance for model development cannot be neglected. These results are in agreement with those of similar studies in the literature (Sadrossadat et al., 2016, Kim, 2004, Sadrossadat et al., 2018b).

Conclusions

In this study, potential of two intelligent methods, i.e. GA and ANN-GA, was evaluated for prediction of M_r of cohesive subgrade soils. GA was used to develop an equation for prediction of M_r of subgrade soils. In addition, a GA was utilized to enhance the predictive capability of the ANN by adjusting the weights and bias.

A comprehensive database was utilized for the development of the models. Nine input parameters including liquid limit (LL), plastic index (PI), percentage of soil particles passing through #200 sieve (P_{200}), optimum moisture content (w_{opt}), degree of saturation (S_r), moisture content (w), unconfined compressive strength (q_u), confining stress (σ_3), and deviator stress (σ_d) were used for model development. The prediction performance of the developed models were compared with existing prediction equations in the literature. A ranking index was used to evaluate the external capability of the proposed model.

Results of this study indicates that both GA and ANN-GA methods can be employed as efficient tools in predicting the M_r of cohesive soils. The R^2 of the predicted and measured values for the ANN-GA model and GA model was 0.97 and 0.87 for both training and testing datasets, which was superior to the available prediction equations. One of the main objectives of the present study was to express the fact that the ANN-GA model can be expressed as explicit formula that can be used for manual calculation purposes. Besides, considering the prediction capability of the developed GA model, it can be concluded that the evolutionary algorithms can be regarded as efficient tools for providing precise and simple equations. The results obtained by ANN-GA and GA models outperformed other existing equations in the literature in terms of precision and accuracy considering several validation criteria.

The obtained results of sensitivity analysis indicated the importance of all input variables for prediction of M_r of cohesive subgrade soils. It should be noted that the capability of the AI-based methods is mostly limited to the range, number and statistical features of the database used for model developments. To resolve this, developed models can be enhanced by increasing the number of datasets. Nonetheless, the models provided in this study could be used for estimation of M_r of subgrade soils without conducting any tests. The proposed models are expected to be useful for preliminary design stages or when the testing is not feasible.

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LIST OF TABLES

Table 1

Available prediction equations for M_r of subgrade soils

Reference	Equation	Affecting variables	Applicability

Seed et al. (1967)	$M_r = k_1\left(\frac{\theta}{P_a}\right)^{k_2}$	θ, P_a, k_2	All soil types
Witczak and Uzan (1988)	$M_r = k_1 P_a\left(\frac{\theta}{P_a}\right)^{k_2}\left(\frac{\tau_{oct}}{P_a}\right)^{k_3}$	θ, P_a, τ_{oct}, k_2, k_3	All soil types
Puppala et al. (1996)	$M_R = k_1 P_a\left(\frac{\sigma_3}{P_a}\right)^{k_2}\left(\frac{\sigma_d}{P_a}\right)^{k_3}$	σ_3, σ_d, P_a, k_2, k_3	All soil types
Kim (2004)	$\frac{M_R}{P_a} = k_1\left[\frac{P_a\sigma_{oct}}{\tau^2_{oct}}\right]^{k_2}$	σ_{oct}, τ_{oct}, P_a, k_1, k_2	A-4 and A-6 soils
Carmichael and Stuart (1985)	$M_r = 37.431 - 0.4566 PI - 0.6179 w - 0.1424 P_{200} + 0.1791 \sigma_3 - 0.3248 \sigma_d + 36.422 CH + 17.097 MH$	PI, w, P_{200}, σ_3, σ_d, MH, CH	Cohesive subgrade soils
AASHTO (2003)	$M_r(MPa) = 17.6(CBR)^{0.64}$	CBR	All soil types
Mohammad et al. (2007)	$M_R = \frac{165.5}{DCPI^{1.147}} + 0.0966\left(\frac{\gamma_d}{w}\right)$	$DCPI$, γ_d, w	A-4 , A-6, A-7-5, and A-7-6 soils
	$M_R = \frac{151.8}{DCPI^{1.096}}$	$DCPI$	
Mohammad et al. (2002)	$\frac{M_R}{\sigma_3^{0.55}} = \frac{1}{\sigma_1}\left(31.79 q_c + 74.81\frac{f_s}{w}\right) + 4.08\frac{\gamma_d}{\gamma_w}$	σ_3, σ_d, q_c, f_s, γ_d, γ_w	Cohesive subgrade soils
Sadrossadat et al. (2018b)	$M_R = \sigma_3 + W_{opt} + (3P_{\#200} + q_u + \sigma_3 - 2\sigma_d - (5LL \times (W_c + S_r + 8\sigma_3 + \sigma_d - 8(PI - W_{opt} + 1.5)^4))/q_u)/W$	w_{opt}, P_{200}, q_u, σ_3, σ_d, LL, w, S_r	A-6 soils

θ: bulk stress ([σ_1 + 2σ_3]); P_a: atmospheric pressure (=101 kPa); σ_{oct}: octahedral normal stress ([σ_1 + 2σ_3]/3); τ_{oct}: octahedral shear stress ([$2^{0.5}$(σ_1 - σ_3)/3); σ_1: major principal stress ([σ_3 + σ_d]; σ_3: minor principal stress; σ_d: deviatoric stress ([σ_1 - σ_3]); PI: plasticity index; w: moisture content; P_{200}: percentage of soil particles passing through #200 sieve; CBR: California bearing ratio; DCPI: dynamic cone penetration index; γ_d: dry density; q_c: cone tip resistance; f_s: sleeve friction resistance; q_u: unconfined compressive strength; S_r: degree of saturation; k_1, k_2, k_3: regression coefficients; CH =1 for CH soil; MH = 1 for MH soil.

Table 2
Descriptive statistics of the variables in the database

Parameter	LL	PI	P_{200} (%)	w_{opt} (%)	S_r (%)	w (%)	q_u (kPa)	σ_3 (kPa)	σ_d (kPa)	M_r (MPa)
Mean	30.83	11.17	77.74	14.01	81.13	13.90	324.87	20.95	41.45	65.34
Median	32.00	11.00	84.00	14.00	85.07	14.00	316.50	20.69	41.37	64.83
Mode	31.00	11.00	56.00	13.40	66.44	14.00	696.13	0.00	41.37	104.00
Standard deviation	4.31	1.72	15.48	2.03	14.23	2.84	178.70	16.36	18.12	31.22
Kurtosis	0.65	1.43	-1.40	0.97	-0.24	-0.61	-0.29	-1.41	-1.05	-0.14
Skewness	-1.10	-1.37	-0.15	-0.85	-0.77	-0.39	0.76	-0.03	-0.02	0.29
Range	16.20	6.10	44.00	8.40	57.08	11.47	607.18	41.40	60.21	167.54
Minimum	21.00	7.00	56.00	9.40	42.92	7.53	88.95	0.00	11.00	11.90
Maximum	37.20	13.10	100.00	17.80	100.00	19.00	696.13	41.40	71.21	179.44

Table 3

Parameter settings for development of GA model

Parameter	Setting
Number of generation	500, 1000
Population size	50, 100, 200, 300, 400
Mutation rate (%)	10, 30
Crossover rate (%)	70, 95
Fitness function	RMSE

Table 4
Optimal coefficients of the developed GA model

Coefficients	Coefficient number									bias (b)
	1	2	3	4	5	6	7	8	9	
w_i	0.103	-0.204	-0.131	0.310	-0.085	-0.273	0.351	0.168	-0.292	0.726
a_i	5.999	0.023	0.961	3.096	0.045	0.862	0.640	0.723	0.148	

Table 5

Parameter settings for development of ANN-GA model

Parameter	Setting
Number of generation	100, 300, 500
Population size	50, 100, 200, 300, 400
Mutation rate (%)	10, 30
Crossover rate (%)	70, 95
Number of hidden layers	1
Number of hidden nodes	3
Activation function	Tangent-sigmoid
Fitness function	RMSE

Table 6
Values of weights and bias for input-hidden layer

Weight (w_{ik})	Neuron (k) Between Input and Hidden Layer		
	1	2	3
W_{1k}	0.519	0.171	-1.909
W_{2k}	0.003	-0.196	-1.203
W_{3k}	-0.252	0.125	0.640
W_{4k}	0.778	0.522	-0.867
W_{5k}	-0.541	-1.093	1.409
W_{6k}	-0.245	0.138	0.960
W_{7k}	0.141	0.103	0.921
W_{8k}	0.035	-0.363	-0.105
W_{9k}	-0.148	-0.067	0.703
$bias_k$	0.787	-1.563	-1.256

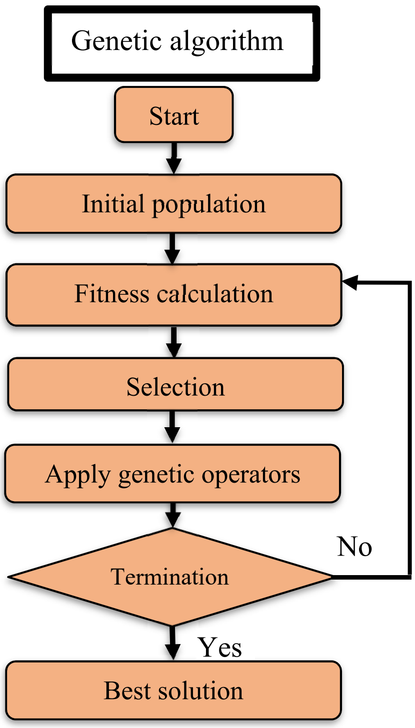
Table 7
Values of the weights and bias of the hidden-output layer

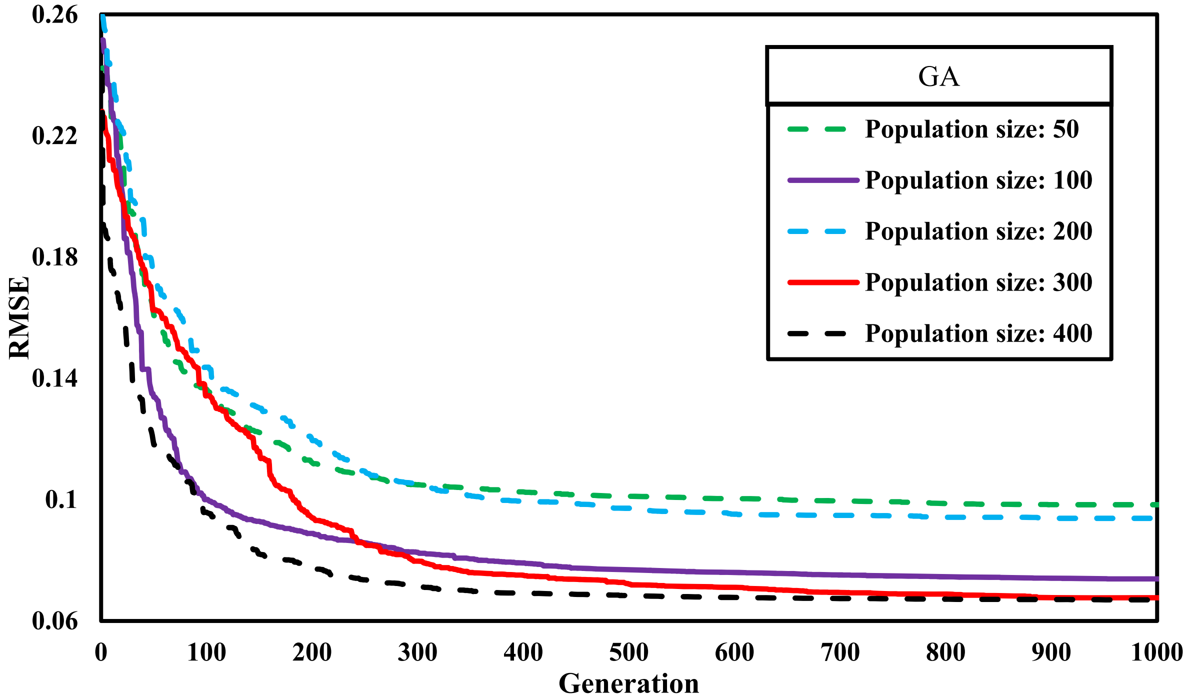
Weight	Neuron (k) Between Hidden and Output Layer			$bias_h$
	1	2	3	
W_k	2.6376	-1.9416	1.6468	-2.4069

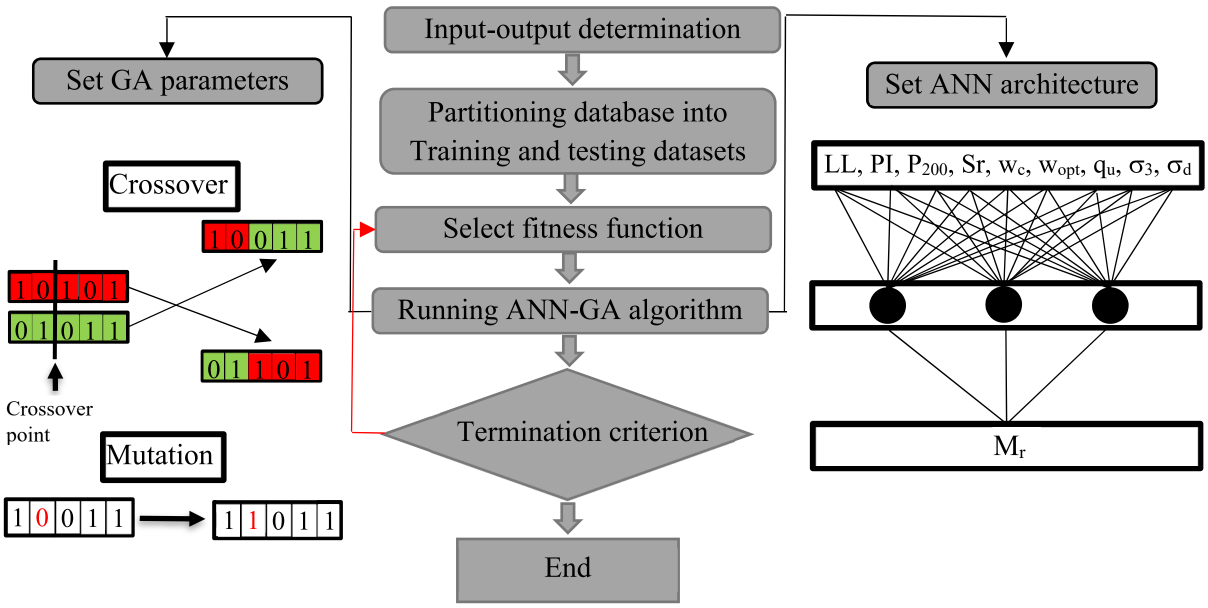
Table 8
Results of statistical analysis of models

Method	Overall performance			Arithmetic calculation of M_{rp}/M_{rm}			Cumulative probability		Coefficient of efficiency		Overall rank	
Model	M_{rfit}/M_{rm}	R^2	R_1	Mean	SD	R_2	M_{rp}/M_{rm} at P_{50}	R_3	E	R_4	RI	Final rank
ANN-GA	1.01	0.97	1	1.01	0.09	1	1	1	0.97	1	4	1
GA	1.01	0.87	2	1.04	0.30	2	1	1	0.85	2	8	2
Sadrossadat et al. (2018b)	0.98	0.83	3	1.06	0.33	3	1	1	0.82	3	11	3
Kim (2004)	0.78	0.56	4	0.77	0.26	4	0.8	4	0.45	4	16	4

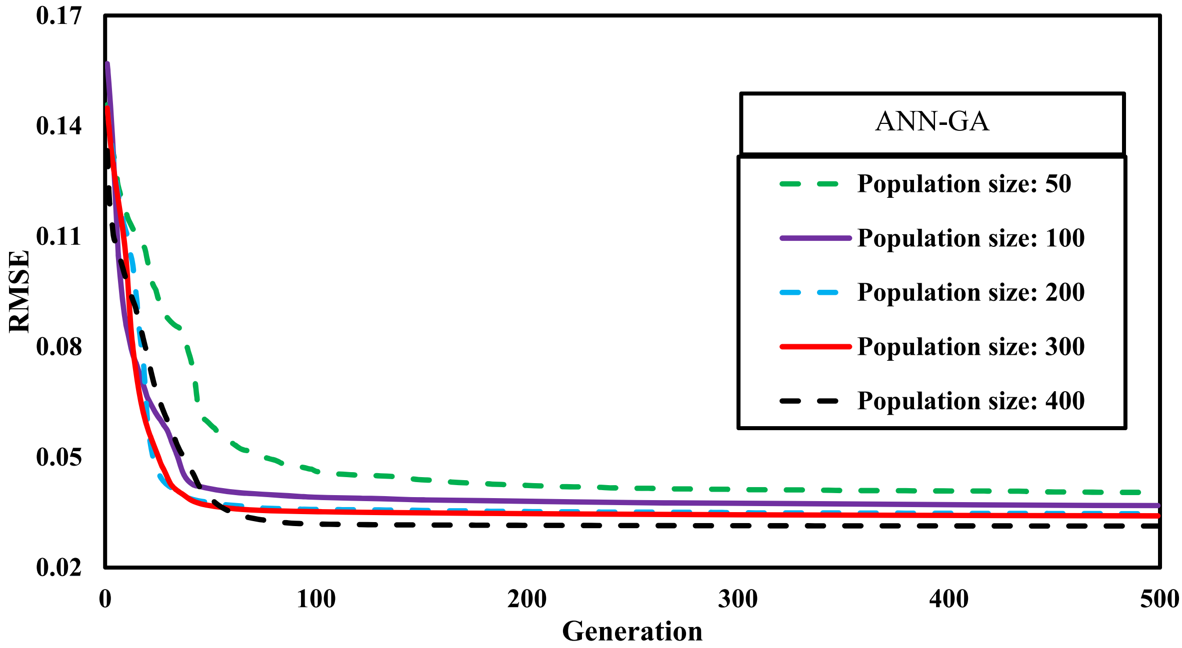
LIST OF FIGURES


Genetic algorithm
Start
Initial population
Fitness calculation
Selection
Apply genetic operators
Termination
No
Yes
Best solution

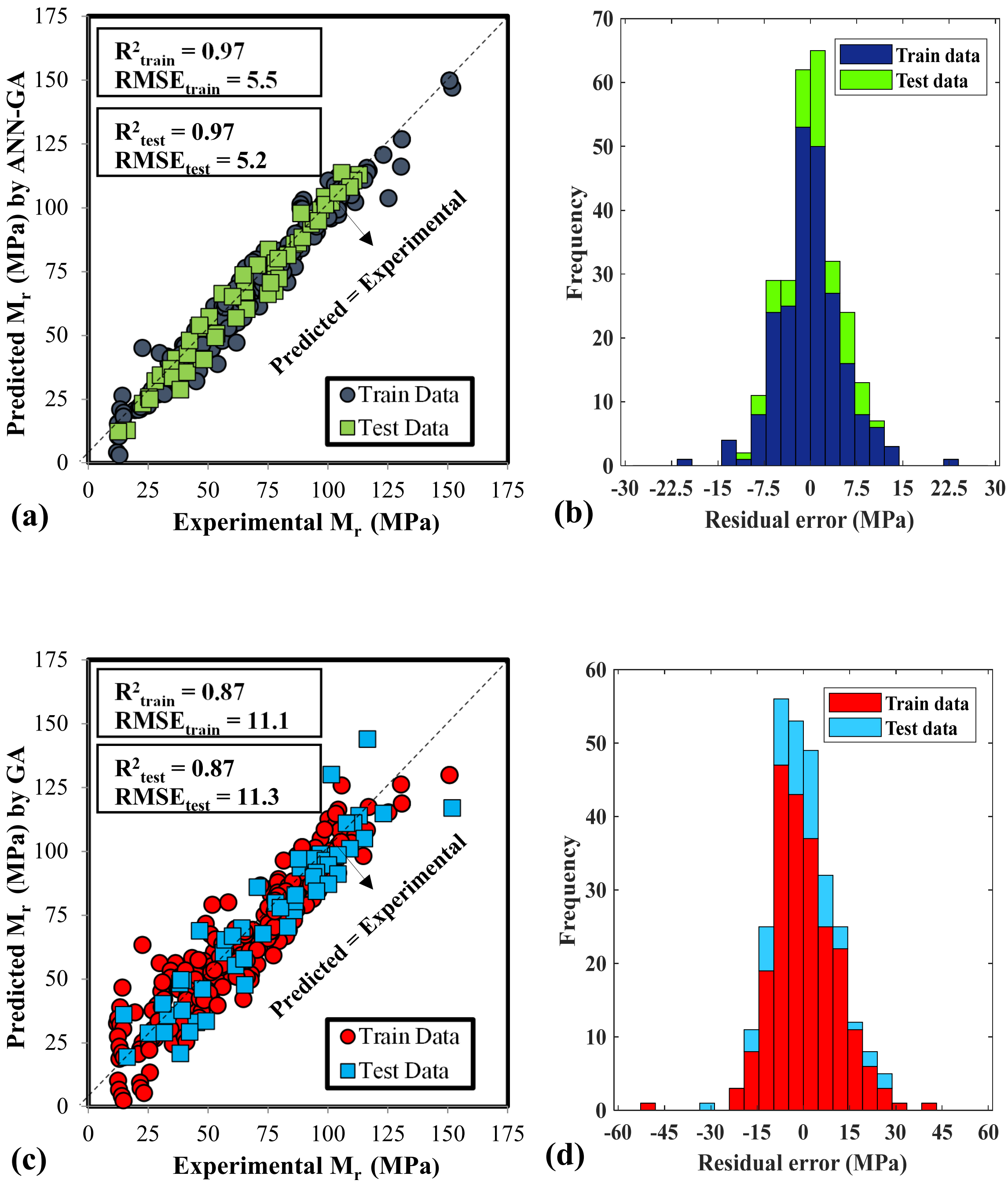

GA
Population size: 50
Population size: 100
Population size: 200
Population size: 300
Population size: 400
RMSE
0.26
0.22
0.18
0.14
0.1
0.06
0
100
200
300
400
500
600
700
800
900
1000
Generation

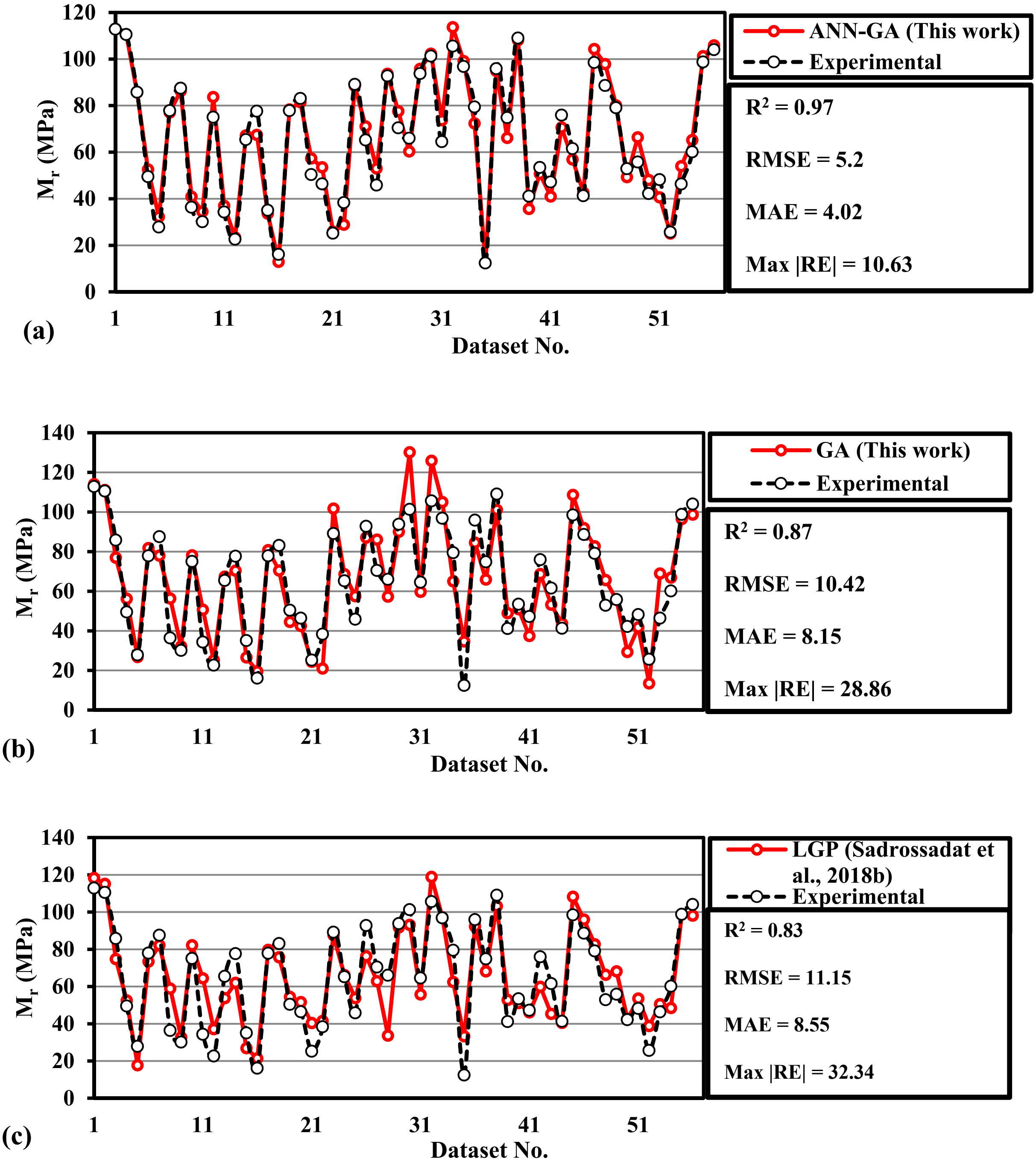


Input-output determination
Partitioning database into Training and testing datasets
Select fitness function
Running ANN-GA algorithm
Termination criterion
End
Set GA parameters
Crossover
1 0 0 1 1
1 0 1 0 1
0 1 0 1 1
0 1 1 0 1
Crossover point
Mutation
1 0 0 1 1
1 1 0 1 1
Set ANN architecture
LL, PI, P_{200}, Sr, w_c, w_{opt}, q_u, σ_3, σ_d
M_r

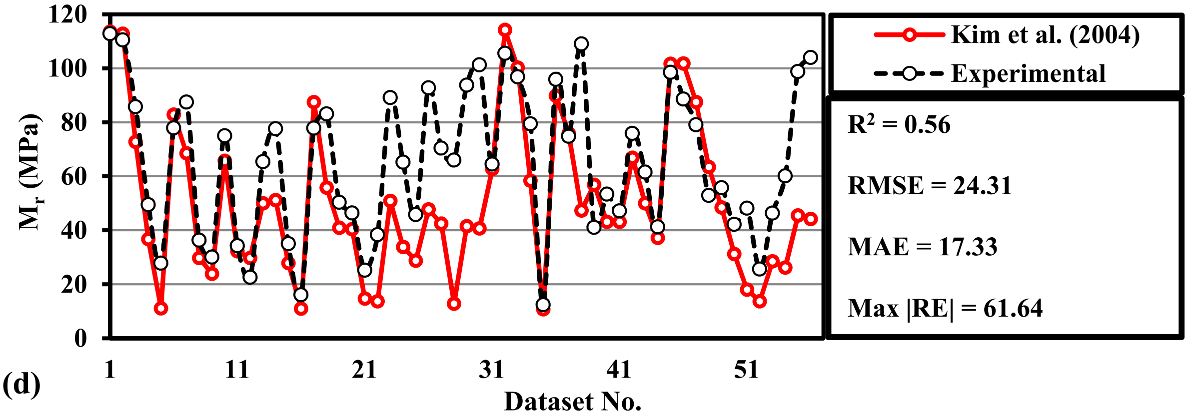


0.17
0.14
0.11
0.08
0.05
0.02
RMSE
0
100
200
300
400
500
Generation
ANN-GA
Population size: 50
Population size: 100
Population size: 200
Population size: 300
Population size: 400


R^2_{train} = 0.97
$RMSE_{train}$ = 5.5
R^2_{test} = 0.97
$RMSE_{test}$ = 5.2
Predicted = Experimental
Train Data
Test Data
Predicted M_r (MPa) by ANN-GA
Experimental M_r (MPa)
(a)
Train data
Test data
Frequency
Residual error (MPa)
(b)
R^2_{train} = 0.87
$RMSE_{train}$ = 11.1
R^2_{test} = 0.87
$RMSE_{test}$ = 11.3
Predicted = Experimental
Train Data
Test Data
Predicted M_r (MPa) by GA
Experimental M_r (MPa)
(c)
Train data
Test data
Frequency
Residual error (MPa)
(d)

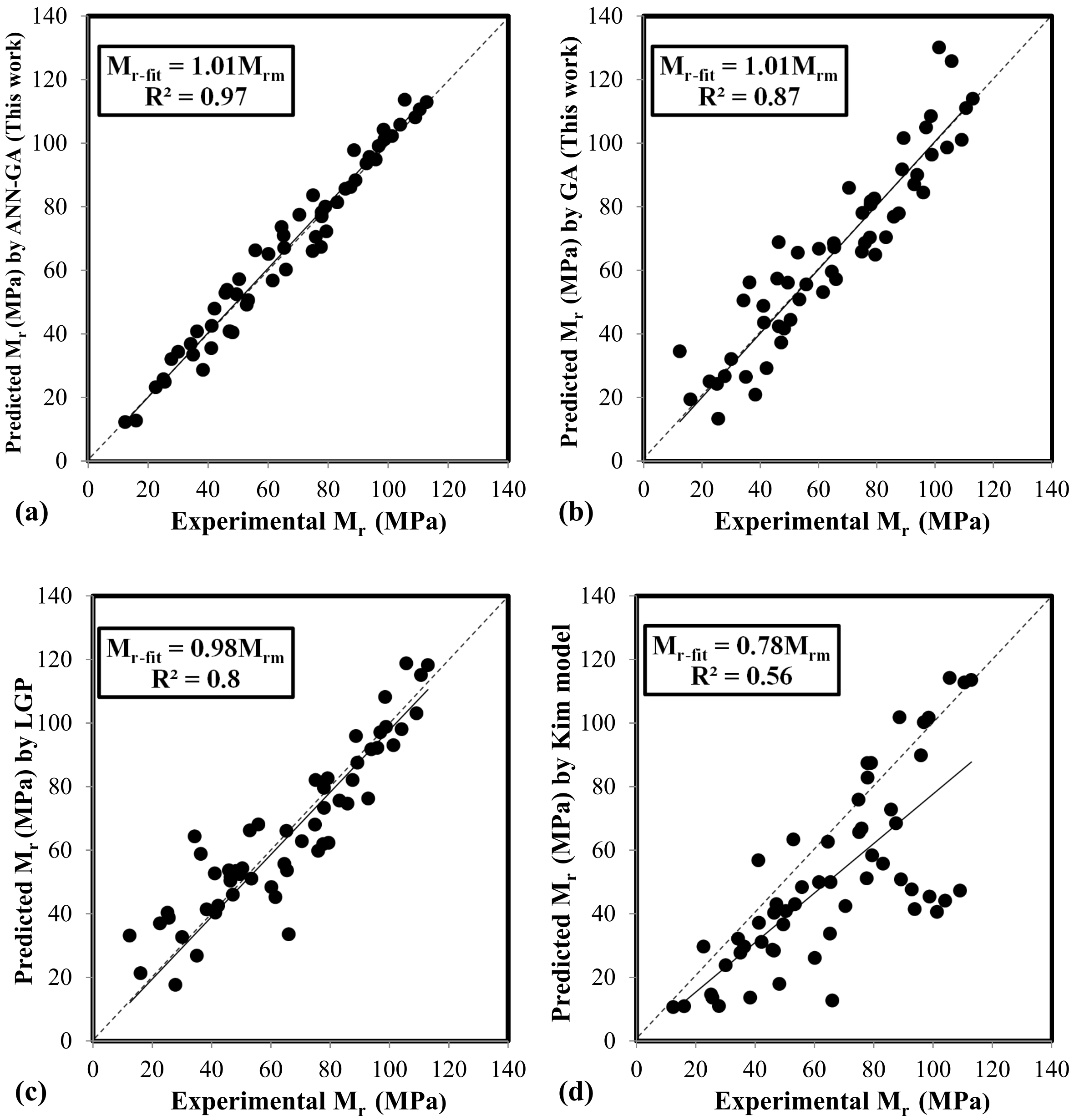


(a)
ANN-GA (This work)
Experimental
R^2 = 0.97
RMSE = 5.2
MAE = 4.02
Max |RE| = 10.63
M_r (MPa)
Dataset No.
(b)
GA (This work)
Experimental
R^2 = 0.87
RMSE = 10.42
MAE = 8.15
Max |RE| = 28.86
M_r (MPa)
Dataset No.
(c)
LGP (Sadrossadat et al., 2018b)
Experimental
R^2 = 0.83
RMSE = 11.15
MAE = 8.55
Max |RE| = 32.34
M_r (MPa)
Dataset No.

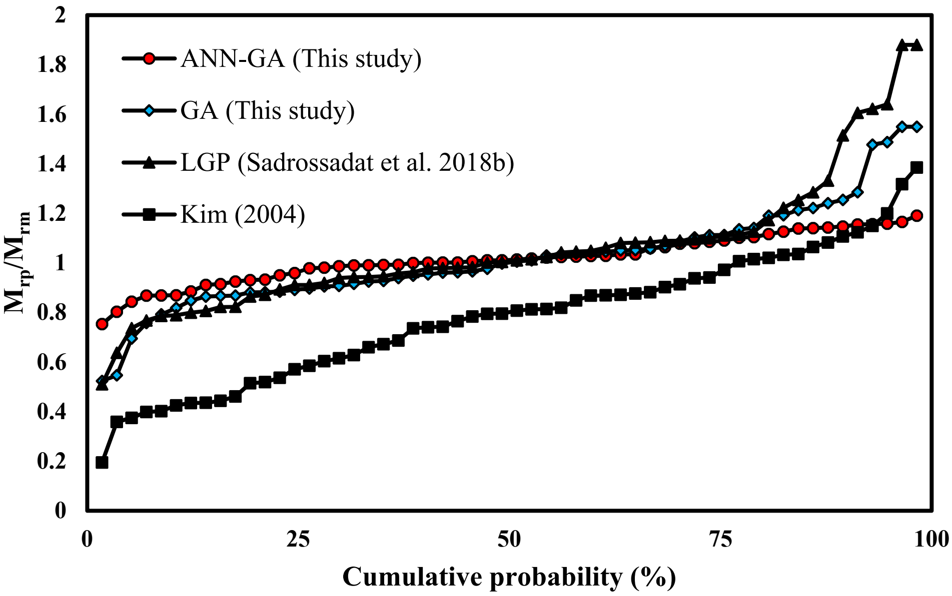


Kim et al. (2004)
Experimental
R^2 = 0.56
RMSE = 24.31
MAE = 17.33
Max |RE| = 61.64
M_r (MPa)
Dataset No.
0
20
40
60
80
100
120
1
11
21
31
41
51

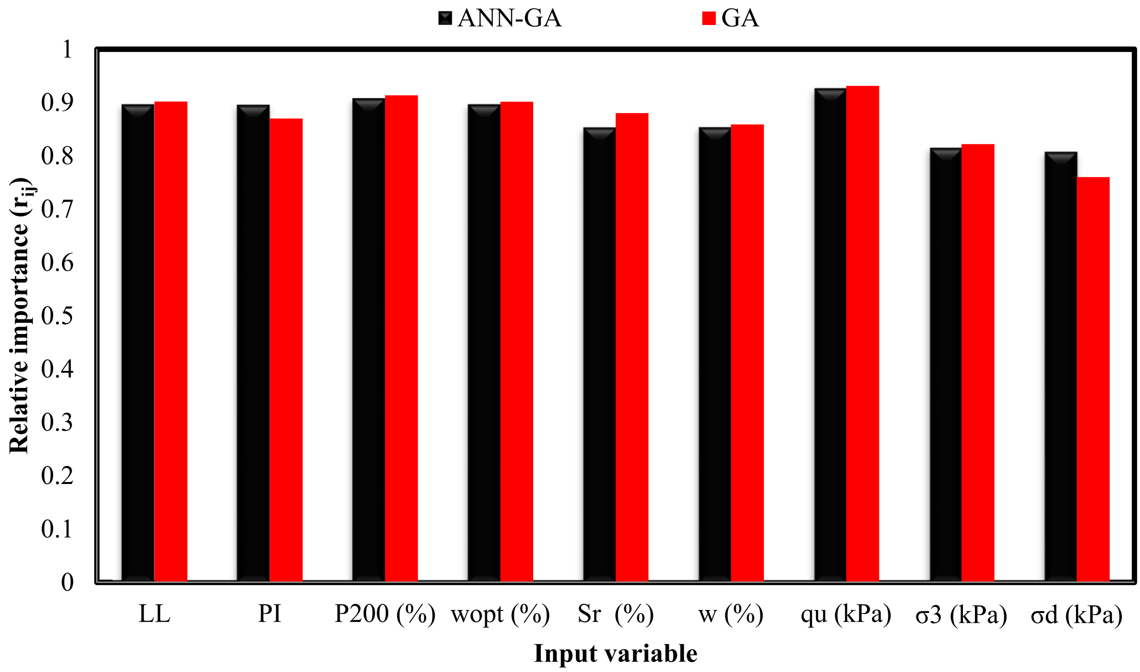
(d)



(a)
$M_{r\text{-}fit} = 1.01M_{rm}$
$R^2 = 0.97$
Predicted M_r (MPa) by ANN-GA (This work)
Experimental M_r (MPa)
(b)
$M_{r\text{-}fit} = 1.01M_{rm}$
$R^2 = 0.87$
Predicted M_r (MPa) by GA (This work)
Experimental M_r (MPa)
(c)
$M_{r\text{-}fit} = 0.98M_{rm}$
$R^2 = 0.8$
Predicted M_r (MPa) by LGP
Experimental M_r (MPa)
(d)
$M_{r\text{-}fit} = 0.78M_{rm}$
$R^2 = 0.56$
Predicted M_r (MPa) by Kim model
Experimental M_r (MPa)



ANN-GA (This study)
GA (This study)
LGP (Sadrossadat et al. 2018b)
Kim (2004)
M_{rp}/M_{rm}
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0
25
50
75
100
Cumulative probability (%)


ANN-GA
GA
Relative importance (rij)
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
LL
PI
P200 (%)
wopt (%)
Sr (%)
w (%)
qu (kPa)
σ3 (kPa)
σd (kPa)
Input variable